Appendix 3: Panel data modeling and estimation process

Our final baseline specification has required adopting a sequential approach in order to deal with number of issues arising with panel data analysis. Our methodology process heavily leans on Cameron and Trivedi's (2009) book *Microeconometrics using Stata*, Christopher F. Baum (2013) lectures of *Financial Econometrics* (Boston College) and Torres' (2007) lectures of *Panel data analysis using Stata* (Princeton). We have chosen to use the conventional notation of Cameron and Trivedi's (2009) book throughout our study because there's no clear consensus among researchers for the use of a universal notation. In addition, the different tests and estimations have been performed under the statistical software Stata®. In this section, we will review in order: (a) the 'heterogeneity bias problem', (b) three basic panel models, (c) different model tests, (d) the error/disturbance structure and (e) the estimation issue.

A. Unobserved heterogeneity

As explicitly mentioned by Cameron and Trivedi (2009) 'the goal of a linear regression is to estimate the parameters of the linear conditional mean $E(y|x) = x'\beta = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \cdots + \beta_k x_k$[1]. However, any regression model may suffer from the 'omitted variable bias', meaning that unobserved individual or time-specific factors might influence the regression outcome beyond the defined regressors (Baum, 2013, Woolridge, 2012). Not controlling for this issue, which amount to assume that units and time periods are homogeneous in levels, can involve misspecification in the model and subsequently biased/inconsistent estimates (Baum, 2013). As mentioned by Baltagi (2005), panel data models can appropriately deal with unobserved heterogeneity and capture his net effect.

B. Basic linear panel data models

We consider three basic panel models; two individual-effects models, namely the covariance model and error component model, and a pooled/ population average model. After being modeled, the different methods will be subject to several statistic tests to determine their relevance for our panel data.

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[1] Cameron, A.C., Trivedi, P.K. (2009), Microeconometrics using Stata, Stata Press Publication, p. 80

1. The covariance model or fixed-effect model (FE)[2]

$$\boldsymbol{y_{it}} = \boldsymbol{a_i} + \boldsymbol{x'_{it}\beta} + \boldsymbol{\varepsilon_{it}} \qquad \text{for } i = 1,\dots,\text{N et } t = 1,\dots,\text{T} \qquad (1)$$

Where: $\beta_0$is a constant term, x_{it} is a (kx1) vector of explanatory variables, a_i are random individual specific effects and ε_{it} are idiosyncratic errors term with $\varepsilon_{it} \sim i.i.d\ (0, \sigma^2)$. The model has (N+K) parameters.

The fixed-effects (FE) model is based on the following assumptions:

a) a_i are permitted to be correlated with the regressors x_{it}
b) We assume strict exogeneity, that is $E(\varepsilon_{it}|a_i, x_{it})$=0.

The fixed effect model accounts for time-invariant unobserved features of the cross-sectional units in order to obtain consistent estimates of the marginal effect of the regressors on $E(y_{it}|a_i, x_{it})$ (Cameron and Trivedi, 2009, Torres, 2007). The FE method control for these differences between cross-sectional units by including individual specific intercepts (=a_i) while assuming a constant variance across individuals (Wooldridge, 2012). The regression is estimated with an Ordinary Least Square (OLS) estimator[3].

2. The error component model or random-effect model (RE)

$$\boldsymbol{y_{it}} = \boldsymbol{x'_{it}\beta} + (\boldsymbol{a_i} + \boldsymbol{\varepsilon_{it}}) \qquad \text{for } i = 1,\dots,\text{N et } t = 1,\dots,\text{T} \qquad (2)$$

In the random effect model, it is assumed that the individual specific effects a_i are variables independently distributed of x_{it}. In order to capture this individual heterogeneity, the RE method estimates error variances specific to cross-sectional units (Park, 2011). Consequently, a_i is treated as a component of the composite error term, which is defined as follow: $u_{it} = \alpha_i + \varepsilon_{it}$ where $a_i \sim i.i.d\ (\alpha, \sigma_\alpha^2)$ and $\varepsilon_{it} \sim i.i.d\ (0, \sigma^2)$ (Cameron and Trivedi, 2009).

[2] Due to the incidental parameter problem, we do not specify a two-way fixed effects model with both individual and time specific effects. Indeed, such a specification would increase the number of parameters to be estimated, which subsequently imply a loss in the degree of freedom and thus less efficient parameters estimates.

[3] It can be performed either with the 'Within', 'LSDV' or 'Between' estimation technique.

The difference among units lies now in their individual disturbance term rather than in their specific intercepts (Park, 2011). Assuming that a_i are purely random presumes the following assumptions about the model

a) a_i are uncorrelated with the regressors x_{it}
b) We assume strict exogeneity, that is $E(u_{it}|x_{it}) = 0$

Although the RE model reduces the number of parameters to be estimated (K instead of K+N in FE model), it will produce inconsistent estimates if a_i and x_{it} are correlated because it would imply that explanatory variables are correlated with the error term (Cameron and Trivedi, 2009). So, the main question surrounding individual-effects models is to determine whether these effects are correlated with regressors (or not) rather than knowing if it needs to be imputed to the intercept or variance component (Greene, 2008). The regression is estimated with a General Least Square (GLS) estimator.

3. Pooled or population-averaged (PA) model:

$$\boldsymbol{Y_{it}} = \boldsymbol{\beta_0} + \boldsymbol{x'_{it}}\boldsymbol{\beta} + \boldsymbol{u_{it}} \qquad \text{for } i = 1,\ldots,\text{N et } t = 1,\ldots,\text{T} \qquad (3)$$

The pooled model could be seen as a natural starting point where the data is pooled all together and individual effects averaged out. Indeed, this basic regression does not include any fixed or random effect but assume a common intercept $\boldsymbol{\beta_0}$ for every cross-sectional units and exogenous regressors x_{it}. The composite error equals $u_{it} = (a_i - \alpha + \varepsilon_{it})$ where individual effects $a_i - \alpha$ are centered on zero (=0) and the idiosyncratic error $\varepsilon_{it} \sim i.i.d.(0, \sigma^2)$ (Cameron and Trivedi, 2009)

The regression is estimated either by a pooled OLS or pooled FGLS technique. Like RE estimators, pooled OLS provides consistent parameters estimates of β if we can be certain that the disturbance term u_{it} is uncorrelated with x_{it} (Baum, 2013, Cameron and Trivendi, 2009).

C. Testing for individual specific effects

In order to choose between the different panel models, we first need to test for the presence of unobserved/individual specific effects (=α_i). Fixed effects are tested with a Fischer (F) test while random effects are explored with a Breusch and Pagan's Lagrange Multiplier (LM) test

(Park, 2011). Following Park (2011), the former F-test settles whether fixed effects or simple pooled OLS better fits our panel data whereas the LM test contrast the random effects with pooled OLS.

C.1. Testing for fixed effects (F-test)

It tests for the null hypothesis that all individual intercepts are equal to zero, i.e. $H_0\colon \alpha_i = 0$ in the regression model $y_{it} = a_i + x_{it}'\beta + \varepsilon_{it}$. More specifically, the result is an F-statistic (N-1, NT-N-K) that quantifies by how much the goodness-of-fit has changed (Park, 2011). By default, Stata's FE estimator command `xtreg, fe` includes the F-test for fixed effects.

```
xtreg spreadgerm corspaaa ca ds debt budgetbal ir outdebt gdpgr , fe

F test that all u_i=0:      F(8, 415) =    11.57     Prob > F = 0.0000
```

Here, the p-value is small enough (at <0.01 level) to reject the null hypothesis. So there is a significant fixed effect and the FE model is thus preferred than a Pooled OLS model.

C.2. Testing for random effects (Breusch-Pagan LM test)

It tests for the null hypothesis that all individual specific variance components are zero, i.e. $H_0\colon \alpha_i = 0$ in the regression model $y_{it} = x_{it}'\beta + (a_i + \varepsilon_{it})$. After having run the random effect model, we test for this specification thanks to Stata's command `xttest0`.

```
xtreg spreadgerm corspaaa ca ds debt budgetbal ir outdebt gdpgr, re

xttest0

Breusch and Pagan Lagrangian multiplier test for random effects

        Test:   Var(u) = 0

                              chibar2(01) =    41.77

                           Prob > chibar2 =   0.0000
```

Here, the p-value us small enough (at <0.01 level) to reject the null hypothesis. So, there's a significant random effect and the RE model is preferred than the Pooled OLS model.

C.3. Testing between FE and RE (Hausman Test)

The distinction between the covariance and error component model is crucial in panel data analysis.

From an economic/ point of view, we have to question ourselves about countries' potential unobserved heterogeneity. Inspired by Reinhart (2010) 'timeline of countries creditworthiness and financial turmoil', we believe factors like countries' (i) serial default (=countries who experienced multiple defaults), (ii) domestic debt (iii) serial pattern in the incidence of international assistance programs, (iv) ramp-up in the short-term debt issuance and (v) national banking crises could be included in the individuals' specific effects and possibly be correlated with the regressors. Furthermore, the nature of the sample' cross-sectional units may also influence our model choice. According to Baum (2013), FE model better fits with observations related to a mutually exhaustive set of cross-sections. Like the fifty states in the United-States, our nine countries nearly comprise the entire population of the Eurozone former states.

From an econometric view, RE estimator have the advantage to secure more-efficient coefficient estimates because it saves N-1 degrees of freedom (=or parameters to be estimated) compared to its FE counterpart. Moreover, it offers the ability to estimate coefficients of time-invariant regressors, a characteristic not shared by the within (=FE) estimator (Cameron and Trivedi, 2009). Nevertheless, the RE model might suffer from the over-identifying restriction which assumes that individual-specific effects are independently distributed. If this additional orthogonality is violated, meaning that cross-sectional characteristics are correlated with explanatory variables, the parameters estimated are inconsistent and biased (Podestà, 2002). So, the crucial issue is to test for the existence of such a correlation between the specific error term α_i and the regressors x_{it}. This will be performed thanks to the Hausman test, which assess the appropriateness of the RE estimator[4].

Indeed, it tests for the null hypothesis that individual-specific are random, i.e. $E(\alpha_i + \varepsilon_{it}|x_{it}) = 0$. More specifically, a Hausman test checks if there are no systematic differences between the coefficient estimators of the two models (Baum, 2013). Under the null

[4] The test is performed conditional on the specification of the model.

hypothesis, both estimators are consistent and estimators should display similar results whereas under the alternative one estimator widely differs from the consistent estimator (Cameron and Trivedi, 2009). In light of this, the RE estimator is consistent and more efficient than the FE estimator under H_0 while only FE remain consistent under the alternative.

The `Hausman` command implements the Hausman test as follows:

```
xtreg spreadgerm corspaaa ca ds debt budgetbal ir outdebt gdpgr,fe

est store FE

xtreg spreadgerm corspaaa ca ds debt budgetbal ir outdebt gdpgr,re

est store RE

hausman FE RE


    Test:  Ho:  difference in coefficients not systematic

                  chi2(8) = (b-B)'[(V_b-V_B)^(-1)](b-B)

                          =       47.13

                Prob>chi2 =      0.0000

                (V_b-V_B is not positive definite)
```

Here, the overall statistic $\chi^2(k)$ has a p=0.0016. This leads to reject the null hypothesis for any confidence level. So, the effects are fixed and the regression model should be an individual FE model.

We now shift to some other complications arising from using paned data structures.

D. Model errors structure

D.1. Structure of the disturbance term

Until now, we have assumed that the idiosyncratic errors were generated in a spherical manner and thus satisfied the classical OLS assumptions about homoscedasticity and correlation (c.f.: i.i.d.). So, we have

a) $E(e_{i,t}) = 0$

b) $Var(e_{i,t}) = \sigma^2$

c) $Cov(\,e_{i,t}|e_{j,s}) = 0$ if t≠s or i≠j

This is equivalent to consider that the default variance-covariance matrix (VCE) of the disturbance terms can be written as (Stata Manual):

$$\mathrm{E}(ee') = \Omega_{d\square fault} = \begin{bmatrix} \sigma^2\mathrm{I} & 0 & 0 \\ 0 & \sigma^2\mathrm{I} & 0 \\ 0 & 0 & \sigma^2\mathrm{I} \end{bmatrix}$$

However, panel data structures often violate these standard assumptions about the error process (Podestà, 2002). So, we need to check for the assumptions concerning homoskedasticity, cross-sectional correlation (=contemporaneous correlation) and autocorrelation within units (=serial correlation). This is of primary importance in order to avoid our findings to be statistical artifacts.

Let us start with the diagnostic of the residuals of our individual FE model.

D.2. Testing for heteroskedasticity

In many panel datasets, the variance among cross-sectional units can differ. Among the reasons responsible for this phenomenon, we can quote differences in the scale of the dependent variable between units. In consequence, we will perform a modified Wald test to detect for the existence of groupwise heteroskedasticity in the residuals of our fixed-effect regression. Under the null hypothesis, the variance of the error is the same for all individuals: $\sigma_i^2 = \sigma^2\ \forall i = 1, \ldots, N$. We test this assumption in Stata thanks the user-written routine `xttest3` developed by C. Baum (2001).

```
.xttest3

Modified Wald test for groupwise heteroskedasticity

in cross-sectional time-series FGLS regression model

H0: sigma(i)^2 = sigma^2 for all i

chi2 (9)   =    13981.60

Prob>chi2 =       0.0000
```

Here, the overall statistic $\chi^2(N)$ has a p=0.0000. This leads to strongly reject the null hypothesis for any confidence level. So, a phenomenon of heteroskedascitcity is present.

D.3. Testing for cross-sectional correlation

A second deviation from i.i.d. errors could result from the contemporaneous correlation of errors across units, i.e. $E(e_{it}e_{jt}) \neq 0 \; pour \; i \neq j$[5]. To test for cross-sectional dependence in the error term, we run a Breusch-Pagan LM test. Under the null hypothesis, the residual correlation matrix is an identity matrix of order N, which means that the error terms are not correlated across entities (Baum, 2001). We test this assumption in Stata thanks the user-written routine `xttest2` developed by C. Baum (2001).

```
. xttest2

Correlation matrix of residuals:


         __e1     __e2     __e3     __e4     __e5     __e6     __e7     __e8     __e9

__e1   1.0000

__e2   0.3363   1.0000

__e3   0.4143   0.3018   1.0000

__e4   0.4273  -0.1871  -0.1178   1.0000

__e5   0.3253   0.5059   0.4809  -0.0427   1.0000

__e6  -0.1811   0.1642   0.3839   0.0159   0.4727   1.0000

__e7   0.3569   0.7009   0.5620  -0.3256   0.6610   0.2769   1.0000

__e8   0.5311  -0.0166   0.0415   0.8932   0.2096   0.1167  -0.1386   1.0000

__e9   0.6979  -0.0880   0.1483   0.7328  -0.0016  -0.2429  -0.0606   0.6785   1.0000


Breusch-Pagan LM test of independence: chi2(36) =   284.279, Pr = 0.0000

Based on 48 complete observations over panel units
```

Here, the overall statistic $\chi^2((N(N-1))/2)$ has a p=0.0000. This leads to strongly reject the null hypothesis for any confidence level. So, the errors exhibit cross-sectional correlation.

D.4. *Testing for autocorrelation within units*

According to Torres (2007), serial correlation is responsible for too optimistic standard errors. To check for this complication, we run a Wald test where the null hypothesis assumes no

[5] According to Baltagi (2005), cross-sectional dependance is a complication particularly specific to long panels.

first-order autocorrelation. Should serial correlation be detected, we may replace the individual identity matrices along the diagonal of $\Omega_{default}$ with more general structures to allow for this correlation (Stata Manual, 2014). We test this assumption in Stata using the user-written routine `xtserial`.

```
. xtserial spreadgerm corspaaa ca ds debt budgetbal ir outdebt gdpgr

Wooldridge test for autocorrelation in panel data

H0: no first-order autocorrelation

    F(  1,       8) =    441.355

           Prob > F =      0.0000
```

The P value (<0.01) leads us to strongly reject the null hypothesis and validate the presence of autocorrelation of first order,

i.e. : $\varepsilon_{i,t} = \rho * \varepsilon_{i,t-1} + \eta_{i,t}$; $\eta_{i,t} \sim iid\left(0, \sigma_\eta^2\right)$ where $\eta_{i,t}$ are incoming 'shocks'.

So, our error structure is characterized by panel heteroskedasticity, autocorrelation and contemporaneous correlation (HPAC) [6] .However, controlling for these standard errors complications depends upon the nature of the panel under study.

In short panels (T fixed, N-> ∞), we can use alternative covariance matrix estimators and obtain valid standard errors (Cameron and Trivedi, 2009). White's (1980) robust standard errors and Rogers' (1983) clustered standard errors are the most popular. Besides being heteroskedasticity-consistent like White's robust SE, the cluster option provides the additional feature to control for arbitrary autocorrelation (Hoechle, 2007)[7]. Yet, some conditions are required for the use of these standard errors, notably that the errors are independent across individuals (often assumed in short panels) as well as the respect of the asymptotic in N (N->∞).

[6] The HPAC acronym is taken over from Blackwell (2005).

[7] In Stata, robust and clustered standard errors are respectively obtained by using the options `vce(robust)` and `cluster(id)`, available in for most estimations commands.

In our case, time periods (T=48) are more numerous than the cross-sectional units (N=9). So, our dataset is temporal dominant and can be characterized as a long panel (N is fixed, T->∞). Since T is relatively larger than N, the asymptotics behind the correct functioning of robust and cluster options is now violated. Consequently, long panels cannot rely on these option methods and require putting some structure on the assumed error process, which is not the case in short panels (Cameron and Trivedi, 2009)[8]. This emphasis on richer and more flexible models of the disturbance term is paramount because it will guide us to different preferred methods of estimation.

E. Estimation issue

Since a covariance (=individual fixed effects) model better fits our panel data (see sub-section C), we need to focus on the following estimation methods: (a) feasible generalized least square (FGLS) estimator (b) OLS with panel corrected standard errors (PCSE) estimation and (c) FE (Within, LSDV) estimator (Blackwell, 2005). Nevertheless, the HAPC structure of our disturbance term (see sub-section D) rules out the simple FE estimators, which do have the appropriate options to deal with non-spherical errors. This leaves us with the two large-T-consistent covariance matrix estimators, namely the Parks-Kmenta's (1986) FGLS approach and the Beck-Katz (1995) PCSE method (Hoechle, 2007)[9].

The former uses an application of the GLS estimation that fits panel data models, namely the FGLS estimator. This estimation strategy has the same optimal properties as GLS for panel data but avoids the GLS assumption that specifies the covariance matrix Ω is known (Podestà, 2002). Instead, it uses an estimate of the variance-covariance matrix $\widehat{\Omega}$ to replace Ω in the next formula, which gives us unbiased estimates of β under very general conditions (Stata Manual):

$$\hat{\hat{\beta}} = \left(X'\widehat{\Omega}^{-1}X\right)^{-1}X'\widehat{\Omega}^{-1}y$$

[8] The error structure modeling will offer the possiblity to relax the over-restricted assumption of independence of spatial units (N). According to Hoechle (2007), panel data are likely to exhibit cross-sectional dependencies, especially in a cross-national context (case of our study). So it is of great advice to deal with this complication.

[9] We deliberately decided not to talk about the Driscoll and Kraay estimator, which applies nonparametric corrections for the contemporaneous correlation. Modeling general forms of spatial dependence is of great interest when the cross sectional dimension N of the panel gets large, which is not the case in our study. Thus implementing this estimation method would render our process unnecessarily tricky.

However, Parks-Kmenta's method requires that T is larger than N. Moreover, Beck and Katz (1995) question the performance of FGLS in finite samples and claim that this method tends to produce overconfident standard errors. So, the authors suggest using a classic OLS estimation method with large-T-based standard errors that are corrected for the HPAC complications, namely the PCSE's (Beck and Katz, 1995).

But, Beck and Katz' (1995) argument of overconfidence in standard errors under FGLS needs to be tempered in our case. First, a general remark is that authors were unable to provide analytic formulae for the degree of overconfidence, which oblige us to settle between the two methods based on Monte Carlo experiments. Second, our panel data structure (N=9, T=48) is one of the most favorable cases for Parks-Kmenta: Following Beck and Katz' (1995) table results of Monte Carlo experiments, Parks becomes more efficient than OLS when the average cross-sectional correlation of the residuals rise to 0.50. Calculated from the correlation matrix of residuals (see sub-section D.3 above) this number stands at 0.4289($\cong$ 0.5) in our case. When we combine this finding with our favorable T/N ratio, the results indicate a 13% efficiency gain in favor of the FGLS estimation relative to OLS. This confirms Cameron and Trivedi's (2009) argument about FGLS estimator efficiency in large T datasets

In conclusion, we will use an FGLS estimation method for our regressions model. `Xtgls` command with the adequate options run this estimation process in Stata.