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Strategic interactions dynamics in New Economic Geography

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Academic year 2022–2023
Master [120] in Mathematical Engineering

Chapter 1

Introduction

In today's interconnected world, the global interactions between countries shape trade and economic development. Nations try to attract skilled workers and be competitively more attractive than others, as individuals optimize their economic well-being. This competition involves factors like trade barriers, diplomatic relationships and economic development. Regions desire to make the most out of the resources available. Maximizing the share of those resources include attracting skilled workers, encouraging economic growth, and thereby making the local area more prosperous.

This master's thesis takes a closer look at regional competition using game theory. The regions are modelled with assumptions from New Economic Geography and in particular from the footloose entrepreneur model, extending the foundational core-periphery model in the field. This framework helps us understand how geography and location impact economic choices. Game theory then permits us to understand the strategies that regions use when dealing with trade barriers and diplomatic ties. To that extent, we propose an objective common to all regions that they will consistently pursue.

The main idea here is that regions, which have different levels of economic strength and diplomatic influence, play a strategic game when picking who their partners and opponents are. They are required to collaborate to assemble more power, to impact the distribution of limited resources to their advantage. Their collaboration changes their own cost structure and that of others, as they try to harm other opposing them by limiting access to their economic output. This is done in order to lure more skilled workers and boost their economies, as a utility differential is created for workers. We look particularly at trade costs, which don't just mean taxes and fees; they also include the diplomatic relationships that can either help or hurt trade.

A big point of this study is that in the game of regional competition, it is what seem to be the weakest player that gets to determine the coalition. Whilst his contribution to a coalition is smaller, he still makes the difference while pursuing its own consistent maximization of attractiveness to workers. That weaker player has no preferences for which region he partners with and will choose the one proposing the most attractive collaboration.

The main aim of this master's thesis is to understand the strategies and decisions that regions use when they compete against each other. By studying how trade barriers relate to diplomatic relationships, standing for rational interactions between players of our game, and how regions with different strengths pick their partners.

Firstly, a contextualization of the work is conducted, which shows the recent development of New Economic Geography, with the footloose entrepreneur model with individual preferences as a starting point. While numerous institutions like the World Bank trust NEG models to provide policy recommendation to governments, none consider how differentiated trade costs standing for diplomatic relations, and subsequent best-responses of regions, change trade and migration dynamics. This forms our contribution to NEG, as we propose a first framework to take dynamic interactions of regions into account to propose more precise policy recommendation. Indeed, only fools take actions without taking those of others into account...

On the other hand, indirect utility can be interpreted as fitness in an evolutionary game. The region with the highest utility grows by attracting more skilled workers to the detriment of others which are pushed to extinction as their share decreases and converges to 0. As we are studying the dynamic decisions of coalitions by countries, facing uncertainty on the individual preferences of their mobile entrepreneurs, with the additional evolutionary game interpretation, we reach new grounds within the literature on dynamic coalition formation. This is bone by designing the whole model and proposing its strategic interactions between players.

Furthermore, modelling assumptions concerning regions behavior and the base model we started with are detailed. Progressively, the base model is transformed with motivated choices towards one that highlights successfully rational interactions between players impacting the distribution of workers. This is done for example by defining that regions compete on the long run for the highest share in the workers mix, which results in them being farsighted rather than myopically optimizing their immediate payoff.

Consequently, results, interpretations and areas of improvement are highlighted in chapter 4. In particular, we tackle how coalitions should emerge based on the state of the system at the time of coalition formation, with a deterministic approach and one that takes individual preferences into account. That last case therefore proposes a probabilistic score for the coalition to succeed. Finally, a short conclusion discusses the results achieved by the master thesis and proposes new research perspectives.

Chapter 2

State of the Art

2.1 Introduction of the field

2.1.1 Economic Geography

While considering welfare and wellbeing, one of the first questions is the economic setting in which an individual evolves. This setting along with the distribution of populations depends on the regional development, as market incentives create migration (Frey et al., 1996, Taylor, 2006). Therefore, understanding today's migrations and related political issues necessitates and understanding of Economic Geography, which is the subfield of human geography covering the spatial distribution of economic activity resulting in further growth and development.

The field also analyses how economic activities affect social and political structures, examines various economic processes such as production, consumption, distribution, exchange of goods and services. Factors such as natural resources, climate, transportation networks, political institutions and culture affecting economic development or globalization are also taken into account. It is therefore an interdisciplinary field drawing on concepts and methods from economics, geography, sociology, anthropology, and environmental studies. It is used to analyze various economic phenomena such as regional disparities, urbanization, globalization, and international trade. (Clark et al., 2018 ; Wood and Roberts ,2012)

2.1.2 New Economic Geography

To summarize, Economic geography is a field that seeks to understand the spatial distribution of economic activity and the factors that influence it. Over time, this field evolved and expanded to incorporate new theoretical frameworks that seek to address even more complex interactions between economic and geographical factors today. For example, the impact of globalization on the spatial distribution of economic activity can not be explained with the basic Economic Geography framework.

This led to the development of New Economic Geography by Paul Krugman in 1991, often shortened NEG. Overall, it reflects the ongoing effort to understand the dynamic relationship between economic and geographical factors that shape the spatial distribution of economic activity. This more specific field, therefore, analyses the emergence of clusters of economic activity. It also examines trade costs that influence the location of economic activity, as well as the movement of people and goods.

In addition to the development of NEG, Paul Krugman also contributed to economic thought with New Trade Theory, closely related to the former. Historically, Economic Geography and past theoretical frameworks explained patterns of trade by comparative advantage, for example in the form of more advanced technologies or production facilities. Recently, those comparative advantages are less relevant as technology is available globally. Therefore, while previously trade occurred between specialized and less-specialized countries, as explained by Adam Smith, today, similar goods are imported and exported as countries dispose of similar expertise. Krugman highlighted this phenomenon in two seminal papers, specifically referencing the automobile industry. This trend continues today: a Japanese consumer might choose a German car over domestically produced alternatives, as evidenced by Gaspar (2020). The phenomenon can be seen as an increased interest of consumers for variety.

Observing and reflecting on the previous led Krugman to use a framework similar to Chamberlin's model (1933) of monopolistic competition, where firms sell differentiated products with similar prices and qualities, which are called horizontally differentiated products. As Krugman's new framework proved to match trade patterns better than previous ones, his trade models became the standard as they were more realistic. As a consequence, his paper "Increasing Returns and Economic Geography" (1991a) published in the *Journal of Political Economy* was cited almost 20.000 times at the time of writing, which highlights its predominant role in the field.

While Economic Geography considered economies of scale to decisively impact the location of economic activity, it could not explain the observation with a mathematically consistent model. This forms another contribution of Krugman, by providing a framework that rigorously support the previous insight while tackling trade and location of economic activity simultaneously. However, analyzing concentration of activity in a precise location was already done previously, for example by

Marshall (1890) or Weber (1929).

To take economies of scale into account within his trade theory framework, Krugman resorted to the Dixit and Stiglitz (1977) model of monopolistic competition. It differs in particular to that of Chamberlin by explicitly modelling consumer's preference for diversity. However, they share four conditions that are common for all monopolositic competition models :

1. lots of firms produce differentiated goods
2. all firms have a decreasing demand curve
3. all firms are small and have a negligible effect on others
4. free entry and exit for firms leading to the absence of profit

In short, Krugman delivered a general equilibrium model of imperfect competition necessary to support increasing returns and economies of scale at the firm level, opening new perspectives for trade models.

Next to monopolistic competition, the framework also tackles transportation costs, whose decrease favors growth of trade. As Samuelson (1954), Krugman chose to model them as iceberg costs, which means that a fraction of transported costs "melts away" while being transported. This results in the *home market effect*, which is that firms concentrate in larger markets because it leads them to fewer transportation and consequently less costs. Without this phenomenon yielding non-constant returns, and because of free entry and exit, each location would provide its own production facilities without transportation. Indeed, not exporting and focusing on local demand would lead to being more competitive than other firms due to no transportation costs being present while selling regionally. However, by incorporating non-constant returns to his framework, Krugman was able to explain clusters of economic activity. This is more precisely developed in the next section detailing his canonical model.

New Trade Theory deeply affected trade policy thanks to its accurate theoretical predictions and welfare analysis. Today, the model by Baldwin and Krugman of 1988 is standardly used by institutions such as the World Bank to evaluate world trade.

Furthermore, it is worth mentioning that Krugman is considered to be one of the most influential economists in the world. He is a Nobel laureate in 2008 for his analysis of trade patterns and location of economic activity. Furthermore, he is a previous professor of economics at MIT and Princeton and the second most cited author on college syllabi for economics, as mentioned by Open Syllabus: Explorer (n.d.).

2.2 Core-periphery and footloose entrepreneur model

In his original paper defining "NEG" published in the Journal of Political Economy, Krugman builds the Core-Periphery model, which is as canonical model for NEG. The core can be referred to as an highly populated and wealthy area while the periphery only supplies agricultural goods and is sparsely populated. The model builds upon basic elements from New Trade Theory described previously in order to explain concentration of economic activity : monopolistic competition, increasing returns and love for variety.

Analyzing the distribution of economic activity, this model shows that extending activity in a region reduces trade barriers locally, but also promotes coordination and innovation. This leads to further increase in geographical concentration of industrial production.

The phenomenon described here is agglomeration, a clustering of economic activity. This movement of production facilities changes local market conditions and boosts the local economy. (Forslid & Ottaviano, 2003). Agglomeration leads to increased returns to scale and is therefore self-reinforcing. Otherwise, subdividing firms with smaller capacities only satisfying local demand, would be preferred. This would lead firms and households to reduce trade and their transportation costs to roughly zero. That situation may be referred to as backyard capitalism, and does not strike to today's empirical reality. (Lafourcade & Thisse, 2011).

However, as we do not observe backyard capitalism, neither are we confronted with full agglomeration. A trade-off between increasing returns due to agglomeration forces and the increased transportation costs as a result is crucial in understanding how economic activities are distributed across different locations. (Koopmans, 1957; Krugman, 1995). To sum it up, reality is a balance between agglomeration and the opposing dispersion forces resulting from increased transportation costs (Papageorgiou and Smith, 1983). NEG tries to understand the nature of those forces and their interactions.

While the core-periphery model , serving as canonical model, is very interesting as its numerous assumptions make it possible to focus on NEG areas of focus, it is analytically intractable. This is why Forslid and Ottaviano proposed modifications making it possible to derive a closed-form solution for the variables. Consequently, the number of equilibria and their global stability could be obtained. The resulting modifications resulted in a new model : the footloose entrepreneur model. While other models , such as that of Baldwin (2001) or Ottaviano et al. (2002) are also analytically tractable, they abandon crucial features of the CP model. As the footloose entrepreneur model by Forslid and Ottaviano proposes slighter modifications with the desired closed form, it is an important contribution in the literature.

Solvability is there obtained by differentiating workers by skillness. Indeed, it has been empirically observed that skilled workers have significantly more mobility (Shields and Shields, 1989). This differentiation by skillness also explains the name, as skilled workers are considered as self-employed entrepreneurs moving across regions. Their migration is drived by their optimization of profit while operating in a certain region.

While Forslid and Ottaviano proposed an economy consisting of two regions, the model is easily adapted to three regions. Delloye and Tharakan (2018) opted for this approach where there are 3 stable equilibria, corresponding to full agglomeration of skilled workers in a single region, and one unstable equilibrium corresponding to an even distribution of workers in all three regions. The main results useful for the scope of this work are explicited in the appendix, however the curious reader is invited to look at both references just mentioned.

2.3 Individual preferences in migration

Following Krugman's work and the advent of the field, several studies were conducted in particular on individual preferences in migration concerning market and non-market settings. (Jacobs, 1961; Hicks, 1963; Rosen, 1979; Roback, 1982; Greenwood, 1985; Glaeser, 2008). They showed that those preferences act as a dispersion force and affect the number and stability of equilibrium population distributions. However, only deterministic procedures were adopted to treat equilibrium stability locally. This results in an equilibrium selection which solely depends on initial conditions.

This is where our main reference paper comes to play (Delloye and Tharakan, 2018), as it introduces uncertainty on adjustment dynamic trajectories resulting from those individual preferences, modelled with a nested logit structure. The heterogeneity in preferences of the agents forces the non-deterministic character of the model which therefore requires stochastic modelling and non-deterministic, probabilistic equilibrium selection. Furthermore, the uncertainty is an improvement compared to the previous studies mentioned as it allows to differentiate the individual agents, and highlights for example different 'home effects' which affect the migration incentive at different rates. The 'home effect' was calibrated using empirical data (Delloye & Thomas, 2021). Such improvement calls for further modelling of individual human behavior concerning migration.

Of course migration is evolution. The evolution dynamics in Delloye and Tharakan (2018) of the population distribution was depicted with a continuous-time Markov chain. Those are of uttermost interest, because as we said in the first paragraph, market incentives create migration. However, the opposite may also be true. Indeed, migration, which will only concerns skilled workers in our model, will affect the economic development and setting of the region. This, as the number of working agents is proportional to the production of goods as shown in the model part.

Another major contribution of Delloye and Tharakan (2018) was the use of a diffusion approximation to reduce the derivative of the transition probability function with respect to time, to a set of nonlinear stochastic differential equations. The necessity of this development is that the Kolmogorov forward equation, the type of the equation being approximated, has a solution that doesn't admit a closed form expression and is not always analytically solvable (Ross 2009). It results that the dynamic change can now be modelled by a continuous process. This is considered as an original contribution to the field of New Economic Geography, where the analytical expression of the derivative of the transition probability function facilitates simulations and result validations.

Simulations of the transition probabilities were then conducted. Those highlighted that dynamic trajectories were not only influenced by equilibrium selection and stability, but also by noise induced transitions. The mere presence of noise influence requires stochastic modelling as trajectories are by definition not deterministic anymore. Furthermore, an analysis on equilibrium switches was also made while highlighting the trajectories from the same original state. This whole forms one of the main contributions of the paper: discovering noice-induced transitions in the spatial dynamics of the skilled workers distribution. It results in the necessity of extending the footloose entrepreneur model.

Firstly, by discussing equilibrium stability in stochastic terms. Secondly, by identifying that equilibrium selection isn't bijective with respect to the initial condition anymore, which logically follows from the previous. Finally, the difference in trajectories. The ability of the new, stochastic, model to differentiate scenario and the use of explicit time units thanks to the Markov chain is an important improvement of the footloose entrepreneur model to support policy recommendations.

The difference in trajectories just highlighted explains that noise, which is called exogenous shocks in Delloye and Tharakan (2018), triggers equilibrium switches. (Robert-Nicoud, 2005). This is a very important topic as the likeliness and nature of each equilibrium influences the design of regional policies, with respect to a certain optimization criterion, which tends to target a certain equilibrium more profitable to the region. Here, equilibrium switches refer to situations where we are deterministically in the basin of attraction of one stable equilibrium of agglomeration, but nevertheless the dynamic system evolves as to converge to another equilibrium. Because of economies of scale, even a small imbalance in share favors the largest share. In previous models, the probability that the system evolves towards total agglomeration in the region with the largest share at the initial condition is 1, while by taking noise into account, this probability decreases. This is shown in the simulations. It remains however always greater than 1/3 independently of the parameters settings, as a favorable initial condition with non-differentiated regions always constitutes an advantage when competing against other regions with our modelling assumptions.

2.4 Differentiating policies to trigger new dynamics

The policies just mentioned are essential to observe the regional interactions. Differentiating them extends the defined migration framework and allows a dynamic response to observed states of the system. . We will formally define in the "Game Theory" section the objective of a region, but it boils down to maximizing its share in the high-skilled workers mix, with the policy as only possible action.

Our main goal will be to define a set of policies and to evaluate which is optimal with respect to their respective maximization of share. The game concludes with either agglomeration or a never-ending stabilization of the equilibrium with equal shares. Regions will design their policies to increase their internal indirect utility and lower that of their competitors. As developed in the modeling part, this makes them more attractive to high-skilled workers which leads to a positive flux towards them.

Furthermore, the rational interactions of regions in specific situations result in correlated strategies that extend the set of possible terminal equilibria from initial conditions. In particular, the unstable equilibrium highlighted in Delloye and Tharakan (2018) can be reached and stabilized by attractive binding contracts between policy designers.

To sum it up, a contribution of this master thesis is to extend the footloose entrepreneur model as developed in Delloye and Tharakan (2018) with a game theory framework, which has not been previously done. The insights generated could support policy recommendations, which enlarge the equilibrium selection possibilities and their respective stability.

2.5 Defining the scope of policies

Having defined that we want to differentiate regional policies, we are now required to define a scope for those. The policy we are searching for should be :

1. a central point of interest in NEG to generate useful insights and confront our model and simulations with other works
2. a dynamic parameter that is easily differentiated and modified, as to respond to the evolution of the system
3. something representing regional interactions as to create rational responses in a game theory framework, required to study those interactions

As said in Lafourcade & Thisse (2011), trade costs are the inherent attribute of exchanges across locations. Therefore, they are a central element in NEG. Because their differentiation can be easily modeled, a first proposition of policy differentiation concerns transportation costs, with respect to neighbouring countries the policy designer is trading with.

As our analysis of differentiated policies will focus on transportation costs, a few insights are welcome before starting any analysis.

First of all, transportation costs, also called iceberg transport costs, stand for 4 different kinds of costs (Lafourcade & Thisse, 2011) :

1. Transaction costs : trading with a country with different customs and practices
2. Tariff and non-tariff costs : regulations restricting trade such as anti-pollution standards
3. Transport costs per se : the cost of physical transport
4. Time costs : the required delay before reaching destination

In our model, as we assume identical regions up to their policy, we assume that the default transportation cost τ_0 englobes all of those. In particular, we will focus on tariff costs which refer to taxes or duties imposed on imported goods by a country. This is considered as a form of trade restriction. In our work, we will consider that regions might vary their export tax within a certain margin, as those regulations are considered to be part of a region's policy.

2.6 Other researches in the field

Because of recent technological improvements regarding transport capabilities and efficiencies, but also improved communication, international trade has experienced a huge boost. This results in a need for increased mobility of workers and therefore, migration. As migration triggers lots of political issues and conflicts, understanding its functioning and stakes is particularly relevant today. (Mosk, 2007)

Other current researches in the field includes increased modelling of the migrants (Badlwin, 2001), for example taking into account the forward-looking expectation of migrating to a region, instead of simply observing the present economical setting of a region. Furthermore, there exists multiple industries and frameworks to apply the work to, such as in Veracierto (2020) : stochastic dynamics with heterogeneous agents are studied for economies with private information. There, a new solution method for the equilibria, using spline coefficients, is introduced and recovers the analytical solution perfectly.

In short, the field is pretty vast due to its range of applications, recent and relevant. While Delloye and Tharakan (2018) has not been reviewed much, other works have been published closely related to the same subject, which indicates that this specific subfield is in full growth.

2.7 Dynamic coalition formation

As the contribution of this paper is to propose a game theory framework to study rational interactions between policy designers in NEG, the need to put the current research in game theory into perspective should be clear.

Game Theory studies interactions and decision-making in situations involving multiple participants, known as players. It provides a framework to analyze how individuals make choices while considering the actions of others. In particular, within this large field in mathematics, we focus on Cooperative Game Theory, which focuses on situations where players can form coalitions and make binding agreements. The goal is to allocate payoffs among players in a fair and stable manner. The reason why players are required to make coalitions here is that agglomeration and stochastic forces described previously cannot be chosen by players. Therefore, the need for an additional force arises, so players may somehow have an impact on the outcome of the game, which can be coalitions as presented throughout the paper.

Because the field of application revolves around a dynamic system, it makes sense to particularly focus on dynamic coalition formation. We chose not to propose how to make binding agreements and differentiate actions within members of the coalitions, which is a notable area of improvement.

However, having delimited more precisely the area of research helps to focus on precise questions. For example, within this subfield, Arnold & Schwalbe (2002) ask the following questions:

"How do coalitions form? How do the players decide on the division of the coalitional payoff? How do coalition structures change over time? Which of the possible coalition structures will the players eventually arrive at, and what will be the resulting allocation?"

Those will be questions partly answered throughout the analysis conducted in this paper. However, while the previous authors tackle dynamic coalition formation with noise, the nature of noise differs with our model. We, indeed observe noise within the dynamical system while their model adds noise at the decision-making level of players. Players can chose for a suboptimal strategy with a small probability.

In general, dynamic coalition formation is a pretty active topic nowadays with numerous applications, for example in mobile computing (Klusch and Gerber, 2002)

However, the prompt "dynamic coalition formation" AND "stochastic system" in Google scholar produces only 3 results, which indicates the limited amount of research conducted in this specific field. The most recent paper being from Asheralieva and Niyato, in 2019, covering a cloud-based Content Delivery Network (CDN). Our paper further differentiates from the existing literature by applying it to a model that is evolutionary, because agent's share evolve towards full agglomeration, extinction or possibly some sort of intermediate equilibrium of coexistence being defined later on. Indeed, fitness here is interpreted as indirect utility, and policy designers at the region level act in order to maximize it, competing with other regions to grow in share and push others to extinction.

To sum it up, we propose differentiated trade costs in the present work. These contribute to NEG via the introduction of a game theory framework. It also contributes to Game Theory since the nature of the applied model is evolutionary and that of a stochastic system, which has not been previously tackled in dynamic coalition formation.

Chapter 3

Model

Developments have been omitted, but this work bases itself mainly on results from Delloye and Tharakan (2018), Forslid and Ottaviano (2003). A list of the constants used is detailed in Appendix. The main results useful for this work are recalled.

3.1 Base model as in Delloye and Tharakan (2018)

We consider a population of workers distributed among 3 regions, which are represented by the set of integers 1,2,3. That population consume differentiated manufactured and agricultural goods, which are traded between regions. Workers produce one unit of labour. High-skilled workers are considered to be only employed in manufacturing sector and can be thought of as mobile entrepreneurs. Low-skilled workers are employed in both sectors. The sum of low-skilled workers is L, and their fixed distribution at a certain time in region i is $l_i(t)$, which is constant over time t. We suppose an even distribution of low-skilled workers which means that $l_i(t) = L/3$. Similarly, the vector $\mathbf{h}$ represents the vector of 3 entries $h_i(t)$, which is the variable amount of skilled-workers, in percent of the total population of workers H, in region i.

Firstly, on the demand side, consumption preferences are represented by a utility function $u(x_i, a_i) = \mu ln(x_i) + (1-\mu) ln(a_i)$, with

$$x_i = \left(\int_M d_i(m)^{(\sigma-1)/\sigma} dm \right)^{\sigma/(\sigma-1)} \tag{3.1}$$

where $\mu \in [0,1]$ is a constant, x_i the individual consumption of manufactured good and a_i is the individual consumption of agricultural goods in the i^{th} region. We integrate over the set of varieties M, while $d_i(m)$ is the individual consumption of specific variety m in a region i. The last constant is σ which is the demand-elasticity and elasticity of substitution between any two varieties. For the sake of simplicity, we will simply refer to it later as demand-elasticity.

On the other hand, on the supply side, we suppose agricultural goods are homogeneous under perfect competition. Returns to scale are fixed because of the fixed population of low-skilled workers. Units are chosen so that one labour unit produces one output unit. While agricultural goods are considered homogeneous, manufactured ones are differentiated, with monopolistically competitive firms under increasing returns to scale (with varying , increasing population of skilled workers in a region). Assuming monopolistic competition simplifies the framework. Indeed, a monopoly is a situation where only a single firm produces a variety. Therefore, there is no price war on any variety sold on the market, firms interaction doesn't need to be taken into account.

We assume a one-to-one relationship between firms and variety. The total cost for the production of $x_i(m)$ units of variety m in region i is :

$$c_i(m) = \gamma y_i + \eta x_i(m) y_i^L \tag{3.2}$$

where y_i^L stands for the wages of low-skilled workers, and y_i for that of high-skilled ones, in a region i. The fixed input requirement in terms of high-skilled worker is γ, while $\eta x_i(m)$ is a variable input in terms of low-skill labour. However, while trading with other regions, manufactured goods are exposed to trade barriers modelled as iceberg costs. Assuming equidistant regions, a constant $\tau \in [1, +\infty]$ is required to leave a producing region, so that one unit arrives at the region of destination.

3.1.1 Indirect utility

This concept refers to the maximum level of utility that a consumer can attain with his given income and the current market prices in their region. Consumers are the set of workers, high-skilled and low-skilled, which only differ by their wages and their ability to migrate. Indirect utility is what drives the most the migration of workers, as it represents the economic welfare and attractiveness of a country. Optimizing consumer utility without income and market constraints, while maximizing firms profit and clearing markets yields the indirect utility, as shown by Forslid and Ottaviano (2003) :

$$U_i(\mathbf{h}) = \ln(\mu^\mu (1-\mu)^{1-\mu} \left[\frac{(\sigma - 1)\gamma^{1/(1-\sigma)}}{\sigma \eta} \right] [h_i + \tau^{1-\sigma}(h_j + h_k)]^{\mu/(\sigma-1)} y_i^*) \tag{3.3}$$

where i, j, k are indexes of the three regions, and y_i^* is the local equilibrium wage of high-skilled workers, implicitly given by :

$$\frac{y_i^*}{\theta} = \frac{y_i^* h_i + L/3}{h_i + \phi(h_j + h_k)} + \frac{\phi(y_j^* + L/3)}{h_j + \phi(h_i + h_k)} + \frac{\phi(y_k^* h_k + L/3)}{h_k + \phi(h_i + h_j)} \tag{3.4}$$

For ease of writing, we use $\phi = \tau^{1-\sigma}$, $\theta = \mu/\sigma$. We detail this system of implicit equations while working with the new model differentiating trade costs in section 5.2 .

3.1.2 Individual relocation dynamics

Having defined regional utilities and that migrants seek to maximize it, the goal is now to mathematically model the process.

It is assumed that workers do not base their decision on expectations about a future state of the system, and are memoryless. Therefore, deciding to migrate is a Poisson process with a decision rate $v_i(\mathbf{h})$. This results in individual relocation dynamics forming a continuous-time Markov chain.

We asume that relocation choices depend from economic incentives being utility differentials, but also from individual (and therefore variable, uncertain) preferences regarding the location of origin, being called the "home effect" in Delloye and Tharakan (2018). This is why we obtain a nested-logit decision process as in Ben-Akiva and Lerman (1985). The product of the probability of leaving region i multiplied by the probability to arrive in region j afterwards is called the individual transition rate $q(j|i,h)$.

3.1.3 Individual transition rate

Assuming that the decision rate is influenced by regional utility differentials, and rises with the difference of utility between other regions and his region, the following transition rates of the Markov chain were proposed.

$$q(j|i,h) = \frac{v}{2} e^{(U_j(\mathbf{h}) - U_i(\mathbf{h}))/\alpha)} \tag{3.5}$$

3.1.4 Transition rates of states

Given original state s and destination state s', we use the functions O, I : { $\mathbf{J}_{.1}$,..., $\mathbf{J}_{.6}$ } $\rightarrow$ {1,2,3} which are respectively the origin and destinations functions, taking as argument a change vector $\mathbf{J}_{.k}$ which highlights a movement of a worker. The full matrix J is explicitly written in the appendix, but consists of entries in [-1,0,1], with -1 referring to the region being left, and +1 to the region of destination. Therefore, the function O, I return the departure or destination region respectively. Given the fact we have 3 regions, we only can have 6 possible movements which justifies the size of matrix $\mathbf{J}$.

$$Q(s'|s) = s_{O(s'-s)} q(I(s'-s)|O(s'-s), s) \tag{3.6}$$

Thanks to this function, we are able to define $\mathbf{Q}$ which is a (6x1) matrix which kth element $\mathbf{Q}_k(s) = Q(s + \mathbf{J}_{.k}|s)$ Finally, we can define $\mathbb{E}(\delta|s(t)) = J \times Q(s(t))$ which can be interpreted as a vector of the deterministic derivative of each component.

3.1.5 Solving the problem with Itô stochastic differential equations

As the derivative of the transition probability function with respect to time doesn't admit a closed-form expression (Ross, 2009), Delloye and Tharakan proposed to use an analytical approximation of this function described by a Kolmogorov-forward equation by considering it as a continuous diffusion process. Intuitively, this relies on the behavior of infinitesimal change of a population evolution that remains small compared to a total population H. Further details are to be found in their paper, but this allows them to rewrite the following differential equation for the state evolution as follows :

$$ds(t) = \mathbf{E}(\delta|s(t))dt + \frac{\mathbf{D}(\delta|(s(t))}{\sqrt{H}} d\mathbf{W}(\mathbf{t}) \tag{3.7}$$

with initial conditions $s(0) = s_0$. This forms a system of Itô stochastic differential equations where $\mathbf{W}(t)$ is a Wiener process and D is a square root of $\mathbf{D}^2$ (Stroock and Varathan,1997). Finally, Delloye and Tharakan define $\mathbf{E}(\delta|s(t))$ as a drift coefficient and $\mathbf{D}(\delta|(s(t))$ as a diffusion coefficient. They are are respectively the expected value and covariance matrix of the instantaneous change vector δ. For the ease of notations, we might refer to the drift coefficient as $\mathbf{E}()$ later in the paper.

Of course, the importance of the stochastic forces generated by the Wiener process reduces while increasing H. By taking the limit of H to $+\infty$, we obtain a deterministic model. Intuitively, we can explain it as on average for large populations, random utility terms average out and yield a zero expected value.

In practice, in our python simulations, the Wiener process was modeled with white noise and the matrix square root couldn't be obtained with the numpy

Cholesky factorization, due to it being semi-positive definite, and not positive definite. Therefore, a small regularization on the diagonal of 10^{-19}, corresponding to the numerical precision of the computer on which the simulations were conducted, was applied, before using scipy.linalg.sqrtm().

3.2 Current model

To study rational interactions between regions, it was necessary to introduce a dynamic differentiation, influencing their state transitions. The differentiation forms their policy, which remains constant over time and is simply a fixed mapping from observed states to decisions. The goal is to simulate maximizations of high-skilled workers share by a single region interacting with others to provide better policy recommendations.

3.2.1 Differentiated transportation costs

As mentioned before, transportation costs are one of the most important concepts in NEG. The original model assumed identical costs for simplicity, but in reality, transportation costs not only depend from the distance, but also from local prices and regional diplomatic relations to fix import and in particular export fees. Considering the interest in the field, it was natural to extend the model in that direction (Redding, 2013).

Consequently, the ϕ_0 constant, valid for all trade, was modified to a variable Φ 3x3 matrix highlighting the cost from a region i to a region j. This means that diagonal entries were meaningless as a region cannot conduct trade with itself, and we suppose there is no additional cost or trade barrier for internal trade within a region. The matrix entries can be modified by the players to some extent, representing their diplomatic relation : collaboration or opposition. This will be developed later. However, the need to have a dynamic differentiation follows from the system's state dynamic evolution, making a permanent response to an initial condition meaningless, as it is rational to change one's policy reacting to the evolution of the system.

This leads us no new implicit equations, obtained by rearranging (Forslid and Ottaviano, 2003). Now, instead of symbolically solve the equations for y_i, this was transformed to a system of implicit wages equations w_i, which computations are detailed in the Appendix. This leads us to a system of the form :

$$\mathbf{w} = A\mathbf{w} + L/3\mathbf{c}$$

where A is a 3x3 matrix and $\mathbf{c}$ a vector of 3 entries obtained by putting the implicit wages equations in vector form.

At every time step, this equation needs to be updated as A and $\mathbf{c}$ depend dynamically from the state s. This can be done as follows :

$$\mathbf{w} = (I_3 - A)^{-1}\mathbf{c}L/3$$

3.2.2 New dynamical equations

Understanding that in equation (3), the utility depends on the current supplies available in the region, we can differentiate the transportation costs ϕ as follows in equation (5) :

$$q(j|i,h) = \frac{v}{2}\left[\frac{h_j + \phi_{ij}h_i + \phi_{kj}h_k}{h_i + \phi_{ji}h_j + \phi_{ki}h_k}\right]^{\mu/(\alpha(\sigma-1))}\left[\frac{Y_j}{Y_i}\right]^{1\alpha} \tag{3.8}$$

As detailed in the appendix, the updated wages are linearly related to Y, which explains why we were required to obtain those before writing this equation. In practice, solving the implicit wages equations is useless as the ratio of any entries of Y is negligible, as showed in the simulations. Therefore, the ratio has been omitted. As the other dynamical situations use $q(j|i,h)$, they do not need to be rewritten. Our final equation resumes to
:

$$q(j|i,h) = \frac{v}{2}\left[\frac{h_j + \phi_{ij}h_i + \phi_{kj}h_k}{h_i + \phi_{ji}h_j + \phi_{ki}h_k}\right]^{p} \tag{3.9}$$

with $p = \frac{\mu}{\alpha(\sigma-1)}$ as to alleviate notations. Without detailing too narrowly the constants, which is done at the start of chapter 4, we'll assume p is positive and greater than 1.

3.3 Game Theory framework

Now, regions are able to differentiate the transportation costs from their region to another one, which are export costs. This stands for diplomatic relations between countries, in the form of trade taxes for example, for expressing their interactions in a setting where they compete for a bigger share of workers. It would be meaningless to allow regions to chose for an import cost, as they would of course minimise it to improve their indirect utility . Indeed, a lower $\tau^{1-\sigma}$ means a higher utility as by definition, the entries of $\mathbf{h}$ cannot be negative.

3.3.1 Objective

What are they striving for ? Their goal is to maximize their share of high-skilled workers to induce economies of scale and increased productivity, as said in the New Economic Geography subsection. At the same time, they thus want to avoid to lose high-skilled workers, as it would diminish their future indirect utility and further create incentives to their high-skilled workers to migrate towards other regions.

A first trivial definition of their payoff is the share of high-skilled workers they are able to capture. Their goal is to maximize it in a 3-player zero-sum game. As we are in the same setting than Delloye and Tharakan (2018), we know there are 4 equilibria, 1 full agglomerations per region and one unstable equilibrium corresponding to a symmetric allocation. However, we are analyzing this system over a large period of time. Indeed, it wouldn't make sense to analyze regional policies over days. While we may use different timeframes through our simulations, this dynamic aspect creates additional complexity and perspectives.

Firstly, the need to account for a balance between immediate and future reward. Are the players farsighted (focused on long term optimization) or myopic (focused on short term) when optimizing ? Defining the payoff R_i^T of a region i with respect to time period T can be done as follows :

$$R_i^T = \sum_{j=0}^{T} \delta^{T-j} r_i^j$$

with $0 \leq \delta \leq 1$ a discount factor expressing the player's patience (the closer to 1, the more myopic the player is , which is aiming at immediate payoffs, while a δ closer to 0 means being patient), j a discrete time index, T the total number of timesteps and r_i^j the reward of player i at time j being his share of high-skilled workers.

Now, how to set δ ? We are looking at governments, which mission is to perform well throughout their mandate as to be reelected. In our theoretical and rather fictitious case, we will still consider this as a long-term vision as we are talking about around a 5 years vision of 365 days. Roughly 1800 days. We arbitrarily propose $\delta = 0,999$ as to enforce that results at the end of the mandate weight double those at the beginning.

Aside from this dynamic aspect, we need to take uncertainty into account as this is a stochastic process. Therefore, players don't maximize their sum of discounted payoffs over time, but their expected sum of discounted payoffs. We can rewrite the objective of a region i at time t as follows :

$$\max_{\pi_i \in \Pi_i} \sum_{j=t}^{T} \delta^{T-j} \mathbb{E}(r_{i,j})$$

with π_i a policy and Π_i the set of policies available for a player i. For the sake of clarity, $\mathbb{E}$ refers to the expected value as we need to take uncertainty into account. Of course, because of the dynamic context, those policies and the respective best replies from other regions are not fixed, but we assume that regions take actions to optimize their payoff at every instant of time, as increasing its share is supposed to be positive at every time for their objective function. While we could be tempted to say that greedily optimizing their immediate payoff could only improve their position due to an increased share, in practice, we will observe that matters are slightly more complicated. Indeed, they will only do so if it does not prevent them for surviving in the long run, as we assume they are farsighted.

3.3.2 Moves and types of players

We start by defining the types of regions. We assume that due to the presence of noise, we will never have perfectly equal shares in high-skilled workers, which allow us to differentiate players, or regional policy designers, by their share.

- Player D : refers to the dominant player, which has the largest share
- Player W : stands for the weakest player, the one with the lowest share.
- Player I : defines the one with an intermediate share

Please note that it is possible to force equal shares at the initial condition. However, we can ignore the case as the situation is reconsidered after a single iteration, and that we force the first coalition formation to happen at the second iteration to avoid the aforementioned issue. As the probability of having an exact noise matching another component is zero, we consider that the situation can be discarded.

As mentioned in the literature concerning NEG and highlighted in simulations, agglomeration forces push the agent with the largest share to entire concentration of workers in his region up to some noise. It seems logical that this situation is profitable to player D, but isn't convenient for W and I. However, next to agglomeration forces, we are working with individual migration preferences acting as dispersion forces. This means that there is a non-zero probability that an equilibrium switch occurs : while we were deterministicly in the basin of attraction of one agglomeration equilibrium, self-reinforcing noise may push the state of the system towards another equilibrium.

Of course, computing exactly this probability and the trajectories associated with this equilibrium switch would allow us to obtain the expected share we were looking for. This can be done numerically as in the simulation section with the appropriated interpretations.

Having defined the types of players and our scope when playing, it's now time to define their moves and strategies. As mentioned in previous sections, players are regional policy designers which can set export transportation costs towards other players as desired within a certain interval. Setting those export costs is their only move to play, and we consider that lower export costs benefit to the destination region, while higher ones reduce the indirect utility in the region and therefore the attractiveness for high-skilled migrating workers.

Lower and higher export costs need to be quantified. Therefore we introduce $\Delta\phi$ which defines the lower and upper bounds of the transportation export costs. Regions collaborating will set transport export costs towards the other to $\phi_0 - \Delta\phi = \phi_{min}$, while they will set $\phi_0 = \phi_{max}$ towards their enemy being the third player. We consider that, as the collaborating regions set high export cost towards the third player, so does he with respect to them. It's not in his interest to increase the indirect utility of regions opposing him, and thereby reducing the difference in attractiveness between his region and the other ones. Indeed, this is a competitive attractiveness as we are working with utility differentials as shown in the base model section.

3.3.3 Policy design

How will those regions interact with this payoff ? We assume regions designing their policies are rational, intelligent agents. This means that they will take actions which are coherent regarding this goal, and that they will be able to analyze the situation at any time and decide how to optimally react as to pursue that goal.

Now, with players moves and types, we can define their policy with respect to the states of the system we observe. We assume there is no variation in time or learning dynamic. Logically, player W and I want to work against the agglomeration forces of player D, by reducing the indirect utility there with increased export transportation costs. This is their only possible way to survive, as agglomeration drives the state derivatives of players to be increasingly negative, until they are zero up to noise. This is detailed in the Appendix and is called extinction. As extinction is the worst possible outcome for a player, resulting in a payoff close to 0, we assume they take opposing actions. Simulations highlight the strength of agglomeration forces and the need for weaker players to collaborate. This leads players I, W to cooperate against player D in the general case where their collaboration is relevant for both of them. Indeed, if they do not oppose D, even a slightly higher share create agglomeration forces, and those reinforce over time.

3.3.4 Interest of the weaker players to engage in a coalition

While avoiding extinction is the main goal for weaker players, once they are doomed and cannot gain any payoff in the long run, they will optimize their short term payoff. This behavior stems from the total payoff that we defined with discount factors. It results in player W preferring to partner with player D rather than player I if the utility differential created by differences in shares cannot be balanced with coalitional play with player I. In other words, a coalition with player I does not prevent W to observe a negative instantaneous change for his share. This is a perfectly possible situation. Let us firstly translate it to mathematical terms :

$$\mathbf{E}[W]_{W,D} > \mathbf{E}[W]_{W,I}$$

with the notation $\mathbf{E}[x]_{i,j}$ referring to the x^{th} component of the drift coefficient E() while there is a coalition of players i and j. This results in the following transportation costs matrix if players i, j, k are represented in chronological order and k is the third opposing member :

$$\begin{bmatrix} / & \phi_{min} & \phi_{max} \\ \phi_{min} & / & \phi_{max} \\ \phi_{max} & \phi_{max} & / \end{bmatrix}$$

Now, let us verify our claim for given ϕ_{min}, ϕ_{max} determined by $\Delta\phi$. For the sake of simplicity , we show that $\mathbf{E}$[W] is related to its own indirect utility using the definition of E(.) and equation 3.6 to obtain :

$$
\begin{aligned}
\mathbf{E}[i] &= s_j q(i,j,s) + s_k q(i,k,s) - s_i q(j,i,s) - s_i q(k,i,s) \\
\mathbf{E}[i] &= 0.5v \left[s_j \frac{e^{U_i(h)}}{e^{U_j(h)}} + s_k \frac{e^{U_i(h)}}{e^{U_k(h)}} - s_i \frac{e^{U_j(h)}}{e^{U_i(h)}} - s_i \frac{e^{U_i(h)}}{e^{U_k(h)}} \right] \\
\mathbf{E}[i] &= 0.5v \left[e^{U_i(h)} \left(\frac{s_j}{e^{U_j(h)}} + \frac{s_k}{e^{U_k(h)}} \right) - s_i \frac{e^{U_j(h)} + e^{U_k(h)}}{e^{U_i(h)}} \right]
\end{aligned}
$$

Clearly, the indirect utility of i multiplies the positive term and divides the negative one. However, we cannot say it is proportional as it depends from the other indirect utilities as well. While the state s is observed and cannot be optimized, all indirect utilities can be affected by player's policy. Let's now compare $\mathbf{E}[W]_{W,D}$ and $\mathbf{E}[W]_{W,I}$ by further developments , assuming that the first one is greater than the second and verifying the claim :

$$
\mathbf{E}[W]_{W,D} \geq \mathbf{E}[W]_{W,I}
$$

Taking the previous expression of **E**[i] where we simplify the constants, and replace $e^{U_i(h)}$ by $h_i + \phi_{j,i} h_j + \phi_{k,i} h_k$ as we are working with positive quantities proportional to the previous, we rewrite **E**[W] as follows :

$$
\mathbf{E}[W] = (h_i + \phi_{j,i} h_j + \phi_{k,i} h_k) \left(\tfrac{s_j}{h_j + \phi_{i,j} h_i + \phi_{k,j} h_k} + \tfrac{s_k}{h_k + \phi_{i,k} h_i + \phi_{j,k} h_j} \right)
$$

$$
- s_i \tfrac{h_j + \phi_{i,j} h_i + \phi_{k,j} h_k + h_k + \phi_{i,k} h_i + \phi_{j,k} h_j}{h_i + \phi_{j,i} h_j + \phi_{k,i} h_k}
$$

Now, engaging in a coalition with D instead of I means that $U_i(h)$ increases as the ϕ_{max} should be applied to the greatest share when maximizing this sequence of terms. Indeed, $\phi_{max} x + \phi_{min} j > \phi_{min} x + \phi_{max} y$ if $x > y$. To be more precise, the term gains $\Delta\phi(h_k - h_j)$ as k corresponds to player D.

While intuitively, we remark that an increased indirect utility for i results in a higher derivative, as $U_i(h)$ multiplies the positive term and divides the negative one, obtaining a formal answer requires some additional computations and assumptions. With a similar reasoning than previously, we can then refer to the new indirect utilities in a coalition of D, W as follows : $U_j(h)$ loses $\Delta\phi h_i$ which is gained in $U_k(h)$ resulting from the change in coalitions.

As mentioned, it is possible to formalize the inequality $\mathbf{E}[W]_{W,D} \geq \mathbf{E}[W]_{W,I}$ with some assumptions, but the computations are quite tedious. For the ease of reading, a numerical solution was preferred which is provided in the appendix. By verifying there that all possible states (discretization with a precision of 10^{-4}) satisfy the claim, we show that it is valid. This is done by plotting the difference of the drift coefficient in both situations, with no sign change to be observed throughout all possible states represented.

Therefore, W will team with I for long-term optimization, as long that it has a real chance of improving its position, which is showed by observing a positive derivative with a non-zero probability. When that is not the case, greedy short-term optimization will be all that is left to do, and result in a coalition with D. Because D therefore increases his utility gap with his next competitor, I, he will happily do so : *Divide et impera.* On the contrary, I needs W to reduce his gap with D. This reasoning shows that W makes the difference and will always be able to choose its partner as long as he doesn't go extinct.

However, player's types evolve over time. If players W and I succeed in opposing agglomeration forces of D, then suddenly that player doesn't have the highest indirect utility, which leads him to have a diminishing share. At some point, its share will be smaller than player I or W, which makes its type change. At every timestep, new types and therefore diplomatic relations, or coalitions, emerge. This is why we are in the context of dynamic coalition formation, with uncertainty of individual preferences.

However, the process of switching diplomatic relations, negotiating and acting is not immediate in reality. This is why we model an additional variable : T_{sw} : which is the number of time steps required before a new coalition can be formed. Next to this parameter, others will be analyzed through simulations to understand in which situations weaker players might overtake a stronger one, and how the system evolves.

Chapter 4

Simulations

The goal of the simulations conducted here is to analyze which situations are relevant for the game we designed to be played, and to understand how each parameter drive equilibrium selection. This is how we will be able to deliver precise policy recommendation. We will start by retrieving past results showing the quality of our personal simulations, before presenting dynamics of the new models and identify situations where the game is relevant to play for all players. Before jumping into results, let us precisely expose the parameter settings used.

4.1 Parameter values

4.1.1 Base model

The following values were overtaken from Delloye and Tharakan (2018) :

1. $\alpha = 10^{-2}$: scale parameter in the nested-logit process
2. $\mu = 0.5$: constant in the utility function
3. $\sigma = 5$: the demand-elasticity
4. $\tau = 1.1504$: the iceberg cost
5. $v = \frac{1}{365}$: the decision rate of individuals evaluating indirect utilities
6. $dt = 1$: the time steps, expressed in days, between state transitions.

4.1.2 New model

Concerning the new parameters introduced, let's dive especially into the range of export transportation costs admitted.

Compared to the standard transportation costs τ_0, $\Delta\phi$ can be seen as additional "tariffs" or "export duties". The World Bank reports an average value for those taxes of 2.6% on its latest report, with an identical 1.69% for EU members (World Band Open Data, s.d.). It can be seen as a measure of protectionism. However, among the 183 countries, 43 apply tariffs of 10% or higher. Given that opposing members in our model exhibit aggression due to competition for increased indirect utility, we have selected the threshold of 10% as indicative of protectionism. This threshold applies to the export trade of one country within the so-called protectionist country.

Assuming that in the previous model transportation costs included tariff cost, as mentioned in the state of the art, we assume it corresponds to the world's average of 2.6 %. Therefore, $\Delta\phi$ should be such that the difference between ϕ_{max} and ϕ_0 corresponds to this difference in transportation costs. Let us compute it !

We assume $\tau_0 = 1.026\tau_-$, $\tau_{max} = 1.1\tau_-$ with τ_- being τ_0 without the tariff costs. This yields τ_{max}= 1.233, and consequently $\phi_{min} : \tau_{max}^{1-\sigma} = 0.4321$. Therefore, $\Delta\phi$ equals $\phi_0 - \phi_{min} = 0.1388$. In short, when a coalition emerges, the opposing player will benefit 24% less from the imported goods and welfare than the standard contribution to his indirect utility.

Next to $\Delta\phi$, the total population H was made to vary as described in the parameters analysis section. However, we assume a standard value of $H = 10^6$. Finally, the minimum time for switching coalitions T_{sw} was also set variably as to observe its impact. However, a standard value of $T_{sw} = 14$ was arbitrarily proposed.

4.2 Base model

As there is no differentiation between players transportation costs , the initial conditions and the stochastic forces determine solely the terminal **h**.

4.2.1 Initial symmetrical condition : $s_0 = [1/3, 1/3, 1/3]$

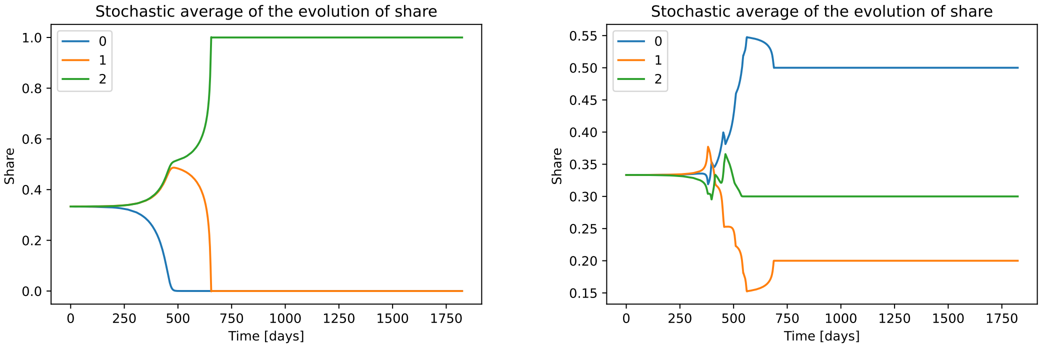
The deterministic version of the Euler method described in equation (3.7) produces the input. Indeed, the situation is purely symmetrical and all components of $\mathbf{E}(.)$ will always be the same, 0, as there is no indirect utility differential between regions.

Therefore, it can be considered as a stable deterministic equilibrium. For the sake of clarity, we precise that a deterministic version of (3.7) is obtained by letting H tend to $+\infty$, thereby cancelling the stochastic term.

Three other equilibria can be identified, corresponding to a complete agglomeration in a region, as in [0,0,1]. Indeed, using scipy.optimize for the sum of absolute values of $\mathbf{E}(.)$, which can be considered as the deterministic derivative. Those equilibria were found starting from a thousand random initial guesses within the domain. The computation may seem futile as results were already known in the literature, but their goal is to double check the validity of our self-made simulations.

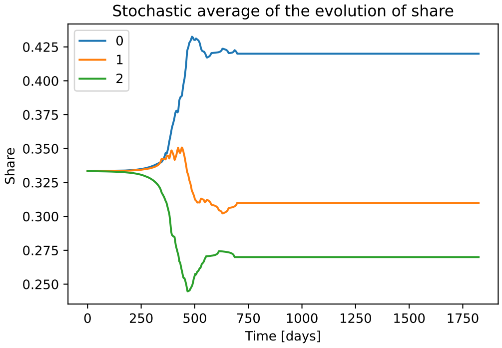
However, more interesting dynamics are triggered when studying the stochastic case, where perturbations will create imbalances and resulting agglomeration. Here is a simulation for this case on the initial condition of this section. A variable number N of stochastic simulations, averaged on the graph, were conducted as to verify the role of that parameter.

As evoked in the state of the art, the timeframe of 5 years is chosen. Numbers $0, 1, 2$ refer to a specific region ordered as in the initial condition.

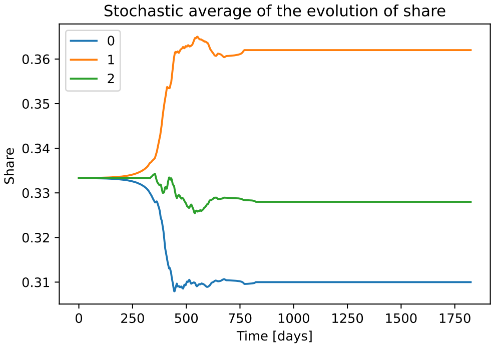


(a) Accelerating divergence of a single simulation : N = 1

(b) Noisy behavior followed by convergence of multiple simulations : N = 10



(a) Reduction in variance of terminal equilibrium : N = 100



(b) Further reduction in variance : N = 1000

Looking at the first graph, we observe the gap between the curves widening at an increasing rate. While the differences between components of **h** are small, their derivative is approximately the same and it is noise that drives the state evolutions as utility differentials and resulting drift coefficient remain small. However, the Brownian motion at hand ultimately creates a higher indirect utility differential between regions resulting in stronger agglomeration forces. Therefore, the deterministic derivative increasing the larger share outplays the stochastic terms and causes the system to terminally select an equilibrium. In our case, this happens when two players out of three are pushed to extinction, which creates a drift coefficient that is close to zero.

Clearly, adding more numerical experiments reduces the variance and highlights a convergence to the initial condition s_0 as perturbations cancel each other out. Furthermore, we can identify that after some time, the definitive distribution of equilibria is attained, with no possibility of equilibrium switching : the system reaches a terminal equilibrium. This can be seen when the curves become entirely flat, while small perturbations at the first iterations highlight the noisy behavior of the different simulations before convergence. Finally, the dynamics highlighted here confirm that the initial condition forms an unstable equilibrium, as all final shares correspond to agglomeration in a single region.

4.2.2 Initial imbalanced condition : s_0 = [0.5,0.4,0.1]

The deterministic case is trivial as the larger region wins with its largerst share. Again, we focus on the stochastic one, using the same parameters, to obtain :

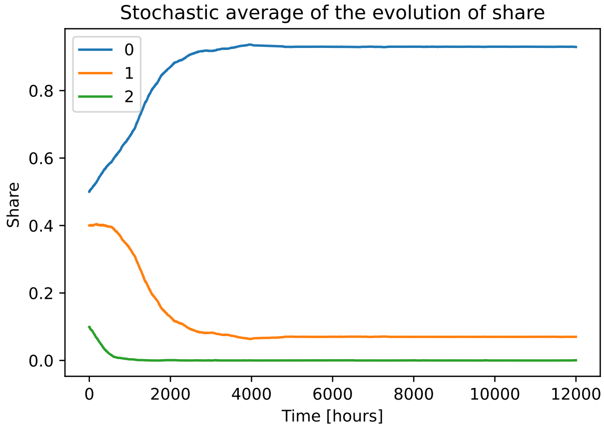


Figure 4.3: Equilibrium switches occurring from region 0 to region 1 with probability of 10%, with N = 100

The dynamics here heavily depend on the balance between deterministic forces and stochastic ones. As made clear by equation (5), increasing the deterministic character of the differential equation can be done by rising the value of dt and that of H. For the simulations conducted here, we did just the opposite by setting $dt = 1/24, H = 10^3$, instead of 1 and 10^6 respectively, for the sake of having non-trivial results (else, equilibrium selection will trivially be the dominant force).

Clearly, equilibrium 0 is mostly chosen as the final share of region 0 is much larger on average than the 1/3 expected on a symmetric initial condition, but equilibrium 1 can be the terminal state as well. Further calibration of the parameters previously mentioned and an increased number of numerical experiments allows equilibrium 2 to be chosen as well. However, this is not the scope of this work, the goal of this section is simply to highlight that we obtain equilibrium switches in certain conditions just as in Delloye and Tharakan (2018), by following their methodology in our own-made program.

4.3 Transportation extension

4.3.1 Relevance of solving the implicit wages equations

As said in the modelling section, solving the implicit wages equations is useless. In Delloye and Tharakan (2018) this was done symbolically with the help of sympy. Considering a setting with as parameters H = 10^7, L = 100*H and $w_0 = 1$, we can use $Y_i = w_0 L/3 + w_i H h_i$ (Forslid and Ottaviano, 2003) and solve numerically the implicit wages equation for the asymmetric initial condition $\mathbf{h}$ = [0.5,0.4,0.4]. This yields :

1. w1 = 6.6×10^5 , Y1 = $2/3 \times 10^5$
2. w2 = 8.3×10^5 , Y2 = $2/3 \times 10^5$
3. w3 = 3.3×10^6 , Y3 = $2/3 \times 10^5$

In practice, we observe differences between the Y_i of order 10^{-8}. With those differences being so small, we compute that the individual transition rate function is identical up to 3.35e-14 %. If we consider a symmetrical initial condition, all the Y become perfectly identical. From there, the choice to omit their ratio from the computations.

Apparently, this comes from the product $w_i h_i$ being constant, which means that the local wages are higher when there is a shortage of workers. This seems a logical conclusion from the law of supply and demand.

4.3.2 Implementing the interactions described in the Game Theory section

Taking what we will further use as a default parameter setting : $H = 10^6, dt = 1$, $\Delta\phi = 0.1388$, we implement the strategy discussed in the game theory section , which consists in a collaboration from players W, I against player D. In our case, player D is the first region, player I the second, and W the last one. This is done as follows for a slightly imbalanced condition $s_0 = [0.35, 0.33, 0.32]$:

- Favoring region 1,2 by putting $\phi_{max} = \phi_0$ towards each other
- Penalizing region 0 by putting ϕ_{min} towards it
- Replying to the previous action by setting ϕ_{min} from region 0 towards the others.

Figure 4.4: Collaborating regions overcome agglomeration forces and push player D to extinction

Trying the same while dynamically updating types and resulting moves and interactions yields :

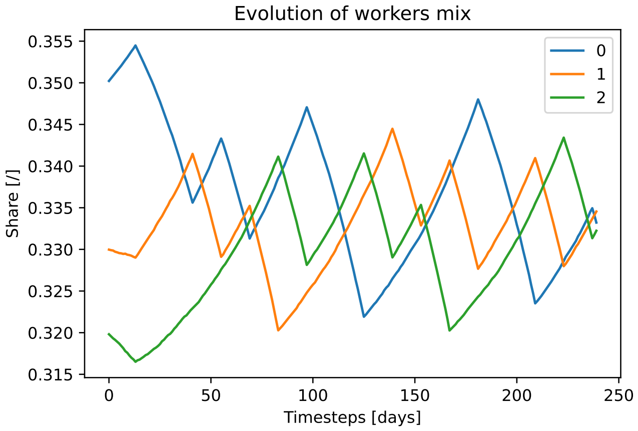


Figure 4.5: A cyclical situation emerges with shares periodically oscillating around 1/3

This simulation shows that, at least in certain settings, it is possible to dynamically change players interactions so that initial conditions and resulting agglomeration forces get balanced by cooperative actions , in order to improve weaker player's position and obtain a symmetrical share (up to small variations due to the reaction time). Indeed, with an immediate response ($T_{sw} = 0$), we obtain :

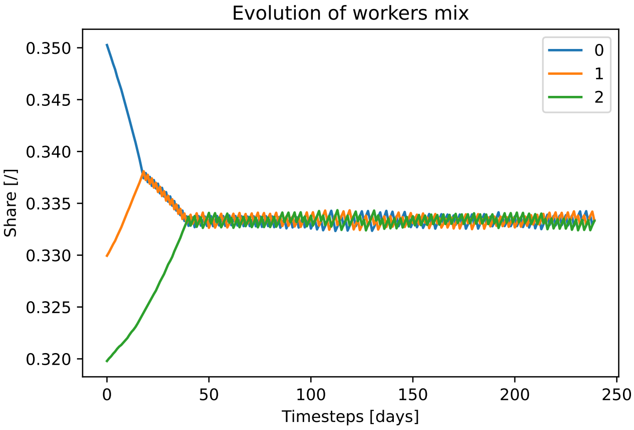
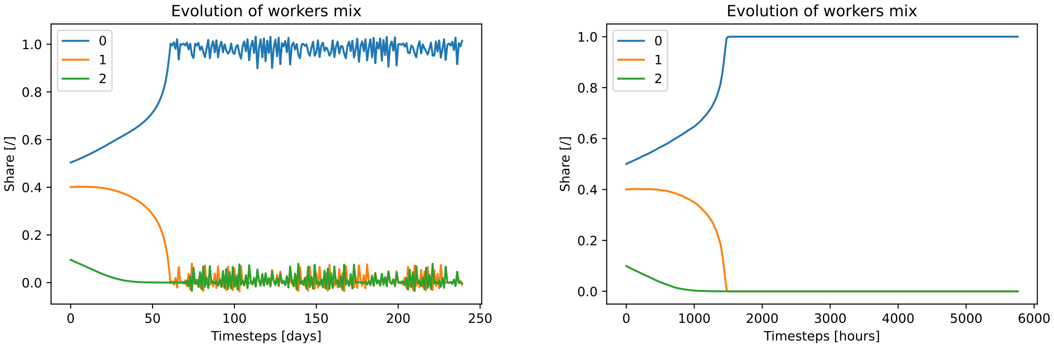


Figure 4.6: Weak amplitude of oscillations around 1/3

Individual preferences interpreted as noise create here almost invisible variations after a transition regime. However, trying with our previous imbalanced condition of $\mathbf{h} = [0.5, 0.4, 0.1]$, the behavior changes radically :



(a) Agglomerations forces win, creating noise, dt = 1

(b) Similar dynamic without instabilities, dt = 1/24

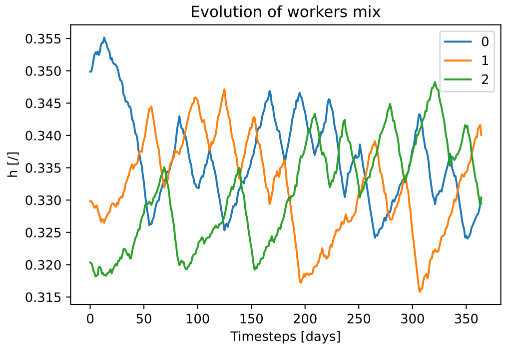
This leads us to think that certain initial states may lead to equilibrium switches, with respect to the initial condition, while others cannot. On the other hand, we observe that the standard value for dt causes numerical instabilities in our implementation. With smaller time steps, the Euler method does not leave the domain while being close to its border. This phenomenon however has no impact on the general outcome of the game, as we simply let the possibility for migrants to move at a certain hour instead of at a certain day. Finally, the possibility that coalitional forces compensate agglomeration depends on different parameters values such as $\Delta\phi$. The goal of the next section is to outline their impact on the game.

4.4 Parameters analysis

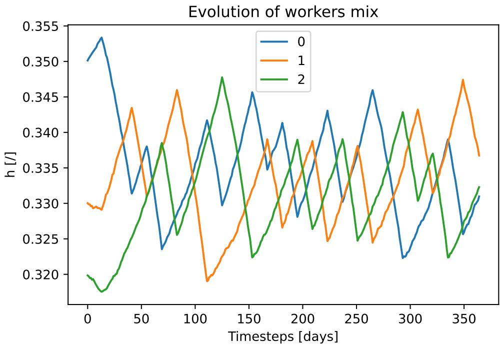
4.4.1 Total population H

We restrain our analysis, when analyzing a set of 3 regions, to a total population of 10^3 to 10^8. Populations with less than some hundreds workers are too exposed to individual preferences, having a huge impact , and no region can be considered to have more than a billion skilled workers empirically.

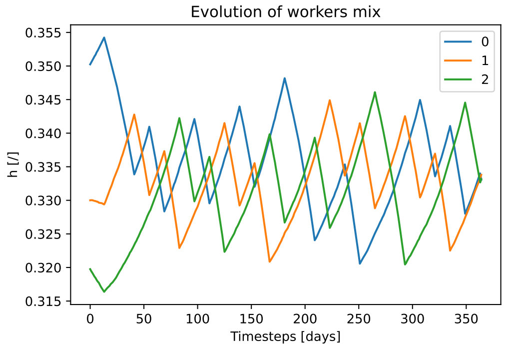
(a) Noise determines the failure of the coalition with $H = 10^3$



(b) Reduced impact, convergence to linear evolution with $H = 10^4$



(c) $H = 10^5$



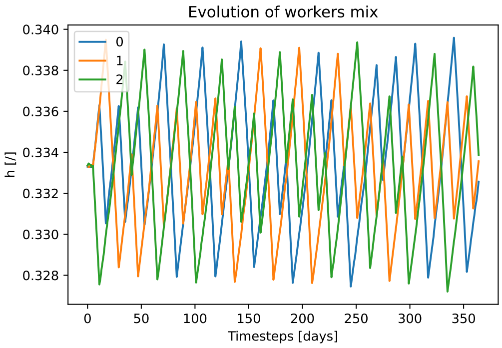
(d) $H = 10^6$

As an increasing H reduces stochastic forces, self-reinforcing noises causing equilibrium switches from the stabilized equilibrium [1/3,1/3,1/3] become less frequent. The deterministic derivatives becomes dominant, and because of a $\Delta\phi$ high enough coupled with a limited difference in share between regions, agglomeration forces are counterbalanced by the differentiated export transportation costs. This is effectively verified here as we see the coalition fails with $H = 10^3$, but succeeds with other values. In practice, the probability to fail can be computed as in section 4.5, but let us assume it is high for the first figure and decreases with higher H. If we were to model situations with higher time frames, we should then take into account small, but not negligible probabilities and would probably conclude that $H = 10^4$ has a reasonable chance of failing at some instant of time as well.

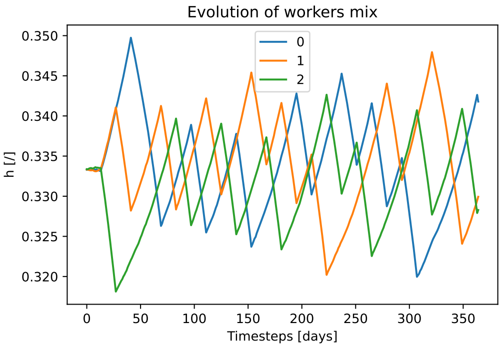
As expected, the dynamic coalition formation creates a cyclical dynamic, as opposing regions change their types in accordance to the evolving shares. Because weaker actors grow together, and that the switching time is non-zero, being able to overcome the most dominant player means that there is some time between the moment where he gets passed by player I, and when the coalition get updated. This means that W can catch up on him during that period. Of course, this depends on this T_{sw} parameter that will be studied as well. Please note that the full defined spectrum of H wasn't analyzed as dynamics do not significantly change for higher values, as the stochastic term is in $H^{-0.5}$. Finally, when the probability of coalition failure is low enough, we say the cyclical dynamic goes on forever. For example, with $t_{End} = 5 \times 365$ being our standard time-frame, we define low enough as $10^{-3} * t_{End}^{-1}$. In words, it means that there is a 0.1% chance that there is one failure in what we consider a large amount of time.

4.4.2 Influence of T_{sw}

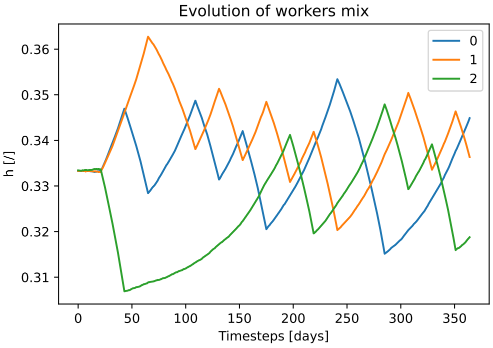
Fixing parameter H to its standard value of 10^6, we focus on the symmetric condition $s_0 = [1/3, 1/3, 1/3]$ as we are particularly interested into the cyclical dynamics changing with T_{sw}:



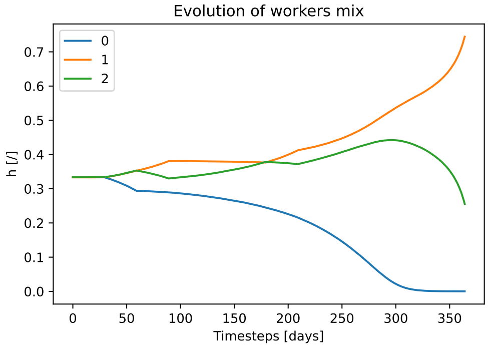
(a) Small oscillations and period $T_{sw} = 6$



(b) Standard behavior $T_{sw} = 14$



(a) Larger period & amplitude $T_{sw} = 22$

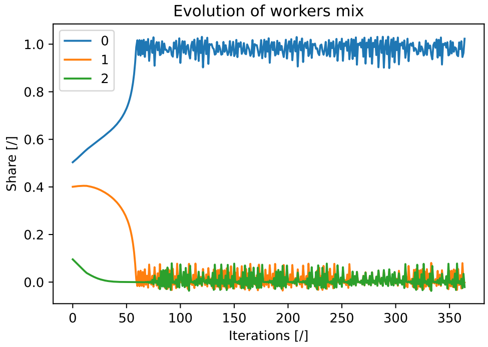


(b) Divergence due to late coalition change $T_{sw} = 30$

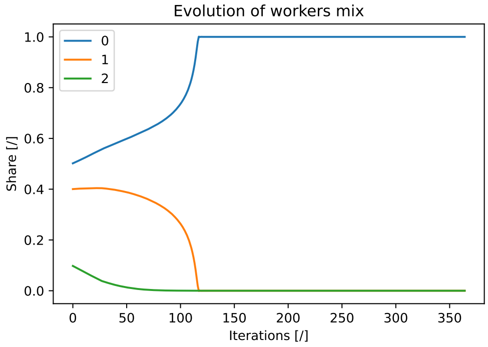
As reactions are slower, agglomeration forces have more time to act and create an edge for regions that cannot be counterbalanced afterwards.

4.4.3 Influence of parameter dt

Because of equation (7), we expect that decreasing dt will lead to slower reactions. While stochastic forces are supposed to have a larger weight, the agglomeration forces are reduced with dt, which means that noise doesn't reinforce as much. Furthermore, we have observed in the previous section that having a *dt* of 1/24 instead of 1 improved the quality of our simulations at the borders of our domain.



(a) Numerical instabilities , dt = 1



(b) Expected behavior, dt = 0.5

Clearly, the reaction speed is linearly related with *dt*, which is confirmed by evaluating the negative powers of 2. This analysis has not been reported for the sake of conciseness. On the other hand, even if a value of $dt = 0.7$ is sufficient to reduce numerical instabilities to almost invisible perturbations, the goal here is to show that we don't need dt to be as small as $1/24$ to solve the problem. However, choosing to express the iterations in hours is considered more elegant than working with parts of days. Again, those issues do not concern the dynamics of the game being played, but rather after that a clear winner has emerged and that all 3 shares at at the border of the domain. Therefore, we do not treat them otherwise than an annoying detail which detracts from the aesthetics of our figure, as our main goal is to describe the game being played.

4.5 When is it relevant to collaborate ?

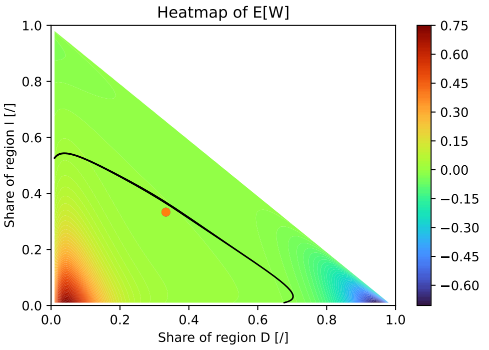
As introduced before the analysis of H, we need to identify in which settings collaborating regions are able to counterbalance the dominant player. This depends from the set of parameters studied previously, but also from the initial conditions. Indeed, there exists settings where the differential equation for the state evolution cannot result in a negative change for player D, and a positive one for the other regions. This results in even higher utility differentials and therefore accelerating agglomeration. Prior to the general case, we observe situations where only the deterministic part of that differential equation cannot oppose agglomeration. Then, it might be possible that noise pushes the system towards a state that does satisfy the previous. Such cases are studied throughout the second subsection.

4.5.1 Deterministic analysis

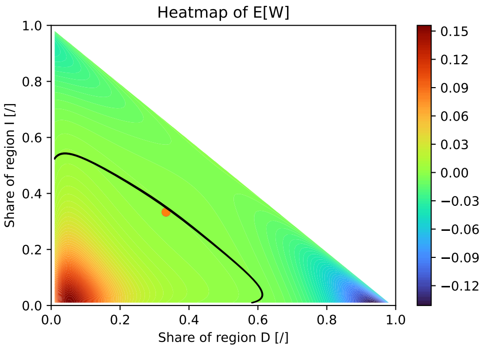
Firstly, we would like to visualize the setting in terms of initial conditions, with fixed parameters. We can evaluate the state evolution in a two-dimensional domain, in function of the share of two regions. This visualization is possible, because we know that the third region is the sum of the two previous.

Setting $\Delta\phi$ at its standard value defined at the start of the chapter, we let I, W be in a coalition against D and intuitively think of evaluating the value of $\mathbf{E}[D]$ and $\mathbf{E}[W]$. We should impose that the first needs to be negative and the second positive in order to have a meaningful coalition. There, an equilibrium switch will occur in favor of player I while being sustainably supported by player W, which also improves his position. However, we know that if $\mathbf{E}[W]$ is positive, so is $\mathbf{E}[I]$ as its share is greater in an identical setting. Therefore, $\mathbf{E}[D]$ needs to be negative as positive $\mathbf{E}$ in a component refer to a higher indirect utility than at least one other region, which is therefore less competitive. This needs to be region D, which experiences a negative $\mathbf{E}[D]$. In short, observing $\mathbf{E}[W] > 0$ is a sufficient condition for the success of the coalition.

We don't bother analyzing T_{sw} as we want to focus on the instantaneous derivative and not time-dependent dynamics. Similarly, determining dt, H isn't interesting as it won't change the sign of the derivative, as dt is a positive constant multiplying it, and we save the stochastic part and considerations such as H for later.



(a) Standard $\Delta\phi$, huge variations and small margin around 1/3



(b) half $\Delta\phi$, smaller variations and margin around the symmetric 1/3.

As expected, the point $s_0 = [1/3, 1/3, 1/3]$, which was highlighted with a red dot, belongs in the negative zone when $\Delta\phi$ is nonzero. The black line stands for a zero derivative, and the graph is not symmetric due to the different ϕ experienced by players I, D. Furthermore, another initial condition which we have tackled previously, $s_0 = [0.5, 0.4, 0.1]$, is in the negative zone, which confirms that with this parameter set coalitions don't trigger equilibrium switches. As expected the negative area grows with a higher $\Delta\phi$.

However, we are surprised at the very high values of $\mathbf{E}[W]$, especially in the first graph. We suspect there is something wrong. Furthermore, even harsher numerical instabilities (for example the black line vanishes) are observed with higher values of $\Delta\phi$. We model the standard $\mathbf{E}[W]$ without coalitions to check.

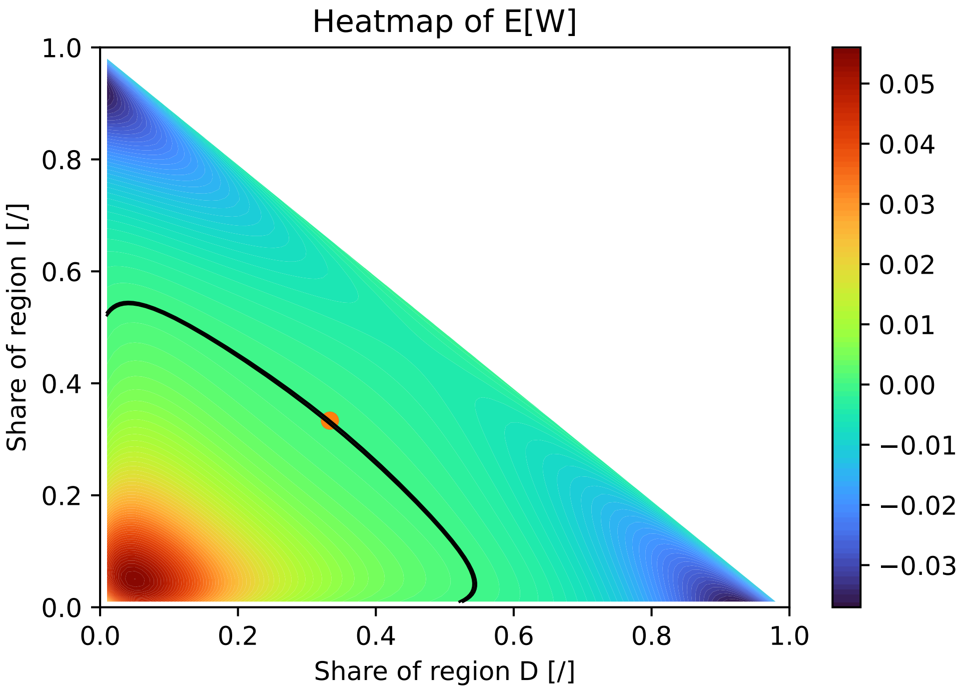


Figure 4.13: No coalitions, a zero derivative coincides with the symmetric distribution of workers

Clearly, those values seem more reasonable for our stochastic differential equation with an explicit Euler method. The previous large values explain our numerical instabilities with $dt = 1$ at the border of our domain, we probably leave the domain by a standard Euler step, which is corrupted by the $\Delta\phi$ we chose ! This means that our assumption at the beginning of the simulations was erroneous, and that in order to gain stability, we need to adopt smaller values of $\Delta\phi$. Indeed, the utility differentials are much greater if the two weakers region don't use the majority of their source of indirect utility, the one opposing them ! This explains the immense negative values for region W. A similar reasoning can be adopted to justify the positive ones.

While the progression seems clear for balanced conditions, unbalanced conditions gain less from a higher $\Delta\phi$. This triggers our curiosity, how can we explain this difference and can we quantify it ?

Firstly, let's intuitively consider an initial condition such as $s_0 = [0.5, 0.49, 0.01]$. There, the indirect utility in region I depends mainly on the share of workers of region D, as a higher share results in more production units brought to the market. Let's picture all of this for this initial condition taking from equation (3) the non constant terms :

$$U_i(h)\ (h_i + \phi_{ji}h_j + \phi_{ki}h_k)$$

Considering the utility in region I, region i is I, j is W and k is D without coalitions, we can say $U_i(h)$ is proportionate to the followings thanks to equation (3.3) :

$$\begin{aligned}
U_i(h)\ (0.49 + \phi_0(0.01 + 0.5) &= 0.781 \\
U_j(h)\ (0.01 + \phi_0(0.49 + 0.5) &= 0.575 \\
U_k(h)\ (0.5 + \phi_0(0.49 + 0.01) &= 0.784
\end{aligned}$$

Creating a coalition of I, W and setting $\Delta\phi$ as standard, we obtain respectively 0.746 , 0.540 and 0.751. This shows that increasing $\Delta\phi$ doesn't help to reduce the utility differential gap. Even worse, the indirect utility in all regions decrease while the gap between with D increases. Actually, the contribution of region W remains too small to have a substantial influence on the other two utilities. This explains the small , but nearly invisible gain for largely imbalanced initial conditions. Furthermore, the indirect utility of region W is in both cases way to small to be competitive with respect to the other two, which means its share will diminish very fast. Shortly, the share of W will further diminish and have an even smaller influence on the game, and we will be back to standard agglomeration forces playing and making D win the game if I hasn't caught up before.

Now, we intuitively understand the phenomenon, but we would like to be be able to quantify it. In order to have a utility higher in region i in the coalition than region k opposing it, one needs to satisfy the following :

$$\phi_{min}(h_j + h_i) + h_k < \phi_{max}h_j + h_i + \phi_{min}h_k$$

Rewriting as to express in function of the shares of W, I with a fixed $\Delta\phi$, and using that $h_k = 1 - h_j - h_i$

$$\begin{aligned}
\phi_{min}(h_j + h_i) + (1 - h_j - h_i) &< \phi_{max}h_j + h_i + \phi_{min}(1 - h_j - h_i) \\
\phi_{min}(h_j + h_i) + (1 - \phi_{min})(1 - h_j - h_i) &< \phi_{max}h_j + h_i \\
\phi_{min}(h_j + h_i) + (\phi_{min} - 1)(h_j + h_i) + 1 - \phi_{min} &< \phi_{max}h_j + h_i \\
(2\phi_{min} - 1)(h_j + h_i) + 1 - \phi_{min} &< \phi_{max}h_j + h_i \\
h_j(2\phi_{min} - 1 - \phi_{max}) + h_i(2\phi_{min} - 2) + 1 - \phi_{min} &< 0
\end{aligned}$$

Defining $C_1 = 2\phi_{min} - 1 - \phi_{max}$, $C_2 = \phi_{min} - 1$, we can rewrite :

$$C_1 h_j < C_2(1 - 2 * h_i)$$

which is a handy relationship equivalent to the visual representation used previously. Quantifying yields C1 = -0.567, C2 = -0.498.

$$\begin{aligned} -0.567h_j &< -0.498(1 - 2 * h_i) \\ 1.138h_j &> (1 - 2 * h_i) \\ 0.138h_j + h_i &> (1 - h_i - h_j) \\ 0.138h_j + h_i &> h_k \end{aligned}$$

This is a very instructive result, as it completes our visual intuition with a quantified objective. By symmetry, we can obtain the same inequality for the other player (if we ask that the derivative of player j is greater than that of player k, which makes the gain due to player j for player i sustainable in order to beat k, not only at one single iteration). In short, we are focusing on the sum of the shares of the collaborating agents compared to one player outside of the coalition. The coalition is then considered to be worth a payoff of 1 in a deterministic context, because it will be able to gain ground on D forever, until it has no more influence on the game or isn't able to trigger equilibrium switches other than total agglomeration in region i, j.

Finally, the methodology developed here is able to deliver very precise policy recommendation to region W. It suffices to simulate the heatmap with a chosen parameter set to decide whether or not W should engage with I in a coalition, or greedily optimize its short term payoff by collaborating with D.

4.5.2 Taking individual preferences into account

After a first deterministic analysis of how collaboration can change equation 3.7, we can now work with noise as well. Firstly, because we are working with white noise, every component of the stochastic term of equation 3.7 is independently random, which means that while studying equilibrium switches caused by flipping signs of the utility differentials, one component of the noise can either exacerbate another or compensate it.

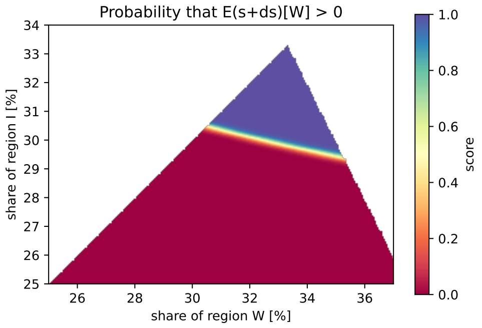
Depending on whether or not noise supports the coalition, the set of initial conditions relevant for the game changes. By supporting, we mean that from a state outside of the negative deterministic derivative zone, noise pushes the system to one inside of it. Alternatively, even if we are inside, noise can drive the state outside of it preventing it to realize the equilibrium switches that should have been possible. For the sake of simplicity, we will not consider consecutive noises that do those kind of operations through multiple iterations. In short, we'll cover the deterministic derivative of the new state obtained after the realization of noise summed with the deterministic derivative at the time of realization.

As our noise components are independent, there is a multitude of different scenarios :

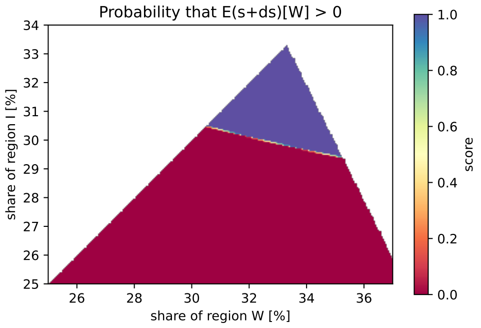
- component W is positive enough to reach the negative zone, while component I doesn't decrease and W doesn't increase.
- component D is negative enough to reach the negative zone, while I, W do not decrease
- ...

Because listing all the possible combinations and their effect at certain thresholds makes no sense, we can resort to simulations to model all the different possible noises and the resulting trajectories. This will in turn validate our previous recommendations or offer new perspectives as to improve them.

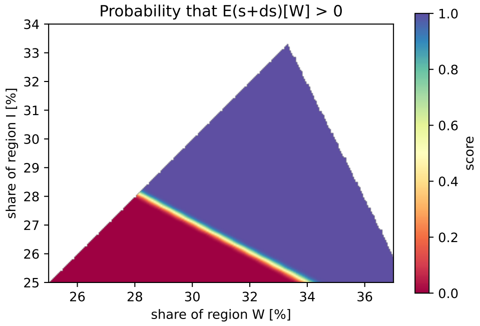
Numerically, 250 white noises were generated and put into equation 3.7, to study when $E(s+ds)[0] > 0$. A score out of 250 was then coined to indicate the probability of a positive derivative for the weaker player. Here are the results, for which we only highlight the subset of states in the domain satisfying that W is the smallest share, followed by I and with D the greatest :



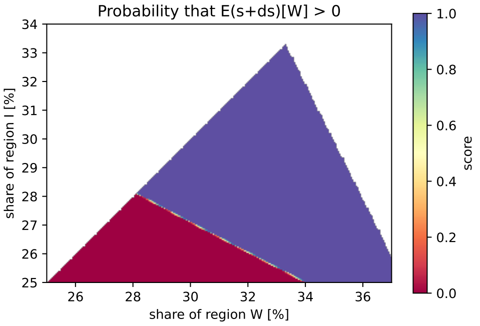
(a) Clear transition zone between zones of 1,0. $H = 10^3$, standard $\Delta\phi$



(b) The transition shrinks with the same $\Delta\phi$, but an H of 10^6

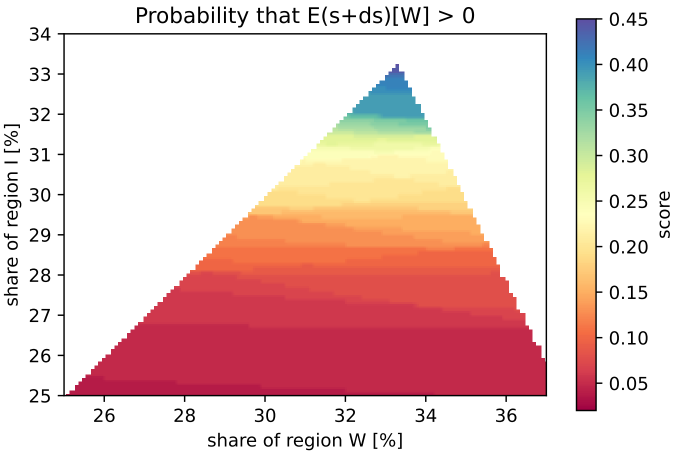


(a) Clear transition zone, greater area favorable to W, $H = 10^3$, $\Delta\phi = 0.471$



(b) Small transition zone with greater area, $H = 10^6$, $\Delta\phi = 0.471$

Therefore, we conclude that an increased $\Delta\phi$ shifts the red zone of a zero probability downwards, which means that there are more situations when collaborating with player I is relevant for player W. On the other hand, increased H decreases the transition zone between a probability of 1 and that of 0. For the sake of better understanding this transition zone, a fully fictitious simulation with $H = 1$ and no coalition was produced.



This allows us to verify that the transition zone is smooth and confirms our understanding. Furthermore, as there is no coalition, at precisely 1/3, the probability of enjoying a positive $\mathbf{E}[W]$ is simply 50%, and decreases with a lower share as expected. Finally, please note that the simulations were conducted with 1000 points for both axes, but that only a slice of the total graph was showed as to zoom into the transition zone.

4.6 Summary and validation of our results

The theoretical results announced by NEG are verified through the simulations. Of course, this is logical as we adopt a NEG framework, however we validate our results with satisfaction. Among others, we observed the following :

1. Decreasing transportation costs leads to a concentration of economic activity
2. The concentration of population in an area leads to an increase in productivity leading to further growth.
3. The trade-off of agglomeration and stochastic forces shapes the evolution of our system
4. Systems with large populations are less exposed to individual preferences
5. Increasing transportation costs decreases the total welfare among the regions
6. Coalitions beats opponents if their total strength (here a weighted sum of shares) exceeds that of their opponents by a certain margin.

Finally, the assumptions we made concerning $\Delta\phi$ are erroneous and cause numerical instabilities. A better understanding of the transportation costs is required to improve the $\Delta\phi$ value. We assume that it is due to the current calibration of other parameters overtaken from Delloye and Tharakan (2018), which did not design their model as to observe even higher utility differentials due to modified export transportation costs.

While there is yet so much to be done, we name some notable areas of improvements for this work :

1. Bargaining between members of a coalition as to redistribute the payoff more equally. This would be done for example by equalizing the indirect utilities of the collaborating regions with differentiated ϕ within the coalition. Even better, favor the weaker one by setting there an indirect utility higher than in the other. This better protects the weaker region against noise and extends the set of states for which he can survive.
2. Calibration of the parameters and in particular $\Delta\phi$ as to stabilize dynamics at the border of the domain
3. Taking into account consecutive realizations of noise and ensuing probabilities of equilibrium switch with repetitive shocks.
4. Further differentiate regions to consistently represent empirical reality

Chapter 5

Conclusion

This study focuses on recommending policies for regions within a dynamic coalition formation framework based on NEG assumption. It contributes to the NEG literature by differentiating transportation costs dynamically by region as to represent diplomatic relations between them, standing for coalition or opposition. The idea to use a game theory framework to tackle this is a novelty , and there is certainly room for further parameter differentiation to produce results closer to empirical observations ; for further model improvement and relevance.

The motivation of this is the preliminary work of Delloye and Tharakan (2018), which makes significant modelling contributions to the footloose entrepreneur model by considering individual preferences and resulting stochastic dynamics with equilibrium switches. Their paper highlights the possibility of policy recommendations in a dynamic context. While discussing with the authors and the supervisor Professor R. Jungers, the need for a game theory framework emerged as to take into account dynamic responses to one's individual policy designs. Optimizing its own payoff without taking into account the rational responses from others will only be interesting in situations where the others are neither intelligent nor rational or where their actions cannot impact the game being played.

While conducting the analysis, several tools and methods have been created or used to better understand the dynamics at hand and propose a relevant poicy recommendation to regions. Firstly, a parameter analysis to gain intuition on their impact on the game. Secondly, a deterministic study of which initial conditions are relevant while playing the game. This was done with the help of heatmaps of the drift coefficient, standing for instanteneous state transitions, which was required to be positive for the members of the coalition and negative for the opposing member. This was followed by a stochastic analysis, which took into account individual preferences of migrating agents, considered as noise. Here, a probabilistic score was given to any initial condition as to evaluate the likeliness of equilibrium switches in a given parameter set.

Both visualizations made it possible to determine in which states the player with the smallest share of workers should team up with who to maximize his expected share over time. He should either collaborate with the dominant player as he is irrevocably pushed to extinction, or fight him off with the third region if he observes a non-zero probability of surviving. This is our policy recommendation.

Other notable observations are that the implicit wages equations do not need to be solved, as their ratio is negligible and that higher transportation costs reduce the total welfare. Finally, we showed that the trade-off of the parameters H and $\Delta\phi$ profoundly affect the transition zone of states between a certainty of success and failure of a coalition involving the two weaker players. In particular, the zone with a certainty of success increased with $\Delta\phi$, while the transition zone became larger by increasing H. This phenomenon is logic as noise then has a more decisive impact on the dynamical system. On the other hand, we showed the degree to which a country might be protectionist extends its possibilities to protect itself from external exposure to more wealthy economies.

Lastly, areas of improvements and further research. Firstly, the addition of bargaining in the game theory modelling, at the coalition formation, should be investigated as to propose more attractive opportunities for the weakest region. Indeed, that region is the one choosing its partner, making the difference and ultimately creating equilibrium switches. Therefore, it should be lured into a coalition by its partner instead of negotiating on equal ground. Furthermore, the set of states relevant for a coalition to emerge without having a player going extinct would grow. Heatmaps show that the margin with a symmetric condition is small and that particular method could improve it. This would make policy recommendation less trivial, but also more fair. As a result, differentiated trade costs would be chosen continuously instead of the binary choice ϕ_{min}, ϕ_{max}. Regions would then extend their set of moves by being able to choose within a range of ϕ, which could trigger new dynamics to be analyzed.

In conclusion, this study took on the challenge of offering policy recommendations for regions in a dynamic coalition formation framework. Therefore, it introduced an innovative approach by adjusting transportation costs based on diplomatic relations between regions. While future enhancements to the model are necessary, this was a first step to capture dynamic responses to policy designs. This study laid the foundation for refining policy recommendation frameworks and advancing our understanding of dynamic economic interactions, in the particular case of evolutionary games with uncertainty applied to the footloose entrepreneur model.

Acknowledgments

I would like to express my gratitude to different people who have contributed to the successful completion of this work.

First and foremost, I extend my appreciation to Professor R. Jungers, my advisor. Every discussion we had provided me with good practices, creative ideas and a critical view of areas for improvements. Despite the initially slow start to the thinking process on my side, Professor Jungers remained patient, supportive and pushed me forward.

In addition to his guidance, I benefited from the expertise of Professors Delloye and Tharakan, who explained their previous work and how I could contribute precisely to the field. Their revision of the draft offered me valuable insights and accelerated the redaction process substantially. I thank particularly Professor Delloye for his time through mails and calls, especially in early April, which boosted and truly kickstarted the work when I was struggling the most.

I am also deeply thankful to my family for their incredible support. The environment in which I live truly boosted my academic journey, and made it so much easier than others. They also, helped me save not months of work, but years of struggle. Additionally, I would like to thank my mother and my grandfather for the revision of the final version.

Furthermore, I acknowledge the benefit derived from the use of ChatGPT-4 and its contribution to refining the language and structure of this thesis occasionally.

Finally, I thank Miguel De Le Court for his informal help. An old friend, who is now pursuing his PhD in Applied Mathematics, Miguel helped me with all my questions in mathematics that I felt too embarrassed to ask others. Additionally, his expertise in python and in particular matplotlib helped me to create some very nice visualizations.

Thank you all for your support and encouragement. I truly enjoyed study at the EPL and conclude with this master thesis. Now, I feel ready to embark on new adventures, confident that the future is bright.

Chapter 6

Appendix

6.1 Notations

6.1.1 Modelling constants

1. $\sigma \in]1, +\inf]$: demand-elasticity
2. α, β s.t. $0 \leq \alpha/\beta \leq 1$: scale parameters of the Gumbell distribution of heterogeneous utilities in the nested-logit in the decision-making process of migration.
3. $\tau \in [1, +\inf]$: an efficiency factor (called iceberg transport cost) describing what needs to leave an origin region to deliver one unit to the destination region. Can also stand for taxes.
4. We define $\phi_0 = \tau^{1-\sigma}$
5. $\mu \in [0,1]$ a constant occuring in the utility function of individuals, necessary to define $\theta = \frac{\mu}{\sigma}$
6. v : base rate parameter : stands for the speed at which an individual enters the Nested Logit cycle of considering migration
7. The jump matrix J : $\begin{pmatrix} -1 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & -1 \end{pmatrix}$, which stands for all infinitesimal state changes, describing the departure region of a worker with -1 and the arrival into another with $+1$.
8. γ : the fixed input requirement in terms of high-skill labour necessary to compute the cost of one production unit

6.1.2 Input values

Those are the values that weren't specifically detailed in [main] and required to be arbitrarily fixed :

1. H,L : total populations of respectively the mobile, high and static low-skilled workers.
2. $\Delta\phi$: the difference between ϕ_{Max} and ϕ_{Min}, representing respectively a collaborative interaction between regions, and an aggressive posture. $\phi_{Mx} = \phi_0$, and ϕ_{min} is obtained by subtracting $Delta\phi$ from ϕ_0.

6.1.3 Variables

This represents changing elements during the dynamical processes.

1. **w** : vector of the 3 local wages
2. **h** : vector of 3 entries describing the allocation of high-skilled workers per region.
3. **s(t)** : state of the system. Equivalent notation than **h** but emphasizes the dynamical aspect.
4. Φ : Matrix of inter-regional transportation costs, with a unit diagonal.

6.1.4 Mathematical notations

1. I_3 denotes the Identity matrix of size 3

6.2 Rewriting equations from Forslid and Ottaviano, 2003

The original equations concern a biregional interaction with unified transportation cost. However, they are the foundation of Deloye and Tharakan (2018) which simply adapted them to a tri-regional model. Using the logic and tools developped there, we are able to adapt the model as to model differentiated trade costs ϕ_{ij}. This stands for a trade cost from region i to region j, or the diplomatic relation of i towards j.

6.2.1 Price equations

Knowing that $n_i = \frac{H_i}{\alpha_{eff}}$ with α_{eff} being an efficiency factor , we can start by observing their equation (10) ; standing for the price observed by the consumer in region j and produced in region i. Because we consider monopolistic competition of Dixit-Stiglitz type (Dixit and Stiglitz, 1977), the conditions for profit maximization yield :

$$p_{ij}(s) = \frac{\tau\beta\sigma}{\sigma - 1}$$
$$p_{ii}(s) = \frac{\beta\sigma}{\sigma - 1}$$

While differentiating trade costs, this τ becomes τ_{ij}, or $\phi_{ij}^{\frac{1}{1-\sigma}}$. Of course, the second equation doesn't contain any τ as there is no transportation losses when staying in the same region. Now, we can go on with their equation (11) :

$$P_i = p_{ii}(s)[n_i + \phi * n_j]^{\frac{1}{1-\sigma}}$$

which represents the CES price in region i, we assume that this price is driven by the supply n_i and the imported good from j. Because it's the price valid in region i, we know it depends from the ressources available in j, which results in that when we use differentiated ϕ, those are with destination i, and with origin corresponding to the n they are associated with. Therefore, and taking into account that we are working with not 2, but 3 regions, we may rewrite :

$$P_i = p_{ii}(s)[n_i + \Phi_{ji}n_j + \Phi_{ki}n_k]^{\frac{1}{1-\sigma}}$$

6.2.2 Total production

Using $x_i = d_{ii}(s) + \tau d_{ij}(s)$ being the total production in location i in the original model, and $d_{ji}(s) = \frac{p_{ji}^{-\sigma}(s)}{P_i^{1-\sigma}}\mu Y_i$ with $i, j = \{1, 2\}$, we are able to obtain the modified version of x_i using previous results and generalizing x_i to three regions :

$$x_i = \frac{\alpha_{eff}(\sigma - 1)}{\beta\sigma}\left(\frac{\mu Y_i}{h_i + \phi_{ji}h_j + \phi_{ki}h_k} + \frac{\mu\phi_{ij}Y_j}{h_j + \phi_{ij}h_i + \phi_{ki}h_k} + \frac{\mu\phi_{ik}Y_k}{h_k + \phi_{jk}h_j + \phi_{ik}h_i}\right)$$

6.2.3 Implicit wages equations

Using $Y_i = L/3 + w_i h_i$ and $w_i = \frac{\beta x_i}{\alpha_{eff}(\sigma-1)}$ we obtain the following implicit equations for w_i :

$$w_i = \frac{\mu}{\sigma}\left(\frac{w_i h_i + L/3}{h_i + \phi_{ji}h_j + \phi_{ki}h_k} + \frac{\phi_{ij}(w_j h_j + L/3)}{h_j + \phi_{ij}h_i + \phi_{ki}h_k} + \frac{\phi_{ik}(w_k h_k + L/3}{h_k + \phi_{jk}h_j + \phi_{ik}h_i}\right)$$

6.3 Extinction criterion

We define that a region is extinct, or doesn't play in the game anymore, if it has no ability to change the outcome of the game. This means that whatever its actions, it will have a negligible probability of provoking an equilibrium switch. By negligible, we mean that it is supposed to be extremely close to 0, but we need to bound the white noise to create a bound for the stochastic term.

We assume that white noise realizations with absolute values of 5 and above can be excluded, as they have a probability of realization of 1.35e-07. Setting this bound allows us define the criterion in function of probabilistic terms : we say that a region is extinct if flipping the sign of the deterministic derivative for any player requires a white noise with an absolute value of 5 on top of its deterministic action. One can argue that for almost symmetric distribution in regions I, D, player W can make a difference with even a very small share, but we showed that it's not rational for him to collaborate with I in such a situation. Therefore, he will not trigger an equilibrium switch and we can discard the case.

Finally, we specify that this concerns firstly player W, followed by player I. Indeed, the weaker share has the highest gap in indirect utility and therefore will firstly converge towards a share of almost zero (noise stills grants some share to extinct players). Once player W is out, another player is doomed to extinction as small imbalances caused by stochastic forces will self-reinforce and lead to an agglomeration equilibrium. Coalition forces won't affect the game since the third player cannot contribute to a meaningfull change in indirect utility. The extinction criterion will not be useful when the probability of success of a coalition is lower or equal than $10^{-3} \times t_{End}$, as mentioned in the simulation subsection covering parameter H.

6.4 Proving $E[W]_{W,D} > E[W]_{W,I}$

As mentioned in the Game Theory section of chapter 2, $\mathbf{E}[W]_{W,D} \geq \mathbf{E}[W]_{W,I}$. This is proved by plotting the difference of those two in a domain defined by the following inequalities representing the types of player 1,2,3 being W,I,D : s1 < s2, s1 < s3 and s2 == 0.

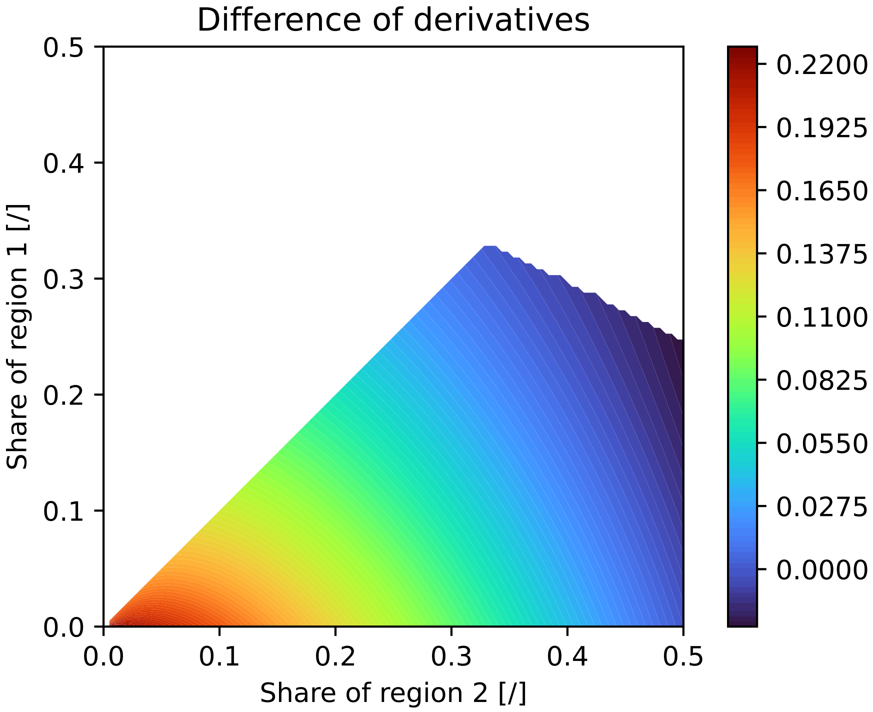


Figure 6.1: The difference is positive, as claimed, within the domain of interest

However, we can add the following constraint : s3 > s2 as we modelled player 3 as the one with the largest share. While we couldn't explain why, a bug occurred when saving the graph, which is why we provided only a screenshot. However, this additional resource allows us to conclude that the negative derivative present at the right part of the previous graph should not be considered, as no negative value appears at the legend. As an additional check, an if-loop was introduced in the code to print for any negative value, but there were none. In other words, in the domain we consider, which is the one defining the configuration of players with roles W, I, D, our claim that $E[W]_{W,D} \geq E[W]_{W,I}$ is always true.

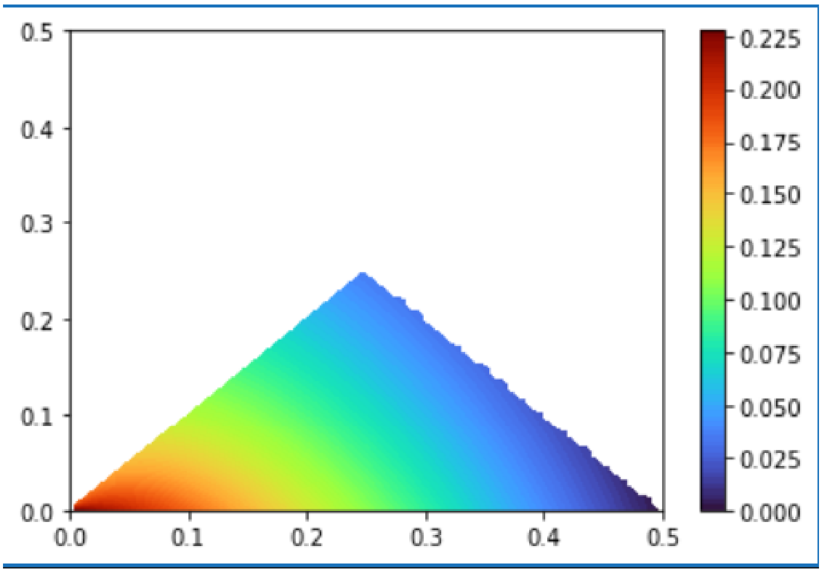


Figure 6.2: This shows the domain of interest, and further proves the claim

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