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Greenfield Transmission Planning for Electric Power Systems

Dissertation presented by
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Abstract

Transmission Expansion Planning is a problem well known by the power system planners. It is a difficult task and therefore still requires attention and researches. New challenges in the sector highlight the need of efficient methods. The integration of renewable energy on larger scales will indeed request the design of new networks to connect the remote generation sites to the consumers. The growing power consumption in some developing countries will also demand specific tools to make decisions about the installation of new transmission lines.

This master's thesis aims at providing solutions to those problems. After an overview of the existing literature on the subject, it therefore develops methods to design efficiently greenfield transmission networks. The work is based on two practical cases corresponding to the two above-mentioned challenges. The first one consists in connecting wind farms in the North Sea to onshore connection points. The second one concerns the design of the grid in Burundi based on the forecasted power levels. The problems are greenfield projects for both lines and substations of the networks. They are both deterministic and do not consider operational costs.

The proposed method for the former case involves a partitioning technique and a heuristic approach to provide a meaningful list of candidate substation nodes. Another approach, based on a systematic meshing of the space, is presented for comparison.

A statistical analysis is also performed in this work to assess the idea of neglecting additional substation candidates by considering only the existing nodes of the grids. The results motivate this choice for the second problem whose *N-1 security* constraint increases the complexity. The proposed method combines a mathematical optimization method (Benders Decomposition) and heuristic techniques. It appears to outperform a purely heuristic approach based on evolutionary algorithms.

The developed methods applied on these two practical cases give substantial results. This motivates the design of these specific tools for providing significant savings.

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Acronyms

AC	Alternative Current
AIS	Artificial Immune Systems
CHA	Constructive Heuristic Algorithm
DC	Direct Current
DE	Differential Evolution
EENS	Expected Energy Not Supplied
GA	Genetic Algorithm
GRASP	Greedy Randomized Adaptive Search Procedure
HVDC	High-Voltage Direct Current
HVAC	High-Voltage Alternative Current
LP	Linear Programming
MEUR	Million Euros
MILP	Mixted-Integer Linear Programming
MINLP	Mixted-Integer Non-Linear Programming
NLP	Non-Linear Programming
OPF	Optimal Power Flow
PSO	Particle Swarm Optimization
SA	Simulated Annealing
SEP	Substation Expansion Planning
TEP	Transmission Expansion Planning
TSO	Transmission System Operator

General Introduction

Considerable challenges are expected in the future in the electricity sector. The integration of large amounts of renewable energy will imply a lot of changes in the energy market. This new generation will include wind farms and solar parks. New transmission networks will be needed in order to transport these future generated powers. This will imply the installation of kilometers of lines to be able to bring gigawatts of power from the remote production sites to the consumers. Significant amounts of money (billions of euros) will be involved and therefore the optimization of the transmission networks should not be taken lightly. Implementing specific tools and methods to design those networks are expected to provide substantial savings in terms of money and resources. They therefore seem to be mandatory in order to integrate efficiently these new sources of energy on larger scales. The alternatives would consist in applying engineering techniques but should be outperformed by an efficient optimization method.

The integration of renewable energies is not the only problem that the electricity sector will face in the next decades. The power consumptions are expected to increase significantly in some areas of the world. These regions include the developing countries of Africa and the Middle-East. This growth of both generation and consumption will also require the design of new networks. Comparatively to the current levels of power involved, the forecasted ones will be enormous and so the existing installations are negligible with respect to the future requirements. Optimization techniques will also be needed to make decisions about the design of those grids.

Both above-mentioned problems are special cases of a more general class known as the *Transmission Expansion Planning (TEP)* problems. They usually aim at expanding, in an optimal way, existing transmission networks, in order to allow the increasing demand to be served in the future. They are well-known problems and thus have already been treated in the literature. However, it is a highly challenging task, therefore no perfect method, working for all the practical cases, has been discovered yet. The subject is then not closed and further researches are still meaningful.

The present work aims at providing methods in order to respond to both problems of electricity transportation described above. The techniques implemented will be tested on practical cases and the results will be discussed. The data of the practical cases were given by **Tractebel**. The methods are implemented in AMPL to be solved with the solver CPLEX, on a computer TOSHIBA L850-1JT dating from mid-2013 (processor Intel® Core™ i7-3630QM - 4-Core - 2.40 GHz - 8GB of RAM). Unless mentioned otherwise, the problems have been solved with the default option values of the solver. The graphs have been generated with MATLAB.

The thesis is divided into three parts.
The first one (part I) gives an overview of the Transmission Expansion Planning problem; presenting the different formulations and methods encountered to tackle this challenge.
The second part (part II) is based on the design of the transmission network to connect offshore wind farms in the North Sea. It then concerns the first challenge mentioned above. It provides two main methods and discusses their implementations and performances. Moreover, a statistical analysis is performed to assess the quality of a generalization of the problem.

The third part (part III) of this work is dedicated to the design of an onshore grid to reach scattered consumption sites. It is based on data of the grid in Burundi. Additional constraints are included in the model and so specific methods are implemented to solve it. Two main approaches are discussed and their results are analyzed.

In both parts (part II and part III), the proposed methods are presented in chapters 3 that constitute then the main core of the work. The most advanced method is presented in chapter 3 of part III. This chapter should then be the one preferred by the reader who lacks time to read through the whole thesis.

This thesis will come to an end with a general conclusion about the provided methods and their generalization to similar problems.

Part I

Literature Review

Introduction

The transmission expansion planning is a well-known problem that has been largely studied in the literature. However, the formulation and the methods used to tackle it might differ greatly among the different researchers. The purpose of this part is the presentation of the various ways of handling this challenge.

First, the model will be the subject of attention. After the introduction of the required notations, a usual version will be presented based on the DC power flow equations. This is one example of the various models that can be found in the literature. Indeed, the formulation is not the same in all research papers. For example, the level of accuracy of the physics laws is a point of divergence. From the simpler model that alleviates the computational burden to the most accurate one, different formulations will be presented. Moreover, the models do not always aim at optimizing the same objective and the constraints included vary from one paper to another. Depending on the specific problem treated and its context, it is meaningful to include some particularities that do not appear in all formulations. The first chapter will then aim at giving an overview on how the TEP problem is formulated in the literature.

The second chapter will address the question of the methods used to tackle the problem. As the reader will be able to see, those are numerous but can be classified in different categories. Some of them will be described in more details because of their importance or the role that they will play in the rest of this work.

Notations

The notations described below are mainly inspired by two papers [1, 2].

Sets

- N : set of nodes.
- E : set of edges - possible connections between nodes: $(i, j) \in E$ if there is a possible connection between nodes i and j. Hence $E \subset N \times N$.
- EL : set of existing cables/lines/circuits.
- CL : set of candidate circuits.

Variables

- x_{ijc} : binary variable deciding if the candidate circuit $c \in CL$ on connection $(i, j) \in E$ is chosen in the expansion plan or not.
- f^0_{ije} : continuous variable - active power flow on existing circuit $e \in EL$ on connection $(i, j) \in E$.
- f^1_{ijc} : continuous variable - active power flow on candidate circuit $c \in CL$ on connection $(i, j) \in E$.
- g_n : positive continuous variable - power generated at node $n \in N$.
- θ_n : continuous variable - voltage angle at node $n \in N$.
- $|V_n|$: positive continuous variable - voltage magnitude at node $n \in N$.
- OC : positive continuous variable - operational cost.

Parameters

- $\bar{g}_n$: maximum power generated at node $n \in N$.
- D_n : demand / power consumed at node $n \in N$.
- C_{ijc}: investment cost of candidate circuit $c \in CL$ on connection $(i, j) \in E$
- $\bar{f}^0_{ijc}$: maximum active power flow capacity on existing circuit $e \in EL$ on connection $(i, j) \in E$.
- $\bar{f}^1_{ijc}$: maximum active power flow capacity on candidate circuit $c \in CL$ on connection $(i, j) \in E$.
- Y^0_{ijc} : admittance of existing circuit $e \in EL$ on connection $(i, j) \in E$. $Y^0_{ijc} = G^0_{ijc} + jB^0_{ijc}$
- Y^1_{ijc} : admittance of candidate circuit $c \in CL$ on connection $(i, j) \in E$. $Y^1_{ijc} = G^1_{ijc} + jB^1_{ijc}$
- G^0_{ijc} : conductance of existing circuit $e \in EL$ on connection $(i, j) \in E$.
- G^1_{ijc} : conductance of candidate circuit $c \in CL$ on connection $(i, j) \in E$.
- B^0_{ijc} : susceptance of existing circuit $e \in EL$ on connection $(i, j) \in E$.
- B^1_{ijc} : susceptance of candidate circuit $c \in CL$ on connection $(i, j) \in E$.
- M_{ijc} : disjunctive parameter for DC Kirchhoff's voltage law for candidate circuit $c \in CL$ on connection $(i, j) \in E$.

Chapter 1

Model

1.1 Typical formulation

This section will present one particular formulation of the TEP problem. Kirchhoff's voltage laws will be expressed using a disjunctive DC model. This model and the other possible ones are discussed in the next section.

$$\min_{x,f^0,f^1,g,\theta} \quad \sum_{(i,j)\in E, c\in CL} C_{ijc}x_{ijc} \quad + \quad OC$$

subject to

$$\sum_{(i,n)\in E, e\in EL, c\in CL} (f^0_{ine} + f^1_{inc}) - \sum_{(n,j)\in E, e\in EL, c\in CL} (f^0_{nje} + f^1_{njc}) + g_n - D_n = 0 \qquad \forall n \in N \quad (1.1)$$

$$g_n \leq \bar{g}_n \qquad \forall n \in N \qquad (1.2)$$

$$f^0_{ije} \leq \bar{f}^0_{ije} \qquad \forall (i,j) \in E, e \in EL \qquad (1.3)$$

$$-f^0_{ije} \leq \bar{f}^0_{ije} \qquad \forall (i,j) \in E, e \in EL \qquad (1.4)$$

$$f^1_{ijc} \leq x_{ijc} \cdot \bar{f}^1_{ijc} \qquad \forall (i,j) \in E, c \in CL \qquad (1.5)$$

$$-f^1_{ijc} \leq x_{ijc} \cdot \bar{f}^1_{ijc} \qquad \forall (i,j) \in E, c \in CL \qquad (1.6)$$

$$f^0_{ije} - B^0_{ije}(\theta_i - \theta_j) = 0 \qquad \forall (i,j) \in E, e \in EL \qquad (1.7)$$

$$f^1_{ijc} - B^1_{ijc}(\theta_i - \theta_j) \leq M_{ijc} \cdot (1 - x_{ijc}) \qquad \forall (i,j) \in E, c \in CL \qquad (1.8)$$

$$-f^1_{ijc} + B^1_{ijc}(\theta_i - \theta_j) \leq M_{ijc} \cdot (1 - x_{ijc}) \qquad \forall (i,j) \in E, c \in CL \qquad (1.9)$$

$$g_n \geq 0 \qquad \forall n \in N \qquad (1.10)$$

$$x_{ijc} \in \{0,1\} \qquad \forall (i,j) \in E, c \in CL \qquad (1.11)$$

In this formulation, the objective function is composed of the investment cost of the lines and some undefined operational cost. The latter is discussed in section 1.3 below.

Equality 1.1 is the power balance constraint. It makes sure that at each node of the network, there is a balance between the power that comes in (through the network or generated) and the one that comes out (through the network or consumed). The reader can note that this constraint, as written above, guarantees that the demand is served at each node, which is not always the case in the different formulations.
Constraint 1.2 limits the power generated at each node while 1.10 ensures its positivity.

Constraints 1.3 and 1.4 limit the absolute value of the active power flow on the existing circuits, while constraints 1.5 and 1.6 do the same for the candidate circuits (if the circuit is not built, then the power flow must be 0).
The DC formulation of the power flow equations (Kirchhoff voltage law) is ensured by equality 1.7 for the existing circuits while it is implemented with inequalities 1.8 and 1.9 for the candidate circuits.
Finally, the binarity of the decision variables is guaranteed with the last constraint. The reader can note that there also exist different ways of formulating those variables. Section 1.7 is dedicated to this discussion.

1.2 Different formulations

The natural formulation of the TEP problem usually relies on the AC (alternative current)-model of the power flow equations, known as the AC optimal power flow (ACOPF). Although this version gives the most accurate representation of the underlying physics of the network [1], it is not the most commonly encountered in the literature due to the complexity it implies [3]. Indeed, the ACTEP is a mixed integer non linear programming (MINLP) problem which makes it a very hard challenge for any solver. It is considered to be one of the hardest class of problems to solve [4]. Furthermore, the problem being not convex, there is little hope of proving the optimality of the solution found for large systems, whatever method is used. Moreover, a simplification of the model might be meaningful since the investment costs associated to the active power (such as for lines) are usually much higher than those for the reactive power (such as for capacitors).

For the above reasons, researchers mainly focused on a simplified flow model known as the direct current power flow (DCPF). This representation of the network is based on the AC-model and, thanks to several assumptions, linearizes the power flow equations.

The equation ruling the active power flow in an AC model is the following for existing circuits [1].

$$f^0_{ije} = |V_i|^2 \cdot G^0_{ije} - |V_i| \cdot |V_j| \cdot (G^0_{ije} \cdot \cos(\theta_i - \theta_j) + B^0_{ije} \cdot \sin(\theta_i - \theta_j)) \qquad \forall (i,j) \in E, e \in EL.$$

Similarly, for the candidate circuits, one can write:

$$f^1_{ijc} = |V_i|^2 \cdot G^1_{ijc} - |V_i| \cdot |V_j| \cdot (G^1_{ijc} \cdot \cos(\theta_i - \theta_j) + B^1_{ijc} \cdot \sin(\theta_i - \theta_j)) \qquad \forall (i,j) \in E, c \in CL.$$

The assumptions, considered in order to simplify these equations, are the following [5] :

- The voltage magnitude on each node is nominal: $|V| \approx 1$
- Line resistance is negligible: $G^0 << B^0$ and $G^1 << B^1$ (approximation valid with high voltage)
- The phase angle differences are small hence the following first order approximations are valid : $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$ and $\cos(\theta_i - \theta_j) \approx 1$ $\forall i, j \in N$

The second assumption implies that power losses are neglected. Moreover, reactive power is ignored in this formulation.

These simplifications lead to a linear formulation of the line power flows (Kirchhoff's voltage law) for existing circuits:

$$f^0_{ije} = B^0_{ijc}(\theta_i - \theta_j) \qquad \forall (i,j) \in E, e \in EL.$$

However, it can be noted that the traditional DC formulation of the TEP still belongs to the MINLP class since the power flow equation implies a product between the binary decision variable and the continuous phase angle variable for the candidate circuits.

$$f^1_{ijc} = x_{ijc} B^1_{ijc}(\theta_i - \theta_j) \qquad \forall (i,j) \in E, c \in CL \tag{1.12}$$

However, the non-linear equalities of Kirchhoff's voltage law can be transformed into linear inequalities through the use of a disjunctive parameter (often named "big-M").

$$-M_{ijc} \cdot (1 - x_{ijc}) \leq f^1_{ijc} - B^1_{ijc}(\theta_i - \theta_j) \leq M_{ijc} \cdot (1 - x_{ijc}) \qquad \forall (i,j) \in E, c \in CL$$

If the disjunctive parameter is high enough, it can be checked that these two inequalities are equivalent to the above equality.

This formulation is sometimes referred to as the *disjunctive DC model.*

The disjunctive model enables the TEP problem to be formulated as an MILP (mixed-integer linear problem) which makes it easier to solve.

There also exist several relaxed versions of the DC model. One can mention the transportation model [6] that neglects Kirchhoff's voltage law. A less relaxed version is the hybrid model [7, 8] that includes this law but only for the existing lines. Such relaxations can be used in first steps before applying a more accurate version [9]. They can also provide valuable information with less computational effort [2] and can, for example, be used to reduce the search space [10].

Other less used formulations can be mentioned for the sake of completeness. Based on the AC-model, an NLP (non-linear programming) problem can be obtained thanks to the relaxation of the integer decision variables, through the introduction of additional constraints or a penalty term in the objective [4].
The AC-model can also be relaxed into a mixed integer semidefinite integer program (MISDP) [11]. The continuous version of this problem can then be solved by a specific method.

The DC formulation has the advantage of alleviating the computational burden. However, since it is only an approximation, the obtained solution might have to be adapted for the AC-operation [12]. Another drawback lies in the impossibility of integrating the reactive power which can lead to savings with the AC formulation [12]. Moreover, the inclusion of the power losses in the model is difficult [13].

Some researches have been conducted in order to keep the computational gain of the DC formulation (MILP) while increasing its accuracy of modeling. The most common technique is the piece-wise linearization of the non-linear terms appearing in the AC-formulation. One can therefore keep both the power losses and the reactive power in the model [1]. The authors of [14] present different ways of performing the piece-wise linearization.

One can also mention RLT (Relaxation-Linear-Techniques) based relaxations [4] with linearization of the product of variables by the introduction of "dummy variables".

1.3 Objective function

The objective function of the transmission expansion planning problem is, of course, a very important choice of modeling and may affect significantly the optimal plan found by the implemented method. It traditionally includes the investment cost of the plan. This simply corresponds to the cost of the construction of the new transmission lines.

Although some papers do not extend it (e.g. [15, 16, 17, 18, 19]), most of them add another term referred to as operational costs. However, these costs include different elements according to the papers. A representative set of operating states might need to be determined in order to include these costs in the model.

One of the most encountered elements included in the operational cost in the literature, is the load curtailment (e.g. [20, 21, 22, 23, 24]). This represents the penalty cost of not serving the demand. If it is not included, the demand is then required to be supplied with a strict constraint which corresponds to the robust approach. The generation cost is also often included (e.g. in [25, 1, 22, 23, 26]). It can be approximated as a quadratic function of the produced power which then requests a piece-wise linearization in order to keep the MILP nature of the problem [14]. Another traditional term is the power losses (e.g. [27, 25, 28, 29, 30]). The problem can also include the variable cost of the lines [27], the cost associated to reserve [24], the wind spillage [24] and the installation of the reactive power devices (if an AC flow is considered)[3].

The reader can note that the costs should be discounted when a multi-stage expansion problem is considered in order to optimize their present values [25, 23, 31].

Notably due to the liberalization of the energy sector, multi-objective expansion plannings are also encountered in the literature. One way to deal with this type of formulations is the implementation of a method that will provide a set of pareto-optimal [1] solutions [32, 33, 34]. Decision analysis can then be applied to the obtained set of non-dominated solutions, in order to select the plan, according to the preferences of the decision maker [35]. A so called *fuzzy satisfying method* can be used to choose the optimal plan [33].
Another way to deal with these problems is to consider the goal programming method that aims at finding the solution that produces the least deviation from targeted objectives [36].
The different objectives usually include the investment cost [32, 33, 36, 35]. They may also comprise a reliability criterion [32, 33, 36, 35], the congestion cost [33], a stability criterion [36] or some measure of the environmental impact [35]. The reliability level is often measured as the Expected Energy Not Supplied (EENS) [33, 36, 35]. It assesses the load curtailment under contingencies [37]. The Correction Cost risk index (CCRI) can be used to extend the EENS and include all the corrective actions undertaken to cope with the contingencies (notably generation rescheduling) [34].

Even in cases of multi-objective models, it is almost impossible to include every planning criterion in the TEP problem. An evaluation phase must then be considered after the optimization problem in order to assess the different requirements for the proposed expansion plan [38]. Techniques that produce several good solutions (such as meta-heuristics: the reader is invited to refer to section 2.3) might then provide an advantage in terms of flexibility since the best plan can then be chosen among the different proposed solutions, considering the criteria not directly included in the model.

1.4 Uncertainties

Most of the papers encountered in the literature ignore the effect of the uncertainties on the TEP problem, probably due to the fact that they increase the inherent complexity of the problem. However, uncertainties do appear in practice and neglecting them might lead to under or over investment [39, 40]. One can mention, as examples, the load forecast, the future economic and environmental policies [12, 36, 18], the fuel cost, the transmission cost,... The increasing amount of renewable energy from wind and photo-voltaic sources also involves uncertainties about the

[1] one objective cannot be improved without affecting negatively another

generation part of the problem.

Moreover, the liberalization of the energy market introduced all new types of uncertainties [36]. Some parts of the generation companies' information can notably be kept secret from the planner. The uncertainties specific to the liberalized configuration include the bidding strategy of the companies involved, the details about the installations, the availability of generators...[18].

The traditional way to deal with the uncertainties is the generation of scenarios [41, 39, 25, 36]. Those scenarios can be produced by Monte-Carlo simulations [25].

One can also resort to Robust Optimization and robust sets to cope with uncertainties. For example, Adaptive Robust Optimization (ARO) expresses the problem with a three-level formulation [23, 42]. This approach tends to minimize the worst case of the determined uncertainty set and escapes scenario generation which might constitute a computational burden. It enables one to avoid the intractability generated by a large number of scenarios [42] but it can be argued that this technique leads to too conservative expansion plans (high cost) to cope with the worst case possible [38].

The authors in [18] suggest another approach. They propose to formulate the TEP problem as a chance constrained programming (CCP) problem which includes random variables in the constraints. The constraints are then guaranteed to be satisfied with a chosen confidence level. They use Monte Carlo simulation method to solve the problem.

Several methods are suggested to solve the stochastic program resulting from the scenario-approach. Benders Decomposition, which is also used for the deterministic version, can be easily extended to the stochastic case [41]. A detailed description of this method can be found in section 2.1.2. Stochastic methods aim at finding the plan with the lowest expected cost.
Minimum regret optimizing techniques can also be considered. In this case, the goal is to minimize the maximum difference between the cost of the chosen plan, under a scenario, and the cost of the optimal plan designed for the specific scenario [40].

1.5 Reliability / Security

The reliability and the security of the transmission network constitute important concerns when an expansion plan is considered. Nevertheless, they are often ignored or barely mentioned in the researches due to the computational cost that they imply. The most known criterion to somehow attest the security and reliability of the network is the *N-1 security* (e.g. [22, 43, 24, 17, 44]) criterion. It guarantees that the system can withstand one failure of any component (generation or transmission). This usually means that no failure should imply load shedding or uncontrolled cascading outage.

Several drawbacks can be mentioned. Indeed, although each failure is considered, the probability of failure is not explicitly taken into account. Moreover, the possibility of more than one failure at a time is ignored [25]. To handle the latter problem, the planner may consider a $N-2$ (or $N-k$) criterion which takes 2 (or k) failures into account at the same time. However, for large systems with numerous components, the complexity burden is already significantly increased with the original version of this security criterion [41]. Extending the number of contingencies might then have a critical impact on the performance. Moreover, increasing the reliability of the network will lead to more expensive investment costs. Therefore, a trade-off should be found between the additional costs and benefits of such a network.

Another approach consists in defining oneself the set of considered contingencies [41, 45]. This can be performed by looking at historical data or considering the most congested lines [46]. A highly relaxed version of the $N-1$ criterion is the steady state security requirement that requests a number of lines connected to each load bus superior or equal to 2 [4] (no load bus can be isolated due to one line failure).

Other security and reliability concerns can be found in the literature. Different indices enable the assessment of the level of reliability or security of a system. If the objective function is somehow impacted by these indices, they will then guarantee a more reliable or secure network. As mentioned in section 1.3, the EENS is a traditional index to assess reliability.

1.6 Planning horizon

The transmission expansion problem has an intrinsic temporal dimension [33]. Indeed not only does it aim at answering the questions *where?*, *what?* and *how many?* lines must be installed but also it has for purpose to find a response to the concern *when?*. Despite this dynamic nature, most researchers neglect this aspect of the problem [47], that might dramatically increase the computational effort, to focus on a relaxed version where the time component is ignored. Indeed, the number of variables is multiplied by the number of stages which may be particularly critical for the integer decision variables. Three different approaches can be distinguished [23]:

1. **Dynamic** planning : The true nature of the problem is preserved. It is then formulated as a multi-stage program and the aim is to minimize the present value of the considered costs [48]. However, since the formulation is not relaxed, the complexity is also intact which makes it a very difficult problem to tackle. Heuristics are often used if the scale of the problem increases [23].

2. **Static** planning : The planner focuses on one stage (one then aims at minimizing the expansion plan for a target network).

3. **Sequential static** planning : The multi-stage formulation is maintained but the problem is divided into different stages that are solved sequentially (one after the other).

Two different techniques are used to tackle the sequential static (pseudo-dynamic) planning: **forward** propagation and **backward** propagation.
In the first case, the different stages are solved sequentially, from the first one to the end of the horizon, considering for each subproblem the lines installed from the previous stage as part of the existing network.
In the latter, the process starts by solving the horizon year and then goes back in time to the first year. The candidate lines available at a stage are then only the ones that are part of the optimal plan of the next stage (in time and so previous solved stage). The backward procedure tends to give better results [47, 12] since the last stages of the planning might be considered the most critical ones. However, one might then need to deal with infeasibility for intermediate stages.

Sequential static procedures might also be used, in a first stage, to reduce the candidate list considered for the dynamic formulation of the problem [48, 49, 46].

1.7 Integer formulation

The TEP problem can be formulated with the help of binary or integer variables. One can either choose to use integer variables, setting the number of transmission lines that will be installed in a given corridor (e.g. in [21, 15, 16, 43, 17]), or binary variables deciding if one transmission line

will be added to the system or not (e.g. in [50, 20, 23, 24, 41]). This is the formulation used in the model described in the beginning of this part and the most suited to the disjunctive DC model.
The binary numeral system can also be used to represent the integer decisions with a restrained number of binary variables [39, 51](decomposition of the integer into bits).

1.8 Candidate line selection

The selection of candidate lines is a topic rarely treated in the literature [50]. Most of the papers assume, indeed, the existence of a candidate line list but few talk about how the latter is constructed.

The initial definition of the list of candidate circuits is based on the Transmission System Operator's own knowledge [22] and congestion analysis. Some lines can be eliminated through environmental and technical analysis [39]. For example, geographical conditions or line lengths may be considered as defining criteria [50].

However, several concerns may be raised about these approaches. Once the techniques listed above have been applied, the set of candidate lines might indeed still be quite large, which might be critical given the complexity of the TEP problem [50]. Moreover, the environmental assessment might reveal itself to be costly and time-consuming [39]. Finally, the TSO has knowledge about its system, located in a restrained area, but the increasing integration of renewable energy, notably, leads to plannings over larger geographical areas [50, 22]. The generation sites might indeed be situated far from the consumption locations. Therefore, the experts' knowledge might be irrelevant for these multiple-region planning problems. Furthermore, when a relatively new network must be designed, one's knowledge about it might be limited.

For all the above reasons, an algorithm to select automatically the candidate lines should be required in some cases. Below are presented some researches on the subject encountered in the literature.

The authors of [22] propose a candidate selection based on a sensitivity analysis. The lines are selected using an index calculated as the quotient of an estimated *"potential benefit"* (based on the dual variables of power balance equations) and the investment cost of the line considered. The reader can refer to section 2.2 (*Heuristics*) in chapter 2 (*Methods*) for a discussion about sensitivity indices. The method also involves a stage of candidate reduction (lines which are not used in the optimal plan of a simplified version of the problem - for example LP relaxation - are restricted).

Other algorithms based on the resolution of linear programs are suggested in [50] and [39].

In order to reduce the search space, a known heuristic method such as CHA [52, 53] or GRASP [54, 55] can also be considered. One can then decide to retain only the candidate lists that are included in the solution(s) of the chosen method. The reader will find a description of these methods in section 2.2 of chapter 2 dedicated to *Methods*.

Partitioning the system may also be considered in order to identify the most promising transmission lines at lower computation cost [56, 51].

In stochastic TEP problems, the search space can be reduced by solving each scenario independently and forming the candidate list with the solutions obtained [10].

1.9 Type of lines

The traditional formulation of the TEP problem involves high voltage alternative current (HVAC) lines. However, another option for transmission expansion has been considered recently by some researchers: HVDC lines (high voltage direct current). The transmission expansion planning problem with addition of either HVDC or HVAC lines has been studied [56, 57, 31]. HVDC lines present several interesting aspects. They allow higher amount of power to be transported with a same transmission corridor width [56, 57] and exhibit low power losses [31]. Moreover, they contribute positively to the stability of the system [56, 31] when the *voltage-source converters* (VSC) are used [2]. It can be noted that the formulation of the power flow equations has to be reviewed with the introduction of HVDC lines since they enable the operator to control the power flow in converters [56]. For the reasons above, HVDC lines might be considered to be the best option to transfer renewable energy from offshore generation sites to onshore load centers [31].

1.10 Substations

Substation expansion planning (SEP) aims at optimally extending the existing substations and/or creating new substations. Similarly to the transmission expansion planning problem with the lines, the SEP methods usually start from a list of candidate buses [58]. The simultaneous optimization of both the TEP and the SEP problems is, to the author's best knowledge, rarely encountered in the literature. This can be explained by the fact that the addition of substation planning increases further the complexity of the problem with the introduction of new binary variables. However, some papers do solve both problems at the same time [58, 59, 60]. The SEP is usually not handled the same way as it will be presented in this work. Instead of defining substations according to the transmission line installations, the expansion or creation of substations usually depends on the forecasted load and geographical locations of distribution centers [59, 60]. The cost of expansion is then not necessarily included in the objective function. Nevertheless, [61, 44] do include the cost of expansion of existing substations in the objective function.

[2]There also exists another technology called *line-commutated current* source converters (LCC).

Chapter 2

Methods

The TEP is a very challenging problem that can quickly become intractable when its size increases. Indeed, it has been shown in [62] that transmission network expansion planning is *NP-hard* by reduction to the 3-SAT problem [1]. This means that there exists no known polynomial-time algorithm that solves it. However, it has been the subject of high interest and the application of diverse techniques has been studied.

The methods implemented in the literature to tackle the problem can be traditionally classified into two categories :

1. **Mathematical optimization**
2. **Heuristics**

In this section, the main features of these two approaches will be presented, as well as examples of the most encountered methods to solve the TEP problem.

2.1 Mathematical optimization

2.1.1 Overview

Optimization methods discover an optimal plan, solution of a determined formulation of the problem, by applying a mathematical procedure. It can be noted that the solution found is dependent of the formulation. Since the TEP problem is often simplified in order to fit to simpler models, the obtained plan is not necessarily the optimal one of the problem that will be implemented in practice [47]. Obviously, the simpler the model, the more unreliable the solution. It can also be mentioned that the result of the algorithm is not necessarily a global optimum but it may lead to a local one if the formulation is not convex.

Numerous optimization methods have been implemented to solve the TEP problem. The more classical ones include :

1. Linear programming (LP) [63, 64]
2. Dynamic programming [65]
3. Non-linear programming (NLP) [4, 13, 66]
4. Mixed-integer linear programming (MILP) [55, 1, 49, 14, 19, 6, 7, 54]

[1] This problem is a special case of the *Boolean satisfiability* problem which consists in determining if the binary variables of a boolean formula can be given values such that the formula is satisfied.

5. Mixed-integer non-linear programming (MINLP) [67]

A traditional algorithm in mixed-integer linear programming is the well-known Branch-and-Bound. It consists in exploring a tree of cases for the integer variables. It obtains lower bounds on the solutions, by solving repeatedly (at each node of the tree) a linear programming relaxation of the problem, and upper bounds with feasible integer solutions. Those bounds are useful to shut down branches of the tree whose further exploration is pointless. The critical considerations for the convergence rely on the selection of the variables for branching and the subproblem to explore [6, 7].
This method has also been applied to solve an MINLP formulation of the problem (including losses) [67] involving an NLP problem solved with the interior point method (IPM) at each node. The Branch-and-Bound algorithm can be modified to include cutting plane methods (adding valid inequalities to the problem) and form Branch-and-Cut algorithms [11, 8].

2.1.2 Benders Decomposition

One mathematical technique commonly applied to the TEP problem will be discussed in this section: Benders Decomposition. First, the method will be presented in the general case. Then its application to the TEP problem will be studied. Finally, the advantages, drawbacks and improvements will be discussed.

Consider the following optimization problem :

$$\min_{x,y} \qquad c^T x + q^T y$$

subject to

$$\begin{aligned} Ax &= b \\ Tx + Wy &= h \\ x, y &\geq 0 \end{aligned}$$

with $x \in \mathbb{R}^{n_1}$ called the first stage variables, $y \in \mathbb{R}^{n_2}$ called the second stage variables, $c \in \mathbb{R}^{n_1}$, $q \in \mathbb{R}^{n_2}$, $A \in \mathbb{R}^{m_1 \times n_1}$, $b \in \mathbb{R}^{m_1}$, $T \in \mathbb{R}^{m_2 \times n_1}$, $W \in \mathbb{R}^{m_2 \times n_2}$ and $h \in \mathbb{R}^{m_2}$

The underlying idea of Benders Decomposition is to split the above problem into two subproblems : a first-stage problem referred to as the *master* and a second stage one, the *slave*. The problem can then be rewritten as :

$$\begin{aligned} \min_{x} \qquad & c^T x + V(x) \\ \text{s.t.} \qquad & Ax = b \\ & x \in \text{dom}V \\ & x \geq 0 \end{aligned}$$

with

$$\begin{aligned} V(x) = \min_{y} \qquad & q^T y \\ \text{s.t.} \qquad Tx + Wy &= h \qquad (\pi) \\ y &\geq 0 \end{aligned}$$

The second problem (slave program) can be expressed with its dual form :

$$\begin{aligned} V(x) = \max_{\pi} \quad & \pi^T(h - Tx) \\ \text{s.t.} \quad & \pi^T W \leq q^T \end{aligned}$$

Moreover, it can be shown [68] that the first stage program, expressed above, is equivalent to

$$\begin{aligned} & \min_{x} \quad c^T x + \theta \\ & \text{s.t.} \quad Ax = b \\ & \sigma^T(h - Tx) \leq 0 \,, \qquad \sigma \in R \\ & \pi^T(h - Tx) \leq \theta \,, \qquad \pi \in V \end{aligned}$$

where V and R are respectively the set of vertices and extreme rays of $\{\pi : \; \pi^T W \leq q^T\}$.

The strategy followed in the Benders Decomposition algorithm consists in solving the *master* and *slave* programs iteratively. The optimal first stage variables of the *master* are passed as parameters to the *slave* whose solution gives a cut to the *master* (with the optimal dual variables or an extreme ray). Actually, the algorithm aims at discovering both the domain and the shape of the function $V(x)$, which is piece-wise linear, by adding cuts to the *master*. The *master* at iteration k is then a relaxation of the latter problem and has the form :

$$\begin{aligned} z_k = & \min_{x} \quad c^T x + \theta \\ & \text{s.t.} \quad Ax = b \\ & \sigma^T(h - Tx) \leq 0 \,, \qquad \sigma \in R_k \subseteq R \\ & \pi^T(h - Tx) \leq \theta \,, \qquad \pi \in V_k \subseteq V \end{aligned}$$

At each iteration the knowledge of the *master* about the function increases thanks to the addition of either a feasibility cut, if the *slave* is infeasible (R_k is extended), or an optimality cut (V_k is increased). Moreover, the solution of the *master* gives a lower bound on the optimal cost (relaxation) and an upper bound can be obtained as $c^T x_k^* + V(x_k^*)$, where x_k^* is the optimal solution of the *master* at iteration k. The iterative process is terminated when the normalized difference between the bounds is below a chosen accuracy. It can be shown that Benders Decomposition, as described above, converges in a finite number of iterations to the optimal solution if the formulation of the *slave* problem is convex considering all variables (second-stage and first-stage).

Benders Decomposition can be applied to the TEP problem by dividing it into the investment and operation problems. The first stage decisions are then the integer variables associated to the construction of the new lines and the second stage variables correspond to the continuous operational variables (power flows and voltage angles). The objective function of the *slave* problem usually includes load curtailment cost [41, 2, 69, 70]. It can be noted that if the load curtailment is not constrained to be 0, the *slave* problem is always feasible and therefore no feasibility cuts will be produced [69]. However, if this requirement is considered and no other operational cost is involved, then only feasibility cuts are produced by the *slave* program [71].

It can be noticed that Benders Decomposition can be helpful to solve an optimization problem when the following conditions are true [68, 41]:

1. If the second-stage constraints are ignored, then the problem is easy to solve (the *master* is then solved efficiently).

2. If the first-stage variables are fixed then the subproblem is easy to solve (the *slave* is then solved efficiently).

3. The two subproblems have a different nature : the separation allows then the use of different and specific methods for each of them.

In the case of the TEP problem, the conditions are met. Indeed, if a DC model is used, once the first-stage decisions are fixed, the *slave* problem is linear and can then be solved efficiently with a well-known linear programming method. The *master* still belongs to the class of the MILP problems (but easier) and can then be solved with a relevant method such as Branch-and-Bound [70] or Branch-and-Cut [41].

However, Benders Decomposition is guaranteed to converge to a global optimum only when the *slave* problem is convex [21]. This condition does not hold with the DC power flow equations written as equality constraints, since they imply products of the phase angle variables with the decision variables (equation 1.12).

For the above reason, the preferred flow model for applying Benders Decomposition to the TEP seems to be the disjunctive DC power flow model [41, 24, 2]. However, one must be careful with the value assigned to the disjunctive parameter *big-M*. Indeed, it should be large enough so no further constraint is added, however, setting it too high implies numerical instabilities [24] and affects convergence [2]. The authors in [55], [2] and [49] suggest different strategies to choose adequate values.

Even if the discussion about the big-M is set aside, Benders Decomposition might require many iterations to converge [2]. Moreover, the *master* program is still an MILP problem and so might require long computational time [21] (up to 88% of the computational time of the algorithm [41]). In order to overcome these drawbacks, many strategies have been implemented. Several of them are presented in the papers [41], [71], [24] and [2].

2.2 Heuristics

A heuristic method aims at finding good solutions by following a set of defined guidelines. The obtained solution is not necessarily the optimal one and in general its presumed optimality is impossible to prove. The evaluation of the optimality gap might also be out of reach [21].

The constructive heuristics constitute a common class encountered in the literature. These kinds of heuristics construct the solution step-by-step, usually guided by some sensitivity index [15]. This index can be defined in different ways.
One can consider the *potential benefit* (π_{mn}) associated to a *susceptance change* as

$$\pi_{mn} = (\pi_m^d - \pi_n^d)(\theta_m - \theta_n)$$

where π_m^d is the dual variable of the power balance constraint [20]. It can be argued that dividing this potential benefit by the investment cost of the line will give a better insight on the financial aspect [22, 48, 72, 19] if the expansion is done for economical reasons.
Another option consists in basing the index on the value of the continuously-relaxed integer decision variable, associated to the construction of the lines, in the optimal solution of a relaxed transportation model [15, 13].
The index can also be defined with the dual variable of the maximum flow constraint [54].
Others indices may be used and a combination of several of them is also encountered [53]. It should be noted that in all cases presented above, the assessment of the sensitivity index requires

to solve an LP relaxation of the TEP problem.

Two common constructive algorithms are presented below.

One of the simplest heuristics applied to the TEP problem is the Constructive Heuristic Algorithm (CHA). It consists in iteratively solving an LP problem to determine the sensitivity index and then adding the most attractive candidate transmission line. After the constructive phase (when there is no more load shedding), the added lines are sorted according to their cost and the removal of the most expensive ones are tested successively (the feasibility of the system is checked with an LP problem) [73].
The forward search method can be identified as a variant of the CHA, however more computationally demanding. It consists in assessing the impact of adding each line of the candidate list (one at a time) and then effectively selecting the best one. Once the line is added, the process is repeated iteratively with the remaining transmission lines. The backward search method has the same underlying idea but starts with all the lines included in the network and then successively removes a line at each step [74].

Another common heuristic is the Greedy Randomized Adaptive Search Procedure (GRASP) for the Transmission Expansion Planning [20]. The construction process is similar to the one conducted by the CHA. However, instead of picking the most attractive line, the added circuit is chosen randomly in a list of candidate lines with the best index (a bias can also be included, for example linear, to increase the greedy character of the algorithm [19]). It also includes a removal phase but then it furthermore involves a local search. This last phase looks for a better solution in the neighbourhood of the current solution. The neighbourhood is defined as all the solutions that are within a given number of swaps of lines (one is removed but another one is included). The exploration might be a difficult task but there exist techniques to avoid exploring the whole neighbourhood and therefore save computational time [20].

Heuristic methods do not usually require an excessive amount of computational work (of course it depends on their complexity) and provide good feasible solutions [21]. Therefore, a heuristic technique might be used to provide a good starting solution for a more complex algorithm, for example a meta-heuristic [15, 72, 75], or to reduce the list of candidate lines [22, 55, 54, 52]. Constructive heuristics such as the CHA can also be used to remove the infeasibility of the solutions generated in the process of more refined methods (e.g. meta-heuristics) [76]. Heuristics can also provide upper bounds that may be useful for the convergence of MILP algorithms such as Branch-and-Bound [19, 54].

2.3 Meta-heuristics

A particular kind of heuristic methods will now be discussed : the meta-heuristics. Etymologically the prefix *meta* comes from an Ancient Greek word and means *beyond* (must be understood as high-level) [77]. They are problem-independent algorithms used to tackle challenging programs [77]. They are based on analogies which come from physics or nature [78]. However, this definition is not well admitted and the border with other heuristics is somehow fuzzy. It can be mentioned that the GRASP presented above is usually classified as a meta-heuristic method [12]. The strengths of meta-heuristics traditionally rely on their flexibility. They do not require some particular properties of the function such as differentiability or continuity. However, this could also be seen as a weakness since they do not take the specific structure of the problems into account. They usually include some mechanisms to avoid local minima (but cannot guarantee optimality either).

The meta-heuristics can be gathered in different categories that are discussed below.

2.3.1 Evolutionary algorithms

Evolutionary algorithms (EA) apply procedures based on analogies with biological evolution. They therefore include operations such as reproduction (with selection), mutation and cross-over or recombination [30]. They start from a population of individuals and through evolution mechanisms improve their fitness (similar to an objective function).

Genetic Algorithm

One of the most commonly encountered meta-heuristics in the literature is the Genetic Algorithm (GA) (e.g. [18, 27, 32, 44, 79]). It traditionally includes all three of the usual evolution mechanisms on a population formed with chromosomes (expansion plans for the TEP) where the genes represent the integer decisions. There exist different methods to code the transmission lines into genes : binary or decimal (considering the number of cables on a corridor all at once) [61].

Starting from a random population, the chromosomes will be reproduced based on a selection criterion. Several techniques can be considered [21, 80]. With the action of the selection operator, the reproduction is an elitist mechanism. The aim of the reproduction operator is to save the most promising individuals while keeping the size of the population constant [33].

The crossover operator selects then randomly pairs of chromosomes, according to a parameter called crossover rate and then, for each, exchanges their gene of a random position. The mutation operator aims at preventing the loss of diversity [21] by randomly modifying genes with small probability (mutation rate).

The traditional stopping criteria include the termination when a certain number of iterations has been reached or a given number of successive generations without improvement for the best fitness found in the population [21]. Those are standard stopping criteria for a large majority of meta-heuristics.

Several parameters were mentioned above and the choice of their values might have a significant impact on the convergence.

The treatment of the constraints might constitute another subject of great attention. Different approaches can be considered. One can decide to penalize the violation of constraints in the fitness or not reproduce the infeasible plans [27]. In [79], it is suggested to penalize only Kirchhoff's voltage law while the other constraints should be satisfied.

The GA and other meta-heuristic techniques are usually well suited for parallel computing. The parallelization of the evolution of independent individuals can indeed decrease significantly the computational time [81].

Other algorithms

There exist other evolutionary algorithms that have been applied to the TEP problem. One can mention Differential Evolution (DE) [36, 45, 72] and Artificial Immune System (AIS) methods [72, 30]. Both methods will be described in part III. There also exist hybrid algorithms, using sensitivity indices specific to the problem [82].

2.3.2 Swarm Intelligence

Swarm Intelligence (SI) methods consist in techniques where the search of good solutions is ensured by the interactions of simple agents inside the population and with the environment [30].

Particle Swarm Optimization

The Particle Swarm Optimization heuristic (PSO) is inspired by the way birds fly in flocks. Indeed, a synchronization of their movement can be observed despite an apparently independent behaviour [30]. The analogy with nature links the different expansion plans (vector of integer decisions) with the positions of the particles in the swarm. In the algorithm, the velocity of a particle is updated with respect to its own velocity, its best position so far and the best solution found by all the particles in the swarm. Concretely, the updating formula is the following :

$$v_i^{t+1} = \omega v_i^t + c_1 r_1 (P_i - x_i^t) + c_2 r_2 (P_g - x_i^t)$$

where v_i^t and x_i^t represent the velocity and position of the particle i at time t, P_i, its best position so far, P_g, the global best position, c_1, c_2 are positive constants, r_1, r_2 are random numbers (distributed according to a standard normal in [30]) and ω is the inertia chosen as a decreasing function of time [30, 28, 43] or constant [28]. The position of the particles is then updated by simply adding the velocity to the current one (as in a uniform motion lasting one unit of time).

The population size is an important parameter for the convergence and the computational burden of the algorithm.

Other algorithms

There exist many meta-heuristics that are part of the class of Swarm Intelligence algorithms and that have been applied to the TEP problem. One can briefly mention Ant Colony Optimization (ACO) [48, 30], Monkey Algorithm (MA) [16], Social Spider (SS) [83], Shuffled Frog Leaping Algorithm (SFLA) [37], Fireflies Algorithm (FA) [84] or Fireflies Optimization (FO) [85] and Artificial Bee Colony (ABC) algorithm [86].

2.3.3 Other meta-heuristics

Simulated Annealing

Simulated Annealing (SA) algorithms are inspired by the real process of annealing in solids. They then involve a cooling process with transitions between different configurations which represent the investment decisions, for this problem.

The basic transition mechanisms include adding a line, swapping two lines (one enters the plan and one gets out) and removing a line. With these basic movements, sequences (such as *add-swap-remove* and *remove-swap-add* [87]) can be created. Those operations are performed sequentially and the sequence is carried on only if the configuration created at one stage is not accepted. A better configuration is always accepted while a worse one is so with a probability depending on the objective gap with the current one and on the current temperature, according to Boltzmann's distribution[2] [87]. Therefore, the probability of transition decreases when the system cools down.

The algorithm consists then in iteratively decreasing the temperature (in a geometric manner) and, at each level of temperature, a given number of sequences are tested, leading to transitions. The initial temperature is chosen such that the probability of transition is high [87]. The number of transitions combined with the decreasing rate, should provide the time for the system to reach

[2] exponential dependence on the objective gap and the inverse of the temperature

thermal equilibrium at each level. The analogy with physics continues since, as the defect-free crystals have more chances to come out from a slow cooling process, so does a good optimal plan from the algorithm. The algorithm can be stopped after a given number of temperature levels or if the objective does not improve for several successive iterations [87]. The number of sequences tested is typically set as a function of the number of candidate lines (for example equal to this number [75]) or varies geometrically throughout the process.

Other methods

There exist other meta-heuristics inspired by diverse mechanisms. For example, Harmony Search can be mentioned, inspired by jazz musicians that search for the perfect harmony through improvisation. This meta-heuristic has also been applied to the TEP problem ([88, 17]). Tabu Search (TS) is a well-known meta-heuristic inspired by mechanisms of human memory [78]. It has been considered to solve the TEP problem as well [79, 89].

Part II

Offshore grid: wind farms in the North Sea

Introduction

After presenting the literature review, the main core of the work will now be tackled. As mentioned earlier, this master's thesis is based on two practical cases. The first case is an offshore grid in the North Sea and will be treated in this part.

The problem consists in designing a transmission network to connect optimally the offshore wind farms to onshore connection points where the power could be routed to the consumption sites. It is illustrated with figure 1 below.[1]

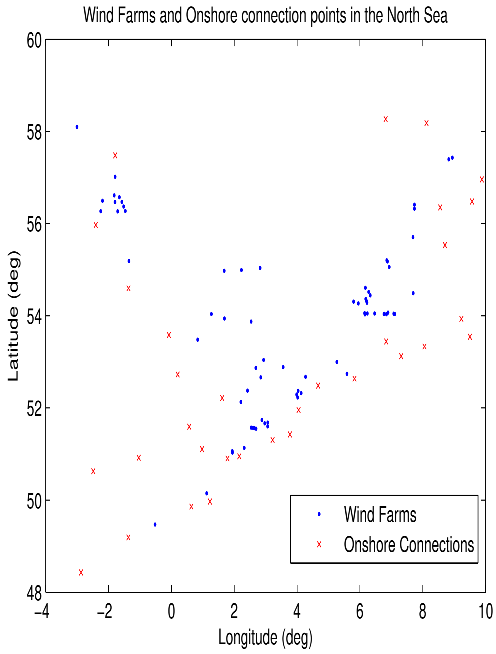


Figure 1: Nodes of the problem

The task consists then in finding the best connections between the blue dots and the red crosses through, possibly, other nodes.

Although there already exist lines in the North Sea, the problem is supposed to be a greenfield project, given the scale of the forecasted increase in powers, comparatively to the current power levels. Hence, the network has to be built from scratch. Moreover, the nodes are not defined beforehand. The planner could indeed imagine building substations (theoretically) everywhere in the North Sea, in order to aggregate the power of multiple wind farms before reaching the onshore stations. This is, of course, associated to a cost.

[1] The reader should note that this figure is just illustrative and is not up to scale. Since the dots represent locations on Earth, the figure should be more and more contracted for the upper part, since the distance between two longitudes will then be lower.

The differences with the general formulation of the TEP problem are mainly due to the two properties listed above. The possibility of substation creation, increases further the complexity of the problem since new binary variables have to be associated to each node. Since a continuous space for substation creations is considered, the set of nodes is not a priori defined. A first task may then consist in determining a reduced, but meaningful, set of considered nodes.

Moreover, since there is no existing lines, one's knowledge about the network is really limited. For example, no information can be obtained from a congestion analysis. Therefore, the candidate list of circuits is not a priori defined either, and every possible connection between the nodes could then be taken into account. It should also be noted that different types of cables (with different properties) are considered. This multiplies further the number of decision variables.

For the reasons listed above, the problem of the offshore grid in the North Sea, is highly combinatorial and so constitutes a real challenge.

Besides the increased computational burden, another difference with the traditional model is the type of cables used to transport the power. Indeed, HVDC lines are considered instead of HVAC ones. A section in the *Literature Review* (part I) is dedicated to these lines and so the reader is invited to refer to it (section 1.9).

For this problem, only investment costs are considered. Moreover, the model handles the extreme case. This means that the network should be able to withstand the maximum power generated at the wind farms. Perfect foresight of the future generation is then assumed and no operational costs with different scenarios are taken into account. The practical example is based on the expected power generations in the North Sea in 2030. Therefore, in order to decrease the computational burden, only a static version of the problem will be treated.

The model tackled for this problem will first be presented. Afterwards, a first approach based on a systematic meshing of the space will be analyzed and discussed. However, this approach is outperformed by the proposed method that will be described in details. The reader who does not have time to read the whole text is then invited to skip chapter 2 and focus on chapter 3. This part will end with a statistical analysis in order to assess the quality of the proposed method and its generalization.

Notations

The needed notations to define the model tackled in this part are presented in this section.

Sets

- W : set of farms.
- S : set of onshore connection points.
- N : set of nodes.
- E : set of edges - possible connections between nodes: $(i, j) \in E$ if there is a possible connection between nodes i and j. Hence $E \subset N \times N$.
- C : set of available cables with given capacity, cost and resistance.

Discrete variables

- x_{ijc} : integer positive variable - number of cables of type $c \in C$ that will be built on edge $(i, j) \in E$.
- y_n : binary variable - deciding if a substation will be built at node $n \in N$.
- z_{ij} : binary variable - deciding if at least 1 cable will be built on edge $(i, j) \in E$.

Continuous variables

- P^f_{ij} : continuous variable - active power flow in connection $(i, j) \in E$ (pu).
- P^l_n : positive continuous variable - power evacuated at node $n \in N$ (pu).
- V_n : continuous variable - voltage at node $n \in N$ (pu).

Parameters

- P^g_n : power generated at node $n \in N$ (pu).
- P^{max}_n : maximum power evacuated at node $n \in N$ (pu).
- w_n : binary parameter defining if the node $n \in N$ corresponds to a wind farm.
- o_n : binary parameter defining if the node $n \in N$ corresponds to an onshore station.
- L_{ij} : Length of connection $(i, j) \in Edges$ (pu).
- Cap_c : maximum capacity of cable $c \in C$ (pu).
- R_c : resistance of cable $c \in C$ (per unit of length).
- C_c : cost of cable $c \in C$ (per unit of length).
- SC : substation cost.
- V^{max} : maximum voltage.
- V^{min} : minimum voltage.
- M : sufficiently high constant.
- $\tilde{M}$: sufficiently high constant.

Chapter 1

Model and bounds

1.1 Model formulation

In this section, the model will first be described. Some remarks on the formulation will then be discussed. First, it should be noted that in order to solve the problem, a finite set of nodes N must be defined. This set is then not a variable of the model described below.

$$\min_{x,y,z,P^f,P^l,V} \qquad \sum_{(i,j)\in E} \sum_{c\in C} x_{ijc} \cdot L_{ij} \cdot C_c \quad + \quad \sum_{n\in N} y_n \cdot SC$$

subject to

$$\sum_{(i,n)\in E} P^f_{in} - \sum_{(n,j)\in E} P^f_{nj} + P^g_n - P^l_n = 0 \qquad \forall n \in N \tag{1.1}$$

$$P^f_{ij} \leq \sum_{c\in C} x_{ijc} \cdot Cap_c \qquad \forall (i,j) \in E \tag{1.2}$$

$$-P^f_{ij} \leq \sum_{c\in C} x_{ijc} \cdot Cap_c \qquad \forall (i,j) \in E \tag{1.3}$$

$$P^l_n \leq P^{max}_n \qquad \forall n \in N \tag{1.4}$$

$$\sum_{(i,n)\in E,\ c\in C} x_{inc} + \sum_{(n,j)\in E,\ c\in C} x_{njc} \ \leq 1 + M \cdot y_n \qquad \forall n \in N:\ o_n = 0,\ w_n = 1 \tag{1.5}$$

$$\sum_{(i,n)\in E} z_{in} + \sum_{(n,j)\in E} z_{nj} \ \leq 2 + M \cdot y_n \qquad \forall n \in N:\ o_n = 0,\ w_n = 0 \tag{1.6}$$

$$\sum_{c\in C} x_{ijc} \leq \tilde{M} \cdot z_{ij} \qquad \forall (i,j) \in E \tag{1.7}$$

$$P^f_{ij} - \sum_{c\in C} \frac{x_{ijc}}{R_c}(V_i - V_j) = 0 \qquad \forall (i,j) \in E \tag{1.8}$$

$$V_n \leq V^{max} \qquad \forall n \in N \tag{1.9}$$

$$V_n \geq V^{min} \qquad \forall n \in N \tag{1.10}$$

$$P^l_n \geq 0 \qquad \forall n \in N \tag{1.11}$$

$$x_{ijc} \text{ integer and } \geq 0 \qquad \forall (i,j) \in E \tag{1.12}$$

$$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in E \tag{1.13}$$

One difference with the formulation found in the *Literature Review* (part I, section 1.1) is here the absence of existing circuits. Nothing is built beforehand and almost everything has to be decided.

The objective function aggregates the cost of line installations and the cost of creation of substations.
Equality 1.1 is the balance constraint at each node.
Constraints 1.2 and 1.3 limit the absolute value of the active power flow on the edges. The reader can note that instead of one power flow per cable, only one variable (and so one constraint) is defined for each edge, collecting the power flows from all the cables built in parallel on this connection.
Constraint 1.4 limits the power evacuated at each node. If the concerned node is not an onshore station, this power is 0. Otherwise, it is limited to the maximum capacity of the onshore station. Constraint 1.11 ensures the positivity of this variable.
Constraints 1.5 and 1.6 make sure the substations are built when needed. The rule normally imposes that a substation is built when at least 2 cables are connected to a particular node. This is the rule applied at the wind farms (1.5). However, no substation has to be built at onshore connection points. Furthermore, this rule does not hold for new (added) nodes. Indeed, in this case, a substation should be built if 3 connections arrive at that node (as in 1.6). Otherwise, this node is just virtual and nothing changes if it is neglected. This rule is illustrated by the examples presented with figure 1.1 below.

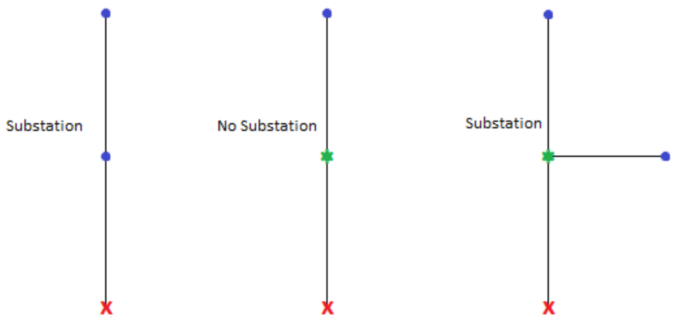


Figure 1.1: Illustrative examples of the substation rule

The blue dots represent wind farms, the red crosses, onshore connection points and the green stars, additional nodes. The lines represent single cables. A substation has to be built when powers are aggregated. Hence, a substation needs to be built at the central node on the left side and on the right side. However, no substation is required for the example in the middle, since the central node is only a virtual one (no aggregation of power and no change before and after). This affirmation still stands if multiple cables are considered. This justifies the use of binary variables.

The definition of the binary variable z is ensured by constraint 1.7 and its binarity by constraint 1.13.

Constraint 1.8 is a formulation of Kirchhoff's voltage law for DC circuits. This expression could be linearized with a disjunctive parameter (similarly as it is done for the AC lines). However, the integer decision variables associated to the cables should then be expressed with binary variables. This would lead to an increase of the computational burden. Since the voltage law is less significant in the case of a DC grid, this constraint will simply be neglected in the rest of this work. This implies that the constraints 1.9 and 1.10, limiting the voltage, can be forgotten, as well as the variable V_n.

Although the name and meaning of the variables and parameters are a little different from the model presented in part I (section 1.1), it can be noticed that some constraints are quite similar. Instead of having a fixed consumption and a variable generation, the roles are reversed with an extreme case for the generation and limitations on the power evacuated. This network can then be seen as a traditional one but with an opposite direction to the power. Since the cable properties are independent of the direction (*isotropic*), it does not bring much change to the model.

The data include the geographic locations of the wind farms and the onshore connection points (latitude and longitude). Hence, a formula must be defined to compute the distances between the different nodes of the network.

In this work, the *haversine formula* is chosen. This formula computes the great-circle distance between two points on a sphere. It is used to compute a distance d on the surface of the Earth and is defined as follows [90]:

$$\boxed{d = 2R \cdot \arcsin \sqrt{\sin^2\left(\frac{\mathrm{Lat}_1 - \mathrm{Lat}_2}{2}\right) + \cos(\mathrm{Lat}_1) \cdot \cos(\mathrm{Lat}_2) \cdot \sin^2\left(\frac{\mathrm{Long}_1 - \mathrm{Long}_2}{2}\right)}}$$

R is the radius of the sphere. Hence, the radius of the Earth should be used. However, the Earth being not a perfect sphere, its radius varies from 6356.7523 km (polar radius: b) to 6378.1370 km (equatorial radius: a) [91]. In this work, a mean value will be chosen, defined as $R = \frac{2a+b}{3} = 6371.0088$ km [91] as in [90].

1.2 Bounds

1.2.1 Lower Bound

When a minimization problem is tackled, it is interesting to look for a lower bound in order to be able to assess the quality of the solutions found. Furthermore, this makes it possible to give a limit to what can be achieved in terms of cost. The tighter the bound, the more information it gives about the problem. If a good bound is known, it might be a decisive argument to decide whether it is meaningful or not to carry on the search for a better solution.

Unfortunately, the problem being highly complex due to the large degree of freedom, finding a tight lower bound seems out of reach. However, thanks to a few simplifications and approximations, it is possible to obtain a true bound on the whole problem, independent of the candidate list of both nodes and edges.

The modifications and definitions added to the model are listed below.

- The set of nodes is defined as the union of the wind farms and the onshore connection points
  $\Rightarrow N = W \cup S$
- The set of edges is defined as the direct connections between the wind farms and the onshore connection points.
  $\Rightarrow E = \{(i, j) : \ i \in W \ , \ j \in S\}$.
- Relaxation of the integrality of the variables x: the number of cables built becomes a continuous variable and so every capacity is reachable.
- The substation cost is neglected and therefore the binary variables z are not useful anymore.
- The costs of the cables is linearly under approximated with respect to their capacity.

Before writing down the problem solved with these modifications, a few new notations have to be introduced.

- x^r_{ij} : continuous positive variable - capacity installed on edge $(i, j) \in E$.
- CC : continuous cost of the cables (per unit of capacity).

With the help of these new notations and the previous ones, the problem can then be described as follows:

$$\min_{x^r, P^f, P^l} \qquad \sum_{(i,j)\in E} x^r_{ij} \cdot L_{ij} \cdot CC$$

subject to

$$P^f_{ij} \leq x^r_{ij} \qquad \forall (i, j) \in E \tag{1.14}$$

$$-P^f_{ij} \leq x^r_{ij} \qquad \forall (i, j) \in E \tag{1.15}$$

$$x^r_{ij} \geq 0 \qquad \forall (i, j) \in E \tag{1.16}$$

$$\text{and} \qquad 1.1, \; 1.4, \; 1.11$$

The linear approximation of the cost of cables still needs to be properly defined. Figure 1.2 presents the costs of the cables with respect to the available capacity.

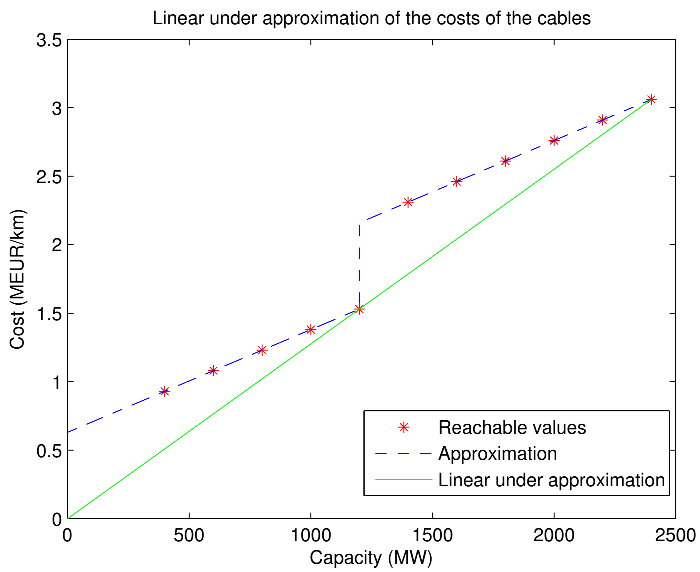


Figure 1.2: Linear under approximation of the costs of cables

It shows the reachable capacities with the 5 different available cables (red stars), the continuous approximation (dashed blue line) and the linear under approximation (green line). The reader can see that this linear cost is always lower or equal to the true cost of the capacity implemented with the available cables. Taking the continuous variables (x^r) with this linear cost (CC), instead of the integer variables (x) with the true costs (C_c) will then only decrease the cost which is the wanted objective in the search of a lower bound.

Moreover, it is obvious that neglecting the substation cost can only decrease the total cost as well. However, one might be concerned about the restriction of the nodes and the edges. The reader might wonder if it is not possible to achieve a lower cost by considering, for example, connections between the farms, or additional nodes. It is nevertheless not the case because of the linear approximation. Indeed, since the cost of cables is linear with respect to the capacity, there is no more gain in aggregating power. In other words, building 2 lines of capacity p or 1 line of capacity $2p$ have the same cost. Therefore, the problem comes down to finding the shortest path from the wind farms to the onshore connection points, which are (by the triangular inequality) the direct ones.

It can be noticed that the problem, as described above, is a linear program. It can then be solved efficiently with a solver such as CPLEX (less than 1 second).

Figure 1.3 shows the plan obtained by solving this problem. The presence of cables in the figure does not give any indication on the capacity installed, but simply that it is not zero. The reader can note that some farms are connected to more than one onshore connection points. This is due to the limitation on the maximum power evacuated at these points which can then be saturated.

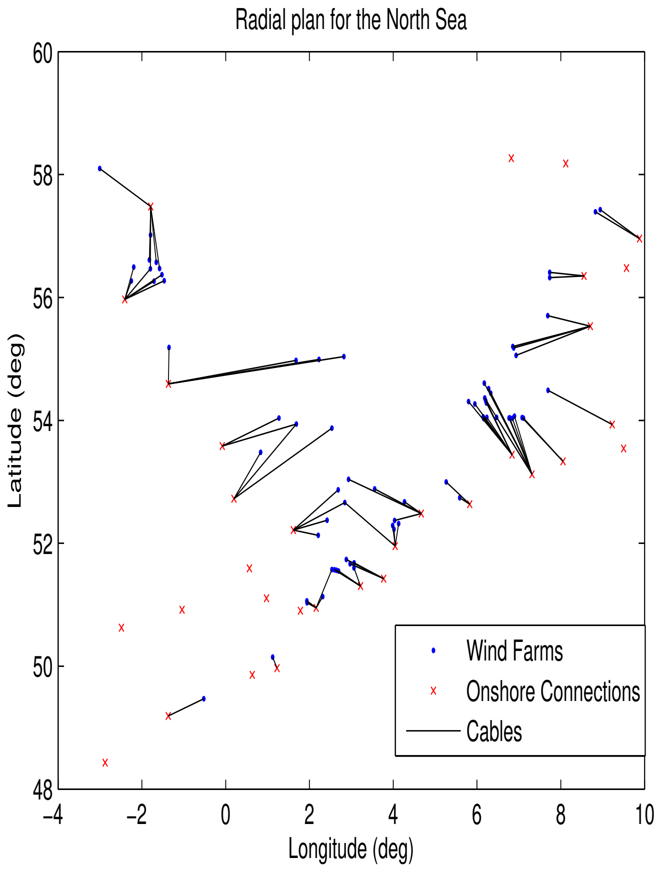


Figure 1.3: Plan with radial connections

The lower bound for this problem is then **4931** MEUR. However, this bound is unreachable since the capacities installed in the obtained plan are not reachable with the available cables.

1.2.2 Upper Bound

Based on the relaxed solution found for the lower bound, a feasible solution for the problem can easily be built. It is indeed sufficient to collect the different capacities installed on each connection and implement a plan to reach at least these capacities with the available cables.

To this end, the easy problem below can be solved.

$$\min_{x} \qquad \sum_{(i,j)\in E} \sum_{c\in C} x_{ijc} \cdot L_{ij} \cdot C_c$$

subject to

$$\sum_{c\in C} x_{ijc} \cdot Cap_c \geq x^r_{ij} \qquad \forall (i,j) \in E \tag{1.17}$$

$$x_{ijc} \text{ integer and } \geq 0 \qquad \forall (i,j) \in E \tag{1.18}$$

The reader can note that the variable x^r is here passed as parameter. The cost associated to substations must then be added to the objective function. This problem belongs to the class of MILP but is almost trivial and so is solved quickly (less than 1 second).

This plan costs **8293** MEUR and will be used to assess the quality of the solutions found by the methods implemented in the rest of this work. It will be referred to as *radial plan* since it only allows direct connections from the wind farms to the onshore connection points.

Chapter 2

Mesh

The approach developed in this chapter is based on a systematic meshing of the space. It is outperformed by the proposed technique in chapter 3. The reader who is only interested in the main method is then invited to skip the current chapter to focus on the next one.

2.1 Description

Since the set of nodes of the network is not determined a priori, the first proposed methodology consists in defining the locations of the nodes systematically. One can then construct a regular mesh and consider connections between the neighbour nodes of the grid.

Several types of mesh could be imagined to achieve this purpose. In this work, two of them were considered: *rectangular* and *hexagonal*.

The *rectangular* mesh is created by placing nodes along the meridians and parallels with a constant step length in terms of latitudes or longitudes. It should therefore be noted that the points are not spaced with the same distance in the two directions. Moreover, since 1 degree in longitude corresponds to different distances, according to the latitude, the spaces in that direction (*East-West*) do not have a constant length [1]. In this work, the same number of nodes are placed along both directions. Moreover, the connections between each node and its 8 closest neighbours are considered for line installation.

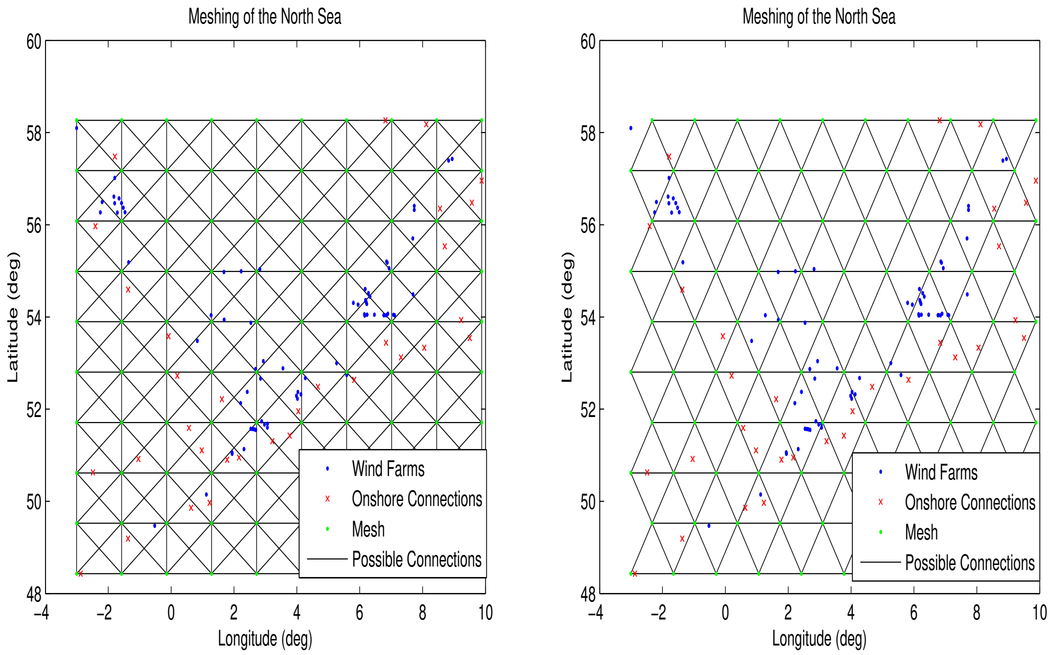
This type of mesh is illustrated with figure 2.1a. This figure shows the nodes of a rectangular mesh of size 10×10 and all the possible connections that can be considered. This mesh is quite coarse regarding the dimensions of the North Sea. However, it already implies a lot of integer variables. The reader can check that for a grid of size $n \times n$, the rectangular mesh with 8 neighbours considers $2 \cdot (n-1) \cdot (2n-1)$ edges. Hence, with 5 different types of cables, the problem includes

- 1710 integer variables for the lines (variables x)
- 342 binary variables for the lines (variables z)
- 100 binary variables for the substations (variables y).

The *hexagonal* mesh is implemented similarly but considers connections between each node and its 6 closest neighbours. It is illustrated with figure 2.1b.

The two different meshes have been tested to solve the problem and the rectangular mesh appears to outperform the hexagonal one in terms of obtained costs, without a significant increase in computational time. Therefore, only the former will be addressed in this report.

[1] The latitudes vary from 48.4300 to 58.2669 and so the relative difference is up to 20.73%



(a) Rectangular mesh

(b) Hexagonal mesh

Figure 2.1: Comparison of the set of edges with the two different meshes of size 10×10

2.2 Standard model

The model formulated in the beginning of this part can be applied directly to the grid created with the rectangular mesh. The nodes of the network are then the nodes of the mesh and the edges, the defined connections (between neighbours). The wind farms and the onshore connections are then not part of the set of nodes N but they can be associated to their closest node. The power generated or evacuated is then directly linked to this node.
However, these additional links between the farms/onshore points and the nodes might create substations, at these nodes, which are not taken into account with the model described in chapter 1. The constraint associated to the substation (1.6) must then be updated to include those extra connections. It can be done by adding the number of farms and onshore stations (nb_n) associated to the node $n \in N$ to the left-hand-side of the inequality. The constraint becomes then the following:

$$\sum_{(i,n)\in E} z_{in} + \sum_{(n,j)\in E} z_{nj} \; + \; nb_n \leq 2 + M \cdot y_n \qquad \forall n \in N \tag{2.1}$$

Once the problem is solved, the cost of linking the farms and onshore connection points to the nodes has to be added. This cost is obtained straightforwardly. Furthermore, substations have to be built at the farms whose connection to their node require more than 1 cable.

Figure 2.2 presents the expansion plan obtained by solving the problem with the rectangular mesh of size 10×10. It was solved with the default options of CPLEX. Again, the lines represent the presence of at least 1 cable but do not give any information on their number.

This problem was solved in 29 seconds. The costs obtained are presented in table 2.1. They are expressed in MEUR and are respectively, the objective of the problem, the cost of linking the farms and onshore stations to their nodes and the total cost. The total cost is the sum of

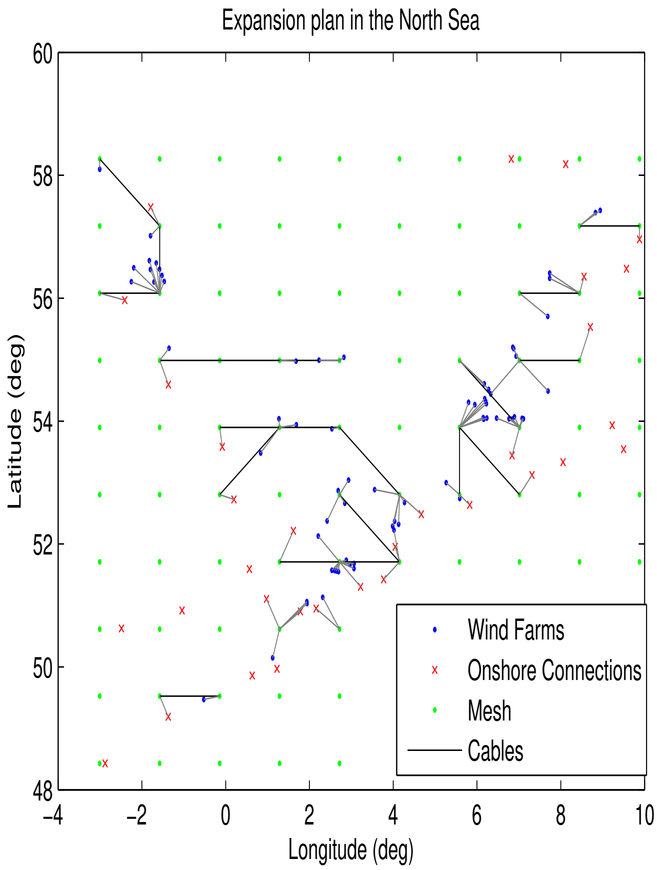


Figure 2.2: Plan obtained with a rectangular mesh of size 10×10, as described in section 2.2

the other two costs increased with the additional cost associated to the substations at the wind farms (cost of 120 MEUR).

Objective (MEUR)	Link Cost (MEUR)	Total Cost (MEUR)
6087	5218	11425

Table 2.1: Costs obtained with a rectangular mesh 10×10

The reader can notice that this plan is highly inefficient since it is 38% more expensive than the *radial plan*. It is mainly due to the very high cost of linking the farms and onshore stations to the nodes. This cost, presented in column 2 of table 2.1, represents 46% of the total cost and is only dependent of the mesh. In order to reduce the part of this cost and so (hopefully) the overall cost, the mesh should be refined. However, it appears that this is the limit size to be able to solve the standard formulation of the problem with a classical solver (CPLEX). Indeed, with a grid of 11×11, the solver did not converge after 5 h and so was stopped (it still presented a relative integrality gap of 10^{-2}, while the default value to reach in order to stop the process is 10^{-4}). Given the poor results obtained with the mesh 10×10 and the increase in computational time when the mesh is refined a little, this approach does not seem to be the most adequate to solve the problem.

Other techniques should be considered in order to refine the mesh but keep the tractability of the problem. One method is proposed in the next section.

2.3 Refining the mesh

2.3.1 Description

The proposed method to refine the mesh and improve the solution involves two techniques: **partition** and **relaxation**.

The poor results obtained in the previous section tended to show that trying to solve the whole problem at once seems too ambitious. Moreover, defining a systematic mesh on the whole space might not be the most clever method, since the different regions are then treated similarly, whether they include farms/onshore points or not.
The presented method of this section, first allocates the farms/onshore points into distinct groups and then solves the problem for each one of them independently, with a more refined mesh of the specific region covered by the group.

The **partition** is made with the help of the *radial plan* presented before (fig 1.3). This plan generates naturally groups distinct from each other. Indeed, the farms and onshore points that are linked in this plan can be grouped together. The defined groups have then a reduced size and the reader can note that the power balance is respected since the partition is generated based on a feasible plan. Moreover, the allocation is meaningful since the farms are grouped with their closest (unsaturated) connection point(s).

The different groups can be solved independently and so possibly in parallel.

The tests revealed that a mesh of size 5×5 can be used for each group. This seems to constitute the limit size for the classical solver to be able to solve the problems efficiently (within a few hours) [2] .

The tests revealed that refining further the mesh gives better results. To this end a **relaxation** of the problem is considered. The modifications are listed below.

- Relaxation of the integrality of the variables x: the number of cables built becomes a continuous variable and so every capacity is reachable. However, a binary variable remains in the model to keep the economy of scale.
- The definition of the binary variables is modified.
- The costs of the cables is approximated as a fixed cost and a variable cost.

Before writing down the relaxed model, a few notations are introduced.

- FC : fixed cost for the cables.
- VC : variable cost for the cables.
- $\hat{M}$: sufficiently high constant.

The reader can recall that the continuous relaxation of the variables x has been introduced in chapter 1, section 1.1 with the variables x^r.
The relaxed version of the model can then be written as follows:

[2] The reader can notice that this mesh involves less nodes than for the whole problem. This can be explained by the fact that, with the reduced groups, a larger part of the generated nodes is meaningful, while a lot of them could be quickly discarded by the solver for the whole space.

$$\min_{x^r,P^f,P^l,z} \quad \sum_{(i,j)\in E} x^r_{ij} \cdot L_{ij} \cdot VC + z_{ij} \cdot FC \quad + \quad \sum_{n\in N} y_n \cdot SC$$

subject to

$$x^r_{ij} \leq \hat{M} \cdot z_{ij} \qquad \forall (i,j) \in E \tag{2.2}$$

$$\text{and} \qquad 1.1,\ 1.4,\ 1.11,\ 1.14,\ 1.15,\ 1.16,\ 1.13,\ 2.1$$

The definition of the binary variable z is ensured by constraint 2.2.
The substation rule is the same as the one used in the standard formulation of the problem with the mesh. Again, substation costs at wind farms have to be added if necessary.
As expressed above, the cost of the cables is replaced by a fixed cost (if non zero capacity is installed) and a variable cost (linear with the capacity). This almost corresponds to the blue approximation of figure 1.2 (without the discontinuous jump). Doing so, the economy of scale is preserved. This means that there might still be a gain in aggregating the powers instead of connecting radially the farms.

After solving the model above, the obtained plan still has to be implemented with the available cables. Indeed, the provided solution only expresses required capacities on the connections (edges) but does not provide integer number of cables. The problem described in section 1.2.2 has then to be solved. Moreover, as before, the cost of connecting the farms/onshore stations to their closest node should be added as well.

2.3.2 Results

The farms and onshore points are allocated into 16 groups. Table 2.2 presents the number of such farms and points in each group.

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
# Farms	8	2	3	4	7	1	1	16	2	4	1	2	2	4	4	11
# Onshore points	2	1	1	1	2	1	1	2	1	1	1	1	1	2	1	2

Table 2.2: Information about the different groups of the partition described in section 2.3

The refinement of the mesh is illustrated with figure 2.3. It presents the number of nodes included in the mesh(es) (if the problem is partitioned in several groups, as it is the case for the method described in this section, the number presented is the sum of the number of nodes included in the different meshes). It starts from the systematic meshing, of size 10×10, of the whole space, presented in section 2.2. The partitioning technique enabled one to create 16 meshes of size 5×5 (one for each group) and neglect the useless parts of the space. The relaxation of the model made it possible to further refine those meshes to 49 nodes (7×7) in each one of them.

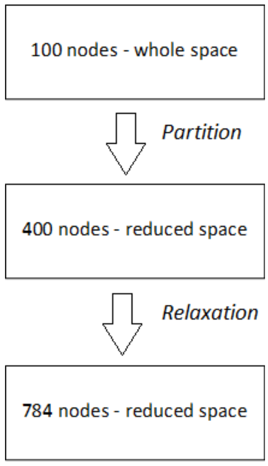


Figure 2.3: Evolution of the number of nodes included in the mesh(es) of the problem through the refinement process

Table 2.3 presents the results obtained with the relaxed model applied on the 16 groups independently. The total costs of the plans are provided for the relaxed model with a rectangular mesh of size 7×7 in each group, alongside the costs of the *radial plans* for the different groups. The reader should recall that the cost of this plan is not a lower bound for the problem but is the cost associated to the plan implementing this bound. The costs are expressed in MEUR. The computational time for the relaxed model is also given in the last column (in seconds). Each row corresponds to the group whose number is given in the first column.

	Cost - Mesh (MEUR)	Cost - Radial (MEUR)	Time - Mesh (s)
1	481	399	29
2	92	70	1770
3	184	193	33
4	516	501	8
5	848	751	5
6	74	74	2
7	23	23	2
8	1506	1991	23
9	127	188	2
10	355	415	26
11	109	109	2
12	86	94	10851
13	130	146	66
14	1290	1145	167
15	1217	1143	13
16	1080	1051	4
Total	8119	8293	13004

Table 2.3: Results of the relaxed rectangular mesh model applied on the partition described in section 2.3

Figure 2.4 presents the obtained plan.

The results show that it is possible to improve the *radial plan* by aggregating the power before reaching the onshore connection points. The relative savings over all the 16 groups is of **2 %**. However, this clustering approach enables one to choose different methods for the different groups. One could then decide to select the best plan among the two proposed (*radial* and *mesh*) for each group independently. Doing so the overall cost is then **7654** MEUR and the improvement increases up to **8 %**.

The method required around 3h30 to solve all 16 groups sequentially. This time could be decreased thanks to parallel computing since the groups are entirely independent. Nevertheless,

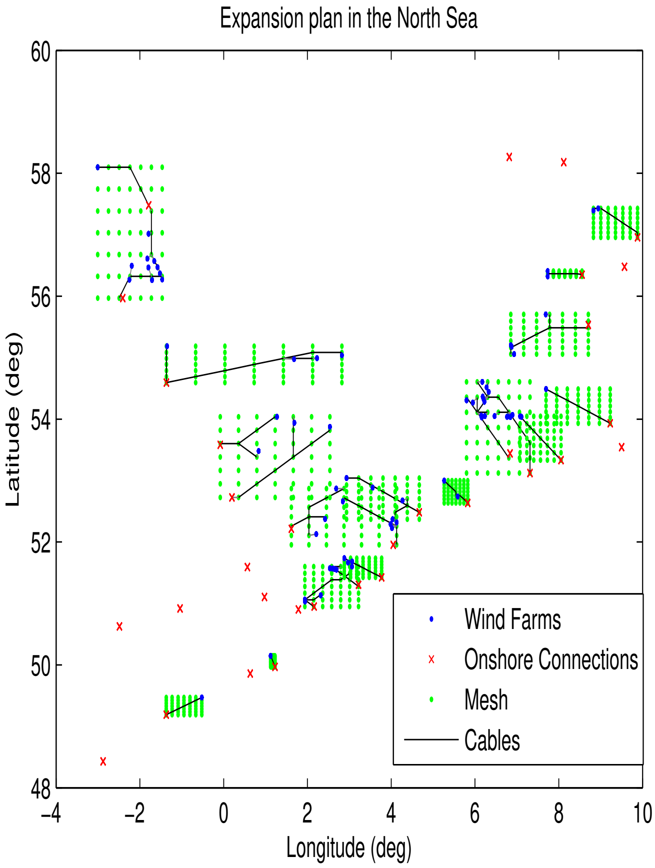


Figure 2.4: Plan obtained with a relaxed model and a rectangular mesh of size 7×7 applied on the partition, as described in section 2.3

the reader should note that the solver did not converge properly for group 12. Indeed, it was stopped by the imposed time limit of 3 hours. The relative integrality gap was then 0.2 which is much higher than the stopping criterion [3]. It is surprising that the method struggles at finding the solution for a group that only includes 2 farms and 1 connection point. But, since the same mesh size is used for every group, the number of nodes and edges are the same and so is the size of the problems.

[3]However, the next chapter will show that the gap to the optimal solution is actually much lower (the optimal solution for this group being 85 MEUR).

Chapter 3

Proposed Method

The proposed method consists in a 3-stage algorithm:

- **Partition**: based on a simple model, allocating the farms and onshore stations into groups that will be treated independently (and so possibly in parallel).
- **Definition of the network**: for each group of the partition, finding possible new substation nodes using an iterative method (heuristics).
- **Resolution** : each group can then be solved independently considering the defined network.

3.1 Partition

The mechanism is quite similar to the one described in section 2.3. However, there are a few differences.

- It is still based on the model considering only direct connections between farms and onshore points (*radial model*).
- One constraint is added: only one connection can reach each farm. It guarantees then that each group contains exactly 1 onshore connection point.

This new constraint reduces the size of the groups which are then more easily handled. Moreover, the power balance is still satisfied inside each group. The different parts can then still be solved independently. The reader can note that this constraint makes also sense since, when the integrality of the number of cables is restored, one may want to avoid multiplying the connections.

The binary variables z are then added to the problem described in section 1.2.1. These variables play a role in the following constraints:

$$x_{ij}^{r} \leq \hat{M} \cdot z_{ij} \qquad \forall (i,j) \in E \tag{3.1}$$

$$\sum_{(f,k)\in E} z_{fk} = 1 \qquad \forall f \in N \ (f \in W) \tag{3.2}$$

Constraint 3.1 defines the binary variable and constraint 3.2 imposes that exactly one connection reaches each wind farm.

The problem, as defined above, can be solved and the obtained plan is presented in figure 3.1. The problem is no longer a linear program since the binary variables z have been introduced. However, it is still an easy problem and so can be solved efficiently (2 seconds) with the solver (CPLEX).

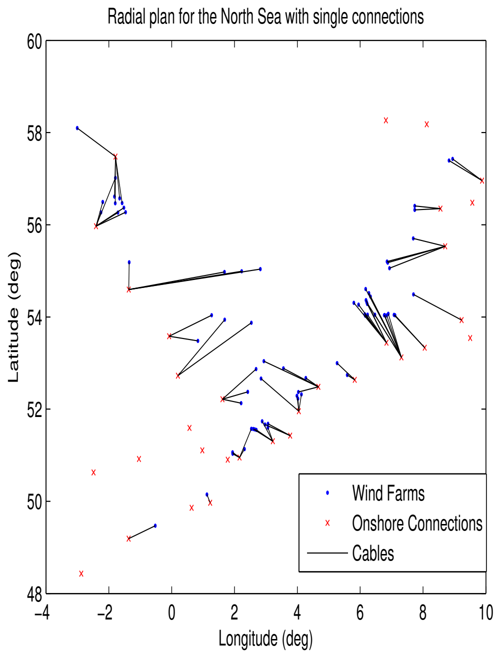


Figure 3.1: Plan with radial connections with the additional constraint of 1 connection per wind farm

As in section 2.3, this plan is the basis to cluster the farms and onshore points into groups by simply grouping the nodes that are linked together.

The number of wind farms in the different groups is provided by table 3.1. As defined with the constraint, there is only one onshore connection point in each group.

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
# Farms	5	2	4	4	2	7	9	2	1	4	2	2	4	1	1	3	2	2	4	6	5

Table 3.1: Number of wind farms in the different groups of the partition described in section 3.1

The reader can notice that the additional constraint reduces significantly the size of the largest groups (from 16 farms and 2 onshore stations to 9 farms and 1 station). The reader can refer to table 2.2 for the comparison.

3.2 Definition of the network

This section aims at defining a method that provides meaningful candidate substation nodes. This method is based on a special case which can be solved easily. This case and the way to solve it will first be described, then the method will be addressed.

3.2.1 Special case

The problem of connecting optimally two farms to one onshore station is an easier task than the general case. Indeed, one only has to find the connection point (if needed) where the substation will be built. Furthermore, this point is known to be in the triangle formed by the 2 farms and the onshore point. Under the particular assumption of equal cost for the different cables (joining the farms to the connection point or this connection point to the onshore station), the best connection point is known as the Fermat point[1] of the triangle [92] and presents some interesting

[1] solution to the problem originally raised by Fermat

geometric properties. Instead of looking for a closed-form solution, the preferred approach in this work is the resolution of the problem with a mesh that gives a satisfactory level of accuracy in a limited amount of time. A mesh can then be created in the triangle and then the point that minimizes the cost of connecting the farms to the onshore point can be identified. This is done straightforwardly. The mesh is illustrated by figure 3.2 below which applies this method to the group 11 of the partition above.

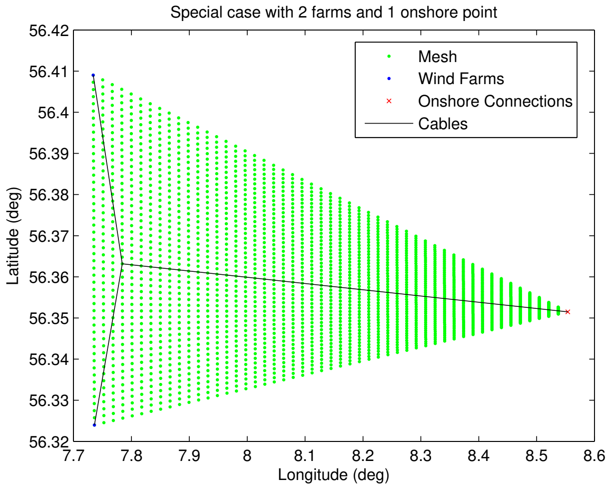


Figure 3.2: Example of the resolution of the special case

The heuristic method, defined below, in order to discover good substation candidates, is based on this case.

3.2.2 Heuristic method

The proposed method consists in treating each pair of farms, one after the other. It has been shown that the problem with 2 farms and 1 onshore station is easy and thus the optimal connection point for each couple of farms can efficiently be obtained (for n farms, there are $\frac{n \cdot (n-1)}{2}$ pairs). This statement inspired the following algorithm.

Algorithm 1: Defining network

Result: Nodes for potential substations
$Nodes = \emptyset$;
while $\#Farms > 1$ **do**
  for $i, j \in Farms$ *not yet considered* **do**
    Find best connection point with a mesh;
    Compute relative improvement;
  end
  Find (f_1, f_2) pair with best relative improvement;
  Merge them at their connection point f_{1+2};
  $Farms \leftarrow Farms \cup \{f_{1+2}\} \setminus \{f_1, f_2\}$;
  $P^g_{1+2} \leftarrow P^g_1 + P^g_2$;
  $Nodes \leftarrow Nodes \cup \{f_{1+2}\}$;
end

The best connection point, for each pair, is found with a mesh as in figure 3.2 and considering all the features of the problem (different types of available cables and substation cost). This requires a negligible amount of time. Once all these connection points are found, the different pairs are compared with the relative improvement cost. This is defined as the difference between the radial connection cost (direct connection to the onshore point) and the cost of joining the two farms to the onshore point through the connection point (including the cost of substation), divided by this radial cost.

$$\Rightarrow \text{Relative improvement} = \frac{RC - BC}{RC}$$

with RC being the radial cost and BC the cost going through the best point.

The pair that provides the highest relative improvement cost is selected and merged at its connection point (which becomes a virtual farm generating the sum of the powers of the pair) and the process is repeated with this new virtual farm but discarding the two merged farms. It stops when there is only one farm left. After each stage, there is one farm less, so the iterative process only requires $(n - 1)$ iterations (with n the number of farms). It is therefore a finite algorithm.

The new virtual farms (connection points) compose then the set of candidate substation nodes. These nodes will then be added to the network to compose the set of nodes (N) alongside the wind farms and the onshore connection points. The reader can note that since at most[2] one node is added to the set at each iteration, at most $n - 1$ nodes are added.

3.3 Resolution

The different groups can be solved independently, and so possibly in parallel, with the set of nodes defined with the above method. The set of edges is chosen to include every possible connection between two nodes in each group.

The reader can note that the problem described in section 1.1 can be solved as such in each group. However, the particular definition of the network allows one simplification. Indeed, since the direct connections from the farms to the onshore stations are considered in the set of edges, additional nodes will only be used to aggregate power from multiple farms and not as simply transit nodes. In other words, this means that the example on the left side of figure 1.1 cannot happen. Therefore, the distinction between the additional nodes and the wind farms is not useful anymore. The two constraints 1.5 and 1.6 can then be merged into one and the binary variables z can be neglected.

The problem can then be written as follows:

$$\min_{x,y,P^f,P^l} \quad \sum_{(i,j)\in E} \sum_{c\in C} x_{ijc} \cdot L_{ij} \cdot C_c \quad + \quad \sum_{n\in N} y_n \cdot SC$$

subject to

$$\sum_{(i,n)\in E,\ c\in C} x_{inc} + \sum_{(n,j)\in E,\ c\in C} x_{njc} \leq 1 + M \cdot y_n \qquad \forall n \in N:\ o_n = 0 \tag{3.3}$$

$$\text{and} \qquad 1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.11,\ 1.12$$

3.4 Results

The three-stage algorithm described has been applied on the problem of the North Sea. A mesh size of 20×20 was chosen to find the best connection points of the pair of farms (a different mesh was created for each pair). The different groups are solved with the classical solver CPLEX and its default options. The largest group of the partition includes 13 nodes and 78 edges. The

[2]The set of nodes remains the same if the best connection point is already included in it.

resulting expansion plan is presented with figure 3.3.

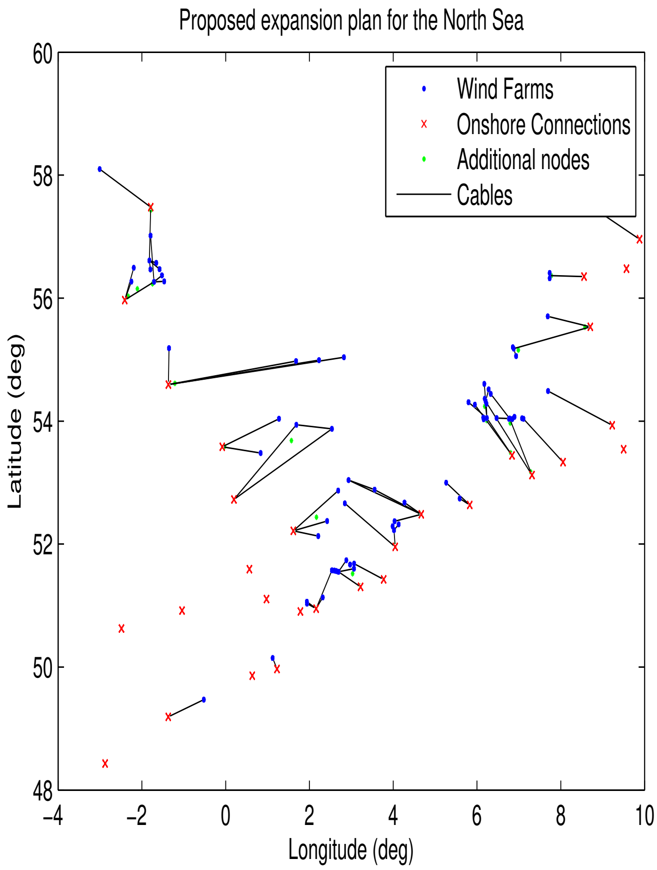


Figure 3.3: Plan obtained with the 3-stage algorithm

Table 3.2 presents the costs obtained for the different groups of the partition. These costs are compared to the ones provided by the *radial* model (refer to chapter 1, section 1.2.2) [3]. The costs are expressed in MEUR. The last column provides the computational time required by the solver for the different groups (last stage of the 3-stage method). It is expressed in seconds.

The results show that the proposed method enables the planner to make significant savings compared to the *radial plan*. The relative improvement is indeed of **18 %**. It should also be noted that the *radial plan* is a feasible solution of the problem for each group and so the proposed method is, at least, as good.

The overall running time, required by the method to solve all the groups sequentially, is around **5 minutes**. Unfortunately most of the time (96%) is dedicated to one single group and so solving the different parts in parallel will not speed up significantly the method.

[3]The reader can notice that the total cost obtained with the *radial* model is lower than the one presented before (in chapter 1, section 1.2.2). This is due to the additional constraint, imposing at most one connection for each farm, that creates another partition of the nodes. These savings confirm the intuition that multiplying the connections, in a relaxed model, has a high probability to decrease the quality of a plan, when it is implemented with the available cables.

	Cost - Proposed Method (MEUR)	Cost - Radial plan (MEUR)	Time (s)
1	213	226	1
2	123	127	< 1
3	269	275	< 1
4	463	501	< 1
5	70	70	< 1
6	506	696	6
7	719	1178	289
8	125	188	< 1
9	109	109	< 1
10	286	415	< 1
11	85	94	< 1
12	121	146	< 1
13	141	141	< 1
14	23	23	< 1
15	74	74	< 1
16	374	374	< 1
17	851	936	< 1
18	226	226	< 1
19	1143	1143	< 1
20	391	445	5
21	504	596	< 1
Total	6817	7985	302

Table 3.2: Results of the 3-stage method

Chapter 4

Summary of the results

Different methods have been considered to tackle the problem of expansion planning in the North Sea. Table 4.1 presents a summary of the main results obtained with the techniques addressed in this part of the thesis. The different approaches are compared in terms of objective functions and computational times.

Model	Lower Bound	Radial Plan	Mesh	Refined Mesh	Proposed Method
Cost (MEUR)	4930	8293	11425	8119	6817
Time	< 1s	< 1s	29s	3h30	5 min

Table 4.1: Summary of the results obtained for the North Sea problem

The main features of the different approaches are summarized with figure 4.1.

The reader should recall that the lower bound presented here does not correspond to a feasible plan.

The results tend to show that the approaches based on a systematic mesh are not the most adequate. Instead of placing the nodes systematically, the current work shows that dedicating a specific method to find candidate nodes is more meaningful. It enables one to restrict the number of supplementary nodes added to the network - and therefore the complexity of the problem - while discovering good candidates.

The work also shows that the partitioning approach should be encouraged when the problem is too complex to be tackled as a whole. The difference, between the results obtained with the mesh of the whole space and with the refined meshes of the small parts, is indeed quite significant.

The proposed method, combining both a partitioning technique and a heuristic algorithm to find good substation candidates, allows significant savings (around **1.5 billion euros**) to be made while solving efficiently the problem (around 5 minutes).

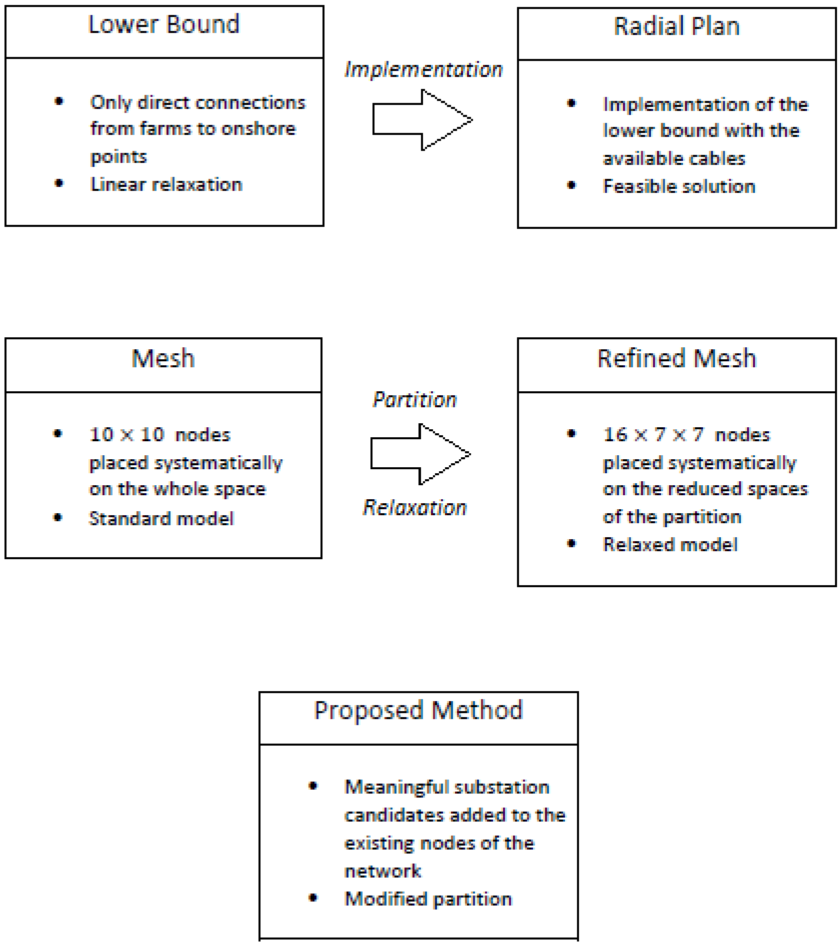


Figure 4.1: Summary of the methods for the North Sea problem

Chapter 5

Statistical Analysis

Chapter 3 proposed a three-stage algorithm to solve the problem of the North-Sea. A heuristic method was presented to find good candidate substation nodes. The refined partition and the size of the problem made it then possible to solve optimally each group with a classical solver and considering all the possible edges. However, in some similar practical cases, the optimal solution might be out of reach because larger groups will be formed by the partitioning technique. In those situations, heuristics or meta-heuristics might be required without the guarantee of coming close to the optimum. Reducing the search space might then be considered in order to guide better the method. Neglecting additional nodes might be a way to perform this reduction.

This chapter aims then at assessing the quality of the optimal solution of the problem restricted to the original set of nodes (in this case wind farms and onshore connection points) compared to the one obtained with its general formulation. To this end, statistical simulations will be run and the relative gain of connecting the wind farms at their optimal connection point, instead of at an existing node, will be determined. Since the ratio between the costs associated to lines and substations is dependent on the problem, the substation cost will be neglected in order to simplify the analysis.

5.1 Simplest case

First, the simple case of two farms connected to one onshore point will be considered.

The easiest case is the one that considers the same cost for the transmission of power from each farm and their sum. It happens when the sum of the powers is lower than the lowest capacity of the available cables. In this case, the problem consists in minimizing the total distance and so finding the point that minimizes the sum of distances to the vertices of the triangle formed by the farms and the onshore point. This particular point is called the Fermat point [92] and presents some interesting geometric properties.

5.1.1 Theoretical bound

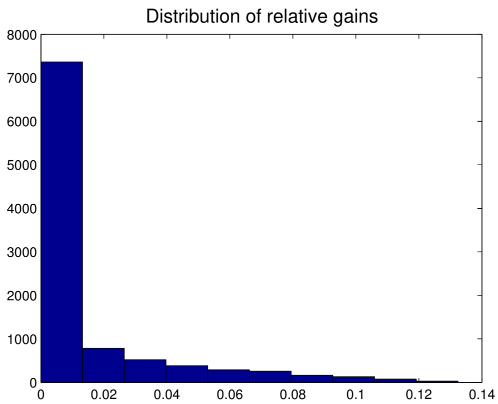
It is quite intuitive that the best case in terms of relative improvement, happens when the triangle is equilateral. In this case the relative gain is $1 - \frac{\sqrt{3}}{2} \approx 13.4\%$. Proposition 1 presents this statement.

Proposition 1. *For the problem of connecting two farms to one onshore connection point without substation cost, under the assumption that the costs of lines are equal for all the connections, the relative gain of adding a new node to the network is upper bounded by* $1 - \frac{\sqrt{3}}{2}$.

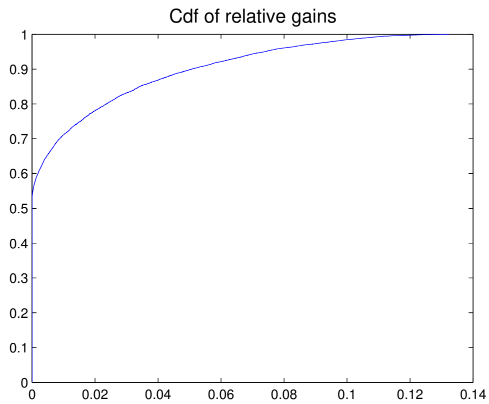
The proof of this proposition is given in appendix A.

5.1.2 Simulations

A bound has then been found on the relative optimality gap of considering only the existing nodes (with equal costs). This is an upper bound but in practice the gap is usually much lower. Figures 5.1 show the results of a simulation of 10 000 runs, choosing the locations of the farms, randomly, in a disc of radius 1 around the onshore point, placed at the origin. The first figure (5.1a) presents an histogram that shows the distribution of the relative gains over the 10 000 runs. The second figure (5.1b) presents the empirical cumulative distribution function (CDF) of these relative gains. The reader can note that, in the simulation, the cost of the plan going through the best connection point, is compared to the minimum cost of connecting the farms through one of them or joining them radially to the onshore station. Instead of a closed-form solution, the best connection point is found with a refined mesh of 100×100 points placed in the triangle formed by the two farms and the onshore point.



(a) Density distribution of relative gains

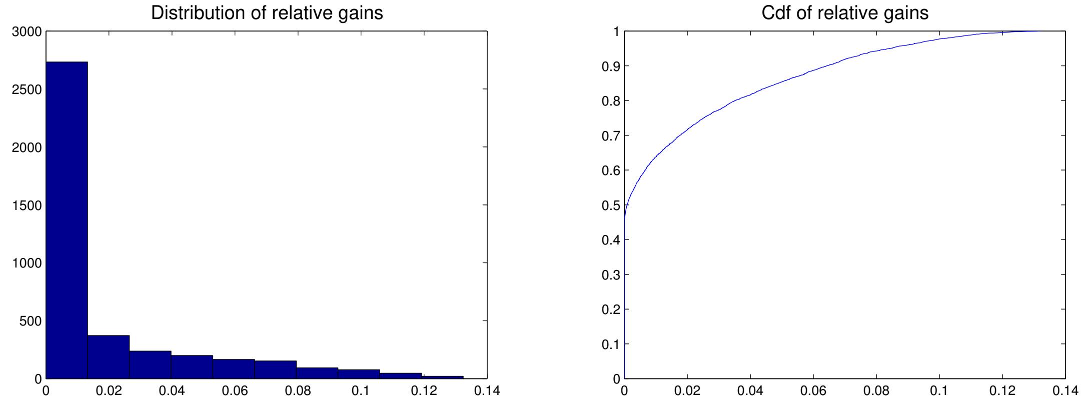


(b) Cumulative distribution of relative gains

Figure 5.1: Results of the simulations with two farms and equal costs

The reader can note that, as expected, the maximum lies close to the theoretical bound. Yet in a large majority of cases, the relative gain is almost negligible. It presents a mean value of 1.3%.

It can however be argued that taking the locations of the farms randomly does not correspond to the practical case since it will include all the cases where the radial connections are the optimal solution and so where the farms are not connected together at all. The next figures (5.2a and 5.2b) show the same results but only keeping the runs that connect the farms together before reaching the onshore point, in the problem without additional node.



(a) Density distribution of relative gains (b) Cumulative distribution of relative gains

Figure 5.2: Results of the simulations with two farms and equal costs (only connected farms)

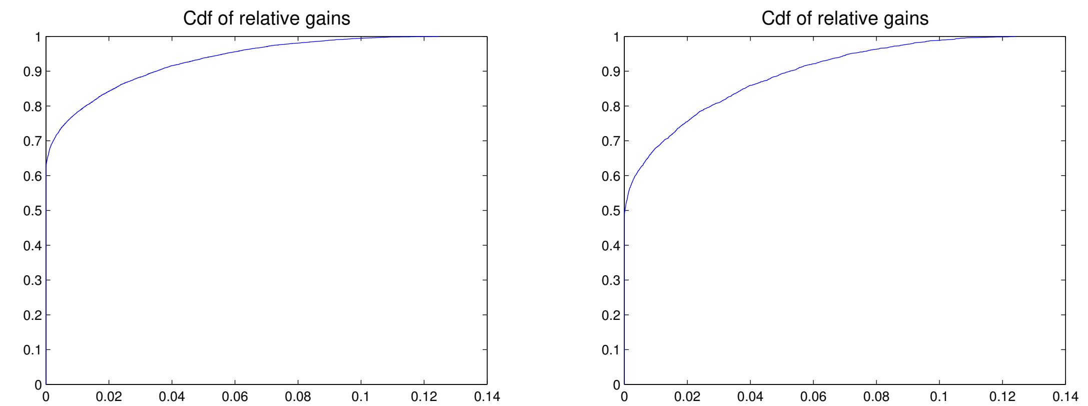
It can be noticed that the distribution is a little more spread out. But still, in almost half the cases, the gain is negligible (0 for 45% of them). As mentioned above, one interesting property of the Fermat point is that if the triangle have an angle larger than 120° then it lies at this vertex. In this case, the gain is then 0.

The mean value for these runs is slightly higher: 1.8%

5.2 Generalization

5.2.1 Unequal costs

The analysis can be repeated with different costs for the different transmissions. One can denote C_1, the cost (per unit of length) of the cable to transport the power from farm 1, C_2 the cost for farm 2 and C_3, the cost for their combined power. Then, without loss of generality (since the locations are chosen randomly), it can be assumed that $C_1 \leq C_2 \leq C_3$. Moreover, $C_3 \leq 2 \cdot C_1$ (otherwise there is no economy of scale and the radial plan is the best one). The costs can then be chosen randomly for each of the 10 000 runs following these two rules. The next figures (5.3) show the cumulative distributions obtained with these different costs. Once again next to the distribution obtained with the 10 000 relative gains (figure 5.3a), the one restricted to the cases where the farms are connected before the onshore station in the problem without additional nodes, is also presented (figure 5.3b).



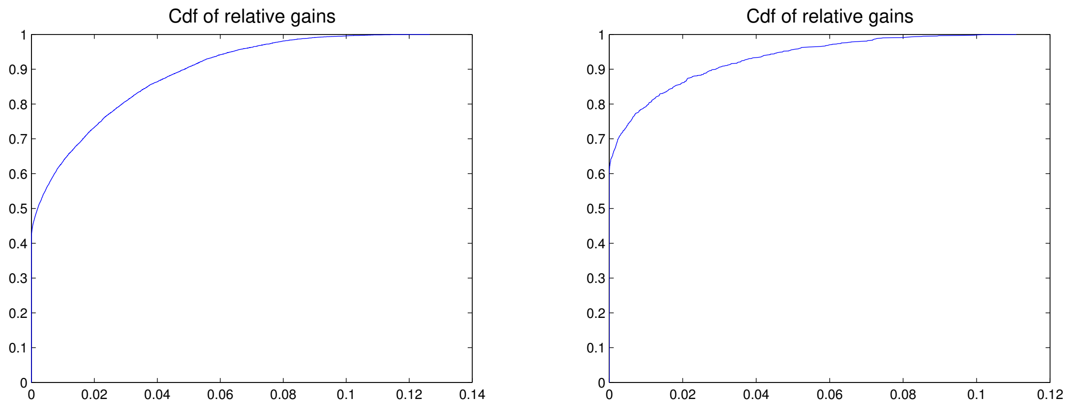
(a) All runs (b) Runs with connected farms

Figure 5.3: Cumulative distributions of relative gains with two farms and different costs

The relative gains are slightly lower in this case compared to the case with equal costs. Moreover, the maximum value is reached with almost equal costs and does not exceed the bound obtained earlier. The mean values for the two distributions (full and restricted) are respectively 0.91% and 1.4%. From the results above, it can be concluded that the specific case of equal costs tends to be the best case for relative gain.

5.2.2 Multiple farms

The analysis can be repeated with more than two farms, but of course, the more farms, the harder it is to find the best plan since the number of possible plans increases. Yet, it is still manageable for three farms, with some simplifications. The reader should note that, in what follows, the possibility of connecting two farms together, at first, and then connecting them to the last one, is not considered. It can however be noticed that this case corresponds to sequentially connecting two farms to an onshore point twice. Equal costs are considered for all transmissions. Similarly to the case with two farms, the figures below first present the distributions of relative gains over the 10 000 runs (figure 5.4a) and then focus on the runs where the three farms are connected together, before reaching the onshore station, in the problem without additional nodes (figure 5.4b). The optimal connection point is obtained with a meshing of the rectangle defined by the minimum and maximum coordinates of the farms with 1000×1000 points.



(a) All runs

(b) Runs with connected farms

Figure 5.4: Cumulative distributions of relative gains with three farms and equal costs

It can be noticed that when all the runs are considered, the distribution is a little more spread out than for the case with 2 farms. This can be explained by the fact that there are more possibilities to connect the farms together and so the gain is less often zero (typically the radial plan is less often optimal). Nevertheless, the maximum is lower (12.65%) and the mean value is comparable (1.4%). Moreover, when one considers only the cases where the three farms are connected together, the distribution of gains is less spread out and on average lower (maximum of 11.08% and mean value of 0.77%).

The last simulation analyzes the best connection of 4 farms to the onshore station. However, given the number of different possibilities to connect the farms together, only one single connection point is considered (4 farms connected together before the onshore point) for the problem with additional nodes. Therefore, only the runs, for which the best solution of the problem without new node is obtained by connecting the 4 farms in a single point, are considered (figure 5.5). However, since this case is less frequent, 100 000 runs are performed in order to get a significant number of gains.

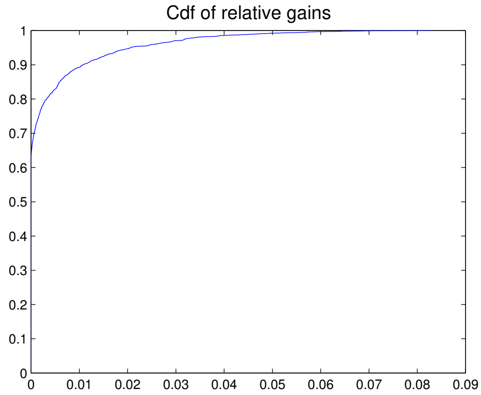


Figure 5.5: Cumulative distribution of relative gains with four farms and equal costs (only connected farms)

The relative gains are lower than for the previous simulations. The maximum relative gain is 8.3% and the average one is 0.34%.

5.3 Analysis and discussion

From the results above, it seems that the best case for relative gains tends to be the simplest one, with 2 farms and equal costs. This appears then to be the worst case for the suggested simplification (most harm done to the optimal cost by neglecting the additional nodes). Both the introduction of unequal costs and the multiplication of the number farms tend to decrease the relative improvement. This conclusion and the results above tend to show that the relative gain is often almost negligible. Therefore, this might provide a validation to the idea of neglecting the additional nodes when the problem is too complicated. This might indeed improve the results obtained by the used heuristic since the search space is reduced and the increase of the optimal cost might not be too significant.

Moreover, the second part of the analysis tends to motivate the method described in chapter 3 to find the additional substation candidates. Indeed, since the best relative gains are obtained with two farms, it might be meaningful to consider, sequentially, the different pairs of farms, in order to define the new nodes.

Conclusion

In this part of the thesis, a problem of transmission expansion planning in the North Sea was tackled. The challenge concerned the design of the transmission network in order to bring, efficiently, the electricity from the offshore wind farms to the onshore connections points.

Two main approaches were studied to tackle the problem: as a whole or with a partition. The work revealed that the latter outperformed significantly the former. Concerning the creation of new substations, again, two different techniques were addressed. One was based on a systematic meshing of the space, the other, proposed a specific technique to discover meaningful candidate substation nodes. The latter outperformed considerably the former leading to conclude that the systematic meshing was not adequate for this problem.

A statistical analysis, based on simulations, was also performed in order to assess the necessity of additional nodes to the network. This work tends to motivate the option of neglecting them for highly complex problems. Moreover, it tends to show that the proposed technique to handle this problem is meaningful.

The methods presented in this part were tested on the offshore grid of the North Sea. However, they can be applied to other similar cases where power has to be brought from remote production sites to the consumption sites. Those kinds of problems include the design of the connecting network to reach the renewable generating sites such as solar parks or wind farms.
These methods can also be used in order to treat the reversed problem of reaching scattered consumption sites. This is of particular interest in some areas of the world - such as Africa or the Middle-East - where the transmission networks are not fully designed yet. Nevertheless, these problems might require additional constraints which might imply significant changes in the network topology and so the need of specific techniques to solve them. The next part tends to respond to this concern by treating another practical case with other constraints and designing suitable methods.

Part III

Onshore grid: network in Burundi

Introduction

The previous part tackled the challenge of the design of an offshore transmission network in the North Sea. The problem was to find the best connections to bring the power from the remote offshore wind farms to the onshore connection points where it could be routed to the consumption sites. This part is dedicated to a similar, although reversed, problem. Instead of reaching the remote production sites, the challenge is now to connect the scattered consuming locations to the network. The practical case studied is the transmission network of Burundi.

It is illustrated with figure 1. The red crosses represent the generating nodes while the blue dots are the consuming nodes. The generating nodes are defined as locations where the maximum generation capacity exceeds its maximum consumption. The definition of the consuming nodes is reversed.

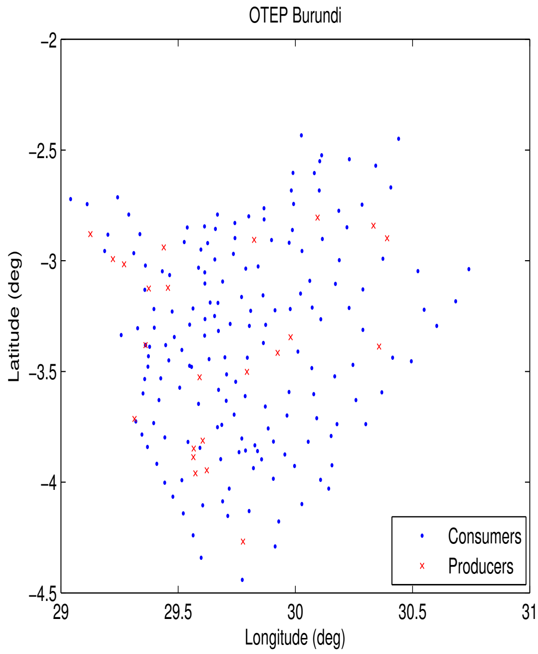


Figure 1: Nodes of the problem

As before, the task consists then in finding the best connections between the blue dots and the red crosses through possibly other nodes.

Although there obviously already exist transmission lines in Burundi, the problem is supposed to be a greenfield project again. Hence, the design of the network has to be defined from scratch. The assumption is not unreasonable given the significant future increase of powers involved. As before, one could imagine building substations (theoretically) everywhere, but of course implying a determined cost.

At first sight, the model for this onshore grid might look quite different from the one treated for the North Sea. However, although the notations will change due to the reversed direction of the power, many parallels can be done between the two problems.

The main differences lie on the network laws, the additional security constraints and the complexity of the problem. Indeed, since AC lines are used in this case, instead of HVDC ones, the power flow equations have to be modified back to their most used formulation (see 1.1 in part I). Moreover, a security constraint, guaranteeing a more reliable network, will be added for this problem (motivated by the consideration of loads). The two changes above and the increased number of nodes imply a more complex problem. Specific methods will then have to be implemented.

Similarly to the North Sea case, only investment costs will be considered and the model will handle extreme cases. This means that the network should be able to deliver, to every node, its maximum power consumption. Perfect knowledge about the problem, both in terms of generation and consumption, is assumed (perfect foresight about the future load and generation). No operational costs with different scenarios are taken into account. Moreover, a static version of the problem is treated.

First, the general model of the problem will be presented. Afterwards, a first approach, based on meta-heuristics, will be studied. However, this approach is outperformed by the proposed technique, relying on Benders Decomposition, that will be described and analyzed in details. The reader who lacks time to read the whole text is then invited to skip chapter 2, to focus on chapter 3 (however, section 2.7.1 is of interest for the proposed method).

Notations

The needed notations to define the model tackled in this part are presented in this section.

Sets

- $Cons$: set of consumers (consuming nodes).
- Gen : set of producers (producing/generating nodes).
- N : set of nodes.
- E : set of edges - possible connections between nodes: $(i, j) \in E$ if there is a possible connection between nodes i and j. Hence $E \subset N \times N$.
- C : set of available lines with given capacity, cost and resistance.
- CL_{ijc} : set of candidate lines of type $c \in C$ on connection $(i, j) \in E$.
- S : set of contingencies.

Discrete variables

- x^e_{ijc} : binary variable - deciding if the line $e \in CL_{ijc}$ (of type $c \in C$) is built (on edge $(i, j) \in E$).
- y_n : binary variable - deciding if a substation will be built at node $n \in N$.
- a_n : positive integer variable - deciding how many extra lines (additional to the two mandatory ones [1]) are connected at node $n \in N$.

Continuous variables

- f^{es}_{ijc} : continuous variable - active power flow on line $e \in CL_{ijc}$ (of type $c \in C$ on edge $(i, j) \in E$) for contingency $s \in S$.
- g^s_n : positive continuous variable - power generated at node $n \in N$ for contingency $s \in S$.
- θ^s_n : voltage angle at node $n \in N$ for contingency $s \in S$.

Parameters

- P^g_n : power generated at node $n \in N$.
- P^c_n : power consumed at node $n \in N$.
- L_{ij} : length of connection $(i, j) \in E$.
- Cap_c : capacity (power) of line $c \in C$.
- C_c : cost of line $c \in C$ (per unit of length).
- X_c : reactance of line $c \in C$ (per unit of length).
- SC : substation cost.

[1] The security constraint imposes that all the nodes are connected with at least two lines. A fixed cost is associated to any additional line connected to a node.

- EC : additional cost for the extra lines at the nodes (cost associated to the variables a_n, independent of the type or length of the lines).

- M_{ijc} : disjunctive parameter for Kirchhoff's voltage law for lines of type $c \in C$ on edge $(i, j) \in E$.

- w_{ijc}^{es} : binary parameter defining if the line $e \in CL_{ijc}$ (of type $c \in C$ on edge $(i, j) \in E$) is working (1) or fails (0) for contingency $s \in S$.

Chapter 1

Model and bounds

1.1 Model formulation

In this part the general model of the problem will be described. It corresponds to the version that would be given to a classical (commercial) solver (such as CPLEX), in order to solve the problem.

A few remarks have first to be addressed before looking at the model.

Due to the changes of notations, the model might appear really different from the one developed for the North Sea. However, if one goes beyond the notations, many similarities can be noticed. Indeed, the consuming nodes can be assimilated to the wind farms in the North Sea problem since their specific power (consumed or generated) is supposed to be known and constant. Moreover, a parallel can be made between the maximum generated power in this case and the maximum evacuated powers by the onshore stations. The deduced constraints are then really close from one version to the other and although the problem is now reversed, one can think about both of them similarly.

However, an important change, with respect to the previous models, is the consideration of a security constraint. This is performed with the help of a set of contingencies. The network has then to be able to withstand any of the given contingencies included in the set and still be able to supply the demand. The well-known *N-1 security* criterion is used in this work. This reliability criterion and other alternatives are presented in section 1.5 of the *Literature Review* (part I). In this work, this constraint guarantees that the demand is served in case of any line failure (but only one at a time). The set of contingencies will then include one element per line considered as candidate circuit. The cardinality of this set is then given by the following formula.

$$\mathrm{card}(S) = 1 + \sum_{(i,j)\in E, c\in C} \mathrm{card}(CL_{ijc})$$

The additional element of the set of contingencies corresponds to the normal case of no failures.

Moreover, the parameters w are defined such that every contingency considers exactly one failure of a candidate circuit and that each candidate circuit failure is taken into account by exactly one contingency.

The set of contingencies affects the continuous variables but not the integer decision variables. The whole problem can indeed be seen as two linked subproblems: the investment problem and the operational problem. The investment problem involves the integer decision variables (*first stage* variables) and the operational problem checks the feasibility of the plans with the continuous variables (*second-stage* variables) for each contingency.[1] Every continuous variable has to be duplicated for each possible contingency. The consideration of the *N-1 security* criterion multiplies then the number of continuous variables by the cardinal of the set S.

Although the problem theoretically allows the consideration of new candidate substation nodes, this work will neglect this possibility. Hence, this implies that

$$N = Gen \cup Cons \tag{1.1}$$

This decision is motivated by two main reasons: the complexity of the problem and the probable low benefit of considering these additional nodes. Indeed, this problem is a real challenge and is more complex than the case of the North Sea. Therefore, considering additional nodes might do more harm than good to the implemented methods and the statistical analysis performed in chapter 5 of part III provides an indication that this simplification is meaningful.

Another subject of attention is the substation rule. Substations have to be built at the nodes, as soon as 2 lines are connected to them. Moreover, a fixed cost is associated to any additional line connected to a node. The reader can notice that with the security constraint considered above, every node connected to the network should be linked with at least two lines. Therefore, substations have to be built at any existing nodes of the network.[2] This reinforces the argument in favor of neglecting the additional candidate nodes since considering them will create new substations without removing any[3](furthermore the cost of substation is quite high for this problem compared to the cost of lines). Hence, with only the producing and consuming nodes in the network, substations have to be built everywhere. Therefore, it is no longer meaningful to include the y variables in the problem (since they will be equal to 1). The additional cost associated to the substations can then be a posteriori added to the objective function. This cost is equal to $SC\cdot$ card(N).

Since only the extreme cases are considered, the power generated or consumed at generating or consuming nodes can be redefined as the absolute value of the difference of maximum power produced and consumed at that node. Therefore, the nodes are then redefined as only producer or only consumer. In other words, the power produced or consumed cannot be both strictly positive at one particular node.

Taking into account the above remarks, the general model for this problem can be written as follows:

$$\min_{x,a,f,g,\theta} \qquad \sum_{(i,j)\in E, c\in C, e\in CL_{ijc}} x^e_{ijc} \cdot L_{ij} \cdot C_c + \sum_{n\in N} a_n \cdot EC$$

subject to

$$\sum_{(i,n)\in E,\ c\in C,\ e\in CL_{ijc}} f^{es}_{inc} - \sum_{(n,j)\in E, c\in C, e\in CL_{ijc}} f^{es}_{njc} + g^s_n - P^c_n = 0 \qquad \forall n \in N,\ s \in S \tag{1.2}$$

$$\sum_{(i,n)\in E,\ c\in C,\ e\in CL_{inc}} x^e_{inc} + \sum_{(n,j)\in E,\ c\in C,\ e\in CL_{njc}} x^e_{njc} = 2 + a_n \qquad \forall n \in N \tag{1.3}$$

[1]Benders Decomposition that will be studied in this work takes profit of this possible separation.

[2]This statement is not necessarily true for the producing nodes but the case of isolated nodes will not be treated in this work.

[3]For the North Sea problem, the creation of a substation at an additional node, usually meant replacing a substation at an existing node. This will not be the case for this problem due to the security constraint.

$$f_{ijc}^{es} \leq x_{ijc}^{e} \cdot w_{ijc}^{es} \cdot Cap_c \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc}, s \in S \quad (1.4)$$

$$-f_{ijc}^{es} \leq x_{ijc}^{e} \cdot w_{ijc}^{es} \cdot Cap_c \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc}, s \in S \quad (1.5)$$

$$g_n^s \leq P_n^g \qquad \forall n \in N, s \in S \quad (1.6)$$

$$f_{ijc}^{es} - \frac{1}{X_c \cdot L_{ij}}(\theta_i^s - \theta_j^s) \leq M_{ijc} \cdot (1 - x_{ijc}^{e} \cdot w_{ijc}^{es}) \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc}, s \in S \quad (1.7)$$

$$-f_{ijc}^{es} + \frac{1}{X_c \cdot L_{ij}}(\theta_i^s - \theta_j^s) \leq M_{ijc} \cdot (1 - x_{ijc}^{e} \cdot w_{ijc}^{es}) \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc}, s \in S \quad (1.8)$$

$$g_n^s \geq 0 \qquad \forall n \in N, \ s \in S \quad (1.9)$$

$$x_{ijc}^{e} \in \{0,1\} \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \quad (1.10)$$

$$a_n \text{ integer and } \geq 0 \qquad \forall n \in N \quad (1.11)$$

The objective function corresponds to the sum of the investment cost to build the lines and the cost of the extra connections at the nodes.
Equality 1.2 is the balance constraint at each node.
Constraint 1.3 makes sure the extra connections to the nodes are counted properly. As mentioned above, the security constraint imposes that at least two lines are connected to each node. Any additional line is then stored with the variables a. The reader can note that a lower inequality instead of an equality might be used but the choice made has a positive impact on the convergence.
Constraints 1.4 and 1.5 limit the absolute value of the active power flow on the lines. The reader can note that, as mentioned above, one flow variable for each line and each contingency is defined. In case of failure of the particular line, the flow is constrained to be zero.
Constraint 1.6 limits the power generated at each node. If the concerned node is not a generating node, this power is 0. Otherwise, it is limited to the maximum power generation at that node. Constraint 1.9 ensures the positivity of this variable.
Constraints 1.7 and 1.8 are the disjunctive formulation of the DC power flow equations with the help of the disjunctive parameters M.
Constraints 1.10 and 1.11 simply define the domain of the variables x and a.

The fact that the different candidate lines in the sets CL are indistinguishable, for the same edge and type of line, might have a negative effect on the convergence of the methods. Therefore, an additional constraint might be added to respond to this concern.

$$x_{ijc}^{e-1} \leq x_{ijc}^{e} \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} : e \neq 1 \quad (1.12)$$

This formulation assumes that the candidate circuits in the sets CL are ordered and simply associated to a number from 1 to the maximum number allowed. The constraint 1.12 enables the lines to be differentiated by imposing that a line can be chosen only if the previous ones - in the set - are.

1.2 Bounds

1.2.1 Lower Bound

Before describing the implemented methods and analyzing their results, it might be interesting to obtain a lower bound on the optimal cost of the problem. In order to do so, the same approach, as the one used for the North Sea problem, is applied.
The modifications and definitions added to the model are listed below.

- The set of edges is defined as the direct connections between the consuming and generating nodes.
  $\Rightarrow E = \{(i,j) : i \in Cons \ , \ j \in Gen\}$.

- Relaxation of the integrality of the variables x: the number of lines built becomes a continuous variable and so every capacity is reachable.
- The cost of additional connections to the nodes is neglected.
- The costs of the lines is linearly under approximated with respect to their capacity.
- The direct connections forbid the creation of loops, hence the power flow equations (Kirchhoff's voltage laws) can be neglected and so can the variables θ.
- The contingencies are not considered anymore.

Before writing down the problem solved with these modifications, a few new notations have to be introduced.

- x^r_{ij} : continuous positive variable - capacity installed on edge $(i, j) \in E$.
- f_{ij} : continuous variable - active power flow on edge $(i, j) \in E$.
- g_n : positive continuous variable - power generated at node $n \in N$.
- CC : continuous cost of the lines (per unit of capacity).

With the help of these new notations and the previous ones, the problem can then be described as follows:

$$\min_{x^r,f,g} \quad \sum_{(i,j)\in E} x^r_{ij} \cdot L_{ij} \cdot CC$$

subject to

$$\sum_{(i,n)\in E} f_{in} - \sum_{(n,j)\in E} f_{nj} + g_n - P^c_n = 0 \qquad \forall n \in N \tag{1.13}$$

$$f_{ij} \leq x^r_{ij} \qquad \forall (i, j) \in E \tag{1.14}$$

$$-f_{ij} \leq x^r_{ij} \qquad \forall (i, j) \in E \tag{1.15}$$

$$g_n \leq P^g_n \qquad \forall n \in N \tag{1.16}$$

$$g_n \geq 0 \qquad \forall n \in N \tag{1.17}$$

$$x^r_{ij} \geq 0 \qquad \forall (i, j) \in E \tag{1.18}$$

The linear approximation of the cost of lines is defined similarly as for the offshore grid (section 1.2.1, figure 1.2).

The reasoning, explaining why the solution of this problem is indeed a lower bound of the true problem, is the same as the one described for the North Sea problem (section 1.2.1). The only difference lies in the security constraint considered for this problem. However, it is obvious that neglecting it can only decrease the cost.

Again the relaxed problem, as described above, is a linear program. It can then be solved efficiently with a solver such as CPLEX (less than 1 second).

Figure 1.1 shows the plan obtained by solving this problem. The presence of lines in the figure does not give any indication on the capacity installed, but simply that it is not zero.

The lower bound for this problem is **14.0** MEUR. However, this bound is unreachable since the capacities installed in the obtained plan are not reachable with the available lines. Furthermore, coming close to this value seems also impossible, since a lot of the capacities involved in this plan are far from those that can be implemented with the lines. A lot of consuming nodes require, typically, very little power compared to the lowest capacity available with the two different lines. Moreover, this plan neglects entirely the security constraint. This will be clearly illustrated with the implementation of this plan to give the upper bound below.

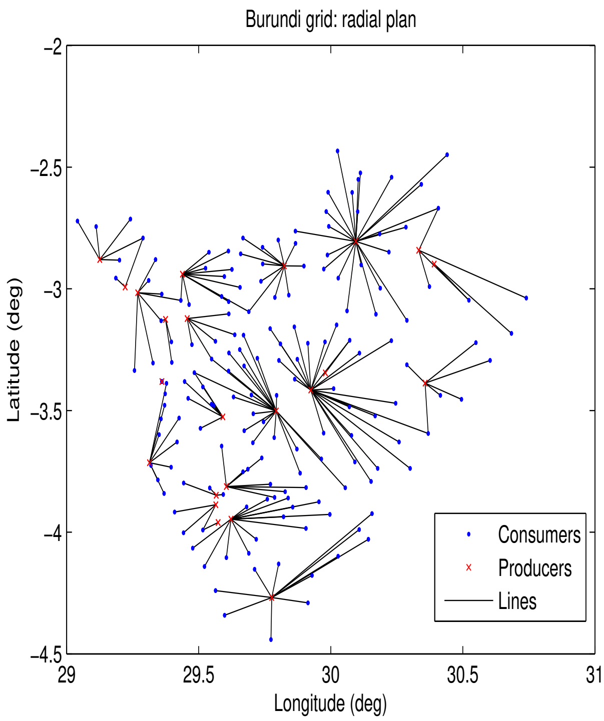


Figure 1.1: Plan with radial connections

1.2.2 Upper Bound

Based on the relaxed solution found for the lower bound, a feasible solution for the problem can easily be built. It is indeed sufficient to collect the different capacities installed on each connection and implement a plan to reach at least these capacities with the available lines.

The problem is exactly the same as the one solved to get an upper bound for the North Sea problem (section 1.2.2). However, each line of the solution to this problem has then to be duplicated in order to respect the security constraint. This cost is then doubled. Moreover, the cost associated to the additional connections to the nodes must then be added to the objective function.

This plan costs **361.8** MEUR and will be used to assess the quality of the solutions found by the methods implemented in the rest of this work. It will be referred to as *radial plan* since it only allows direct connections from the consuming to the generating nodes. In order to have the real total cost, one has to add the mandatory substation cost which is equal to 297 MEUR.

Comparing the costs of the lower and upper bounds allows one to draw some conclusions. The high difference between the two costs (the upper bound is 12.9 times as expensive as the lower bound), tends to imply that the lower bound is not really meaningful since it will probably be impossible to come close to it. Moreover, this gap gives hope that significant savings can be made from the *radial plan*.

Chapter 2

Meta-heuristics

The problem described in chapter 1, is highly challenging and therefore a classical solver [1] (such as CPLEX) fails at providing a good solution, as soon as the number of nodes considered becomes large. The first implemented approach consists then in stepping out of the mathematical optimization to look for other methods. However, since the network is designed from scratch, power flow problems (without expansion lines but only the operational variables) will not give meaningful information and therefore heuristics based on sensitivity indices (based on the resolutions of these problems) are not expected to be implemented efficiently.
Different meta-heuristics have been successfully applied to the TEP problem and therefore this class of heuristics will be considered in this chapter. A more detailed discussion about those techniques can be found in the *Literature Review* (part III, section 2.3). The main advantage of these techniques is their flexibility. Furthermore, they can provide reasonably good solutions even when classical methods fail at doing so and are therefore considered for highly complex problems. However, the quality of the obtained solutions is often hard to assess due to the lack of bounds provided by the method. Moreover, they usually include multiple parameters whose tuning might affect highly the process. Tuning properly the different parameters for a specific problem might be complex.

The meta-heuristics usually include different mechanisms for both *diversification* and *intensification*. *Diversification* helps avoiding getting stuck at local (but not global) optima by making sure that a certain level of diversity stays in the populations during the process. *Intensification* consists in searching in depth the local spaces in order to find the local optima. The methods have then to find a compromise between the two mechanisms. Indeed, if the intensification is privileged over the diversification, the process has a high probability to converge early to a local optimum, meaning that all the individuals in the population will be in the same region of the search space. On the other hand, if the diversification overshadows the intensification, the process is expected to explore the whole space but not in depth and so will fail at discovering the local optima. The balance between the two mechanisms will often be ruled by the choice of the parameters of the methods. This explains why this decision might be crucial.

Different meta-heuristics have been tested to solve this problem. The two most promising ones, among them, will first be described and then discussed below.

The two described meta-heuristics will ultimately be outperformed by the proposed technique in chapter 3. The reader who is only interested in the main method is then invited to skip the current chapter and focus on the next one. Section 2.7.1 is, however, useful for the method developed in chapter 3.

[1] based on a Branch-and-Bound strategy

2.1 Description of the methods

In this section the two chosen meta-heuristics will be described and their application to the problem will be addressed.

2.1.1 Differential Evolution

The first meta-heuristic considered in this work is Differential Evolution (DE).

This method is an evolutionary algorithm that has been applied to the TEP problem [36, 45, 72]. It starts from an initial population of a given size and associates a vector to each individual. For this problem, the vector is created with the integer variables that decide how many lines are installed on the different connections. To each vector is associated a fitness corresponding to the cost of the proposed plan. This cost includes also the extra cost due to additional connections at the nodes, although those decision variables do not appear in the vector. Moreover, if the plan is not feasible, unserved demand is accompanied with a very high penalty cost [2].
The method consists in iteratively applying mutations on the decision vectors. Those mutations in the DE are not generated with random perturbations (as in the genetic algorithm) but with linear combinations of the other individuals of the populations [30]. The mutation rule is the following:

$$v_i^{G+1} = x_{n_1}^G + F \cdot (x_{n_2}^G - x_{n_3}^G)$$

where v_i^{G+1} is the result of the mutation, F (typically $\in [0, 2]$) is a scaling factor and $x_{n_1}^G, x_{n_2}^G, x_{n_3}^G$ are randomly chosen individuals in the current generation G (but different from the parent i) [45]. The vector is then rounded and brought back to the considered domain (the number of lines have to be positive and an upper limit of 4 is set for each type of line on each connection).
Then the crossover operator will determine for each component of the vector, randomly according to a recombination rate, if the offspring will take its component from the mutated result or from the parent. The selection is then made between the parent and the offspring comparing their fitness. And a new mutation cycle is repeated with the new population.

The parameters of the method include then, the population size, the scaling factor F and the recombination rate CR. The stopping criterion used in this work is the number of generations.

2.1.2 Artificial Immune Systems

The Artificial Immune System (AIS) methods are based on an analogy with the Natural Immune System (NIS). One of the most common algorithms of this class is the CLONALG [30] and it is the one implemented in this work. The expansion plan is, in this case, represented by an antibody and each investment decision is a position of the antibody (again, only the lines are part of the decision vector).
The first process involved in the algorithm is the reproduction: each antibody is cloned without modification. It means that each *clone* will have the same associated decision vector as its *parent*. Then the hypermutation procedure takes place and adds a standard normal distributed perturbation to each clone. This perturbation is weighted by a factor, referred to as mutation magnitude (σ) [72]. The mutation rule is then the following:

$$x_{i,c}^G = x_i^G + \sigma \cdot Z$$

[2]The penalty cost is set arbitrarily to 10^9 kEUR/MW in this work (methods not really sensitive to this value).

where x_i^G is the decision vector associated to the individual i at generation G, $x_{i,c}^G$ is the vector associated to the clone c of this individual and Z is a random vector with the same length as the decision vector and whose components follow independent standard normal distributions. Again, the resulting vector has to be rounded and brought back to the considered domain.

The first selection process compares the antibodies of each group, formed by a parent and its clones, and keeps the best distinct ones (the number of clones selected is a parameter of the method). Finally, the last phase merges all the best obtained antibodies (selected in each group) and selects the overall best distinct ones to repeat the evolution strategy with a constant size population. The selection procedure includes then two successive steps in order to encourage diversity in the population. The individuals are compared with their fitness which is defined as above.

The parameters of the method include then, the population size, the number of clones, the number of antibodies selected in each group and the mutation magnitude. The stopping criterion used in this work is, again, the number of generations.

2.2 Model

As explained above, the meta-heuristics do not solve the model presented in section 1.1. Instead, plans are proposed by the evolutionary mechanisms and their feasibility is tested by solving a linear program. The cost associated to a plan is then the sum of the actual cost of the plan and the objective function of the linear problem which aims at minimizing the unserved demand with integer decisions kept constant. This load curtailment is associated to a very high penalty cost in order to always privilege a feasible plan over an infeasible one.

Before presenting the linear program solved, a few notations need to be introduced.

- x_{ijc}^m : positive integer parameter - number of lines of type $c \in C$ built on edge $(i, j) \in E$ (the reader can note that it is now a parameter since it is determined by the evolutionary mechanisms).
- f_{ij}^{ms} : continuous variable - active power flow on edge $(i, j) \in E$ for contingency $s \in S$.
- r_n^s : positive continuous variable - unserved demand at node $n \in N$ for contingency $s \in S$.
- PC : penalty cost due to unserved demand.
- b_{ijc}^s : binary parameter - defines if there is a failure of a line of type $c \in C$ on edge $(i, j) \in E$ for contingency $s \in S$.

The problem can then be written as follows:

$$\min_{r, f^m, g, \theta} \qquad PC \cdot \sum_{n \in N, s \in S} r_n^s$$

subject to

$$\sum_{(i,n)\in E} f_{in}^{ms} - \sum_{(n,j)\in E} f_{nj}^{ms} + g_n^s + r_n^s - P_n^c = 0 \qquad \forall n \in N,\ s \in S \tag{2.1}$$

$$f_{ij}^{ms} \leq \sum_{c\in C}(x_{ijc}^m - b_{ijc}^s)\cdot Cap_c \qquad \forall (i,j) \in E, s \in S \tag{2.2}$$

$$-f_{ij}^{ms} \leq \sum_{c\in C}(x_{ijc}^m - b_{ijc}^s)\cdot Cap_c \qquad \forall (i,j) \in E, s \in S \tag{2.3}$$

$$g_n^s \leq P_n^g \qquad \forall n \in N, s \in S \tag{2.4}$$

$$f_{ij}^{ms} - \sum_{c\in C}\frac{(x_{ijc}^m - b_{ijc}^s)}{X_c \cdot L_{ij}}(\theta_i^s - \theta_j^s) = 0 \qquad \forall (i,j) \in E, s \in S \tag{2.5}$$

$$g_n^s \geq 0 \qquad \forall n \in N,\ s \in S \tag{2.6}$$

$$r_n^s \geq 0 \qquad \forall n \in N,\ s \in S \tag{2.7}$$

The reader can note that this is indeed a linear program and so it can be solved efficiently with a classical linear solver.

The set of contingencies is not the same as the one described in section 1.1. Indeed, one does not have to consider each possible failure of a given circuit in the sets CL but only one per type of line and per edge. Moreover, it is useless to consider failures for type of lines that are not included in the plan proposed by the evolutionary mechanisms. Therefore, the cardinality of the set S is given by the following formula.

$$\mathrm{card}(S) = 1 + \mathrm{card}(\{(i,j) \in E, c \in C:\ x_{ijc}^m > 0\})$$

This reduces the number of contingencies and so the size of the problem.

The cost of the proposed plan is then given by

$$PC \cdot \sum_{n\in N, s\in S} r_n^s + \sum_{(i,j)\in E, c\in C} x_{ijc}^m \cdot L_{ij} \cdot C_c + \sum_{n\in N} a_n \cdot EC$$

where a_n, $n \in N$ are now parameters defined with

$$a_n = \max\left(\sum_{(n,j)\in E, c\in C} x_{njc}^m + \sum_{(i,n)\in E, c\in C} x_{inc}^m - 2, 0\right) \qquad \forall n \in N \tag{2.8}$$

2.3 Partition

The size and complexity do not give much hope of finding a good solution for the whole problem at once, even with meta-heuristics. The tests revealed indeed that in order for the methods to provide good solutions, the problems must not be too large.

Similarly to the problem in the North Sea, it is then proposed to cluster the nodes before solving the groups independently. As before, the partition is made with the help of the *radial plan* presented in section 1.1 (fig 1.1). This plan generates naturally groups distinct from each other. The consuming and generating nodes that are linked in this plan can be grouped together. The defined groups have then a reduced size and the reader can note that the power balance is respected since the partition is generated based on a feasible plan. Moreover, the allocation is meaningful since the consuming nodes are grouped with their closest (unsaturated) producer(s).

For this problem, the generating and consuming nodes are then allocated into 11 groups. Table 2.1 shows the number of the different nodes in each group.

Group	1	2	3	4	5	6	7	8	9	10	11
# Producers	2	4	3	1	1	2	3	1	3	1	2
# Consumers	6	29	29	8	13	3	28	11	30	3	15

Table 2.1: Information about the different groups of the partition described in section 2.3

In order to reduce further the search space to guide better the methods, not all the possible edges are considered. For each node, only the connections to the three nearest neighbours and the generating nodes are kept in the set. The number of nodes and edges in each group is presented in table 2.2.

Group	1	2	3	4	5	6	7	8	9	10	11
# Nodes	8	33	32	9	14	5	31	12	33	4	13
# Edges	21	165	137	22	34	10	135	26	147	6	40

Table 2.2: Nodes and edges per group of the partition described in section 2.3

2.4 Initialization

The initial population might have a large impact on the performances of the methods. Instead of a perfectly random initialization, a more clever one was implemented in this work.

In order to make sure that the methods provide solutions at least as good as the *radial plan*, the first individual of the population is given this plan as initial vector. The other vectors are initialized by applying random changes to this plan. They are generated by an iterative and random algorithm. At each step, the process carries on with a given probability that decreases geometrically. If it does, a change to the plan is operated randomly: a random line is removed with probability $\frac{1}{4}$, a random line is added with probability $\frac{1}{4}$ and two lines are swapped (one enters the plan and one leaves it) with probability $\frac{1}{2}$. The number of steps and so the number of changes are then random numbers. However, at least one change is operated (the first step has a probability 1 to happen). The reader can note that the expected number of lines of the different plans is the same as in the *radial plan*. It is meaningful to do so because there is a high probability that the optimal plan contains a number of lines close to it. The common ratio chosen in this work, for the geometrical decrease of probabilities, is 0.95.

2.5 Parameter analysis

The parameters of the meta-heuristics can have a significant impact on the results obtained. The sensitivity of the methods to the parameters was tested on the group 8 of the partition mentioned in section 2.3 (see tables 2.1 and 2.2). This group was chosen because it presents a good compromise in terms of size. It is indeed large enough, to provide meaningful conclusions, but not too large, to allow several runs in a reasonable amount of time. The values of the parameters were changed one at a time (keeping the others constant).

The results, presented in appendix B, revealed that the two meta-heuristics are highly sensitive to the values of some of their parameters.

2.6 Results

This section provides the results of the two methods applied on the 11 groups of the partition. Table 2.3 compares the costs obtained thanks to the two meta-heuristics with the one associated to a *radial* model.[3] Only one run was performed for each group and with the best values of the parameters resulting from the analysis. The costs are expressed in MEUR. The first column provides the index of the groups.

	Radial	DE	AIS
1	8.1	4.0	4.1
2	43.2	40.2	43.2
3	63.2	51.1	63.2
4	13.7	3.9	3.3
5	25.6	11.6	7.5
6	4.2	1.5	1.7
7	60.6	49.7	60.6
8	25.0	9.7	9.3
9	52.6	44.9	52.6
10	2.5	1.1	1.1
11	24.1	15.4	14.9

Table 2.3: Comparison of the results of the meta-heuristics (AIS and DE) on the partition defined in section 2.3 (costs expressed in MEUR)

The total costs of the methods are respectively 232.9 and 261.3 MEUR for the DE and AIS. The DE provides then **36 %** savings compared to the *radial plan.* The DE and AIS required respectively 3599 and 3887 seconds to solve all groups sequentially with a maximum of 599 and 850 seconds for a single group.
The DE seems to outperform the AIS for the whole problem. Analyzing the results, one can see that the AIS fails at improving the *radial* plan for large groups, getting stuck at that local optimum. However, it tends to perform better for groups with similar size as group 8, for which the parameters are tuned. It can then be concluded that the parameters of the AIS are more sensitive to the group size than the DE's. Yet, even the DE method performs significantly better for small groups than larger ones. It is then meaningful to refine the partition in order to limit the sizes of the problem and keep them in the range where the methods perform well and for which the parameters are tuned.

2.7 Refined partition

2.7.1 Description

The previous section showed that the methods struggle at providing good solutions when the size of the problems is too large. This conclusion tends to imply that the methods would provide better solutions if the partition was further refined.

Similarly as for the problem in the North Sea, the first step consists in creating groups with only one generating node. This is done exactly the same way by adding the following two constraints (2.9 and 2.10) to the *radial* problem of section 1.2.1

[3] This plan is not exactly the same as the one presented in section 1.2.2. Instead of first solving an LP relaxation of the problem, it directly considers the combinatorial formulation but with only direct connections from consuming to producing nodes. As before, the contingencies are not taken into account but the lines are duplicated afterwards.

$$x_{ij}^{r} \leq \hat{M} \cdot z_{ij} \qquad \forall (i,j) \in E \tag{2.9}$$

$$\sum_{(i,k)\in E} z_{ik} = 1 \qquad \forall i \in N \ (i \in Cons) \tag{2.10}$$

where z are binary variables.

Doing so, an MILP problem is defined, easy to solve with a classical solver (such as CPLEX). The obtained plan is illustrated with figure 2.1.

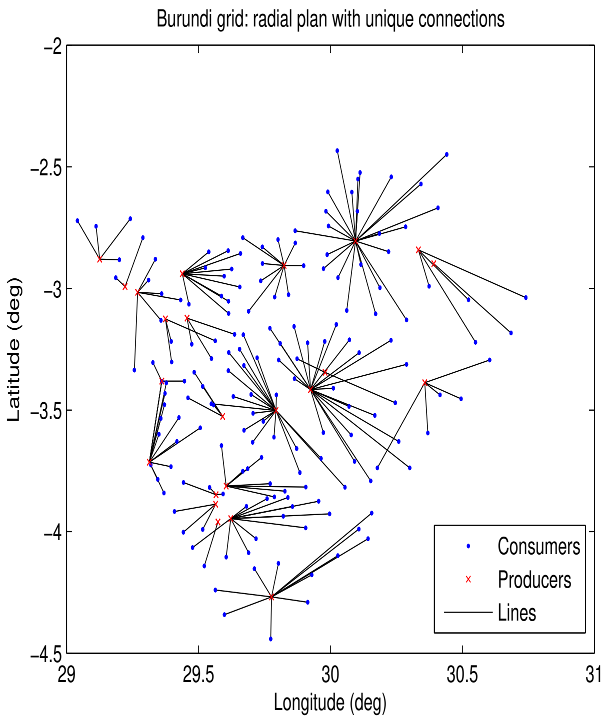


Figure 2.1: Plan with radial connections with the additional constraint of 1 connection per consuming node

This new *radial* plan leads to smaller groups. However, some of them are still quite large and so might lead to poor performances. It might be meaningful to define a clustering technique that limits the size of the different groups.

In this work, the proposed technique allocates the consuming nodes of the groups, into subgroups of limited size, according to their angular position with respect to the (unique) generating node of the (large) group. The subgroups can then be solved independently by including the generating node of the larger group.

In order to differentiate the partitions in the rest of this work, the following terminology will be used.

- *Groups*: refers to the allocation of the partition based on a *radial* plan as in section 2.3 or solving the problem described in this section.
- *Subgroups* : refers to the partition generated by algorithm 2. The *subgroups* are then subsets of the *groups*.

Algorithm 2: Defining subgroups

Result: Subgroups of limited size

Initialize $maxSize$;

for $gr \in Groups$ **do**

  Initialize angular distances to $+\infty$;

  Initialize subgroups (SG) with individual consumers ;

  Compute angular distance for every pair of consumers ;

  Compute minimum angular distance: $minDist$;

  Identify closest subgroups: (n_1, n_2) ;

  while $minDist < +\infty$ **do**

    if $card(SG_{n_1}) + card(SG_{n_2}) \leq maxSize$ **then**

      Merge SG_{n_1} and SG_{n_2}

        $SG_{n_1+n_2} \leftarrow SG_{n_1} \cup SG_{n_2}$;

      Define distances to $SG_{n_1+n_2}$

        $dist(SG_n, SG_{n_1+n_2}) \leftarrow \min\left(dist(SG_n, SG_{n_1}), dist(SG_n, SG_{n_2})\right) \quad \forall n;$

      Discard SG_{n_1} and SG_{n_2};

    end

    else

      $dist(SG_{n_1}, SG_{n_1}) \leftarrow +\infty$;

    end

    Compute $minDist$ and find (n_1, n_2) ;

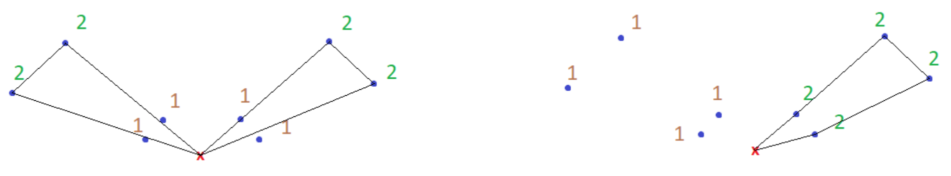
  end

end

Starting from subgroups with individual consuming nodes, the algorithm consists in iteratively merging the closest subgroups unless the size of their union exceeds the size limit. If it does, the algorithm makes sure that this pair is not selected again by setting their distance to $+\infty$. The algorithm stops when there is no more subgroups that can be merged without exceeding the limit. This process is repeated for each large group independently.

A few topics can be discussed about the definition of the distance between the subgroups.

First, the reader will notice that instead of a distance based on absolute locations, the angular distance, with respect to the generating node, is used in this work. This distance is then based on the angle formed by the line segments joining the producing node to the consuming nodes. Since the goal is to connect the consumers to the producing node, the former are more likely to be connected together if they come from the same direction (with respect to the producer). This statement is illustrated by the example on figure 2.2.



Absolute distance   Angular distance

Figure 2.2: Illustrative example motivating the choice of the angular distance

The example considers a group of 8 consumers and imposes a size limit of 4 for each *subgroup*. The consumers are represented by blue dots and the generating node by a red cross. The numbers correspond to the index of the *subgroup* in which the node is allocated. The lines of the optimal plan for the *subgroup* 2 are presented. The clusters formed by the absolute distance are obviously not optimal since the lines from the nodes of *subgroup* 2 will pass close to the nodes of *subgroup* 1 to reach the producer but without being able to go through them (as illustrated in the figure on the left). It is then more meaningful to allocate the nodes as in the figure on the right and so with an angular distance.

Moreover, as illustrated with the figure, this choice of distance will cluster the nodes in distinct regions of the space without possible line crossings between them in the generated plans.

The second point that requires attention is the choice of the way the distances are updated, when the subgroups are merged. In this work, the distance to a pair of subgroups is defined as the minimum distance to either one among this pair. This corresponds to the clustering technique known as *single-linkage clustering*. This choice is motivated by the topology of the solutions. The generated plans of the North Sea problem presented a *tree-topology*, aggregating the power in one node before connecting it to the station. In that case, the distance to a central point (*centroid*) could be more meaningful and so an update based on an average distance could be preferred. However, the solutions of the problem of this onshore grid tend to privilege large loops instead of trees (because of the security constraint). Therefore, subgroups will be more likely to be linked together if there exists a small edge between them. A clustering technique based on the minimum distance makes then more sense for this case.

2.7.2 Results

Applying the previous method on the specific problem of the Burundi network, with a limit size of 10 consumers per group, one obtains 23 large groups and 33 subgroups.

Each subgroup can then be solved independently with the meta-heuristics developed. Table 2.4 presents the results obtained with both methods. However, the subgroups with strictly less than 5 nodes are solved with the classical solver since it can be done efficiently. The costs are expressed in MEUR. The first provided cost is the one obtained with the meta-heuristics (but without the mandatory substation cost). The cost associated to an improved plan is then provided. Since the meta-heuristics do not guarantee to provide the optimal solutions, one might indeed try to improve the solution by replacing the expensive lines by cheaper ones (with lower capacity) and testing the feasibility of the obtained network. The computational times needed to solve all subgroups sequentially are presented in the next column and the last one gives the maximum running time for a single subgroup (limit time for parallel computing). The times are expressed in seconds.

	Cost (MEUR)	Improved Cost (MEUR)	Total time (s)	Max time (s)
DE	146.3	139.7	4216	251
AIS	133.0	129.9	4090	249

Table 2.4: Results of the meta-heuristics (DE and AIS) on the refined partition as described in section 2.7

Although the DE performed better on the large groups, these results show that the AIS outperforms it when the size of the subgroups is limited.
It can be noticed that significant savings can be made by refining the partition. The best plan provided by the AIS is indeed **44 %** cheaper than the previous best one.
The computational time is quite similar for both methods. It could be significantly decreased by the implementation of parallel computing.

The plan obtained with the AIS method is illustrated with figure 2.3.

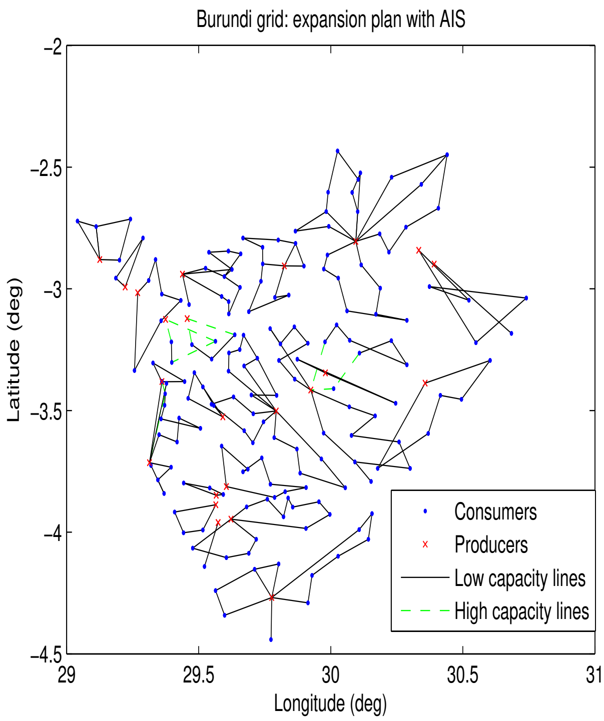


Figure 2.3: Plan obtained with the AIS method applied on the refined partition described in section 2.7

The solution could be improved by increasing the complexity of the problem (population size and number of generations). However, it will not be done in this work because the performances of the meta-heuristics are outperformed by the method described in the next chapter.

Chapter 3

Proposed Method

In this chapter, the proposed method, based on a Benders Decomposition, will be described and analyzed. First, the application of the decomposition to this specific problem will be studied. Its results will then be presented and discussed. Finally, a heuristic method to improve the obtained plan will be addressed.

3.1 Benders Decomposition

3.1.1 Model

In this section, the specific Benders Decomposition applied to the problem considered will be presented. The general formulation of this decomposition can be found in the *Literature Review* (part I, section 2.1.2).

The approach consists in separating the problem into an investment problem and several operational problems. One operational problem can indeed be defined for each of the contingencies and be solved independently. The investment problem is called the *master* and includes the integer decision variables (x and a). The operational problems are referred to as *slaves* and aim at checking the feasibility of the plans proposed by the *master* and generating cuts.

The *master* problem can then be written as follows:

$$\min_{x,a} \quad \sum_{(i,j)\in E, c\in C, e} x^e_{ijc} \cdot L_{ij} \cdot C_c + \sum_{n\in N} a_n \cdot EC$$

subject to

$$\sum_{(i,n)\in E,\ c\in C,\ e\in CL_{inc}} x^e_{inc} + \sum_{(n,j)\in E,\ c\in C,\ e\in CL_{njc}} x^e_{njc} = 2 + a_n \qquad \forall n \in N \tag{3.1}$$

$$x \in \cap_{s\in S} \mathrm{dom} V_s \tag{3.2}$$

$$x^{e-1}_{ijc} \leq x^e_{ijc} \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} : e \neq 1 \tag{3.3}$$

$$x^e_{ijc} \in \{0,1\} \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \tag{3.4}$$

$$a_n \text{ integer and } \geq 0 \qquad \forall n \in N \tag{3.5}$$

The *slave* problem associated to contingency $s \in S$ is presented below.

$$V_s(x) \quad = \quad \min_{f,g,\theta} \quad 0$$

subject to

$$\sum_{(i,n)\in E,\ c\in C,\ e\in CL_{ijc}} f_{inc}^{es} - \sum_{(n,j)\in E, c\in C, e\in CL_{ijc}} f_{njc}^{es} + g_n^s - P_n^c = 0 \quad \forall n \in N \qquad (\lambda_n) \tag{3.6}$$

$$f_{ijc}^{es} \leq x_{ijc}^e \cdot w_{ijc}^{es} \cdot Cap_c \quad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \quad (\mu_{ijce}^+) \tag{3.7}$$

$$-f_{ijc}^{es} \leq x_{ijc}^e \cdot w_{ijc}^{es} \cdot Cap_c \quad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \quad (\mu_{ijce}^+) \tag{3.8}$$

$$g_n^s \leq P_n^g \quad \forall n \in N \qquad (\kappa_n) \tag{3.9}$$

$$f_{ijc}^{es} - \frac{1}{X_c \cdot L_{ij}}(\theta_i^s - \theta_j^s) \leq M_{ijc} \cdot (1 - x_{ijc}^e \cdot w_{ijc}^{es}) \quad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \quad (\nu_{ijce}^+) \tag{3.10}$$

$$-f_{ijc}^{es} + \frac{1}{X_c \cdot L_{ij}}(\theta_i^s - \theta_j^s) \leq M_{ijc} \cdot (1 - x_{ijc}^e \cdot w_{ijc}^{es}) \quad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \quad (\nu_{ijce}^-) \tag{3.11}$$

$$g_n^s \geq 0 \qquad \forall n \in N \tag{3.12}$$

The reader can note that the constraints 3.6, 3.7, 3.8, 3.9, 3.10, 3.11 and 3.12 are equivalent to the constraints 1.2, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9. The only difference lies in the fact that the latter are repeated for each contingency while with this decomposition, the contingencies are separated in different problems.

The dual variables of the constraints - $(\lambda, \mu^+, \mu^-, \kappa, \nu^+, \nu^-)$ - are introduced at the end of the corresponding lines into brackets. They enable one to write the *slave* problem with its dual form.

$$\begin{aligned} V_s(x) = \max_{\lambda,\kappa,\mu^+,\mu^-,\nu^+,\nu^-} \quad & \sum_{(i,j)\in E,c\in C,e\in CL_{ijc}} \Bigg(x_{ijc}^e \cdot w_{ijc}^{es} \cdot Cap_c \cdot (\mu_{ijce}^+ + \mu_{ijce}^-) \\ & \qquad + M_{ijc} \cdot (1 - x_{ijc}^e \cdot w_{ijc}^{es}) \cdot (\nu_{ijce}^+ + \nu_{ijce}^-) \Bigg) \\ & + \sum_{n\in N} \lambda_n \cdot P_n^c + \kappa_n \cdot P_n^g \end{aligned}$$

subject to

$$\lambda_j - \lambda_i + \mu_{ijce}^+ - \mu_{ijce}^- + \nu_{ijce}^+ - \nu_{ijce}^- = 0 \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \tag{3.13}$$

$$\lambda_n + \kappa_n \leq 0 \qquad \forall n \in N \tag{3.14}$$

$$\begin{aligned} & \sum_{(n,j)\in E,c\in C,e\in CL_{ijc}} -\frac{1}{X_c \cdot L_{nj}}(\nu_{njce}^+ - \nu_{njce}^-) \\ + & \sum_{(i,n)\in E,c\in C,e\in CL_{ijc}} \frac{1}{X_c \cdot L_{in}}(\nu_{ince}^+ - \nu_{ince}^-) = 0 \qquad \forall n \in N \end{aligned} \tag{3.15}$$

$$\mu^+,\ \mu^-,\ \nu^+,\ \nu^-,\ \kappa \leq 0 \tag{3.16}$$

In this case, no optimality cuts will be added to the *master* (since the primal form of the *slaves* does not have any cost associated to it). The feasibility cuts have the following form:

$$\begin{aligned} & \sum_{(i,j)\in E,c\in C,e\in CL_{ijc}} x_{ijc}^e \cdot w_{ijc}^{ek} \cdot Cap_c \cdot (\bar{\mu}_{ijce,k}^+ + \bar{\mu}_{ijce,k}^-) + M_{ijc} \cdot (1 - x_{ijc}^e \cdot w_{ijc}^{ek}) \cdot (\bar{\nu}_{ijce,k}^+ + \bar{\nu}_{ijce,k}^-) \\ + & \sum_{n\in N} \bar{\lambda}_{n,k} \cdot P_n^c + \bar{\kappa}_{n,k} \cdot P_n^g \leq 0 \end{aligned} \tag{3.17}$$

with $(\bar{\lambda}, \bar{\mu}^{+}, \bar{\mu}^{-}, \bar{\nu}^{+}, \bar{\nu}^{-}, \bar{\kappa})$ being an extreme ray of the domain of a given *slave* problem and k being the index of the cut corresponding to the associated contingency.

Similarly to the problem treated with the meta-heuristics, one does not have to consider the failures of all the candidate lines but only the ones that are part of the plan proposed by the *master*. Therefore, the set S varies at each iteration and its size is given by the following formula:

$$\mathrm{card}(S) = 1 + \mathrm{card}(\{(i,j) \in E, c \in C, e \in CL_{ijc} : \; x^{e}_{ijc} = 1\})$$

This decreases significantly the number of contingencies considered.

At each iteration of the process, the *master* problem is solved, its solution is passed as parameter to the *slaves* and cuts are generated from them. It can then be noted that at each iteration multiple cuts can be produced from the *slave* problems (one from each). If the proposed plan (optimal solution of the *master* for the current iteration) is feasible for all the *slaves*, the process stops. Moreover, since the formulation of the *slaves* is convex, this plan is the optimal solution of the given problem.

The solutions of the successive *master* problems are then lower bounds of the problem. However, no feasible solution is produced before the end of the convergence and so the optimal one. This constitutes one drawback of the method since if it is stopped before convergence, due to time limitation, it does not provide any feasible plan.

The tests revealed that constraint 3.3 speeds up significantly the convergence of the method by differentiating the identical candidate circuits. This motivates then its inclusion in the model.

3.1.2 Disjunctive parameter

Setting the values

In this section, a technique is presented to set the value of the disjunctive parameter *"big-M"*. Setting an appropriate value is important since high values of the parameter could imply numerical instabilities [2] while low values might reduce the search space (and so cut out the optimal solution).

The authors in [2] propose to solve a longest path problem to set the values for disconnected nodes. However, it is a hard problem (NP) and so it is suggested to enumerate the different possible paths. For this problem, since all the nodes are originally disconnected, the enumeration is out of reach for large groups (its complexity is $\mathcal{O}(n!)$ for n nodes). Hence, an over approximation of this longest path is used in this work. The reasoning is described below.

As a reminder, the disjunctive formulation of Kirchhoff's voltage law can be written as follows (constraints 3.10 and 3.11):

$$\left| f^{es}_{ijc} - \frac{1}{X_c \cdot L_{ij}} (\theta^{s}_{i} - \theta^{s}_{j}) \right| \leq M_{ijc} \cdot (1 - x^{e}_{ijc}) \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc}$$

If $x^{e}_{ijc} = 1$ then the left-hand-side should be equal to 0 and so the disjunctive parameter does not have any influence.

If $x^{e}_{ijc} = 0$ then constraints 3.7 and 3.8 impose that $f^{es}_{ijc} = 0$ and thus the constraint becomes:

$$\left| \theta^{s}_{i} - \theta^{s}_{j} \right| \leq M_{ijc} \cdot (X_c \cdot L_{ij}) \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc}.$$

The value of the parameter has to be chosen such that this constraint is satisfied for any feasible solution.

Imagining a path in a feasible solution from node i to node j of length m: $p_1, p_2, ..., p_m$ such that $p_1 = i$ and $p_m = j$, the difference of phase angles can then be written as:

$$(\theta_i^s - \theta_j^s) = \sum_{k \in \{2,..,m\}} (\theta_{p_k}^s - \theta_{p_{k-1}}^s) \Rightarrow \left|\theta_i^s - \theta_j^s\right| \leq \sum_{k \in \{2,..,m\}} \left|\theta_{p_k}^s - \theta_{p_{k-1}}^s\right|.$$

However, since the nodes p_k and p_{k-1} are directly connected, their phase angle must satisfy Kirchhoff's voltage law. Let $c^* \in C$ and $e^* \in CL_{p_k p_{k-1} c^*}$ such that $x_{p_k p_{k-1} c^*}^{e^*} = 1$ then one has:

$$(\theta_{p_k}^s - \theta_{p_{k-1}}^s) = f_{p_k p_{k-1} c^*}^{e^* s} \cdot (X_{c^*} \cdot L_{p_k p_{k-1}}).$$

An upper bound to this expression can then be found by introducing the maximal capacity of the lines.

$$|\theta_{p_k}^s - \theta_{p_{k-1}}^s| \leq \max_{c \in C}(Cap_c \cdot X_c) \cdot L_{p_k p_{k-1}}$$

Hence, one can write:

$$\left|\theta_i^s - \theta_j^s\right| \leq \max_{c \in C}(Cap_c \cdot X_c) \cdot \sum_{k \in \{2,..,m\}} L_{p_k p_{k-1}}.$$

If the nodes i and j do not belong to the same connected component of a feasible solution, then the value of the parameter does not matter. Otherwise, there exists a path from i to j of length at most $n - 1$ (n being the number of nodes). Hence, if the set of $n - 1$ longest edges is defined as E_{max}, the disjunctive parameter for connection $(i, j) \in E$ and line $c \in C$ can be chosen as:

$$M_{ijc} = \frac{1}{X_c \cdot L_{ij}} \cdot \max_{c^* \in C}(Cap_{c^*} \cdot X_{c^*}) \cdot \sum_{(o,p) \in E_{max}} L_{op}.$$

As shown, this choice does not reduce the search space. However, this value could be further decreased by solving a real longest path problem instead of using an over approximation.

Discussion

The previous section presented a technique to set a meaningful value for the disjunctive parameters associated to the different lines. One might wonder if the values have a significant effect on the convergence of the method. For the formulation of the problem with optimality cuts, it is clear that increasing the value assigned to these parameters will make those cuts less tight and therefore have a negative effect on the convergence. However, the impact of this increase on the feasibility cuts is less obvious. The main objective of these cuts is to remove the infeasible solutions of the *master* from its domain. This goal will be achieved, whatever value is chosen for the disjunctive parameter, as long as it is large enough. It can then be expected that the convergence of the method will be less sensitive to this choice for this problem.

Nevertheless, the feasibility cuts might also rule out other infeasible solutions from the *master*'s domain and in this case the values of the *big-M* might matter. This is illustrated by the following example.

The example involves a network with two consuming nodes and one generating node. For the sake of simplicity, only one type of line is considered with unitary capacity. The powers consumed are also unitary and there is enough power generated to serve the demand. One edge linking the generating node to a consumer is 1.2 times as long as the other two. The length of the edges and the powers involved are presented with figure 3.1 (not up to scale). The generating node is represented by a red cross, while the consuming nodes are blue dots.

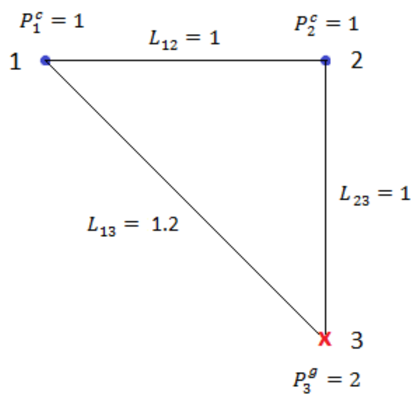


Figure 3.1: Example network for the discussion about the *big-M*

The security constraint is not considered in this example. It can be checked that, for this network, the plan consisting in installing one line on each edge is not feasible ($x_{12} = x_{13} = x_{23} = 1$). Table 3.1 presents an extreme ray of the *slave* problem.

$\bar{\mu}_{12}^{+} = 0$	$\bar{\mu}_{13}^{+} = 0$	$\bar{\mu}_{23}^{+} = 0$	$\bar{\mu}_{12}^{-} = 0$	$\bar{\mu}_{13}^{-} = 0$	$\bar{\mu}_{23}^{-} = -1$
$\bar{\nu}_{12}^{+} = -0.3125$	$\bar{\nu}_{13}^{+} = 0$	$\bar{\nu}_{23}^{+} = -0.3125$	$\bar{\nu}_{12}^{-} = 0$	$\bar{\nu}_{13}^{-} = -0.375$	$\bar{\nu}_{23}^{-} = 0$
$\bar{\lambda}_1 = 0.375$	$\bar{\lambda}_2 = 0.6875$	$\bar{\lambda}_3 = 0$	$\bar{\kappa}_1 = -0.375$	$\bar{\kappa}_2 = -0.6875$	$\bar{\kappa}_3 = 0$

Table 3.1: Extreme ray of the *slave* problem in the example for the disjunctive parameter

The extreme ray is, in this case, independent of the value of the disjunctive parameters. The feasibility cut can then be expressed as follows:

$$-x_{23} - 0.3125 \cdot M_{12} \cdot (1 - x_{12}) - 0.375 \cdot M_{13} \cdot (1 - x_{13}) - 0.3125 \cdot M_{23} \cdot (1 - x_{23}) + 1.0625 \leq 0 \tag{3.18}$$

The reader can check that setting $M_{23} = 2.2$ is large enough. Doing so, the feasibility cut also rules out the solution $x_{12} = x_{13} = 1$ and $x_{23} = 0$ from the domain of the *master*. This is not the case for values higher than 3.4. Therefore, the value of the parameter does have an impact in this case.

The convergence of the method might then be dependent on the values chosen. However, the tests revealed that they have a very limited impact on the convergence for the problems of the refined partition described in section 2.7. This can be explained by the fact that, given the topology of the solutions (large loops) and the security constraint, the power flow equations are less significant (since the topology of the optimal solution in cases of contingencies is often a tree).

3.1.3 Speed-up techniques

Several techniques have been tested to speed up the convergence of Benders Decomposition. Two of them are presented below. They rely on the fact that it is not necessary to have the optimal solution of the *master* in order to generate feasibility cuts.

Suboptimal master

The first technique consists in increasing the relative integrality gap tolerance (*mip gap*) to stop the solver quicker for the resolution of the *master* problems [41]. If the proposed solution is feasible then the *mip gap* is reset to its default value (10^{-4}) to check if the method has reached the optimal solution.
Different *mip gap* values have been tested with the 33 subgroups of the refined partition presented in the previous chapter (section 2.7) with a maximal size of 10 consumers per subgroup. A maximum of 3 identical lines are considered for installation on each connection. The results obtained with the 5 groups that require the longest times (more than 1 minute with the classical Benders method) are presented below in table 3.2. The other subgroups are solved too quickly and so no meaningful conclusion can be drawn from them. The running times and the number of iterations for the different values of the parameter are presented alongside the results for the classical Benders method (mipgap = 10^{-4}).

Time (s)	mipgap = 10^{-4}	214	442	266	1356	78
	mipgap = 10^{-3}	116	360	152	1301	70
	mipgap = 10^{-2}	106	323	145	964	70
	mipgap = 10^{-1}	98	257	119	838	71
	mipgap = 0.2	97	574	127	749	75
Nb iterations	mipgap = 10^{-4}	73	89	79	85	72
	mipgap = 10^{-3}	57	87	81	81	65
	mipgap = 10^{-2}	57	86	82	73	65
	mipgap = 10^{-1}	58	108	83	92	70
	mipgap = 0.2	67	187	84	129	87

Table 3.2: Results of Benders Decomposition with suboptimal master, applied on 5 representative subgroups

The results show that setting the relative mip gap to 10^{-1} decreases consistently the computational times. The required number of iterations is higher but is compensated by the quicker resolutions of the *master* problems.

Local search

Another technique consists in only considering plans in a neighbourhood of the previous solution of the *master* [71]. This reduces the search space of the *master* problem and so enables faster resolutions. The true *master* problem is solved periodically to keep the global optimality and not always stay in the same region of the search space. This method involves two parameters: the size of the neighbourhood and the number of local searches between two optimal resolutions of the *master*. The size of the neighbourhood is defined as the number of changes (additions or removals) of lines compared to the previous solution.

This technique is implemented by making small modifications to the *master* problem. If the previous solution of the *master* is named - *xold* - then the constraint ensuring the local search can be expressed as follows (3.19):

$$\sum_{(i,j)\in E, c\in C, e\in CL_{ijc}} \left| x^e_{ijc} - xold^e_{ijc} \right| \leq \text{size} \tag{3.19}$$

However, this constraint being not linear, another formulation would be preferred. This is done by introducing additional variables, replacing the absolute values. If abs^e_{ijc} is defined as a binary variable $\forall(i,j) \in E, c \in C, e \in CL_{ijc}$, the constraint 3.19 can be expressed with the 3 following constraints added to the *master*:

$$x^e_{ijc} - xold^e_{ijc} \leq abs^e_{ijc} \qquad \forall(i,j) \in E, c \in C, e \in CL_{ijc} \tag{3.20}$$

$$xold^e_{ijc} - x^e_{ijc} \leq abs^e_{ijc} \qquad \forall(i,j) \in E, c \in C, e \in CL_{ijc} \tag{3.21}$$

$$\sum_{(i,j)\in E, c\in C, e\in CL_{ijc}} abs^e_{ijc} \leq \text{size} \tag{3.22}$$

Constraints 3.20 and 3.21 define the binary variable while 3.22 is the real constraint that guarantees the local search.

The rest of this section is dedicated to a parameter analysis in order to determine the best values for both of them.
The first parameter studied is the size of the neighbourhood. The true *master* problem (without constraints 3.20, 3.21 and 3.22) is solved every 10 iterations and when a feasible solution is found (in order to check if the local solution is the global optimum). The performances in terms of computational time and number of iterations are presented in table 3.3.

Time (s)	size = 2	231	721	153	2352	93
	size = 4	222	257	89	527	56
	size = 6	67	145	110	485	55
	size = 8	187	335	107	404	45
Nb iterations	size = 2	141	303	157	251	125
	size = 4	121	171	120	115	101
	size = 6	75	121	113	102	85
	size = 8	71	121	89	82	71

Table 3.3: Results of Benders Decomposition with local search, applied on 5 representative groups: impact of the size of the neighbourhood

The results confirm the intuition. The larger the size, the less iterations are required (in general) but the more computationally heavy they are. The best compromise seems to be reached with a neighbourhood of size 6. The results also show that significant time savings can be made with this technique (almost 3 times quicker than the classical method for the hardest group).

The second parameter of this technique is the number of local search iterations (resolutions of the *master with* constraints 3.20, 3.21 and 3.22) between two true optimizations of the *master* (*without* constraints 3.20, 3.21 and 3.22) .
The tests revealed that the method is less sensitive to this parameter (for values chosen between 5 and 20). Moreover, the best value depends on the problem. As expected the number of required iterations tends to increase with the period but the average running time of an iteration decreases (since less true *masters* are solved).

Aside the time savings, another advantage of using this technique is that in case of slow convergence, the method could still provide a feasible and good solution if the best solution found, during the local search iterations, is stored.

3.1.4 Results

The plan obtained with Benders Decomposition applied on the refined partition is presented with figure 3.2.
This plan costs **110.7** MEUR. The method guarantees that it is optimal for the defined partition. The subgroups of the partition required 1015 seconds to be solved sequentially with Benders Decomposition with the local search speed-up technique. The neighbourhood size was 6

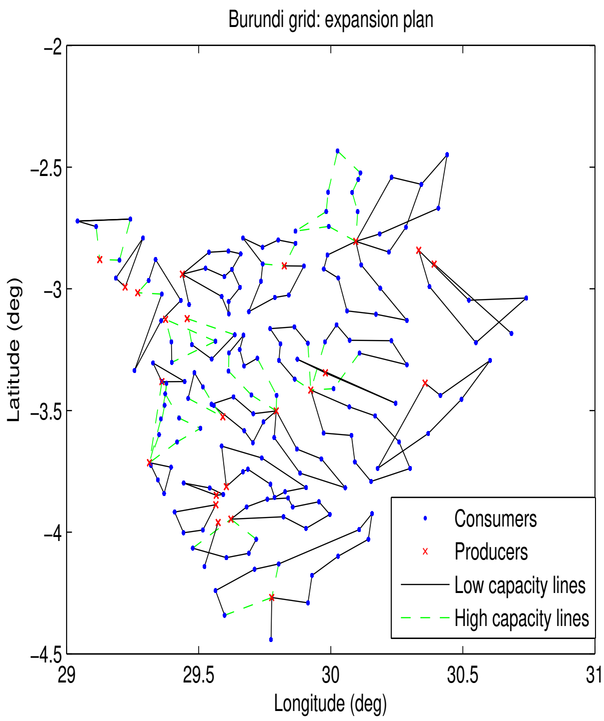


Figure 3.2: Plan obtained with Benders Decomposition on the refined partition

and the period 10. The hardest subgroup resolution required 485 seconds.

3.1.5 Line crossings

Looking at figure 3.2, one might have concerns about the apparent line crossings. The crossings between the different *groups* or *subgroups* might reveal the suboptimality of the plan due to the defined partition. This might then support the idea of exploring possible connections between the *subgroups*. However, it can be noticed that line crossings do appear inside *subgroups* (twice). This does not indicate suboptimality, though, since the plan is optimal for the refined partition. This proves then that line crossings can be present in the optimal solution. The particular subgroups, in which it occurs, are not easy to analyze since they are not the smallest ones. An easier example is presented in appendix C.

3.1.6 Comparison with the classical solver

This section is dedicated to the comparison of Benders Decomposition and the classical solver based on a Branch-and-Bound strategy.

The main difference between the two methods is the decomposition strategy used in the Benders. This strategy allows each contingency to be solved independently. This implies then smaller linear programs that can be solved efficiently. But the main advantage relies on the fact that the decomposition enables one to only consider the failures of lines in the plan proposed by the *master*, instead of taking into account any possible failure (which is a much larger number).

In the decomposition, the fact that several cuts can be generated, with only one solution of the *master*, also helps to speed up the convergence.

These distinctions between the two methods lead to significant differences in the performances. The classical solver fails at providing good solutions when the size of the groups increases.
For example, a subgroup with 9 nodes (8 consumers and 1 producer) only reached a relative integer gap of 0.442 after 1 hour, while Benders Decomposition found the optimal solution after 2 seconds. The classical solver behaves even more poorly for larger groups. It is indeed still stuck at the root node of the Branch-and-Bound tree after 1 hour and presents a relative mip gap of 0.9922, for a subgroup with 11 nodes. Benders Decomposition, however, finds the optimal solution of this subgroup in less than 4 minutes.

3.2 Improvements

In this section, several techniques will be used to improve the plan obtained with the partition. They aim at finding better solutions by getting rid of the limitations imposed by the partition.

3.2.1 Merging

Description

The first proposed method consists in merging the smallest groups defined by the partition. If a size limit is imposed on the merged groups, the problems will still be tractable. Therefore, the groups created will still be solved optimally and so the technique can only improve the solution.

The algorithm consists in iteratively merging the groups whose generating nodes are the closest, unless the size limit is reached. Again, a *single-linkage clustering* technique is used to update the distances (based on the minimum distance). An additional constraint is added to obtain meaningful clusters: the distance between the groups does not have to exceed a maximum value. This value is given by the maximum value (among the two groups to merge) of the average edge length.[1]

The reader can note that this process does not affect the *subgroups* of a larger *group* (since they cannot be merged without reaching the size limit).

Results

The size limit, in this work, is set as before to a maximum of 10 consuming nodes in a subgroup/merged group.

The plan obtained by solving the merged groups independently is presented in figure 3.3a. This plan costs **105.8** MEUR. The computational time was similar to the one obtained with the previous partition (the merged ones are not the most time consuming groups).

3.2.2 Local Search between subgroups

Description

In this subsection, a heuristic method is introduced in order to improve the solution inside the large groups. This technique consists in performing successive local searches, starting from the individual optimal plans of the subgroups, and considering a few additional connections between them.

The candidate list of edges is created with the connections used in the individual solutions (of the separated subgroups) and by adding the smallest edges between nodes of different subgroups.

[1] This constraint is not mandatory but aims at avoiding that some groups that are far from each other (and so have no chance to be connected together) are merged.

In this work, the two smallest edges between each pair of subgroups are considered.

The local search consists in iteratively finding the optimal solution in the neighbourhood of the current solution. In order to do so, a Benders Decomposition is used. The modifications to the model are similar to those performed in section 3.1.3. If $xref$ is defined as the decision variables of the current solution, then with binary variables, *abs*, the following constraints can simply be added to the *master*:

$$x_{ijc}^{e} - xref_{ijc}^{e} \leq abs_{ijc}^{e} \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \tag{3.23}$$

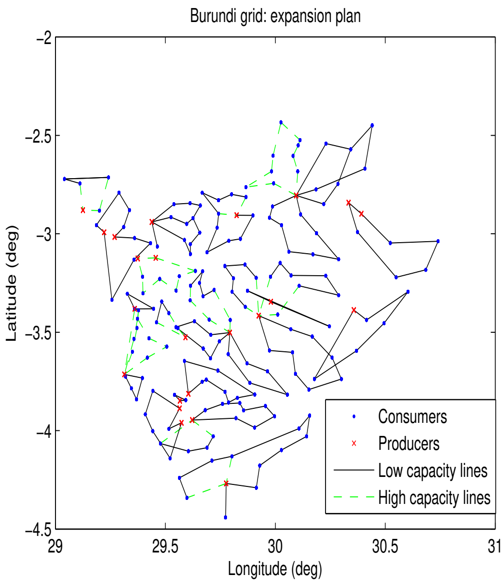
$$xref_{ijc}^{e} - x_{ijc}^{e} \leq abs_{ijc}^{e} \qquad \forall (i,j) \in E, c \in C, e \in CL_{ijc} \tag{3.24}$$

$$\sum_{(i,j) \in E, c \in C, e \in CL_{ijc}} abs_{ijc}^{e} \leq \text{size} \tag{3.25}$$

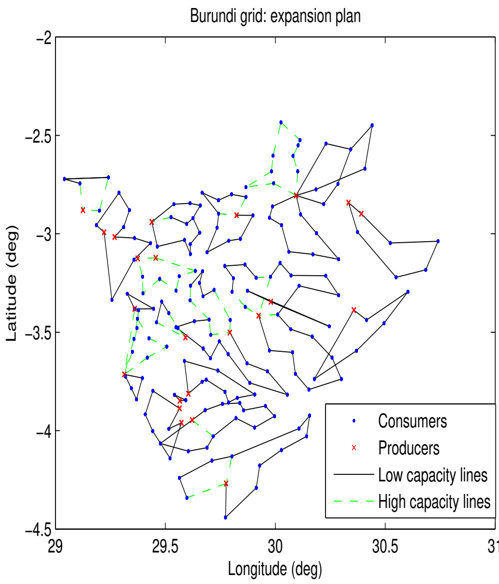
The first reference solution ($xref$) is the union of the individual solutions of each subgroup. The process stops when no better solution is found during the current *iteration of the local search* (not to be confused with an iteration of the decomposition). The reader should note that each iteration of the search requires a resolution of the problem with Benders Decomposition. The process differs then from the local search speed-up technique presented in section 3.1.3, that updated the reference solution (called *xold* in that section) at each iteration of the decomposition. The approach described in this section, only updates the parameters $xref$ after full convergence of Benders Decomposition.

Results

The solution obtained, with the local searches performed inside the large groups, with a neighbourhood of size 6, is presented with figure 3.3b. This plan costs **102.1** MEUR. The local search was performed in 364 seconds for all 7 large groups (that contain more than 1 subgroup). It was done with the *suboptimal master* speed-up technique (section 3.1.3) with a *mip gap* set to 10^{-1}. The maximum needed number of iterations of local search was 3 and the maximum time for one group was 182 seconds.



(a) Merged groups



(b) Merging and local search between subgroups

Figure 3.3: Plan obtained with Benders Decomposition on the refined partition with improvements

3.2.3 Local Search between large groups

Description

Several concerns might be raised, looking at the plan of figure 3.3b, about the line crossings between the groups. Although it has been shown in section 3.1.5 that a line crossing does not necessarily mean suboptimality, some of them seem improvable.

The following technique aims at fixing it.
It starts by detecting suspicious line crossings and then it merges the groups involved. Not all crossings are retained, but only those that involve a node whose two closest neighbours are in another group than its own.

A local search (as described in section 3.2.2) will then be performed on the different merged groups independently. The first reference solution ($xref$) is the union of the individual solutions of each group. The set of edges is defined as the connections included in the individual plans and some additional edges. The definition of those additional edges is illustrated with figure 3.4. The figure presents two groups and their optimal plan (with black solid lines). Since a line crossing is detected, the two groups will be merged and solved as one.

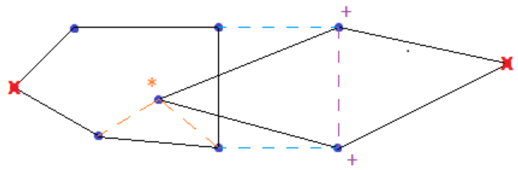


Figure 3.4: Example of the set of edges for local search (the additional edges are represented with dashed lines).

The nodes, whose two nearest neighbours are not in the same group as they are (possibly involved in a crossing), are detected ($*$ in the figure) and connections to these two neighbours are added to the candidate list (orange lines). Moreover, additional connections between the nodes (+ in the figure) linked to these interesting nodes are also added (allows one to close the loop: purple line) and connections between those nodes and their nearest neighbour in the other groups are considered as well (allows one to extend the loop: blue lines in the figure).

Results

As before, the local search is performed iteratively with a neighbourhood of size 6 and Benders Decomposition. Four line crossings between the groups have been detected implying three merged groups (two line crossings involved a same group). The obtained plan is presented with figure 3.5.

This plan costs **99.3** MEUR. The local search was performed in 836 seconds for all 3 merged groups. It was done with the *suboptimal master* speed-up technique (section 3.1.3) with a *mip gap* set to 10^{-1}. The maximum needed number of iterations of local search for one merged group was 6 and the maximum time was 779 seconds.

These last sections showed that improvements could be made by going beyond the limitations of the partitions with local searches.

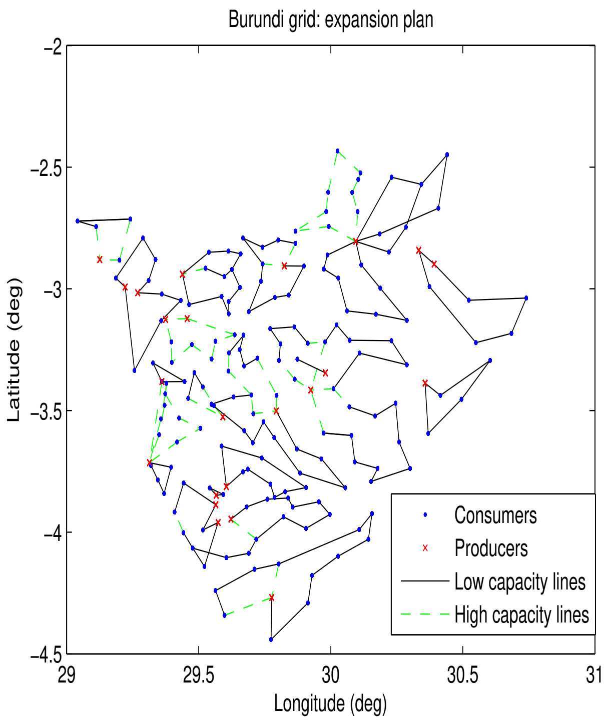


Figure 3.5: Plan obtained with Benders Decomposition on the merged groups combined with local searches between *subgroups* and *groups*

3.3 Further work

In this chapter, the proposed method has been applied to the given problem. However, this method could be considered to solve a modified version of the model. For example, the generation costs might be included. This might imply modifying the partitioning technique in order to take this new feature into account.
Another option would consist in keeping the same partition as before but considering the generation costs for the local search between the groups. With the introduction of this operational cost, Benders Decomposition would then involve both feasibility and optimality cuts.
Although both approaches might not imply highly complex modifications to the method, they will not be presented in this work. They would indeed require additional work to provide the suitable adjustments and analyze the results.

Chapter 4

Summary of the results

Different methods have been considered to tackle the problem of transmission network design in Burundi. Table 4.1 presents a summary of the main results obtained with the techniques addressed in this part of the thesis. The different approaches are compared in terms of objective functions and computational times.

Model	Lower Bound	Radial	DE on p.	AIS on r.p.	Benders	Proposed Method
Cost (MEUR)	14.0	361.8	232.9	129.9	110.7	99.3
Time	< 1s	< 1s	≈ 1h	≈ 1h	17 min	37 min

Table 4.1: Summary of the results obtained for the onshore grid in Burundi

The main features of the different approaches are summarized with figure 4.1.

The reader should recall that the lower bound presented here does not correspond to a feasible plan. Moreover, it should be noted that the costs given above do not include the mandatory cost associated to the substations.

The results tend to show, once again, that the partitioning approach is meaningful when the problem is too complex to be solved as a whole. Partitioning the nodes and even keeping control of the group sizes allow significant savings to be made. (The notations *p.* and *r.p.* refer respectively to the partition of section 2.3 and refined partition of section 2.7).

Moreover, although the refined partition enabled one to improve considerably the solution obtained by the meta-heuristics, they still failed at finding the optimal solution of the subgroups and so were outperformed by Benders Decomposition.

The proposed method combines a partitioning technique that keeps the groups tractable, a Benders Decomposition that solves optimally the partition and an iterative local search to improve the solutions obtained. It allows the planner to make significant savings (around **260 millions of euros**) while solving efficiently the problem (around 40 minutes).

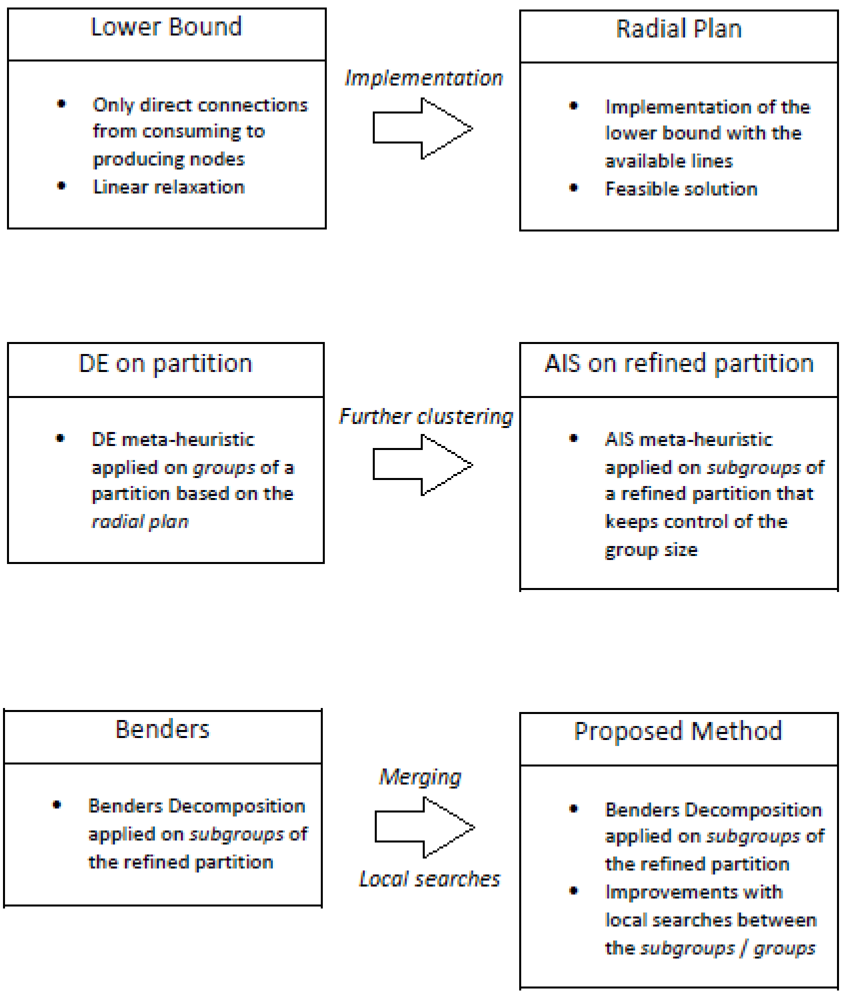


Figure 4.1: Summary of the methods for the onshore grid in Burundi

Conclusion

In this part, another problem of transmission expansion planning was tackled. It consisted in designing the network of an electricity network to serve power to scattered consumption sites. The methods were tested on the practical case of the Burundi grid. The problem included the additional constraint of the *N-1 security* criterion in order to guarantee a higher level of reliability.

The problem being more complex than the one handled in the offshore case, specific methods had to be implemented in order to find satisfactory solutions. Once again, the partitioning techniques revealed themselves to be really useful. Due to the high complexity of the problem, the approaches based on a refined partition outperformed significantly the others. The proposed technique enables one to keep control of the group size and therefore the tractability of the problems.

Two types of methods were presented in this work: meta-heuristics and Benders Decomposition. The advantage of the former is their flexibility and their ability of finding a feasible solution, even for highly complex problems. However, the complex tuning of the parameter and their sensitivity to the problem size constitute significant challenges. The partitioning technique provided a respond to this concern. Nevertheless, the methods struggled at reaching the optimal solutions and assessing the quality of the obtained solutions is not an easy task.
For these reasons, the proposed method is based on Benders Decomposition. Since it is a mathematical optimization method, the optimal solution is then guaranteed to be reached. This strategy enabled one to solve the groups of a refined partition optimally and provided significant savings. Moreover, two techniques were presented and discussed in order to speed-up the convergence.

A heuristic approach, based on an iterative local search, was also presented and analyzed in order to go beyond the limitations imposed by a strict partition. The individual plans of the groups of the partition can then be used as reference solution to be improved by the method.

The methods were tested on the particular problem of the Burundi grid. Nevertheless, their formulation is general and so they can be applied to similar problems of network designs. These problems might though involve other constraints and so adjustments might need to be made.

General Conclusion

This master's thesis aimed at providing an answer to two hard challenges that the sector of the transmission of electricity will face in the next years and decades. The first task is the efficient integration of powers from renewable sources that are far from the consumption sites. The second is the quick growth of the electricity networks in some developing areas of our planet. Both problems consist in designing transmission networks almost from scratch, with minimal number of elements defined beforehand.

In order to develop suitable methods to solve those problems, the general class of problems, to which they belong, was firstly the subject of attention. The *Transmission Expansion Planning* problem was then studied through a comprehensive literature review. An overview of the different formulations and methods found in the literature was then the first part of this work.

In the light of this review, a practical case was then tackled. It consisted in designing the transmission network in order to connect the offshore wind farms to the onshore connection points, in the North Sea. Through this example, several conclusions could be drawn. This work revealed that the meshing strategy, consisting in defining locations of potential substations systematically, was not adequate. One should rather develop specific methods to define a substation candidate list. Moreover, the partitioning techniques showed very promising results, outperforming significantly the approach of solving the problem as a whole. This work provided both a method to find good substation candidates and a meaningful partitioning technique. A classical solver was then used to solve the problem for the different groups.

The second problem concerned an onshore network and aimed at connecting the scattered consumption sites to the producing locations. It was based on the grid in Burundi. This problem showed the limitations of the previous method since it could not deal with the increased complexity. The modified model, notably, added a security constraint to provide a higher level of reliability of the network.
A statistical analysis was performed to assess the possibility of neglecting the additional substation candidates. It led to conclude that the simplification could indeed be considered to reduce the search space for complex problems. The security constraint might also reinforce this statement. Nevertheless, the tackled problem showed that even with this simplification, the task might still be difficult. Therefore, this work provided another partitioning technique that enables one to keep control of the group sizes.

If their formulation is suitable, Benders Decomposition might then be considered to solve the problems associated to the groups, instead of the classical solver. This work showed that this method could provide good performances when a *N-1* security constraint is included in the model.
If the complexity of the problem increases further, meta-heuristics, such as evolutionary algorithms, could be implemented. However, one should then be careful about the tuning of the parameters and their sensitivity to the problems.

This thesis also provided an approach to improve the solutions obtained with the partition through local searches.

This work revealed that it is meaningful to dedicate time and resources to implement efficient methods in order to design these transmission networks. With appropriate techniques, significant savings could indeed be made.

Obviously, the models solved with the implemented methods do not take everything into account. For example, the costs are approximated, the topographies of the areas treated are not considered, the physics laws are simplified,... Therefore, the plans obtained thanks to the methods cannot be implemented straight away in practice, but require a second phase of assessment and modifications. This stage should, of course, be the subject of high attention and might require further researches to be performed efficiently. However, this goes beyond the topic of this master's thesis.

Appendices

Appendix A

Proof of proposition 1

This appendix provides the proof of proposition 1, presented in chapter 5, part II. The proposition is recalled below.

Proposition 1. *For the problem of connecting two farms to one onshore connection point without substation cost, under the assumption that the costs of lines are equal for all the connections, the relative gain of adding a new node to the network is upper bounded by* $1 - \frac{\sqrt{3}}{2}$.

Before providing the proof, notations are firstly introduced with figure A.1.

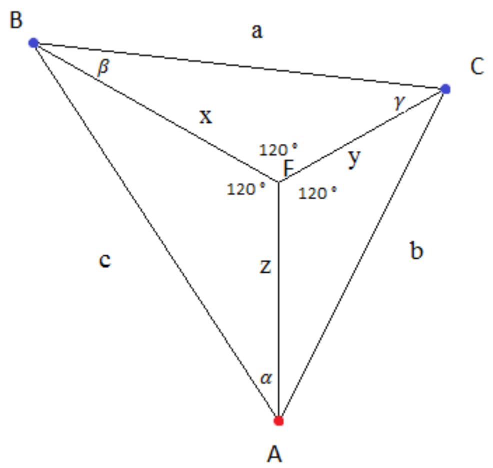


Figure A.1: Introduction of the notations for the proof of the theoretical bound

It has been proven that if all the angles of the triangle have an amplitude lower than 120°, then the Fermat point is inside the triangle and the line segments joining the vertices form angles of 120°. Otherwise, the Fermat point is on the vertex that presents the obtuse angle [92]. In the latter case, the relative gain is 0. Hence, in order to get the upper bound, it is assumed, in what follows, that the angle with the largest amplitude γ is such that $60° \leq \gamma < 120°$. Moreover, the shortest path, restricted to the existing nodes, will go through the vertex associated to this angle. Therefore, with the notations of figure A.1 and assuming a unitary cost per unit of length, the best cost that can be achieved to join the farms B and C to the onshore point A, without additional node, is given by $a + b$. Furthermore, the cost of joining those farms through their best connection point F is given by $x + y + z$.

With these notations, proposition 1 can be written as follows:

$$\boxed{(x + y + z) \geq \frac{\sqrt{3}}{2}(a + b)}$$

The proof can now be given, with the help of the presented notations.

Proof. The following formulas can be deduced.

$$\begin{aligned}
(x+y+z)^2 &= x^2 + y^2 + z^2 + 2(xy + yz + xz) && \text{(A.1)}\\
&= \frac{1}{2}(a^2 + b^2 + c^2) + \frac{3}{2}(xy + yz + xz) && \text{(A.2)}\\
&= \frac{1}{2}(a^2 + b^2 + (a^2 + b^2 - 2ab\cos(\gamma)) + \frac{3}{2}(xy + yz + xz) && \text{(A.3)}\\
&= a^2 + b^2 - ab\cos(\gamma) + 2\sqrt{3}\text{Area(ABC)} && \text{(A.4)}\\
&= a^2 + b^2 - ab\cos(\gamma) + \sqrt{3}ab\sin(\gamma) && \text{(A.5)}\\
&\geq a^2 + b^2 - \frac{1}{2}ab + \frac{3}{2}ab && \text{(A.6)}\\
(x+y+z)^2 &\geq a^2 + b^2 + ab = (a+b)^2 - ab && \text{(A.7)}\\
(x+y+z)^2 &\geq \frac{3}{4}(a+b)^2 && \text{(A.8)}
\end{aligned}$$

One goes from line A.1 to line A.2 by applying the law of cosinus on the small triangles ABF, ACF and BCF. For example, for the triangle BCF, one has

$$a^2 = y^2 + x^2 - 2yx\cos(\widehat{BFC}) = y^2 + x^2 - yx$$

The next equality is obtained by applying this law of cosinus to the triangle ABC.
The following line (A.4) is found by noticing that the area of the triangle ABC can be written as follows.

$$\text{Area(ABC)} = \frac{1}{2}xy\sin(\widehat{BFC}) + \frac{1}{2}yz\sin(\widehat{BFA}) + \frac{1}{2}xz\sin(\widehat{AFC}) = \frac{\sqrt{3}}{4}(xy + yz + xz)$$

Equality A.5 results from another expression of the area of the triangle ABC.
Inequality A.6 is obtained by taking the minimum value of the expression for the domain of the angle $\gamma \in [60°, 120°[$. This minimum is obtained with $\gamma = 60°$ (both minimum value of the sinus and maximum value of the cosinus).
A.8 is derived from A.7 by noticing that $(a+b)^2 \geq 4ab$ since $(a+b)^2 - 4ab = (a-b)^2 \geq 0$.
A.8 implies trivially that

$$(x + y + z) \geq \frac{\sqrt{3}}{2}(a + b)$$

which was the thesis.

□

Appendix B

Parameter analysis of the meta-heuristics

This appendix provides some details of the parameter analysis mentioned in section 2.5, chapter 2, part III.

The tests were performed on the group 8 of the partition presented in section 2.3.

In order to be able to compare the results, the complexity of the methods is chosen constant. It corresponds to the resolutions of 10 000 linear programs for one run.

B.1 Differential Evolution

In this section, the impact of the parameters of the Differential Evolution meta-heuristic is studied. The two first parameters discussed are a weighting factor (*mutation factor* F) and the recombination rate (CR). The latter corresponds, in this work, to the probability of taking the components of the vector from the parent (and not from the mutated vector).

The tests revealed that the method is highly sensitive to the recombination rate. This sensitivity is illustrated by table B.1. It analyzes the effect of the recombination rate CR while keeping the mutation factor constant to 0.9. The number of individuals is set to 100 and so is the number of generations. The costs are expressed in MEUR and do not include the constant substation costs (but do include the costs of extra connections at nodes). They represent the best, worst, average and standard deviation of the costs obtained over 10 different runs.

This section will not present the running times of the methods because it is not its purpose. Those considerations are discussed in the section dedicated to the results (section 2.6).

	Best cost	Worst cost	Average cost	Standard dev.
$CR = 0.1$	18.1	23.4	21.5	1.81
$CR = 0.3$	16.0	23.3	20.8	1.60
$CR = 0.5$	18.8	21.3	19.5	1.86
$CR = 0.7$	14.0	18.3	16.5	1.40
$CR = 0.75$	13.9	17.4	15.6	1.39
$CR = 0.8$	13.7	16.1	14.8	1.37
$CR = 0.85$	12.6	16.5	14.5	1.26
$CR = 0.9$	12.8	17.0	15.6	1.28
$CR = 0.95$	14.5	19.1	16.8	1.45
$CR = 0.99$	19.5	20.9	20.3	1.94

Table B.1: Results DE - impact of the recombination rate (costs expressed in MEUR)

It can be noticed that the difference is quite significant between the different runs and thus the population size or the number of generations should be increased in order to obtain more stable results. However, this would increase the computational time and this is not the purpose of this analysis. From the results above, the best value for the recombination rate seems to be 0.85 which provides the best mean and standard deviation.

A similar analysis was performed for the mutation factor (F) which led to conclude that 0.7 is a good value for this parameter.

In order to improve the solutions obtained by the method, the size of the population or the maximum number of generations may be increased. However, this would also raise the computational time of the method. Changing the ratio of those two parameters, with their product constant, might affect the solutions, while keeping the same complexity. Table B.2 presents the results obtained on the same group as above, with different values of those two parameters. Their product is however kept equal to 10 000 as above. The values given in the first column correspond to the ratios between the maximum number of generations and the population size. As before, the costs are expressed in MEUR.
The other parameters are kept constant and set to their best found value ($F = 0.7$ and $CR = 0.85$).

	Best cost	Worst cost	Average cost	Standard dev.
$ratio = 0.25$	14.6	18.6	16.7	1.02
$ratio = 1$	12.3	15.8	14.2	1.13
$ratio = 4$	10.1	13.9	11.9	1.22
$ratio = 25$	9.3	11.7	10.3	0.75
$ratio = 100$	9.9	18.0	13.1	2.69

Table B.2: Results DE - impact of the ratio: number of generations/population size (costs expressed in MEUR)

The results tend to show that this ratio is a crucial choice for the performance of the method. The best value found is a ratio of 25 (which corresponds to 500 generations and 20 individuals in the population).

B.2 Artificial Immune Systems

In this section the impact of the parameters of the CLONALG method is studied.

The tests revealed that the method is highly sensitive to the mutation magnitude (σ). Its impact is analyzed in table B.3. The other parameters are then kept constant. In order to have comparable computation times, the size of the population, the number of clones and the number of generations are chosen such that their product (number of LP problems to solve for 1 run) is the same (10000). They are set to 20, 10 and 50. The number of clones selected in each group is set to 5.

	Best cost	Worst cost	Average cost	Standard dev.
$\sigma = 0.1$	25.0	25.0	25.0	0
$\sigma = 0.2$	10.1	13.5	11.6	1.24
$\sigma = 0.3$	9.5	11.9	10.3	0.79
$\sigma = 0.4$	11.2	14.4	13.2	0.98
$\sigma = 0.5$	14.5	17.6	16.7	1.21
$\sigma = 0.7$	19.1	24.8	22.0	1.86
$\sigma = 0.9$	21.6	25.0	24.2	1.19

Table B.3: Results AIS - impact of the mutation magnitude (costs expressed in MEUR)

The best value seems to be 0.3 (also presents the lowest standard deviation among the good values for the parameter).

Other similar analyses were performed for the other parameters. The method appears to be less sensitive to changes in their values. However, they still do have an impact on the results. These parameters are the number of selected clones and the ratios between the number of individuals (population size) and the number of clones, and between the number of generations and this population size.

The number of selected clones in each group is a good example of the compromise between diversification and intensification. A high value for this parameter will imply a high intensification since a lot of clones of a same good solution can be included in the population. Low values will privilege diversification.
The best found value for this parameter seems to be 3 for $\sigma = 0.3$ and the other parameters set as above.

If one wants to keep the complexity of the problem almost constant, the number of individuals and the number of clones must be linked together. Three ratios were tested (2:1, 1:1, 1:2) and the best one, among them, appears to be 1:1. For a problem of equivalent complexity as the one above, this gives a population size and a number of clones equal to 14.

The same analysis can be performed with the number of generations and the number of individuals. Table B.4 presents the results with different values of these parameters. The number of clones is set equal to the population size and the number of selected clones to $\frac{3}{10}$ of this value. The mutation magnitude is kept constant to 0.3. The values given in the first column, correspond to the ratio of the maximum number of generations and the squared value of the population size (product of the population size and the number of clones). The product of the number of generations, the population size and the number of clones is kept (almost) constant to 10 000. The complexity is then stable.

	Best cost	Worst cost	Average cost	Standard dev.
$ratio = \frac{1}{16}$	9.3	12.2	10.8	0.99
$ratio = \frac{1}{4}$	9.0	11.1	9.8	0.76
$ratio = 1$	8.4	10.7	9.6	0.76
$ratio = 4$	8.8	9.8	9.2	0.33
$ratio = 16$	9.0	10.3	9.4	0.39

Table B.4: Results AIS - impact of the ratio: number of generations/(population size)^2 (costs expressed in MEUR)

The results tend to show that this ratio is not the most significant factor for this method. However, its influence on the performances is not negligible. The best value found is a ratio around 4 (which corresponds to 200 generations and 7 individuals in the population).

B.3 Commentary on the analysis

This section aims at discussing some remarks about the parameter analysis presented above.

The first point concerns the chosen methodology to find the best values of the parameters. The impacts of the different parameters of the methods were tested independently and sequentially. Yet, their interactions might also affect the performances of the methods and so the best value for one parameter might also depend on the values of the others. A more comprehensive analysis would then include variations of the different parameters simultaneously, in order to assess their influence on one another. One could also consider an iterative methodology where one parameter

is optimized after the other, in a loop repeated several times.
This work does not include any of the two proposed methods because of two main reasons. This more complete analysis would severely increase the time consumption and since the proposed method does not include any meta-heuristic, it does not seem essential in the present work.

Secondly, the reader should note that the tests were only performed on one group of the partition (and therefore only one problem size). The best values for the parameters might vary though, according to the problems that are treated and their size. Nevertheless, tuning the parameters for all the different groups seems to be out of reach. In chapter 2 of part III, the best values found for group 8 are then used for all the groups. This is then expected to impact the performances of the method. However, another approach is presented that will partially respond to this concern (section 2.7).

Finally, the choice of the best values for the parameters may need to be addressed briefly. In this work, the decisions were obvious since the chosen values both presented the best mean and the lowest standard deviation. However, if it is not the case, one might need to design a rule to choose the *best* values. This rule might depend on the average behavior and the standard deviation obtained over the different runs.

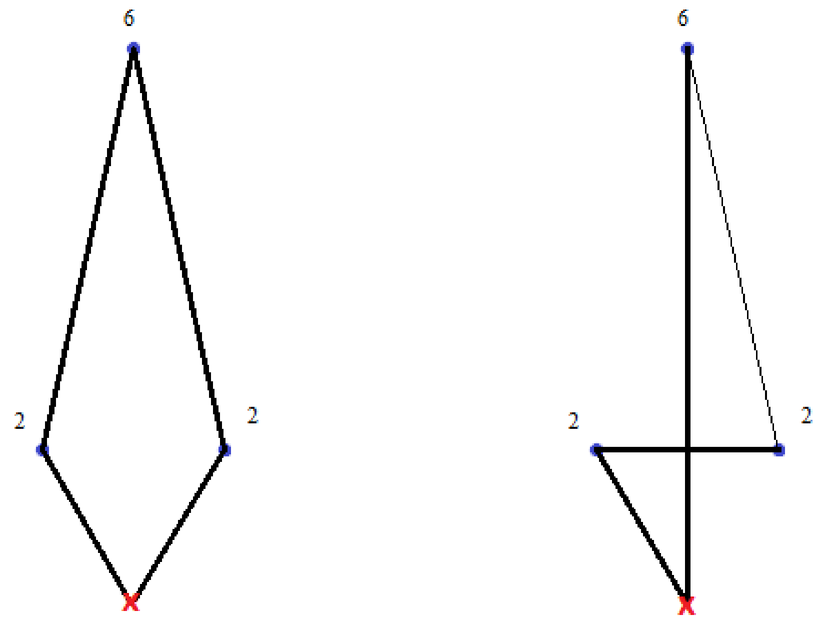
Appendix C

Example of line crossing

In this appendix, an example of problem, for which the optimal solution contains a line crossing, is presented. This appendix responds to the concern raised in section 3.1.5, chapter 3, part III.

For the problem in Burundi, two types of lines are available with capacity of 7.5 and 21.3 MW. If the sum of consumed powers in the subgroup is lower than 7.5 MW then the problem is simply a Travelling Salesman Problem. This is a well-known combinatorial problem that consists in finding the shortest Hamiltonian cycle (cycle that goes through every node exactly once). If the capacity limit is not reached, the problem indeed comes down to finding this shortest cycle and installing only lines of 7.5 MW. In this case, the optimal plan will not include any line crossings.
However, if the sum of powers exceeds the lowest capacity of the lines, this topology is possible in the optimal solution. This is illustrated with figures C.1 below. Assuming that the scale is such that it is more expensive to have additional lines at the nodes (and so pay the extra cost) than using lines with higher capacity, the two figures present the optimal plans for the given group, without and with line crossings. The consumed powers (in MW) are presented in the figure and the lines of 21.3 MW are drawn with a larger line. As before the consuming nodes are blue dots and the producer is a red cross.

It can be shown that because of the security constraint, the lines of 7.5 MW must be replaced by lines of 21.3 MW for most of the connections. The figure on the right shows that connecting directly the larger consumer to the producer allows one to save one high-capacity line. Depending on the difference of costs and distances, this saving can compensate the cost due to increased distance (for example, one can consider the extreme case of a small equilateral triangle, formed by the three nodes at the bottom, far away from the largest consumer). The optimal plan, in this case, does include a line crossing, as illustrated in the figure.



(a) Optimal plan without crossing (b) Optimal plan with crossing

Figure C.1: Example of line crossings

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