# APPENDIX D

# PCB: RADIUS OF CONNECTION BETWEEN THE LOOPS

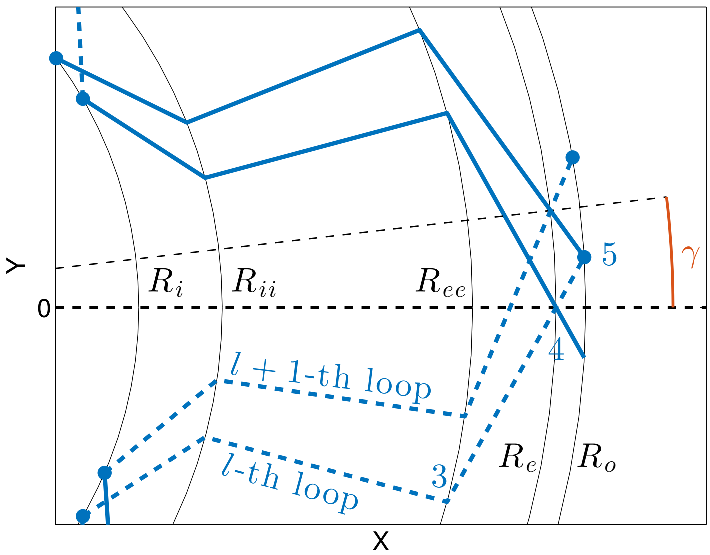



Fig. D.1: Zoom on the neutral axis of the tracks $l$ and $l+1$.

In order to ensure the series connection of the different loops of one phase, the tracks of the first lobe of the $N_l$ loops of each phase are extended. In this way, as shown in Fig. D.1, the return conductor of the $l$-th loop is connected to the forward conductor of the $(l+1)$-th loop through a via placed on the point 5. This point is located on a radius $R_o > R_e$ at an angle equal to $\gamma/2$. Hence, in polar coordinates, the fifth point is given by:

$$\begin{aligned} x_5 &= R_o \cos(\gamma/2) \\ y_5 &= R_o \sin(\gamma/2) \end{aligned} \tag{D.1}$$

The straight line passing through the points 3 and 4, given in (4.3), is:

$$\begin{aligned} y &= m(x - R_e) \\ &= \frac{R_{ee}\sin(\pi/2p)}{R_e - R_{ee}\cos(\pi/2p)} \cdot (x - R_e) \end{aligned} \tag{D.2}$$

Hence, given that the fifth point belongs to this line, the radius $R_o$ is:

$$R_o = \frac{mR_e}{m\cos(\gamma/2) - \sin(\gamma/2)}. \tag{D.3}$$

## APPENDIX E

# PCB: CONSTANT TRACK WIDTH

The track width has to be calculated so that the distance between each consecutive track remains constant and equal to the minimal insulation distance $m_s$. Again, the track of the three segments winding can be described using only eight points, four for the left side and four for the right side of the track. Taking advantage of the rotational revolution of the PCB, the calculations are simplified by choosing an appropriate frame in each case. Then, the points describing the polygons of the tracks can be calculated using rotation matrices for the complete PCB.

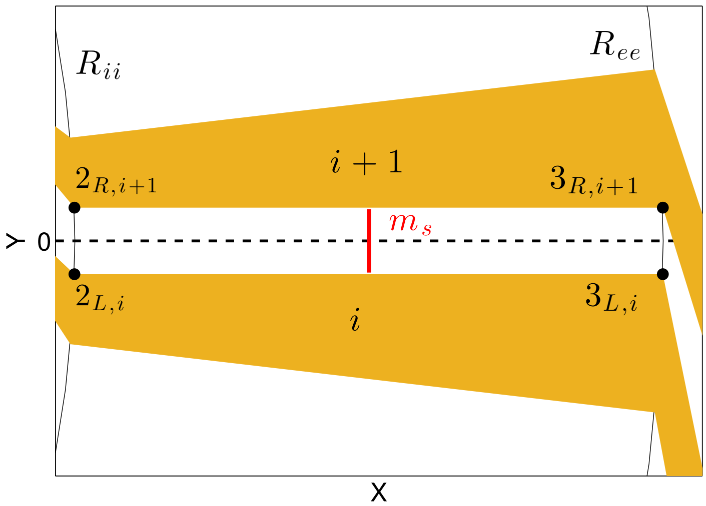



Fig. E.1: Zoom on the second segment of the tracks $i$ and $i+1$.

Let us start with the second segment. As shown in Fig. E.1, the $X$-axis of the frame is placed between two consecutive tracks $i$ and $i+1$. The points $2_{R,i+1}$ and $2_{L,i}$ are located on the radius $R_{ii}$ and the points $3_{R,i+1}$ and $3_{L,i}$ are located on the radius $R_{ee}$. Hence, knowing that the distance between both tracks is $m_s$, the four points are easily determined in polar coordinates, yielding:

$$
\begin{aligned}
2_{L,i} &= \{R_{ii}\,;\,-\sin^{-1}(m_s/(2R_{ii}))\} \\
2_{R,i+1} &= \{R_{ii}\,;\,+\sin^{-1}(m_s/(2R_{ii}))\} \\
3_{L,i} &= \{R_{ee}\,;\,-\sin^{-1}(m_s/(2R_{ee}))\} \\
3_{R,i+1} &= \{R_{ee}\,;\,+\sin^{-1}(m_s/(2R_{ee}))\}
\end{aligned}
\tag{E.1}
$$

Let us now consider the third segment. The $X$-axis of the frame is placed on the symmetry axis of the loop $i$, as shown in Fig. E.2. Among the four points describing the third segment tracks, only the point $3_{R,i+1}$ is entirely known as it has been calculated above. Indeed, in order to obtain a constant distance between two consecutive tracks for this segment, the point $3_{L,i}$ is replaced by the point $3_{Ln,i}$ which has to be placed on a

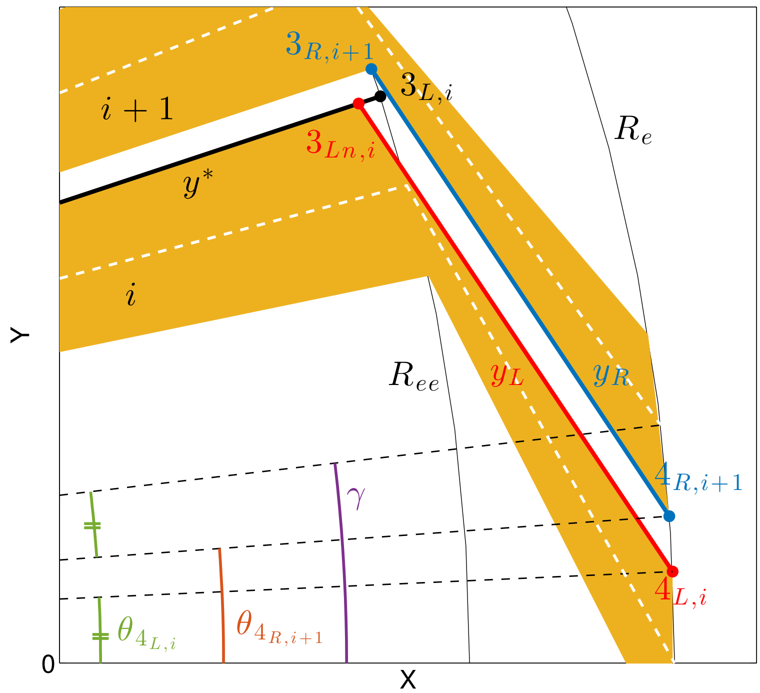



Fig. E.2: Zoom on the third segment of the tracks $i$ and $i+1$.

radius smaller than $R_{ee}$. Defining the slope of both straight lines $y_R$ and $y_L$ as $m$, the distance $m_s$ separating them can be linked to their respective intercepts $p_R$ and $p_L$ through the following relation:

$$p_R - p_L = m_s\sqrt{m^2+1}. \tag{E.2}$$

Supposing that the angle $\theta_{4_{R,i+1}}$ and the distance $m_s$ are small, the distance between the points $4_{R,i+1}$ and $4_{L,i}$ along the radius $R_e$ is considered to be equal to the vertical distance separating them, namely $p_R - p_L$. The former assumption is verified when the product of the number of pole pairs, number of phases and number of loops is high. The latter depends on the technology but is minimised to increase the fill factor. Hence, the angle $\theta_{4_{L,i}}$ is calculated as:

$$\theta_{4_{L,i}} = \frac{1}{2}\left(\gamma - \frac{m_s}{R_e}\sqrt{m^2+1}\right). \tag{E.3}$$

It allows to find the angle $\theta_{4_{R,i+1}}$:

$$\theta_{4_{R,i+1}} = \gamma - \theta_{4_{L,i}} = \frac{\gamma}{2} + \frac{m_s}{2R_e}\sqrt{m^2+1}. \tag{E.4}$$

The radius of the point $4_{R,i+1}$ being $R_e$ and considering the angle $\theta_{4_{R,i+1}}$ as being small, its cartesian coordinates are given by:

$$\begin{cases} x_{4_{R,i+1}} = R_e\cos(\theta_{4_{R,i+1}}) \approx R_e \\ y_{4_{R,i+1}} = R_e\sin(\theta_{4_{R,i+1}}) \approx R_e\theta_{4_{R,i+1}} \end{cases} \tag{E.5}$$

Hence, the slope $m$ of the straight line $y_R$ can be rewritten as:

$$\begin{aligned} m &= \frac{y_{4_{R,i+1}} - y_{3_{R,i+1}}}{x_{4_{R,i+1}} - x_{3_{R,i+1}}} \\ &= \frac{\frac{\gamma R_e}{2} + \frac{m_s}{2}\sqrt{m^2+1} - y_{3_{R,i+1}}}{R_e - x_{3_{R,i+1}}}. \end{aligned} \tag{E.6}$$

This relation can be rearranged so as to obtain:

$$\begin{aligned} m^2 \left( (R_e - x_{3_{R,i+1}})^2 - \frac{m_s^2}{4} \right) + 2m(R_e - x_{3_{R,i+1}}) \left( y_{3_{R,i+1}} - R_e \frac{\gamma}{2} \right) \\ + \left( \left( y_{3_{R,i+1}} - R_e \frac{\gamma}{2} \right)^2 - \frac{m_s^2}{4} \right) = 0. \end{aligned} \tag{E.7}$$

Solving this equation yields the slope $m$. The points $4_{R,i+1}$ and $4_{L,i}$ being located on the radius $R_e$, they can be calculated knowing the relations (E.4) and (E.3) respectively. The point $3_{Ln,i}$ corresponds to the intersection between the straight lines $y^* = m^* x + p^*$ and $y_L$. The former is determined through the points $2_{L,i}$ and $3_{L,i}$. The slope of the latter is already known as being $m$ and its intersect $p_L$ is found through the relation (E.2), yielding:

$$\begin{aligned} p_L &= p_R - m_s\sqrt{m^2+1} \\ &= (y_{3_{R,i+1}} - m x_{3_{R,i+1}}) - m_s\sqrt{m^2+1} \end{aligned} \tag{E.8}$$

Finally, the point $3_{Ln,i}$ is given by:

$$\begin{cases} x_{3_{Ln,i}} = -\dfrac{p^* - p_L}{m^* - m} \\ y_{3_{Ln,i}} = m x_{3_{Ln,i}} + p_L \end{cases} \tag{E.9}$$

The points describing the first segment, shown in Fig E.3, can be determined using exactly the same principle. Indeed, in order to obtain a constant distance between two consecutive tracks for this segment, the point $2_{R,i+1}$ is replaced by the point $2_{Rn,i+1}$ which has to be placed on a radius larger than $R_{ii}$. The $X$-axis of the frame is placed along a radius passing through the intersection between the neutral axis of the first segment of the $i$-th track and the radius $R_i$. Hence, following the same assumptions on the values of $m_s$ and considering $\theta_{1_{L,i}}$ as being small, the latter is found as:

$$\theta_{1_{L,i}} = \frac{1}{2}\left( \gamma - \frac{m_s}{R_i}\sqrt{m^2+1} \right). \tag{E.10}$$

It allows determing the cartesian coordinates of this point:

$$\begin{cases} x_{1_{L,i}} = R_i \cos(\theta_{1_{L,i}}) \approx R_i \\ y_{1_{L,i}} = R_i \sin(\theta_{1_{L,i}}) \approx R_i \theta_{1_{L,i}} \end{cases} \tag{E.11}$$

Hence, the slope $m$ of the straight line $y_L$ can be rewritten as:

$$\begin{aligned} m &= \frac{y_{1_{L,i}} - y_{2_{L,i}}}{x_{1_{L,i}} - x_{2_{L,i}}} \\ &= \frac{\frac{\gamma R_i}{2} - \frac{m_s}{2}\sqrt{m^2+1} - y_{2_{L,i}}}{R_i - x_{2_{L,i}}}. \end{aligned} \tag{E.12}$$

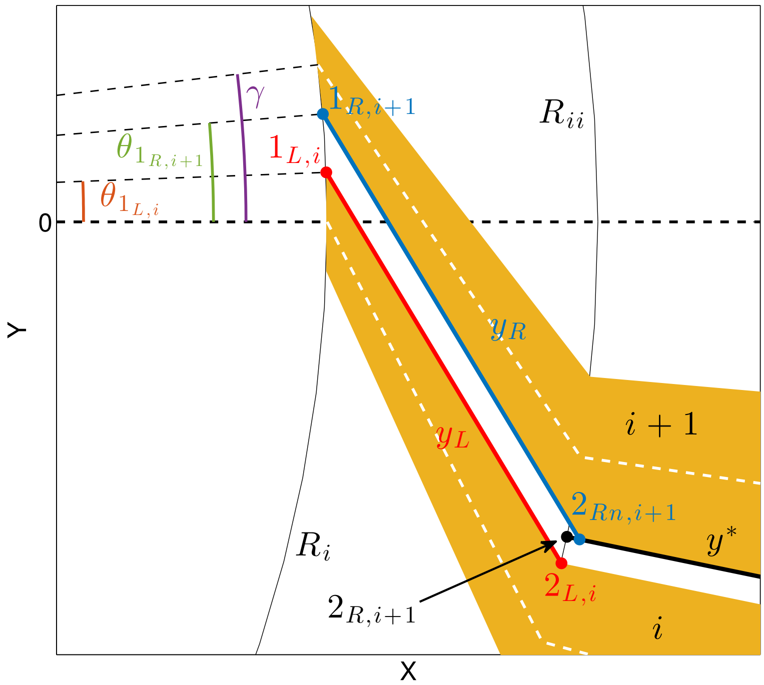



Fig. E.3: Zoom on the first segment of the tracks $i$ and $i+1$.

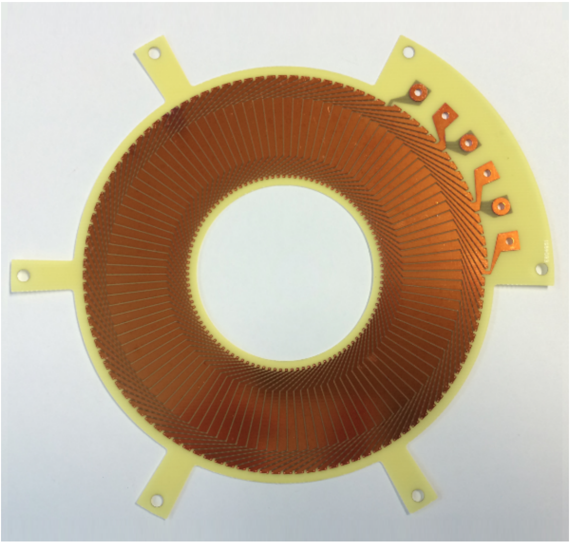
Rearranging this relation yields:

$$m^2\left((R_i - x_{2_{L,i}})^2 - \frac{m_s^2}{4}\right) + 2m(R_i - x_{2_{L,i}})\left(y_{2_{L,i}} - R_i\frac{\gamma}{2}\right) + \left(\left(y_{2_{L,i}} - R_i\frac{\gamma}{2}\right)^2 - \frac{m_s^2}{4}\right) = 0. \tag{E.13}$$

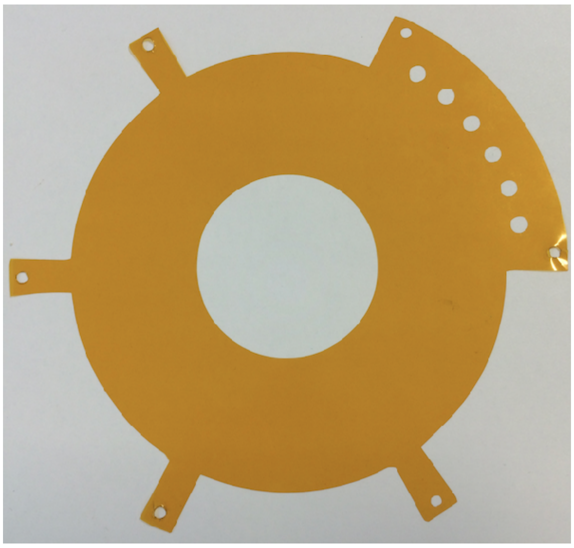
The slope $m$ is found by solving this equation. The points $1_{R,i+1}$ and $1_{L,i}$ being located on the radius $R_i$, they can be calculated knowing the angle that separates them and the relation (E.10). The point $2_{Rn,i+1}$ corresponds to the intersection between the straight lines $y^* = m^*x + p^*$ and $y_R$. The former is determined through the points $2_{R,i+1}$ and $3_{R,i+1}$. The intercept of the latter is calculated in the same way as for the third segment.

## APPENDIX F

# PCB: ASSEMBLY



(a) PCB.



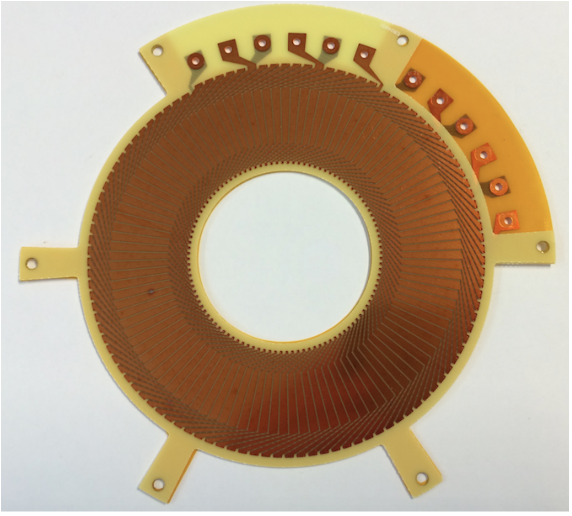
(b) Insulation layer.

Fig. F.1: PCB winding assembly: parts.

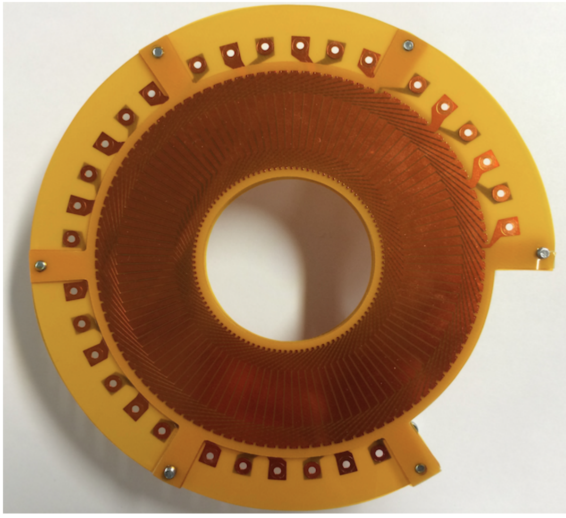
Figs. F.1a and F.1b show respectively one single PCB constituted of three phases of six loops and one insulation layer. The complete assembly of each part of the armature winding is carried out in three steps. First, the stack composed of the five boards and six insulations is assembled, layer by layer, as shown in Fig. F.2a. Fig. F.2b shows one resulting stack.

Second, these stacks have to be baked at 180 degrees during two hours, leading to the reticulation of the insulation which allows to hold the different layers together and to rigidify the whole system. During the baking, the stack must be maintained pressed to favour this reticulation. To this end, it is squeezed between two compliant layers surrounded by two wooden pieces, the whole system being screwed as shown in Figs. F.2c and F.2d. The resulting stack, shown in Fig. F.2e, is far more rigid, which is really important as it allows to transmit the electrodynamic force without bending. It can be remarked that the stack is lighter in the center, meaning that it has been better pressed. This is due to the fact that the system used to compress the stack does not yield an identical pressure on the whole surface of the PCB.

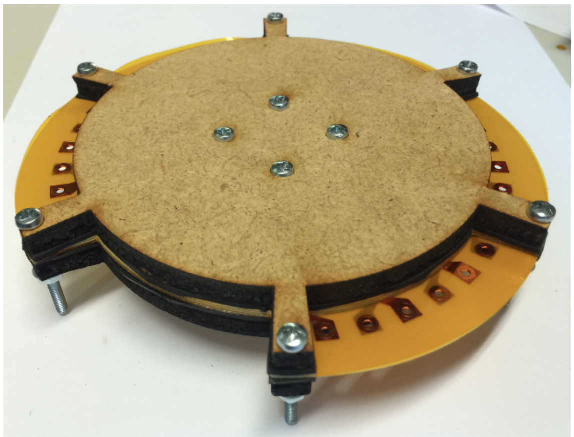
Third, as shown in Fig. F.2f, the five boards are connected in series, phase by phase. There are thus only six terminals (two for each phase) for the final stack.



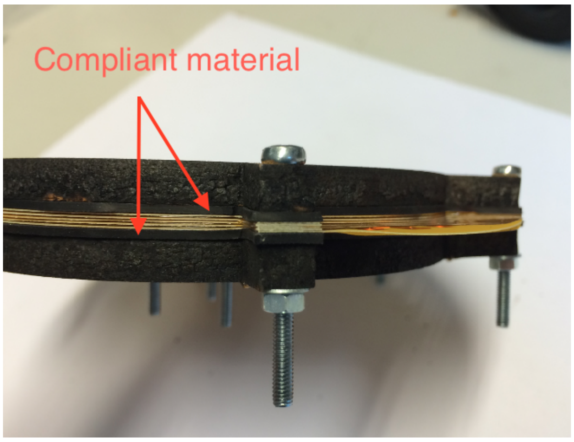
(a) Two PCBs.



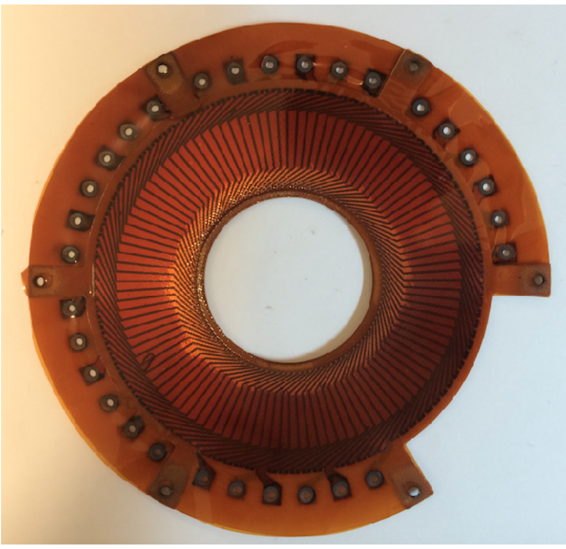
(b) Stack of five PCBs.



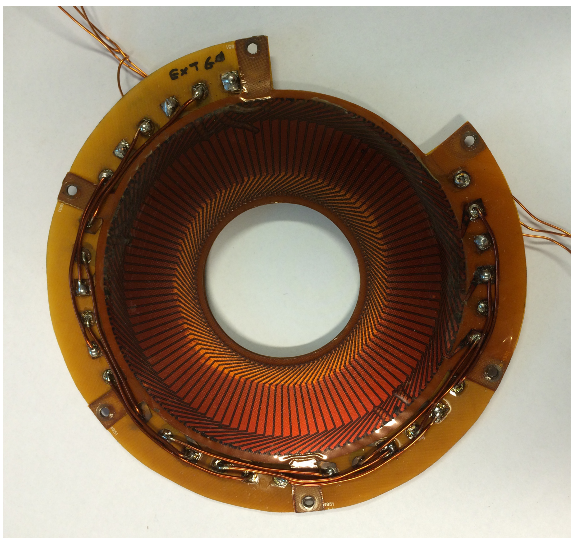
(c) Compressed stack: upside view.




(d) Compressed stack: lateral view.



(e) Stack after baking.



(f) Stack with the five boards connected.

Fig. F.2: PCB winding assembly: steps.

APPENDIX G

# PCB: AUTOMATIC BOARD GENERATION

## G.1 Parameters

As explained in section 4.1.3, for the three segments wave winding, all the equations yielding the points defining the tracks and the vias depend on a set of ten parameters, namely:

- $p$: number of pole pairs;
- $N$: number of phases;
- $N_l$: number of loops per layer;
- $R_i$: inner radius;
- $R_{ii}$: inner radius of the second segment;
- $R_{ee}$: outer radius of the second segment;
- $R_e$: outer radius;
- $d_i$: minimal insulation distance;
- $D_d$: minimal drill diameter;
- $D_v$: minimal via diameter.

Based on these parameters, a MATLAB code was developed to compute all the points constituting the PCB, allowing to test different combinations and to produce the corresponding figures, as those shown in section 4.1.3. As mentioned earlier, all the calculations are made for the first lobe of the first phase and are then rotated, using a rotation matrix, to obtain the complete phase windings.

## G.2 Script file

The MATLAB code also creates a script file (`.scr`) containing all the information about the PCB. This file is readable by the software EAGLE, allowing to generate automatically the board file (`.brd`) that is required by the manufacturers as *Eurocircuits*. Let us now present the main commands that are used in the scripts to create the PCB windings. The interested reader will find additional information in the *Edit commands* part of the *Help* in EAGLE.

### G.2.1 Global parameters

First of all, global parameters are defined:

```
grid ON mm 0.1 ALT mm 1;
set WIRE_BEND 2;
```

The `grid` parameter indicates the normal and alternative grid sizes while `WIRE_BEND` set to 2 specifies that the wires are constituted of straight lines between the different points. In a classical PCB, the connections between the electrical components are often made using simple wires whose width is constant. As explained above, the tracks of the three segments winding are composed of polygons to increase the layer fill factor.

```
change POUR SOLID;
change ORPHAN ON;
```

The `POUR` parameter set to `SOLID` specifies that the polygons are completely filled. The `ORPHAN` parameter allows to keep polygons that are not connected to electrical components, which is often the case for PCB windings.

### G.2.2 Polygons

To create a polygon, the following command has to be used:

```
poly 'name' 0.0 (x1 y1) ... (xn yn) (x1 y1)
```

The `name` of the signal as well as the list of the different points of the polygons are given. It will not have escaped the careful reader that EAGLE uses a cartesian frame to position the different points. The value `0.0` indicates the width of the edges of the polygon. It should also be remarked that the last point of the list is identical to the first one so that the polygon is closed. Besides, it is compulsory to assign the same `name` to two polygons that have to be connected. Otherwise, EAGLE removes parts of the polygons to separate them.

As there are more than one layer, it is important to be able to choose on which layer the track must be created:

```
change LAYER number
```

The parameter `number` is replaced by `1` for the top layer and `16` for the bottom layer. This command has to be used before creating the polygon.

### G.2.3 Vias

The following command allows to add vias:

```
via 'name' Dv shape (x1 y1) ... (xn yn)
```

The parameter `Dv` specifies the external diameter of the vias while `(x1 y1) ... (xn yn)` indicate their positions. The parameter `shape` can be replaced by `ROUND`, `SQUARE`, etc. It is again compulsory that the tracks and the vias that have to be connected present the same `name`. For each via, the drill diameter `Dd` can be changed using this command before adding the via:

```
change DRILL Dd;
```

### G.2.4 Screw holes

The screw holes can be added with:

```
    hole Dh (x1 y1) ... (xn yn)
```

The parameter `Dh` specifies the diameter of the holes while `(x1 y1) ... (xn yn)` indicate their positions.

### G.2.5 External limits

The external limits of the PCB are defined using a `continuous wire` of `0` width placed on the layer `20` (dimensions), as shown hereinafter:

```
    change LAYER 20;
    change STYLE CONTINUOUS;
    wire 'name' 0 (x1 y1) ... (xn yn)
```

### G.2.6 Final commands

The script file should be ended with the following commands:

```
    ratsnest
    window FIT
```

The first one verifies that all the polygons, wires and vias having the same `name` are connected together. Otherwise, it creates airwires showing unconnected parts. The second command only fits the complete board on the screen for convenience.

### G.2.7 Example

As the entire script of the PCB winding final version takes more than 1300 lines, the following code only resumes the most important parts. In this way, only one lobe of each phase and the corresponding vias are created.

```
# Date : 01-Apr-2016
# Author : JVV

# General settings
grid ON mm 0.1 ALT mm 0.01;
set WIRE_BEND 2;
change POUR SOLID;
change ORPHAN ON;
change SHAPE ROUND;

[...]

# Phases
change LAYER 1;
poly 'phase1' 0.0 (20.859928 12.464084) (21.460832 12.823132)
    (28.785165 8.450696) (42.913576 12.680465) (49.987090 1.136170)
    (50.850266 -0.300996) (52.050245 -0.308098) (54.236460 9.132709)
     (59.167048 9.962955) (59.790770 5.006379) (54.808206 4.589181)
    (52.037717 -1.182779) (49.987090 -1.136170) (43.749844
    10.533336) (29.298137 7.108218) (21.835574 12.174060) (21.224177
     11.833186) (20.859928 12.464084)
```

```
change LAYER 16;
poly 'phase1' 0.0 (20.859928 -12.464084) (21.460832 -12.823132)
    (28.785165 -8.450696) (42.913576 -12.680465) (49.987090
    -1.136170) (50.850266 0.300996) (51.550254 0.305139) (51.480781
    2.692758) (50.781737 2.656194) (49.987090 1.136170) (43.749844
    -10.533336) (29.298137 -7.108218) (21.835574 -12.174060)
    (21.224177 -11.833186) (20.859928 -12.464084)

change DRILL 0.35
via 'phase1' 0.7 ROUND (21.347526 12.325000)

change DRILL 0.35
via 'phase1' 0.7 ROUND (51.196992 0.653046) (51.148346 2.324899)

change DRILL 1.8317
via 'phase1' 4.5794 ROUND (57.050079 7.179030)

[...]

change LAYER 1;
poly 'phase2' 0.0 (15.338953 18.846923) (15.780816 19.389839)
    (24.158899 17.786163) (35.988596 26.593047) (46.583906
    18.164242) (47.886567 17.108972) (49.016607 17.512714)
    (48.642820 25.668581) (53.064895 28.002088) (55.210424
    23.490616) (50.609555 21.533065) (49.775225 15.223334)
    (48.627695 14.872371) (47.361092 16.028941) (37.508792
    24.861426) (25.100089 16.700093) (16.354954 18.908080)
    (15.897015 18.378654) (15.338953 18.846923)

change LAYER 16;
poly 'phase2' 0.0 (23.864889 -4.577892) (24.552354 -4.709765)
    (29.939516 1.904049) (44.662545 2.761568) (47.361092 16.028941)
    (47.680674 17.674659) (48.337030 17.917962) (47.455133
    20.137829) (46.810752 19.864383) (46.583906 18.164242)
    (44.714018 5.065230) (29.962397 3.341013) (24.682501 -3.971668)
    (23.991391 -3.860461) (23.864889 -4.577892)

change DRILL 0.35
via 'phase2' 0.7 ROUND (15.844715 18.882996)

change DRILL 0.35
via 'phase2' 0.7 ROUND (47.886081 18.124065) (47.268561 19.678456)

change DRILL 1.8317
via 'phase2' 4.5794 ROUND (51.926941 24.694996)

[...]

change LAYER 1;
poly 'phase3' 0.0 (7.967874 22.956546) (8.197401 23.617845)
    (16.618713 24.976356) (24.722861 37.298114) (37.562016
    33.001438) (39.147040 32.455345) (40.070843 33.221236)
    (38.149138 39.618724) (41.617241 43.220426) (45.068447
    39.608522) (41.312743 36.307812) (41.566725 31.329384)
    (40.608436 30.607109) (39.022648 31.260725) (26.743627
```

```
    36.190861) (17.874600 24.277690) (8.901685 23.361507) (8.652438
    22.707385) (7.967874 22.956546)

change LAYER 16;
poly 'phase3' 0.0 (23.991391 3.860461) (24.682501 3.971668)
    (27.482719 12.029139) (41.024552 17.870515) (39.022648
    31.260725) (38.760088 32.916497) (39.293646 33.369615)
    (37.705695 35.153981) (37.193699 34.676634) (37.562016
    33.001438) (40.285022 20.052854) (27.012750 13.387268)
    (24.552354 4.709765) (23.864889 4.577892) (23.991391 3.860461)

change DRILL 0.35
via 'phase3' 0.7 ROUND (8.430797 23.163423)

change DRILL 0.35
via 'phase3' 0.7 ROUND (38.799402 33.409054) (37.687490 34.658500)

change DRILL 1.8317
via 'phase3' 4.5794 ROUND (41.572933 39.723308)

# External limits
change LAYER 20;
change WIDTH 0;
change STYLE CONTINUOUS;
wire 'PCBe' 0 (68.000000 0.000000) (67.998655 0.427681) [...]
    (67.998655 -0.427681) (68.000000 -0.000000)

wire 'PCBi' 0 (22.500000 0.000000) (22.499555 0.141512) [...]
    (22.499555 -0.141512) (22.500000 -0.000000)

hole 3 (32.500000 56.291651) (-32.500000 56.291651) (-65.000000
    0.000000) (-32.500000 -56.291651) (32.500000 -56.291651)
    (65.000000 0.000000)


ratsnest
window FIT
```

# APPENDIX H

# PCB: THEORITICAL RESISTANCE

The winding phase resistance $R$ is calculated using Pouillet's law:

$$R = \rho_c \int_0^L \frac{dx}{S(x)}, \tag{H.1}$$

where $\rho_c$ is the copper electrical resistivity, equal to $1.68 \times 10^{-8}$ [$\Omega$m] at 25°. More precisely, the resistance $R_f$ of one forward conductor of one lobe, divided into three segments, is determined and then multiplied by $2pN_{l,t}$ to obtain the resistance of one stack of PCBs. The factor 2 allows to take into account the return conductors while the factor $p$ is for the $p$ lobes that constitute each phase loop. The resistance of the vias, the wires between the boards and the external wires between both parts of the armature winding must also be added.

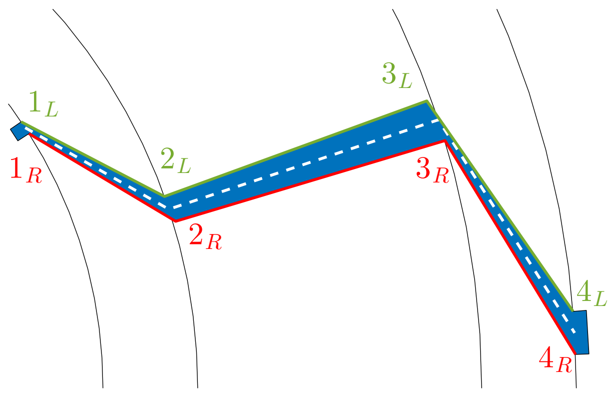



Fig. H.1: Forward track defined through eight points.

Let us determine the section $S_i(x) = w_i(x)t$ along the $i$-th segment, where $x$ follows the neutral axis of the tracks, $w_i(x)$ is the track width and $t$ is its thickness. The latter has been measured on the manufactured PCB and is roughly equal to 130 [$\mu$m]. Considering that the neutral axis, defined by the points in (4.3), is in the center of the tracks, their width can be calculated as two times the distance between this neutral axis and one outer edge of the track. As a reminder, the distance $d$ between a point $\{x\,;\,y\}$ and a straight line $y = mx + p$ is given by:

$$d = \frac{|mx - y + p|}{\sqrt{m^2 + 1}} \tag{H.2}$$

Fig. H.1 shows the eight points describing one forward track. As both edges of the tracks are straight lines, the tracks widths are also described by straight lines, defined as $w_i(x) = a_i x + b_i$. The first section of each segment is located at $x = 0$ and its end section at $x = l_i$, $l_i$ being the length of one edge of the track of the $i$-th segment. Hence, the resistance of the forward conductor is:

$$R_f = \frac{\rho_c}{t} \sum_{i=1}^{3} \int_0^{l_i} \frac{dx}{a_i x + b_i} = \frac{\rho_c}{t} \sum_{i=1}^{3} \frac{\ln(a_i l_i + b_i) - \ln(b_i)}{a_i} \tag{H.3}$$

For each segment, the function $w_i(x)$ can be determined either with the left edge (in green) or the right edge (in red). Hence, the final value of $R_f$ is calculated as the mean of the value obtained with both edges. On the one hand, it allows to take into account the fact that the neutral axis is not exactly in the center of the tracks as the points $2_R$ and $3_L$ are not on the radii $R_{ii}$ and $R_{ee}$ respectively. On the other hand, it allows to consider the fact that both edges do not have the same length and that the mean of both lengths is close to the length of the neutral axis.

For each of both parts constituting the armature winding, the twelve wires connecting the different boards between them are 3 [cm] long and their diameter is 1 [mm], yielding a resistance $R_w$ equal to 7.7 [m$\Omega$]. Likewise, the six external wires are 40 [cm] long and their diameter is 0.65 [mm], yielding a resistance $R_{ew}$ equal to 121.5 [m$\Omega$]. Finally, the 525 vias can be considered as hollow tubes whose inner and outer radii are respectively 0.35 and 0.45 [mm] and the height is 0.2 [mm], yielding a resistance $R_v$ equal to 28.1 [m$\Omega$]. Hence, the total resistance $R$ is given by:

$$R = 2(2pN_{l,t}R_f + R_w + R_{ew} + R_v) = 5.32\ [\Omega]. \tag{H.4}$$