

Percolation theory and complementary currencies

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Abstract

Complementary currencies experienced for the last three decades a remarkable development. Originally created in situations of economical crisis they are also used today to promote local, social or commercial purposes. New complementary currency systems are frequently started all over the world, with or without the expected success. For example, in the small Belgian municipality of Ottignies-Louvain-la-Neuve, a complementary currency was first launched in 2014 without much success and relaunched only recently in 2017. On another note, network theory is increasingly used today to model and study economic networks. Markets are modelled as networks of bilateral exchanges in order to understand better their functioning. However, the literature covering applications of network theory tools to complementary currency systems is limited. There is to our point of view a lack of network theory approach in the complementary literature. This master thesis first extracts from the complementary currency literature the contributions with a *network theory* approach. It then uses percolation theory to build a *viability threshold* for complementary currency systems. A key element consists in the need of circularity, for a complementary currency system to prosper.

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| Introduction

This master thesis was led by the idea that graph theory, as a tool, can help to understand better economic phenomenons. This is not a new idea and many mathematicians, economists and scientists have used network theory for this purpose. Entire books have been written on the subject as for example the one of Matthew O. Jackson, *Social and Economic Networks* [31]. Here, however, we will use a specific mathematical tool in order to tackle a subject usually left to practitioners. Indeed, complementary currencies have not been the objects of study of many academics, and even less of many mathematicians. Yet, we believe complementary currency systems not only deserve a great interest, but also deserve the attention of graph theoreticians.

Not all mathematical tools are suited for all subjects. The use of mathematical models in social fields such as economics should be done with many precautions and criticism. Indeed, if modeling social phenomenons leads, in most cases, to a greater knowledge, one should always remember that every model, as detailed as possible, will never reflect the entire reality. This master thesis falls within this reflexion of the utility, but also the limitations, of the use of mathematical tools to study social subjects. We hope the reader will realize that network theory can and should be used for a better understanding of complementary currency systems. However, we also ask the reader to remember that the results presented in this thesis will leave aside a huge part of the reality behind complementary currencies. That being said, we will not insist further on these precautions and the rest of this master thesis will focus on possible links between the field of complementary currencies and the one of network theory.

In order to do so, we will first present the state of the art of both fields. We will also restrict our attention to specific aspects of each topic. Concerning complementary currencies, we will be interested in papers with a *network theory* approach. We will also present a few elements about the functioning of conventional monetary systems. Concerning network theory, our whole focus will be on percolation theory. These two sections, namely section 1.1 *What are complementary currencies?* and section 1.2 *Networks* represent the majority of the first part of this master thesis, which is a literature review. The third topic covered in the literature review allows a complete picture of the current network-based approaches to complementary currencies and is not directly used later on. It briefly outlines the network approach to the question of a better resilience of economic networks thanks to complementary currencies.

The second part of this master thesis develops a *viability threshold* that should ensure a complementary currency to prosper. This is first done theoretically by using percolation theory and random graphs. Then, we use a data set to test the *viability threshold* developed. Finally, we criticize the results and conclude with the contributions of this thesis and the future possible work.

1 | Literature Review

1.1 What are complementary currencies?

A large literature exists about complementary currencies, as well as many platforms aiming to promote them. As an illustration, the Research Association on Monetary Innovation and Community and Complementary Currency Systems (RAMICS) is an international association that aims to link the scholars as well as practitioners working on complementary currencies [47]. They also link international academic elements that already exist, such as the International Journal of Community Currency Research (IJCCR), a peer-reviewed online forum about complementary currencies [29]. But many other platforms have been created by complementary currency promoters, such as the Complementary Currency Ressource Center. However, despite the plentiful information, it is not an easy task to define complementary currencies. One of the main reason is the various forms they can take, every complementary currency system obeying its own set of rules and organisation. Flourishing especially since the last three decades, some 310 complementary currency systems have registered on the Complementary Currency Resource Center website [9]. By doing so, the organizers of the complementary currencies recognize their project as part of a larger movement. However, their number is estimated to more than 5000 in the world [1]. Everywhere, these moneys aim to *complement* the existing monetary system, not replace it. In order to gain insight on complementary currencies we will start by a short description of our conventional monetary system.

1.1.1 Conventional monetary systems

Before considering the important literature about complementary currencies, understanding the history and the fundamentals of conventional money systems is an indispensable prerequisite. First, because we need to be able to distinguish between conventional and complementary currency systems. Second because money is an everyday life element that is rarely understood by non-economists. However, this work is not meant to explain deeply the history of money systems, its functions and challenges as this would fall outside the frame of this master thesis. We will thus only try to extract the relevant and primordial information needed for a better comprehension of complementary currencies.

The functions of money

Money has taken various aspects since the history of man and has almost always been part of human systems. From wheat grains to the flight miles, its features evolved with time and space. Money has thus served several functions. The functions of money were listed in the book *Money and the Mechanism of Exchange* written by William Stanley Jevons in 1875 to the number of four [32]. Since then, many economists [27, 28, 37] have used the analysis of Jevons in order to list the principal functions of money, now recorded in the economic literature to three.

Probably the first function that comes to mind is money as a *medium of exchange*, allowing more than bilateral barter. When two people exchange wood and bread in a bilateral way, there is no money implied in the process. As soon as we consider three individuals with cyclic preferences (individual A wants products of individual B, B of C and C of A) the only possibility allowing an exchange is the creation of money. If we introduce time in a bilateral barter (A needs what B produces during summer, B needs what A produces during winter), there is a need for keeping the information in a journal book, thus the introduction of money is also necessary. Bilgram and Levy [3] consider any medium of exchange that goes beyond the barter process (the two individuals need what the other possess) as money [16].

A second function of money is the one of *unit of account*, as money allows to compare prices. Note that money sometimes fail to fulfill this function, as during periods of high inflation or regression where money does not provide safe information. As an example, let us consider the hyperinflation of the Weimar Republic during which the prices at the end of year 1923 were on average 1.26 *trillion* times higher than they had been in 1913 [17].

The last of the three main functions attributed is money as a *store of value*, in space and in time. Here we see that all the different kinds of money do not fulfill the functions of money with the same manner. Grains of weads are unable, by nature, to store value for a long time period whereas gold will never decay. The ability to store value in time also allows savings and hoardings.

Some authors such as Thomas Greco [22] have highlighted the contradiction inherent to these three functions. Indeed, the aim to store value in time enters in contradiction with the function of money as a means of exchange. Each dollar kept outside of the circulation system prevents from using it. Also, the function of measure of value should not, according to Greco, only be held by money. Indeed, a currency should not by itself be a measure of value as it could then be totally disconnected from real values. It then hurts the exchange function as it distorts the information. We can also see through history that when a money fails to fulfill one of its function, people find alternatives to repare this gap. For example, countries with a high inflation rate and thus a poor unit of account function, often use another money, usually the US dollar, to exchange goods.

A brief history of money

In his latest book, *The end of money and the future of civilization* [23], T. Greco explains the evolution of money and of our monetary systems. He relates how reciprocal exchange evolved from barter (only two people or groups involved) to commodity money (grains, shells then precious metals) and then to symbolic money (claim checks for goods on deposits somewhere). The acceptability of symbolic money simply comes from the fact that, at any time, it is redeemable for the commodity it represents. Such symbolic money systems existed worldwide and since many centuries. For Greco, though these types of money (barter - commodity money - symbolic money) are different, they form a *continuous* period. The main characteristic shared by these three types of money is the fact that they are always related directly to *real* goods.

The next step in the evolution of money, called credit money, represents to Greco's point of view the major leap done by our civilization in the way to exchange goods. It consists in the ability for banks to give notes not only to those who had deposited metals or other commodities but also to those who came to borrow banknotes. Here, we must highlight the huge difference between a loan coming from a friend or an enterprise and the one coming from a bank. Of course, humans have always borrowed money. For example, someone gives you 50 dollars and trusts you for giving back these 50 dollars later on.

The difference here is that the issuer of the loan had the 50 dollars. On the contrary, with the credit money system, banks do not have to possess 50 thousands dollars in order to lend it. They are able to create it *from nowhere*, given some limits. It is evident that banks do not just give loans anyhow to anyone. They take some collateral assets (houses, merchandises, etc) in exchange. But once the loan was made, the banknotes attributed against collateral assets were, just as the ones given in exchange of gold, redeemable for gold. This monetary system, making every banknote redeemable for gold, is called as the Gold Standard. The credit money has a great advantage : it allows to exchange a lot more than the finite amount of gold disponible on Earth. However, the ability to create liquidity (banknotes) from unliquid things (collaterals) makes the system extremely vulnerable to worthless or fake collaterals, swelling the monetary total value from the real total value. Greco also notes the problem of the manipulation of interests that 'enforce' people to borrow money with low interest rates but then are raised, causing impossibilities to reimburse the debts. A good example of such interest effects is the mortgage subprime crisis of 2007 - 2008.

«When the banker issued notes in amounts greater than the amount of metal in his vault and made all notes redeemable on demand for metal, the stage was set for trouble.» Extrait de Thomas Greco, *The End of Money and the Future of Civilization*, p.157

At some point, the redeemability of money was abandoned. During World War I many countries abandoned for a moment the Gold Standard. With the 1929 crisis some European countries completely gave up and after World War II, all EU countries had given up. The solution brought took the form of the well-known Bretton-Woods system adopted in 1944. As all the gold had flown to the United-States during the wars, the latter were the only one able to keep the Gold Standard. The redeemability was thus kept for US dollars but all the other currencies would be measured in reference to the dollar. Instead of keeping gold as fractional reserve, european banks now had to keep US dollars. This system is called the Gold-Exchange Standard. Let us note that the convertibility of European currencies into dollars was not possible until the end of the 1950's.

The last step in the evolution of money is, according to Greco, the emergence of credit clearing. It was introduced around the mid-nineteenth century when people began to make a regular use of checks in order to make payments. The use of checks and deposits thus decreased the necessity to use paper money : many transactions could be done directly at the bank by a clearing process. Thus the transition from banknotes (that could only be created into central bank) to deposits allowed banks to circumvent government's limitations on the issuance of banknotes. The role of commercial banks became the control of requests for credit and the clearing of credits among all customers of the banks. However, this process implies much more confidence as the cheques were not legal tender, as coins and banknotes. Today, with all purchases and salaries now being made electronically, the role of banks consists in increasing someone's account when the salary is received, decreasing the account when a purchase is made. It amounts to changing numbers in different accounts. This credit clearing system is extremely convenient while also efficient as it covers exactly the same roles as the credit money described above. The difference is that by creating and making use of a new type of money that is not a legal tender (cheques) the commercial banks were now able to create money easily, without central bank's limitations.

Now, this credit clearing can be seen in different ways. On the one hand, it can be seen as if each account was a fluctuating wave around zero : at each time step we make the

difference between sales and purchases which gives a positive or a negative balance. The important thing being that an account cannot be always negative. On another hand, the credit clearing system can present money as if banks actually owned the money before creating the loan. This is how banks act today : charging interests if they add a positive line in an account (credit). Let us use Greco's example to illustrate this difference. A group of companies A,B,C and D trade with one another so that each company owes 100 dollars to another, forming a cycle. We also suppose for the sake of the example that all companies have an empty account. In today's credit clearing system, one or more of the companies will have to borrow money to a bank in order to pay the company it owes. Let us consider the simple case of A borrowing money to the bank. A then pays B, who pays C, who pays D, who pays A again. At the end, A has to repay back to the bank the principal (the 100 dollars) and some interests. However, when the bank made the loan it did not create the interests, neither did the exchanges between all companies. In order for company A to pay back the interests, the latter must compete in the market to capture that extra money. But there is no reason for the market to produce the interests of the ABCD circuit. Thus this will leave a deficit somewhere. They will never be enough money for all borrowers to pay back. The only solution is to borrow more money. This explains the graph 1.1.

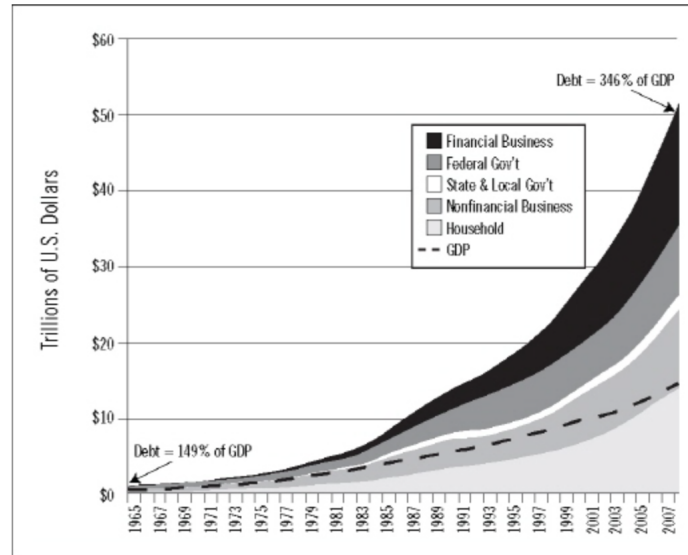


Figure 1.1: Actual growth of debt in the United States - T. Greco

The other vision consists in companies paying each others with an IOU (I owe you) created without interests. It is the one promoted actually by credit clearing circles.

« *Direct credit clearing among buyers and sellers obviates the need to use any third party currency or credit instrument (money) as a payment medium.* »

Extrait de Thomas Greco, *The End of Money and the Future of Civilization*, p.161

Money creation in the modern economy

Let us now present another paper explaining more in details today's monetary system. The Bank of England's report *Money creation in the modern economy* [44] explains how money is created by banks making loans, paying attention in deconstructing some misconceptions. Some of the explanations below have already been mentioned in section 1.1.1 about money history but in this section we look at money creation in more details. A first misconception is that banks act simply as intermediaries, lending the deposits

from savers to other borrowers. Actually savings do not increase the amount of money or funds available for banks to loan. Banks do not need deposits to lend money. Instead of lending out deposits, the banks create them. This means that when one goes to the bank and asks for a loan of 10 000 euros, the bank creates a new line in the customer balance sheet, and thus money is created. The bank created simultaneously a debt (IOU from the customer to the commercial bank) as the customer will have to repay the loan and a new deposit (IOU from the commercial bank to the customer). Another misconception regards the money-multiplier approach. The latter states that central banks choose the quantity of reserve (the fractional reserve percentage), which then determines how much money can be lent. The money is then multiplied by economic agents in their transactions [Blanchard]. However, for the theory to hold, the reserves must be a binding constraint on lending, which is today not the case. Instead, central banks usually make use of the interest rates. Commercial banks then choose how much they lend depending on the profits they can make, which depend on the interest rates of the central bank.

Let us now define two terms widely used when talking about money. The monetary base, also called base money, consists in the total currency circulating in a country added to the commercial banks' reserves. When talking about banks reserves we refer to a special kind of IOU that can be used between banks only. Reserves are an IOU from the central bank to commercial banks. It is also highly liquid as all banks' reserves can be swapped into currency at the Central Bank. When talking about currency, we refer to banknotes and coins, thus an IOU from the central banks to every holder of the note. The term broad money is meant to aggregate the deposits of households and companies in banks as well as the currency. In 2013 in the UK, of these two types, deposits consisted in the 97% of the circulating or broad money [44]. Everytime a bank makes a loan, it creates the money *out of nowhere*. It does not take it in the reserve, which can only be lent between banks. And when someone repays a loan, it destroys money, just in the same way it created it. We thus see that banks do not act just as intermediaries and that they create money by making loans. But does it mean they are able to create money without any limits ? It is thankfully not the case and we describe shortly hereafter the three main reasons why.

Banks themselves face limits on how much they can lend.

Without any external constraints, banks are not able to lend as much as they would like in order to be profitable and safe from risks. What makes a commercial bank profitable ? Of course, when a bank makes a loan that is repaid entirely and with interests, the bank makes a profit. One important part of a bank's strategy thus relies in making profitable loans. So far, there is thus no reason why a bank should also attract deposits. Indeed, we just explained that banks could make loans without having a specific amount of deposits. If it is the case, what happens if a bank makes a loan without having the equal amount of deposits ? At first, nothing. But at the end of the day, the central bank will debit the commercial banks accounts with the loans made during the day. Thus the reserves of all commercial banks will decrease. If a bank has no more reserves, it has to borrow reserves on the *wholesale deposits market* from another bank with exceeding ones. Borrowing reserves is cheap but still more expensive than deposits. Banks thus need deposits in order to balance their sheets. And this holds even if deposits cost them. The business model of a bank is thus based on receiving higher interest rates for loans than the interests payed on deposits. Even if lowering the interest rates on loans allows to make people borrowing more, banks are limited in doing so. If the interests on loans decrease too much, the bank might become non-profitable. This explains the first reason why banks themselves face limits on how much they can lend. The second reason is that banks must minimize the risks associated with making loans. In order to face the

liquidity risk, a commercial bank must attract long term deposits but the latter are more costly and thus impact the profitability of the firm, which in turn impact the interests on loans and the total amount of money lent. In the same way, each bank business model anticipates that some clients will never repay their loans. In order to compensate this credit risk, interests charged on loans include the expected losses and are thus higher, which discourage households or companies to borrow.

Constraints arising from the response of households and companies.

Another reason why banks are not able to create as much money as they want is that the newly created money is then used by households and companies in a way not controlled by banks. There are two possibilities when a new deposit is created : it is used to pay back another debt and the money created thus quickly vanishes or it is spent and circulates among people with positive accounts who will not destroy the money created by repaying loans. This constraint affects the total amount of created money in the economy. It does not directly constraints the behaviour of individual banks.

Monetary policy.

The last constraints on money creation are monetary policies. The central bank can always influence the behaviour of banks by changing the interest rates on the reserves owned by the commercial banks. The reserves are actually an IOU from the Central Bank to commercial banks that the latter can also exchange among each other. Thus commercial banks place their reserves at the central bank which is the 'bank of the banks' and get interests on these reserves. If the bank rate on reserves is lowered, banks are encouraged to lend more to households or companies even if they are not sure to have enough reserves. For example, due to the financial crisis, the interest rate on reserves was lowered to 0.5%. Of course, the bank rate has to stay positive. Otherwise, commercial banks would draw all of their reserves and the central bank would have no control at all on the economy. It is thus important to notice that the central bank monetary policy is not to choose a quantity of reserves that in turn influences the interest rates. Rather, the opposite happens and the central bank is able to do so because of its monopoly. The Bank of England's report thus seems in conflict with the Keynesian school and the monetary approach found in conventional textbooks. Following the approach of the Bank of England, the inverse of the usually explained *money multiplier* effect happens.

The three constraints described above show how money is created and with which limits. We have also seen that money today is just an IOU based on confidence : everyone in the economy believes it will be accepted by others. If money - whatever the form (currency, bank deposits or bank reserves) is just a simple IOU based on confidence, we understand how easy it could be to create a new type of money but in the same time, how hard it is to make that money be trusted by a large network, without the support of the State. For example, e-money allows to make transactions online but are generally not accepted for all transactions. In the same manner, local currencies or complementary currencies aim to replace the national currency but for a limited amount of transactions. The limitation is here intentional which is usually not the case of e-money. Finally, digital money such as the Bitcoin do not usually serve as a medium of exchange and have a limited supply.

1.1.2 Complementary currencies

Let us notice the variety of terms used to describe complementary currencies. Local, social, alternative or complementary currency, each specific adjective reflects a specific feature of the money created. However, we prefer the term of complementary currency. It allows, to our point of view, to cover the entirety of the monetary systems except the

national currencies. The term *local* aims to emphasize a geographic limitation but the latter can vary from a neighborhood to a whole country or sometimes several countries which make it confusing. The terms *social* or *alternative* in some way suggest that the monetary system should be a factor of social change or have a societal impact. However, many complementary currencies, such as travel miles, do not at all match this feature [1].

The term of complementary currency is thus interesting as it allows a broad definition. However, it does not allow to distinguish between the bitcoin and a local citizen corporate. We will thus distinguish between complementary currencies similar to the *local exchange trading systems* (LETS) even when the *local* angle is not preeminent, monetary systems substitutes such as fidelity cards, miles and coupons, and finally the virtual currencies [10]. Our interest focus mainly on complementary currencies such as LETS. Let us present two complementary currencies relevant for this work as they have been studied more than others, with sometimes a 'network theory' point of view.

The WIR Bank in Switzerland

Probably the most complete work about WIR has been realized by Dr. Tobias Studer in 1997 and is a report about "the *modus operandi* and the relate and purpose of WIR in the Swiss economy" ¹. Relating the economic circumstances accompanying the birth of WIR and its evolution with time, Studer's work [50] allows to retrieve the main features of the WIR. The WIR Economic Circle was founded in 1934 only five years after the market crash of October 1929. The crisis affected countries all over the world. To counter the recession, each country tried different policies, each of them with its own strengths and limitations. Switzerland experienced the heart of the crisis relatively late compared to the other countries and therefore had a range of examples of policies at its disposal. Amid all the various divergent opinions in the debates, the solutions brought were finally wage reductions, "crisis taxes", or protectionist measures. There was a lack of buying power for consumers but also for firms, which urgently needed an increase in turnover and faced underutilized productivity due to the lack of buying power. This problem was at the heart of the WIR Economic Circle creation. The startup created by 16 businesses grew rapidly and aimed to promote exchanges within the Circle, catching up the lack of liquidity by the creation of a new money : the WIR. The rules of the Circle, after a pioneer phase with different experiments from 1934 to 1952, stayed more or less the same from 1952 to 1988. During that period, the Circle experienced a near-constant growth. Today, the WIR Circle has become the WIR Bank and the credit clearing concept is not the same anymore. The WIR Bank is open to individual consumers and treats both currencies (WIR and Swiss francs). We summarize here after the concept and rules followed from 1952 to 1988.

- When a business becomes a member of the WIR Circle, the business can establish a positive account by one of the two following ways. Either by selling goods to another member of the WIR Circle or by buying WIR money with Swiss francs. The second option is however unusual as it negates partly the advantage of the WIR credit clearing system. Indeed, the WIR money served as a solution to the lack of liquidity of Swiss francs. All positive accounts are interest-free. Let us note that both options do not generate any money creation.
- A business is also able to become a member of the WIR Circle and to establish a negative account by taking loans. Short-term WIR loans are allowed without interests by the central office against collaterals. Long-term WIR loans can be

¹Studer J., (1997). WIR and the Swiss National Economy, p.1

granted to specific members but in return the loan is granted with negotiated interest fees, in any case lower than normal banks as a WIR loan is less universally useful than a Swiss franc loan.

- All transactions are made without banknotes or coins. The central office records all purchases and accordingly increases or decreases the members accounts.
- In order to guarantee a large range of goods and services, each member must accept payment in WIR for at least 30% of the first 2000 francs of the selling price. All loan holders must also sell goods and services in WIR in order to repay his or her debt.
- Participants are all listed in a catalogue so that all members have direct access to the different offers of goods and services.

In practice, when two businesses are willing to exchange goods in WIR, the payment is made thanks to a check that is sent to the WIR central office. The latter credits the account that receives the payment and takes a transaction fee. So when business A has found in the catalogue an interesting item from business B and desires to buy it, A must discuss with B of the percentage of the payment that will be made in WIRs (from 30% to 100%). Business A then writes a check for the payment in WIR and give it to business B. The rest of the payment, if any, is settled as usually in cash or with a credit card. Business B then sends the nominative check to the WIR central office that credits business B's account. The check is then *destroyed*, each new transaction requiring a new check [15].

Here it is of extreme importance for us to highlight a few features of the Circle. First, let us notice that the benefit in entering the Circle, besides interest-free loans, was also to enter the economic network and consequently benefit from contacts and solidarity between the members. This solidarity can be seen as a promise : if I am part of the Circle, then all the members interested in my production or services will choose my business before another that is not in the Circle. Besides, government policies aiming for an increase in buying power brought by monetary instruments such as an expansion of the monetary base, have the disadvantage to require a time lap and an uncertainty due to the agents reactions to the measure. The Economic Circle could offer to individual firms an immediate effect, proportionnal to their own implication. A key concept that allowed the success of the Circle is the WIR credit clearing system. The money created by loans never leaves the Circle but besides, there is no global limit on the possible money creation. Usual barter systems function as a central office with an administrative task consisting in making sure that every participant follows the rule. Usual barter central offices do not create any money and the total amount of negative accounts is only influenced by the total amount of positive ones. With the WIR credit clearing system, the central office plays an active role as it also grants loans. The clearing office thus controls the expansion or not of the WIRs circulating. Moreover, as the WIR bank bears the credit risks itself, it will be very carefull in granting loans. Finally, the loan business, besides generating profit, is the driving force behind the barter process, making the WIR Circle the largest complementary currency system in the world.

The Sardex network

Another complementary currency that specially caught our attention is the Sardex, very similar to the WIR and created in Sardinia in 2010 in response to the recession. Electronic-only, the Sardex is not exchangable with the Euros. It is a business to business local currency that cannot be used outside of Sardinia. Already eight other regions in

Italy replicated the Sardex concept, notably because so far the transactions volume and membership grew continuously.

The Yale Institute for Network Science in partnership with a member of the Sardex startup, realized a *Network Analysis of the Sardex Community Currency* [30] which explains why we focus on this particular complementary currency even if it is pretty similar to the WIR. Their work consists in a short analysis of the Sardex network. They model the economic exchanges as a directed weighted graph with economic agents represented by nodes and directed edges representing the transactions. The analysis relates some of the Sardex network properties. The network is made of a unique component of a bit more than 2000 nodes and 17000 trading relationships.

Conclusion

We conclude this section by insisting on some common features shared by most complementary currency systems. These common features are drawn from the WIR and Sardex examples. The example of the *Talent* in Louvain-la-Neuve, not described in this literature review also helped. The best solution in order to understand complementary currencies is to try to extract their main characteristics from some practical examples. One can notice that complementary currencies seem to share the characteristic of a limited convertibility. WIRs are officially not exchangeable into Euros. If a black market allowing to do so exists, there is a certain cost in the latter transaction. Sardex are not exchangeable in euros either and it is also the case for the Talent in Louvain-la-Neuve. In all these complementary currencies, one should not be interested in any sparing. Indeed, the banks do not provide interests for the positive accounts which encourage the agents to spend rather than spare their money. One last remark about the complementary currency networks concerns the size of the networks considered. While the WIR and Sardex networks are broader than the Talent network in Louvain-la-Neuve, their size is still limited.

1.2 Networks

We now have a small picture of complementary currency systems and their main characteristics. The purpose of this master thesis is to use tools coming from network theory in order to understand better complementary currency networks. If there is a wide range of possible network theory tools to use, we have decided in this master thesis to focus on the theory of percolation. As the reader will discover through this section 1.2, percolation theory allows to determine thresholds from which some network properties hold.

Let us consider all the economic exchanges within a specific set of agents as a network. For example, let us consider all economic exchanges in Sardinia. The set of exchanges made in complementary currencies will form a subnetwork of the network of all economic exchanges. If we stick to the same example, the exchanges made with Sardex money will consist in a subnetwork of the total network of all exchanges in Sardinia. Without any strong assumptions about the type of networks considered, the subnetwork of Sardex exchanges might be obtained by removing or deleting edges from the network of all exchanges in Sardinia.

Percolation theory asks whether some properties hold or not in the subnetwork obtained by deleting edges or nodes. It is thus possible to use percolation theory in order to determine whether some properties hold in the subnetwork of complementary currency exchanges.

The reader should not at this point understand deeply how we are going to use percolation theory in order to define the *viability threshold* mentioned in the introduction of this master thesis. However, one should understand the motivation behind our interest in percolation theory.

1.2.1 A brief history of percolation

Historically, percolation theory has its genesis in chemistry. More precisely, it comes from Broadbent and Hammersley, two scientists who were interested in the way a fluid could flow in a porous medium with randomly blocked channels [7, 35, 53]. The word *percolation* refers to the movement of fluids through porous media, just as our coffee percolators. Broadbent and Hammersley asked particularly which characteristics of the medium were necessary for a fluid to make its way, hole to hole, from the top of the medium to the bottom. In 1957, they published a paper studying how the random properties of a medium could influence the diffusion of a fluid [25]. In order to make the difference with the usually studied diffusion theory, focusing on the properties of the fluid rather than on the medium ones, they decided to call their approach *percolation*. Already at that time, they saw how the practical case they were at first interested in could, with a bit of abstraction, be expressed in a more mathematical form, leading to other applications such as studying diseases infecting a community. Their pioneering work shows that the characteristics of the medium, together with some external laws, determine the progress of the fluid through the medium. Specifically, they introduced the concept of a random crystal and by doing so they formalized percolation as a mathematical discipline. The random crystal is made of atoms or sites linked by bonds or edges. If an edge between to sites is open, the liquid can flow from one site to the other. Each edge has an independent probability p to be open and to let the fluid pass and a probability $1 - p$ to be closed. What is the probability that there exists an open path from the top of the medium to the bottom, for a given p ? This problem they first introduced is called **bond percolation**. It differs slightly from **site percolation**, in which sites (and not bonds) have independent probability to be open. Broadbent and Hammersley could not at that time find the critical probability p allowing the liquid to pass through the medium. However, they proved that the critical probability was non-trivial with $0 < p_c < 1$ and introduced a *connectivity constant* giving a lower bound to the critical probability p_c .

1.2.2 Percolation on a lattice

Even if Broadbent and Hammersley qualified their results as somewhat scanty, their methodology actually gave rise to and inspired many works afterwards. Béla Bollobas and Olivier Riordan in their book *Percolation* retrace how these two scientists paved the way to a rich theory with applications going far beyond the understanding of natural physical processes [7]. Indeed, the link between a real crystal and an abstract graph can be made easily by considering the graph $\mathbb{Z}^3$. The latter, as the random crystal described by Broadbent and Hammersley, is made of sites linked by edges but with three dimensions. Once the abstraction is made, the problem can also be extended to any lattice $\mathbb{Z}^d$. Bollobas and Riordan relate that the first spectacular mathematical result in percolation theory for lattices was proved by Kesten and Harris in 1980. Kesten then gave a lower bound on the critical probability for bond percolation on the square lattice $\mathbb{Z}^2$, which proved this critical probability to be $1/2$ as Harris had already proved the opposite bound to be $1/2$. The Harris-Kesten theorem has been a key result in percolation theory. Figure 1.2 shows an example of a $\mathbb{Z}^2$ square lattice with probability $p = 0.51$. There is thus a critical value at which the behaviour of the lattice changes suddenly. The notion of a critical threshold was already introduced in our presentation of Broadbent

and Hammersley's work with the percolation of a liquid through a medium. However, we must insist on this sudden change of behaviour as it defines the field of percolation theory.

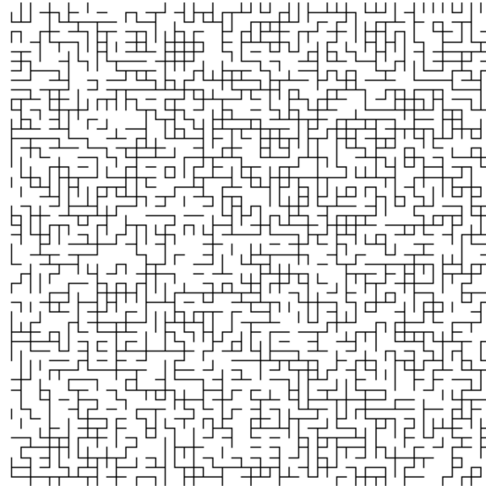


Figure 1.2: Square lattice in two dimensions with percolation probability $p = 0.51$ ²

Another important result, related in Bollobas and Riordan's book, comes from the theorems of Menshikov and Aizenman & Barsky. Both proved independently the exponential decay of the clusters size distribution. Here a cluster is simply a maximal connected subset of (open) edges. When p is smaller than the critical p_c at which percolation occurs, the cluster size distribution has an exponential tail : it goes from really small and grows exponentially [49]. Also, for the square lattice, it can be easily found that the behaviour of the cluster size above the critical probability is also exponential. This is done by looking at the dual graph of the square lattice which is also a square lattice. An example of such a dual two-dimensional lattice is shown on figure 1.3. Yet another interesting result of Aizenman, Kesten and Newman states that under certain conditions, there is a unique infinite open cluster above the critical probability. However, we will not deepen this point. Let us only refer the reader to figure 1.4 to see more concretely the behaviour of the probability function for percolation. The percolation probability as a function of the site vacancy.

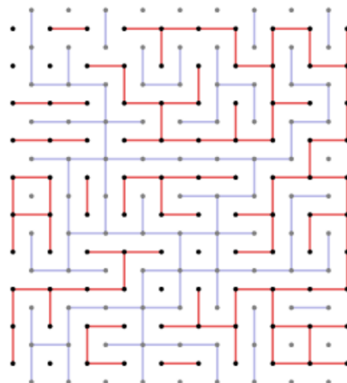


Figure 1.3: Square lattice and its dual. The black nodes and red edges form the original graph. Each node of the dual graph is placed in the middle of the squares and linked to another only if it does not cross an red edge. Requiring that the red edges are open with probability p amount to require that the dual blue edges are open with probability $1 - p$.³

Let us recall the elements that are relevant for us. Percolation on lattice $\mathbb{Z}^2$ happens suddenly at probability $p_c = 1/2$ and it is possible to find this value theoretically. Moreover,

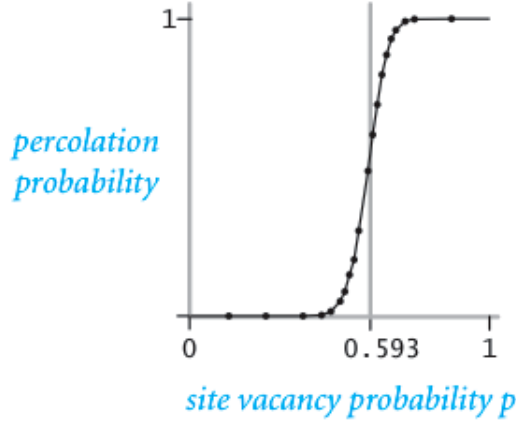


Figure 1.4: The percolation probability as a function of the site vacancy. Bond percolation looks similar⁴.

we also learned about the behaviour of the lattice before and after the critical probability. The different clusters composing the entire lattice have a size growing exponentially with p .

One could also wonder if results similar as the ones stated above hold in the case of a directed lattice. Unfortunately, it is not the case, the directed case being much more complicated. However, many mathematicians gave estimates for the critical value in a directed lattice $\mathbb{Z}^2$ that are extremely precise : Guttmann and De'Bell for example found in 1988 a critical probability equal to 0.644701 ± 0.000001 [7].

Due to the important historical result of the critical probability for bond percolation on $\mathbb{Z}^2$, many other results followed. As percolation theory can be applied to a huge amount of graphs, results exists for $\mathbb{Z}^d$ lattices, triangular lattices, hexagonal ones, etc. However, as mentioned at the beginning of this section 1.2 about networks, our interest in percolation is motivated by our willingness to define *viability thresholds* for complementary currency networks. If percolation on a lattice allows to understand the very beginnings of percolation theory it does not seem sufficient for this master thesis. Indeed, it would be a strong hypothesis to assume that an economic network looks like a lattice. The results about the Sardex complementary currency found by the Yale Institute for Network Science show that it is indeed not the case [30]. Let us then focus on percolation for random graphs and not especially lattices.

1.2.3 Percolation for random graphs

The firsts to study percolation in random graphs were Erdős and Rényi [12–14], even if Erdős had already used probabilistic methods to treat problems in graph theory and if many others, beginning with Paley and Zygmund in the early 1930's had studied random series of functions. Also, different authors, in quite the same timing, got interested in statistical aspects of graphs but their approach was not based on probabilistic developments and led to much fewer results [6]. Just as Broadbent and Hammersley paved the way for percolation theory in lattices, Erdős and Rényi are at the origin of important results in percolation for the particular case of random graph model they developped. When refering to an Erdős-Rényi model, one actually refers to one of the two following models for generating random graphs.

- The first model, the one exposed by Erdős and Rényi, is written $G(n, N)$. The graph G is a random graph with a fixed number of nodes n and edges N , chosen at

random among all the possible $\binom{n}{2}$ edges so that all $\binom{n}{2} = C_{n,N}$ possible choices of random graphs have equal probability $1/C_{n,N}$ to be chosen.

- The second one, introduced independently and at the same time by Gilbert, is written $G(n, p)$. Here, G is the random graph constructed by connecting a fixed number of nodes n . Each edge has probability p to be included in the graph, independently of the other edges [20]. It is thus closer to the formulation given by Broadbent and Hammerley.

As the reader guesses, if these two models refer both to the Erdős-Rényi model, they must be linked in some way. It is indeed the case. Due to the law of large numbers, a $G(n, p)$ graph will have an amount of edges approximately equal to the expected number of edges $\binom{n}{2}p$. If $N = \binom{n}{2}p$ then both graphs $G(n, p)$ and $G(n, N)$ behave the same way, at least for a large number of edges. Therefore, provided $n \rightarrow \infty$, $G(n, p)$ and $G(n, \binom{n}{2}p)$ will behave similarly. However, not all properties of both graphs will be the same. Erdős and Rényi studied the behaviour of $G(n, N)$ as N increases. They found that starting with $N = 0$ and increasing the number of edges step by step, there was for a certain value of N a drastic change in the size of the largest component. Under that specific value, called threshold, the size of the largest component is $O(\log n)$ while above the threshold it is in $\theta(n)$. At the threshold, the size of the giant component is of order $n^{2/3}$, but this has only been found almost twenty years after by Łuczak [41] after Bollobas [5] had given an upper bound in 1984.

Let us now define the threshold function, following the definition of Erdős and Rényi. The function $f(n, p)$ is a threshold function for a monotone property A if

$$\lim_{n \rightarrow \infty} \Pr(G(n, p) \text{ has } A) = \begin{cases} 1 & \text{if } \frac{p}{f(n, p)} \rightarrow \infty \\ 0 & \text{if } \frac{p}{f(n, p)} \rightarrow 0 \end{cases}$$

A few remarks need to be made on this definition. The fact that the property A must be monotone will be explained more in details later on. It means that if A holds for a subgraph of a graph, then it holds also for the graph. Also, the conditions $\frac{p}{f(n, p)} \rightarrow \infty$ and $\frac{p}{f(n, p)} \rightarrow 0$ can be understood, respectively, as $p \gg f(n, p)$ and $p \ll f(n, p)$. Above the threshold, the probability to have A is 1 while under it is 0.

Erdős and Rényi found the value of the threshold for the apparition of a giant component, which is $N(n) = \frac{n}{2}$. If we want to make the link with the $G(n, p)$ model, this gives the critical threshold

$$p = N \frac{1}{\binom{n}{2}} = N \frac{2}{n(n-1)} \approx \frac{1}{n}$$

They also found the value of the threshold for other properties. For example, the threshold function for the property that a graph contains a cycle of order k , for each value of $k \geq 3$ is n . In another paper published just after in 1961, Erdős and Rényi studied not only the apparition of a giant component but also the connectivity of $G(n, N)$. They found a threshold function of $N(n) = \frac{1}{2}n \log n$. Above the giant component phase and under this threshold, the graphs are almost surely connected. Here *almost surely* means that the probability tends to 1 as n tends to infinity. Finally, for $N(n) \sim n \log nw(n)$ with $w(n) \rightarrow \infty$, the graphs are not only almost surely connected but the orders of all points are almost surely asymptotically equal. They were thus able to characterize the evolution of a random graph $G(n, N)$ that we summarize here after :

- *Phase 1.* corresponds to the range $N(n) = o(n)$. The graph consists almost surely exclusively of small tree-like components.

- *Phase 2.* corresponds to the range $N(n) \sim cn$ with $0 < c < \frac{1}{2}$. The graph consists almost surely exclusively of many components with most of all that are trees but some components also contain cycles.
- *Phase 3.* corresponds to the range $N(n) \sim cn$ with $c \geq \frac{1}{2}$. At that threshold, the structure of the graph changes abruptly and a giant component appears. This giant component has approximately $n^{2/3}$ vertices and a complex structure. This giant component is also unique : the other components are still small trees.
- *Phase 4.* corresponds to the range $N(n) \sim cn \log n$ with $c \leq \frac{1}{2}$. The graph almost surely becomes connected.
- *Phase 5.* corresponds to the range $N(n) \sim n \log w(n)$ with $w(n) \rightarrow \infty$. The graph becomes *asymptotically regular*.

Again, Bollobás in his book *Random Graphs* [6] presents a clear overview of the different researches preceding the work of Erdős and Rényi and refers in particular to Kahane [33] for a more complete review of the subject. Much work also followed the findings of Erdős and Rényi. For example, Chung, Horn and Lu made a good work generalizing the findings of Erdős and Rényi for general graphs. They found that, if G satisfies some conditions depending on its spectral gap and higher moments of its degree sequence, it was possible to find a threshold for the apparition of a giant component [8]. Indeed, the Erdős and Rényi model is only a special case where the host graph is K_n . Chung, Horn and Lu considered a very general graph such as a contact graph. Their model is thus similar to the Erdős and Rényi model but more general. If the host graph is G the random subgraph G_p is obtained by retaining each edge of G with probability p . Chung, Horn and Lu asked for which value of p a giant component appears. We have seen that for the special case of the Erdős and Rényi model, this threshold was $p = \frac{1}{n}$. Chung, Horn and Lu define the second order average degree

$$\tilde{d} = \sum_v d_v^2 / (\sum_v d_v)$$

where d_v denotes the degree of v . They proved that for any $\epsilon > 0$, if $p > (1 + \epsilon)/\tilde{d}$ then asymptotically almost surely G_p has a giant component. The threshold thus depends on the structure of G . For more examples, we refer the reader to Grimmett [24] for a review of the subject, even if it does not cover more recent results.

1.2.4 Phase transition theorem

As it appeared that many graph properties have a threshold as the number of vertices tends to infinity, many wondered if it was a general rule. Bollobas and Thomason [4] proved that every non-trivial monotone increasing property of subsets of a set has a threshold function.

This remarkable result gave rise to a *theory of threshold functions*. Notably, many authors focused their researches not only on the existence of a threshold function but on its behaviour. The main question consisted in asking whether, given a particular property, its associated threshold function was *sharp*. The *sharpness* of a threshold function can be intuitively understood as the width of the phase transition.

We here use the work of Friedgut [18] to describe the notion of sharpness of a threshold function. Let us recall that we defined in 1.2.3 a threshold function using the notation of Erdős and Rényi as follows. The function $f(n, p)$ is a threshold function for a monotone

property A if

$$\lim_{n \rightarrow \infty} \Pr(G(n, p) \text{ has } A) = \begin{cases} 1 & \text{if } \frac{p}{f(n, p)} \rightarrow \infty \ (p \gg f(n, p)) \\ 0 & \text{if } \frac{p}{f(n, p)} \rightarrow 0 \ (p \ll f(n, p)) \end{cases}$$

Using the same notations, a sharp threshold function $f(n, p)$ is such that

$$\lim_{n \rightarrow \infty} \Pr(G(n, p) \text{ has } A) = \begin{cases} 1 & \text{if } p > (1 + \epsilon)f(n, p) \\ 0 & \text{if } p < (1 + \epsilon)f(n, p) \end{cases}$$

It appears that most threshold functions are sharp. Friedgut and Kalai [19] proved this important conjecture of Linial that every monotone graph property has a sharp threshold, based on the fact that symmetry implies sharp thresholds. We will not go further into the theory of threshold functions but we insist only on the remarkable fact that every monotone graph property has a sharp threshold.

As this result of Friedgut and Kalai is based on the theorem of Bollobas and Thomason about monotone properties, the rest of this section ?? is devoted to the latter interesting result. The reader should however be able to follow the rest of this master thesis without going into these mathematical details.

Let $X = X_n = 1, 2, \dots, n$ be a finite set and $\mathcal{P} = \mathcal{P}(X)$ be the set of subsets of X . A property Q is identified with the set of subsets of X having Q , which is thus a subset of $\mathcal{P}$. Q is called *monotone increasing* or *monotone* if $A \in Q$ and $A \subset B \subset X$ imply $B \in Q$. Applied to graphs, this would mean that if a subgraph A possesses some property, the graph B obtained by adding edges also possesses the property. Moreover, if $\emptyset \neq Q \neq \mathcal{P}$ then the property is said to be non-trivial. Let us now define $X^{(k)}$ as the set of k -subsets of X , $0 \leq k \leq n$ and for $Q \subset \mathcal{P}$ define $Q_k = Q \cap X^{(k)}$. The probability that a random k -set of X has Q is then $P_k(Q) = |Q_k|/|X^{(k)}|$.

Almost every k -subset of X has Q , or equivalently a k -subset of X almost surely has Q , if $\lim_{n \rightarrow \infty} P_k(Q) = 1$. If Q is a non-trivial monotone increasing property, $P_0(Q) = 0$, $P_n(Q) = 1$ and $P_k(Q)$ is a monotone increasing function of k . As we have seen, a function, here called $m^* = m^*(n)$ and not $f(n, p)$ is said to be a threshold function if for $m/m^* \rightarrow 0$ almost no m -subset has Q while for $m/m^* \rightarrow \infty$ almost every m -subset has Q .

This definition is similar to the Erdős and Rényi one. The idea of the proof is the following. Firts, Bollobas and Thomason prove the theorem 1, which is intuitive but we do not enter in the details of the proof that are quite long.

Theorem 1. *Let Q be a monotone decreasing property of subsets of a set X . Then, for $0 \leq j < k \leq n = |X|$ we have $P_j(Q)^k = P_k(Q)^j$.*

The corollary 1 follows, where one should notice that if Q is monotone increasing then $Q^* = A \in \mathcal{P}(X) : X \setminus A \in Q$ is monotone decreasing and $P_k(Q) = P_{n-k}(Q^*)$.

Corollary 1. *If Q is monotone increasing then*

$$P_{k_1}(Q)^{(n-k)/(n-k_1)} \leq P_k(Q) \leq P_{k_2}(Q)^{(n-k)/(n-k_2)}$$

Their final result is the following theorem 2.

Theorem 2. *Let Q be a monotone increasing non-trivial property of subsets of a set X , $|X| = n$. Then, for $0 \leq j < k \leq n = |X|$. Let also $m^*(n) = \max l : P_l(Q) \leq 1/2$ and $\omega(n) \geq 1$.*

$$m \leq m^*/\omega(n) \Rightarrow P_m(Q) \leq 1 - 2^{1-\omega}$$

and

$$m \geq (m^* + 1)\omega(n) \Rightarrow P_m(Q) \geq 1 - 2^{-\omega}$$

which implies that m^* is a threshold function of Q .

Proof.

$$m \leq m^*/\omega(n) \Rightarrow P_m(\neg Q) \geq P_{m^*}(\neg Q)^{1/\omega} \geq 2^{1-\omega}$$

and

$$m \leq m^*/\omega(n) \Rightarrow P_m(\neg Q) \leq P_{m^*+1}(\neg Q)^\omega \leq 2^{-\omega}$$

□

An idea of the proof, expressed differently by Luczak, is given in [42]. The theorem is stated as follows in 3.

Theorem 3. *Each monotone property A has a threshold function. More precisely, for every $\epsilon > 0$, there exists k such that if*

$$Pr(G(n, p) \text{ has } A) \geq \epsilon$$

then,

$$Pr(G(n, kp) \text{ has } A) \geq 1 - \epsilon$$

The idea of the proof consists in seeing $G(n, kp)$ as the union of k independent copies of $G(n, p)$, so that

$$Pr(G(n, kp) \text{ has } A) \geq 1 - (1 - \epsilon)^k \geq 1 - \epsilon.$$

As k depends only of ϵ and not of A , the threshold exists for every monotone non-trivial property.

1.2.5 Directed networks

We have seen they were great results in percolation theory for random undirected graphs. We have also seen that every non-trivial monotone increasing property of subsets of a set has a threshold function. This suggests such threshold functions would exist for properties in directed graphs.

Unfortunately, much fewer results exist for percolation thresholds in the case of directed networks. Tomasz Luczak in *The phase transition in the evolution of random digraphs* [41] addresses the question. Of course, as the structure of the undirected graph influences the directed graph properties, we can apply some results from the undirected percolation theory. For example, we know a directed graph with n nodes and M edges $\vec{D}(n, M)$ contains no large connected component for $M/n < 0.5$. What do we mean by *connected component*? In the case of directed network, we need to distinguish between two ways to describe the connectedness of a network. Indeed, as the edges now have a particular direction, can we say that node A is connected to node B when there is an edge from A to B ? Or should we ask node A to be also reachable from B ? A strongly connected component is such that every node can be reached from any other node. On the contrary, a weakly connected component is obtained by taking the undirected version of the graph and looking for the connected components. Two nodes are connected if there is a path from A to B **or** from B to A . In the previous example the graph $\vec{D}(n, M)$ contains no large weakly or strongly connected component for $M/n < 0.5$. Indeed, for a component to be strongly connected, it must first be weakly connected.

But can we go further than applying results coming from undirected graphs? A result proved in 1980 by Karp and Tarjan states that when M/n is greater than a large constant

$c(\epsilon)$ then almost surely $\vec{D}(n, M)$ contains a strongly connected component of size at least $(1 - \epsilon)n$. Luczak's main result consists in proving that a phase transition phenomenon also occurs for directed networks when one looks at the apparition of the directed version of a giant component. A giant component in a directed networks is defined as a strongly connected component with a size at least greater then a fraction of the number of nodes n . At first the network is composed of small cycles then at some point an unique giant component appears while all other components stay small. Independently, the result was found by Richard Karp and published in the same year [34].

Both investigate a $\vec{G}(n, p)$ digraph and the structure of the strongly connected components. The graph is constructed exactly as $G(n, p)$ but the edges are now directed so that each edge among the $n(n - 1)$ edges has probability p to be chosen, independently of the other edges. Also, let us recall that in a directed graph a strong component, or strongly connected component, is just a set of nodes such that each of them can be reached from everywhere in the set. Karp and Luczak showed that when n is large and $np = c > 1$, almost surely all but one of the strong components are very small, and the unique large strong component contains about $\theta^2 n$ vertices, where θ is the unique root in $[0, 1]$ of the equation $1 - x - \exp^{-cx} = 0$. However, not much more work have been done since then and the remaining work concerns mostly different models of random graphs, with different probabilities than in $G(n, p)$ and $G(n, N)$.

1.2.6 Graphs and digraphs with a given degree sequence

An example of such a different model for constructing random graphs consists in graphs with a given degree sequence. Molloy and Reed first studied the presence of a giant component in random graphs with a given degree sequence [45, 46] Here we must define briefly the notions of *degree sequence* and *degree array*. A degree sequence consists in an array of non-negative integers whose sum is even. A graph with degree sequence $D_n = (d_1, d_2, \dots, d_n)$ is such that each vertex v_i has degree d_i . A random graph with given degree sequence is then chosen at random among all possible graphs with degree sequence D_n . The problem faced here is that a degree sequence is defined for a finite n while percolation theory asks what happens when $n \rightarrow \infty$. This problem led to the definition of a degree array. The degree array $D = d_0(n), d_1(n), \dots$ represents the degrees of the vertices as functions of n . It is such that

- $d(i) = 0$ for all $i \geq n$ and
- $\sum_{i \geq 0} d_i(n) = n$.

Molloy and Reed's model allows, thanks to the definition of a degree array, to define a percolation threshold for the apparition of a giant component. We present here after their model and results but we should warn the reader that this model of random graph will not be used when we will define the *viability threshold*. This warning also holds for the directed version of the model, developed by Cooper and Frieze as well as Alessandra Graf. However, we present their results as it allows to give a complete picture of the state of the art in percolation theory.

Molloy and Reed's model consists in graphs with n vertices such that they are approximately $\lambda_i n$ vertices of degree i , with $\lambda_0, \lambda_1, \dots$ a sequence of non-negative real numbers summing to 1. The value of λ_i is defined by the following condition on the degree array. A degree array is called *smooth* if there exists constants λ_i such that $\lim_{n \rightarrow \infty} d_i(n)/n = \lambda_i$. In their first paper (1995), the authors showed that if $\sum i(i - 1)\lambda_i > 0$ then the graph almost surely has a giant component while if $\sum i(i - 1)\lambda_i < 0$ then almost surely all

components in the graph are small. The size of the largest component before the apparition of a giant component is showed to be $O(\omega^2 \log n)$ where ω is the highest degree in the graph. When the giant component appears, its size is almost surely in $\Theta(n)$. In their second paper (1998), the authors precise their arguments for the size of this giant component. More precisely, they impose more conditions on the degree sequence, defining a *well-behaved* degree sequence, and under these additional conditions, find a size for the giant component of $\epsilon_D n + o(n)$. The value of ϵ_D depends on the degree sequence $\lambda_0, \lambda_1, \dots$. Again, their work inspired many others after them. For example, Janson and Luczak (2009) studied the same phenomenon and Bollobas and Riordan even improved Molloy and Reed's result (Bollobas & Riordan, 2015). And again, if many authors followed Molloy and Reed in the study of random graphs with a given degree sequence, few results are known in the directed case.

Still, Cooper and Frieze (2002) gave results for sparse random digraphs specified by a degree sequence, defining a *proper* degree sequence. Alessandra Graf (2016) in her thesis improved the conditions on the degree sequence first given by Cooper and Frieze. As her work is pretty recent and reviews the results obtained before, we will focus on the latter. A degree sequence for an undirected graph is a sequence D_n of n non-negative integers whose sum is even. Let us call $\mathcal{G}(D_n)$ the set of all graphs with degree sequence D_n . The definition of a degree sequence for directed graphs is quite the similar. It is a double sequence (D_n^-, D_n^+) such that $\sum D_n^- = \sum D_n^+$ with n_{ij} the number of vertices with in-degree i and out-degree j . Cooper and Frieze also found a value for the threshold of the parallel to the giant component in random graphs that is the largest strongly connected component (SCC) with linear size. They also found that the threshold function was linked to the in- and out- degree sequences. They thus found conditions on the degree sequence, defining the *proper degree sequence* as follows.

Definition 1. A degree sequence D_n is proper if

- There is an absolute constant A_1 such that $\theta = \frac{1}{n} \sum_{i \geq 0} \sum_{j \geq 0} i n_{ij} = (1 + o(1))A_1$,
- Every item of the sequence is at most $\Delta \leq n^{1/12} / \log n$
- $\max(\sum_{i \geq 0} \sum_{j \geq 0} i^2 \frac{n_{ij}}{n}, \sum_{i \geq 0} \sum_{j \geq 0} j^2 \frac{n_{ij}}{n}) \leq A_3$ for some absolute constant A_3 ,
- $\sum_{i \geq 0} \sum_{j \geq 0} i j \frac{n_{ij}}{\theta n} = (1 + o(1))A_2$ where $A_2 \leq 1 - \epsilon$ or $A_2 \geq 1 + \epsilon$ with A_2 and ϵ absolute constants,
- For $\rho = \max(\sum_{i \geq 0} \sum_{j \geq 0} i^2 j \frac{n_{ij}}{\theta n}, \sum_{i \geq 0} \sum_{j \geq 0} j^2 i \frac{n_{ij}}{\theta n})$, if $\rho \rightarrow \infty$ with n then $\rho = o(\Delta)$.

Theorem 4. Let D_n be a proper degree sequence and $\lambda = \sum_{i \geq 0} \sum_{j \geq 0} i j \frac{n_{ij}}{\theta n}$.

- If $\lambda < 1$, then a.a.s. every strongly connected component of $G \in \mathcal{G}(D_n)$ has size $O(\Delta^2 \log n)$
- If $\lambda < 1$, then a.a.s. there exists in $G \in \mathcal{G}(D_n)$ a unique giant strongly connected component.

Alessandra Graf provided a new proof of the theorem ... using well-behaved degree sequence instead of proper ones which allows to increase for example the maximum degree in the sequence. This new result is for us the achievement of the long way made since Broadbent and Hammersley initiated percolation theory.

1.3 Resilience and complementary currencies

We end this literature review by providing the reader a short insight of the literature about the resilience brought by complementary currencies.

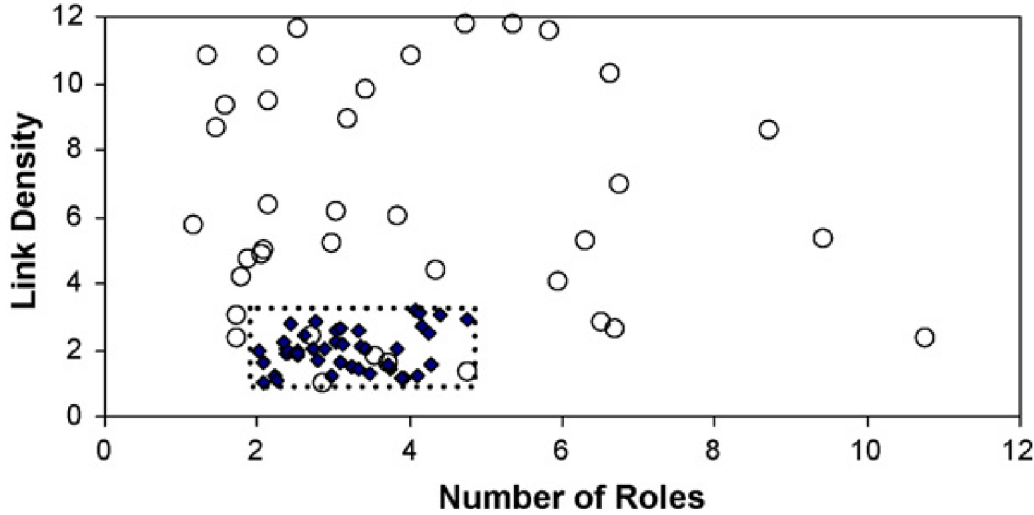


Figure 1.5: Random networks (circles) and empirically estimated ecosystems networks (dark squares) represented with their link density c and number of effective roles n .⁵

Lietaer and Ulanowicz approach

The approach of Bernard Lietaer and Robert Ulanowicz is explained intuitively in [39] and [40]. The mathematical framework that underlies their idea comes from [52] and [21]. Their research focus on the economic stability that can be brought by complementary currencies. Like the few papers that link complementary currencies and network theory, Lietaer and Ulanowicz represent the economic agents by nodes and their transactions by edges.

More specifically, the authors observe that economic breakdowns and crashes occurred under many different regulatory environments, development levels, in different countries and at different times. Indeed, besides the 2008 crisis, 96 other banking crises and 126 monetary crises happened since the early 1970's and even before, we record 48 crashes between the tulip mania that collapsed during the XVII century and the 1929 crash. For the authors, this suggests that a *structural* fault is involved in the apparition of such crashes. Unlike what have mostly been done so far in economy, the authors thus decided to investigate the structure of economic networks in order to find the cause of instabilities.

They used developments in information theory and ecology in order to develop a single measure of *sustainability* for economic networks. They intuitively define the latter as a measure that takes into account the *efficiency* A as well as the *interconnectivity* Φ of a system.

Zorach and Ulanowicz [56] calculated for existing networks of ecosystems as well as random networks the values of Φ and A . They plotted the results on transformed axes of Φ and A , namely $c = e^{\Phi/2}$ and $n = 2^A$. Figure 1.5 illustrating their results shows that all the existing ecosystems networks cluster into a specific rectangle. The value of c represents the logarithmic average degree and n , measures on logarithmic average how many transfers a quantum of medium makes before leaving the system.

Ulanowicz explains three of the four sides of the bounding rectangle as follows. The side $c \geq 1$ implies that average connectivity is greater than one thus that the network is weakly connected. The fact that $n > 2$ is explained but we will not deepen this point.

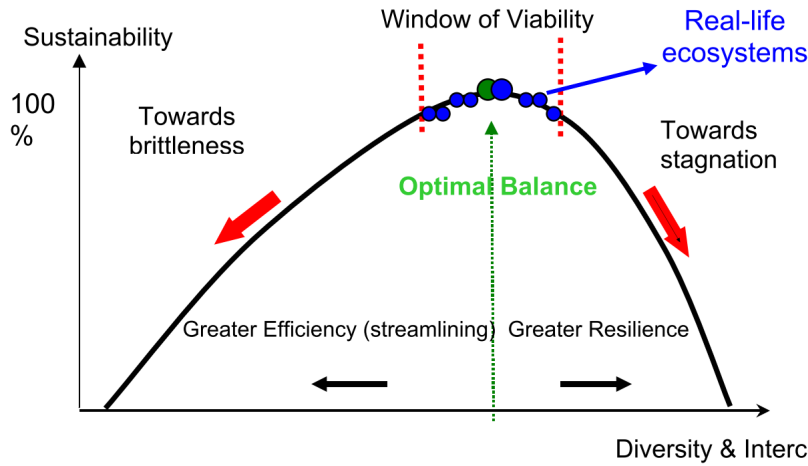


Figure 1.6: The *Window of Viability* in which all sustainable natural ecosystems operate.⁶

The frontier $c < 3.01$ is particularly noteworthy as it relates to the May-Wigner stability hypothesis.

May-Wigner theorem

In 1972, Robert May found, based on analytic work developed by Wigner [26], a stability theorem for random systems. He worked on the stability of random systems in the frame of his researches on ecological networks and stability of populations. Using the notions of connectivity C and interaction strength α , May's main result is that there is a sharp transition from unstable to stable and inversely as C and α exceed a certain value [43].

More precisely, a too large connectivity C for a too large interaction strength α lead to instability. Conversely, a weak connectivity and a weak interaction strength leads also to instability. This suggests that agents that interact with many other agents should do it weakly (small α) and the ones that have only a few connections should have strong interactions.

Reconciliation

Unlike most scientists who interpreted the May-Wigner theorem as a warning that too much diversity causes instability, Lietaer and Unlanowicz consider that May-Wigner's theorem allows for more diversity but against a loss of *efficiency*. This led them to represent the need of an equilibrium between diversity and efficiency with the help of figure 1.6.

The approaches of Lietaer - Ulanowicz and of May thus come from two different approaches of a system's stability [11]. The first one uses information theory while the second one is based on simulations of shocks in financial systems. However, the two approaches might appear complementary or even overlapping. Dominique Dron for example calls for a modelisation of both approaches that could help to understand better the relation between both approaches.

2 | Percolation for economic networks

2.1 Motivation and goals

We have seen there was a large literature in both network theory and in particular percolation theory as well as for complementary currencies. Complementary currencies are used in a large variety of economic networks, going from small local communities to large *business to business* circles. Even if new complementary currencies are launched regularly today, there is a lack of network theory approach in the way such economic systems are studied. In addition, the vocabulary used in the complementary currency literature regularly refers to words used in network theory. The idea to use tools coming from network theory in order to analyse complementary currency networks thus seems to us extremely relevant. Many different applications can be made when using graph theory tools for an economic networks study. One question that arises when studying complementary currencies concerns the viability of such economic networks. What makes a local currency project flourish ? Is it the number of initial participants involved or only the qualities of the project initiators that matters ? Here, we will investigate which conditions should hold for a complementary currency to flourish. Is it possible, given an economic network, to determine if the network is *well-suited* for complementary currencies ? This chapter addresses the issue by applying percolation theory to economic networks, thus looking for a *viability* threshold behind which the complementary currency will prosper with high probability.

The goal of this chapter is threefold. We want to represent economic networks without specifically having access to any real data. We also want to create models generating subgraphs from a specific economic network. Each subgraph then represents the complementary network that is included in the larger economic network. Finally, we want to use percolation theory to *test* whether some *properties* hold or not, for the different subgraphs.

This methodology leaves room for different choices. Indeed, even for the representation of the network many different situations may appear. If one has access to the data representing a real economic network, no choice has to be made. However, depending on the level (local, regional, international) of the economic network represented we will have extremely different types of graphs. The type of economic agents studied also impacts the shape of the graph studied.

Let us take a few examples. A shop, that sells fruits for example, will have only a few outgoing edges as it must pay mainly its suppliers and a rent for the shopping surface. In contrast, the shop has many incoming edges, representing here the clients coming to buy some fruits. A citizen probably sits at the other end of the scale. Indeed, the citizen makes lots of little purchases that are outgoing edges and receives money once a month when his salary is paid. Finally, businesses that transform raw material into another form and sell it only to a few trade partners are probably going to have as much outgoing than

incoming edges. Now if one does not have access to any data set, one will need to use random networks, trying to choose the right representation, sticking to reality as much as possible.

In the previous section, we encountered different types of graphs : lattices, networks with a specific degree-sequence, undirected and directed graphs, weighted graphs and some that are not. We focused notably on the Erdős-Rényi model to generate random graphs. There are also many ways to generate random subgraphs, given an initial network. One possibility consists in deleting more and more edges from the initial graph. Another one could be to delete nodes and their adjacent edges. Many other possibilities exist. One could for example select a node randomly and then delete a specific percentage of its adjacent edges, repeating the procedure step by step until no more edges from the initial graph are left. Finally, many properties of the different subgraphs generated may be studied, going from the connectedness to the degree distribution.

In this work, limited to an introduction of the application of percolation theory for economic networks, we will not cover all types of graphs or many types of subgraph generating models. We will instead look at two types of graphs.

- We will first build the *viability threshold* based on Erdős-Rényi type of random graphs where each edge has the same probability to exist. The graphs will be considered in their undirected and directed versions, with or without weights on the edges.
- In a second phase, we will apply the *viability threshold* on a network coming from a virtual-life data set.

We will generate subgraphs by deleting edges chosen at random among all possible edges.

This choice is not only justified by the impossibility to treat all types of graphs and subgraph generating models. In addition, this choice seems relevant to us as it enables to treat, amongst others, directed weighted graphs. The latter represent best, in our view, real monetary transactions that are indeed directed and weighted. The choice to generate subgraphs by deleting edges chosen at random is led by the fact that we obtain then Erdős-Rényi random graphs. As we have many known results in percolation theory for Erdős-Rényi random graphs this will allow us to make use of these results.

Let us now investigate which properties should hold for a complementary currency to prosper. The complementary currency network being a subgraph of the total economic network, we will then test for different subgraphs if the set of properties hold.

2.2 Giant component

Our first property consists in the existence, in the complementary currency subgraph, of a giant component. The apparition of a giant component in a network has been the most studied topic in percolation theory. Great results have been found for undirected networks and some results exist for directed networks. Let us consider how these tools can be used for economic networks.

First, let us recall that a giant component in an undirected network is such that the maximal connected component (with maximal number of nodes) contains a constant fraction of the total number of nodes. For a directed network, the giant component must be such that the maximal strongly connected component contains a constant fraction of the total number of nodes.

The hypothesis that the subgraph of complementary currency exchanges must have a giant component seems realistic. Indeed, let us take the case of a graph without any giant component. In the case of random Erdős-Rényi networks, percolation theory informs us of the following :

- For undirected graphs, with high probability, the size of the components is in $O(\log n)$ before the apparition of a giant connected component.
- For directed graphs, even if less results are known, we can apply some of the results found for undirected graphs. Here for example, if we consider the undirected version of a directed graph, we know that before the apparition of a giant connected component, the size of the components will also be in $O(\log n)$. Thus the size of the directed components (weakly or strongly connected) is also in $O(\log n)$.

This size of $O(\log n)$ is pretty small. We see that for both undirected and directed networks, without the apparition of a giant component, a network taken at random using the Erdős-Rényi model is made of many small components, not connected with each other. If node i decides to create a complementary currency, even if all its economic partners decide to join the adventure, the complementary currency will never reach nodes outside node i 's component and thus will not spread among the economic agents.

Percolation theory shows furthermore that behind a specific threshold, the structure of the graph changes brutally.

- For undirected graphs, the threshold at which a giant component appears is $p_c = \frac{1}{n}$.
- For directed graphs, the apparition of a giant strongly connected component is harder to analyse. However, Luczak and Karp proved that a phase transition phenomenon also occurs for directed networks. More specifically, when n is large and $np = c > 1$, almost surely all strong components but one are very small and the unique large component contains $O(n)$ vertices.

For a complementary currency to spread among an economic network, and reach at least a constant fraction of the economic agents, the initial subgraph must be such that a giant component exists.

- In the undirected case we know that the giant component is reached when $p > 1/n$. For a hundred nodes, we have $p > 0.01$. This means that over the approximately $100 \times 100 = 10000$ possible edges, the network must contain 100 edges thus each agent, in average, should make at least one transaction in complementary currency. If we simulate an undirected network with more and more edges, we can test when the giant component phase appears. Figure 2.1 and 2.2 show two cases : the threshold at which half of the nodes are in the giant component, as well as the connectedness threshold for a growing number of nodes. We see that the theoretical threshold $p_c = 0.01$ is more or less obtained in the simulation for a hundred nodes.
- It is also possible to illustrate what the *giant component* constraint implies in the directed case. The graph 2.3 shows the percolation threshold for the apparition of a giant component in a directed graph. It calculates, for a growing density, the proportion of graphs that passed the giant component phase. The giant component phase is also reached when half of the nodes are in the giant component. It uses Tarjan's algorithm to detect the maximal strongly connected component [38]. For a hundred nodes, the percolation threshold is approximately 0.01 as in the undirected case but we see that while the undirected threshold is just below 0.01, the directed threshold is just above.

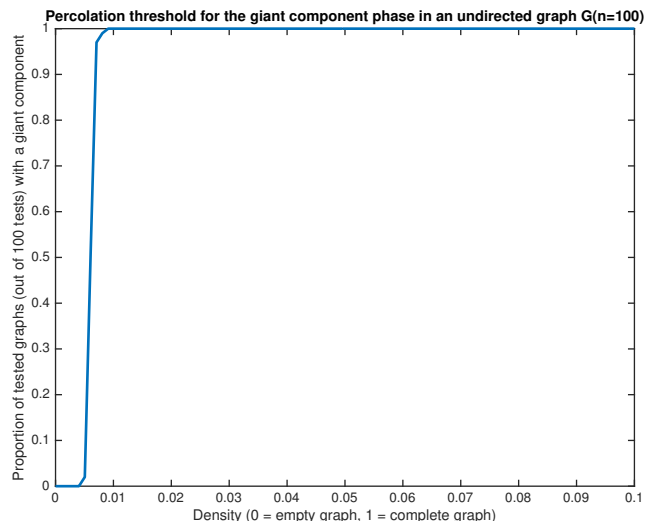


Figure 2.1: Percolation threshold for the giant component phase which is reached when half of the nodes are in the giant component. The number of tested graphs for each density is 100 and the number of nodes is 100 too.

2.3 Cycles

2.3.1 A need for circularity

We have seen that a necessary condition for the implementation of complementary currencies was that the subnetwork of exchanges in complementary currency contains a giant component. Is this condition sufficient? It is easy to find examples of connected networks that are not well-suited for complementary currency exchanges. The first example that comes to mind is the one of a chain. Let us consider an economic network with a subnetwork composed of a chain as on figure 2.4 A). Let us call u the first agent of the chain and v the last one. The first agent u buys with national currency the same amount of complementary currency. He is able to do it as a feature of most complementary currencies is that agents are able to *buy* them with national currency. He also knows that he will be able to exchange the complementary currencies with goods or services from another economic agent, open to complementary currencies. The complementary currency will thus travel through the chain, each time being exchanged against some goods or services. At the end of the chain (we consider a finite network), last agent v will be unable to spend the complementary currency. If he is able to exchange the complementary currency into national currency, the problem is solved. But a specific feature of complementary currencies is that they cannot be easily exchanged into national currency. Let us take the example of WIR in Switzerland. A black market appeared so that agents could exchange their WIRs into Swiss francs, but the value of the currency is depreciated. Instead, of exchanging 100 WIRs in 100 Swiss francs, agents get approximately 60 Swiss francs. Agent v , at the end of the chain, will thus be unable to use his money in complementary currency. The problem also arises for tree-like graphs. We can draw three conclusions out of this example.

- The connectedness condition does not seem sufficient, at least when considering a weak connectedness.
- Even if last agent of the chain v is connected to the first one u as on figure 2.4 B), the circulation of complementary currencies is very limited as it can only pass through one specific cycle. The connectedness condition while considering strong connectedness may thus be not sufficient.

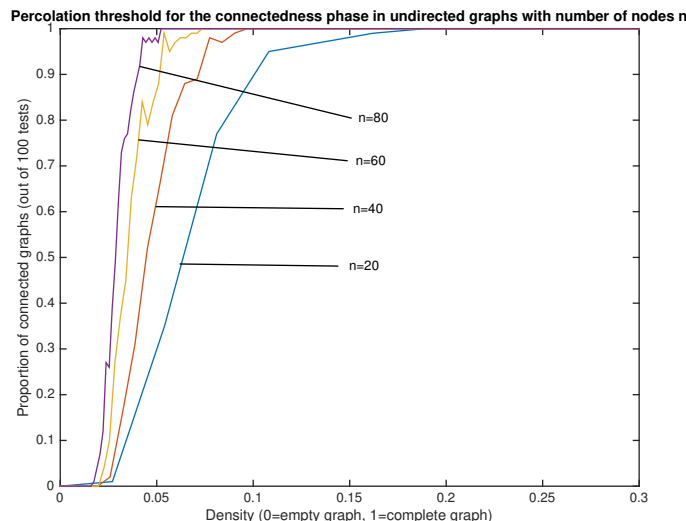


Figure 2.2: Percolation threshold for the connectedness phase. The number of tested graphs for each density is 100 and the number of nodes is [20, 40, 60, 80]

- The presence of cycles in the graph seems a key-feature that allows a complementary currency to flow through a network.

To illustrate differently the weakness of a single cycle, we can make a comparison with the traffic model. Many scientists tried to explain why and how congestions or *traffic jams* occur on a road. Of course, accidents or constructions that cause bottlenecks are a cause of congestion. However, Yuki Sugiyama et al. showed in [51] an experimental evidence for the physical mechanism of the formation of jams without bottlenecks. The latter consists in an experiment of a circular road of 230m of circumference with cars driving around the circle. They showed that when the density of cars increases, even if the cars could technically all drive at the same speed, small fluctuations appear and grow. After a period of time, the fluctuations disturb the flow so much that cars cannot move homogeneously and finally some vehicles are even forced to stop. A *stop-and-go* wave is observed. Thus everything starts because small fluctuations do exist in the natural movement of vehicles. Indeed, if a driver changes just a little bit his speed, the fact that the following driver must first see the change and then adapt his behaviour, causes the fluctuations to grow. With a monetary circle, a similar phenomenon could occur as fluctuations are even greater. When a citizen gets his salary, the latter does not spend it directly and smoothly but waits and spends money step-by-step in an irregular way. An economic circle would probably act in the same manner as the experimental traffic circle and money would stop and go, hurting the efficiency of the economic exchanges.

Let us summarize our point. The main idea in this section is the need of circularity for complementary currencies. Indeed, if a national currency can always be changed into another one, it is rarely the case for complementary currencies. Often, one can buy complementary currencies with national currency but not the inverse. This implies that the complementary currency network should be able to work *on his own*, thus without anyone being unable to spend his or her complementary currency. The subgraph of complementary currency exchanges should thus have a particular structure and we think that the presence of many cycles is primordial.

Going back to the example given by T. Greco while explaining how credit money function, we can also illustrate the need of circularity in complementary currency networks. The example given was the one of four economic agents A, B, C and D with A needing to buy

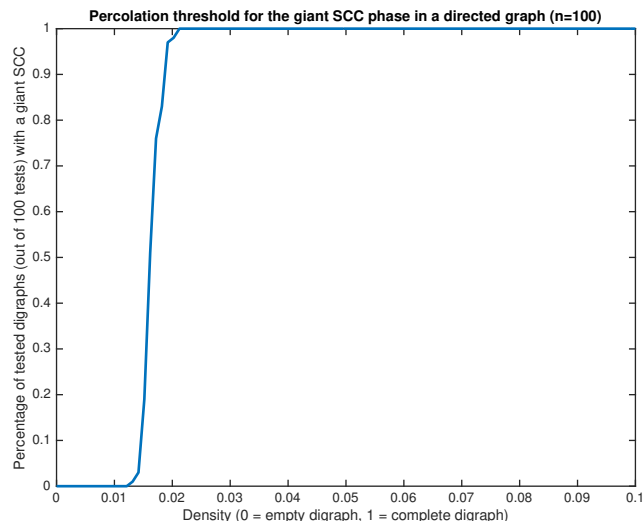


Figure 2.3: Percolation threshold for the giant component phase which is reached when half of the nodes are in the giant strongly connected component. The number of tested graphs for each density is 100 and the number of nodes is 100 too.



Figure 2.4: Illustration of the chain problem

for 100 dollars of B products, B needing 100 for dollars of C products, C of D and D of A. T. Greco highlighted the fact that, thanks to a complementary currency, the exchanges could be made easily. For example, A can write an IOU to B and B can transfer it to C until the IOU comes back to A. This requests everybody to trust A's IOU but if it is the case then the only constraint is the ability for A's IOU to return to A and thus to be *destroyed*. If A's IOU remains in the hands of B they are only two possibilities. First, the ability for B to exchange this IOU into a national currency, then destroying the IOU but creating national currency. This possibility exists but often, complementary currency systems are such that the convertibility is limited or costs something. Second, B is not able to exchange A's IOU into a complementary currency. He will thus refuse A's IOU, hurting the complementary currency system. We see again through this simple example that circularity is a key feature for complementary currency networks.

2.3.2 Circuit rank and cycle rank

How can we express a *circularity threshold* that would ensure enough circularity for a complementary currency network? One possible way to do that is to measure the number of cycles, as the density of edges grow. The circuit rank is one possible way to calculate the number of cycles in an undirected graph. More precisely, it defines the number of edges that should be removed in order to obtain an acyclic graph. It is calculated as follows, where m is the number of edges, n the number of nodes and c the number of connected components.

$$\text{Circuit rank } r = m - n + c$$

Clearly, when the graph becomes connected, $c = 1$ and r just grows linearly with the number of edges. A condition on r thus amounts to a condition on the number of edges. A possible condition on r could be to ask $r \geq n$. This would represent an average of two edges by node, allowing each agent to exchange with at least two partners. This condition seems realistic and also necessary. Otherwise, if the only partner of an agent stops using complementary currencies, the agent has no more possible partner at all. This strengthens the first condition developed about the giant component phase. Instead of having a least one exchange in complementary currency, agents should have at least two.

The parallel measure for directed graphs, the cycle rank, is not easily computable. The cycle rank, just as the circuit rank, measures how many edges should be removed to make a directed graph acyclic. It can be written as follows.

$$\text{Cycle rank } r'(\vec{G}) = \begin{cases} 0 & \text{if the graph is acyclic} \\ 1 + \min_{v \in V} r'(\vec{G} - v) & \text{if the graph is strongly connected and the} \\ & \text{set of edges non-empty} \\ \max_{SCC_i} r'_i & \text{among all strongly connected components} \\ & SCC_i, i = 1 \dots c \text{ if the graph is not strongly} \\ & \text{connected} \end{cases}$$

The difficulty here is that it is not easy to calculate $\min_{v \in V} r'(\vec{G} - v)$. Indeed, also known as the *minimum feedback arc set problem*, this problem is NP-complete. Of course, we are able to simulate the removal of randomly chosen edges and count the number of strongly connected components, but this would not give the *minimal* number of edges that needs to be removed if one wants an acyclic network.

2.3.3 Maximum cyclic flow

So far, we have developed a measure of circularity that does not take into account the weights that can be assigned to networks. Monetary networks are however weighted. Let us take this feature into account by looking at the maximal cyclic flow in a network. The flows on the directed edges represent the amount of money exchanged between two agents. It is possible to distinguish in a network between the cyclic and the acyclic flow. The cyclic flow consists of the symmetric exchanges (cycles of length 2) as well as any other cycles. The value of the flow in a cycle of m edges will be equal to the minimum flow between the m edges. Once we have removed all cyclic flows, only acyclic ones remain in the network. There exists many ways to remove all the cyclic flows in a network. Our definition of the maximal cyclic flow, based on [36] chooses the cyclic decomposition that allows, of course, a maximal cyclic flow. But the solution might still not be unique. In that case, the decomposition chosen is such that no cycle is favoured over another. The solution is spread out as much as possible among all the possible cycles.

Let us investigate if one could expect to be able to apply percolation theory for a condition related to the maximal cyclic flow. The transition phase theorem addressed in subsection 1.2.4 informs us that every non-trivial monotone increasing property of subsets of a set has a threshold function. Figure 2.5 shows that the ratio of maximal cyclic flow compared to the one in the initial digraph increases monotonically with the density of a graph.

A condition on the maximal cyclic flow allows us to choose subnetworks for which the cyclic flow is important. How can we express what *important* means? One way to do so is to compare the cyclic flow of the economic subnetwork to the total cyclic flow. Let us define the property of having a maximal cyclic flow of at least half the one of the initial digraph. By doing so, one gets a measure of the reached exchanges among all possible

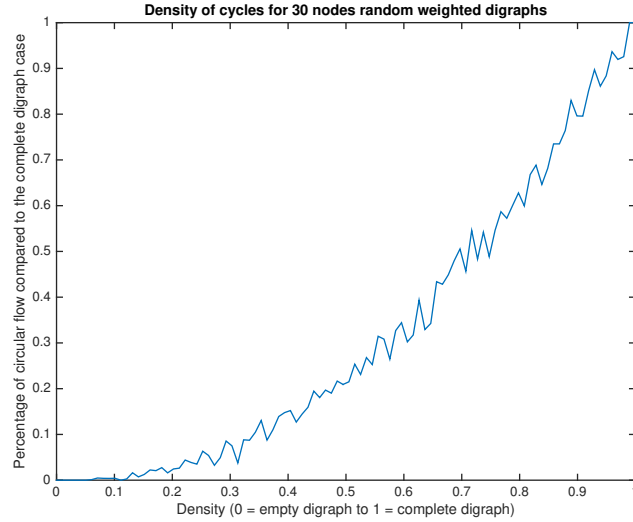


Figure 2.5: Percentage of maximal cyclic flow compared to a complete random digraph, for a growing number of edges and 30 nodes.

exchanges in an economic network. Figure 2.6 shows when the complementary currency subnetwork reaches half of the initial cyclic flow in a weighted, complete, directed network. We are using an algorithm that separates, for any directed graph, the symmetric, cyclic, and asymmetric exchanges and that was developed by Jean-Charles Delvenne et al. in [36]. The percolation threshold is a lot more higher than the ones obtained for the giant component phase or for the number of cycles. However, by decreasing the *objective*, the threshold decreases too, allowing realistic constraints. Figure 2.7 shows the percolation threshold if the objective is to reach one fifth of the possible cyclic flow.

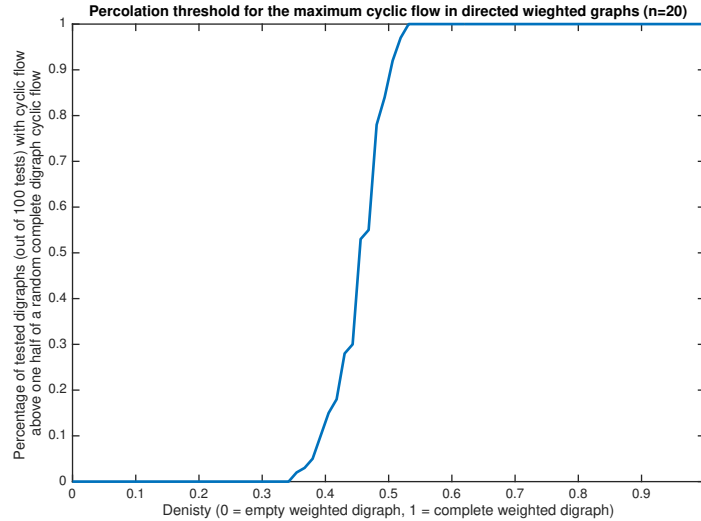


Figure 2.6: Percolation threshold for the maximum cyclic flow in directed weighted graphs of 20 nodes with growing density. A digraph percolates when the cyclic flow amounts for one half of the cyclic flow in a complete random directed graph with the same vertex set.

We thus developed so far two different notions of circularity. The first one is based on the circuit rank and tackles, in undirected versions of graphs the number of cycles. Once a network is connected and with a fixed number of nodes, the circuit rank evolves linearly with the number of edges. The second notion of circularity developed is the

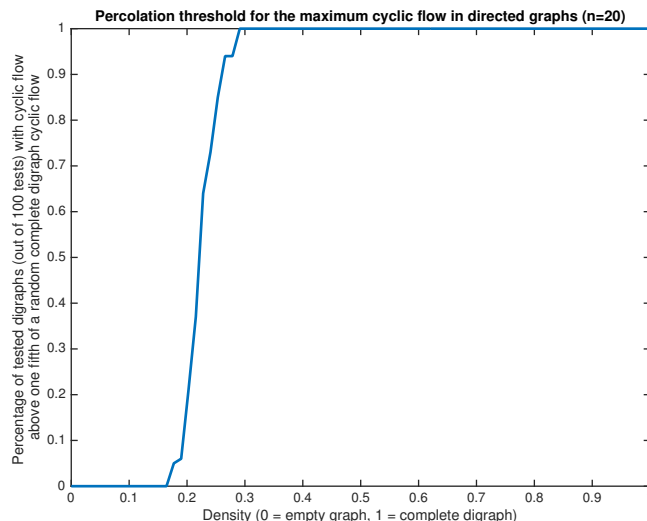


Figure 2.7: Percolation threshold for the maximum cyclic flow in directed weighted graphs of 20 nodes with growing density. A digraph percolates when the cyclic flow amounts for one fifth of the cyclic flow in a complete random directed graph with the same vertex set.

one of maximal cyclic flow. The latter investigates how much money could flow in a network without leaving the network. Both seem relevant. Otherwise, we could have many cycles in a complementary currency network but with very small transactions. Correspondingly, we could have few cycles in the complementary currency networks but with edges representing an important flow of money. Following the purposes behind the complementary currency, one could be happy with only one of the two possibilities. However, the ideal complementary currency network shares, in our view, both the fact to imply many different cycles and the fact to treat a relatively important amount of money.

Are these two notions of circularity related in some way ? If one considers a directed network with unit weight on all its edges, we can build a parallel between the two notions. Indeed, the number of independent cycles in the undirected version of the network will represent an upper bound on the number of independent directed cycles and thus an upper bound on the maximal cyclic flow.

2.3.4 A formula for the number of cycles?

In this last subsection about circularity we aim to find a formula for the expected number of cycles in a directed graph. When considering graphs with unit weight on all edges, the maximal cyclic flow is also a measure of the number of directed cycles. We would like to find a formula that expresses mathematically the curve obtained for the percentage of maximal cyclic flow and shown on figure 2.5.

When do cycles appear in a digraph? Can we characterize these cycles? In order to study the apparition of cycles in digraphs we must understand when the latter are created. Actually, small cycles can already be created at a very low density of edges. Indeed, even if p is low and the giant strongly connected component phase has not been reached yet, small cycles can appear in a randomly created directed graph. Can we find a function that approximates best this curve ?

The first basic cycles are just composed of two nodes. These can be seen as economic exchanges between two agents. They are thus (compared to triangles or squares) specific

exchanges as, if two agents decide to exchange in complementary currency, they know that the cycle is complete.

The expected number of cycles of length 2 is obtained by looking at the probability for two nodes to form a cycle, which is p^2 . Then we must multiply over the total amount of possible pairs of nodes. Once the nodes i and j are chosen, there is only one possible cycle.

$$\mathbb{E}(\#2\text{-cycles}) = p^2 \frac{n(n-1)}{2}$$

The expected number of cycles of length 3, as with cycles of length 2, consists in the probability for three nodes to form a cycle, multiplied by the possible triplets of nodes. The probability for three nodes i , j and k to form a cycle is p^3 . The number of possible triplets of nodes is $\binom{n}{3}$. Given the three nodes, they can only form 2 different directed cycles. We thus obtain the following expected number of cycles of length 3.

$$\mathbb{E}(\#3\text{-cycles}) = p^3 \times 2 \times \frac{n(n-1)(n-2)}{6}$$

The expected number of cycles of length 4 is obtained with the same logic. However, with four nodes, they are six number of different possible cycles.

$$\mathbb{E}(\#4\text{-cycles}) = p^4 \times 6 \times \frac{n(n-1)(n-2)(n-3)}{4!}$$

The total number of cycles in a graph is of course equal to the number of cycles of any length in the graph. If we continue with this series of expectations we can approximate the total number of cycles in a graph with a Taylor development.

$$\mathbb{E}(\text{cyclic flows}) \simeq \frac{p^2 n^2}{2} + \frac{p^3 n^3}{3} + \epsilon$$

We already see that for $pn \ll 1$, the leading term is quadratic. Generalizing this formula, with k the length of the cycles, leads to

$$\mathbb{E}(\text{cyclic flows}) = \sum_{k=2}^n \frac{n(n-1)\dots(n-k+1)}{k} (k-1)! p^k$$

We could also look at another special limit case : the one with $p = 1 - \epsilon$. In this case, all the possible graphs are almost complete. As we delete edges from the complete graphs, the first cycles that will be eliminated are cycles of length 2 then of length 3 etc... Indeed, let us consider the complete graph. We delete the edge going from u to v . Of course, one could say that the cycle being destroyed is for example a Hamiltonian cycle, or a triangle, or any other cycle. But we could also decide that the flow destroyed was the one going from u to v back to u . In both cases, if we make the hypothesis of a graph with equal weight 1 on all edges, the cyclic flow destroyed weights 1. What if when deleting the edge from u to v , the one from v to u is already destroyed ? We then decide to destroy a triangular cycle passing through u and v . The new graph created with the destroyed edges is actually constructed in the exact same way of creating a new graph from an empty edge set. Thus, how is the cyclic flow impacted when we remove edges ? In a complete directed graph with weight 1 for each edge, the maximal flow is just $\frac{n(n-1)}{2}$. As we remove small cycles, the total cyclic flow decreases. If $q = 1 - p$ we have :

$$\mathbb{E}(\text{cyclic flows}) = \frac{n(n-1)}{2} - \mathbb{E}(\#2\text{-cycles}) - \mathbb{E}(\#3\text{-cycles}) - \dots$$

$$\mathbb{E}(\text{cyclicflows}) \simeq \frac{n(n-1)}{2} - \frac{q^2 n^2}{2} - \frac{q^3 n^3}{3} - \epsilon$$

We thus have here again a quadratic behaviour.

If we try to plot a quadratic function next to the evolution of cycles density, it confirms the previous intuitions : the density is mainly quadratic. Figure 2.8 shows in red the function x^2 in the density graph.

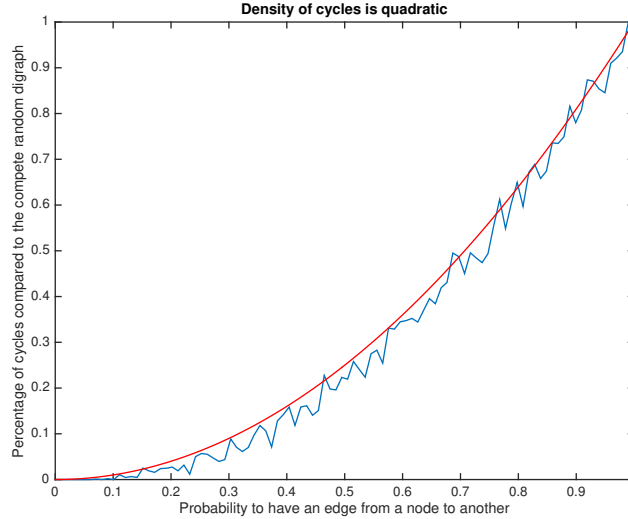


Figure 2.8: Density of cycles for 30 nodes random weighted digraphs. The weights are distributed randomly between 0 and 1. The density of nodes goes from 0 (empty graph) to 1 (complete graph). The function p^2 is added to the figure in red.

One should then ask why the quadratic behaviour holds even when pn or $(1-p)n$ is not below 1. Indeed, taking $p = 0.5$ makes pn and $(1-p)n$ way above 1 for a reasonable amount of nodes. The quadratic term should not be the leading one. One possible explanation could come from the fact that the algorithm used to detect the cyclic flow from the non-cyclic one, first *cleans* the network by removing all symmetric exchanges. As we work here with edges having all the same weight 1, removing the symmetric flow actually removes most of the edges. The symmetric exchanges thus dominate the other cyclic exchanges as we choose to look at them first.

Conclusion

In the last two sections we developed a set of three conditions that could represent indicators for the viability of a complementary currency. We tried to build the conditions based on intuitive developments but that rest on coherent observations about complementary currencies. The first condition states simply that the agents must be almost connected : at least, there must be a constant fraction of agents strongly connected with each other. The second condition is more restrictive. Based on the idea that each agent should, on average, be able to exchange complementary currencies in two different ways, it states that the number of edges should be twice as large as the number of nodes. Last condition might seem even more restrictive. Indeed, it asks that the maximum cyclic flow of the complementary currency subgraph forms a constant fraction of the maximum cyclic flow of the total graph. However, just like with the giant component simulation, by asking a *constant fraction* to represent one half or one fifth of the set of nodes, the constraint becomes more or less restrictive. Following the choice of the *constant fraction*, the third condition regarding the maximal cyclic flow will generate a threshold smaller or greater than the threshold defined by the condition about the cycle rank.

2.4 Data analysis

2.4.1 Presentation of the data set and sampling

The reflexion developed so far aims to characterize a few conditions allowing a complementary currency network to prosper. It would thus be interesting to test the conditions on graphs coming from real economic networks, instead of randomly created ones. As we do not possess such data sets, we will use virtual-life data, as it is often proposed when one does not have access to real-life data. The data we will use comes from the game EVE Online [48]. The latter is a **multiplayer online game** which means that many players are able to be in the game simultaneously, interacting with each other. Moreover, the game consists of an universe reachable only thanks to a special network and it is a persistent game which implies that the universe evolves even when players are not connected. In our case, the universe of EVE Online consists of a huge galaxy governed by a capitalist economic system. The game is also a **ROLE PLAYER GAME** thus each player creates an avatar and receives a spaceship that allows him to go out in search of fortune and power. They obtain the latter by collaborating through alliances, making commercial exchanges and fighting others. The relevant part for us is the commercial one. In order to get richer and buy new spaceships and weapons, players choose between the following possibilities. They may transport material for other players against a monetary contribution, which is acted with a *courier* contract. They may also exploit asteroids and moons in order to extract valuable substances and potentially create an industry that transforms the raw material into weapons or other goods. Finally, they are encouraged to exchange their items either on a market through *exchange* contracts or through auctions that are limited to a certain group of players and are acted with *auction* contracts. All contracts have been grouped by pair of issuer and acceptor of the contract and the data set looks as table 2.1.

IssuerID	AcceptorID	Volume	Count	MoneyValue
90000036	92870802	1.605019e+05	2.0	0.000000e+00

Table 2.1: Head of the data file

The data is thus directed from issuer to acceptor. For each directed pair, the column COUNT expresses the number of contracts going from *IssuerID* to *AcceptorID*. The volume is recorded as not all exchanges are made with money. Indeed, lots of players proceed to symmetric exchange contracts, exchanging goods directly against other goods. This explains why some pairs have a zero *MoneyValue* and thus why it is relevant to give the volume of the transaction, as well as the number of contracts.

All contracts amounted to 33 million contracts but, when grouped by pair of issuer and acceptor of the contracts, this reduces to approximately 10 million economic relationships. The average number of contracts between two players is thus approximately three. Precisely, if we take the mean of the column *Count* we obtain 3.0735 contracts by pair. Even if there is an average number of contracts between players of 3, as many pairs of issuer and acceptors have a very high count of contracts, this implies that many relationships are made of one-to-one contracts between players that will not exchange goods anymore in the game. More precisely, this type of relationship amounts, out of the 10 million economic relationships, to more or less 7 million. A large majority of economic relationships consist in stranger to stranger relations rather than economic partnerships.

Even if the transactions grouped together amount for 10 millions of contracts instead of 33 millions, this is still way too large to be treated with a single computer and a limited time. We will thus need to analyse only a few samples out of the whole data set. If

we choose a maximal number of 20 000 edges, this still allows us to create an adjacency matrix from the edge list obtained. We will thus take samples of edges of size smaller than 20 thousands. The way by which we choose the data set will of course influence the results. We will thus be cautious with the interpretation of the results.

One first possible thing to do is to compare the amount of edges having a monetary value with the ones that do not. Indeed, by taking only the graph with money value zero we get rid of almost all symmetric exchanges. Out of the 10806948 edges, if we keep only the ones with a non-zero money value, the number of edges already drops to 6477073. This gives us the information that a huge amount of contracts even if they are expressed under the form issuer to acceptor are actually symmetric exchanges. We may now choose to consider the symmetric network or the asymmetric one. In this work we will consider the asymmetric one where items are exchanged against money instead of other items. Indeed, it fits a lot better to reality and to the purpose of this work. Among the subnetwork formed, we will choose samples of less than 20 thousands edges in different ways. And among the many methods that we could have chosen we focus on the following ones.

- A first possibility for drawing 20 000 edges out of a set of six millions consists in drawing them randomly among all possible edges, without repetition.
- Another possibility is to choose a few starting nodes and then extend the network step by step so that each node is taken with a percentage of its neighbours. This gives a more local picture of the graph.
- A last possibility we used is to sort the edges thanks to the *Count* column, considering for example only economic partners with a certain amount of exchanges, without exceeding the limit of 20 thousands edges.

We thus obtain three different subnetworks of approximately 20 thousand edges. In the following subsection, we use these subnetworks in order to test the percolation tools developed earlier in this section.

2.4.2 Random sampling analysis

Let us first analyse the sample of edges chosen at random among the set of edges with non-zero monetary value. We must note that the sample represents only 1/500 of the whole set of edges and 1/300 of the edges with non-zero monetary value. Figure 2.9 shows the in-, out- and total degree distribution of our subnetwork. We notice that the distributions are similar and skewed but with a huge majority of nodes with low degree. Maximum in-degree is 20 while maximum out-degree is 30. Mean degree is ≈ 0.63 which is pretty low. We already see that edges are not connected with each other. The number of nodes implied is 31720, which is close to 40000, the number of nodes for totally disconnected edges. Most edges are just by their own, not connected to any other edge. This implies of course that the giant component phase has not been reached yet. The graph is also acyclic and the clustering coefficient is thus zero. The largest weakly connected component contains 70 nodes and 69 edges and is represented on figure 2.10. There is no strongly connected component at all. Should we however conclude that the whole graph is similar to our sampling ? Surely not because of the size of the sample and the way it was chosen. The average degree of the whole data set can for example be calculated by taking the ratio between twice the number of edges and the number of nodes.

$$av_{deg}(G) = \frac{2 \times 10806948}{1342671} \simeq 16.098$$

We see that the sample is far to look similar to the initial graph, even if we are not able to compute the average path length or the clustering coefficient for the initial data set.

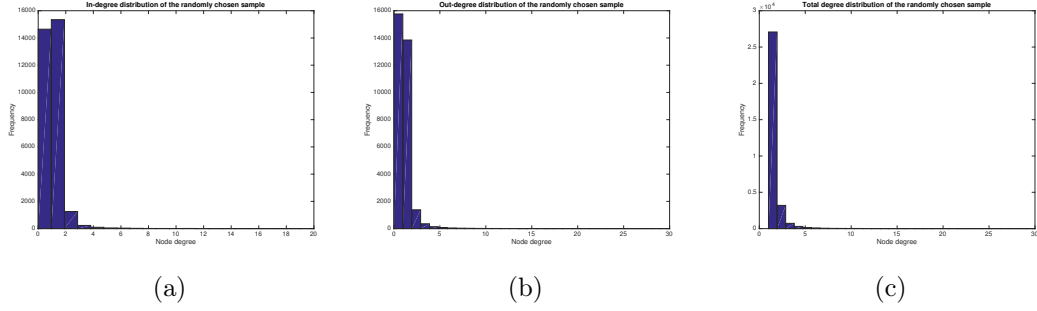


Figure 2.9: In- (a), out- (b) and total (c) degree distributions of the randomly chosen sample.

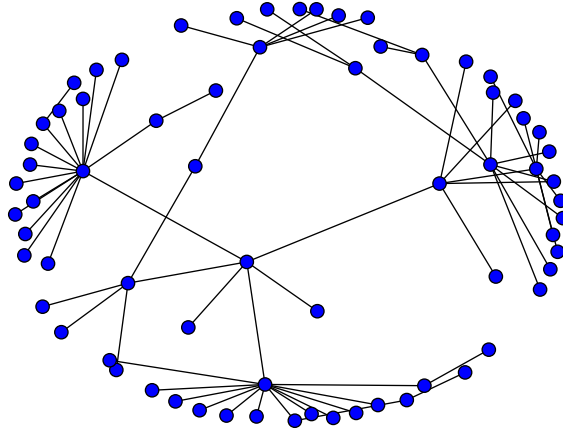


Figure 2.10: Largest weakly connected component of the random sample

2.4.3 Most used edges analysis

We have seen that most of the exchanges were made between *strangers* instead of *economic partners*. Let us analyse the subgraph formed with only frequently used edges, thus between *economic partners*. If we select only the edges that were used more than fifty times, we obtain a subgraph of 18420 edges, just below the limit of 20 thousands fixed. The number of nodes is 22664 which already shows that the density is higher than in the random case. Again, the degree distribution is shown on figure 2.11. The maximal in-degree is now 152 and the maximal out-degree is 90. However, the mean degree raises only a little. It is 0.8127 which is still below one and far from 16.098. The clustering coefficient found as the mean of the local clustering coefficients of each node is $\frac{465.7101}{22664} \approx 0.02$.

One may ask if this coefficient is close to the one we would obtain in the case of random graphs as the Erdős-Rényi ones. The local clustering coefficient of a random graph $G(n, p)$ will depend on the probability p to have an edge between two nodes. The expected local clustering coefficient (written here as l) for node u can be expressed as

$$\mathbb{E}(l(u)) = \sum_{d=0}^{n-1} P[\deg(u) = d] \times \mathbb{E}(l(u) | \deg(u) = d)$$

The expectation of the local clustering coefficient knowing the degree can be obtained by looking at all possible pairs among the d neighbours. The pairs that allow to form a triangle are linked with probability p so the expected number of triangles is the number of pairs multiplied by p .

$$\mathbb{E}(l(u)|deg(u) = d) = \frac{p \binom{d}{2}}{\binom{d}{2}} = p$$

The expected local clustering coefficient thus reduces to

$$\mathbb{E}(l(u)) = p \sum_{d=0}^{n-1} P(deg(u) = d) = p$$

In our case, knowing that the number of nodes is 22664, the total number of possible edges is $\frac{22664 \times 22663}{2}$. As we have only 18420 edges, the value of p would be the following one.

$$\frac{18420}{256817116} \approx 7.17 \times 10^{-5}$$

We thus realize that our sample has a clustering coefficient much higher than the equivalent random graph. If we separate the cyclic flow from the acyclic one, we find that 0.2388 of the total edges are part of cycles. The largest weakly connected component has 3875 edges and 3031 nodes and the largest strongly connected component consists in 10 nodes linked with 27 edges. The average path length of the largest strongly connected component is 1.8333.

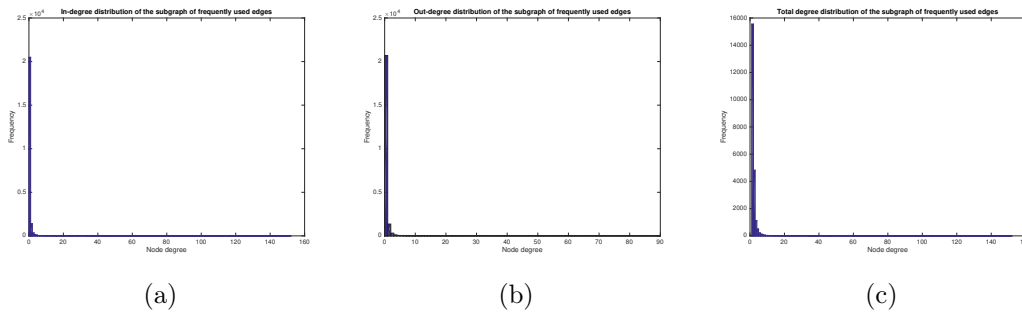


Figure 2.11: In- (a), out- (b) and total (c) degree distributions of the sample consisting in the most used edges (more than 50 times used).

2.4.4 Neighbour to neighbour analysis

The neighbour to neighbour sampling method allows to get a much more *local* picture of the graph. Indeed, we have chosen at random an edge between all possible edges thus defining two starting nodes. Then the neighbours of these nodes were chosen with a probability 0.01. After only 8 iterations we obtain a maximal number of nodes under the constraint of 20 thousands nodes. The degree distribution is similar to the previous ones. Maximum in-degree is 262 and maximum out-degree is 155. The mean degree is 1.121 which is still very low compared to 16 but now the whole sample is weakly connected. The clustering coefficient is $10.6442/16112 = 0.00066$. The largest strongly connected component has 112 nodes, 161 edges and average path length 10.6861. The ratio of edges that are part of cycles is 0.0063.

2.4.5 Percolation on virtual-life economic networks

In order to have an overview of the different samples we summarize their main characteristics in table 2.2.

	Random	Frequency	Neighbour
Mean degree	0.63	0.8127	1.121
Clustering coefficient	0	0.02	0.00066
Average path length of the largest SCC	X	1.8333	10.6861
Ratio of edges that are part of cycles	0	0.2388	0.0063

Table 2.2: Characteristics of the different subgraphs

Out of the different samples analysed, the random sample is far from being close to the real graph. Indeed, it has a very low average degree when the initial data set has average degree close to 16 and the Sardex network has average degree 8.521. Moreover, as the samples are designed to represent the initial economic network, it does not allow to test the percolation thresholds. Indeed if the initial graph did not pass the giant component phase itself, we cannot do much with its subgraphs.

Out of the two other samples, both have interesting characteristics. The graph made of the most used edges has a higher clustering coefficient but a lower average path length for the strongly connected component. The number of nodes in the SCC is also pretty small which is problematic.

The last sample obtained with the neighbour to neighbour sampling method has the advantage to have a higher average degree. However, the clustering coefficient is very low. The strongly connected component is interesting as it consists in a set of 112 nodes which allow to test the percolation tools developed here above.

Let us use the last two examples as initial networks.

Percolation on the sample formed by the frequently used edges:

Let us consider our sample as representative of the economic network of the game EVE online. The goal is now to take different subgraphs of this economic network and to test whether the conditions that we defined on sections 2.2 and 2.3 hold on the different subgraphs. The first condition we defined asked for the existence of a giant component. The largest strongly connected component in our sample has only 10 nodes : we cannot consider it as a giant component. We see that the first condition is already not met. The second condition investigates the average number of edges per node and asks that the number of edges is twice the number of nodes. The sample considered consists of 22664 nodes and 18420 edges which implies that the second conditions will also never be met. Finally, last condition is about the maximum cyclic flow. We constructed the condition by asking the subgraph considered to represent a constant fraction of the maximum cyclic flow. The maximum cyclic flow being the one of the sample, it will always be possible to find a percentage of edges in the sample such that the cyclic flow of the subgraph reaches a certain percentage of the maximum one.

Percolation on the neighbour to neighbour sample:

The giant component is defined as a connected component that contains a constant fraction of the nodes. In our case, the giant component of the total sample contains only 112 nodes which represents less than 0.01 of the total number of nodes. In the previous simulations, we considered that the giant component phase was reached when half of the nodes were members of the giant component. With our given sample, this condition is too restrictive and will never be met. Figure 2.12 shows the percolation threshold if we ask that the giant component contains 1/1000 of the total number of nodes. We see

that even with this relaxed condition, the percolation threshold is high. Almost every exchange of the economic network should be done in complementary currencies.

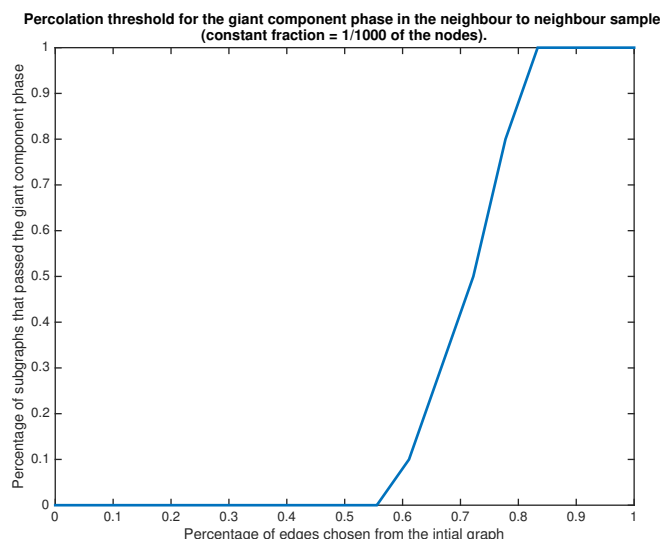


Figure 2.12: Percolation for the giant component

Let us now consider the two other conditions. The *cycle rank* condition asks that the number of edges represents twice the number of nodes. The sample consists of 16112 nodes and 18061 edges. The second condition is thus not respected even if the whole sample uses complementary currencies. The third condition, as explained, will always be met for a certain percentage of edges.

Conclusion

What conclusions can we draw from our trial on the virtual-life data set? First did we apply a tool, created for Erdős-Rényi random graphs, to graphs that resemble to Erdős-Rényi ones? It is hard to determine if the EVE Online network looks like an Erdős-Rényi one as we do not know much about it. Now, are our samples similar to Erdős-Rényi graphs? We already saw that the clustering coefficient of our samples were a lot higher than the one that could be obtained with an Erdős-Rényi graph of the same density. Moreover, the edges chosen were never (in all three sampling methods) chosen at random among all possible edges as it is the case for Erdős-Rényi networks. Even the first sampling method that consisted in choosing edges randomly did it among the existing ones. Our sample seem far from similar to Erdős-Rényi graphs.

Another question that could be raised is the one of the similarity between the EVE Online network and real-life economic networks. Again, we are not able to give to the reader a complete answer to the question. If both networks should *a priori* be very different, one should study the characteristics of real-life economic networks and then of virtual-life ones in order to answer the question.

We are now able to conclude that the viability conditions are far from being all met in the different samples. However this can be explained by the difference between the samples considered and Erdős-Rényi graphs. Moreover, the density of the three samples is much lower than the one of the EVE Online network.

The assumptions taken while constructing the *viability threshold* are far from respected in the samples of data. This implies that we are unable to answer to the question of the relevance of the conditions developed based on the data. However, we are able to assert

that, according to our viability threshold, no subnetwork could serve as a complementary currency network. Subnetworks with 80% of the edges of the neighbour to neighbour sample are the closer to be able to serve as complementary currency networks. However, can we really consider a money that represents 80% of the economic exchanges as a complementary currency ?

| Conclusion

The idea to use network theory in order to understand economical systems is not new. The one of using network theory tools on complementary currency networks is much more recent. However, some authors investigated tools coming from network theory and aiming to understand better complementary currency networks. Particularly, Lietaer and Ulanowicz used information theory and ecology in order to develop a notion of sustainability. Complementary currencies are, in this context, presented as a tool allowing to offer more diversity in economic networks. The Yale Institute for Network Science also recently started a network analysis of the Sardex complementary currency network. If network theoreticians are beginning to take an interest in complementary currency networks, we believe there is still much to learn in the field of economic networks and particularly complementary currency ones.

In this thesis, we tried to decrease the gap between network theoreticians and complementary currency promoters. We decided to focus on one particular field in network theory, namely percolation theory. By reviewing the state of the art of percolation theory and the complementary currency literature, we were able to extract an interesting challenge. The latter was to define, thanks to percolation, a viability threshold for complementary currencies. We also tried to provide the reader with a complete picture of the current network-based approaches to complementary currencies.

Afterwards, we started to create the *viability threshold* using Erdős-Rényi random graphs. We established three conditions allowing a complementary currency system to prosper. The first one addresses the need for a complementary currency to spread among the network by asking that at least a giant component should be present in the complementary currency subnetwork. The second and third ones focus on the need of circularity in complementary currency systems. All three conditions lead to a *threshold* in the density of edges that must be exceeded for the three conditions to hold.

Once we established this *viability threshold*, we used a virtual-life economic network in order to test the three conditions. As the virtual-life data set was composed of a too large number of edges, we took different samples of the data set. We then applied the viability conditions on the sampled subnetworks. It appeared that the three conditions were never met together in any of the three samples. However, as the assumptions taken to construct the viability threshold were not met in the sampled data networks, we could not conclude that the threshold was or not relevant based on the data analysis. However, we concluded that no sample could, based on our viability threshold, host a complementary currency subnetwork. This did not tell us however anything about the virtual-life network's ability to host a complementary currency subnetwork as the samples differ in many ways from the total data set.

This work's contributions are thus threefold. First, we achieved a preliminary work that extracted from the complementary currency literature the interesting papers that looked at complementary currencies with a *network theory* approach. Secondly, we showed that

percolation theory was a specific tool, coming from network theory but that was not exploited so far. Finally, we used percolation theory to develop a *viability threshold* for complementary currencies inside a specific economic network.

Doing so, we encountered some struggles. The most important one consists in our difficulty to have a realistic image of an economic network. This implied that the *viability threshold* developed so far did not pass any *confirmation process* that would ensure its validity. This work only gives insights to directions of further exploration.

And the possible future work is therefore huge. First, one could try to refine and sharpen the threshold developed in this thesis as well as test it on real-data sets. One could also use past examples of complementary currencies to show why some of them prospered whereas some others failed. For example, one could show to which extent the threshold developed in the frame of Erdős-Rényi networks holds when considering real economic networks, that might not look as Erdős-Rényi ones. Secondly, one should try to use the percolation threshold in order to understand better many aspects of the complementary currency systems. Indeed, if our work showed that percolation theory could be used while investigating economic networks, we probably haven't made the best use of it. We believe that percolation theory could be used for many other purposes than defining a viability threshold. Also, this work has not investigated deeply the notion of *resilience* and *diversity* used by Lietaer. We have briefly seen in the literature review that the connectivity of a network had an important impact on its stability. However, complementary currencies are presented in the literature as a means to provide diversity in an economic network. One could thus try to express thanks to network theory tools a notion of diversity and investigate the extent to which complementary currencies increase the diversity.

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A | Codes MATLAB

```

1 function [] = perco_giant_comp(B, tests)
2
3 % This function plots, for a growing density of edges, the
4 % proportion of
5 % graphs that passed the 'giant component' phase (with a fixed
6 % number of
7 % graph tested). The giant component phase is here reached when
8 % half of the
9 % nodes are in the giant component. It uses Tarjan's algorithm
10 % to detect
11 % the maximal strongly connected component in the directed case.
12 % The undirected case can also be taken into account.
13 % Finally, function might also be used to plot the connectivity
14 % threshold.
15 %
16 % INPUT : - B : An initial adjacency matrix of economic
17 %          exchanges
18 %          - tests : The number of tests used
19
20 profile on
21
22 if nargin==0
23     n = [20, 40, 60, 80];
24     nv = 100;
25     tests = 100;
26     B = ones(nv,nv);
27 end
28
29 %% Used to plot the threshold function for different number of
30 % nodes.
31 % for ii=1:length(n)
32 %     nv = n(ii);
33 %     B = ones(nv,nv);
34
35 q = 0.5; % fraction of vertices needed to be contained in the
36 % giant component
37 % q = 1; % if we look for the connectedness
38 B = B - diag(diag(B));
39 [IB,JB,~]=find(B);
40 s = length(IB);

```

```

34 %k=500; % allows a fastest run of the code
35 k=20;
36 c = ceil(s/k);
37 prop_true = zeros(c, tests); % vector containing the proportion
    of graphs
38 % that passed the gaint component phase
39
40 for i = 1:c
41     disp 'step '
42     for j=1:tests
43         kept = randperm(s);
44         ind = [IB(kept(1:k*i)), JB(kept(1:k*i))];
45         A = zeros(nv, nv);
46         l = length(ind(:, 1));
47         for m=1:l
48             A(ind(m, 1), ind(m, 2)) = B(ind(m, 1), ind(m, 2));
49             %A(ind(m, 2), ind(m, 1)) = B(ind(m, 1), ind(m, 2)); %
                Uncomment for
                %the undirected case.
50         end
51         [S, ~] = tarjan(A);
52         if length(S{1}) > nv*q || length(S{1}) == nv*q
53             prop_true(i, j) = 1/tests;
54         end
55     end
56 end
57 p = sum(prop_true, 2);
58 if p(i) > 0.999
59     disp 'stop '
60     break
61 end
62 end
63 prop_true(i+1:end, :) = 1/tests;
64
65 percentage = sum(prop_true, 2);
66 x = linspace(0, (s-1), length(percentage))./(s-1);
67 plot(x(1:end), percentage(1:end), 'linewidth', 2)
68 ylim([0, 1])
69 xlim([0, 0.1])
70
71 %% Uncomment to plot the threshold functions for different
    number of nodes.
72 % hold on
73 % end
74
75 % Create xlabel
76 xlabel({'Density_(0=_empty_digraph, _1=_complete_digraph)'});
77
78 % Create ylabel
79 ylabel({'Percentage_of_tested_digraphs_(out_of_100_tests)_with_a
    _giant_SCC'});
80

```

```

81 % Create title
82 title({'Percolation_threshold_for_the_giant_SCC_phase_in_a_
      directed_graph_(n=100)'});
83
84
85 end

```

```

1 function [] = perco_maxflow(B, seuil, tests)
2
3 % This function plots, for a growing density of egdes, the
4 % proportion of
5 % graphs for which the maximal cyclic flow amounts for half of
6 % the maximal
7 % cyclic flow of the initial graph. In order to find the maximal
8 % cyclic
9 % flow of a graph, we use the function 'separate_cyclic(adj)'
10 % created by
11 % Jean-Charles Delvenne et al.
12 %
13 % INPUT : - B : An initial adjacency matrix of economic
14 %           exchanges
15 %           - seuil : A real value between 0 and 1 that indicates
16 %           the
17 %           percentage of maximal cyclic flow that must be
18 %           reached, compared
19 %           to the initial network maximal cyclic flow.
20 %           - tests : The number of tests used
21
22
23 profile on
24
25 if nargin==0
26     B = sprand(20,20,1);
27     seuil = 0.5;
28     tests = 100;
29 end
30
31 nv = size(B,1);
32 B = B - diag(diag(B));
33 [IB,JB,~]=find(B);
34 s = length(IB);
35 %k=100; % For different speeds of execution (with more or less
36 % precision).
37
38 k = 10;
39 c = ceil(s/k);
40 prop_true = zeros(c,tests);
41 [B_C, ~] = separate_cyclic(B);
42 ncc = sum(sum(B_C));
43 for i = 2:c
44     disp 'step'
45     for j=1:tests
46         kept = randperm(s);

```

```

37     ind = [IB(kept(1:k*(i-1))), JB(kept(1:k*(i-1)))] ;
38     A = zeros(nv,nv);
39     l = length(ind(:,1));
40     for m=1:l
41         A(ind(m,1),ind(m,2)) = B(ind(m,1),ind(m,2));
42         %A(ind(m,2),ind(m,1)) = B(ind(m,1),ind(m,2)); %
43         %Uncomment for
44         %the undirected case.
45     end
46     if isempty(A)==0
47         [A_C, ~] = separate_cyclic(A);
48         cycles = sum(sum(A_C));
49         if cycles > seuil*ncc
50             prop_true(i,j) = 1/tests;
51         end
52     end
53     p = sum(prop_true,2);
54     if p(i)>0.999
55         disp 'stop'
56         break
57     end
58 end
59 prop_true(i+1:end,:) = 1/tests;
60
61 percentage = sum(prop_true,2);
62 x = linspace(0,(s-1),length(percentage))./(s-1);
63 plot(x(1:end),percentage(1:end),'linewidth',2)
64 ylim([0,1])
65 xlim([0,1])
66
67 % Create xlabel
68 xlabel({'Density_(0=_empty_weighted_digraph,_1=_complete_
69         weighted_digraph)'});
70
71 % Create ylabel
72 ylabel({'Percentage_of_tested_digraphs_(out_of_100_tests)_with_
73         cyclic_flow', '_above_one_half_of_a_random_complete_digraph_
74         cyclic_flow'});
75
76 % Create title
77 title({'Percolation_threshold_for_the_maximum_cyclic_flow_in_
78         directed_weighted_graphs_(n=20)'});
79
80 end

```

```

1 function [cycles] = percentage_maxflow(nv, D, steps)
2
3 % This function generates a random weighted digraph with nv
4 % vertices and a
5 % density of edges D between 0 (empty graph) and 1 (complete

```

```

    graph).
5 % A is the adjacency matrix representing the digraph. The
    function then
6 % generates subgraphs by removing edges step by step from the
    initial
7 % graph. The maximal cyclic flow is computed thanks to the
    function
8 % 'separate_cyclic(adj)' created by Jean-Charles Delvenne et al.
9 %
10 % INPUTS : - nv : number of nodes of the considered graph
11 %           - D : density of the initial graph
12 %           - steps : number of steps to remove all edges from
    the initial
13 %           graph
14
15 if nargin==0
16     nv = 50;
17     D = 1;
18     steps = 100;
19 end
20
21 A = sprand(nv, nv, D); % random weighted digraph
22 %% We continue by applying the next functions :
23 % A = ones(nv,nv); % to remove the weights
24
25 deleted = linspace(0,1,steps);
26 cycles = zeros(steps,1);
27 non_cycles = zeros(steps,1);
28 for i=1:steps
29     d = deleted(i);
30     S = take_subgraph(A,d);
31     if isempty(S)==0
32         [A_C, A_NC] = separate_cyclic(S);
33         cycles(i) = sum(sum(A_C));
34         non_cycles(i) = sum(sum(A_NC));
35     end
36 end
37 if cycles(1)>0
38     cycles = flipud(cycles./cycles(1));
39 end
40 axis([0 1 0 1])
41 plot(deleted, cycles)
42 hold on
43 plot(deleted, (deleted.^2), 'r')
44
45 %CREATEFIGURE(X1, Y1)
46 % X1: vector of x data
47 % Y1: vector of y data
48 X1 = deleted;
49 Y1 = cycles;
50

```

```

51 % Create figure
52 figure1 = figure;
53
54 % Create axes
55 axes1 = axes('Parent',figure1);
56 box(axes1,'on');
57 hold(axes1,'on');
58
59 % Create plot
60 plot(X1,Y1);
61
62 % Create xlabel
63 xlabel({'Density_(0==empty_digraph_to_1==complete_digraph)'});
64
65 % Create ylabel
66 ylabel({'Percentage_of_circular_flow_compared_to_the_complete_digraph_case'});
67
68 % Create title
69 title({'Density_of_cycles_for_30_nodes_random_weighted_digraphs'
70       '});
71 end

```

```

1  %[X] = csvimport('JCD.csv', 'columns', {'issuerID', 'acceptorID',
2      'Volume','count', 'MoneyValue'});
3
4  load X.mat
5
6  IssuerID = X(:,1);
7  AcceptorID = X(:,2);
8  Volume = X(:,3);
9  Count = X(:,4);
10 MoneyValue = X(:,5);
11 D = [IssuerID , AcceptorID , MoneyValue];
12
13 %% Only monetary exchanges
14
15 indices = find(not(MoneyValue));
16 NewIss=IssuerID;
17 NewAcc=AcceptorID;
18 NewMV=MoneyValue;
19 NewCount=Count;
20 for i=1:length(indices)
21     NewIss(indices(i))=0;
22     NewAcc(indices(i))=0;
23     NewCount(indices(i))=0;
24 end
25 NewIss(NewIss==0)=[];
26 NewAcc(NewAcc==0)=[];
27 NewMV(NewMV==0)=[];
28 NewCount(NewCount==0)=[];

```

```

27
28 e1 = [NewIss,NewAcc,NewMV];
29
30 %% Randomly chosen 20000 nodes %%
31
32 EX0 = datasample(e1,20000,'Replace',false);
33
34 %% Only counts above a value %%
35
36 Cindices = find(NewCount>50);
37 NewI=NewIss(Cindices);
38 NewA=NewAcc(Cindices);
39 NewM=NewMV(Cindices);
40
41 EX1 = [NewI,NewA,NewM];
42
43 %% Neighbour to neighbour %%
44
45 EX2={};
46 start = datasample(e1,100);
47 [row1,col1]=find(ismember(e1,start(:,1)));
48 [row2,col2]=find(ismember(e1,start(:,2)));
49 ex2 = [e1(row1,:);e1(row2,:)];% first neighbours
50 ex2 = datasample(ex2,floor(length(ex2)/1000));
51 ex2 = unique([start;ex2],'rows');
52 EX2{1}=ex2;
53 for i=1:1000
54     [row,~]=find(ismember(e1,ex2(:,1)));
55     [row_bis,~]=find(ismember(e1,ex2(:,2)));
56     v = [e1(row,:);e1(row_bis,:)];
57     ex2 = datasample(v,floor(length(v)/1000));
58     if size(EX2{i},1)+length(ex2) > 20000
59         disp 'stop'
60         break
61     end
62     EX2{i+1} = unique([EX2{i};ex2],'rows');
63 end

```