

APPENDIX

1. BASIC MODEL RESULTS

Complete network

QUANTITIES

$$q_1 = q_2 = q_3 = \frac{4(a - c)\gamma}{-3 + 16\gamma}$$

EFFORTS

$$e_1 = e_2 = e_3 = \frac{-a + c}{-3 + 16\gamma}$$

PROFITS

$$\pi_1 = \pi_2 = \pi_3 = \frac{(a - c)^2\gamma(-1 + 16\gamma)}{(3 - 16\gamma)^2}$$

$$PS = \frac{3(a - c)^2\gamma(-1 + 16\gamma)}{(-3 + 16\gamma)^2}$$

CONSUMER SURPLUS

$$CS = \frac{72(a - c)^2\gamma^2}{(-3 + 16\gamma)^2}$$

SOCIAL WELFARE

$$SW = \frac{3(a - c)^2\gamma(-1 + 40\gamma)}{(3 - 16\gamma)^2}$$

Star network

QUANTITIES

$$q_{hub} = \frac{4(a-c)\gamma(1+2\gamma)}{-1-2\gamma+32\gamma^2}$$

$$q_{spokes} = \frac{8(a-c)\gamma^2}{-1-2\gamma+32\gamma^2}$$

EFFORTS

$$e_{hub} = \frac{(a-c)(1+2\gamma)}{-1-2\gamma+32\gamma^2}$$

$$e_{spokes} = \frac{4(a-c)\gamma}{-1-2\gamma+32\gamma^2}$$

PROFITS

$$\pi_{hub} = \frac{(a-c)^2\gamma(1+2\gamma)^2(-1+16\gamma)}{(1+2\gamma-32\gamma^2)^2}$$

$$\pi_{spokes} = \frac{16(a-c)^2\gamma^3(-1+4\gamma)}{(1+2\gamma-32\gamma^2)^2}$$

$$PS = \frac{(a-c)^2\gamma(-1+12\gamma+28\gamma^2+192\gamma^3)}{(-1-2\gamma+32\gamma^2)^2}$$

CONSUMER SURPLUS

$$CS = \frac{8(a-c)^2\gamma^2(1+6\gamma)^2}{(-1-2\gamma+32\gamma^2)^2}$$

SOCIAL WELFARE

$$SW = \frac{(a-c)^2\gamma(-1+20\gamma+124\gamma^2+480\gamma^3)}{(1+2\gamma-32\gamma^2)^2}$$

Partially connected network

QUANTITIES

$$q_{isolated} = \frac{4(a-c)(-1+\gamma)\gamma}{3-17\gamma+16\gamma^2}$$

$$q_{linked} = \frac{(a-c)\gamma(-3+4\gamma)}{3-17\gamma+16\gamma^2}$$

EFFORTS

$$e_{isolated} = \frac{3(a-c)(-1+\gamma)}{3-17\gamma+16\gamma^2}$$

$$e_{linked} = \frac{(a-c)(-3+4\gamma)}{2(3-17\gamma+16\gamma^2)}$$

PROFITS

$$\pi_{isolated} = \frac{(a-c)^2(-1+\gamma)^2\gamma(-9+16\gamma)}{(3-17\gamma+16\gamma^2)^2}$$

$$\pi_{linked} = \frac{(a-c)^2(3-4\gamma)^2\gamma(-1+4\gamma)}{4(3-17\gamma+16\gamma^2)^2}$$

$$PS = \frac{(a-c)^2\gamma(-27+128\gamma-194\gamma^2+96\gamma^3)}{2(3-17\gamma+16\gamma^2)^2}$$

CONSUMER SURPLUS

$$CS = \frac{2(a-c)^2\gamma^2(-5+6\gamma)^2}{(3-17\gamma+16\gamma^2)^2}$$

SOCIAL WELFARE

$$SW = \frac{(a-c)^2\gamma(-27+228\gamma-434\gamma^2+240\gamma^3)}{2(3-17\gamma+16\gamma^2)^2}$$

2. EX-ANTE DIFFERENT FIRMS RESULTS

Empty network

QUANTITIES

$$q_1 = \frac{4}{13}(a - c - 9x - 5y)$$

$$q_2 = \frac{4}{13}(a - c + 4x - 5y)$$

$$q_3 = \frac{4}{13}(a - c + 4x + 8y)$$

EFFORTS

$$e_1 = \frac{3}{13}(a - c - 9x - 5y)$$

$$e_2 = \frac{3}{13}(a - c + 4x - 5y)$$

$$e_3 = \frac{3}{13}(a - c + 4x + 8y)$$

PROFITS

$$\pi_1 = \frac{7}{169}(-a + c + 9x + 5y)^2$$

$$\pi_2 = \frac{7}{169}(a - c + 4x - 5y)^2$$

$$\pi_3 = \frac{7}{169}(a - c + 4x + 8y)^2$$

$$PS = \frac{7}{169}(3a^2 - 6ac + 3c^2 - 2ax + 2cx + 113x^2 - 4ay + 4cy + 114xy + 114y^2)$$

CONSUMER SURPLUS

$$CS = \frac{8}{169}(3a - 3c - x - 2y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{169}(93a^2 + 93c^2 + 799x^2 + 830xy + 830y^2 + 62c(x + 2y) - 62a(3c + x + 2y))$$

Complete network

QUANTITIES

$$q_1 = \frac{1}{13}(4a - 4c - 10x - 7y)$$

$$q_2 = \frac{1}{13}(4a - 4c + 3x - 7y)$$

$$q_3 = \frac{1}{13}(4a - 4c + 3x + 6y)$$

EFFORTS

$$e_1 = \frac{1}{52}(4a - 4c - 10x - 7y)$$

$$e_2 = \frac{1}{52}(4a - 4c + 3x - 7y)$$

$$e_3 = \frac{1}{52}(4a - 4c + 3x + 6y)$$

PROFITS

$$\pi_1 = \frac{15(-4a + 4c + 10x + 7y)^2}{2704}$$

$$\pi_2 = \frac{15(4a - 4c + 3x - 7y)^2}{2704}$$

$$\pi_3 = \frac{15(4a - 4c + 3x + 6y)^2}{2704}$$

$$PS = \frac{15(24a^2 - 48ac + 24c^2 - 16ax + 16cx + 59x^2 - 32ay + 32cy + 67xy + 67y^2)}{1352}$$

CONSUMER SURPLUS

$$CS = \frac{8}{169}(3a - 3c - x - 2y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{104}(72a^2 + 72c^2 + 73x^2 + 97xy + 97y^2 + 48c(x + 2y) - 48a(3c + x + 2y))$$

Star network (firm 1 hub)

QUANTITIES

$$q_1 = \frac{4}{29}(3a - 3c - 5x - 4y)$$

$$q_2 = \frac{2}{29}(4a - 4c + 3(x - 5y))$$

$$q_3 = \frac{2}{29}(4a - 4c + 3x + 14y)$$

EFFORTS

$$e_1 = \frac{1}{29}(3a - 3c - 5x - 4y)$$

$$e_2 = \frac{1}{29}(4a - 4c + 3x - 15y)$$

$$e_3 = \frac{1}{29}(4a - 4c + 3x + 14y)$$

PROFITS

$$\pi_1 = \frac{15}{841}(-3a + 3c + 5x + 4y)^2$$

$$\pi_2 = \frac{3}{841}(4a - 4c + 3(x - 5y))^2$$

$$\pi_3 = \frac{3}{841}(4a - 4c + 3x + 14y)^2$$

$$PS = \frac{3}{841}(77a^2 - 154ac + 77c^2 - 102ax + 102cx + 143x^2 - 128ay + 128cy + 194xy + 501y^2)$$

CONSUMER SURPLUS

$$CS = \frac{2}{841}(14a - 14c - 4x - 9y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{841}(623a^2 + 623c^2 + 530cx + 461x^2 + 888cy + 726xy + 1665y^2 - 2a(623c + 265x + 444y))$$

Star network (firm 2 hub)

QUANTITIES

$$q_1 = \frac{2}{29}(4a - 4c - 3(6x + 5y))$$

$$q_2 = \frac{4}{29}(3a - 3c + x - 4y)$$

$$q_3 = \frac{2}{29}(4a - 4c + 11x + 14y)$$

EFFORTS

$$e_1 = \frac{1}{29}(4a - 4c - 18x - 15y)$$

$$e_2 = \frac{1}{29}(3a - 3c + x - 4y)$$

$$e_3 = \frac{1}{29}(4a - 4c + 11x + 14y)$$

PROFITS

$$\pi_1 = \frac{3}{841}(-4a + 4c + 18x + 15y)^2$$

$$\pi_2 = \frac{15}{841}(3a - 3c + x - 4y)^2$$

$$\pi_3 = \frac{3}{841}(4a - 4c + 11x + 14y)^2$$

$$PS = \frac{3}{841}(77a^2 - 154ac + 77c^2 - 26ax + 26cx + 450x^2 - 128ay + 128cy + 808xy + 501y^2)$$

CONSUMER SURPLUS

$$CS = \frac{2}{841}(14a - 14c - 5x - 9y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{841}(623a^2 + 623c^2 + 358cx + 1400x^2 + 888cy + 2604xy + 1665y^2 - 2a(623c + 179x + 444y))$$

Star network (firm 3 hub)

QUANTITIES

$$q_1 = \frac{2}{29}(4a - 4c - 18x - 7y)$$

$$q_2 = \frac{2}{29}(4a - 4c + 11x - 7y)$$

$$q_3 = \frac{4}{29}(3a - 3c + x + 2y)$$

EFFORTS

$$e_1 = \frac{1}{29}(4a - 4c - 18x - 7y)$$

$$e_2 = \frac{1}{29}(4a - 4c + 11x - 7y)$$

$$e_3 = \frac{1}{29}(3a - 3c + x + 2y)$$

PROFITS

$$\pi_1 = \frac{3}{841}(-4a + 4c + 18x + 7y)^2$$

$$\pi_2 = \frac{3}{841}(4a - 4c + 11x - 7y)^2$$

$$\pi_3 = \frac{15}{841}(3a - 3c + x + 2y)^2$$

$$PS = \frac{3}{841}(77a^2 - 154ac + 77c^2 - 26ax + 26cx + 450x^2 - 52ay + 52cy + 118xy + 118y^2)$$

CONSUMER SURPLUS

$$CS = \frac{2}{841}(14a - 14c - 5x - 10y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{841}(623a^2 + 623c^2 + 1400x^2 + 554xy + 554y^2 + 358c(x + 2y) - 2a(623c + 179(x + 2y)))$$

Partially connected network (firm 1 isolated): NEVER EXISTS

QUANTITIES

$$q_1 = -2(2x + y)$$

$$q_2 = \frac{1}{4}(2a - 2c + 8x + y)$$

$$q_3 = \frac{1}{4}(2a - 2c + 8x + 5y)$$

Partially connected network (firm 2 isolated)

QUANTITIES

$$q_1 = \frac{1}{4}(2a - 2c - 7x + y)$$

$$q_2 = 2(x - y)$$

$$q_3 = \frac{1}{4}(2a - 2c - 3x + 5y)$$

EFFORTS

$$e_1 = \frac{1}{8}(2a - 2c - 7x + y)$$

$$e_2 = \frac{3(x - y)}{2}$$

$$e_3 = \frac{1}{8}(2a - 2c - 3x + 5y)$$

PROFITS

$$\pi_1 = \frac{3}{64}(2a - 2c - 7x + y)^2$$

$$\pi_2 = \frac{7}{4}(x - y)^2$$

$$\pi_3 = \frac{3}{64}(2a - 2c - 3x + 5y)^2$$

$$PS = \frac{1}{32}(12a^2 - 24ac + 12c^2 - 60ax + 60cx + 143x^2 + 36ay - 36cy - 178xy + 95y^2)$$

CONSUMER SURPLUS

$$CS = \frac{1}{8}(2a - 2c - x - y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{32}(28a^2 + 28c^2 + 76cx + 147x^2 - 4a(14c + 19x - 5y) - 20cy - 170xy + 99y^2)$$

Partially connected network (firm 3 isolated)

QUANTITIES

$$q_1 = \frac{1}{4}(2a - 2c - 7x - 10y)$$

$$q_2 = \frac{1}{4}(2a - 2c - 3x - 10y)$$

$$q_3 = 2(x + 2y)$$

EFFORTS

$$e_1 = \frac{1}{8}(2a - 2c - 7x - 10y)$$

$$e_2 = \frac{1}{8}(2a - 2c - 3x - 10y)$$

$$e_3 = \frac{3}{2}(x + 2y)$$

PROFITS

$$\pi_1 = \frac{3}{64}(-2a + 2c + 7x + 10y)^2$$

$$\pi_2 = \frac{3}{64}(-2a + 2c + 3x + 10y)^2$$

$$\pi_3 = \frac{7}{4}(x + 2y)^2$$

$$PS = \frac{1}{32}(12a^2 - 24ac + 12c^2 - 60ax + 60cx + 143x^2 - 120ay + 120cy + 524xy + 524y^2)$$

CONSUMER SURPLUS

$$CS = \frac{1}{8}(2a - 2c - x - 2y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{32}(28a^2 - 56ac + 28c^2 + 147x^2 + 540xy + 540y^2 - 76a(x + 2y) + 76c(x + 2y))$$

2.1 Only firms 2 and 3 remaining

Complete network

QUANTITIES

$$q_2 = \frac{1}{7}(3a - 3c - 5y)$$

$$q_3 = \frac{1}{7}(3a - 3c + 2y)$$

EFFORTS

$$e_2 = \frac{1}{21}(3a - 3c - 5y)$$

$$e_3 = \frac{1}{21}(3a - 3c + 2y)$$

PROFITS

$$\pi_1 = \frac{8}{441}(-3a + 3c + 5y)^2$$

$$\pi_2 = \frac{8}{441}(3a - 3c + 2y)^2$$

$$PS = \frac{8}{441}(18a^2 - 36ac + 18c^2 - 18ay + 18cy + 29y^2)$$

CONSUMER SURPLUS

$$CS = \frac{9}{98}(2a - 2c - y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{882}(612a^2 + 612c^2 + 612cy + 545y^2 - 612a(2c + y))$$

Empty network

QUANTITIES

$$q_2 = \frac{3}{7}(a - c - 4y)$$

$$q_3 = \frac{3}{7}(a - c + 3y)$$

EFFORTS

$$e_2 = \frac{2}{7}(a - c - 4y)$$

$$e_3 = \frac{2}{7}(a - c + 3y)$$

PROFITS

$$\pi_2 = \frac{5}{49}(-a + c + 4y)^2$$

$$\pi_3 = \frac{5}{49}(a - c + 3y)^2$$

$$PS = \frac{5}{49}(2a^2 - 4ac + 2c^2 - 2ay + 2cy + 25y^2)$$

CONSUMER SURPLUS

$$PS = \frac{9}{98}(2a - 2c - y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{14}(8a^2 + 8c^2 + 8cy + 37y^2 - 8a(2c + y))$$

2.2 Only firms 1 and 3 remaining

Complete network

QUANTITIES

$$q_1 = \frac{1}{7}(3a - 3c - 5(x + y))$$

$$q_3 = \frac{1}{7}(3a - 3c + 2(x + y))$$

EFFORTS

$$e_1 = \frac{1}{21}(3a - 3c - 5x - 5y)$$

$$e_3 = \frac{1}{21}(3a - 3c + 2x + 2y)$$

PROFITS

$$\pi_1 = \frac{8}{441}(-3a + 3c + 5(x + y))^2$$

$$\pi_3 = \frac{8}{441}(3a - 3c + 2(x + y))^2$$

$$PS = \frac{8}{441}(18a^2 - 36ac + 18c^2 - 18ax + 18cx + 29x^2 - 18ay + 18cy + 58xy + 29y^2)$$

CONSUMER SURPLUS

$$CS = \frac{9}{98}(2a - 2c - x - y)^2$$

SOCIAL WELFARE

$$SW = \frac{1}{882} (612a^2 + 612c^2 + 612c(x + y) + 545(x + y)^2 - 612a(2c + x + y))$$

Empty network

QUANTITIES

$$q_1 = \frac{3}{7} (a - c - 4(x + y))$$

$$q_3 = \frac{3}{7} (a - c + 3(x + y))$$

EFFORTS

$$e_1 = \frac{2}{7} (a - c - 4x - 4y)$$

$$e_3 = \frac{2}{7} (a - c + 3x + 3y)$$

PROFITS

$$\pi_1 = \frac{5}{49} (-a + c + 4(x + y))^2$$

$$\pi_3 = \frac{5}{49} (a - c + 3(x + y))^2$$

$$PS = \frac{5}{49} (2a^2 - 4ac + 2c^2 - 2ax + 2cx + 25x^2 - 2ay + 2cy + 50xy + 25y^2)$$

CONSUMER SURPLUS

$$PS = \frac{9}{98} (2a - 2c - x - y)^2$$

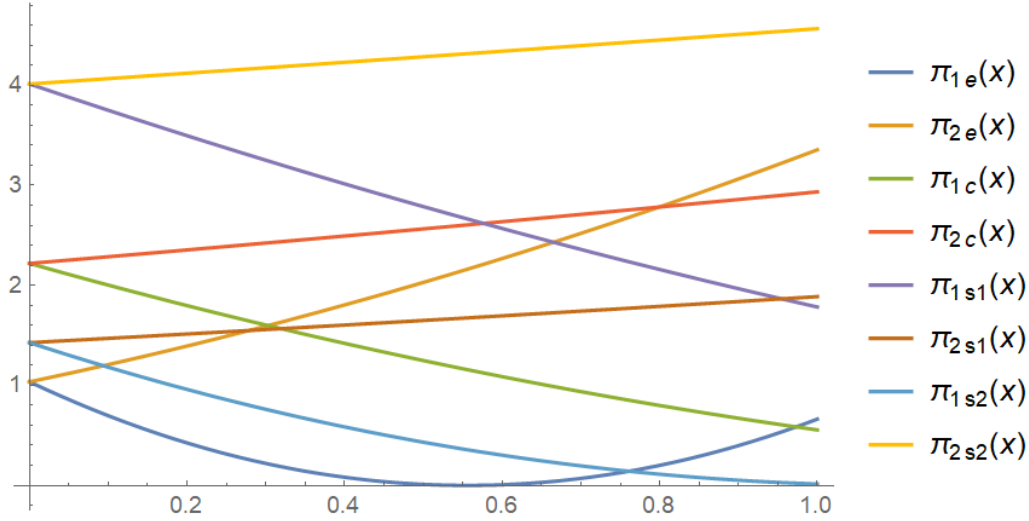
SOCIAL WELFARE

$$SW = \frac{1}{14} (8a^2 + 8c^2 + 8c(x + y) + 37(x + y)^2 - 8a(2c + x + y))$$

3. EXAMPLES OF PROFITS' COMPARISON TO FIND THE STABLE NETWORK

Case of $y = 0$

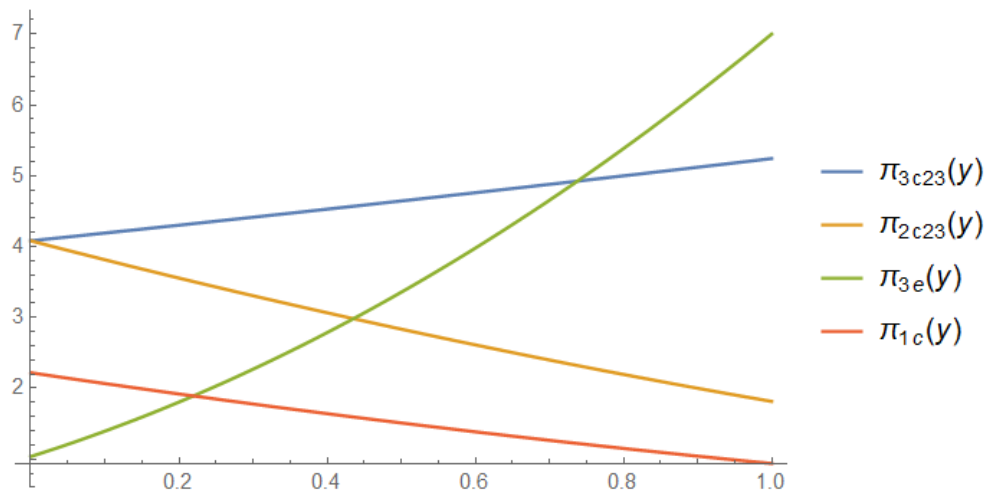
`Plot[{ $\pi_{1e}[x]$ ,  $\pi_{2e}[x]$ ,  $\pi_{1c}[x]$ ,  $\pi_{2c}[x]$ ,  $\pi_{1s1}[x]$ ,  $\pi_{2s1}[x]$ ,  $\pi_{1s2}[x]$ ,  $\pi_{2s2}[x]$ }, { $x$ , 0, 1}, PlotLegends → "Expressions"]`



π_{1e} = profits of firm 1 in the empty network, π_{2e} = profits of firm 2 in the empty network, π_{1c} = profits of firm 1 in the complete network, π_{2c} = profits of firm 2 in the complete network, π_{1s1} = profits of firm 1 in the star network with firm 1 hub, π_{2s1} = profits of firm 2 in the star network with firm 1 hub, π_{1s2} = profits of firm 1 in the star network with firm 2 hub, π_{2s2} = profits of firm 2 in the star network with firm 2 hub

Case of $x = 0$

`Plot[{ $\pi_{3c23}[y]$ ,  $\pi_{2c23}[y]$ ,  $\pi_{3e}[y]$ ,  $\pi_{1c}[y]$ }, { $y$ , 0, 1}, PlotLegends → "Expressions"]`



π_{3c23} = profits of firm 3 in the complete network with only firms 2 and 3 remaining, π_{2c23} = profits of firm 2 in the complete network with only firms 2 and 3 remaining, π_{3e} = profits of firm 3 in the empty network, π_{1c} = profits of firm 1 in the complete network