

École polytechnique de Louvain

Torsional Response of U-shaped Walls

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Abstract

Whether it is for structural or architectural reasons, many modern buildings are based on the presence of reinforced concrete central cores. These cores take up the lateral loads and are known to have a much bigger stiffness than any other typical elements in the building due to their geometry and larger section size in comparison to gravity load resisting elements (i.e. columns). Such sections are often so-called U-shaped sections, as they are used for lift shafts or staircases and therefore require openings over their height. Moreover, these cores are frequently moved to the periphery of the building for architectural purposes. The consequence of the previous considerations is that in configurations where the U-shaped RC core is placed on the perimeter, center of rigidity (also called the stiffness centroid) will be located close to the core. Knowing that the external resulting lateral forces (e.g., wind and earthquake) will be applied at the center of mass, the result will be a plan asymmetry that gives the configuration the potential to create a torsional response (twist).

The RC U-shaped walls are extensively used in millions of RC buildings around the world, they act as a backbone for the entire building, their location on the perimeter has the capacity to create a torsional response that may lead to important damages and in spite of that there is a really poor amount of researches and experiments that have been focusing on the torsional response of those walls.

It is in this context that this thesis humbly aims to contribute to a better understanding of these crucial elements of modern construction. This contribution is expressed through a comprehensive review of the state of the art, followed by the development of a new mechanical model capable of predicting two central values of the response of walls: the torque at the onset of the first cracks and the ultimate torque before the failure of the element.

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The realisation of this thesis during the past year would have been impossible without the support of a number of people, whom I would like to particularly thank.

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Without the help of all the people in the LEMSC laboratory, this work would only have been theoretical. This is why I would like to thank all of them not only for answering my hundreds of questions but also for their daily good humour which turned the different tasks into a real pleasure.

The people I want to thank now have not just supported me this year, but throughout my life and experiences. I am of course talking about my family, without whom none of this would have been possible. I would especially like to thank my mother, who has helped me to such an extent that it is difficult to put into words. I could perhaps write a thesis on this, but I will confine myself to these few lines to thank her from the bottom of my heart. I don't forget my brothers either, with whom I spent countless exam sessions offering each other support.

This work will also mark the end of my university journey, during which I have met some incredible people. Whether they are friends, colleagues or teachers, I wanted to dedicate these closing lines to thank them.

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Part I

Context

Introduction and Thesis Organisation

“ *Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less.*

— Marie Curie

1.1 Motivation

When designing a building, there is a very wide range of actions that need to be taken into account. From gravity loads (permanent loads due to the weight of the structure or the furniture for example), live loads (due to occupancy or snow for example), accidental loads to even seismic loads and many more. However, all of those loads can be divided into two big categories: vertical loads and horizontal loads. The vertical load path, meaning how the vertical load is transmitted through the structure, is pretty much straightforward and well known. In a few words, in a frame structure with floor slabs, beams and columns, the load is first applied on the floor slab which transfers the load to the beams which in turn transfer it to the different columns down to the foundations. Obviously, other type of structural concepts exist, but often the columns will act as the main vertical bearing element and their behaviour is very well understood.

The horizontal load path, on the other hand, can become a bit trickier. Like the vertical load path, the first step is the transmission of the lateral forces by the floors also called diaphragms. The difference is that this time there are a lot of possibilities regarding the structural system that will be used to withstand those lateral loads. Among all the possibilities, three commonly used structural systems are the moment-resisting frame, the bearing-wall system and the frame/shear-wall hybrid system (Fig 1.1)

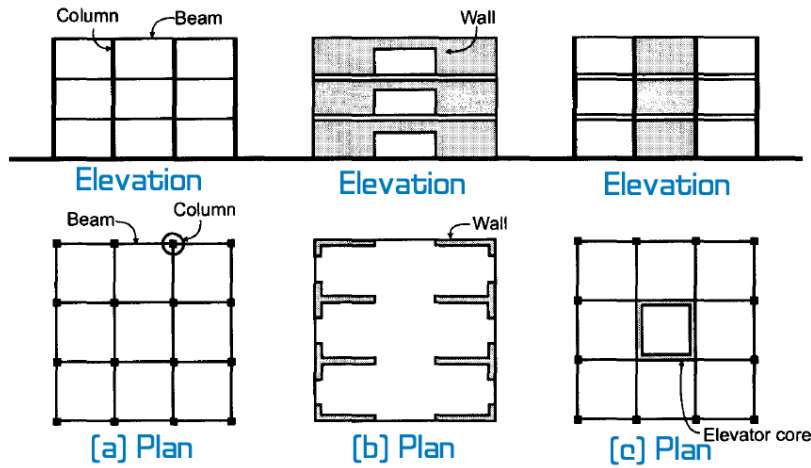


Fig. 1.1.: Figure example: (a) moment-resisting frame, (b) bearing-wall system; (c) frame/shear-wall hybrid system [7]

Accordingly to what is presented on the Fig 1.1, different systems can be combined just like the frame/shear-wall hybrid system. It appears that a very large number of buildings and even more tall buildings are based on this third structural concept. The reinforced concrete (RC) core really acts as the backbone of the entire building and for that reason is a crucial element. The location of which is also a critical point in the design and what is observed is that this element is often placed on the perimeter of the building (Fig 1.2). This is due to the fact that there is an always increasing demand driven by the architects for a more efficient use of the building area. Also linked to the architecture, the shape of the cores is often what is called U-shaped as it is at least partially open on one side. The reason is quite simple: they are used for the lift shaft or as a staircase and need openings for circulation

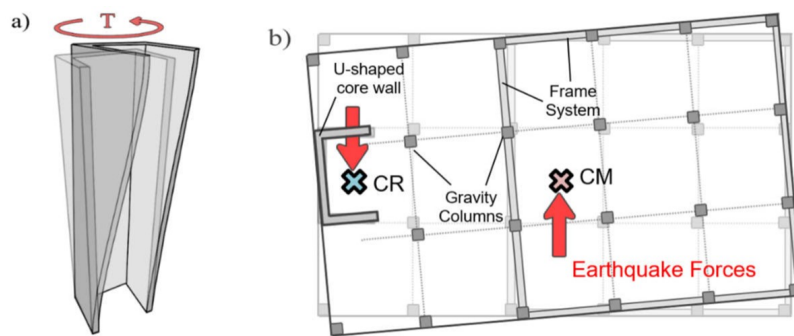


Fig. 1.2.: Outside perimeter RC core [8]

Moreover, RC walls are known to have a much bigger stiffness than any other typical elements in the building due to their geometry and larger section size in comparison to gravity load resisting elements (i.e. columns). This means that in a configuration where the U-shaped RC core is placed on the perimeter, center of rigidity (also called the stiffness centroid) will be located close to the core. Knowing that the external resulting lateral forces

(e.g., wind and earthquake) will be applied at the center of mass (Fig 1.2), the result will be a plan asymmetry that gives the configuration the potential to create a torsional response (twist).

The RC U-shaped walls are extensively used in millions of RC buildings around the world, they act as a backbone for the entire building, their location on the perimeter has the capacity to create a torsional response that may lead to important damages and in spite of that there is a really poor amount of researches and experiments that have been focusing on the torsional response of those walls. Before Dr. Hoult's important contribution to the subject ([10], [8], [12], [11], [9]) the only experiments available were focusing either on U-shaped beams ([32], [19]) or on specimens with different cross-section shapes (rectangular [24] or H-shaped [22]).

Therefore, given their predominant presence in international construction, research on U-shaped reinforced concrete cores are of paramount importance. Even more, considering the ever-present uncertainties regarding their torsional response.

1.2 Objectives

It is in this context that a new research programme has been developed at GCE/LEMSC with the main objective to better understand the behaviour of U-shaped walls under various load cases. This will take the form of, among other things, full-scale experimental tests. As it will be explained in a later section, some techniques used for these tests represent world firsts and have required a great deal of upstream work.

The aim is not only to better understand the torsional response, but also to predict it. The experimental results will therefore be compared to a mechanical model that will be set up based on different pre-existing mechanical models that will be modified to fit the LEMSC, UCLouvain specimen. As with the small number of experiments carried out on U-shaped walls, the majority of the mechanical models previously developed focused on beams ([18], [4], [32]). The difficulty will therefore lie in developing a mechanical model capable of predicting the torsional response not for beams but for walls. In order to understand the influence of certain parameters on the results that will be obtained, a parametric study will be conducted as a final step before concluding this thesis.

1.3 Thesis structure

The present work will be divided into 5 main parts:

1. Part I Context aim to explain the motivation of this thesis subject but also the objectives pursued and how these will be achieved.
2. Part II State of the art seeks to introduce the main theoretical concepts that will be used in the rest of the document but also the different mechanical models available. Theoretical concepts of Saint Venant's torsion and warping torsion will thus be introduced first, followed by some material-related aspects. This will contribute to a better understanding of the different mechanical models of Krpan and Collins [18], Xu et al. [32], Chen et al. [4] and Ryan Hoult [8] which will be presented. Among these, one will be chosen and modified to be used in order to compare the different experimental results presented in the following section with the adapted model.
3. Part III Experimental tests introduces the various experimental programmes that have been set up to better understand the behaviour of U-shaped sections and to compare and refine the predictions of mechanical models.
4. Part IV Analytical formulation presents the modifications and new contributions to the model of Xu et al. which was chosen as the basis for the new mechanical model capable of predicting results for beams like the different models presented but also for the experimental programme carried out at LEMSC, UCLouvain. The various modifications and contributions concern the calculation of the two torques studied, i.e. the cracking torque and the ultimate torque, but also the new boundary conditions provided by a wall. In order to understand the results in greater detail, a parametric study will be carried out following these results on the different parameters with the greatest influence.
5. Part V Concluding remarks and Outlook will ultimately provide the general conclusions of this thesis, but also some leads that can be deepened in the future in order to further develop the knowledge of these U-shaped sections.

Part II

State of the Art

Theoretical Concepts

“*The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them.*

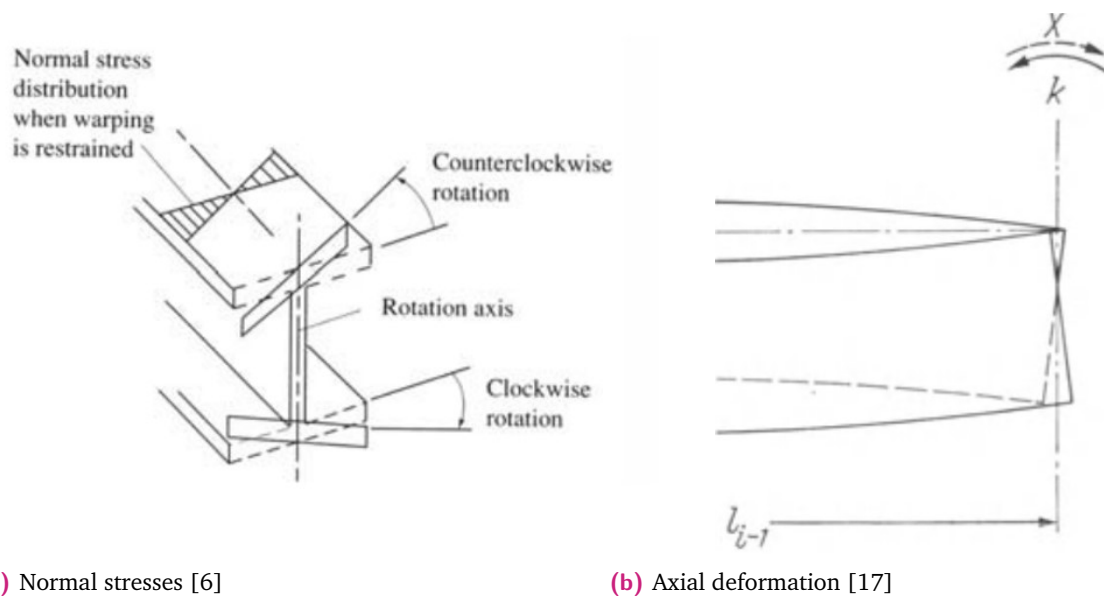
— William Henry Bragg

2.1 Torsion

Since the aim is to analyse the torsional response of U-shaped walls, it is important to discuss some theoretical aspects related to the torsion of these elements that are certainly common but so poorly studied when it comes to torsion. Two aspects greatly affect torsional behaviour and these are the geometry and the boundary conditions. Indeed, these two parameters alone can completely transform the behaviour and therefore the results

Firstly, the geometry of the cross-section has a big influence because, as is well known, for elements with a symmetrical closed cross-section (e.g., circular tube, T-shaped, X-shaped, and L-shaped cross-sections) subjected to a torsional stress, they remain undeformed in their axial direction. This means that all the torsion is resisted by shear stresses that develop in the section's plan according to the Saint Venant's theory described among others by George G. Penelis and Gregory G. Penelis [23] that will be explained in further details in Section 2.1.1. On the contrary, for members with composite non-symmetrical cross-sections (e.g., U-shaped open section) submitted to torsional external moments, they develop axial stresses and strains leading to what is called “warping of the cross-section”. This phenomenon is therefore visible as the section is axially deformed in Fig 2.1b.

Not all sections have warping and even those that do, may have no effect in terms of internal stresses depending on the second aspect which is the boundary conditions. Indeed, if no axial restraint is present, the section will be able to deform freely. On the other hand, being axially restraint will lead to axial stresses (Fig 2.1a).



(a) Normal stresses [6]

(b) Axial deformation [17]

Fig. 2.1.: Warping effects

The link between axial forces and torsional strength is not obvious at first sight. These forces that are not even in the same plane as the torsional shear forces could lead to the wrong conclusion that they have no influence on the torsional strength. However, according to the principle of reciprocity of shear forces [25], they actually generate additional shear forces in the section's plan (Fig 2.2). These will therefore counterbalance the external torsional moment along with the torsional resistance of the Saint Venant's shear stress state.

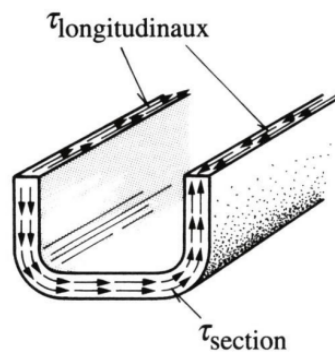


Fig. 2.2.: Reciprocity of stresses [21]

In conclusion, one part of the external moment of torsion $T_{(z)}$ will be resisted by those shear stresses related to the warping effect, $T_{w(z)}$ while the other part will be resisted by the Saint Venant's shear stresses $T_{v(z)}$ that appear again in the section's plan when submitted to torsion (Fig 2.3). To illustrate this effect of shear forces in the section plane in a simple way, one can simply place a hand on a flat surface such as a table and apply a rotation with the hand. this effect should therefore be felt on the palm of the hand.

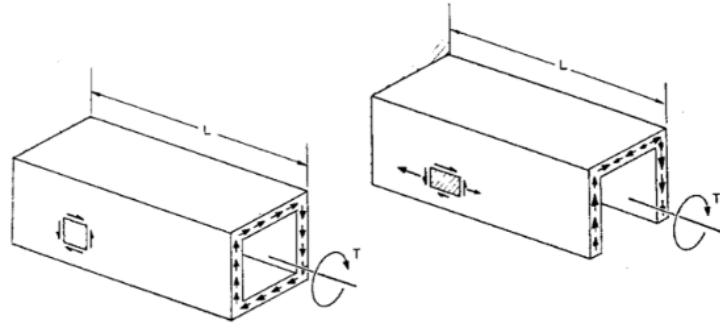


Fig. 2.3.: Warping vs St Venant [18]

The torsional equilibrium equation can therefore be written as follows:

$$T_{(z)} = T_w(z) + T_v(z) \quad (2.1)$$

2.1.1 St Venant

The torsional response of an elastic member when submitted to circulatory torsion can be described using Adhémar Jean Claude Barré de Saint-Venant's theory, as stated before. Clarification on the term elastic is very important and will be covered in the following sections when the different mechanical models are discussed. For the time being, attention will be paid to the theory intensively detailed by Saint Venant, but also subsequently by numerous works ([17], [18], [28], [15], [30]). To do this, one can take the case explained by George G. Penelis and Gregory G. Penelis [23] of a circular and closed thin-wall cross-section cantilever (Fig 2.4) as a basis, but applied to a U-shaped section to obtain the desired equations.

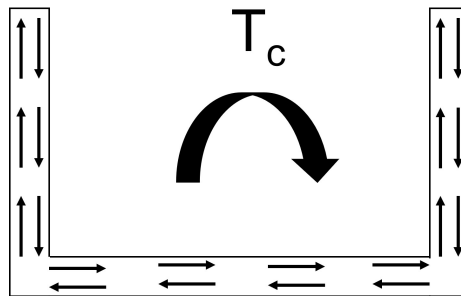


Fig. 2.4.: Idealized shear stresses from circulatory torque for a U-shaped section

The angle of rotation for a distance z from the base of a cantilever U-shaped section is expressed as follows:

$$\theta_z = \frac{T_c \cdot z}{GK} \quad (2.2)$$

Where:

- T_c = twisting moment applied
- z is the longitudinal position to which the twist is referred
- G is the shear modulus of elasticity of the material considered
- K is the torsional constant that can be approximated as $K = \frac{\sum x_i^3 y_i}{3}$ using Bach's equations for a section that consists of narrow rectangles. In which x_i and y_i = the smaller and larger dimensions of the rectangle.

Equation (2.2) can also be rewritten in terms of the derivative of the twist angle:

$$\frac{\partial \theta}{\partial z} = \frac{T_c}{GK} \quad (2.3)$$

Or:

$$T_v(z) = GK \frac{\partial \theta}{\partial z} \quad (2.4)$$

On the other hand, the shear stresses in the section's plan presented in Fig 2.4 can also be expressed in terms of the twisting moment:

$$\tau_{max} = \frac{T_c \cdot x}{K} = \frac{3 \cdot T_c \cdot x}{\sum x_i^3 y_i} \quad (2.5)$$

Uniform or Saint Venant's torsion produce only pure shear stresses that vary from zero at the center line of the section, to maximum value at the perimeter and do not produce any longitudinal stress.

It is important to note that the previously introduced equations are only valid under two strong assumptions:

1. Plane cross-sections in the undeformed state remain plane when torque is applied
2. Cross-sections remain undistorted in their own plane

Typically, for open thin-walled sections, the first of the assumptions is no longer valid and leads to what was presented before, namely warping torsion. Which is in addition to the effects of the Saint Venant's torsion that have just been introduced.

2.1.2 Warping torsion

In 1959 Vlasov published his theory about warping torsion and more precisely the response of elastic members in warping torsion ([31] in Russian and [30] in English). This theory has been the founding basis for many of the very good articles that have followed throughout the years (e.g., [15], [28], [23]). Among these, Krpan and Collins, in their paper on the torsional response of a fixed-fixed U-shaped beam [18], beautifully summarised the various equations related to warping torsion through a comparison with a more widely understood concept, namely the engineer's theory of bending (ETB). It is therefore on the basis of the latter document that warping torsion will be explained in this section.

For a U-shaped wall fixed at the base submitted to an external torque, the distribution of the warping torsion is completely analogous to the distribution of the shear stresses if a lateral force would be applied at the top of the wall as shown in Fig 2.5.

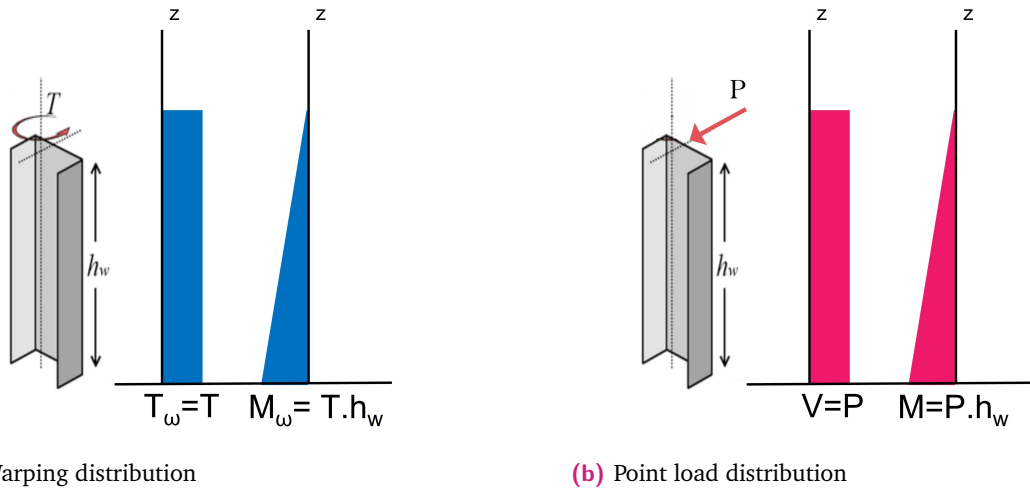


Fig. 2.5.: Load distribution: a) warping torsion and warping moment b) shear force and bending moment distribution

This analogy will continue in the equations, starting with the well-known relation linking the shear force to the moment by the derivative of the latter:

$$V_y = \frac{\partial M}{\partial z} \quad (2.6)$$

This relationship therefore indicates that the shear force is present provided that the moment varies along the length. The shear force consequently suggests that the longitudinal strains also vary along the length, as the following equation applies:

$$\sigma_b = \frac{M_y \cdot y}{I_{yy}} \quad (2.7)$$

Where:

- y = the vertical coordinate of the point considered (see Fig 2.6)
- I_{yy} = the moment of inertia given by :

$$I_{yy} = \int_A y^2 dA \quad (2.8)$$

That can be approximated for thin-walled sections as a line integral:

$$I_{yy} = \int_s y^2 t ds \quad (2.9)$$

In which t = the thickness of the wall and s = a coordinate measured along the length of the section (Fig 2.6)

If the moment induces longitudinal stresses and that to exist the shear has a need of change in the moment, it can only mean that the longitudinal stresses will also change along the length of the element. Of course, the distribution of those stresses follows the assumption that plane sections remain plane leading to a distribution of the longitudinal strains given by:

$$\epsilon = -\phi \cdot y \quad (2.10)$$

Where:

- ϕ = rate of change of the slope in the sectional plane along the wall, also called curvature

For small deflections, ν , Eq (2.10) can be approximated by:

$$\epsilon = \frac{\partial^2 \nu}{\partial z^2} \quad (2.11)$$

This need of variation in longitudinal deformations expressed in Eq (2.6) is very similar to the variation caused by the warping introduced earlier. This analogy is therefore expressed in the following equation, for which the need for longitudinal strain variation in the case of warping is linked to a term called the warping moment:

$$T_\omega = \frac{\partial M_\omega}{\partial z} \quad (2.12)$$

The longitudinal stresses are again related to the moment:

$$\sigma_\omega = \frac{M_\omega \cdot \omega}{I_{\omega\omega}} \quad (2.13)$$

Where:

- ω = principal sectorial coordinate. This coordinate is defined on the center line of the section and for every specific point is calculated as twice the area swept out by the mobile radius vector, POS (Fig 2.6), moving along the centreline from the sectorial origin (S_0) to the position that is under consideration (S)[8]:

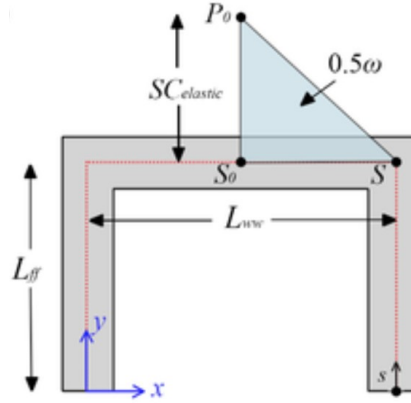


Fig. 2.6.: Principal sectorial coordinate [8]

- $I_{\omega\omega}$ = the sectorial moment of inertia given by:

$$I_{\omega\omega} = \int_A \omega^2 dA \quad (2.14)$$

That can be approximated for thin-walled sections as a line integral:

$$I_{\omega\omega} = \int_s \omega^2 t ds \quad (2.15)$$

In which t = the thickness of the wall and s = a coordinate measured along the length of the section (Fig 2.6)

The consequence of those longitudinal stresses is a strain distribution over the cross-section resulting from warping torsion given by:

$$\epsilon = -\theta'' \cdot \omega \quad (2.16)$$

Where:

- θ'' = the rate of change of twist. Once again, the similarity with the bending theory in Eq (2.10) is clear.

Using this last equation (2.16) together with equation (2.13) and the definition of the young's modulus which is $E = \frac{\sigma}{\epsilon}$ [14], the following relation is obtained:

$$\frac{\partial^2 \theta}{\partial z^2} = -\frac{M_\omega}{EI_{\omega\omega}} \quad (2.17)$$

Knowing that beforehand, Eq (2.12) had been introduced, the equation of warping torsion is finally deduced:

$$T_\omega(z) = -EI_{\omega\omega} \frac{\partial^3 \theta}{\partial z^3} \quad (2.18)$$

In this section, the equation for warping torsion has been progressively explained through a comparison with the engineer's theory of bending (ETB). This comparison revealed an analogy between these two concepts, which is summarised in the following table:

	engineer's theory of bending (ETB)	Warping theory
Moment relation	$V_y = \frac{\partial M}{\partial z}$	$T_\omega = \frac{\partial M_\omega}{\partial z}$
Longitudinal stresses	$\sigma_y = \frac{M_y \cdot y}{I_{yy}}$	$\sigma_\omega = \frac{M_\omega \cdot \omega}{I_{\omega\omega}}$
Inertia	$I_{yy} = \int_s y^2 t \, ds$	$I_{\omega\omega} = \int_s \omega^2 t \, ds$
Strain distribution	$\epsilon = -\phi \cdot y$	$\epsilon = -\theta'' \cdot \omega$

Tab. 2.1.: Analogy between the engineer's theory of bending (ETB) and the warping torsion

2.1.3 Mixed torsion

As introduced in the beginning of this torsion's section, the external moment of torsion $T(z)$ will be resisted by those shear stresses related to the warping effect, $T_w(z)$, while the other part will be resisted by the Saint Venant's shear stresses $T_v(z)$. The torsional equilibrium equation,

$$T(z) = T_w(z) + T_v(z)$$

can therefore be rewritten using the different equations introduced for the St Venant's (2.1.1) torsion and the warping torsion (2.1.2):

$$T(z) = -EI_{\omega\omega} \frac{\partial^3 \theta}{\partial z^3} + GK \frac{\partial \theta}{\partial z} \quad (2.19)$$

2.2 Reinforced concrete behaviour

The specificity of the wall geometry has just been discussed. Now it is interesting to analyse the specificity of the material used. Indeed, not only the geometry and the boundary conditions will influence the torsional response but also the material that will have a great influence on the elaboration of an accurate mechanical model.

Nowadays, in developed countries, all concrete walls are in fact composite walls, as they are made of concrete but also of steel reinforcements bars. The reason these two materials are put together can be deduced from the respective stress-strain graphs (Fig 2.7). Looking at the concrete (Fig 2.7a), it is clear that its tensile strength is negligible compared to its compressive strength. Indeed, the tensile to compression ratio between the two is generally 10 to 15 percent. Knowing that an average concrete will have a compressive strength of around 30 MPa, the resulting tensile strength will be around 3 MPa. This poor behaviour in tension is not present for the steel rebars that have the same behaviour in both tension and compression. This remark is valid at least up to the yield point because beyond that the behaviour of steels in compression and tension may differ. Indeed, a steel that has been put in tension and then in compression could express a buckling effect during the latter, as shown in Fig 2.7b. In any case, provided there is a good bond between the two materials, it is therefore possible to use the good tensile behaviour of steel to compensate for that of concrete in a serviceable state. This serviceability clarification is important because if the load is extreme, it is expected that the bond will dissipate in areas where the steel yields and behaves inelastically.

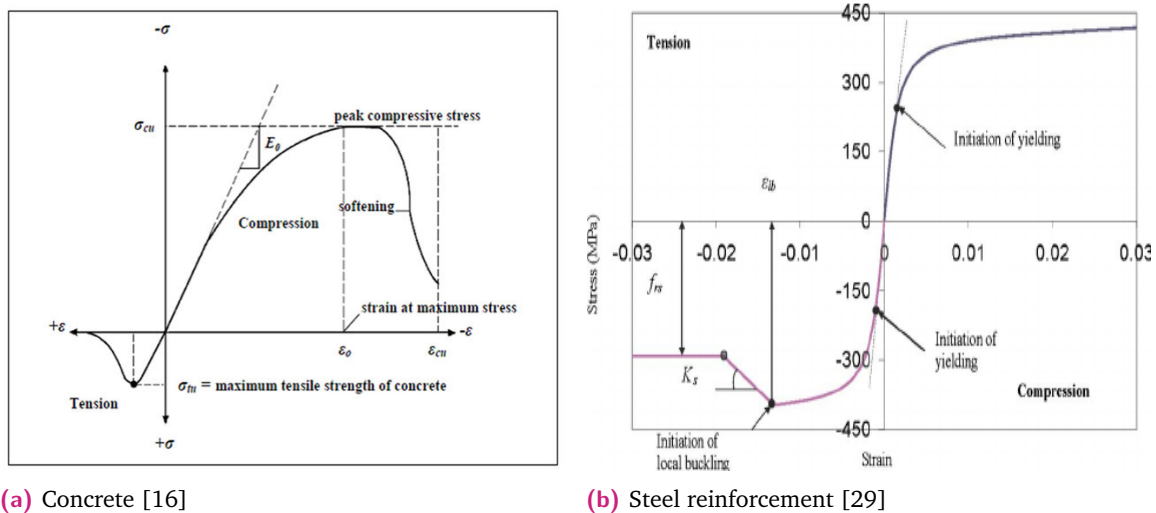


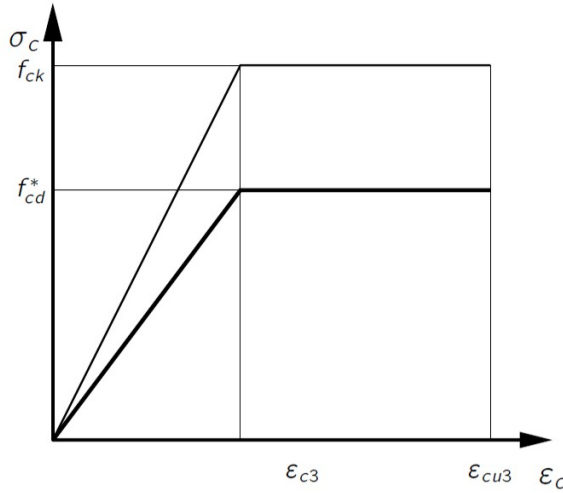
Fig. 2.7.: Stress strain curves for both concrete and steel reinforcement

That being said, when designing, it is not the actual curves that are used but simplified/approximated curves. There are various models in the literature. Regarding the concrete, it goes from the simplest, such as a bi-linear stress-strain relation (Fig 2.8a), to more accurate

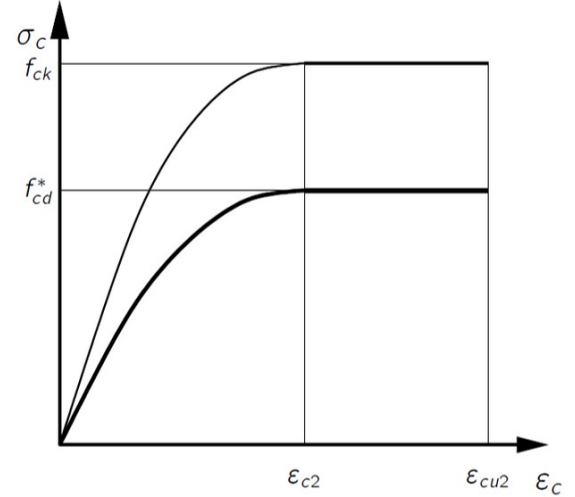
approximations like a parabola rectangle relation (Fig 2.8b). Those are used for design purposes and present in the Eurocode 2 [26].

$$\sigma_c = \begin{cases} f_{cd} \cdot \frac{\epsilon_c}{\epsilon_{c3}}, & \text{for } 0 \leq \epsilon_c \leq \epsilon_{c3} \\ f_{cd}, & \text{for } \epsilon_{c3} \leq \epsilon_c \leq \epsilon_{cu3} \end{cases}$$

$$\sigma_c = \begin{cases} f_{cd}^* \cdot \left[1 - \left(1 - \frac{\epsilon_c}{\epsilon_{c2}} \right)^n \right], & \text{for } 0 \leq \epsilon_c \leq \epsilon_{c2} \\ f_{cd}^*, & \text{for } \epsilon_{c2} \leq \epsilon_c \leq \epsilon_{cu2} \end{cases}$$



(a) Bi-linear stress strain relation



(b) Parabola-rectangle stress strain relation

Fig. 2.8.: Stress strain approximation's models [26]

As for the idealized curves of steel, similarly, different models exist except that the majority are linear models. Most often, they are bi-linear models like those used in the Eurocodes (Fig 2.9a). In order to take into account a phenomenon called the yield plateau, some models are trilinear (Fig 2.9b). This last phenomenon is not present in all cases and depends on the manufacturing process, which is why the Eurocodes do not take this model into account.

Now that the individual material behaviours are clear, it remains to analyse the most important ones, namely the behaviour of the combination of the two. To illustrate this, let's start with the most basic example possible, i.e. an isostatic beam bent on two supports by means of a point force at mid-span (fig 2.10). The bottom half of the beam is in tension while the top half is in compression.

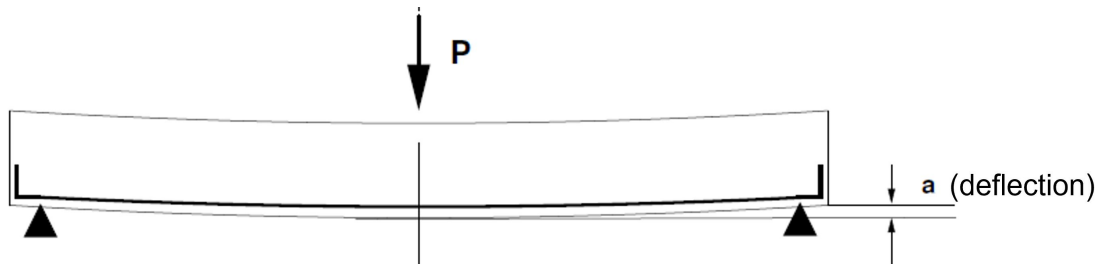
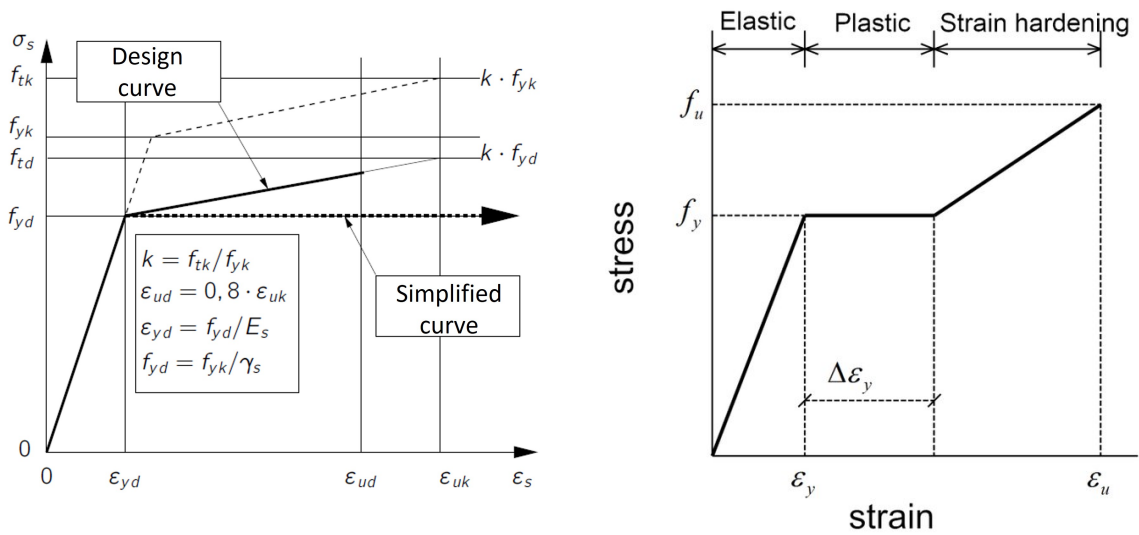


Fig. 2.10.: Bent beam on 2 supports [3]



(a) Bi-linear stress strain relation [26]

(b) Trilinear stress strain relation [20]

Fig. 2.9.: Stress strain approximation's models

Regarding the tension zone, once the beam has cracked, the assumption is that the concrete is fully bonded to the steel and provides negligible tensile capacity. It goes without saying that in reality the concrete also participates in traction, as shown in the following graph in which the bending moment as a function of the vertical deflection is represented:

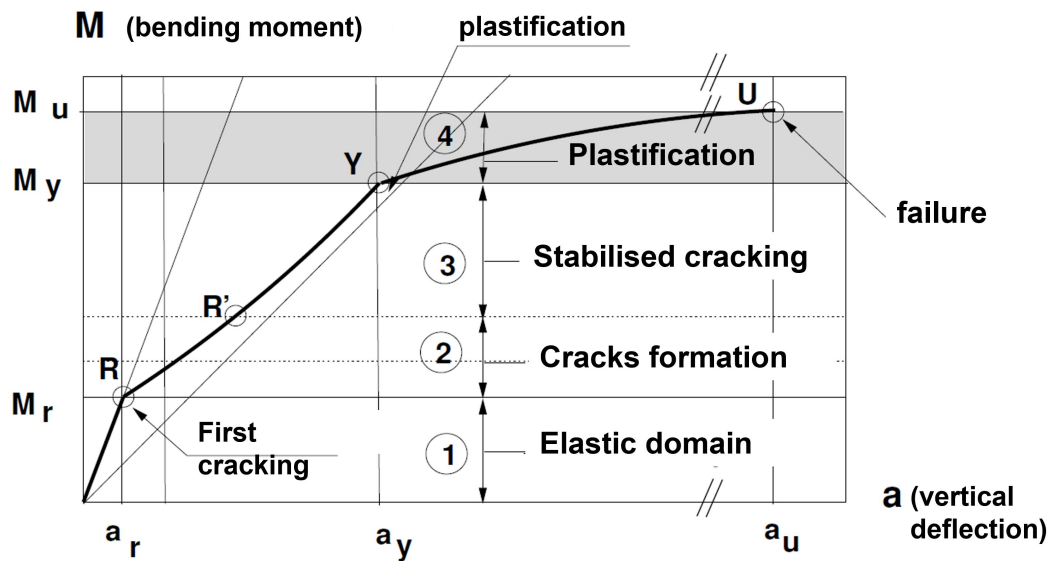


Fig. 2.11.: Moment-deflection curve [3]

The beam will go through 4 different stages:

1. Elastic domain: at this point even though the concrete has a very poor tensile strength, it's still sufficient and no cracks appear. The behaviour is completely elastic and reversible (i.e. no damage)

2. Cracks formation: the tensile strength of the concrete is exceeded and cracks begin to appear with an increasing number
3. Stabilised cracking: the number of cracks is now stable and only the length and the width of those are increasing
4. Plastification: the bars that were taking the major part of the tension enter in their plastic domain until the failure

From the standpoint of compression, concrete is of course not neglected this time. However, as shown earlier with the help of the behaviour curve (fig 2.8b), concrete is not linear throughout the loading. Indeed, for low stresses, the behaviour of the concrete in compression can be considered as linear. While at higher stresses, it's not the case any more. The behaviour and the different phases through which a reinforced concrete element passes will be further detailed in the following section.

Mechanical Models

“It is strange that only extraordinary men make the discoveries, which later appear so easy and simple.

— Georg C. Lichtenberg

Now that the different theoretical concepts related to torsion and the behaviour of the material in question are clearer, it is possible to look at the different existing mechanical models. These models aim at predicting the torsional behaviour which represents, among other things, the angle of rotation at the different stages, the torsional moment at the first cracks or the ultimate torque before total failure.

As briefly explained in the introduction, the number of experiments as well as mechanical models on U-shaped reinforced concrete walls is very low. Moreover, whether the experiments or the mechanical models, most of them dealt in fact not with walls but with beams. Nevertheless, all this research will be useful when developing the mechanical model. Indeed, there is a strong analogy between an embedded beam and a vertical wall with the same section (fig 3.1). The differences will be found in the boundary conditions, but also in the loading, where the axial load differs greatly. For a beam there is usually no axial load, whereas for a wall this represents a large part of the load.

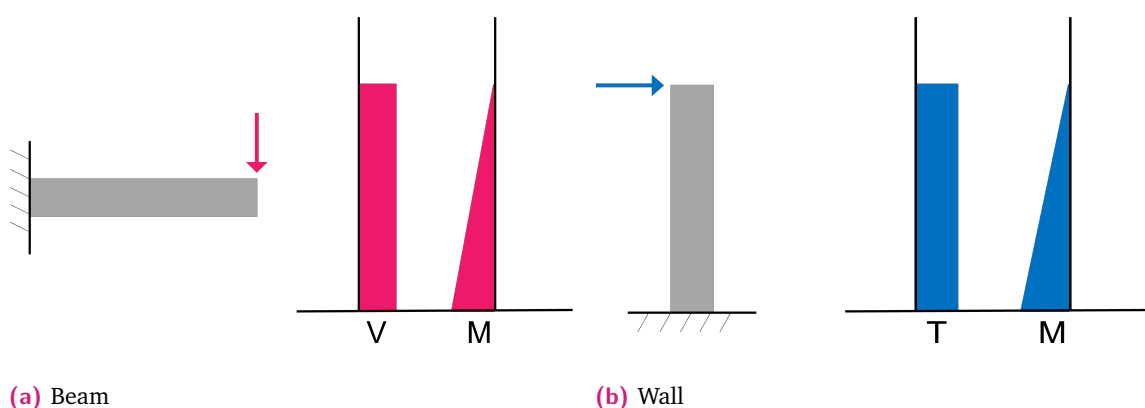


Fig. 3.1.: Beam-wall analogy

The difficulty will therefore lie in modifying one of the existing mechanical models, which will be introduced below. This is possible because although the boundary conditions and loading differ, the basis itself remains the same and can be used.

3.1 Krpan and Collins (1981)

This model has in fact already been partially explained in the section explaining the mixed torsion (section 2.1.3). The subdivision into different phases is also taken from this book [18], which will be explained in further details.

The basis of this model is thus that the torsional response depends on the phase the wall is in. Based on what was said in Section 2.2 about the interaction between concrete and steel as a function of increasing stress, the different phases are as follows:

1. Uncracked elastic: at low stresses, both the steel or the concrete in compression or in tension resist linearly. The response is thus elastic
2. Cracked elastic: at intermediate stresses, the tension, or compression withstand by the steel is still small enough (smaller than f_y) and the compression in the concrete as well (smaller than $0.5f'_c$ so that it can still be considered as linearly elastic.
3. Cracked inelastic: at high levels of stresses, both the concrete and the steel become inelastic.

Uncracked elastic

At this first stage, the equations describing the torsional response of the wall are exactly the ones used in the mixed torsion theory section 2.1.2 and won't be explained again. Let's remember the hypotheses and the final equation.

HYPOTHESES:

Warping component:

- The cross-sectional shape does not change during deformation
- The shear strains on the middle surface of the section are negligibly small

Saint Venant's component:

- Plane cross-sections in the undeformed state remain plane when torque is applied
- Cross-sections remain undistorted in their own plane

$$T(z) = -EI_{\omega\omega} \frac{\partial^3 \theta}{\partial z^3} + GK \frac{\partial \theta}{\partial z}$$

That can be rewritten as follows:

$$\frac{\partial^4 \theta}{\partial z^4} - \frac{GK}{EI_{\omega\omega}} \frac{\partial^2 \theta}{\partial z^2} = \frac{m(z)}{EI_{\omega\omega}} \quad (3.1)$$

Where:

- $m(z) = -\frac{\partial T}{\partial z}$ = the applied distributed external torque per unit of length

Looking at equation (3.1) there is actually a fairly significant variable or rather ratio that can be highlighted. This is the ratio by which the contribution of circulatory torsion increases with distance from the constrained support. It's often expressed as a characteristic length that has the following expression:

$$a = \sqrt{\frac{EI_{\omega\omega}}{GK}} \quad (3.2)$$

This value can be extremely useful both for understanding which phenomenon is dominant in the section and for experiments. Indeed, if the aim is to analyse the warping torsion of a beam or a wall only, it will be convenient to use this characteristic length and to make sure that it is big enough. Because, the longer it is, the more dominant the warping will be over a longer distance. For example, Krpan and Collins explain in their paper [18] for a fixed-fixed beam that at around two or three times this characteristic length of the support and mid-span, almost all the external torque will be resisted by the Saint Venant's torsion.

The evolution of the contribution of the Saint Venant's torsional and the warping torsional effects as a function of the distance from the support(s) for a beam unrestrained against warping can be seen in Fig 3.2. This evolution is analysed for different characteristic lengths, showing the impact that this can have on the results. For a very high L/a ratio (thus a very small characteristic length), the Krpan and Collins hypothesis [18] is confirmed with a very quickly negligible warping contribution near the supports or the mid-span. On the contrary, for a very low L/a ratio (thus a very large characteristic length), the contribution of the circulatory torsion is almost negligible compared to the warping torsion. The impact of the characteristic length on the results will also be analysed more specifically for the specimen tested at LEMSC, UCLouvain in Section 8.4.

As the beam is not long enough in the case of Krpan and Collins experiment [19], the characteristic length is never reached, and it can already be stated that the warping will be present all along the element. On the contrary, if an element was constructed so that the characteristic length was excessively small, almost all the torque would be resisted by the so-called Saint Venant's torsion and warping could be neglected over a large part of the element.

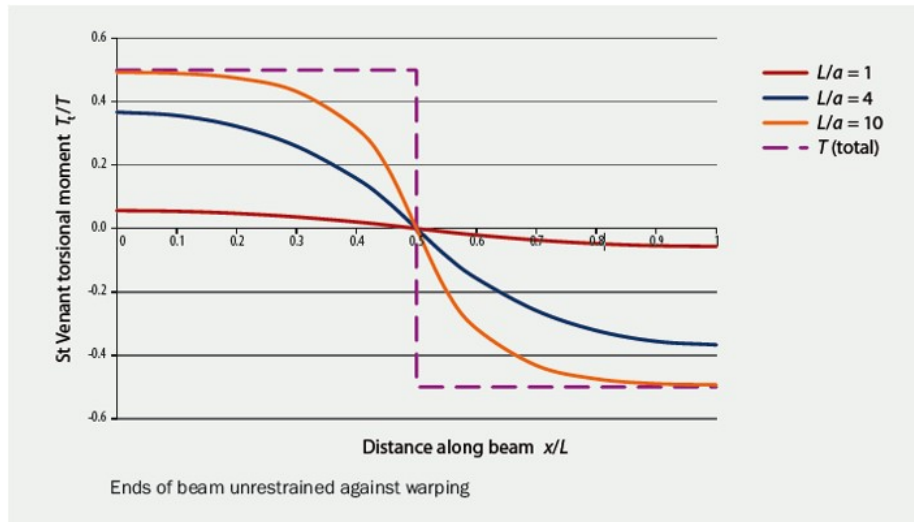


Fig. 3.2.: Variation of warping moment and St Venant's torsional moment in a beam subjected to unit torque at mid-span [13]

Finally, Krpan and Collins [18] make a small correction to one of the assumptions, which says that in the middle of the section the shear forces are negligible. In fact, they are small but not equal to zero. This means that they will also contribute to the torsional response of the member by increasing the final twist. This contribution, $\frac{\partial \theta_\gamma}{\partial z}$, can be quantified and is expressed by the following equation:

$$\frac{\partial \theta_\gamma}{\partial z} = \frac{T_w(z)}{GK_\gamma} \quad (3.3)$$

Where:

- K_γ = the equivalent torsion constant for twist due to shear strains

Cracked elastic

As soon as the tensile stresses in the concrete exceed its strength, cracks appear. However, as already explained, the level of stress remain quite low and the general behaviour of concrete and steel is still considered elastic. This means that the equations introduced for an elastic uncracked section are still valid at this stage.

HYPOTHESES:

- Both the concrete and the steel are linearly elastic
- Concrete in the tensile zone only acts as a shear connector
- In the cracked state, the response of the beam to M_ω does not depend on the value of T_ω and the response of the beam to T_ω does not depend on the value of M_ω

This second assumption leads to some modifications in the previous equations. The first is the reduction of the so-called circulatory torsional stiffness, calculated as the product of the shear modulus and the torsional constant of Saint Venant (GK). Indeed, as the concrete contribution in the cracked tensile zones is neglected, the section to be considered is no longer the same and the sectional variables such as the Saint Venant's constant must be adapted accordingly. The cracked circulatory torsional stiffness is computed as follows:

$$GK_{cr} = \frac{E_s}{2} \frac{4A_0^2}{p_0} \sqrt{\frac{A_l}{p_0} \frac{A_h}{sp}} \quad (3.4)$$

Where:

- E_s = Young's modulus of the steel
- p_0 = perimeter of the shear flow path
- A_l = cross-sectional area of the longitudinal steel rebars
- A_h = cross-sectional area of one leg of the closed hoops
- sp = spacing between hoops
- A_0 = Area enclosed by the shear flow q . This shear flow, is simply the shear stress multiplied by the thickness of the section and is a representation of the circulation of shear stresses caused by circulatory torsion through the section much like a liquid flow (fig 3.3)

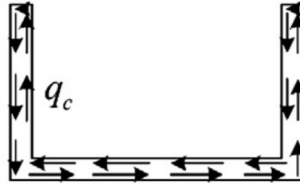


Fig. 3.3.: Shear flow due to circulatory torsion [4]

The equation of the shear flow resulting from a shear force, V , is given by:

$$q = \frac{VQ}{I} \quad (3.5)$$

In which Q = static moment of area given by $Q_y = \int_0^s y \cdot t ds$

This circulatory torsion therefore leads to a reduction in the circulatory torsional stiffness, but this is not the only consequence it brings. Indeed, for cracked elastic elements, circulatory torsion is resisted by means of concrete diagonal compressive stresses between the transverse stirrups giving rise to a spiral all around the element (fig 3.4).

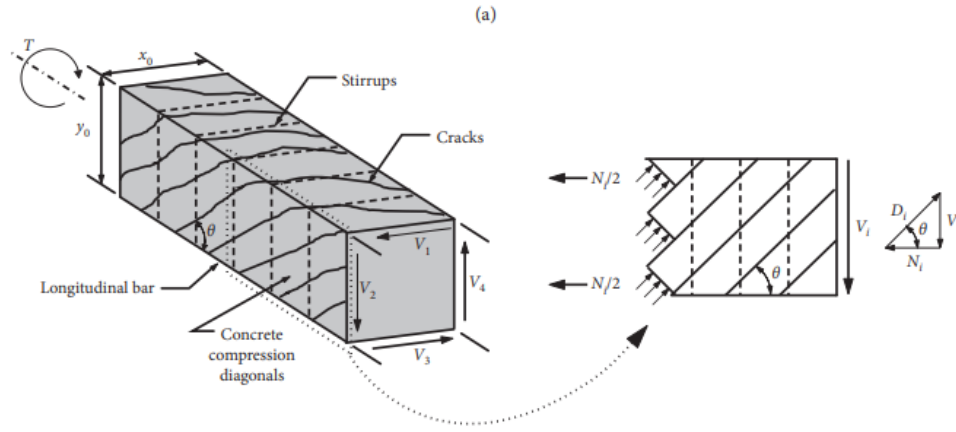


Fig. 3.4.: Diagonal compressive stresses [5]

These diagonal compression forces are in turn resisted by the transverse stirrups and longitudinal reinforcements, in which tensile forces are generated to balance the compression. This tensile stress, f_h , in the stirrups can be calculated using the following equation:

$$f_h = \frac{T_c \cdot sp \cdot \tan \alpha}{2A_0 A_h} \quad (3.6)$$

Where:

- α = diagonal compressive stresses angle, which is given by:

$$\tan^4 \alpha = \frac{1 + \frac{E_c \cdot t_d \cdot P_0}{E_s A_l}}{1 + \frac{E_c \cdot t_d \cdot sp}{E_s A_h}} \quad (3.7)$$

In which t_d = the effective wall thickness.

Regarding the changes to the warping component, it is again the second hypothesis that plays a major role. As a matter of fact, the concrete in tension will be kept neglected while the steel bars will be transformed into equivalent areas of concrete by multiplying their respective areas by the ratio of the moduli of elasticity of the steel and the concrete $\frac{E_s}{E_c}$. As the section changes with less concrete considered, the principal sectorial coordinate ω does too. The implications of this change are extremely important since the three unknowns related to the sectorial coordinate system, namely the position of P_0 (in x and y) and a value of s for which $\omega = 0$, will also change. These values are obtained by means of three integrals in Eq (3.8) which will be used to obtain convergence as the section changes with the cracks.

$$\int_A \omega \cdot dA = 0; \quad \int_A \omega \cdot x \cdot dA = 0; \quad \int_A \omega \cdot y \cdot dA = 0 \quad (3.8)$$

The compression zones (negative values) become smaller, while tensile zones (positive values) become bigger. These variations in values will also have an impact on the sectorial moment of inertia $I_{\omega\omega}$ (Eq (2.14)) which will unsurprisingly decrease and be named $I_{\omega\omega cr}$.

Similar to circulatory torsion, which causes a shear flow that leads to the compressive diagonals, warping torsion also causes a shear flow that has the following equation:

$$q = \frac{T_{\omega} Q_{\omega}}{I_{\omega\omega}} \quad (3.9)$$

In which Q_{ω} = sectorial static moment of area given by $Q_{\omega} = \int_0^s \omega \cdot t ds$

This will have the same consequences, namely, compression diagonals as well as tensile forces in the stirrups for which the stress is given by:

$$f_h = \frac{q \cdot sp}{A_v} \tan \alpha \quad (3.10)$$

Where:

- A_v = area of the transverse steel within the spacing s

The torsional equilibrium equation,

$$T(z) = -EI_{\omega\omega} \frac{\partial^3 \theta}{\partial z^3} + GK \frac{\partial \theta}{\partial z}$$

Can therefore be rewritten for the cracked elastic stage as:

$$T(z) = -EI_{\omega\omega cr} \frac{\partial^3 \theta}{\partial z^3} + GK_{cr} \frac{\partial \theta}{\partial z} \quad (3.11)$$

The contribution of the shear strains due to warping torsion are yet again taken into account. Eq (3.3) can be slightly adapted with a new cracked effective shear modulus

$$(GK_{\gamma})_{cr} = \frac{0.5 E_c t I_{\omega\omega cr}^2}{\left[1 + \sqrt{\left(1 + \frac{E_c t p}{E_s A_l} \right) \left(1 + \frac{E_c t s}{E_s A_v} \right)} \right] \int_s Q_{\omega cr}^2(s) ds} \quad (3.12)$$

that lead to this new shear strain contribution equation:

$$\frac{\partial \theta_{\gamma}}{\partial z} = \frac{T_{\omega}(z)}{(GK_{\gamma})_{cr}} \quad (3.13)$$

Cracked inelastic

As it has been shown in section (2.2), at certain level of stresses both the steel and concrete express nonlinear inelastic behaviours. The response is said to be cracked inelastic and is the last state through which the element will pass before it fails completely. Like the two previous stages, this one is supported by some assumptions:

HYPOTHESES:

- Circulatory torsion will be small enough so that only warping torsion is taken into account
- The warping moment M_ω does not depend on the value of the warping torsion T_ω
- The cross-section is subdivided into small steps of area along the centreline of the section

The linear equation linking stress to strain being no longer valid, new relationships are introduced:

$$f_s = E_{secant,s}(\epsilon_s)\epsilon_s \quad f_c = E_{secant,c}(\epsilon_c)\epsilon_c \quad (3.14)$$

Where:

- $E_{secant,s}, E_{secant,c}$ = secant moduli of the steel and the concrete, which are functions of the strain

These relationships will be at the heart of the iterative process arising from the last hypothesis on which this stage is based. These iterations actually aim to identify 3 important parameters which are the elastic modulus given by the ACI expression (E_{cACI}), the inelastic sectorial coordinate (ω_{inel}) and the inelastic sectorial moment of inertia ($I_{\omega\omega_{inel}}$) which are all involved in the longitudinal deformation distribution equation given by:

$$\epsilon_s = \epsilon_c = \frac{M_\omega}{E_{cACI} \cdot I_{\omega\omega_{inel}}} \cdot \omega_{inel} \quad (3.15)$$

For the first iteration, the cracked values of ω and $I_{\omega\omega}$ are used as first approximation. This allows the deformations to be calculated a first time and the secant moduli to be deduced using the introduced stress-strain relations. These moduli make it possible to transform the steel and concrete areas into equivalent concrete areas whose modulus is precisely E_{cACI} . As the section has been modified, new ω_{inel} and $I_{\omega\omega_{inel}}$ values can be calculated in order to repeat the process until convergence is achieved. When this is achieved, the rate of change of twist can be calculated using the final values:

$$\theta'' = \frac{\partial^2 \theta}{\partial z^2} = \frac{M_\omega}{E_{cACI} \cdot I_{\omega_{inel}}} \quad (3.16)$$

Knowing that for a given T the distribution of M_ω is known and that the relation with the rate of change of twist has just been introduced, the transformation towards a T- θ'' relation is immediate.

However, it is possible (not often dominant) that the failure is not caused by the warping moment but by the warping torsion and more precisely by the shear flow, which can no longer be transmitted through the cracked concrete. This shear flow would result in stresses in the compressive diagonals, whose values are given by:

$$f_d = \frac{q}{t} \left(\tan \alpha + \frac{1}{\tan \alpha} \right) \quad (3.17)$$

The stress in the transverse steels, f_h , is however still given by the Eq (3.10).

The angle of inclination of the compression diagonals is for its part given by:

$$\tan^2 \alpha = \frac{\epsilon_d + \epsilon_h}{\epsilon_d + \epsilon_l} \quad (3.18)$$

3.2 Chen *et al.* (2016)

In their paper, Chen *et al.* [4] performed four large U-shaped thin-walled RC beams tests with both ends restrained under pure torsion. This configuration is the same as the one that has been used in the past by Krpan and Collins, allowing their model to be easily compared. Apart from the experimental results, they also presented a different method from Krpan and Collins to calculate the ultimate and cracking torque. Based on the experimental results, it was possible to show that these torques were dominated by warping moment. For the cracking torque, the basis remains similar since the warping moment is expressed according to the Vlasov theory [30]. However, the expression of the warping moment is presented differently:

$$M_\omega = \frac{T_\omega \cdot C_{fixed}(z)}{2} \quad (3.19)$$

Where:

- C = constant that depends on the boundary conditions. In the case of a fixed-fixed beam, this constant is given by:

$$C_{fixed}(z) = \frac{\cosh(az) - \cosh[a(\frac{l}{2} - z)]}{a \sinh(\frac{al}{2})} \quad 0 \leq z \leq \frac{l}{2} \quad (3.20)$$

Cracking torque

In a very similar way to the Eurocodes, the American Concrete Institute (ACI) establishes equations for the different failure modes. As is well known, when subjected to warping torsion, the member will develop longitudinal compressive and tensile stresses due to the warping moment as would be seen for a beam in bending. According to ACI 318-05, the first transverse flexural crack appears when the maximum warping stress exceeds the tensile strength of the concrete given by:

$$f_{ct} = 0.63\sqrt{f'_c} \text{ (MPa)} \quad (3.21)$$

In which f'_c = compressive strength of the concrete.

Knowing that those longitudinal stresses can be calculated with Eq (2.13) that we recall,

$$\sigma_\omega = \frac{M_\omega \cdot \omega}{I_{\omega\omega}}$$

the cracking load can be obtained by substituting the warping moment Eq (3.19) in Eq (2.13) and equalized this with eq (3.21):

$$T_{cr} = \frac{1.26 \cdot \sqrt{f'_c} \cdot I_{\omega\omega}}{\omega_{max} \cdot C_{fixed}} \quad (3.22)$$

Ultimate torque

The method is based on the fact that the strength of the element is not calculated on the whole U-shaped element but rather on the calculation of three narrow rectangles which are the two flanges and the web. Using this assumption, it is sufficient to analyse the respective resistance of the three elements and then take the minimum value among the three.

Taking as example the right flange, in the same way as the bending moment due to a bending force given by $M_z = \int_A y \cdot \sigma_x dA$, the bending moment due to warping torsion in either of the three rectangles is given by:

$$M_{fla,r} = \int_A y \cdot \sigma_\omega dA = \frac{M_\omega}{I_{\omega\omega}} \int_A y \cdot \omega dA = \frac{M_\omega}{I_{\omega\omega}} H_{\omega y} \quad (3.23)$$

Where:

- $H_{\omega y}$ = sectorial linear static moment

As for the left flange and the web, the bending moment is given by:

$$M_{fla,l} = \frac{M_{\omega}}{I_{\omega\omega}} \cdot H_{\omega y} \quad (3.24)$$

$$M_{web} = \frac{M_{\omega}}{I_{\omega\omega}} \cdot H_{\omega x} \quad (3.25)$$

Looking now at the stress state of a flange (fig 3.5), several components are to be taken into account in the analysis of the section. Those are the concrete in compression, the top compressive bars, the tensile bars at the bottom and the distributing bars located beyond 1.5 times the height of the compressive zone. This obviously follows from the assumptions that (a) in compression the distributing bars are neglected (b) concrete in tension is neglected (c) due to the lower stress condition, only a fraction of the distributing bars are taken into account.

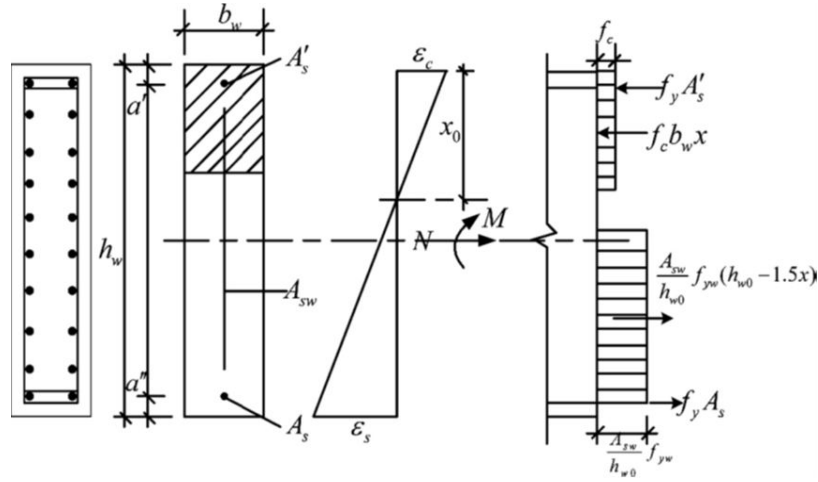


Fig. 3.5.: Stress state of a flange [4]

It is therefore possible to write these two equilibrium equations:

Axial load equilibrium:

$$N = f_c b_w \chi - f_{yw} \frac{A_{sw}}{h_{w0}} (h_{w0} - 1.5\chi) + f'_y A'_s - f_y A_s = 0 \quad (3.26)$$

Moment equilibrium:

$$M_{fla,r} = f_{yw} \frac{A_{sw}}{h_{w0}} (h_{w0} - 1.5\chi) \left(\frac{h_{w0}}{2} + \frac{\chi}{4} \right) + f'_y A'_s \left(\frac{\chi}{2} - a' \right) + f_y A_s \left(h_{w0} - \frac{\chi}{2} \right) \quad (3.27)$$

Where:

- f_c = compressive strength of the concrete from prism
- b_w = thickness of the element
- χ = height of the compressive zone
- f_{yw}, f'_y, f_y = ultimate strength of distributing bars and main bars
- A_{sw} = total area of distributing bars
- $h_{w0} = h_w - a''$
- A'_s, A_s = area of top compressive bars and bottom tensile bars

As part of their experiment, the different steels had the same strength $f_{yw} = f'_y = f_y$ and for top and bottom steel the same area $A'_s = A_s$. By isolating χ in the equation (3.26) and substituting this value in the equation (3.27) while imposing that $\xi = \frac{f_c b_w h_{w0}}{f_{yw} A_{sw}}$, Eq (3.27), can be rewritten as:

$$M_{fla,r} = f_y A_s (h_{w0} - a') + f_{yw} A_{sw} h_{w0} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{1.5 + \xi} \right) - \frac{3}{8} \left(\frac{1}{1.5 + \xi} \right)^2 \right] \quad (3.28)$$

The ultimate torque that the right flange can withstand is then obtained by substituting Eq (3.23) and Eq (3.19) into this last equation (3.28):

$$T_{fla,r} = \frac{2I_{\omega\omega}}{H_{wy}C} \left\{ f_y A_s (h_{w0} - a') + f_{yw} A_{sw} h_{w0} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{1.5 + \xi} \right) - \frac{3}{8} \left(\frac{1}{1.5 + \xi} \right)^2 \right] \right\} \quad (3.29)$$

The calculation of the ultimate torque in the left flange and in the web is identical and leads to the following final equations:

$$T_{fla,l} = \frac{2I_{\omega\omega}}{H_{wy}C} \left\{ f_y A_s (h_{w0} - a') + f_{yw} A_{sw} h_{w0} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{1.5 + \xi} \right) - \frac{3}{8} \left(\frac{1}{1.5 + \xi} \right)^2 \right] \right\} \quad (3.30)$$

$$T_{web} = \frac{2I_{\omega\omega}}{H_{wx}C} \left\{ f_y A_s (h_{w0} - a') + f_{yw} A_{sw} h_{w0} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{1.5 + \xi} \right) - \frac{3}{8} \left(\frac{1}{1.5 + \xi} \right)^2 \right] \right\} \quad (3.31)$$

Finally, the ultimate resisting torque of the U-shaped element is obtained by taking the minimum value among those three $\{T_{fla,r}, T_{fla,l}, T_{web}\}$.

3.3 Xu et al. (2018)

The experiments that were carried out and presented in this article [32] have many similarities with the paper presented just before. Starting with the tested elements that are again U-shaped beams for which both ends are fixed. The difference here is in the type of loading according to which the beams will be assessed. Where previously only pure torsion was applied, it is now a combination of bending and torsion. The ratio varies between 1:5, 1:1 and 1:0 (pure torsion) for each specimen. The presence of this additional force will obviously have an influence on the equations governing the mechanical model that will be presented in the following section.

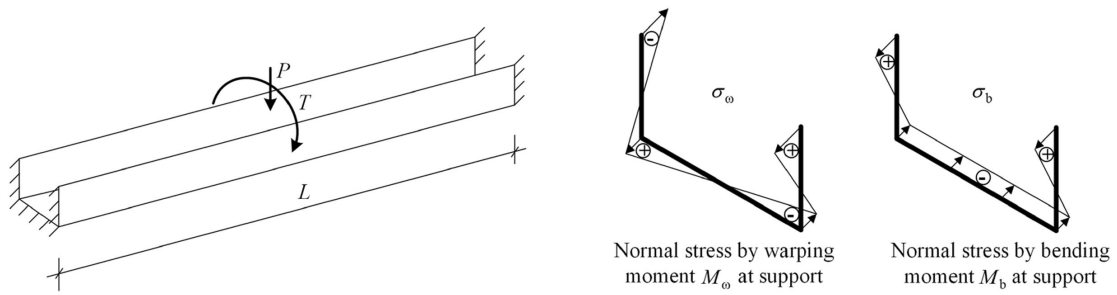


Fig. 3.6.: Warping and bending normal stresses [32]

Cracking torque

Once again, behaviour will be governed by longitudinal stresses. This is supported by the extremely small stirrup strains obtained in the tests, suggesting that the shear force was non-dominant. Those longitudinal stresses appear due to warping, but this time also due to the bending. On this basis and in the same way as [4], the first crack torque can be evaluated. The difference is that this now has to include the effect of bending that adds normal stresses due to bending to already existing normal stresses from warping (fig 3.6). The longitudinal bending stresses are given by:

$$\sigma_b = \frac{M_b \cdot y}{I} \quad (3.32)$$

In which I = inertia of the section.

Using the same equations presented in the Chen et al. model, namely, Eqs (2.13), (3.19), (3.21) and adding this last Eq (3.32) lead to the cracking torque equation for which the

different calculation steps are presented in order to better understand where it comes from:

$$\begin{aligned}
 \sigma_{max} &= \frac{M_b \cdot y_{max}}{I} + \frac{T_{cr} \cdot C_{cantilever} \cdot \omega_{max}}{2 \cdot I_{\omega\omega}} \\
 \sigma_{max} &= \frac{R \cdot T_{cr} \cdot y_{max}}{I} + \frac{T_{cr} \cdot C_{cantilever} \cdot \omega_{max}}{2 \cdot I_{\omega\omega}} \\
 \sigma_{max} &= T_{cr} \cdot \left(\frac{R \cdot y_{max}}{I} + \frac{C_{cantilever} \cdot \omega_{max}}{2 \cdot I_{\omega\omega}} \right) \\
 \sigma_{max} \cdot 2 \cdot I \cdot I_{\omega\omega} &= T_{cr} \cdot \left(R \cdot y_{max} \cdot 2 \cdot I_{\omega\omega} + C_{cantilever} \cdot \omega_{max} \cdot I \right) \\
 \boxed{\frac{0.63\sqrt{f'_c} \cdot 2 \cdot I \cdot I_{\omega\omega}}{R \cdot y_{max} \cdot 2 \cdot I_{\omega\omega} + C_{fixed} \cdot \omega_{max} \cdot I} = T_{cr}} & \quad (3.33)
 \end{aligned}$$

Where:

- R= ratio between the bending moment and the torque applied

Ultimate torque

From the figure (3.6) it is quite obvious that one half of the U-shaped element is more critical than the other. Indeed, in one case the forces due to warping add up to those due to bending, while in the other case they oppose each other. The ultimate bearing capacity will therefore be determined by this so-called doubly loaded half and consequently, the determination of this capacity will be based on a model taking only half of the U (fig 3.7). On this half, the warping forces can be considered instead of forces along the whole half, with two resultants applied to the neutral axis producing the same effect (fig 3.7). These are an equivalent moment $M_{eq} = \int_A \sigma_{\omega} y_1 dA$ and a normal force $N_{eq} = \int_A \sigma_{\omega} dA$. In the case where a bending force is added, it is considered that half of this force is taken up by half of the U while the other half is taken up by the other part of the U. It is therefore necessary to adapt the resultant moment at the neutral axis to take this into account $M_{tot} = M_{eq} + 0.5M_b$.

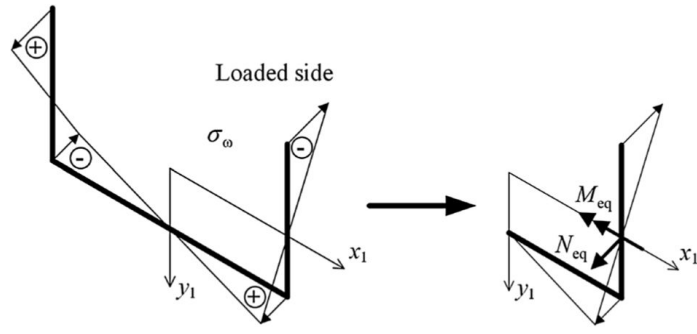


Fig. 3.7.: Half U-section model [32]

As in the previous model, based on the stress distribution of the section (fig 3.8) it is possible to derive two equilibrium equations. The assumptions are globally the same, namely (a) in compression the distributing bars are neglected (b) concrete in tension is neglected (c) due to the lower stress condition, only a fraction of the distributing bars are taken into account. However, an additional assumption is added here, which is that (d) all tensile bars at the bottom got ultimate strength and the warping normal stress is considered uniform on the bottom slab.

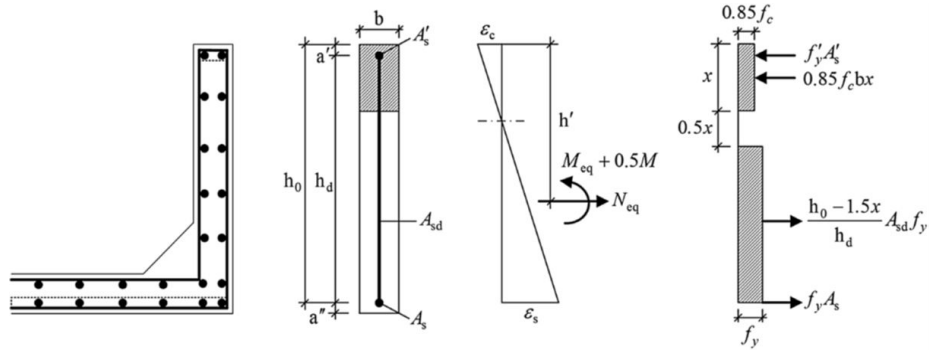


Fig. 3.8.: Stress distribution of the half U-section model [32]

The equilibrium equations can therefore be derived as follows:

Axial load equilibrium:

$$N_{eq} = f_y A_s + f_y A_{sd} \frac{(h_0 - 1.5\chi)}{h_d} - 0.85 f_c b \chi - f_y' A_s' \quad (3.34)$$

Moment equilibrium:

$$\begin{aligned} (M_{eq} + 0.5M_b) + N_{eq}(h' - 0.5\chi) &= f_y A_s(h_0 - 0.5\chi) + f_y A_{sd} \frac{(h_0 - 1.5\chi)}{h_d} (0.5h_0 + 0.25\chi) \\ &\quad + f_y' A_s'(0.5\chi - a') \end{aligned} \quad (3.35)$$

Where:

•

$$M_{eq} = \int_A \sigma_{\omega} y_1 dA = \int_A \frac{M_{\omega} \omega}{I_{\omega\omega}} y_1 dA = \frac{TC}{I_{\omega\omega}} \eta_1 \quad (3.36)$$

In which $\eta_1 = \int_A \omega y_1 dA$

•

$$N_{eq} = \int_A \sigma_{\omega} dA = \int_A \frac{M_{\omega} \omega}{I_{\omega\omega}} dA = \frac{TC}{I_{\omega\omega}} \eta_2 \quad (3.37)$$

In which $\eta_2 = \int_A \omega dA$

In the equations presented, only χ and T are unknown. By substituting the expression for χ obtained in Eq (3.34) into Eq (3.35), the ultimate torque is easily found.

3.4 Hoult (2021)

The mechanical model developed by Krpan and Collins is based on the theory established by Vlasov. In turn, Dr. Hoult [8]- who takes part in the tests carried out at the LEMSC - has used the Krpan and Collins model to adapt the calculations to U-shaped walls. This new model is fundamentally the same in the sense that the 3 distinct phases of uncracked elastic, cracked elastic and cracked inelastic still form the basis of the procedure. This new model is innovative in that the Krpan and Collins model is not only improved in terms of speed of compilation, but also because it can now be applied to walls in addition to beams.

The speed of execution is undoubtedly due to the more advanced technologies available, as well as to the iterative procedure used to calculate the ultimate torque that is illustrated in Fig 3.9.

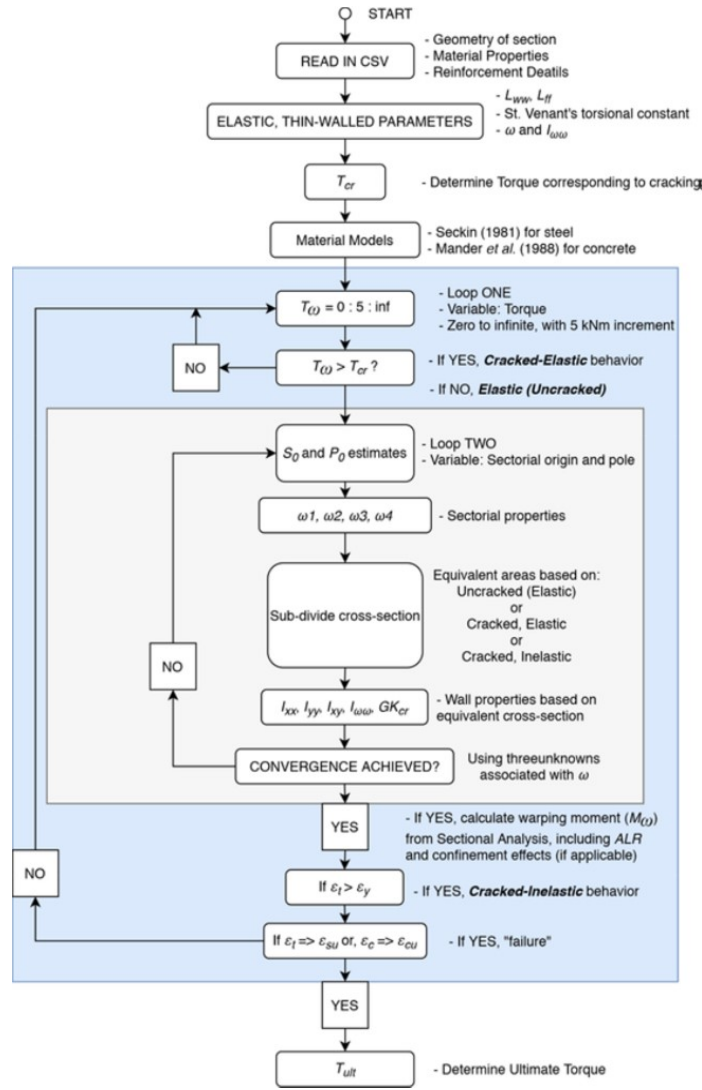


Fig. 3.9.: Flow chart calculations programmed in MATLAB to determine the ultimate torque capacity [8]

Whether it is for the calculation of the cracking torque, the ultimate torque or the rotation associated with these, each equation of the Krpan and Collins model must be adapted to match the new boundary conditions and load cases. This publication by Dr. Ryan Hoult [8] brings together all these new equations established by leading researchers before him. Among these, is the equation for the $C_{cantilever}$ coefficient that is given by Zbirohowski-Kościa [34] which will then be used in Section 6.2.1 to develop a new model

$$C_{cantilever} = \frac{\sinh(\alpha h_w)}{\alpha \cosh(\alpha h_w)} \quad (3.38)$$

This coefficient is similar to the one introduced in Section 3.2 except that the boundary conditions are different, meaning that the expression changes accordingly.

But also the equations for the rotation of the uncracked wall and the flexural cracking torque

$$\Theta = \frac{T_{\omega} h_w^3 \cdot (1 + \frac{3}{\beta})}{3E_c I_{\omega\omega} \cdot (1 + \frac{\beta}{12})} \quad (3.39)$$

$$T_{cr} = \frac{f_{ct} I_{\omega\omega}}{h_w \omega_{max}} \quad (3.40)$$

Part III

Experimental Tests

Experimental Programme at LEMSC, UCLouvain

“ *What we know is a drop, what we don't know is an ocean.*

— Isaac Newton

Developing mechanical models to predict the behaviour of beams or walls is of paramount importance. But this only makes sense if these models are compared with experimental results. This comparison makes it possible not only to check the accuracy of the models, but also to refine them on the basis of experimental observations. The following sections therefore aim to present the main experiments that have been carried out on U-shaped sections, starting with those carried out here at LEMSC. Within the framework of this experimental programme, an international competition has been created so that anyone could try to make predictions about the response of the various specimens. It is in this context that a file gathering the main information on these tests has been written [1]. The following sections gather the information concerning the UW2 specimen taken from this document.

Three specimens are to be experimentally tested under different flexure-to-torsion (M-T) ratios. The three M-T ratios are 1:0 (i.e., pure flexure), 0:1 (i.e., pure torsion), and 1:1 (i.e., combined actions). Importantly, for this thesis, only the results for the second unit will be considered, corresponding to the wall subjected to pure torsion. An axial load is also applied. The test specimens represent the approximate bottom 1.5 stories of the 6-story prototype structure, at half scale (fig 4.1). Application of an overturning moment, shear force, and axial force on the top of the test specimen simulated the effects of the gravity and lateral loads acting on the upper 4.5 stories of the wall. For the second unit, denoted UW2, subjected to pure torsion, only horizontal loads (i.e., twisting) and a constant axial load were applied.

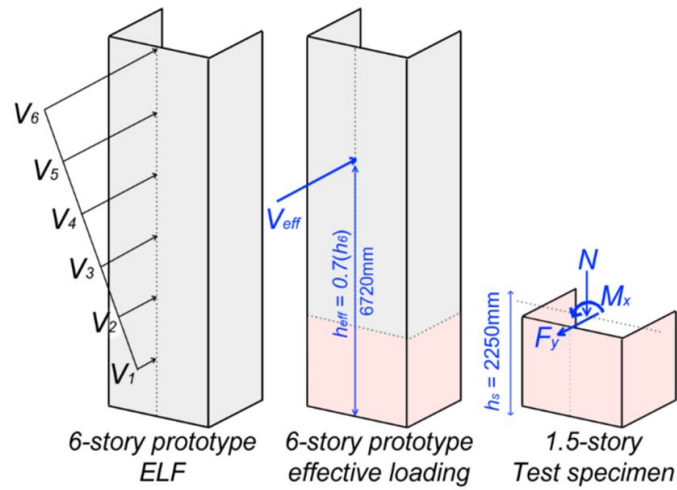


Fig. 4.1.: Six-story prototype core-wall structure

4.1 Description of Test Units

All the specimens are identical in geometry and design detailing, with the cross-section of the wall and reinforcement shown in Figure 4.2. The U-shaped specimens have a flange and web thickness (t_w) of 100 mm, web length (L_w) of 1300 mm, and flange length (L_f) of 1050 mm. Lumped longitudinal reinforcement, consisting of 12 mm diameter (d_{bl}) bars, is used in the boundary ends, while distributed $d_{bl}=6$ mm bars are used in the web. Note also that in the flanges (near the web), two longitudinal rebars of 8 mm diameter were used. Confinement reinforcement is provided in the boundary ends using 6 mm bars spaced at 75 mm, while the transverse (shear) reinforcement consists of 6 mm bars spaced at 150 mm.

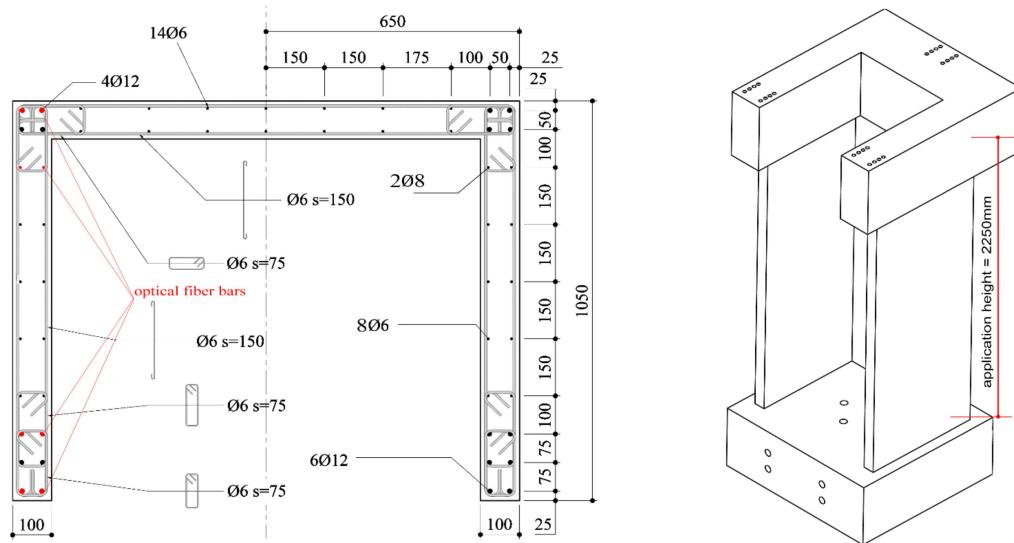


Fig. 4.2.: Test units: cross-section and reinforcement layout (dimensions in mm) and elevation view

Figure 4.2 also illustrates that the specimen height (h_s) to the application of the horizontal actuators is 2250 mm, while the simulated effective height of the wall (h_{eff}) is 6720 mm. The footing and the collar of the wall are detailed with a high volume of reinforcement using 20 mm and 12 mm diameter bars, respectively, such that they effectively remain elastic during testing. For unit UW1, the size of the footing was 1400 mm x 1500 mm x 450 mm. For unit UW2 and UW3, the width of the footing was slightly increased to 1900 mm x 1500 mm x 450 mm which is the only difference in terms of conception. The collar of the wall has a variable thickness over the flanges and in the web, increased to fasten the actuators to the wall unit. A collar was used, rather than a thick top slab, due to the warping restraints [2], which are particularly important for specimen UW2 subjected to pure torsion. The full set of reinforcement detailing and plan drawings of the wall units are available at the end of this document in Appendix .1.

4.2 Material Properties and Casting

Steel reinforcement properties and rebar cage

As per the requirements of Eurocode 8 [27] to ensure ductility and energy dissipation, Class C steel was used for the longitudinal and transverse reinforcing bars in the wall. Class B reinforcement was used for the other reinforcement in the foundation and collar of the wall. The salient mechanical properties of the Class C reinforcing steel, as obtained from tensions tests, are given in Table 4.1: f_y and f_u are the yield and ultimate stress; ϵ_{sy} , ϵ_{sh} , ϵ_{su} are the strain values corresponding to yield, strain hardening, and ultimate strain (respectively); and E_s is the Young's modulus.

d_{bl}	f_y	f_u	ϵ_{sy}	ϵ_{sh}	ϵ_{su}	E_s
[mm]	[MPa]	[MPa]	[mm/mm]	[mm/mm]	[mm/mm]	[GPa]
6	550	676	0.0027	0.010	0.095	207
8	538	664	0.0027	0.024	0.120	196
12	580	690	0.0029	0.021	0.101	199

Tab. 4.1.: Mechanical properties of the Class C reinforcement

The rebar cages were assembled in a precast production factory. Although a maximum guidance and control was given, it was not possible to guarantee in absolute detail the respect of the construction plans. Some of the features that are judged potentially more relevant to the response of the walls are reported below, to the best of the authors' observations,

judging from photos taken immediately prior to the casting. Firstly, it should be noted that the units are cast with its longitudinal axis in the horizontal position, with the flanges positioned vertically and the web on the top (i.e., in the position □). Unlike the first unit where the absence of spacers led to a deflection of the reinforcement cage during pouring of the concrete, the use of spacers for UW2 guaranteed a high-precision concrete cover and placement of the reinforcement cage, according to the plans. However, it was observed that the fourth layer of longitudinal rebars in the South boundary elements of both flanges (i.e. at 275 mm from the South face) were 8 mm diameter, instead of 6 mm.

Concrete properties and casting

The desired concrete strength (f'_c) on the day of the test was 30 MPa. To achieve it, and in view of the impossibility of the precast construction factory to change its high cement quality, and to use vibrating engines, an air-entrained (5-6 %) self-consolidating concrete was employed. This is likely to reduce the density of the concrete from its usual value, which is reported below

Specimen	Day from casting	Cylinder compressive strength				Cube compressive strength			Tensile strength		
		f'_c (MPa)	E_c (MPa)	ρ (kg/m ³)	n	f'_c (MPa)	ρ (kg/m ³)	n	f_{sp} (MPa)	f_t (MPa)	n
UW2	10	-	-	-	-	23.5	2150	1	-	-	-
	28(day of test)	37.1	-	2204	4	36.5	-	4	3.2	2.88	4
	45	41.4	25891	-	1	-	-	-	-	-	-

Tab. 4.2.: Mean values of mechanical properties of the concrete for UW2

The f'_c indicated in the table above is the average value resulting from typical compressive tests on a number of cylinder and cube samples (n). The splitting strength f_{sp} was determined from cylinders. All cylinders had standard diameter x length of 150 mm x 300 mm, and the cubes were also standard with sides of 150 mm. The tensile strength (f_t) indicated in the tables below was computed simply as $f_t \approx 0.9f_{sp}$.

In addition to this, compression tests were conducted on two smaller cylinders of 103 mm in diameter and 176 mm in length. At the time of writing of this document, only units UW1 and UW2 were tested so Figure 4.3a illustrates the stress-strain behaviour of the concrete for those units, which was achieved by a displacement-controlled axial compression test on these additional cylinders. The two smaller cylinders were instrumented with strain gages and LVDTs (Figure 4.3b), which were used to determine the strain of the cylinders in the pre- and post-peak branches. These latter tests were conducted several days after the testing

of the wall units, hence the slight increase in compression strength indicated in Figure 4.3a in comparison to the indicated compressive strength given in Table 4.2 at the day of the tests. The lateral deformation of the smaller cylinders of UW1 was also measured using a circumferential extensometer (see Figure 4.3b), which was used to calculate the Poisson's ratio as a function of the axial strain in Figure 4.4. The Poisson's ratio (ν) of 0.143 was calculated as the average of the values within the axial strains between zero and $0.5\epsilon_{c0}$, where ϵ_{c0} is the axial strain corresponding to peak compression stress ($\epsilon_{c0} = 0.25\%$).

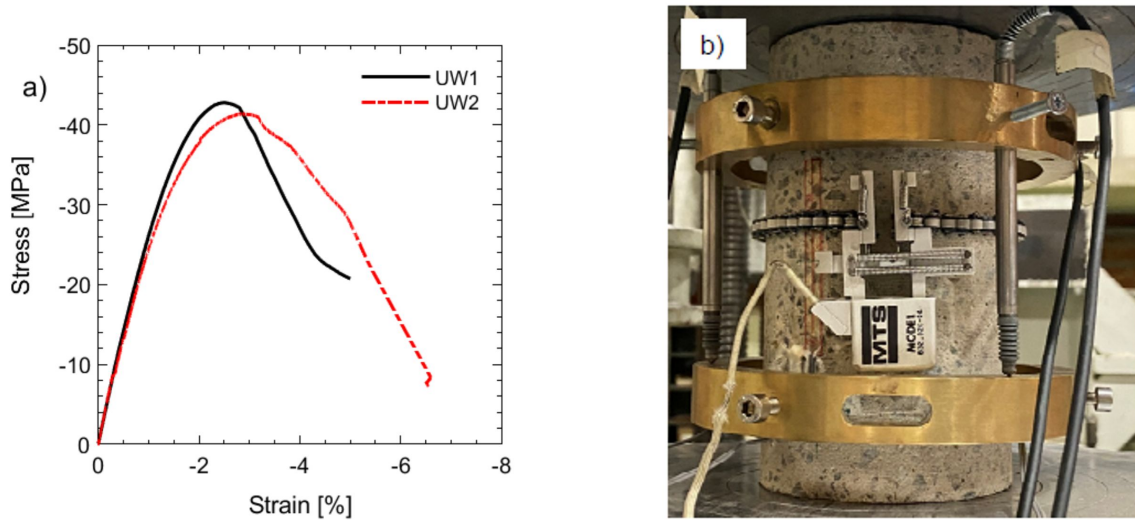


Fig. 4.3.: (a) stress-strain behaviour of concrete used for unit UW1 (from 3 LVDTs and 3 strain gauges; thicker line corresponds to average on the pre-peak phase, and proposed post-peak) (b) instrumentation used to derive pre- and post-peak stress-strain curve

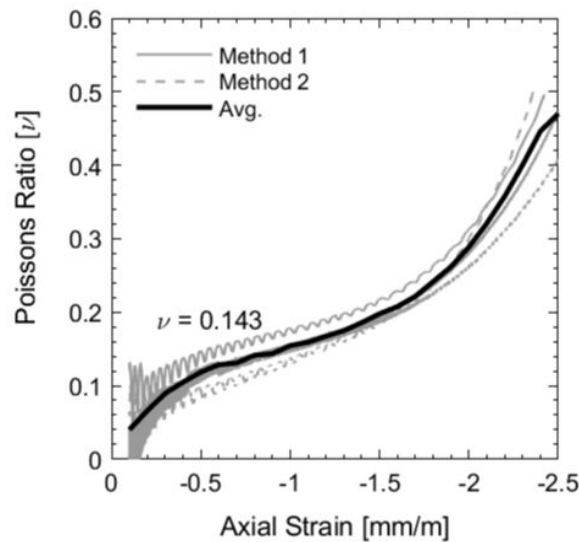


Fig. 4.4.: Poisson's ratio as a function of the axial strain determined from a test of a small cylinder of the concrete for unit UW1 (from diametrical extensometer)

4.3 Test Setup

In total, five actuators are used to simulate the behaviour of the half-scale, 6-story RC core wall. Figure 4.5 illustrates the laboratory setup used to test the wall specimens. The three vertical actuators are used to apply the overturning moment, simulating a wall with a larger shear span, as well as the axial load. A constant axial load ratio (ALR) of 5% is used throughout testing for both units. The applied axial force (N) can be calculated using Equation (4.1). One-third of the axial force (i.e., $N/3$) will be applied by each of the vertical actuators on the wall.

$$ALR = \frac{N}{f'_c A_g} \quad (4.1)$$

In which A_g = gross cross-sectional area of the wall ($=320,000\text{mm}^2$).

With the given f'_c value indicated in Table 4.2 for unit UW2, the axial load (N) used experimentally came to 593.6 kN. As discussed above, this total axial load value was distributed evenly to the three vertical actuators (i.e., $N/3$).

Two horizontal actuators apply the shear force for specimen UW1 and the torque required to follow a pre-determined cyclic rotation history for specimen UW2. As depicted in Figure 4.5, the horizontal actuators are connected to a steel beam, which is subsequently connected to the collar of the wall. The lever arm distance between the application points of the two horizontal actuators is 2400 mm. The unit foundation was fixed to the strong floor using a series of post-tensioned, threaded rods. Two additional foundation blocks on either side of the test unit were also used to restrain the wall from sliding and twisting at its base (Figure 4.5), connected to the foundation of the wall through four post-tensioned threaded rods. Finally, the foundation blocks were also fixed to the strong floor, again through post-tensioned threaded rods.

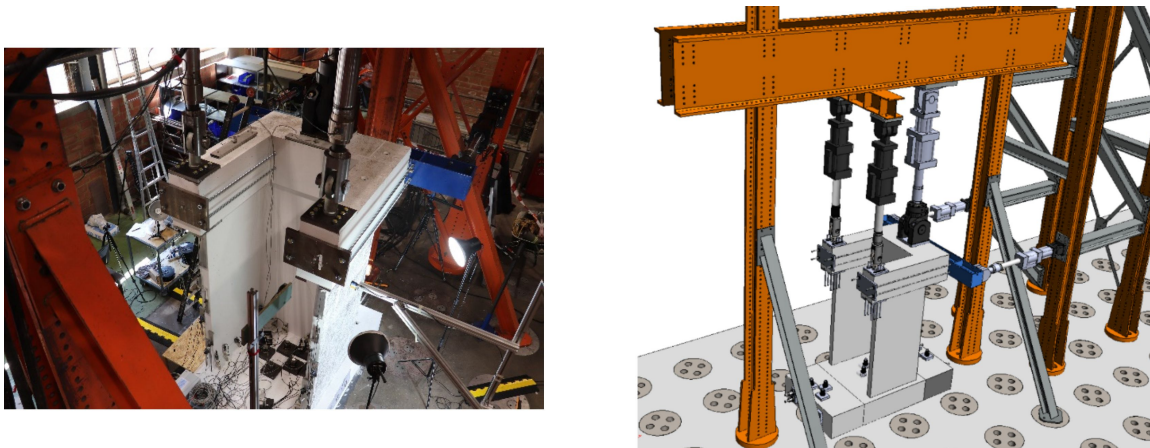


Fig. 4.5.: Test setup at LEMSC, UCLouvain

kN in each direction (i.e., positions D and C, Figure 4.6b). The other series of pushes, for rotations equal to or greater than 10 mrad, will be displacement controlled. A push will be applied to the wall at the following rotation levels: 0 mrad, 10 mrad, 20 mrad, 30 mrad. It is assumed that the failure of the wall will not occur before a rotation of 30 mrad; if failure is attained after 30 mrad, no further push will be applied not to influence the response of the wall in torsion. Each push will be applied in both directions (i.e., positions D and C, Figure 4.6b) and returned to the original position of the wall prior to the push. During each applied twist, the vertical actuator forces will be kept with their values before twisting, which (for this wall) is equivalent to the applied axial loads.

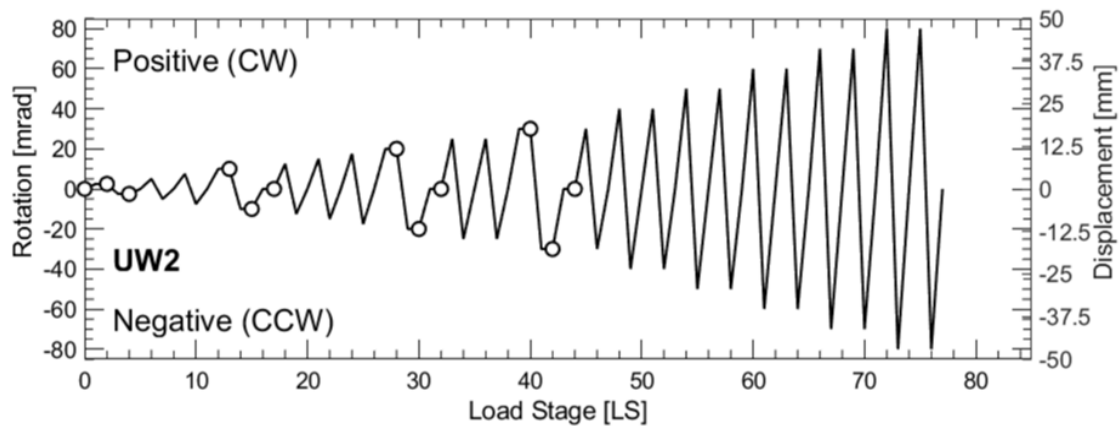


Fig. 4.7.: Loading protocol for specimen UW2, under pure torsion. The circles indicate the load stages (LS) where the small push, to determine the translational stiffness, will be applied to the wall.

4.5 Instrumentation

4.5.1 Extensometers

Piezoelectric sensors and Linear Variable Differential Transformers (LVDTs) are the two main types of extensometers that were used on the wall. For the latter, no less than 21 LVDTs were positioned on the wall as shown in Fig 4.8. These extensometers allow to obtain a linear deformation measurement between two precise points. These are the point of attachment to the wall and the end of the rod resting on the bracket. Taking the example of the LVDT 100A on the right-hand side of Fig 4.8, this extends to a height of 600 mm meaning that the linear deformation measurement corresponds to the deformation of this segment from 0 to 600 mm in height.

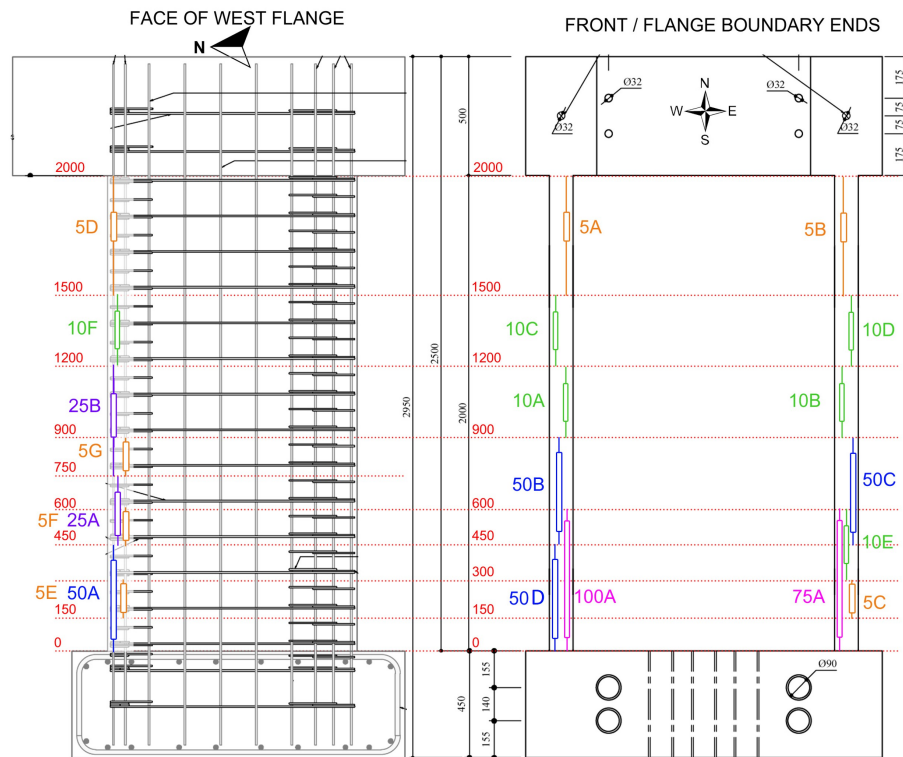


Fig. 4.8.: Linear Variable Differential Transformers arrangement

As for the piezoelectric sensors, there were 16 of them positioned at the base of the wall (Fig 4.9). They allow the measurement of smaller deformations, proportionally converted into electrical signals thanks to its sensitive element. This sensor is insensitive to electromagnetic fields and has a very high dynamic response. However, like an LVDT, the piezoelectric sensor only allows linear deformation measurement.

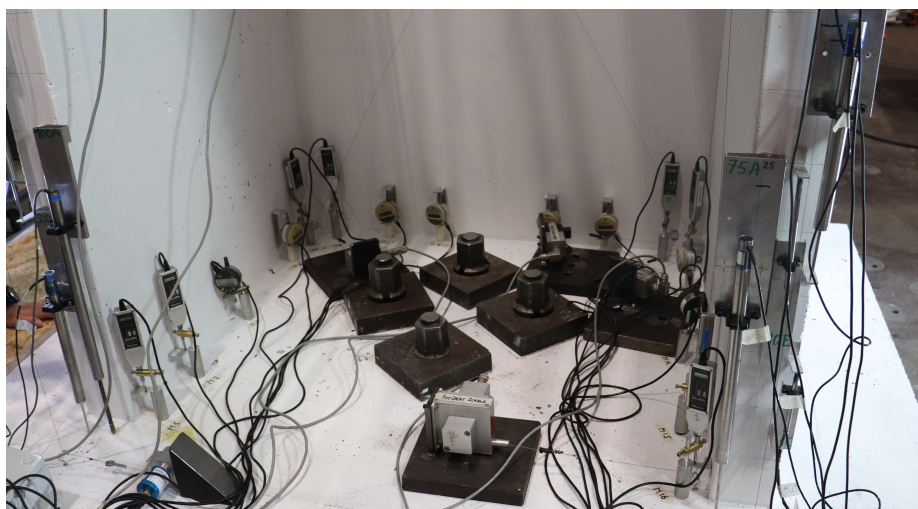


Fig. 4.9.: Piezoelectric sensors arrangement

4.5.2 Potentiometers

The top displacements of the two wall flanges, $\Delta 1$ and $\Delta 2$, will be measured along the North-South direction by a pair of potentiometers (i.e., string pots), each connected to the South extremity of the top wall collar. These string pots are connected at the same height of the imposed horizontal loads (=2250 mm) from the wall base, and are represented in Figure 4.2b. For specimen UW2, the rotation of the wall (in milliradian) will be calculated as such:

$$\theta_{exp} = \frac{\Delta 1 - \Delta 2}{1.2} \quad (4.2)$$

Where:

- $\Delta 1$ and $\Delta 2$ = the displacement (in mm) recorded by string pot 1 and 2 respectively
- 1.2 = the lever arm distance (in m) between the two flanges (center-to-center)

4.5.3 Digital Image Correlation

A speckle pattern for digital image correlation (DIC) measurements is applied on two outside surfaces of the wall (web and East flange) covering the full height of the wall from the base up to 2 meters high. The speckle pattern is applied by randomly painting a pattern to produce black dots with an approximate diameter between 3-5 mm. Two sets of two-camera systems are used, depicted in Figure 4.10, each set capturing the three-dimensional displacement field of the surfaces of the wall. The cameras recorded black and white photographs to decrease file size and simultaneously have an increased capacity for sensor dimensions, resulting in high resolution images. The images will be processed using Instra4D (V4.6) from Dantec Dynamics, where the displacement and strain fields for the perimeter of the wall (i.e., web and east flange) are computed. The speckle pattern on the East flange and web of unit UW1 is shown in Figure 4.11.

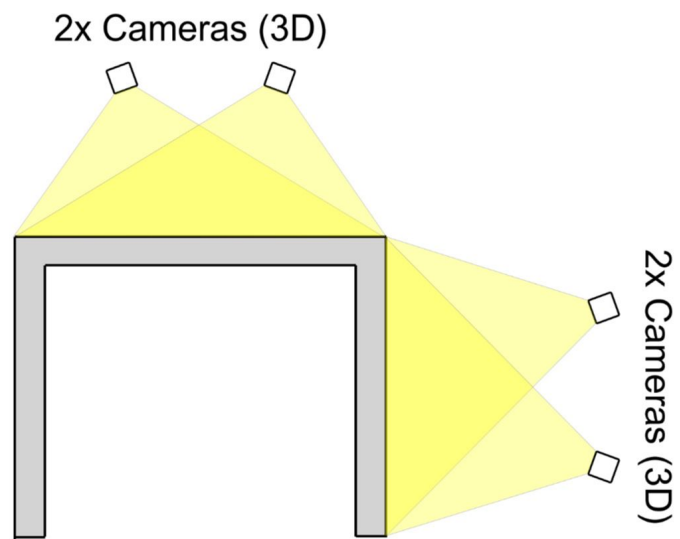


Fig. 4.10.: Setup and locations of the cameras for digital image correlation techniques



Fig. 4.11.: Speckle pattern on the East flange and Web of UW1 at LS0 (i.e., prior to loading)

4.5.4 Optical Fibers

State-of-the-art high-definition fibre optic sensing is used to accurately determine the strains of some reinforcing bars (Figure 4.12). A LUNA optical distributed sensor interrogator 4-channel ODiSI 6000 Series system is used to compile the data. High resolution (≈ 0.65 mm) measurements of strain along the sensor fiber are recorded throughout the testing of unit UW2 at an acquisition frequency of ≈ 3 Hz. Importantly, eight longitudinal reinforcing bars, as indicated in Figure 4.2a, had optical fibers glued to derive the strain profiles up the height of the wall. A current limitation of this technology, in practice, appears to be a maximum allowable strain of 1.0-1.5% being recorded by the system. Thus, while the strains from the fiber optics will be recorded through the entire test, resolution will be likely lost in the plastic regions of the wall with strains greater than 1.0%. The fiber used was polyimide coated sensing fiber housed in polytetrafluoroethylene tubing. After some initial trials, a general-purpose adhesive (“Loctite 401”, cyanoacrylate technology) was used to glue the fiber to the bar, along within a previously cut 1 mm slit. An epoxy (“ESK-50”) was used to provide a coating of protection between the glued fiber and the concrete. The full step-by-step procedure which was developed at the LEMSC is available in Appendix .2.



Fig. 4.12.: Close-up of the 12 mm reinforcing bar with optical fiber attached

4.6 Results

A section on results can be very large. In addition, as highlighted in Section 4.5, a very large amount of instruments were used. Therefore, thousands of data were recorded during this test, leading to information on deformations and displacements at several places on the wall and even in the wall with the optical fiber. It is then all about selecting the right results to best understand and illustrate the behaviour of the wall during the tests. In the context of an experimental protocol designed to test a specimen in torsion, it is only natural that the best representation of the results is provided by the torque-rotation curves that are represented in Fig 4.13. These are crucial in observing the different phases through which

the wall transits. These same uncracked elastic, cracked elastic and cracked inelastic phases that were introduced in Section 3.1 of the Krpan and Collins mechanical model [18]. By carefully observing these curves it is therefore possible to obtain the two torques that are the subject of the predictions, i.e. the cracking torque at the transition between the first two phases and the ultimate torque just before the rupture at the end of the last phase.

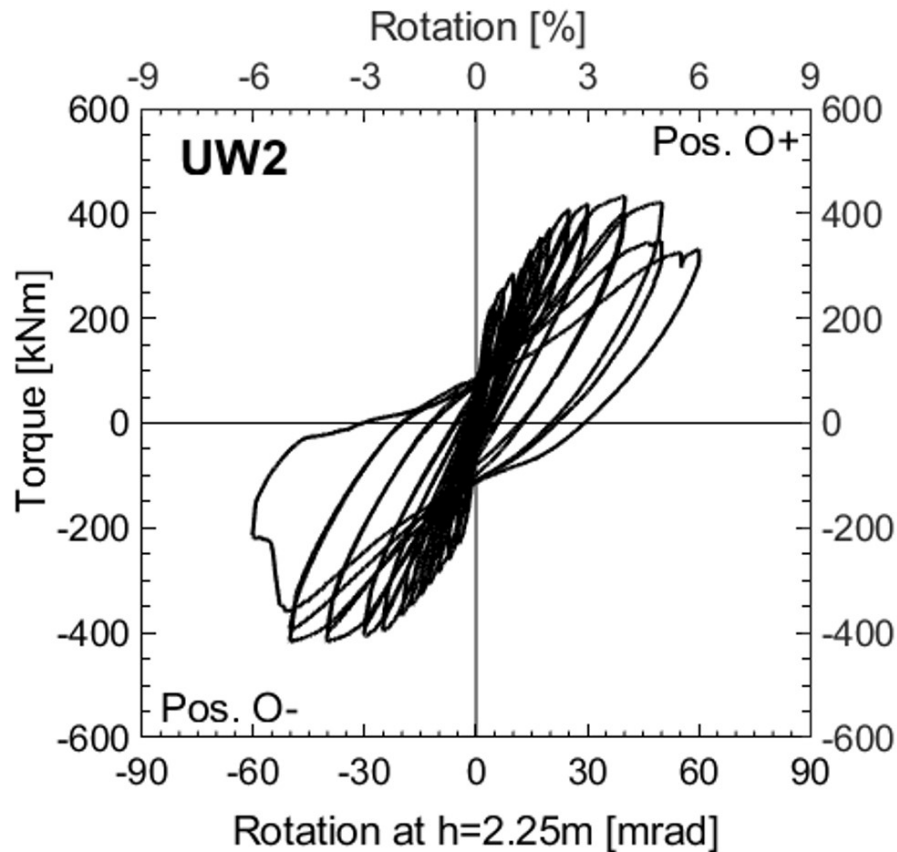
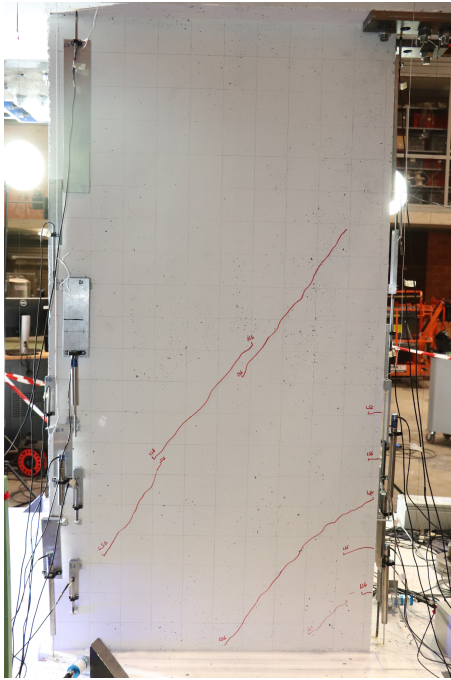
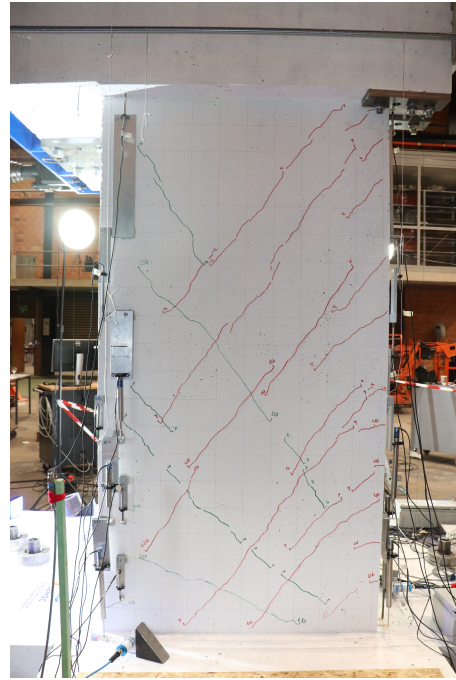


Fig. 4.13.: Torque-Rotation curves of specimen UW2

Throughout the loading stages, particular attention was paid to the different cracks. This is because they are immediately observable and make it easy to understand the patterns of breakdown. How these were recorded will be explained in the following section, but the cracks at different load stages can already be seen in Fig 4.14 and Fig 4.15. Monitoring the evolution of cracking with the help of photographs is precisely the principle on which DIC is based. However, the pattern of points that must be applied to the faces makes it difficult to see the cracks. Keeping a completely blank face is therefore a good way to visually follow the cracks and even compare with the DIC afterwards.

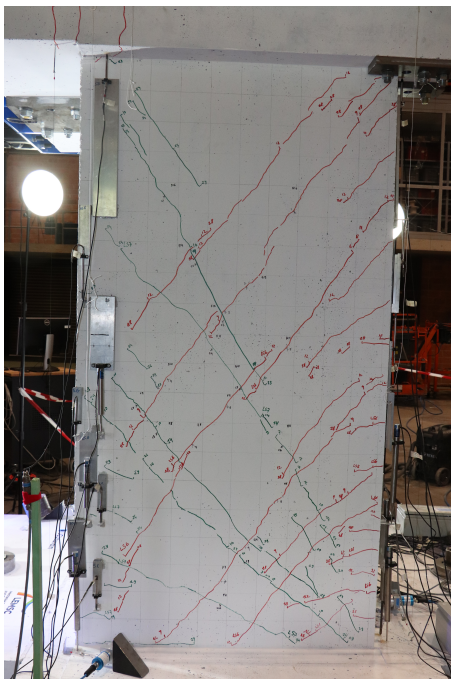


(a) Load stage 6

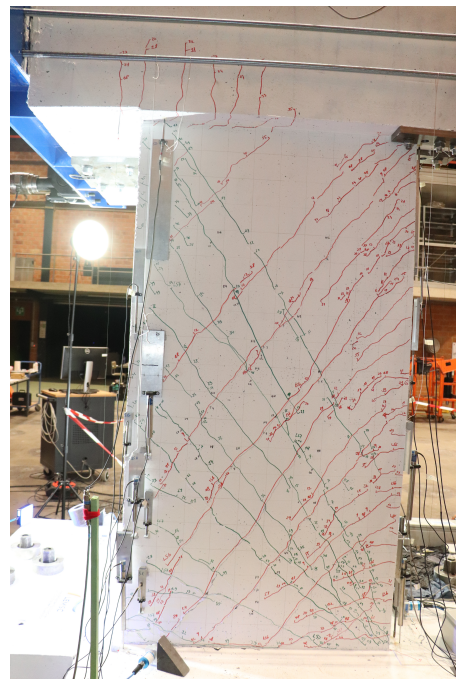


(b) Load stage 12

Fig. 4.14.: Crack patterns for load stages 6 and 12 during UW2 specimen test



(a) Load stage 20



(b) Load stage 56

Fig. 4.15.: Crack patterns for load stages 20 and 56 during UW2 specimen test

4.7 Personal contribution

Preparing such an experimental protocol is obviously not something that can be done overnight or at a moment's notice. After the design on paper, the different specimens had to be prepared many days before the test day, given all the work that had to be done on them. Some of the work was done before the wall was ready to be concreted, while the rest was done after the wall was delivered to the laboratories of LEMSC, UCLouvain.

4.7.1 Before casting of the specimen

The various specimens were made by a company independent of the LEMSC laboratory. However, some reinforcing steels and the steels with the optical fibre had to be made in the laboratory and then passed on to the company because this had to be done by hand.

Steel reinforcement

Starting from straight reinforcement of several metres, more than 800 elements had to be cut, bent and welded. These were mainly stirrups and shear reinforcement. The longitudinal bars in which the optical fibre was to be installed were also included in the package. These can be seen not only in Fig 4.2 but also in the annexed drawings .1.

Optical fiber installation

Once the rebars were cut, bent and welded they were ready for the optical fiber installation procedure Fig 4.16. As the fibers had never been placed inside the reinforcement bars as was done, the procedure had to be continuously refined. To get an idea, it took no less than 5 full days with 2 people on it to prepare the 4 pairs of bars for the first specimen. This time could obviously be reduced afterwards, but was still important given the rigour required for the optical fiber installation. Despite all the rigour of the world once transmitted to the company, unforeseen events can always happen which is why 2 fibres were cut at their ends in the concreting process for the first specimen, UW1. Fortunately, a severed fibre is not totally lost and with the help of a specific gel, it is possible to recover a signal.



Fig. 4.16.: Reinforcement bars ready to receive the optical fiber

4.7.2 Once the specimen was delivered

Once the wall is delivered, a multitude of steps still need to be carried out in order to make it more aesthetic on the one hand and on the other hand to be able to work on it (cf. marking of cracks)

Plaster

When the wall is brought into the laboratories, it is not completely perfect. In some places, holes can be seen making the wall unaesthetic and some surfaces are not perfectly smooth. If the first point is purely superficial, the second must be absolutely corrected for certain faces such as the one on which the transverse beam of the two horizontal actuators is fixed. For these reasons, a very thin layer of plaster must be applied in certain areas

Paint

The next natural step is painting. In fact, there are three reasons why the wall should be painted in white. The first remains the aesthetic reason, while the second is related to the cracks. Indeed, as can be seen in Fig 4.17, when the cracks appear they are highlighted with colours and numbers. In order to make the cracks and the markings more visible, it is important to paint the wall white. Note that the paint chosen and the number of coats should be selected so that the result is not too thick. Indeed, if this is the case, some very small cracks may appear but not be visible because the paint layer is too thick and hides

them. The last reason is that DIC requires a black point pattern, Fig 4.11, that is clearly visible to the cameras. Black dots on white paint offers a better contrast than black dots on grey surface.

LVDT's

The different LVDTs shown in Fig 4.8 had to be installed one by one. For each LVDT, a support had to be made to fix it and a bracket to adjust the stroke of the rod. A two-component glue was used to stick the support and the bracket to the wall. The advantage and disadvantage is that the glue hardens very quickly, which means that the mixture has to be made for each piece, i.e. a minimum of 42 times for one wall. Time and rigour were once again required.

DIC pattern and calibration

The final result of the dots pattern can be seen in Fig 4.11 for specimen UW1. To achieve this, it was necessary to use dots that were neither too large nor too small and above all to avoid creating a repetitive pattern. Indeed, the cameras should not mistake the area because of the similarity of the arrangement of the dots. The pattern must therefore be random and uniform. Once the pattern is applied to the two chosen sides, the various cameras and software can be calibrated. It is during the calibration that the importance of certain factors such as brightness or pattern is really revealed. It was necessary to play with these parameters many times and for several hours in order to obtain a good calibration and consequently a good mesh. This has to be done at the beginning of each test day and as the tests are spread over several days, it also takes a lot of time.

Cracks marking

Two of the three sides of the walls were used for the DIC, leaving only one side available to observe the cracks with the naked eye. Of course, a very small portion of this face was occupied by LVDTs, but this did not prevent the observation of cracks at the end of each load stage (Fig 4.17). The inscription to identify a crack was composed of a colour and a number. The colour corresponds to the direction of rotation applied, i.e. red for positive (clockwise) and green for negative (counterclockwise). The number is simply the number of the load stage in order to know when the crack appeared. In addition, the thickness of certain cracks was measured sporadically to observe their evolution.

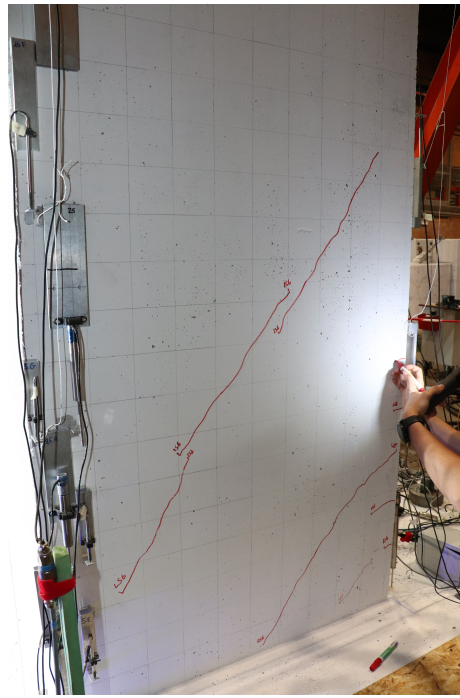


Fig. 4.17.: Marking of cracks between two load stages

In addition to the preparatory work required for the three specimens, these last two points, the DIC and the marking of the cracks, represent a large part of the author's contribution during the course of the tests. Expressing itself through the calibration of the DIC, running the program at the beginning of each load stage and marking the cracks at the end of each load stage.

Other Experimental Tests

“Everything should be made as simple as possible, but not simpler

— Albert Einstein

The tests carried out here at LEMSC were obviously not the first to be carried out on open U-shaped sections. Before that and as already mentioned, other experiments on this subject had been carried out but in a very limited number and especially not on walls but on beams. Not only is there a certain behavioural similarity between beams and walls, but also from the perspective of experimental implementation. Therefore, it seems appropriate to present some of these experiments carried out in the past. Furthermore, an analytical formulation will be presented and used to predict the results obtained in these experiments, but also those obtained at LEMSC.

5.1 Krpan and Collins (1981)

In the same way that the tests carried out in the LEMSC laboratories aimed to fill in the blur surrounding U-shaped reinforced concrete walls, the tests carried out at the time by Krpan and Collins [19] aimed to clarify the blur surrounding the behaviour of open thin walled reinforced concrete beams in pure torsion. There is therefore no question of applying additional bending to the element, which is a fixed-fixed beam, for which the torque is applied at mid-span.

5.1.1 Description of Test Unit

Only one unit was tested and had overall dimensions, cross-section and reinforcement that are shown in Figure 5.1. Those dimensions are in feet and inches, but once expressed in a metric system, lead to flange and web thickness (t_w) of 76.2 mm, web length (L_w) of 838.2 mm, and flange length (L_f) of 647.7 mm. The length of the element is about 7.62 m. These different values were imposed in order to respect the characterisation of the section as thin walled. This was achieved by taking the width and the height of the cross-section ten times bigger than the wall thickness, while the length was one hundred times this thickness. Figure 5.1 also presents the reinforcement, which was uniform throughout the length of

the beam. It consisted of 24-#5 (15.875 mm diameter) longitudinal bars arranged in the 4 corners and 9-#2 (6.35 mm diameter) bars distributed this time in the middle of the 3 sides to control cracking. The transverse (shear) reinforcement consists of #2 bars spaced at 76.2 mm.

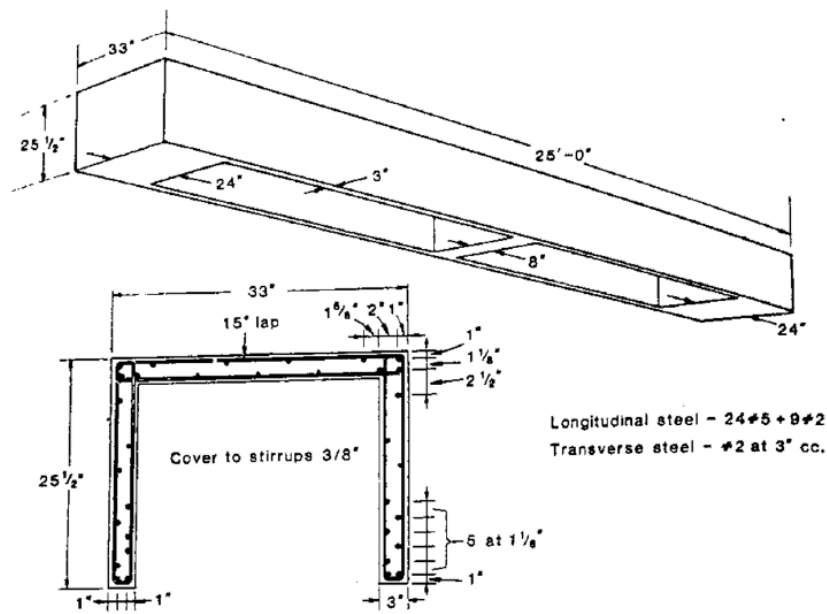


Fig. 5.1.: Dimensions of beam tested in Krpan and Collins experiment [19]

5.1.2 Steel reinforcement and concrete properties

The properties of the steels used and their stress-strain curve are summarised in Figure 5.2. While, the mean values of the two concrete batches supplied for the beam are given in Table 5.1.

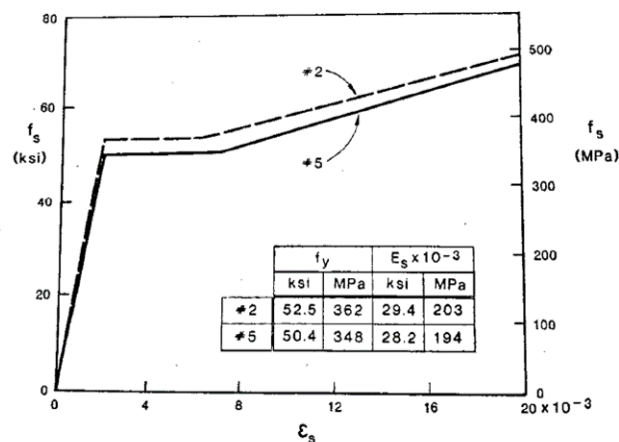


Fig. 5.2.: Stress strain curve of the rebars in Krpan and Collins experiment [19]

Specimen	Day from casting	Cylinder properties				
		Aggregate size (mm)	Slump test (mm)	f'_c (MPa)	Peak strain (mm/mm)	n
Collins-1:0	50(day of test)	10	75	52	0.003	2

Tab. 5.1.: Mean values of mechanical properties of the concrete for Collins-1:0 specimen

5.1.3 Test Setup

As introduced, there are many similarities between the tests carried out at LEMSC and those carried out in the programme currently described. These are particularly visible when the test arrangement provided in Fig 5.3 is analysed. The rigid foundation blocks introduced in Section 4 are here replaced by restraining solid blocks and torsional reaction frames on both ends. The horizontal actuators connected to a steel beam, which is subsequently connected to the collar of the wall are replaced by two 180 kN vertical jacks connected to two vertical rods and a torsional loading frame

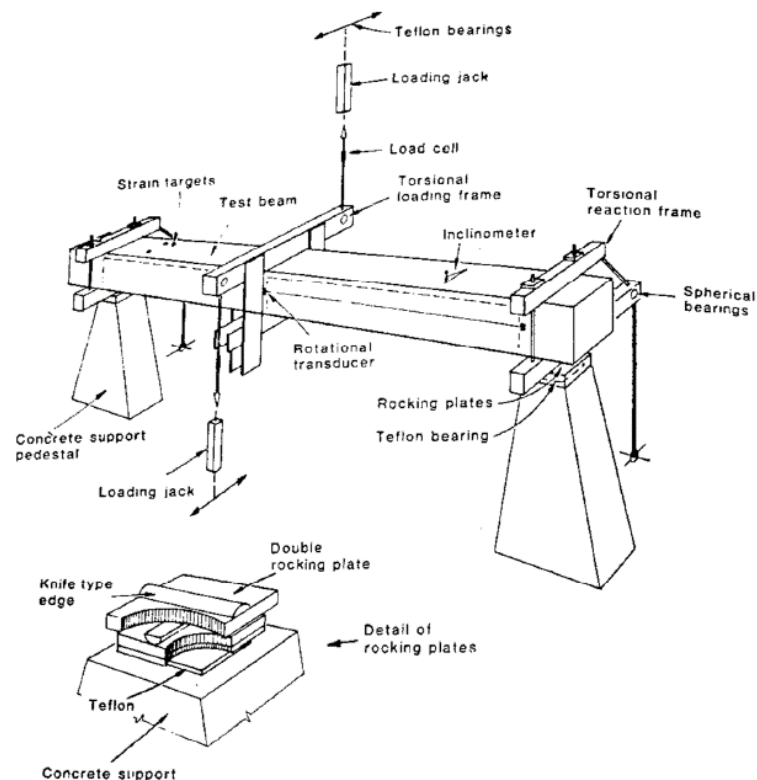


Fig. 5.3.: Test setup for Collins-1:0 [19]

5.1.4 Results

The experimental results obtained by Krpan and Collins [19] are presented in almost the same format. These are still represented as a torque-rotation curve, but in this case the rotation is evaluated at mid-span. Moreover, not all load stages are visible, but only a part of them for which the rotation obtained for the applied torque is visible. Also displayed on this graph are the different transitions between the 3 phases introduced in Section 3.1 and the different associated torques.

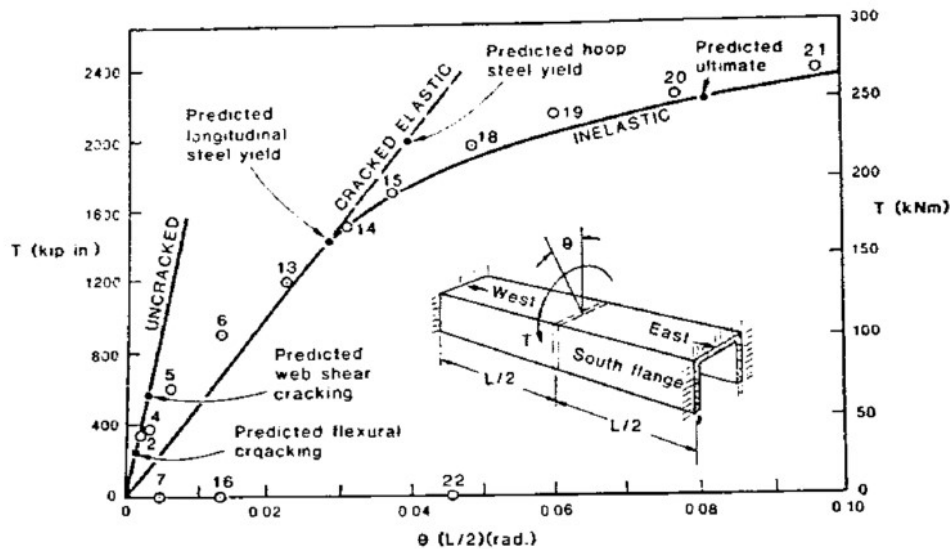


Fig. 5.4.: Torque-Rotation at mid-span for the beam tested by Krpan and Collins [19]

5.2 Chen et al. (2016)

Although Krpan and Collins pioneered the testing and calculation of U-shaped beams in 1981, research on this subject has remained very limited in the following years. Therefore, years later, the extensive use of these elements in railways prompted Chen et al. to initiate an experimental programme on U-shaped reinforced concrete beams [4]. Four U-shaped members, denoted as MEM-1, MEM-2, MEM-3, MEM-4 respectively, were tested and once again were doubly fixed and loaded at mid-span in order to apply pure torsion.

5.2.1 Description of Test Units

The overall dimensions of the 4 specimens and details of reinforcement for MEM-4 are shown in Fig 5.5.

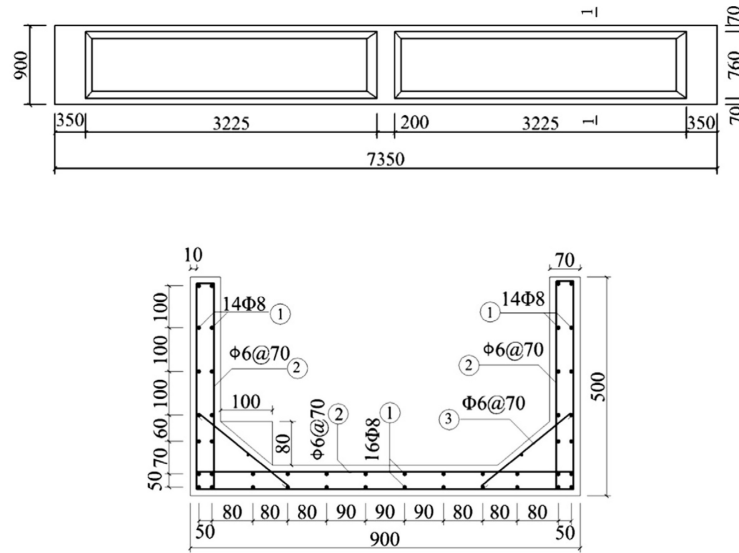


Fig. 5.5.: Overall dimensions and cross-section for Chen et al. experiments [4]

Although the general dimensions of these 1:4 scale walls are identical for each specimen, their reinforcement is not completely the same. Indeed, the overall dimensions and arrangements of stirrups were the same, but the longitudinal reinforcements are different for the first two and the other two. As a matter of fact, for the first two the reinforcement was of diameter $d_{bl}=10$ mm while the following ones were tested with longitudinal reinforcement of diameter $d_{bl}=8$ mm as it's shown 5.5.

5.2.2 Steel reinforcement and concrete properties

Apart from the diameter of the reinforcement which differs between the walls, the properties for the same diameter also vary when it comes to stirrups as shown in the Table 5.2

Steel	Specimen	d_{bl} [mm]	f_y [MPa]	f_u [MPa]	E_s [GPa]
Stirrup	MEM-1 and MEM-2	6	365.00	645.00	200
	MEM-3 and MEM-4	6	276.70	446.70	200
Longitudinal	MEM-1 and MEM-2	10	576.25	623	200
	MEM-3 and MEM-4	8	353.33	573.33	200

Tab. 5.2.: Steel properties for MEM-1, MEM-2, MEM-3, and MEM-4

For the concrete, the same composition and procedure was used for all 4 specimens. Namely, a mixture made of natural sand, natural gravel, Portland cement, fly ash and admixtures.

In addition to this, maximum aggregate diameter of 10 mm and the use of high speed vibrators was used to ensure a good quality of concrete. The combination of these different elements led to the average values for the four specimens that are summarised in Table 5.3. Unfortunately, compressive strength of concrete cylinder specimens for MEM-1 and MEM-2 were not tested, which explains the missing values.

Specimen	Cylinder compressive strength	Cube compressive strength	Elastic modulus
	(MPa)	(MPa)	(GPa)
MEM-1	-	50.95	40.00
MEM-2	-	50.95	38.5
MEM-3	42.56	63.83	42.2
MEM-4	40.92	51.57	34.38

Tab. 5.3.: Concrete properties for MEM-1, MEM-2, MEM-3, and MEM-4

5.2.3 Test Setup

As shown in Figure 5.6, the test setup developed in this experimental programme is basically the same as the one designed by Krpan and Collins. Indeed, the ends are used as anchor blocks, but also the two vertical cylinders connected to a loading frame at mid-span of the beam in order to apply pure torsion.

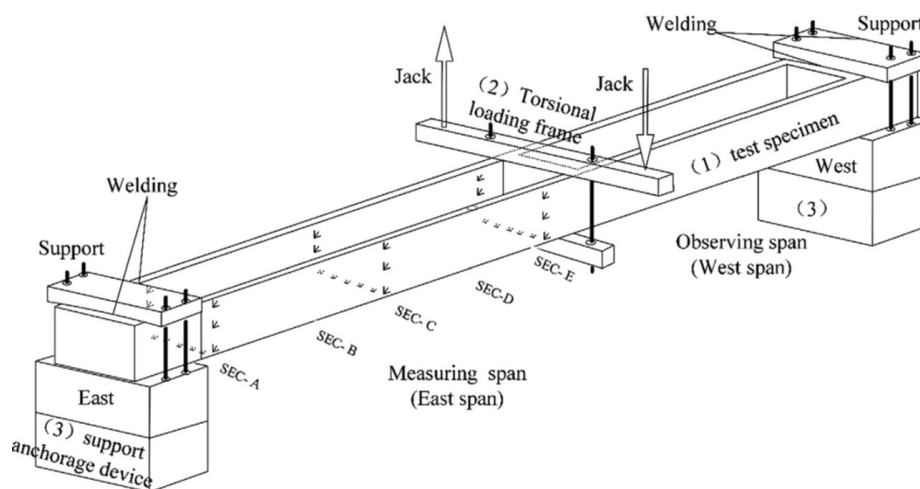


Fig. 5.6.: Test setup for MEM-1, MEM-2, MEM-3, and MEM-4 [4]

5.2.4 Results

The way in which the results are presented remain based on the torque-rotation curve because of its simplicity, but with the only difference that this time not one but four specimens were tested. As the first two specimens have the same characteristics and the same applies to the last two, it is not surprising to see that the results are almost overlapping for MEM-1 and MEM-2 and for MEM-3 and MEM-4 in Fig 5.7.

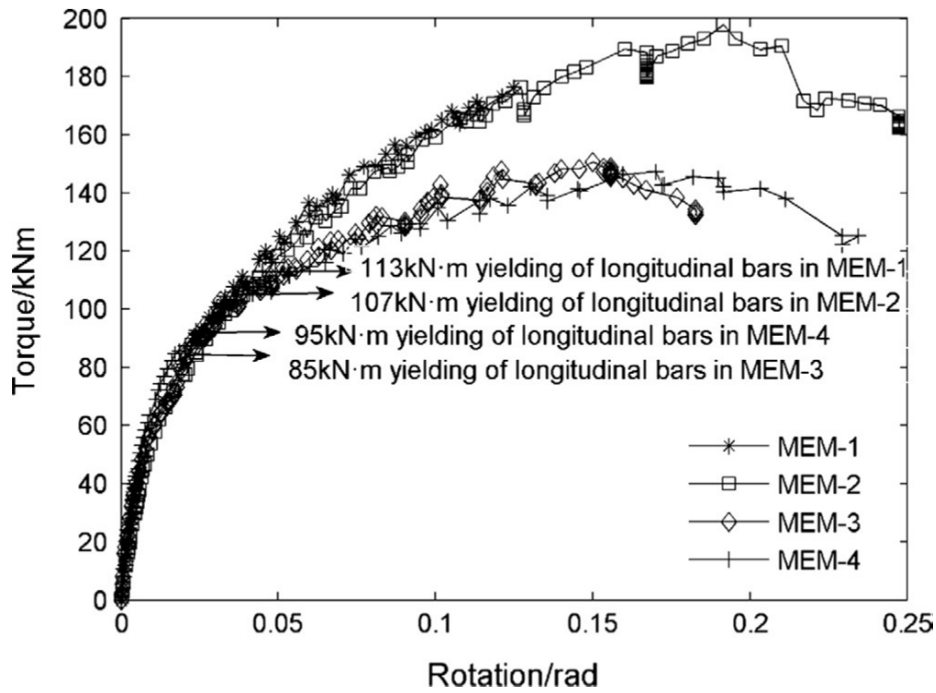


Fig. 5.7.: Torque-Rotation at mid-span for the beams tested by Chen et al. [4]

5.3 Xu et al. (2018)

In this new experimental program [32], some stakeholders are the same as for the tests presented in the previous section. There is therefore a certain continuity in the processes used as well as in the objective. Indeed, where previously only pure torsion was applied, here a bending moment will be added. This is done with the aim of subjecting the walls (which in the real world undergo not only torsion but also bending and shear) as realistically as possible. Three identical walls were therefore set up and tested with different torque–bending moment ratios (T–M ratios) of 1:5, 1:1 and 1:0.

5.3.1 Description of Test Units

The continuity in the experimental programs is obvious when looking at the general dimensions and the reinforcement plan (Figure 5.8) of the 3 respective specimens named MEM-1:5, MEM-1:1 and MEM-1:0 according to their T-M ratios applied. It can be seen that the three specimens are identical to the last two specimens in the previous experimental program

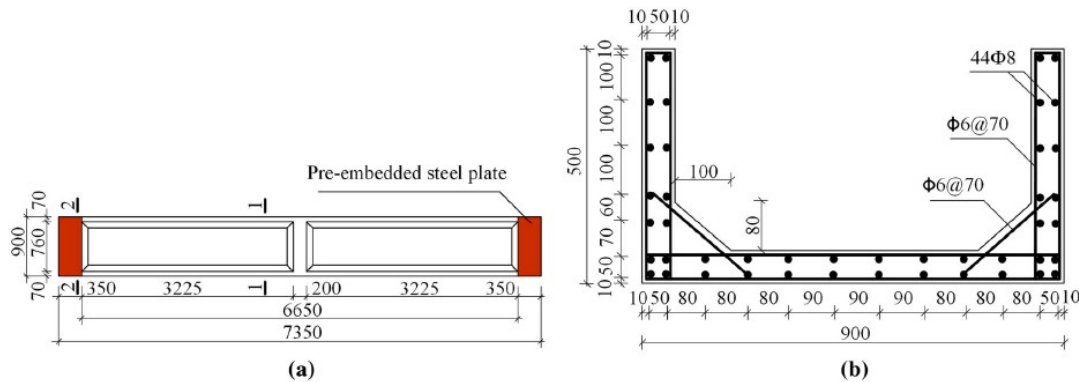


Fig. 5.8.: Overall dimensions and cross-section for Xu et al. experiments [32]

5.3.2 Steel reinforcement and concrete properties

Not only are the dimensions and reinforcement the same, but also the mechanical properties of the steels. These are already present in Table 5.2 for the specimens MEM-3 and MEM-4 but are again summarized in Table 5.4, valid for the all specimens present here

Steel	dbl (mm)	f_y (MPa)	f_u (MPa)	E_s (GPa)
Stirrups	6	276.7	446.7	200
Longitudinal	8	353.33	573.33	200

Tab. 5.4.: Steel properties for MEM-1:5, MEM-1:1, and MEM-1:0

As for the mechanical properties of the concrete used for the three specimens, these are listed in Table 5.5 in which only the cylinder strengths are present

Specimen	Cylinder compressive strength	Elastic modulus
	(MPa)	(GPa)
MEM-1:5	39.62	36.7
MEM-1:1	35.40	34.5
MEM-1:0	40.92	34.4

Tab. 5.5.: Concrete properties for MEM-1:5, MEM-1:1, and MEM-1:0

5.3.3 Test Setup

The test setup follows the rest and hardly changes from the previous experimental program as shown in Figure 5.9. The only notable changes are linked to the need of bending moment in addition to the torsional moment. For MEM-1:5 and MEM-1:1, the two jacks are no longer actuated simultaneously in opposite directions, but only one of the two cylinders is actuated with a specific eccentricity leading to the desired T-M ratio

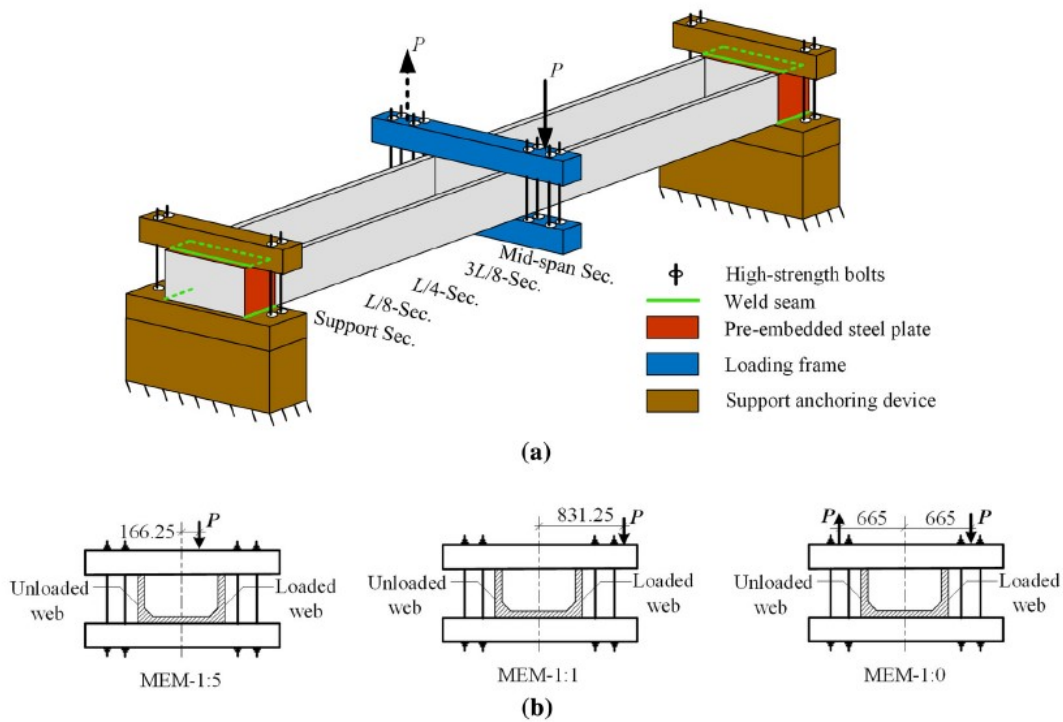


Fig. 5.9.: Test setup for MEM-1:5, MEM-1:1, and MEM-1:0 [32]

5.3.4 Results

Until now, whether for one or several specimens, the results were presented in the form of the torque-rotation graph. However, in this latest experimental protocol, a bending force

was added to the torsion force for 3 of the 4 specimens. This is why Fig 5.10 shows not only the torque-rotation curve as well as a bending moment-deflection curve for the concerned specimens.

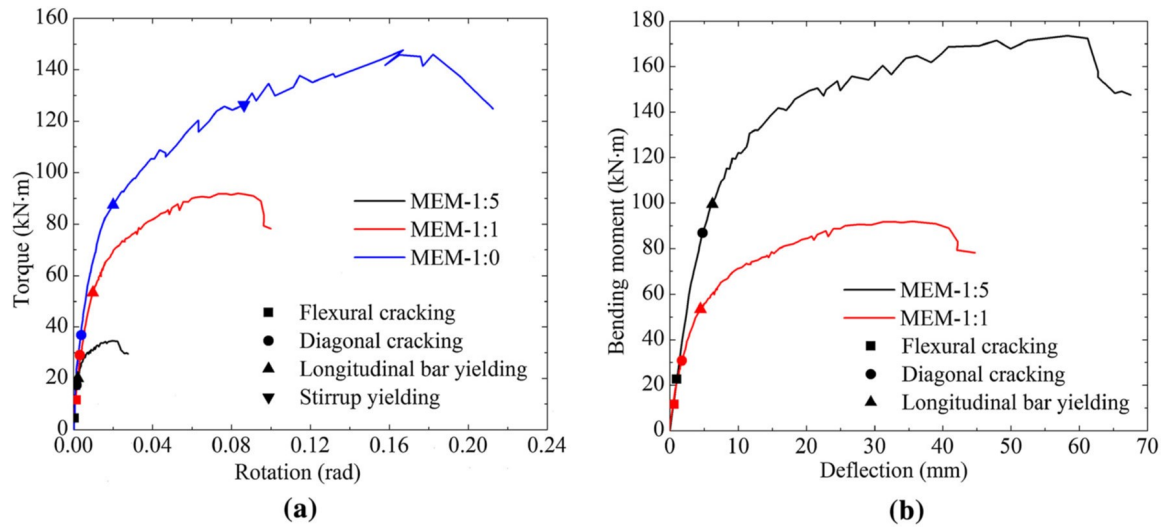


Fig. 5.10.: Torque-Rotation and bending moment-deflection curves at mid-span for the beams tested by Xu et al. [32]

Part IV

Analytical Formulation

Adaptation of Analytical Formulation to RC Walls

“ *Scientists have become the bearers of the torch of discovery in our quest for knowledge.*

— Stephen Hawking

As briefly discussed in the introduction to Section 3, understanding the different existing models is the first step. The first step towards adapting/developing a new model based on the work already done. To this end, the model chosen as a starting point was the one developed by Xu et al. [32]. Firstly, because unlike the Krpan and Collins procedure [18] which requires many iterative loops and a deep conceptual understanding, this one requires less knowledge and can therefore be handled by a wider range of engineers. Less knowledge, in the sense that the estimation of the cracking torque and the ultimate torque requires only 3 equations. Secondly, this choice was also made because the work of Xu et al. is in fact already a continuity of a model that was set up by Chen et al. Finally, in their article, a section is dedicated to the explanation of the calculation details based on the proposed procedure allowing to compare and verify the accuracy of the new model

It is therefore not a question of starting from a blank page but of making changes which, in the author's opinion, are necessary in order to be closer to reality. A whole work of adaptation to walls is also necessary, since the initial model only aimed at predicting the behaviour of fixed-fixed beams. It is again this strong similarity between an embedded beam and a free head wall that allows us to first modify the model by staying on the beam prediction before making the modifications that will allow the analysis of walls.

6.1 Modifications to the existing model

A model simplified to the maximum while keeping interesting results is what makes it a good choice at first sight. However, some calculation assumptions raise a few concerns. These questions have been discussed with the authors of the original paper, to whom thanks are reiterated here. A number of the issues are discussed below, while the rest of the remarks addressed are available in the appendix .3.

The different equations to obtain the value of the torque leading to the first flexural cracks have not been modified in any way and are therefore the ones presented in section 3.3.

Changes were made mainly to the calculation of the ultimate torque, as the assumptions did not seem to be adequate. Firstly, the way of calculating the equivalent forces introduced in Section 3.3 will be slightly adapted due to the impossibility of deriving the results announced in the scientific paper of Xu et al. [32]. Subsequently, comments and modifications will be made to one of the assumptions regarding steel strains in the web. Finally, a new iterative loop is proposed to calculate the different steel areas in compression and tension. These data are necessary to find the depth of the compression zone and to evaluate the equilibrium equations to obtain the ultimate torque.

6.1.1 Equivalent actions

As a reminder, these equivalent efforts were given in Eq (3.36) and Eq (3.37) in the following forms:

$$M_{eq} = \int_A \sigma_{\omega} y_1 dA = \int_A \frac{M_{\omega} \omega}{I_{\omega\omega}} y_1 dA = \frac{TC}{I_{\omega\omega}} \eta_1$$

In which $\eta_1 = \int_A \omega y_1 dA$

$$N_{eq} = \int_A \sigma_{\omega} dA = \int_A \frac{M_{\omega} \omega}{I_{\omega\omega}} dA = \frac{TC}{I_{\omega\omega}} \eta_2$$

In which $\eta_2 = \int_A \omega dA$

These two parameters η_1 and η_2 are therefore calculated as the integral over the entire area of the section. However, in the theoretical article by Krpan and Collins the evaluation of these same integrals is described and leads to a null result. The reason is that these integrals are actually used to find the three unknowns associated with the ω coordinate system

$$\int_A \omega dA = 0; \quad \int_A \omega x dA = 0; \quad \int_A \omega y dA = 0 \quad (6.1)$$

It is therefore not possible to obtain the non-zero values announced in the article by Xu et al. The choice was therefore made to evaluate the integrals not over the whole area, but only over half the U-shaped section. Just as the equivalent actions resulting from loading half of the U. This modification made it possible to strongly approximate the results presented in the article

- $\eta_1 = \int_{A/2} \omega y_1 dA$
- $\eta_2 = \int_{A/2} \omega dA$

After discussion with the authors, it became clear that the evaluation was indeed done on half the area. Note that in reality only η_1 really influences the final results while η_2 has very little impact. This is due to the fact that η_1 represents the effect of equivalent moment while η_2 represents the effect of equivalent axial force, and the effect of equivalent moment is dominating. Hence, the ultimate load depends mainly on η_1 (Xu et al., Personal communication, June 27, 2022).

6.1.2 Steel strain hypothesis in the web

Among the calculation assumptions, one of them has been further investigated. The assumption that all tensile bars at the bottom got ultimate strength considering that at the ultimate state, it was found that a strain gauge in the extreme tension fibre region measured a strain equivalent to the ultimate conditions of the longitudinal reinforcing bars. In the details of the calculations, it is therefore not f_y that has been used, despite what is shown in Figure 3.8, but f_u . This choice is significant and has a huge impact on the results, as will be shown in the section 8.2, but it does not seem to be the closest choice to reality for specimen UW2. Indeed, both a quick theoretical calculation and the experimental measurements show that the steel in tension in the web is actually extremely close to f_y .

Using as a first approximation that the depth of the compression zone in the web is equal to $0.2 \times L_w$ and knowing the ultimate strain in the concrete, it is possible to evaluate the strain profile over the web's length

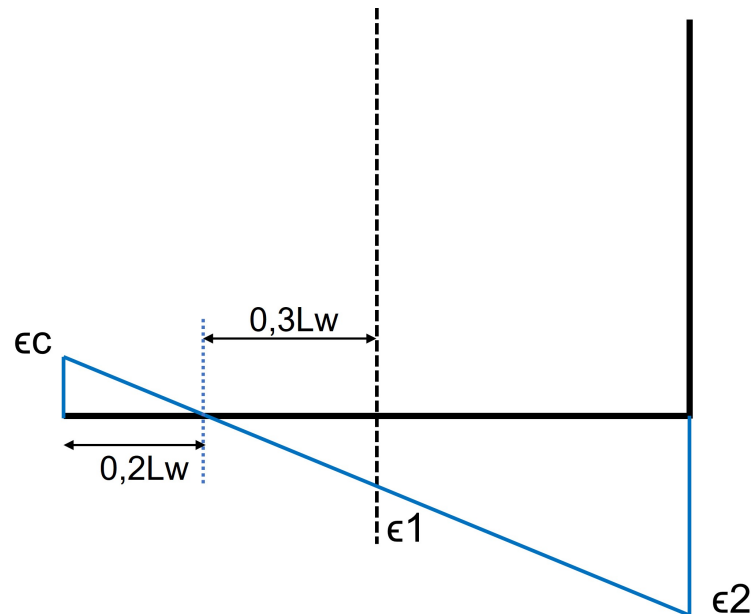


Fig. 6.1.: strains in the web (ϵ_c = concrete's peak deformation, ϵ_1 = steel's strain at mid-span of the web, ϵ_2 = steel's strain at the extremity of the web and L_w = web's length)

Depending on the value of the peak deformation of the concrete that is set, the values of ϵ_1 and ϵ_2 will change. There are roughly three possibilities for which the following results are obtained

For $\epsilon_c = 0.004$ = unconfined ultimate strain:

- $\epsilon_1 = \frac{\epsilon_c \cdot 0.3 \cdot Lw}{0.2 \cdot Lw} = \frac{0.004 \cdot 0.3}{0.2} = 0.006$
- $\epsilon_2 = \frac{\epsilon_c \cdot 0.8 \cdot Lw}{0.2 \cdot Lw} = \frac{0.004 \cdot 0.8}{0.2} = 0.016$

For $\epsilon_c = 0.01$ = confined ultimate strain:

- $\epsilon_1 = \frac{\epsilon_c \cdot 0.3 \cdot Lw}{0.2 \cdot Lw} = \frac{0.01 \cdot 0.3}{0.2} = 0.015$
- $\epsilon_2 = \frac{\epsilon_c \cdot 0.8 \cdot Lw}{0.2 \cdot Lw} = \frac{0.01 \cdot 0.8}{0.2} = 0.04$

The first two cases use current theoretical values. For the confined case, it is possible using the model of Mander et al. to obtain a more valuable value of the ultimate strain of confined concrete for our wall. To do this, a sequence of equations in which the various geometric data are involved is necessary. These are presented below, but only the result of the last one is of interest to us

$$\left\{ \begin{array}{l} \epsilon_{c0} = (1.8 + 0.0075 * f_{c0})/1000 \\ pcc = \sqrt{pwx^2 + pwy^2} \\ Ai = (w^2/6) * (n - 1) \\ st2 = st - (2 * dbt) \\ Ae = ((bc * dc) - Ai) * (1 - (st2/(2 * bc))) * (1 - (st2/(2 * dc))) \\ ke = Ae/(bc * dc * (1 - pcc)) \\ fl = (pcc * fy) \\ fl2 = fl * ke \\ fcc = fc0 * (-1.254 + (2.254 * \sqrt{1 + (7.94 * (fl2/fc0))}) - (2 * (fl2/fc0))) \\ ecc = ec0 * (1 + 5 * ((fcc/fc0) - 1)) \end{array} \right. \quad (6.2)$$

Where:

- $fc0$ = unconfined concrete strength, MPa

- f_y = yield strength of the longitudinal bars, MPa
- bc = length of the boundary end of the wall, mm
- dbt = diameter of the transverse reinforcement, mm
- dbl = diameter of the longitudinal reinforcement, mm
- st = spacing of the transverse reinforcement ties, mm
- w = spacing of longitudinal reinforcing bars in the boundary end, mm
- dc = thickness of the wall, mm
- n = total number of longitudinal bars in the boundary end (e.g., 2 layers of 3 bars = 6)
- pwx = transverse reinforcement ratio in the x (typically parallel to the thickness of the wall, e.g., 0.01)
- pwy = transverse reinforcement ratio in the y (typically parallel to the length of the wall, e.g., 0.01)
- f_{yh} = min yield strength of bars used; this is used to calculate ϵ_{cmax} (max concrete strain)

By successively calculating these different equations on the basis of the geometric data of the UW2 wall, the confined concrete strain at peak is $\epsilon_c = 0.007$. It's a bit less than the theoretical value used before and leads to the following values of strains in the web

For $\epsilon_c = 0.007$ = real confined ultimate strain:

- $\epsilon_1 = \frac{\epsilon_c \cdot 0.3 \cdot Lw}{0.2 \cdot Lw} = \frac{0.007 \cdot 0.3}{0.2} = 0.0105$
- $\epsilon_2 = \frac{\epsilon_c \cdot 0.8 \cdot Lw}{0.2 \cdot Lw} = \frac{0.007 \cdot 0.8}{0.2} = 0.028$

Using the idealised trilinear stress strain curve presented in Section 2.2 and the deformations calculated for the three previous cases, it is possible to assess the accuracy of the assumption that the steels are closer to f_y in our case than f_u . The four values needed to check this are the start (ϵ_{sy}) and end (ϵ_{sh}) values of the plastic yield plateau, ϵ_1 and ϵ_2 from the real confined ultimate strain of the concrete. These first two vary according to the characteristics of the steel, as summarised previously in the Table 4.1 mechanical properties of the class C reinforcement that is recalled below.

d_{bl}	f_y	f_u	ϵ_{sy}	ϵ_{sh}	ϵ_{su}	E_s
[mm]	[MPa]	[MPa]	[mm/mm]	[mm/mm]	[mm/mm]	[GPa]
6	550	676	0.0027	0.010	0.095	207
8	538	664	0.0027	0.024	0.120	196
12	580	690	0.0029	0.021	0.101	199

If the values of ϵ_1 and ϵ_2 are between those 2 limits, the assumption of a constant elastic limit reached for half the web and not the ultimate limit would therefore be consistent. If not, then it makes sense to see what impact those strain values will have on the value to be considered for the strength of the steel.

For the smallest diameter ($d_{bl} = 6$ mm), both ϵ_1 and ϵ_2 are above the end limit, ϵ_{sh} corresponding to strains outside the plastic yield plateau. This also means that the stresses associated with these deformations are higher than the yield stress and are not equal. But what are the stresses associated with these deformations? Is it wise to take the ultimate strength? Xiang Yun and Leroy Gardner [33] developed two different models to predict the stress-strain curves of steels. Their bilinear plus nonlinear hardening model can be used to know the stresses corresponding to the displacements before the plateau, on it and after it

$$f(\epsilon) = \begin{cases} E\epsilon, & \text{for } \epsilon \leq \epsilon_y \\ f_y, & \text{for } \epsilon_y < \epsilon \leq \epsilon_{sh} \\ f_y + (f_u - f_y) \left\{ 0.4 \cdot \left(\frac{\epsilon - \epsilon_{sh}}{\epsilon_u - \epsilon_{sh}} \right) + 2 \cdot \left(\frac{\epsilon - \epsilon_{sh}}{\epsilon_u - \epsilon_{sh}} \right) / \left[1 + 400 \cdot \left(\frac{\epsilon - \epsilon_{sh}}{\epsilon_u - \epsilon_{sh}} \right)^5 \right]^{1/5} \right\}, & \text{for } \epsilon_{sh} < \epsilon \leq \epsilon_u \end{cases} \quad (6.3)$$

The different stresses resulting from the deformations grouped in Table 6.1 show that the stress distribution is no longer rectangular, but trapezoidal. A solution to see the impact of this increase is to calculate the respective areas represented by the distributions. Thanks to this, it is possible to realise that in reality the impact is very small and, above all, very far from the impact that a rectangular distribution with the ultimate effort could have. Indeed, it can be seen that this new distribution is very close to the new proposed hypothesis. In order to be perfectly precise, it was decided to multiply the chosen value of f_y by the ratio

obtained by making the ratio of the areas, even if this only changes the results slightly

d_{bl}	ϵ_1	$f(\epsilon_1)$	ϵ_2	$f(\epsilon_2)$	Rectangular distribution	Trapezoidal distribution	Ratio
[mm]	[mm/mm]	[MPa]	[mm/mm]	[MPa]	[Area]	[Area]	[/]
6	0.0105	551.78	0.028	612.38	357500	377133.49	1.055
8	0.0105	538	0.028	550.60	349700	353794.97	1.012
12	0.0105	580	0.028	603.09	377000	384504.94	1.02

Tab. 6.1.: Strain, stress, and ratio between the rectangular distribution hypothesis and the suggested trapezoidal distribution for the different diameters of rebars

Another way to check if this new hypothesis is acceptable is to look at the experimental data captured by the different LVDTs during the test. The arrangement of these introduced during the description of the tests at LEMSC in Section 4.5.1 provides just such a measurement in the corner of the West flange. This value can be used for the web because it is adjacent to it. Thanks to this value, it is possible to compare the theoretical deformation ϵ_2 for diameter $d_{bl}=12\text{mm}$ with the observed deformation. At failure, the LVDT 50A indicates a strain equal to 0.023. This is slightly below the theoretical value but even closer to ϵ_{sh} , which confirms that the resistance to consider is extremely close to f_y and not f_u in the case of UW2.

6.1.3 Areas of steel

Another potential problem with the original methods proposed by Xu et al. [32] for estimating T_{ult} is the calculation involved for the different areas of reinforcing steel. Before doing so, it is worth recalling what the unknowns in the various equations are. In this case, they are the depth of the concrete compression zone and the ultimate torque. There are therefore two equations with two unknowns which could be solved very simply. However, the areas of steel in compression or tension are themselves dependent on this compression zone. Setting these areas and then solving the system of equations and finding the compression zone seems rather unsuitable.

Indeed, the preferred approach is based on an iterative loop (Figure 6.2), the principle of which is to set a value for the compression zone (starting at zero), deduce the different steel areas and finally evaluate the equations. When the equations are not verified, the depth of the compression zone is incremented by a certain step. Once convergence is reached, the compression zone, the steel areas and the ultimate torque are known.

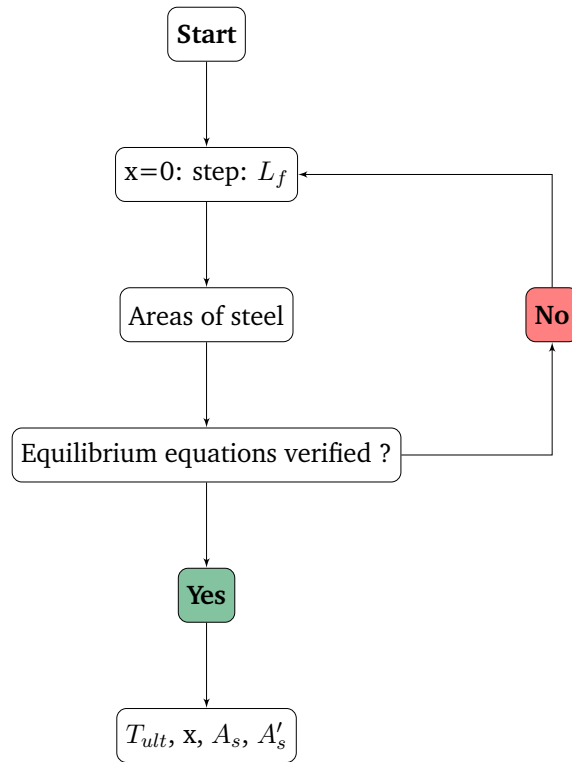


Fig. 6.2.: Flowchart of ultimate torque procedure (x = depth of compression zone, L_f = flange's height, A_s = area of steel in tension and A'_s = area of steel in compression)

6.2 New model improvements

The modifications presented in the previous section are only adaptations of concepts already present in the basic model. However, in order for the model to be used to predict walls performances, it has been necessary to add completely new elements to the model. These mainly concern the new boundary conditions and the axial load that are now present. The first part aiming at calculating the different geometrical data such as area, inertia, sectorial coordinates, etc. do not change. The calculations are later differentiated according to whether it is a beam or a wall. This differences will mostly come from the distribution of efforts being different.

6.2.1 Flexural cracking torque

Specifically, three cases are analysed for the calculation of the cracking torque: if it is a beam, if it is a wall without axial load and if it is a wall with axial load

For a beam

In this case, nothing changes and the Equation (3.33) introduced in Section 3.3 is used. Note that if no bending force is applied, the variable R is equal to zero and the equation reverts to that introduced in the model presented by Chen et al. (Equation (3.22)).

For a wall without Axial Load Ratio

The modifications for a wall without axial load are rather simple and only involve changing the boundary conditions. These will first remove the two factors two present in the equation for a beam. The second change concerns the coefficient for warping moment, C, that was adapted by Zbirohowski-Kościa [34] for a cantilever wall and presented in the work of Ryan Hoult [8] in Section 3.4. These two changes lead to the following cracking torque equation, for which the different calculation steps are presented in order to better understand where it comes from

$$\begin{aligned}\sigma_{max} &= \frac{M_b \cdot y_{max}}{I} + \frac{T_{cr} \cdot C_{cantilever} \cdot \omega_{max}}{I_{\omega\omega}} \\ \sigma_{max} &= \frac{R \cdot T_{cr} \cdot y_{max}}{I} + \frac{T_{cr} \cdot C_{cantilever} \cdot \omega_{max}}{I_{\omega\omega}} \\ \sigma_{max} &= T_{cr} \cdot \left(\frac{R \cdot y_{max}}{I} + \frac{C_{cantilever} \cdot \omega_{max}}{I_{\omega\omega}} \right) \\ \sigma_{max} \cdot I \cdot I_{\omega\omega} &= T_{cr} \cdot \left(R \cdot y_{max} \cdot I_{\omega\omega} + C_{cantilever} \cdot \omega_{max} \cdot I \right) \\ \boxed{\frac{\sigma_{max} \cdot I \cdot I_{\omega\omega}}{R \cdot y_{max} \cdot I_{\omega\omega} + C_{cantilever} \cdot \omega_{max} \cdot I} &= T_{cr}}\end{aligned}\tag{6.4}$$

Where:

- R= ratio between the bending moment and the torque applied
- $\sigma_{max} = 0.63\sqrt{f'_c}$

For a wall with Axial Load Ratio

For the latter case, starting from the previous one, the effect of the axial load must be added. This is half of the total load applied to the total section, as we are working with only half the area of the U-shaped section

$$\sigma_{max} = \frac{M_b \cdot y_{max}}{I} + \frac{T_{cr} \cdot C_{cantilever} \cdot \omega_{max}}{I_{\omega\omega}} - \frac{F_a}{A/2}$$

$$\sigma_{max} + \frac{F_a}{A/2} = \frac{M_b \cdot y_{max}}{I} + \frac{T_{cr} \cdot C_{cantilever} \cdot \omega_{max}}{I_{\omega\omega}}$$

$$\sigma_{max} \cdot A/2 \cdot I \cdot I_{\omega\omega} + F_a \cdot I \cdot I_{\omega\omega} = R \cdot T_{cr} \cdot y_{max} \cdot I_{\omega\omega} \cdot A/2 + T_{cr} \cdot C_{cantilever} \cdot \omega_{max} \cdot I \cdot A/2$$

$$\sigma_{max} \cdot A/2 \cdot I \cdot I_{\omega\omega} + F_a \cdot I \cdot I_{\omega\omega} = T_{cr} \cdot \left(R \cdot y_{max} \cdot I_{\omega\omega} \cdot A/2 + C_{cantilever} \cdot \omega_{max} \cdot I \cdot A/2 \right)$$

$$\frac{\sigma_{max} \cdot A/2 \cdot I \cdot I_{\omega\omega} + F_a \cdot I \cdot I_{\omega\omega}}{\left(R \cdot y_{max} \cdot I_{\omega\omega} \cdot A/2 + C_{cantilever} \cdot \omega_{max} \cdot I \cdot A/2 \right)} = T_{cr}$$

$$\boxed{\frac{0.63\sqrt{f'_c} \cdot A/2 \cdot I \cdot I_{\omega\omega} + F_a \cdot I \cdot I_{\omega\omega}}{\left(R \cdot y_{max} \cdot I_{\omega\omega} \cdot A/2 + C_{cantilever} \cdot \omega_{max} \cdot I \cdot A/2 \right)}} = T_{cr} \quad (6.5)$$

Where:

- A= area of the section
- Fa= axial load applied on the half U-section

6.2.2 Shear cracking torque

One phenomenon that was not taken into account at all in the basic procedure was shear cracking. Indeed, the various beams tested were designed and strengthened in such a way that warping dominated the response. Since the shear force is not critical, there is no need to calculate its value. However, not all beams or walls are designed in this way and including the calculation of the shear cracking torque is essential. The different equations are based on the concepts of shear flow resulting from a shear force (6.6) and shear flow from the warping torsion (6.8).

$$q = \frac{V}{Q \cdot I} \quad (6.6)$$

Where:

- V= Shear

- Q = Static moment of area given by:

$$Q_y = \int_0^s y \cdot t ds \quad (6.7)$$

- I = Moment of inertia

$$q = \frac{T_\omega}{Q_\omega \cdot I_{\omega\omega}} \quad (6.8)$$

Where:

- T_ω = Warping torsion
- Q_ω = Sectorial static moment given by:

$$Q_\omega = \int_0^s \omega \cdot t ds \quad (6.9)$$

- I = Sectorial moment of inertia

Using these equations and the ACI indicating that shear cracks appear when shear stress exceeds $0.33\sqrt{f'_c}$, it is possible to obtain the torque inducing the first cracks for the two different cases

For a fixed-fixed beam

This calculation must be carried out for both the web and the flange, for which the only difference will be the parameter Q_y whose values will be different.

$$T_{cr \text{ shear}} = \frac{0.33 \cdot \sqrt{f'_c} \cdot I_{\omega\omega} \cdot t_w}{|Q_y| \cdot 2} \cdot (1 - Swell) - \left[\frac{2 \cdot 0.33 \cdot \sqrt{f'_c} \cdot K}{t_w} \right] \quad (6.10)$$

Where:

- $Swell$ = coefficient that takes into account the coexistence of circulatory torsion and warping torsion. Indeed, the contribution of circulatory torsion will reduce the shear stresses caused by warping torsion

For a cantilever wall

For the calculation of acceptable shear stresses, the axial load has no influence. The only modification will come from the boundary conditions, which leads to the suppression of the factors two present in the previous equation

$$T_{cr\ shear} = \frac{0.33 \cdot \sqrt{f_c} \cdot I_{\omega\omega} \cdot t_w}{|Q_y|} \cdot (1 - S_{well}) - \left[\frac{0.33 \cdot \sqrt{f_c} \cdot K}{t_w} \right] \quad (6.11)$$

6.2.3 Ultimate torque

The principle introduced in Section 6.1.3 remains the same. Namely, an iterative procedure until the equilibrium equations introduced for their part in Section 3.3 are verified. Like the procedure for the flexural cracking torque, the calculation of the ultimate torque is divided into three different cases. For each case, the equations are adapted accordingly.

For a beam

The equations presented in section 3.3 were for a beam and are therefore still applicable. Since the aim is to obtain a value for the ultimate torque, this can be isolated in equation (3.34) leading to the following expression

$$T = \left[f_y A_s + f_y A_{sd} - 0.85 f_c b \chi - f'_y A'_s \right] \cdot \frac{2 \cdot I_{\omega\omega}}{C_{fixed} \cdot \eta_2} \quad (6.12)$$

This expression is then injected into equation (3.35) and the equilibrium is evaluated. There are several ways to evaluate convergence. The choice was made to group all the parameters from this equation on the same side, in order to have one term (E_{ql}) minus a second one (E_{qr})

$$E_{ql} = \left[f_y A_s \cdot (h_0 - 0.5\chi) + f_y A_{sd} \cdot (0.5h_0 + 0.25\chi) + f'_y A'_s \cdot (0.5\chi - ap) \right] \quad (6.13)$$

$$E_{qr} = T \left[\frac{C_{fixed} \cdot \eta_1}{2I_{\omega\omega}} + 0.5R + \frac{C_{fixed} \cdot \eta_2}{2I_{\omega\omega}} \cdot (h' - 0.5\chi) \right] \quad (6.14)$$

$$E_{ql} - E_{qr} = 0 \quad (6.15)$$

At each iteration of the loop, the ratio between E_{ql} and E_{qr} is evaluated. When this ratio is equal to 1 it means that the convergence is reached.

For a wall without Axial Load Ratio

In the case of a wall that is not subjected to any axial force, the only changes are due to the boundary conditions. Since in Eq_l only the internal loads appear, this term remains the same as no loads have been modified compared to the previous case. As for the convergence condition, it will never change regardless of the case. This leaves the equations of T and Eq_r for which the changes concern the coefficients of the warping moment and those related to the new stress distribution

$$T = \left[f_y A_s + f_y A_{sd} - 0.85 f_c b \chi - f'_y A'_s \right] \cdot \frac{I_{\omega\omega}}{C_{cantilever} \cdot \eta_2} \quad (6.16)$$

$$Eq_r = T \left[\frac{C_{cantilever} \cdot \eta_1}{I_{\omega\omega}} + 0.5R + \frac{C_{cantilever} \cdot \eta_2}{I_{\omega\omega}} \cdot (h' - 0.5\chi) \right] \quad (6.17)$$

For a wall with Axial Load Ratio

In this last case, the axial load must now be taken into account. As this is an external force, it is included in the variable Eq_r , so once again there is no change to Eq_l . Since the modifications linked to the boundary conditions are already taken into account in the last case, only the axial force must be added with a variable called F_a that appears in both the torque and Eq_r equations

$$T = \left[f_y A_s + f_y A_{sd} - 0.85 f_c b \chi - f'_y A'_s + F_a \right] \cdot \frac{I_{\omega\omega}}{C_{cantilever} \cdot \eta_2} \quad (6.18)$$

$$Eq_r = T \left[\frac{C_{cantilever} \cdot \eta_1}{I_{\omega\omega}} + 0.5R + \frac{C_{cantilever} \cdot \eta_2}{I_{\omega\omega}} \cdot (h' - 0.5\chi) \right] - F_a \cdot (y_{max} - 0.5\chi) \quad (6.19)$$

Application of the Model to Experiments

“ *Everything is theoretically impossible, until it is done.*

— Robert A. Heinlein

Before going into the details of the results, let us retrace a little of the process followed in this thesis. The first step was obviously to address the theoretical foundations. These are obviously essential to the understanding of the various models introduced afterwards. These models were obviously confronted with experimental results, which is why the next step was to describe the various tests. Once the overview was finally clear, it was possible to choose the model that would form the basis of the new model. This model was modified in some aspects, but was also supplemented with completely new elements. This thread leads us naturally to apply this model to the different tests introduced and to obtain, as they do, the first cracking torque and the ultimate torque

7.1 Cracking torque results

In total 7 specimens tested have enough data to make a sufficiently consistent prediction. Indeed, Chen et al. tested 4 specimens and not 2, but unfortunately the concrete strengths of the first two are not provided. As will be shown in Section 8.1, this value greatly influences the results, and it is therefore unwise to attempt to predict the behaviour of the first two walls. For the first six specimens, the experimental value was available in the various articles summarising these experiments ([19], [4], [32]). For the UW2 wall, the way in which the experimental value was obtained will be explained later, as well as the prediction which required a little more attention.

As shown in Table 7.1, the cracking torque predictions are very good for almost all specimens. This is true regardless of whether a bending moment is present in addition to the torsion. Indeed, except for MEM-3 and Collins-1:0, all other specimens show predictions with a deviation of less than 8% which is totally acceptable. It should be noted that for the first 6 specimens, the cracking torque is due to so-called flexural cracking. The authors and the results clearly indicate a domination of the warping effects in comparison with the shear force, thanks to the crack patterns observed.

Specimen	Test	Models calculation	Author's calculation	Test/Author's prediction
MEM-1:5	4.5	4.8	4.75	0.95
MEM-1:1	9.6	10.5	10.46	0.92
MEM-1:0	17.3	16.8	16.84	1.03
MEM-3	19.3	18.13	17.18	1.12
MEM-4	17.29	17.78	16.84	1.03
Collins-1:0	23	29	26.94	0.85
UW2	48	/	58.97	0.81

Tab. 7.1.: Cracking torque results in kNm

For the UW2 specimen tested at LEMSC, there is obviously no other mechanical model that has attempted to predict the cracking torque. The only comparison of the author's prediction will therefore be with the experimental results. In this case, however, particular attention must be paid to the shear force, unlike the other specimens where the effect was clearly not dominant. Indeed, very early in the loading stages, consequent diagonal cracks representative of shear cracks could be observed (Fig 4.14). It is therefore necessary to analyse in more detail the so-called flexural and shear cracking torques. The results obtained from equations 6.5 and 6.11 are 58.97 kNm and 147.22 kNm respectively. Given those results, is it therefore conceivable that bending cracks have appeared before without being visible? To verify this hypothesis, it is possible to observe the torque-rotation curve during the first loading cycle.

The value circled Fig 7.1 has not been chosen randomly. Indeed, after this value, the curve changes its slope. This can be seen by plotting the slope between the abscissa of 0 to 0.5 mrad and that between 0.5 and 1. A softening slope is akin to a crack in the wall that would reduce its rigidity. This is all the more plausible because a sudden increase in rotation without an increase in torque is clearly visible at this point. Looking at the value of the torque corresponding to this rotation, it appears to be of approximately 50 kNm. Coupling the graph in Fig 7.1 with the predictions made, it appears that the first crack appears at around 50 kNm which is very close to the predicted flexural cracking torque. It is therefore also not a shear type cracking that first took place like the other specimens, and this despite very early shear cracks. It is very likely that this flexural cracking went unnoticed because of the paint, which turned out to be very thick.

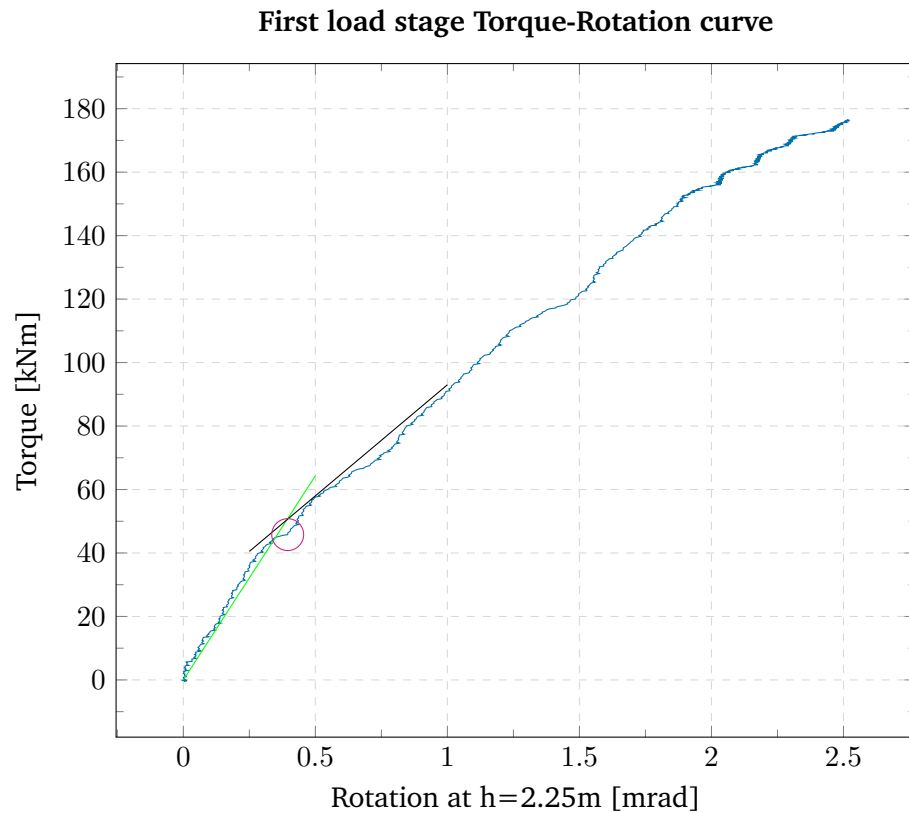


Fig. 7.1.: First load stage torque rotation curve

7.2 Ultimate torque results

For the ultimate torque, only 2 of the 4 specimens tested by Chen et al. will be analysed, for the same reasons as above. The Tab 7.2 shows the different values of ultimate torques and their accuracy with respect to the experimental results.

Specimen	Test	Model calculation	Author's calculation		Test/Author's calculation	
			With f_y	With f_u	With f_y	With f_u
MEM-1:5	34.8	35.5	34.03	48.71	1.02	0.71
MEM-1:1	92	88.5	77.09	112.02	1.19	0.82
MEM-1:0	147	139	113.09	165.68	1.3	0.89
MEM-3	151	138.99	113.17	165.53	1.33	0.91
MEM-4	147	138.33	113.09	165.69	1.3	0.89
Collins-1:0	266	249	246.74	/	1.08	/
UW2	432.81	/	348.74	368.34	1.24	1.17

Tab. 7.2.: Ultimate torque results in kNm

The propensity that emerges from this table is that the author's predictions tend to overestimate the ultimate torque of the various specimens when using the ultimate strength of the steel (f_u) and to underestimate when using the yield strength (f_y). This is certainly true

for the first five, which also leads to an inaccuracy in the Test/Prediction ratio that is not as accurate as for the cracking torque. The aim of these predictions is obviously to obtain results as close as possible to the tests, but not only. Indeed, in the case of disparities in the results, the most important point is to be able to identify the possible reasons.

For the first five specimens, this reason could come from one of the hypotheses introduced in Section 3.3. This one advises everyone not to use f_y for tensioned armatures in the web but rather f_u . As detailed in Section 6.1.2, this assumption does not hold at all for specimen UW2 tested at LEMSC. However, for the predictions of the first five specimens and following the recommendation in their article [32] for those tests, this assumption has been kept for their specimens. However, for the purpose of comparison, predictions using yield strength have also been produced. The reason is that some uncertainties persist in this hypothesis. Firstly, this hypothesis says that ONE of the bars would have reached f_u in the web. However, what about the rest of the bars in the web ? Moreover, the graphs that they give that are supposed to prove that the ultimate strength has been reached for a bar in the web seems to stop way before reaching deformations close to the ultimate deformation for this type of steel (Fig 7.2). Wouldn't using f_u for all of them be the reason why the predictions overestimate the resistance ? As will be shown in Section 8.2, the value used for the tensile strength of the steel greatly influences the results, but since using f_y underestimates the ultimate torque and using f_u overestimates it seems fairly clear that a value between the two gives the expected values. A better knowledge of the distribution of deformations in the core would undoubtedly help to refine the results and answer this question.

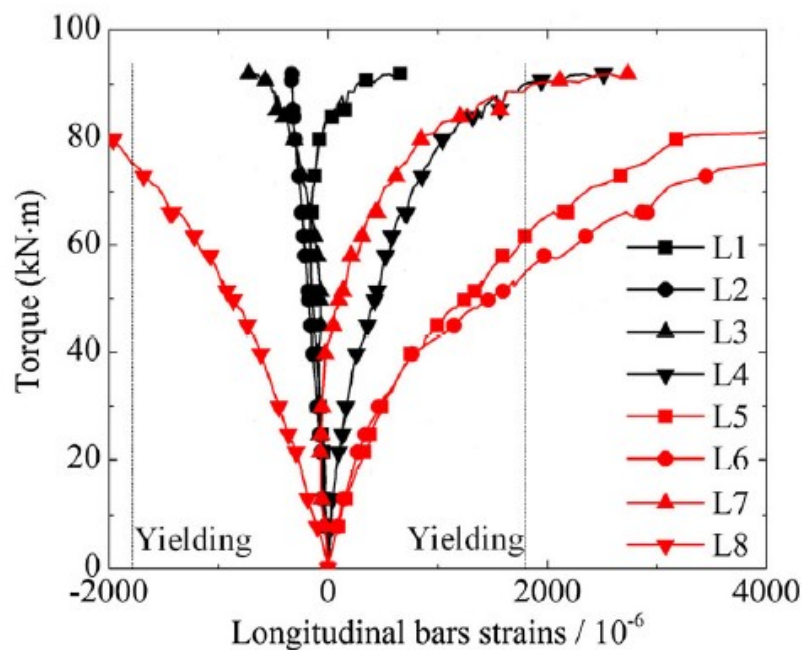


Fig. 7.2.: Reinforcement strains for MEM-1:1 at mid-span [32]

For the specimen tested by Krpan and Collins, the prediction is quite good. The difference could be explained by the different units used and the details provided. The specimen description document was written using the imperial system of measurement, which when translated into the international system of measurement may result in slight differences. Furthermore, the reinforcement details are excessively brief, again leading to some inaccuracies. However, the prediction remains very close to the experimental value.

As for specimen UW2, this time it greatly underestimates the ultimate torque. Such a large difference cannot be explained with just convergence inaccuracies or approximations on the characteristics of the materials used. To find the beginning of an explanation, it is necessary to look at the diagram of the internal forces. More precisely, the warping moment diagram must be examined. The distribution introduced in Section 2.1.2 is in fact an approximation of the real distribution. It is in practice non-linear as shown in Fig 7.3. What is important is the general appearance and to understand that at the foot of the wall the value is max and that at the top of the wall the diagram converges to zero. The total height of the wall logically intervenes in the different equations based on this distribution.

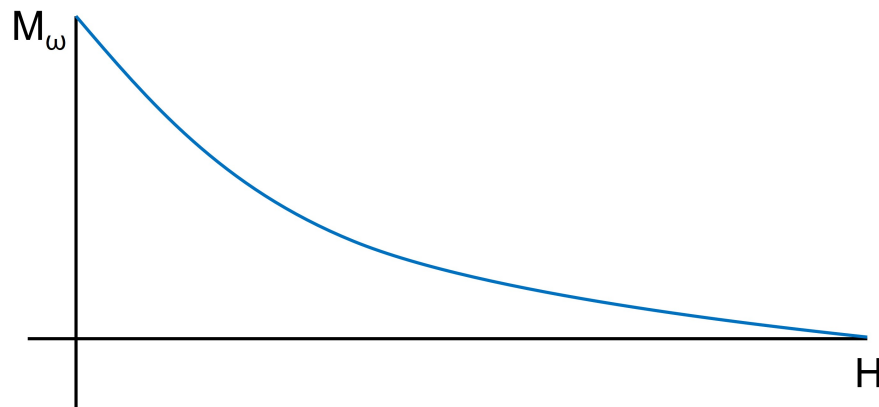


Fig. 7.3.: Assumed warping moment distribution

The fact is that during the experimental tests on UW2, many cracks appeared at the head of the wall, as can be seen in Fig 7.5. If the actual distribution during testing matched the theoretical distribution, these cracks should not be present.

A zero moment cannot create cracks, which is why the author hypothesises that the distribution of the warping moment would rather look like Fig 7.4. Under perfect conditions, i.e. a completely free wall at the top, the first distribution is completely correct. What creates the second distribution is the inability to recreate these boundary conditions. As the head of the wall is not completely free, the distribution of the warping moment changes

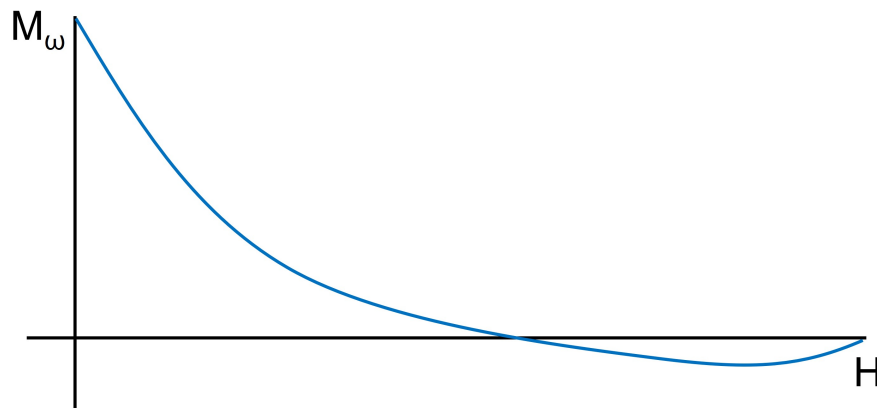


Fig. 7.4.: Suggested warping moment distribution

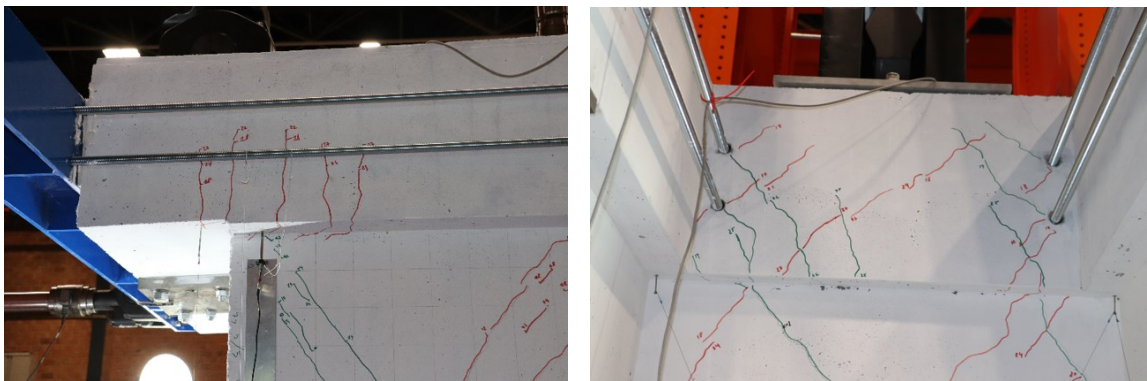


Fig. 7.5.: Cracks in the collar

This is obviously not without influence on the results, as from now on it is a fraction of the height that would be used in the calculations. The impact of the height will be discussed further in Section 8.3, but to give a rough idea, taking $0.8 \times H$ would increase the ultimate torque from 348.74 kNm to 424.53 kNm. The exact coefficient to be applied has not been calculated in the context of this thesis but could be in order to improve the results considerably.

Parametric Study

” *The good thing about science is that it’s true whether or not you believe in it.*

— Neil deGrasse Tyson

When looking at all the equations required and involved for the calculation of the cracking and ultimate torques, it becomes clear that a lot of parameters are involved. However, not all of these dozens of parameters have the same influence on the results. Those that have barely any influence are obviously not interesting to study. On the other hand, by making the different predictions for the several specimens, some parameters quickly appeared to have a great influence on the results. The aim of this study is therefore to focus on the UW2 specimen tested at LEMSC and to vary the parameters of concrete strength, steel strength, wall height, characteristic length and axial load ratio to observe the evolution of the results. It is also good to recall the actual values used to obtain the results of the Tables 7.1 and 7.2 in order to have a basis from which to start.

Concrete strength	Steel strength	Height	Characteristic length	Axial Load Ratio
[MPa]	[MPa]	[m]	[m]	[%]
37.1	580	2.25	4.59	5

Tab. 8.1.: Initial values of UW2 parameters

8.1 Influence of the concrete resistance

Some parameters introduced in the model, such as the strength of the concrete, are in fact the average of different tested samples. These tests on cylinders and cubes are used to evaluate the strength of the concrete. So there is obviously at least one test performed on the day of the test on UW2 in order to have the most accurate value possible. However, as the final value is an average, the exact strength value could be slightly different from the value used in the model. Fig 8.1 shows the evolution of the two torques as a function of the concrete strength specified in the model. A zero value does not make much sense, so the graph starts with a strength of 5 MPa.

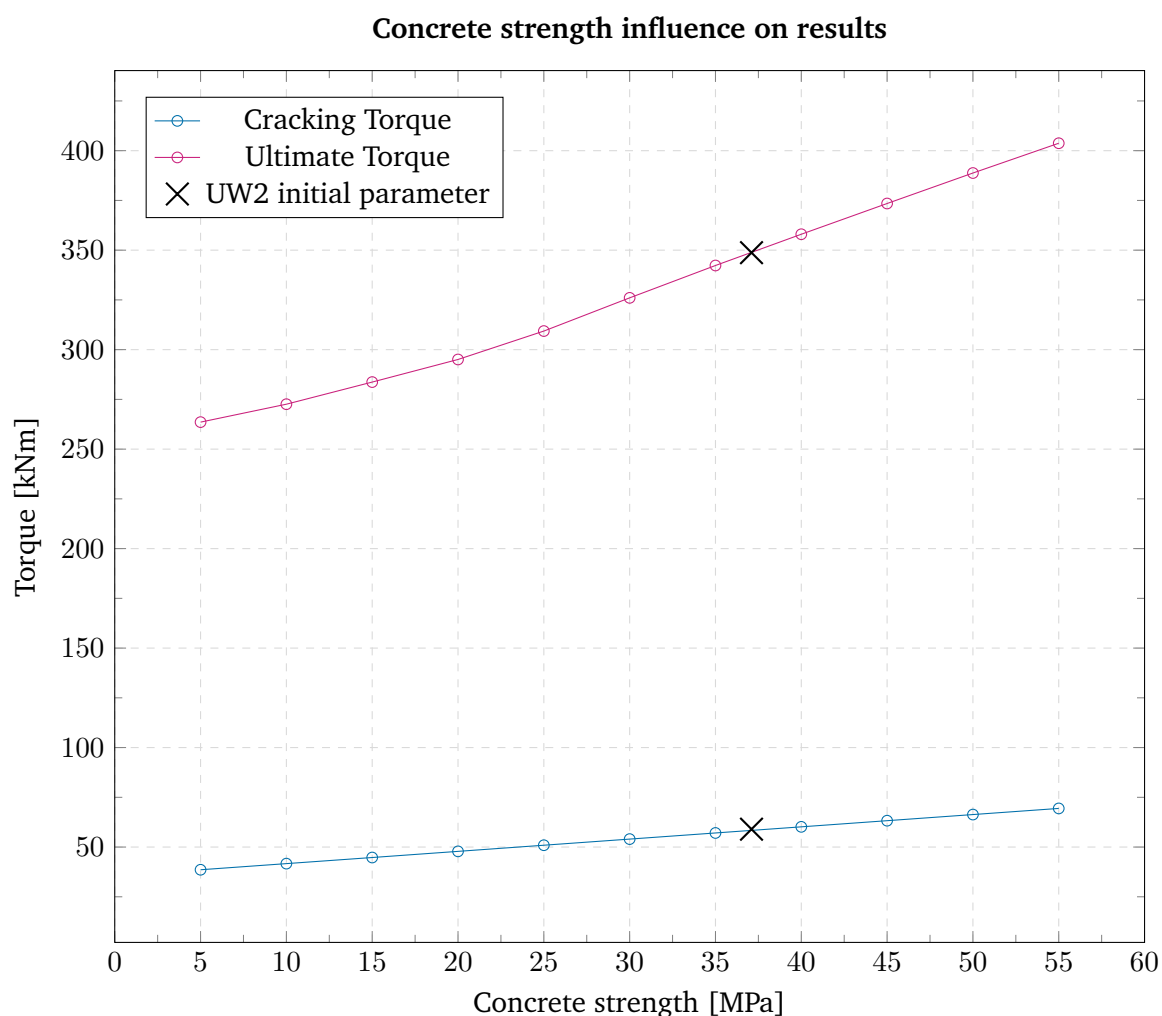


Fig. 8.1.: Concrete strength influence on results

Quite intuitively, both cracking torque and ultimate torque increase with increasing concrete strength. This is not meaningless, considering that more and more codes are advising the use of higher strength concretes. In terms of increase, it is more important for the ultimate torque than for the cracking torque. In fact, for the former, an increase of 3 MPa in the concrete resistance results in an increase in torque of about 15 kNm. Whereas for the cracking torque it is rather 6 kNm. If the strength of the concrete were to be increased, this would therefore be beneficial in terms of torsional strength.

8.2 Influence of the yield strength of the steel

The other parameter related to the mechanical properties of the materials is of course the strength of the steel rebars. This time only the ultimate torque will be influenced, since the strength of the steel is not involved in the calculation of the cracking torque. However,

similar to the strength of the concrete, an increase in the strength of steel results in an increase in the ultimate torque.

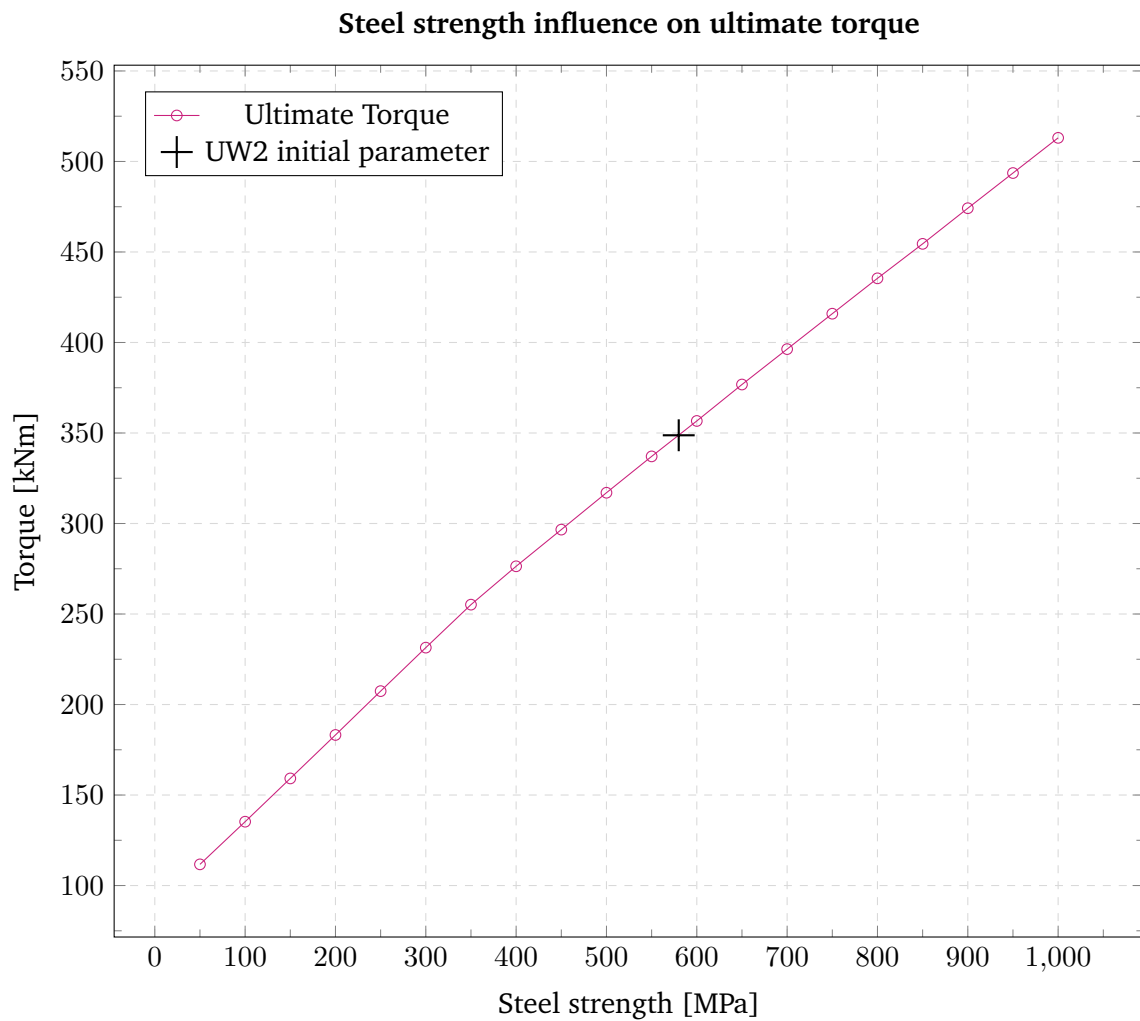


Fig. 8.2.: Steel strength influence on ultimate torque

As with the strength of concrete, knowing that increasing the strength of steel would have a positive impact on the ultimate torque is valuable information as the quality of steel continues to improve. This impact is expressed as an increase of 20 kNm for each 50 MPa increment in the strength of the steel

8.3 Influence of the height

Doing a parametric study on the height of the wall seemed quite natural in view of the hypothesis introduced in Section 7.2. Thanks to Fig 8.3 it is possible to see that this parameter, which is only taken into account in the calculation of the variable C, is very influential. At least for small values of height. This is because the height is used as an argument for the

hyperbolic cosine as well as the hyperbolic sine. These functions tend well towards infinity, but for large values slow down strongly.

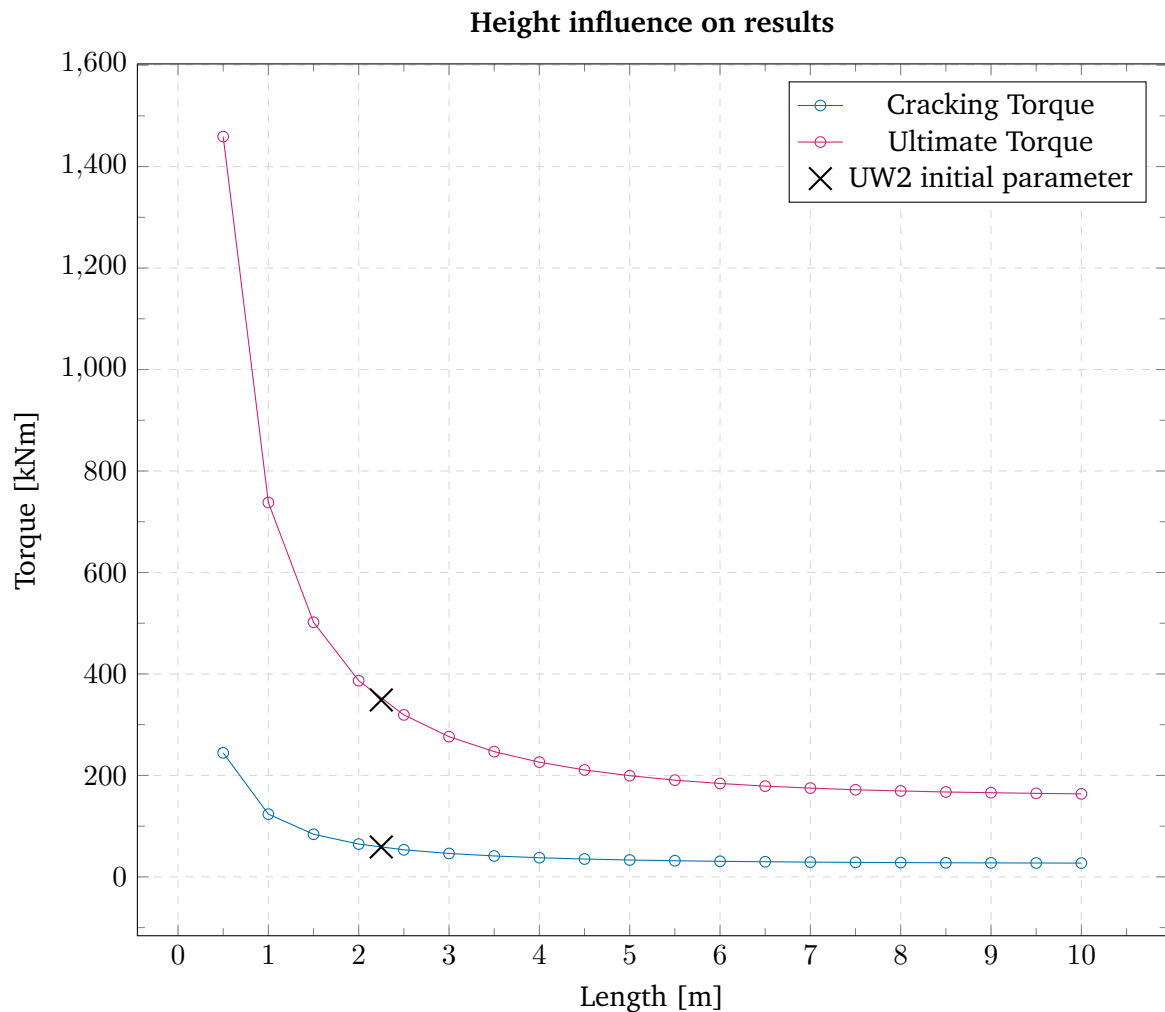


Fig. 8.3.: Height influence on results

It should be noted that the impact of the height is visibly more important on the ultimate torque, whose variations are much greater than the cracking torque for the same increment of height. What emerges from this study is that it is better to avoid making the walls too high, as this will have a strong impact on the torques. This can be explained by the equations in which the height is involved, but also by the distribution diagram of the warping moment. Indeed, as the latter is very similar to the diagram of the moment of a point load at the head of the wall, it is easy to understand that the higher the wall is, the more the warping moment at the base of the wall will be consequent.

8.4 Influence of the characteristic length

This somewhat specific parameter has been detailed in Section 3.1 but as a reminder it corresponds to the ratio by which the contribution of circulatory torsion increases with distance from the constrained support. At 2 or 3 times this characteristic length, almost all the torque will be resisted by the circulatory torsion. In other words, beyond that, warping can be neglected. Fig 8.4 shows this effect for very small values of characteristic length. With the effect of warping neglected, the torque values increase. On the contrary, for large values of characteristic length, the warping effect is added and reduces the cracking and ultimate torque values. It is once again possible to understand this curve by turning to the equation of the coefficient C . Indeed, just like the height of the wall, the characteristic length is involved in the arguments of the hyperbolic sine and cosine functions. When the characteristic length is very small, the coefficient C itself becomes very small and thus leads to high values of torques. This is due to the fact that the coefficient is involved in the denominator of the torque equations.

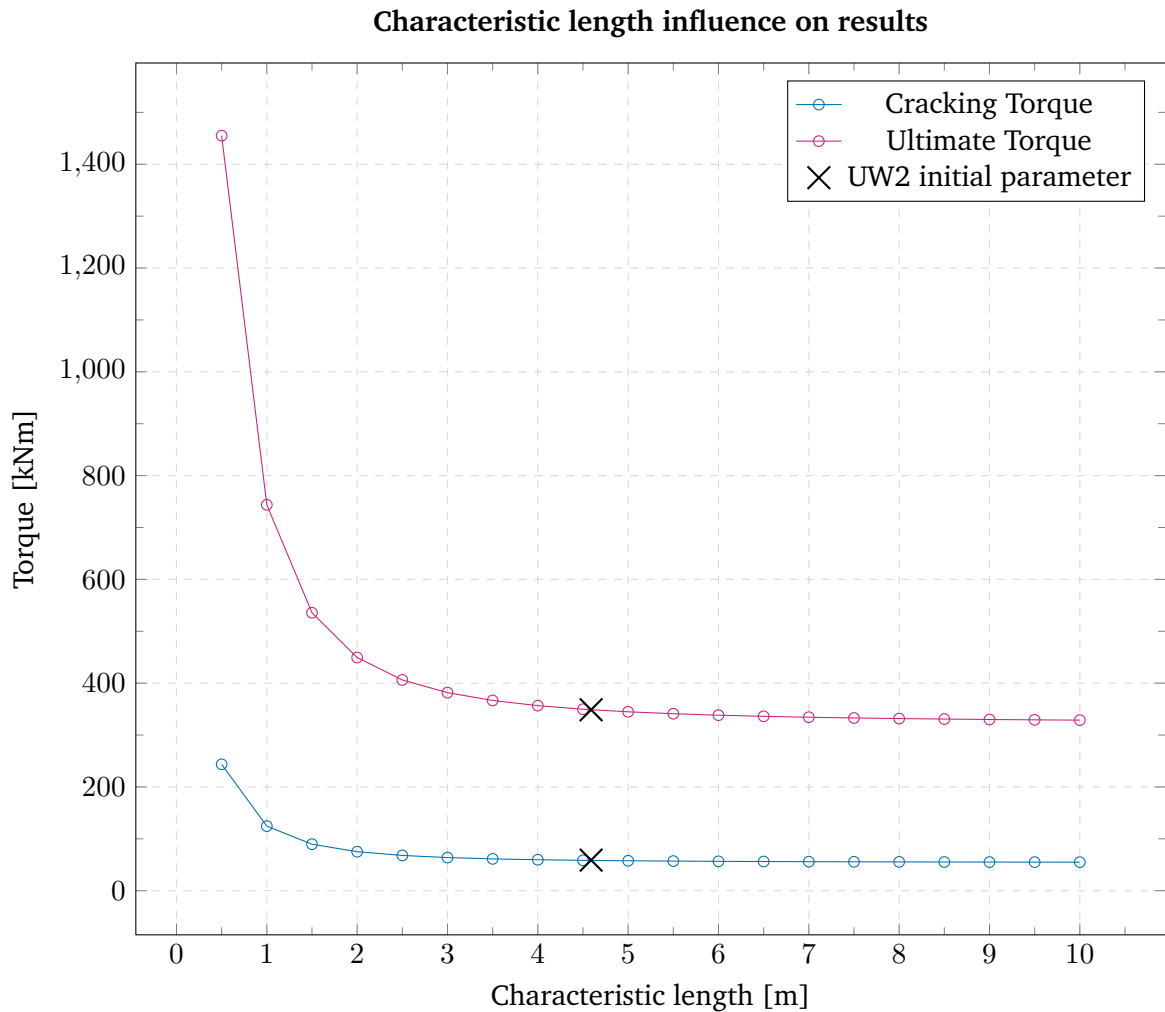


Fig. 8.4.: Characteristic length influence on results

8.5 Influence of the ALR

The last parameter studied is a phenomenon more specific to walls, namely the axial load. As it is rightly not taken into account in the basic model for beams, its consideration here provides ample justification for examining its influence on the results. A value of zero simply means that there is no additional axial load, whereas a value of 1 corresponds to an axial load equal to the limit axial load of the element. However, in Fig 8.5 below, only the range [0;0.5] is considered because higher values are rarely encountered in practice.

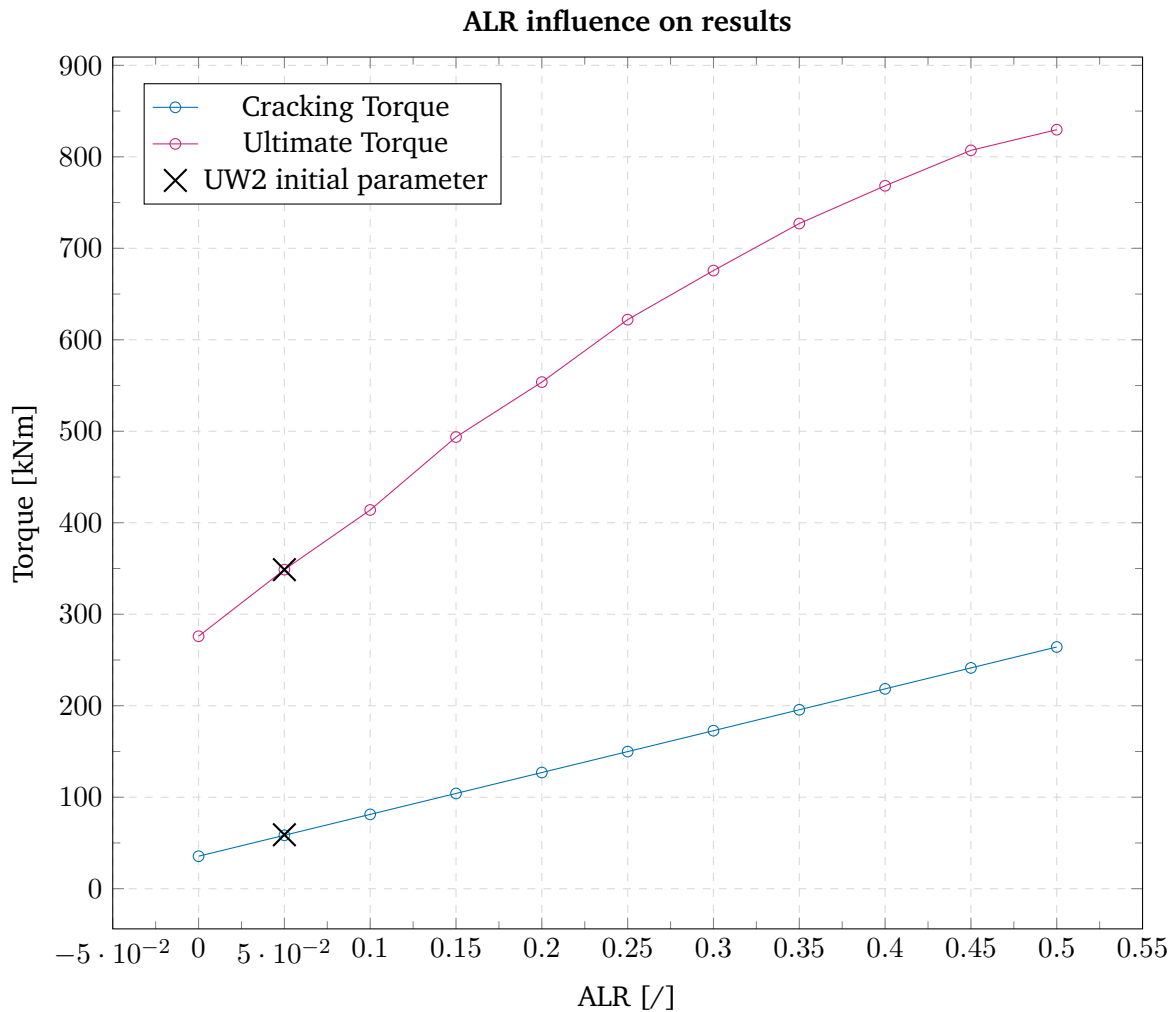


Fig. 8.5.: ALR influence on results

As described previously, cracking can occur in two main ways. The most common being that the tensile strength of the concrete is exceeded, therefore adding compressive stress will in fact reduce the tension in the concrete and therefore increase the cracking torque. In terms of ultimate torque, the reflection is not as straightforward, but the increase in ALR allows a greater depth of compression zone to be used and higher values of ultimate torque to be achieved. In order to understand this conclusion, it should be remembered that the

convergence is evaluated by the subtraction of two terms, E_{ql} and E_{qr} . This last term is in fact also composed of two terms in the case of a wall with ALR. The first is the torque T multiplied by a factor and subtracted by the second term, the axial load. Thanks to the latter, the torque term T can be much higher while still verifying the balance $E_{ql}-E_{qr}=0$.

8.6 Interpretations

This parametric study has brought to light elements that can guide future engineers to a certain extent in the choice of some parameters. Indeed, if the objective was simply to obtain the highest cracking and ultimate torques, then choosing the highest possible concrete and steel strengths and reducing the wall height and characteristic length as much as possible while keeping a high axial load ratio would be the solution. However, apart from the first two parameters, a number of limitations arise for the rest of the parameters. If the height were to be reduced too much, failure would more than likely occur due to the shear force. Care must therefore be taken not to create walls described as squat walls, which are extremely sensitive to shear stress. Regarding the reduction of the characteristic length, which depends on values such as the Saint Venant's torsion constant, this would mean changing the type of cross-section, which represents a significant change. Finally, the axial load ratio is not easily adjustable and is generally between values of 0.2 and 0.5 in the construction standards.

Part V

Concluding Remarks and Outlook

Conclusion

When the aim of a piece of work is to extend knowledge on a subject, it is essential to take on board all the work previously done. Accordingly, this thesis could be reorganised into two main parts. The first part deals extensively with what is already known, and the second part deals with what is being explored. What is already known being the response of U-shaped beams under torsion, and what is new being the response of U-shaped walls.

With this in mind, a thorough review of the literature was presented. On the one hand on the different existing mechanical models and on the other hand on the different experimental protocols associated with them. This allowed to see the different limits of application and to define precisely the path that was going to be followed to reach the desired goal of finding a method as simple as possible that could help any engineer wanting to predict the response of U-shaped wall submitted to torsion in the future. It also helped to define which of these existing models would form the best basis for achieving this objective.

Once this base was found and adapted to make the new model, it had to be confronted with the experimental results in order to have a calibration as accurate as possible. In this respect, the results of the new mechanical model proved to be extremely accurate for the calculation of the cracking torque with an average ratio between the experimental value and the predicted one of 0.96.

The case of ultimate torque prediction is slightly different. Indeed, the results obtained with the initial parameters and the parametric study that followed highlighted a very important point. The method is certainly simple to understand and apply, but to obtain accurate results it requires the removal of many unknowns that are not easily obtained. Whether it is the precise characteristics of the materials, the distribution of the deformations in the steels, the distribution of the forces along the wall or the beam or the boundary conditions, a small approximation in one of these parameters leads to a large variation in the results. This leads to the observation that it is difficult to provide reasonably accurate estimates of the ultimate torque of non-planar sections using a simplified method, as it has been done in this research.

Fortunately, difficult does not mean impossible, which is why the following section discusses some of the paths that can be explored in order to go even further.

Future developments

There are different outlooks for anyone who wants to go deeper into the subject and the method presented in this thesis. Foremost, by staying with U-walls subjected to torsion only, the distribution of deformations in the steels as well as the force diagrams along the element, which have been presented as being rather capricious, could be given more attention. If an

equally straightforward method could be incorporated into the present model, the results of the model would improve considerably. This can be seen from the discussion of the assumption of another warping moment distribution in Section 7.2 and the strength of the steel to be considered in Section 6.1.2. To be able to accurately predict the effective height of the specimen or the strength of the steel to be considered would be a considerable improvement.

Another interesting prospect would be to test the model on U-shaped walls subjected to various load cases including torsion. The present model being adapted to load cases involving torsion and bending force. Moreover, not only are the load cases often more complicated than pure torsion, but also the cross-sections are often more complex than a simple U. In the future, it would be interesting to look at sections similar to the U-section but with a little more complexity.

The final area for improvement, which would apply to all of the above, would be to further improve the precision of the convergence of the procedure. Indeed, the latter is not perfect, given that it does not always remain constant and therefore leaves room for improvement.

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Part VI

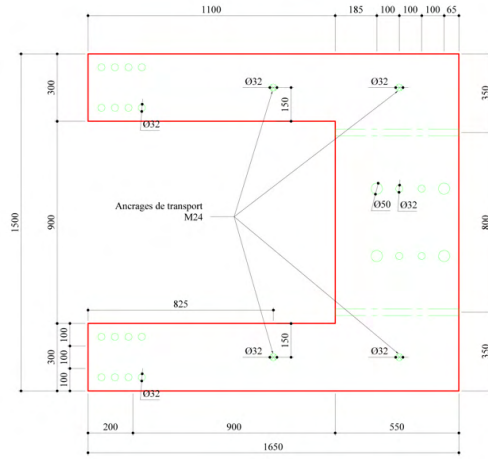
Appendices

Torsional behaviour of RC U-shaped walls

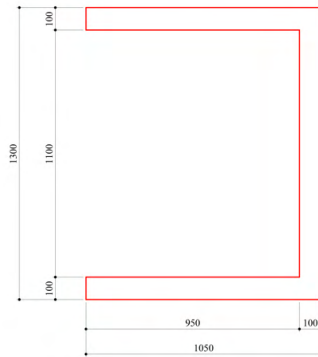
Drawings

NUM	NOM DU PLAN	RACCOURCI
1	Vue isométrique et plans du spécimen	UW1_PlansGlobaux
2	Vues latérales du spécimen	UW1_CoupesGlobales
3	Plan de renforcement de la fondation du mur	UW1_FondationDétailée
4	Coupes de renforcement du spécimen	UW1_CoupesDétailées
5	Plan de renforcement du mur	UW1_MurDétailé
6	Plan de renforcement de la couronne supérieure	UW1_PlanCouronne
7	Coupe du renforcement de la couronne - 1	UW1_CoupeCouronne
8	Coupe du renforcement de la couronne - 2	UW1_CoupeCouronne2
LISTE 1	Liste de renforcements pour la fondation du mur	UW1_Renforcements1
LISTE 2	Liste de renforcements pour la couronne supérieure	UW1_Renforcements2
LISTE 4	Liste de renforcements pour le mur	UW1_Renforcements4
LISTE 5	Liste de renforcements pour le mur (Class B)	UW1_Renforcements5

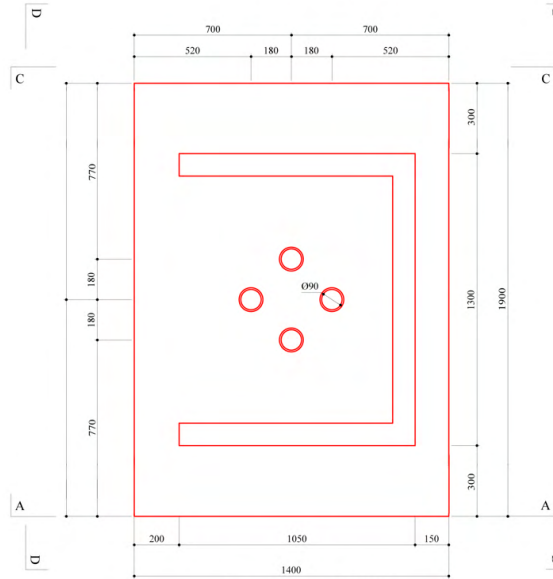
Plan 4 - 1:20



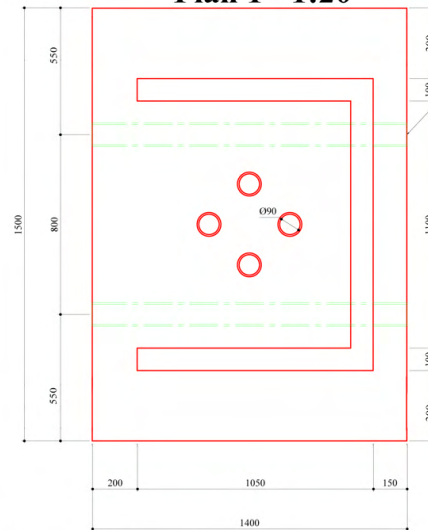
Plan 3 - 1:20



Plan 2 - 1:20

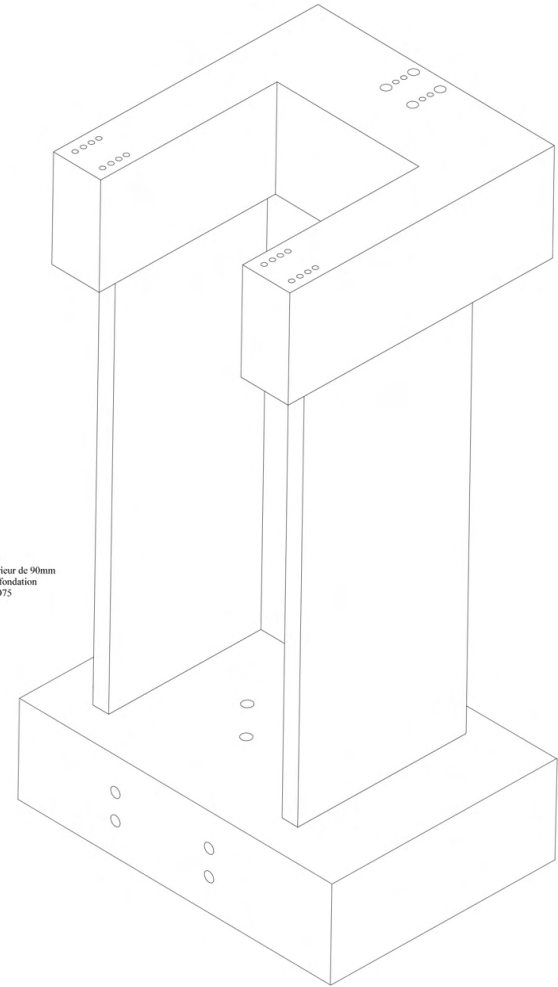


Plan 1 - 1:20



Tube avec le diamètre intérieur de 90mm pour la précontrainte de la fondation avec des barres Macalloy Ø75

Isometrie



UCLouvain/iMMC
Torsional behaviour of RC U-shaped walls

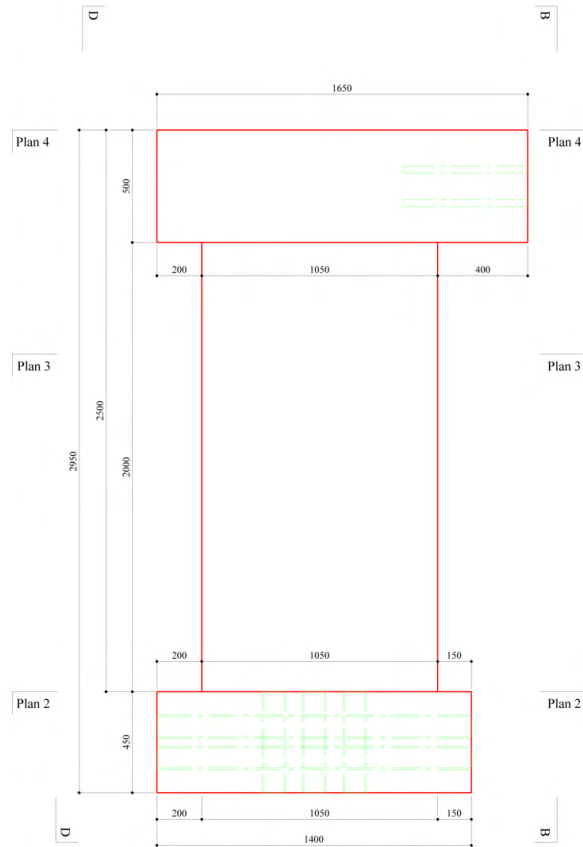
Plan de coffrage UW1
Vue isométrique et plans du spécimen

Remarques:

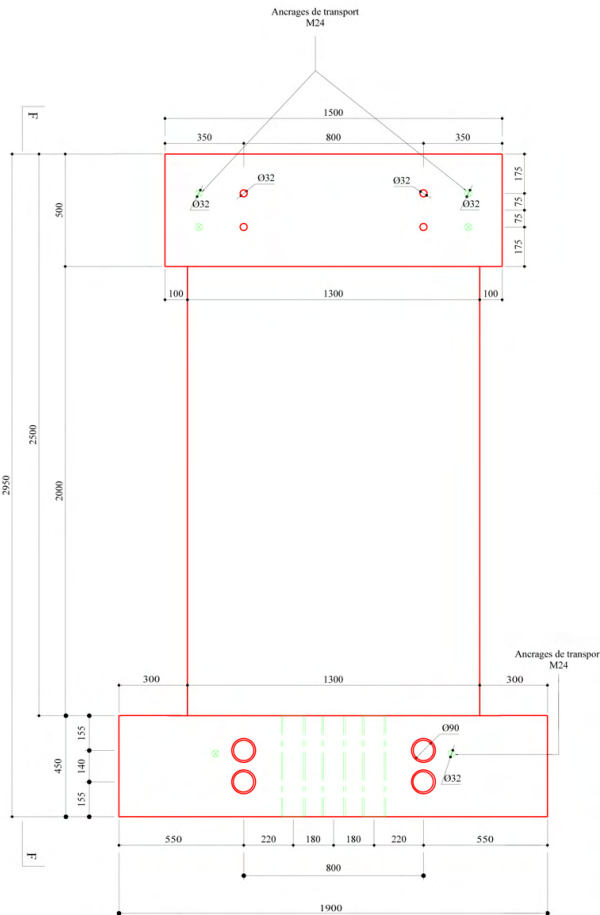
- Volume: 1,845 m³
- Masse: 4640 kg
- Enrobage: 15mm pour la fondation du mur et le paroi de manchon
- Béton: C25/30

Dessin N°: UW1_PlansGlobaux Plan: 1 Date: 01.11.2021 Echelle: marque sur le dessin

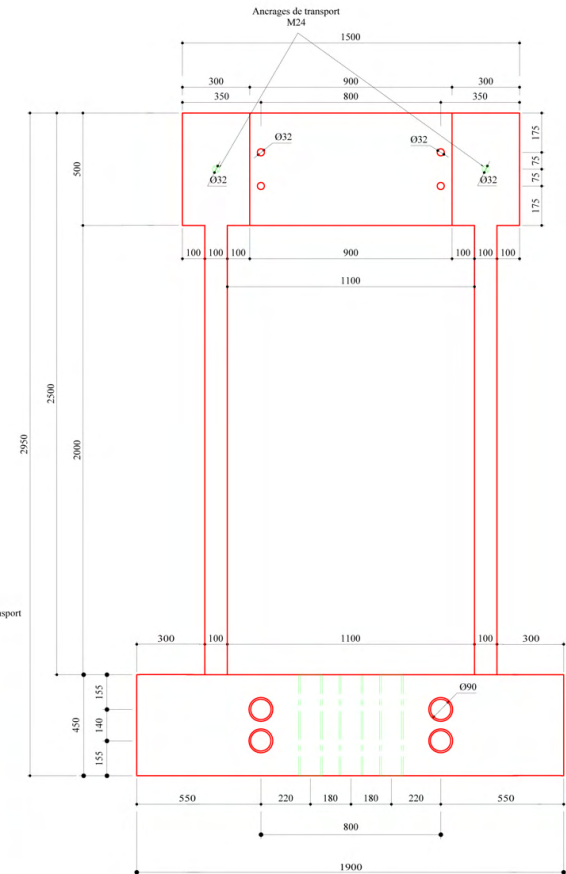
Coupe A-A 1:20



Coupe B-B 1:20



Coupe D-D 1:20

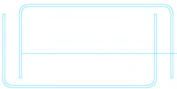


UCLouvain/iMMC
Torsional behaviour of RC U-shaped walls
Plan de coffrage UW1
Vues latérales du spécimen

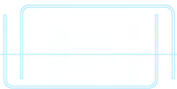
Dessin N°: UW1_CoupesGlobales Plan: 2 Date: 01.11.2021 Echelle: manque sur le dessin

Plan 1 - 1:10

102 10x 2 Ø20



101 10x 2 Ø20



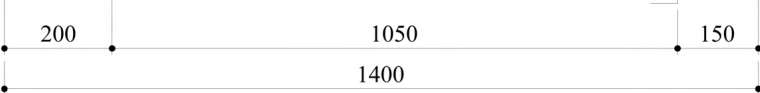
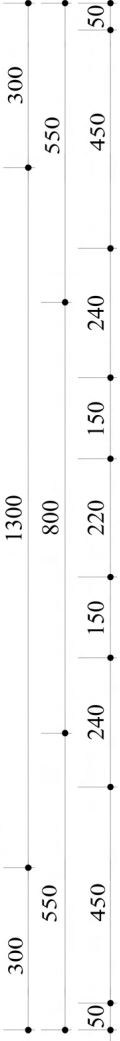
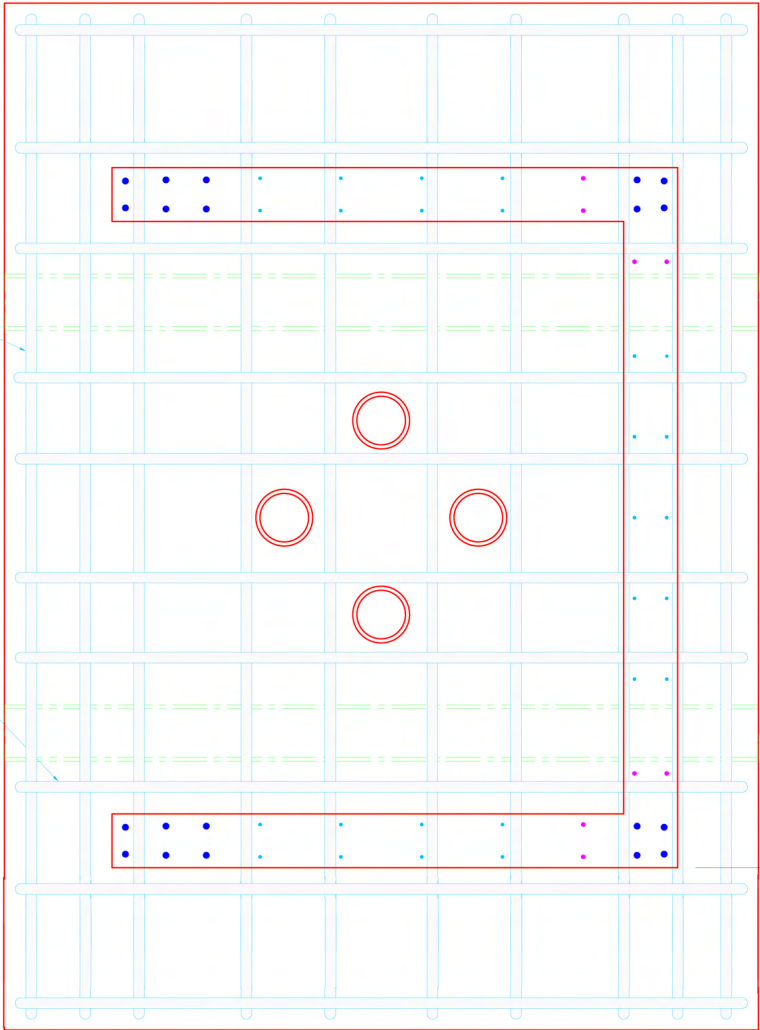
Remarques:
- Enrobage de béton pour la fondation du mur: 20mm
- Liste de renforcement: Liste 1

A

A

B

B



UCLouvain/iMMC			
Torsional behaviour of RC U-shaped walls			
Plan de ferrailage UW1			
Plan de renforcement de la fondation du mur			
Dessin Nr.: UW1_FondationDétailée	Plan: 3	Date: 01.11.2021	Echelle: marque sur le dessin

Technical drawing of a reinforced concrete slab (L.01) showing a plan view with dimensions and reinforcement details. The drawing includes a grid of reinforcement bars (Barras) and a cross-section view (L.01) showing the slab thickness and reinforcement layout. The plan view shows a rectangular slab with dimensions 10x2.020. The cross-section view shows a slab with a thickness of 100mm and a reinforcement layout with bars of diameter 10mm and spacing of 150mm. The drawing also includes a table of reinforcement details for Class B and Class C bars.

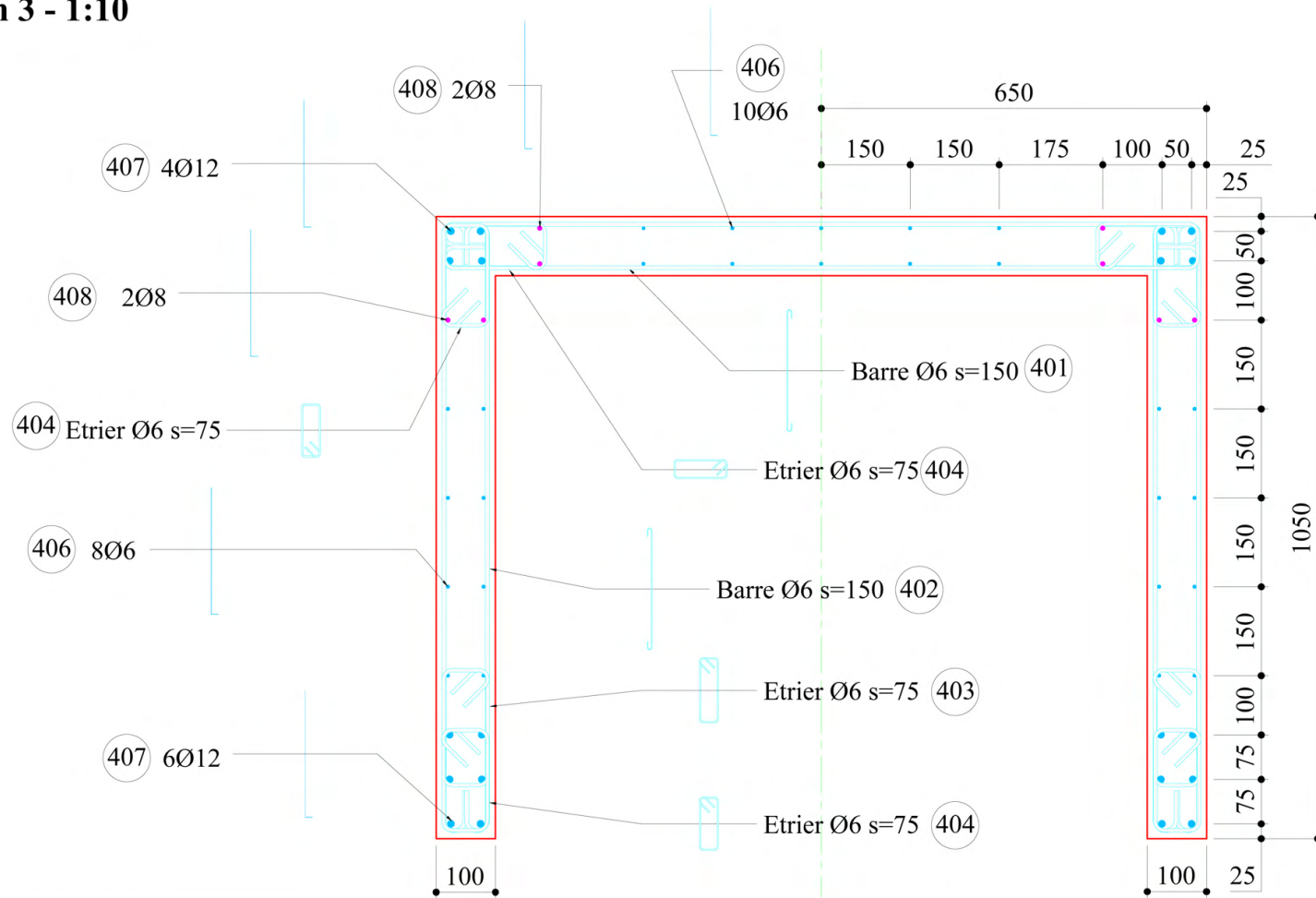
Reinforcement Details:

- Class B:**
 - 502 Etrier Ø6 s=75
 - 502 Etrier Ø6 s=75
 - 401 Barras Ø6 s=150
 - 402 Barras Ø6 s=150
- Class C:**
 - 404 Etrier Ø6 s=75
 - 404 Etrier Ø6 s=75
 - 401 Barras Ø6 s=150
 - 402 Barras Ø6 s=150

Dimensions:

- Overall dimensions: 10x2.020
- Slab thickness: 100mm
- Reinforcement spacing: 150mm (Barras), 75mm (Etrier)

Plan 3 - 1:10



UCLouvain/iMMC
Torsional behaviour of RC U-shaped walls

Plan de ferrailage UW1 Plan de renforcement du mur

Dessin Nr.: UW1_MurDétailé

Plan: 5

Date: 01.11.2021

Echelle: marque sur le dessin

Remarques:

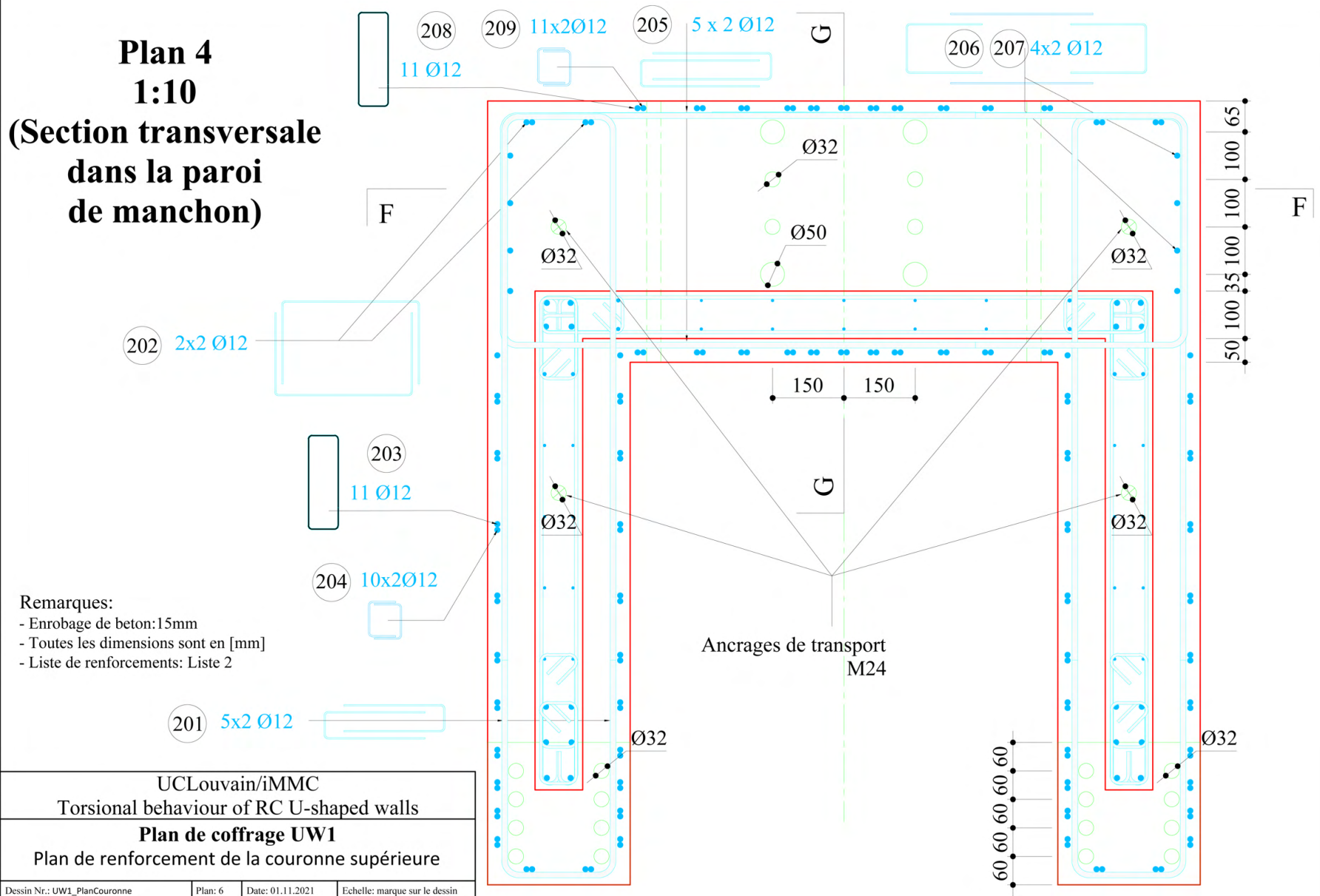
- Enrobage béton: 10mm
- Toutes les dimensions sont en [mm]
- Liste de renforcement: Liste 4
- Béton C25/30

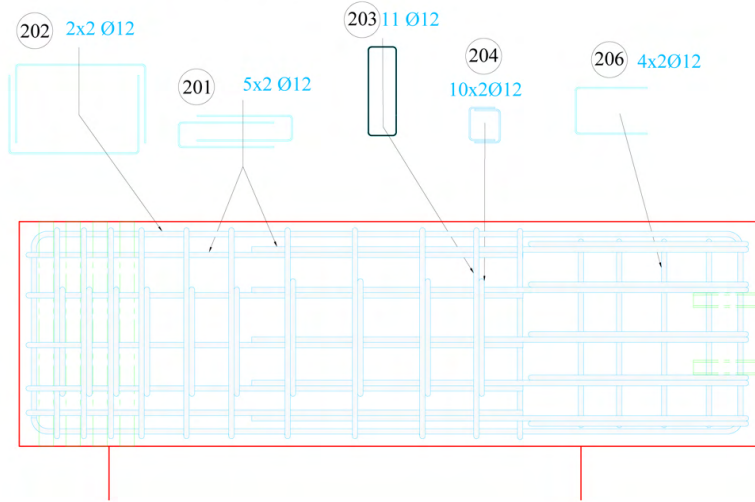
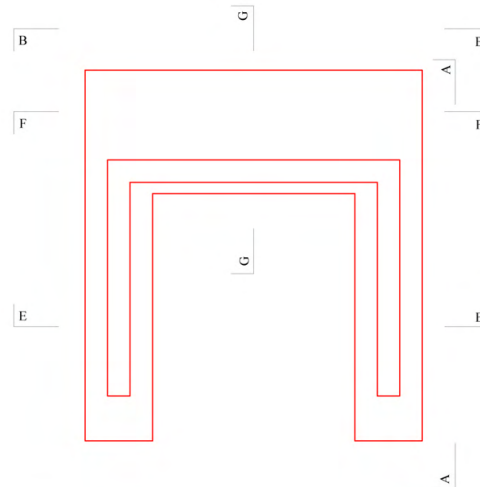
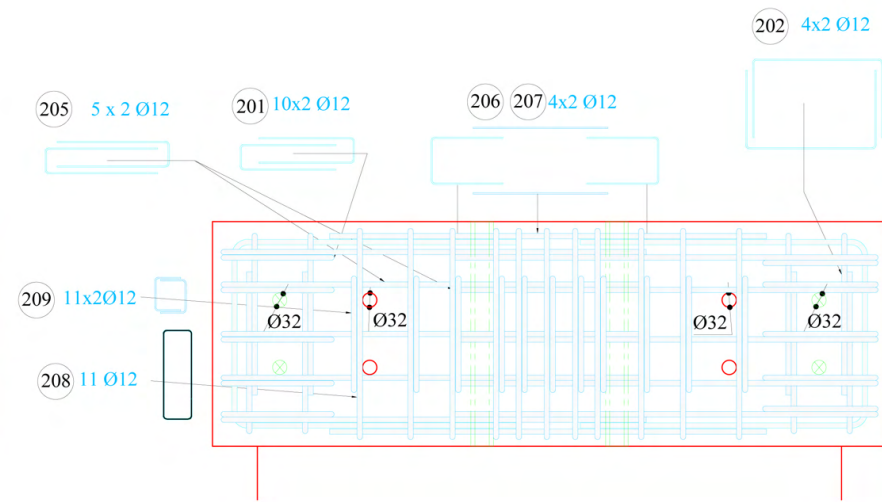
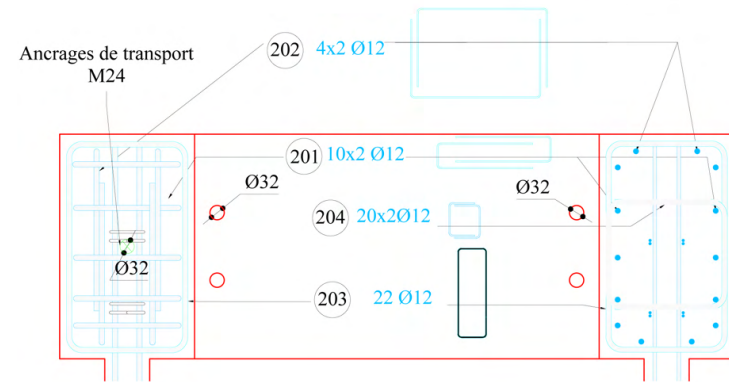
Remarques:

- Barre de 6 et 12 mm de diamètre.
- Barres verticales "Classe C".
- Barres et étriers horizontaux "Classe B".

Plan 4

1:10



Coupe A-A 1:10**Coupe B-B 1:10****Coupe E-E 1:10**

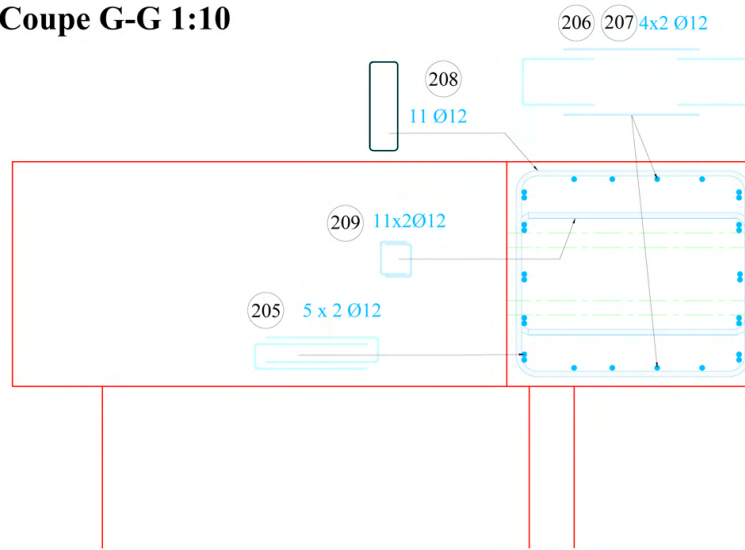
UCLouvain/iMMC
Torsional behaviour of RC U-shaped walls
Plan de coffrage UW1
Coupe du renforcement de la couronne - 1

Remarques:

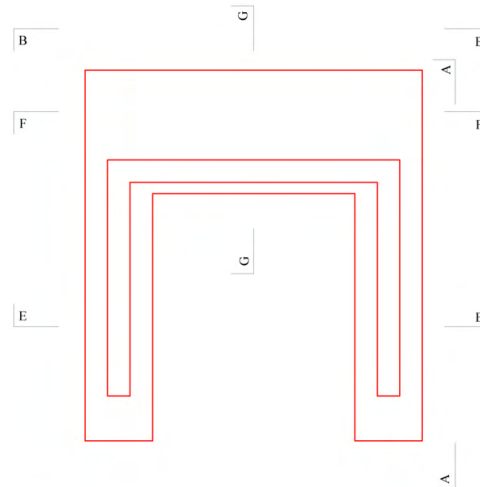
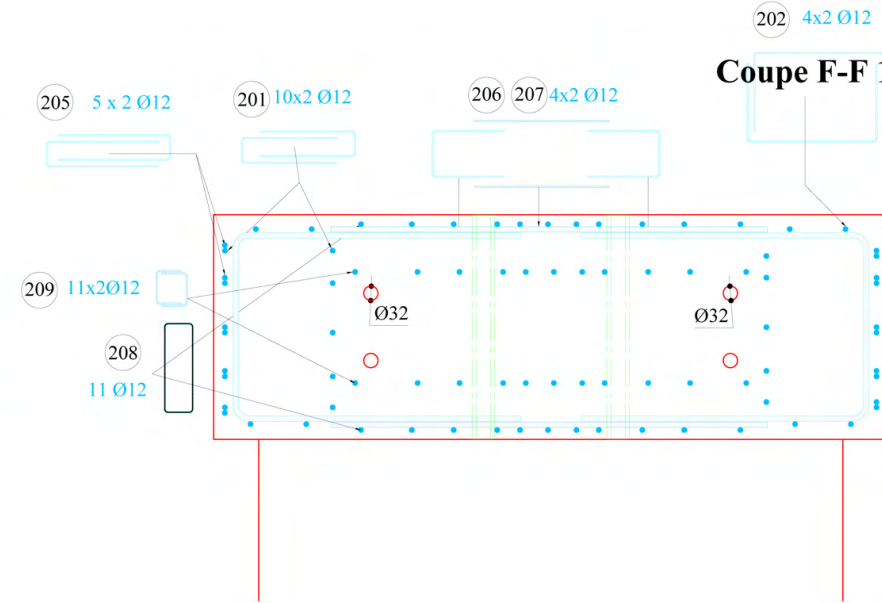
- Enrobage de béton: 15mm
- Toutes les dimensions sont en [mm]
- Liste de renforcement: Liste 2

Dessin N°: UW1_CoupeCouronne | Plan: 7 | Date: 01.11.2021 | Echelle: marque sur le dessin

Coupe G-G 1:10



Coupe F-F 1:10

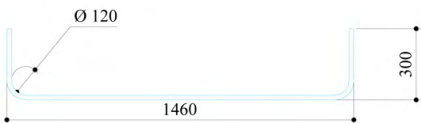


UCLouvain/iMMC
Torsional behaviour of RC U-shaped walls
Plan de coffrage UW1
Coupe du renforcement de la couronne - 2

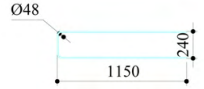
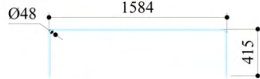
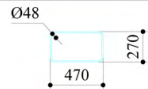
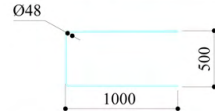
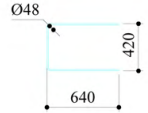
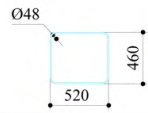
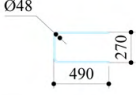
Remarques:

- Enrobage de beton: 15mm
- Toutes les dimensions sont en [mm]
- Liste de renforcement: Liste 2

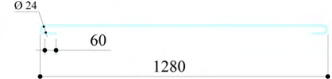
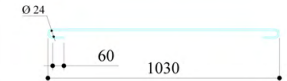
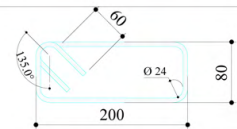
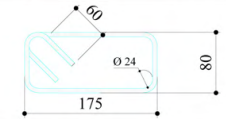
Dessin Nr.: UW1_CoupeCouronne2 Plan: 8 Date: 01.11.2021 Echelle: marque sur le dessin

Position	Façonnage pièce	Acier	Diamètre [mm]	Longueur [mm]	Quantité	Poids total [kg]
101		B500B	20	2040	20	100
102		B500B	20	2060	20	101

UCLouvain/iMMC			
Torsional behaviour of RC U-shaped walls			
Liste de renforcements pour la fondation du mur			
Liste 1			
Dessin N°: UW3_Renforcements1	Liste: 1	Date: 01.11.2021	Echelle: pas à l'échelle

Position	Façonnage pièce	Acier	Diamètre [mm]	Longueur [mm]	Quantité	Poids total [kg]
201		B500B	12	2540	20	45
202		B500B	12	2415	8	17
203		B500B	12	1480	22	24
204		B500B	12	745	40	26
205		B500B	12	2500	10	22
206		B500B	12	2120	8	15
207		B500B	12	1000	8	7
208		B500B	12	1960	11	19
209		B500B	12	1250	22	24

UCLouvain/iMMC			
Torsional behaviour of RC U-shaped walls			
Liste de renforcements pour la couronne supérieure			
Liste 2			
Dessin N°: UW3_Renforcements2	Liste: 2	Date: 01.11.2021	Echelle: pas à l'échelle

Position	Façonnage pièce	Acier	Diamètre [mm]	Longueur [mm]	Quantité	Poids total [kg]
401		B450C	6	1450	36	12
402		B450C	6	1200	72	19
403		B450C	6	680	34	9
404		B450C	6	630	102	26
406		B450C	6	2860	26	16
407		B450C	12	2860	20	50
408		B450C	8	2860	8	9

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Torsional behaviour of RC U-shaped walls

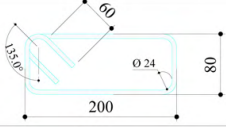
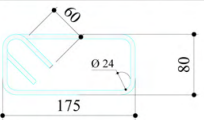
Liste de renforcements pour le mur
Liste 4

Dessin N°: UW3_renforcements4

Liste: 4

Date: 01.11.2021

Echelle: pas à l'échelle

Position	Façonnage pièce	Acier	Diamètre [mm]	Longueur [mm]	Quantité	Poids total [kg]
501		B450B	6	680	32	9
502		B450B	6	630	96	26

UCLouvain/iMMC			
Torsional behaviour of RC U-shaped walls			
Liste de renforcements pour le mur Liste 5			
Dessin N°: UW3_Renforcements4	Liste: 4	Date: 01.11.2021	Echelle: pas à l'échelle

.2 Optical fiber installation procedure

This document aims to describe the procedure of the optical fiber installation inside the steel reinforcement bars (or rebars) that will be placed inside the concrete walls. Here is the list of the material needed:

Material	References
Rebars	/
High-Definition Fiber Optic Strain Sensors	HD6S
Optical Distributed Sensor Interrogator	ODISI 6000 SERIES
Funnel	EAN: 8711293011360
Heat-shrinkable polymer	Any kind
Compressed air duster	HG nettoyant à air comprimé
Parts cleaner	Loctite 7063
Conditioner A mild phosphoric-acid	M-Prep Conditioner A (MCA-2)
Neutralizer 5A ammonia-based liquid	M-Prep Neutralizer 5A (MN5A-2)
Cyanoacrylate	M-Bond 200 Adhesive
Epoxy glue	Wurth colle epoxy ESK 50
Cyanoacrylate Adhesive Activator	Loctite 7457
Brushes	Any kind
Adhesive tape	Any kind
Butyl paste	Any kind
Sealing paste	Any kind
Wooden stick	All kind

Rebars assembly

Even before preparing the bars for the installation of the optical fiber, the individual straight bars must be bent and welded to obtain the final shape of the bar. However, as can be seen in the following 3 figures, the welding process results in a covering/filling of the groove. This adds a preliminary step to the cleaning of the bars, namely reopening the groove and sanding off the excess welding material. Indeed, leaving hard relief points along the bar would lead to local stress concentrations which would be detrimental to the general behaviour



Figure 1



Figure 2



Figure 3

The procedure is quite simple and can be summarised in 2 steps:

- 1) Sand off the excess welding filler material to restore the bar to its original level



Figure 4

- 2) Come up with a small disc cutter and gently join the existing grooves



Figure 5

Rebars preparation:

The first steps are intended for cleaning and preparing the steel before installing the optical fiber and applying the glue. Once the steel arrives on the table, it's placed on wood supports that are made to avoid contact with the table:



Figure 6

After that the following steps are followed to fully clean the steel before contact with the optical fibers:

- First clean with the Loctite 7063 and a small brush to remove the biggest particles
- Drying with compressed air or just time
- Second cleaning process with a brush and the M-Prep Conditioner A that acts as a mild etchant and accelerates the cleaning process
- Immediately followed by the M-Prep Neutralizer 5A that is used to neutralize the chemical reaction introduced by the acid solution
- Drying with compressed air or just time

Optical fiber installation:

Now that the steel is completely clean, the next step is to put the optical fiber inside the groove. Cyanoacrylate will be used to glue the fiber but also to protect it as it will be installed inside concrete that could damage the fiber. **Great caution should be taken as it is very fragile once there is an angle in it.** Here are the different steps:

- Installing spacers to give a counter deflection to the steel ensuring a perfect contact between the fiber and the steel along the whole length



Figure 7

- Unroll the HD6S fiber for the length needed
- Use adhesive tape to properly place the fiber inside the groove on the first centimetres of the fiber where there is a protective sleeve



Figure 8

- Continue placing the fiber inside the groove using small wooden sticks of the size of the groove to properly press it against the rebar at certain locations like the beginning, the end or where the fiber rises

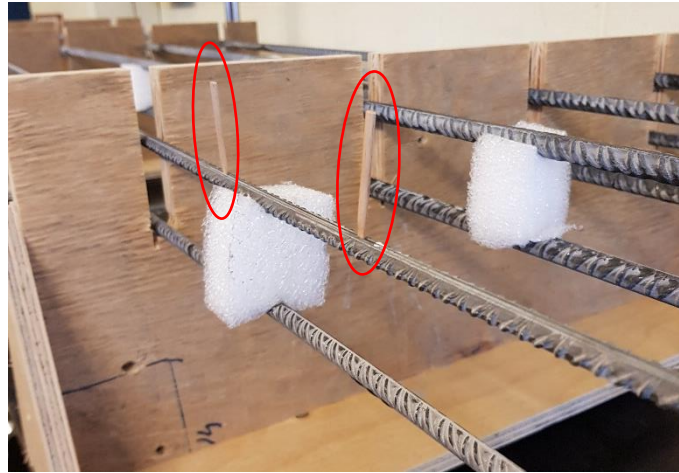


Figure 9

at the end of the return leg, if the length of fiber is longer than what is needed, a solution would be to let the bobbin hang with the excess fiber still rolled up before turning the whole thing over when gluing this other side



Figure 10

- At the end and the beginning put a small drop of cyanoacrylate immediately followed by a shot of spray of the activator. Thanks to that it will instantly polymerized and keep the tension in the fiber to keep the pressure against the rebar while the rest is progressively glued.
- Apply the cyanoacrylate all along the rest of the groove. Be careful **there is a termination at the end of the fiber that needs to be immaculate**. If there is an excess of length, there is no problem you can apply the cyanoacrylate all along the rebar. If not, then stop the glue before so that the end is not in the glue



Figure 11

- Wait polymerisation of the cyanoacrylate
- Apply again and repeat until the groove is filled
- Return the bar and to all these steps again for the other side

Protective funnel piece

The goal is to have access to the beginning and the end of the fiber even after concreting. This funnel will enable you to do that and store the excess length of fiber that you have. The steps might be a bit different depending on the diameter of the rebars used.

- Unroll the remaining length of fiber
- Cut a sufficient length of heat-shrinkable polymer (around 90 mm) and put it around the the fiber (beginning and end). This polymer will also go over the funnel to form a tight seal.

- Put on the funnel around everything. Again, both the beginning and the end part of the fiber must be inside it. It is also interesting to slightly notch the funnel in order to slide it more easily around the set. Once the funnel is correctly placed, put the heat-shrinkable polymer over it and heat it to seal everything



Figure 13



Figure 12

- Although the polymer already partially prevents relative movement between the funnel and the bar, some twisting is still possible at this stage. This is very dangerous for the fiber, which breaks excessively quickly in such situations. To remedy this, the solution found was

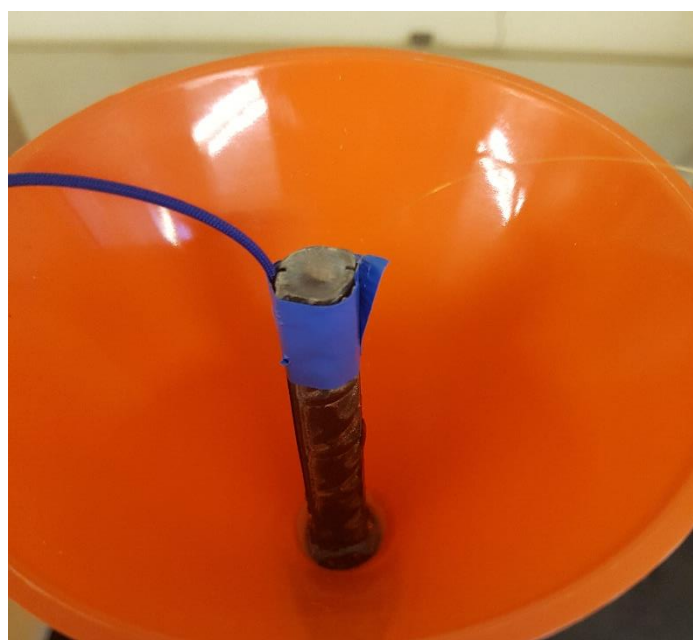


Figure 14

to fill the funnel with epoxy glue at least to the widening of the funnel. Once dry, it totally prevents any movement

- Once the glue has dried, all that remains is to roll up the remaining length of fibre and hold it in place with string. The choice of string and not something more rigid/solid is to avoid damaging the fiber



Figure 15

- Close the funnel with a cap to protect the fiber while the pouring of the concrete is made

If you did everything written in this document, you should have a rebar with optical fiber protected all around it and have access to the beginning and the end of this fiber through a protective funnel closed with a lid.

Placing of gages:

The only thing needed to place gages along the fiber inside the software is a compressed air duster. The procedure to do that is as followed:

- Connect the Optical Distributed Sensor Interrogator to the beginning of the fiber inside the funnel
- Tare the software
- Identify the point of interest along the rebar

- Open the gage management window
- Point by point blow compressed air on the rebar. The air coming out is cold and lead to a local deformation of the rebar (and the fiber). This deformation is visible and expressed as a local peak deformation in the software
- Press add gage after the location of the deformation is clear and find manually

Test setup:

On the day of the tests, there is only one thing to do which is to open the funnel caps and connect the fibers to the acquisition boxes. In order to have a simple and organised access to these boxes and to the fibers, it is preferable to glue wooden boards on the test element in order to screw the different boxes near the fibers. A colour code on the different cables can also simplify the organisation and placement of the different elements



Figure 16

.3 Discussion on Xu et al. paper

CONTRIBUTION BY SACHA AWAD, RYAN HOULT, AND JOÃO PACHECO DE ALMEIDA

The paper presenting experimental studies and analytical techniques to determine the torsional performance of reinforced concrete (RC) U-shaped beams by Xu *et al.* (2018) is a timely and important article for the engineering community. The methods proposed in the research by Xu *et al.* (2018) could be extrapolate to, say, RC U-shaped core walls subjected to both bending and torsion, the subject of which has a paucity of research. However, several of the assumptions and calculations used in Xu *et al.* (2018) for beams require further clarification.

In Xu *et al.* (2018), some example calculations for the methods proposed are given in the Appendix. These example calculations use the parameter values corresponding to the experimentally tested specimen of MEM-1:1 in Xu *et al.* (2018). This Appendix serves as an important piece of supplementary material for understanding, and also validating, the methods and calculations proposed in Xu *et al.* (2018). In calculating the cracking torque (T_{cr}), Xu *et al.* (2018) propose the following equation:

$$\sigma_{max} = \frac{M_b}{I} y_{max} + \left(-\frac{TC}{2I_{\omega\omega}} \omega_{min} \right) \quad 1$$

where σ_{max} is the tensile strength of the concrete ($f_t = 0.63\sqrt{f'_c}$), M_b is the bending moment, y_{max} is the distance from the neutral axis to the extreme tensile fibre region (=0.342 m), C is a parameter based on the theory proposed by Vlasov (1961) for the warping of a thin-walled fixed-fixed section subjected to a concentrated torque T at mid-span (=1.376 m), $I_{\omega\omega}$ is the principal sectorial moment of inertia ($=\int_A \omega^2 dA=0.00034 \text{ m}^6$), in which ω is the principal sectorial coordinate.

For the illustrated example in the appendix, the MEM-1:1 specimen is used (i.e., where $M_b = T$), and substituting in the values calculated in Xu *et al.* (2018) gives:

$$0.63 * \sqrt{35.40} = \frac{T}{0.0029} 0.342 + \left(-\frac{1.376T}{2 \times 00034} 0.1187 \right) \quad 2$$

In the appendix of the article, Xu *et al.* (2018) have the right-hand side (RHS) of Equation 2 as equating to $367.1T$. However, it is believed that this should be $358.1T$. This results in a different estimated T_{cr} in comparison to those calculated in Xu *et al.* (2018) for all beam specimens, albeit a small difference but a difference none the less. The different values of T_{cr} for the four beam specimens used for validation purposes in Xu *et al.* (2018) are given in Table 1. Can the authors comment on whether this was just a small mistake, or if something else has been employed in the calculation of T_{cr} ?

Table 1 Comparison of test cracking torque and calculated cracking torque

Beams	Test	Xu et al. (2018)	Updated Calculations	Test/(Updated Calculations)
MEM-1:5	4.5	4.7	4.75	0.95
MEM-1:1	9.6	10.2	10.46	0.92
MEM-1:0	17.3	16.9	16.84	1.03
MEM-Colins-1:0	23.0	18.5	30.5	0.8

Two parameters (η_1 and η_2) are needed for the ultimate torque (T_u) with the methods proposed in Xu *et al.* (2018). These two parameters can be calculated with Equation 3 and 4.

$$\eta_1 = \int_A \omega \cdot y \cdot dA \quad 3$$

$$\eta_2 = \int_A \omega \cdot dA \quad 4$$

However, in its current form as presented in Xu *et al.* (2018), it would appear that the calculations of η_1 and η_2 are based on the entire U-shaped section [i.e., using the geometric properties and principle sectorial coordinates given in the figure presented in the Appendix of Xu *et al.* (2018)]. If this is the case, then Equation 3 and 4 should equal zero. For example, the methods proposed in Krpan and Collins (1981) use Equations 3 and 4 as part of the convergence criteria for an uncracked and cracked section, assuming these two integrations are equal to zero. Therefore, it is more likely that Xu *et al.* (2018) have used the elastic sectorial coordinates corresponding to the “loaded half U-section” only. If the sectorial coordinates corresponding to the loaded half section are considered, then [using the same values of ω given in Xu *et al.* (2018)] one obtains $\eta_1 = -4.4934 \times 10^{-4} \text{ m}^5$ and $\eta_2 = 3.5989 \times 10^{-4} \text{ m}^4$. However, the values given in the Appendix of Xu *et al.* (2018) are $\eta_1 = 4.738 \times 10^{-4} \text{ m}^5$ and $\eta_2 = 1.581 \times 10^{-4} \text{ m}^4$. Given that the calculation of T_u using the proposed method is highly dependent on these two parameters, could the authors elaborate and provide more insight into these calculations and resulting values of η_1 and η_2 ?

The longitudinal reinforcing steel of the loaded half U-section is also separated into different regions: A_{sd} is the area of all distributing bars, A_s is the area of the bars in tension, and A'_s is the area of the bars in compression. In the Appendix of Xu *et al.* (2018), the values of these three separate areas of steel are given. However, one wonders how this is possible, given that one of the unknowns to solve is the compression depth (x). Was an initial assumption made for the neutral axis to calculate A'_s (and the other areas of steel) prior to determining “ x ”? The values of the bar areas given in the Appendix of Xu *et al.* (2018) correspond to $A_{sd} = 703.36 \times 10^{-6} \text{ m}^2$ (i.e., 14 bars), $A_s = 301.44 \times 10^{-6} \text{ m}^2$ (i.e., 6 bars), and $A'_s = 100.48 \times 10^{-6} \text{ m}^2$ (i.e., 2 bars). Could the authors confirm which of the longitudinal bars of the loaded half U-section were used for each of the three separate bar areas? A diagram or figure could aid in illustrating the bars used for the different areas.

53 **REFERENCES**

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57 shaped thin-walled RC beams under combined actions of torsion, flexure and shear. *International Journal of*
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