

École polytechnique de Louvain

Spin-diode effect in vortex based nano-oscillators for hardware artificial intelligence applications

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Abstract

A hardware implementation of neural networks is considered through the study of artificial neurons and synapses. Firstly spin-transfer nano-oscillators have been investigated as physical neurons. Particular focus is made on vortex-based oscillators excited by an out-of-plane spin-polarized current. An analytical model describing the vortex dynamics is proposed based on Thiele equation. Nonlinearities related to the vortex core displacement are taken into account. Micromagnetic simulations are performed with mumax³ to allow comparison with theoretical results. Two oscillation regimes are identified, both falling in radiofrequency range. Below a critical current, relaxation is obtained and the device frequency evolves proportionally to the current density. Above this threshold value gyrotropic precessions are observed, leading to a nonzero steady orbit radius and nonlinear frequency evolution. Moreover a significant impact of the Ampère-Oersted field on the dynamics is shown. Using these simulations data-driven corrections are made to the model. These fittings allow to derive with great accuracy the vortex properties both in transient and steady-state regime. Finally spin-diode effect in magnetic tunnel junctions is investigated. The semi-analytical model predictions are compared to experimental results obtained by partners. An unprecedented agreement is demonstrated. Secondly a memristor device mimicking synaptic behaviors has been explored. The latter is based on an interconnected silver nanowire network. Following a technique developed in past decades at UCLouvain a track-etched polycarbonate membrane is filled with Ag wires by electrodeposition. After this step the sample is fragmented by plasma etching-induced stresses. This allows to create gaps between Ag wire segments. Then voltage sweeps up to 100 V are performed. Characteristic memristive hysteresis is present in the I-V curves. Remanence is also observed. Sample resistance, in the $G\Omega$ range, decreases with successive sweeps. The creation of conductive Ag filament in the gaps separating segments is suggested. Those would result from field-induced ion migration along the nanopore walls. Spontaneous breaking of the bridging is remarked with time. Evidences of electron direct tunneling and field emission are shown in the first voltage scan. After tens of sweeps a dramatic change of properties is measured. The sample presents a resistance more than three orders of magnitude below its original value. Very peculiar behavior is shown in this regime, as the sample conductance is minimum for the largest applied bias.

Keywords: Spintronics, Nanomagnetism, Nano-oscillator, Vortex, Memristance, Tunneling, Artificial intelligence, Neuromorphic computing.

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List of scientific communications

Communications to be submitted during the next 2 months.

- [1] Flavio Abreu Araujo, Chloé Chopin, and **Simon de Wergifosse**. *Ampère-Oersted field splitting of the nonlinear spin-torque vortex oscillator dynamics*. Ready to be submitted to *Scientific Reports*.
- [2] Chloé Chopin, **Simon de Wergifosse**, Nicolas Marchal, Pascal Van Velthem, Luc Piraux, and Flavio Abreu Araujo. *Memritive and tunnelling effects in 3D interconnected Ag nanowires*. Writing is under progress. (Target: *Appl. Phys. Lett.* or *Scientific Reports*)
- [3] Flavio Abreu Araujo, **Simon de Wergifosse** and Chloé Chopin. *Data driven Thiele equation approach for solving nonlinear spin-torque vortex oscillator dynamics*. Writing is under progress. (Target: *Nature Comm.*)

Communications to be submitted before the end of 2021.

- [4] Flavio Abreu Araujo, Alex Jenkins, Chloé Chopin, **Simon de Wergifosse**, Julie Grollier... *Spin-diode effect inversion in vortex based nano-oscillators*. Writing is under progress. (Target: *Nature Physics*)
- [5] Chloé Chopin, Flavio Abreu Araujo, and **Simon de Wergifosse**, *Combining the spin-diode effect and machine learning to detect RF signals using vortex based nano-oscillators*. Writing is under progress. (Target: *Nature Nanotechnology*)
- [6] Flavio Abreu Araujo, Chloé Chopin, and **Simon de Wergifosse**, *Optimizing the spin-diode effect in vortex based nano-oscillators with a fast Thiele equation approach tool*. Research under progress. (Target: *Appl. Phys. Lett.*)

Nomenclature

List of acronyms

3D	Three dimensional
AC	Alternating current
AF	Antiferromagnetic
Ag	Silver
Al	Aluminum
AP	Antiparallel
Au	Gold
B	Boron
BWD	Backward
CÉCI	Consortium des équipements de calcul intensif
CF	Conductive filament
CGS	Centimeter-gram-second system of units
CMOS	Complementary metal oxide semiconductor
CPP	Current perpendicular to the plane
CPU	Central processing unit
Co	Cobalt
Cr	Chromium
Cu	Copper
DC	Direct current
DOS	Density of states
Fe	Iron
FFT	Fast Fourier transform
FLT	Field-like torque
FM	Ferromagnetic
FN	Fowler-Nordheim
FRS-FNRS	Fond de la recherche scientifique
FWD	Forward

GMR	Giant magnetoresistance
GPU	Graphics processing unit
INL	International Iberian nanotechnology laboratory
IP	In-plane
LL	Landau-Lifshift
LLG	Landau-Lifshift-Gilbert
LTD	Long-term depression
LTP	Long-term potentiation
MgO	Magnesium oxide
MR	Magnetoresistance
MRAM	Magnetic random access memory
MTJ	Magnetic tunnel junction
NaOH	Sodium hydroxide
Ni	Nickel
NM	Nonmagnetic
NP	Nanoparticle
NW	Nanowire
ODE	Ordinary differential equation
OF	Oersted field
OOP	Out-of-plane
P	Parallel
PC	Polycarbonate
Pt	Platinum
Py	Permalloy ($\text{Ni}_{80}\text{Fe}_{20}$)
RF	Radiofrequency
RKKY	Ruderman–Kittel–Kasuya–Yosida
SEM	Scanning electron microscope
SI	International system of units
SNIFA	Simple numerical instantaneous frequency approximation
SVA	Single vortex ansatz
STD	Spin-torque diode
STNO	Spin-transfer nano-oscillator
STPD	Spike timing-dependent plasticity
S(T)T	Spin(-transfer) torque
STVO	Spin-torque vortex oscillator

TEA	Thiele equation approach
TiO ₂	Titanium oxide
TMR	Tunnelling magnetoresistance
TVA	Two vortex ansatz

List of important variables

α	Spin asymmetry coefficient
α_G	Gilbert damping parameter
α_L	LL damping parameter
β	Bulk spin asymmetry coefficient
γ_e	Gyromagnetic ratio
γ_G	Gilbert gyromagnetic ratio
γ_L	LL gyromagnetic parameter
Γ	Time constant in the vortex harmonic oscillator model
δ^D	Global correction parameter for D
δ^G	Global correction parameter for G
ϵ	Spin-diode sensitivity
Θ	Core profile angle
κ	Magnetic hardness factor
κ^{ST}	Perpendicular Slonczewski force amplitude
$\kappa_{\parallel}^{\text{ST}}$	Parallel Slonczewski force amplitude
$\kappa_{\text{FLT}}^{\text{ST}}$	Field-like torque force amplitude
λ	Exchange coupling coefficient
ξ	Aspect ratio
μ	Magnetic permeability
μ_0	Vacuum permeability
μ_B	Bohr magneton
μ_e	Electron mobility
ρ	Resistivity
ρ_c	Vortex core diameter
$\rho_{\uparrow}$	Majority electron resistivity
$\rho_{\downarrow}$	Minority electron resistivity
τ	Mean scattering time
ϕ	Planar angle
ϕ	Flux linkage
χ	Magnetic susceptibility
ω	Frequency
ω_0	Relaxation frequency
a_J	Slonczewski torque amplitude
A	Exchange stiffness
b_J	Field-like torque amplitude
$\mathbf{B}$	Magnetic induction

$\mathbf{B}_{\text{Oe}}$	Oersted field magnetic induction
C	Vortex chirality
d_x	Vortex core translation in the x -axis
d_y	Vortex core translation in the y -axis
D	Damping term
D_0	Damping constant
$\bar{\mathbf{D}}$	Damping tensor
D_s	Critical domain wall size
e	Electron charge
E	Energy
E_A	Anisotropy energy
E_{ex}	Exchange energy
E_F	Fermi energy
E_{ms}	Magnetostatic energy
E_Z	Zeeman energy
f	Frequency
f_0	Resonant frequency
f^D	Polynomial fitting function for D
f^G	Polynomial fitting function for G
f_{rf}	Radiofrequency
$\mathbf{F}^D$	Damping force
$\mathbf{F}^{\text{dip}}$	Dipolar force
$\mathbf{F}^{\text{ext}}$	External forces
$\mathbf{F}^G$	Gyrotropic force
$\mathbf{F}^R$	Restoring force
$\mathbf{F}^{\text{ST}}$	Spin-torque forces
$\mathbf{F}_{\perp}^{\text{ST}}$	Perpendicular component of the Slonczewski spin-torque force
$\mathbf{F}_{\parallel}^{\text{ST}}$	Parallel component of the Slonczewski spin-torque force
$\mathbf{F}_{\text{FLT}}^{\text{ST}}$	Field-like spin-torque force
G	Gyrotropic term
G_0	Gyroconstant
$\bar{\mathbf{G}}$	Gyrotropic tensor
g_e	Electron g-factor
$G_{\uparrow\uparrow}$	Conductance of majority to majority electron channel transition
$G_{\uparrow\downarrow}$	Conductance of majority to minority electron channel transition
$G_{\downarrow\uparrow}$	Conductance of minority to majority electron channel transition
$G_{\downarrow\downarrow}$	Conductance of minority to minority electron channel transition
G_{AP}	Conductance for an AP configuration of FM layers
G_{P}	Conductance for a P configuration of FM layers
h	Height (or thickness)
$\hbar$	Reduced Planck constant
$\mathbf{H}$	Magnetic field
$\mathbf{H}_d$	Demagnetizing field
$\mathbf{H}_{\text{eff}}$	Effective magnetic field
$\mathbf{H}_{\text{ext}}$	External magnetic field
H_{an}	Annihilation field

H_n	Nucleation field
H_s	Saturation magnetic field
i	Current
I	Current
I_{ac}	Alternating current
I_{max}	Maximum current
j	Current density
J	Current density
J_{ac}	Ac current density
J_{cr}	Critical current
J_{c1}	Lower critical current
J_{c2}	Upper critical current
J_{dc}	Dc current density
$j_{\uparrow}$	Majority electron current density contribution
$j_{\downarrow}$	Minority spin current density contribution
k	Spring-like constant
K	Anisotropy constant
k^{CPP}	Degree of non-adiabaticity
k^{ex}	Exchange spring-like constant
k^{ms}	Magnetostatic spring-like constant
k^{Oe}	Oersted field spring-like constant
k^T	Thermal spring-like constant
k^Z	Zeeman spring-like constant
l_{ex}	Exchange length
l_{sf}	Spin-diffusion length
l_w	Bloch wall thickness
$\mathbf{m}$	Magnetic moment
$\mathbf{m}$	Reduced magnetization
M	Memristance
$\mathbf{m}_{ref}$	Reduced reference magnetization
m^*	Effective mass
$\mathbf{M}$	Magnetization
$\bar{\mathbf{M}}$	Mass tensor
$\mathbf{M}_{fixed}$	Magnetization of the fixed layer
$\mathbf{M}_{free}$	Magnetization of the free layer
$\mathbf{M}_{ref}$	Magnetization of the reference layer
M_s	Saturation magnetization
M_z	Out-of-plane magnetization
$n(E)$	Density of states
$n(E_F)$	Density of states at Fermi level
$\mathbf{p}$	Polarization direction
p_J	Spin polarization
p_z	Out-of-plane component of the polarization
P	Vortex polarity
P_{in}	Input power
q	Charge

r	Radius
R	Radius
R	Resistance
$\overline{R}$	Average resistance
$R_{\uparrow}$	Majority electron resistance
$R_{\downarrow}$	Minority electron resistance
$R_{\min}$	Minimum resistance
$R_{\max}$	Maximum resistance
R_{AP}	Resistance for a AP configuration of FM layers
R_{P}	Resistance for a P configuration of FM layers
s	Reduced vortex core position
s_0	Steady-state vortex core position
t	Time
T	Temperature
$\mathbf{T}_{\text{ST}}^{\text{Slonc.}}$	Slonczewski spin torque
$\mathbf{T}_{\text{ST}}^{\text{Z\& L}}$	Zhang and Li spin torque
v	Voltage
V	Voltage
V	Volume
$\mathbf{V}$	Volume
V_{th}	Threshold voltage
W	Potential magnetic energy
W^{ex}	Exchange energy
W^{ms}	Magnetostatic energy
W^{Oe}	Oersted field energy
W^{T}	Thermal energy
W^{Z}	Zeeman energy
X	Vortex core position in the x -axis
$\mathbf{X}$	Vortex core position
$\dot{\mathbf{X}}$	Vortex core velocity
$\ddot{\mathbf{X}}$	Vortex core acceleration
$\mathbf{X}_{\text{I}}$	Image vortex core position
Y	Vortex core position in the y -axis
Z_0	Impedance of a transmission line

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Introduction

A tremendous number of new scientific discoveries have been made possible through the rise of nanotechnologies in past decades. Progress in manufacturing processes and intense research efforts have allowed to shrink devices and to exploit physical phenomena occurring at reduced scale. Numerous new applications have emerged from these studies. Among them all the development of artificial intelligence has had a big impact on modern society. Through deep learning or other algorithmic frameworks, machines have proven to be able to resolve complex problems. Various practical implementations have been considered up until now, generally based on CMOS technologies. However a new approach called neuromorphic computing has attracted a lot of interest recently in the scientific community. [1]

Neuromorphic computing is inspired from the functioning of the human brain. A typical neuromorphic neural network is composed of two main components, namely the neurons and the synapses. Those act like their biological equivalents. Signals are processed by nodes, i.e. the artificial neurons, which can communicate thanks to synapses. As in other artificial intelligence systems the connections between nodes are adjusted during the learning process by weighting. Such network presents the main advantage of being much more energy efficient compared to classical computers. [2] This thesis focuses on the study of the hardware implementation of both these components.

The emergence of spintronics has led to one of the most promising device mimicking neuronal properties, i.e. spin-transfer nano-oscillators (STNOs). Indeed applying spin-polarized currents in magnetic tunnel junctions allows to create a torque acting on the magnetization of one of the ferromagnetic layers. In particular conditions it happens that this torque induces steady precessions of the magnetization. Considering magnetoresistance effects in ferromagnetic/nonmagnetic/ferromagnetic trilayers one can produce oscillations of the device resistance. As a consequence emission of signals in the microwave frequency range can be obtained. The nonlinear response presented by these junctions under an external solicitation justifies the comparison with biological neurons. Moreover in this thesis we will exploit an exotic magnetic groundstate of the free layer, namely the magnetic vortex. This particular configuration leads to what are called spin-torque vortex oscillators (STVOs). Those have proven to be capable of being successively used to resolve complex tasks, such as voice recognition. [3, 4] In addition to signal generation and neuromorphic applications STNOs can also be used for frequency detection. [5] This can be achieved by using the so-called spin-diode effect. This phenomena is at the opposite of the signal generation mentioned previously. It is characterized by the conversion of an alternating input signal into a constant voltage. Depending on the

applied frequency the magnitude of the output signal can vary, allowing to find back the alternating current (ac) frequency after calibration. This thesis aims at studying STVO properties, with a particular focus on the magnetization dynamics. Several approaches will be used. First an analytical model will be developed, based on vortex theoretical background. At the same time, micromagnetic simulations will be performed. Those are known to describe reliably magnetic systems, at the expense of high computational time and power. That is where lies the main interest of an analytical model, i.e. to speed-up calculations. Those developments will allow to compare analytical predictions to experimental results with quantitative agreement.

The second part of this work is dedicated to investigate a hardware implementation of synapses. Different physical systems have been explored in past decades to try to reproduce artificially synaptic behaviors. Among those, memristance based networks have attracted attention. [6] Theorized by Chua in 1971, a memristor is the forth basic component of any electrical circuit, along with the capacitor, the inductor and the resistor. [7] A memristance is similar in principle to a classical resistance except that it keeps tracks of past applied currents and voltages. As a consequence, its value evolves with the electrical charge, diverging from standard ohmic behavior. Many devices can present memristive effects. In the framework of this thesis we will focus on silver nanowire networks. Various groups have shown that these systems could present a field-induced evolution of resistance. [8, 9, 10] The main reason explaining these phenomena would be a migration of surface Ag ions, creating conductive bridges between adjacent nanowires. Depending on the wire geometry, those effects could be reversible or persistent. This thesis includes thus an experimental section dedicated to study 3D crossed nanowire networks. Those are fabricated following a process developed in past decades at UCLouvain. It consists in filling track-etched thin polymer membranes with metallic wires thanks to an electrodeposition step. [11] This technique has led to many interesting applications, e.g. in thermoelectricity and spintronics. [12, 13] As the nanopores present an angle with respect to the direction normal to the film, connections between Ag wires are created. This allows current to flow. However thanks to a plasma etching step, heat-generated stress is induced in the membrane. As a result a fragmentation of the network is obtained, by the creation of tiny gaps in the wires. This prevents efficient electron flow in the device. However by applying voltage ramps, we will show that it is possible to provoke ionic migration within the network leading to the creation of conductive filaments. Dramatic decrease of resistance can then be observed after several sweeps.

This report, divided in three main sections, presents all results obtained during this year of research. A first chapter reviews the state of the art in the various scientific fields concerned by this work. It provides basic notions in spintronics, nanomagnetism and memristive behavior. The next part is dedicated to the study of spin-transfer nano-oscillators. An analytical model describing the magnetic vortex dynamics is developed. It is later compared to results obtained by simulations. Moreover its prediction are used to explain experimental data of spin-diode effect in magnetic tunnel junctions. A third section investigates the properties of crossed silver nanowire networks. The sample preparation is detailed as well as the electrical characterization procedure. Obtained results are analyzed and discussed. Particular emphasis is made on memristive and

tunneling behaviors. Finally a short conclusion closes up this report. This thesis is the result of a joint effort of the neuromorphic group from the BSMA department in UCLouvain. It has been achieved in close collaboration with Chloé Chopin and Flavio Abreu Araujo. Before each section, a short paragraph summarises the respective contributions to this work.

State of the art

This section presents an overview of the theoretical notions that are needed to fully understand the core of this thesis. Indeed the designing of spin-vortex vortex oscillators (STVOs) required to combine together advanced knowledge in different fields of material sciences. First a general introduction about spintronics and its major discoveries will be given. Then magnetism at the nanoscale will be explained, with a specific focus on the vortex state and its dynamics. Those explanations will allow to consider a spin-transfer nano-oscillator (STNO) structure. This will let us outline one of its application of interest, the spin-diode effect. Finally information about memristors will be provided. Particular emphasis will be put on silver nanowire networks switching resistive devices.

1.1 Notions of spintronics

Spintronics (or spin electronics) is a quite recent scientific field, as it has only been introduced in the second half of the 20th century. It emerged when Albert Fert, one of its precursor, confirmed in the 70's the existence of spin-polarized currents in ferromagnetic materials. This followed theoretical predictions made by Mott years earlier. [14] On the contrary to classical electronics which is only based on the manipulation of charges to transport currents, spintronics also considers the electronic spin.

Many important physical phenomena are related to spintronics. Some of them, such as giant magnetoresistance (GMR) or tunneling magnetoresistance (TMR) will be detailed in following sections. Those have led to important industrial applications in past decades. The most notable one has been the dramatic increase of the reading sensitivity in magnetic data recording.

1.1.1 Spin dependent electronic transport

In ferromagnetic materials, the electronic transport properties are spin-dependent. Indeed the electronic and magnetic behavior of transition metals is ruled by their *s* and *d* bands. It happens that for Ni, Fe and Co an imbalance appears between two half-bands, one for spin up and one for spin down. This is due to the fact that the density of states in the *d* band is large and allows to fulfill the Stoner criterion:

$$\mu_B^2 \mu_0 \lambda n(E_F) > 1 \quad (1.1)$$

With μ_B the Bohr magneton, μ_0 the vacuum permeability, λ the exchange coupling coefficient and $n(E_F)$ the density of states (DOS) at the Fermi level. This criterion considers a balance in energy between two terms in the event of a band splitting. The first one is a reduction in energy due to the spin interaction with the created exchange field. The second one is the cost in energy for each electron transferred above the Fermi level. If the sum of both terms is negative, a splitting is observed.

This situation is illustrated in Figure 1.1. On the right, one can see the conduction band for a paramagnetic material (e.g. Cu). In this case, both half-bands present the same DOS and no spin dependent conduction is observed. On the contrary one can note a clear band splitting for 3*d*-ferromagnetic metals, on the left. In this example a

majority of spin up (in green) is observed. Indeed the Fermi level E_F crosses the d -band of the spin down DOS (in blue). This leaves empty levels compared to the spin up part, creating an imbalance and a net magnetic moment.

The Drude and Lorentz theory gives the mobility μ_e of electrons in perfect metals as follows:

$$\mu_e = \frac{e\tau}{m^*} \quad (1.2)$$

With e the electron charge, τ the mean time between scattering events and m^* the effective mass. Due to the band curvature the effective mass of s electrons is much smaller compared to the d ones. Therefore, most of the electric current is transported by s electrons.

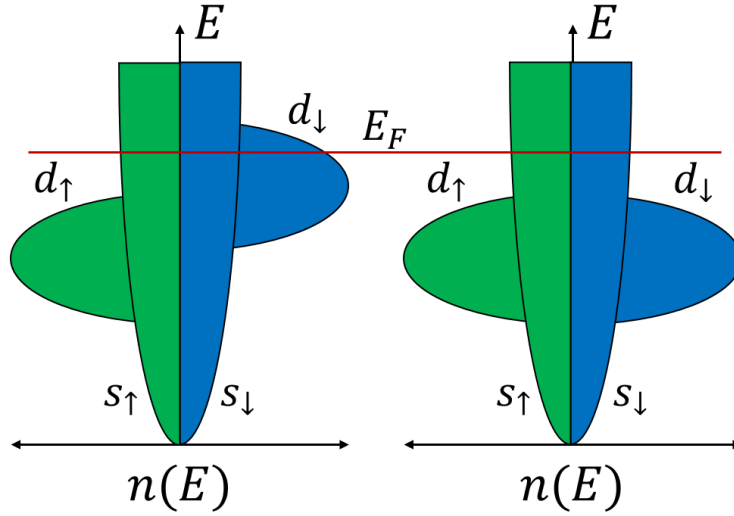


Figure 1.1: Schematic of the density of states $n(E)$ in the case of (left) a magnetic transition metal (Ni, Fe, Co) and (right) another metal (Cu). Spin up is represented in green, spin down in blue. The Fermi level E_F is indicated by the red line. Both $4s$ and $3d$ bands are depicted.

The electrical conduction is governed by the scattering probability of the electrons at the Fermi level. In $3d$ -ferromagnetic metals this probability depends on the spin, because it is linked with the DOS. For the minority electrons, lots of empty states are available in the d -band. The time between scattering events is then reduced. For the majority electrons, the Fermi level only crosses the s -band. Less empty states are available as only $s-s$ transition are allowed. Therefore, the scattering probability is much lower.

As the resistivity is directly linked to the mean time between scattering events τ , one can conclude that the conductivity of majority electrons will be higher than the one of minority electrons. This conclusion allowed Mott to propose a two-channel model for the electric current. The latter is depicted in Figure 1.2. The total current density j flowing in the material is the sum of two contributions, one for each spin ($\downarrow$ and $\uparrow$):

$$j = j_\downarrow + j_\uparrow \quad (1.3)$$

Each channel is modeled by a resistivity, which depends on the spin. Two spin-asymmetry coefficients are commonly used to compare between compounds this resistivity imbalance:

$$\alpha = \frac{\rho_{\downarrow}}{\rho_{\uparrow}} \quad \text{and} \quad \beta = \frac{\rho_{\downarrow} - \rho_{\uparrow}}{\rho_{\downarrow} + \rho_{\uparrow}} \quad (1.4)$$

Due to previous considerations, the resistivity of majority electrons is lower compared to minority electrons. As both resistors are placed in parallel, the total resistivity ρ is derived as:

$$\rho = \left(\frac{1}{\rho_{\downarrow}} + \frac{1}{\rho_{\uparrow}} \right)^{-1} \quad (1.5)$$

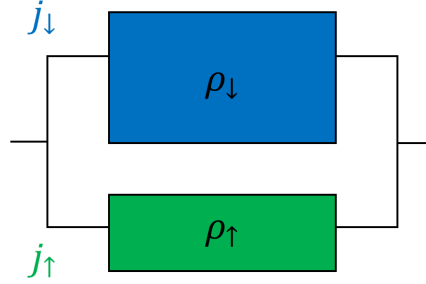


Figure 1.2: Two-current model. The resistivity of the spin down channel is higher than for the spin up channel in this example.

However this model is only valid if the spin is conserved over a large distance compared to the mean free path of the electrons. If it was not the case, a short circuit between both branches of the circuit should be added as some electron spins would switch rapidly. Therefore the notion of spin diffusion length l_{sf} must be considered. It defines the mean distance that an electron travels before a spin flip event. Two major causes of spin-flip have been identified, magnons and spin-orbit scattering. Magnons, which are quantized excited magnetic waves in the crystal lattice, are the predominant source of flips at high temperature.

Generally the spin-diffusion length is greater than the mean free path, up to several orders of magnitude. As the density of magnons decreases with temperature, it is also enhanced at lower temperature. Some experimental l_{sf} values of important metals are resumed in Table 1.1 (at different temperatures).

Material	l_{sf} [nm]	Temperature [K]
Ni	21 ± 2	4.2
Co	38 ± 12	300
Fe	8.5 ± 1.5	4.2
Py (=Ni ₈₄ Fe ₁₆)	3	293
Al	350 ± 50	293
Cu	1000 ± 200	4.2

Table 1.1: Spin-diffusion length in some important metals and alloys, at different experimental temperatures. Extracted from [15].

1.1.2 Giant magnetoresistance (GMR)

The giant magnetoresistance (GMR) effect has been discovered by Albert Fert and Peter Grünberg in 1988. In 2007, they were awarded with the Nobel Prize in Physics for this scientific breakthrough. This effect appears in artificial multilayered nanostructures which are composed of a stack of thin magnetic layers separated by a nonmagnetic spacer. It is characterized by an important decrease of resistance of such a sample under a magnetic field $\mathbf{H}$. It is based on a coupling between the successive magnetic layers, causing the diffusion of a spin polarized current.

The parameter of importance influencing this phenomenon is the relative orientation of the magnetization of the magnetic layers. For simple cases, the latter can either be parallel (P) or antiparallel (AP). In a parallel configuration, the magnetization vectors of the magnetic layers are aligned in the same direction. In an antiparallel system those are rather alternatively opposed. This is illustrated in Figure 1.3 by the experimental results obtained by Fert *et al.* [16]

In this example, iron is the ferromagnetic (FM) material and chromium the non-magnetic (NM) spacer. When no external field is applied, the system is in an AP configuration. No spin channel is therefore promoted and the resistance is maximum. Both spin currents are blocked similarly. However if an external magnetic field is applied the stack switches to a P configuration. Indeed all magnetization vectors will be aligned with the field. One spin channel is therefore promoted over the other one. This induces a reduction of electrical resistance in the sample. If the magnetic field is applied in the opposite direction the same result is obtained. What should be highlighted here is that the magnitude of the effect is dependent on the spin diffusion length. This is why it is easier to observe at low temperature as the latter will be enhanced.

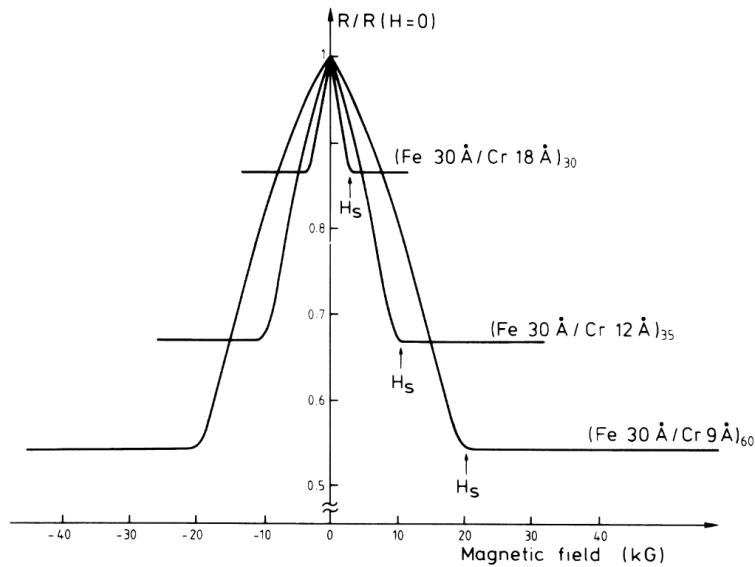


Figure 1.3: Magnetoresistance in Fe/Cr superlattices, for three different non-magnetic layer thicknesses at 4.2 K. H_s indicates the field required to reach saturation. Reproduced from [16].

Mathematically one can define the GMR ratio. Its value characterizes the importance of the phenomenon. It is obtained as follows:

$$\text{GMR} = \frac{R_{\text{AP}} - R_{\text{P}}}{R_{\text{P}}} \quad (1.6)$$

With R_{AP} and R_{P} the resistance in an AP and P state, respectively. Again one can use to two-current model to qualitatively understand the electron progress in the layers. In a parallel state the majority spin current is favored as its electrons are preferentially transmitted. Indeed the scattering probability is lowered for this channel, creating a low resistance shortcut. On the contrary the opposed spin encounters a preferential reflection at the interface causing backscattering. The perceived resistance for this channel is higher. Conversely if the system is in an antiparallel state the two channels are blocked. Majority and minority electron currents both face layers with opposed magnetization to their spin. No channel is favored and the total resistance is higher. Figure 1.4 illustrates the electron diffusion depending on the magnetic state. The values of resistance R_{AP} and R_{P} for this example can be obtained as follows:

$$R_{\text{P}} = \frac{2R_{\downarrow}R_{\uparrow}}{R_{\downarrow} + R_{\uparrow}} \quad \text{and} \quad R_{\text{AP}} = \frac{R_{\downarrow} + R_{\uparrow}}{2} \quad (1.7)$$

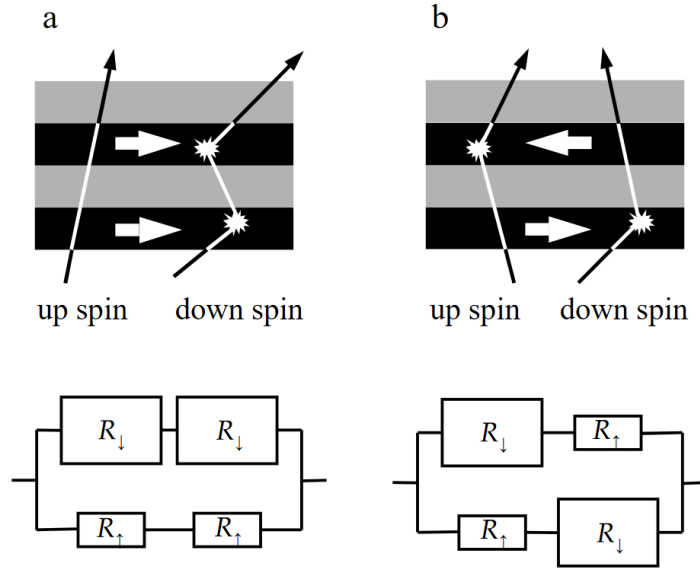


Figure 1.4: Two-current model for a GMR stack. On the left, the system is in a parallel state. The spin up current is favored because its scattering probability is lower. It encounters successive small resistance states compared to the spin down channel. On the right, the system is in an antiparallel state. Spin up and spin down currents encounter successively high and low resistance layers. No spin channel is favored. Reproduced from [17].

As explained previously GMR can only be obtained if the system can switch from an AP to a P state. For a simple stack of alternating FM and NM layers, this cannot always be achieved. In fact the thickness of the nonmagnetic layer determines the type of

coupling between successive FM layers. The experimental results of Mosca *et al* depict this principle in Figure 1.5. [18] One can observe oscillations in the MR ratio depending on the Cu layer thickness. This effect is one of the main prediction of the RKKY¹ interaction although the period of oscillation is much larger than anticipated. For some values, an antiferromagnetic (AF) coupling is obtained. This leads to GMR as an AP state is created. However for other values a ferromagnetic coupling is noticed. All FM layers are then aligned when no external field is applied. No GMR can be achieved.

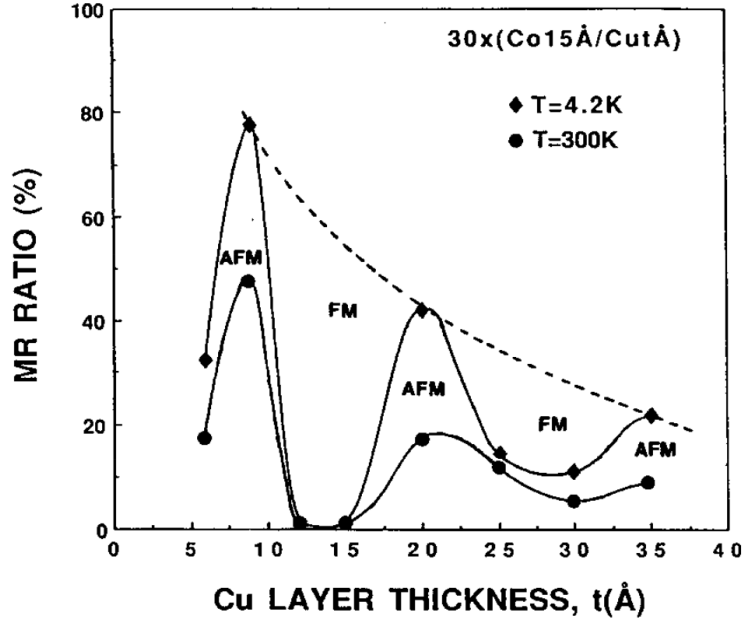


Figure 1.5: Oscillations in the MR ratio for a Co/Cu sample depending on the thickness of the Cu layer. If an AF coupling is obtained, GMR is achieved. For a FM coupling, no MR is observed. Measurements were done at 4.2 and 300K (diamond and circle points, respectively). Reproduced from [18].

To resolve this problem a solution, extensively used in experiments and applications, has been found to ensure a possible switch between an AP and a P state. This system, presented in Figure 1.6, is called a spin valve. An antiferromagnetic layer is placed above a first FM layer. At the interface an exchange anisotropy appears. This induces the apparition of an exchange bias, preventing the FM layer from switching freely if the latter is not overcome. This layer is said to be blocked (or pinned). Then a second FM layer is placed below the NM spacer. This one is called the free layer as the magnetization aligns itself with the external magnetic field direction. The interest of this type of structure is highlighted when looking at its hysteresis curve. First the external field is aligned with the blocked layer magnetization. The free layer follows and the system is in a P configuration of minimum resistance. Let us now consider a magnetic sweep towards a positive field. Just above $\mathbf{H} = 0$, the free layer magnetization switches rapidly to the opposed direction. The exchange bias prevents the blocked layer to behave similarly. An AP configuration is then obtained and the resistance is maximum. If the field increases further it is possible to overcome the energy barrier created by the exchange anisotropy.

¹RKKY stands for Ruderman–Kittel–Kasuya–Yosida, the four scientists who proposed this theory.

The first FM layer is forced to switch and another P configuration is reached. Generally this extreme case is not encountered as a spin valve is mainly used under small external field. [17]

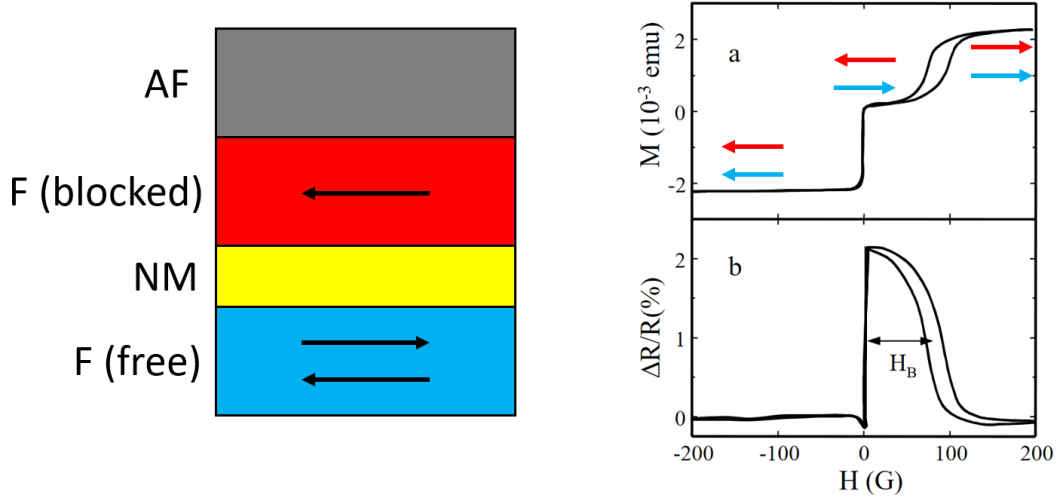


Figure 1.6: On the left, schematic illustration of a spin valve stack. An antiferromagnetic (AF) layer blocks the magnetization in a first FM layer thanks to an exchange bias. Another FM layer is placed below the NM spacer. Its magnetization is aligned with the external magnetic field. On the right are presented experimental results on such system. If the external field is aligned with the blocked FM layer, a P configuration is obtained. If a small field is applied in the opposite direction, the magnetization in the free layer switches. At much higher field the magnetization in the fixed layer can still be reversed. H_B is the exchange bias. Adapted from [17], after [19].

1.1.3 Tunneling magnetoresistance (TMR)

Instead of separating the two magnetic layers by a nonmagnetic spacer, another possibility is to place an insulating barrier between those. This thin layer (a couple of nanometers) is generally made of an oxide (e.g. MgO). This creates a so-called magnetic tunnel junction (MTJ). This configuration presents the advantage of greatly increasing the MR ratio (up to 1000%) as it will be presented hereafter.

The physical principle behind this phenomenon differs from GMR as quantum tunneling is required for the electrons to travel through the insulating layer. Actually this difference led to naming this effect tunneling magnetoresistance (TMR). However it is also dependent on the relative orientation of the magnetization between successive magnetic layers. A schematic representation of the electronic travel through the barrier is given in Figure 1.7.

In a parallel state a matching between the DOS at the Fermi level of both spin currents gives rise to a low resistance state. Indeed the current is proportional to the sum of the channel conductances. We have thus:

$$G_P = G_{\uparrow\uparrow} + G_{\downarrow\downarrow} \quad (1.8)$$

With G_P the total conductance in a parallel state, $\uparrow$ and $\downarrow$ denoting majority and minority spin, respectively. In this case, if the two FM layers are made of the same material, the number of available states corresponds for both channels between FM₁ and FM₂. Conductance is high. The situation is different for the AP configuration. In the example of Figure 1.7, lots of states are available in FM₂ to receive a low density of spin up electrons which can travel. On the contrary the DOS is large for spin down electrons in FM₁ but the number of available state in FM₂ is limited. Those two effects combined give rise to an inefficient transfer of electrons between both FM layers. The conductance is low:

$$G_{AP} = G_{\uparrow\downarrow} + G_{\downarrow\uparrow} \quad (1.9)$$

With G_{AP} the total conductance in an antiparallel state. Other parameters influencing the conductance are the thickness of the oxide layer and the height of the potential barrier.

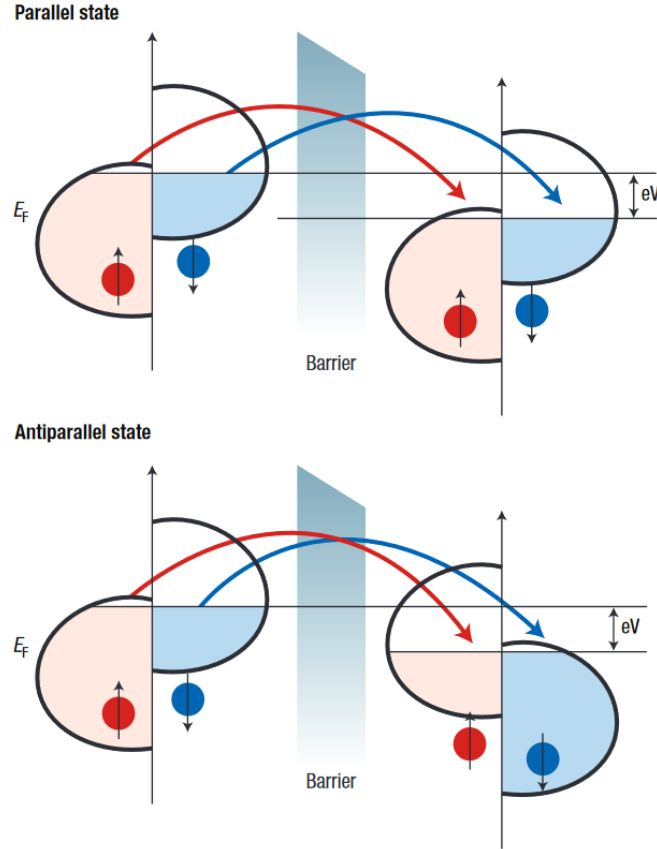


Figure 1.7: Schematic illustration of the electron travel in a magnetic tunnel junction. In a parallel state (top) the DOS of both channels in the two FM layers is matched. A high conductance is obtained. In an antiparallel state (bottom) lots of free states are allowed to receive a limited number of electrons or inversely, depending on the channel. A low conductance is obtained. Reproduced from [20].

Experimental TMR loops obtained by Lee *et al* are presented in Figure 1.8. [21] A pseudo spin-valve configuration is used. This means that different coercivity electrodes (e.g. hard and soft FM) are used instead of using the exchange bias property. This

explains why the MR curve is symmetrical in this case. At low field, an AP state is observed. As explained previously, conductance is low and the observed resistance is maximum. When increasing the field the magnetization in all FM layers is aligned and a P state is obtained. The transfer of electrons in both current channels is more efficient and the resistance is minimum.

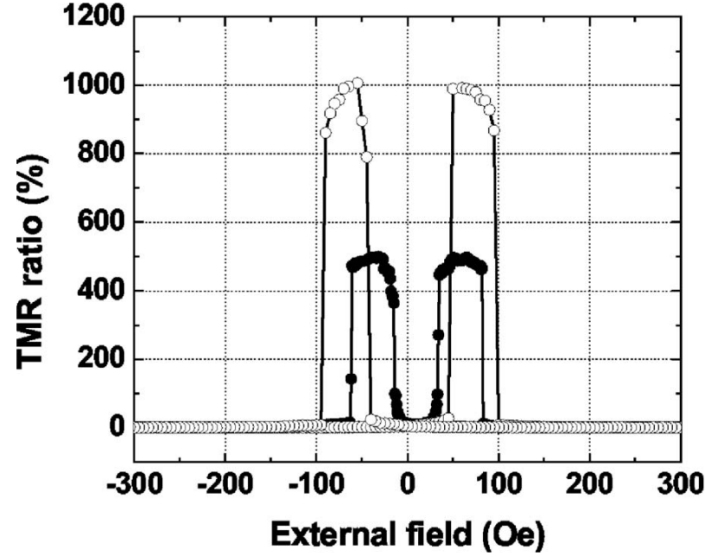


Figure 1.8: TMR curves of a sample made from two $(\text{Co}_{25}\text{Fe}_{75})_{80}\text{B}_{20}$ FM electrodes and MgO oxide barrier. Experiments were made at room temperature (black circles) and 5K (white circles). Reproduced from [21].

1.1.4 Spin-transfer torque (STT)

A schematic of a nanostructure leading to the generation of a spin-transfer torque (STT) is presented on Figure 1.9. Generally it is made of a multilayer of ferromagnetic/nonmagnetic/ferromagnetic materials. The first ferromagnet (FM_1) is considered thicker than the second one (FM_2). In a spin valve configuration the magnetization in FM_1 ($\mathbf{M}_{\text{fixed}}$) is fixed and free in FM_2 ($\mathbf{M}_{\text{free}}$).

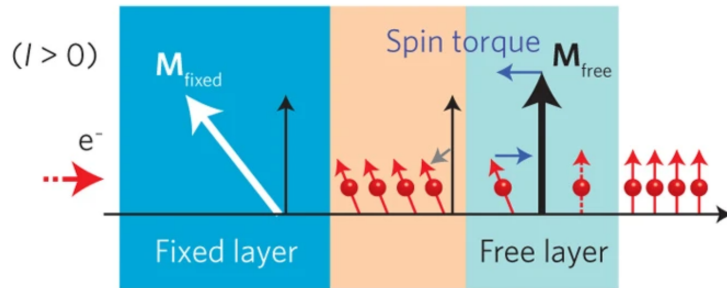


Figure 1.9: Schematic illustration of a multilayered structure made of a thick ferromagnet, a nonmagnetic material and a thin ferromagnet. The fixed layer acts as a spin polarizer. When passing through the free layer the electrons align their spin with its magnetization. This will generate a spin-transfer torque by conservation of angular momentum in the free layer. Reproduced from [22].

Let us consider that initially the magnetization direction is different in the two FM layers. Then a current is injected in the structure. FM_1 will act as a spin polarizer when the electrons are passing through it. The spin of the transferred electrons is therefore not aligned with $\mathbf{M}_{\text{free}}$ initially. But when crossing the free layer those align themselves to the free layer magnetization by exchange interaction. The transverse component of the spin polarized current is transferred to the free layer magnetization. By conservation of angular momentum, a torque is generated in FM_2 . The latter is called spin-transfer torque. It is important to note that the dimensions of the NM and free layers must be small compared to typical spin conservation lengths. The current density must also be large enough to allow the rotation of $\mathbf{M}_{\text{free}}$ ($\sim 10^7 \text{ A cm}^{-2}$). [22]

As explained, this torque allows to act on the magnetization of the free layer without the application of any external magnetic field. The use of this attribute led to one of the major spintronic application, the magnetic random access memory (MRAM). As seen previously, the relative orientation in successive FM layers gives rise to two possible states. Those can be associated to binary bits. When the system is antiparallel, the resistance is high (bit '1'). On the contrary a parallel state leads to a low resistance (bit '0'). Reading the information is therefore trivial. For the writing part the STT principle is exploited. The operation is illustrated in Figure 1.10. To write a '0', the electrons are first injected through the thick layer. Spins parallel to the fixed magnetization are then preferentially transmitted. Opposite spins are reflected at the interface. The free layer magnetic moment is then forced to align itself to the polarizer by angular momentum conservation. To write a '1', the electrons first pass through the free layer. When arriving to the fixed FM, an antiparallel spin polarized current is reflected. The latter creates the spin-transfer torque required to switch the free layer magnetization.

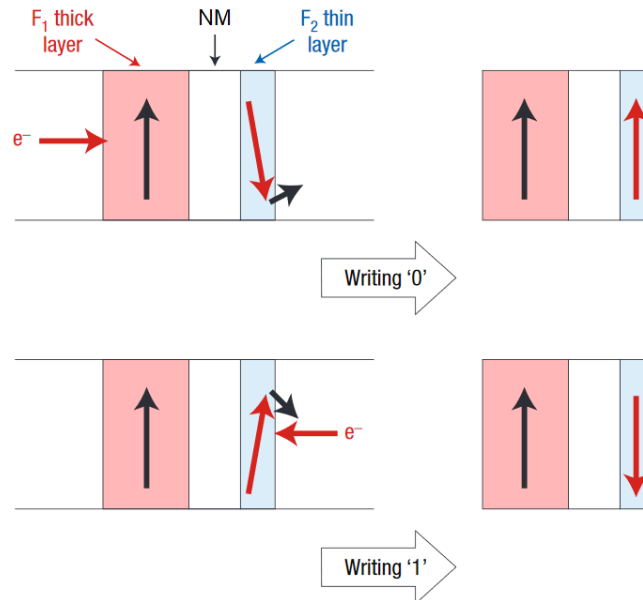


Figure 1.10: Schematic representation of the MRAM writing principle. Depending on the current direction an AP or P state is created using spin-transfer torque. Reading is possible thanks to the difference of electrical resistance depending on the magnetic configuration. Adapted from [20].

1.2 Nanomagnetism basics

In this work artificial structures of small dimensions will be considered ($\sim \text{nm}$). As for other types of properties nanoscale magnetism differs from the typical phenomena encountered in bulk materials. Indeed those are composed of multiple magnetic domains. Therefore when reducing the size of a structure its dimensions can approach the critical limit to consider it as a single domain. For spin-diode applications various magnetic states have been explored. Though most studies work with a uniform magnetization we will consider in the thesis a more exotic state, the so-called magnetic vortex. As it is explained further, the latter presents a non-uniform magnetization. This fact is in principle incompatible with the mono-domain concept. However the strong connection between all magnetic moments constituting the vortex allows such a consideration.

1.2.1 Concepts of magnetism

Let us first recall some basic concepts concerning magnetism. This fundamental field of science studies all physical phenomena involving magnetic interactions. This thesis focuses mainly on magnetic material properties but their interaction with external fields must also be considered.

The global magnetic moment $\mathbf{m}$ in a material can be obtained by integrating the magnetization $\mathbf{M}$ over the volume $\mathbf{V}$:

$$\mathbf{m} = \iiint \mathbf{M} d\mathbf{V} \quad (1.10)$$

It is expressed in A m^2 in the International System (SI). The magnetization, such as the magnetic field $\mathbf{H}$, is given in A m^{-1} . From those quantities one can define the magnetic susceptibility, which has then no dimension, by the following relationship:

$$\chi = \frac{|\mathbf{M}|}{|\mathbf{H}|} \quad (1.11)$$

Depending on the internal configuration of the magnetic moments different classes of materials have been identified. Those able to present a nonzero global magnetic moment are presented on Figure 1.11. The magnetic susceptibility of a ferromagnet is several orders of magnitude greater than the one of a paramagnetic material.

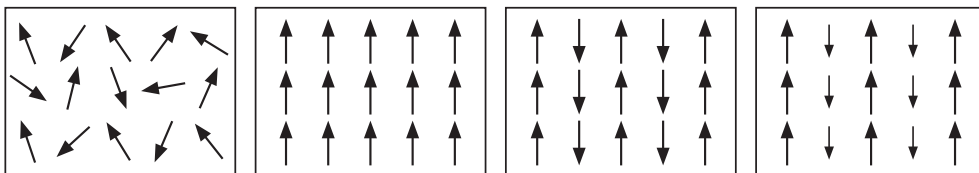


Figure 1.11: Schematic representation of (from left to right) magnetic moments in paramagnetic, ferromagnetic, antiferromagnetic and ferrimagnetic materials.

Ferromagnetic materials are especially interesting because those can present a net magnetic moment even when no external field is applied. In such materials all microscopic magnetic moments are ordered parallel to each others. But this property only

exists below a specific temperature called the Curie temperature T_c . Above the latter the magnetic order vanishes and the ferromagnet becomes paramagnetic. For antiferromagnetic materials the same principle is used to define the Néel temperature T_N . Those property modifications can be associated to phase transitions. Note that above T_c the application of an external field $\mathbf{H}$ can bring back the magnetic order in a ferromagnet.

The magnetic induction $\mathbf{B}$, or magnetic flux density, is linked to the magnetic field $\mathbf{H}$ and the magnetization $\mathbf{M}$ by:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad (1.12)$$

Where μ_0 is the vacuum permeability and equals $4\pi \cdot 10^{-7} \text{ H.m}^{-1}$. In all generality the magnetic permeability is defined as:

$$\mu = \frac{|\mathbf{B}|}{|\mathbf{H}|} \quad (1.13)$$

With $\mathbf{B}$ expressed in Tesla (T). Despite the use of SI units, the centimeter-gram-second (CGS) unit system is still extensively used nowadays. Equation 1.12 can then be rewritten as:

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} \quad (1.14)$$

With $\mathbf{B}$ in Gauss (G), $\mathbf{H}$ in Oersted (Oe) and $\mathbf{M}$ in emu.cm^{-3} .

As introduced before bulk magnetic materials are composed of multiple magnetic domains. Applying a magnetic field $\mathbf{H}$ will first cause the motion of the domain walls which separate distinct domains. A saturation is attained when all walls have disappeared to form a single domain, the magnetization and the external field being eventually parallel. These considerations are only valid above a critical size D_s . Below this length it becomes energetically favorable for the (nano)material to only present a mono-domain, even in the absence of magnetic field. Its value can be estimated using parameters specific to each material:

$$D_s = \frac{72\sqrt{AK}}{\pi M_s^2} \quad (1.15)$$

Where A is the exchange stiffness, K the anisotropy constant and M_s the saturation magnetization (in CGS units). [23] Typical values for 3d ferromagnetic materials are listed in Table 1.2.

Material	M_s [emu/cm^3]	K [10^4 erg/cm^3]	A [10^{-6} erg/cm^3]	D_s [nm]
Fe	1710	45	1	16.8
Co	1430	430	1	31.6
Ni	483	4.5	1	59.6

Table 1.2: Critical diameter and several magnetic parameters for 3d transition metals. Extracted from [23].

From those material parameters one can also extract two values important when dealing with nanostructured materials. The first one is called the exchange length l_{ex}

and is used to measure the strength of the exchange energy:

$$l_{\text{ex}} = \sqrt{\frac{A}{2\pi M_s^2}} \quad (1.16)$$

The second is the Bloch wall thickness l_w which give insights about the length of the transition between magnetic domains:

$$l_w = \sqrt{\frac{A}{K}} \quad (1.17)$$

1.2.2 Magnetic vortices

A nanopillar can give rise to a particular magnetic topology, called a magnetic vortex, for certain values of its aspect ratio. The latter is defined as $\xi = h/2R$, where h is the thickness of the cylinder and R its radius. In a vortex state the magnetization presents a typical non-uniform distribution separated in two regions. First one can identify a vortex core featuring an important out-of-plane magnetization. The second zone, around the vortex center, is characterized by a curling planar magnetization. Figure 1.12a depicts this situation.

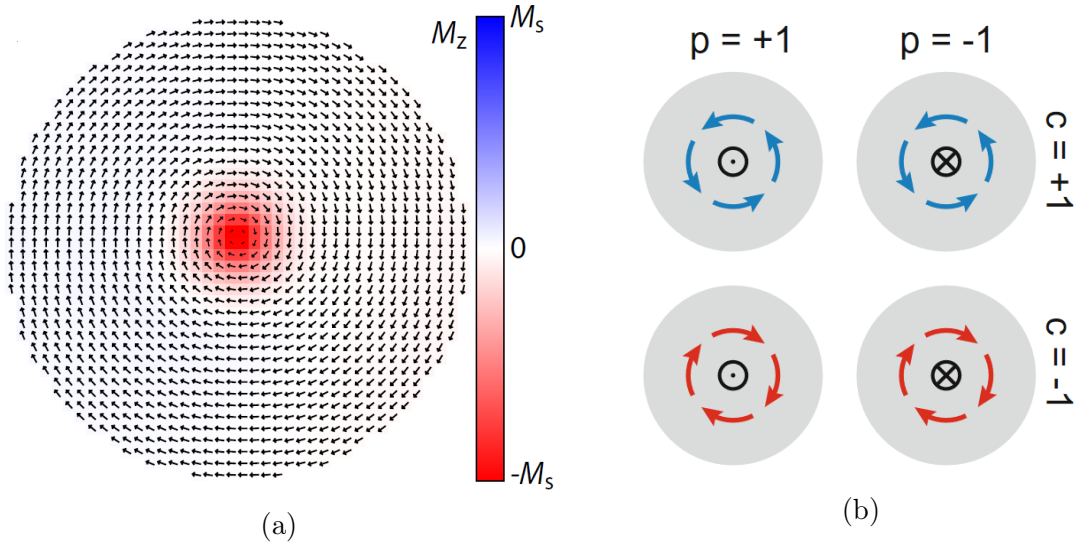


Figure 1.12: (a) Distribution of the magnetization in a vortex state. The colorbar indicates the out-of-plane magnetization M_z . Small black arrows represent the curling planar component. Reproduced from [24] (b) Schematic illustration of the four possible vortex configurations. The polarity ($P = \pm 1$) tells if the vortex core is pointing up or down. The chirality $C = \pm 1$ tells if the planar curling direction is counterclockwise or clockwise. Reproduced from [25].

As presented on Figure 1.12b, two main parameters are used to describe a vortex. First, the polarity P indicates if the core magnetization points upwards ($P = +1$) or downwards ($P = -1$). Then the chirality informs on whether the planar curling direction is counterclockwise ($C = +1$) or clockwise ($C = -1$). Four possible configurations

result from the combination of those two parameters. In addition to vortices ($q = +1$), antivortices of opposed charge ($q = -1$) may also arise in microstructures but will not be discussed here.

Influence of the aspect ratio on the magnetic ground state

As introduced previously the aspect ratio ξ of a nanodot plays an important role on the magnetic ground state. Indeed some energy contributions, especially shape anisotropy, may promote a single domain uniform magnetization. Let us consider two extreme cases. If the thickness is too small compared to the radius ($h \ll R$), a uniform planar magnetization is obtained. This is due to the generation of a strong demagnetizing field perpendicular to the plane in a thin film. If now the radius becomes small with respect to the thickness ($R \ll h$), a uniform out-of-plane magnetization is promoted. In this case a demagnetizing field aligned with the cylinder bases is created. Between those bounds, intermediate values of ξ give rise to the nucleation of a magnetic vortex as ground state.

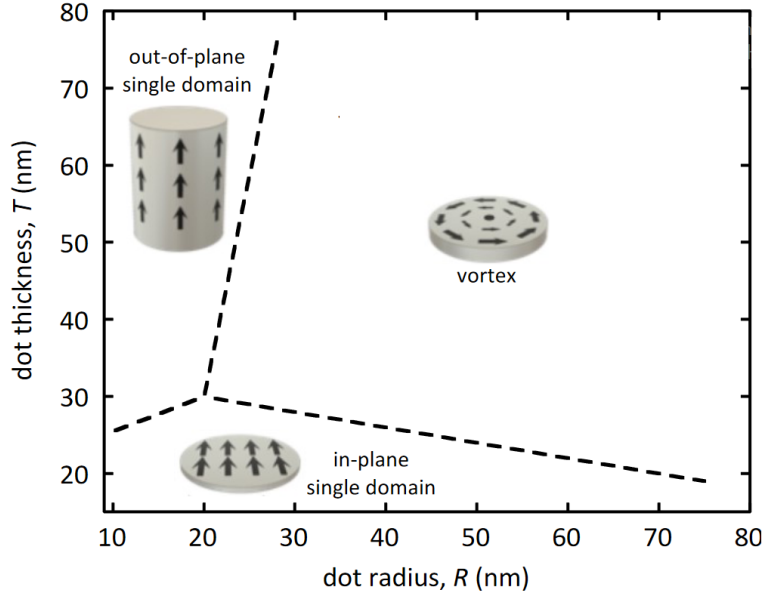


Figure 1.13: Schematic phase diagram of the magnetic ground state for $\text{Ni}_{80}\text{Fe}_{20}$ nanodots, depending on their aspect ratio. The dashed lines separating the three different states were obtained based on theoretical and experimental results. Adapted from [26].

Several works have allowed to draw magnetic phase diagrams as a function of the thickness and the radius of nanodots for different materials. The case of permalloy samples ($\text{Ni}_{80}\text{Fe}_{20}$), studied by Goiriena *et al*, is represented in Figure 1.13. [26] One can observe that a vortex is stable only for a given region of the phase space corresponding to intermediate aspect ratio values. Otherwise the magnetization is uniform, either in-plane or out-of-plane. The dashed lines separating the different possible ground states have been determined by both experimental and theoretical results. One can also generalize this type of phase diagram for all materials. To do so it is necessary to reduce the thickness and radius variables to remove material dependent behaviors. This can be done by dividing those by the exchange length ($l_{\text{ex}} = 5.7 \text{ nm}$, for permalloy).

Vortex Ansätze (in-plane and out-of-plane magnetization profile)

For different reasons developed in further sections it can happen that the vortex core is no more situated on the geometrical center of the dot. Guslienko *et al* developed two models to describe the magnetization in such case, namely the Single Vortex Ansatz (SVA) and the Two Vortex Ansatz (TVA). [27, 28, 29, 30] Those will be presented hereafter. Let us start for simplicity by the situation where the core is situated at the center of the dot. The reduced magnetization $\mathbf{m} = \mathbf{M}/M_s$ at a point $\mathbf{r} = (r, \theta)$ can be characterized by two angles, as seen in Figure 1.14a. The first one is the planar angle ϕ associated with the curling magnetization. The other one, the core profile angle Θ , is related to the shape of the latter. Those are given by:

$$\begin{cases} \Theta = \arccos(Pf(\rho)) \\ \phi = \theta + C\frac{\pi}{2} \end{cases} \quad (1.18)$$

With P the polarity, C the chirality and $f(\rho)$ a function to be proposed which models the out-of-plane component of the magnetization m_z .

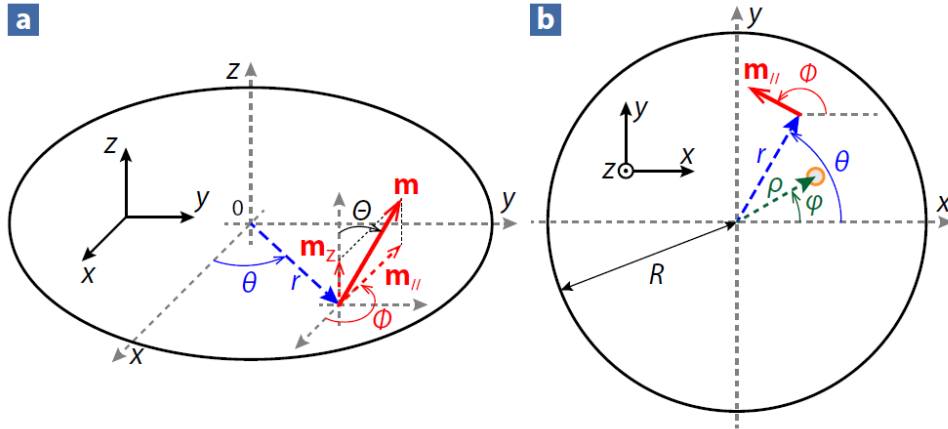


Figure 1.14: Schematic illustration of (a) the reduced magnetization $\mathbf{m} = \mathbf{M}/M_s$ at a point $\mathbf{r} = (r, \theta)$ (in spherical coordinates) in a magnetic nanopillar and (b) an off-centered vortex with its core in $\mathbf{X} = (\rho, \varphi)$ (in polar coordinates). Reproduced from [24].

Let us now consider an off-centered vortex with a core situated in $\mathbf{X} = (\rho, \varphi)$, as represented in Figure 1.14b. The SVA has been the first model proposed to describe this configuration. Its main advantage relies on its simplicity. The vortex core is translated from the origin to its new position, with no modification of its shape. The magnetization is then given as $\mathbf{m}(\mathbf{r} - \mathbf{X}(t))$. Due to this shape conservation side charges are created at the dot edges because the condition $(\mathbf{m} \cdot \mathbf{n})_s = 0$ is no more respected. The vector $\mathbf{n}$ represents the normal to the edge surface. A second model, namely the two vortex ansatz, has been theorized to resolve this problem. The latter consists in the superposition of two rigid vortices comparable to those assumed in the SVA. The first one is real and its center is situated in $\mathbf{X} = (\rho, \varphi)$. The second vortex is virtual and is an image of the first

one, with a center in $\mathbf{X}_I = (R^2/\rho, \varphi)$. The angles from Eq.1.18 are now derived as:

$$\begin{cases} \Theta = \arccos(Pf(\rho)) \\ \phi = \arg(\mathbf{r} - \mathbf{X}) + \arg(\mathbf{r} - \mathbf{X}_I) - \varphi + C\frac{\pi}{2} \end{cases} \quad (1.19)$$

Thanks to this correction one can prove that no charges are created on the edges. [24]

Influence of external magnetic fields

The addition of an external magnetic field will modify the vortex behavior. Three different cases will be studied hereafter. First in-plane and out-of-plane magnetic fields will be considered. Besides one can also take into account the Oersted field induced by a current injected in the nanopillar.

Let us start by analyzing the consequences of an in-plane magnetic field sweep. An example of experimental hysteresis loop is given in Figure 1.15. At high negative field $\mathbf{H}$ the nanodot magnetization reaches saturation. All magnetic moments are aligned to form a single magnetic domain. When the field increases one can remark a sudden step in magnetization. The latter corresponds to the nucleation of a vortex state which becomes energetically favorable at H_n , the nucleation field. Then one can find a region corresponding to the displacement of the vortex core. The latter moves perpendicularly to $\mathbf{H}$. At zero field, the vortex core is at the center of the dot and no net magnetization appears on the curve. Finally saturation is attained at a positive value H_{an} called the annihilation field. The core has reached the dot edges and the vortex is expelled from the dot. Another single domain state, aligned with the external field, is obtained. [31] When decreasing the field from positive saturation the same stages are obtained with a sudden demagnetization at nucleation and annihilation at high negative field.

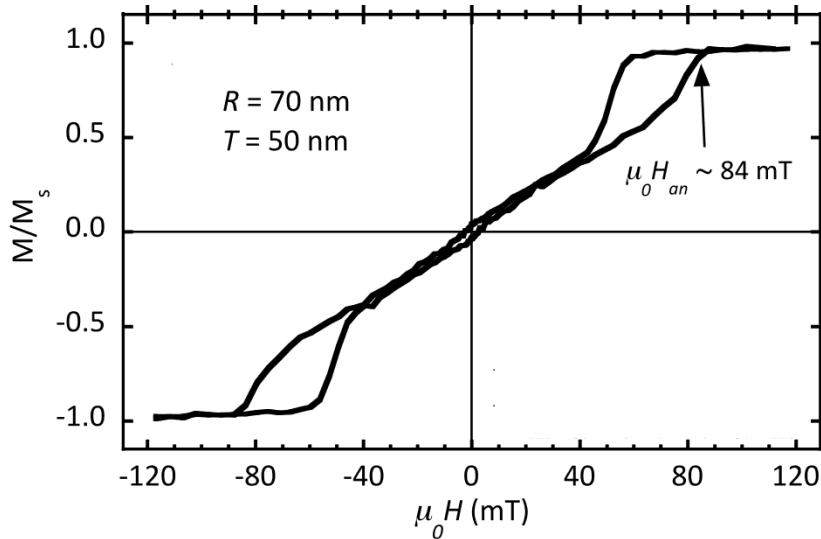


Figure 1.15: Hysteresis loop of a permalloy nanodot of radius $R = 70$ nm and thickness $T = 50$ nm. The external magnetic field is applied in-plane. The annihilation field is denoted by H_{an} . Adapted from [26].

Fried *et al* have conducted studies over the impact of out-of-plane (OOP) magnetic fields on the magnetization distribution of nanodots in vortex state. [32] In Figure 1.16 one can remark three major consequences resulting from the OOP field. The latter is applied in the direction of the vortex polarity. First the reduced z -component of the magnetization m_z is offset towards a value corresponding to the applied field. An enlargement of the vortex core is also noticed. The stronger the field, the larger the latter will be. Finally one can observe a slight increase of m_z at the dot edges. This effect is due to the canting of surface spins.

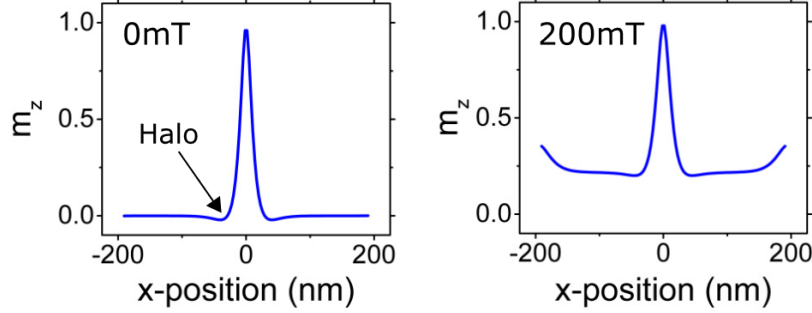


Figure 1.16: Z -component of the magnetization in a nanodot with $R = 192$ nm, $h = 30$ nm and $M_s = 800$ kA/m, obtained from micromagnetic simulations. (left) No out-of-plane field is applied. (right) With a uniform out-of-plane field of 200 mT. The profile results from a cutting at the center of the dot. Reproduced from [32].

As it will be developed in further sections an OOP electrical current is often injected into a nanopillar in practical applications. As in any other medium (e.g. conducting wires) this current will induce a magnetic field both inside and outside of the dot. The latter is called the Ampère-Oersted field, as the paternity of its discovery is discussed between the two scientists. In the context of this thesis we will call it Oersted field for purposes of simplicity. Its value is calculated by equaling an integral of the field along a curve C to the current inside this closed curve I_{enc} :

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \quad (1.20)$$

Inside the nanopillar one can derive its expression as follows, with $J = I/(\pi R^2)$ the current density:

$$\mathbf{B}(r) = \frac{\mu_0 J r}{2} \mathbf{u}_\theta \quad (1.21)$$

One observes a linear evolution of the field magnitude with the radius r inside the dot. Outside the latter one can demonstrate that it decreases proportionally to $1/r$.

From Eq.1.21 one can remark that the Oersted field possesses the same curling behavior as the planar magnetization component of a vortex. Depending on the respective orientation of the chirality with respect to the current direction one identifies two situations. If the Oersted field is opposed to the chirality the vortex is destabilized. On the contrary a stabilization occurs if both components are parallel. Those effects are dependent on the magnitude of the current as well as on geometrical parameters, as studied by Urazhdin *et al.* [33]

1.3 Vortex dynamics

Different approaches may be used to describe the vortex dynamics. At first, we will present the Landau-Lifshitz-Gilbert-Slonczewski equation which gives a precise description of the evolution of the magnetization with time. It is of tremendous importance for the study of vortices as it is the equation used by micromagnetic solvers to perform simulations. However it requires to work with finite element methods as it has no analytical solution for arbitrary geometries. This explains why another framework, based on the so-called Thiele equation, has been explored in past decades. This analytical model presents the main advantage of providing analytical results after solving a system of differential equations.

1.3.1 Landau-Lifshitz-Gilbert-Slonczewski equation

Landau and Lifshitz (LL) have studied the motion of the magnetization $\mathbf{M}$ caused by an effective magnetic field $\mathbf{H}_{\text{eff}}$ in ferromagnets. [34] The detailed expression of this field will be given in a further subsection. As spin moments of the electrons are considered in those the field $\mathbf{H}_{\text{eff}}$ will induce a precessional movement of the magnetization $\mathbf{M}$ around it, resulting from a torque of value $-\gamma_e \mathbf{M} \times \mu_0 \mathbf{H}_{\text{eff}}$. In this expression the gyromagnetic ratio $\gamma_e = g_e \mu_B / \hbar$ is a parameter comparing the magnetic moment of the electron with respect to its angular momentum. In SI units its value is $1.761 \cdot 10^{11} \text{ rad s}^{-1} \text{ T}^{-1}$. If no damping is considered the evolution of the magnetization with time can be written as:

$$\frac{d\mathbf{M}}{dt} = -\gamma_G \mathbf{M} \times \mathbf{H}_{\text{eff}} \quad (1.22)$$

Where γ_G , the Gilbert gyromagnetic ratio, is defined as $\gamma_G = \mu_0 \gamma_e$.

However the theoretical behavior predicted by the LL equation encounters discrepancies with several experimental observations. For example hysteresis loops of ferromagnets always present saturation at some point. The fact that the magnetization ends up aligned with the applied magnetic field is not predicted by Eq.1.22. Gilbert resolved this phenomenological issue by adding a supplementary damping term. [35] The LL equation was thus adapted to:

$$\frac{d\mathbf{M}}{dt} = -\gamma_G \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha_G}{M_s} \mathbf{M} \times \frac{d\mathbf{M}}{dt} \quad (1.23)$$

Where α_G represents the dimensionless Gilbert damping parameter. This expression is then known as the Landau-Lifshitz-Gilbert (LLG) equation.

Another way, called explicit version, to express the motion of $\mathbf{M}$ is given hereafter:

$$\frac{d\mathbf{M}}{dt} = -\gamma_L \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha_L}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}) \quad (1.24)$$

Where γ_L is the LL gyromagnetic parameter and α_L the LL damping parameter. These parameters are related to the Gilbert constants by the following relations:

$$\gamma_L = \frac{\gamma_G}{1 + \alpha_G^2} \quad (1.25)$$

$$\alpha_L = \frac{\alpha_G \gamma_G}{1 + \alpha_G^2} \quad (1.26)$$

One can demonstrate that the Eq.1.23 & 1.24 are similar if $\alpha_G \ll 1$.

The spin-transfer nano-oscillators (STNOs) under study in this thesis cannot be fully described by the LLG equation only. As a magnetic tunnel junction is used a spin-transfer torque will be exerted in the free layer which lies in a vortex state (cf. next section). Thus one needs to add supplementary terms to Eq. 1.23. Two improvements have been added by first Slonczewski [36] and Berger. [37] Later Zhang & Li proposed a refined version of the Berger term. [38] The latter is also called field-like torque (FLT). The complete expression for STNO applications is therefore given as follows:

$$\frac{d\mathbf{M}}{dt} = -\gamma_G \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha_G}{M_s} \mathbf{M} \times \frac{d\mathbf{M}}{dt} + \mathbf{T}_{\text{ST}}^{\text{Slonc.}} + \mathbf{T}_{\text{ST}}^{\text{Z\&L}} \quad (1.27)$$

Where the spin-transfer terms can be explicitly developed for the particular case of a magnetic tunnel junction in which the current is flowing perpendicular to the plane (CPP) [39]:

$$\mathbf{T}_{\text{ST}}^{\text{Slonc.}} = -\gamma_G \frac{a_J}{M_s^2} \mathbf{M} \times (\mathbf{M} \times \mathbf{M}_{\text{ref}}) \quad (1.28)$$

$$\mathbf{T}_{\text{ST}}^{\text{Z\&L}} = -\gamma_G b_J (\mathbf{M} \times \mathbf{M}_{\text{ref}}) \quad (1.29)$$

Where $\mathbf{M}_{\text{ref}}$ is the magnetization direction of the polarizer, i.e. the fixed layer. The parameters a_J and b_J are related to the amplitude of the Slonczewski and field-like torque, respectively. Those can be derived as:

$$a_J = \frac{\hbar J P}{2heM_s} \quad (1.30)$$

$$b_J = k^{\text{CPP}} a_J \quad (1.31)$$

Where J is the uniform CPP current density, P the spin polarization of the current and h the thickness of the free layer. The factor k^{CPP} represents the degree of non-adiabaticity.

Mumax³ the software used in the framework of this thesis to simulate the dynamics of vortices uses the LLG equation including the STT terms. It allows to compute the magnetization for each element of a mesh and its evolution with time.

1.3.2 Micromagnetic approach

For a complete description of magnetic systems one should take into account all atomic spins and their interactions. Fortunately one can obtain an accurate description of the properties of magnetic materials even when approximating those as a continuum. The micromagnetic approach has been developed in accordance with this assumption to model the evolution of the magnetization $\mathbf{M}$ at nanoscopic length scales. The atomic structure is therefore neglected but the typical lengths considered are much smaller than the size of the studied material. This framework is then well suited for the analysis of magnetic vortices in sub-micrometer dots.

A minimization of the energy allows to derive the magnetization at any spatial point $\mathbf{r}$. Let us start by considering a system at stationary equilibrium. The total energy can

be decomposed in a sum of terms, namely the exchange energy E_{ex} , the magnetostatic energy E_{ms} , the anisotropy energy E_A and the Zeeman energy E_Z :

$$E = E_{\text{ex}} + E_{\text{ms}} + E_A + E_Z \quad (1.32)$$

Other effects such as magnetoelastic or domain wall energy will be neglected here. An expression of each term is given hereafter, as detailed by Kittel. [40] The reduced magnetization $\mathbf{m} = \mathbf{M}/M_s$ is used in the equations, as well as V the volume.

The exchange energy is given as:

$$E_{\text{ex}} = A \int_V (\nabla \mathbf{m})^2 dV \quad (1.33)$$

Where A is the exchange stiffness.

The magnetostatic energy is given as:

$$E_{\text{ms}} = -\frac{\mu_0 M_s}{2} \int_V \mathbf{m} \cdot \mathbf{H}_d dV \quad (1.34)$$

Where $\mathbf{H}_d$ is the demagnetizing field created by the magnetization.

The anisotropy field is given as:

$$E_A = \int_V K e_A(\theta) dV \quad (1.35)$$

Where K is the anisotropy constant and e_a a function which depends on θ the angle between an easy axis and the magnetization. [24]

The Zeeman energy is given as:

$$E_Z = -\mu_0 M_s \int_V \mathbf{m} \cdot \mathbf{H}_{\text{ext}} dV \quad (1.36)$$

Where $\mathbf{H}_{\text{ext}}$ is the external magnetic field.

Assembling these terms allows to rewrite Eq.1.32 as:

$$E = \int_V \left(A(\nabla \mathbf{m})^2 - \frac{\mu_0 M_s}{2} \mathbf{m} \cdot \mathbf{H}_d + K e_A(\theta) - \mu_0 M_s \mathbf{m} \cdot \mathbf{H}_{\text{ext}} \right) dV \quad (1.37)$$

By taking $\frac{\mu_0 M_s^2}{2}$ out of the integral one obtains:

$$E = \frac{\mu_0 M_s^2}{2} \int_V \left(l_{\text{ex}}^2 (\nabla \mathbf{m})^2 - \frac{1}{M_s} \mathbf{m} \cdot \mathbf{H}_d + \kappa e_A(\theta) - \frac{2}{M_s} \mathbf{m} \cdot \mathbf{H}_{\text{ext}} \right) dV \quad (1.38)$$

This operation reveals two material parameters. In the first term the exchange length l_{ex} appears. The magnetic hardness factor κ is also identified as:

$$\kappa = \frac{2K}{\mu_0 M_s^2} \quad (1.39)$$

The latter measures the relative importance of the anisotropy compared to the magnetostatic effects. [24] Another way to express it is by using the Bloch wall width which gives $\kappa = (l_{\text{ex}}/l_{\text{w}})^2$.

To minimize the system energy the magnetization $\mathbf{M}(\mathbf{r})$ must be aligned with the effective field $\mathbf{H}_{\text{eff}}(\mathbf{r})$. This field includes the contributions of the different magnetic fields related to the energy terms developed hereabove. It can be calculated as the derivative of the total energy density with respect to the magnetization vector:

$$\mathbf{H}_{\text{eff}} = -\frac{1}{\mu_0 M_s V} \frac{\partial E}{\partial \mathbf{m}} \quad (1.40)$$

Using the expression found in Eq.1.37 one obtains:

$$\mathbf{H}_{\text{eff}} = \frac{2A}{\mu_0 M_s} \nabla^2 \mathbf{m} - \frac{1}{\mu_0 M_s} \frac{\partial E_A}{\partial \mathbf{m}} + \frac{\mathbf{H}_d}{2} + \mathbf{H}_{\text{ext}} \quad (1.41)$$

This effective field is of capital importance to capture the dynamics of vortices. Indeed the micromagnetic approach will be used theoretically to describe equations of motion which are later used in micromagnetic simulations.

1.3.3 Thiele equation approach

As presented previously, the vortex topology is the magnetic groundstate for a range of cylindrical dots of specific aspect ratio. Without any external contribution the vortex is generally at rest, i.e. its core remains at the center of the dot. However if the vortex is subject to a sufficient solicitation, such as an external field or a current injection, its core position $\mathbf{X}(t)$ varies thus with time so will the magnetization distribution. For spin-diode applications we will only consider the gyrotropic mode, which is the most studied and simple regime of oscillation.

The dynamics of vortices has been widely explored in the literature. With time one framework has become dominant to study them analytically. The latter originates from the Thiele equation, developed in the seventies. [41, 42] Successively Huber adapted it to the specific case of vortices. [43] This approach is based on the LLG equation. The vortex, seen as a quasi-particle, dynamics will be studied here through the evolution of its core position $\mathbf{X}(t)$ and its derivatives with time. The Thiele equation is given as:

$$\bar{\mathbf{M}} \cdot \ddot{\mathbf{X}} + \bar{\mathbf{G}} \cdot \dot{\mathbf{X}} + \bar{\mathbf{D}} \cdot \dot{\mathbf{X}} = \frac{\partial W(\mathbf{X})}{\partial \mathbf{X}} + \mathbf{F}^{\text{ext}} \quad (1.42)$$

Let us detail each term composing this equation.

The total (potential) magnetic energy W is decomposed in different contributions:

$$W = W^{\text{ms}} + W^{\text{ex}} + W^{\text{Oe}} + W^{\text{Z}} + W^{\text{T}} \quad (1.43)$$

Where W^{ms} is the magnetostatic energy, W^{ex} the exchange energy, W^{Oe} the Oersted field energy, W^{Z} the Zeeman energy and W^{T} the thermal energy.

The external force vector $\mathbf{F}^{\text{ext}}$ is also composed of different terms:

$$\mathbf{F}^{\text{ext}} = \mathbf{F}^{\text{ST}} + \mathbf{F}^{\text{dip}} + \dots \quad (1.44)$$

Where $\mathbf{F}^{\text{ST}}$ is the force related to spin-transfer torque and $\mathbf{F}^{\text{dip}}$ is the force created by dipolar coupling (e.g. created by another oscillator). Other sources of external force could be added.

Using tensorial notations one can calculate the components of the mass tensor $\bar{\bar{\mathbf{M}}}$, the gyrotropic mode tensor $\bar{\bar{\mathbf{G}}}$, the dissipation, i.e. damping, tensor $\bar{\bar{\mathbf{D}}}$ and the spin-transfer torque force vector $\mathbf{F}^{\text{ST}}$. [44]

$$M_{\alpha\beta}(\mathbf{X}) = \frac{1}{\gamma_G^2} \int d^3\mathbf{r} \left(\partial_\alpha \times \frac{\partial \mathbf{m}}{\partial \dot{X}_\beta} \right) \cdot \mathbf{m} \quad (1.45)$$

$$G_{\alpha\beta}(\mathbf{X}) = \frac{M_s}{\gamma_G} \int d^3\mathbf{r} (\partial_\alpha \mathbf{m} \times \partial_\beta \mathbf{m}) \cdot \mathbf{m} \quad (1.46)$$

$$D_{\alpha\beta}(\mathbf{X}) = -\alpha_G \frac{M_s}{\gamma_G} \int d^3\mathbf{r} (\partial_\alpha \mathbf{m} \cdot \partial_\beta \mathbf{m}) \quad (1.47)$$

$$F_\alpha^{\text{ST}}(\mathbf{X}) = M_s a_J \mathbf{m}_{\text{ref}} \cdot \int d^3\mathbf{r} (\mathbf{m} \times \partial_\alpha \mathbf{m}) \quad (1.48)$$

Where $\mathbf{m} = \mathbf{M}/M_s$ is the reduced magnetization and $\mathbf{m}_{\text{ref}} = \mathbf{M}_{\text{ref}}/M_s$ is the direction of the magnetization in the polarizer.

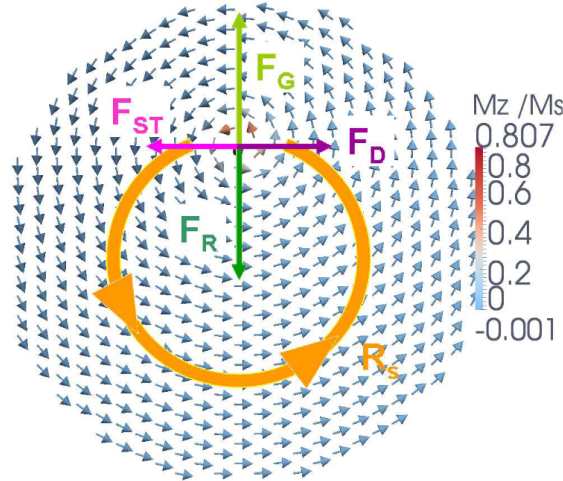


Figure 1.17: Free layer of a MTJ with a moving magnetic vortex (view from above). The radius of steady oscillations is represented by R_s . The different forces acting on the vortex are represented with arrows. Those are the spin-torque $\mathbf{F}_{\text{ST}}$, damping $\mathbf{F}_{\text{D}}$, gyrotropic $\mathbf{F}_{\text{G}}$ and restoring $\mathbf{F}_{\text{R}}$ forces. Reproduced from [44].

In this thesis the mass tensor contribution will be neglected as it is negligible for small aspect ratios $\xi < 0.15$. [45] The spin-transfer torque will also be the only external force taken into account as single STNOs will be considered, suppressing the dipolar term. Following these simplifications one can represent the remaining forces acting on the vortex core, as illustrated in Figure 1.17. In steady-state oscillations the vortex rotates along a circle of radius $R_s = |\mathbf{X}|$. The spin transfer force $\mathbf{F}^{\text{ST}}$ is parallel to the velocity vector and opposed to the damping term $\mathbf{F}^{\text{D}}$. The restoring force $\mathbf{F}^{\text{R}}$ is pointing towards the center of the dot and counterbalanced by the gyrotropic force $\mathbf{F}^{\text{G}}$. The

frequency of oscillation f , i.e. the number of vortex core rotations around the dot center per second, will be a central point of discussion in this report.

The analysis of the equilibrium between the two couples of collinear forces allows to deduce two fundamental properties of vortex oscillators, namely the critical current J_c and the gyrotropic frequency ω_0 .

Relaxation frequency

The relaxation frequency of vortices in cylindrical dots of thickness h and radius R has been studied by Gusliencko, based on the equilibrium between the gyrotropic and restoring forces. [46] Starting from the Thiele equation he showed that the motion of an off-centered vortex in a nanopillar could be written as:

$$\mathbf{G} \times \frac{d\mathbf{X}}{dt} - \frac{\partial W(\mathbf{X})}{\partial \mathbf{X}} = 0 \quad (1.49)$$

Where $\mathbf{G} = -G\hat{\mathbf{z}}$ is the gyrovector, with $G = 2\pi PhM_s/\gamma_G$.

Within the framework of the two vortex ansatz one can develop the potential energy as a Taylor expansion if the core is slightly shifted from the dot center:

$$W(\mathbf{X}) = W(0) + \frac{1}{2}k|\mathbf{X}|^2 + O(|\mathbf{X}|^4) \quad (1.50)$$

Where k is the spring-like restoring constant associated with the total potential energy ($\partial W/\partial \mathbf{X} = k\mathbf{X}$). It can be found by summing all energy term spring constants:

$$k = k^{\text{ms}} + k^{\text{ex}} + C J k^{\text{Oe}} + k^{\text{Z}} + k^{\text{T}} \quad (1.51)$$

Each of these terms will be developed in Section 2.2. One should notice that the magnetostatic term is generally dominant. On another note the Oersted field constant will gain importance with increasing current density.

Finally one can demonstrate that the vortex relaxation frequency can be calculated as:

$$\omega_0 = \frac{k}{G} \quad (1.52)$$

The value of ω_0 generally lies in the radiofrequency range, i.e. microwave, with variation depending on the geometry.

Finally Gusliencko states that, within the TVA approach, the volume averaged magnetization would be proportional to the displacement of the vortex core by a factor $2/3$, so that $\langle \mathbf{m}(\mathbf{r}) \rangle_V = -2/3C(\hat{\mathbf{z}} \times \mathbf{X}/R)$.

Critical currents

Gusliencko *et al* calculated the conditions to be stipulated on the injected CPP current to obtain stable oscillations of the vortex core position. [44] They obtained two critical currents such as $J_{c1} < J < J_{c2}$ giving rise to a steady-state. Stating that the damping

tensor is diagonal in circular geometry ($\bar{\mathbf{D}} = D\bar{\mathbf{I}}$) and posing a reduced radius $s = |\mathbf{X}|/R$ one can derive the following equation:

$$d(s)|G|\omega_G(s)sR = F^{\text{STT}} \quad (1.53)$$

Where $d(s)$ is the dimensionless damping parameter $d = -D/|G| > 0$. It can be modeled by a parametric second order non-linear approximation by $d(s) = d_0 + d_1s^2$, where d_0 and d_1 are geometry dependent. The vortex gyrotropic frequency ω_G is given as $\omega(s) = \omega_0 + \omega_1s^2$, with $\omega_0(\beta) = (20/9)\gamma_G M_s \beta(1 - 4\beta/3)$ and $\omega_1(\beta) \simeq 4\omega_0(\beta)$ (valid for a large range of aspect ratio $\beta = h/R$). If the contribution of the Oersted field is taken into account one finds $\omega_0 = \omega_0(\beta) + \omega_{\text{Oe}}J$, where $\omega_{\text{Oe}} = (8\pi/15)(\gamma_G/c)C[1 + 15(\ln 2 - 1/2)R_c/8R]$. The latter expression is calculated for finite core radius $R_c \ll R$. In the linear approximation a balance between the damping and the spin-transfer forces allows to find a first critical current J_{c1} above which oscillations can happen:

$$J_{c1} = \frac{d_0\omega_0}{\gamma_G\sigma/2 - d_0\omega_{\text{Oe}}} \quad (1.54)$$

Where $\sigma = a_J/J$.

Thus the value of the steady-state vortex radius increases for higher current densities and writes:

$$s(J) = \lambda \sqrt{\frac{J}{J_{c1}} - 1}, \quad J > J_{c1} \quad (1.55)$$

With λ given as:

$$\lambda^2 = \frac{1}{2} \frac{\gamma_G\sigma J_{c1}}{(d_1\omega_0(J_{c1}) + d_0\omega_1)} \quad (1.56)$$

From those results the gyrotropic frequency can be found as:

$$\omega_G = \omega_0(\beta) + \omega_{\text{Oe}}J + \lambda^2(J/J_{c1} - 1)\omega_1 \quad (1.57)$$

Excessive current density ($J > J_{c2}$) can lead to two different processes stopping steady state oscillations. Either the vortex is expelled because its core position goes beyond the dot radius ($s = 1$) or its velocity $|\dot{\mathbf{X}}|$ reaches a critical value ν_c . This leads to a reversal of the vortex polarity preventing excitation and causing damping to the dot center. This condition is translated into the following equation $\omega_G(J)s(J)R = \nu_c$. Those two conditions lead to two maximal currents, J_{c2} and J'_{c2} . The minimum value between both expressions for a given dot geometry is retained as the limiting condition.

1.4 Spin-transfer nano-oscillators (STNOs)

Combining the principles from spintronics and nanomagnetism a new kind of oscillator device has emerged in the early 2000's, the so-called spin-transfer nano-oscillators (STNOs). It allows to transform an injected dc current into an ac voltage from the spin-transfer torque and magnetoresistance phenomena in a nanoscale structure.

An example of that kind of devices is presented in Figure 1.18a. A typical trilayer nanopillar is considered. Their radius can be as small as 10 nm. [3] The junctions are

composed of two ferromagnetic layers separated by a nonmagnetic spacer. A first thick FM layer acts as a polarizer. Its magnetization $\mathbf{M}$ is fixed. The second FM layer is free and usually much thinner. Its magnetization $\mathbf{m}$ direction will thus vary with time. To do so a dc current perpendicular to the plane is injected into the junction. The latter is polarized by the fixed layer magnetization. A spin-transfer torque is therefore exerted on the free layer. As introduced in previous sections sustained precessional movements of the magnetization $\mathbf{m}$ can be obtained if the current density is large enough. Consequently the relative orientation of the magnetization between the two layers is varying with time. This leads to the apparition of a magnetoresistance effect in the junction. An oscillating voltage of shape $v = v_0 \cos(\omega t + \varphi)$ can thus be measured across the junction, as illustrated in Figure 1.18b.

First STNOs were based on the GMR phenomenon leading to picowatt range emitted power. A switch to TMR magnetic tunnel junction allowed to increase this power to microwatts. [22] STNOs present many interesting features notably important for telecommunication applications. Their signal frequency lies in the microwave range and can be tuned from hundreds of megahertz to tens of gigahertz. [3] Those devices are also CMOS compatible and their nanoscale allows implementing them by millions on a single chip. Current researches are mainly focused on improving their spectral performances.

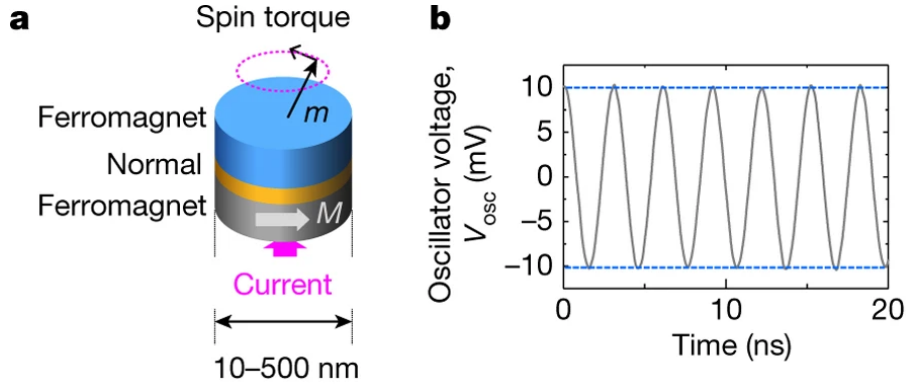


Figure 1.18: Schematic illustration of (a) a spin-torque nano-oscillator made of a stack of FM/NM/FM layers. The first FM of fixed magnetization M is the polarizer. The second FM layer is free. An injected dc current causes spin-transfer torque thus inducing magnetization m precessions. (b) An ac voltage is measured across the structure due to magnetoresistance. Reproduced from [3].

In the framework of this report a specific type of STNO, based on the vortex state, is considered. This category has been named the spin-torque vortex oscillators (STVOs). In this case the free layer presents a magnetic vortex topology. Above a critical current J_c its core will start to oscillate around the center of the dot. As a result the mean planar magnetization direction changes with times, allowing the desired MR effect to occur.

1.5 Spin-torque diode effect

Vortex-based nano-oscillators are one of the most promising configurations for future practical implementations. Indeed their nonlinear behavior is crucial with regards of neuromorphic computing and hardware artificial intelligence applications as those can mimic neuronal properties. In addition to neural networks, STVOs have found use in a completely different field of applications, i.e. frequency detection. Based on the spin-diode effect, presented hereafter, one is able to associate the response of the oscillator to the frequency of an alternating input signal. Moreover these junctions can be used in a passive way, i.e. without any dc bias, which is a tremendous advantage as far as energy consumption is concerned.

1.5.1 Introduction

Conversely to radiofrequency generation with STNOs one can also inject an ac signal into a MTJ to obtain a rectified dc voltage component as an output. This effect, based on resonance mechanisms between the input current and the eigenfrequency of spin oscillations in the free layer, is called the spin-torque diode (STD) effect.

This phenomenon relies on the fact that the alternating injected current will cause oscillations in the magnetization of the free layer. Indeed a spin-transfer torque is created by the polarizer ($\mathbf{M}_{\text{fixed}}$). It is noteworthy that the rotation direction of $\mathbf{M}_{\text{free}}$ depends on the current sign. For a positive (resp. negative) current $\mathbf{M}_{\text{free}}$ rotates towards (resp. away from) $\mathbf{M}_{\text{fixed}}$. [47] As the respective magnetization orientation between the two FM layers changes with time, magnetoresistance is obtained. Moreover the junction resistance evolves at the same frequency with respect to the ac current which leads to the measure of a dc voltage output across the junction. The frequency of the input signal ω is of tremendous influence on the magnitude of the output voltage. If it is close enough to the natural frequency of spin oscillation in the free layer the effect is amplified through a resonance mechanism.

The first experimental results related to the STD effect were obtained by Tulapurkar *et al* in 2005, on a magnetic tunnel junction. [5] The latter was composed of a CoFeB/MgO/CoFeB structure, one of the CoFeB being pinned by AF coupling to create a fixed layer. Their experimental curves are presented in Figure 1.19. One can observe that the measured dc voltage across the junction depends on the frequency of the input signal. Close to the eigenfrequency of the free layer a large amplification is obtained. An external magnetic field was also applied at an angle of 30° from the in-plane $\mathbf{M}_{\text{fixed}}$ direction. This leads to a shift of the amplification peak to higher frequencies.

The limitation of the presented hereabove spin-diode is its limited sensitivity ϵ . The latter is defined as follows:

$$\epsilon = \frac{\overline{\Delta V}}{P_{\text{in}}} \quad (1.58)$$

Where $\overline{\Delta V}$ is the dc output voltage and P_{in} the input power. It is generally expressed in mV mW⁻¹.

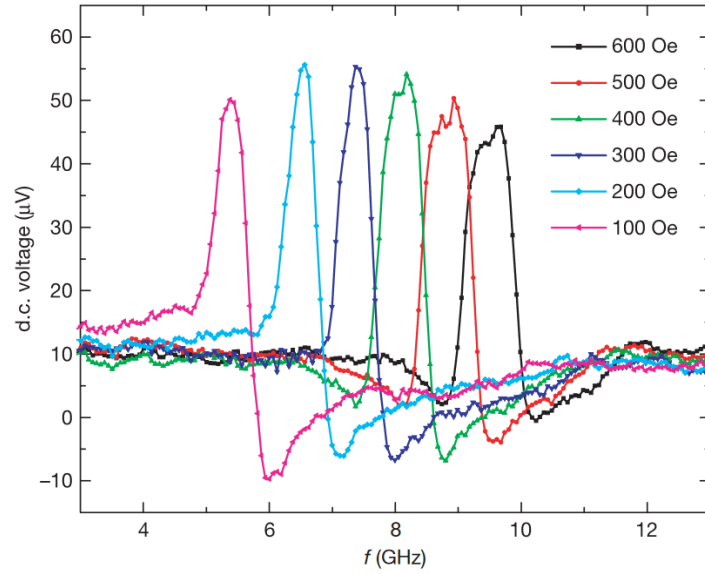


Figure 1.19: Spin diode effect in a CoFeB/MgO/CoFeB MTJ. The dc output voltage is plotted with respect to the frequency of the ac current injected (0.55 mA). Different values of external perpendicular magnetic field are applied. Reproduced from [5].

The sensitivity of spin-torque diodes is a relevant parameter to characterize their performances. Many works have shown that STDs could surpass classical Schottky diodes, with experimental values of up to tens of thousands of mV mW^{-1} . [47] These results could lead to high-end applications (e.g. radar detectors).

1.5.2 Vortex-based RF signal detection

Nanopillars presenting a vortex as the magnetic groundstate of the free layer have also been studied, especially for low frequency applications (below 1 GHz). Those present many advantages, among them an increased sensitivity for low power input signals and the lack of need to apply an external magnetic field in some configurations. [47] Those properties could lead to passive frequency detection devices. Indeed once the STD has been characterized it is possible to associate an output dc voltage to a given radiofrequency with high precision.

Skirdkov *et al* have investigated the impact of a dc bias on the sensitivity of vortex-based STDs. [48] As introduced in Section 1.3.3 a critical current J_c is required to obtain steady-state oscillations of the vortex core around its equilibrium position. By applying a dc bias approaching this critical value ($J = J_{dc} + J_{ac} \sin(\omega t)$), the oscillations produced by the rf signal are amplified more acutely. Their analytical results on the sensitivity of the STD with respect to the frequency for different dc biases are presented in Figure 1.20a. One can note that an increased sensitivity is obtained for a dc bias close to the critical current. Above J_c the sensitivity is still improved. However over this current threshold an injection-locking is often required to match the frequency of oscillation with the rf signal, diminishing the interest.

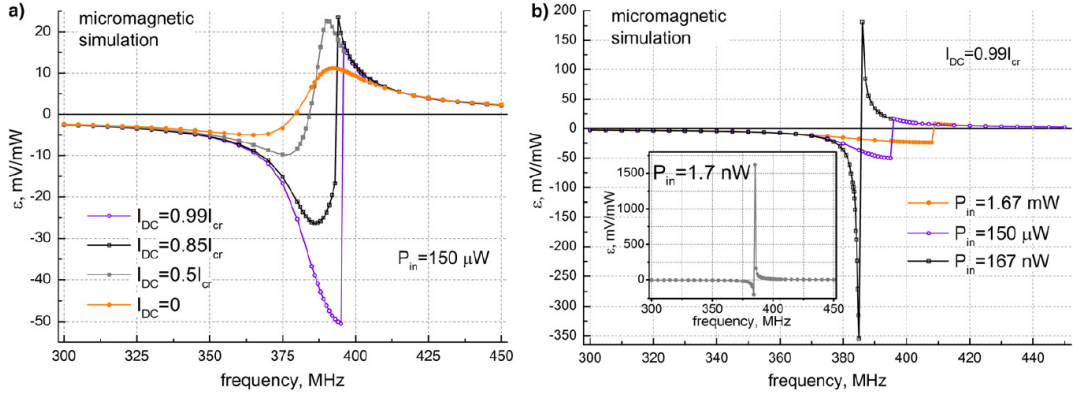


Figure 1.20: Micromagnetic simulations of the sensitivity with respect to the frequency of a MTJ with a free layer made of permalloy, of thickness $h = 9$ nm and radius $R = 100$ nm. (a) For different input biases at fixed power. (b) For different input powers at fixed bias. Reproduced from [48].

Another important conclusion from their work is that the sensitivity increases for low power input signals (Fig.1.20b). Nevertheless one should take into account noise and accessible precision on measurement which are neglected by simulations. Those considerations balance considerably the practical reachable sensitivity for ultra small input power.

Jenkins *et al* demonstrated another method for radiofrequency detection. [49] The latter is based on the expulsion of the vortex core for high enough rf currents. Even if it requires important input power and an external magnetic field contribution it is still worth mentioning. Its principle repose on the fact that the radius of oscillation of the vortex core increases for higher currents. Also this effect is amplified for rf frequencies close to the resonant frequency of the vortex. As a consequence the vortex core can be expelled from the nanopillar for a right combination of those parameters. After the vortex core expulsion the free layer presents a novel magnetic distribution called the C-state. The interest of this relies on the fact that the resistance of the MTJ is strongly modified by this state modification. As a result a sensitivity of up to $40\,000 \text{ mV mW}^{-1}$ has been obtained.

Starting from the Thiele equation they developed an analytical model for the evolution of the radius of oscillation ρ_{osc} as a function of the input current magnitude and frequency. For a rf current density J_{rf} with a dc bias J_{dc} they obtained:

$$\rho_{osc} = \frac{J_{rf} \sqrt{\Lambda_{Slong.}^2 + \Lambda_{FLT}^2}}{2G \sqrt{(\omega - \omega_0)^2 + \frac{(-\pi \sigma M_s h J_{dc} + D\omega)^2}{G^2}}} \quad (1.59)$$

Where $\Lambda_{Slong.}$ and Λ_{FLT} correspond to the in-plane forces created by the spin-transfer torques divided by the current density ($F_{Slong.||} = \Lambda_{Slong} J$, $F_{FLT||} = \Lambda_{FLT} J$). The natural frequency of the oscillator is referred as ω_0 .

From micromagnetic simulations they confirmed the expected behavior of the vortex core. For a free layer of permalloy ($R = 250$ nm) they obtained the transient motion of

the vortex core for an input rf current of 1.6 mA at 100 MHz. As expected it reaches the dot edges and is eventually expelled. As a consequence a C-state magnetization results, leading to an abrupt change in resistance.

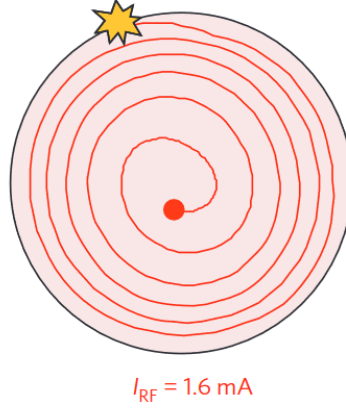


Figure 1.21: Simulation of the transient motion of the vortex core for an input rf current of 1.6 mA at 100 MHz. The free layer of the MTJ ($R = 250$ nm) is made of permalloy. Reproduced from [49].

1.6 Memristive effect

As spin-transfer nano-oscillators have now been introduced, let us now focus on the hardware implementation of artificial synapses. As a reminder the role of these components is to ensure the communication between neurons. One of the main property of biological synapse is called plasticity, i.e. the ability to control the importance of particular connections and to make it evolve. It has been demonstrated that various physical systems can mimic this effect. In this work we focused our efforts on the study of memristances. As a first approximation, these may be seen as resistance which value evolves with past currents and voltages. As it will be explained hereafter, those constitute promising devices regarding neuromorphic applications.

1.6.1 Introduction

To describe any electrical circuit four variables are required. Those are the voltage v , the current i , the charge q and the flux linkage ϕ . As a consequence a total of six two-variable relationships is possible. Three of them are given by the basic circuit components which are the capacitor (q, v), the inductor (ϕ, i) and the resistor (v, i). Two others are given as follows:

$$q(t) = \int_{-\infty}^t i(t') dt' \quad (1.60)$$

$$\phi(t) = \int_{-\infty}^t v(t') dt' \quad (1.61)$$

One missing equation should therefore link ϕ and q . In 1971 Chua proposed the introduction of a last basic component resolving this problem, namely the memristor. [7] Its

names originates from the fact that it behaves similarly to a resistor with the difference that it keeps a memory of past applied currents and voltages. This results in a nonlinear resistance value which can change with time.

A charge-controlled memristor is described by the following equation:

$$v(t) = M(q(t))i(t) \quad (1.62)$$

Where M is a positive value given by:

$$M(q) = \frac{d\phi(q)}{dq} \quad (1.63)$$

It is then expressed in Wb C^{-1} which is equivalent to ohm. Its value evolves as it depends on the charge which depends itself on time. When no current is flowing into the circuit M does not change. This confers the desired memory property to the system. Under an oscillating periodic voltage a typical pinched hysteresis curve is obtained. The latter is depicted in Figure 1.22. One can clearly notice that $i = 0$ if $v = 0$. At higher frequencies the loop narrows and the hysteresis disappears.

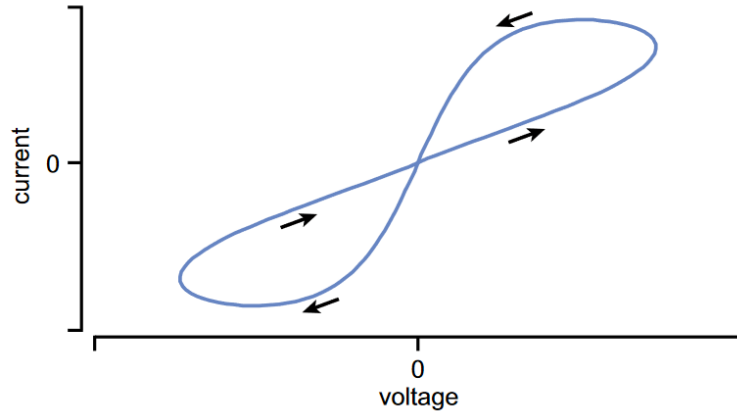


Figure 1.22: Ideal memristive curve under a periodic voltage excitation. A typical pinched hysteresis loop is obtained. Reproduced from [6].

1.6.2 Practical implementation

The first experimental proposal of a memristor system was exposed in 2008 by a team from HP Labs. [50] Their device was made of a thin semiconductor film based on TiO_2 terminated by two electrodes. This film of total thickness D is composed of two distinct regions modeled by two resistors in series. The first one, of length w , is doped and presents a low resistance R_{ON} . The second one is undoped and is therefore a better electrical insulator of resistance R_{OFF} . Applying a voltage difference $v(t)$ across the device causes the dopants to move and a displacement of the separation between the two regions. A schematic of the device is given on Figure 1.23 (left). The following $i - v$ relationship is then obtained:

$$v(t) = \left(R_{\text{ON}} \frac{w(t)}{D} + R_{\text{OFF}} \left(1 - \frac{w(t)}{D} \right) \right) i(t) \quad (1.64)$$

The evolution of the frontier between regions with time is expressed as:

$$\frac{dw(t)}{dt} = \mu_V \frac{R_{ON}}{D} i(t) \quad (1.65)$$

Where μ_V is the dopant mobility. The integral of this equation can be combined with previous developments to give the final memristance relation:

$$M(q) = R_{OFF} \left(1 - \frac{\mu_V R_{ON}}{D} q(t) \right) \quad (1.66)$$

This result is obtained with the assumption that $R_{ON} \ll R_{OFF}$.

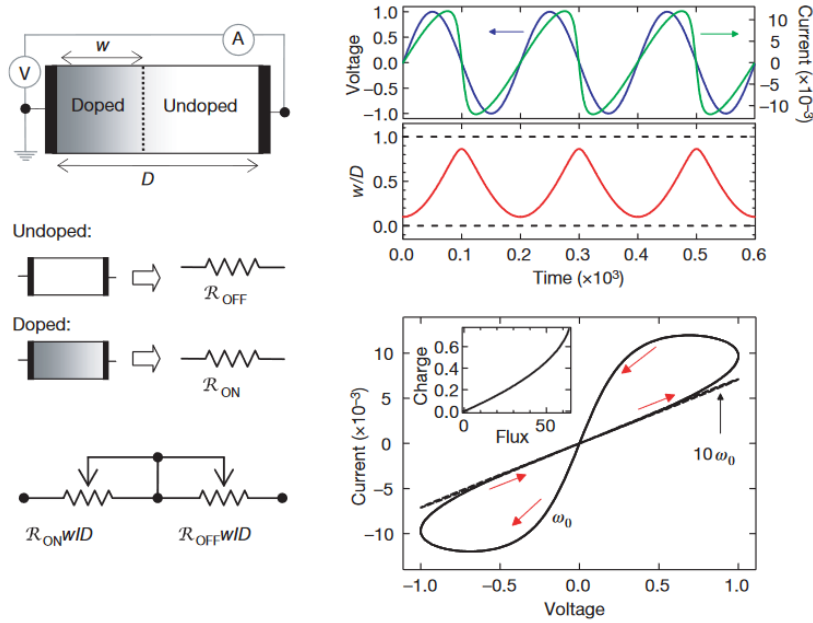


Figure 1.23: Left : schematic illustration of the memristor device with the two regions (doped and undoped). The two resistors are modeled and an equivalent circuit is given. Right, top : applied voltage $v = v_0 \sin(\omega_0 t)$ (blue) and obtained current (green). The evolution of the w/D ratio is depicted (red). Right, bottom : corresponding hysteresis loop. A straight line is obtained at $10 \omega_0$. Reproduced from [50].

Experimental results are presented on Figure 1.23 (right). A potential $v = v_0 \sin(\omega_0 t)$ is applied, displacing the frontier between the doped and undoped regions. The resulting current allows to draw an $i - v$ curve. On the latter one can notice the validity of previous mathematical developments. A pinched hysteresis loop is obtained. The team also demonstrates that this hysteresis collapses at higher frequencies. A straight line appears after a sweep in frequency to $10 \omega_0$.

Following this publication many other material memristor designs have been proposed. However an intense debate exists in the scientific community on whether or not a real memristance device has been demonstrated practically, corresponding to the original 1971 definition. Many criticisms have emerged regarding the 2008 HP labs paper

since it would rather correspond to a memristor system than a real memristor. [51] The difference between the two cases relies on the fact that M should only depend on q . Here M has been proven to also be a function of the current. Nevertheless the dispute does not prevent from thinking about possible applications, at least from a theoretical standpoint.

1.6.3 Neural network applications

Biological neural networks are composed of two constitutive components, namely the neurons and the synapses. The latter serves as a neuronal connector. In hardware neural network applications one desires to reproduce physically those two functions. As developed in previous sections STNOs can be compared to neurons. This is mainly due to their ability to respond to an input by a nonlinear output. However, until the emergence of memristors, no practical artificial device was found to mimic a synaptic behavior. Those possess peculiar properties which cannot be implemented easily with classical passive electrical components. Thomas resumed three fundamental characteristics of synapses, namely long-term potentiation (LTP), long-term depression and retention (LTD) and spike timing-dependent plasticity (STPD). [6] He showed that a MTJ-based memristor could fulfill those requirements.

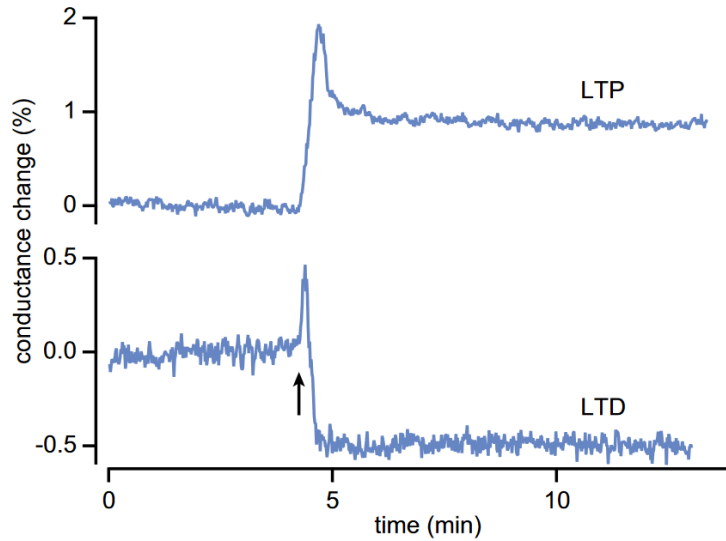


Figure 1.24: LTP (top) and LTD (bottom) behavior in a memristive magnetic tunnel junction. The black arrow indicates a stimulus dispensed after 4 min. Reproduced from [6].

The long-term potentiation describes the long lasting strengthening of a connection after a stimulus. It possesses three major properties; cooperativity, associativity and input-specificity. This implies different considerations. First a signal of minimal strength is required. Above this threshold LTP is triggered. Other than that an addition of several weak signals can still allow LTP to occur. Finally only the synapse receiving an input is affected. Long-term depression can be seen as an opposite effect to LTP. Indeed after a stimulus a persistent decrease of the strength of a connection is observed. The

last effect, spike timing-dependent plasticity (STPD), determines whether LTP or LTD is applied. Depending on the time lag between different input signals, especially if it is positive or negative, one of both mechanism is executed.

Figure 1.24 illustrates how an artificial material structure can mimic those behaviors. Here a MTJ-memristor is able to reproduce both LTP and LTD characteristics. The strengthening or weakening of the connection is associated to a change of conductance in the device.

1.7 Nanowire networks

In the framework of this thesis our memristor device will be based on a network of interconnected nanowires (NW) made of silver. The reasons of this choice will be explained hereafter.

1.7.1 Introduction

A nanowire is a cylinder of which diameter is of the order of several nanometers. Researchers have started to be interested in that kind of structures several decades ago. Their large aspect ratio allows indeed to highlight some important physical properties. The Pr. Piroux group from UCLouvain has developed a strong expertise in the field over the years, especially regarding an interconnected architecture. [52, 53, 54] Starting from a porous polymer template it is possible to create an artificial structure of intersecting nanowires.

This specific configuration presents many advantages. Firstly the polymer matrix ensures the flexibility of the device. It can also be removed by chemical etching to reveal only the metallic structure (e.g. for SEM imaging). Moreover the multiple contacts between the wires provide electrical conductivity. The latter can be tuned by adjusting template parameters, i.e. the wire density or their diameter. These interesting characteristics have allowed to use them for intense research purposes as well as for promising future industrial applications. [52]

The fabrication process uses an electrodeposition technique. Starting from a nanoporous template (Fig.1.25a) an electrolytic solution containing metallic ions of interest is reduced by a potential application. This step allows the filling of the pores. The most simple configuration is to reduce only one species of ions. When doing so homogeneous wires are obtained (Fig.1.25b). However more complex structure may also be achieved, such as nanotubes (Fig.1.25c) and multilayered networks (Fig.1.25d). This can be done by varying the fabrication parameters. For example a multilayered structure is obtained from a solution mixing different ions. Varying the electrical potential with time allows to pass below or above the respective deposition potentials resulting in the successive reduction of different compounds.

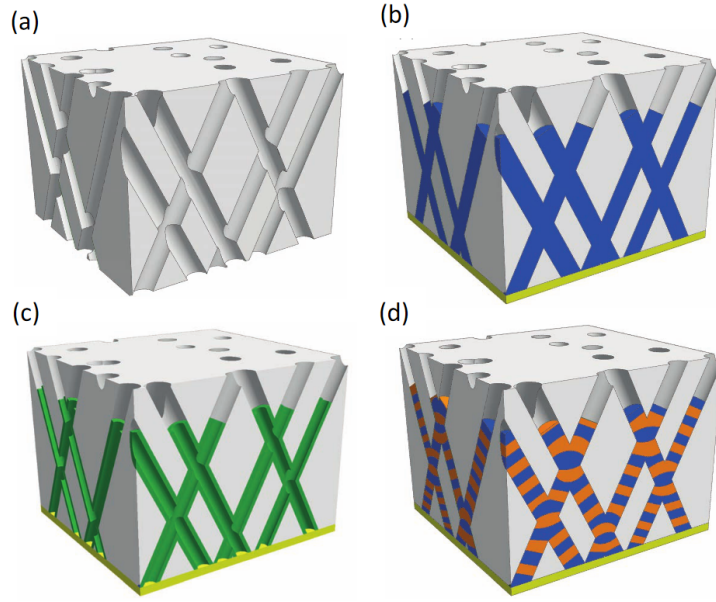


Figure 1.25: Schematic illustration of a (a) 3D nanoporous template, (b) interconnected nanowire network, (c) interconnected nanotube network and (d) multilayered nanowire network. Reproduced from [52].

As these networks feature great control on the composition, shape and density of the nanowires it has been possible to use them in research to highlight fundamental physical phenomena. As an example magnetic nanowires have been extensively studied, notably by Piroux *et al.* [52]. Due to their high aspect ratio and controlled orientation those can highlight magnetic anisotropy and magnetoresistance. Let us take a homogeneous nanowire network made of a ferromagnetic material. Due to shape anisotropy the magnetization in the NW is preferentially aligned with the cylinder longest axis. Neglecting exotic behaviors occurring at the intersections between wires those can be considered as monodomains. Therefore different hysteresis loops can be obtained depending on the direction of application of an external field. If the NWs are assumed to be nearly parallel to the template out-of-plane direction the latter can be considered as an easy axis of magnetization. On the contrary an in-plane external field follows a hard axis of magnetization. Considering now a multilayered nanowire network one can produce magnetoresistance effects. Indeed if those are composed of successive stacks of ferromagnetic/nonmagnetic/ferromagnetic layers we find back the appropriate structure giving rise to magnetoresistance. As it has been seen previously an antiferromagnetic coupling is required between the FM for GMR to occur. It is possible to control this coupling by varying the thickness of the thin nonmagnetic layer (Fig.1.5).

1.7.2 Ag NW networks switching resistors

Ag NW networks have attracted a lot of interest recently in the field of switching resistive devices. Based on different physical mechanisms developed hereafter it happens that the conductance of such network can vary. One of the most promising approaches consists in modifying the wires connections by applying electric fields and currents. This could be used in many different applications, notably as a nonvolatile memory or as a

memristance. [10]

Even if silver is well-known for its excellent electrical conductivity, the resistance of overlapping individual Ag NWs is usually very important. This is due to the important contact resistance between adjacent wires. A method to permanently enhance the conductance of this type of system has been studied by Vafaie *et al* [55] as well as by Song *et al* [56]. It is based on a local electrowelding of nanowire junctions via Joule heating. At first Ag NWs, forming a network connecting two electrodes, are simply overlapping each other. Due to the presence of contaminants on the wire surface, e.g. surfactants or oxides, and the shape of the contacts the resistance is initially very high. Open circuit can even be considered. This connection can be modified by applying a current through the device. This induces a heating of the contacts by Joule effect. A hotspot is created and causes Ag ions to migrate to the adjacent NW. This melting remains local as the contact resistance of the junctions is much greater than the one of the wires themselves. After electrowelding a good contact is obtained, as illustrated in Figure 1.26a. The wire with the smaller diameter, less stable, is generally the one which deforms itself the more. This treatment reduces greatly the network global resistance, up to several orders of magnitude. It presents the main advantage of only damaging locally the NWs, i.e. at the contact points, compared to other methods. However the modification remains permanent and cannot be switched back to the initial state.

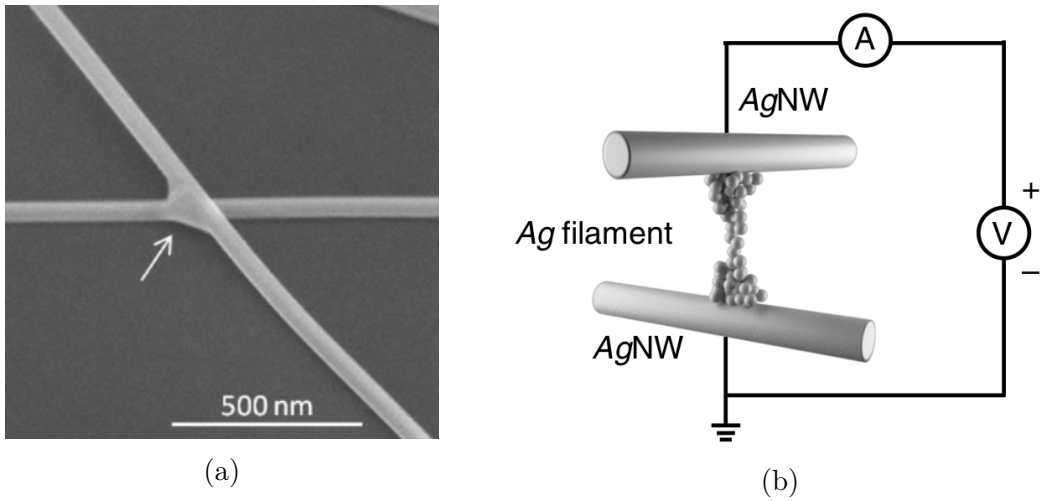


Figure 1.26: (a) SEM image of Ag NWs after electrowelding treatment. Adapted from [55]. (b) Schematic illustration of the growth of an Ag filament under a voltage application to the electrodes. Reproduced from [57].

Moreover Ag NWs confined in organic matrix can present dynamical conductance properties under an electrical field. These effects have been studied by White *et al*. [8] They assume that applying a voltage to wires separated by a gap could lead to the formation of a conductive Ag filament between those. This phenomenon is illustrated in Figure 1.26b. The tiny gaps between wires could cause a significant enhancement of the local electrical field. This would provoke the migration of ions to a filament joining neighbouring wires. As a result new connections are created in the network, leading to a significant increase of conductance. [10] One should notice that those bridging effects

can depend on many experimental parameters. The nature of the matrix is critical as Ag must present good solubility in it to allow the mobility of surface ions. Local heating caused by Joule effect could promote dissociation.

Conversely to electrowelding, conductive filament (CF) bridging is reversible. Its lifetime depends on surface diffusion, as detailed by Wang *et al.* [57] They reported that the CF can break spontaneously if the electrical field is removed. This can be explained by a minimization of the system energy. Indeed surface atoms on the CF present vacancies compared to bulk atoms in crystalline configurations. With time those atoms tend thus to outdiffuse from the filament to form back separate clusters, as shown in Figure 1.27. This phenomenon is associated to Rayleigh instabilities, or Thomas-Gibbs theorem. [10] The latter states that the chemical potential of surface atoms increases for important curvature. As a consequence the bridging volatility is highly dependent on the CF aspect ratio. Larger diameter tends to stabilize the filament which is less prone to surface diffusion. A large variability of CF lifetime can thus be observed, from ns up to years. A tunable outdiffusion time could be of great interest both in volatile and non-volatile memory applications.

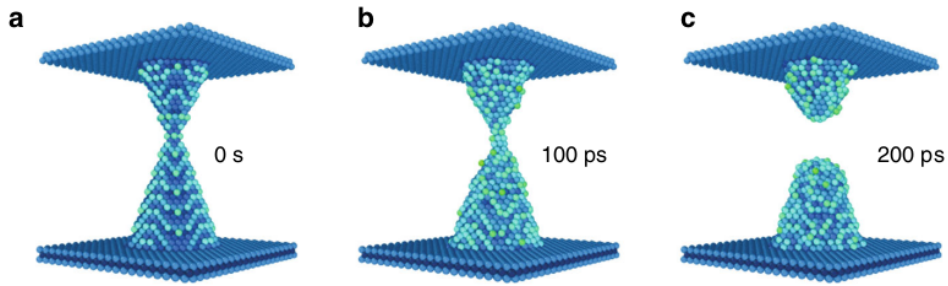


Figure 1.27: Molecular dynamics simulation of the spontaneous rupture of an Ag nanofilament, at different times. Surface atoms, having higher free energy, are shown in light blue. Those tend to diffuse more easily to minimize the system energy, splitting the filament in two clusters. More stable atoms are depicted in dark blue. Reproduced from [57].

In addition to these exotic electrochemical behaviors, more common quantum phenomena have also been observed in Ag NWs. Wan *et al* have reported direct electron tunneling and field emission in fragmented NW networks. Starting from standard Ag NWs they exposed those to a UV/ozone irradiation. As a result part of the wires become oxidized to AgO_x compounds. If a thermal annealing is performed after this step it is possible to obtain back only Ag NW segments, separated by Ag nanoparticles (NP). The latter originate from oxide decomposition. As gaps exist between NPs, the network can be considered as an insulator. However if a sufficient voltage is applied between the electrodes there is a probability of quantum tunneling though this energy barrier. Therefore a nonzero current is obtained for small voltage applied. At high field, i.e. above a threshold voltage V_{th} , one can even observe field electron emission. These effects are illustrated by the $I - V$ curve as well as by a Fowler-Nordheim plot in Figure 1.28. In direct tunneling regime $\ln(I/V^2)$ is proportional to $\ln(1/V)$ while it is proportional to $1/V$ in field emission. This results from a modification of the energy barrier shape with

increasing potential, i.e. from rectangular to triangular. [58]

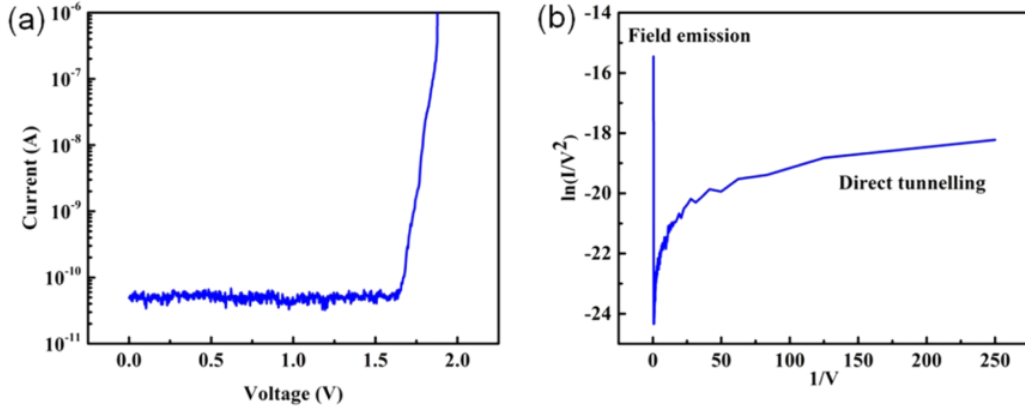


Figure 1.28: (a) $I - V$ curve of a fragmented Ag NW network. The diameter of the wires is 20 nm. Gaps exist between Ag segments and act as energy barriers preventing conduction. However tunneling is possible and allows limited current to flow. At high voltage field emission appears. (b) Fowler-Nordheim plot illustrating the two regimes. Reproduced from [9].

The properties of Ag NW networks have led to consider those as promising devices for mimicking synaptic behaviors in hardware neuromorphic computing. [10] Indeed the possibility to obtain switching resistive behavior of tunable lifetime can be related to the previously described STP, LTP and STDP phenomena.

Designing of a fast spin-torque vortex oscillator model: a new tool for studying STVO based neurons

Contributions: (Note by Flavio ABREU ARAUJO, advisor)

The results exposed in the following chapter were obtained by the team composed of Flavio ABREU ARAUJO (F.A.A.), Chloé CHOPIN (C.C.), and Simon de WERGIFOSSE ([S.d.W.](#)).

F.A.A. created and designed the analytical model. F.A.A. designed the micromagnetic simulations and [S.d.W.](#) performed almost all the micromagnetic simulations. F.A.A., [S.d.W.](#), and C.C. analyzed the micromagnetic simulations. The three coauthors contributed almost equally to the redaction of ref [1] (see page iii).

F.A.A. designed the data-driven Thiele equation approach. [S.d.W.](#) and F.A.A. performed the micromagnetic simulations. F.A.A., [S.d.W.](#), and C.C. analyzed the micromagnetic simulations. The three coauthors are contributing almost equally to the redaction of ref [3] (see page iii).

The experimental data showing the spin-diode inversion has been provided by our partner from the International Iberian Nanotechnology Laboratory (INL, thanks to Dr. Alex JENKINS). The UCLouvain teams (F.A.A., [S.d.W.](#), and C.C.) used the fast data-driven model from ref [3] to explain and understand the experimental data. F.A.A. performed the numerical evaluation of the data-driven model and [S.d.W.](#) & C.C. contributed to the analysis. The UCLouvain teams (F.A.A., [S.d.W.](#), and C.C.) is working on the first draft of the ref [4] before it's submitted to the partners.

C.C. & F.A.A. designed the RF detection scheme based on a combination of the spin-diode effect and standard machine learning approach. [S.d.W.](#) is involved in this study but the results cannot be exposed here as they are part of a classified European project and the approval from the European Commission is mandatory. C.C. & F.A.A. are preparing a deliverable that will also be published as mentioned in ref [5] (see page iii).

The optimization of the spin-diode effect for making the best experimental design suffers from the same constraint as ref [5] (see page iii) and will give rise to ref [6] (see page iii) after approval from the European commission. This work is also using the new tool for making the adequate optimization and [S.d.W.](#) is of course also involved.

2.1 Introduction

Neurons are the most elementary components of a human brain. Those are responsible for generating or transmitting signals across all the nervous system. In the neuromorphic computing framework, one tries to mimic that kind of behavior to resolve complex computational tasks with great energy efficiency. In any neural network system, the neurons are modelled as nodes described by a given mathematical expression. In fact these transfer functions are required to be nonlinear for efficient learning. [1]

Spintronics has emerged as a very promising field to create artificial neurons. [2] Among all possible hardware neuronal implementations, spin-transfer vortex oscillators have already proven to be capable of achieving complex problem resolutions. For example the nano-oscillators have been used successfully in voice recognition tasks within a neural network architecture. [3, 4] However the models used to describe their dynamics are often oversimplified. Those are not quantitatively comparable to experimental results obtained on real magnetic tunnel junctions. There is therefore a need to develop new ways of acquiring the complex properties of STVOs. This new approach should be both fast and accurate.

Micromagnetic simulations are well-known to give accurate results in various problems involving magnetic interactions. However two main problems arise from this approach. First the simulations are extremely time consuming and require important computational power. Then it does not allow to capture well the physics of the system as only raw data are extracted, with few tuning possibilities. On the other hand, various analytical developments have been proposed in the literature. [59, 60, 61] The main problem of these existing models is their limited validity when compared to simulations or experimental results. This is due to a lack of consideration of the nonlinear behaviors of several parameters influencing the dynamics. In addition some energy and spin-torque terms are sometimes completely neglected.

In this chapter, we provide a new pioneering approach to compute the evolution of the vortex properties with time. Based on the Thiele equation, we will first provide an analytical model describing the vortex dynamics. The geometry considered will be a nanopillar of $R = 100$ nm and $h = 10$ nm, as it is the most studied configuration in the literature. Then a hybrid approach, including a limited set of micromagnetic simulations, will allow to provide corrections to some of its parameters. We will then show that it allows to compute with great precision many different results. Indeed its predictions will match micromagnetic simulations, both in transient and steady-state regime.

Despite neuromorphic computing applications, spin-diode effect will also be considered. We will see that our model is able to explain experimental data obtained on real magnetic tunnel junctions. This constitutes very promising results for frequency detection applications as well as for junction optimization.

Part of the developments and results presented in this section should be published in the future in five different reviewed scientific communications.

2.2 Thiele equation approach

Let us restart from the general Thiele equation presented in Section 1.3.3 to particularize it to our geometry. The expression describing the vortex core position $\mathbf{X}$ is given as:

$$\bar{\bar{\mathbf{M}}} \cdot \ddot{\mathbf{X}} + \bar{\bar{\mathbf{G}}} \cdot \dot{\mathbf{X}} + \bar{\bar{\mathbf{D}}} \cdot \dot{\mathbf{X}} = \frac{\partial W(\mathbf{X})}{\partial \mathbf{X}} + \mathbf{F}^{\text{ext}} \quad (2.1)$$

Where $\bar{\bar{\mathbf{M}}}$, $\bar{\bar{\mathbf{G}}}$ and $\bar{\bar{\mathbf{D}}}$ are the mass, the gyrotropic and the dissipation tensors, respectively. The total potential energy is given by $W(\mathbf{X})$ and the different external forces acting on the system are referred as $\mathbf{F}^{\text{ext}}$.

The total energy is a sum of several contributions:

$$W = W^{\text{ms}} + W^{\text{ex}} + W^{\text{Oe}} + W^{\text{Z}} + W^{\text{T}} \quad (2.2)$$

Where W^{ms} is the magnetostatic energy, W^{ex} the exchange energy, W^{Oe} the Oersted field energy, W^{Z} the Zeeman energy and W^{T} the thermal energy. The system is considered at 0 K and no external magnetic field is applied. Those assumptions allow to cancel out the last two terms.

In a circular magnetic nanopillar the mass and damping tensors are diagonal, so that $\bar{\bar{\mathbf{M}}} = M\bar{\bar{\mathbf{I}}}$ and $\bar{\bar{\mathbf{D}}} = D\bar{\bar{\mathbf{I}}}$, with $\bar{\bar{\mathbf{I}}}$ the identity tensor (see Eqs.1.45 & 1.47). The gyrotropic tensor is also transformed into the gyrovector $\bar{\bar{\mathbf{G}}} = G\mathbf{e}_z$ (see Eq.1.46). The latter points out of the plane of the magnetic free layer. Moreover the only external force taken into account here is the spin torque force $\mathbf{F}^{\text{ST}}$. Above a critical dc current the latter can compensate the damping force allowing steady-state oscillations to occur. All developments mentioned hereabove allow to modify the Equation 2.1 to:

$$M\ddot{\mathbf{X}} + G(\mathbf{e}_z \times \dot{\mathbf{X}}) + D\dot{\mathbf{X}} = \frac{\partial(W^{\text{ms}} + W^{\text{ex}} + W^{\text{Oe}})}{\partial \mathbf{X}} + \mathbf{F}^{\text{ST}} \quad (2.3)$$

It is worth noting that M , G and D are not constant. Indeed those variables are scalar functions which depends on the vortex core position. To keep all further developments as general as possible, we will define the reduced norm of the vortex core position $s = |\mathbf{X}|/R$ so that M , G and D are functions of s .

The spin-torque force is composed of three different contributions in a cylindrical dot. Two of them are associated with the Slonczewski term and the third one is related to the Zhang & Li field-like torque:

$$\mathbf{F}^{\text{ST}} = \underbrace{\mathbf{F}_{\perp}^{\text{ST}} + \mathbf{F}_{\parallel}^{\text{ST}}}_{\text{Slonczewski (damping-like)}} + \underbrace{\mathbf{F}_{\text{FLT}}^{\text{ST}}}_{\text{Zhang\&Li (field-like)}} \quad (2.4)$$

The perpendicular component of the Slonczewski force is well-known as [31]:

$$\mathbf{F}_{\perp}^{\text{ST}} = \kappa^{\text{ST}} J(\mathbf{e}_z \times \mathbf{X}) \quad (2.5)$$

Where J_{dc} is the current density and $\kappa^{\text{ST}} = \pi a_J M_s \hbar p_z$ with $a_J = \frac{p_J \hbar}{2|e|M_s^{\text{ref}}}$ the spin-transfer torque amplitude and $p_z = \sin \theta$ the z -component of the normalized magnetization direction of the polarizer. M_s and M_s^{ref} are the saturation magnetization of the free

and reference layers, respectively. The spin polarization of the current is noted as p_J . The parallel component of the Slonczewski force is expressed as:

$$\mathbf{F}_{\parallel}^{\text{ST}} = \kappa_{\parallel}^{\text{ST}} J_{\text{dc}} \mathbf{p}_{x,y} \quad (2.6)$$

Where $\kappa_{\parallel}^{\text{ST}} = -\pi \ln(2) a_J M_s h \rho_c C / R$, with $\mathbf{p}_{x,y} = (\cos \theta \sin \phi, \cos \theta \cos \phi)$. Those derivations come from Dussaux *et al* [59]. In these $\mathbf{p}_{x,y}$ is the in-plane unit-vector of the magnetization direction of the polarizer, ρ_c is the vortex core diameter and C is the vortex chirality.

Finally the Zhang & Li term writes:

$$\mathbf{F}_{\text{FLT}}^{\text{ST}} = \kappa_{\text{FLT}}^{\text{ST}} J_{\text{dc}} (\mathbf{e}_z \times \mathbf{p}) \quad (2.7)$$

Where $\kappa_{\text{FLT}}^{\text{ST}} = -\pi C \delta M_s h b_J$ with $b_J = k^{\text{CPP}} a_J$ is the field-like torque amplitude. [59] The degree of non-adiabaticity k^{CPP} is generally comprised between 0.1 and 0.4, depending on the type of MTJ. [48] The factor δ represents the connection between the in-plane magnetization $\mathbf{M}$ of the free layer and the vector $\mathbf{X}$. As far as the TVA is considered it gives $\delta = \frac{2}{3}$. In all generality the coordinates of the normalized direction of the magnetization in the polarizer $\mathbf{p}$ are given as $(\cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta)$. Due to the cylindrical symmetry it can be simplified considering a polarizer orientation in the xz -plane. In this case $\phi = 0$ and $\mathbf{p} = (\cos \theta, 0, \sin \theta)$ which allows to derive $(\mathbf{e}_z \times \mathbf{p}) = -\cos \theta \mathbf{e}_y$.

One final simplification consists in assuming that the mass term is negligible for small aspect ratios $\xi < 0.15$. [45] For a free layer of radius $R = 100$ nm and thickness $h = 10$ nm one obtains $\xi = 0.05$. Considering all previous developments the vortex core dynamics is expressed as:

$$G(\mathbf{e}_z \times \dot{\mathbf{X}}) + D\dot{\mathbf{X}} = \frac{\partial(W^{\text{ms}} + W^{\text{ex}} + W^{\text{Oe}})}{\partial \mathbf{X}} + \mathbf{F}^{\text{ST}} \quad (2.8)$$

2.2.1 Harmonic oscillator (homogeneous system)

First we will only consider the case of a perpendicular polarizer, so that $p_z = \pm 1$ and $\theta = \pm \pi/2$. As a result the only remaining spin-transfer term is the perpendicular Slonczewski force:

$$\mathbf{F}^{\text{ST}} = \mathbf{F}_{\perp}^{\text{ST}} + \cancel{\mathbf{F}_{\parallel}^{\text{ST}}} + \cancel{\mathbf{F}_{\text{FLT}}^{\text{ST}}} = \kappa^{\text{ST}} J_{\text{dc}} (\mathbf{X} \times \mathbf{e}_z) \quad (2.9)$$

The derivative of the potential energy can be expressed using the spring-like restoring constants:

$$\frac{\partial(W^{\text{ms}} + W^{\text{ex}} + W^{\text{Oe}})}{\partial \mathbf{X}} = (k^{\text{ms}} + k^{\text{ex}} + k^{\text{Oe}} C J_{\text{dc}}) \mathbf{X} \quad (2.10)$$

Where the chirality $C = \pm 1$ and the current density J_{dc} have been taken out of the Oersted constant for the sake of understanding their influence on the dynamics.

Eventually one obtains the final expression ruling the core motion, derived from the Thiele equation:

$$G(\mathbf{e}_z \times \dot{\mathbf{X}}) + D\dot{\mathbf{X}} = (k^{\text{ms}} + k^{\text{ex}} + k^{\text{Oe}} C J_{\text{dc}}) \mathbf{X} + \kappa^{\text{ST}} J_{\text{dc}} (\mathbf{e}_z \times \mathbf{X}) \quad (2.11)$$

Where G and D are functions of s . Those are modeled as $G(s) = G_0 f^G(s)$, with the gyroconstant $G_0 = G(0) = -2\pi P M_s h / \gamma_G$ [62] and $D_0 = D(0) = -\alpha_G \eta |G_0|$. [46] The parameter η is given as $0.5 \ln(R/(2l_{\text{ex}})) + 3/8$. One can deduce that $f^G(0) = f^D(0) = 1$.

One finally needs the expression of the different spring-like parameters. As far as the literature is concerned various propositions exist to compute those. Though, these parameters have often been calculated for small oscillation amplitude. Here we will only retain the ones that are valid for the largest range of reduced radius s , giving rise to the most general model. Let us start with the exchange interaction. Gaididei *et al* [60] derived the following expression for k^{ex} , based on Gusliencko *et al* works [62]:

$$k^{\text{ex}}(s) = -2\pi h M_s^2 \left(\frac{l_{\text{ex}}}{R} \right)^2 \frac{1}{1-s^2} \quad (2.12)$$

Using an order 8 Taylor expansion based on the expression of the energy associated to the Oersted field, Abreu Araujo calculated analytically the Oersted force constant [24]:

$$k^{\text{Oe}}(s) = \frac{8\pi^2}{75} M_s R h \left(1 - \frac{4}{7}s^2 - \frac{1}{7}s^4 - \frac{16}{231}s^6 - \frac{125}{3003}s^8 \right) \quad (2.13)$$

This approximation is valid for a wide range of radius values ($s < 0.8$). Moreover the vortex is generally unstable, i.e. expelled or the polarity is reversed, for a radius of oscillation above this threshold. The imprecision for large s has thus limited consequences on the model precision.

Finally Gaididei *et al* developed an expression for the potential energy related to a displacement of the vortex core. [60] Under the TVA it is given as:

$$W_{\text{ms}}(\xi, s) = \frac{4M_s^2 h^2 R s^2}{\pi} \Theta(\xi, s) \quad (2.14)$$

Where Θ is the following multiple integral:

$$\Theta(\xi, s) = \int_0^1 dz \int_0^1 d\eta_1 \int_0^1 d\eta_2 \int_0^{2\pi} d\varphi_1 \int_0^{2\pi} d\varphi_2 \Psi(z, \eta_1, \eta_2, \varphi_1, \varphi_2, \xi, s) \quad (2.15)$$

Where Ψ is a function given in [60]. The parameters $\eta_1 = r_1/R$ and $\eta_2 = r_2/R$ are related to the position of the real and image vortices, with φ_1 and φ_2 the angles associated. It appears that an analytical solution exists for this integral only for $s = 0$ and $\xi = 0$. Abreu Araujo resolved this limitation by using a non-deterministic Monte-Carlo integration scheme for $\Theta(\xi = 0.05, s)$ with guaranteed absolute precision of 10^{-4} . [24] After fitting, the following expression is obtained, for our specific aspect ratio (valid for $0 \leq s \leq 0.8$):

$$k_{\xi=0.05}^{\text{ms}}(s) = \frac{8M_s^2 h^2}{R} 1.594(1 + 0.087s^2 + 0.024s^4 - 0.017s^6) \quad (2.16)$$

As all parameters have been explained, one can proceed to further developments. Using basic cross product and after switching to matrix format, Eq.2.11 can be transformed to:

$$\begin{bmatrix} D & -G \\ G & D \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} C J_{\text{dc}} k^{\text{Oe}} + k^{\text{ex}} + k^{\text{ms}} & -J_{\text{dc}} \kappa^{\text{ST}} \\ J_{\text{dc}} \kappa^{\text{ST}} & C J_{\text{dc}} k^{\text{Oe}} + k^{\text{ex}} + k^{\text{ms}} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (2.17)$$

One can isolate the vector $\dot{\mathbf{X}}$ by multiplying both sides of the equation by the inverse of the matrix containing D and G :

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \frac{D}{D^2+G^2} & \frac{G}{D^2+G^2} \\ -\frac{G}{D^2+G^2} & \frac{D}{D^2+G^2} \end{bmatrix} \begin{bmatrix} C J_{dc} k^{Oe} + k^{ex} + k^{ms} & -J_{dc} \kappa^{ST} \\ J_{dc} \kappa^{ST} & C J_{dc} k^{Oe} + k^{ex} + k^{ms} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (2.18)$$

By multiplying the two matrices that do not include any X or Y one finally obtains:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \frac{D(C J_{dc} k^{Oe} + k^{ex} + k^{ms}) + G J_{dc} \kappa^{ST}}{D^2 + G^2} & \frac{-D J_{dc} \kappa^{ST} + G(C J_{dc} k^{Oe} + k^{ex} + k^{ms})}{D^2 + G^2} \\ \frac{D J_{dc} \kappa^{ST} - G(C J_{dc} k^{Oe} + k^{ex} + k^{ms})}{D^2 + G^2} & \frac{D(C J_{dc} k^{Oe} + k^{ex} + k^{ms}) + G J_{dc} \kappa^{ST}}{D^2 + G^2} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (2.19)$$

Our STVO dynamics can thus be modeled by an oscillator described by a homogeneous system of linear first order differential equations (ODE). Defining two parameters Γ and ω allows to write the system as follows:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \underbrace{\begin{bmatrix} \Gamma & -\omega \\ \omega & \Gamma \end{bmatrix}}_{\bar{\bar{\Omega}}} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (2.20)$$

Where Γ and ω are given as:

$$\Gamma = \frac{D(C J_{dc} k^{Oe} + k^{ex} + k^{ms}) + G J_{dc} \kappa^{ST}}{D^2 + G^2} \quad (2.21)$$

$$\omega = \frac{D J_{dc} \kappa^{ST} - G(C J_{dc} k^{Oe} + k^{ex} + k^{ms})}{D^2 + G^2} \quad (2.22)$$

The ODE system can be resolved by finding its eigenvalues λ . The characteristic equation of the matrix $\bar{\bar{\Omega}}$ writes:

$$\lambda^2 - 2\Gamma\lambda + (\Gamma^2 + \omega^2) = 0 \quad (2.23)$$

The two solutions of this equation are $\lambda = \Gamma \pm i\omega$. Using Euler's formula, one finds the position of the core center $(X(t), Y(t))$ as:

$$X(t) = e^{\Gamma t} [(C_1 + C_2) \cos(\omega t) + i(C_1 - C_2) \sin(\omega t)] \quad (2.24)$$

$$Y(t) = e^{\Gamma t} [-i(C_1 - C_2) \cos(\omega t) + (C_1 + C_2) \sin(\omega t)] \quad (2.25)$$

Where Γ and ω are considered being constant with respect to time. This assumption is verified when a steady-state (SS) regime of oscillation is reached. As it will be shown later, we can in fact gather important informations about our STVO dynamic behavior and properties only from the steady-state values. If the two constants C_1 and C_2 are posed such as $C_1 = -C_2 = -\frac{i}{2} \|\mathbf{X}\|$ one can prove that:

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \|\mathbf{X}\| e^{\Gamma t} \begin{bmatrix} \sin(\omega t) \\ -\cos(\omega t) \end{bmatrix} \quad (2.26)$$

In a steady-state regime the vortex core orbit should evolve no more if a dc current is applied ($J_{dc} > J_{cr}$), implying $\Gamma = 0$. One can also remark that the gyrotropic frequency of our STVO vortex is simply obtained as $f^{\text{STVO}} = \omega/(2\pi)$. A polarity $P = +1$ (resp.

$P = -1$) gives rise to a positive (resp. negative) frequency of oscillation.

The parameter Γ can also be used to derive a theoretical expression of the critical current J_{cr} . Indeed the latter may be seen as the first current presenting a non-zero Γ value for $s = ||\mathbf{X}||/R$ still being equal to zero. Mathematically this condition is expressed as:

$$\Gamma(s=0) = \frac{D_0 (C J_{\text{cr}} k_0^{\text{Oe}} + k_0^{\text{ex}} + k_0^{\text{ms}}) + G_0 J_{\text{cr}} \kappa^{\text{ST}}}{D_0^2 + G_0^2} = 0 \quad (2.27)$$

Which allows to give the following expression for the critical current:

$$J_{\text{cr}} = \frac{-D_0(k_0^{\text{ex}} + k_0^{\text{ms}})}{D_0 C k_0^{\text{Oe}} + G_0 \kappa^{\text{ST}}} \quad (2.28)$$

2.2.2 Design of the data-driven evaluation of $D(s)$ and $G(s)$

As the calculation of $D(s)$ and $G(s)$ is a very difficult task, no convincing expression exists in the literature describing their evolution with respect to the core position to the best of our knowledge. Therefore we selected another approach. These functions will be deduced from a set of simulations. By fitting micromagnetic results to our model it will be possible to considerably enhance its precision. This step is performed by following the procedure described hereafter.

As the values of Γ and ω can be obtained from micromagnetic simulations, we are able to find the s dependence of G and D . Using the matrix $\bar{\bar{\Omega}}$ the following system is derived:

$$D(s)\Gamma - G(s)\omega = C J_{\text{dc}} k^{\text{Oe}}(s) + k^{\text{ex}}(s) + k^{\text{ms}}(s) \quad (2.29)$$

$$D(s)\omega + G(s)\Gamma = J_{\text{dc}} \kappa^{\text{ST}} \quad (2.30)$$

As $\Gamma = 0$ in steady-state oscillation, $G(s)$ and $D(s)$ can be calculated as follows:

$$G(s) = -\frac{C J_{\text{dc}} k^{\text{Oe}}(s) + k^{\text{ex}}(s) + k^{\text{ms}}(s)}{\omega} \quad (2.31)$$

$$D(s) = \frac{J_{\text{dc}} \kappa^{\text{ST}}}{\omega} \quad (2.32)$$

The only unknown parameter in these equations is the vortex pulsation ω . However it can be simply extracted from micromagnetic simulations arrived in the steady-state regime as $\omega = 2\pi f^{\text{STVO}}$. Thus one can easily model the s dependence of G and D after performing a fit on several simulation results. Let us note that in mumax³ the working variable will be the input current applied. We will then generally obtain $\omega(J_{\text{dc}})$ as well as $s(J_{\text{dc}})$, giving rather $D(J_{\text{dc}})$ and $G(J_{\text{dc}})$ than $D(s)$ and $G(s)$.

As introduced previously the gyrotropic and damping term will be modeled as $G(s) = G_0 f^G(s)$ and $D(s) = D_0 f^D(s)$, respectively. In these expressions $f^G(s)$ and $f^D(s)$ are

orbit-dependent functions to be fitted which respect $f^G(s=0) = 1$ and $f^D(s=0) = 1$. The Equations 2.31 & 2.32 are thus modified to:

$$\delta^G f^G(s) = -\frac{C J_{\text{dc}} k^{\text{Oe}}(s) + k^{\text{ex}}(s) + k^{\text{ms}}(s)}{G_0 \omega(J_{\text{dc}})} \quad (2.33)$$

$$\delta^D f^D(s) = \frac{J_{\text{dc}} \kappa^{\text{ST}}}{D_0 \omega(J_{\text{dc}})} \quad (2.34)$$

Where we have also introduced two global correction factors δ^G and δ^D . Those absorb any impreciseness of the model resulting from assumptions made, even at $s = 0$ (e.g. on the z -profile of the vortex core). Those discrepancies could originate from the analytical expressions of D_0 and G_0 as well as from the other terms at $s = 0$, i.e. k_0^{Oe} , k_0^{ex} , k_0^{ms} or κ^{ST} . Finally the functions $f^G(s)$ and $f^D(s)$ to fit are supposed to be even power expansions of s :

$$f^G(s) = 1 + a_G s^2 + b_G s^4 + c_G s^6 + d_G s^8 \quad (2.35)$$

$$f^D(s) = 1 + a_D s^2 + b_D s^4 + c_D s^6 + d_D s^8 \quad (2.36)$$

Where the parameters for the gyrotropic term, i.e δ^G , a_G , b_G , c_G , d_G , and for the damping term, i.e δ^D , a_D , b_D , c_D , d_D , are to be computed from micromagnetic simulations.

2.2.3 Harmonic oscillator (nonhomogeneous system)

As explained in Section 1.5, spin-diode effect relies on the torque created by the polarizer on the free layer magnetization. One can easily understand that the previously described configuration, i.e. $p_z = 1$, cannot give rise to this phenomenon. Indeed magnetoresistance arises for a changing respective orientation of the magnetization in the two ferromagnetic layers. For a perpendicular polarization a variation of the in-plane magnetization of the free layer with time has no impact on the resistive state of the junction. Therefore one must tilt $\mathbf{m}_{\text{ref}}$. Practically this operation is often done by applying an in-plane external magnetic field $\mathbf{H} = (H_x, H_y)$. We will thus consider its impact on the dynamic of our STVO. Moreover tilting the polarizer causes the other components of the spin-torque forces to become effective.

Using similar developments than for the homogeneous case one can obtain a more general expression of the ODE system describing our oscillator. An additional vector Υ has to be added to Eq. 2.20 which gives:

$$\dot{\mathbf{X}} = \bar{\bar{\Omega}} \mathbf{X} + \Upsilon \quad (2.37)$$

Where Υ is given as:

$$\Upsilon = \begin{bmatrix} \frac{D(H_0 H_y + J_{\text{dc}} \kappa_{y||}^{\text{ST}} + J_{\text{dc}} \kappa_{y\text{FLT}}^{\text{ST}})}{D^2 + G^2} + \frac{G(-H_0 H_x - J_{\text{dc}} \kappa_{x||}^{\text{ST}} + J_{\text{dc}} \kappa_{x\text{FLT}}^{\text{ST}})}{D^2 + G^2} \\ \frac{D(-H_0 H_x - J_{\text{dc}} \kappa_{x||}^{\text{ST}} + J_{\text{dc}} \kappa_{x\text{FLT}}^{\text{ST}})}{D^2 + G^2} - \frac{G(H_0 H_y + J_{\text{dc}} \kappa_{y||}^{\text{ST}} + J_{\text{dc}} \kappa_{y\text{FLT}}^{\text{ST}})}{D^2 + G^2} \end{bmatrix} \quad (2.38)$$

With H_0 defined as $H_0 = \pi R h M_s \delta C$. [63] The parameters κ_x and κ_y refer to the x and y components of the spin-torque forces. Their relative importance depends on the

polarization direction, following Eq. 2.6&2.7. As our matrix $\bar{\bar{\Omega}}$ is exactly the same as for a perpendicular polarizer the homogeneous solution of our system is not modified. As a consequence the decay parameter Γ as well as the vortex pulsation ω are not affected by the external field and the reference magnetization tilting. However an additional particular solution must be added to the general solution to take into account the impact of the vector Υ :

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = ||\mathbf{X}||e^{\Gamma t} \begin{bmatrix} \sin(\omega t) \\ -\cos(\omega t) \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix} \quad (2.39)$$

Where $[d_x, d_y]$ is the particular solution of the ODE system. In fact this vector has a precise physical meaning. It defines the new equilibrium position of the center of oscillation which was previously situated at the center of the dot $(0,0)$. It is now moved to the coordinates (d_x, d_y) . Those can be found by considering the equation $\bar{\bar{\Omega}} \cdot [d_x, d_y]^T = \Upsilon$. This can be developed as:

$$\begin{bmatrix} \Gamma & -\omega \\ \omega & \Gamma \end{bmatrix} \begin{bmatrix} d_x \\ d_y \end{bmatrix} = \begin{bmatrix} \Gamma d_x - \omega d_y \\ \Gamma d_y + \omega d_x \end{bmatrix} = \Upsilon \quad (2.40)$$

Finally after multiple simplification steps one obtain the expressions of d_x and d_y :

$$d_x = \frac{H_0 H_x J_{dc} \kappa^{ST} - H_0 H_y k_{rest} + J_{dc}^2 \kappa^{ST} (\kappa_{x||}^{ST} - \kappa_{xFLT}^{ST}) - J_{dc} k_{rest} (\kappa_{y||}^{ST} + \kappa_{yFLT}^{ST})}{J_{dc}^2 (\kappa^{ST})^2 + k_{rest}^2} \quad (2.41)$$

$$d_y = \frac{H_0 H_x k_{rest} + H_0 H_y J_{dc} \kappa^{ST} + J_{dc}^2 \kappa^{ST} (\kappa_{y||}^{ST} + \kappa_{yFLT}^{ST}) + J_{dc} k_{rest} (\kappa_{x||}^{ST} - \kappa_{xFLT}^{ST})}{J_{dc}^2 (\kappa^{ST})^2 + k_{rest}^2} \quad (2.42)$$

Where $k_{rest} = C J k^{Oe} + k^{ms} + k^{ex}$. One can observe that the translation depends on the current as well as on the external magnetic field. Let us notice that even in the absence of current, i.e. $J_{dc} = 0$, the vortex core is still displaced.

2.3 Micromagnetic simulations

This section is dedicated to present the results obtained by micromagnetic simulations. Those aim at being compared with our theoretical predictions, to confirm the validity of our analytical model. Moreover it will allow to make data-driven corrections to the damping and gyrotropic term appearing in our equations. At first our methodology is presented, followed by detailed interpretation of our main results.

2.3.1 Methodology

The methodology we followed is described in the next subsection. At first our micromagnetic solver and our geometry will be introduced. After this, explanations on the methods used to derive the vortex radius and frequency of oscillation are provided.

Introduction to mumax³

Our simulations were performed using mumax³, an open-source micromagnetic solver developed at Ghent University. [64] Using finite-element methods this program calculates the time-dependent magnetization in small size structures. It presents the main advantage to be GPU-based which allows much faster computation time compared to CPU calculations, i.e. up to two orders of magnitude. It holds many features, adapted to the resolution of different magnetic problems. What is important in our case is that this solver falls in the LLG micromagnetic formalism with the possibility to add the spin-transfer torque components, i.e. Slonczewski & Zhang-Li terms. In mumax³, the time derivative of the reduced magnetization is calculated as:

$$\frac{\partial \mathbf{m}(\mathbf{r}, t)}{\partial t} = \boldsymbol{\tau}(\mathbf{r}, t) \quad (2.43)$$

Where $\boldsymbol{\tau}$ represents the torque exerted on the magnetization vector. It combines the contributions theorized by LLG, Slonczewski and Zhang-Li.

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Simulation parameters

The free layer of our magnetic tunnel junction was simulated as a magnetic dot presenting a radius R of 100 nm and thickness h of 10 nm. This geometry was subdivided into 2.5×2.5 nm² cells in the xy plane. In the z direction two layers of 5 nm each were created. These choices ensure that the dimensions of the micromagnetic cells are below the exchange length $l_{\text{ex}} = \sqrt{A/(2\pi M_s^2)}$ of permalloy (Ni₈₀Fe₂₀), giving rise to more accurate simulations. Typical parameters for this material were selected. The saturation magnetization M_s used is 800 emu/cm³, the exchange stiffness constant A is $1.07 \cdot 10^{-6}$ erg/cm and the Gilbert damping parameter α_G is 0.01. Magneto-crystalline anisotropy as well as thermal fluctuations are neglected in this study ($T = 0$ K).

For our aspect ratio $\xi = 0.05$ the magnetic groundstate is a vortex, as illustrated in Figure 2.1. Mumax³ allows to define a vortex as the input magnetic state. Actually the latter is defined within the Single Vortex Ansatz framework within the solver. Fortunately this has no effect on the steady-state value that will be extracted as energy relaxation will occur with time. We will consider a perpendicular orientation of the polarizer, so that $p_z = 1$. A moderate spin-current polarization $p_J = 0.2$ is also chosen. A dc current density J_{dc} is injected in the positive z direction, i.e. $p_z = +1$. To obtain steady-state oscillations the two following conditions must be met concurrently:

$$J_{\text{dc}} P p_z < 0 \quad \text{and} \quad |J_{\text{dc}}| > |J_{\text{cr}}| \quad (2.44)$$

Where P is the vortex polarity. As the other parameters have already been provided we need to fix $P = -1$ for all simulations. Otherwise a damping would always be observed. We have performed simulations in three different configurations, highlighting the impact of the Oersted field on the vortex properties:

- without taking the Oersted field into account;
- with the Oersted field parallel to the curling planar magnetization of the vortex ($C = +1$);
- with the Oersted field antiparallel to the curling planar magnetization of the vortex ($C = -1$).

For the three cases, a dc current density range from 5 to 10 MA/cm² has been injected into the junction. This will allow exploring different behaviors. Below J_{cr} a damping will be observed. Above J_{cr} a steady-state oscillation orbit should be reached. Finally above a given current density, the vortex will become unstable. Either a damping or an expulsion should be observed at this point. The current range to be explored was decided following theoretical calculations of J_{cr} for our study parameters, based on Eq. 2.28. Those are presented in Table 2.1 and will be compared in a further section to micromagnetic simulation results.

Polarity	Polarization	Chirality	J_{cr} [MA/cm ²]
$P = -1$	$p_z = +1$	$C = +1$	6.269
$P = -1$	$p_z = +1$	$C = -1$	5.542

Table 2.1: Theoretical values of critical current for our STVO geometry and topological parameters.

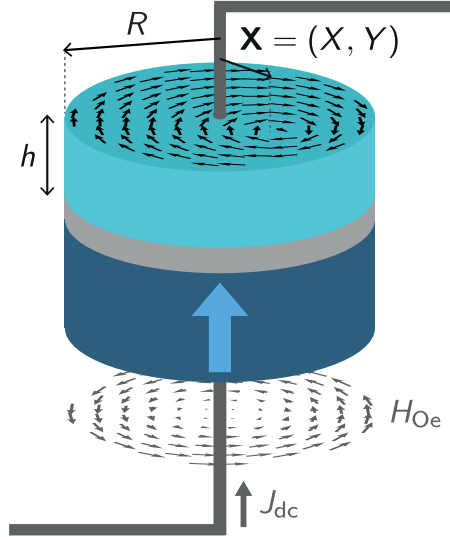


Figure 2.1: Schematic illustration of the MTJ under study. The free layer is made of permalloy. The nonmagnetic spacer is typically MgO. The polarizer, a synthetic antiferromagnet, direction is represented by the big arrow.

Oersted field integration

The Oersted field is not automatically generated in mumax³ when a current is passing through the magnetic dot. To implement it we had to create an input file seen by each

cell as a local external field which reproduces its effects, as seen in Figure 2.2. We used the standard format with which mumax3 is working when dealing with vector fields, i.e. an ovf file. To keep this field as generic as possible the components of the vectors are still to be multiplied by the current density. This allows to obtain the correct Oersted field for each simulation, i.e. for each value of dc current, while keeping the same input file. To confirm the validity of our implementation we compared the magnitude of the vectors generated by the program to the theoretical value at the center of each cell. The latter is given as:

$$\frac{||\mathbf{B}_{\text{Oe}}||}{J_{\text{dc}}} = \mu_0 \frac{r}{2} \quad (2.45)$$

Where μ_0 is the vacuum permeability and r the distance from the dot center. The relative error between simulation and theory has been calculated. We observed a perfect accuracy, as the maximum error for a cell is only $1.38 \cdot 10^{-5} \%$.

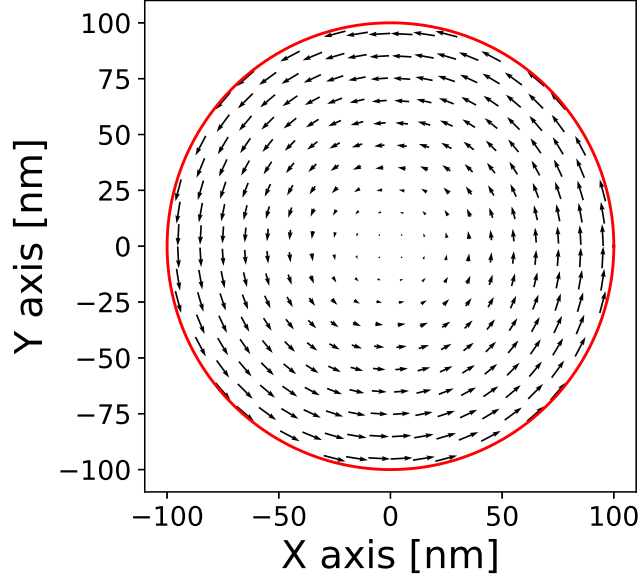


Figure 2.2: Oersted field generated by mumax³ for an arbitrary current value in our magnetic dot ($R = 100$ nm). The magnitude of the field is given by the relative length of the black arrows.

Reduced radius derivation

For every simulation performed we followed the same procedure. The latter can be separated in three steps. First we define a vortex and we confer it defined properties (e.g. chirality, polarity). Subsequently a dc current is applied and we wait for the vortex to reach its steady-state orbit. As it will be explained hereafter the total time required to attain equilibrium depends on many parameters. One of the most critical is the initial starting position of the vortex core. Indeed the closer it is from its steady-state value to quicker it reaches it. Starting from $s = 0$ is thus rarely an adequate choice. The next section will be dedicated to find a method providing a reasonable guess for this starting value (cf. "Input radius prediction"). Moreover we do not have to actually wait for

perfect steady-state. If a sufficient number of data points have been generated it will be possible to estimate with great accuracy the final radius of oscillation thanks to a fitting function. This estimation is part of the last step consisting in treating the generated data to extract properties of interest, i.e. the gyrotropic frequency and the orbit.

Mumax³ can output different physical parameters which are calculated at each time step. Generally we kept six of those: each component of the mean reduced magnetization ($\langle m_x \rangle_V, \langle m_y \rangle_V, \langle m_z \rangle_V$) as well as the vortex core position and polarization (X, Y, Z). The mean magnetization is calculated over all cells composing the magnetic dot, resulting in a single value for each component. The mean magnetization in the z direction is thus usually small as the vortex core, i.e. where the out-of-plane magnetization is prominent, covers only a tiny fraction of the dot surface. As far as the vortex core position is concerned, it is derived internally in the solver. Their algorithm presents a resolution smaller than the cell size (< 1 nm) which is sufficient for our studies. A time step of 10 ps was chosen between each recording. Knowing the vortex core coordinates one can easily derive the reduced radius of oscillation:

$$s(t) = \frac{\|\mathbf{X}(t)\|}{R} = \frac{\sqrt{X(t)^2 + Y(t)^2}}{R} \quad (2.46)$$

Gusliencko *et al* proposed an analytical expression for the evolution of the vortex orbit with time. [61] It is given as follows:

$$s_0^2(J_{dc}) = s^2(t, J_{dc}) \left[1 + \left(\frac{s_0^2(J)}{s^2(0)} - 1 \right) \exp \left(-\frac{t}{\tau(J_{dc})} \right) \right] \quad (2.47)$$

Where s_0 is the steady-state reduced orbit, s the reduced orbit at a given time and $s(0)$ the starting value. The decay constant $1/\tau(J_{dc})$ is the parameter of interest here. It is proportional to $(J_{dc}/J_{cr} - 1)$. Below J_{cr} an exponential decrease to the center of the dot will be observed. Above J_{cr} the orbit will reach a non-zero steady value with time. What has to be noticed is that the closer we are from the critical current the slower s approaches its steady-state value. The total simulation time we provided to mumax³ had thus to take this effect into account. Far above or below J_{cr} 1000 ns were enough to obtain a steady-state. However close to it 1500 to 2000 ns were necessary.

To illustrate our approach the results from two of our simulations are presented on Figure 2.3. The radius of oscillation is depicted with respect to time. The chirality was set to $C = -1$ and the Oersted field was taken into account. On the left one can observe a clear case of damping to the dot center. The injected dc current, of 5.2 MA/cm², is therefore below the critical current. On the right however a non-zero orbit of roughly 74 nm is obtained for $J_{dc} = 8$ MA/cm². One can also notice that the asymptotic value is reached rapidly, indicating a dc current far superior to J_{cr} . Furthermore one can remark quick and intense oscillations near $t = 0$ ns for both simulations. Those have no physical meaning. Indeed this effect comes from the fact that the vortex must relax to a more energetically stable state just after its creation. This results from its imperfect definition within the Single Vortex Ansatz framework. As introduced previously we fitted our simulation results with a function to extract the steady-state orbits. This serves two purposes. First it allows to obtain a precise value, as some noise is present

on the data. Moreover it helps to save computational time and power. Depending on the GPU model chosen 1000 ns simulated correspond to roughly 90 min of real time for a dot with a radius $R = 100$ nm. More than 100 simulations were performed in total for this study only.

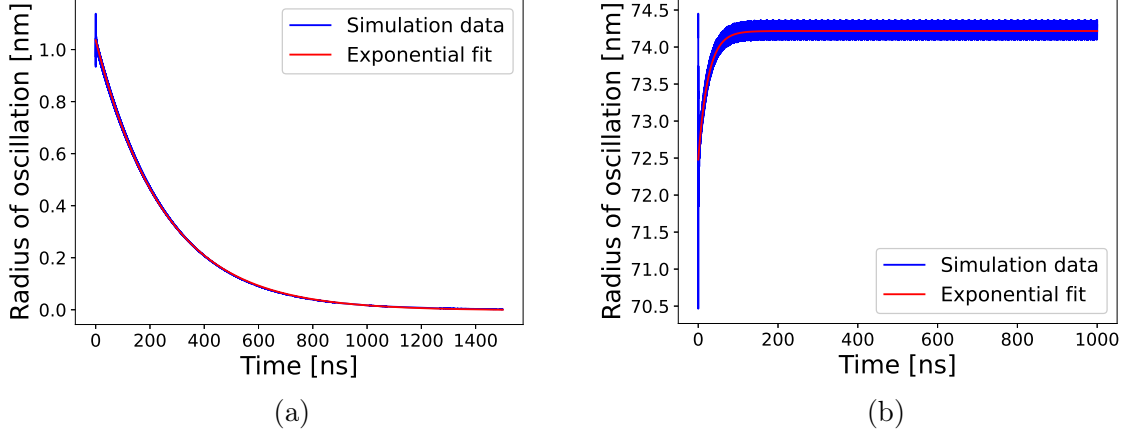


Figure 2.3: Radius of oscillation of the vortex with respect to time for $C = -1$ with Oersted field. Data from the micromagnetic simulations are plotted in blue. The fitting functions are represented in red (a) A current of $5.2 \text{ MA/cm}^2 < J_{\text{cr}}$ is injected. Damping is observed. (b) A current of $8 \text{ MA/cm}^2 > J_{\text{cr}}$ is injected. The orbit reaches a steady-state value after roughly 200 ns.

Our fitting function, a simplified version of Eq.2.47, has the following form:

$$s_{\text{fit}}(t) = p_1 \exp\left(-\frac{t - t_0}{p_2}\right) + p_3 \quad (2.48)$$

Where p_1 , p_2 and p_3 are three parameters to be optimized. Usually t_0 is fixed at 0. However one could conceive to start the fitting after several ns to suppress the impact of the oscillations at the beginning of the simulation. Actually we think that the decay rate of the exponential, fitted by p_2 , could also be time dependent. This effect will be neglected here. For $t = \infty$ one should recover $s_{\text{fit}} = s_0$, e.g. the steady-state orbit value. It is then trivially given by the parameter p_3 .

As introduced in Section 1.3.3, Guslienko predicted in another work that $s_0(J_{\text{dc}}) \propto \sqrt{J_{\text{dc}}/J_{\text{cr}} - 1}$. [44] The derivative of this function is $1/(2J_{\text{cr}}\sqrt{J_{\text{dc}}/J_{\text{cr}} - 1})$, which diverges to infinity for $J_{\text{dc}} = J_{\text{cr}}$. As a consequence for small differences in input current close to the critical current one obtains large variations of s_0 . For this kind of situation we developed another method to get an accurate steady-state orbit value, based on the previous one. It is presented in Figure 2.4. Instead of using one data set we base our fitting on two simulations. The only difference between the two is the starting position of the vortex core, all other parameters are kept similar. One of them is initiated at an orbit superior to the guessed steady-state value, the other one is starting from below.

Two fitting are then undertaken, one for each situation. The fitting functions write:

$$s_{\text{fit},\uparrow}(t) = p_{1,\uparrow} \exp\left(-\frac{t - t_0}{p_{2,\uparrow}}\right) + p_{3,\uparrow} \quad (2.49)$$

$$s_{\text{fit},\downarrow}(t) = p_{1,\downarrow} \exp\left(-\frac{t - t_0}{p_{2,\downarrow}}\right) + p_{3,\downarrow} \quad (2.50)$$

As the predicted value of $s_{\text{fit},\uparrow}$ should be similar to the one from $s_{\text{fit},\downarrow}$ for $t = \infty$ we force $p_{3,\uparrow}$ to be equal to $p_{3,\downarrow}$. A very precise steady-state orbit can be extrapolated using this technique.

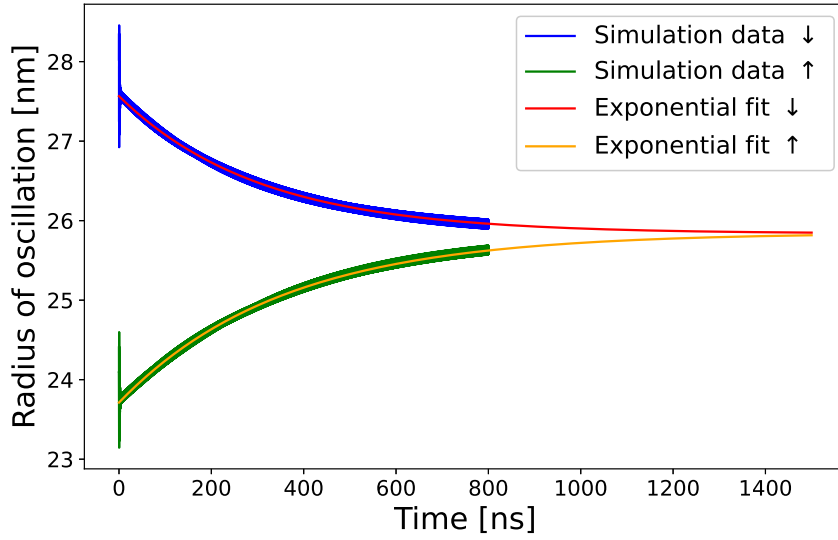


Figure 2.4: Radius of oscillation of the vortex with respect to time for $C = -1$ with Oersted field. Two simulations are performed, for a same 5.96 MA/cm^2 dc current. The first one (resp. the second one), plotted in blue (resp. green), starts from above (resp. below) the guessed steady-state orbit value. The fit functions, represented in red and orange respectively, are forced to adopt the same asymptotic value.

Input radius prediction

As introduced previously finding an approximate guess for the initial translation of the vortex core is crucial for a quick approach of the steady-state orbit. However if this starting point matches perfectly s_0 the fit would become imprecise as an exponential function would have to capture the behavior of a straight line. We found a method to approximate the characteristic evolution of s_0 with respect to the current, requiring only four test simulations. Our fitting function is given as follows:

$$s_{0,\text{fit}}(J_{\text{dc}}) = (a_0 + a_1 J_{\text{dc}} + a_2 J_{\text{dc}}^2) \sqrt{\frac{J_{\text{dc}}}{J_{\text{cr},\text{fit}}}} - 1 \quad (2.51)$$

This function is composed of a second order polynomial multiplied by the radical function proposed by Guslienko *et al.* [44] It presents four degrees of freedom, namely the

parameters a_0 , a_1 and a_2 as well as a guessed critical current $J_{\text{cr,fit}}$. One can notice that this equation is not defined for $J_{\text{dc}} < J_{\text{c,fit}}$. As a result only the excited part of the characteristic is modeled, i.e. not the damping to the dot center for low currents. A minimum of four test simulation results are required for the fit to be performed. Those should be arrived in steady-state and the applied current must be greater than J_{cr} .

After fitting on those test values a starting point for any new simulations can be proposed. Our approach has been the following. For any current $J_{\text{dc}} < J_{\text{cr,fit}}$ the starting translation is fixed to 1 nm. A damping to $s_0 = 0$ nm is observed in any case. For currents $J_{\text{dc}} > J_{\text{cr,fit}}$ the initial orbit was chosen based on Eq. 2.51. As this guessed value is close but not exactly the same as the real s_0 an exponential decay will nevertheless be observed, allowing the fitting proposed in Eq. 2.48.

Evaluation of the vortex gyrotropic frequency

As far as the vortex frequency is concerned different methods could be used to extract it. Instead of using the position of the vortex core $(X(t), Y(t))$ to derive the frequency we decided to work with the mean magnetizations $\langle m_x \rangle$ and $\langle m_y \rangle$ as their signals are less noisy. Oscillations of the vortex will induce a sinusoidal evolution of the mean magnetization. This is illustrated in Figure 2.5 by the results of our micromagnetic simulations. In steady-state, and in the absence of any in-plane external magnetic field, one can see that $\langle m_x \rangle$ and $\langle m_y \rangle$ present similar phase-shifted oscillations, i.e. the amplitude and the frequency are the same. As in many other problems requiring frequency analysis one could treat our discrete data with a Fast Fourier Transform (FFT) method. However that kind of algorithm presents some limitations, especially for our problem. First the frequency resolution is limited by the number of data points as well as by the sampling frequency. But most importantly such technique is not adapted to frequency-varying signals. [24] Another method based on geometrical considerations on the circle formed by plotting $\langle m_x \rangle$ with respect to $\langle m_y \rangle$ (Fig. 2.5b) will be explained hereafter.

During his PhD thesis Abreu Araujo developed an algorithm to compute the instantaneous oscillation frequency of vortices. [24] This method is called Simple Numerical Instantaneous Frequency Approximation (SNIFA) and is now available as a public python package. [65] This technique is able to calculate the frequency of an oscillator based on only two data points, so also its evolution with time. Its limitation is that it only retains the dominant frequency. It is thus more adapted to mono-frequency signals, which is respected by our simulations having reached steady-state. The instantaneous frequency f is given as:

$$f = \frac{\alpha}{2\pi(t_{n+1} - t_n)} \quad (2.52)$$

Where $(t_{n+1} - t_n)$ is the difference between two successive timesteps. The variable α is the angle formed between two successive mean reduced magnetization vectors defined between the origin of the circle of Figure 2.5b and the coordinates $(\langle m_x(t_n) \rangle, \langle m_y(t_n) \rangle)$ and $(\langle m_x(t_{n+1}) \rangle, \langle m_y(t_{n+1}) \rangle)$. This angle can be obtained using the al-Kashi's theorem on the triangle formed by the two vectors. To obtain a refined value of the gyrotropic frequency one can perform an average of the instantaneous frequency over several periods of oscillation.

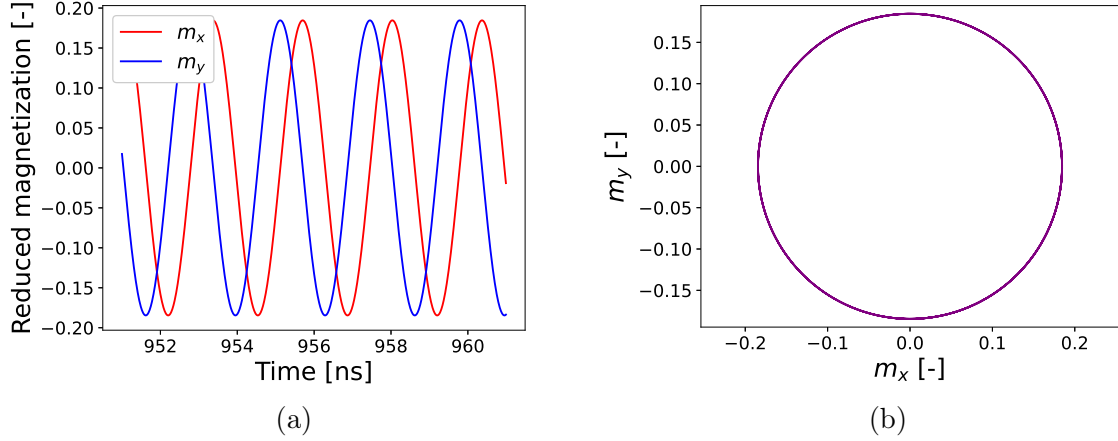


Figure 2.5: Micromagnetic simulation of a $C = -1$ vortex with a current of 8 MA/cm², the Oersted field is taken into account. (a) Reduced mean magnetization with respect to time for a vortex arrived in steady-state. The x (resp. y) component is given in red (resp. blue). (b) Circle formed by plotting $\langle m_x \rangle$ with respect to $\langle m_y \rangle$.

2.3.2 Results and discussion

Our results are presented and discussed in the next subsection. Firstly three characteristic curves describing our STVO dynamics will be shown. Those relate the current density, the vortex radius and its frequency. Then details about the data-driven fitting of the gyrotropic and damping terms will be provided. Finally a comparison between our semi-analytical model predictions and simulations in transient and steady-state regime will be displayed.

Evolution of the intrinsic vortex properties (core position and frequency)

Using the methodology presented previously we were able to obtain characteristic curves of our STVO dynamics. Those describe the relation between current density, reduced radius and gyrotropic frequency. From now on when mentioning the reduced radius the notation s will be used instead of s_0 as all simulations are considered to have reached steady-state. The reduced radius s with respect to the current density is depicted on Figure 2.7 for our three different study cases. A cubic interpolation is performed between the points for which $J_{dc} > J_{cr}$. This fitting does not take into account data points presenting a $s > 0.8$, as our analytical model stops to be valid above this limit.

One can observe that our results respect to square root behavior predicted by Guslienko *et al*, i.e. $s(J_{dc}) \propto \sqrt{J_{dc}/J_{cr} - 1}$. [44] Such evolution has been widely confirmed in many other works involving micromagnetic simulation of vortex-based nanopillars. [48, 59, 61, 60] A clear appearance of critical currents is also noticed. For each curve a systematic damping to $s = 0$ is attained for any current below a threshold value. Above J_{cr} on the contrary the vortex can be excited. For the C^- case one can observe that $s = 0$ for $J_{dc} \geq 8.6$ MA/cm². This follows another development from Guslienko *et*

al on the existence of a second critical current. [44] Indeed for too important current densities the vortex core is expelled when oscillating at too large radii ($s \approx 0.8$). As a consequence a new vortex with reversed polarity nucleates. The condition for stable oscillation $J_{dc} P p_z < 0$ is no more respected and a damping results. We obtained a similar behavior for the C^+ and noOF studies for currents $J_{dc} > 9.8$ and 9 MA/cm^2 , respectively. Gaididei *et al* also observed this polarity switch for large currents in micromagnetic simulations. [60]

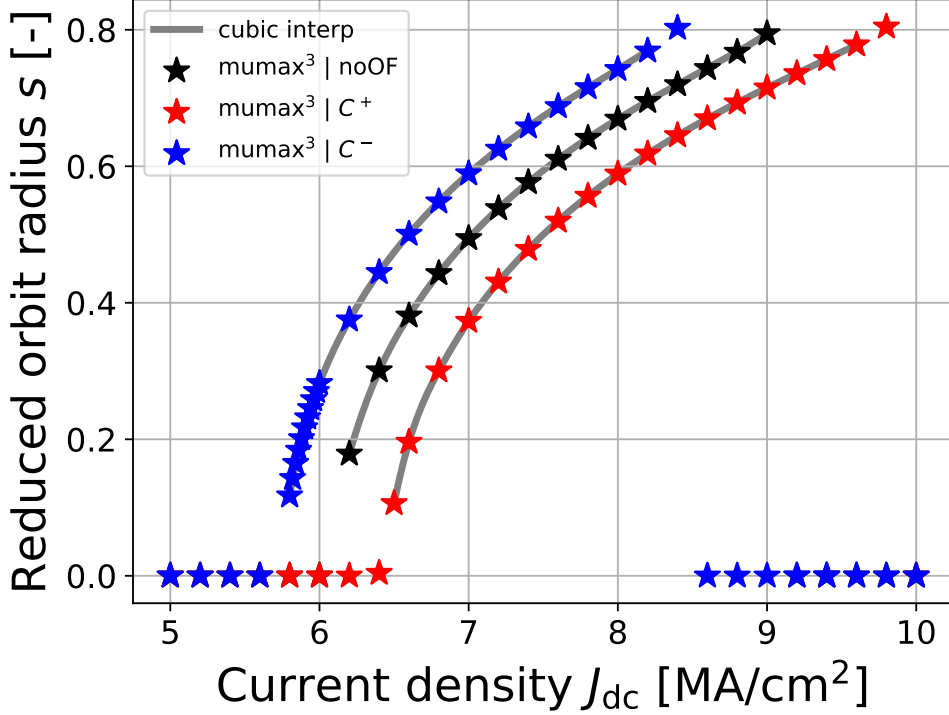


Figure 2.6: Vortex reduced radius of oscillation s with respect to the input dc current density J_{dc} . Three situations are studied ; one without the contribution of the Oersted field (noOF) in black, one with OF and $C = +1$ (C^+) in red and one with OF and $C = -1$ (C^-) in blue. Simulated data points are represented by stars. A cubic interpolation (grey line) is performed between all values presenting $0 < s < 0.8$. A clear splitting due to the OF can be observed.

The most remarkable conclusion that can be drawn from these curves is the major influence of the Oersted field on the vortex dynamics, depending on its orientation with respect to the chirality. Due to the difficulty to obtain an analytical expression valid for a large range of s values describing its impact very few papers in the literature have considered its influence. [59, 61, 66, 67] Other works generally neglect it, obtaining only one characteristic curve per geometry. [48, 60] These group's curves are equivalent to our noOF results, depicted in black in Figure 2.6. However not taking the OF into account makes any comparison with experimental data difficult as the simulation does not reflect the physics of the system. Indeed in practical STVO implementation a magnetic field will always be generated inside the junction by the current flowing through it. In our

study we show a clear impact of the chirality on the dynamics, especially concerning the critical current value. We can observe that $J_{\text{cr}}^{C^-} < J_{\text{cr}}^{\text{noOF}} < J_{\text{cr}}^{C^+}$. This phenomenon was described by Dussaux *et al.* [59] If the Oersted field is parallel (resp. antiparallel) to the vortex chirality ($C = +1$) (resp. $C = -1$) a higher (resp. lower) J_{cr} is observed due to a switch of sign of the confinement induced by the Oersted field. The precise values of critical current will be detailed further in this section. Moreover the magnitude of this splitting depends directly on the spin polarization of the current p_J . For larger p_J the splitting increases. Practically this parameter is difficult to estimate in experimental MTJ. One could therefore envisage to use that kind of characteristic curve, which would have been previously calibrated, to obtain an estimation of the quality of the current spin polarization by reverse engineering.

The vortex frequency with respect to the current density is depicted in Figure 2.7. As expected sub-GHz frequencies are obtained. Let us note that the absolute value of the frequency is depicted here, i.e. $|f| = |\omega|/(2\pi)$, as its sign depends on the vortex polarity. Two distinct regimes can be identified. Below J_{cr} a relaxation frequency is obtained. Above J_{cr} the indicated frequency corresponds to steady gyrotropic precessions of the magnetization.

Let us start by considering the part below the critical current. Considering relaxation and taking into account that $D^2 \ll G^2$ [59] one can simplify the expression of ω given in Eq. 2.20 to $\omega = -(k^{\text{ms}} + k^{\text{ex}} + k^{\text{Oe}}CJ_{\text{dc}})/G$. If we consider k^{ms} , k^{ex} , k^{Oe} and G to be constant in this regime, as $s = 0$, we find the following linear equation:

$$\omega = \omega_J J_{\text{dc}} + \omega_0 \quad (2.53)$$

Where ω_0 is $-(k^{\text{ms}} + k^{\text{ex}})/G$ and the slope $\omega_J = -Ck^{\text{Oe}}/G$. These two parameters have been calculated for our three case studies, both theoretically and from simulation results and are given in Table 2.2. For the simulations a linear fit was performed on the data for which $J_{\text{dc}} < J_{\text{cr}}$. One can remark a good agreement between theory and simulations. As expected the sign of the slope depends on the chirality, resulting in a frequency splitting for non-zero currents. Its magnitude is proportional to k^{Oe} and thus directly linked to the Oersted field. For the noOF case the frequency is almost independent of the current, as predicted by TEA. One can notice that ω_0 is almost similar for noOF, C^+ and C^- . This behavior was predicted by Eq. 2.53 as the three curves are supposed to have the same value for $J_{\text{dc}} = 0$. This eigenfrequency shift in the relaxation regime due to the OF has also been observed by Choi *al et.* [66]

	noOF		C^+		C^-	
	TEA	Fit	TEA	Fit	TEA	Fit
$\frac{\omega_0}{2\pi}$ [MHz]	-448.84	-449.51	-448.84	-449.56	-448.84	-449.56
$\frac{\omega_J}{2\pi}$ [1/G]	0	-0.03	-4.69	-4.74	4.69	4.82

Table 2.2: Comparison of $\frac{\omega_0}{2\pi}$ and $\frac{\omega_J}{2\pi}$ between theory (i.e. Thiele equation approach) and linear fit from micromagnetic simulations.

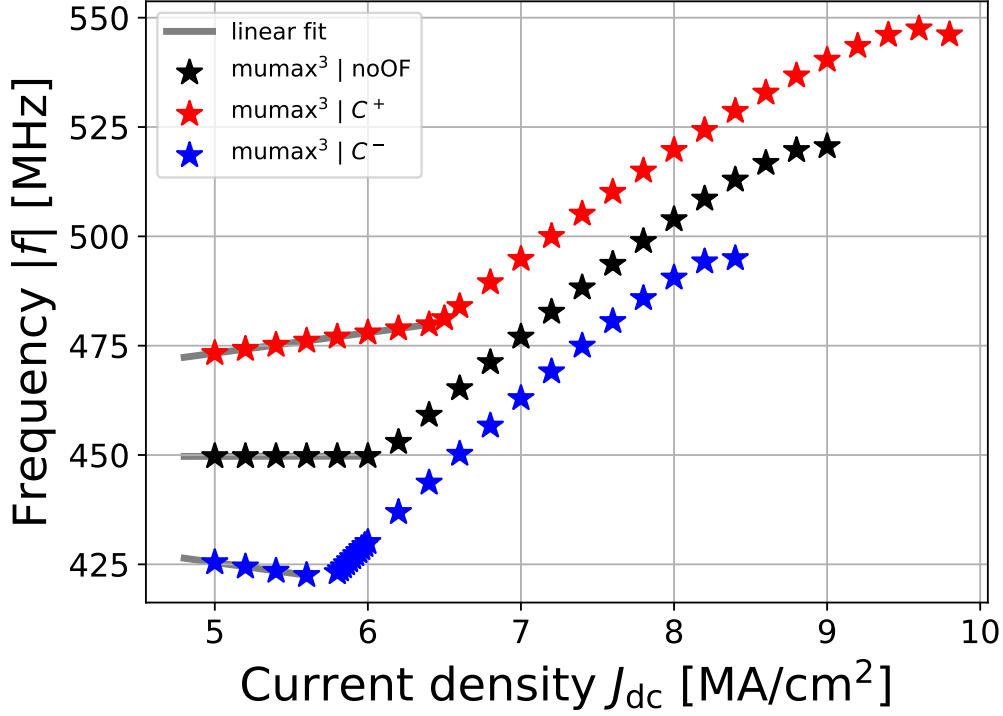


Figure 2.7: Vortex absolute frequency $|f| = |\omega|/(2\pi)$ with respect to the input dc current density J_{dc} . Three situations are studied ; one without the contribution of the Oersted field (noOF) in black, one with OF and $C = +1$ (C^+) in red and one with OF and $C = -1$ (C^-) in blue. Simulated data points are represented by stars. A linear fit (grey line) is performed between all values presenting a current $J_{dc} < J_{cr}$. A clear splitting due to the OF can be observed.

A second regime appears for currents above J_{cr} . The precessional frequency of steady oscillations of our STVO is obtained. As expected this regime starts at different currents for the three curves resulting from the shift in critical current due to the chirality. The frequencies associated to the critical currents are $f = -478.5$, -449.1 and -423.1 MHz for C^+ , noOF and C^- , respectively. The switching point between the two regimes has in fact a precise physical meaning. It is defined as the equilibrium between the forces induced by spin-transfer torque and linear damping. [59] The switching frequency is therefore given as the intersection of two linear equations. The first one is the equation describing the relaxation regime, i.e. $\omega_J J + \omega_0$, and the second the linear damping, i.e. $J_{dc} \kappa^{ST}/D$. Moreover equaling the damping and relaxation equations allows to retrieve the critical current expression given in Eq. 2.28.

In this gyrotropic regime the frequency seems at first to evolve proportionally to the current density (Fig. 2.7). At higher currents however higher-orders terms become non-negligible and non-linearities start to appear. Indeed many parameters involved in the vortex dynamics are orbit dependent (e.g. spring-like constants). As s is itself function of the current, the more the latter increases the more the frequency behavior will deviate from a linear equation. The force associated to the Oersted field seems to

have an important influence on this phenomenon. In fact the chirality modifies the sign of its contribution. For the noOF curve this contribution even disappears. One can also remark the apparition of a maximum in the curves after which the frequency decreases with J_{dc} . Similar results were obtained by Guslienko *et al.* [61] This maximum occurs at $f = -547, -520$ and -494 MHz for C^+ , noOF and C^- , respectively.

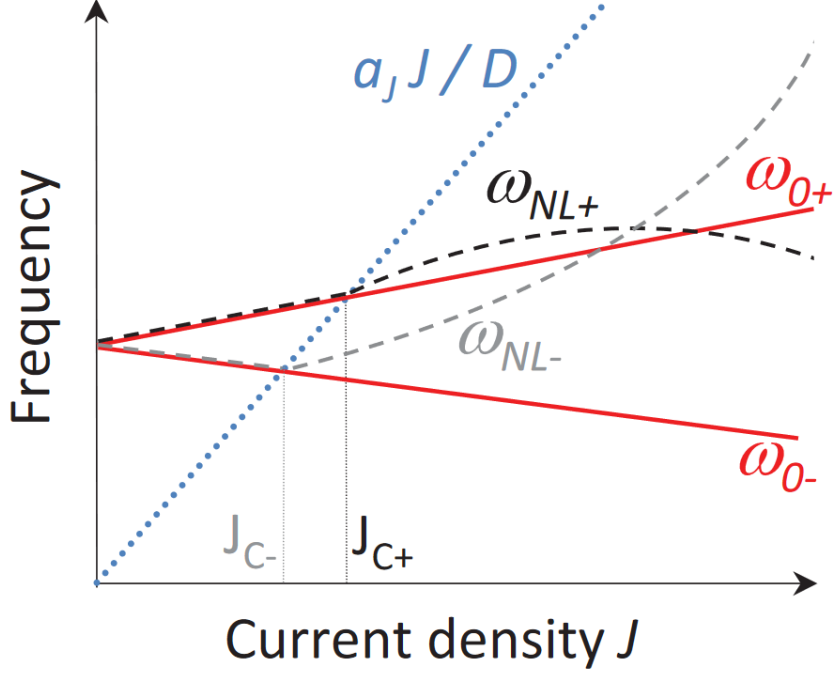


Figure 2.8: Vortex frequency with respect to the current density, predicted by Dussaux *et al.* The symbol + (resp. -) refers to the case where the chirality is parallel (resp. antiparallel) to the OF. The frequency evolution is separated in two regimes. First a linear part ω_0 which stops when the spin-transfer forces become greater than the damping. The latter is given by the linear equation $a_J J / D$. After this switching point a non-linear evolution of the frequency ω_{NL} is noticed. Reproduced from [59].

Lack of studies in the literature captures the whole current range and even fewer consider the Oersted field influence. Generally only one regime is examined, i.e. relaxation or gyrotropic oscillations. [48, 61, 66] This results in an incomplete picture of the vortex dynamics. In this report we present a comprehensive analysis of the frequency evolution with the current, from relaxation for $J_{dc} < J_{cr}$ to the vortex polarity reversal at high current. Dussaux *et al* have been one of the few groups to explore to impact of the chirality for any current value on the vortex frequency, though only under a schematic view. [59] Their results are given in Figure 2.8. Their model is globally compatible with our simulations but some discrepancies are still observed. In the relaxation regime they also predict a linear evolution of the frequency with a slope sign which is chirality dependant. This lasts until crossing with the straight line defined by $a_J J / D$. This intersection corresponds to a current value, i.e. critical current J_{cr} , above which spin-transfer forces become greater than damping. Let us note that their a_J parameter is equivalent to our κ^{ST} . For $J_{dc} > J_{cr}$ they anticipate a nonlinear evolution of the frequency. They even postulate that for large currents and a chirality parallel to the Oersted field the nonlinear

frequency $\omega_{\text{NL}+}$ could drop below the resonant frequency ω_{0+} . This would be a consequence of nonlinear effects related to the Oersted field. Such effect is not observed in our simulations (Fig. 2.7) as the vortex is expelled before, even if the frequency decreases after a maximum. Moreover they predict that, in the antiparallel case, $\omega_{\text{NL}-}$ increases continuously. This constitutes a major difference with our results. Indeed we show that the evolution of the nonlinear frequency for C^- does not present this behavior, as its growth is slowing down at high currents. This difference could be due to the fact that their analytical derivation only takes k^{ms} and k^{Oe} into account. In addition their Taylor expansions are limited to s^2 terms which limits the validity of their expressions for large J_{dc} .

Finally the vortex absolute frequency of oscillation is depicted with respect to the reduced radius in Figure 2.9. A highly non-linear behavior is noticed. Even if this graph is just a combination of previously presented information it allows to highlight better the general vortex behavior as well as the OF splitting. Indeed plotting the frequency with respect to s has more physical meaning compared to J_{dc} as all parameters of our model are defined as functions of s . This means that those only indirectly depend on J_{dc} as the current magnitude modifies the steady-state orbit s which is the actual variable of interest.

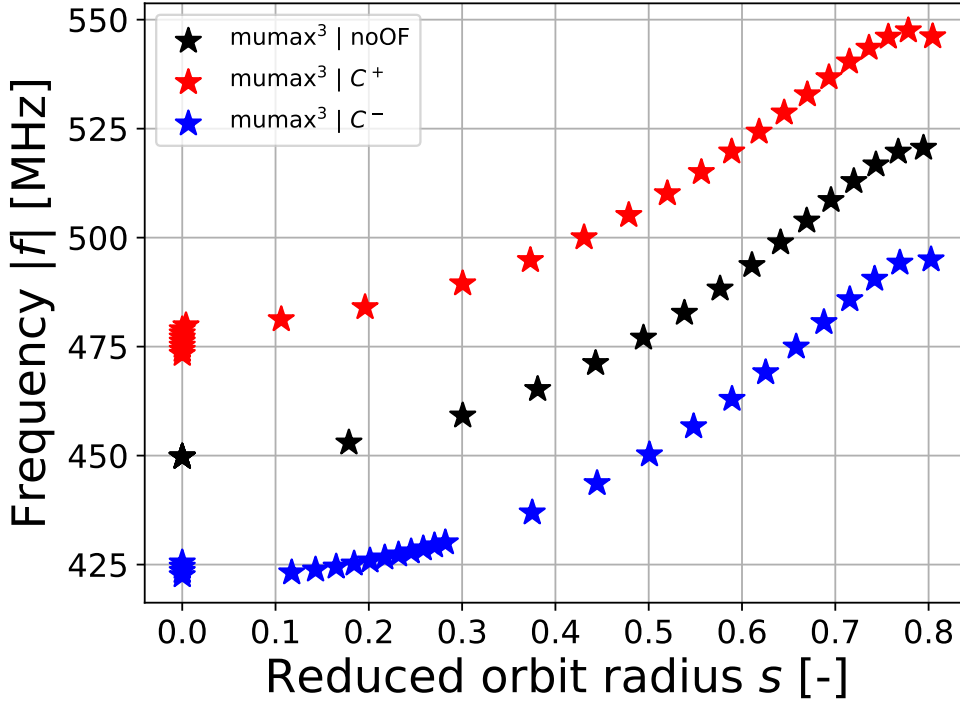


Figure 2.9: Vortex absolute frequency $|f| = |\omega|/(2\pi)$ with respect to the reduced radius s . Three situations are studied ; one without the contribution of the Oersted field (noOF) in black, one with OF and $C = +1$ (C^+) in red and one with OF and $C = -1$ (C^-) in blue. Simulated data points are represented by stars. A clear splitting due to the OF can be observed.

One can observe several points in $s = 0$. Those correspond to the relaxation regime where the steady-state position of the vortex is the center of the dot. Different frequencies are obtained depending on the current applied. For $J_{\text{dc}} = 0$ the same frequency, roughly 449 MHz, would be observed for the three curves. Then if the current increases the frequency deviates from this values with the current if the OF is taken into account, i.e. C^+ and C^- curves. This lasts until $J_{\text{dc}} = J_{\text{cr}}$ when the vortex enters the gyrotropic precession regime as s starts to be non-zero. From this point the frequency increases with the reduced radius, as seen in Dussaux *et al* paper. [59] Depending on the applied current one can tune the frequency of our STVO on a range of roughly 75 MHz, for each our the study case. A maximum appears just before the vortex polarity reversal which happens at $s \approx 0.8$. A clear splitting due to the OF field is again noticed. Its magnitude, calculated as $|f_{C^+} - f_{C^-}|$, decreases slightly for increasing s . Our model predicts this behavior as k^{Oe} , the spring-like constant associated to the OF, diminishes constantly with s .

Data-driven evaluation of $D(s)$ and $G(s)$

As a reminder we will introduce data-driven corrections to our analytical model in order to improve its precision. Those concern the gyrotropic and damping terms. Generally existing models only deal with G_0 and D_0 , i.e. their value at the center of the dot. This limits the validity of the calculations to small orbits of oscillations. In this work we assume that G and D are functions of s . Their expressions are given as $G(s) = G_0 f^G(s)$ and $D(s) = D_0 f^D(s)$, with $f^G(0) = f^D(0) = 1$. The second operation is to add global correction factors δ^G and δ^D . Those aim to absorb any imprecision in the expression of G_0 and D_0 as well as in the spring-like constants k_0^{Oe} , k_0^{ex} , k_0^{ms} and κ^{ST} .

Let us start with the gyrotropic term. Using the expression of $G(s)$ (see Eq. 1.46) we have previously deduced that:

$$\delta^G f^G(s) = -\frac{C J_{\text{dc}} k^{\text{Oe}}(s) + k^{\text{ex}}(s) + k^{\text{ms}}(s)}{G_0 \omega(J_{\text{dc}})} \quad (2.54)$$

Where $f^G(s) = 1 + a_G s^2 + b_G s^4 + c_G s^6 + d_G s^8$. This function, multiplied by δ^G , will be fitted to our micromagnetic results. The frequency obtained from mumax³ was originally expressed as $\omega(J_{\text{dc}})$. As we have obtained the evolution of s with J_{dc} , we can easily express Eq. 2.54 either as being s or J_{dc} dependent. To increase the number of data points we first performed a cubic interpolation on simulation results before fitting. Of course a set of parameters is obtained for each of the study case. As a result of the critical current shift we used different current ranges depending on the chirality. Data points starting from 6, 6.2 and 6.6 MA/cm² were examined for C^- , noOF and C^+ , respectively. All fitting were stopped at 8 MA/cm². Our results are depicted in Figure 2.10.

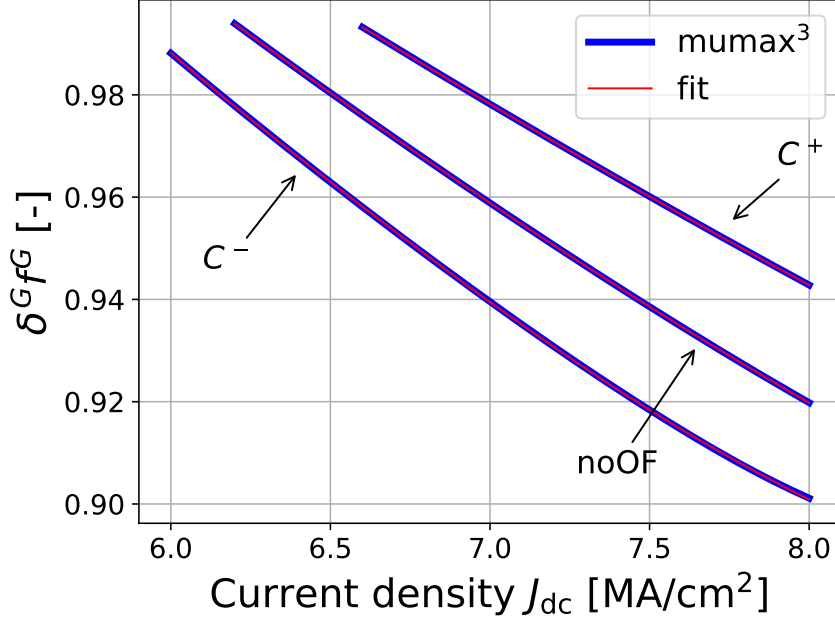


Figure 2.10: $\delta^G f^G$ with respect to the current density J_{dc} for our three study cases, identified with labels and arrows. First a cubic interpolation was performed on simulated points (blue lines). Then these data are fitted to obtain the evolution of G with s (red lines).

The obtained fitting parameters are presented in Table 2.3. The maximum error between fitting and simulated points, very low for the three cases, is also indicated. Our G_0 can be rectified thanks to the δ^G factor. The analytical value G_0 is adapted by -0.13, -0.13 and +0.22 % for C^+ , noOF and C^- , respectively. We also observe that the gyrotropic term $G(s)$ decreases for increasing current density and so for increasing orbit s . This diminution reaches nearly 9 % between 6 and 8 MA/cm² for the C^- curve. This shows the importance of providing data-driven corrections to our model. Different behaviors are also observed depending on the chirality. While the C^+ curve is almost linear, greater non-linearities appear for noOF and even more for C^- . Moreover the distance between the C^+ /noOF curves and between noOF/ C^- is not the same. This fact could indicate that we indeed absorb other imprecisions coming from the definition of our different parameters.

	δ^G	a^G	b^G	c^G	d^G	max(err) [%]
C^+	0.9987	-0.1411	-0.0301	-0.1311	0.1428	0.001
noOF	0.9987	-0.1505	-0.0268	-0.1527	0.1845	0.003
C^-	1.0022	-0.1841	0.1547	0.0067	0.1648	0.028

Table 2.3: Obtained fitting parameters of Eq. 2.54 allowing to derive the evolution of G with respect to s , for our three study cases. The maximum relative error between fitting and micromagnetic simulations is also indicated.

As far as the damping term is concerned the identical procedure is applied. We start

in this case from the expression of $D(s)$ (see Eq.1.47) which allowed to find:

$$\delta^D f^D(s) = \frac{J_{dc} \kappa^{ST}}{D_0 \omega(J_{dc})} \quad (2.55)$$

Where $f^D(s) = 1 + a_D s^2 + b_D s^4 + c_D s^6 + d_D s^8$. This function, multiplied by δ^D will be fitted to our micromagnetic results. Again we just need to extract $\omega(J_{dc})$ from mumax³. The same range of current as for the gyrotropic term is used. A comparison between fitting and simulation is shown in Figure 2.11.

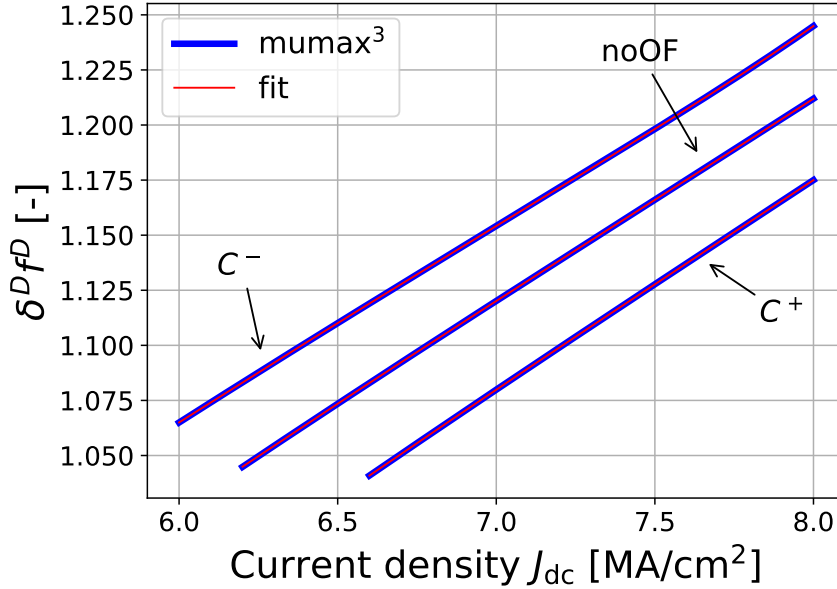


Figure 2.11: $\delta^D f^D$ with respect to the current density J_{dc} for our three study cases, identified with labels and arrows. First a cubic interpolation was performed on simulated points (blue lines). Then these data are fitted to obtain the evolution of D with s (red lines).

The resulting fitting parameters are presented in Table 2.4. The maximum error between fit and simulations is again very low. A quite important correction is provided to D_0 by the δ^D factor. The analytical value of D_0 is adapted by +2.70, +3.50 and +4.30 % for C^+ , noOF and C^- , respectively. We notice an increase of $D(s)$ with the current. This damping enhancement reaches nearly 20 % for the range of current explored. This confirms that by simply considering D_0 , i.e. without data-driven correction, the dynamics of our STVO can not be rendered accurately. Conversely to the gyrotropic curves, a nearly linear behavior is observed for all study cases. Those are almost parallel. However the distance between them is again different between C^+ /noOF and noOF/ C^- .

	δ^D	a^D	b^D	c^D	d^D	$\max(\text{err}) [\%]$
C^+	1.0270	0.3469	0.1423	0.1499	0.0268	0.001
noOF	1.0350	0.2961	0.1547	0.0067	0.1648	0.007
C^-	1.0430	0.2552	0.1301	0.0109	0.1281	0.003

Table 2.4: Obtained fitting parameters of Eq. 2.55 allowing to derive the evolution of D with respect to s , for our three study cases. The maximum relative error between fitting and micromagnetic simulations is also indicated.

Besides the evolution of $G(s)$ and $D(s)$ we were also able to extract a precise value of the critical current from micromagnetic simulations. As the reduced orbit radius s evolves proportionally to $\sqrt{(J_{\text{dc}}/J_{\text{cr}}) - 1}$ in the gyrotropic regime the critical current can be derived by fitting our results. It should correspond to the value in $s = 0$. We could have used to same function that the one presented in Eq. 2.51 but the latter includes a square-root of the current. This is not optimal for least-square minimization algorithms as it could contain negative values to be calculated. A simple solution to this problem is to inverse our function. As a consequence the square-root disappears and enhanced precision is obtained. The following expression is proposed:

$$J_{\text{dc}}(s) = J_{\text{cr}} \left(\left(\frac{s}{a_0 + a_1 J_{\text{dc}} + a_2 J_{\text{dc}}^2} \right) + 1 \right) \quad (2.56)$$

Where a_0 , a_1 , a_2 and J_{cr} are the fitting parameters. The fact that our function is now implicit does not pose any issue for numerical calculations as s and J_{dc} values are known. To increase the number of data points we performed a cubic interpolation on simulation results. After this initial step fitting was performed on our three study cases. The $J_{\text{dc}}(s)$ curves as well as their corresponding fit are presented in Figure 2.12.

The obtained fitting parameters are summarized in Table 2.5. As expected we observe again that $J_{\text{cr}}^{C^-} < J_{\text{cr}}^{\text{noOF}} < J_{\text{cr}}^{C^+}$. Those critical currents, extracted from simulations, can be compared to the theoretical values predicted by Eq.2.28. We found $J_{\text{cr}} = 6.269$ (-2.99%), 5.883 (-3.60%) and 5.542 MA/cm² (-3.80%) for C^+ , noOF and C^- , respectively. We notice a systematic underestimation of the critical current by our analytical model compared to simulations, up to several percents.

	$J_{\text{cr}} [\text{MA/cm}^2]$	a_0	a_1	a_2	$\text{avg}(\text{err}) [\%]$
C^+	6.4615	2.5735	-0.2654	0.0118	0.09
noOF	6.1025	2.9612	-0.3692	0.0186	0.12
C^-	5.7613	2.8924	-0.3654	0.0191	0.05

Table 2.5: Obtained fitting parameters of Eq. 2.56 allowing to derive the critical current, for our three study cases. The average relative error between fitting and micromagnetic simulations is also indicated.

Adding data-driven corrections to the theoretical critical current expression allows to find results closer to simulations. The damping and gyroconstant are thus modified

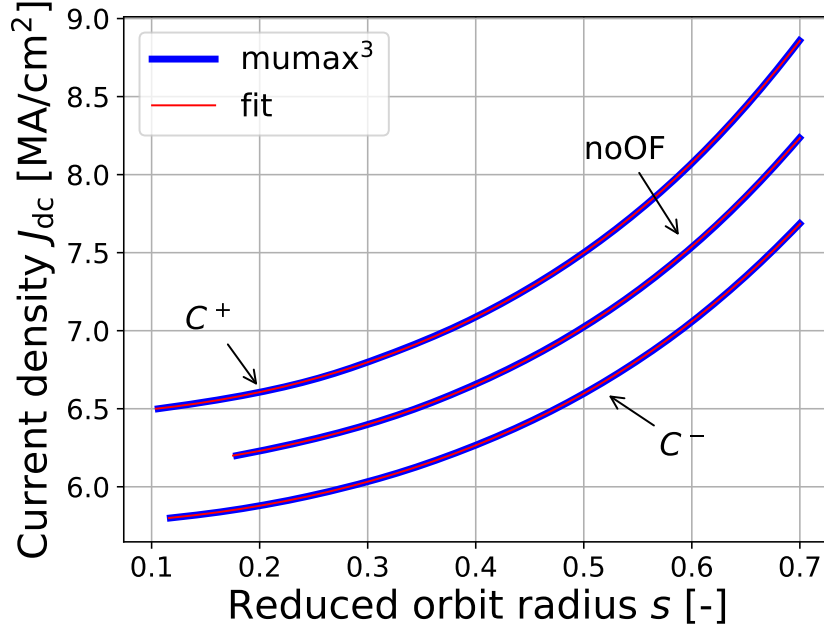


Figure 2.12: Reduced orbit radius s with respect to the current density J_{dc} for our three study cases, identified with labels and arrows. First a cubic interpolation was performed on simulated points (blue lines). Then these data are fitted to obtain a precise value of the critical current (red lines).

to $\delta^D D_0$ and $\delta^G G_0$. Taking those considerations into account, the expression of Eq. 2.28 is adapted to:

$$J_{cr} = \frac{-\delta^D D_0 (k_0^{ex} + k_0^{ms})}{\delta^D D_0 C k_0^{Oe} + \delta^G G_0 \kappa^{ST}} \quad (2.57)$$

Using this equation, we obtain $J_{cr} = 6.4591$ (-0.04%), 6.0972 (-0.09%) and 5.7547 MA/cm² (-0.11%) for C^+ , noOF and C^- , respectively. Those results are much closer to the ones presented in Table 2.5. This confirms the utility of adding data-driven corrections to our semi-analytical model.

Using $D(s)$ and $G(s)$ in the transient regime

As it will be shown hereafter our model is able to predict accurately the transient vortex dynamics after the data-driven fitting on $D(s)$ and $G(s)$. This is a tremendous achievement as the latter was performed using only steady-state values.

For practical applications, especially in neuromorphic computing, the steady-state regime of oscillation is of limited interest. Indeed the input signal, here in the form of a current, is rarely constant. For an arbitrary current presenting rapid amplitude variations, i.e. faster than the time required to reach a stable orbit, our STVO could constantly undergo a transient regime. To render with great precision the vortex under such solicitation one could again use micromagnetic simulations. This method however shows rapidly its limits for signals extending in time. Indeed micromagnetic simulations require important computation time and power (e.g. up to hours of calculation for only

1 μs of simulated time). Therefore, for large scale computations, one needs to derive analytical models fitting the vortex physics the best way possible. Various attempts have been conducted in the literature in recent years to propose such model, usually starting from Thiele equation. However these results are of limited interest, as only steady-state values are generally predicted. [48, 60, 66] If transient regime evolution is presented prediction can not be made for new input parameters as only simple fitting is performed on simulation results. [59]

The imprecision of existing models is due to the lack of consideration of high-order terms involved in the vortex dynamics. Those are often completely neglected giving analytical expressions only valid at the center of the dot (e.g. for the spring-like constants). Other developments, more precise, take into account the position dependency but only up to s^2 or s^4 expansions. Adding the fact that some potential energy or spin-torque components are sometimes completely omitted gives rise to difficulties in making predictions on the transient regime. This is especially true when high orbits are considered, as high-order terms become non-negligible. As a consequence comparison between analytical and simulation results are difficult to find in the literature. Guslienko *et al* have been one of the few groups to present such results, as illustrated in Figure 2.13. [61] Their analytical solution is based on Eq.2.47. A relatively good general behavior is obtained but discrepancies are noted when looking at details. One can observe a systematic underestimation of the relaxation time compared to simulations. Moreover the predicted steady-state orbit always ends below simulated points, up to several percents.

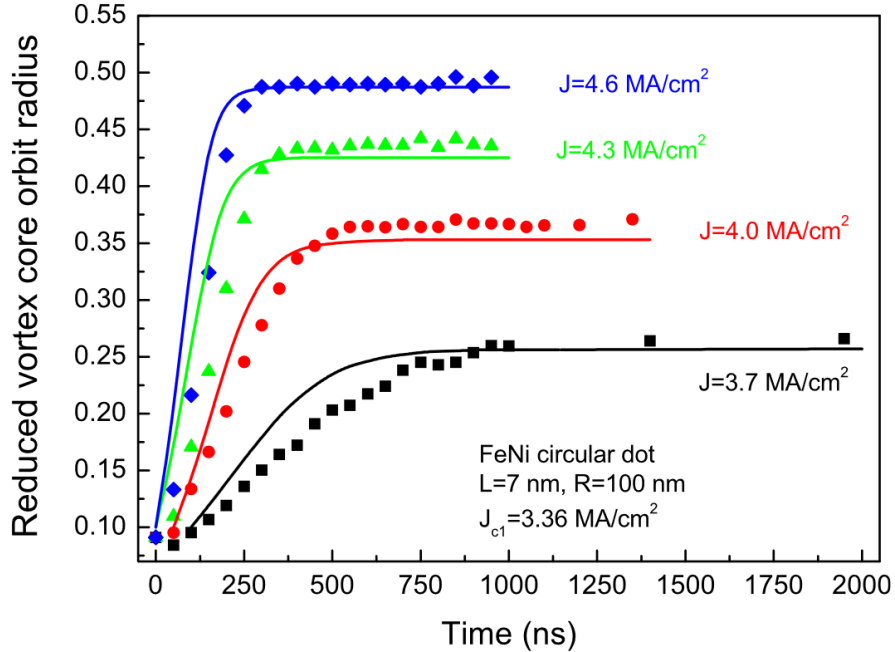


Figure 2.13: Comparison of the evolution of the reduced vortex core orbit radius between an analytical solution and micromagnetic simulations, performed by Guslienko *et al* for different input currents. Solid lines represent the theoretical solutions. The simulated points are depicted with symbols. Reproduced from [61].

As it will be shown our model is able to predict with great precision the transient

regime of oscillation for any new input current value as well as initial core translation. Two main reasons can explain this accuracy. First we used Taylor expansions up to s^8 terms adapted to our aspect ratio, leading to a validity on a large range of reduced core position, i.e. $0 < s < 0.8$. Such precision on the spring-like constants had never been explored, especially for k^{ms} and k^{Oe} . This only consideration would have led to an increased validity of our model compared to existing ones. However we added data-driven corrections on the gyrotropic and damping terms $G(s)$ and $D(s)$ as well as a global correction factor δ . This allowed to fit perfectly the dynamics obtained by micromagnetic simulations. However let us note that these rectifications are only valid for a given geometry. If one wants to adapt our model to another aspect ratio, additional simulations should be performed to fit the dynamics. This means also that our model is only semi-analytical, as it is partially based on simulated data. However once a small number of simulations have been performed and parameters adjusted one just need to resolve an EDO system to obtain the response to any input signal.

A comparison between simulation and our model prediction for the reduced vortex core position with respect to time is given in Figure 2.14. The input current were chosen to be superior to the critical current and cover a large range of steady orbit values. A perfect agreement between mumax³ and analytical curves is observed. The steady-state regimes are completely overlapped for both methods. This is the case even for a current $J_{\text{dc}} = 9 \text{ MA/cm}^2$ which leads to $s \approx 0.8$, i.e. where is situated its validity limit. As far as transient regime is concerned, very satisfying behaviors are also obtained. Its shape respects the one acquired from simulations and relaxation times are consistent. A negligible shift is remarked for the lower currents curves. Let us note that we worked here with the noOF configuration, i.e. without considering the Oersted field, to allow comparison with Guslienko's work. Similar agreement would have been obtained for C^+ and C^- .

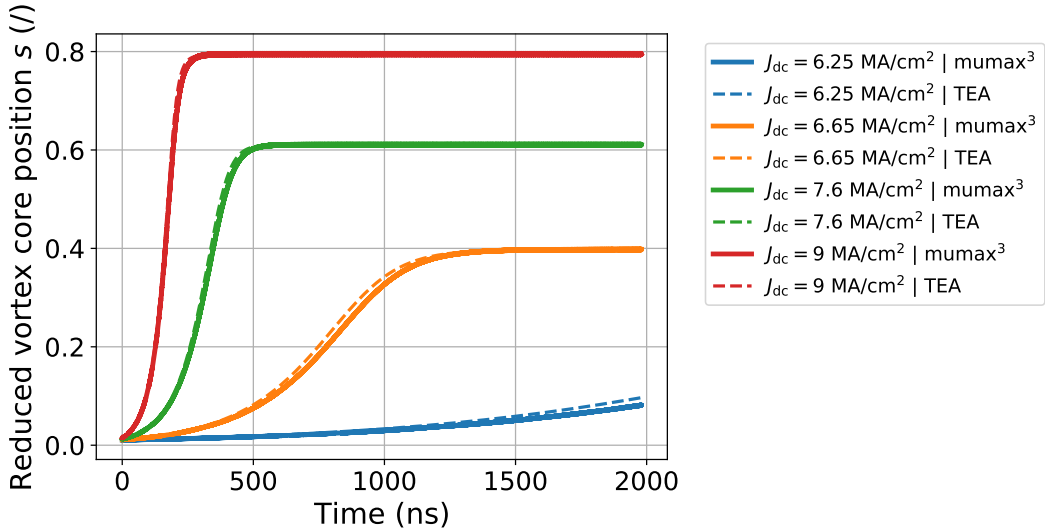


Figure 2.14: Comparison between mumax3 (solid lines) and our TEA model (dashed lines) of the reduced vortex core position s with respect to time. Different J_{dc} are used, all superior to J_{crit} (noOF case). All simulations were started at $s = 0.01$.

Using our SNIFA technique we also computed the evolution of the instantaneous vortex frequency with time, as illustrated in Figure 2.15. Those results would have been impossible to obtain using standard FFT algorithms. All curves start at $f \approx -450$ MHz. This frequency corresponds to a core position of $s = 0.01$, i.e. the starting point of our simulations. As this orbit is close to the center of the dot the frequency is almost equal to $\omega_0/(2\pi)$ (see Table 2.2). With time increasing the frequency evolves to its steady-state value, corresponding to the input current imposed. Again we observe very good agreement between simulations and TEA approach. For the $J_{dc} = 9$ MA/cm² curve, approaching the validity limit of $s = 0.8$, we notice a slight underestimation of the frequency by our model. Again a minor shift is also observed for the $J_{dc} = 6.65$ MA/cm² case. A very good correspondence in relaxation time is nevertheless computed.

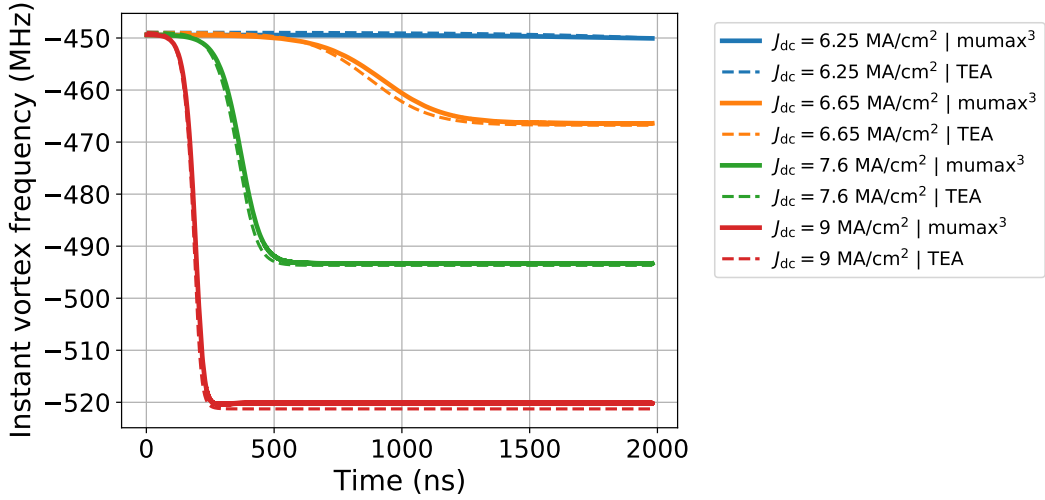


Figure 2.15: Comparison between mumax3 (solid lines) and our TEA model (dashed lines) of the instant vortex frequency with respect to time. Different J_{dc} are used, all superior to J_{crit} (noOF case). All simulations were started at $s = 0.01$.

2.4 Spin-diode effect

This section presents experimental spin-diode effect results, obtained by a partner research group. Those are then compared to our semi-analytical model predictions for comparable parameters. Detailed explanations are provided on peaks observed in the curves, appearing at particular frequencies.

2.4.1 Experimental INL results

We obtained experimental data of MTJ presenting spin-diode effect from our partners working at the International Iberian Nanotechnology Laboratory (INL). Their junctions present different geometrical parameters, i.e. $R = 500$ nm and $h = 9$ nm, compared to the one studied previously. Moreover the free layer is composed of two materials. A 7 nm thick layer of permalloy is placed on top of a 2 nm thick CoFeB layer, itself positioned above the NM spacer. As low power consumption applications are targeted a passive device is considered. This means that no dc biasing current is applied. As

a result the input current density is defined as $J = J_{ac} \sin(2\pi ft)$, with J_{ac} the signal amplitude and f its frequency. As far as the junction resistance is concerned they gave us very approximative values. The average resistance would be $\bar{R} \approx 50 \Omega$ and the difference between AP and P configuration $\Delta R = R_{AP} - R_P \approx 10 \Omega$. Moreover a planar polarizer is considered here. A weak in-plane external field is also applied to displace the vortex core from the dot center and facilitate oscillations. INL experimental curves are shown in Figure 2.16. An ac power between 0 and -5 dBm is applied, for both chiralities. A conversion from dBm to mW is possible thanks to $P_{dBm} = 10 \log(P_{mW}/P_0)$ with $P_0 = 1\text{mW}$. Several observations can be made on this graph. A resonant frequency of approximately 65 MHz is noticed. For an input frequency far above the latter very low spin-diode voltage is obtained. The maximum output voltage is around 6.5 mV. It corresponds to the input signal of greatest power. One can also remark that the absolute amplitude of the C^+ curves is superior to the one of C^- at same input power. Finally a particular behavior can be noted around 33 MHz as an indent appears in the highest amplitude curves. We will try to explain these observations by comparing the experimental results to our analytical predictions.

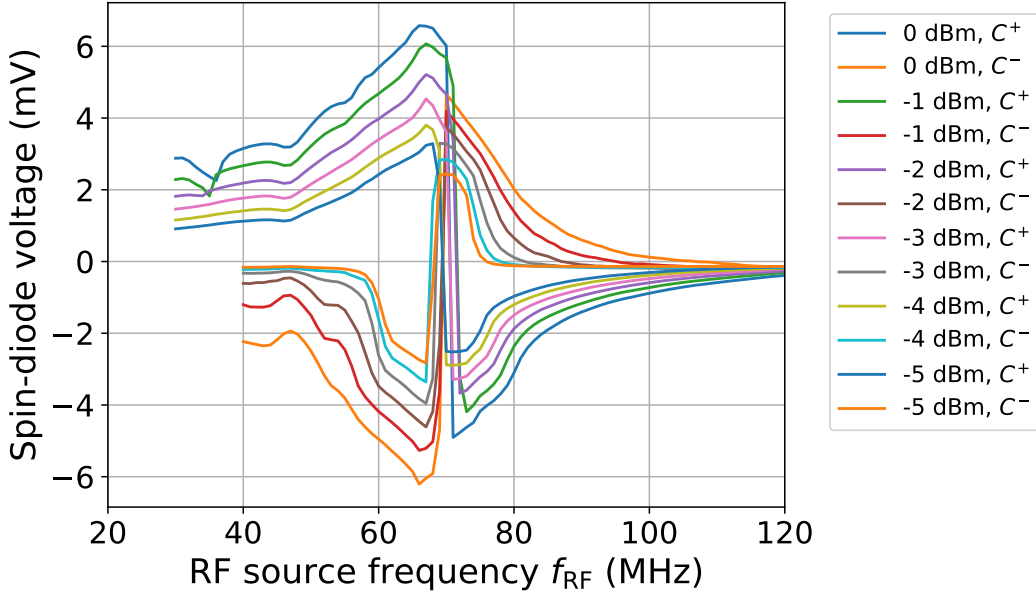


Figure 2.16: Experimental spin-diode voltage with respect to the rf source frequency. Different rf powers are applied, for both chiralities. Data were provided by partners from INL (Braga, Portugal). A resonant frequency around 65 MHz is obtained as well as a maximum output voltage of roughly 6.5 mV.

2.4.2 Using our data-driven model to understand experimental data

Using our TEA model it was possible to compute a graph similar to the one presented in Figure 2.16. As it will be shown very noteworthy curves are obtained, even with important assumptions. To acquire quantitative results comparable to the one from INL, we had to adapt slightly their geometric and material parameters. All these modifications are compatible with possible experimental indetermination and are detailed

hereafter. In any case our semi-analytical model had to be adapted. The free layer dimensions have changed compared to our study case, i.e. $R = 100$ nm and $h = 10$ nm. A larger nanopillar is studied here, i.e. $R = 525$ nm and $h = 8.55$ nm. Our aspect ratio is thus modified to $\xi = 0.008$. As a consequence the spring-like constant k^{ms} , associated to magnetostatic energy, had to be recalculated as it depends heavily on the latter. To gain time and avoid a calculation using the Monte-Carlo integration method we used a rough approximation here. As Abreu Araujo had also calculated $k_{\xi=0.0}^{\text{ms}}(s)$ during his PhD thesis [24] we used a simple rule of three between $k_{\xi=0.0}^{\text{ms}}(s)$ and $k_{\xi=0.05}^{\text{ms}}(s)$ to obtain $k_{\xi=0.008}^{\text{ms}}(s)$. Moreover the data driven corrections brought were the same as for the $R = 100$ nm case. So $G(s)$ and $D(s)$ as well as the global correction factors δ^G and δ^D were not adapted to the particular INL geometry. As it will be shown these assumptions have small influence on the quality of the results. The average resistance was set at $\bar{R} = 65 \Omega$ and ΔR at 9Ω . The saturation magnetization proposed is $M_s = 650$ emu/cm³

Skirdkov *et al* have introduced equations allowing to calculate the STD effect in MTJs. [48] As far as passive junctions are considered the input power P_{in} is expressed as:

$$P_{\text{in}} = \frac{\bar{R}}{8Z_0} \bar{R} I_{\text{ac}}^2 \quad (2.58)$$

Where $Z_0 = 50 \Omega$ is the impedance of the transmission line and $I_{\text{ac}} = J_{\text{ac}} \pi R^2$ the rf current. The spin-torque diode sensitivity ϵ can be calculated as:

$$\epsilon = \frac{\overline{\Delta V}}{P_{\text{in}}} \simeq -\frac{\Delta R I_{\text{ac}}}{2P_{\text{in}}} \int_{T_1}^{T_2} \langle (\mathbf{m} \cdot \mathbf{e}_p) \rangle \frac{\sin(2\pi f t)}{T_2 - T_1} dt \quad (2.59)$$

Where $\overline{\Delta V}$ is the constant output voltage, $\langle (\mathbf{m} \cdot \mathbf{e}_p) \rangle$ is the scalar product between the unit magnetization vector and the unit vector representing the polarization direction averaged over the dot volume, f is the input ac frequency and T_1, T_2 are two moments in time. Those should be chosen far from transient regime and separated by a sufficient duration. We chose $T_1 = 2500$ ns and $|T_2 - T_1| = 500$ ns. As a planar polarizer is used $\langle (\mathbf{m} \cdot \mathbf{e}_p) \rangle$ is reduced to $(2/3)Y(t)C$ within the TVA approach, with C the chirality and $Y(t)$ the vortex core position in the y axis. The evolution of the vortex core position with time can be calculated analytically using the same method as for the previous section, dedicated to transient regime predictions. The difference here is that a small external magnetic field will be applied to displace our vortex core, even at rest. This field is defined as $\mathbf{H} = (H_x, 0)$ and its magnitude is 12 G. It will displace our vortex core to a new position (d_x, d_y) which will be the starting point of our analytical resolution. The value of d_x and d_y can be calculated very precisely thanks to the nonhomogeneous solution of our model (see Eqs 2.41 & 2.42). Considering all these aspects we were able to obtain the spin-diode graph presented in Figure 2.17. To allow comparison with INL experiments signals of similar power were used, for both chiralities. The ac current to inject for each case can be derived from Eq. 2.58.

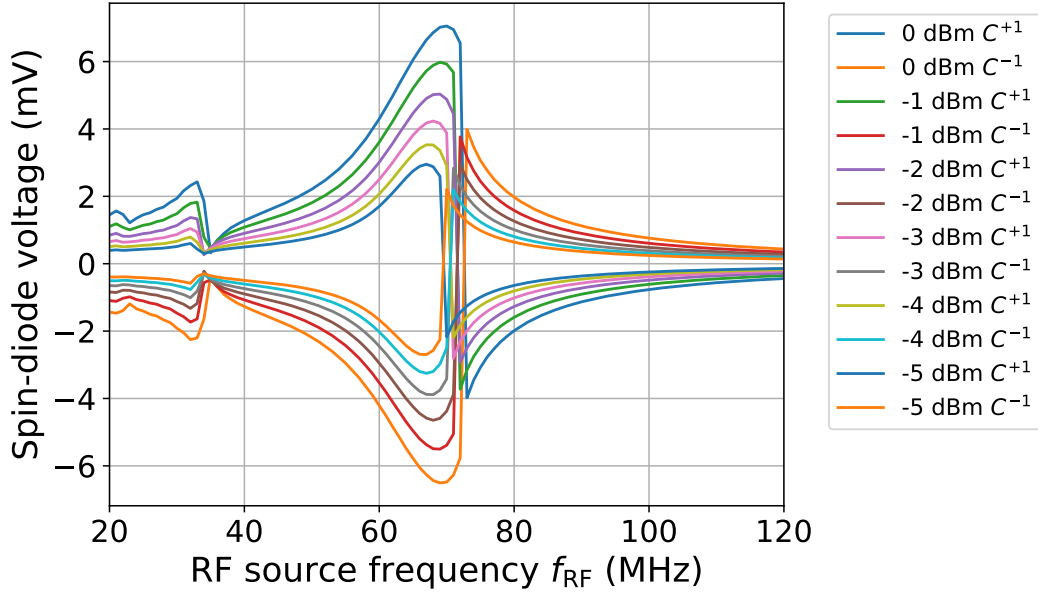


Figure 2.17: Theoretical spin-diode voltage with respect to the rf source frequency, predicted by our TEA model. Different rf powers are applied, for both chiralities. Similar results to INL experiments are obtained, i.e. the resonant frequency and the maximum output voltage are well predicted.

A very good agreement was observed between analytical predictions and experimental results. The resonant frequency f_0 and the maximum output voltage are well estimated. As expected input signals with frequency far above f_0 give rise to low spin-diode voltage. As far as the absolute amplitude of the curves is concerned one can again notice that the C^+ configuration leads to higher voltages than its C^- equivalent for an equivalent power. This can be understood by separating the spin-diode voltage in two major components. The first one is associated to the Oersted field and represents the major contribution of $\overline{\Delta V}$. Its sign depends on the chirality, explaining the difference between the curves for a same input power. The second component, much weaker, is linked to the spin-torque and its sign is unrelated to the chirality. This explains the slight shift in maximum absolute amplitude.

The important oscillations in the curves at 22, 33 and 65 MHz can be explained by our model, supported by Figure 2.18. Indeed these variations are also observed when looking at the most extreme $Y(t)$ position with respect to the input rf frequency (Fig. 2.18a). The peak at 65 MHz is the easiest to understand as this correspond to the resonant frequency of our oscillators. The peaks at lower frequencies are the result of a partial synchronization between the vortex frequency and harmonics of f_0 (Fig. 2.18b). This explains the peaks at $f_0/2 \approx 33$ MHz and $f_0/3 \approx 22$ MHz.

One of the most remarkable aspects of our model is how much it speeds up computations compared to micromagnetic simulations. Only 40s were required to acquire the data presented in Figure 2.17. Taking into account the simulation time alone, the same precision would have required 3.05 years of computation using mumax³ with the best commercially available GPU (RTX3090). This corresponds to a six order of magnitude

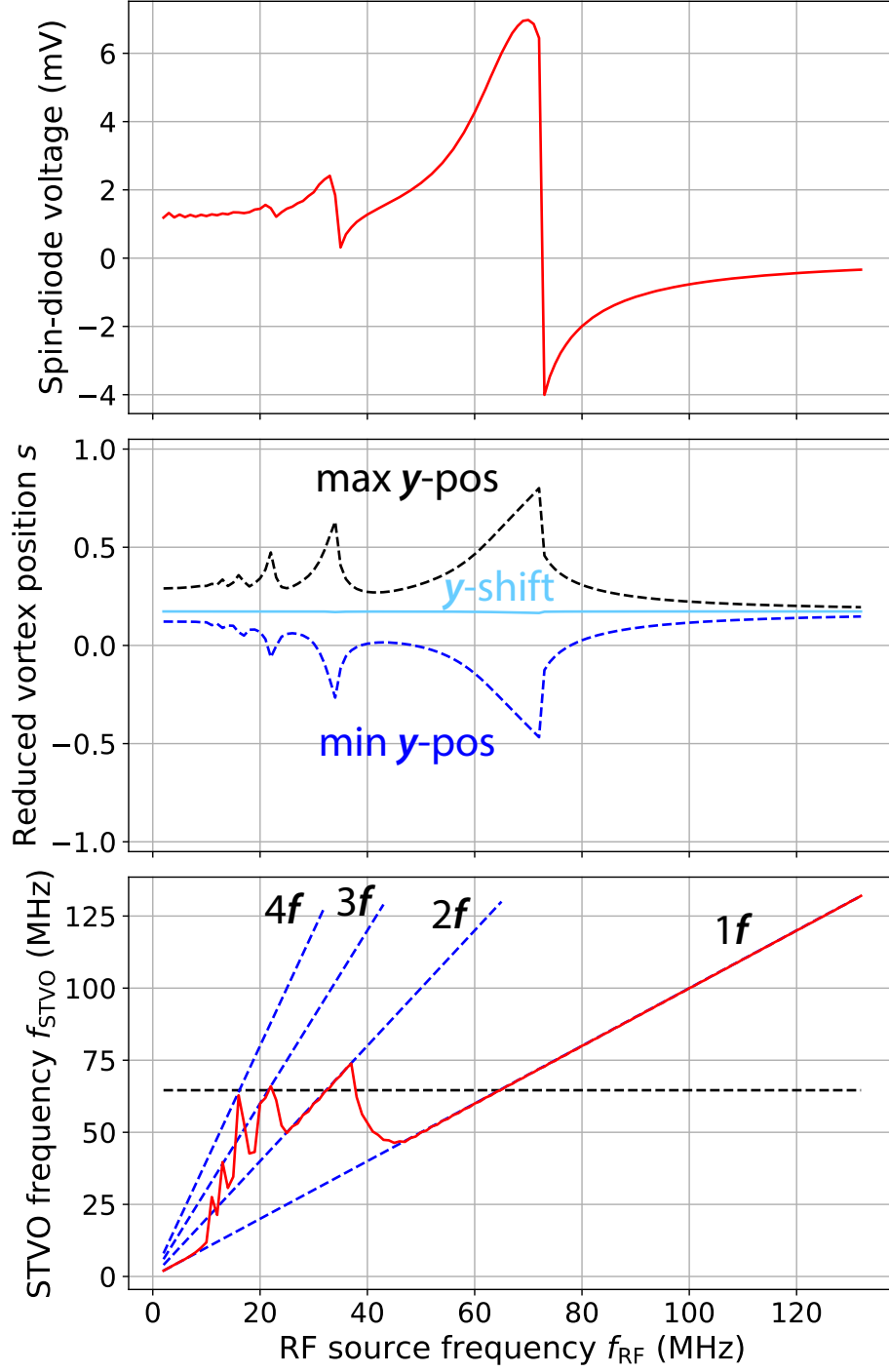


Figure 2.18: (a) Spin-diode voltage for a $P_{\text{in}} = 0$ dBm in the C^+ case (b) Vortex reduced core position with respect to the rf input frequency. The Y -shift is indicated as well as the maximum and minimum position of $Y(t)$. Oscillations are observed at f_0/n with n an integer. (c) Vortex frequency with respect to the rf input frequency. At low frequencies we observe a partial synchronization. The linear equations in blue dashed lines correspond to $f_{\text{stvo}} = n f_{\text{rf}}$ for $n = 1, 2, 3, 4$. Above this we find back a $f_{\text{stvo}} = f_{\text{rf}}$ behavior. The black dashed line indicates f_0 .

speed up. Those numbers are really promising concerning possibility of optimization of existing MTJ. As new theoretical results can be obtained very quickly one could tune input physical parameters and observe the resulting effects on spin-diode curves. As an example changing material parameters (e.g. M_s , A) or the geometry could improve the STD sensitivity as well as the resonant frequency. Such optimization process would be impossible to consider in reasonable time based on micromagnetic simulations.

2.5 Conclusion and perspectives

An analytical model describing the vortex dynamics was developed. It allows fast computation of the vortex core position by resolving an ODE system. Both perpendicular and arbitrary direction of polarization have been considered. Moreover micromagnetic simulation were performed, using mumax3 solver. The vortex core position and frequency have been analyzed with respect to the input current density. An important impact of the Oersted field was noticed. Two regimes were identified, i.e. relaxation and nonlinear gyrotropic precessions. These simulation data were used to adjust the expressions of $D(s)$ and $G(s)$, the damping and gyrotropic terms. This allowed predicting with great precision the vortex dynamics in transient regime. Finally our model has proven to be capable of understanding experimental results of spin-diode effect in magnetic tunnel junctions.

Despite the impressive results already obtained with the current version of our model we are aware that it could still be improved further. First those refinements could concern the potential energy terms included in the Thiele equations and especially the spring like constants. We emit doubts on the expression of k^{ex} found by Gaididei *et al.* [60] Depending on whether they calculate it within the SVA or TVA, its expression sign varies. This fact seems unforeseen and could be investigated. Then more terms could be taken into account such as thermal energy or dipolar terms. We also predict that a demagnetizing field could be generated by our dot itself and could impact our results. Finally all papers from the literature in the vortex field state that the volume averaged magnetization of the dot would be linked to the core position by a factor $\delta = 2/3$. All along this thesis we have used this value. We suspect that it could be erroneous by a slight margin as discrepancies have been observed with simulations. Moreover it would depend on the vortex core positions itself meaning that it would not be constant.

Concerning more specifically our spin-diode results, a new data-driven process should be initiated for the particular geometry of INL. This would provide values of $G(s)$ and $D(s)$ characteristic of the new aspect ratio. Finally the value of k^{ms} should also be adapted. Rather than using a basic rule of three, a proper Monte-Carlo integration scheme would give us better precision.

Memristive and tunneling effects in Ag crossed nanowires: towards hardware synapse design

Contributions: (Note by Flavio ABREU ARAUJO, advisor)

The results exposed in the following chapter were obtained by the team composed of Flavio ABREU ARAUJO (F.A.A.), Pascal VAN VELTHEM (P.V.V.), Chloé CHOPIN (C.C.), Nicolas MARCHAL (N.M.), Simon de WERGIFOSSE ([S.d.W.](#)), and Luc PIRAUX (L.P.).

L.P. created and designed the device. P.V.V. performed the challenging task of electrodepositing Silver (Ag) nanowires in porous templates. N.M. made the plasma etching steps to partially remove the gold electrode for the electrical measurements. F.A.A. designed the experimental set-up and performed the transport measurements. [S.d.W.](#) & C.C. also performed experimental measurements. The analysis was mainly performed by C.C. and [S.d.W.](#) who are also the main contributors of the first draft of ref [2]. (see page iii)

3.1 Introduction

Neurons without any interconnections are of limited interest. The aim of the neuromorphic approach is to allow them to communicate to resolve complex problems that an individual node could not deal with. In real biological systems, such as in the human brain, this signal transfer is ensured by synapses. Those basic components are responsible for regulating the relative importance of the connections to allow what is called plasticity. [6] Synapses present essential properties related to this phenomenon, i.e. long-term potentiation, long-term depression and spike timing-dependent plasticity (see Section 1.6.3). To be considered as a valid substitute to be integrated in a neural network, an artificial hardware device should be able to mimic these effects. For their ability to switch their conductance state with past applied electrical charges, memristance have rapidly attracted a lot of interest in this field. [68, 69, 70]

As a reminder, a memristance is the forth basic component of any electrical circuit. It behaves similarly to a classical resistance except that its value can evolve when a current is imposed. Discrepancies with Ohm's law are thus observed as well as typical pinched hysteresis curves. Among the various possibilities explored in the literature, we have chosen to work with silver nanowires for physical implementation. Through diverse processes those have proven to be able to switch between ON and OFF resistance states. [9, 55, 56] The most interesting phenomenon for our application, because of its reversibility, is the field-induced ionic migration of Ag surface atoms. [8, 10, 57] It allows to create new conduction bridges across the network between adjacent wires, decreasing its global resistance. Moreover these conductive filaments are supposed to disappear with time, allowing the system to find back its original state.

Researchers generally chose to work with Ag nanowires randomly dispersed inside polymer matrices. In this work we propose a different approach. Based on a process developed in past decades at UCLouvain by Pr Piraux's group, we will create a 3D crossed nanowire network with defined properties. [52, 53, 54, 71] A polymer membrane will be used as a flexible support. These thin film present an organized nanoporous network, following a chemical etching after ion bombardment. An electrodeposition allows then to grow metallic nanowires inside the pores. As interconnections exist between the wires, current is allowed to flow. However, before any resistance measurement, we will add a supplementary step. A damaging of our network will be obtained through a plasma etching process. This will allow to fragment the wires and create gaps in those. This has two major consequences. First it increases considerably the network resistance, i.e. near an open circuit state, due the poor conductivity inside the sample. In addition it allows to create zones favorable to an field-induced ionic migration process. The aim of this experimental part is thus to observe the effects of voltage sweeps on the network resistance state. This will be done by proceeding to multiple voltage sweeps and analysis of the results.

The results presented in this chapter should be published in a future reviewed scientific communication.

3.2 Method

The section details the method used to perform the experiments. At first the sample fabrication is presented. Then SEM images of our networks are displayed, allowing its characterization. Finally explanations about the measurement procedure used to obtain I-V curves are given.

3.2.1 Fabrication of 3D crossed nanowire samples

The subsection is dedicated to the fabrication process. Informations about the templates are provided. Then the electrodeposition procedure is described. Eventually the plasma etching step is justified.

Synthesis of 3D nanoporous polycarbonate templates

Polymer templates are used as structural matrix containing the nanowire networks. Those are received ready to be filled from the it4ip company. [72] Nevertheless let us detail their fabrication process. A polycarbonate (PC) membrane of 22 μm of thickness is bombarded with heavy ions of high energy. Those are accelerated thanks to a cyclotron. The traces left by the ion paths in the thin film are then chemically etched by an NaOH solution. As a result the diameter of the pores is enlarged.

Three principal parameters can be chosen depending on the application. Those are the angle formed between the ions tracks and the out-of-plane direction of the film, the porosity and the diameter of the nanopores. These parameters can have an important impact on the properties of the final device. For our application the angle formed between the ions tracks and the direction normal to the film plane is of $\pm 25^\circ$. It ensures a connectivity between the pores in the templates as those present a non-zero probability of being interconnected. An angular dispersion of $\pm 5^\circ$ is reported with respect to the desired angle. [12] The film undergoes an irradiation sweep in four different directions, as depicted in Figure 3.1. The porosity of our membranes is of less than 1 %. Very few connections are therefore created between the nanopores. The latter present a diameter of 30 nm.

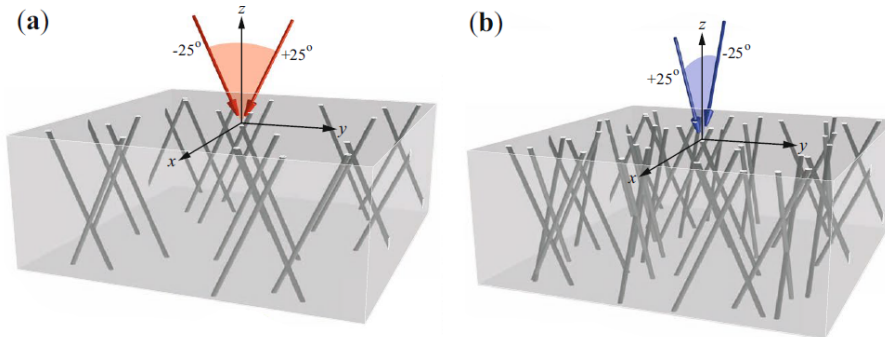


Figure 3.1: Schematic illustration of the successive irradiations at an angle of $\pm 25^\circ$ from the out-of-plane direction in a polycarbonate membrane. (a) Sweeps in the yz plane. (b) Sweeps in the xz plane. Reproduced from [53].

One must notice that the distribution of the nanopores is random. Even if the previously mentioned parameters are well controlled two different templates always present disparities, at least from a microscopic point of view. All variables used to describe the templates must thus take into account this statistical behavior.

The last step to be performed before electrodeposition is to cover one of the template face with a metallic cathode. In our case a 700 μm film of gold is sputtered, obstructing one extremity of the nanopores. It will allow the future growth of the NWs. A thin 3 nm chromium layer is placed prior to Au with the same technique to promote its adhesion to PC. As the dimensions of a template are originally quite large a punch is used to extract a circular sample of 1.3 cm of diameter

Electrodeposition of nanowires

Electrodeposition is used to create the NW network. This step was performed by Pascal Van Velthem¹. The experimental set-up is presented in Figure 3.2. To fill-up the nanopores a potential is applied to the Au electrode. As a consequence the metallic ions M^{n+} contained in the electrolytic solution are reduced at the cathode. This reduction starts at a potential specific to each metal. The general reduction reaction writes:



The reaction kinetics depends among other parameters on the solution concentration. A more concentrated solution gives rise to a quicker deposition. [13] Moreover one must notice that this deposition is not perfectly uniform along the film. Some wires could grow quicker compared to others. This is especially true for small pore diameters, such as in our case. A proportion of pores could not be filled at all. [11] As a result, even if our pore density was of about 1 %, a lower packing density of Ag wires could be obtained at the end of the process.

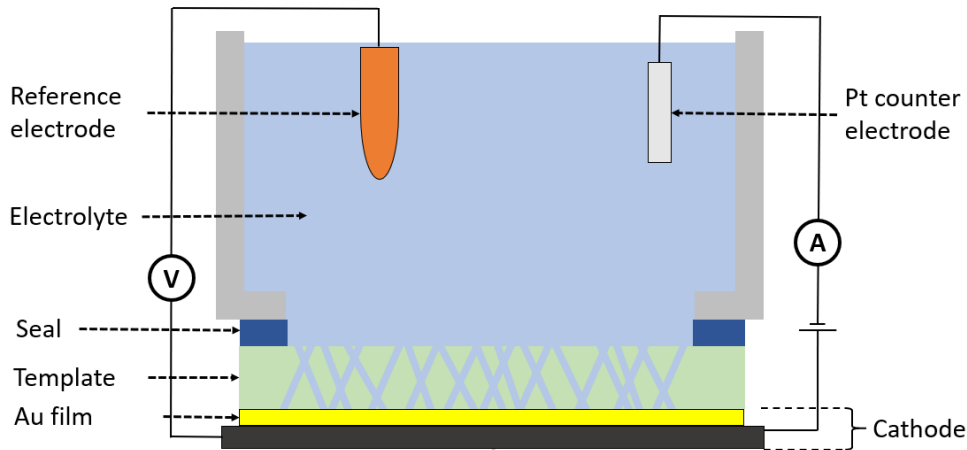


Figure 3.2: Schematic illustration of the experimental set-up used to fill the polycarbonate template with metallic nanopores. The electrolyte is placed in a teflon support. Electrodeposition occurs when a difference of potential is applied between the cathode and the counter electrode. A reference electrode ensures a constant potential.

¹IMCN, UCLouvain. Place Croix du Sud 1, Louvain-la-Neuve (Belgium).

Our electrodeposition process is performed in potentiostatic mode which means that the applied potential stays constant while the current can vary. As seen in Figure 3.2 the electrolyte is placed inside a teflon cell which lies above the nanoporous template. A rubber ring is placed between the two parts to ensure proper sealing. Applying a difference of potential superior to the cell equilibrium potential provokes then the deposition of metal in the pores. Consequently a current appears across the cell between the cathode and a counter electrode made of Pt. A high-stability reference electrode is also used to ensure a constant potential.

A typical curve of the current with respect to deposition time is depicted in Figure 3.3. When electrodeposition begins a transient regime is first observed. Metallic Ag starts to be reduced on the Au cathode and a large current is obtained. After this the deposition continues on the NWs. A steady-state current is observed. When the pores are completely filled a sudden increase in current is remarked. It happens because the deposition surface is considerably enhanced by the formation of metallic heads at the surface. Deposition should be stopped as soon as this jump is noticed. If one pursues the reaction, saturation is finally reached as a thin film is created on the surface.

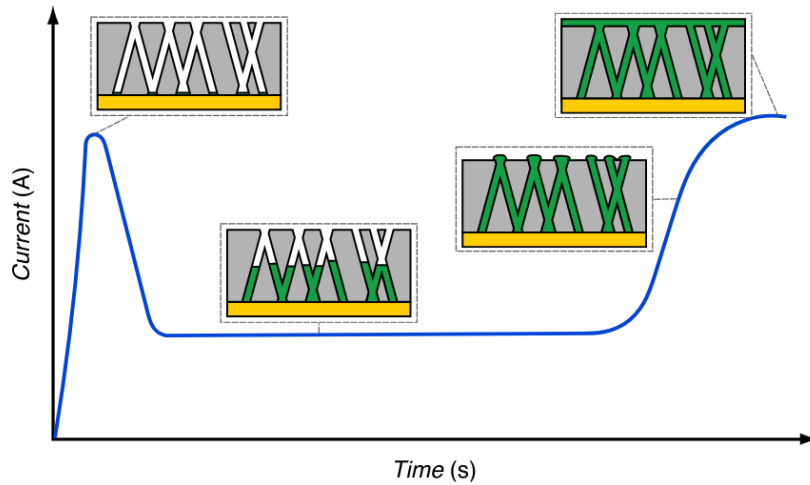


Figure 3.3: Schematic illustration of the current with respect to time during a potentiostatic electrodeposition process. At the start a high transient current is obtained because the metal is reduced on the cathode. Then a steady-state current is observed while the electrodeposition occurs on the NWs. Eventually the pores are completely filled, increasing the deposition surface. The current rises thus until saturation. The latter happens after the apparition of a film covering all the template surface. Reproduced from [71].

As far as the electrolyte is concerned we used a commercial SilverBright100 solution, bought from the Metakem company. [73] It consists of a cyanide-based bath containing 45 g/L Ag (0.4M) as well as additives. Its important pH-value, i.e. 11.5, is of critical importance for our application as it prevents formation of oxides at the NW surface according to Ag Pourbaix diagram. [74] Otherwise it would prevent conductive filament formation in the gaps separating neighbouring NWs. [8] A constant potential of -1.5 V is applied between the electrodes in the electrochemical cell. For about 120 s of deposition time the total charge Q was 97 mC. Reaction has been stopped as soon as the current

started to increase after the steady-state regime. This prevents any connection between NWs at the template surface.

Sample preparation using plasma etching

A plasma etching is performed on the electrodeposited templates using a Plassys MP500S platform. This step, achieved by Nicolas Marchal², serves two main purposes. First it allows to create two electrodes from the Au cathode (Fig. 3.4). In addition it damages the Ag wires as gaps can be created by heat generated stresses. This last consideration is essential for possible memristive behaviors as the NWs must present breaks to be reconnected by ion migrations under an applied voltage. A glass mask presenting a 2 mm-wide linear opening at its center is placed on the sample. An Ar plasma with a working pressure of 5 mTorr is then created. The power is set to 80 W and the dc bias to 1.126 kV. Six 10 min steps of etching are performed. Those are interrupted by 10 min of relaxation time. Indeed heat is generated by the plasma treatment which can cause excessive damages in the membrane and the wires. Sensors in the instrument also become less effective after too long exposure time. A pause between steps permits thus to counteract those phenomena. A total of 60 min of etching time is sufficient to completely remove the 700 μm cathode on the exposed area (Fig.3.4b) as well as damaging partially the Ag wires.

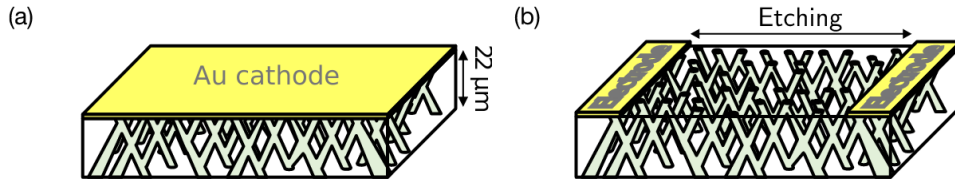


Figure 3.4: Schematic illustration of an interconnected NW network created by electrodeposition from a 22 μm thick PC porous membrane with (a) the original Au cathode and (b) the two Au electrodes resulting from a local etching. Reproduced from [71].

3.2.2 Sample characterization using scanning electron microscope

A JEOL FEG SEM 7600 scanning electron microscope (SEM) was used to characterize the morphology of our networks. The images obtained hereafter were obtained by Pascal Van Velthem and Etienne Ferrain³. The resolution of the microscope is of about 10 nm. Before any image acquisition the PC membrane had to be chemically etched. This serves two purposes. First it allows to see the metallic network across all its thickness. In addition it avoids charging effects on the polymer film as the latter is insulating. Those could blur the images. An acceleration voltage of 15 kV was used for the electrons. The working distance was roughly 8 mm.

Figure 3.5 depicts high magnification images of crossed nanowires. The wires composing this network are made of nickel and were not used in the framework of this thesis.

²IMCN, UCLouvain. Place Croix du Sud 1, Louvain-la-Neuve (Belgium).

³IMCN, UCLouvain. Place Croix du Sud 1, Louvain-la-Neuve (Belgium).

However the membrane used had the same parameters than in our process, i.e. a diameter of 30 nm and a packing factor of less than 1%. These examples are thus relevant illustrating support. One can remark that the wires present a slightly higher diameter compared to the membrane specifications. Various reasons can explain this observation. First we approached the limits of the microscope resolution. Moreover variations can result from the chemical etching process used after ion bombardment used to enlarge the nanopores. Even with a precise control on etching time this step remains critical, especially for small pore diameters. On the lower magnification image one can observe connections between the wires. However, due to the small diameter and the low packing factor those are not easy to perceive. After dissolution of the membrane the network remains partially self-supported even if it appears to be fragile. Indeed the nanostructure of the sample changed after removing PC as some wires collapsed. In theory those should be aligned with the four different directions of ionic bombardments. One can clearly see that those angles are no more respected here.

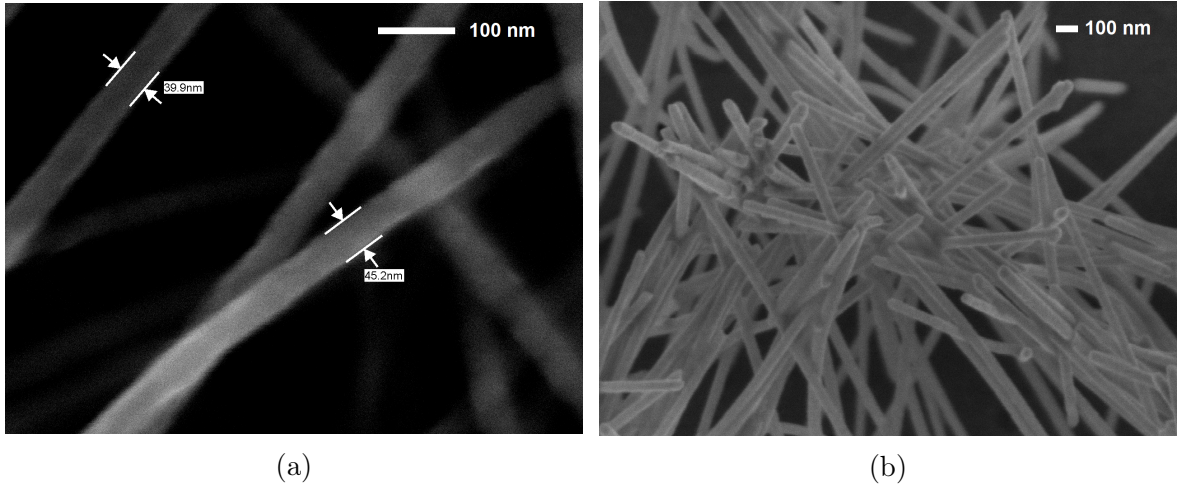


Figure 3.5: SEM images of Ni interconnect nanowire networks. Those were grown using the same membrane as for our Ag-based sample. The designed diameter of the nanopores was 30 nm. The packing factor is of less than 1 %. (a) High-magnification image showing individual wires presenting a diameter of roughly 40 nm. (b) Lower magnification image of the same network.

Lower magnification images were obtained on our Ag-based crossed nanowire network. Those are displayed in Figure 3.6. Again one can observe a self standing structure. However a clear modification of the nanostructure is noticed after dissolving the membrane. This is once more attributed to the low packing factor and the small wire diameter. The high aspect ratio of the wire is remarked. Considering the enormous number of pores in the membranes connections are nonetheless created between wires during the synthesis process. Let us mention that these images were acquired before any plasma etching step. The damages resulting from this procedure would have prevented to retain a self-standing structure. Therefore it is important to note that these images do not represent the final state of our network before electrical measurements.

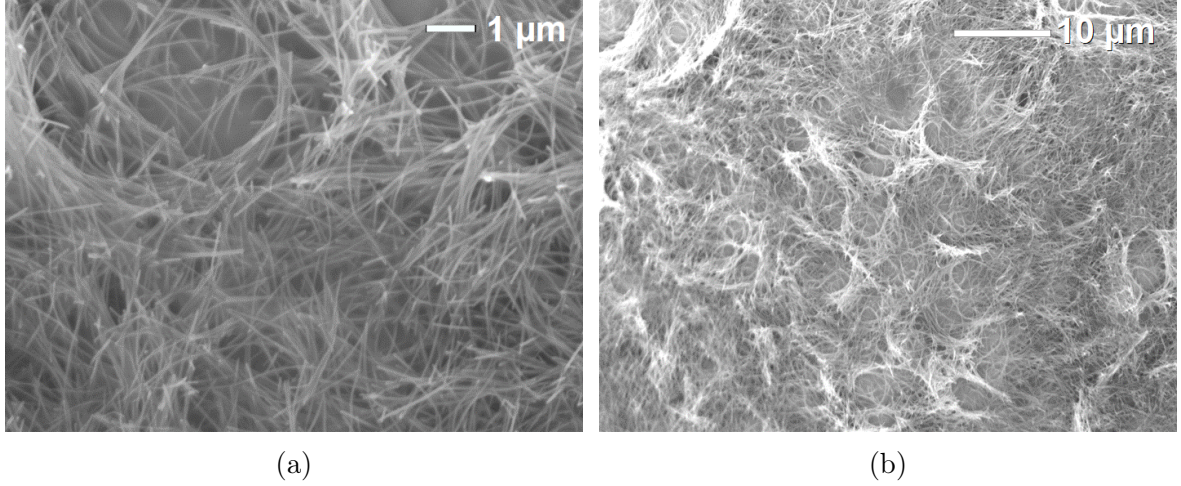


Figure 3.6: SEM images of Ag interconnect nanowire networks. The designed diameter of the membrane nanopores was 30 nm. The packing factor is of less than 1 %. (a) Low-magnification image showing the nanowire structure. (b) Lower magnification image of the same network.

3.2.3 Measurement procedure

One should be aware that all results presented in the next section originate from only one sample. This is especially important to notice as its electrical behavior will evolve considerably with experiments. After plasma etching two Au electrodes have been created on the template. To facilitate measurements our sample has been fixed to a holder. An electrical contact is made between two pins of this holder and each sample electrode by applying silver paint. This was done by Pr Luc Piraux. The current will thus be flowing in the plane of the PC film. Considering the important resistance of our NW network, i.e. $\sim \text{M}\Omega$, there is no need to perform four contact measurements. Indeed the contact resistance is completely negligible compared to the sample resistance. $I - V$ curves were obtained using a Keithley K617 programmable electrometer controlled with Labview. The device allows to measure currents up to the pA range with great accuracy and resolution. All results were obtained under ambient conditions, i.e. atmosphere and temperature. In all cases we imposed a voltage and measured the resulting current in our sample. We separated our data acquisition in three steps. We first performed four voltage sweeps $0 \rightarrow 100 \rightarrow 0 \text{ V}$. Those curves are analyzed in Section 3.3.1. After this we imposed numerous sweeps in the purpose of modifying the internal structure of the network. We aimed to create conducting filaments to lower the sample resistance. A series of six sequences $0 \rightarrow 50 \rightarrow 0 \text{ V}$ was followed by six others $0 \rightarrow 50 \rightarrow -50 \rightarrow 0 \text{ V}$ scans. The results of this procedure are analyzed by looking at four final $0 \rightarrow 50 \rightarrow -50 \rightarrow 0 \text{ V}$ sweeps, as shown in Section 3.3.3. In all sequences detailed hereabove the voltage increment was fixed at 1 V with a 1 s timestep between each value. Considering the exotic effects that have been observed and the large resistances measured we performed control tests on our sample holder. A first test was done between two pins without any sample between, i.e. in open circuit. The second one was carried out on an empty template without any Ag NWs. Both tests allowed to conclude that the observed effects are effectively coming from the NW

network itself. The presence of any artifact due to the holder or the PC template has been ruled out.

3.3 Results and discussion

Our results are presented and discussed in this section. We start by describing the $I - V$ curves obtained for the four first scans, corresponding to a high resistance regime. Then explanations about the appearance of a threshold voltage in the first scan are given. After this the low resistance regime is analyzed. For these last sweeps some exotic behaviors are observed and detailed.

3.3.1 High-resistance regime

The first four scans from 0 to 100 V are exhibited in Figure 3.7. All sweeps display very low current, i.e. nA order, indicating resistance in the $G\Omega$ range. A highly nonlinear behavior is observed, far from classical ohmic law. A typical memristive opening can be seen in each curve. This hysteresis seems to increase with successive scans, except for the second one. In addition the maximum current is never obtained for the maximum voltage value. This indicates remanence in the system and dynamic behavior. All sweeps present at first very low currents i.e. pA range. At a given point, namely the threshold voltage V_{th} current starts to raise dramatically with increasing voltage. This happens at much higher bias for the first sweep than for the others.

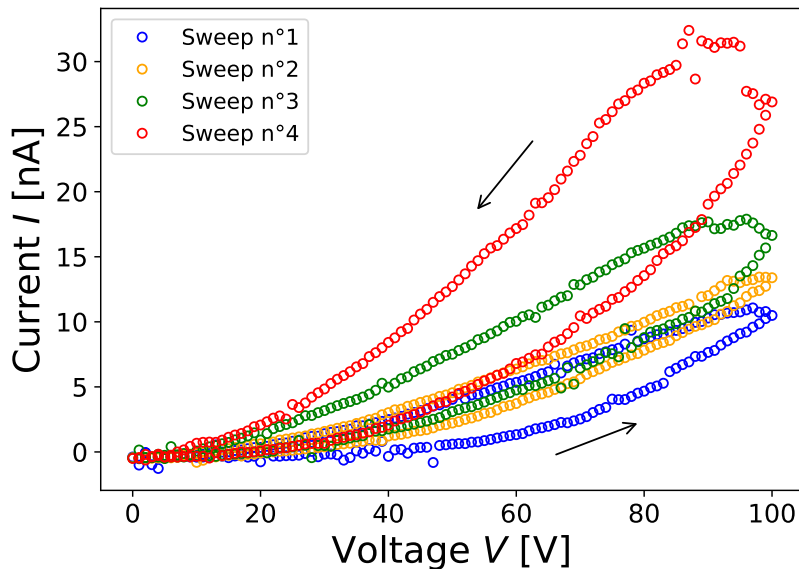


Figure 3.7: $I - V$ curves of the first four sweeps performed on the sample. Hysteresis is observed and seems to increase with consecutive scans. A remanence behavior is noticed as the maximum current does not arise for the maximum voltage. Black arrows helps identifying which data were acquired first.

All these characteristics could be insights of conductive filaments formation in the gaps separating neighbouring wires under applied voltage. Various other works involving Ag NWs have reported similar phenomena. [8, 10, 57] Indeed an electric field could provoke the migration of surface Ag atoms resulting in bridging between parts of the network. [10] New connections are thus created with time, diminishing the sample resistance. However depending on the filament properties, and especially its diameter, those effects could be reversible. If the source stops to deliver current those CF may break spontaneously due to Rayleigh instability. As a matter of facts atoms situated at the surface of those CF are less stable than in the bulk of the NWs due to atomic vacancies. A higher chemical potential results and CF is separated back to two atomic clusters to minimize system energy. [57] We suppose here that atoms preferentially migrate along the walls of the nanopores which have been left empty after electrodeposition and plasma etching. The ions could also diffuse directly through the PC template but the latter represents a much higher energy barrier. This volatility explains why the descending part of a sweep does not match with the ascending part of the following one. As some bridges have been destroyed with time conductance has decreased. Moreover CF formation could explain the remanence effects if the migration kinetic is slower than the sweep rate. Finally it would be easier to reform CF after multiple scans. [8] Those would become more robust. This explains why V_{th} decreases starting from the second scan. It also allows to understand why higher currents can be obtained with sequential sweeps.

In Figure 3.8a we normalized the current I by the maximum value of each sweep I_{max} . This aimed to identify if the behavior is reproducible between scans. We also unfolded our curves by plotting the data point index rather than the voltage ramp as x-axis. This allowed to identify that the ascending and descending part of our $I - V$ is far from symmetrical. One can see that the maximum current is obtained at a bias lower than 100 V, after the voltage peak. This indicates again remanence. The descending part of all sweeps seems very consistent on the contrary to the ascending part. The major difference between scans in the latter lies in the V_{th} value. The threshold voltage seems delayed in the first sweep compared to the others. Section 3.3.2 is dedicated to explain this difference. The evolution of the sample resistance is exposed in Figure 3.8b. A focus is made on the highest conductance values. Resistance up to the T Ω range has been measured for low voltage bias. This means that, within a same sweep, the perceived resistance of our sample can vary by several orders of magnitude. The general trend of decreasing resistance with successive scans emerges again. As explained previously we propose that the creation of multiple Ag bridging filaments across the network would open new paths for current to flow. Again remanence is noticed from the shift of resistance curve towards the descending part of the voltage ramp. This could imply that ions take some time to diffuse and form conductive filaments while still feeling the effects of past applied electric field.

The maximum current as well as the minimum resistance calculated during the four sweeps are reported in Table 3.1. The general trend of decreasing resistance and increasing currents is confirmed with consecutive sweeps. However one can still notice very important resistance values. We assume that this is due to three main factors. First the resistance of a wire is defined by $\rho L/A$ with ρ the resistivity, L the length and A the cross section. As our wires present a really high length to section aspect ratio,

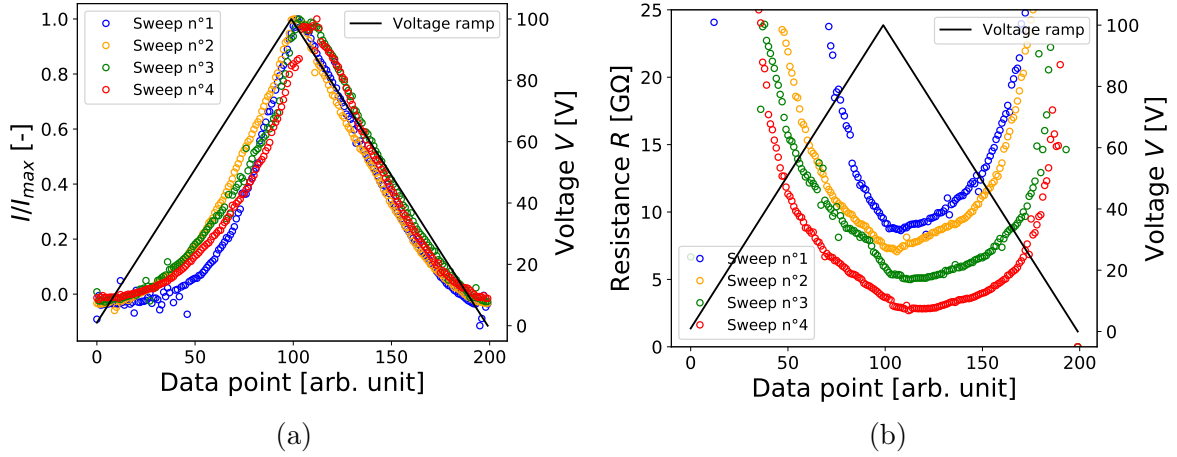


Figure 3.8: (a) Current I normalized by its maximum value $I_{\max}$ as well as the voltage ramp and (b) the sample resistance R with respect to the data point index. Each data point corresponds to a 1 V increment (or decrement) with a 1s timestep. Different behaviors are obtained for the voltage ascents ($0 \rightarrow 100$ V) while the descents ($100 \rightarrow 0$ V) are similar. Sample resistance decreases with consecutive scans.

their resistance is large. Then we used membranes with very low porosity. This reduces the number of possible interconnections between NWs. Finally the plasma etching step is supposed to have damaged our networks, as intended. Even if a few new connections are created through CF bridging, our sample thus still present an important resistance.

	$I_{\max}$ [nA]	$R_{\min}$ [GΩ]
Sweep n°1	11.7	8.34
Sweep n°2	13.4	7.06
Sweep n°3	17.9	4.99
Sweep n°4	32.4	2.69

Table 3.1: Maximum current and minimum resistance for the first four sweeps. Current increases with scans while the opposite effect is observed for resistance.

3.3.2 Conduction by tunneling effect

We identified a precise physical explanation for the threshold voltage V_{th} presented by our sample in the first 0 – 100 V sweep. Without considering any Ag migration behavior it originates from a transition between two different electron tunneling regimes. Indeed the gaps existing between Ag NWs resulting from its fabrication process can be seen as insulating barriers. When a voltage is applied there is a probability for electrons to travel across this barrier. As a consequence current increases with the dc bias. This corresponds to the direct tunneling regime. At a given point, namely the threshold voltage, a dramatic jump in current is noticed. The electronic transport has switched to field emission tunneling, also called Fowler-Nordheim (FN) tunneling. As it will be explained this is due to a change in the shape of the energy barrier with increasing field. Similar behavior was obtained by Wan *et al* in Ag NW segment separated by Ag

nanoparticles, however at much lower bias. [9] Other groups also reported comparable results in molecular electronic junctions. [75, 76] A Fowler-Nordheim plot of our experimental results illustrates this phenomena in Figure 3.9. One can observe a change of regime in $1/V = 0.0184$, giving thus $V_{th} = 54.3$ V. Let us note that we used a small correction to be able to calculate $\ln(I/V^2)$. Indeed due to experimental conditions it happened that the first current values were negative. We thus offset I to $I + |\min(I)|$ to be able to plot the FN plot. This has no consequences on the following conclusions.

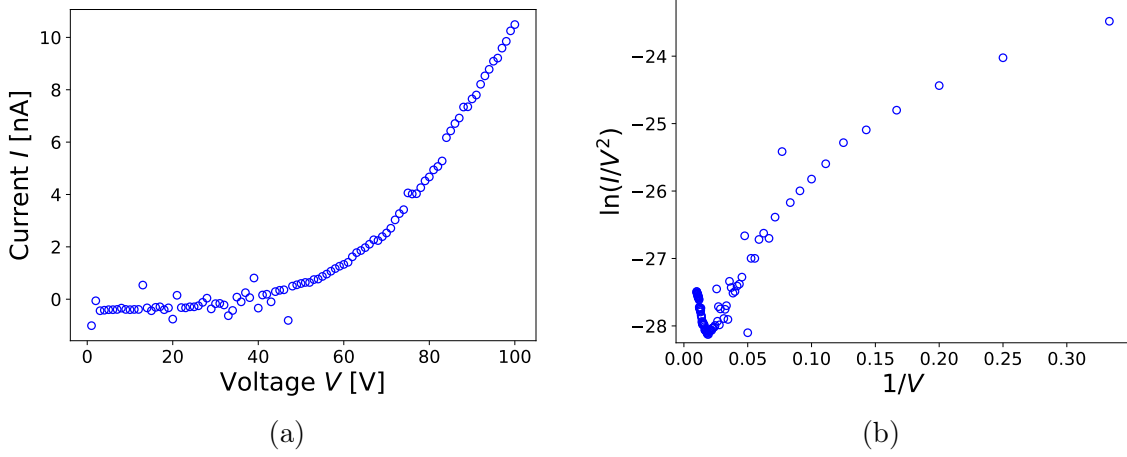


Figure 3.9: (a) $I - V$ curve of the first 0 – 100 V sweep performed (b) Fowler-Nordheim plot of the same data points, i.e. $\ln(I/V^2)$ with respect to $1/V$. One can identify two regimes, namely direct tunneling and field emission.

Simmons proposed a theoretical model allowing to understand qualitatively the transition between tunneling regimes. [77] He derived $I(V)$ formula for junctions presenting similar electrodes separated by a thin insulating barrier. In the low voltage range the potential barrier presents a rectangular shape. The direct tunneling current can be obtained as:

$$I \propto V \exp\left(-\frac{2d\sqrt{2m^*\phi}}{\hbar}\right) \quad (3.2)$$

Where d and ϕ are the width and the height of the barrier, respectively and m^* the effective mass of the electrons. One can easily deduce that $\ln(I/V^2)$ is proportional to $\ln(1/V)$ in this case. [58] If the voltage is further increased the barrier starts to adopt a trapezoidal shape. At a given point ($V > \phi/e$) it even become triangular, given rise to Fowler-Nordheim tunneling. In the regime the current is given as:

$$I \propto V^2 \exp\left(-\frac{4d\sqrt{2m^*\phi^3}}{3\hbar eV}\right) \quad (3.3)$$

It implies that $\ln(I/V^2)$ is proportional to $-1/V$ in field emission. Our experimental measurements fall in perfect agreement with these equations. Strong evidences of field emission and direct tunneling are obtained. In Figure 3.10a a focus is made on the high field part of the FN plot. A linear regression has been performed with excellent R^2 value. As expected a negative slope is observed. In Figure 3.10b the x-axis has been modified

to highlight the logarithmic behavior at low voltage. Again a linear regression has been derived with great R^2 value. The anticipated positive slope appears.

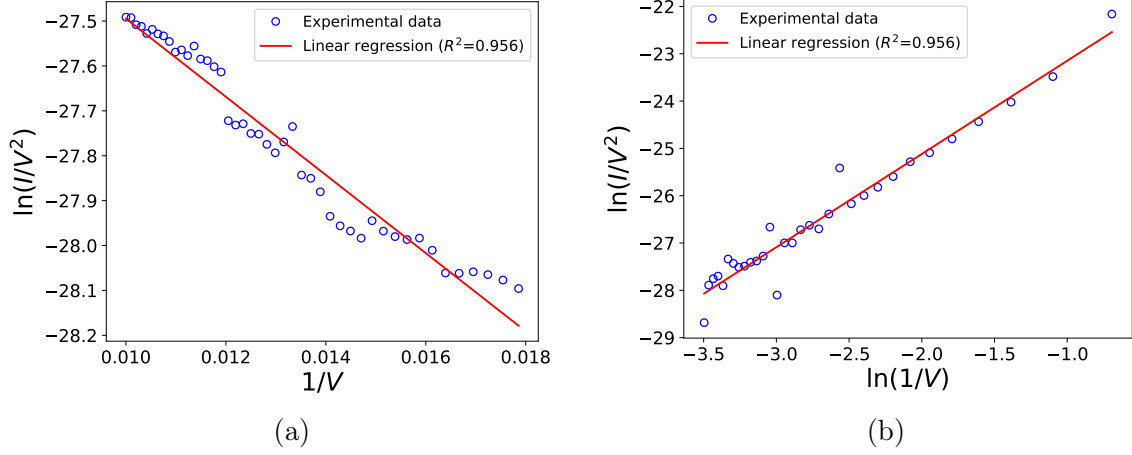


Figure 3.10: Focus at high and low field of the Fowler-Nordheim plot presented in Figure 3.9. A linear regression is performed in both cases. (a) Evidence of field emission at high field. (b) Evidence of direct tunneling at low field. The x-axis was modified to $\ln(1/V)$ to highlight logarithmic behavior.

One should notice that these tunneling effects can only be observed due to important initial resistance of the sample. In fact, starting from the second sweep where conductive filaments are supposed to already have started to be created the FD plot show less clear evidences of tunneling. We propose then a two-contribution electron transport in our NW network. The first one is tunneling between gaps in the sample while the second corresponds to classical conduction through NW contacts. At first very few connections would exist, due to our low packing factor and breaks caused by plasma etching. We suppose that after multiple sweeps tunneling did not disappear but it become negligible compared to classical ohmic conduction.

3.3.3 Low-resistance regime

After the four initial scans analyzed in previous sections, multiple scans have been performed as explained in Section 3.2.3. Those aimed to change radically our sample behavior. As it will be presented this objective has been fully achieved. The maximum current values obtained after this procedure are more than three orders of magnitude greater compared to the one presented in Table 3.1. Such ON/OFF state transition has been observed in many other results involving neuromorphic networks. [8, 10, 78]

The four $I - V$ curves consecutive to this treatment are exposed in Figure 3.11. On the contrary to former results we also exposed the sample to a negative voltage ramp. The purpose of this was to seek for any impact of the current polarization on the network response. Such effect has not been observed. The maximum absolute voltage bias was also lower, i.e. 50 V. Even considering this diminution one can directly notice a complete change in the range of currents obtained compared to Figure 3.7. A much

lower resistance regime is noticed ('ON' state). This difference is in no case comparable to the change of resistance between successive sweeps within a same regime. In Table 3.1 ('OFF' state) a $\sim 3 : 1$ ratio could be observed between $R_{\min}$ of scans n°1 and 4. Here a current of hundreds of μA is measured, compared to only tens of nA previously. This tremendous switch could be explained by the apparition of filaments presenting extended lifetime, much larger than the experiment timescale. [8] This would confer new properties to the system, in particular improved conductance. This behavior is often associated to a nonvolatile memory which could be used in data storing applications. [10] Stable bridging could maintain new connections paths for the current within the network. In facts the shape of the filament would influence greatly its lifetime. Spontaneous instability is supposed to occur more easily for smaller diameter filaments, as studied by Wang *et al.* [57] Repetition of voltage sweeps could create larger and thus more robust connections between Ag NWs. It should also be noted that strong local field enhancement could appear due to our sample geometry, facilitating this process. [8] Irreversible CF could present extended retention time, up to years. Another possibility, quite similar in principle, could be local electrowelding of Ag NWs by Joule effect. This would create persistent junctions between them, as explored by Song *et al* and Vafaei *et al.* [55, 56]

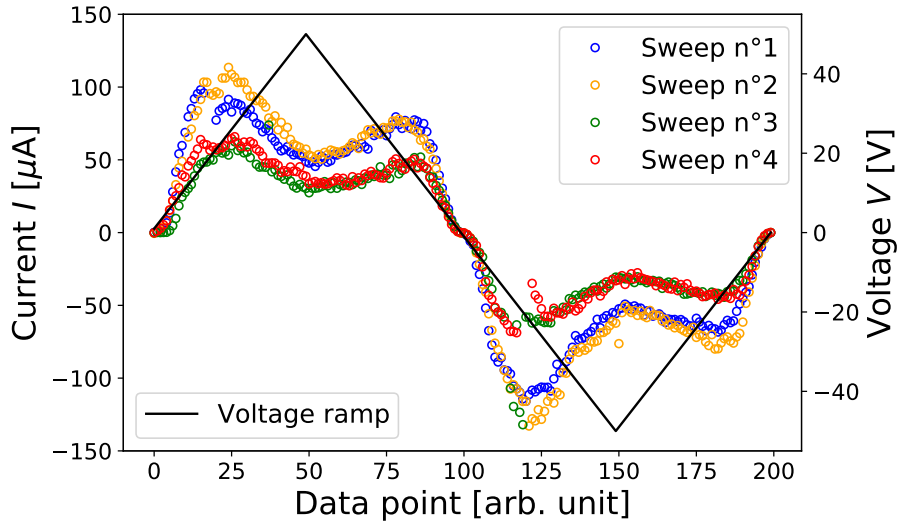


Figure 3.11: Current I as well as the voltage ramp with respect to the data point index. Each data point corresponds to a 1 V increment (or decrement) with a 1s timestep. Current is first rising with increasing voltage. After it decreases until the maximum bias is imposed. A similar jump is noticed in the descending part of the voltage ramp. Same results are observed for reversed polarization.

Following this preliminary discussion on the switch from high to low resistance regime, a particular focus can be made on the behavior of the four sweeps presented. Again strongly nonlinear curves are presented. However very exotic behaviors can be pointed out compared to the OFF state of Section 3.3.1. At first current seems to rise with increasing voltage. A first local maximum appears at roughly half of the ascending part of the voltage ramp. Then current decreases for the second half, which is not ex-

pected at all. As far as the descending part is concerned I starts to grow again to reach a second local maximum. The amplitude of the latter is smaller than the first one. On the contrary to previous results, the general trend tends to a diminution of the current with consecutive sweeps. All previous observations are valid for a reverse polarity, i.e. the negative voltage ramp. Up to now we found no convincing physical explanations for these peculiar phenomena. Our very specific sample design makes any comparison with works from literature difficult. Similar results have never been observed to the best of our knowledge.

The evolution of the sample resistance is depicted in Figure 3.12. Values in the $M\Omega$ range are obtained. One can remark great agreement between both polarizations. Bell curves can be observed. At first the resistance shows a minimum value. Then it rises progressively to a maximum, matching with the maximum voltage bias. After this R diminishes to find back a value close to the one seen at the beginning. Again finding that the resistance curves follow the amplitude of the voltage ramp was not expected. In addition lower conductance is monitored for sweep n°3 and 4 compared to 1 and 2. For voltage close to 0 V a jump in resistance is also noticed. Values up to tens of $M\Omega$ can be obtained.

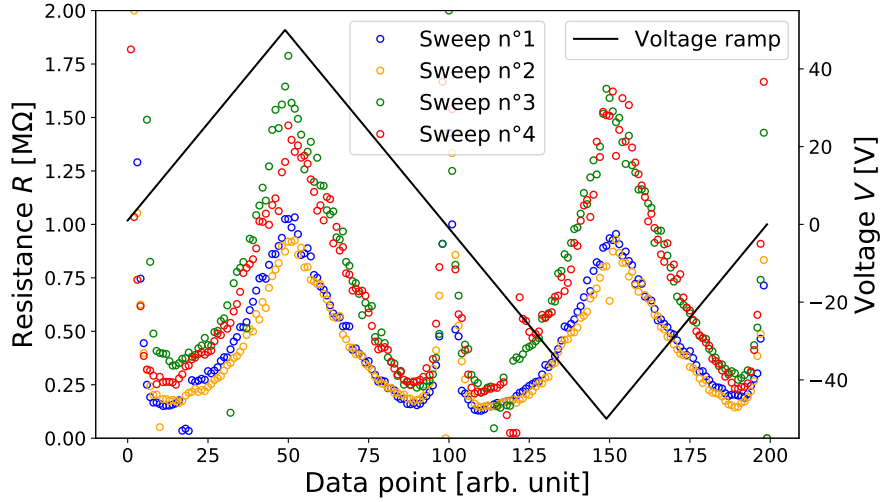


Figure 3.12: (a) Sample resistance R as well as the voltage ramp with respect to the data point index. Each data point corresponds to a 1 V increment (or decrement) with a 1s timestep. Bell shape curves are obtained for the resistance.

The minimum and maximum resistance for both positive and negative voltage ramps can be found in Table 3.2. Only the transition regions have been considered, so data points close to $V = 0$ have been excluded from the calculations. Moreover the minimum resistance values are generally located outside of the general trend of the curves, at lower values. In any other system we would have considered those values as outliers. However the dynamic and complex behaviors displayed by our sample justifies to keep these. One can see good consistency between the two polarizations. As discussed previously both minimum and maximum resistance values are higher for later sweeps than for the firsts.

Up to two orders of magnitude can separate $R_{\min}$ and $R_{\max}$ within the same scan. Again compelling reasons justifying these exotic observations have yet to be found.

	$R_{\min}^{\text{fwd}}$ [M Ω]	$R_{\min}^{\text{bwd}}$ [M Ω]	$R_{\max}^{\text{fwd}}$ [M Ω]	$R_{\max}^{\text{bwd}}$ [M Ω]
Sweep n°1	0.034	0.129	1.033	0.955
Sweep n°2	0.052	0.144	0.925	0.929
Sweep n°3	0.119	0.047	1.788	1.634
Sweep n°4	0.249	0.024	1.463	1.622

Table 3.2: Minimum and maximum resistance presented by the sample in the transition regions. Those have been considered between data points from 5 to 95 and 105 to 195. The acronym fwd (resp. bwd) refers to positive (resp. negative) voltage ramp. Good agreement is obtained between both polarizations.

3.4 Conclusion and perspectives

Track-etched polycarbonate membranes were filled with silver crossed nanowires by electrodeposition. A plasma etching step allowed to damage partially the network by heat-induced stresses. As a result we postulate that segmented wires were created, separated by tiny gaps. Then $I - V$ curves were obtained by applying voltage sweeps. At first a high resistance regime was obtained. A decrease of resistance was remarked with successive scans as well as a variation of the resistance within a same sweep. Typical memristive hysteresis was also highlighted. We postulate that these effects are due to the creation of field-induced conductive filaments in the gaps separating the wires. New connections in the network are therefore produced, leading to an improved conductance. These effects, at first reversible, were made permanent by a series of several sweeps. A new regime, of low resistance, was then obtained. The resistance decreased by more than three orders of magnitude compared to the first sweeps. In this state, very peculiar behaviors were observed. The calculated resistance was maximum for highest voltage amplitude. No valid physical explanation was found up to now to understand these phenomena.

This experimental section has been started at the very end of this Master thesis. Due to limited time, we thus only focused on one sample. Even if interesting results have been obtained, many future improvements could help to understand part of the complex behavior displayed by our network. In fact we could play on several parameter to capture different properties. At first we could compare the curves obtained for samples processed in the exact same condition, i.e. pore diameter, electrodeposition, etching. As explained previously when considering track-etched membranes, one must take into account their statistical parameters. In our case, very small differences between samples could have important consequences on their properties. Comparison between 'similar' samples could then show in which degree their behavior are reproducible. In addition to these basic tests we could use nanowires of different diameters. As explained previously the conductive filament lifetime is supposed to be dependent on its aspect ratio, and thus on its diameter. As a consequence membranes with larger wires could present more rapidly irreversible bridging. In addition the experimental conditions could be

improved. Taking into account the extremely small currents measured, any perturbation could interfere with the results. For example placing our sample holder under a bell could prevent the possible effects of air turbulences. Finally, we should control more precisely the time between successive sweeps. As our samples present dynamic effects and relaxation of the conductance with time, we could study the effects of waiting time between scans.

Conclusion

The Master thesis has explored the hardware implementation of neural networks through two of its main components. Spin-transfer vortex oscillators (STVOs) and segmented Ag nanowire (NW) networks were investigated as artificial neurons and synapses, respectively. This work is in line with the neuromorphic computing approach. The latter consists in mimicking biological brain functions to resolve complex machine learning tasks with high energetic efficiency.

Spin-transfer vortex oscillators (STVOs) were first studied. An analytical model describing the vortex dynamics under an out-of-plane spin-polarized current was first developed, based on Thiele equation. Three potential energy components were retained, namely the exchange, the magnetostatic and the Oersted-field energy. Unlike existing models, generally valid only for small vortex core displacements, high order non-linear terms were taken into account. This implies that the different potential energy contributions are functions of the reduced core radius. Initially a perpendicular polarizer was assumed. This allowed to consider only the perpendicular Slonczewski component as the force associated to the spin-transfer torque. From this, a homogeneous system of linear first order differential equations was derived. The latter allows to calculate the evolution of the vortex core position with time. An analytical expression of the precession frequency was found from these developments, as well as the critical current density. This threshold value corresponds to the minimum current required to obtain a nonzero steady radius of oscillations. A more complex situation was explored after these initial derivations. Indeed a tilted polarizer is required for spin-diode effect to occur. Moreover, a small planar magnetic field is generally applied in practical implementations. A nonhomogeneous part has then been added to the system. It renders the impact of the planar Slonczewski and field-like torques as well as the magnetic field. In addition a translated vortex core equilibrium position has been found.

These analytical results were compared to micromagnetic simulations, using mumax³ solver. The studied system was a nanopillar made of permalloy of radius and thickness equal to 100 and 10 nm, respectively. A current density range of 5 to 10 MA/cm² was explored. The steady-state radii were obtained by fitting the transient free layer dynamics to a decreasing exponential functions. The SNIFA technique was used to calculate the vortex frequency. A strong impact of the Oersted field on the dynamics was demonstrated, as the critical current shows a dependence on the chirality. For vortex radii above ~ 0.8 a polarity reversal was noticed, leading to damping. Concerning the frequency, two regimes were identified. Below the critical current it evolves linearly with the current density. Above it nonlinear gyrotropic precessions are observed. Those

are caused by the evolution of all parameters influencing the dynamics with the oscillation radius. Again a splitting is observed due to the Oersted field. Finally, based on simulation results, a fitting of the theoretical model has been performed, making it semi-analytical. It allowed to correct the expression of the damping and gyrotropic term. This led to a more precise description of the dynamics and to a refined evaluation of the critical current. These adjustments allowed to observe an excellent agreement between simulation results and our analytical predictions, both in transient and steady-state regime.

Finally we analyzed experimental results of spin-diode effect in nanofabricated magnetic tunnel junctions. Those data were provided by INL, a partner research laboratory. These curves were compared to our model predictions, for comparable parameters. Excellent agreement was found, proving that our model is able to describe qualitatively complex STVO properties. Moreover explanations were given on some peaks observed at particular frequencies in the spin-diode voltage. Those correspond to a partial synchronization between the vortex frequency and harmonics of the input signal. In addition we remarked how our method could speed up spin-diode calculations compared to micromagnetic simulations, i.e. up to several orders of magnitude. Eventually potential improvements which could be added to our model were mentioned.

The second part of the work was dedicated to the study of a memristance device. The latter is based on a segmented Ag nanowire network, deposited in a polycarbonate membrane. Following a method developed at UCLouvain a thin polymer film presenting nanopores of 30 nm was filled with metallic silver. The membrane, previously track-etched after ionic bombardment, was provided by the it4p company. Electrodeposition in potentiostatic mode was used to fill the membrane. As the pores present an angle of $\pm 25^\circ$ with respect to the normal to the film, the wires are interconnected. However few connections are created as the packing factor is of less than 1%. After this, a plasma etching step was carried out. First it allowed to separate the Au cathode, which had been sputtered before deposition to allow the reduction of metallic ions, in two distinct electrodes. In addition it damaged to NW network due to stresses caused by the heating of the membrane. As a result, small gaps have been created in the wires which are thus fragmented.

After this four first voltage ramps up to 100 V were applied to the sample. Very low currents were observed, i.e. nA range. A memristive opening was obtained in the $I - V$ curves. Moreover the sample resistance decreased with successive scans. Remanence effects were also noticed. In addition the sample resistance decreased within a same scan, for increasing bias. All these phenomena were attributed to the formation of field-induced conductive filaments in the gaps separating Ag NW segments. Those would result from surface ionic migration, creating new path for the current to flow. However this bridging is reversible due to Rayleigh instabilities but would be facilitated by consecutive scans. In addition evidences of electron tunneling were provided for the first scan. It can only be observed due to the important original resistance value, i.e. several G Ω . Gaps between wires would act as energy barrier across which electron have a given probability of tunneling. At low bias direct tunneling is observed. At higher

voltage, a switch to a field-emission regime is noticed. Those effects were remarked thanks to a Nordheim-Fowler plot and explained by a deformation of the energy barrier with bias.

Consecutively to these initial scans, tens of sweeps were applied to the sample. This aimed to induce modifications of the sample properties, again supposed to be caused by conductive filament formation. An analysis on four final sweep, ranging from -50 to 50 V allowed to highlight a dramatic change in resistance regime. Three orders of magnitude higher conductance was noticed. This is supposed to be caused by the apparition of persistent filaments presenting extended lifetime. No effect on the current polarization was observed in the curves. On the contrary to the high resistance regime, very peculiar behavior was observed in the current and resistance evolution. For maximum voltage amplitude the current was much lower than for intermediate bias. Moreover the resistance seems to be enhanced by increasing voltage and to be at its minimum value for bias near 0 V. Up to now, no convincing explanations were found to explain these phenomena.

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