École polytechnique de Louvain

Design of a steam boiler for cashew nut processing

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Abstract

This work is part of the Biostar project that targets small and medium-sized companies in West Africa to help them through the energetic transition, by implementing bioenergy production units from their organic waste. One of these companies is ANASAM, a cashew nut processing company located in Burkina Faso. To generate the steam needed to process the cashew nuts, ANASAM uses a steam boiler heated by a furnace in which empty cashew nut shells are pyrolyzed.
The NITIDAE company that produces the boiler of the ANASAM company would like to reduce the pressure inside the boiler for safety (less risks when empty heating of the boiler), economic (less and/or cheaper material) as well as environmental (shorter heating time) reasons. Therefore the aim of this thesis is to *design a steam boiler for cashew nut processing*.

In the current work, we first modeled the thermodynamic behaviour of the content of the boiler throughout the operational steps of the actual boiler, by using a Python code that we developed to obtain an estimation of the outgoing mass flow rate of the generated steam. Next, we simulated the influence of several parameters of the boiler (quality of the heat exchange, height of the boiler, volume of the boiler, initial water volume inside the boiler) on the maximal pressure that is needed to obtain the same mean outgoing mass flow rate.

Improving the heat exchange between the hot source and the inside of the boiler will decrease the maximal pressure needed to obtain the same outputs and at the same time reduce the material needed as well as the heating time.
While increasing the volume of the boiler decreases the maximal pressure needed and thereby the risks when empty heating, it requires more material to build the boiler and increases the heating time.
Increasing the height of the boiler has a positive impact on all aspects. Yet, the impact is smaller than the impact of improving the heat exchanger and more difficult to implement as it requires the replacement of the boiler.
The initial water volume can be optimized for every boiler design to minimize the maximal pressure needed to obtain the same outputs. Its effect is minimal, but, as it does not present any disadvantage, it is always interesting to ensure that the optimal water volume is used.

We can conclude that, to meet all three concerns raised by NITIDAE, the most interesting parameters of the actual boiler to adapt are the heat exchanger and the initial water volume pumped inside it. In case the boiler has to be replaced or the effects of the improvement of the heat exchange are insufficient, changing the height could add benefit.

Acknowledgements

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Contents

Nomenclature

$\dot{m}_{mean}$	Mean mass flow rate $[kg/s]$
$\dot{m}_{out}$	Outgoing mass flow rate $[kg/s]$
ϵ	Rugosity of the outlet pipe $[m]$
γ	Ratio of specific heats $[/]$
λ	Friction factor $[/]$
$\mathscr{R}$	Universal gas constant $[/]$
μ	Dynamic viscosity $[N.s/m^2]$
μ_{ref}	Reference dynamic viscosity $[N.s/m^2]$
ρ	Density $[kg/m^3]$
ρ_{out}	Density of the gas at the end of the outlet pipe $[kg/m^3]$
σ_i	Principal stress $[Pa]$
σ_{yield}	Yield strength $[Pa]$
A_p	Internal section of the outlet pipe $[m^2]$
C	Cost [€]
c	Speed of sound $[m/s]$
$c_{v,av}$	Specific calorific capacity at constant volume, averaged over time $[J/kg.K]$
D	Internal diameter of the boiler $[m]$
D_s	External diameter of the smoke tubes $[m]$
D_p	External diameter of the outlet pipe $[m]$
$f(M)$	Critical length function at Mach number M $[/]$
F_p	Partial closure factor $[/]$
F_{heat}	Heat flux factor $[/]$
F_{yield}	Temperature depending factor multiplying the yield strength $[/]$

g	Acceleration due to gravity $[m/s^2]$
H	Height of the boiler $[m]$
h	Specific enthalpy $[J/kg]$
$h_{exhaust}$	Convective heat transfer coefficient of the exhaust gases $[W/m^2.K]$
h_{gas}	Convective heat transfer coefficient of gas $[W/m^2.K]$
H_{water}	Water height inside the boiler $[m]$
k_{steel}	Conductive heat transfer coefficient of steel $[W/m.K]$
L_{eq}	Equivalent pipe length $[m]$
M	Mach number $[/]$
m	Mass $[kg]$
M_m	Molar mass $[kg/mol]$
P	Heat flux from the exhaust gases to the inside of the boiler $[W]$
p	Pressure $[Pa]$
p_0	Reservoir pressure $[Pa]$
P_{av}	Heat flux from the exhaust gases to the inside of the boiler, averaged over time $[W]$
$p_{max,yield}$	Maximal pressure to avoid yielding $[Pa]$
p_{water}	Hydrostatic pressure $[Pa]$
Q	Heat transferred from the exhaust gases to the inside of the boiler $[J]$
r	Radius of the boiler $[m]$
R_{cond}	Conductive thermal resistance $[K/W]$
R_{conv}	Convective thermal resistance $[K/W]$
$r_{smoke,ext}$	External radius of the smoke tubes $[m]$
$r_{smoke,int}$	Internal radius of the smoke tubes $[m]$
R_{th}	Thermal resistance $[K/W]$
Re_D	Reynolds number relative to the diameter of the outlet pipe $[/]$
S	Sutherland constant $[K]$
SF	Safety factor $[/]$
T	Temperature $[K]$

t	Thickness of the walls of the boiler $[m]$
T_0	Reservoir temperature $[K]$
t_p	Thickness of the wall of the outlet pipe $[m]$
t_s	Thickness of the wall of the smoke tubes $[m]$
$T_{boiler,av}$	Temperature of the inside of the boiler, averaged over time $[K]$
T_{boiler}	Temperature of the inside of the boiler $[K]$
$T_{exhaust,in}$	Temperature of the exhaust gases at the inlet of the smoke tubes $[K]$
$T_{exhaust,out}$	Temperature of the exhaust gases at the outlet of the smoke tubes $[K]$
$T_{f,heat}$	Temperature inside the boiler at the end of the closed boiler heating $[K]$
t_{heat}	Heating time before opening of the valve $[s]$
$T_{i,heat}$	Temperature inside the boiler at the start of the closed boiler heating $[K]$
T_{ref}	Reference temperature $[K]$
U	Internal energy $[J]$
u	Speed $[m/s]$
u_{out}	Speed of the outgoing gas at the end of the outlet pipe $[m/s]$
V	Volume $[m^3]$
V_{boiler}	Volume of the boiler $[m^3]$
V_{mat}	Volume of material $[m^3]$
$V_{water,min}$	Minimal initial volume of water $[m^3]$
$V_{water,opt}$	Initial water volume that reduces the initial pressure in the boiler $[m^3]$
V_{water}	Initial water volume pumped into the boiler $[m^3]$
X	Specific cost $[€/kg]$
X_v	Volumic cost $[€/m^3]$
OS	Ordinary steel
SS	Stainless steel

Chapter 1

Introduction

1.1 ANASAM : cashew nut processing

Cashew nuts are the seeds of the cashew apple and grow underneath the fruit (Figure 1.1a). These cashew apples grow on the cashew tree (Figure 1.1b) in tropical countries such as India, Brazil or Burkina Faso. The cashew nut is covered by a hard shell that needs to be removed manually to avoid damage to the fragile nut (Figure 1.1c). To facilitate the shelling, the raw cashew nuts are humidified with hot and pressurized steam to weaken their shells. After the nuts are removed from their shells, the skin is peeled off (Figure 1.1c) before the final product can be roasted and eaten.

(a)

(b)

(c)

Figure 1.1: The cashew nuts (green) grow underneath the cashew apples (yellow) (a) that are carried by the cashew tree (b)[17]. The cashew nut is covered by a very hard shell and a thin skin (c)[10].

ANASAM is a cashew nut processing company based in Burkina Faso. The raw nuts are purchased from local farmers, to be processed to the finished product by ANASAM. The different steps of this process take place on different locations on the ANASAM site as represented in the flow chart in Figure 1.2 and discussed below:

1. A pyrolysis furnace coupled to a steam boiler and a cooker. The furnace produces heat by pyrolysis of empty cashew nut shells. This heat is transferred to the boiler

that will produce the steam used to humidify the cashew nuts in order to weaken their shells in the cooker.

2. A hangar where the nuts are manually shelled after being cooled.

3. A hangar where the empty shells are stored before being used in the pyrolysis furnace or being sent off-site.

4. A room where the shelled nuts are humidified thanks to steam produced by the boiler and then dried to loosen the skin around them.

5. A room where the skin of the dry nuts is manually removed.

6. A room where the peeled nuts are stored.

7. A room used for sorting and packaging the finished product before being distributed for sale.

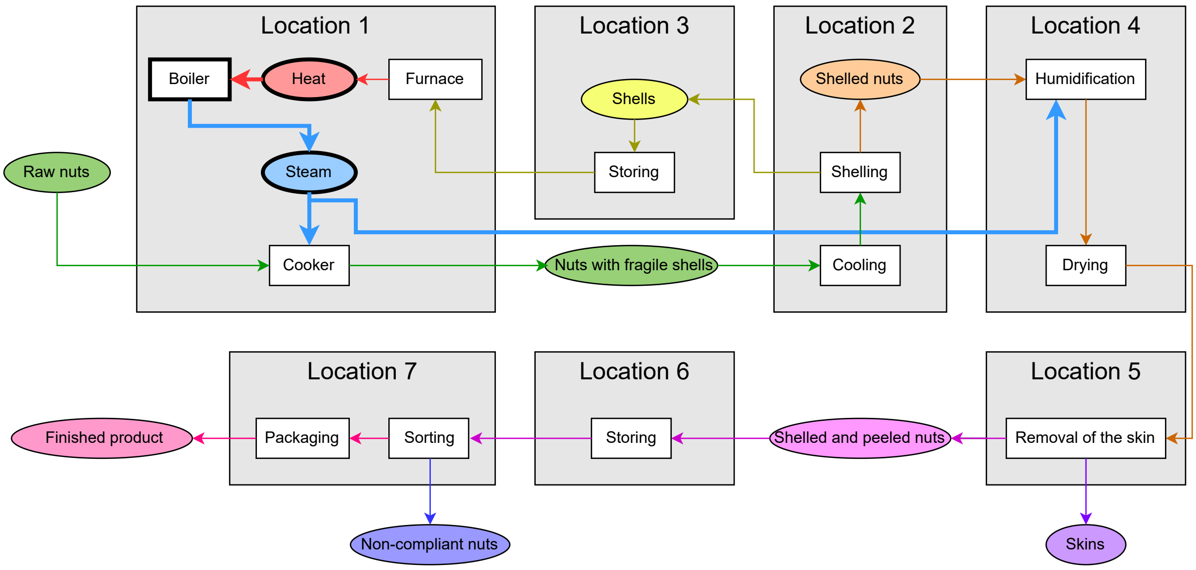


Figure 1.2: Flow chart of the ANASAM cashew nut processing chain. The raw nuts go into the cooker where their shells are weakened. Next, they are cooled, shelled, humidified and dried before their skin is removed. After that, they are stored, sorted and packaged before being distributed. The cooker and humidifier are supplied with steam that is generated by the boiler which is supplied in heat by the furnace burning empty cashew nut shells. The bold arrows indicate the process that is relevant for this work.

1.2 Objective and incentives

The NITIDAE company, which is the producer of the ANASAM boiler, wants to optimize the boiler by reducing the maximal pressure inside it. Therefore, this work focuses on the boiler from location 1 discussed in Section 1.1.

To produce a sufficient steam mass flow, this boiler operates at a maximal pressure of 11 *bar*. The aim of this thesis is to reduce this maximal pressure while keeping the same boiler performance, i.e. producing sufficient steam to allow the cooker to weaken the cashew nut shells and to allow the humidifier to loosen the cashew nut skins.

To achieve this objective, we will proceed step wise:

1. In Chapter 2, the operation of the actual boiler will be numerically modeled using a Python code that we developed. The thermodynamic quantities of the inside of the boiler will be modeled for each operational step and compared to some measures taken on the actual boiler.

2. In Chapter 3, we will use the model generated in Chapter 2, to study the influence of several parameters of the boiler (e.g. its volume or the quality of the heat transfer) on the pressure. These results will be analyzed and discussed in relation to the motives the NITIDAE company has to decrease the pressure inside the boiler:

   - they want to reduce the safety risks when the operator forgets to pump water into the boiler and the boiler is empty heated;
   - they want to reduce the thickness and quality requirements of the material as well as the frequency by which the pieces of the boiler have to be replaced due to wear, and thus reduce the cost of the boiler;
   - they want to reduce the heating time to reduce the burned amount of fuel and reduce their environmental impact.

   These incentives will be further explained and analyzed in Section 2.12, when the boiler has been modeled.

Chapter 2

Model of the actual boiler operation

In this chapter, the actual boiler operation will be modeled. To do so, we will start by describing the geometry of the boiler. Next, we will describe its general operation and expose the measurements taken on it. Finally, we will model the thermodynamic behaviour of the inside of the boiler throughout each step of its operation by using a Python code that we developed, and some theoretical and numerical tools that will be explained throughout the chapter.

The desired outcome of the model is to obtain an estimation of the evolution of the outgoing mass flow rate during the different operational steps of the boiler, in order to reproduce the function of the actual boiler in the new boiler that will be designed in Chapter 3.

This model is not implemented to be as precise as possible, as a lot of strong assumptions will be made by lack of data and by simplicity. Yet, this is not considered to be an issue as the actual boiler operation is not an exact procedure, and the output can vary from one operational cycle to another.

2.1 Geometry of the boiler

As illustrated in Figure 2.1a, the boiler is an ordinary steel cylindrical vessel with $10mm$ (t) thick walls. It has a volume of $0.8m^3$ (V_{boiler}), a diameter of $0.84m$ (D) and a height of $1.43m$ (H) and is insulated by a $10cm$ thick layer of glass wool.
The boiler is crossed vertically by 18 smoke tubes that have walls with a thickness of $4.45mm$ (t_s) and an external diameter of $8.89cm$ (D_s) (Figure 2.1b).
At the bottom of the boiler, there is an inlet pipe through which the fresh water is pumped inside the boiler. Its dimensions are unknown but are not relevant in this work.
At the top of the boiler there is an outlet pipe through which the outgoing steam and air mass flow leaves the boiler to the cooker and the humidifier. The outlet pipe is also made of steel, has an external diameter of $33mm$ (D_p) and its walls have a a thickness of $2.5mm(t_p)$ (Figure 2.1c). By supposing middle-quality steel, the rugosity of this kind of pipes is $\approx 1mm$ [13].

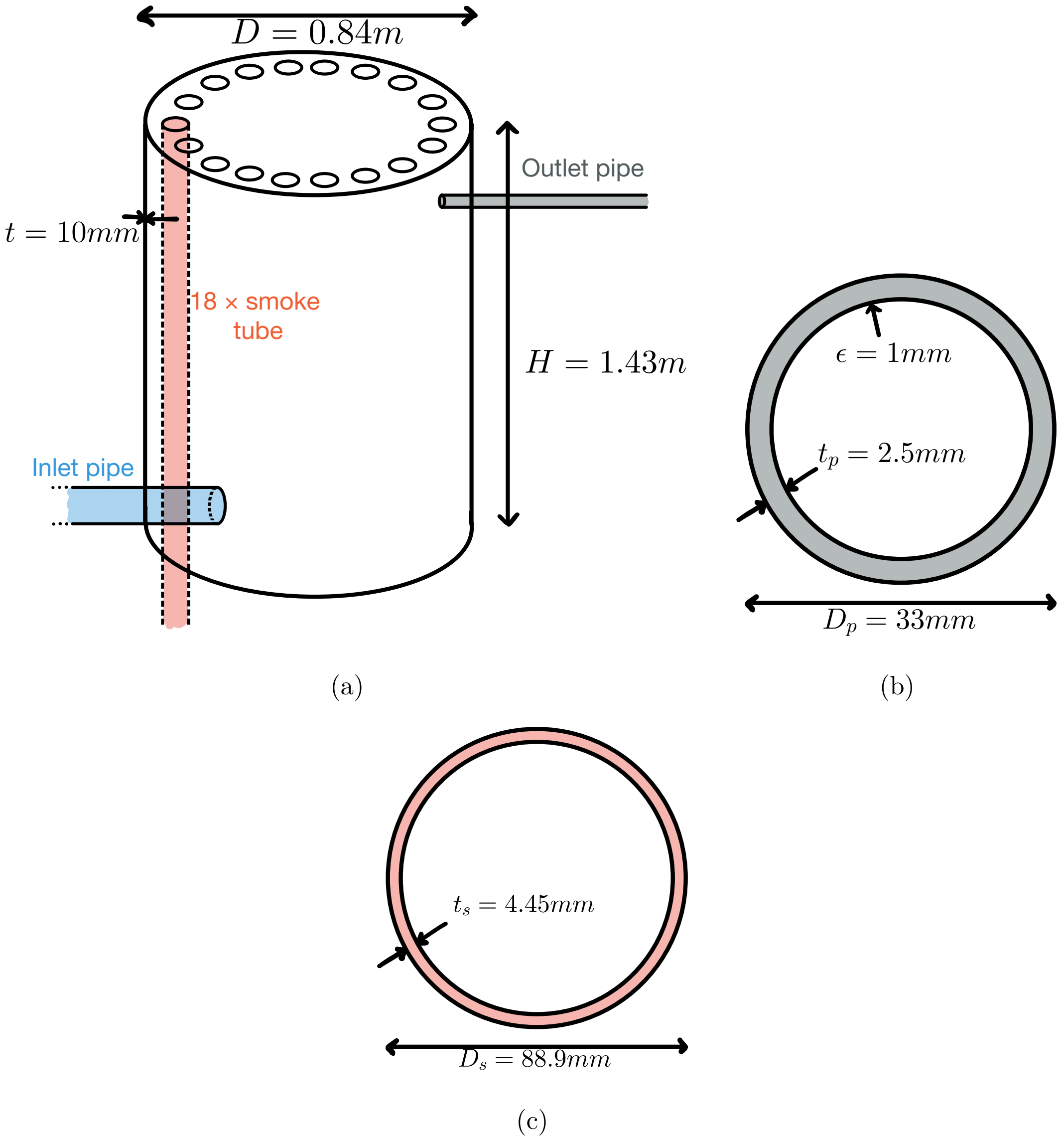


Figure 2.1: Geometry and dimensions of the boiler (a), the smoke tubes (b) and the outlet pipe (c).

2.2 General operation of the boiler

The general operation of the boiler can be summarized in four steps that are represented in Figure 2.2.

Step 0: Inlet of the water

$0.5m^3$ of water at $26°C$ and at atmospheric pressure, are pumped into the boiler. At the end of this step, the inside of the boiler is at atmospheric pressure and at $26°C$, we suppose that the gas phase above the water is composed of air only.

Step 1: Closed tank heating

Once the $0.5m^3$ of water are pumped into the boiler, the heating of the inside of the boiler is started.

Under the boiler, a furnace ensures the pyrolysis of empty cashew nut shells. This pyrolysis produces charcoal and gases, that will undergo combustion and thereby produce hot exhaust gases. This furnace is continuously supplied in shells during the whole process -even during the next steps- to keep the heat generation as constant as possible.

The heat is transferred to the inside of the boiler thanks to 18 smoke tubes connected to the top of the furnace and passing vertically through the boiler (Step 1 of Figure 2.2) allowing the exhaust gases to leave the furnace through the boiler. At the inlet of the tubes, the smoke has an average temperature of approximately $832°C$ and at the outlet of the smoke tubes, it has an average temperature of $\approx 742°C$ [2].

Step 2: Opening of the valve

When the desired pressure of $11bar$ is reached inside the boiler, the operator opens the outlet valve.

A flow composed of steam and some atmospheric air, flows through the outlet pipe to the cooker to fragilize the shells and to the humidifier to humidify the cashew nuts (blue arrows in Figure 1.2). The most important function of the flow is the weakening of the cashew nut shells in the cooker that will allow the easy extraction of the cashew nuts from their shells. In the actual process, this step requires a flow during 18 minutes.

Step 3: Partial closure of the valve

After 15 minutes, the operator partially closes the outlet valve. As a consequence, the mass flow rate leaving the boiler through the outlet pipe is decreased. The decrease in mass flow rate will be quantified later (see Section 2.6). The aim of this partial closure is to reduce the amount of steam injected in the cooker when the cashew nut shells are considered "almost ready".

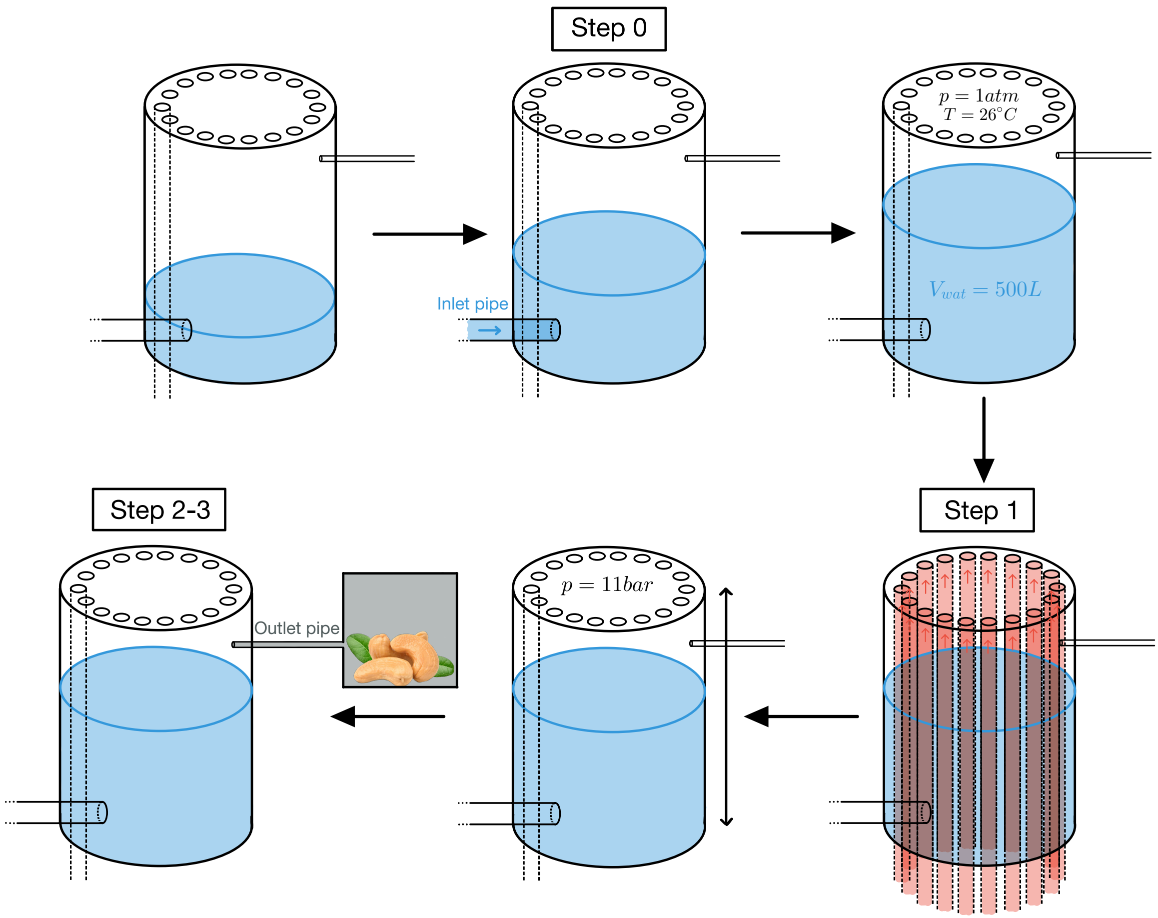
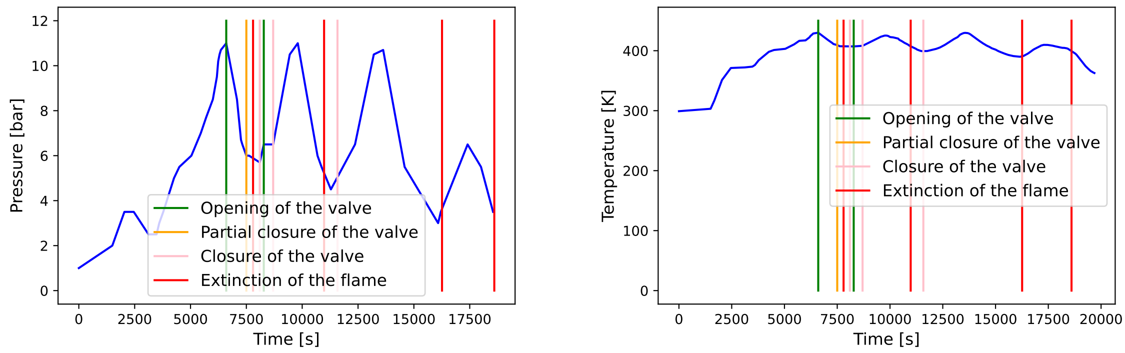


Figure 2.2: Schematic representation of the general operation of the boiler. After pumping $0.5m^3$ of water inside the boiler (step 0), the inside of the boiler is heated until it reaches a pressure of $11bar$ (step 1). Next, the outlet valve is opened and the mixture of steam and air flows onto the cashew nuts (steps 2 and 3).

2.3 Measures

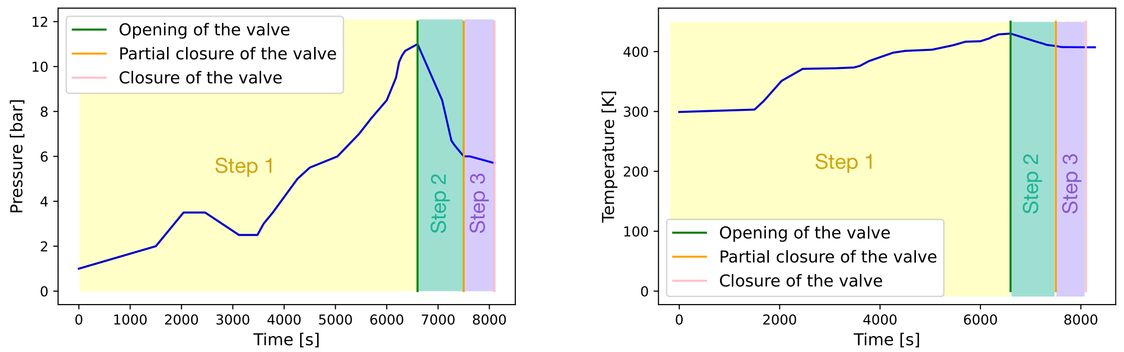
Some measures have been taken on the boiler and sent to us by the NITIDAE company. The measures of the temperature and the pressure in the boiler will be used to compare and verify the accuracy of our model, and are plotted in Figure 2.3.



(a) (b)

Figure 2.3: Measured pressure (a) and temperature (b) inside the boiler as a function of time.

We can see that the graphs present a lot of cyclic variations, caused by the different operational steps of the boiler. As one cycle represents the entire process described in Figure 2.2, from now on, we can consider only one cycle that can be repeated as many times as required. This first cycle is shown in Figure 2.4.



(a) (b)

Figure 2.4: The measured pressure (a) and temperature (b) as a function of time during the first operational cycle of the boiler. Before the opening of the valve, we can see the closed boiler heating (step 1) which implies an increase in pressure and temperature. After the opening of the valve (step 2), a flow composed of steam and atmospheric air flows out of the boiler, generating a decrease in the pressure and in the temperature inside the boiler. When the valve is partially closed (step 3), the outgoing mass flow rate is reduced and so is the decrease in pressure and in temperature.

The graphs in Figure 2.4 visualize the three main operational steps of the boiler.
First, before opening the valve, we have the closed boiler heating. The volume of the boiler being constant, the temperature and the pressure inside the boiler increase.
After opening the valve, when steam and atmospheric air flow out of the boiler, the pressure and the temperature of the inside of the boiler decrease. At a certain time, the operator partially closes the valve, which will reduce the outgoing mass flow. During this step, the decrease in temperature and in pressure of the inside of the boiler is also reduced.
Now that these three steps are decribed we can start modelling them. Step 0, discussed in Section 2.2, will not be modeled as it has no influence on the evolution of the thermodynamic quantities in the boiler.

2.4 Step 1 : Closed boiler heating

In this section, the evolution of the thermodynamic quantities of the inside of the boiler throughout step 1 are modeled. During step 1, the boiler is kept closed and heated from the initial conditions ($26°C$ and atmospheric pressure) until it reaches a pressure of $11bar$.

First, to allow us to implement a model, some assumptions have to be made. Next, the average heat flux from the exhaust gases to the inside of the boiler is estimated from the measures of Section 2.3 and the model of step 1 is obtained. Finally, the two major corrections that led us to the final model are justified.

2.4.1 Assumptions

To compute the heat flux going from the exhaust gases of the furnace in the 18 smoke tubes to the inside of the boiler, we need to make some assumptions :

- The heat flux is uniformly distributed in the volume of the boiler. In reality, the heat flux is distributed over the surface of the smoke tubes.
- There is no heat loss through the boiler's walls. This assumption can be made because the boiler is insulated by a $10cm$ thick layer of glass wool.

2.4.2 Estimation of the heat flux

To compute the average heat flux during the closed boiler heating, we use the measured initial and final conditions of the inside of the boiler as well as the time needed until the final conditions are reached ($t_{heat} = 1h50 = 6600s$).
The inside of the boiler is composed of water, steam and air. The air cannot be neglected as discussed in Section 2.4.3.
The inside of the boiler is not a perfect gas, meaning that the calorific capacity at constant volume (c_v) varies with the temperature and thus changes during the heating. To take this variation into account, an average calorific capacity ($c_{v,av}$) is calculated by computing the c_v for each intermediate temperature and taking the average of all these values.
Finally we compute:

$$Q = c_{v,av} \times m \times (T_{f,heat} - T_{i,heat}) \tag{2.1}$$

$$P_{av} = \frac{Q_{av}}{t_{heat}} \tag{2.2}$$

$$= 42.65kW \tag{2.3}$$

where

- Q is the total heat transferred from the exhaust gases of the furnace to the inside of the boiler;
- m is the mass of the content of the boiler;
- $T_{i,heat}$ is the temperature of the inside of the boiler at the start of the closed boiler heating;

- $T_{f,heat}$ is the temperature of the inside of the boiler at the end of the closed boiler heating;
- P_{av} is the average heat flux going from the exhaust gases to the inside of the boiler during the closed boiler heating.

We use this average heat flux to compute the evolution of the thermodynamic state of the inside of the boiler as a function of time in Figure 2.5. The closed boiler heat addition in the model is stopped when the inside of the boiler reaches a pressure of $11bar$, the measured pressure at the end of the heating step (Figure 2.4a). The decision to adapt the model to the measured pressure and not to the temperature is justified in Section 2.4.3.

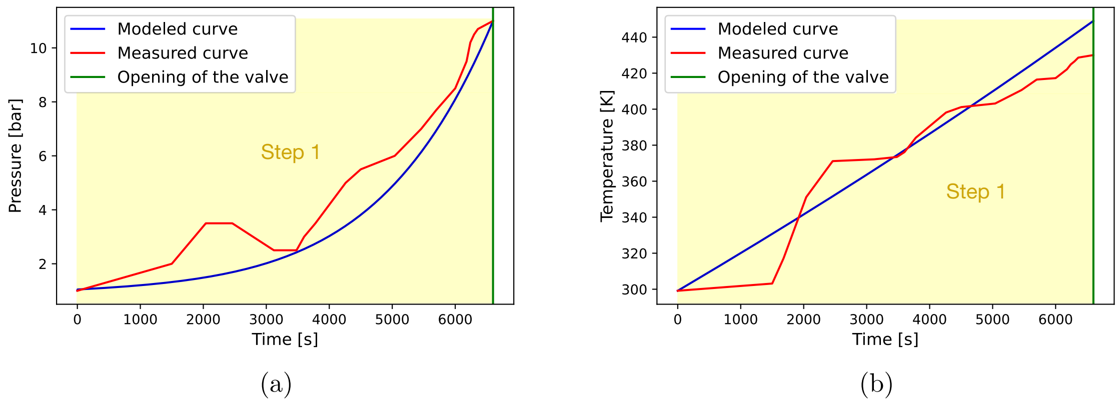


Figure 2.5: Comparisons of the modeled with the measured pressure (a) and temperature (b) inside the boiler, as a function of time during the closed boiler heating.

We can see that the modeled pressure follows a curve similar to the one of the measured pressure (Figure 2.5a). The difference between the two curves can be explained by the assumption of a constant heat flux. In reality, the temperature of the exhaust gases is not constant and the heat flux varies over time.

The modeled temperature curve is higher than the measured curve (Figure 2.5b). This can be explained by the fact that the temperature measures are taken at the inlet of the outlet pipe and thus, close to the boiler wall. Indeed, the modeled temperature is assumed to be uniform inside the boiler, while the actual temperature is higher in the vicinity of the heat source and lower close to the boiler walls.

2.4.3 Corrections

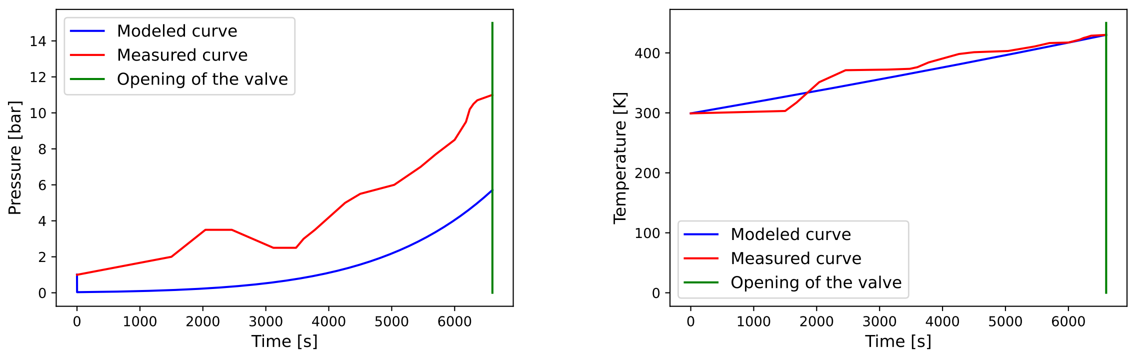
In this section, the two major corrections that are made on the model of the closed boiler heating are justified. At first, we modeled the closed boiler heating by neglecting the air inside the boiler and by adapting the end of the heating to the measured temperature. The obtained model not being accurate, we took the air in boiler into account but still adapted to the measured temperature. The model still being inaccurate, we decided to adapt to the measured pressure instead of the measured temperature. These two corrections are detailed in the two sections below.

Air in the boiler

During step 0 of the operation of the boiler, the entire boiler volume is not filled with water. Only $0.5m^3$ of water are initially pumped in the $0.8m^3$ tank. This leaves $0.3m^3$ of air in the tank.

At first, the effects of the air present in the boiler were neglected. This is equivalent to assuming that the volume that is above the water right after the water has been pumped into the boiler and before the heating is started, is void.

In this case, by stopping the closed boiler heating step in the simulation when the simulated temperature reaches the maximum measured temperature, the graphs of Figure 2.6 are obtained.



(a) (b)

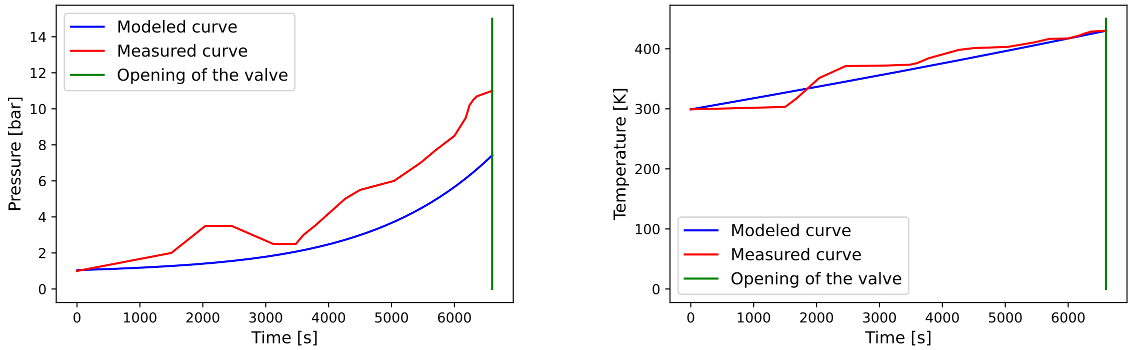
Figure 2.6: Comparisons between the measured and modeled pressure (a) and temperature (b) inside the boiler when the air in the boiler is neglected, and when we match the modeled temperature to the measured temperature.

In these graphs, we see that when we match the temperature of the actual measurements, the computed pressure is far under the measured pressure.
A possible reason for this difference is the partial pressure of air in the boiler that has to be taken into account. This is done by making the following changes:

- The total pressure becomes the sum of the partial pressures of air and of steam.
- The outgoing mass flow is composed of a fraction of air and a fraction of steam equivalent to their volumic fractions.
- The specific enthalpy, specific internal energy and calorific capacities are the massic weighted sum of these values for the air and for the steam.

Adaptation to the pressure

After adding the effects of the air on the conditions in the boiler, the graphs of Figure 2.7 are obtained.



(a) (b)

Figure 2.7: Comparisons between the measured and modeled pressure (a) and temperature (b) inside the boiler when the air is not neglected, and when we match the modeled temperature to the measured temperature.

The maximum modeled pressure is still far under the measured pressure (Figure 2.7a). This can be explained by the position of the temperature measuring device. Even if the simulation supposes a uniform temperature inside the boiler, it is not the case because the heat is not added uniformly in the volume and there are heat losses at the walls of the tank.
However, if we neglect the hydrostatic pressure due to the water level, the pressure is constant in the tank. In our case, the pressure measures are taken in the gas phase of the boiler, so the hydrostatic pressure has no effect on the measures. That is why, in the simulation, it is a better idea to stop the heating when we obtain a computed pressure equal to the maximal measured pressure. By doing so, the final plots of Figure 2.5 are obtained.

2.5 Step 2 : Opening of the valve

In this section, the evolution of the thermodynamic quantities inside the boiler throughout operational step 2 of the boiler is modeled.
At the end of step 1, when the inside of the boiler reaches a pressure of $11bar$, the outlet valve is opened and a mixture of steam and atmospheric air flows out of the boiler.

To allows us to implement this model, we first have to make some assumptions. Next, the acquisition of the state at each instant from the state at the previous instant is computed, and the model is explained by the used algorithm. The theoretical and numerical tools used in this section, will be detailed in the following sections.

2.5.1 Assumptions

To model the thermodynamic behaviour of the inside of the boiler after the opening of the valve, the following assumptions are made:

- atmospheric air behaves as a perfect gas;
- the outgoing mass flow, composed of atmospheric air and steam, behaves as a perfect gas;

- a new equilibrium is obtained instantly at every perturbation of the old equilibrium.

2.5.2 New equilibrium

When the boiler reaches a pressure of $11bar$ thanks to the closed boiler heating (Section 2.4), the valve is opened by the operator. At each instant, a certain amount of gas flows out of the boiler. From the conservation of mass, the outgoing mass flow ($\dot{m}_{out}$) is the same at the inlet of the outlet pipe and at the outlet, and can be computed with:

$$\dot{m}_{out} = u_{out} \times \rho_{out} \times A_p. \tag{2.4}$$

The outlet conditions u_{out} (speed of the gas at the end of the outlet pipe) and ρ_{out} (density of the gas at the end of the outlet pipe) are computed with the Fanno flow theory that will be detailed in Section 2.7.3, and A_p is the area of the section of the outlet pipe.
To obtain the new conditions at the next instant, we use the conservation of internal energy:

$$\frac{dT}{dt} = \frac{\gamma - 1}{Rm}\left(\sum_i \frac{dm_i}{dt} h_i - \frac{RT}{\gamma - 1}\sum_i \frac{dm_i}{dt} + \frac{dQ}{dt} - p\frac{dV}{dt}\right), \tag{2.5}$$

where

- $\frac{dT}{dt}$ is the derivative of the temperature inside the boiler with respect to time;
- $\frac{dQ}{dt} = P$ is the incoming heat flux;
- γ is the ratio of specific heats of the contents of the boiler;
- $\frac{dm_i}{dt}$ is the i^{th} incoming (positive) or outgoing (negative) mass flow;
- h_i is the enthalpy of the fluid of the i^{th} incoming or outgoing mass flow;
- T is the temperature inside the boiler;
- $R = \frac{\mathscr{R}}{M_m}$ where $\mathscr{R} = 8.314[J/mol.K]$ is the universal constant of perfect gas and M_m is the molar mass of the inside of the boiler;
- $\frac{dV}{dt}$ is the derivative with respect to time of the volume of the boiler;
- m is the mass of the content of the boiler;
- p is the pressure inside the boiler.

The incoming heat flux varies with the water level inside the boiler and has to be computed again at each instant. The computation of the heat flux as a function of the water level in the boiler, is detailed in Section 2.8.

The volume of the boiler is constant, so the last term of the equation vanishes, and $\frac{R}{\gamma - 1} = c_v$:

$$mc_v \frac{dT}{dt} = \sum_i \frac{dm_i}{dt} h_i - c_v T \sum_i \frac{dm_i}{dt} + P. \tag{2.6}$$

The only incoming or outgoing mass flow is the mass flow through the outlet pipe $-\dot{m}_{out} = \frac{dm}{dt}$ with an enthalpy h_{out}:

$$\rightarrow c_v \left(m \frac{dT}{dt} + T \frac{dm}{dt} \right) = -\dot{m}_{out} h_{out} + P \tag{2.7}$$

$$\rightarrow \frac{d(c_v m T)}{dt} = -\dot{m}_{out} h_{out} + P \tag{2.8}$$

By definition of the internal energy, $\frac{d(c_v m T)}{dt} = \frac{dU}{dt}$.

$$\rightarrow \frac{dU}{dt} = -\dot{m}_{out} h_{out} + P \tag{2.9}$$

With the internal energy differential $\frac{dU}{dt}$, we can compute the new internal energy of the boiler. Then, with the new internal energy, all the thermodynamic values at the new equilibrium can be computed by using CoolProp. CoolProp is a numerical tool defined in Section 2.9.

2.5.3 Algorithm

In practice, the evolution of the thermodynamic conditions inside the boiler during the time the valve is open, is computed with Algorithm 1. At each instant, the outgoing mass flow rate is computed with the Fanno flow theory (Section 2.7), then the new internal energy of the boiler is computed with Equation 2.9. The new temperature cannot be obtained directly from the internal energy; we have to perform a dichotomic search to obtain it. With this temperature, all the other quantities can be computed.
This algorithm is repeated at each instant until the valve is closed.

Algorithm 1 Computation of the new equilibrium at time t with equilibrium at time $t-1$

Inputs: Conditions of the inside of the boiler at time $t-1$.

Outgoing mass flow
Computation of $\dot{m}_{out}$ with Algorithm 2 and values at equilibrium $t-1$.
Computation of $m_{air}(t)$ and $m_{water}(t)$.

Internal energy
Computation of the heat flux P with Equation 2.53 and dU/dt with Equation 2.9.

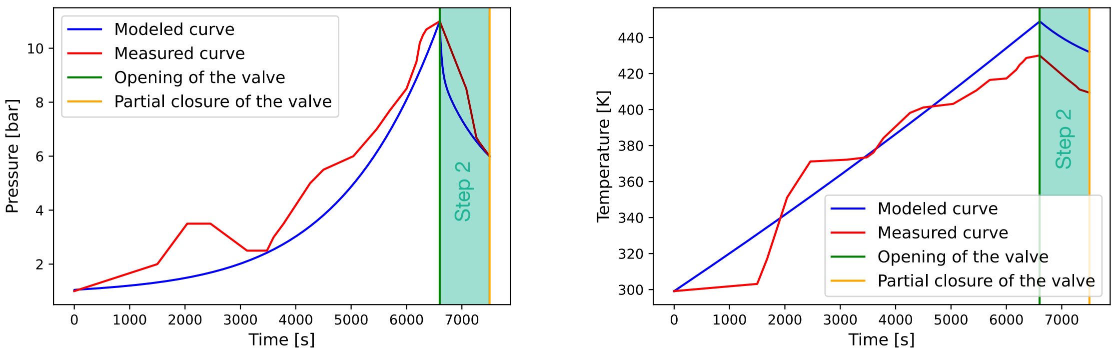
$U_{guess} = 0$
$T_{guess} = T[t-1]$
T_{high}=[ ], T_{low} = [ ]
while $abs(U_{guess} - U) > error$ **do**
  Dichotomic search for new temperature
  if $U_{guess} < U$ **then**
    $T_{guess} \to T_{low}$
    if T_{high} empty **then**
      $T_{guess} = T_{guess} - 10$
    else
      $T_{guess} = (T_{guess} + T_{high}[-1])/2$
    end if
  else
    $T_{guess} \to T_{high}$
    if T_{low} empty **then**
      $T_{guess} = T_{guess} - 10$
    else
      $T_{guess} = (T_{guess} + T_{low}[-1])/2$
    end if
  end if
  Computation of steam quality : $x(T_{guess}) = \dfrac{v_{wat} - v_{liq}(T_{guess})}{v_{vap}(T_{guess}) - v_{liq}(T_{guess})}$
  Computation of masses of steam and water:
    $m_{steam} = x \times m_{water}$
    $m_{liq} = (1-x) \times m_{water}$
  Computation of V_{liq} and V_{gas}.
  Computation of specific internal energy of water, steam and air.
  Computation of U_{guess}.
end while

Computation of the values at time t with temperature T_{guess}.
Computation of $p_{water}(T)$ with CoolProp (Section 2.9) and p_{air} with the perfect gas equation.
Computation of the total pressure: $p_{tot} = p_{water} + p_{air}$.
Computation of the specific enthalpy of the inside of the boiler h_{tot} with CoolProp (Section 2.9).

return Conditions inside the boiler at time t.

2.5.4 Equivalent pipe length

The singular pressure losses as well as the lengths and the rugosity of the downstream pipes -including the outlet and all subsequent connected pipes- being unknown, all these factors are included in an equivalent pipe length L_{eq}. This length is computed by adapting the outgoing mass flow until the simulated pressure curve matches the measured pressure curve at the end of step 2 (right before the valve is partially closed). We obtain an equivalent length of $296.6m$ (L_{eq}) and the graphs represented in Figure 2.8.



(a) (b)

Figure 2.8: Comparisons between the measured and modeled pressure (a) and temperature (b) while the outlet valve is open.

We can see that the modeled pressure curve presents a more steep decrease than the measured curve. This difference has three possible explanations:

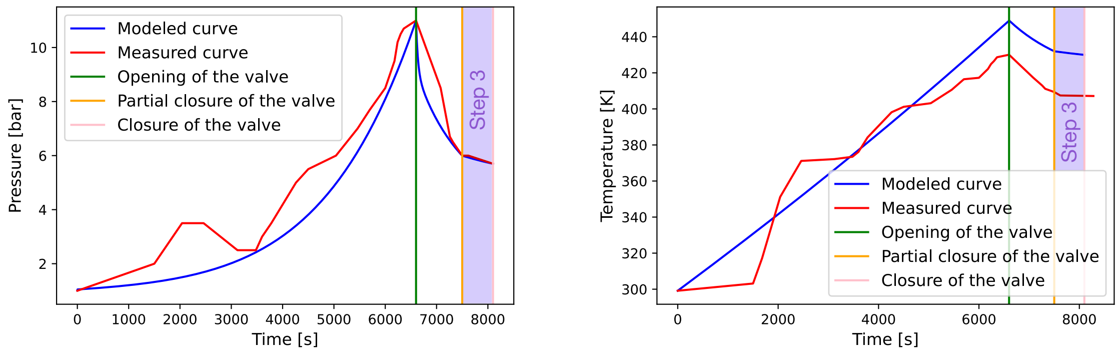
- The valve is opened manually by an operator and is a quarter turn valve, so the valve does not go from close to open instantly.
- The Fanno flow equations are valid for stationary flows, while the real flow goes through a transitory phase after the valve is opened.
- The downstream piping is more complex than the equivalent straight pipe that is used in the model.

The modeled temperature curve has the same shape as the measured curve with an offset between them. The modeled temperature is higher than the measured temperature because of the position of the temperature measuring device (as explained in Section 2.4.2).

2.6 Step 3 : Partial closure of the valve

In this section, the evolution of the thermodynamic quantities inside the boiler throughout operational step 3 of the boiler is modeled. Fifteen minutes after the opening of the valve, the valve is partially closed and we move on to the last operational step of the boiler.

To model this partial closure, we use the same model as for step 2. While keeping the equivalent pipe length computed in Section 2.5.4, we multiply the outgoing mass flow rate $\dot{m}_{out}$ by a partial closure factor $F_p < 1$. This factor is computed by equalizing the pressure at the end of the step, when the valve is closed, to the measured one. We obtain $F_p = 0.706$ and the graphs represented Figure 2.9.



(a) (b)

Figure 2.9: Comparisons between the measured and modeled pressure (a) and temperature (b) after the partial closing of the valve.

We see that the modeled and measured pressure follow the same curve. However, the modeled temperature stays above the measured temperature. This can, once again, be explained by the fact that the temperature measuring device is located at the inlet of the outlet pipe, where the temperature is lower than in the center of the boiler.

2.7 Fanno Flow

The Fanno flow [5] is a theory used to model the behaviour of a certain type (described in Section 2.7.1) of compressible flows, by taking into account the effects of friction between the fluid and the wall of the pipe. We consider that the outgoing mass flow rate of the boiler can be modeled by the Fanno flow theory, and we use it in our model.

2.7.1 Assumptions

To use the Fanno flow model on the outgoing flux, some assumptions have to be made:

- the flow is adiabatic;
- the pipe has a constant cross-section;
- the fluid is a perfect gas;
- the flow is compressible.

2.7.2 Equations

To compute the outlet conditions of the outlet pipe, we need to compute the outlet Mach number of the flow. To do so, several equations are needed.

Mach number

The Mach number is an adimensional number used to characterise the speed of a flow and is defined as

$$M = \frac{u}{c}, \tag{2.10}$$

where $u[m/s]$ is the speed of the flow and $c[m/s]$ is the speed of sound in the gas, and can be computed with

$$c = \sqrt{\gamma \times R \times T}, \tag{2.11}$$

where

- γ is the ratio of specific heats of the gas;
- $R = \frac{\mathscr{R}}{M_m}$ with $\mathscr{R} = 8.314[\frac{J}{mol.K}]$ being the universal gas constant and M_m the molar mass of the gas;
- T is the temperature of the gas.

Compressible flow equations

At any moment in a compressible flow, the following relations hold:

$$\frac{T_0}{T} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right), \tag{2.12}$$

where T_0 is the reservoir temperature that, since the flow is adiabatic, is conserved along the flow and

$$\frac{p_{0,x}}{p} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}, \tag{2.13}$$

where $p_{0,x}$ is the reservoir pressure at the location in the flow where the Mach number M and the pressure p are taken. The reservoir pressure is not conserved along the flow.

Critical length function

To obtain the Mach number at the inlet M_{in} of the pipe when the Mach number at the outlet M_{out} is known, we use the critical length function. The critical length function is:

$$f(M) = \frac{1}{\gamma}\left(\frac{(1 - M^2)}{M^2} + \frac{(\gamma + 1)}{2} \log\left(\frac{\frac{(\gamma+1)}{2} M^2}{\left(1 + \frac{(\gamma-1)}{2} M^2\right)}\right)\right) \tag{2.14}$$

where λ is the friction factor of the flow.

The difference between two critical length functions at two different positions in the flow can also be computed:

$$f(M_1) - f(M_2) = \lambda \frac{x_2 - x_1}{(D_p - 2t_p)},$$

where $x_2 - x_1$ is the distance between the two considered points.
If the distance between the inlet of the pipe and the outlet of the pipe is L_{eq}, we have:

$$f(M_{in}) = f(M_{out}) + \lambda \frac{L_{eq}}{(D_p - 2t_p)}. \tag{2.15}$$

Friction factor

To obtain the friction factor λ, we use the Colebrook formula [14]. This equation is implicit and needs an iteration to be solved:

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{2.51}{Re_D}\frac{1}{\sqrt{\lambda}} + \frac{1}{3.71}\frac{\epsilon}{(D_p - 2t_p)}\right). \tag{2.16}$$

In this formula:

- ϵ is the rugosity;
- D_p is the external diameter of the pipe;
- t_p is the thickness of the wall of the outlet pipe;
- Re_D is the Reynolds number relative to the diameter of the pipe:

$$Re_D = \frac{\rho \times u \times (D_p - 2t_p)}{\mu}; \tag{2.17}$$

- μ is the dynamic viscosity of the fluid.

The Colebrook formula can be used if:

1. $4000 < Re_D < 10^8$: In our case, the flow out of the boiler has Reynolds number Re_D with a minimal value of 95.9×10^3 and a maximal value of 172.1×10^3.
2. $0 < \epsilon/D_p < 0.05$: In our case, the outlet pipe of the boiler has a constant ϵ/D_p that is equal to 0.036.

To determine the dynamic viscosity of the fluid, we use the Sutherland's formula [6]:

$$\frac{\mu(T)}{\mu_{ref}} = \left(\frac{T}{T_{ref}}\right)^{3/2}\frac{(T_{ref} + S)}{(T + S)}, \tag{2.18}$$

where μ_{ref} is the reference dynamic viscosity, T_{ref} is the reference temperature and S is the Sutherland constant. For air and steam, we use the following values of these constants:

	$\mu_{ref}[\frac{N.s}{m^2}]$	$T_{ref}[K]$	$S[K]$
Air	1.716×10^{-5}	273	110
Steam	1.12×10^{-5}	350	1064

We approximate the equivalent dynamic viscosity by the sum of the dynamic viscosities of air and steam, each term weighted by the corresponding volumic fraction.

Sonic conditions

The sonic conditions are used to obtain the sonic reservoir pressure from the outlet pressure and the outlet Mach number, and to obtain the inlet reservoir pressure from the sonic reservoir pressure and the inlet Mach number.
The sonic conditions are obtained when $M = 1$ and are marked with an $*$. We have:

$$\frac{p_0^*}{p^*} = \left(\frac{(\gamma+1)}{2}\right)^{\frac{\gamma}{(\gamma-1)}} \tag{2.19}$$

$$\frac{T_0^*}{T^*} = \frac{\gamma+1}{2} \tag{2.20}$$

$$= \frac{T_0}{T^*} \text{ since } T_0 \text{ is conserved} \tag{2.21}$$

$$\rightarrow \frac{T}{T^*} = \frac{T}{T_0}\frac{T_0}{T^*} \tag{2.22}$$

$$= \frac{\frac{(\gamma+1)}{2}}{\left(1+\frac{(\gamma-1)}{2}M^2\right)} \tag{2.23}$$

$$\frac{u}{u^*} = \frac{M\sqrt{T}}{M^*\sqrt{T^*}} \tag{2.24}$$

$$= M\sqrt{\frac{\left(\frac{\gamma+1}{2}\right)}{\left(1+\frac{\gamma-1}{2}M^2\right)}} \tag{2.25}$$

By conservation of the mass flow rate, we also have:

$$\rho u = \rho^* u^* \tag{2.26}$$

$$\rightarrow \frac{\rho}{\rho^*} = \frac{u^*}{u} \tag{2.27}$$

$$= \frac{1}{M}\left(\frac{\left(\frac{\gamma+1}{2}\right)}{\left(1+\frac{\gamma-1}{2}M^2\right)}\right)^{-1/2} \tag{2.28}$$

Next by using the perfect gas equation

$$p = \rho R T, \tag{2.29}$$

we obtain:

$$\frac{p}{p^*} = \frac{\rho}{\rho^*}\frac{T}{T^*} \tag{2.30}$$

$$= \frac{1}{M}\left(\frac{\frac{(\gamma+1)}{2}}{\left(1+\frac{(\gamma-1)}{2}M^2\right)}\right)^{1/2} \tag{2.31}$$

$$\frac{p_0}{p_0^*} = \frac{p_0}{p}\frac{p}{p^*}\frac{p^*}{p_0^*} \tag{2.32}$$

$$= \frac{1}{M}\left(\frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{\frac{(\gamma+1)}{2}}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \tag{2.33}$$

2.7.3 Algorithm

In practice, the outlet Mach number is obtained through an iterative process: an initial outlet Mach number is guessed, then the reservoir pressure needed to achieve this Mach number is computed and, with this value and the actual value of the reservoir pressure p_0, the guess for the outlet Mach number is adapted. Algorithm 2 gives a more detailed explanation of this computation.

Algorithm 2 Computation of the output Mach number

Inputs: Pipe geometry, reservoir conditions, γ, R.

$p_{0,\text{guess}} = 2bar$
$M_e = 0.1$
$M_{e,\text{low}} = [\,]$, $M_{e,\text{high}} = [\,]$
while $|p_{0,\text{guess}} - p_0| > \text{error}$ **do**
    Dichotomic search for the exit Mach number
    if $p_{0,\text{guess}} > p_0$ **then**
        $M_e \rightarrow M_{e,\text{high}}$
        if $M_{e,\text{low}}$ empty **then**
            $M_e = M_e/2$
        else
            $M_e = (M_e + M_{e,\text{low}}[-1])/2$
        end if
    else
        $M_e \rightarrow M_{e,\text{low}}$
        if $M_{e,\text{high}}$ empty **then**
            $M_e = M_e \times 2$
        else
            $M_e = (M_e + M_{e,\text{high}}[-1])/2$
        end if
    end if

    Outlet conditions
    Computation of the outlet conditions with Equations 2.12, 2.29, and 2.10.

    Friction factor
    Computation of μ with Equation 2.18 and Re with Equation 2.17.
    Computation of λ with Equation 2.16 (implicit equation $\rightarrow$ iteration).

    Sonic conditions
    Computation of p_0^* with Equations 2.31 and 2.19.

    Inlet conditions
    Computation of $f(M_e)$ with Equation 2.14.
    Computation of $f(M_i)$ with Equation 2.15.
    Computation of a new value of $p_{0,\text{guess}}$ with Equation 2.33.
end while
return Outlet conditions.

2.8 Influence of the water level on the heat flux

In this section, we want to obtain an equation that gives the heat flux as a function of the water level inside the boiler.

The heat transfer from the exhaust gases to the inside of the boiler is not the same when the smoke tube is in contact with gas or with water.
During step 2 and 3 of the operation of the boiler, steam and air flow out of the boiler, and water evaporates into steam thereby reducing the water level in the boiler. If the water level varies, so does the heat flux going from the exhaust gases to the inside of the boiler.

In Section 2.4.2, the average heat flux as well as the evolution of the temperature of the inside of the boiler have been computed in a situation where the water level in the boiler is known. We also know the temperature of the exhaust gases at the inlet and at the outlet of the smoke tubes: $T_{exhaust,in} = 1105K$ and $T_{exhaust,out} = 1015K$ [2].

With these values, we can compute the convective heat transfer coefficient of the exhaust gases $h_{exhaust}$ and, compute the heat flux going from the exhaust gases to the inside of the boiler when the water level is different.

2.8.1 Estimation of the smoke's convection coefficient

LMTD method

To approximate the value of the equivalent thermal resistance of the heat transfer between the exhaust gases and the inside of the boiler, we use the LMTD method [3]:

$$P = \frac{1}{R_{th}} \frac{\Delta T_2 - \Delta T_1}{\log(\Delta T_2 / \Delta T_1)} \tag{2.34}$$

$$\Delta T_1 = T_{exhaust,in} - T_{boiler} \tag{2.35}$$

$$\Delta T_2 = T_{exhaust,out} - T_{boiler} \tag{2.36}$$

During the closed boiler heating, we know the average (over time) heat flux received by the inside of the boiler: $P_{av} = 42.65kW$. To use the LMTD method with the average heat flux, we use the average value of the right hand side of the equation by taking the average value over time of T_{boiler}:

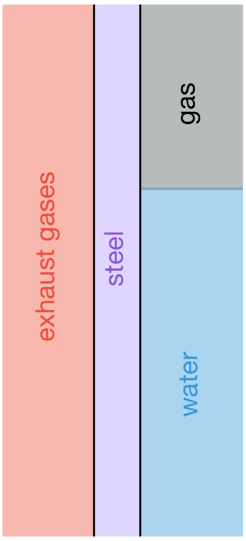
$$P_{av} = \frac{1}{R_{th}} \frac{T_{exhaust,out} - T_{exhaust,in}}{\log((T_{exhaust,out} - T_{boiler,av})/(T_{exhaust,in} - T_{boiler,av}))} \tag{2.37}$$

The temperature of the smoke and of the inside of the boiler is also known during the closed boiler heating, so we can compute the equivalent thermal resistance:

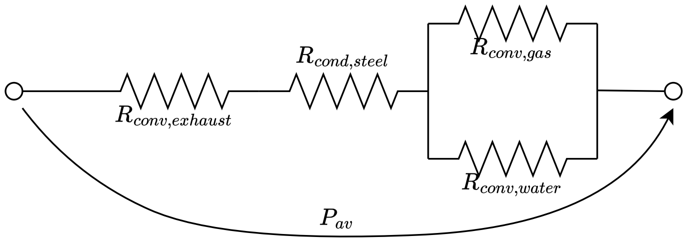
$$R_{th} = \frac{1}{P_{av}} \frac{T_{exhaust,out} - T_{exhaust,in}}{\log((T_{exhaust,out} - T_{boiler,av})/(T_{exhaust,in} - T_{boiler,av}))}. \tag{2.38}$$

Equivalent thermal resistance

To isolate the convective heat transfer coefficient of the exhaust gases, we need the expression of the equivalent thermal resistance of the heat transfer through the 18 smoke tubes. In Figure 2.10a, we can see that we have convective heat transfer through the exhaust gases, then conductive heat transfer through the steel walls of the tube and finally convective heat transfer either through the water or through the gas composed of steam and air. This leads us to the equivalent thermal circuit of the heat transfer on Figure 2.10b.



(a)



(b)

Figure 2.10: The heat flux goes from the exhaust gases to the inside of the boiler by three successive modes: a convective transfer mode through the smoke, a conductive transfer mode through the steel walls of the pipe and a convective heat transfer through gas or water. The schematic of this situation (a) brings us to the equivalent thermal circuit (b) where the heat flux crosses the convective thermal resistance of the smoke, then the conductive thermal resistance of the steel and then the convective thermal resistances of the gas and the liquid water inside the boiler in parallel.

From the equivalent thermal circuit of Figure 2.10b, the equivalent thermal resistance for one smoke tube $R_{th,1}$ is obtained:

$$R_{th,1} = R_{conv,exhaust} + R_{cond,steel} + \frac{1}{\dfrac{1}{R_{conv,gas}} + \dfrac{1}{R_{conv,water}}} \tag{2.39}$$

where

- $R_{conv,exhaust}$ is the convective thermal resistance of the exhaust gases;
- $R_{cond,steel}$ is the conductive thermal resistance through the steel wall of the tube;
- $R_{conv,gas}$ is the convective thermal resistance of the gas inside the boiler;
- $R_{conv,water}$ is the convective thermal resistance of the water inside the boiler.

To have the equivalent thermal resistance for the 18 tubes R_{th}, we divide the thermal resistance for 1 tube by 18:

$$R_{th} = \frac{1}{18} R_{th,1} \tag{2.40}$$

We are in a cylindrical configuration, so these thermal resistances can be expressed as:

$$R_{cond,steel} = \frac{\log\left(\dfrac{r_{smoke,ext}}{r_{smoke,int}}\right)}{2\pi \times H \times k_{steel}} \tag{2.41}$$

$$R_{conv,gas} = \frac{1}{2\pi \times r_{smoke,ext} \times (H - H_{water}) \times h_{gas}} \tag{2.42}$$

$$R_{conv,water} = \frac{1}{2\pi \times r_{smoke,ext} \times H_{water} \times h_{water}} \tag{2.43}$$

$$R_{conv,exhaust} = \frac{1}{2\pi \times r_{smoke,int} \times H \times h_{exhaust}} \tag{2.44}$$

where

- $r_{smoke,ext}$ is the external radius of the smoke tubes;
- $r_{smoke,int}$ is the internal radius of the smoke tubes;
- H is the height of the boiler;
- H_{water} is the water level inside the boiler;
- k_{steel} is the conductive heat transfer coefficient of steel;
- h_{gas} is the convective heat transfer coefficient of the gas phase inside the boiler;
- h_{water} is the convective heat transfer coefficient of the water inside the boiler.

To obtain a value for h_{gas}, we need the values of the other heat transfer coefficients:

- The conductive heat transfer of steel is a property of the material and is: $k_{steel} \approx 15[W/m.K]$ [4][7].
- The convective heat transfer coefficients not only depend on the type of fluid, they also depend on the velocity and the temperature of the fluid. The natural convection coefficient of water is in the range $50 \rightarrow 100[W/m^2K]$[9]. We know that the water in the boiler can have very high temperatures and is quite still so we assume that its convection coefficient is in the lower values of this range: $h_{water} \approx 50[W/m^2K]$.
- The range for the natural convection coefficient of gases is $5 \rightarrow 10[W/m^2K]$[9]. By following the same reasoning for the gas in the boiler as for the water, and knowing that the convection in gas should be much lower than the coefficient in water, we use $h_{gas} \approx 5[W/m^2K]$.

The values of the internal radius $r_{smoke,in}$ and the external radius $r_{smoke,out}$ of the smoke tubes, are easily obtained with the geometry given in Figure 2.1b:

$$r_{smoke,ext} = \frac{D_s}{2} = 44.45mm \tag{2.45}$$

$$r_{smoke,int} = \frac{D_s}{2} - t_s = 40mm \tag{2.46}$$

With these values, we compute:

$$R_{cond,steel} = 28 \times 10^{-6}[K/W] \tag{2.47}$$
$$R_{conv,gas} = 87.94 \times 10^{-3}[K/W] \tag{2.48}$$
$$R_{conv,water} = 5.28 \times 10^{-3}[K/W] \tag{2.49}$$
$$\rightarrow R_{conv,exhaust} = 11.06 \times 10^{-3} \tag{2.50}$$
$$= \frac{1}{2\pi \times r_{smoke,int} \times H \times h_{exhaust}} \tag{2.51}$$
$$\rightarrow h_{exhaust} = 15.97[W/m^2K] \tag{2.52}$$

This value is quite low and could easily be increased by using smoke tubes with a higher surface rugosity. Indeed increasing the surface rugosity of an exchange surface, increases friction and so the heat exchange through this surface [12].

2.8.2 Heat flux as a function of the water level and temperature in the boiler

To compute the heat flux as a function of the water level and the temperature in the boiler, we use the LMTD method of Equation 2.37.

The equivalent thermal resistance is given by Equation 2.39 and the thermal resistances are functions of the water level given by Equations 2.41, 2.42, 2.43 and 2.44.
ΔT_1 and ΔT_2 are functions of the temperature inside of the boiler (Equations 2.35 and 2.36).
In reality, the temperature of the exhaust at the outlet of the tube also varies $T_{exhaust,out}$. Yet, this variation is neglected in our model because of the high temperature of the smoke and the small relative difference between the temperature at the inlet of the pipe and at the outlet of the pipe during the closed boiler heating.

We obtain:

$$P(T_{boiler}, H_{water}, H) = R_{th} \times \frac{T_{smoke,out} - T_{smoke,in}}{\log\left(\dfrac{T_{smoke,out} - T_{boiler}}{T_{smoke,in} - T_{boiler}}\right)} \tag{2.53}$$

$$R_{th}(H_{boiler}, H_{water}) = \left(R_{cond,steel} + R_{conv,smoke} + \frac{1}{\dfrac{1}{R_{conv,gas}} + \dfrac{1}{R_{conv,water}}}\right) \tag{2.54}$$

2.9 CoolProp

To model the operation of the boiler, CoolProp has been used to compute the thermodynamic quantities in the boiler.
CoolProp is a C++ library that was wrapped into Python. It allows to compute physical properties for a large range of pure fluids as well as for some mixtures.
During this work, it has been used to compute the different thermodynamic properties of air, steam and water during the process. The function that has been mostly used is the following:

```
CoolProp.CoolProp.PropsSI(param1,param2a,param2b,param3a,param3b,param4)
```

where the parameters are defined in Table 2.1.

Table 2.1: Definition and data type of the parameters of the `CoolProp.CoolProp.PropsSI` function.

	Type	Definition
`param1`	string	Quantity that will be returned by the function
`param2a`	string	Known thermodynamic quantity
`param2b`	float	Value of the thermodynamic quantity of param2a
`param3a`	string	Known thermodynamic quantity
`param3b`	float	Value of the thermodynamic quantity of param3a
`param4`	string	Nature of the fluid

As an example, we compute the density (`'D'`) of water (`'water'`) when it has a pressure(`'P'`) of `P_val`[Pa] and a vapour quality (`'Q'`) of `Q_val`[/]:

```
CoolProp.CoolProp.PropsSI('D','P',P_val,'Q',Q_val,'water')
```

2.10 Error sources of the model

The difference between the obtained model and the measurements can be explained by a few weaknesses of the model:

- The uniform temperature assumption: the model considers a uniform temperature inside the boiler. In reality, this is not the case. The temperature inside the boiler will be higher close to the smoke tubes and decrease when approaching the walls of the boiler.

- The uniform pressure assumption: the model considers a uniform pressure inside the boiler which is not true either. The pressure in the gas should be uniform but the pressure inside the liquid water increases as we go deeper in the water. This additional pressure Δp_{water} can be computed with :

  $$\Delta p_{water} = \rho_{water} \times H_{water} \times g \tag{2.55}$$

  where H_{water} is the water height, ρ_{water} is the density of the water and g is the gravitational acceleration.
  This value is maximal at the bottom of the boiler (where H_{water} is the highest) when the pressure inside the boiler is at its lowest (when ρ_{water} is the highest) and is equal to:

  $$\Delta p_{water,max} \approx 0.085bar \tag{2.56}$$

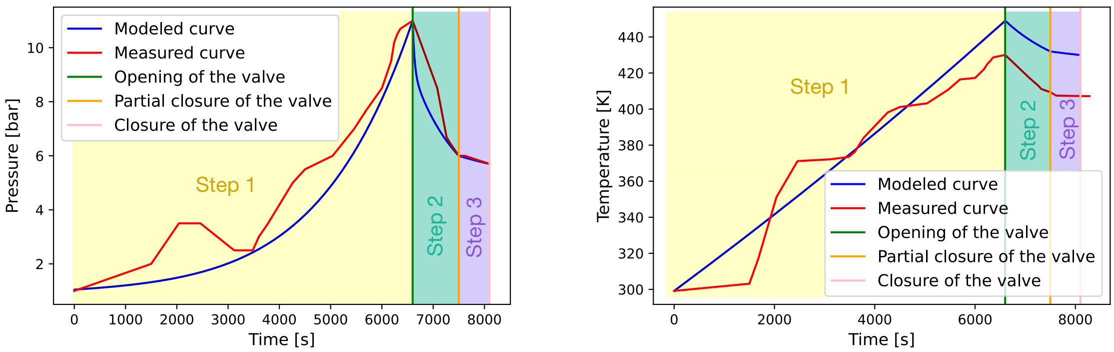
  This value is much smaller than the pressure, and the error induced by neglecting it is not considerable.

- The ideal gas approximation: the whole model considers dry air as an ideal gas which is quite a good approximation. However, when using the Fanno flow equations, the mixture of dry air and steam is also considered as an ideal gas; this assumption is much less valid.

2.11 Summary : Final results of the simulation

To conclude this chapter that aimed at modeling the current operation of the boiler, all the obtained results are combined and summarized in this section.

The pressure and the temperature of the inside of the boiler have been modeled throughout the three major operational steps of the boiler: the closed boiler heating in Section 2.4 (step 1), the opening of the valve in Section 2.5 (step 2) and the partial closure of the valve in Section 2.6 (step 3). The final graphs are given in Figure 2.11.



(a) (b)

Figure 2.11: Measured and modeled pressure (a) and temperature (b) inside the boiler during one cycle of the boiler functioning. The three major steps are highlighted : step 1 (closed boiler heating), step 2 (opening of the valve) and step 3 (partial closure of the valve).

To obtain these graphs, an equivalent pipe length (L_{eq}) has been used to model all the pressure losses of the outgoing mass flow and a partial closure factor (F_p) has been used to model the effect of the partial closure of the valve on the outgoing mass flow:

$$L_{eq} = 296.6m \tag{2.57}$$

$$F_p = 0.706 \tag{2.58}$$

Using these results, it is also possible to obtain the graph of the outgoing mass flow rate after the opening of the valve (Figure 2.12), which will be a very important quantity in the following chapter.

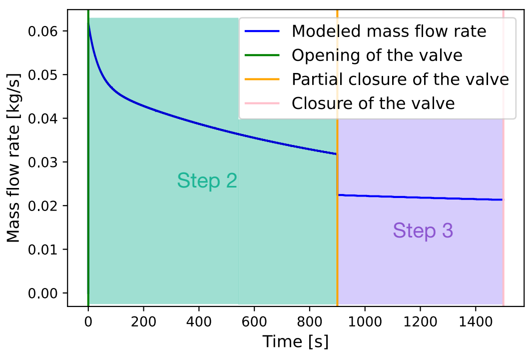


Figure 2.12: Model of the outgoing mass flow rate during step 2 (opening of the valve) and step 3 (partial closure of the valve) of the boiler's functioning.

This mass flow rate is composed of a mixture of steam and air. We can decompose it in two distinct mass flow rates: a mass flow rate of steam (blue curve in Figure 2.13) and a mass flow rate of air (red curve in Figure 2.13).

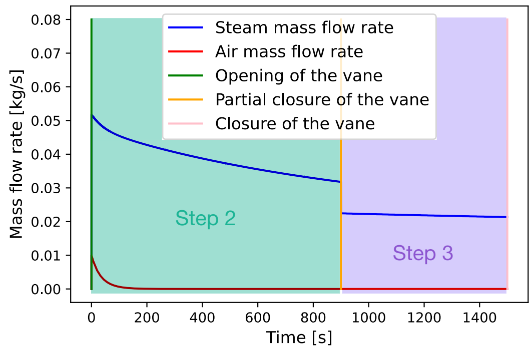


Figure 2.13: Model of the outgoing mass flow rates of steam (blue curve) and of air (red curve) during step 2 (opening of the valve) and step 3 (partial closure of the valve) of the boiler's functioning.

We can see that the air mass flow rate is very small compared to the mass flow rate of steam, and tends towards zero in a couple of minutes.

2.12 Analysis of the motives

As explained in Section 1.2, there are three principal motives for the reduction of the maximal pressure in the boiler:

- the reduction of the risks when empty heated (safety);
- the reduction of the thickness of the walls of the boiler and/or the material requirements in order to reduce the cost of the boiler, as well as of the replacement of the parts of the boiler (economy);
- the reduction of the heating time before opening of the valve, and therefore the reduction of their environmental impact.

Now that the boiler is modeled, these three motives can be analyzed and quantified.

2.12.1 Risks when empty heating

The studied boiler being completely hand-operated, there are risks of human errors. One of those risks is the operator forgetting to pump the water in the boiler before starting the heating.
If this happens, the only heat absorber in the boiler will be the air in it, which could heat up a lot more than it should and the pressure could become very high. The boiler not being conceived to endure high pressure and high temperature, could be damaged or even explode.

We modeled the evolution of the pressure and the temperature inside the boiler when it is empty heated (blue curves on Figure 2.14). We see that they quickly reach constant values.
This can be explained by the temperature of the smoke in the smoke tubes. When the temperature of the inside of the boiler approaches the temperature of the smoke that heats the boiler, the heat flux decreases until reaching a zero heat flux when the temperature inside the boiler is equal to the temperature of the smoke.
At the inlet of the smoke tubes, the smoke has a temperature of $\approx 1105K$ [2], so the temperature of the inside of the boiler cannot exceed this temperature. If the temperature in the boiler does not increase, the pressure also stays the same and has a maximal value of 3.75 bar.

The maximal pressure that the tank can hold without being plastically deformed can be computed with the Tresca yield criterion [8]:

$$\frac{\sigma_{yield}}{2} > \frac{max\{\sigma_1, \sigma_2, \sigma_3\} - min\{\sigma_1, \sigma_2, \sigma_3\}}{2} \tag{2.59}$$

The material of the boiler is ordinary steel, its yield strength is $\sigma_{yield}(0°C) = 305MPa$[1] at $0°C$. However, the yield strength of steel varies with its temperature. When empty heated, the boiler reaches a maximal temperature of $1105K$, at this temperature the yield strength of steel is equal to approximately 20% of its value at $0°C$ [16]. We obtain:

$$\sigma_{yield}(1105K) = \sigma_{yield}(0°C) \times 0.2 \tag{2.60}$$

$$= 61MPa \tag{2.61}$$

By using the principal stresses for a cylindrical pressure vessel (computed in Annex A), we have:

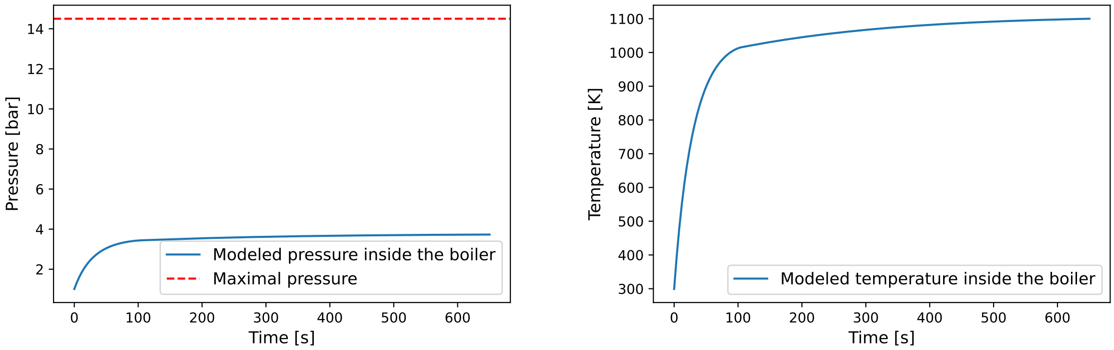
$$\frac{1}{2}\left(\frac{pr}{t} - 0\right) < \frac{\sigma_{yield}(1105K)}{2} \tag{2.62}$$

$$\rightarrow p < \frac{61 \times 10^6 \times t}{r} \tag{2.63}$$

The actual boiler has a radius of $r = 0.42m$ and a wall thickness of $t = 10mm$ (Section 2.1). This gives a maximal pressure of:

$$p_{max,yield} = 14.5bar \tag{2.64}$$

We compare this maximal pressure with the modeled pressure curve on Figure 2.14a:



(a) (b)

Figure 2.14: The modeled pressure (a) and the temperature (b) inside the boiler as functions of time when the boiler is heated while being empty. The modeled pressure inside the boiler stays below the maximal pressure that the boiler can hold without being damaged or explode (represented by the red dotted line).

On the graph of Figure 2.14, we see that the pressure inside the boiler stays below the maximal pressure that it can hold without being damaged or explode.

However, when empty heated, the boiler reaches temperatures much higher than during the usual operation of the boiler and thus the steel will expand more than during its normal operation and will create internal stresses. These internal stresses will add up to the stresses created by the pressure, and could exceed the yield strength. We do not have enough data to compute these stresses.
Given this uncertainty, we conclude that, by decreasing the pressure inside the boiler, we will indeed reduce the risks when empty heating.

2.12.2 Material quantity and quality

Another reason why the ANASAM company wants to decrease pressure, is to reduce the amount of material needed to make the boiler or to replace parts of the boiler, and thus reduce its cost.

At constant boiler dimensions, if the pressure in the boiler is reduced, the material quality requirements and/or the thickness of the walls of the boiler can also be reduced, which will reduce the volume of material needed and/or the cost of the boiler.

The boiler is made out of ordinary steel, the specific cost of ordinary steel is $X = 1.23[€/kg]$ and the density is $\rho_{steel} = 7.81 \times 10^3[kg/m^3]$ [1], with the geometry of the boiler given in Section 2.1, we can compute the volume of material V_{mat} and the cost C :

$$V_{mat} = 2\pi r H t + 2 \times \pi r^2 t \tag{2.65}$$
$$= 0.0488 m^3 \tag{2.66}$$
$$C = X \times \rho_{steel} \times V_{mat} \tag{2.67}$$
$$= 469[€] \tag{2.68}$$

2.12.3 Reduction of the heating time

As explained in Section 2.2, the inside of the boiler is heated while the boiler is kept close, before an outgoing mass flow is released. The maximal pressure in the boiler is reached right before opening the valve.
If this maximal pressure is reduced at a constant boiler volume and water volume inside the boiler, the heating time before the opening of the valve is also reduced, which means that less fuel has to be burned, less smoke is released outside the boiler and the environmental impact of the boiler is reduced.

For this motive, other factors that we will try modifying during the design of the new boiler (Chapter 3) have to be taken into account:

- The volume of the boiler: If the volume of the boiler increases, a bigger volume will need to be heated before the opening of the valve, and the heating will thus be longer.
- The initial volume of water inside the boiler: Water having a higher calorific capacity than air, the heating will be longer to reach a same pressure if a larger volume of water is pumped into the boiler.
- The quality of the heat transfer: If the heat transfer is improved, a higher heat flux will be added to the inside of the boiler and the heating time will be shorter to reach the same pressure.

To be able to make comparisons, we have the actual heating time before opening the valve:

$$t_{heat} = 1h50min$$

We want to reduce this heating time as much as possible while also respecting the other motives.

Chapter 3

Design of a new boiler

The aim of this chapter is to model a new boiler in which the maximal pressure is decreased while maintaining identical boiler performance, as described in Section 1.2. To do so, we will vary several parameters of the boiler one by one -keeping all the others fixed at the actual boiler parameters- and analyze the influence of this change on the maximal pressure inside the boiler.

We start by enumerating a few constraints that this new design has to respect.
After that, we analyze the effects of:

- the heat flux;
- the height of the boiler;
- the volume of the boiler;
- the initial water volume in the boiler.

Next, we select the material that is the most adapted for our design and we express the thickness of the walls of the new boiler as a function of its material, its maximal pressure and its dimensions.
Finally, we look at the results obtained with several combinations of the four parameters.

3.1 Constraints

During the design of a new boiler, a few constraints need to be taken into account. These constraints will always be respected in the rest of this section.

3.1.1 Downflow conditions

In the actual boiler, the downflow conditions are represented by an equivalent pipe length, computed in Section 2.5.4. The actual downflow conditions are unknown because:

- the singular pressure losses are unknown;
- the actual pipe length is unknown.

It is difficult to change this kind of uncertain factors and therefore, we keep the downflow conditions constant by fixing the pipe length at the equivalent pipe length computed in Section 2.5.4:

$$L_{eq} = 296.6m.$$

Moreover, the aim of this work is to modify the boiler and not its downstream conditions.

3.1.2 Exit mass flow rate

To complete its first and principal function, i.e. the weakening of the cashew nut shells, the exit flow of the boiler is used during the 18 first minutes after the opening of the valve. As we can see in Figure 3.1, this mass flow rate peaks at the moment of the opening of the valve and then decreases slowly. After 15 minutes, when the weakening of the shells is almost finished, the valve is partially closed and the mass flow rate becomes almost constant.

The peak in the actual exit mass flow rate not being necessary for the weakening of the shells, we constrain the mean exit mass flow rate $\dot{m}_{mean}$ to stay equal to the actual mean mass flow rate over the 18 first minutes after the opening of the valve:

$$\dot{m}_{mean} = \frac{\int_0^{18min} \dot{m}_{out} dt}{\int_0^{18min} dt} \tag{3.1}$$

$$= 0.0366kg/s. \tag{3.2}$$

where $\dot{m}_{out}$ is the outgoing mass flow rate modeled in Chapter 2.

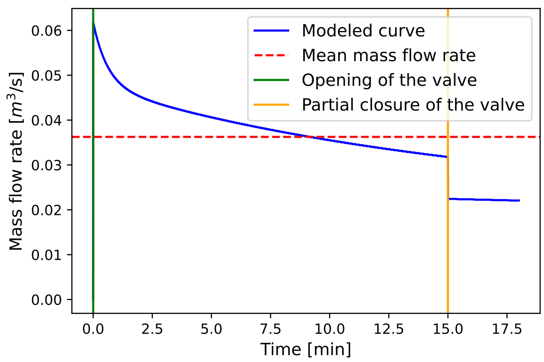


Figure 3.1: The outgoing mass flow rate of the actual boiler during the 18 first minutes after the opening of the valve. The mass flow rate peaks before slowly decreasing. After 15 minutes, the valve is partially closed. The mean mass flow rate of the new boiler must be the same as the mean mass flow rate of the actual boiler (red dashed line).

3.1.3 Minimal initial volume of water

The cashew nut shells need to be fragilised for 18 minutes. If we want to achieve a mean mass flow rate of $\dot{m}_{mean} = 0.0366[kg/s]$, there needs to be a minimal volume of water $V_{water,min}$ initially in the boiler :

$$V_{water,min} = \frac{1}{\rho(T_{in})} \times \dot{m}_{mean} \times 18 \times 60 \tag{3.3}$$

$$= 0.039[m^3] \tag{3.4}$$

This value being very small and not optimal (Section 3.5), it will never be used. Moreover, often the operation of the boiler is repeated several times before water is pumped into the boiler again.

3.1.4 Partial closure of the valve

The sudden decrease in mass flow rate caused by the partial closure of the valve is not needed for the boiler to complete its functions, the only utility of the partial closure being the reduction in the mass flow rate when the cashew nut shells are almost sufficiently weak. That is why no partial closure is performed in the model for the design of the new boiler.

3.2 Influence of the heat exchange

In this section, we study the influence of the heat exchange on the maximal pressure inside the boiler.

The actual heat exchanger (smoke tubes) is very basic. It could very easily be ameliorated, by e.g. adding fins on the tubes, increasing the number of tubes or increasing their rugosity. To model an improved heat exchange, the heat flux is multiplied by a factor $F_{heat} > 1$.

Parameters

To simulate the effect of the heat flux on the maximal pressure in the boiler, the following parameters are set to their actual values:

- the volume of the boiler: $V_{boiler} = 0.8m^3$;
- the geometry of the boiler: $r = 0.42$ and $H = 1.43$;
- the volume of water initially pumped in the boiler: $V_{water} = 0.5m^3$.

Results of the simulation

The maximal pressure in the boiler as a function of the heat flux factor F_{heat} is plotted in Figure 3.2.

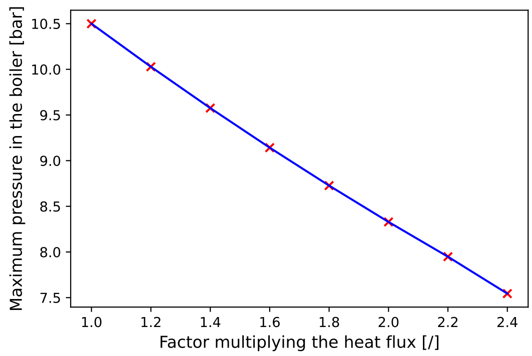


Figure 3.2: Influence of the heat flux factor on the maximal pressure in the boiler. The maximal pressure in the boiler decreases when the heat flux factor increases.

As we can see in Figure 3.2, the maximal pressure in the boiler decreases with an increasing factor multiplying the heat flux.

Indeed, if the heat flux is increased, the derivative of the internal energy $\dfrac{dU}{dT}$ computed with Equation 2.9 will be less negative, and the internal energy will decrease less. This can be confirmed by looking at the evolution of internal energy over time for different heat flux factors (Figure 3.3).

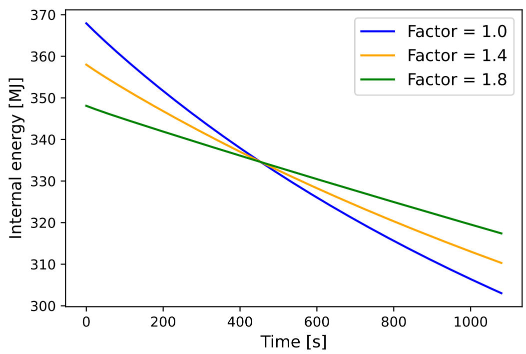


Figure 3.3: Internal energy as a function of time for several heat flux factors. The internal energy decreases less when the heat flux factor is higher.

The temperature of the inside of the boiler depends only on its internal energy U. If the internal energy decreases less, the temperature also decreases less. We verify this idea with the evolution of temperature inside the boiler for different heat flux factors (Figure 3.4).

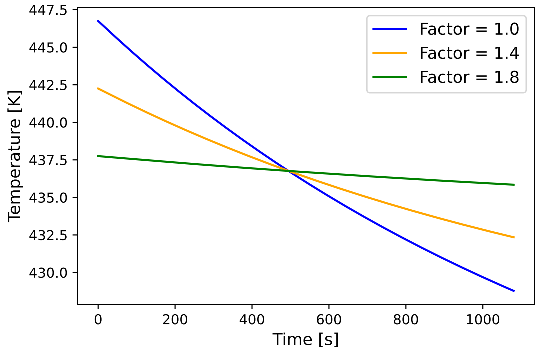


Figure 3.4: Temperature inside the boiler as a function of time for several heat flux factors. The temperature presents a smaller decrease when the heat flux is higher.

As we are in saturated steam conditions, the pressure of the inside of the boiler is directly linked to its temperature. The pressure follows a similar curve and decreases less when the temperature decreases less, and thus the pressure decreases less with increasing heat flux (Figure 3.5).

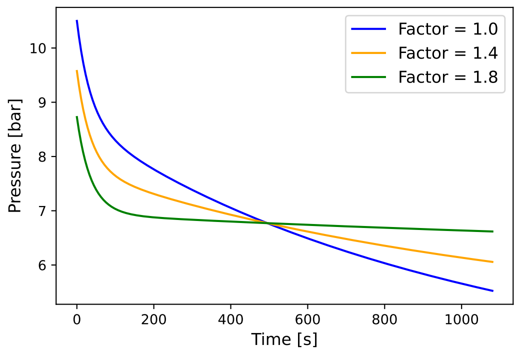


Figure 3.5: Pressure inside the boiler as a function of time for different heat flux factors. The pressure presents a smaller decrease when the heat flux is higher.

Finally, the outgoing mass flow depends on the pressure inside the boiler: when the pressure decreases, the mass flow also decreases. So, if the pressure decreases less, the outgoing mass flow rate also decreases less (Figure 3.6) and, to obtain the same mean mass flow rate, the initial mass flow rate can be smaller for higher heat flux factors. Thus, if the initial mass flow rate is smaller, both the initial and maximal pressure is also smaller.

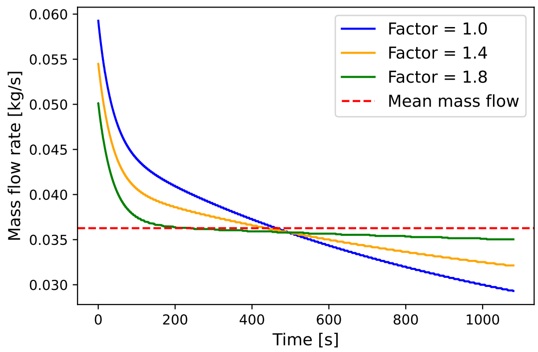


Figure 3.6: Outgoing mass flow rate as a function of time for different heat flux factors. The mass flow rate presents a smaller decrease when the heat flux is higher.

With these results, we can obtain the heating time and the needed volume of material to build the boiler (by considering that the material is ordinary steel) for several heat flux factors in Table 3.1.

Table 3.1: Heating time and volume of material for several heat flux multiplying factors.

F_{heat}	$t_{heat}[min]$	$V_{mat}[m^3]$
1	108	0.0488
1.4	75	0.0454
1.8	56	0.0415

Conclusion

The more the heat exchange is increased, the smaller the maximal pressure inside the boiler needs to be to keep the same mean mass flow rate. The increase of the heat exchange can be easily implemented by modifying the heat exchanger only, by e.g. adding more smoke tubes, adding fins on the smoke tubes or increasing the rugosity of the smoke tubes. The increase of the heat exchange also decreases the heating time as the heat is added faster inside the boiler. And finally, the maximal pressure decreases while keeping the same boiler dimensions, so the volume of material needed to build the boiler also decreases.

3.3 Influence of the height of the boiler

In this section, we study the influence of the height of the boiler (H_{boiler}) on the maximal pressure inside the boiler. To do so, we keep the volume of the boiler constant but we vary its height and thus its radius.

If we do so, and we keep the same initial volume of water that is pumped inside the boiler, the water height and the length of the smoke tubes will automatically change. As we saw in Section 2.8.2, the water level in the boiler has an influence on the heat flux between the smoke in the tubes and the inside of the boiler, and should thus also have an effect on the maximal pressure in the boiler.

Parameters

To model the effect of the variation of the height of the boiler on the maximal pressure inside the boiler, the following parameters are fixed:

- the volume of the boiler: $V_{boiler} = 0.8m^3$;
- the volume of water initially pumped into the boiler: $V_{water} = 0.5m^3$;
- the design of the heat exchanger: 18 tubes with the same dimensions as the actual tubes, except their height that will vary with the height of the boiler.

Results of the simulation

The maximal pressure inside the boiler decreases when the height of the boiler increases (Figure 3.7).

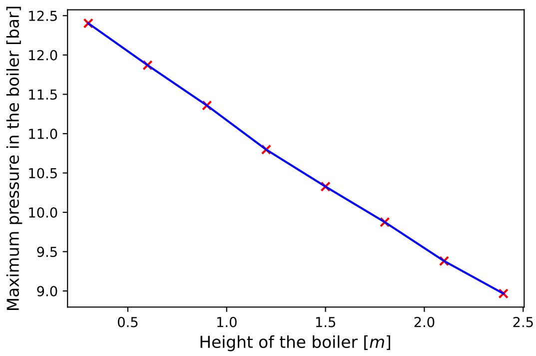


Figure 3.7: Maximal pressure in the boiler as a function of the height of the boiler. When the height of the boiler increases, the maximal pressure inside the boiler decreases.

When the height of the boiler increases, the heat exchange surface between the exhaust gases and the inside of the boiler increases, and so does the heat flux (Figure 3.8).

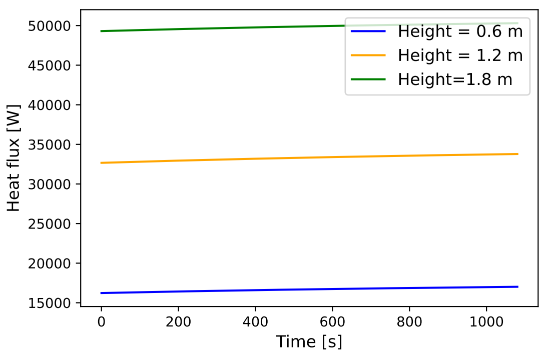


Figure 3.8: Heat flux going from the exhaust gases to the inside of the boiler for different boiler heights. When the height of the boiler increases, the heat flux also increases.

If the heat flux going from the exhaust gases in the boiler increases, we use Equation 2.9 to deduce that the derivative of the internal energy with respect to time will be less negative, and thus the internal energy inside the boiler decreases less (Figure 3.9).

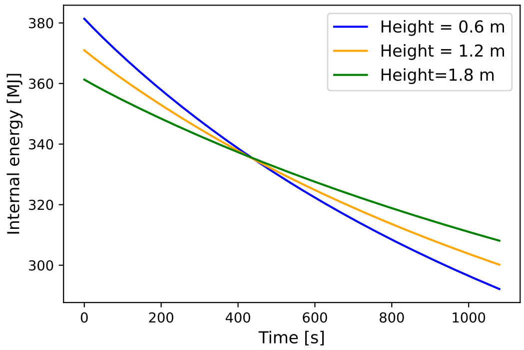


Figure 3.9: Internal energy inside the boiler as a function of time for different boiler heights. When the boiler is higher, the internal energy inside the boiler decreases less fast.

The temperature inside the boiler depends only on its internal energy. If the internal energy decreases less, the temperature also decreases less (Figure 3.10). The pressure inside the boiler only depends on its temperature and thus, if the temperature decreases less, so does the pressure (Figure 3.11)

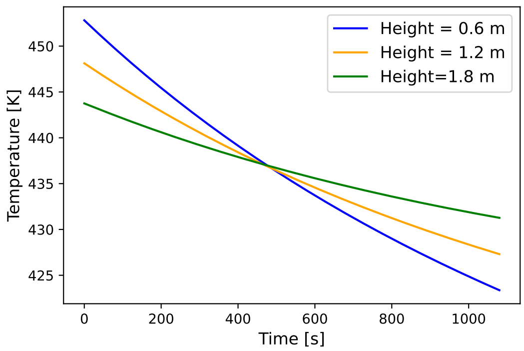


Figure 3.10: Temperature inside the boiler as a function of time for different boiler heights. When the boiler is higher, the temperature inside the boiler decreases less fast.

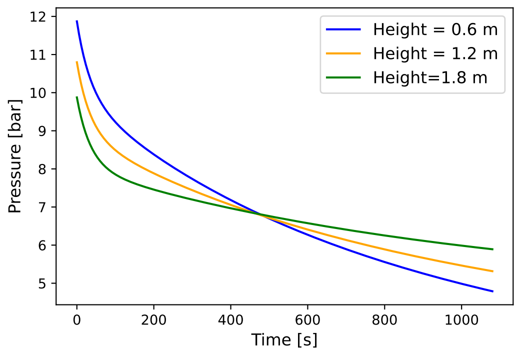


Figure 3.11: Pressure inside the boiler as a function of time for different boiler heights. When the height of the boiler increases, the pressure inside the boiler decreases less fast.

The outgoing mass flow depends on the pressure inside the boiler, when the pressure decreases, the mass flow also decreases. Thus, if the pressure decreases less fast, the mass flow rate also decreases less fast (Figure 3.12). This means that, to obtain the same mean mass flow rate, the initial mass flow rate can be lower for higher boilers and if the initial mass flow rate can be smaller, so can the pressure.

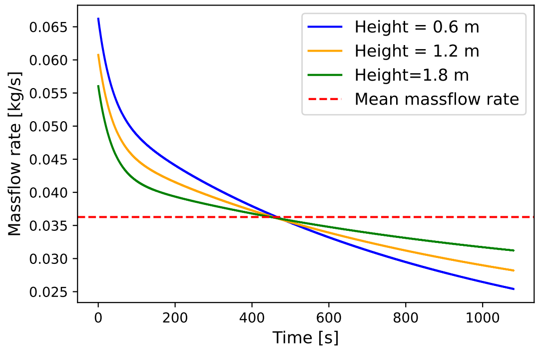


Figure 3.12: Outgoing mass flow rate as a function of time for different boiler heights. When the height of the boiler increases, the mass flow rate decreases less fast.

With these results, we can obtain the heating time and the needed volume of material to build the boiler (by considering that the material is ordinary steel) for several heights of the boiler in Table 3.2.

Table 3.2: Heating time and volume of material for several boiler heights.

H_{boiler}	$t_{heat}[min]$	$V_{mat}[m^3]$
0.6	268	0.0919
1.2	131	0.0566
1.8	84	0.045

Conclusion

The maximal pressure inside the boiler, the heating time and the needed volume of material are lower for the same mean mass flow rate with a higher boiler.
This parameter is not easy and expensive to implement, as the whole boiler needs to be replaced. However, is the boiler is replaced and the location of the boiler allows it, increasing the height is beneficial.

3.4 Influence of the volume of the boiler

In this section, we analyze the influence of the volume of the boiler. This parameter is not easy to vary in reality because, again, the entirety of the boiler would have to be replaced. However, if the influence is very promising, we could consider modifying it.

Parameters

To model the influence of the boiler volume on the maximal pressure in the boiler, we fix the following parameters:

- the volume of water initially pumped into the boiler: $V_{water} = 0.5m^3$;

- the height of the boiler: $H = 1.43m$;
- the design of the heat exchanger: 18 smoke tubes with the same dimensions as the actual tubes.

Results of the simulation

We compute the maximal pressure inside the boiler for different boiler volumes (Figure 3.13). The maximum pressure in the boiler decreases with an increasing volume.

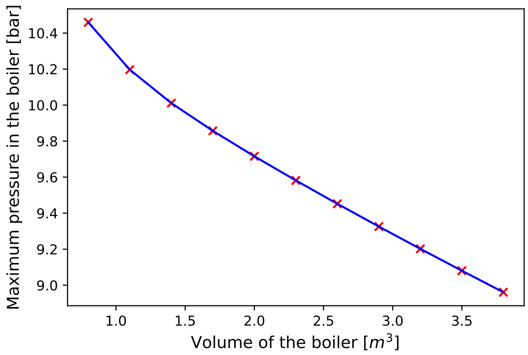


Figure 3.13: Maximal pressure inside the boiler as a function of the boiler volume. When the volume of the boiler increases, the maximal pressure inside the boiler decreases.

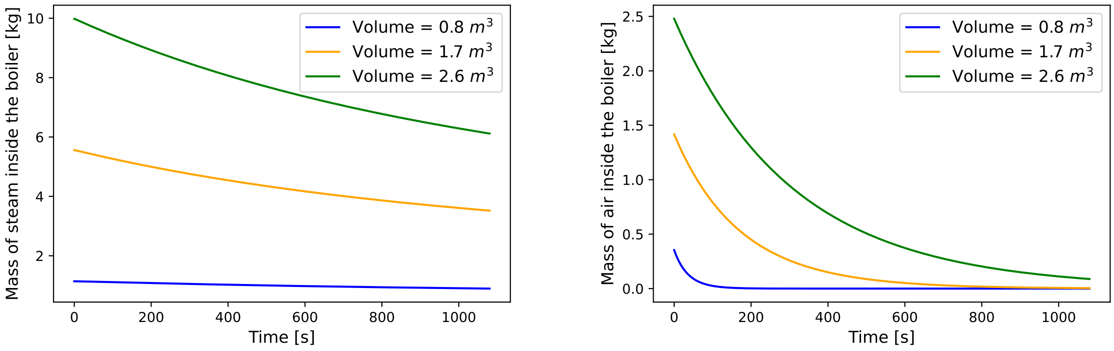
To explain this graph, we will prove that for a same maximal (initial) pressure, the mean mass flow rate increases with the volume.
If we keep a constant maximal pressure (p_{tot}), the sum of the partial pressures of steam (p_{steam}) and air (p_{air}) also stays constant.

$$p_{tot} = p_{steam} + p_{air} = \text{constant} \tag{3.5}$$

The partial pressures of steam and air only depend on the temperature ($p_{steam}(T)$, $p_{air}(T)$). For a same maximal total pressure, we will thus have the same maximal temperature and both pressures will also stay constant. By using the perfect gas assumption for air and steam, we obtain:

$$p_{air}(V_1) = p_{air}(V_2) \tag{3.6}$$
$$\rightarrow \frac{m_{air,1}}{V_1} = \frac{m_{air,2}}{V_2} \tag{3.7}$$
$$\rightarrow \text{if } V_2 > V_1, \text{ then } m_{air,2} > m_{air,1} \tag{3.8}$$
$$p_{steam}(V_1) = p_{steam}(V_2) \tag{3.9}$$
$$\rightarrow \frac{m_{steam,1}}{V_1} = \frac{m_{steam,2}}{V_2} \tag{3.10}$$
$$\rightarrow \text{if } V_2 > V_1, \text{ then } m_{steam,2} > m_{steam,1} \tag{3.11}$$

These equations prove that before the opening of the valve, the initial masses of air and steam in the boiler will be higher for a higher volume (Figure 3.14).



(a) (b)

Figure 3.14: Mass of steam (a) and air (b) inside the boiler after the opening of the valve for different boiler volumes that are all at the same initial pressure. When the volume is larger, the initial mass of steam inside the boiler is also higher.

We recall the equation of the derivative with respect to time of the internal energy (Equation 2.9):

$$\frac{dU}{dt} = -\dot{m}_{out} \times h_{out} + P$$

When the volume is larger, the volumic fraction of air in the gas is higher (Figure 3.15). The enthalpy of air being higher than the enthalpy of steam, h_{out} is smaller for larger volumes. However, if we assume that $\dot{m}_{out}$ is higher for larger volumes (which we are demonstrating), we can understand why $\frac{dU}{dt}$ is almost the same for all the volumes, and the internal energies follow curves with the same slope (Figure 3.16).

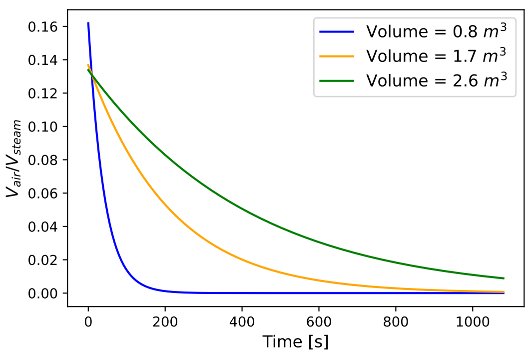


Figure 3.15: Volumic fraction of air in the gas phase as a function of time for different boiler volumes. When the volume of the boiler increases, so does the volumic fraction of air.

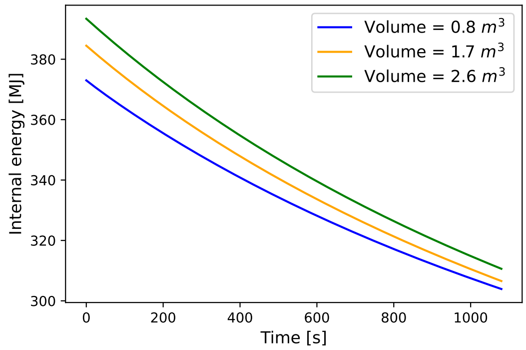


Figure 3.16: Internal energy of the boiler after the opening of the valve for different boiler volumes that are all at the same initial pressure. The initial internal energy as a function of time follows a curve with approximately the same slope, independent of the volume of the boiler but with a higher offset for larger volumes.

The temperature inside the boiler is directly dependent on its internal energy. If the internal energy follows curves with similar slopes, so will the temperature. The temperature having the same initial value for all the volumes, curves will be overlapping (Figure 3.17).

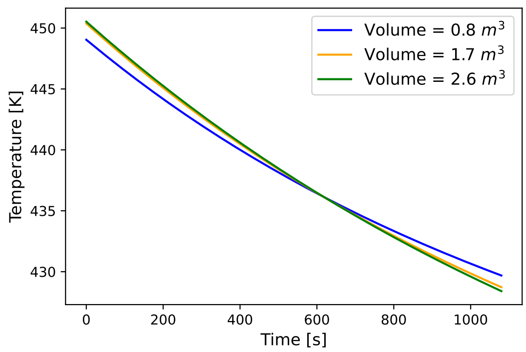


Figure 3.17: Temperature inside the boiler after the opening of the valve for boilers that have different volumes but are all at the same initial pressure. The temperature follows a similar curve independent of the volume of the boiler.

The steam being at saturated conditions, the partial pressure of steam only depends on its temperature. If the temperature curve does not depend on the volume of the boiler, the curve of the partial pressure of steam does not either (Figure 3.19a).

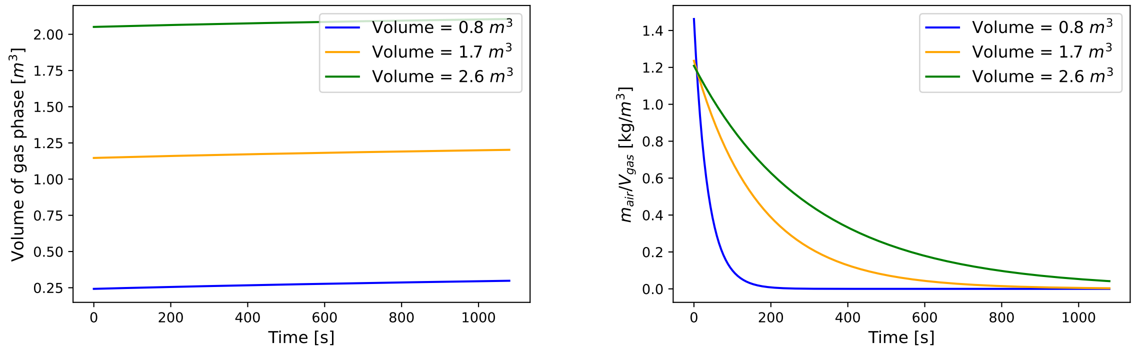
The partial pressure of air is estimated by the ideal gas law:

$$p_{air} = \frac{m_{air} R T}{V_{gas}}. \tag{3.12}$$

We know that the evolution of the temperature does not depend on the volume of the boiler and neither does R, which is only dependent on the nature of the gas. The total pressure being the same at the opening of the valve for all the volumes, so is the ratio $\frac{m_{air}}{V_{gas}}$. We know that:

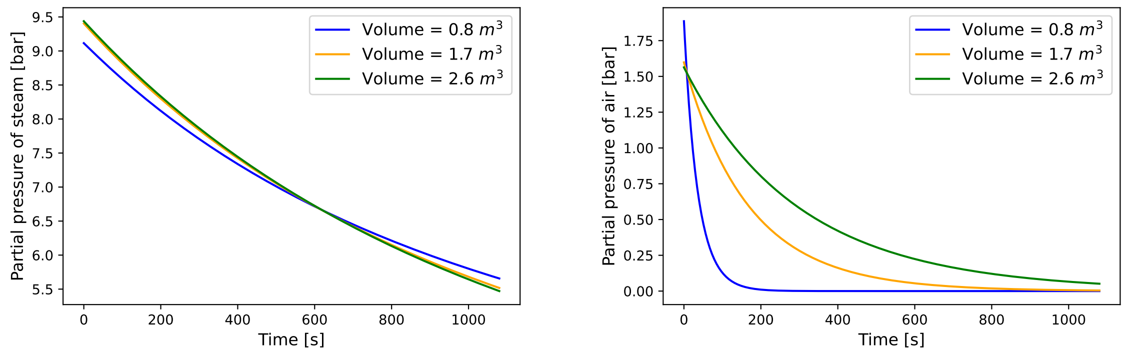
- The mass of air inside the boiler (m_{air}) decreases less for higher volumes (Figure 3.14b);
- the volume of the gas phase (V_{gas}) stays approximately constant as gas leaves the boiler at each instant through the outlet pipe but new gas is also generated at each instant by evaporation of the water inside the boiler (Figure 3.18a).

Thus, the $\frac{m_{air}}{V_{gas}}$ ratio will decrease less for larger volumes (Figure 3.18b) and so will the partial pressure of air (Figure 3.19b).



(a) (b)

Figure 3.18: Volume of the gas phase (a) and ratio of the mass of air on the volume of the gas phase (b) as functions of time for different boiler volumes. The volume of the gas phase stays approximately constant over time for all the boiler volumes, and the ratio of the mass of air on the volume of the gas phase decreases less when the volume of the boiler increases.



(a) (b)

Figure 3.19: Partial pressure of steam (a) and of air (b) inside the boiler as functions of time for different boiler volumes. The curves of the partial pressure of steam are similar for every boiler volume while the partial pressure of air decreases less when the volume of the boiler is larger.

The total pressure inside the boiler is the sum of the partial pressure of steam and the partial pressure of air. The partial pressure of steam does not vary significantly with the volume of the boiler (Figure 3.19a) while the partial pressure of air increases (Figure 3.19b), and therefore the total pressure also increases (Figure 3.20).

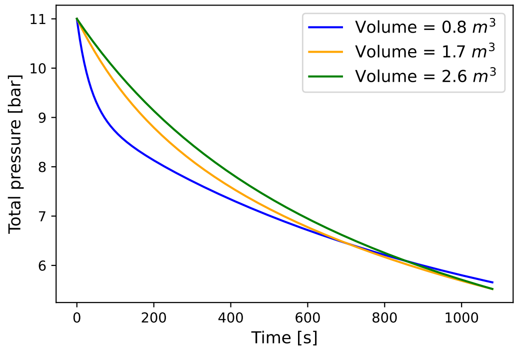


Figure 3.20: Pressure inside the boiler after the opening of the valve for boilers that have different volumes but are all at the same initial pressure. The pressure has a less steep decrease when the volume is higher.

If the pressure decrease is less steep, the decrease of the mass flow rate will also be less steep, and the mean mass flow rate will be higher (Figure 3.21).

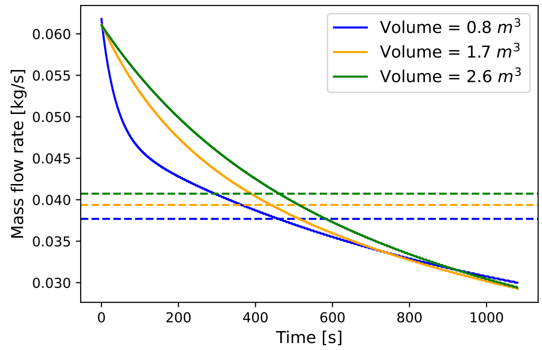


Figure 3.21: Outgoing mass flow rate after the opening of the valve for different boilers that have the same initial pressure but different volumes. When the initial volume is larger, the decrease of the mass flow rate is less steep, and thus the mean mass flow rate (dashed lines) is higher.

In Figure 3.21, we clearly see that, for a fixed (maximal) pressure at the moment of the valve opening, the mean mass flow rate increases with an increasing volume of the boiler. We can deduce that, to have the same mean mass flow rate, the maximal pressure in the boiler can be lower for larger volumes.

With these results, we can obtain the heating time and the needed volume of material to build the boiler (by considering that the material is ordinary steel) for several volumes of the boiler in Table 3.3.

Table 3.3: Heating time and volume of material for several boiler volumes.

$V_{boiler}[m^3]$	$t_{heat}[min]$	$V_{mat}[m^3]$
0.8	108	0.0493
1.7	130	0.1089
2.6	145	0.1736

Conclusion

By increasing the volume of the boiler, the maximal pressure inside the boiler to have the same mean mass flow rate, is lower.
On the other hand, as the volume of the boiler increases, the needed volume of material and the heating time also increase. Moreover, the modification of the volume of the boiler requires the replacement of the boiler and is expensive and complicated to implement.

3.5 Influence of the initial water volume in the boiler

In this section, we study the influence of the initial water volume pumped into boiler before starting the closed boiler heating, on the maximal pressure inside the boiler.

This parameter is very interesting to analyze as it is very easy to modify in reality. There are no modifications to be made on the actual design of the boiler; the operator only has to wait a longer or shorter time before stopping the pumping of the water into the boiler (Step 0 of Figure 2.2).

Parameters

To model the influence of the volume of water initially pumped into the boiler, the following parameters are fixed:

- the volume of the boiler: $V_{boiler} = 0.8m^3$;
- the geometry of the boiler: $r = 0.42m$ and $H = 1.43m$;
- the design of the heat exchanger: 18 smoke tubes with the same dimensions as the actual smoke tubes.

Results of the simulation

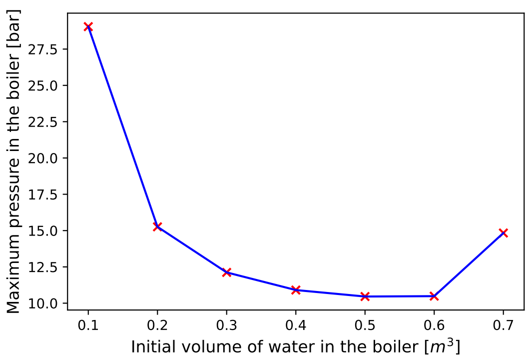


Figure 3.22: Maximal pressure in the boiler as a function of the initial volume of water pumped into the boiler.

On the graph of the maximal pressure inside the boiler as a function of the initial water volume (Figure 3.22), we see that at low initial water volumes, the maximal pressure decreases with an increasing water volume. At a certain initial water volume around

$0.5m^3$, the maximal pressure reaches a minimal value. Once the water volume exceeds $0.5m^3$, the maximal pressure starts to increase with increasing water volumes.

The maximal pressure in the boiler as a function of the initial water volume is the result of a trade-off between the influence of the heat flux and the influence of the mass of air inside the boiler.

As discussed in Section 3.2, when the heat flux from the exhaust gases to the inside of the boiler increases, the mass flow rate decreases less with time during the operation of the boiler.
If the water level in the boiler is lower, there will be a greater exchange surface in contact with gas and a smaller exchange surface in contact with water. The convection coefficient of water being considerably higher than the convection coefficient of the gas (Section 2.8.1), the heat flux will be smaller when the initial water volume is smaller (Figure 3.23) and induce a decrease in the mass flow rate at small initial water volumes.

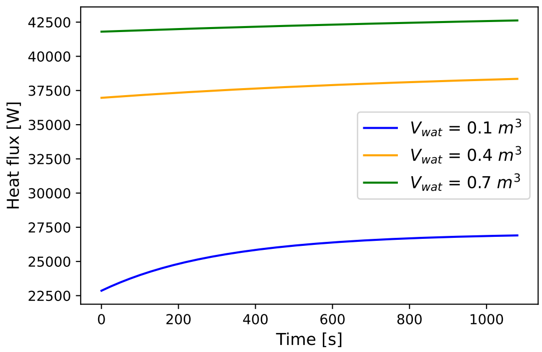


Figure 3.23: Heat flux from the exhaust gases to the inside of the boiler as a function of time for different initial water volumes. When the water volume increases, the heat flux is higher.

As discussed in Section 3.4, when the mass of air inside the boiler increases, the mass flow rate decreases less with time.
If the water volume is higher but the volume of the boiler and its geometry are constant, the water level is higher and there will be less air inside the boiler, inducing a lower mean mass flow rate at high initial water volumes (Figure 3.24).

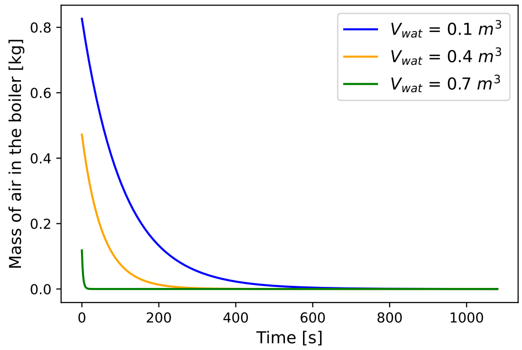


Figure 3.24: Mass of air inside the boiler as a function of time for different initial water volumes. When the water volume increases, the mass of air is lower and decreases more.

We put those two elements together to understand that the mass flow rate will present a steep decrease with a low initial water volume because of the low heat flux, and a steep decrease at high water volumes because of the low mass of air inside the boiler (Figure 3.25). This implies that there exists an initial water volume that minimizes the decrease of the mass flow rate during the operation of the boiler. If the decrease in mass flow rate is smaller, it can have a smaller initial value to reach the same mean value, and thus a smaller initial and maximal pressure.

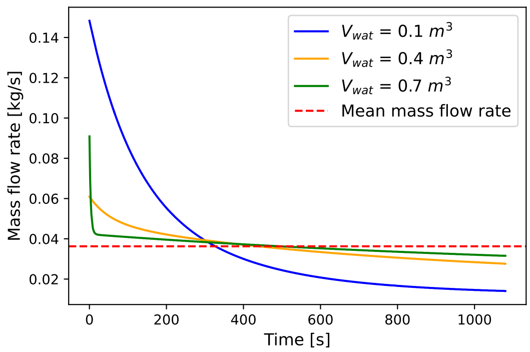


Figure 3.25: Mass flow rate leaving the boiler as a function of time for different initial water volumes. The initial water volume that induces the mass flow rate with the lowest initial volume, is the result of a compromise. At low initial water volumes, the heat exchange is low and at high initial water volumes, the mass of air inside the boiler is low.

With these results, we can obtain the heating time and the needed volume of material to build the boiler (by considering that the material is ordinary steel) for several initial water volumes in Table 3.4.

Table 3.4: Heating time and volume of material for several initial water volumes.

$V_{wat} [m^3]$	$t_{heat} [min]$	$V_{mat} [m^3]$
0.1	46	0.166
0.4	93	0.0527
0.7	139	0.0703

Conclusion

If no other parameter of the boiler is changed in the new design, the actual initial water volume is close to its optimal value.
On the other hand, for every other combination of the three previous parameters of the boiler (F_{heat}, H_{boiler} and V_{boiler}) there exists an other optimal initial water volume ($V_{water,opt}$) that reduces the maximal pressure inside the boiler that is needed to obtain the same mean mass flow rate. Although the effect of working at the optimal water volume is not significant, its modification has no disadvantage and is very easy to implement. The optimization of the initial water volume also minimizes the needed volume of material but the heating time is the lowest when the water volume is the lowest.

3.6 Material selection and dimensioning

One of the incentives for reducing the pressure inside the boiler, is to reduce the cost of it. This is done by dimensioning and choosing the material of the boiler with the objective of minimizing the cost.

The cost C of the boiler can be expressed as:

$$C = X_v \times V_{mat} \tag{3.13}$$

$$= X_v \times \left(2\pi Hrt + 2t\pi r^2\right) \tag{3.14}$$

where X_v is the volumic cost of the used material and V_{mat} is the volume of material of the boiler. The material of the outlet pipe and of the smoke tubes is not included in this computation. We consider that, even if the design of the boiler is modified, the thicknesses of the outlet pipe or the smoke tubes will not be modified.

While we fix the volume, radius r, height H and pressure inside the boiler, the thickness of the walls can be changed.
The boiler has to be able to hold this pressure without being deformed. With the stresses computed in Appendix A and the Tresca criterion (Equation 2.59), we obtain:

$$\sigma_{yield} \geq \frac{pr}{t} \rightarrow t \geq \frac{pr}{\sigma_{yield}} \tag{3.15}$$

We consider that the environment, loads and stresses are uncertain, so we use a safety factor SF of 4, considering that the boiler is subjected to cyclic loadings, we multiply this safety factor by 1.5 to obtain $SF = 6$ [11]. By taking the smallest possible thickness, we have:

$$t = SF \times \frac{pr}{\sigma_{yield}} \tag{3.16}$$

$$\rightarrow C = X_v \times \left(SF \times 2\pi Hr\frac{pr}{\sigma_{yield}} + SF \times 2\frac{pr}{\sigma_{yield}}\pi r^2\right) \tag{3.17}$$

$$= SF \times \frac{X_v}{\sigma_{yield}}\pi pr^2 \left(2H + 2r\right) \tag{3.18}$$

Parameters p, r and H are fixed, so we need to minimize $\frac{X_v}{\sigma_{yield}}$.

To do so, we use the Granta Edupack software [1].

3.6.1 Material selection

We start by plotting all the existing materials on a graph of σ_{yield} as a function of X_v (Figure 3.26).

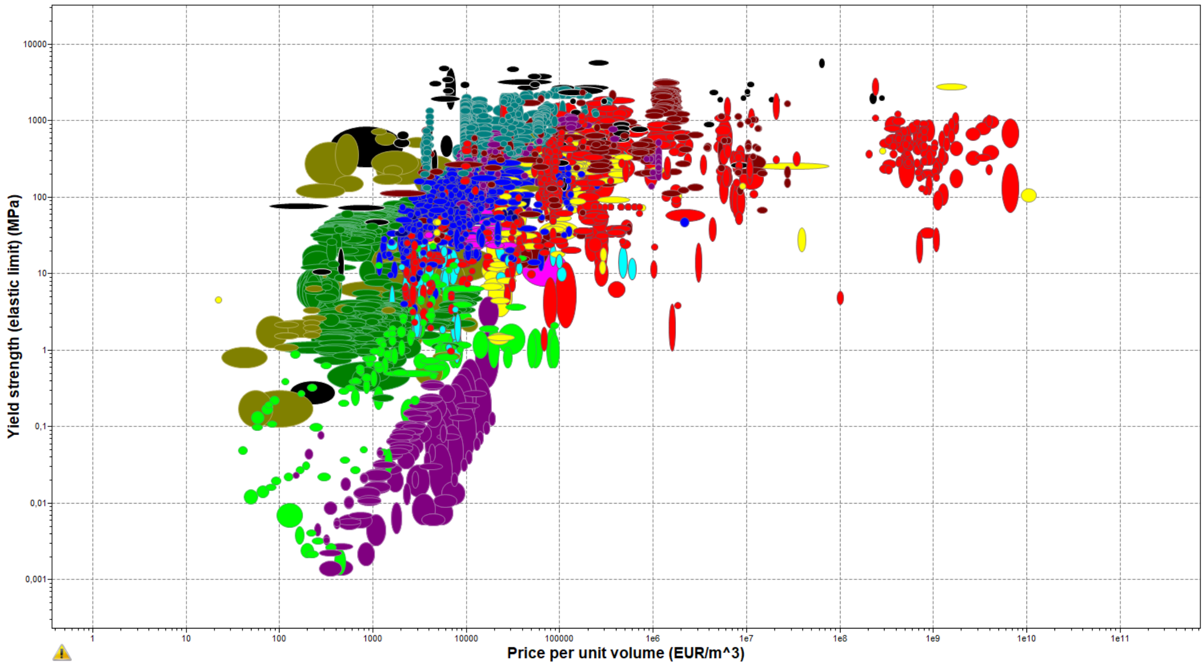


Figure 3.26: The yield strength as a function of the price per unit volume for all the existing materials.

Next, for the material to be appropriate in the application of the boiler, we exlude all the materials that do not have the following properties:

- Water resistant
- UV radiation resistant
- Non-flammable
- Good weldability

The materials that do not meet these criteria are put in grey in the graph of Figure 3.27.

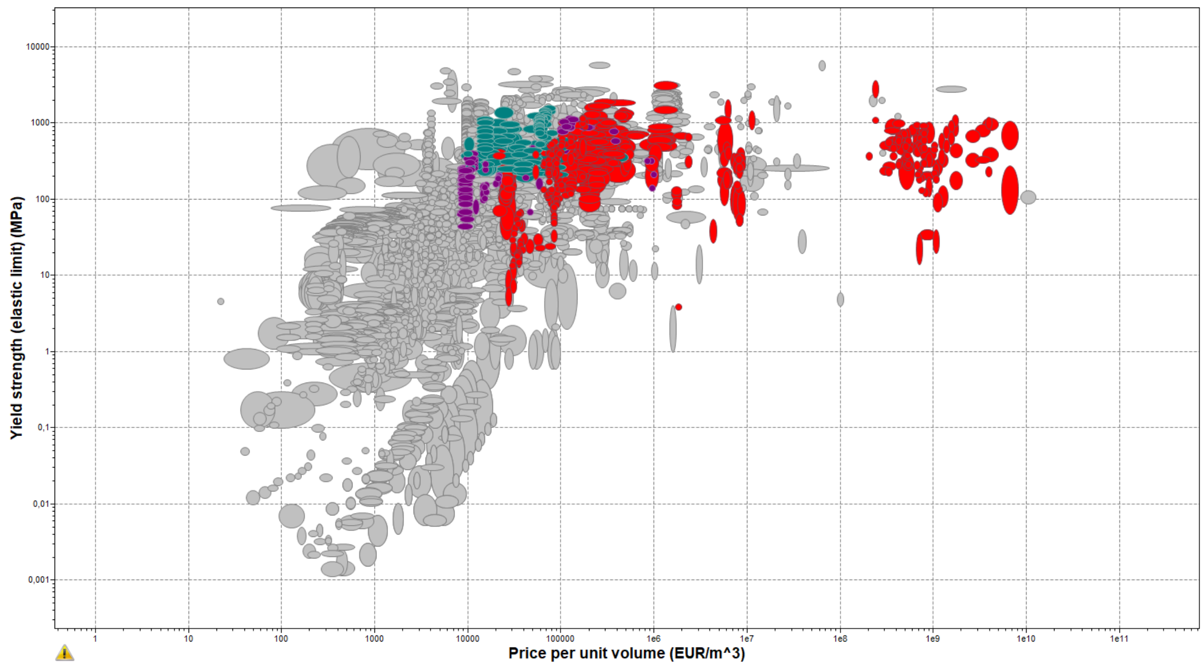


Figure 3.27: The yield strength as a function of the volumic price for all the existing materials that are suitable for our application (in color).

To obtain the material that minimizes $\frac{X_v}{\sigma_{yield}}$, we plot a line with slope 1 and we increase the offset as much as possible. The last material remaining above the line is the material that minimizes $\frac{X_v}{\sigma_{yield}}$ and the cost of the boiler.

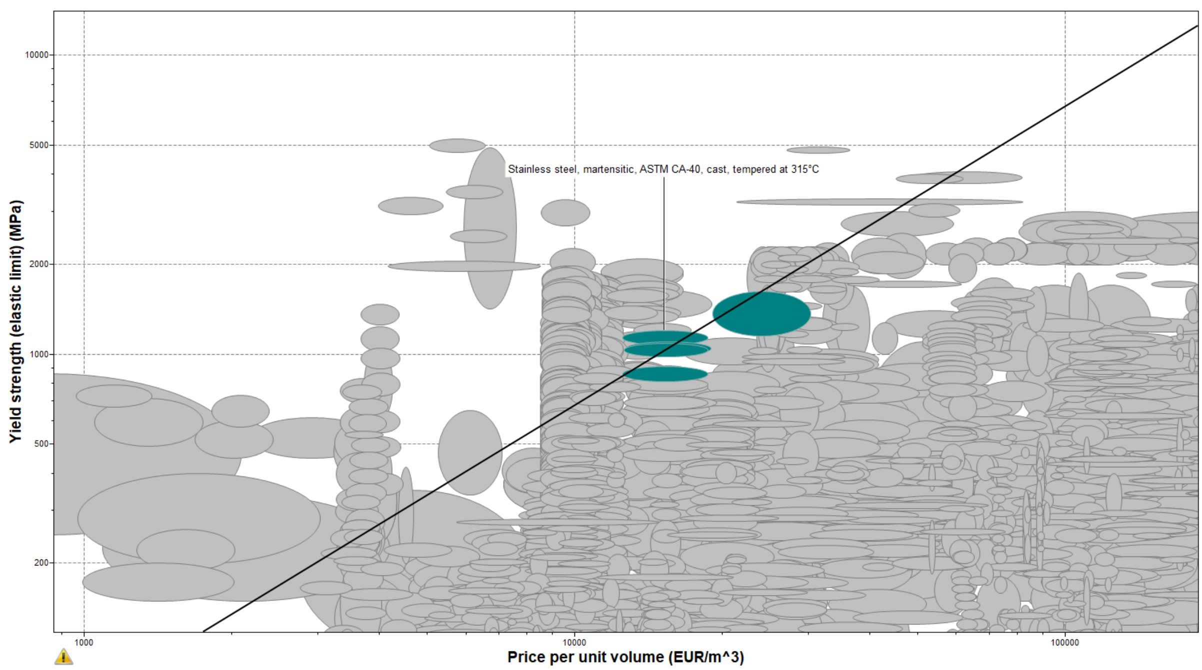


Figure 3.28: The yield strength as a function of the volumic price for all the existing materials that are suitable for our application. The material with the lowest ratio of its volumic price over its yield strength, is the last material that remains above a line with slope 1 when the offset of the line is increased.

On Figure 3.28, we see that the best material is stainless steel, whereas ordinary steel is actually being used.
After discussions with an engineer from the NITIDAE company, we learned that this decision was taken for economic reasons, as ordinary steel was much cheaper than stainless steel, even if ordinary steel is not water resistant.
By using ordinary steel, the frequency with which boiler parts need to be replaced is greatly increased. The savings made by using ordinary steel are thus questionable in the long term.

The actual boiler design requires some parts to be welded. If the boiler could be redesigned to eliminate the need for welding, a much cheaper and stronger material could be used, as shown in Appendix B.

3.6.2 Influence of the material on the thickness and on the price

The different properties of stainless steel are [1]:

- $\sigma_{yield}(0°C) = 1.15 \times 10^3 [MPa]$
- $X_v = 1.5 \times 10^4 [€/m^3]$

For each maximal temperature inside the boiler, σ_{yield} will have a different value. We enumerate the different factors F_{yield}, corresponding to different temperatures, that have to be multiplied by $\sigma_{yield}(0°C)$ to obtain the yield strength at that temperature [16]. The values of F_{yield} with the corresponding yield strengths of ordinary steel and stainless steel are given in Table 3.5.

Table 3.5: Factor multiplying the yield strength for different temperatures and the resulting yield strengths of steel where SS refers to stainless steel and OS refers to ordinary steel

	$T = 273.15K$	$T = 457K$	$T = 438K$	$T = 436K$	$T = 430K$
F_{yield}	1	0.8	0.84	0.84	0.86
$\sigma_{yield,SS}[MPa]$	1150	920	966	966	989
$\sigma_{yield,OS}[MPa]$	305	244	256	256	262

We can compute the thickness of the boiler walls, the volume of material needed, and the cost of the boiler for different values of pressure, radius and length with the following formula:

$$t = SF \frac{pr}{\sigma_{yield}} \tag{3.19}$$

$$V_{mat} = \left(2\pi Hrt + 2t\pi r^2\right) \tag{3.20}$$

$$C = X_v \times \left(2\pi Hrt + 2t\pi r^2\right) \tag{3.21}$$

Again, the price of the boiler can only be used for comparison, as it only represents the price of the raw material. If the actual boiler would be replaced (same values of p, r, and H but stainless steel as material), we could obtain the thickness, volume of material and price given in Table 3.6.

Table 3.6: Values of the thickness of the walls, the volume of material and the cost of the boiler if we keep the same boiler than the actual case but replace the material with stainless steel.

	$t[mm]$	$V_{mat}[m^3]$	C [€]
Actual case $\rightarrow p_{max} = 11bar$ $\rightarrow T_{max} = 457°C$ $\rightarrow r = 0.42m$ $\rightarrow H = 1.43m$	3.1	0.015	227

3.7 Combination of parameters

To conclude this section, we make a few combination of the following parameters :

- the heat flux factor (F_{heat});
- the height of the boiler (H);
- the volume of the boiler (V_{boiler});
- the initial water volume pumped into the boiler (V_{water}).

These combinations are not optimisations, but only examples to understand what could be obtained with some reasonable designs.

For each of these designs, we model the maximal pressure and temperature inside the boiler, and we compute the heating time, the thickness of the boiler walls and volume of material used to build the boiler. With these values, we can visualize the extent to which these combinations match the incentives formulated in Section 1.2. The designs are defined and the results are given in Table 3.7.

Table 3.7: Definition of four new designs of the boiler combining the four previously analyzed parameters, and the results obtained with these designs.

	Actual boiler	Design 1	Design 2	Design 3	Design 4
$F_{heat}[/]$	1	2	1	2	2
$H[m]$	1.43	1.43	2	2	2
$V_{boiler}[m^3]$	0.8	0.8	2	2	0.8
$V_{water}[m^3]$	0.5	0.43 ($V_{water,opt}$)	0.94 ($V_{water,opt}$)	0.46 ($V_{water,opt}$)	0.24 ($V_{water,opt}$)
Material	OS	OS	SS	SS	SS
$p_{max}[bar]$	11	8.3	8.6	7.2	6.3
$T_{max}[K]$	457	436	438	430	424
$t_{heat}[min]$	110	45	145	41	20
$V_{mat}[m^3]$	0.0488	0.0399	0.0269	0.0225	0.008
C[€]	469	383	404	338	120

In this table, OS is ordinary steel, SS is stainless steel and $V_{water,opt}$ is the initial water volume that reduces the maximal pressure inside the boiler while keeping the same mean outgoing mass flow at the chosen volume of the boiler V_{boiler}.

To compare these four new designs with the actual design, we make an evaluation chart with five criteria:

- complexity of implementation;
- frequency of replacements due to wear;
- maximal pressure inside the boiler;
- heating time;
- needed volume of material.

Each design is evaluated for each criterion by a number between 1 and 4, a 4 meaning that the criterion is not met and a 1 meaning it is met.
At the end, the design for which the sum of the grades is the lowest, is considered to be the best design. The evaluation metric is detailed in Appendix C. The evaluation chart is given in Table 3.8.

Table 3.8: Evaluation chart of the four designs defined in Table 3.7. Design 4 obtains the best score.

	Actual situation	Design 1	Design 2	Design 3	Design 4
Complexity	1	2	3	4	4
Wear	4	4	1	1	1
Maximal pressure	4	3	3	2	1
Heating time	3	1	4	1	1
Cost	4	2	3	1	1
Occupied space	1	1	4	4	1
Total	17	13	18	13	9

The design that best meets our requirements is design 4, which combines an increased heat transfer, an increased height and an optimal water volume, while keeping the actual boiler volume. The following two best designs are equally design 1 and design 3. The common ground of these three designs is that they all have an improved heat exchange. If design 4 is judged too complex because of the replacement of the boiler and/or the high space it requires, and the results of design 1 are sufficient, this design can also be a good alternative.

We conclude this chapter by bringing to the fore that the most efficient and simple way to achieve the goal of this thesis, is the improvement of the heat exchanger. If, together with the initial water volume that always can be optimized, this is the only modified parameter (as in design 1) the boiler does not need to be replaced. However, if the replacement of the boiler is not a problem and the obtained results are not sufficient, the height of the boiler can additionally be increased (as in design 4) to obtain better results. Moreover, the results of the improvement of the heat exchange and the height of the boiler are both very promising, as they considerably reduce the heating time, the volume of material needed to build the boiler and the maximal pressure inside the boiler, and so the risks when empty heating.

Chapter 4

Conclusion

To design a new boiler that meets the goals of the project, the actual boiler used in the ANASAM company has first been modeled. Even if strong assumptions have been made, an estimation of the output of the boiler has been obtained.

Next, several parameters of the boiler have been varied one by one, while keeping the others fixed and preserving the outputs of the boiler, and the influence of these parameters on the maximal pressure inside the boiler has been analyzed.

The first parameter that has been analyzed is the heat exchange between the exhaust gases of the pyrolysis furnace and the inside of the boiler. This can be improved by increasing the number of smoke tubes, by adding fins on the inside or/and on the outside of the tubes or by increasing the rugosity of the smoke tubes, for example. The increase of the heat exchange implies a significant decrease in the maximal pressure inside the boiler, in the heating time and in the material quantity needed to build the boiler, and thus the improvement of the heat exchanger would be a very interesting modification to be made on the boiler.

The second analyzed parameter of the boiler is its height. This parameter also has a positive effect on the maximal pressure, the needed volume of material and the heating time. Yet, the height has to be improved a lot to have effects as beneficial as the improvement of the heat exchanger. Moreover, the increase of the height requires the replacement of the boiler.
If the boiler needs to be replaced or the effects of the increase of the heat exchange are insufficient, additionally increasing the height of the boiler is a good solution.

Next, the increase of the volume of the boiler is studied. Although the increase of the volume of the boiler induces a reduction of the maximal pressure, it also has a negative effect on the needed volume of material and the heating time. In addition, the increase of the volume also requires the replacement of the boiler.

Finally, the influence of the initial water volume is studied. We found that for every combination of the three first parameters, there is an optimal initial water volume that decrease the maximal pressure inside the boiler. Even though the effects of this optimization are not very significant, it does not require any modification to be made on the actual boiler and there is no disadvantage.

The choice of ordinary steel for the material of the boiler is questioned. Even though it is much less expensive than stainless steel, it is also less strong and thus the walls need to be thicker. Moreover, the replacements of the boiler due to the wear caused by water would be much less frequent when using stainless steel.

To conclude, the increase of the heat flux and the optimization of the initial water volume inside the boiler are the two easiest solutions that will, at the same time, reduce the maximal pressure inside the boiler -and so the risks when empty heating-, reduce the economic impact of the boiler by reducing the material quantity requirements, and reduce the environmental impact of the boiler by decreasing the heating time. If the effects of these two parameters are insufficient, the height of the boiler can be increased which will also have beneficial effects on the three concerns.
If the decrease of the maximal pressure is considered more important than the two other motives, the volume of the boiler can also be increased at the expense of the economical and environmental impacts of the boiler.

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Appendix A

Stresses in a cylindrical pressure vessel [15]

A.1 Circumferential stress

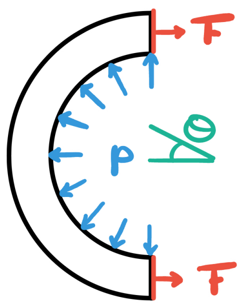

Figure A.1: Forces acting circumferentially on the vessel.

Equilibrium of forces :

$$2F = \int_0^{2\pi} p \times H \times r \times \cos\theta d\theta \tag{A.1}$$

$$\rightarrow F = pHr \tag{A.2}$$

Definition of stress :

$$F = \sigma_{\theta\theta} \times A_\theta \tag{A.3}$$

$$= \sigma_{\theta\theta} \times H \times t \tag{A.4}$$

$$\rightarrow \sigma_{\theta\theta} = \frac{pr}{t} \tag{A.5}$$

where A_θ is the surface of application of the force F, p is the pressure inside the vessel, r is the radius of the vessel, H is the length of the vessel and t is the thickness of the walls.

A.2 Longitudinal stress

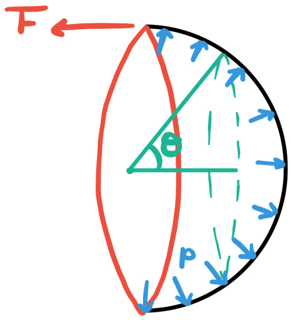

Figure A.2: Forces acting longitudinally on the vessel.

Equilibrium of forces :

$$F = \int_0^{2\pi} p \times 2\pi \times \sin\theta \times r \times \cos\theta d\theta \qquad (A.6)$$
$$= p\pi r^2 \qquad (A.7)$$
$$(A.8)$$

Definition of stress :

$$F = \sigma_{zz} \times A_z \qquad (A.9)$$
$$= \sigma_{zz} \times 2\pi \times t \times r \qquad (A.10)$$
$$\rightarrow \sigma_{zz} = \frac{pr}{2t} \qquad (A.11)$$

where A_z is the surface of application of the force F, p is the pressure inside the vessel, r is the radius of the vessel and t is the thickness of the wall.

A.3 Radial stress

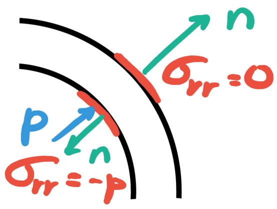

Figure A.3: Forces acting radially on the vessel.

In this case, the radial stress at the inside of the vessel is $\sigma_{rr} = p$ and at the outside of the vessel $\sigma_{rr} = 0$. Which means:

$$-p < \sigma_{rr} < 0$$

In the case that $\frac{r}{t} << 1$, we have:

$$\sigma_{rr} << \sigma_{zz}, \sigma_{\theta\theta} \qquad (A.12)$$

We obtain the stress tensor of pressure vessel in function of its radius r, the thickness of its walls t and the pressure inside the vessel p:

$$\begin{bmatrix} \approx 0 & 0 & 0 \\ 0 & \frac{pr}{2t} & 0 \\ 0 & 0 & \frac{pr}{t} \end{bmatrix}$$

Appendix B

Material selection and dimensioning for a boiler with no welding

B.1 Material selection

If the boiler does not need to be welded, we exclude the materials that do not have the following properties:

- Water resistant
- UV radiation resistance
- Non-flammable

The graph becomes:

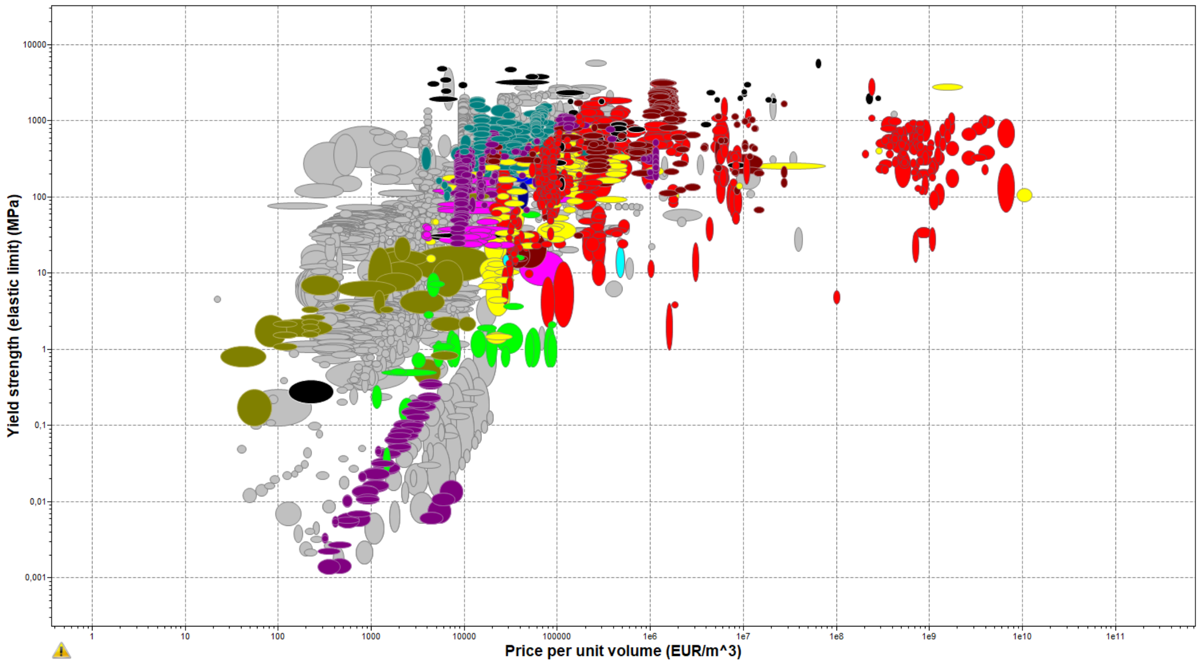

Figure B.1: The yield strength as a function of the volumic price for all the existing materials that are suitable for our application.

Again, to obtain the material that minimizes $\dfrac{X}{\sigma_{yield}}$, we plot a line of slope 1 and we increase the offset as much as possible. The last material remaining above the line is the material that minimizes the cost of the boiler:

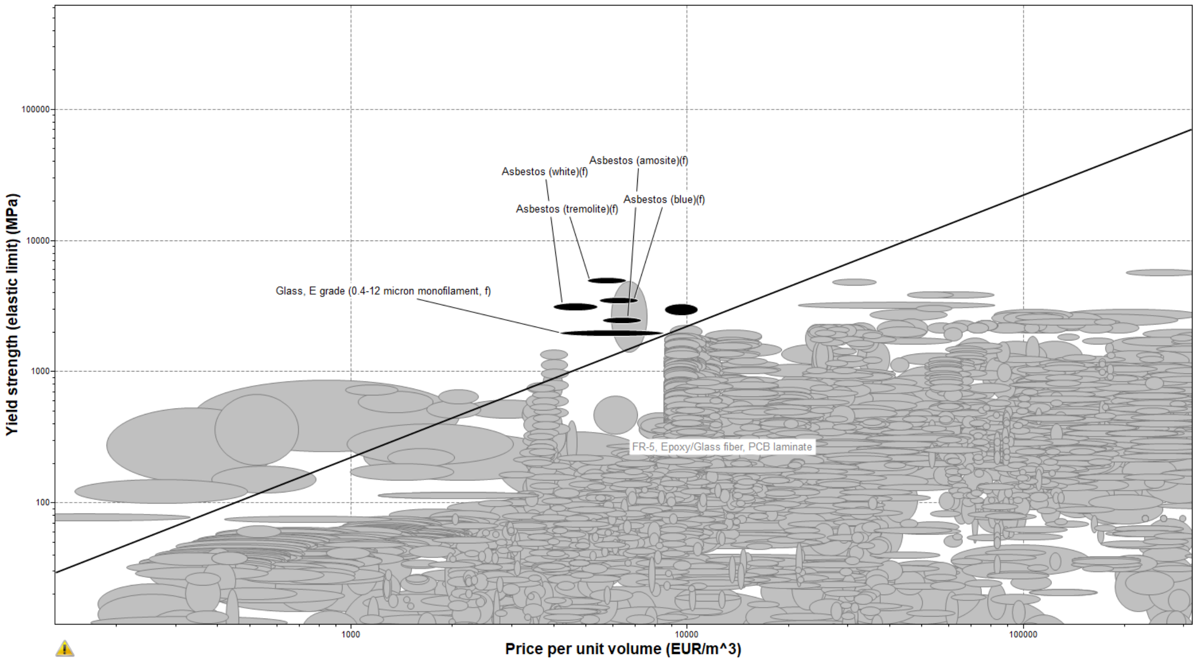

Figure B.2: The yield strength as a function of the volumic price for all the existing materials that are suitable for our application. The material with the lowest ratio of its volumic price over its yield strength, is the last material that remains above a line with slope 1 when the offset of the line is increased.

We see that the best material is asbestos.
Asbestos being toxic and the water contained in the boiler being sprayed on cashew nuts, we exclude this material. We look for the second best material which is "Glass,E grade".

Appendix C

Evaluation metric for the choice of a new design

Table C.1: Evaluation matrix for the choice of the new design of the boiler

	1	2	3	4
Complexity	Nothing is replaced.	Only the heat exchanger is replaced.	Only the boiler is replaced.	The boiler and the heat exchanger are replaced.
Frequency of replacements due to wear	The material is water resistant, less frequent replacements.	/	/	The material is not water resistant more replacements.
Maximal pressure	$p_{max} \leq 7bar$	$7bar < p_{max} < 8bar$	$8bar < p_{max} < 9bar$	$p_{max} \geq 9bar$
Heating time	$t_{max} \leq 60min$	$60min < t_{max} < 90min$	$90min < t_{max} < 120min$	$t_{max} \geq 120min$
Cost	$C \leq 350$€	350€ $< C <$ 400€	400€ $< C <$ 450€	$C \geq 450$€
Occupied space	$V_{boiler} \leq 1m^3$	$1m^3 < V_{boiler} < 1.5m^3$	$1.5m^3 < V_{boiler} < 2m^3$	$V_{boiler} > 2m^3$

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