APPENDIX A

MODEL: TRANSFORMED FLUX VECTOR

The transformed magnetic flux vector is obtained through (3.17) and (3.28):

$$\mathbf{\Phi^s} = \mathbf{U^{-1}\Phi}. \tag{A.1}$$

Knowing the identity:

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}, \tag{A.2}$$

the vector $\mathbf{\Phi}$ can be expressed as:

$$\mathbf{\Phi} = zK_\phi(\mathbf{G} + \mathbf{D}), \tag{A.3}$$

where the components of $\mathbf{G}$ and $\mathbf{D}$ are:

$$\begin{aligned} G_k &= \frac{e^{j\left(\frac{2\pi}{N}(k-1)+p(\delta_0-\alpha)\right)}}{2}; \\ D_k &= \frac{e^{-j\left(\frac{2\pi}{N}(k-1)+p(\delta_0-\alpha)\right)}}{2}, \end{aligned} \tag{A.4}$$

with $k = \{1, \ldots, N\}$. Let us calculate the components of $\mathbf{U^{-1}G}$ and $\mathbf{U^{-1}D}$ separately:

$$\begin{aligned} \left[\mathbf{U}^{-1}\mathbf{G}\right]_k &= \sum_{i=1}^{N} U_{ki}^{-1} G_i \\ &= \frac{e^{-jp(\alpha-\delta_0)}}{2\sqrt{N}} \sum_{i=1}^{N} e^{j\frac{2\pi}{N}(i-1)(k-1)} e^{j\frac{2\pi}{N}(i-1)} \\ &= \frac{e^{-jp(\alpha-\delta_0)}}{2\sqrt{N}} \sum_{i=1}^{N} e^{j\frac{2\pi}{N}(i-1)k} \\ &= \frac{\sqrt{N}}{2} e^{-jp(\alpha-\delta_0)}, \qquad \text{if } k = N \end{aligned} \tag{A.5}$$

and 0 otherwise. The term $\mathbf{U}^{-1}\mathbf{D}$ is derived in a similar way:

$$\begin{aligned} \left[\mathbf{U}^{-1}\mathbf{D}\right]_k &= \sum_{i=1}^{N} U_{ki}^{-1} D_i \\ &= \frac{e^{jp(\alpha-\delta_0)}}{2\sqrt{N}} \sum_{i=1}^{N} e^{j\frac{2\pi}{N}(i-1)(k-1)} e^{-j\frac{2\pi}{N}(i-1)} \\ &= \frac{e^{jp(\alpha-\delta_0)}}{2\sqrt{N}} \sum_{i=1}^{N} e^{j\frac{2\pi}{N}(i-1)(k-2)} \\ &= \frac{\sqrt{N}}{2} e^{jp(\alpha-\delta_0)}, \qquad \text{if } k = 2 \end{aligned} \tag{A.6}$$

and 0 otherwise. Substituting (A.5) and (A.6) into (A.1) yields:

$$\mathbf{\Phi}^{\mathbf{s}} = z\frac{K_{\Phi}\sqrt{N}}{2}\begin{bmatrix} 0 \\ \exp(jp(\alpha-\delta_0)) \\ 0 \\ \vdots \\ 0 \\ \exp(-jp(\alpha-\delta_0)) \end{bmatrix}. \tag{A.7}$$