

École polytechnique de Louvain

Numerical study of the aerodynamics and wakes of VAWTs in complex situations: double-rotor and offshore single rotor

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List of Symbols

The next list describes several symbols that are used throughout the body of the document.

Physical parameters

		q_0	Free stream pressure $0.5\rho U_\infty^2$	$[Pa]$
		D_h	Heave disc diameter	$[m]$
		D_c	Upper column diameter	$[m]$
c	Chord length	$[m]$		
D	Rotor Diameter	$[m]$		
R	Rotor radius $R = D/2$	$[m]$		
x	Transverse direction	$[m]$		
y	Spanwise direction	$[m]$		
z	Streamwise direction	$[m]$		
U_∞	Free stream velocity	$[\frac{m}{s}]$		
u	Transverse velocity	$[\frac{m}{s}]$		
v	Spanwise velocity	$[\frac{m}{s}]$		
w	Streamwise velocity	$[\frac{m}{s}]$		
ω	Rotational speed	$[\frac{rad}{s}]$		
RPM	Rotational speed	$[\frac{rev}{min}]$		
α	Angle of attack	$[deg]$		
α_0	Inwards pitch angle	$[deg]$		
θ	Azimuthal angle	$[deg]$		
ρ	Fluid density	$[\frac{kg}{m^3}]$		
ρ_w	Water density	$[\frac{kg}{m^3}]$		

Non-dimensional numbers

Re_D	Diameter based Reynolds number
C_z	Streamwise load coefficient
C_t	Tangential force coefficient
C_n	Normal force coefficient
C_d	Column drag coefficient
C_a	Column added mass coefficient
Re_c	Chord based Reynolds number
λ	Tip Speed Ratio $\frac{\omega R}{U_\infty}$
TSR	Tip Speed Ratio $\frac{\omega R}{U_\infty}$
AR	Aspect ratio $\frac{H}{D}$
σ	Solidity $\frac{Nc}{D}$
N	Number of blades
n	Number of bodies
C_x	Transverse load coefficient

Introduction

The world energy mix is facing one of its greatest transition since the end of the industrial era. While 10 years ago, nuclear, coal and natural gas power plants provided up to 80% of Europe's total power generation, the share of renewable energies -namely hydro, solar and wind power- has almost doubled from 9% in 2005 to 16.7% in 2015. The target of 20% established by the European Union (EU) by 2020 is within reach [1].

Economic growth and power consumption have always gone along. This trend is not ready to settle yet and has lead to an overwhelming contribution to the greenhouse effect. Over the past few years, climate change has become one of the most worrying subjects for our society. Key challenges for the 21st century will be the mitigation and adaptation to this global warming. With the question of energy production at the core of those challenges, the European Union ratified both the Paris Agreement and the European Energy Package, extending the share of renewable energies to 27% by 2030. The REmap analysis conducted by IRENA indicates that, with current and future technologies, the EU could even ambition a share of 34% by the same year [1, 2, 3]. The energetic future appears technically challenging, and the urge to meet these requirements will definitely give some research momentum in renewable energies. Among the latter, wind energy is one of the fastest-growing renewable energy technology, and offers a very low carbon intensity per energy consumed ratio [4].

In order to respond to the increasing demand and enhance the power output, horizontal-axis wind turbines (HAWTs) have never ceased to grow in size. However, wind energy is weather and location dependent. Onshore regions of high wind velocities and very uniform winds have seen wind farms sprout, reducing the left available area, while offshore regions, where space is not an issue, tackle other technical challenges. Innovative turbine designs are needed to efficiently extract power from wind, along with research in the practical implementation and infrastructures of far offshore -or farshore- wind farms.

In that sense, recent studies have shown a renewed and growing attention towards vertical-axis wind turbines (VAWTs). Their aerodynamics is inherently unsteady, which explains the underdevelopment of proposed models for VAWTs and an intermittent research [10]. Wakes from VAWTs indeed results from more complex mechanisms compared to HAWTs, with a strong interaction between the forming wake and the blades



Figure 1: The TwinFloat concept by Nenuphar: two closely spaced vertical-axis wind turbines on a single semi-submersible platform. Credits [63].

in the downwind part of the turbines. However, VAWTs have regained interest as they offer certain advantages. Among them, one can mention the insensitivity to the wind direction, the vanishing of the yawing actuation and the ability to place the drive-train at the base of the turbine, leading to more stability. Also, VAWTs heights are not limited by practical considerations, as it is the case for their horizontal counterparts, where turbine's size and blade's radius are directly proportional. Moreover, the promises of higher power extraction densities and faster wake recoveries have motivated many researches on wind farms composed of closely spaced VAWTs. Mutual interactions between rotors are said to result in more intense wake decay mechanisms and enhanced power extractions [6]. Besides, new types of floating platforms have been designed to lower the swell while reaching greater ocean depth and larger distances to shore. The semi-submersible platform type offers little torque and yaw displacements, and seems to be the easiest to install in very deep water [41]. A promising concept has been proposed by *Nenuphar*, combining two closely spaced vertical-axis wind turbines and a single semi-submersible platform, shown in figure 1.

The present work concerns a complete numerical study of the vertical-axis wind turbines in two complex situations: double-rotor and floating offshore single rotor. Two numerical tools, both developed at UCL, are used for this purpose. The aerodynamic solver *VPM*, is an efficient lifting-line-enabled vortex particle-mesh method. The multibody solver, *ROBOTRAN*, is a symbolic generator of the equations of motion and is used to study the interactions between turbines and platforms.

We therefore identify two main objectives for this Master's thesis:

1. Elucidating the gains offered by closely-spaced vertical-axis wind turbines through the analysis of their aerodynamics and wakes.

2. Clarifying the influence of the hydrodynamic loads of a semi-submersible platform on the performances of a vertical-axis wind turbine.

This document is articulated as follows. Chapter 1 provides a rapid description of the numerical tools. Chapter 2 presents the double-rotor concept, and applies a methodical two-dimensional analysis on its aerodynamics and wakes. Chapter 3 extends the former analysis to three dimensions. Finally, Chapter 4 exposes the effects of the aforementioned platform's loads and motions on the aerodynamics and wakes of a single vertical-axis wind turbine. Conclusions as well as perspectives for possible further research are discussed in the very last part.

Abstract per chapter

Chapter 1: Numerical tools

This chapter presents the numerical tools that were used in this document. It first describes the aerodynamic solver, which consists in a vortex particle-mesh (VPM) method. Thereafter, the symbolic software used to generate the equations of motion and analyze multibody systems (MBS) is presented. For both tools, the underlying equations and methodology are exposed and briefly explained. Finally, the coupling between the aerodynamic solver and the multibody system solver is introduced.

Chapter 2: Double-rotor vertical-axis wind turbines: a 2D study

This chapter focuses on the aerodynamic and wakes of the double-rotor VAWTs by performing a two-dimensional analysis. First, the double-rotor concept is explained and its different layouts are studied. Thereafter, the effect of turbine inter-distance is investigated. Finally, the link between turbines performances and wind inflow directions is assessed.

Chapter 3: Double-rotor vertical-axis wind turbines: a 3D study

This chapter extends the former analysis to a 3D one. Its first part analyses the performances of the different double-rotor configurations. The second part provides the reader with insights of the effect of the number of blades. The third and last part discusses the influence of the aspect ratio on the wake behaviour and wake recovery.

Chapter 4: Floating offshore wind turbine

This chapter concerns the study of offshore vertical-axis wind turbines. A full floating platform and its vertical-axis wind turbine are implemented in the symbolic software, along with a statistical description of ocean waves. The setup permits all possible displacements and orientations. The chapter presents the resulting aerodynamic and hydrodynamic forces acting on the platform and turbine. It thereafter describes the influence of the wave's motion and loads on the performances.

Chapter 1

Numerical Tools

The era of data processing and computers has completely changed the tools engineers use to explain and mostly predict their environment. New sciences and theories have been developed in the last 50 years, presenting the complex behaviours of numbers, numerical schemes and data acquisition. Numerical computation has been beneficial: its use in most scientific fields has enabled to apprehend physical phenomena where analytical solutions were not provided, or very time-consuming to obtain.

However, the excessive use of these new computational resources comes with several drawbacks. As the availability of automated software increases, people are less prone to questioning their results or their feasibility. Unfortunately, simulation results can be quite remote from the truth. Furthermore, their ease of use makes the user less aware of the real physics behind a phenomena. Numerical computation is a powerful tool, but is limited by the input of the users. A single ill-calibrated parameter can deviate the results far from the reality. Therefore, a thorough comprehension of the models and numerical schemes behind a software is essential to fully develop their potential.

This chapter aims to provide the reader with an overview of the numerical tools used throughout this document. First, the aerodynamic solver, a vortex-particle mesh (VPM) method, will be presented. Then, a symbolic generator for solving multibody systems will be introduced. Finally, the coupling between both software will be explained.

The interested reader is referred to [11, 12], [14] and [16] for deeper details about the flow solver, and [49, 50] for the symbolic generation of multibody systems.

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1.1 Aerodynamic solver

Two of the fields that most benefit from numerical computations are undoubtedly the fluid dynamics and aerodynamics of flows. Demonstrating the existence and unicity of the solution of the Navier-Stokes equations is still one of the *millenium problems*. Many models suitable for numerical computations have since been proposed to approximate the physics of flows, and how they evolve around bodies.

The flow solver used in this work is being developed by the Thermodynamics and Fluid mechanics Lab (TFL) at the Université Catholique de Louvain (UCL). Running on massively parallel architectures (MPI), it combines two approaches :

- *Vortex particle* methods, based on the Lagrangian discretization of the vorticity-velocity formulation of the Navier-Stokes equations. These methods are well suited for convection dominated vortical flows, such as those presented in this work. However, they are prone to inaccuracies because of the Lagrangian distortion phenomena.
- *Mesh-based* methods, enabling the efficient numerical computation of differential operators and optimized solvers for the Poisson equation.

The combination allows for highly efficient computations of incompressible and unsteady vortical flows as in the present work. Finally, the VPM method accounts for slender lifting bodies in the flow by means of immersed lifting-lines. The latter operate as vortex generators immersed in the flow.

1.1.1 The Navier-Stokes equations

The Navier-Stokes equations are vectorial partial differential equations (PDEs) describing the conservation of mass, momentum and energy of a Newtonian fluid. Under the assumption of an incompressible flow, the mass and momentum equations are :

$$\nabla \cdot \mathbf{u} = 0 \quad (1.1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} \quad (1.2)$$

where ν represents the kinematic viscosity, a fluid-dependent proportionality factor between stresses and deformation rate. This formulation is called the *velocity-pressure* form of the equations. Another form, the *vorticity-velocity* form, can be obtained by taking the curl of equation (1.2).

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} \quad (1.3)$$

also called the *vorticity transport* equation. $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{u} \cdot \nabla)$ is the material derivative, here applied to the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$. The relation between vorticity and velocity field is primordial, as the latter, after a Helmholtz decomposition $\mathbf{u} = \mathbf{u}_\omega + U_\infty$, may be recovered from the former by solving the Poisson equation

$$\nabla^2 \mathbf{u}_\omega = -\nabla \times \boldsymbol{\omega} \quad (1.4)$$

A direct consequence of the vorticity definition is that its divergence inevitably equals zero.

$$\nabla \cdot \boldsymbol{\omega} = \nabla \cdot (\nabla \times \mathbf{u}) = 0 \quad (1.5)$$

The physical meaning of equation (1.5) is that vortex filaments, *i.e.* connected vortex points of same vorticity, always form closed curves. This is called the *solenoidal* property. This property should be fulfilled by the numerical schemes discretizing the vorticity-velocity form of the Navier-Stokes equations. Choosing one form or another to numerically solve the Navier-Stokes equations leads to different approaches and inaccuracies, as the discretization of the involved operators are implemented differently.

1.1.2 The vortex particle-mesh approach

The VPM method used here combines a vortex particle approach for the advection of vorticity and a mesh for all other differential operators. For the sake of clarity, the method is denoted as VPM, whereas the code used for this work is denoted as *VPM*.

Vortex particle method

A vortex-particle (VP) method discretizes the vorticity field into particles (referred to by subscript $\mathbf{p}$), which are quantities defined by a position $\mathbf{x}_\mathbf{p}$, a volume V_p and a

strength $\alpha_{\mathbf{p}}$. The strength of the particle being the vorticity integral over its volume :

$$\alpha_{\mathbf{p}} = \int_{V_p} \omega d\mathbf{x} \simeq \omega_{\mathbf{p}} V_p \quad (1.6)$$

where $\omega_{\mathbf{p}}$ is the mean vorticity norm over the volume V_p . Dirac distributions of the particles over the field does not preserve the total vorticity $\Omega = 0$, instead, a certain distribution function ξ_p for each particle is needed to compute the vorticity elsewhere, being influenced by all neighbouring particles.

$$\omega(\mathbf{x}, t) = \sum_p \alpha_{\mathbf{p}} \xi_p(\mathbf{x} - \mathbf{x}_{\mathbf{p}}(t)) \quad (1.7)$$

It corresponds to a diffused view of the vorticity field, including the influence of all particles. These particles are free to move in the bulk flow, and their carried vorticity varies with time and position. A set of Ordinary Differential Equations (ODEs) is established knowing a particle is convected by the flow field. The corresponding strength $\alpha_{\mathbf{p}}$ is subjected to vortex stretching and diffusion.

$$\frac{d\mathbf{x}_{\mathbf{p}}}{dt} = \mathbf{u}_{\mathbf{p}} = \mathbf{u}(\mathbf{x}_{\mathbf{p}}, t) \quad (1.8)$$

$$\frac{d\alpha_{\mathbf{p}}}{dt} = \int_{V_p} [(\omega \cdot \nabla) \mathbf{u} + \nu \nabla^2 \omega] d\mathbf{x} \quad (1.9)$$

$$\simeq [(\omega \cdot \nabla) \mathbf{u}(\mathbf{x}_{\mathbf{p}}) + \nu \nabla^2 \omega(\mathbf{x}_{\mathbf{p}})] V_p \quad (1.10)$$

The VP method promises a straightforward implementation and a robust adaptivity, it also affords larger time steps than explicit integration schemes for advection, waiving the classical CFL condition as it avoids the discretization of the convection term. Unfortunately Lagrangian methods lack in accuracy due to Lagrangian distortion. Particles tend to deplete certain regions of the flow and/or regroup in others. Moreover, retrieving the velocity field from the vorticity field through equation (1.5) implies solving an elliptic problem, where the Green's function solution requests more computational resources than optimized mesh-based solvers.

Vortex particle-mesh method

Remeshing has been proposed to avoid the aforementioned problems. It consists in the periodic redistribution of the particles on a mesh via high-order interpolation.

$$\alpha_{\mathbf{q}} = \sum_p \alpha_{\mathbf{p}} W\left(\frac{\mathbf{x}_{\mathbf{q}} - \mathbf{x}_{\mathbf{p}}}{h}\right) \quad (1.11)$$

where the subscript $\mathbf{q}$ stands for locations on the mesh. The used interpolation kernel is represented by W , and h is the mesh spacing. The use of a mesh and particles forms a hybrid discretization and is the core of VPM methods.

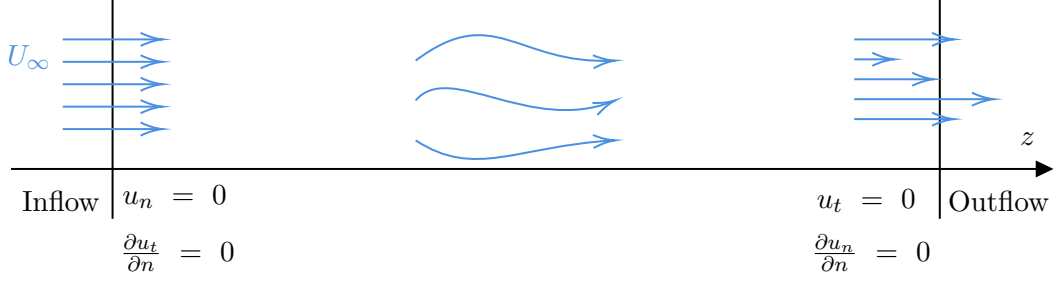


Figure 1.1: Boundary conditions : Inflow and outflow conditions imposed.

The presence of the mesh is further exploited to improve computational efficiency, with higher time steps than for purely Eulerian methods. Once particle-to-mesh interpolation was performed using high-order interpolation kernels, standard Finite Differences discretization schemes can be used on the mesh to evaluate the field quantities and differential operators.

$$\omega_{\mathbf{q}} = \sum_p \frac{\alpha_{\mathbf{p}}}{h^d} W\left(\frac{\mathbf{x}_{\mathbf{q}} - \mathbf{x}_{\mathbf{p}}}{h}\right) \quad (1.12)$$

d being the dimension of the interpolation method. The mesh is also used to efficiently solve the Poisson equation instead of the Green's function solution, by use of fast Poisson solvers. VP methods are not divergence-free, the spurious divergence has to be treated to reconnect the vortex lines. The solenoidal condition on the vorticity field is explicitly enforced through a relaxation in the Fourier space. Finally, mesh quantities can be interpolated back onto the particles locations. The back and forth interpolation forming the communication channel between mesh and particles is one of the foundations of the presented VPM method. The time-integration is finally performed using a low-storage third-order Runge-Kutta scheme (RK3).

The VPM code used here enables both periodic and unbounded boundary conditions, which are exactly solved using Fast Fourier Transforms (FFT) on an extended domain. The use of unbounded conditions in directions transverse to the incoming flow as in the present work enables leakage from the sides of the domain. The streamlines are therefore not constrained between the domain limit and possible bodies in the flow. Periodic conditions permit to adapt the initially three-dimensional solver into pseudo two-dimensional domains in the transverse directions. The inflow and outflow conditions are imposed in the streamwise direction, denoted as z , as shown in figure 1.1.

1.1.3 Accounting for bodies : immersed lifting-lines

The VPM method being developed at the UCL-TFL may take into account the effect of lifting bodies in the flow. It is commonly used for flows around slender bodies as airplane blades and wind turbine blades. To model the slender bodies, immersed lifting-lines are used as sources of vorticity. The lifting-line model was proposed by Ludwig Prandtl in

1918 to predict the lift distribution along a finite wing, without solving the Navier-Stokes equations around the wing sections themselves.

3D aerodynamics of slender bodies and Prandtl's lifting-line theory

Any body immersed in a flow generates a wake, and vorticity at its surface. Vorticity is related to the circulation Γ , which is the contour integral of the velocity

$$\Gamma = - \oint_{\partial S} \mathbf{u} \cdot d\mathbf{l} = - \int_S \boldsymbol{\omega} \cdot \mathbf{n} dS \quad (1.13)$$

The strength of an infinitesimal wing section attached vorticity is quantified by the bound circulation around the airfoil, and is related to the lift per unit span L by the Kutta-Joukowski theorem under the assumption of steady flows

$$L = \rho U_{\text{rel}} \Gamma \quad (1.14)$$

where U_{rel} is the relative velocity as seen by the airfoil section and ρ the fluid density. Shrinking the surface but keeping the circulation value, we can define a vortex filament, grouping adjacent points of equal circulation. The second Helmholtz theorem stipulates that such a filament may not vanish spatially. It is either sent to infinity, or forms closed paths.

The idea behind the lifting-line is, regarding the above relation, to model a wing with a bound vortex filament, being a material line going through the quarter-chord line of each wing section. For a constant circulation, one gets a *horseshoe* vortex. But the finite aspect of a wing does not enable lift production at the tip of the blades, and hence circulation must drop to zero at both ends. In between, lift is produced and circulation takes a non-zero value. A finite wing may be seen as infinitesimal airfoil sections next to each other, where the strength of the vortices associated to each section composes a circulation distribution along the span of the wing. The result is an infinite superposition of *horseshoe*-like vortices. Each free vortex filament composing the *horseshoes* forms a vortex sheet, shed in the flow behind the wing. A schematic is provided in figure 1.2.

For a fluid initially at rest, there is no free stream velocity, no circulation around the wind and hence no lift. From a Lagrangian point of view, Kelvin's theorem stipulates that the material derivative of circulation is conserved, and equals 0: $\frac{D\Gamma}{dt} = 0$. As a consequence, any temporal variation of the circulation distribution on a accelerating wing, say $\Gamma(t + \Delta t)$, introduces in the flow a starting vortex of strength $\Gamma(t + \Delta t) - \Gamma(t)$. The kinematics of the body and its orientation within the flow induces both temporal and spatial variations of the circulation. The distribution evolution across the span of the slender body results in a complex shed vortex sheet.

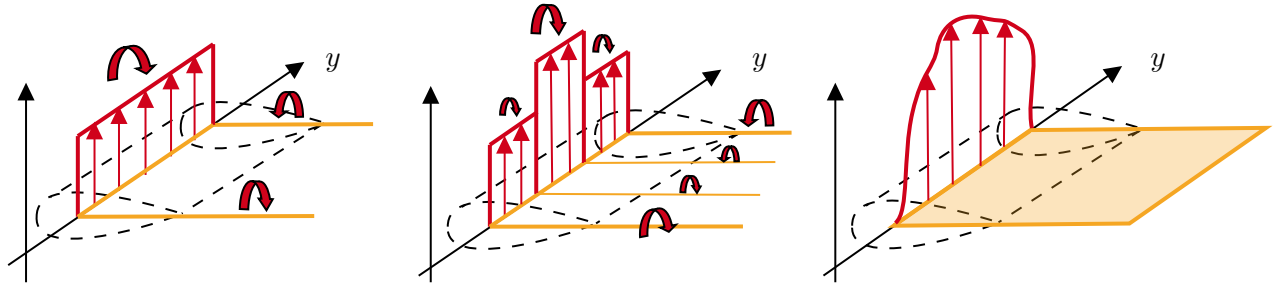


Figure 1.2: Visualization of the lifting-line theory from left to right : horseshoe vortex, superposition of two horseshoe-like vortices, infinite superposition of horseshoe-like vortices. The circulation is given in red, while the vortex filaments are in yellow. The resulting vortex sheet is the yellow shaded area. Final circulation distribution is not representative of any existent one.

Lifting-lines as sources of vorticity

The lifting-line theory merged with the Kutta-Joukowski condition enables to write down a compatibility equation for the circulation across the span. The latter equation permits to predict the wing performances when imposing the circulation distribution. Here, the equations are used in another way. Polars of the airfoil used to model the slender body are considered as an input. Lift and drag coefficients, C_L and C_D , are given along with the local chord based Reynolds number $Re_c = \frac{\rho U_{rel}}{\nu}$ for each angle of attack α . The value of lift on each wing section gives the circulation distribution associated $\Gamma(y, t)$ at each time step through the Kutta-Joukowski condition. Finally, the vortex sheet can be retrieved with the solenoidal property of ω . Even though the Kutta-Joukowski condition is classically derived for two-dimensional potential flows, it is also valid for a separated flow in a time-averaged sense [17].

The trailing vortices have to be added to the flow. This is done via Lagrangian tracers positioned along the lifting-line. They construct in the bulk flow a singularly thin vortex sheet by means of vortex particles captured over a single mesh point. This singularity is then handled with a mollification process: vortex particles are added to smoothen the singular strength in the thickness direction. The use of tracers enables to compute the shed vortex sheet at any time step, especially for a multistep time integration scheme. The shed vorticity is finally interpolated onto the existing particles in the bulk flow through a particle-to-particle interpolation.

Diagnostics of a lifting body

In VPM, lifting bodies are defined as a set of lifting-lines. Each of them has a specified polar, geometric dimensions and orientation with respect to the body-defined frame. The code accounts for multiple bodies in the flow, where their position and orientation are defined with respect to the inertial frame. Also, kinematics may be applied independently

to the bodies, by specifying, for example in the case of wind turbines, the angular velocity vector. Lifting bodies disturb the flow by means of the lifting-lines attached to it.

Aerodynamic performances, as power and torque, are finally given as outputs to the user. This comes with data at every time step of the diagnostics and performances of the lifting-line sections, as the positions of the blades $\mathbf{x}(t)$, the local angle of attack $\alpha(\mathbf{x}, t)$ and the local lift and drag forces respectively $L(\mathbf{x}, t)$ and $D(\mathbf{x}, t)$. A single structure is given per body, enabling the efficient post-processing of the results.

1.1.4 Large Eddy Simulations

The flows of interest are typically complex turbulent flows at high Reynolds numbers. Large Eddy Simulation (LES) is a mathematical model for turbulence. It modifies the Navier-Stokes equations via low-pass filtering. This greatly reduces the computational resources needed. However, the effect of the small scales on the resolved ones then has to be modeled. If these small scales are assumed to be isotropic, a simple model can be applied. In the code, a sub-grid scale (SGS) model is implemented to account for the unresolved scales, through an eddy viscosity model acting only on the very small-scale fields. Equation (1.3) is adapted to

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \cdot \mathbf{T}^M \quad (1.15)$$

$$\mathbf{T}^M = \nu_{\text{SGS}} \left(\nabla \boldsymbol{\omega}^s + \nabla^t \boldsymbol{\omega}^s \right) \quad (1.16)$$

where $\mathbf{T}^M$ is the SGS model, $\boldsymbol{\omega}^s$ the small-scale part of the vorticity field, obtained by high-pass filtering, and ν_{SGS} the eddy viscosity. The latter is evaluated using the complete velocity field. We refer to [14] for more details.

1.1.5 Accounting for VAWTs dynamics

The present work aims at studying the complex aerodynamics and wakes of closely spaced vertical-axis wind turbines (VAWTs). Their aerodynamics is inherently unsteady, because of the constantly varying angle of attack of the blades over a rotor's revolution. The blades are prone to a phenomena called *dynamic stall*, characterized by a delayed lift gain followed by a rapid drop and a massive flow separation. The lift overshoot is caused by high-velocities in the leading-edge shed vortex on the suction side. When the vortex passes the trailing edge, the flow is fully detached and the blade is in normal stall conditions. The account for dynamic stall is made through a Leishman-Bedeoes model. Parameters for the model are tuned and added to the polar information of the lifting-lines.

Finally, the lifting-line model implemented is not able to capture the flow curvature effects. For a blade in rotation through straight streamlines, the airfoil behaves as an

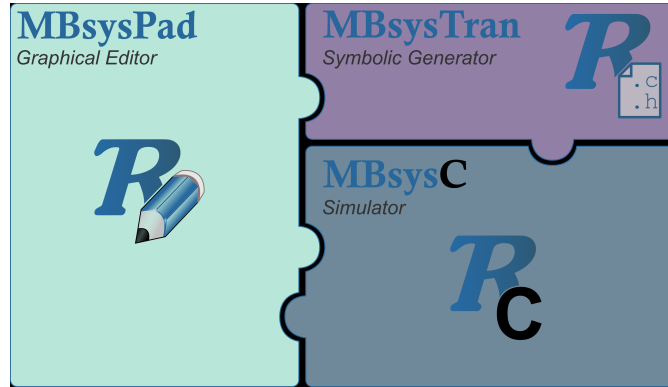


Figure 1.3: Interaction between ROBOTRAN modules [49].

airfoil with an additional camber [30]. To account for this additional camber, the blades are pitched inwards by an angle α_0 :

$$\alpha_0 = \arctan \left(\frac{1 - \cos(\frac{\beta}{2})}{\sin(\frac{\beta}{2})} \right) \quad (1.17)$$

where β is given by $\beta = 2 \arctan \left(\frac{c}{2R} \right)$.

1.2 Multibody system solver

The last part of this Master's thesis is concerned with the behavior of a floating offshore wind turbine as well as the impact of the sea-induced motion on the turbine's wake. With that in mind a multibody system (MBS) software, ROBOTRAN, is coupled with the above explained aerodynamic solver.

The ROBOTRAN environment was developed by the UCL-MEED research group. It generates the multibody system's equations (kinematics and dynamics) via the symbolic approach, a powerful tool to drastically simplify mathematical expressions and confer the equations a high portability.

Three modules compose the software. *MBsysPad* is a graphical editor where the user is able to define his MBS (dimensions of the bodies, type of articulations, ...). *MBsysTran* is the symbolic translator, generating the MBS equations in a few milliseconds. Those are then automatically interfaced with the simulator, *MBsyslab* and *MBsysC*. After the simulations, 3D animations can be performed thanks to the *MBsysPad* editor. For the sake of clarity, interaction between those modules are illustrated in figure 1.3.

The following sections summarize the general topology of an MBS by presenting bodies and joints characterization. Then, it briefly introduces the MBS formalisms and explains

how the dynamic system is solved. Readers interested in further information are invited to refer to [50].

1.2.1 Topology of a multibody system

A general multibody system is a mechanical system composed of N *bodies* connected by *joints*. Those systems can be very simple, hence easily represented in a tree-like structure (the number of bodies is equal to the number of joints, see figure 1.4a). In practise, MBS can become quite complex, containing lots of interconnections (closed-loop structure). In the latter case, the system is temporary simplified into a tree-like structure. Either a body or a joint is virtually cut into two parts (figure 1.4b). Constraints are subsequently added to close the problem.

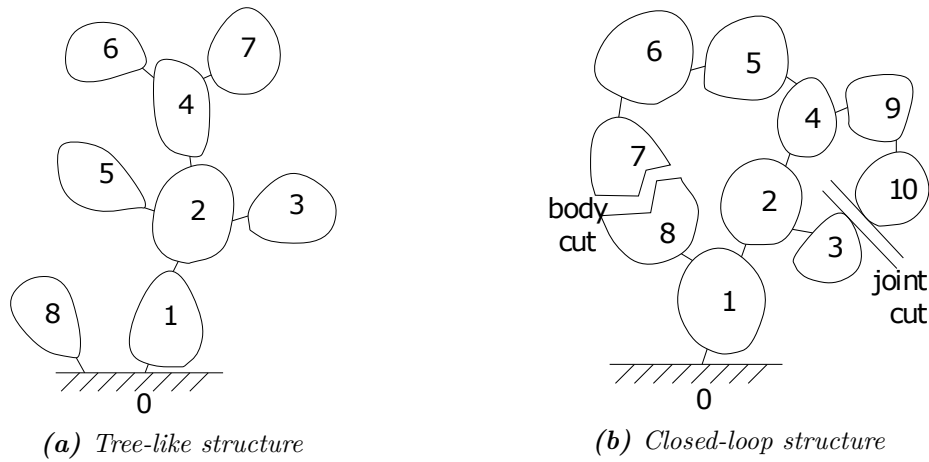


Figure 1.4: System topology [49]

Bodies characterization

In ROBOTRAN, a rigid body i is characterized by a -unique- parent body h and by its children bodies, namely j and k in figure 1.5. By convention, a joint preceding a body will take the same index as this body and any joint i is defined with two anchor points: the first one, O^i , on the parent body h and the second one, O'^i , on the body i [49]. Each body i is described by means of the following quantities:

- the joint vector, $\mathbf{d}^j = [\hat{\mathbf{X}}^i]^t d^j$, which locates the body point O^j of joint j with respect to the connection point O'^i of joint i . As bodies are considered rigid, the components of $\mathbf{d}^j$ are constant in the frame $[\hat{\mathbf{X}}^i]$ of the body i . Joint vectors are defined for each of the children $j, k, \dots$ of body i ;
- the position vector of the center of mass CM^i of body i , $\mathbf{l}^i = [\hat{\mathbf{X}}^i]^t l^i$, expressed in the reference frame of the body, $[\hat{\mathbf{X}}^i]$;

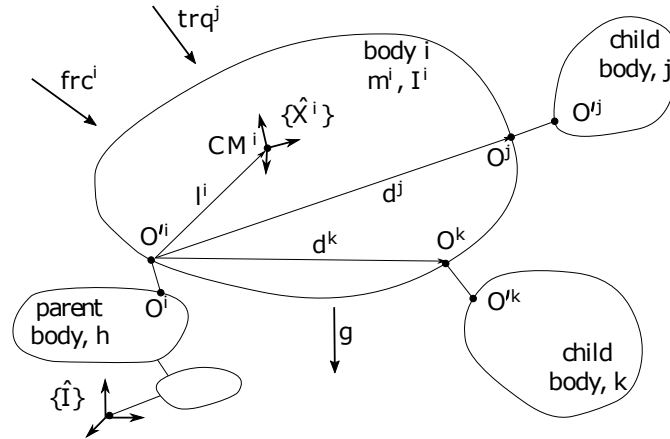


Figure 1.5: Body characterization [49]

- the mass of the body, m^i ;
- the symmetric inertia tensor, $\mathbf{I}^i = [\hat{\mathbf{X}}^i]^t I^i [\hat{\mathbf{X}}^i]$, with respect to the center of mass in the body frame $[\hat{\mathbf{X}}^i]$.

The only way to take into account flexible bodies is to apply a finite segment approach, consisting in replacing the body by a ensemble of N rigid bodies connected by equivalent springs. While incorporation of an elastic tool to investigate the structure's response to aerodynamic and hydrodynamic loads could enhance accuracy of the results, it certainly is beyond the scope of this work. The structure is thus assumed to be rigid.

Bodies are also subject to forces and torques. In ROBOTRAN, they are divided into three categories:

- The *gravitational force* is taken apart in the formalism. It is computed from the gravity vector $\mathbf{g}$ whose three components are defined in the inertial frame and it acts on all the bodies of the MBS.
- The *joint forces/torques* act on a body via joint connections between its parent and its child. Their contribution, expressed Q^i for joint i , to the equation of motion will be described in section 1.2.2.
- The *external forces/torques* represent all other kind of forces acting on a body and gather them into a resultant force vector $\mathbf{frc}^i$ and into a resultant torque vector $\mathbf{trc}^i$ with respect to the center of mass CM^i of the body.

Joint characterization

Only two types of joint are implemented in ROBOTRAN: the *prismatic* (denoted T) joints and the *revolute* (denoted R) joints (see figure 1.6). Both of them only have one

degree of freedom, namely a relative displacement or a relative rotation, but all types of connections can be modelled by a smart combination of those two.

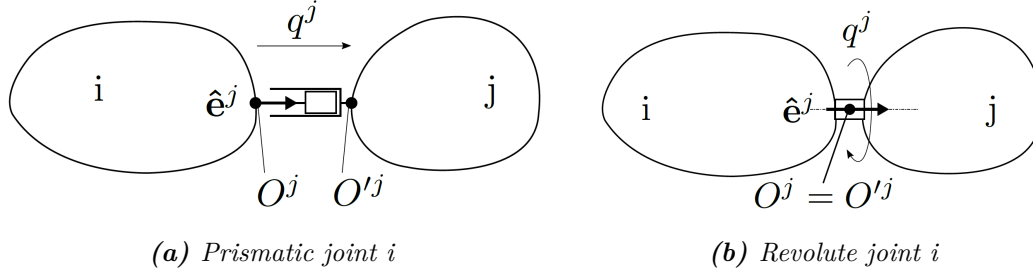


Figure 1.6: Elementary joints [49]

1.2.2 Multibody formalisms

In all the different multibody approaches, MBS direct dynamics is always required. Direct dynamics is defined as the computation of the generalized accelerations $\ddot{\mathbf{q}}$ (joint accelerations) of the system in response to forces and torques. In an explicit form, the problem can be expressed as,

$$\ddot{\mathbf{q}} = f(\dot{\mathbf{q}}, \mathbf{q}, \mathbf{frc}, \mathbf{trq}, \mathbf{g}) \quad (1.18)$$

or in a semi-explicit and more common form,

$$\mathbb{M}(\mathbf{q}, \delta) \ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}, \delta, \mathbf{frc}, \mathbf{trc}, \mathbf{g}) = \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}) \quad (1.19)$$

where $\mathbb{M}$ is the symmetric generalized mass matrix of the system, $\mathbf{c}$ is the nonlinear dynamic vector containing gyroscopic, centrifugal, gravity terms and contributions of external forces and torques. $\mathbf{q}$ represents the relative generalized coordinates whilst $\mathbf{Q}$ corresponds to the associated generalized joint forces and torques.

ROBOTRAN solves the above equations through several formalisms [52]. Among them: the recursive *Newton-Euler* (N-E) method and a symbolic generator. The former obtains the semi-explicit form (equation (1.19)) while the latter generates the explicit direct dynamics model (equation (1.18)).

The N-E algorithm consists in two recursive computations: a forward recursion and a backward one. The first one computes bodies velocities and accelerations going from the base body to the leaf ones. The backward recursion, from the leaf bodies to the base, rewrites the equations of motion of each body and extracts forces and torques acting on each joint required to compute Q , the generalized joint forces/torques. Repeating these steps recursively delivers equation (1.19) in the matrix form.

The symbolic generator is the key characteristic of this MBS solver. It results in high efficiency while maintaining a low computational complexity. Somewhere between a *by-hand* approach and a numerical implementation, it combines advantages of both methods, which are arithmetic and trigonometric simplifications and automation of the computational process. Symbolic generation manipulates analytic expressions, organizes and prioritizes them in a tree structure. Depending on the nature of the expression, this structure is recursively optimized to achieve a minimum number of operations. In addition to removing useless terms, entire equations can be detected as being superfluous. Moreover, final equations are generated only once, but are keeping the same level of generality, giving symbolic approach a great advantage against their numerical counterparts.

Other features are available in ROBOTRAN such as resolving dynamic of constrained multibody systems by means of a coordinate partitioning, inverse dynamics, reaction dynamics, ... but as those are not used in the present work, they will not be presented. Interested readers are invited to refer to [50] and [52, 53].

In conclusion, Robotran is a highly efficient, compact tool, which, coupled with VPM, offers a numerical tool for modeling floating platforms at a moderate computational complexity.

1.3 Coupling of the flow and MBS solvers

A coupling interface between the two previous tools was implemented and validated by S. Buffin. It is currently further developed in TFL [53]. Therefore, this chapter only consists in a small review on the fundamental choices that have been made.

There are two ways to combine simulation tools: either by a *strong* coupling or by a *weak* coupling. Strong coupling generally implies development and mixing of equations, resulting in a new numerical tool, whereas weak coupling implements a junction interface between the two existing codes. The latter method has been chosen, mainly for limiting the number of modifications in the original codes, yet offering a high modularity and leaving the computational time unchanged.

The weak coupling strategy corresponds to a co-simulation of codes running independently and exchanging information. Since non-synchronized evaluations lead to numerical issues, the key challenge of the interface is to provide an efficient communication between simulations. Communication is synchronized by means of a master-slave relation between codes. The leader code, *VPM*, is the one ruling the co-simulation.

Aerodynamic forces computed by *VPM* are sent to ROBOTRAN, which treats them as external forces. Afterwards, the MBS software takes care of the dynamics of the entire system and sends the updated generalized position and velocity, $(\mathbf{q}, \dot{\mathbf{q}})$ back to the aerodynamic solver.

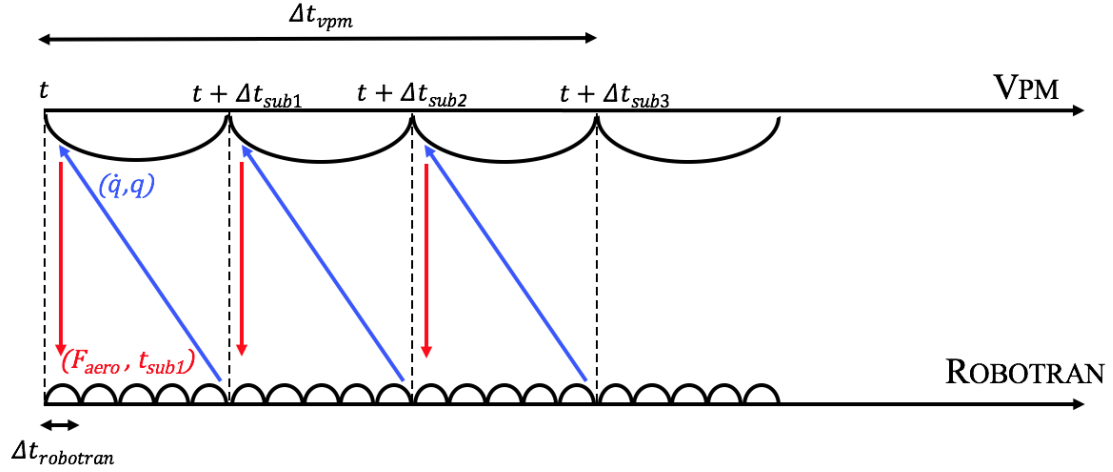


Figure 1.7: Weak strategy in the form of a co-simulation for the coupling between ROBOTRAN and VPM with synchronized evaluations [53].

To enhance accuracy, information exchange is executed at each substep of the VPM integration scheme. Loads, considered as constant during the entire call of ROBOTRAN, are then updated three times for a time step of VPM, instead of one. Since the time step of VPM may be adaptative, an adaptative time-step integration has been selected for Robotran. Coupling operations are illustrated in figure 1.7. This resulting numerical tool is called *Vir*.

Chapter 2

Double-rotor vertical-axis wind turbines: a 2D study

This chapter aims at providing the reader with a thorough vision of the double-rotor VAWTs concept and their aerodynamics and performances. The analysis is done in three main sections. The first part is based on what will be referred to as the *Delft* setup. It is a two-dimensional analysis of the double-rotor concept and its wakes. Based on the same setup, the second and third sections explain, respectively, the effects of the rotor spacing and of the inflow wind angle on the performances.

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2.1 Introduction

Throughout the years, several models have been proposed to model a single vertical-axis wind turbine [9]. The blade element momentum (BEM) theory, for example, combines momentum theory across an actuator with blade element theory. Studies about VAWTs have tried to adapt this concept where each cross section is approximated by an actuator disk at each constant height. But this model poorly represents the geometry of a vertical-axis wind turbine. This theory was extended to the multiple streamtube theory (MST), where the VAWTs cross section is cut into many streamtubes. Further research led to the double multiple streamtube theory, where two actuator disks are used to describe the upwind and downwind parts of the cylinder swept by the blades. The latter has given birth to the actuator cylinder theory: an actuator surface coinciding with the swept area of the blades, rapidly followed by the actuator line model [7, 8].

Besides, a two-dimensional panel/vortex model was developed by Ferreira [10], where bodies are modelled using a distribution of constant strength sources and doublets in the code *U2DiVA*. Vortex particle methods (VP) have also contributed to the analysis of vertical-axis wind turbines. The code *ARDEMA2D* uses a conformal mapping approach to generate the profiles, whereas the wake is modelled as a free vortex particle wake. The addition of the bodies in the flow is done by a shed vorticity layer. This code and its three-dimensional counterpart, *ARDEMA3DS*, was used by *Nenuphar* to study the *TwinFloat* concept [63].

Finally, a vortex particle-mesh method (VPM) with immersed lifting-lines is being developed at the *Thermodynamics and Fluids Lab* (TFL). This code has been described in chapter 1 and is used for the first time to simulate the double-rotor concept.

Following the experiments of Dabiri in South California on closely spaced vertical-axis wind turbines arrays [6], some have attempted to study experimentally [24], or numerically the performances and wakes of closely spaced vertical-axis wind turbines. Turbine spacing is a critical parameter for wind farms, where a compromise between occupied area and power losses due to wakes needs to be found. However, Dabiri *et al.* suggested that paired configurations of VAWTs could benefit from positive interactions of their respective induced velocities. This finding could in fact enhance by an order of magnitude the power output per unit land compared to already existing wind farms consisting of HAWTs. Dabiri also emphasized the fact that no individual efficiency improvement was needed to enhance the global performances. Only a closer turbine spacing and a sufficient incoming wind flux was sufficient to obtain larger power outputs for the setup.

Numerical studies of closely spaced VAWTs have confirmed Dabiri's findings. Araya *et al.* [18] proposed a low-order model of the mean flow through vertical-axis wind turbine arrays using *leaky Rankine bodies*. The same year, Feng *et al.* [19] developed a free vortex method combined with empirical wake models. Further research on double-rotors was carried out by *Nenuphar* [63] using 2D and 3D vortex methods. Their results were compared with 2D CFD simulations with a URANS (Unsteady Reynolds-Averaged

Navier-Stokes) model. Ning [20] applied the actuator cylinder theory to two turbines in close proximity and also compared his results with 2D URANS simulations. The latter method has been extensively used by Giorgiotti *et al.* [21], and Zanforlin *et al.* [22] and showed that the efficiency of staggered pair of turbines could not surpass that of in-row turbines. They all predicted a power enhancement between 5 to 10%. More recently, De Tavernier *et al.* [23] gave some clarification about the double-rotor concept and its aerodynamics using the 2D panel/vortex code *U2DiVA*. They performed parametric studies on the rotor spacing and the wind inflow angle. Finally, wind tunnel experiments were performed on H-type Darrieus VAWTs at the Politecnico di Milano by Vergaerde *et al.* [24]. Their findings showed a strong and repeatable spontaneous synchronization of the turbines: their rotational speeds tend to equal and the phase differences converged to a mean null value.

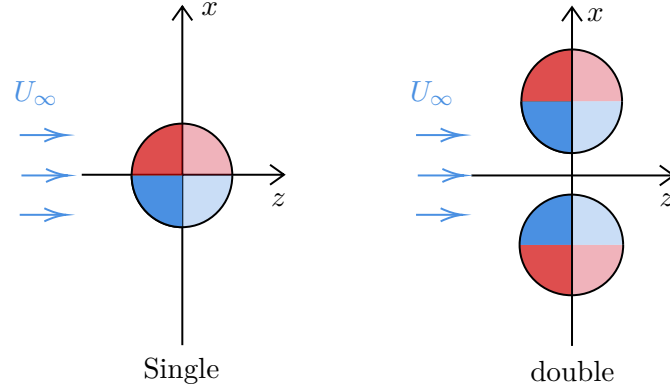
Most of the numerical computations performed on double-rotor VAWTs were restricted to two-dimensional analyses. We here propose to study this concept using *VPM* to compare the findings of De Tavernier *et al.* in 2D.

2.2 The double-rotor concept

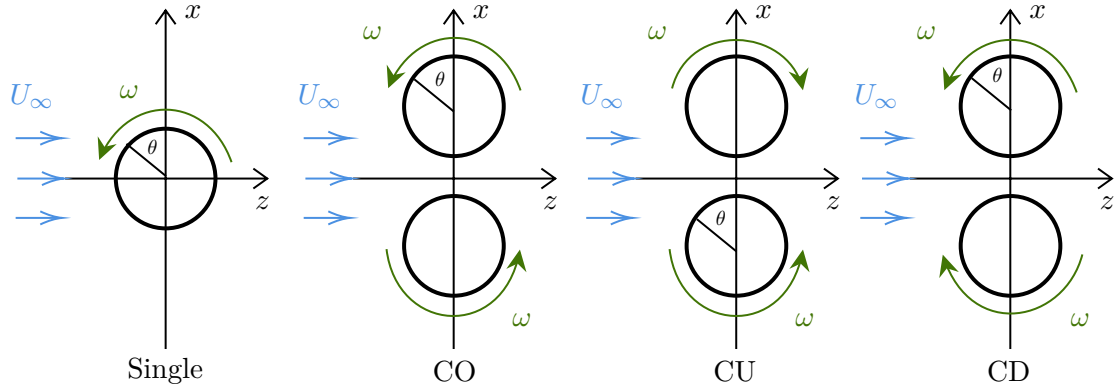
The double-rotor concept consists of two closely spaced vertical-axis wind turbines. This setup can be implemented onshore, as for the experiments of Dabiri [6], or offshore, as the *TwinFloat* prototype developed by *Nenuphar* shown in figure 1.

The double-rotor can be implemented in three configurations: the *co-rotating* configuration (CO), the *counter-up* configuration (CU) and the *counter-down* configuration (CD), the last two configurations being purely symmetric. The terms *up* and *down* refer to the orientation of the blade's tangential velocity compared to the incoming wind velocity U_∞ . The cylinder swept by the blades of each rotor can be divided into parts: upwind and downwind, from either sides of the x -axis linking the rotor's centers, and inner and outer, indicating the region in between or outside the rotor's centers. For the single configuration however, inner denotes for the upper-half and outer for the lower-half of the cylinder. The regions define four quarters: outer-upwind, inner-upwind, inner-downwind and outer-downwind. The azimuth angle θ is defined to start at the beginning of the upwind path for a turbine. In this chapter and all upcoming chapters, the z -axis is defined as the streamwise direction, while the y - and x -axes are respectively the spanwise and transverse directions. A schematic of the definition of quarters along with a schematic of the possible configurations and azimuth angle convention are given in figures 2.1a–2.1b.

The reference turbine is taken as the counter-clockwise rotating turbine for all configurations. Note that quarters definition and associated azimuth angles change for the (CU) configuration compared to the others. As a convention throughout this report, θ is given in degrees.



(a) Schematic of the quarts: outer parts are in red, inner parts are in blue, upwind parts are opaque and downwind parts translucent.



(b) Schematic of the configurations from left to right: single (SI), co-rotating (CO), counter-up (CU) and counter-down (CD) configurations. The azimuth angle θ is assigned to the counter-clockwise turbine, taken as reference. The rotational speed of each turbine is denoted by ω (green).

Figure 2.1: Schematics of the double-rotor concept and the definition of quarts.

We consider that the two turbines rotate in phase, meaning that the blades are reaching the downwind and upwind parts at the same time. This choice is made based on the synchronization findings of Vergaerde *et al.* [24] where the phase differences converge towards a mean zero value.

2.3 Co- and contra-rotating configurations

2.3.1 Methodology and setup

The VAWTs used in the numerical study of De Tavernier *et al.* consist of H-shaped Darrieus wind turbines. Each turbine is made of two straight blades ($N = 2$) with a NACA0015 profile. Even though Ferreira proved that a slight camber on the airfoils could enhance the performances, most of the H-shaped turbines are composed of symmetric wing sections [10]. A symmetry in the lift-to-drag ratio is desired as the blade experiences incoming velocities on both sides of its airfoil sections over a revolution. Positive angles of attack α are obtained for velocities coming from outside the cylinder swept by the blades, *i.e.* mainly in the upwind part of the rotor.

Concerning the aerodynamics of external flows, the flow and the geometric characteristics of bodies are commonly described by dimensionless numbers. The governing parameters are the Reynolds number Re , the tip-speed ratio λ and the Mach number Ma ;

$$Re = \frac{\rho U L}{\mu} \quad \lambda = \frac{\omega R}{U_\infty} \quad Ma = \frac{\omega R}{a} \quad , \quad (2.1)$$

ρ and μ are the fluid density and dynamic viscosity respectively. $U \left[\frac{m}{s} \right]$ is a characteristic velocity, taken as the incoming wind velocity U_∞ or the relative velocity as seen by the blades U_{rel} . $L \left[m \right]$ is a characteristic length of the problem. The rotational speed is denoted by $\omega \left[\frac{rad}{s} \right]$, while $R \left[m \right]$ is the radius of the turbine and a is the speed of sound $\left[\frac{m}{s} \right]$. The parameter L being any characteristic length, two Reynolds numbers can be defined: one at the rotor scale Re_D , and one at the blade scale Re_c , the subscript referring to the diameter D and the chord c respectively. Experiments at full dynamic similarity carried out by Miller M. [28] suggested that the chord-based Reynolds number $Re_c = \frac{\rho U_{rel} c}{\mu}$ is the best definition for wind turbines to accurately capture the behaviours and performances. Concerning the geometrical characteristics, a non-dimensional number can be defined to describe the loading of a wind turbine, namely the solidity $\sigma = \frac{Nc}{D}$. It shows the ratio between the overall area of the blades and the area swept by the turbine. The aspect ratio, $AR = \frac{H}{D}$, is defined as the ratio of the blade's height to the rotor diameter. It is a critical parameter when studying wake recoveries of 3D flows. Its impact is discussed in section 3.4. The rotor spacing, defined as the normalized distance in between the rotors centers $d_{int} = \frac{D_{int}}{D}$, is inherent to the double-rotor concept and has here been set to 1.2 as to match the numerical implementation of De Tavernier *et al.* [23].

2.3. Co- and contra-rotating configurations

$U_\infty \left[\frac{m}{s}\right]$	$D \text{ [m]}$	$c \text{ [m]}$	N	d_{int}	Re_c	σ	λ	AR
20	1	0.05	2	1.2	$6.67 \cdot 10^4$	0.1	2.5	∞

Table 2.1: Physical parameters of the present work simulations with the corresponding non-dimensional numbers.

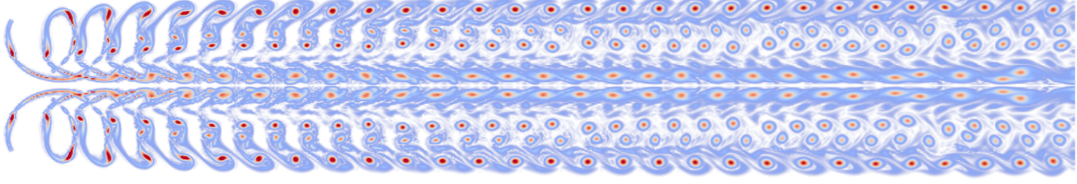


Figure 2.2: H-type VAWTs with infinite aspect ratio (two-dimensional): top view of the CU configuration by a volume rendering of the vorticity magnitude $||\omega||$. The setup parameters are provided in table 2.1.

In order to efficiently compare our results, the numerical setup here matches their configuration for all dimensionless numbers. The table 2.1 contains the main parameters of our simulations. One should note that, as no controller is implemented in the present work, the rotational speed of the turbines remains constant throughout the computation.

All three configurations are being tested and compared to the single rotor in the flow. Simulations were performed over $\hat{t} = \frac{tU_\infty}{D} = 100$ non-dimensional times within a computational domain of $[4D \times 18D]$ in the transverse and streamwise directions respectively. The maximum time step is set to $5 \cdot 10^{-4} s$, which corresponds to an azimuth position increment $\Delta\theta$ of less than 3° . In order to obtain a pseudo two-dimensional solver, the spanwise direction sees a periodic condition, and the height of the blades exactly equals the spanwise computational length, as to mimic an infinite aspect ratio $AR = \infty$. The mesh contains $N_D = \frac{D}{h} = 50$ points per rotor diameter. The whole domain is described by means of more than 7 million particles. The resulting diagnostics and wakes are solved and as an illustration the obtained vorticity field of the counter-up configuration (CU) is provided in figure 2.2.

Several computational and implementation aspects differ from the simulations of De Tavernier *et al.* First, the *U2DiVA* code is based on a vortex/panel method, where airfoils are represented by constant strength sources and doublets and the far wake by vortex points. Then, as stated by Ferreira [10], the unsteady aerodynamics commonly results in the dynamic stall of the blades. Dynamic stall is characterized by a lift overshoot and a massive flow separation, caused by the constantly varying angle of attack α . We recall that a Leishman-Beddoes dynamic stall model is implemented in the aerodynamic solver to correct the aerodynamic behaviour of the lifting-lines polars. Its parameters are based on the NACA0015 at a chord Reynolds number $Re_c = 8 \cdot 10^5$. Finally, standard lifting-lines are not able to capture flow curvature effects. As an adjustment, the blades are pitched inwards by an angle α_0 . This rectification for the lifting-lines has been

2.3. Co- and contra-rotating configurations

validated against experimental data and brings a notable improvement on the results at moderate tip-speed ratios [30]. Based on the geometric characteristics, an inward pitch angle of $\alpha = 1.4312^\circ$ has been imposed to the blades of the H-type VAWTs.

Figures 2.3a–2.3d show the time-averaged streamwise velocity $\frac{\bar{w}}{U_\infty}$ normalized by the free stream velocity for the 4 configurations. Statistics are taken over $\hat{t} = 50$ non-dimensional times, beginning at $\hat{t} = 30$ non-dimensional times in order to reach steady-state conditions.

Because of the presence of the neighbouring turbine, the flow is forced in the narrow inner zone and reaches higher velocities than the free stream velocity U_∞ . The flow is accelerated, which results in a higher mass flow in the inner-upwind parts of the turbines. The wake of each turbine is highly asymmetric, and the highest velocity deficit is found behind the outer part of the turbines for the single, counter-down and co-rotating configurations, but in the inner part of the turbines for the counter-up configuration. Another smaller lobe is found in the wakes, forming two-lobed structures. The wakes seem very similar to the single configuration. We notice few interactions between the wakes of the paired turbines, even after reaching 16 diameters of depth behind the rotors. This is also observable in terms of the vorticity field given in figure 2.2. Furthermore, a slight deficit can be seen in front of the rotors, coinciding with the upstream start of the main lobes.

2.3.2 Diagnostics and performances

Diagnostics about the rotors and the lifting-lines are provided by *VPM* at each time step. In terms of diagnostics, non-dimensional coefficients are defined in order to compare configurations having different physical parameters. The power coefficient C_p is the ratio of the extracted power by the turbine against the power contained in the free stream. The thrust coefficient C_T relates the total thrust provided by both blades to the free stream dynamic pressure $q_0 = \frac{1}{2}\rho U_\infty^2$ acting on the turbine extraction plane $A = HD$, H being the blade's span.

$$C_p = \frac{\tau\omega}{q_0 U_\infty A} \quad C_T = \frac{F_t}{q_0 A} \quad (2.2)$$

where τ is the moment acting on the rotor and ω its rotational speed. For the double-rotor layout, the global power coefficient of the setup is taken as the mean value of the power coefficients of the individual rotors. For 2D simulations, these parameters are defined per unit span. The tangential and normal force coefficients per unit span represent the local forces against the free stream dynamic pressure acting on a chord length.

$$C'_t = \frac{f_t}{cq_0} \quad C'_n = \frac{f_n}{cq_0} \quad (2.3)$$

2.3. Co- and contra-rotating configurations

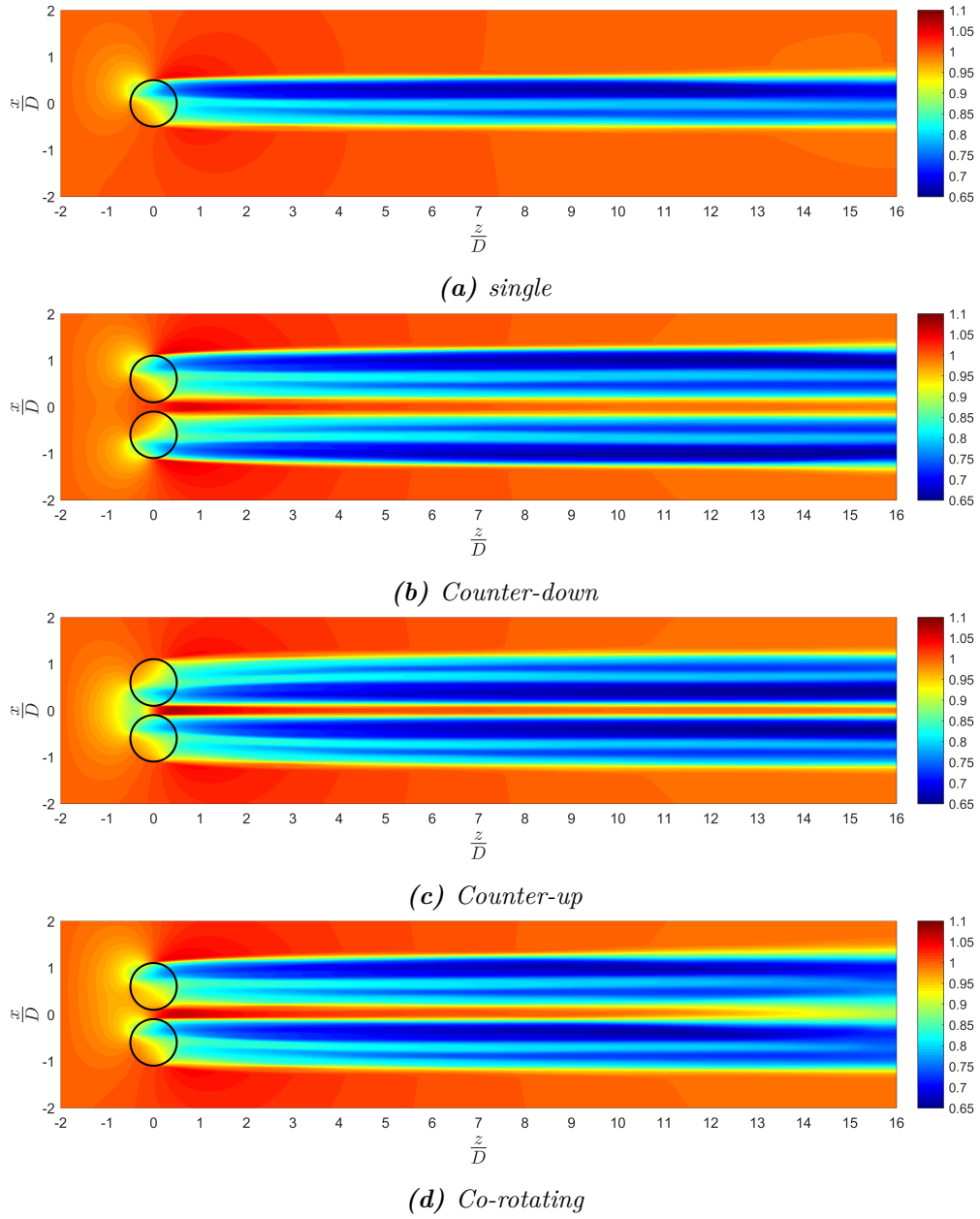


Figure 2.3: Time-averaged streamwise velocity field $\frac{\bar{w}}{U_\infty}$ around H-shaped rotors for the 4 configurations. The streamwise velocity is normalized with the free stream velocity U_∞ . Simulation parameters are those of table 2.1. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

2.3. Co- and contra-rotating configurations

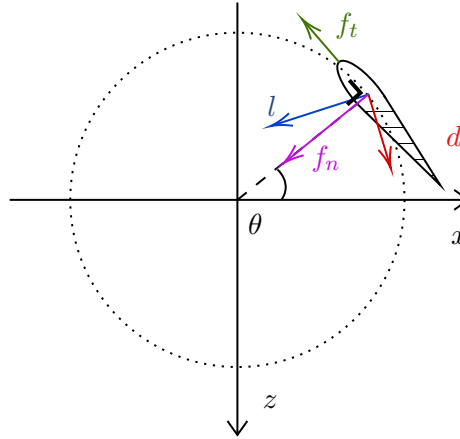


Figure 2.4: Local lift force l , drag force d , tangential force f_t and normal force f_n acting on an airfoil section of a straight-bladed vertical-axis wind turbine.

	single	CO	CU	CD
C_p	0.1415	0.1397	0.1408	0.1395
$C_{p,\text{rotor}}$	0.1415 /	0.1397 0.1397	0.1408 0.1408	0.1395 0.1395
$C_{p,\text{norm}}$	1	0.9873	0.9949	0.9857

Table 2.2: H-VAWT type global performances for all configurations. The output power coefficient C_p is given for each setup and normalized with the power coefficient of the single configuration. The contribution of each individual rotor is given for the top (positive x -axis) and bottom (negative x -axis) turbine. Setup parameters are those of table 2.1.

These forces are obtained by mapping the local lift and drag acting at the aerodynamic center of the airfoil, onto the normal and tangential directions of the wing section. figure 2.4 recalls the discussed forces acting on the blade of a vertical-axis wind turbine.

Dimensionless coefficients given in (2.2) and (2.3) can be expressed as a combination of the non-dimensional numbers presented in equation (2.1). This means that different combinations of physical parameters provide the same output power and thrust coefficients for identical Reynolds numbers, solidities and tip-speed ratios, as long as the Mach number Ma is kept below the compressibility limit. Table 2.2 contains the global performances of the different configurations. The contribution of each rotor to the global power coefficient is given for the double-rotor layouts, where $C_{p,\text{rotor}}$ gives the values of the top (positive x -axis) and bottom (negative x -axis) rotor respectively. Except for the power coefficient curve combining both rotors, the diagnostics over the azimuth position for the double-rotor layout (CO, CU and CD) are given for a single blade of the reference rotor.

A first hint of the poor interaction existing between the rotors in this case is that the rotors of the co-rotating configuration yield an identical contribution. Indeed, this configuration is the only double-rotor layout which is not symmetric. In fact, no gain is

2.3. Co- and contra-rotating configurations

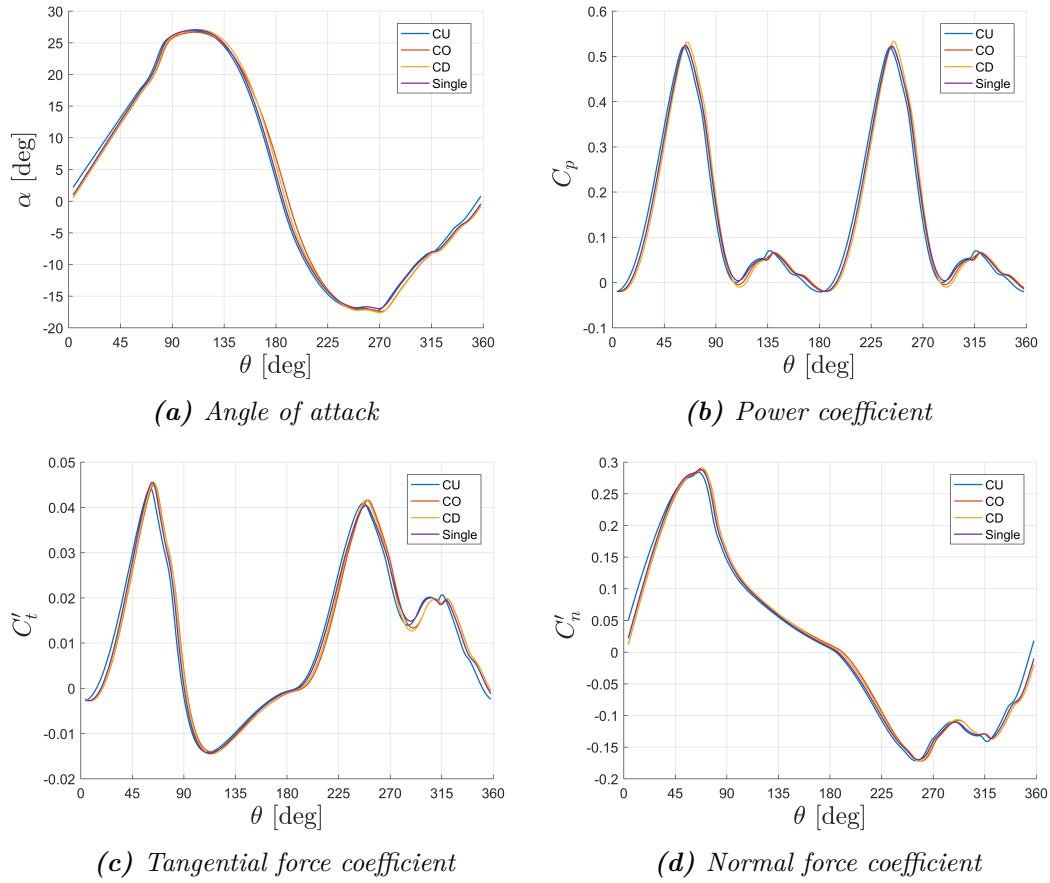


Figure 2.5: Diagnostics of the H-type VAWTs for all 4 configurations. Simulation parameters are those of table 2.1. Values are averaged over 10 rotor revolutions. Except for the power coefficient combining both rotors (b), other diagnostics are given for a single blade of the reference rotor.

observed on the power output for the double-rotor layouts compared to the single rotor. This result is a clear departure from former numerical computations [18, 19, 20, 21, 22], all using a two-dimensional approach. This difference is explained by taking a deeper look into the evolving aerodynamic quantities over a revolution.

Figures 2.5a–2.5d respectively provide the evolution of the angle of attack α , the global power coefficient evolution C_p for the layout, and the normal and tangential force coefficients, C'_n and C'_t acting on one blade. The normal force is directed inwards, while the tangential force points towards the leading edge. The values are averaged over 10 rotor revolutions and given for the reference rotor.

A positive angle of attack corresponds to a relative velocity coming outside from the cylinder swept by the blades. While the blade swings through the air, it encounters a higher incoming velocity during the outer part and a lower during the inner part.

Moreover, the downwind part already is in the wake of the upwind part. The evolution of the angle of attack is thus not symmetric over a revolution. The angle of attack reaches its maximum just after the most upstream position, *i.e.* right after $\theta = 90^\circ$. It then rapidly drops to negative values in the downstream part, and weakly oscillates while increasing after its minimum value in the downwind part, around $\theta = 270^\circ$. These oscillations are physical as the blades in the downwind part encounters the shed wake of the blades in the upwind part, and produces at the blade level variations of the relative velocity. The oscillations in angle of attack induce oscillations in the loadings in the last quarter and thus in the power coefficient. The latter being given as the mean of each rotor's individual C_p , we observe two peaks for a H-shaped turbine. A negative power coefficient is encountered when the rotors add energy to the flow, at constant rotational speed. Its two optima coincides with those of the blade loadings, given by the tangential force coefficient in figure 2.5c. The rapid drop after $\theta = 50^\circ$ and $\theta = 250^\circ$ is an indicator of a stalled region, occurring at low tip-speed ratios. Indeed the angle of attack α has not yet reached its maximal value when the loading collapses. Moreover, the angle of attack increases even faster right after the blade is stalled. This is due to the blockage effect characterizing the upstream region.

The tip-speed ratio has a huge impact on the performances of vertical-axis wind turbines. Low TSRs result in stall due to weak blade velocities ωR and large angles of attack α . Higher TSRs come with higher blade velocities and hence lower angles of attack, potentially still in the linear behaviour of the lift coefficient. For each set of non-dimensional numbers, an optimum tip-speed ratio exists. Especially, for a fixed value of λ , the blade's solidity σ greatly influences the rotor's performances. Rezaeiha *et al.* [27] showed the invariance of the optimal tip-speed ratio λ_{opt} to a newly introduced parameter $\sigma\lambda^3$. They derived a correlation with 95% confidence levels to estimate the optimal tip speed ratio as a function of the solidity for vertical-axis wind turbines.

$$\lambda_{\text{opt}} \simeq 2.693\sigma^{-0.329} - 1.605 \quad (2.4)$$

Based on equation (2.4), the optimal tip-speed ratio with the present solidity of $\sigma = 0.1$ should be around 4.2. This value is significantly higher than the 2.5 imposed by De Tavernier *et al.* in their simulations.

Regarding the obtained results, we identified possible causes explaining the absence of gain for the double-rotor layouts simulated with this setup:

1. For a solidity of $\sigma = 0.1$, the operating tip-speed ratio is far lower from its optimum value $\lambda_{\text{opt}} \simeq 4.2$. The lower tip-speed ratio enlarges the dynamic stall phenomenon, decreasing the performances.
2. The parameters for the Leishman-Beddoes model are here tuned for a chord Reynolds number Re_c of $8 \cdot 10^5$, which is an order of magnitude higher than the operating Reynolds number $Re_c = 6.67 \cdot 10^4$. The latter is relatively small and magnifies the flow separation by dynamic stall.

2.3. Co- and contra-rotating configurations

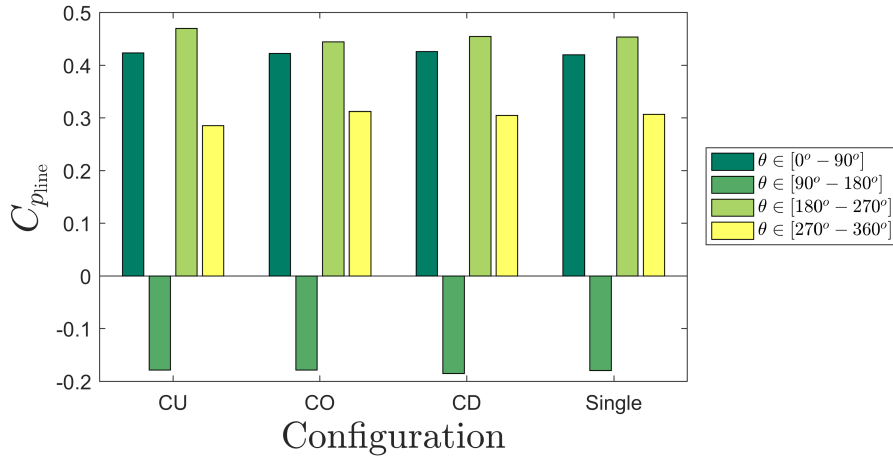


Figure 2.6: Quartile distribution of the power coefficient of a single blade for all configurations. The parameters are those given by table 2.1.

Even though the flow is effectively accelerated in between the rotors, few interactions exist between the rotors and so for all configurations. This is quite obvious regarding the almost perfect superposition of all diagnostics over a revolution: the rotors are weakly influenced by the presence of their neighbour. De Tavernier *et al.* suggested that the superposition of both induced velocity fields and the presence of a neighbouring rotor cause transverse velocities to cancel in the inner-upwind part of the flow, leading to a slightly higher angle of attack and loading. However, this gain is cancelled by the same induced velocities in the outer-upwind part. The net contribution of a paired layout in the upwind part of the flow is unchanged. Very few differences in loading are observed for $\theta = 0^\circ$ to 180° . The inner-downwind part is characterized by a wake contraction and an increase in mass flow. The flow is constrained between the two wakes, cancelling the transverse velocity and opening the angle of attack. The inner-downwind contribution is the key of the double-rotor concept. figure 2.6 presents the distribution per quartile of the loading on a single blade. The inner-downwind part, being the third quartile, effectively seems to contribute the most to the output power of the blade, producing for all configurations about 45% of the power. Still no significant difference is observable between the different layouts: each blade identically behaves in its respective quart of the revolution. The upwind part indeed is left unchanged, while the downwind part has minor differences between the configurations. Nevertheless, the counter-up layout seems to be the most promising configuration for the double-rotor concept.

2.3.3 Wakes

Wakes are caused by any solid body exerting a force on a flow and their characteristics can partially be captured by the Reynolds number Re , comparing the inertial effects to the viscous effects inside a flow. The very near wake of a blade is dominated by

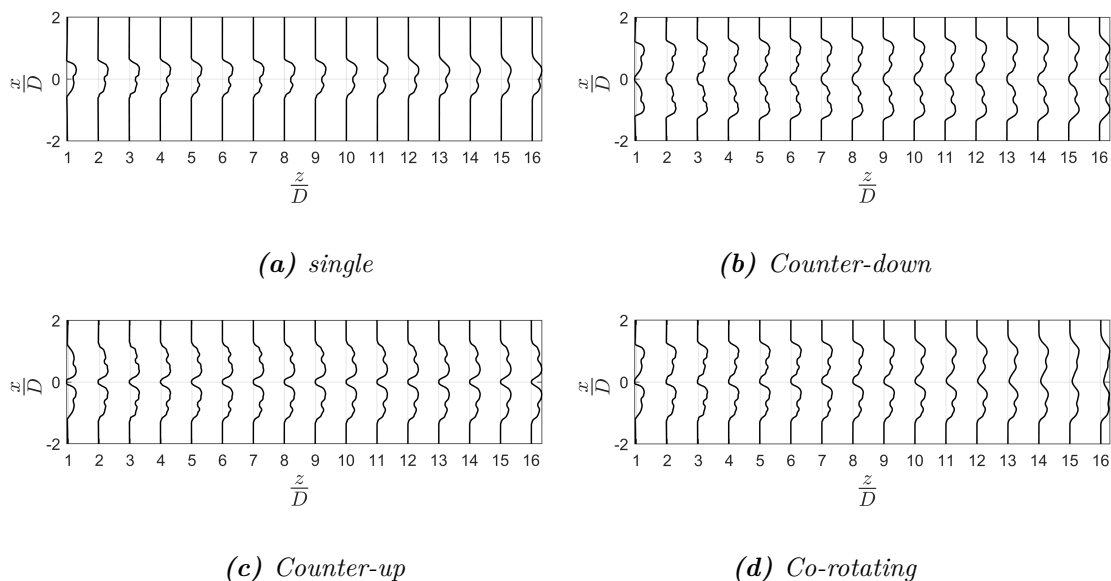


Figure 2.7: Time-averaged streamwise velocity deficit $(1 - \frac{\bar{w}}{U_\infty})$ around H -shaped rotors for the 4 configurations. Simulation parameters are those of table 2.1. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

viscous effects, and adds vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ in the flow. The unsteadiness of the relative velocity impinging the blade over a revolution implies a time variation of the circulation of the blades in the case of VAWTs. The blades constantly shed vortex sheets of varying strength, along with tip-vortices at their sides. VAWT wakes create complex vortical structures. Interaction between them give rise to fast-growing vortical instabilities and vortex reconnections, resulting in a faster transition to turbulence [30]. For a two-dimensional approach of turbine wakes, no tip vortices are shed in the wake. This lack of vertical mixing predicts a lower recovery rate for 2D analysis and is presumably the most important limitation. 2D analysis should be performed for turbines with a large enough aspect ratio AR so that blade tip effects can be neglected [22].

Dabiri *et al.* [6] showed that for very large vertical-axis wind turbine farms, one of the main benefits of such turbine arrays could be a reduction of the wake loss through the farm and hence the upkeep of high-speed flows and large power extractions, even with closely spaced rotors. An attempt to numerically demonstrate this phenomenon has been done by Giorgetti *et al.* [21] using *Magnus* spinning cylinders. However they concluded that schematizing a permeable turbine with spinning cylinders is not realistic, as a turbine enables the flow to cross through it. Secondly, spinning cylinders cause a massive wake bending and strong vortex interactions when in paired configurations, which is less observed for real vertical-axis wind turbines in direct vicinity.

Figures 2.7a–2.7d present the time-averaged streamwise velocity deficit $\left(1 - \frac{\bar{w}}{U_\infty}\right)$ located from 1 to 15 diameters downstream of the turbines for all four configurations. The two-lobed structure corresponding to high loaded regions is clearly observable in figure 2.7a. The structure is conserved until the end of the computational domain, as no merging of the wakes occurs. Figures 2.7b–2.7d indeed present an accelerated flow in between the rotors. The inner sides of the paired configurations experience a wake contraction, whereas the outer half of the wake does not change appreciably. This contraction is more pronounced for the counter-up configuration, as the wake’s bendings caused by the rotor’s rotation tend to push the wakes together. The wake bending effect will be predominant at higher tip-speed ratios when studying the full three-dimensional wake in section 3. As it can be seen in figure 2.2, the vortical structures in the 2D wakes are composed of a helical vortex sheet with periodic eddies where stopping vortices occur due to dynamic stall, at $\theta = 60^\circ$ and 260° . The stopping vortices are advected downstream through the whole computational domain. For symmetric configurations (counter-up and counter-down) in very close proximity, the merging of those counter-rotating eddies could cancel one another by their overlapping in the far wake. This phenomenon, not observable here due to an off-design tip-speed ratio and a lack of vertical mixing, could increase the availability of kinetic energy in the downwind part and further in the wake. This will be investigated in section 3.2.

Following the work of Chatelain *et al.* [30], we modified their definition of displacement and momentum surfaces to the present context. The wake of the single configuration is highly asymmetric, making the definition of a velocity deficit evolution based upon a single characteristic unsuitable. Moreover, the analysis of the different double-rotor layouts needs to account for the presence of two bodies disturbing the flow. The definitions of the non-dimensional displacement width L_1 and momentum width L_2 are given as:

$$L_1(z) = \frac{1}{nD} \int_{-\infty}^{\infty} \left(1 - \frac{\bar{w}(x, z)}{U_\infty}\right) dx \quad (2.5)$$

$$L_2(z) = \frac{2}{nD} \int_{-\infty}^{\infty} \frac{\bar{w}(x, z)}{U_\infty} \left(1 - \frac{\bar{w}(x, z)}{U_\infty}\right) dx \quad (2.6)$$

where n is the number of bodies of diameter D present in the flow. The displacement and momentum widths correspond to the integral of flux quantities in the transverse (x) direction located at a distance $z = i \cdot D$, $i = 1, 2, \dots, 15$ downstream of the turbine axis. Practically, the approximate values for L_1 and L_2 were obtained through numerical integration on a transverse width of $4D$. We recall that the vortex particle-mesh code used here enforces unbounded conditions, and hence allows for a transverse mass flow to *leave* the computational domain. The displacement width L_1 corresponds to the mass flow deficit compared to a uniform flow of free stream velocity U_∞ . This quantity actually quantifies the blockage effect caused by the turbines on the flow. Regarding L_1 values in figure 2.8, no significant difference is to be observed between the different double-rotor layouts: all begin at a low value indicating a small velocity deficit in the rotors plane, and keep increasing with the streamwise distance z . The globally decelerating flow reaches a

2.4. Effect of the turbine inter-distance

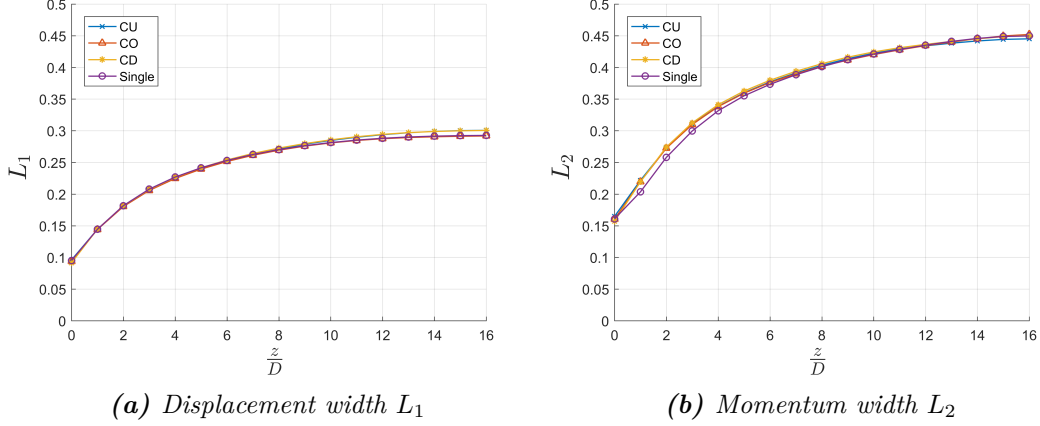


Figure 2.8: Displacement and momentum width L_1 and L_2 based on the statistical streamwise velocity deficit at each diameter downstream. The parameters of the simulation are those of table 2.1.

plateau after 15 diameters downstream, suggesting the velocity profile will qualitatively remain the same in the very far wake if no merging of the wakes takes place, or no other instabilities occur. This is not unexpected: due to the small tip-speed ratio, the wake envelope angle is narrow and the instability growth within the wake is very slow. The wakes resulting from both turbines do not merge within the computational domain, disabling the transition to a well-mixed fully turbulent wake. For this 2D analysis, no recirculation bubble is present for any of the configurations. Recirculation bubbles are characterized by an overshoot of the displacement width $L_1 > 1$ and are typical of 3D flows of vertical-wind turbines. Especially with small aspect ratios AR and higher tip-speed ratios λ , which is quite the opposite implemented in the present case. The momentum width L_2 quantifies the deficit in the flux of momentum at each position downstream the rotors, compared to a uniform flow of free stream velocity U_∞ . Again, no major differences is observed between the possible rotor configurations, compared to the single rotor. Our results for two-dimensional flows based on the *Delft* setup do not illustrate the predictions of Dabiri.

2.4 Effect of the turbine inter-distance

Insights of the key physics behind the double-rotor configuration and its wake have been presented in the previous section. Several major operating conditions still have to be analyzed in order to globally optimize closely spaced vertical-axis wind turbine farms. But what should the turbine inter-distance, *i.e.* the distance between the rotor's centers, be to benefit from a better wake recovery and enhanced power performances ? The present section aims at studying the effect of the spacing on the aerodynamics and wakes of the counter-up configuration, the latter being the most promising in terms of performances.

2.4. Effect of the turbine inter-distance

	CU_1	CU_2	CU_3	CU_4	CU_5
d_{int}	1.022	1.0307	1.0429	1.0598	1.0834
	CU_6	CU_7	CU_8	CU_9	CU_{10}
d_{int}	1.1163	1.1623	1.2264	1.3158	1.4405

Table 2.3: Turbine inter-distance d_{int} . Other setup parameters are left unchanged (see table 2.1).

2.4.1 Methodology and setup

The setup is globally the same as for the previous section, and the main parameters are taken from table 2.1. The computational domain is not enlarged and keeps its initial width of $4D$ as *VPM* exactly computes the unbounded conditions. The main interest is the performances evolution of the rotors instead of their wake physics. Therefore, to save computational resources, the depth of the domain was shrunk to 7 diameters downstream of the rotor's centers. The turbine spacing considered in the simulations ranges over $d_{\text{int}} = 1.022$ to $d_{\text{int}} = 1.4405$. Ten simulations, named CU_1 to CU_{10} , were performed in total to capture the effect of spacing on the setup performances. The inter-distances used are based on a geometric progression, tuned here as to obtain increasing differences between successive spacings. The exact values of the inter-distances are provided in table 2.3.

A turbine spacing very close to 1, as it is the case for the CU_1 simulation is practically not feasible: the blades themselves also have a certain width and cannot collide. Nevertheless, this layout should see the strongest interactions caused by the induced velocities of both rotors.

When putting the two rotors closer to one another, the inner sides of the wakes get obviously closer and closer. An accelerated flow in between the rotors is still observable, and the blockage effect is enhanced for small spacings. The permeability reduction for closer arrangements compared to the single turbine emphasizes the velocity deficit upstream of the wind turbines. The inflow condition also slightly influences the phenomenon. Even in very close vicinity, the effect of a low tip-speed ratio is still predominant and no merging of the two inner lobes occur. One expects for the performances of each rotor to gradually converge to the performance of the isolated rotor while increasing the turbine inter-distance. For all coming diagnostics and wake analysis, four spacings will be considered: CU_1 , CU_3 , CU_7 and CU_{10} . The dimensionless time-averaged streamwise velocity $\frac{\bar{w}}{U_\infty}$ is shown for each considered setup in figures 2.9a–2.9d. The configurations are chosen as to contain the first unfeasible arrangement, a very close but feasible arrangement, a close arrangement and a large arrangement. The layout studied in section 2.3 has a rotor spacing of $d_{\text{int}} = 1.2$, which lies in between CU_7 and CU_8 in terms of rotor spacing.

2.4. Effect of the turbine inter-distance

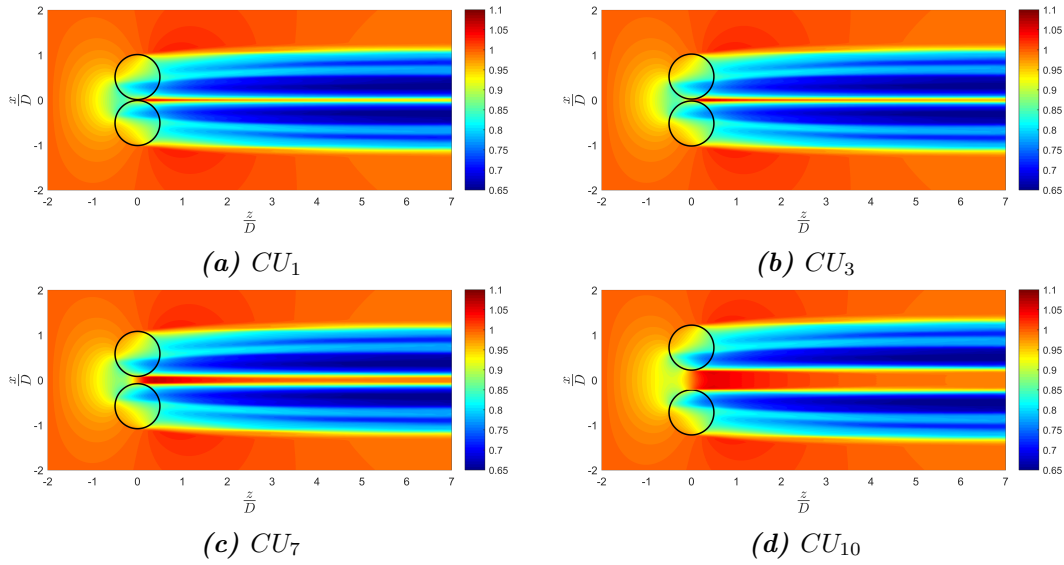


Figure 2.9: Time-averaged streamwise velocity field $\bar{w}$ around H-shaped rotors for 4 turbine spacings. The velocity field is normalized with the free stream velocity U_∞ . Simulation parameters are those of table 2.1 and corresponding spacings from table 2.3. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

2.4.2 Diagnostics and performances

Table 2.4 contains the global power coefficients of the different rotor arrangements. The contribution of each rotor to the power coefficient is also provided, with $C_{p,\text{rotor}}$ being successively the top (positive x -axis) and bottom (negative x -axis) turbine. Note that as all setups are composed of counter-up configurations, the reference rotor for diagnostics is the bottom turbine. The contribution of each rotor is expected to be the same by symmetry.

		CU_1	CU_2	CU_3	CU_4	CU_5
C_p		0.1434	0.1432	0.1431	0.1431	0.1428
$C_{p,\text{rotor}}$		0.1434 0.1434	0.1432 0.1432	0.1431 0.1431	0.1431 0.1431	0.1428 0.1428
$C_{p,\text{norm}}$		1.0064	1.0055	1.0044	1.0047	1.0027

		CU_6	CU_7	CU_8	CU_9	CU_{10}
C_p		0.1427	0.1425	0.1425	0.1422	0.1422
$C_{p,\text{rotor}}$		0.1427 0.1427	0.1425 0.1425	0.1425 0.1425	0.1422 0.1422	0.1422 0.1422
$C_{p,\text{norm}}$		1.0017	1.0004	1	0.9986	0.998

Table 2.4: H-VAWT type global performances for different turbine inter-distances d_{int} . The output power coefficient C_p is given for each setup and normalized with the power coefficient of the single configuration. The contribution of each individual rotor is given for the top (positive x -axis) and bottom (negative x -axis) turbine. Setup parameters are those of table 2.1.

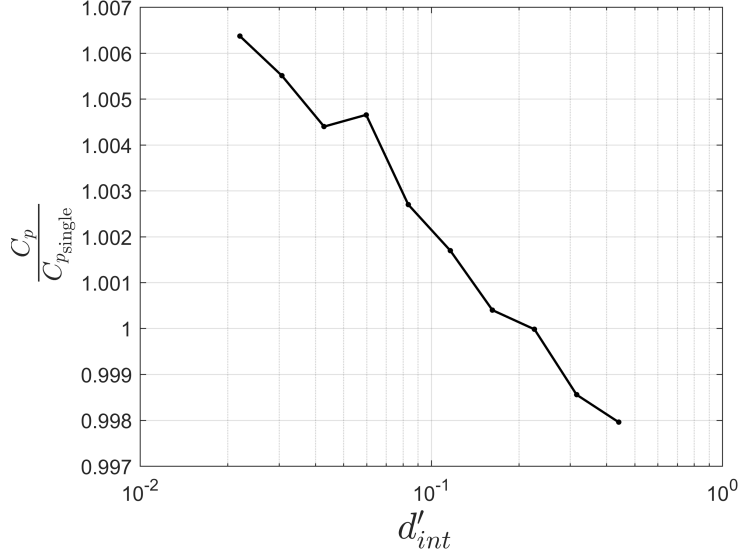


Figure 2.10: Normalized power coefficient with respect to rotor spacing $d'_{int} = d_{int} - 1$ in logarithmic scale for the counter-up configuration. Simulation parameters, except for turbine inter-distance, are those of table 2.1.

These results are consistent. We observe in figure 2.10 a gradually increasing gain compared to the single configuration when reducing the distance between the turbines. Even if those gains remain very small due to the off-design tip-speed ratio, it shows that closely spaced vertical-axis wind turbines, when correctly calibrated, benefit from interactions between their rotors. The closer the turbines, the stronger they sense the induced velocities on one another, and the stronger the interactions. This behaviour is very different from that of horizontal-axis wind turbines (HAWTs), where each turbine has to be spaced several diameters apart to optimize the farm performances. Note that the obtained power coefficients for the single configurations in the present section and the former one, show a relative difference of approximately $e_{rel} \simeq 1.5\%$, all other parameters unchanged. This indicates the small yet present sensitivity with the outflow conditions implemented in the aerodynamic solver. At the outlet of the computational domain, transverse velocities are forced to 0 on odd positions and no normal velocity gradients may remain on even positions:

$$u_t = u_x = 0 \quad \frac{\partial u_n}{\partial n} = \frac{\partial u_z}{\partial z} = 0 \quad . \quad (2.7)$$

The Poisson equation solved to retrieve the velocity field everywhere is biased by this boundary condition, and influences the rotor's performances.

Studies conducted by Zanforlin *et al.* [22] showed that the peak power coefficient with respect to tip-speed ratio λ is shifted to higher values for paired configurations. In the present analysis, the tip-speed ratio has been kept constant and set to $\lambda = 2.5$ for all

2.4. Effect of the turbine inter-distance

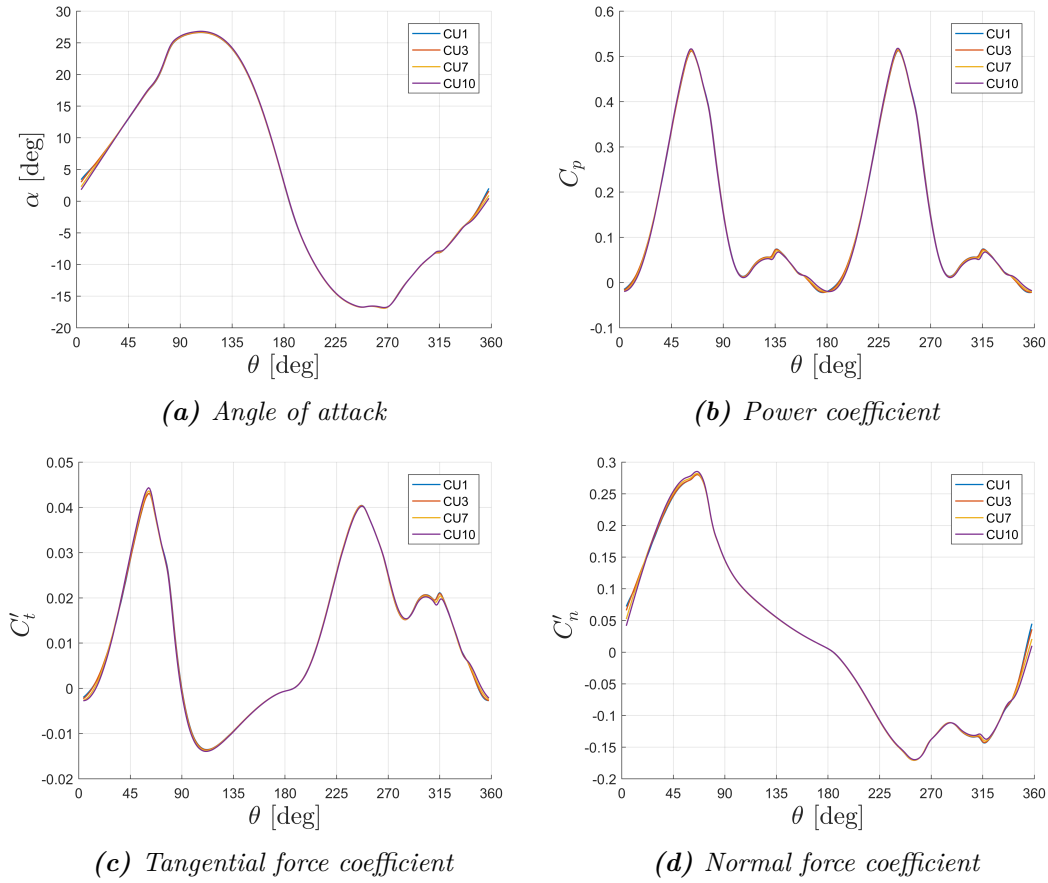


Figure 2.11: Diagnostics of the H-type VAWTs for different turbine inter-distances. Simulation parameters are those of table 2.1. Values are averaged over 10 rotor revolutions and given for the reference rotor.

arrangements. Further studies should investigate the sensitivity of optimal tip-speed ratio λ_{opt} with turbine inter-distance.

Figures 2.11a–2.11d provide the evolution of the angle of attack α , the global power coefficient evolution C_p for the four studied spacings, and the normal and tangential force coefficients, C_t' and C_n' . All different spacings show mostly identical behaviours. The dynamic stall occurring around $\theta = 60^\circ$ and $\theta = 260^\circ$ is still observed with the collapsing of the tangential force coefficient C_t' . The aerodynamic interaction cancels transverse velocities in the inner part, both upwind and downwind, and modifies the absolute flow approaching the blades, making it more favourable to generate lift and torque. As the turbines are placed closer to one another, the flow through the downwind path loses its trend to diverge laterally and the inner parts of the contracted wakes remain parallel to the configuration symmetry plane.

2.4. Effect of the turbine inter-distance

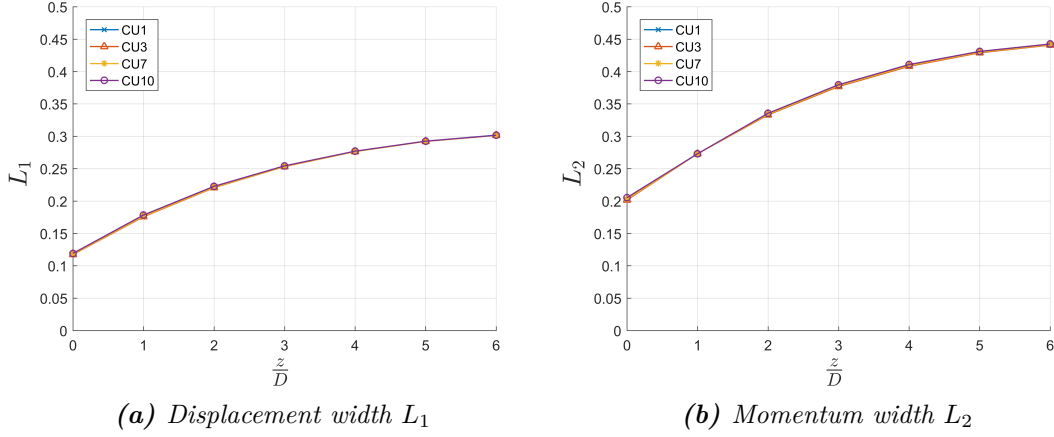


Figure 2.12: Displacement and momentum width L_1 and L_2 based on the statistical streamwise velocity deficit at each diameter downstream for different turbine spacings. The parameters of the simulation are those of table 2.1.

2.4.3 Wakes

The previous section presented the positive interactions in terms of performances of the counter-up layout regarding spacing between the turbine's centers. Along with the aerodynamic performances, it remains relevant to analyze the wake diagnostics for further optimization of wind farms. Intuitively, it can be understood that, from closer arrangements, result stronger induced velocity fields and closer wakes, which can later merge and develop to a well-mixed fully turbulent wake. Moreover, closer turbines benefit from enhanced vortex shedding suppression. The overlapping of the starting vortices after dynamic stall in the inner part is facilitated and more likely to occur sooner in the wake. But closer turbines arrangements also raise the velocity deficit in the upwind part and lowers the peak velocity in between the turbines, because of a lower permeability of the setup. This further reduces the availability of kinetic energy downstream in the inner parts of the wakes.

Regarding the displacement width L_1 , few differences are observable between the different spacings. This almost perfect superposition is mainly due to low wake interactions caused by an off-design tip-speed ratio. But two contrary spacing effects are also suggested for a similar computational domain with exact unbounded conditions:

1. A bigger turbine inter-distance lowers the wake contraction and increases the permeability of the setup. More flow is allowed to pass in between the turbines at higher streamwise velocities, lowering the integral value of L_1 . Even though the wake bending is favourable in the case of counter-rotating layouts, *i.e.* the wakes are deflected towards each other, the outer parts of the wakes redirect the outer high speed flow outside the computational domain, resulting in an increase of the flow deficit and L_1 .

2. Lowering the turbine inter-distance lowers the permeability and the available high-speed flow in between the turbines. It forces some of the flow to pass through the turbines, and accelerates the flow at the outer parts of the turbines. This accelerated flow however experiences a deflection with small transverse velocities near the domain limits. Consequently, flow loss associated to the unbounded conditions is reduced, as are the values of L_1 .

The two contrary effects and the poor wake interactions explain the superposition of the wake diagnostics for different spacings. A more relevant comparison should be performed by scaling the domain width with the largest turbine inter-distance d_{int} . As for section 2.3, the two wakes do not merge within the computational domain, disabling the transition to a well-mixed fully turbulent wake. The flow is globally decelerating, and will also reach a plateau further downstream, showing qualitatively steady streamwise velocity profiles further downstream up to the point where the wakes will eventually merge. Surprisingly, for L_1 , a relative difference of approximately 20% is observed after 5 diameters downstream, compared to figure 2.8a. The curve seems to plateau faster when using a shorter computational domain, again suggesting an influence of the outflow conditions 7 diameters downstream on the performances and wake diagnostics. Care has to be taken regarding the exact numerical values for the displacement and momentum widths.

2.5 Effect of the wind inflow angle

One of the main advantages of vertical-axis wind turbines compared to their horizontal counterpart is their insensitivity to the wind direction. This also suggests a more robust efficiency in turbulent conditions. Where HAWTs need to align with the incoming wind, VAWTs do not need this yawing actuation, which goes along with reduced installation and maintenance costs.

Unfortunately, vertical-axis wind turbines in double-rotor layouts are very sensitive to the wind direction [6], [22]. This study aims to elucidate the effect of wind direction on the double-rotor configuration. As seen from table 2.4, each rotor performs equally when the configuration is symmetric. But if the wind direction changes, the performances of both rotors fall out of balance and one will perform better than the other. This behaviour is crucial regarding the optimization of wind farms containing many rotors, exposed to varying winds. Dabiri *et al.* [6] performed their measurements on many farm layouts, testing two, three and six-VAWT's configurations. Giorgietti *et al.* [21] performed numerical simulations including 4 turbines in a trapezoidal pattern composed of a counter-up configuration at the basis, co-rotating layouts on the sides and a counter-down configuration at the top. Before investigating wind farms, a thorough comprehension of the effect of wind inflow angle on the double-rotor layout is needed.

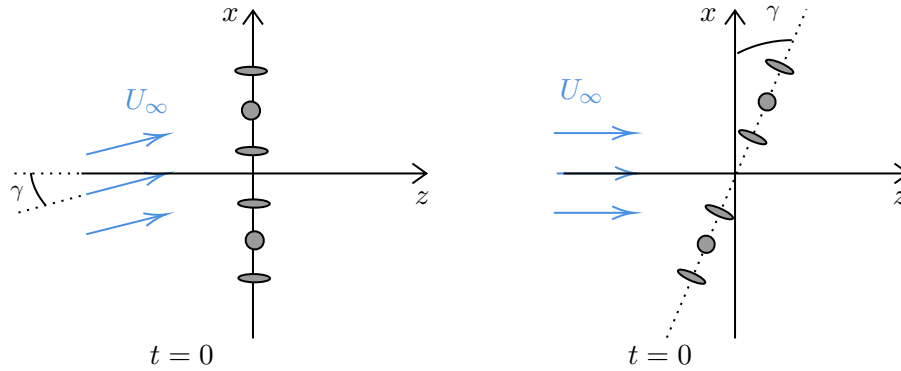


Figure 2.13: Concept of wind direction with an inflow angle γ and its practical implementation in the aerodynamic solver VPM. The shaded areas are the vertical-axis wind turbines in counter-up configuration. Turbine's centers are moving along a circle of radius $R = \frac{d_{int}}{2}$.

2.5.1 Methodology and setup

Nineteen simulations were here performed, with a flow angle ranging from $\gamma = 0^\circ$ to $\gamma = 90^\circ$ by increments of $\Delta\gamma = 5^\circ$. These will be referred to as CU_{00} to CU_{90} setups. As for the previous parametric study on the turbine spacing, the counter-up configuration is chosen, showing overall better performances compared to the co-rotating and counter-down configurations. A yaw angle γ of zero degrees represents the former cases, where the incoming wind velocity direction is perpendicular to the axis linking the rotor's centers. The practical implementation into the aerodynamic solver needs to adjust the turbines positions instead of the incoming wind velocity. The figure 2.13, shows that to reproduce an inflow angle γ , the turbines are yawed while their spacing remains constant. A complete wind rose hence represents turbines moving along a circle with a radius of $R = \frac{d_{int}}{2}$.

The top rotor -or north rotor- (positive x -axis), gradually moves downstream in the wake of the bottom rotor -or south rotor- (negative x -axis). They will also be referred to as the back and front rotor respectively. Note that the blades are aligned with the axis joining the rotors centers at time $\hat{t} = 0$ and that a constant tip-speed ratio is applied throughout the simulations. This imposed kinematics can not capture any phase effects that would occur with wind direction. The synchronization found by Vergaerde *et al.* [24] may be lost and needs further investigation. Other geometrical considerations are provided in table 2.1.

The aspect of interest is the global performance of the double-rotor layout against wind direction. The simulations were performed with the same computational domain as that of the previous parametric study, *i.e.* a width of $4D$ in the transverse direction and a depth of $9D$ in the streamwise direction. The domain is discretized by approximately 3.5 million particles. For all coming diagnostics and wake analysis, some wind inflow angles γ are chosen to represent the associated behaviours. Note also that, as the configuration

2.5. Effect of the wind inflow angle

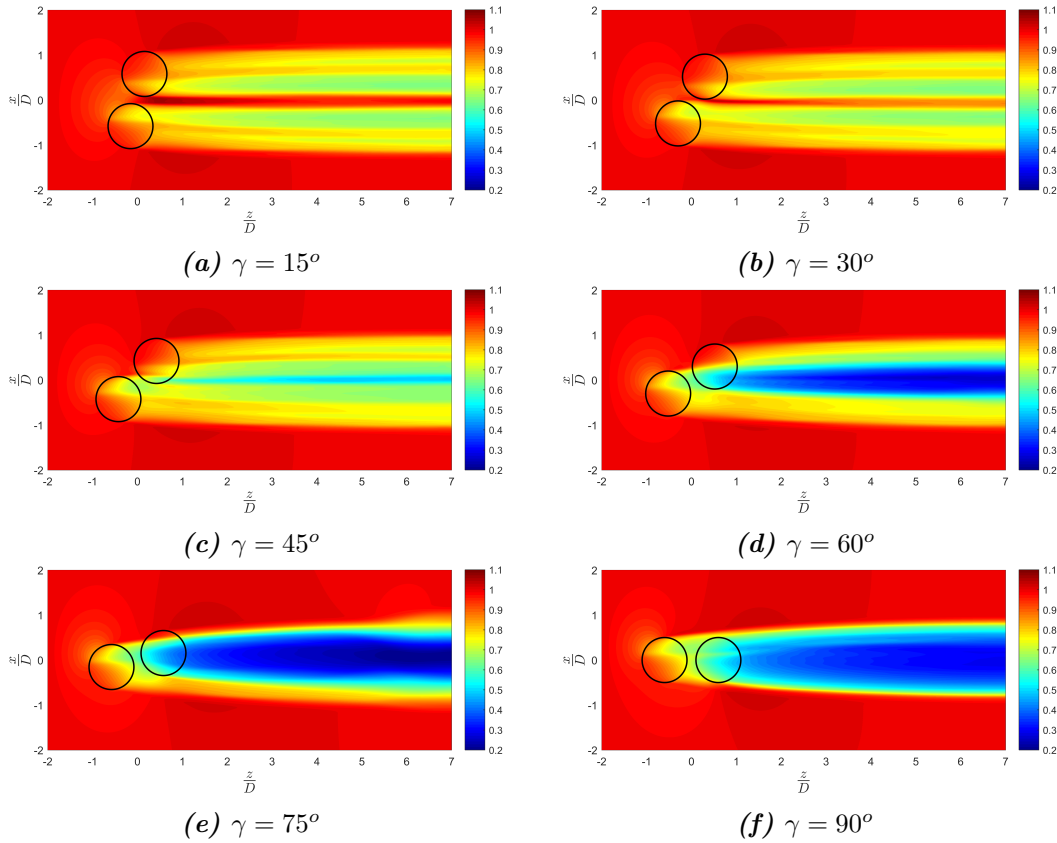


Figure 2.14: Time-averaged streamwise velocity field $\bar{w}$ for the counter-up configuration given 6 wind directions. The streamwise velocity is normalized with the free stream velocity U_∞ . Simulation parameters are those of table 2.1 and corresponding inflow angles γ are given. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

is counter-up, the front rotor is taken as reference. Further analysis should include the aerodynamics of the back rotor as well.

Figures 2.14a–2.14f present the time-averaged streamwise velocity field $\frac{\bar{w}}{U_\infty}$ for 6 different inflow angles. As the top rotor moves backward, *i.e.* as γ increases, the rotor gradually enters the wake of the front turbine. For $\gamma = 15^\circ$ and $\gamma = 30^\circ$, we observe that the inner sides of the wake are more and more bent. The fall out of balance of the performances is mainly due to blockage effects on both turbines. This bending occurs up to the limit where there is no opening left for the incoming flow. Starting from $\gamma \simeq 45^\circ$, the wake of the upstream rotor impinges on the back rotor one, leading to a drastic reduction of the performances. The wakes of each rotor are composed of two lobes associated with regions of high loading, in the inner-upwind and outer-downwind parts. For $\gamma \simeq 90^\circ$, the main lobe of each wake disjoins, enabling the back rotor to experience higher inflow velocities from the stalled part of the front turbine. The upstream blockage effect of the turbines also becomes less significant and tends to that of the single rotor

2.5. Effect of the wind inflow angle

		CU_{05}	CU_{10}	CU_{15}	CU_{20}	CU_{25}	CU_{30}
C_p		0.1425	0.1425	0.1423	0.1418	0.1417	0.1413
$C_{p,rotor}$		0.1427 0.1422	0.1432 0.1418	0.1431 0.1415	0.1424 0.1413	0.1421 0.1412	0.1414 0.1412
$C_{p,norm}$		1.0001	1.0003	0.9990	0.9957	0.9944	0.9918

		CU_{35}	CU_{40}	CU_{45}	CU_{50}	CU_{55}	CU_{60}
C_p		0.1411	0.1401	0.1389	0.1379	0.1367	0.1358
$C_{p,rotor}$		0.1408 0.1414	0.1384 0.1419	0.1354 0.1424	0.1328 0.1430	0.1298 0.1437	0.1272 0.1445
$C_{p,norm}$		0.9903	0.9836	0.9750	0.9678	0.95989	0.9535

		CU_{65}	CU_{70}	CU_{75}	CU_{80}	CU_{85}	CU_{90}
C_p		0.1331	0.1307	0.1264	0.1240	0.1245	0.1280
$C_{p,rotor}$		0.1200 0.1462	0.1150 0.1463	0.1054 0.1474	0.0997 0.1484	0.0997 0.1493	0.1063 0.1497
$C_{p,norm}$		0.9344	0.9172	0.8874	0.8704	0.8741	0.8987

Table 2.5: *H-type VAWTs global performances for different wind directions γ . The output power coefficient C_p is given for each setup and normalized with the power coefficient of the single rotor. The contribution of each individual rotor $C_{p,rotor}$ is given for the back and front rotor respectively. Setup parameters are those of table 2.1.*

configuration. As the back rotor moves downwards, the global shape made by the two turbines gets sharper, and the outer sides of the wakes experiences a smaller deviation.

2.5.2 Diagnostics and performances

The suggestions made earlier are confirmed by the values of power coefficient obtained for the different wind directions. Table 2.5 provides for each setup the global performances and the power coefficient obtained by each rotor. The first value given of $C_{p,rotor}$ is that of the back rotor.

Up to a certain deflection angle, the back rotor performs better than the front rotor. The normalized power coefficient for the average, north and south rotors, are given with respect to the wind direction in a polar plot in figure 2.15. For a yaw angle γ of 5° and 10° , the double-rotor layout performs better than the single configuration. The mechanism responsible for the better performance of the back rotor is the upstream turbine blockage. Because of the presence of the front turbine, the outer flow is accelerated and this high-velocity flow impinges on the upwind part of the back rotor. Besides, due to the presence of the back rotor, the front rotor's wake is unable to fully develop, and is forced to deviate when facing the back rotor (see figure 2.14b). From that angle, the two wakes are forced to merge and the performances of the front rotor are enhanced. This increase is slightly reduced about $\gamma = 60^\circ$ due to the backpressure generated by the downstream turbine, attenuating the mass flow in the inner-downstream part. Globally, the losses of the back rotor outbalances the potential gain of the front rotor, causing a lower power coefficient for the setup. For a counter-up configuration, the induced velocities of the back rotor tend to reduce the mass flow of the front rotor for angles above $\gamma = 60^\circ$, as they are directed towards negative x -values. The performances of the front rotor should hence decrease. The increase encountered here can partially be explained by the low tip-speed ratio, unable to induce large velocity fields compared to the incoming free stream velocity. An insight on the blade loadings elucidates the gains of the front rotor.

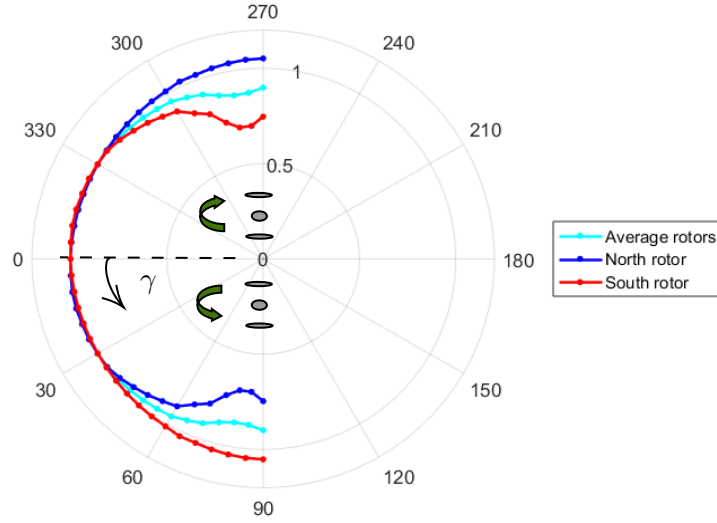


Figure 2.15: Effect of wind direction on power coefficient for counter-rotating layouts. Power coefficients are given for each setup and each rotor, all normalized by the single configuration.

According to our values, the worst performance is obtained for an inflow angle of $\gamma \simeq 80^\circ$, where the back rotor lies in the unstalled part $\theta \in [0^\circ, 90^\circ]$ of the upstream rotor. As γ increases further, the two main lobes disconnect and the normally unstalled region of the back rotor encounters the stalled region of the front rotor and hence higher velocities. This explains the global increase in power coefficients for those angles. Note that the above findings only count for off-design tip-speed ratios, causing significant dynamic stalls in the upstream ($\theta = 60^\circ$) and downstream ($\theta = 260^\circ$) regions.

Figures 2.16a–2.16d provide the evolution of the angle of attack α , the global power coefficient evolution C_p for some of the studied wind directions, and the normal and tangential force coefficients, C'_t and C'_n acting on a single blade. Inflow angles are chosen as to pinpoint specific behaviours of the double-rotor configuration, namely the slight power gain for low values of γ , moderate values of γ for which the back rotor enters the wake of the front rotor, and large values of γ where the front rotor outperforms the back rotor. For the sake of clarity, only three setups are shown for the global power coefficient evolution with azimuth position. We recall that the figures provided take the front rotor as reference, being counter-clockwise. Few differences are observable regarding the angle of attack for a large range of azimuth positions. The angle of attack is slightly shifted towards lower azimuth values in the inner-upwind part for perpendicular wind directions, resulting in higher loadings in the region $\theta \in [0^\circ, 60^\circ]$. The blockage effect of the back turbine does not cancel the transverse velocity anymore as it moves downwards, and the streamlines encountering the inner part of the front rotor are more deflected. The main differences occur in the downstream region, where angles of attack reach lower absolute

2.5. Effect of the wind inflow angle

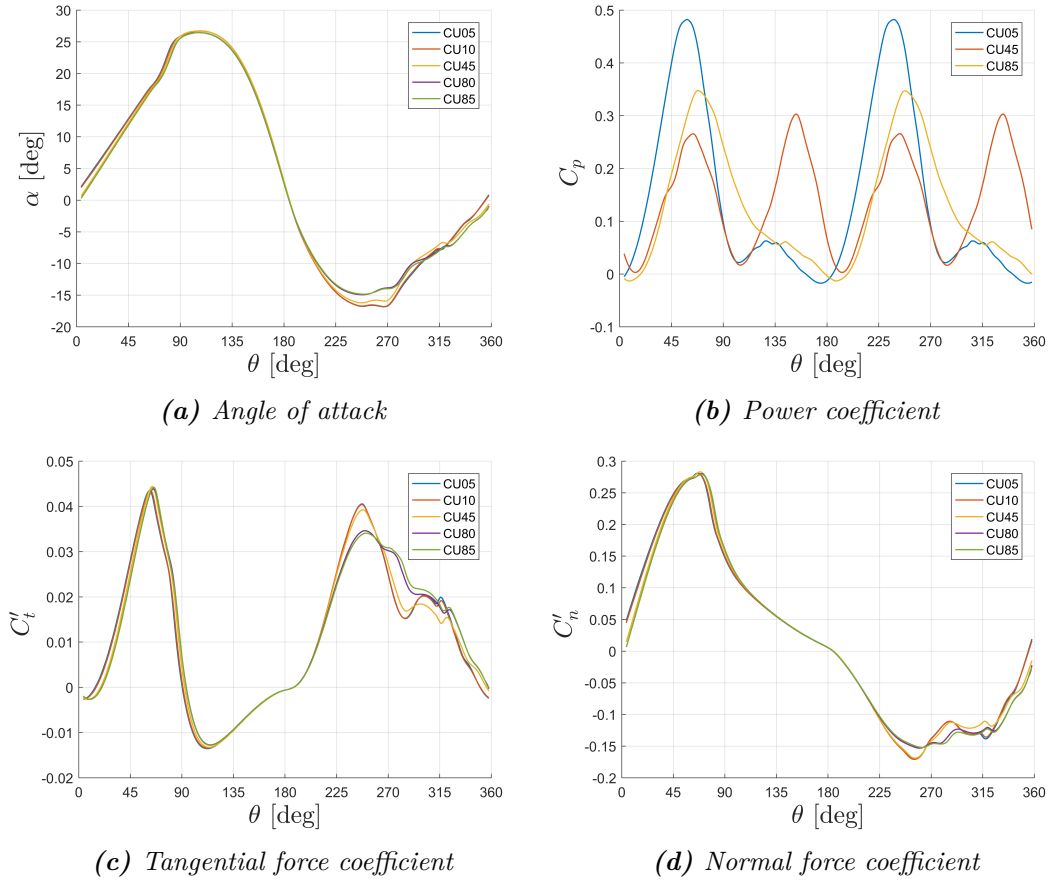


Figure 2.16: Diagnostics of the H-type VAWTs for different wind directions. Simulation parameters are those of table 2.1. Values are averaged over 10 rotor revolutions and given for the reference rotor.

2.5. Effect of the wind inflow angle

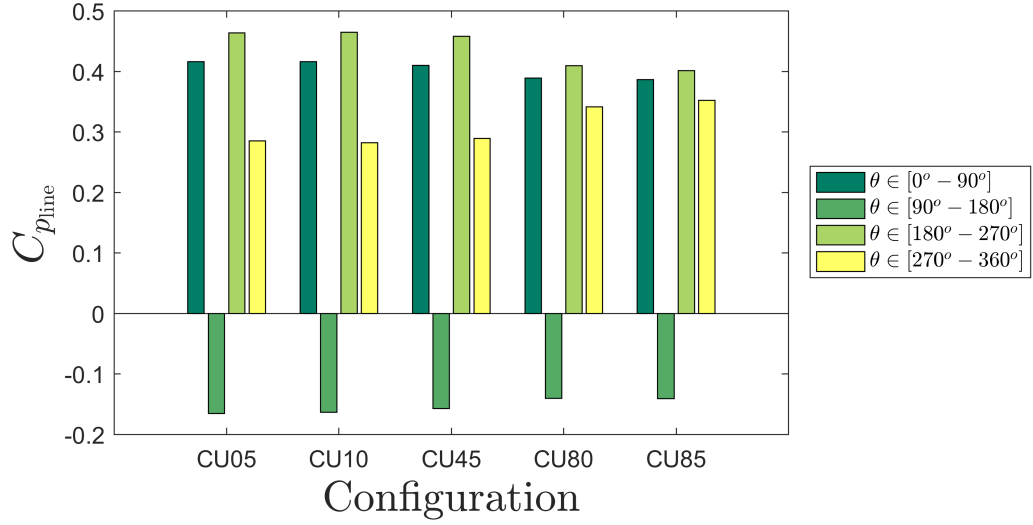


Figure 2.17: Quartile distribution of the power coefficient of a single blade for different wind directions. Setup parameters are those of table 2.1.

values when the back rotor is directly behind the front rotor. This lower angle of attack is beneficial for the front rotor, as it delays the dynamic stall observed in figure 2.16c. The backpressure of the back rotor further compresses the detaching flow on the blades of the front rotor in the downstream region. This phenomenon explains, for large γ angles, the better performances of the front rotor characterized by low tip-speed ratios and hence large dynamic stalls. This may however not be the case for rotors at their optimum tip-speed ratio (TSR), as suggested by [23], where the induced velocities in the x -direction become significant enough compared to the upstream velocity U_∞ . This is confirmed by figure 2.17, showing the quartile distribution of power produced by each blade of the front rotor. The produced power in the upwind part hardly changes from $\gamma = 5^\circ$ to $\gamma = 85^\circ$, whereas power production in the last quarter is greatly improved by the backpressure of the north rotor. The latter also causes a decrease in power production in the third quarter by reducing the massflow through the outer-downwind part, but the attenuation of dynamic stall yields higher gains.

The latter phenomenon also delays the peak power production compared to the CU_{05} configuration. The maximum C_p values spread across a larger azimuth range. For the CU_{45} setup, four different production peaks are present. This occurs because of a phase difference in loading with respect to the wind direction. starting from $\gamma = 0^\circ$, the turbines and hence peaks are in phase, but when increasing the inflow wind angle, the front and back rotor will see their alignment with the upstream wind delayed, creating a delay in the respective regions of loading. This is not captured by the angle of attack values because of the chosen definition of θ . The power peaks split into two smaller peaks up to $\gamma = 45^\circ$, and joins back together afterwards. A schematic is provided in figure 2.18.

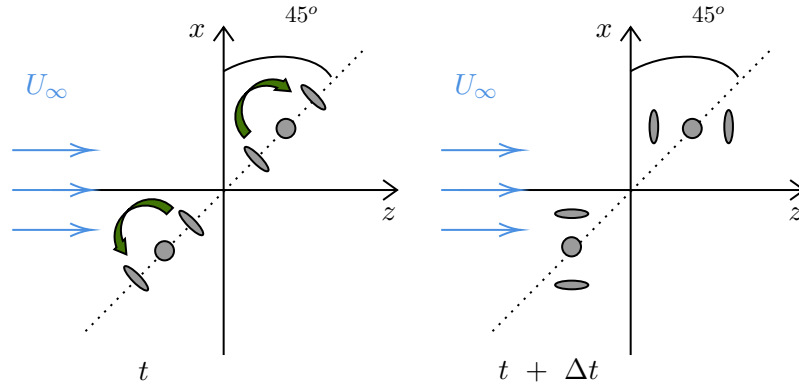


Figure 2.18: Phase differences in orientation with respect to the upstream velocity causing a delay in loading and peak power production between the turbines.

Spacing and wind directions are crucial when studying the double-rotor layouts. Indeed one directly impacts the other, as the available window where the back rotor is not in the wake of the front rotor depends of the turbine inter-distance. The design of a double-rotor configuration, especially in a farm, should be seen as a global optimization problem balancing the loss caused by the wind directions, and the gain obtained for smaller spacings. This study has been partially conducted by Ning et al [20]. The double-rotor layout is also site dependent: for a uniform wind-rose, *i.e.* a wind equally probable in all directions, figure 2.15 suggests an expected value around 95% of the power output given by a single turbine in the same flow conditions. The double-rotor layout is the most efficient for very directional winds and close turbine inter-distances. Finally, the present analysis enforced same tip-speed ratios on both turbines. Enabling the back rotor to adapt its TSR with the wind direction should also be part of the global optimization. The previous discussion could also give some momentum in designing creative wind farms subjected to equally probable wind directions: instead of staggered or structured grid, one could imagine placing turbines on the vertices of pentagons, hexagons, ...

2.5.3 Wakes

In order to analyze the feasibility of the proposed farm designs, the wakes and wake recoveries have to be understood. When changing the wind direction, we loose the symmetry of the wakes, and even enforce their merging. Mixing is imposed by the wind direction and will likely cause more intense stretching and distortion of the shedded vortices in the wakes. This fastening transition to turbulence has to be compared against the loss in velocities caused by the back rotor, which further extracts power from the flow. Figures 2.19a–2.19b present the non-dimensional displacement and momentum widths L_1 and L_2 .

Values at zero and one diameter downstream have to be taken with care, as the extend of the blades may influence the obtained values up to 1.7 diameters downstream for the

2.5. Effect of the wind inflow angle

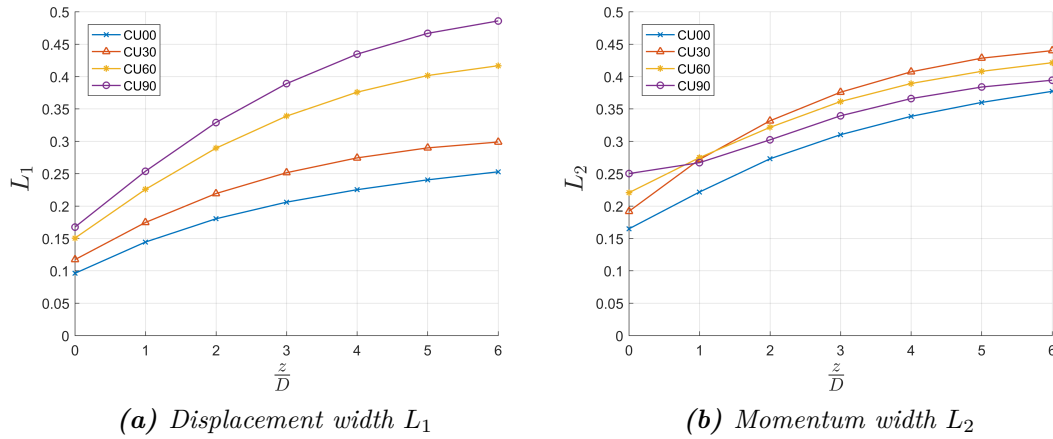


Figure 2.19: Displacement and momentum width L_1 and L_2 based on the statistical streamwise velocity deficit at each diameter downstream for different wind directions γ . The parameters of the simulations are those of table 2.1.

worst case, *i.e.* when the rotors are aligned ($\gamma = 90^\circ$). Also, the narrow wake envelope resulting from a low TSR and the short computational domain does not allow to observe any benefits of forced mixing in the far wake, leading to fully-turbulent flows. We observe instead in the near wake a higher velocity deficit for each diameter downstream when the turbines get aligned with the wind direction. The behaviour is the one expected: the values of L_1 are proportional to the mean velocity deficit on slices taken at each diameter downstream. The presence of the back turbine in the direct wake of the front turbine highly decelerates the flow down to 20% of the free stream velocity U_∞ compared to 65% at $\gamma = 15^\circ$. The curves show similar trends as in sections 2.3 and 2.4, except for the momentum width value of CU_{90} in the slice of the front turbine's plane: not having a direct neighbour in the flow anymore, the momentum width L_2 yields higher values, while still accounting for the number of rotors in the flow.

2.6 Conclusions

Based on the same parameters as De Tavernier *et al.* [23], a set of two-dimensional simulations were performed to understand the physics behind the double-rotor layout. The three main configurations, -namely counter-up (CU), counter-down (CD) and co-rotating (CO)- were studied and compared against a single rotor in the flow. Additionally, the effect of turbine inter-distance and wind direction on the counter-up layout has been explained. At this point, several conclusions are drawn:

- The double-rotor layout promises higher power coefficients, without individual improvement of the rotor's efficiency. The interactions between them increase their loading, especially in the inner-downstream part.
- The counter-up configuration yields the highest power output.
- The double-rotor layout is sensitive to turbine inter-distance and wind direction. A close arrangement combined with directional winds is advised to enhance the performances.
- The low tip-speed ratio and the ill-calibrated polar resulted in regions with important dynamic stall, greatly influencing the performances and the wake behaviours. The off-design tip-speed ratio prevented to obviously highlight the positive interactions between the turbines. Nevertheless, we explained that the backpressure of the back rotor delays the dynamic stall on the blades of the front rotor, yielding higher power outputs for large wind inflow angles.

Further analysis could be insightful to fully understand the complex aerodynamics and wakes of vertical-axis wind turbines in double-rotor layouts. The effect of spacing, wind direction and individual tip-speed ratios should be part of a global optimization. Moreover, two-dimensional analysis lacks in accuracy as no tip-vortices are shed by the blades. The lack of vertical mixing partly prevents a fast transition to fully turbulent wakes.

Chapter 3

Double-rotor vertical-axis wind turbines: a 3D study

This chapter is a fully three-dimensional analysis of the double-rotor concept, based on what will be referred to as the *VUB* setup. The chapter is composed of three parts: in the first part, the wakes and performances of each configuration are studied, in the second part, the effect of blade number is explained while the last part highlights the influence of the aspect ratio.

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3.1 Introduction

The previous two-dimensional analysis has a main limitation compared to fully three-dimensional resolved flows around vertical-axis wind turbines: the periodic condition previously imposed in the spanwise direction suppresses tip blade effects. The normally shed tip vortex at the edges of the blades induces velocities in the spanwise and transverse direction over a revolution. These new induced velocities, absent from two-dimensional models, greatly modify the flow field in the very near wake and the performances of the turbines, but also add a vertical mixing in the mid wake. Moreover, the tip vortices, when advected downstream, create complex vortical structures and interact with the shed vortex sheet, leading to reconnection events and ultimately to the transition to turbulence. Wake recoveries predicted by the two-dimensional analysis are usually lower than for the full 3D cases [22].

Most research done about vertical-axis wind turbines used a two-dimensional approach, where few investigations have attempted to fully solve their aerodynamics and wakes in three dimensions. Among others, Porté-Agel *et al.* [7] investigated the wake behind a vertical-axis wind turbine (VAWT) in a three-dimensional turbulent flow, using a turbine model and Large Eddy Simulation (LES). Shortly after, they imposed as inflow condition an undisturbed and precomputed atmospheric boundary layer to their original setup, getting closer to real external flow conditions [8]. Boudreau and Dumas used a three-dimensional Delayed Detached-Eddy Simulations (DDES) model and the Unsteady Reynolds-averaged Navier-Stokes equations to compare the behaviour of horizontal- and vertical-axis wind turbines [29]. Finally, Chatelain *et al.* used the present VPM code to investigate the aerodynamics and wakes of VAWTs in both uniform and turbulent inflows [30], [15]. To the authors knowledge, it is the first time that VPM methods are used to analyze the aerodynamics and wakes of the different double-rotor layouts presented in chapter 2, fully solved in the three dimensions.

The present analysis is inspired by the experimental setup of Vergaerde *et al.* [24], from now on denoted as the *VUB* setup. First, using their parameters, we compare the three double-rotor configurations, -namely counter-up (CU), counter-down (CD) and co-rotating (CO)- to the single rotor in the flow. Then, the effect of blade number is inspected. Finally, and to conclude on the double-rotor layout, the influence of the aspect ratio (AR) of the turbines is assessed, and compared to the two-dimensional case when the blade's height becomes theoretically infinite.

3.2 Co- and contra-rotating configurations

3.2.1 Methodology and setup

The experiments conducted at the wind tunnel lab of the Politecnico di Milano concern two closely spaced H-type Darrieus vertical-axis-wind turbines. The latter are scaled-down models of a full-scale VAWT design, with similar geometrical layout. Each turbine is made of two straight blades with a NACA0018 profile and blades are connected to the mast with two-inclined struts, as for the *Twinfloat* concept presented earlier (figure 1). They focused their research on the two symmetric configurations: counter-down and counter-up, and compared their performances to a single rotor in the wind tunnel.

In order to repeat the experimental results, the same non-dimensional quantities -chord-based Reynolds number Re_c , tip-speed ratio λ , solidity σ and aspect ratio AR - were implemented in our numerical simulations. The tip-speed ratio λ is set to 3. This value coincides with the highest power coefficient of their experimental results for the counter-rotating layout. This tip-speed ratio is very close to the optimum one, that is to say $\lambda_{\text{opt}} = 2.968$, found by the fitting of Rezaeiha *et al.* [27] relating optimal tip-speed ratio and solidity (see equation (2.4)). Unlike the *Delft* setup presented in section 2.3, we also chose equivalent geometrical characteristics of the turbines. The parameters used for our simulations and the resulting non-dimensional numbers are given in table 3.1.

The turbines modeled are small turbines with a medium solidity σ . The turbine spacing d_{int} , which we have found the double-rotor layout is very sensitive to, is set to 1.3. This value is slightly larger than for the two-dimensional analysis conducted in chapter 2. Simulations for the three double-rotor layouts and the single rotor were performed over $\hat{t} = 100$ non-dimensional times, whereas the computational domain is a 3D box of $[8D \times 8D \times 24D]$ in the transverse, spanwise and streamwise directions respectively. The box is large enough to minimize the influence of the inflow and outflow conditions. The computational mesh has a spatial increment of $h = 0.025 \text{ m}$, resulting in 20 points per rotor diameter N_D and more than 12 million particles. The time step used in *VPM* is adaptive and its maximum value is set to $\Delta t = 5 \cdot 10^{-4} \text{ s}$ corresponding to an azimuth increment of $\Delta\theta = 3.44$ degrees. One of the main findings of the experiments conducted by Vergaerde *et al.* is that the rotors in counter-rotating configurations tend to synchronize. The phase difference between the two rotors oscillates around a null value. Therefore in the present work the 2 blades of each rotor are aligned on the x -axis at the very beginning of the computation. The blades are also pitched inwards by an angle $\alpha_0 = 2.85^\circ$, to add a fictive camber. This correction on the lifting-lines permits

$U_\infty \left[\frac{m}{s} \right]$	$D \text{ [m]}$	$c \text{ [m]}$	$H \text{ [m]}$	N	d_{int}	Re_c	σ	λ	AR
10	0.5	0.05	0.8	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	1.6

Table 3.1: Geometrical parameters of the Vergaerde *et al.* and UCL simulations with the corresponding non-dimensional numbers

to capture flow curvature effects, and results and diagnostics are very sensitive to the latter correction. The parameters for the Leishman-Beddoes dynamic stall model are in the present case tuned from a NACA0015 airfoil to fit the static behaviour of the used polar, at a relative chord Reynolds number $Re_c = \frac{U_{rel}c}{\nu}$ of 150 000, comparable to the experimental Re_c .

The experiments performed in the wind tunnel of the Politecnico di Milano are achieved in a confined milieu compared to the outside environment where full-scale VAWTs are meant to operate. Their results were corrected to account for the blockage effect caused by the finite environment. In *VPM* however, the unbounded directions are exactly solved, and no blockage correction on the upstream velocity U_∞ is needed. Other differences remain between their experimental setup and our numerical computations. Among them, we can point the presence of the struts and masts, not implemented in *VPM*, also generating wakes and aerodynamic loads. Besides, their turbines are controlled in rotational speed as to reach and maintain a design TSR, whereas constant rotational speed is imposed in *VPM* from the start.

Figures 3.1a–3.1d present the time-averaged streamwise velocity $\bar{w}$ normalized by the free stream velocity U_∞ at mid blade for all configurations. Statistics are taken over $\hat{t} = 50$ non-dimensional times, beginning at $\hat{t} = 30$ non-dimensional times in order to reach steady-state conditions. Compared to the previous two-dimensional analysis, a clear difference is the merging of the two wakes within the computational domain and the disappearance of the two-lobed structure. Instead, a recirculation bubble is found about 5 diameters downstream for all four layouts, and its shape strongly depends on the configuration. The highest recirculation values are found for the single configuration. The wakes of symmetric configurations exhibit symmetric behaviours, whereas the co-rotating configuration shows a very asymmetric time-averaged velocity field. The wake bending effect is important. The wakes of counter-clockwise rotating turbines tend to deviate to the positive x -axis, and the clockwise rotating turbines to the negative x -axis. This causes for the single and co-rotating configurations to exhibit a wake heading to the left side of the domain. The counter-down and counter-up expose diverging behaviours. The wake bendings of their turbines pushes the wakes apart and towards each other, respectively. The counter-down configuration is expected to exhibit a wake about the same height as the single configuration, whereas the wakes of the counter-up layout being pushed together will stretch in the spanwise direction. For the latter case, the velocity deficit is elongated in the streamwise direction while for the former, it is dilated in the transverse direction. Another velocity deficit is found in the upstream part caused by blockage effects, close to the most upstream position of the blades for every configuration, while double-rotor layouts also exhibit an accelerated flow at the inner sides of the turbines, being the most intense for the CU configuration.

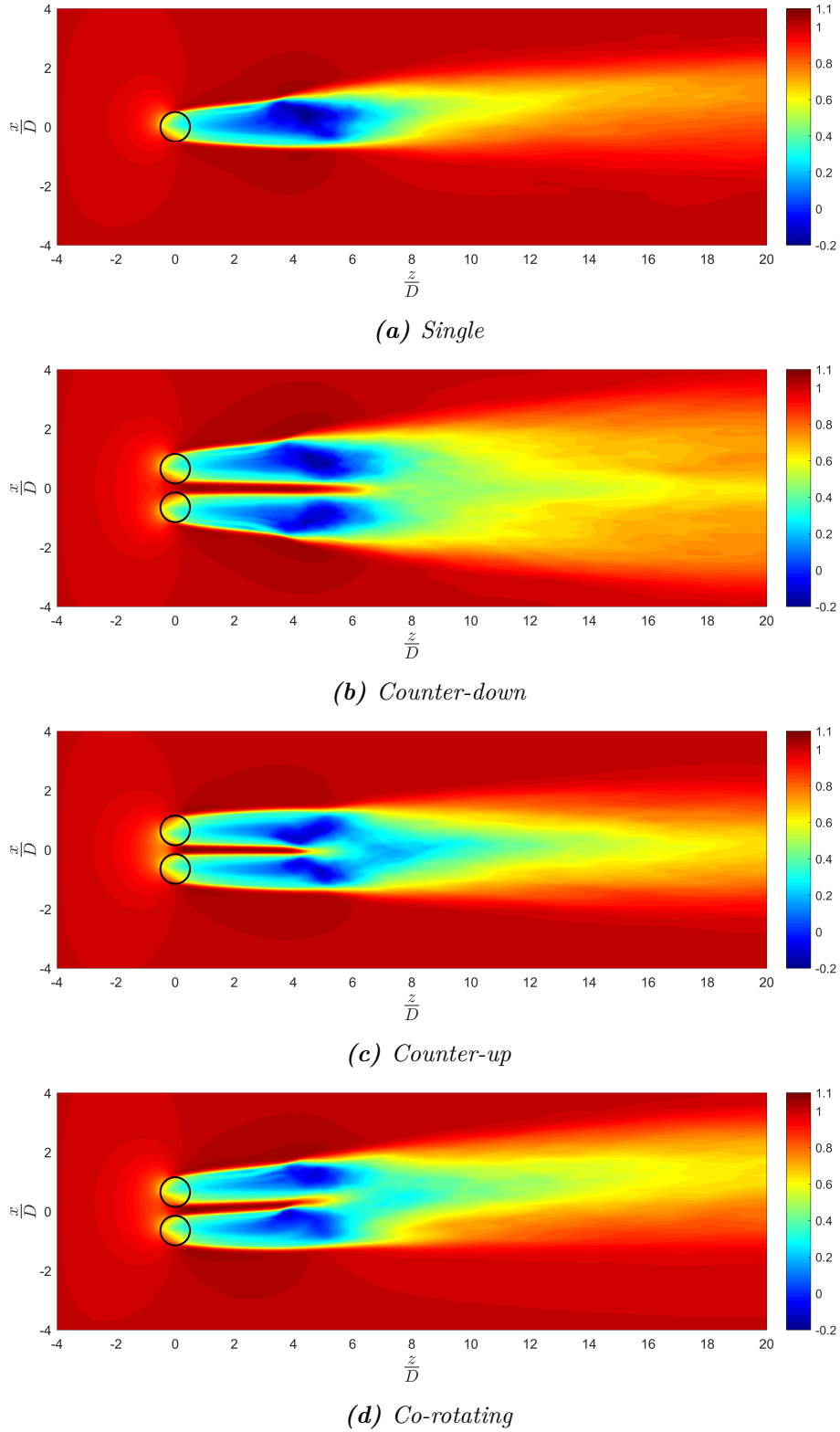


Figure 3.1: Time-averaged streamwise velocity field around H-shaped rotors for the 4 configurations. The streamwise velocity $\bar{w}$ is normalized with the free stream velocity U_∞ . Simulation parameters are those of table 3.1. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

3.2.2 Diagnostics and performances

Diagnostics about the rotor and the lifting-lines are provided by *VPM* at each time step. The non-dimensional numbers characterizing the performances (equation (2.2)) are adapted for three-dimensional flows, where the actual height H of the turbines is used in the expression of the turbine extraction plane $A = H \cdot D$.

$$C_p = \frac{\tau\omega}{\frac{1}{2}\rho U_\infty^3 A} \quad C_T = \frac{F_t}{\frac{1}{2}\rho U_\infty^2 A} \quad (3.1)$$

We keep however the definitions of the tangential and normal force coefficients per unit span, but retrieve the information locally at mid blade.

$$C_t = \frac{f_t}{\frac{1}{2}\rho U_\infty^2 c} \quad C_n = \frac{f_n}{\frac{1}{2}\rho U_\infty^2 c} \quad (3.2)$$

Table 3.2 contains the global power coefficients for the different configurations. The power coefficient for a double-rotor layout is the averaged power coefficients of each individual rotors, which are also given for the top (positive x -axis) and bottom (negative x -axis) respectively. The performances of the individual rotor is denoted as $C_{p,\text{rotor}}$. Finally, the normalized power coefficient $C_{p,\text{norm}}$ compared to the single configuration is provided. All values are averaged over the 10 last rotor revolutions, and the maximal standard deviation in the averaging process, associated with a specific azimuth position, is given by σ_{max} .

The different double-rotor layouts all present a gain compared to the single rotor. The gain is the lowest for the counter-down configuration and the highest for the counter-up layout, ranging from approximately 6% to 8%. Those values are in good agreement with the predictions of Dabiri *et al.* [6] but diverge from the gains experimentally obtained by Vergaerde *et al.* [24]. Their measured values indicated gains of about 17% and 14% for the counter-up and counter-down configurations respectively, but with absolute C_p values around 0.18 instead of the 0.28 obtained in the present simulations. We suggest numerical simulations overestimates the power coefficients as neither the mast and struts are modelled. Moreover, they mentioned some uncertainties due to their data acquisition

	single	CO	CU	CD
C_p	0.2682	0.2848	0.2905	0.2792
$C_{p,\text{rotor}}$	0.2682 /	0.2789 0.2907	0.2905 0.2904	0.2794 0.2789
$C_{p,\text{norm}}$	1	1.0618	1.083	1.0408
σ_{max}	0.007	0.0065	0.0107	0.0068

Table 3.2: *H*-type VAWTs global performances for all configurations. The output power coefficient C_p is given for each setup and normalized with the power coefficient of the single configuration. The contribution of each individual rotor is given for the top (positive x -axis) and bottom (negative x -axis) turbine. the maximum standard deviation is denoted by σ_{max} . Setup parameters are those of table 3.1.

instruments and the processing of this data [24]. For the co-rotating configuration, we observe a different contribution of each rotor to the global power coefficient. The rotor laying on the negative x -axis outperforms his neighbour by more than 4%. This highlights the interactions occurring between closely spaced turbines. For symmetric configurations each rotor participates equivalently to the power output, even though small differences remain. More importantly, each individual rotor of any configuration performs better than the single rotor in the flow. The interactions between neighbouring turbines are mutually beneficial.

The diagnostics over the azimuth position θ are given for one blade of the reference rotor, being the counter-clockwise rotating one in each configuration. We briefly recall that the cylinder swept by the blades is divided in four quarts: inner-upwind, inner-downwind, outer-upwind and outer-downwind, as shown in figure 2.1a and we refer to figure 2.1b for the definitions of the layouts, the azimuth angle θ and the rotor of reference. Figures 3.2a–3.2d respectively provide the evolution of the angle of attack α , the global power coefficient C_p and the normal and tangential force coefficients C_n and C_t acting at mid span on one blade over the azimuth position θ . The normal force acting on the blade is directed towards the center of the rotor, while the tangential force points towards the leading edge (see figure 2.4).

A positive angle of attack α corresponds to a relative velocity oriented from the outside to the inside of the cylinder swept by the blades. The evolution of the angle of attack is not symmetrical with respect to the azimuth position. The angle of attack reaches its maximum right after its most upstream position $\theta = 90^\circ$, and drops rapidly afterwards. In the downstream part, it increases slightly while oscillating. The blade in the downwind part already encounters the wake of the blade in the upwind part, and this causes variations in relative velocities in the downwind part, creating the oscillations visible from $\theta = 220^\circ$ in figure 3.2a. These oscillations give rise to variations of the lift and drag acting on the blades in the downwind part, and ultimately lead to fluctuations of the loadings C_t and C_n . A negative power coefficient is obtained when the rotors add energy to the flow at constant rotational speed ω . The power coefficient is here given as an average between both rotor's power coefficients, taking into account the contributions of each blade. This results in two main production peaks as no phase differences exist between the rotors. The rapid drop in tangential loading after its most upstream region is an indicator of a stalled region. Indeed the angle of attack increases even further after the tangential force acting on the blade vanishes. However, compared to an off-design tip-speed ratio as in section 2.3, the region where the dynamic stall occurs is delayed further in azimuth position and its extend is shortened, enhancing the performances. The region where the blades are stalled is clearly observable on the time-averaged streamwise velocity field in figures 3.1a–3.1d at the beginning of the second quarter swept by the blades. Without loading, no power is extracted from the flow and the streamwise velocity does not show any deficit. Unlike before, no stalled region is to be found in the downstream part of the turbines, the tip-speed ratio and hence tangential speed ωR being sufficient enough

3.2. Co- and contra-rotating configurations

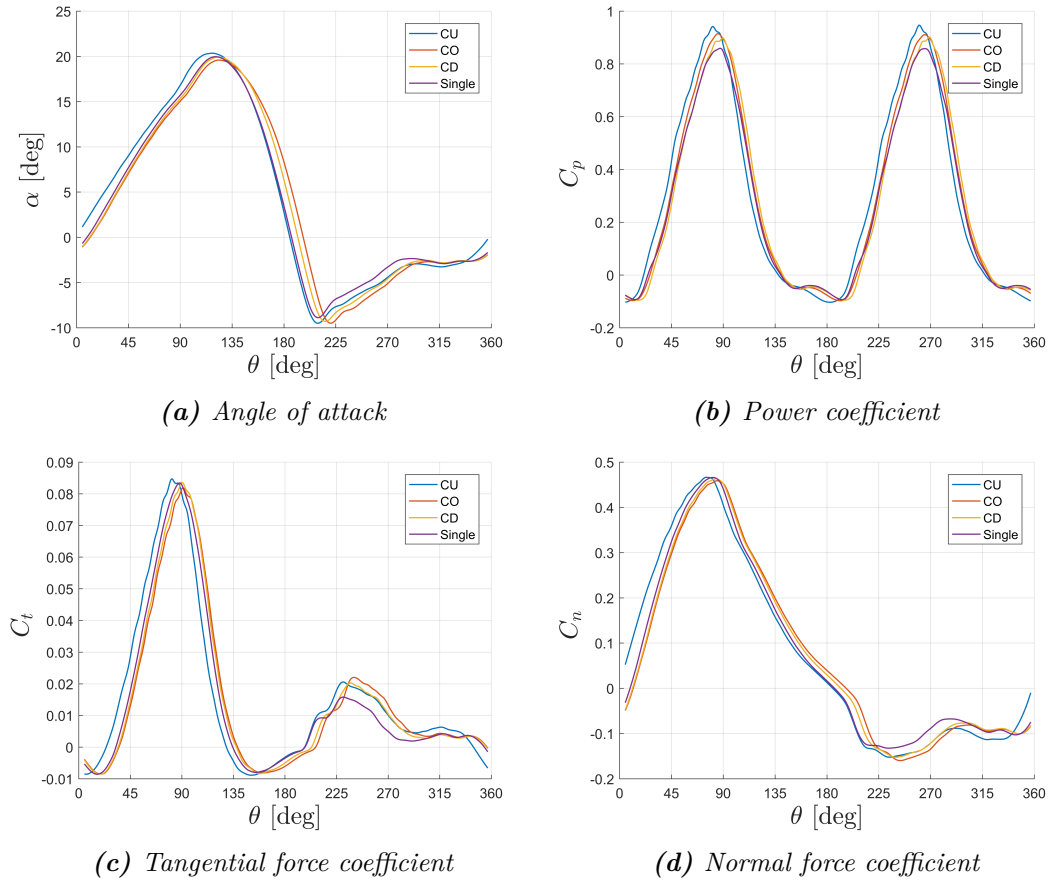


Figure 3.2: Diagnostics of the H-type VAWTs for all 4 configurations. Simulation parameters are those of table 3.1. Values are averaged over 10 rotor revolutions. Diagnostics are given for a single blade of the reference rotor.

compared to the streamwise velocity in the wake coming from the upstream part. This result in lowers absolute values of α , especially in the downstream region.

The induced velocities of the neighbouring rotor are globally beneficial for each configuration, but through different mechanisms. The presence of the neighbouring turbine influences both the transverse (x -axis) velocity u and the streamwise (z -axis) velocity w in between the turbines, yet to a lower extend. This effect is predominant for the counter-up configuration, where the streamlines are prevented to diverge laterally and are even tightened together, as suggested in figure 3.4c presenting the time-averaged transverse velocity $\frac{\bar{u}}{U_\infty}$. This causes a favourable opening of the angle of attack in the inner-upwind part: the angles of attack are shifted towards lower azimuth positions. This translates to a direct gain in torque and thus in power for this region. The induced velocity also causes an increase in transverse velocities for the outer-upwind part, but less pronounced because of a higher distance. The angles of attack of the CU configuration

and the single configuration are almost superimposed in the outer-upwind region. The counter-up configuration produces most of its power in its first quarter of revolution. For the reference rotor of the counter-down configuration in its first quarter, the streamlines are not tightened. The flow is free to diverge in the transverse direction without any neighbouring turbine. Moreover this outer-upwind part of the flow feels the induced velocities of the other rotor, further increasing the transverse velocity. This results in a less favourable angle of attack and a decreased power production. For their inner upwind parts, the angle of attack is increased but mainly due to the blockage effect, and this phenomenon is less pronounced so it leads to a mean null value of the transverse velocity in figure 3.4b. The angles of attack are shifted to higher azimuth positions. For the co-rotating layout, only the bottom turbine benefits from the induced velocities in the inner-upwind region, explaining the fall out of balance in power coefficients of table 3.2. The upper turbine, being the rotor of reference, hence shows a behaviour similar to that of the single turbine. The co-rotating and counter-down configurations yield for the second quarter $\theta \in [90^\circ, 180^\circ]$ better power output compared to the single configuration due to the upstream blockage effect.

As a summary, figure 3.3 presents the distribution per quartile of the power achieved by a single blade for all configurations. For all double-rotor layouts, the induced velocities and the blockage effect have a predominant influence on the upwind parts of the turbines, whereas the downwind part is less affected. This is not unexpected as the wake of the upstream part disturbs in a comparable way for all configurations the relative velocities as seen by the blades in the downwind part, and hence their performances.

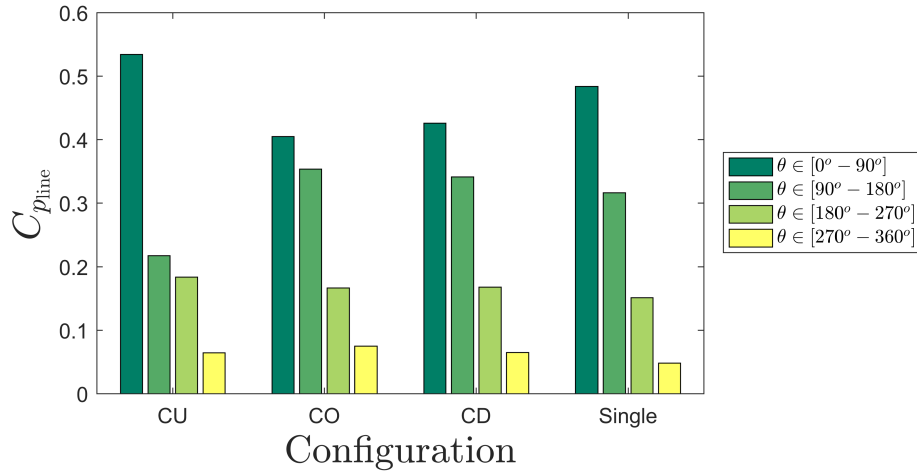


Figure 3.3: Quartile distribution of the power coefficient of a single blade of the rotor of reference. The parameters are those given by table 3.1.

3.2. Co- and contra-rotating configurations

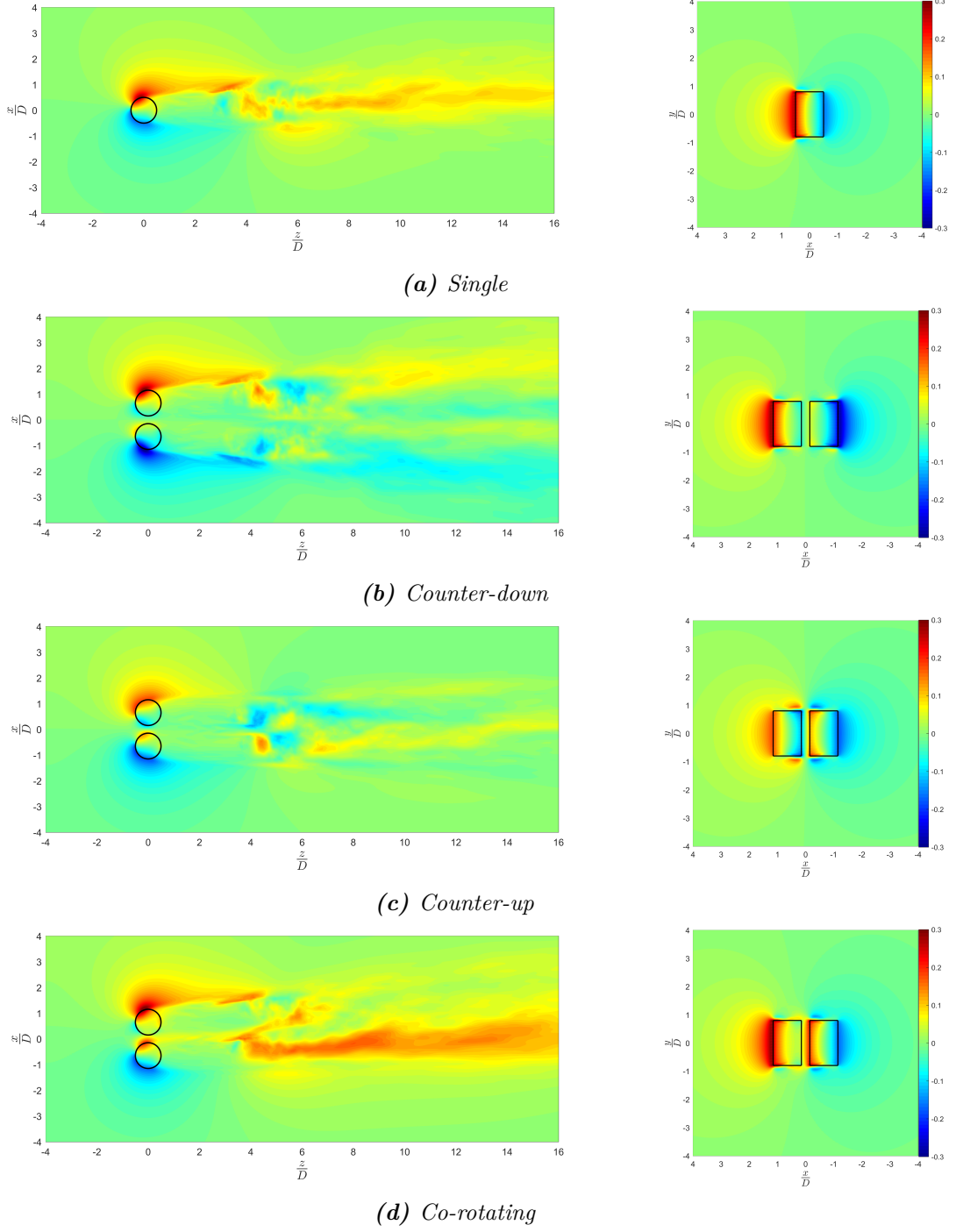


Figure 3.4: Time-averaged transverse velocity $\bar{u}$, normalized by the free stream velocity U_∞ for all configurations in the $x-z$ and $x-y$ plane of the turbines. Simulation parameters are those of table 3.1. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

3.2.3 Wakes

Three-dimensional wakes are far more complex than two-dimensional wakes, due to the tip vortices continuously shed at the edges of the blades. The wakes are composed of two vortical structures: tip vortices and vortex sheets. The tip vortices constitute the top and bottom sides of the wakes, whereas the vortex sheet forms the lateral sides. According to Prandtl's lifting-line theory presented in section 1.1, both types of vortices are due to the spatial and temporal variation of circulation Γ . The circulation varies over a revolution and results in complex wakes and induced velocities in the flow. Figure 3.5 presents for both three-dimensional and two-dimensional cases the orientation of the vortices shed by a blade in its first quarter of revolution $\theta \in [0^\circ, 90^\circ]$. The tip induced velocities are oriented towards the rotor's center over a revolution, and their magnitude depends on the azimuth position of the blades.

Figures 3.6a–3.6d present different views of the instantaneous vorticity magnitude $\|\boldsymbol{\omega}\|$ obtained by volume rendering for all configurations. The instant visualization is taken at $\hat{t} = 80$ non-dimensional times. When advected downstream, the vortical structures become unstable and interact. Mainly at the corner of the wakes, *i.e.* the region where the two types of vortical structures intersect. For a single turbine, their interactions cause intense stretching, as the stronger vortices distort the weaker ones. This highly contributes to the transition to a turbulent flow. Starting from the corners, the flow reaches a fully turbulent state when the instabilities grow from the corners towards the mid plane and ultimately merge downstream. The distance to obtain a fully-developed turbulent flow is governed by the tip-speed ratio, opening (high TSR) or closing (low TSR) the wake envelope angle, and by the aspect ratio (AR) of the turbines. The former is common to vertical- and horizontal-axis wind turbines: a higher tip-speed ratio shed vortex separations that are spatially closer than for a low-tip speed ratio. Differently said, the observed frequency between successive eddies is higher. This reduces the time needed for two successive vortices to reconnect and conditions the growth rate of the instabilities. The effect of aspect ratio will be discussed in section 3.4.

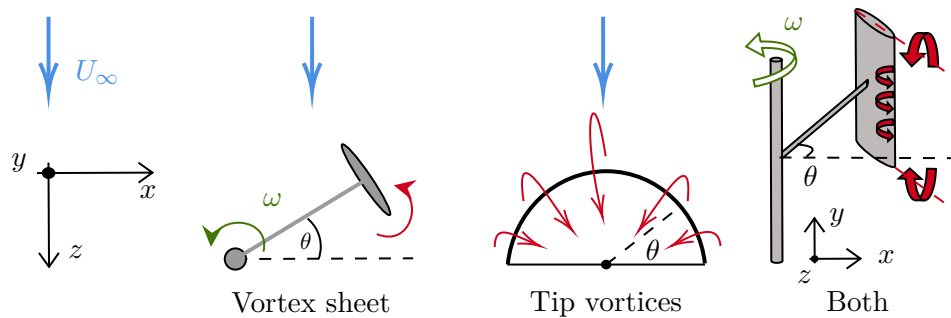
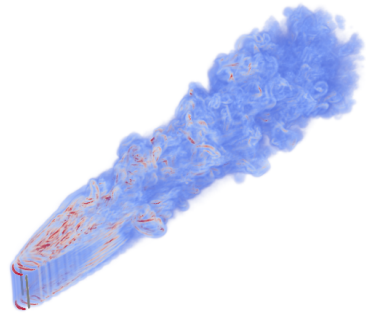
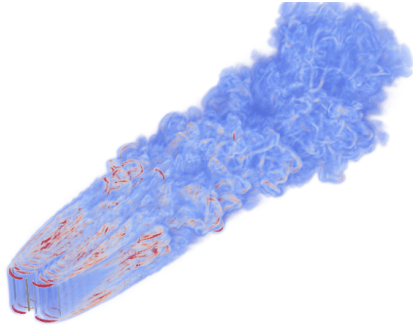
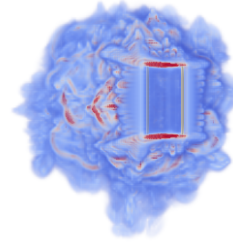


Figure 3.5: Orientation of vortices shed by a blade in its first quarter of revolution. From left to right: axis convention and incoming free stream velocity, vortex sheet orientation, tip vortices on the upstream part of the turbine and combination of both in a three-dimensional view.

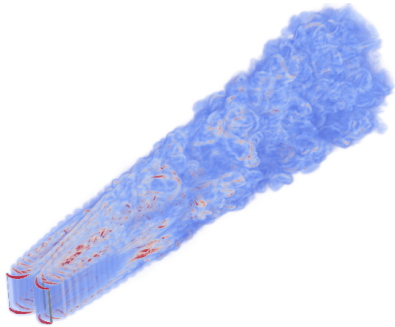
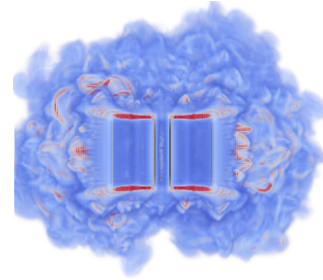
For double-rotor layouts however, the presence of the neighbouring turbine influences the wakes behaviour. The wake bending effect enforces the wakes of the counter-up and co-rotating configurations to merge, whereas for the counter-down configuration, the wakes are free to diverge from each other. As a consequence, the wakes of the counter-up configuration being pushed together tend to stretch in the spanwise direction, resulting globally in a single slender wake compared to the counter-down configuration. When accounting for atmospheric boundary layers and no-through conditions at the bottom for modeling the ground, this spanwise deviation will greatly affect the wake behaviour of the counter-up layout, and for the co-rotating layout in a smaller extend. It can be seen from figures 3.4c and 3.4d that it enables higher streamwise velocity deficits in the far wake compared to the counter-down and the single configuration. These small velocities are however very local. Even though the wakes merge sooner, this does not mean their transition to a fully-turbulent flow is fastened. A predominant effect of the induced velocities in a three-dimensional analysis is their contribution to vertical mixing. The already-turbulent flows from the corners grow and merge together in the mid plane of the turbine. After it occurs, the wake fully develops in a turbulent flow. The spanwise velocities in the mid plane of the turbine ($y = 0$) are thus indicators of where the merging of the corners of each wake, or the merging of both wakes, occur. Figures 3.7a–3.7d present the time-averaged spanwise velocity field $\bar{v}$ in the $y = 0$ plane, normalized with the free stream velocity U_∞ for all configurations. Statistics are taken over $\hat{t} = 50$ non-dimensional times, beginning after $\hat{t} = 30$ non-dimensional times. The maximum statistical spanwise velocity achieved for the counter-down configuration is slightly higher than for the single rotor and twice higher for the other double-rotor layouts. In the mid plane, no spanwise fluxes are present until the corners of the individual wakes or both wakes have merged. This occurs sooner for the single rotor, followed by the CO configuration, compared to the two symmetric layouts. At this point, no conclusion may be taken on which configuration yields the best wake recovery. A more quantitative flux and momentum budget is needed to compare the wake recoveries of the different layouts.



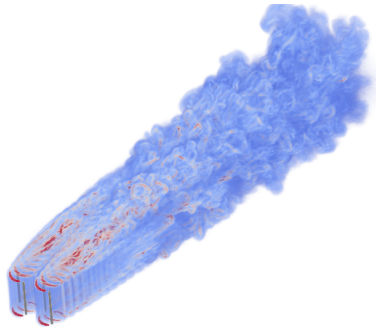
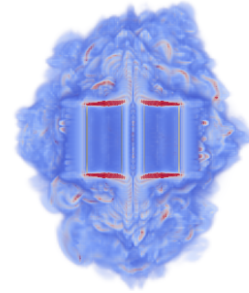
(a) *Single*



(b) *Counter-down*



(c) *Counter-up*



(d) *Co-rotating*

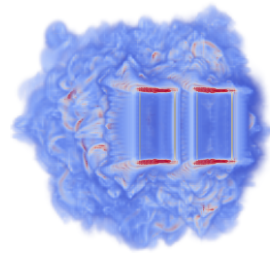


Figure 3.6: *H*-type VAWTs with finite aspect ratios: multiple views by volume rendering of the instantaneous vorticity magnitude for all configurations. Simulation parameters are those of table 3.1.

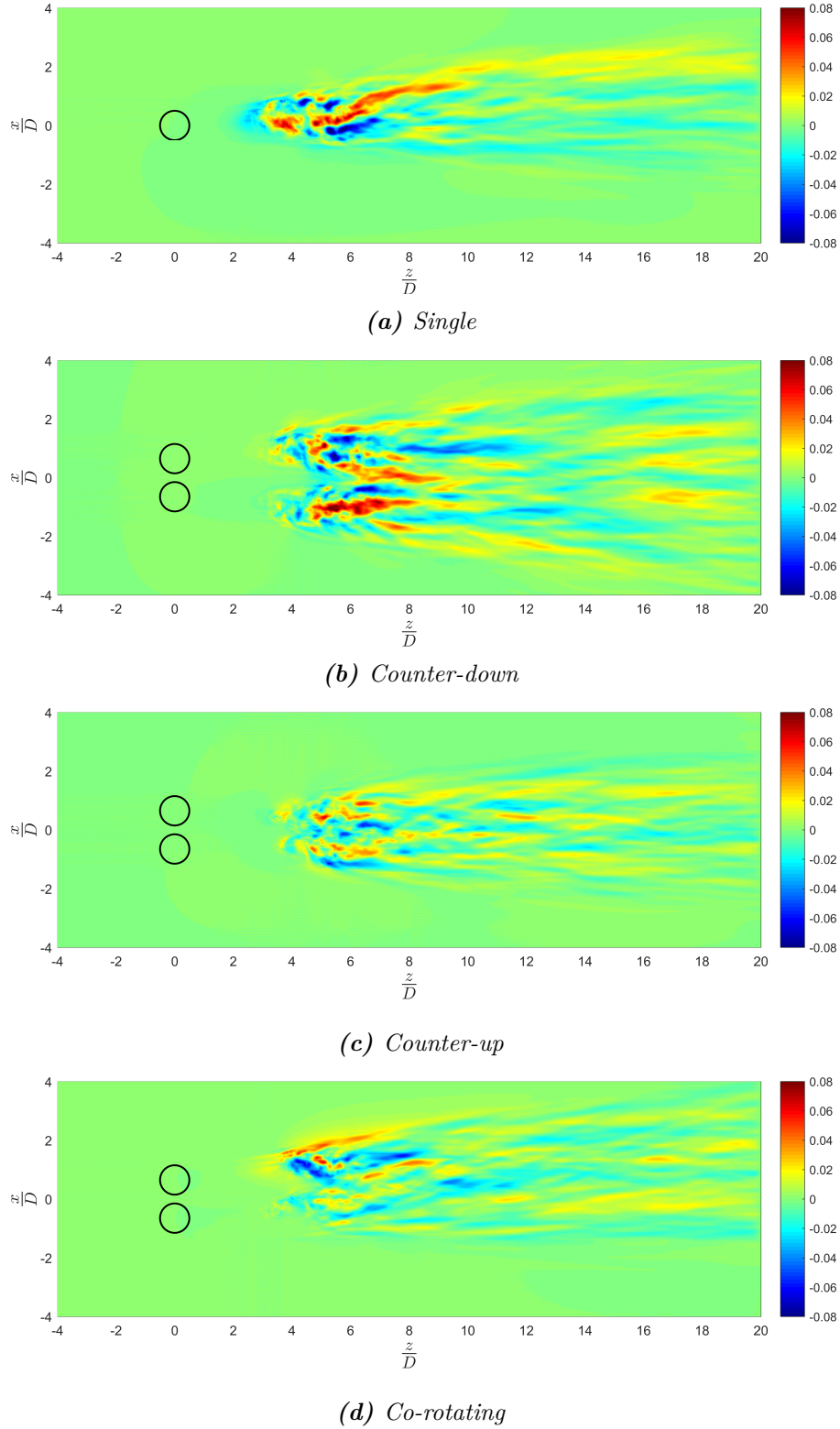


Figure 3.7: Time-averaged spanwise velocity $\bar{v}$, normalized by the free stream velocity U_∞ for all configurations in the $x - z$ plane of the turbines. Simulation parameters are those of table 3.1. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

3.2. Co- and contra-rotating configurations

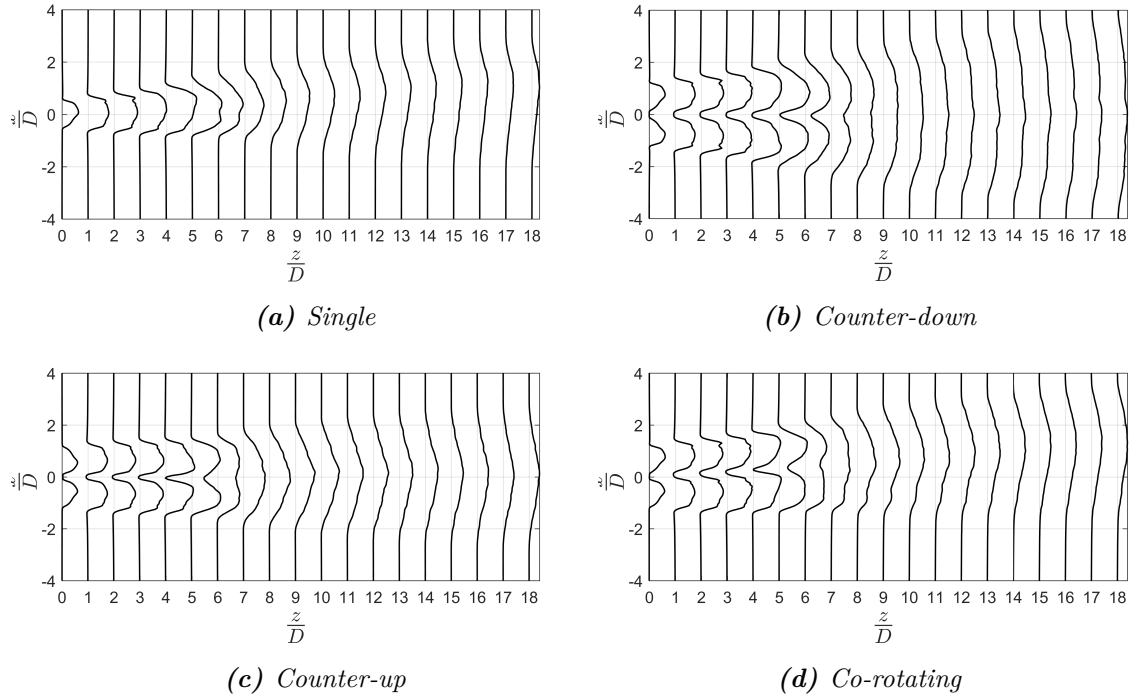


Figure 3.8: Time-averaged streamwise velocity deficit $\left(1 - \frac{\bar{w}}{U_\infty}\right)$ around H-shaped rotors for the 4 configurations at each diameter downstream. Simulation parameters are those of table 3.1. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

Figures 3.8a–3.8d present for the four configurations the local streamwise velocity deficit $\left(1 - \frac{\bar{w}(x)}{U_\infty}\right)$ at each diameter downstream. The velocity values are time-averaged over $\hat{t} = 50$ non-dimensional times. The downstream position where the wakes are merging can be noticed by the rapid deceleration of the flow in between the turbines. The two lobes of velocity deficit are attenuated and diffuse across the entire transverse direction, mainly for the counter-down configuration. The wake bending effect is also clearly observable for the single and co-rotating layouts, globally deviating to the left side of the computational domain.

In order to quantitatively observe the wake recoveries for the different configurations, the displacement and momentum widths L_1 and L_2 are adapted for three-dimensional flows to displacement and momentum surfaces S_1 and S_2 .

$$S_1(z) = \frac{1}{nD} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \frac{\bar{w}(x, y, z)}{U_\infty}\right) dx dy \quad (3.3)$$

$$S_2(z) = \frac{2}{nD} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\bar{w}(x, y, z)}{U_\infty} \left(1 - \frac{\bar{w}(x, y, z)}{U_\infty}\right) dx dy \quad (3.4)$$

where n is the number of bodies in the flow, *i.e.* the number of turbines. The above definitions take into account the velocity deficits and momentum deficits everywhere

in the flow at each diameter downstream $z = i \cdot D$, $i = 1, 2, \dots$ whereas, in the former definition, the values of L_1 and L_2 were integrated only in the transverse direction at mid span of the blades. Practically, the values of L_1 and L_2 were obtained through 1D numerical integration on a transverse width of $8D$, whereas the values of S_1 and S_2 were obtained through 2D numerical integration on a cross-section of $[8D \times 8D]$. The ratio between turbine extraction plane and the integrated area $\frac{n \cdot DH}{16 \cdot D^2}$ is equal to $\frac{1}{5}$ for the double rotor layouts and $\frac{1}{10}$ for the single rotor. This ratio is set small enough to minimize any leakage in the transverse direction, while still enforcing an unbounded condition so that wall blockage effects do not alter the obtained values. Indeed, we observe very low values of transverse velocities near the side walls in the single and counter-down layouts. For the counter-up and co-rotating configurations, the induced velocities in the outer-upwind part are more pronounced, deviating the fluid away from the turbines. We expect the displacement width and surface to reach lower values in the near wake, as they are more prone to leakage effects. In the far wake, we however expect the values to settle at a higher plateau, especially for S_1 compared to L_1 . Moreover, and especially for the counter-up configuration, the spanwise stretching in the far wake eventually causes leakage not only in the transverse direction but also in the spanwise direction. Even though the displacement and momentum width were defined for a two-dimensional case, their comparison with the displacement and momentum surfaces highlights different wake recovery rates across the turbine's span. In that sense, figures 3.9a–3.9d present the displacement and momentum widths in vis-à-vis of the displacement and momentum surfaces.

Unlike in the 2D analysis, major differences are observable between the different configurations, while showing globally an equivalent behaviour. Regarding the displacement quantities, all begin at low values for L_1 and S_1 indicating a small mean velocity deficit in the turbines plane $z = 0$ and especially at mid span of the blades. The flow then decelerates in the near wake and reaches values above $L_1 = 1$ and $S_1 = 1$. This overshoot is an indicator of a recirculation region, not present in the 2D analysis of section 2.3. The highest overshoot for L_1 is achieved for the single configuration, closely followed by the symmetric counter-down layout. This correlates with the intensity and shape of the recirculation bubble observable in figures 3.1a and 3.1d. The counter-down layout having the second-highest recirculation is not unexpected, as the wake bending deviates the wakes apart from each other. The wake of each individual rotor of this layout resembles the closest to the wake of the single turbine in the flow. For the counter-up configuration, the recirculation bubble is stretched and sent further downstream, and a near stagnation region is found directly downstream the recirculation bubbles. Finally, the co-rotating configuration has asymmetric recirculation bubbles in its wakes with a smaller intensity. The asymmetry of the layout causes the upper recirculation bubble to be stretched in the streamwise direction. This diffuses the velocity deficit integral L_1 and S_1 over a larger range of streamwise positions and no significant overshoot is present. Based on S_1 , we observe that, even though the recirculation bubble is stronger for the single configuration, its recovery rate is better than that of the other configurations. At 12 diameters down-

3.2. Co- and contra-rotating configurations

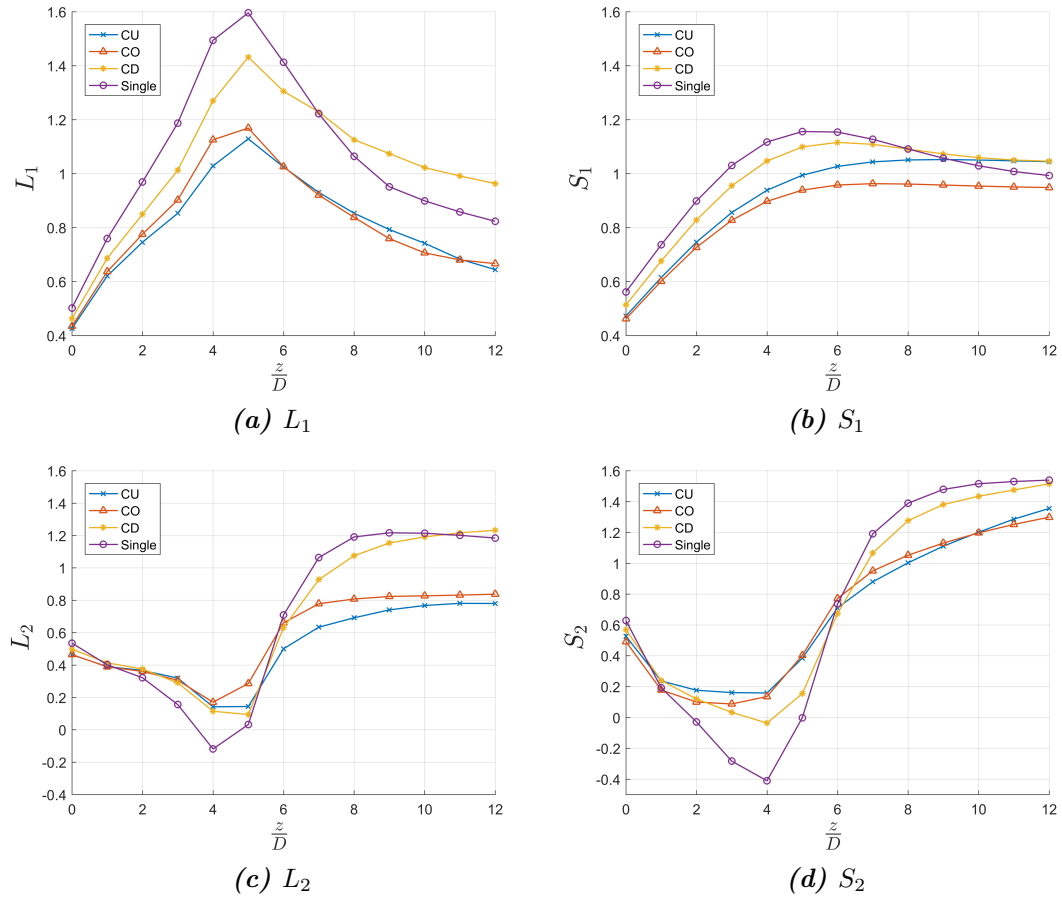


Figure 3.9: Displacement and momentum widths L_1 and L_2 and surfaces S_1 and S_2 based on the statistical streamwise velocity deficit at several diameters downstream. The parameters of the simulation are those of table 3.1.

stream, it settles below the symmetric configurations, whereas the co-rotating layout yields the lowest mean velocity deficit over the cross section. Comparing L_1 and S_1 , we imagine that the curves obtained through the cross-sectional integration will not keep the plateau going, and will ultimately drop in the very far wake.

3.3 Effect of the number of blades

From now on we have only considered two-bladed H-shaped vertical-axis wind turbines. The complex aerodynamics and wakes of vertical-axis wind turbines are strongly influenced by their geometrical characteristics, whose effects must be understood. Firstly, the effect of a given geometrical characteristic can be studied individually. Thereafter, the combined effects of the various parameters can be studied as a global optimization process focusing on performances, wake behaviour and structural loads. Among the geometrical characteristics, we can mention the number of blades, airfoil shape, surface roughness, struts, pitch angle, aspect ratio and mast diameter. The present study concerns the effect of the number of blades of H-type Darreius vertical-axis-wind turbines. The number of blades is not known to have a significant impact on the turbines performances, especially when keeping equivalent tip-speed ratios and solidities [27]. However the number of blades participates in the turbine design in the sense of modulating the uniformity of torque and output power, but also structural loads and vibrations. Of course, cost is one but global parameter of the design of a VAWT, but the latter is not accounted for in pure numerical studies. Most of the numerical studies performed to analyze the effect of number of blades on lift-type vertical-axis wind turbines used conformal meshes with URANS equations, and their approach were essentially two-dimensional. We here propose a full three-dimensional analysis of the effect of blade number using VPM

3.3.1 Methodology and setup

Because the counter-up configuration is the most promising in terms of performances compared to the single turbine, we choose the former simulation concerning this layout to be our reference case. Three different number of blades will be studied: the general two-bladed H-shaped turbine of the former section, a three-bladed turbine and four-bladed one. The different layouts will be denoted N_2 , N_3 and N_4 respectively. The blades are straight and have a NACA0018 airfoil shape across their whole span.

The number of blades has a direct impact on the solidity of a rotor σ if we keep the diameter to chord ratio $\frac{D}{c}$ constant. In the present case, we decided to study the impact of blade number while maintaining the main non-dimensional quantities constant for all three cases. This comes with a certain scaling of the physical characteristics of the rotor. We here chose to keep the chord length c constant to maintain the corresponding chord-based Reynolds number $Re_c = \frac{U_\infty c}{\nu}$ unchanged. This raises the diameter of each turbine by the number of blades ratio between each configuration. The main characteristics of each layout and the corresponding non-dimensional numbers are provided in table 3.3.

The only non-dimensional number that is not conserved is the diameter based Reynolds number Re_D . Keeping the chord-based Reynolds number along with the tip-speed ratio λ unchanged keeps the relative chord Reynolds number $Re_{c,rel} = \frac{(U_\infty + \omega R)c}{\nu} = \frac{U_\infty(1+\lambda)c}{\nu}$ constant. The latter is about $1.333 \cdot 10^5$. Because the diameter between the different layouts increases, the actual rotational speeds of the turbine lowers at constant free stream

3.3. Effect of the number of blades

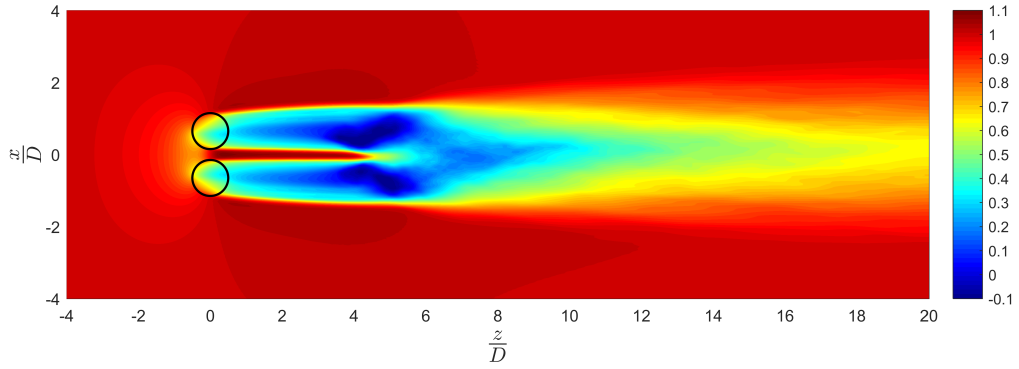
velocity U_∞ . Hence the computational domain has its physical dimensions increased, but keeps the shape of a $[8D \times 8D \times 24D]$ box with 20 points per diameter N_D . All three simulations are discretized by means of more than 12 million particles and are run over $\hat{t} = 100$ non-dimensional times. The flow curvature correction only depends on the diameter to chord ratio, which is here changed between the different layouts. The value of the inwards pitching angle of the blades is for, N_2 , N_3 and N_4 respectively, $\alpha_0 = 2.8553^\circ$, 1.907° and 1.4312° . The parameters for the Leishman-Beddoes model of dynamic stall are the same as for the former section. They are tuned based on a NACA0015 profile and a relative chord-based Reynolds number of $1.5 \cdot 10^5$.

The dimensionless time-averaged streamwise velocity field is given in figures 3.10a–3.10c for the three different layouts. Several aspects are noticeable. The most obvious difference between the different layouts is the shape of the recirculation bubble. It appears that for the four-bladed turbine, the width of the recirculation bubble is narrowed and slightly stretched in the streamwise direction. The shapes of the wakes themselves are sharper. The most negative streamwise velocity achieved however is lower in absolute value for the four-bladed turbines than for the classic two-bladed ones and the three-bladed turbines, going approximately from 5 to 11% of the free stream velocity U_∞ between the different layouts. This may be caused by global Reynolds number effects. For the four-bladed layout, at the diameter scale, the Reynolds number Re_D is doubled compared to the two-bladed turbine, showing the relative importance of inertial effects above viscous effects. At the blade level however, the chord-based Reynolds numbers Re_c are identical and, for a same solidity σ and tip-speed ratio λ , we do expect few fluctuations of the output power coefficient between the layouts. Globally, the used chord and diameter based Reynolds numbers are relatively small, and Reynolds number effects are expected in any case. It is also caused by a more significant dynamic stall on the three- and four-bladed turbines, where higher velocities pass through the turbines in the outer-upwind parts and prevent the wakes to expand laterally, stretching the bubbles in the streamwise direction. Surprisingly, for the three-bladed and four-bladed turbines, the symmetry is lost. The lower recirculation bubble for N_3 appears tightened, and the accelerated flow in between the turbines appears to be deviated to the negative x -axis. The opposite occurs for the four-bladed rotor, where the upper recirculation bubble is narrowed. For all layouts, the upstream part of the flow field remains the same, as every geometrical parameter between the layouts are identically scaled, producing the same upstream blockage.

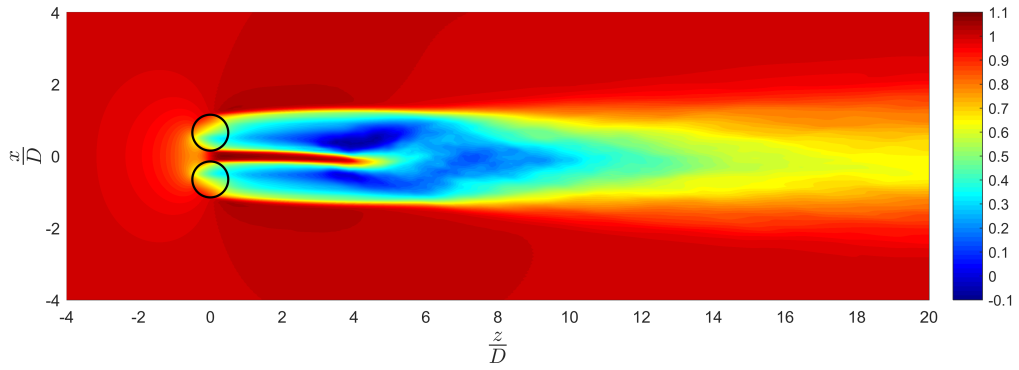
	$U_\infty \left[\frac{m}{s} \right]$	$D \text{ [m]}$	$c \text{ [m]}$	$H \text{ [m]}$	N	d_{int}	Re_c	σ	λ	AR
N_2	10	0.5	0.05	0.8	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	1.6
N_3	10	0.75	0.05	1.2	3	1.3	$3.33 \cdot 10^4$	0.2	3.0	1.6
N_4	10	1	0.05	1.6	4	1.3	$3.33 \cdot 10^4$	0.2	3.0	1.6

Table 3.3: Geometrical parameters of the different layouts composed of 2, 3 and 4 blades with the corresponding non-dimensional numbers.

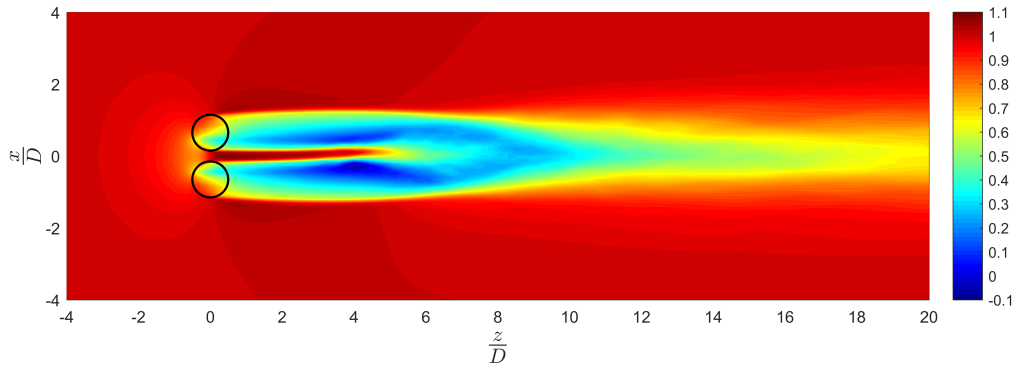
3.3. Effect of the number of blades



(a) Two-bladed turbine N_2



(b) Three-bladed turbine N_3



(c) Four-bladed turbine N_4

Figure 3.10: Time-averaged streamwise velocity field around H-shaped rotors for the 3 layouts. The streamwise velocity $\bar{w}$ is normalized with the free stream velocity U_∞ . Simulation parameters are those of table 3.3. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

3.3.2 Diagnostics and performances

If the tip speed ratio for the different layouts is changed, *i.e.* if no scaling on rotational speed ω is performed with an increase of diameter, this would automatically change the frequency at which vortex separation occurs. Differently said, with the appropriate scaling, any blade of a N -bladed turbine sees in the downwind part the exact same vortex structures advected from the upwind part. It should hence result in a globally comparable wake. Regarding the performances of the different layouts, we present the output power coefficients C_p and the contribution of each rotor $C_{p,\text{rotor}}$ to the power coefficient for the top (positive x -axis) and bottom (negative x -axis) rotor respectively in table 3.4. The values are obtained by averaging the power output over the ten last revolutions of the turbines. The maximum standard deviation σ_{max} for this averaging process and hence associated to a specific azimuth position is also provided.

We observe that the mean performances of the turbines remain almost unchanged, with a relative difference of approximately 1.2%. The maximal standard deviation encountered through the averaging process over 10 revolutions is of the same order of magnitude, showing good confidence in the results. We note that this maximal standard deviation σ_{max} is decreased with a greater number of turbines. This is explained by the fact that all blades equally contribute to the power output of a turbine, and hence, over a revolution N power peaks are produced. These peaks are of smaller amplitude as the total power output remains the same, but lowers the individual fluctuations.

For a perfect scaling, we expect the other aerodynamic quantities, -namely α , C_t and C_n - to remain globally unchanged between the different layouts. Figures 3.11a-3.11d present the angle of attack α , the global power coefficient C_p and the local tangential and normal force coefficients C_n and C_t acting at mid span on the blade, over the azimuth position θ . As mentionned, the values are averaged over 10 rotor revolutions. Except for the global power output, the diagnostics are given for the rotor of reference, which is the bottom rotor (negative x -axis) for the counter-up configuration. The normal force acting on the blade is directed towards the rotor's center while the tangential force points towards the leading edge.

	N_2		N_3		N_4	
C_p	0.2766		0.2751		0.2786	
$C_{p,\text{rotor}}$	0.2761	0.2771	0.2742	0.2760	0.2790	0.2782
σ_{max}	0.0071		0.0041		0.0031	

Table 3.4: H-type VAWTs global performances for all configurations. The output power coefficient C_p is given for each setup. The contribution of each individual rotor is given for the top (positive x -axis) and bottom (negative x -axis) turbine, and is denoted by $C_{p,\text{rotor}}$. the maximum standard deviation is denoted by σ_{max} . Setup parameters are those of table 3.3.

3.3. Effect of the number of blades

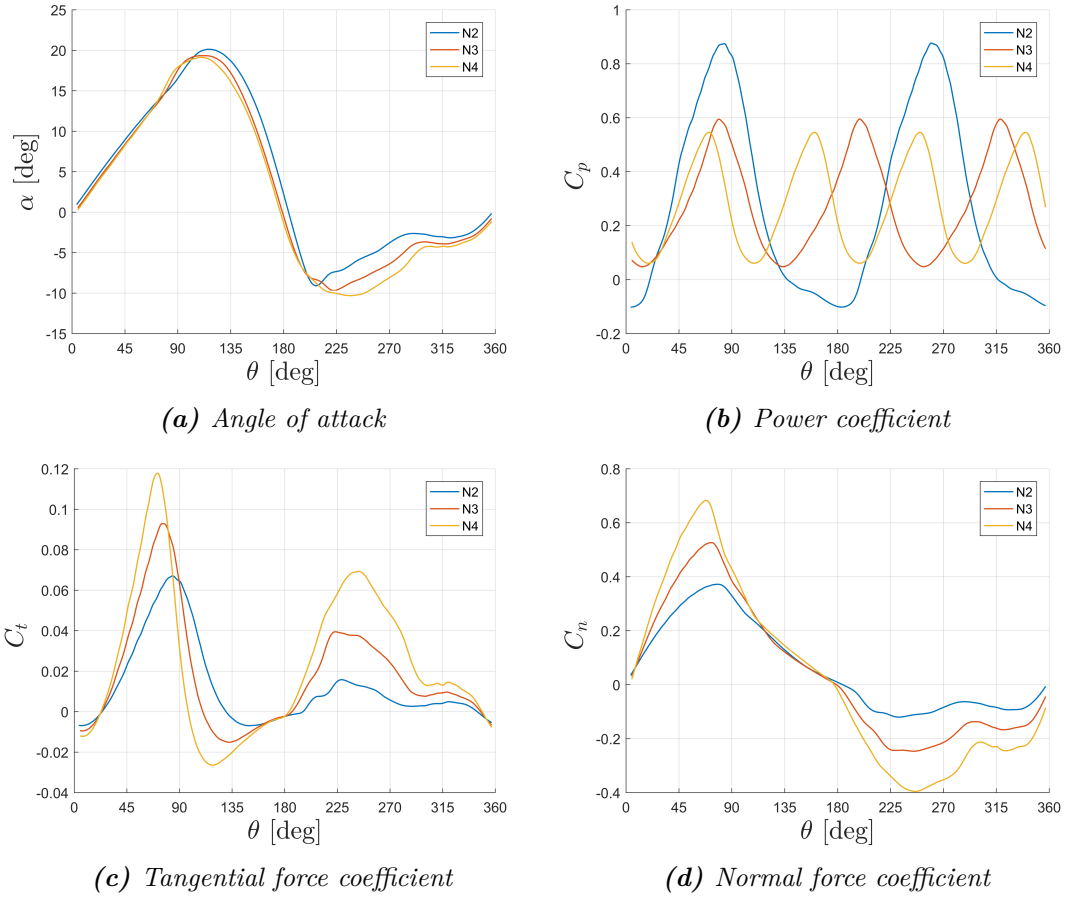


Figure 3.11: Diagnostics of the H-type VAWTs for all configurations. Simulation parameters are those of table 3.3. Values are averaged over 10 rotor revolutions. Diagnostics are given for a single blade of reference rotor.

The layouts show quite different behaviours in terms of angle of attack and loading, which is unexpected for configurations having the exact same non-dimensional numbers. First of all we note that the blades of the three- and four-bladed turbines present a slight shift in the angle of attack in the upwind part, and much higher absolute values in the downwind part. Dynamic stall occurs sooner, around $\theta \in [70^\circ, 80^\circ]$ instead of $\theta \simeq 90^\circ$ for N_3 and N_4 compared to the two-bladed turbine, N_2 . This stall is associated with a rapid drop in tangential forces, and a rapid increase in angle of attack. Correctly calibrated as for the base case, the blade exploits a delayed dynamic stall and its lift overshoot, while smoothly decreasing when entering the retreating region. The deep stall encountered for N_3 and N_4 in the upwind part modifies the behaviour of the downwind part: the blades initially experiences the high-velocities of the stalled upstream region, opening the angle of attack and extracting further power from this flow. The loading increases up to the point where it encounters the wake of the unstalled part, dropping rapidly after.

The differences observed for the different diagnostics may be explained by the only different parameter between the layouts: the inward pitch angle α_0 for the curvature correction. While it has been shown to accurately improve the results in the validation of Chatelain *et al.* [30], we suggest this correction, even adapted, is not scalable when comparing the diagnostics over a revolution of N -bladed turbines. This correction represents the angle that has to be applied to obtain a zero lift on the blades when swirling through a stagnating fluid ($U_\infty = 0$). But the upwind and downwind part of the turbines see very different incoming flows and velocities, while the pitching angle remains constant over a revolution. In fact, the correction is scalable only in terms of *averaged* diagnostics over many revolutions, as it is the case for the global power coefficients presented in table 3.3. This pitch angle really is a correction: using the same setup but leaving the angle α_0 of the two-bladed rotor, we observed a global power loss of almost 50% when compared to the actual values of table 3.3.

As expected, we encounter N peaks of power production for a N -bladed turbine, and the amplitude of each peak is reduced with an increasing number of blades. Increasing the number of blades on a rotor tempers the oscillations of torque and power production and is known to reduce the structural loads on the rotor. Regarding the normal force acting over a revolution at mid span of a blade, increasing the number of blades increases the amplitude of the force for the most upstream part $\theta = 90^\circ$ and around $\theta = 250^\circ$ in the downwind part. C_n and C_t being non-symmetric over the whole revolution, the resulting load felt by the mast of the turbines will not vanish, also for turbines composed of an even number of blades. We assumed that for straight blades, as in the present case, the resulting transverse (x -axis) and streamwise (z -axis) forces F_x and F_z acting on the mast holding the blades, is the sum of the local forces, integrated along the span, of each individual blade. We define two non-dimensional coefficients C_x and C_z , being the ratio of the corresponding forces F_x and F_y against the equivalent force of the free stream pressure $q_0 = \frac{1}{2}\rho U_\infty^2$ acting on the turbine extraction plane $A = D \cdot H$. The F_z does not

3.3. Effect of the number of blades

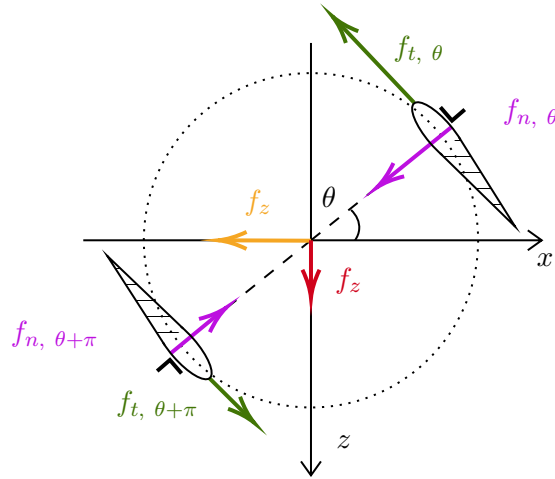


Figure 3.12: Local force components f_x and f_z acting on the mast as a resultant of the local aerodynamic forces on each blade. Blades and forces are not to scale for the sake of clarity.

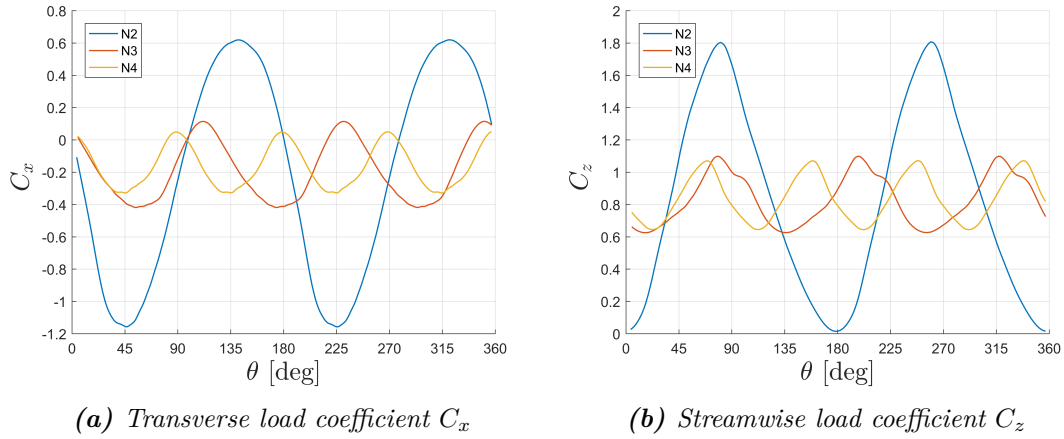


Figure 3.13: Streamwise and transverse load coefficients C_x and C_y acting on the mast. Simulation parameters are those of table 3.3. Values are averaged over 10 rotor revolutions

take into account the drag directly acting on the mast caused by the free stream velocity.

$$C_x(\theta) = \frac{\sum_{i=1}^N \int_{-\frac{H}{2}}^{\frac{H}{2}} f_x^i dy}{\frac{1}{2} \rho U_\infty^2 A} \quad C_z(\theta) = \frac{\sum_{i=1}^N \int_{-\frac{H}{2}}^{\frac{H}{2}} f_z^i dy}{\frac{1}{2} \rho U_\infty^2 A} \quad (3.5)$$

where $\mathbf{f}^i(y, \theta) = (f_x, f_y, f_z)$ is the local vector force acting on the i -th blade. A schematic is provided in figure 3.12, showing the resulting forces on the mast for a two-bladed turbine at an azimuth position of θ . Note that no bending moments are here considered, neither on the mast, nor on the blades and should be the object of design considerations in future analyses.

Figures 3.13a–3.13b present the evolution of the non-dimensional force coefficients acting on the mast of the reference turbine. We observe that C_x oscillates around a

3.3. Effect of the number of blades

	N_2	N_3	N_4
$C_{x,\text{mean}}$	-0.1973	-0.1877	-0.1630
$C_{z,\text{mean}}$	0.8318	0.8301	0.8384

Table 3.5: Mean force coefficients acting on the mast in the transverse (x -axis) and streamwise (z -axis) directions for the N -bladed turbines.

negative mean value whereas C_z oscillates around a positive mean value. These mean values are given in table 3.5 for the different turbines. Non-zero mean values implies that the mast is always under loading, and these loads fluctuates. The fluctuations however are significantly smaller for the three- and four-bladed turbines. The oscillations cause fatigue and vibrational loadings on the rotating shaft. The frequency of the loadings and their intensity should be taken into account in the preliminary design of the turbines. The number of peaks is concordant with the number of blades for each setup, and the azimuth position for the first encountered peak of C_z coincides with the optimum location of the normal force coefficient acting at mid blade C_n . These results are coherent with the discussion above: at the same solidity and tip-speed ratio, a higher number of blades enables a less fluctuating torque, power production and loadings on the mast. Note however that the blockage effect of the neighbouring turbine do change the tangential and normal forces acting on the blades. Compared to the single turbine, the different double-rotor layouts experience mainly a shift in azimuth position of the values C_n and C_t as can be seen in figures 3.2c–3.2d. The amplitudes of those forces globally remain unchanged, and hence we do not expect the type of double-rotor layout to have a significant effect on the loadings on the mast of each turbine.

3.3.3 Wakes

We stated before that having the same non-dimensional numbers, except for Re_D , we would obtain the same aerodynamic behaviours and wakes. This is obviously not the case, regarding the time-averaged streamwise velocity $\bar{u}$ in figures 3.10a–3.10c. We suggested that the main differences occur because of the curvature correction, being an adequate correction for mean values as the global power coefficient, but not scalable for the detailed diagnostics over a rotor’s revolution. Although the inward pitch angle α_0 affects directly the aerodynamic performances, obviously for the angle of attack, it is its consequences and scaling issues that here affect the wakes. For the N_3 and N_4 layouts, the applied curvature correction implies a significant stall in the upstream part region. This explains the stretching of the lobes of low velocities composing the wakes, presented in an extreme case for the two-dimensional analysis performed in chapter 2. In the former analysis, we observed a two-lobed structure caused by an off design tip-speed ratio. In this case, the flow curvature correction acts as if the layouts N_3 and N_4 were operating below their optimum tip-speed ratio. Off-design tip-speed ratios are associated with a decreasing opening envelope angle of the respective wakes, and a stopping vortex being advected through the rotor where deep stall occurs. Contrary to the former two-dimensional analysis, we do observe the merging of the two wakes within the computational domain.

3.3. Effect of the number of blades

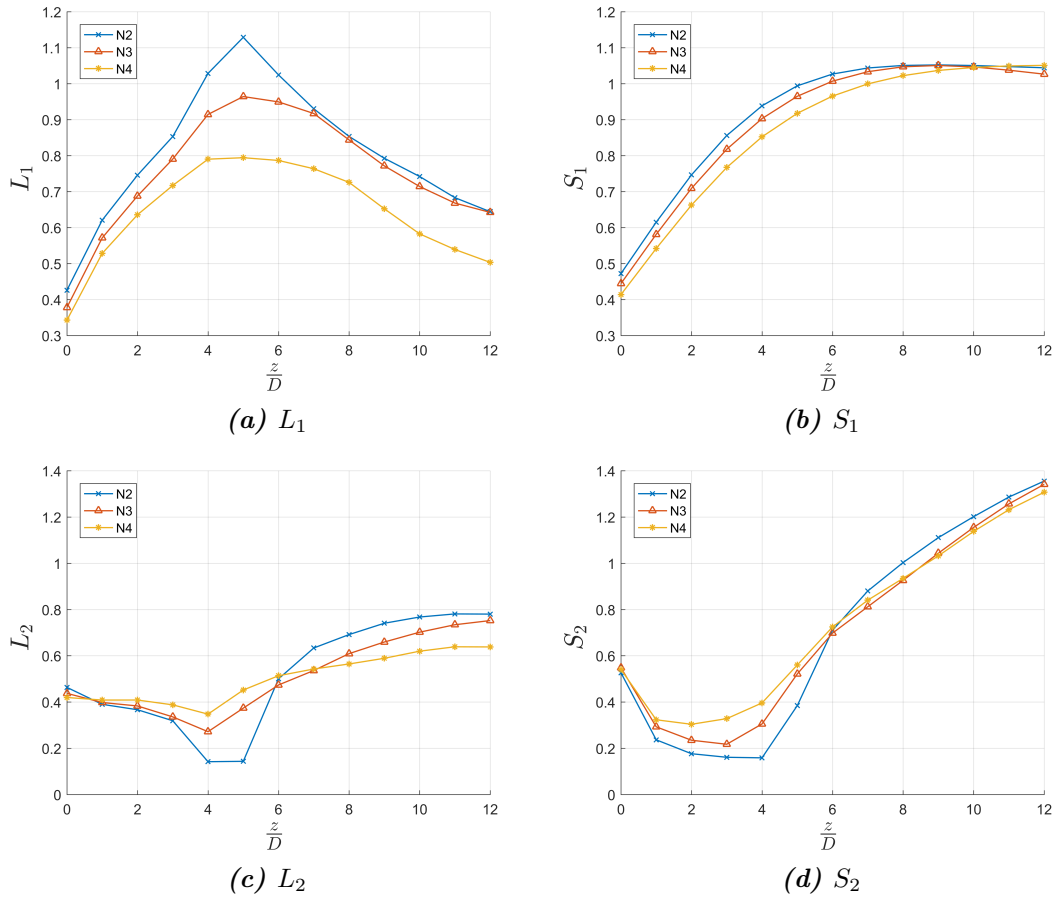


Figure 3.14: Displacement and momentum widths L_1 and L_2 and surfaces S_1 and S_2 based on the statistical streamwise velocity deficit at several diameters downstream. The parameters of the simulation are those of table 3.3.

Figures 3.14a–3.14d present the displacement and momentum widths L_1 and L_2 along the displacement and momentum surfaces S_1 and S_2 . We recall that the respective definitions diverge in the number of dimensions being integrated. For L_1 and L_2 , the velocity deficit and momentum deficit are integrated across the transverse direction only, and at mid span of the blades. For S_1 and S_2 , the whole $x - y$ plane is considered at each diameter downstream.

These results are coherent with the above discussion. Regarding the displacement width L_1 , we do not observe an overshoot $L_1 > 1$ as no significant recirculation bubble is to be found for the three- and four-bladed turbines. However, the downstream location of highest velocity appears to be the same for all layouts. The relatively low value of L_1 for the four-bladed turbine suggest a lower leakage due to blockage in the very far wake, as the solver allows for unbounded direction exactly. We recall however that L_1 is an

integrated value only across the transverse direction at mid blade. This is not confirmed by the displacement surface S_1 , showing a better superposition of the three configurations and an equivalent mass deficit budget in the far wake. However, the four-bladed turbine shows lower values of S_1 in the near wake but with a comparable increase rate. This illustrates the smaller velocity deficits in the whole velocity field instead of just in the mid plane ($y = 0$) of the turbines, but to a smaller extend. Indeed, the deep stall occurring in the upstream part of the turbines enables higher velocities to pass through the rotors.

3.4 Effect of the aspect ratio

One of the advantages of vertical-axis wind turbines compared to their horizontal counterparts lies in practical considerations: the VAWTs extraction plane $A = D \cdot H$ does not need to be equally proportioned between its diameter and its height. In contrast, for horizontal-axis wind turbines, a growth in radius automatically implies a comparable growth in size of the mast and whole turbine. Increasing the HAWTs rotor swept area increases its footprint, whereas it is possible to increase the swept area of a VAWT without changing its footprint, *i.e.* we can increase $A = H \cdot D$ by increasing H only. This motivates the use of slender VAWTs with high aspect ratios $AR = \frac{H}{D}$ in wind farms, to achieve the needed power while reducing the land area per turbine ratio [6]. Finally, increasing the turbine's height does not appreciably changes the tangential and normal loadings felt by the blades, compared to HAWTs blades experiencing high centrifugal loadings. The design requirements associated will hence be less restrictive and enables cheaper manufacturing processes.

Changing the aspect ratio greatly influences the three-dimensional mechanisms behind the performances and wakes of the turbines. It has been shown that increasing the aspect ratio slightly increases the output power coefficient and significantly affects the wake topology [29, 30]. We here propose to study the effect of aspect ratio on the aerodynamics and wakes of the double-rotor concept, going from low aspect ratios to the theoretical two-dimensional case with an infinite aspect ratio.

3.4.1 Methodology and setup

Once again, as the counter-up configuration (CU) is the most promising one in terms of aerodynamic performances compared to the single turbine and other double-rotor layouts, we chose that configuration as our reference-case. Besides, we study four other aspect ratios, changing from $AR = 1$ to the theoretical two-dimensional case where $AR \rightarrow \infty$. The different layouts are denoted by AR_{05} , AR_{08} , AR_{11} , AR_{14} and finally AR_{2D} , in reference of their respective height. The reference case is AR_{08} . For each layout, the turbines used are H-shaped Darrieus turbines, having 2 straight blades with a NACA0018 profile. Apart from the height and hence aspect ratio, all other non-dimensional numbers and geometrical characteristics are left unchanged. Table 3.6

3.4. Effect of the aspect ratio

	$U_\infty [\frac{m}{s}]$	$D [m]$	$c [m]$	$H [m]$	N	d_{int}	Re_c	σ	λ	AR
AR_{05}	10	0.5	0.05	0.5	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	1
AR_{08}	10	0.5	0.05	0.8	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	1.6
AR_{11}	10	0.5	0.05	1.1	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	2.2
AR_{14}	10	0.5	0.05	1.4	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	2.8
AR_{2D}	10	0.5	0.05	∞	2	1.3	$3.33 \cdot 10^4$	0.2	3.0	∞

Table 3.6: Geometrical parameters of the different layouts having different blade heights with the corresponding non-dimensional numbers.

provides the main characteristics for all 5 layouts along with the corresponding non-dimensional numbers.

We briefly recall that the computational parameters used are the same as in section 3.2. The computational domain is a rectangular box of $[8D \times 8D \times 24D]$, in the transverse (x -axis), spanwise (y -axis) and streamwise (z -axis) directions respectively. The number of points per diameter $N_D = \frac{D}{h}$ is set to 20, leading to more than 12 million particles discretizing the flow. The simulations are run over $\hat{t} = 100$ non-dimensional times, with a maximum time step of $\Delta t = 5 \cdot 10^{-4} s$. The flow curvature correction yields a pitching angle $\alpha_0 = 2.8553^\circ$ for all the different layouts, and parameters for the Leishman-Beddoes model of dynamic stall are tuned based on a NACA0015 profile and at a relative chord-based Reynolds number of $1.5 \cdot 10^5$.

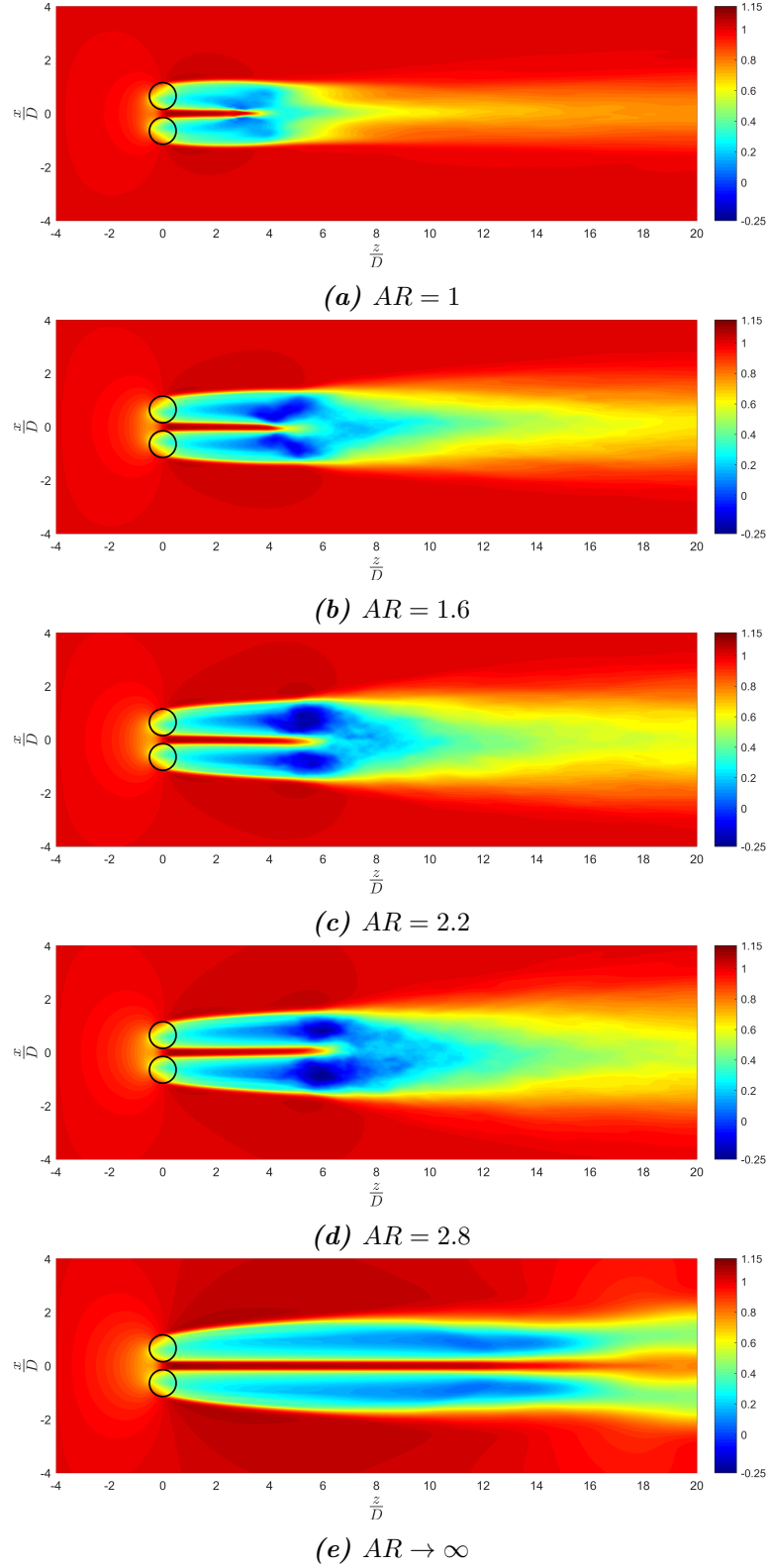


Figure 3.15: Time-averaged streamwise velocity field around H-shaped rotors for the 5 configurations. The streamwise velocity $\bar{w}$ is normalized with the free stream velocity U_∞ . Simulation parameters are those of table 3.6. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

3.4. Effect of the aspect ratio

The dimensionless time-averaged streamwise velocity field $\frac{\bar{w}}{U_\infty}$ is given in figures 3.15a–3.15e for each configuration. We observe that increasing the aspect ratio increases the span of the velocity deficit and moves the lowest velocity zone to further downstream positions. The resulting wakes are longer, and the intensity of the recirculation bubble is increased. No recirculation bubble is observable for the lowest aspect ratio $AR = 1$ and for the fully two-dimensional case. The latter achieves its lowest value further downstream and reaches 5% of the free stream velocity U_∞ . The shape of the bubbles themselves highly depends on the aspect ratio and we expect to observe major differences for the decay diagnostics. The configurations are more alike in the very near wake, suggesting few differences in aerodynamic performances. All configurations preserve their symmetry. Another noticeable behaviour is that the very far wake of the two-dimensional case starts to diverge after 15 diameters downstream. This may be explained by a bouncing effect due to an insufficient bending of the wakes. They will eventually encounter, but the transverse velocities are unable to efficiently merge the two wakes. This may trigger greater instabilities in the very far wake and lead to a fully-developed turbulent flow further downstream. This phenomenon may be magnified by the outflow condition. The aspect ratio also dictates the intensity of the accelerated outer flow and hence the widths of the resulting wakes. The higher the aspect ratio the more the spanwise blockage is influential and the wider the outer flow is accelerated, reaching 110% of the free stream velocity for the two-dimensional case.

3.4.2 Diagnostics and performances

Based on the qualitative discussion regarding the time-averaged normalized streamwise velocity, we do not expect the different aerodynamic behaviours to change appreciably. Table 3.7 provides the global power coefficients C_p for the different layouts, and the power coefficients of each individual rotor, beginning with the top rotor (positive x -axis) and denoted by $C_{p,\text{rotor}}$. The values are averaged based on the 10 last rotor's revolutions, and the maximal standard deviation σ_{max} in this averaging process is given, for a specific azimuth position θ .

The output power coefficient C_p varies slightly from one layout to the other, where the smallest aspect ratio yields the highest power coefficient compared to all other configurations. It outperforms the AR_{11} layout by 3% whereas the associated standard deviation in the averaging process is of the same order of magnitude. The latter is quite

	AR_{05}		AR_{08}		AR_{11}		AR_{14}		AR_{2D}	
C_p	0.2823		0.2766		0.2742		0.2757		0.2793	
$C_{p,\text{rotor}}$	0.2826	0.2819	0.2761	0.2771	0.2732	0.2752	0.2768	0.2745	0.2791	0.2794
σ_{max}	0.0076		0.0071		0.0060		0.0058		0.0147	

Table 3.7: H-type VAWTs global performances for all configurations. The output power coefficient C_p is given for each setup. The contribution of each individual rotor is given for the top (positive x -axis) and bottom (negative x -axis) turbine. the maximum standard deviation is denoted by σ_{max} . Setup parameters are those of table 3.6.

high for the AR_{2D} simulation, reaching 5% of its power coefficient value. Qualitatively, the rotors all produce power with a comparable efficiency, which could be expected. Quantitatively however, the values averaged over the 10 last rotor revolutions show relatively high maximum standard deviations. This disables to highlight the precise effect of aspect ratio on the power coefficients. This may also explain the fall out of balance in the contribution of each rotor to the global power coefficient C_p , for pure symmetric configurations. Compared to the power output of a single turbine having the layout of the one in section 3.2, all present configurations yield a higher averaged power coefficient. This shows that even smaller turbines, normally known to yield lower power coefficients, outperforms the single rotor in the flow when disposed in the counter-up configuration.

Figures 3.16a–3.16d present the angle of attack α , the global power coefficient C_p and the local tangential and normal force coefficients C_n and C_t acting at mid span on the blade, over the azimuth position θ . The values are averaged over ten rotor revolutions and given for the rotor of reference (bottom rotor on negative x -axis) except for the global power coefficient C_p . The normal force acting on the blade is directed towards the center while the tangential force points towards the leading edge.

All graphs show good agreement in regard with the above discussion. All configurations are counter-up layouts and benefits from the same mechanisms presented in section 3.2. Nevertheless, some minor differences are observable. For the evolution of the angle of attack α with the azimuth position, we notice a slightly different behaviour of the AR_{05} and the AR_{2D} configurations compared to the others. Globally, lower aspect ratios lead to larger amplitude variations of the angle of attack in the upwind part of the turbines. This may be explained by the lower distance between the shed tip vortices and their associated induced velocities acting on the mid span of the blades. The circulation on the blades varies with space and time, and reaches its maximum values around the most upstream position $\theta \simeq 90^\circ$. This coincides with the sharp rise in angle of attack α just before the blade is in stalled conditions. It causes for the tip vortices generated in the upstream half and especially at $\theta = 90^\circ$ to be predominant compared to the remaining part of the revolution. When looking at the tangential forces acting at mid span of the blade, we observe that the rapid drop occurs around the same azimuth position θ for each layout. In fact, the lower aspect ratios benefit from a more pronounced delayed stall effect and lift overshoot, increasing the turbines performances in the upwind half. For the downwind part of the turbines, the two-dimensional case globally exhibits a shifted behaviour towards lower azimuth positions compared to the three-dimensional layouts. This is observable regarding the time-averaged transverse velocity ratios in figures 3.18a–3.18e, showing high transverse velocities towards the negative x -axis in the outer-downwind quart of the bottom turbine for the AR_{2D} compared to the other configurations. This restrains the effective angle of attack for the third quarter. Once the blades enter the third quarter, *i.e* $\theta \geq 270^\circ$, the angle of attack α of the two-dimensional layout behaves as the other 3D configurations, and the curves superimpose.

3.4. Effect of the aspect ratio

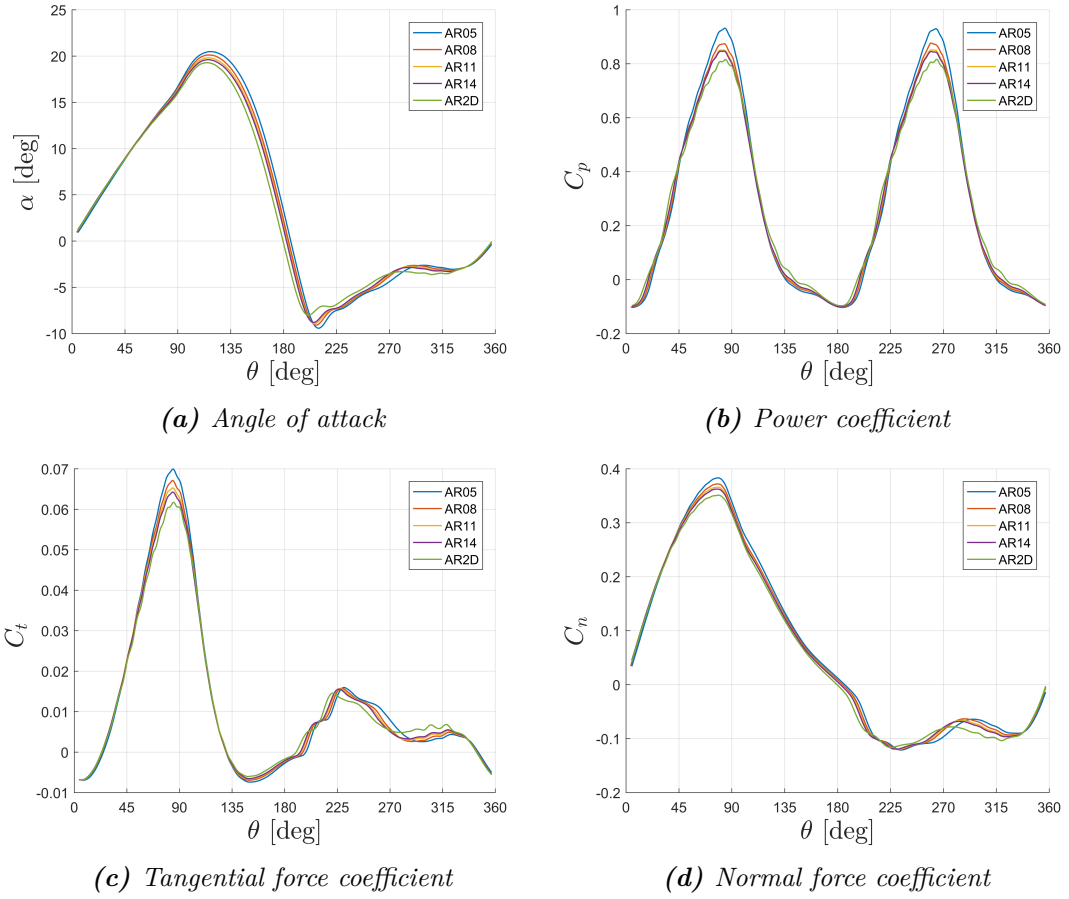


Figure 3.16: Diagnostics of the H-type VAWTs for all 5 configurations. Simulation parameters are those of table 3.6. Values are averaged over 10 rotor revolutions. Except for the power coefficient combining both rotors (b), other diagnostics are given for a single blade of the reference rotor.

3.4. Effect of the aspect ratio

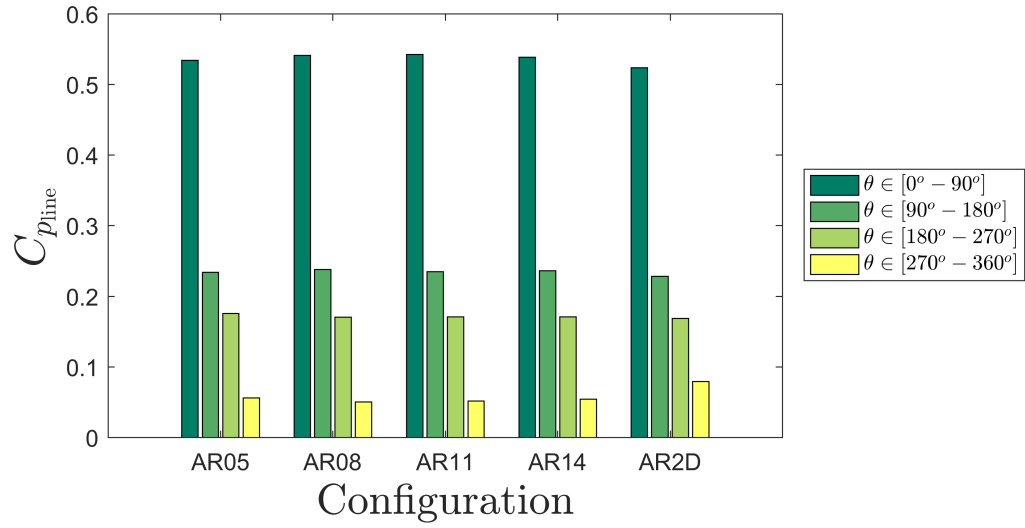


Figure 3.17: Quartile distribution of the power coefficient of a single blade for all configurations. The parameters are those given by table 3.6

To show that the aspect ratio does not fundamentally change the global behaviour of the aerodynamic quantities, we present figure 3.17. We recall that the reference rotor is the bottom rotor (negative x -axis) for a counter-up configuration. Each blade of the different layouts contributes equivalently to the power coefficient of its respective turbine. Only one difference is noticeable: the contribution of the first quarter $\theta \in [0, 90^\circ]$ of the two-dimensional layout AR_{2D} is decreased whereas its last is increased, the outer part of the turbine showing identical behaviours. Finally, figures 3.18a–3.18e show the effect of aspect ratio on the transverse velocities, being induced or due to upstream blockage, in the outer-upwind part of the turbines at mid blade. It also highlights the total absence of transverse mixing for the AR_{2D} configuration compared to the other layouts, and especially for the smallest aspect ratio were the transverse velocities are significant enough to force the merging of the two wakes.

3.4. Effect of the aspect ratio

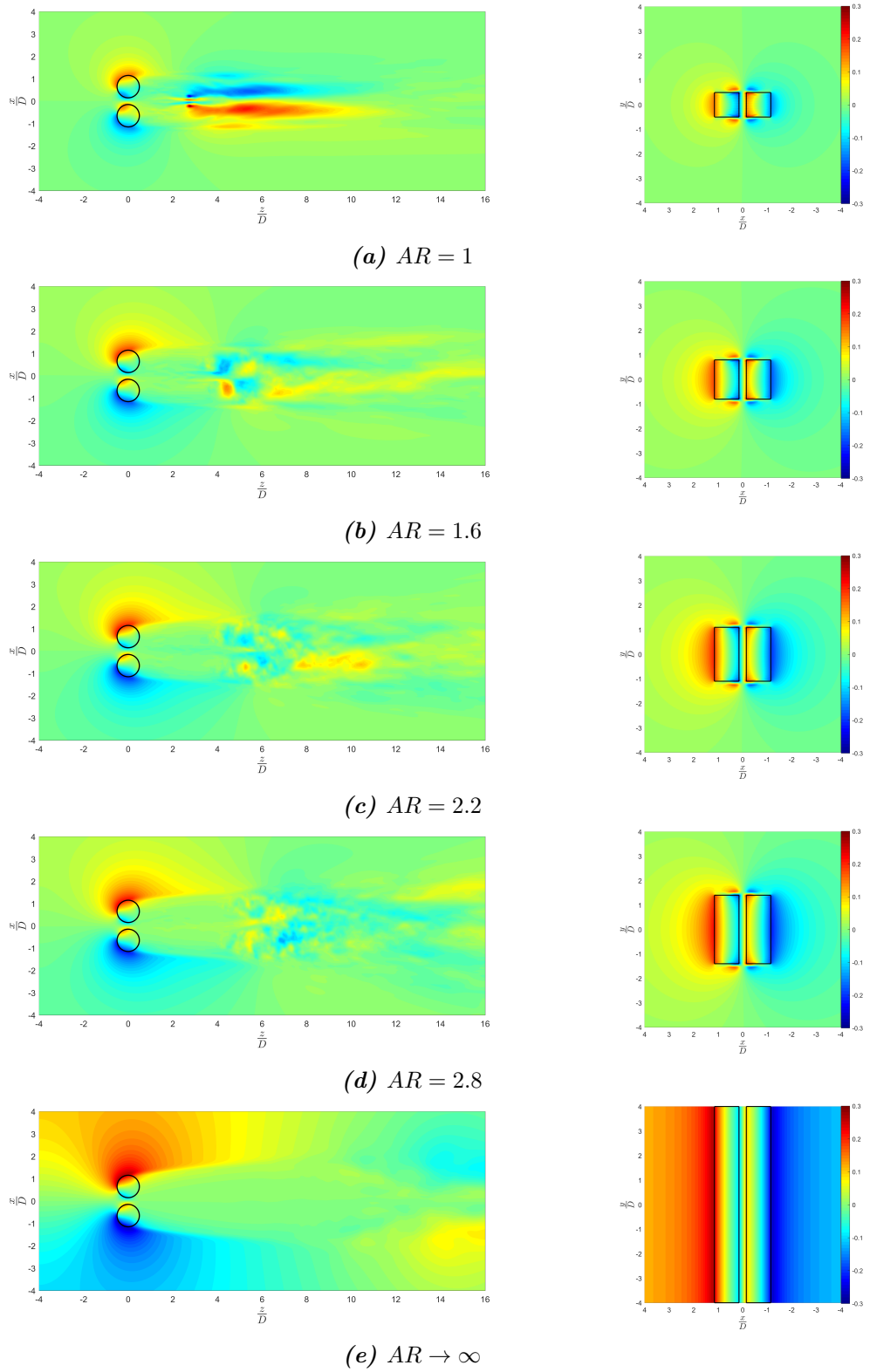


Figure 3.18: Time-averaged transverse velocity $\bar{u}$, normalized by the free stream velocity U_∞ for all configurations in the $x - y$ and $x - z$ plane of the turbines. Simulation parameters are those of table 3.6. Statistics are taken over $t = 50$ non-dimensional times.

3.4.3 Wakes

As stated above it is mainly in the wakes and wake recoveries that the effect of aspect ratio is significant. The derivative of the blade's bound circulation tends to zero around the most upstream position leading to a near zero net circulation of the shed vortex sheet around this position. At this same position, where the highest angle of attack α without stall of the blade is reached, the strongest tip vortex occurs. The downstream part is characterized by low varying angles of attack, which will result in low bound circulations for the AR_{2D} but also for the other aspect ratios. The lack of tip vortices for the AR_{2D} case induces a lack of vertical mixing in the near and far wake. For the smaller aspect ratio, the distance between the mid plane and tips of the blades is shorter. Hence the intense tip vortex generated at $\theta = 90^\circ$ will have its predominant effect at nearer distances downstream. However, the intensity of the induced velocities is greater for higher aspect ratios. This phenomenon is visible in figures 3.19a–3.19d, showing the time-averaged normalized spanwise velocity $\frac{\bar{v}}{U_\infty}$ in the $y = 0$ mid plane of the turbines. The slice for the intermediate aspect ratio AR_{11} is not presented as it is in the continuity of the expected behaviour. We observe non-zero spanwise velocities 3 diameters downstream for the smallest aspect ratio whereas we have to wait up to 5 diameters downstream to observe a vertical mixing in the mid plane of the blades for the highest aspect ratios. For the pure two-dimensional case, the mixing occurs after 12 to 15 diameters, coinciding with the region where the aforementioned *bouncing* phenomenon is observable.

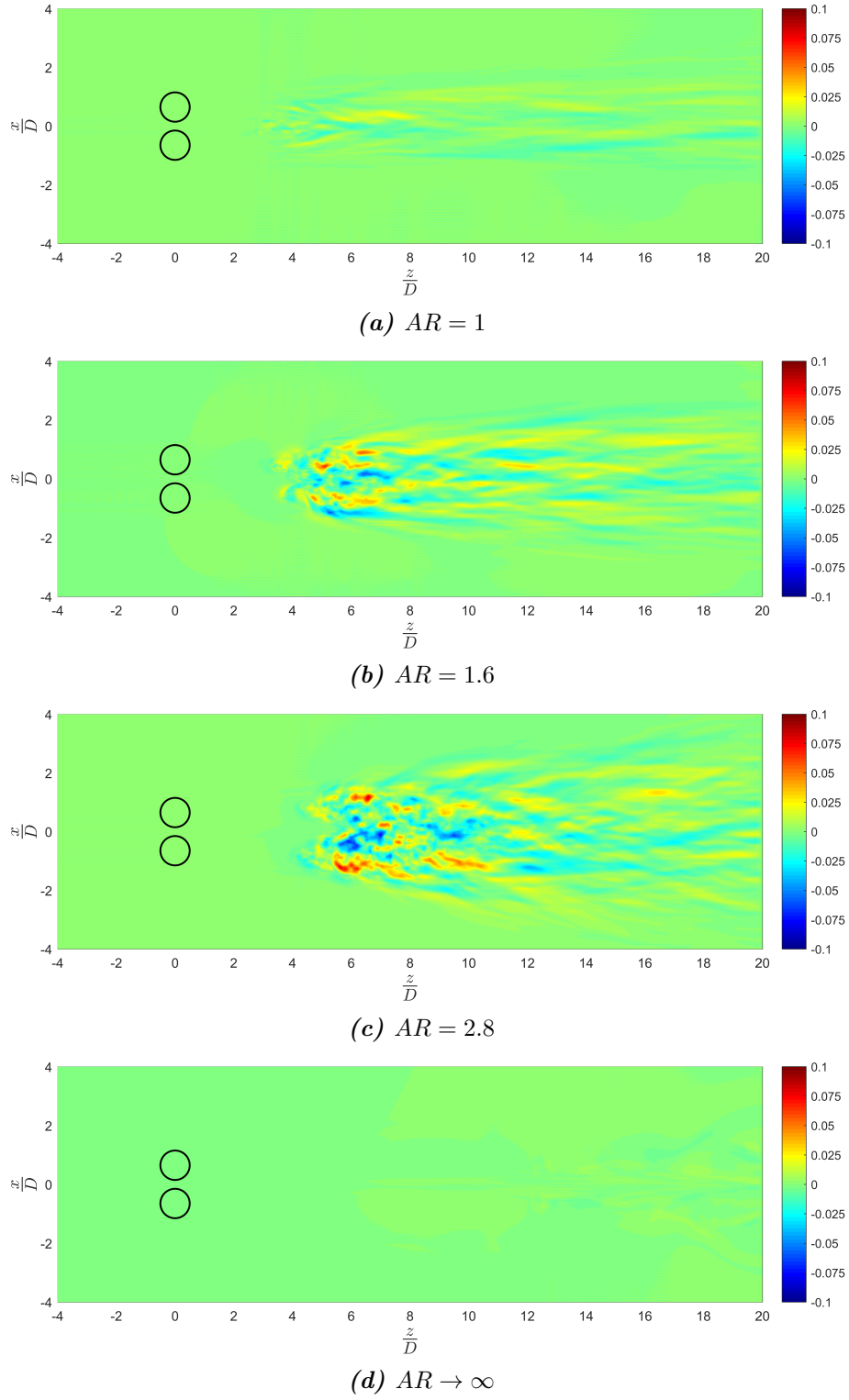


Figure 3.19: Time-averaged spanwise velocity $\bar{v}$, normalized by the free stream velocity U_∞ for four configurations in the $x - z$ plane of the turbines. Simulation parameters are those of table 3.6. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

We have seen that increasing the intermediate aspect ratios pushes the recirculation bubble further downstream, while the AR_{05} and AR_{2D} do not show any recirculation region. For the latter, the absence of vertical mixing induced by the tip vortices does not trigger any early transition to turbulence. In fact, the mechanism behind this transition is different for the two-dimensional case and the other three-dimensional cases. For the two-dimensional case, the transition to turbulence is triggered by an instability in the eddies composing the shed vortex sheet over a revolution, and the stopping vortex due to dynamic stall. This instability comes from the outer sides of the wakes as the main eddies are produced during the retreating part of the blade's motion. When two successive eddies are spatially diffused enough to interact, the instabilities grow and propagate. This brings the main contribution to the transition to turbulence for the two-dimensional layout. Another instability is triggered from the inner sides of the wakes, being pushed together. Eddies of opposite directions are advected downstream, and, at $z = 10D$, the strongest eddy distorts its neighbour, and triggers a vortex street with a constant frequency, again ultimately leading to the transition to turbulence. For the three-dimensional cases however, we have previously seen that, for a single turbine, the instabilities arise from the corners, being the intersection of the tip vortices and the vortex sheet. The tip vortices tend to roll the vortex sheet in the direct vicinity up, and leads to reconnection events and intense stretching, with the vortex sheet but also between successive shed vortices. The wake reaches its fully turbulent state when the turbulent regions growing from the corners meet together in the mid plane $y = 0$ of the turbine, being pushed by the tip induced velocity occurring at $\theta = 90^\circ$. The different behaviours described above can be seen in figures 3.20a–3.20b showing the instantaneous spanwise vorticity component $\frac{\omega_y D}{U_\infty}$ for the smallest aspect ratio AR_{05} and the two-dimensional case AR_{2D} .

Furthermore, figures 3.20a–3.20b show the important wake bending that occurs for the AR_{05} case compared to the full two-dimensional layout. The wake bending forces the two wakes to interact and participates to the transition to turbulence. Regarding all of what was previously mentioned, we can wonder what is the predominant driver to the transition to turbulence in the double-rotor layout. We recall that, as the considered layout is counter-up, the wake bendings are directed towards each other, enforcing their mixing. Besides, the turbulent regions growing from the corners of each individual turbine develop a fully turbulent flow once they have merged, pushed by the tip induced velocities. Figures 3.21a–3.21b propose insights of this answer. It presents, for the smallest aspect ratio AR_{05} and the highest aspect ratio AR_{14} , the time-averaged streamwise velocity $\frac{\bar{w}}{U_\infty}$ along with an instantaneous contour of the transverse vorticity component $\frac{\omega_x D}{U_\infty}$ in the $x = \frac{d_{\text{int}}}{2}$ plane, *i.e.* at the center of the rotor on the positive x -axis. It appears that for small aspect ratios, the merging of the two wakes and the merging of the corners of each individual wake occur about the same downstream position $z = 4D$. Nevertheless, for higher aspect ratios, even though the corners merge further downstream, this event occurs before the wakes themselves are mixed. For higher aspect ratios, we can conclude that the transition to turbulence is not caused by the double-rotor layout but instead by

3.4. Effect of the aspect ratio

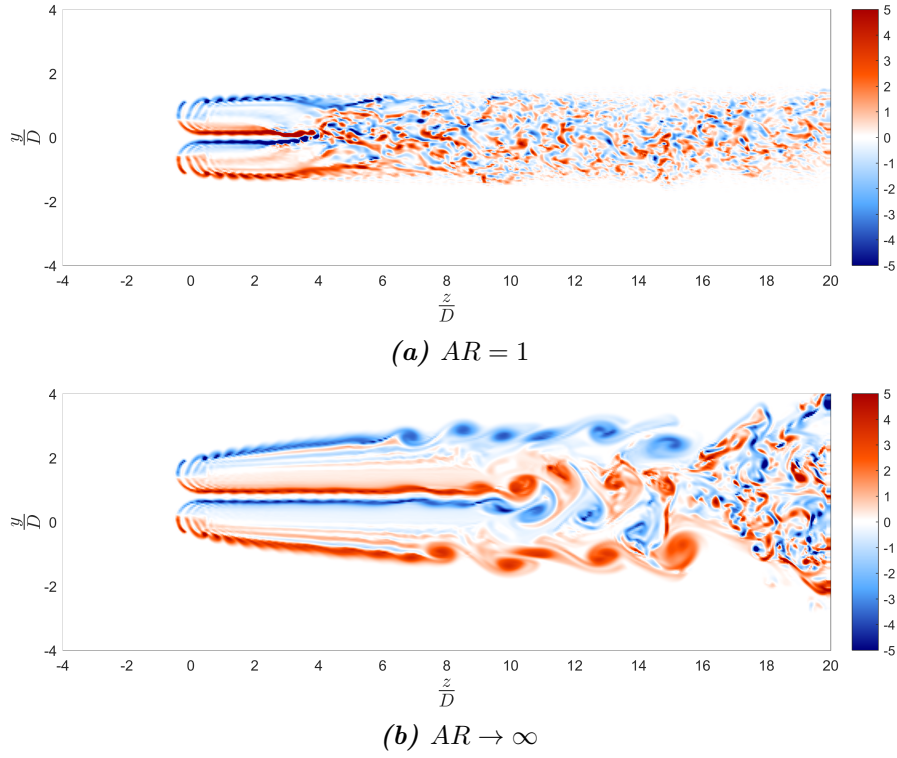


Figure 3.20: Instantaneous spanwise vorticity component $\frac{\omega_y D}{U_\infty}$ in the $y = 0$ plane. Simulation parameters are those of table 3.6. Statistics are taken over $\hat{t} = 50$ non-dimensional times. Colors for the instantaneous vorticity fields are saturated for the sake of clarity.

the transition to turbulence of each individual turbine, triggering an efficient mixing of the two wakes.

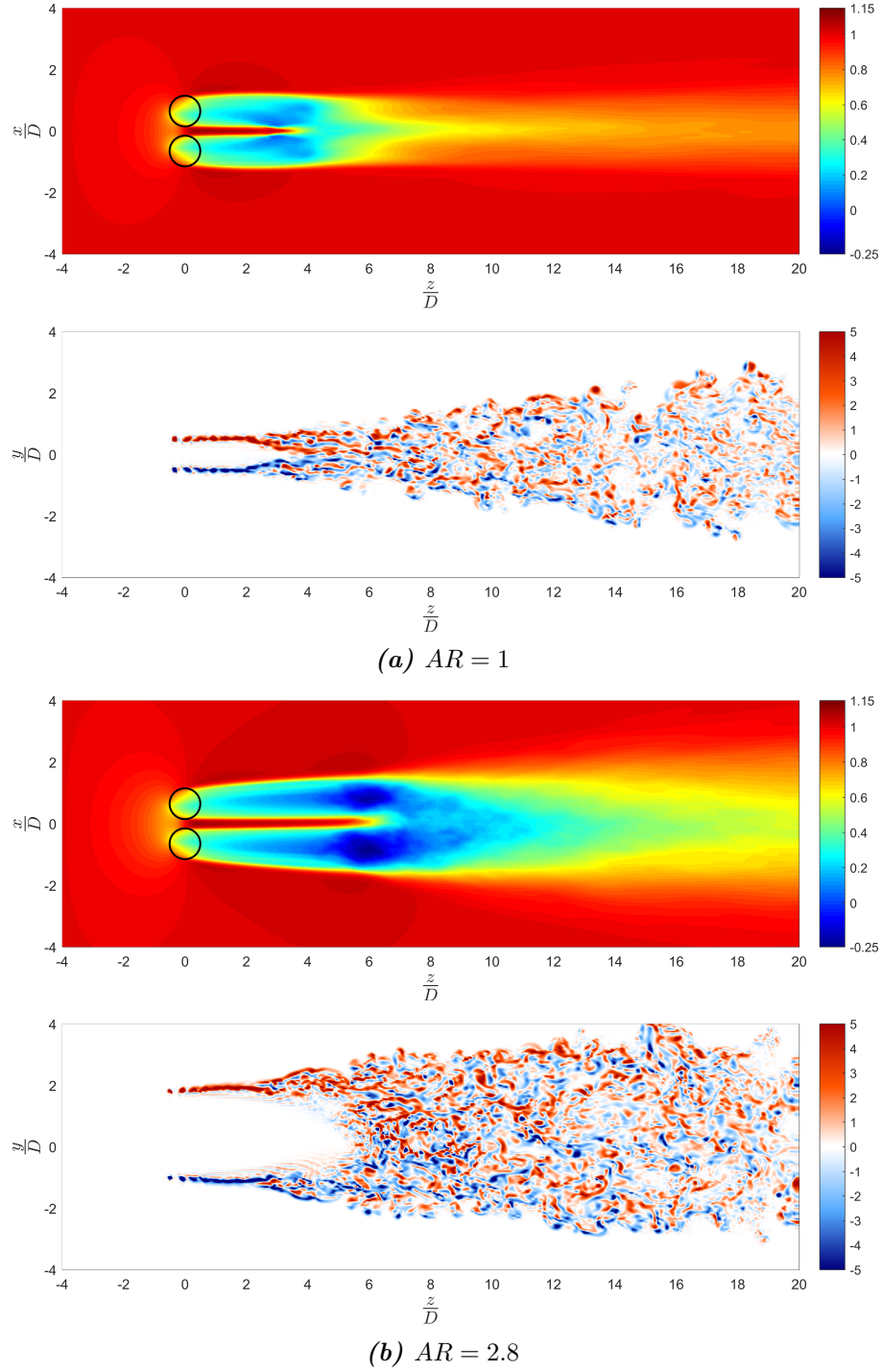


Figure 3.21: Time-averaged streamwise velocity $\frac{\bar{w}}{U_\infty}$ in the $y = 0$ plane, and associated instantaneous cross stream vorticity component $\frac{\omega_x D}{U_\infty}$ in the $x = \frac{d_{int}}{2}$, i.e. at the top turbine center. Simulation parameters are those of table 3.6. Statistics are taken over $\hat{t} = 50$ non-dimensional times. Colors for the instantaneous vorticity fields are saturated for the sake of clarity.

3.4. Effect of the aspect ratio

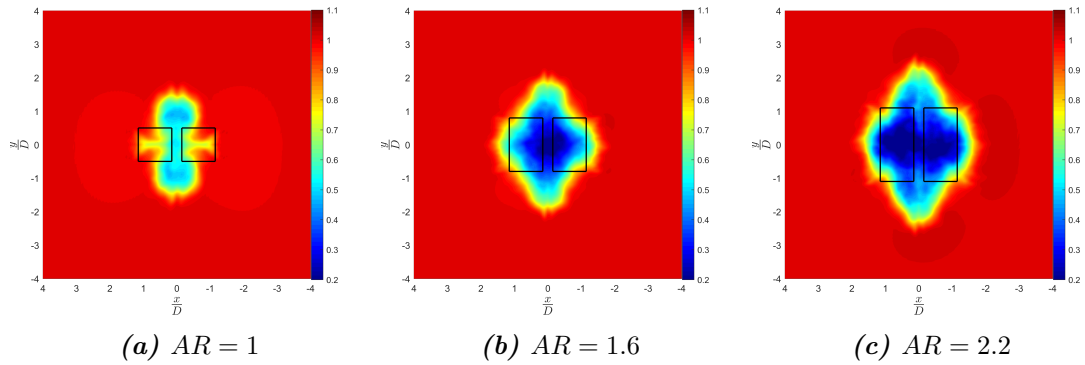


Figure 3.22: Time-averaged norm velocity field $\frac{\|\mathbf{u}\|}{U_\infty}$ in the cross-sectional plane at $z = 7D$ downstream. Setup parameters are those of table 3.6. Statistics are taken over $\hat{t} = 50$ non-dimensional times.

This analysis only applies to the counter-up layout. The direction of the wake bending does play a role in the transition to turbulence for the double-rotor concept. We suggest that for the counter-up configuration, the velocities induced by the tip vortices around $\theta = 90^\circ$ have to act against the growing wake in the spanwise direction caused by the respective wake bendings. This phenomenon is enhanced for smaller aspect ratios and could explain the lower values of spanwise velocities found in figure 3.19a compared to the layouts with higher aspect ratios. Ultimately, the cross-sectional time-averaged velocity magnitude will look like a cross-shaped or rhombus-shaped wake downstream, as it can be seen in figures 3.22a–3.22c. Note that for the counter-down configuration, the wake bending is directed outwards. We expect the transition to turbulence for this configuration especially to be mainly triggered by the instabilities of the individual turbines, and not to be due to interactions between their wakes.

Finally, figures 3.23a–3.23d present the displacement and momentum widths L_1 and L_2 along with the displacement and momentum surfaces S_1 and S_2 . We recall that the respective definitions diverge in the number of dimensions being integrated. For L_1 and L_2 , the velocity deficit and momentum deficit are integrated across the transverse direction only, and at mid span of the blades. For S_1 and S_2 , the whole $x - y$ plane is considered at each diameter downstream.

The obtained displacement width L_1 is in good agreement with the effect of aspect ratio discussed above. It is an indicator of the position and intensity of the recirculation bubble. For AR_{05} and AR_{2D} , no circulation bubbles are present in the flow, and the overshoot is less pronounced. Also, as the solver allows unbounded directions exactly, some of the mass flow is able to *leave* the computational domain. This leakage caused by the blockage effects and quantified by the values of S_1 in the far wake is, as expected, more predominant for higher aspect ratios, within a same box size of $[8D \times 8D \times 24D]$. Knowing that the counter-up configuration tends to push the wakes towards each other, leakage through the up and bottom surfaces is enhanced with the increasing height of

3.4. Effect of the aspect ratio

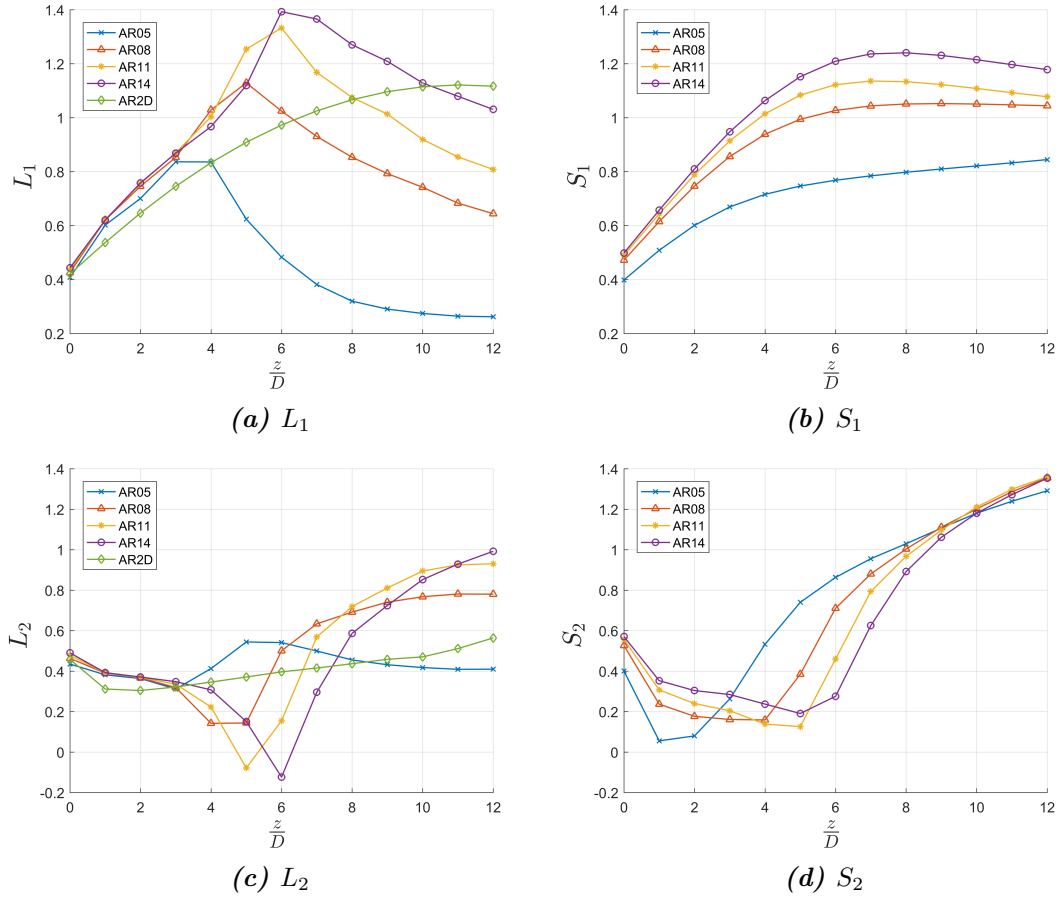


Figure 3.23: Displacement and momentum widths L_1 and L_2 and surfaces S_1 and S_2 based on the statistical streamwise velocity deficit at several diameters downstream. The parameters of the simulation are those of table 3.6.

the turbines. The overshoot is however less pronounced for the cross-sectional integral S_1 compared to the 1-dimensional transverse integral L_1 . Comparatively, we can observe that the smaller aspect ratio maintains a lower velocity deficit throughout the cross sections at each diameter downstream, but is not yet accelerating after 12 diameters downstream, whereas the highest aspect ratio has a decreasing mean velocity deficit after only 8 diameters downstream, and 7 diameters downstream for the intermediate aspect ratio AR_{11} .

3.5 Conclusions

Based on the same parameters as the experimental setup of Vergaerde *et al.* [24], a set of three-dimensional simulations were performed to extend the findings of chapter 2 by adding 3D effects, namely the spanwise blockage and the tip vortices. The different double-rotor layouts were studied and compared against a single rotor in the flow. Additionally, the effect of the number of blades and the influence of the aspect ratio on the aerodynamics and wakes were assessed. At this point, several conclusions are drawn:

- The interactions between turbines of each double-rotor layout are mutually beneficial.
- The numerical simulations overestimate the power coefficient experimentally obtained by Vergaerde *et al.*, but underestimate the gains the double-rotor concept offers compared to a single rotor.
- The wakes of each layout behave differently. For a same turbine inter-distance, the wake bending deviations dictates the distance needed to fully merge the individual wakes of the turbines. Moreover, the spanwise stretching of the wakes for the counter-up configuration will strongly interact with any ground surface, not taken into account in the present simulations.
- The counter-up and co-rotating configurations present the lowest mean velocity deficit in the far wake, but the single rotor yields in the same region a better recovery rate. Tip-induced velocities bring a significant spanwise mixing, fastening the transition to turbulence compared with a two-dimensional case.
- The numerical results are very sensitive to the curvature correction α_0 . This latter parameter is not scalable when comparing the diagnostics evolution over azimuth position of N -bladed turbines with identical non-dimensional numbers.
- Mean aerodynamic loadings are decreased with an increased number of blades. The associated fluctuations over a revolution are attenuated, decreasing the structural fatigue loadings on the rotor's mast.
- The aspect ratio has little influence on the aerodynamic quantities. However, its effect on the wake's behaviours is significant. Increasing the aspect ratio pushes

any recirculation region further downstream and dictates the distance needed for each individual wake to reach a fully-turbulent state.

- The transition to a fully-turbulent flow for the double-rotor layout, and especially the counter-up configuration, is caused by the merge of individual wakes through wake bending effects for very low aspect ratios. For higher aspect ratios, the transition to turbulence for the layout is caused by the already turbulent wakes of each individual turbine.

Performances being equivalent, choosing an aspect ratio for a wind farm is a trade-off between higher recovery rates, and lower average velocity deficits. In terms of wake recovery diagnostics, the most influencing parameters are the tip-speed ratio and the aspect ratio of the turbines. For a double-rotor layout in counter-up configuration, the wake bending direction is a new parameter that has to be taken into account regarding decay diagnostics, which in turn is dictated by the turbine inter distance d_{int} . Optimizing the wake recovery for double-rotor layouts goes with optimizing tip-speed ratio λ , aspect ratio AR and turbine inter-distance d_{int} simultaneously.

Chapter 4

Floating offshore wind turbine

This chapter provides an overview of the dynamics behind a floating vertical-axis wind turbine. This chapter is composed of three parts: in the first part, a description of the entire system is offered, in the second part, the numerical implementation is described while in the last, an analysis of the interactions between the floating platform and the VAWT aerodynamics and wakes is performed.

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4.1 Introduction

Oceans represent an immense and until now under-used renewable power resource, mainly due to high cost-involved, technical and environmental challenges. However, a new generation of wind turbines (WT), the floating ones, overcoming those issues. These turbines are expected to play a key role in moving towards a sustainable energy system in Europe.

Up to recently, ocean wind turbine's implementation was not feasible, mainly because of the foundation structure. Above 30 m of depth, fixed-bottom structures become extremely expensive, making this technologies completely noncompetitive in terms of capital and operational expenses. Nevertheless, offshore WTs present some advantages, including stronger and steadier winds. As the wind power curve follows an exponential for moderate wind speeds, a small increase in wind speed will generate a higher gain in energy production. Furthermore, they are also able to overcome drawbacks faced by ground wind turbines: lack of space as well as noise and visual pollution.

With the apparition of floating technologies, exploiting ocean wind fields becomes achievable at moderate costs. In case of floating systems, foundation structures are actually substituted by mooring systems, which enable a reduction of the investment. Additionally, compared to bottom-fixed foundations, they are suitable for a wider range of distance to shore while having lower installation, maintenance and removal costs. They also reduce loads acting on the mast. But this design comes with new challenges such as minimizing the wave induced-motions, understanding and modelling the complete system and analyzing the effect of the coupling on the support structure and on the wind turbine.

Models of floating platform already exist in other fields such as oil extraction, yet the incentives regarding their design are different: floating structures for wind turbines must achieve a high rotor stability while dealing with the overturning thrust loads. For now, three types of design stand out: the Spar class, the Semi-Submersible class and the Tension-Leg Platform (TLP) class. All of them are depicted against bottom-fixed structures in figure 4.1.

The spar class concept stabilizes the wind turbine thanks to a separation of the center of buoyancy and the center of gravity. The gravity center is adjustable and well below the sea water level (SWL), allowing a fine tune of the stability. To ensure performances whatever the atmospheric conditions, mooring lines are added. Semi-submersible class, also maintained by anchor cables, uses buoyancy distribution across the entire platform, principle inspired by catamaran boats. Basically, it consists in 3 or 4 floaters maintaining the wind turbine at the water surface. Finally, the TLP design freezes the wind turbine thanks to a strong tension of the mooring lines. An excessive buoyancy of the hull

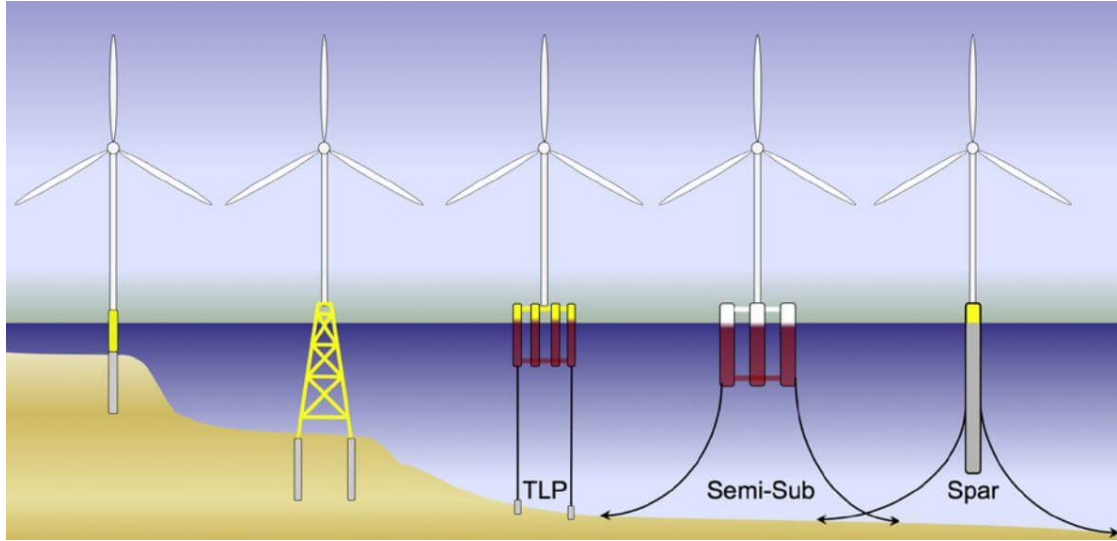


Figure 4.1: Types of offshore platforms [38]

stretches cables to the maximum. Practically, any floating platform can be defined as a combination of the above ones to ensure the required balance [41].

Advantages of VAWTs against the HAWTs are amplified when it comes to offshore applications. Location of the drive-train, which is more modular in the case of the vertical ones, allows not only to change the gravity center of the entire structure (rotor, tower and platform), but also to ease the access to major components. A center of mass positioned near the surface will reduce the structure tilting moment and costs implied in the substructure stability. A facilitated access to drive-train parts (generator, gearbox, ...) lowers repair and replacement charges. Finally, large VAWT rotors are subjected to combination of centrifugal and aerodynamic loads, but are not constrained to cyclical gravitational loads, in contrast to HAWTs. Enlarging the size of wind turbines, proportional to their energy production, then becomes an option [39]. Currently, offshore wind technologies are still quite expensive compare to onshore wind turbines but in a long term view the gap between those two should vanish with the increase of the technology maturity.

Floating VAWTs concept is quite a recent research field, its state-of-the-art is mainly composed by validations of numerical tools throughout fitting of experimental data, code-to-code verification and comparison between different types of platforms.

The Offshore Code Comparison Collaboration (OC3) project constitutes some of the greatest work achieved in the field of floating wind turbines. This project, gathering several research group, aimed to (1) discuss strategy of modelling, (2) develop a suite of benchmark models and simulations, (3) run the simulations and process their results,

and (4) compare and discuss the results. They have mainly compared a fixed bottom foundation against a spar-buoy platform, named *Hywind - Spar* [42]. Continued with OC4 (investigation of a semi-submersible platform) and OC5 (code-to-data validation), this project ambitions to give insights of simplified or more complex model to understand mechanisms of the coupling between aerodynamics, hydrodynamics and elastics loads. The final objective is to develop a highly sophisticated aero-hydro-servo-elastic code (coupling of aerodynamic, hydrodynamic, control system and structure-dynamic computation). For the moment, they are only considering horizontal-axis wind turbines [43] [44].

Besides, Leroy *et al.* [46] compared two solvers (coupling hydrodynamics and aerodynamics) in terms of accuracy and computational cost. Their first simulations were computed with a Double Multiple Streamtube method (DMS) along with *InWave*, a seakeeping software composed of a hydrodynamic data base and a multibody-solver while their second simulations were performed using a Free Vortex Wake (FVW) solver. DMS solver assumes quasi static flow on the turbine whereas FVW code is inherently unsteady, resulting in higher accuracy but also higher computational cost. This method is more realistic as the six degree of freedom (DOF) of the platform induce complex motions of the rotor, influencing its aerodynamics. The set up was a 2-straight-bladed VAWT on the *OC3Hywind* SPAR platform in the OC3 load cases. Results show good agreement for small amplitude motion but diverge slightly for higher amplitude ones, DMS solver showing a lower aerodynamic damping [47, 46]. At the same time, Ying *et al.* [48] studied aerodynamics and motion performance of a H-type wind turbine equipped with a truss spar foundation. Surge-heave-pitch nonlinear coupling equations were established and coupled with a DMS theory. After observing that the motions of the floating foundation are of large amplitude, they compared the behavior of a H-type floating VAWT against that of the Φ -type floating VAWT and concluded that H-type VAWT's are less sensitive to sea-induced motions.

Even though experimental validation might be of need, the present aero- and hydrodynamic solver gives a first overview on impact of sea-induced motions on VAWT aerodynamics and wakes. This chapter runs through the selected platform and its main properties. It then details the different loads present in such a system and their current numerical implementation. The conclusion eventually discusses the model performances.

4.2 Floating platform properties

In the present work, it has been decided to study floating properties of an existing industrial setup. Several reasons motivated this choice but data availability was the decisive one. With that in mind, the studied design is greatly inspired by the *TwinFloat* project [40]. This French concept aims at producing a new type of wind farm: floating offshore wind farm of VAWTs in double-rotor configuration, making it perfectly in accordance with the two Master's thesis objectives. Figure 4.2 reproduces an example of what the entire wind farm might look like.

Since physics of double-rotor configuration has already been widely described in chapters 2 and 3, we will solely focus on underlying physics of the floating effects. Therefore, we adapt the *TwinFloat* to transform it into a single H-type floating one. Apart from that, the other main design drivers remain unchanged. Sketch of the final selected model is illustrated in figure 4.3a.



Figure 4.2: *Nenuphar - TwinFloat offshore wind farm project, from [40]*

The H-type VAWT is thus mounted on a semi-submersible structure, maintained thanks to three catenary mooring lines. The height of the entire system is 120 m above sea water level (SWL), and the corresponding weight is approximately 1300 ton. The structure goes down to 20 m below the SWL. Platform properties come from [43] while VAWTs characteristics were found in [60]. Main dimensions are depicted in figure 4.3b. Note that the semi-submersible structure has two non-negligible advantages for our case. A wide range of reachable water depth gives more flexibility for the choice of wind farm site whereas an ease of installation makes it more adaptable for a double VAWT configuration.

The next subsections give an insight of each structure components, namely wind turbine, platform and mooring lines.

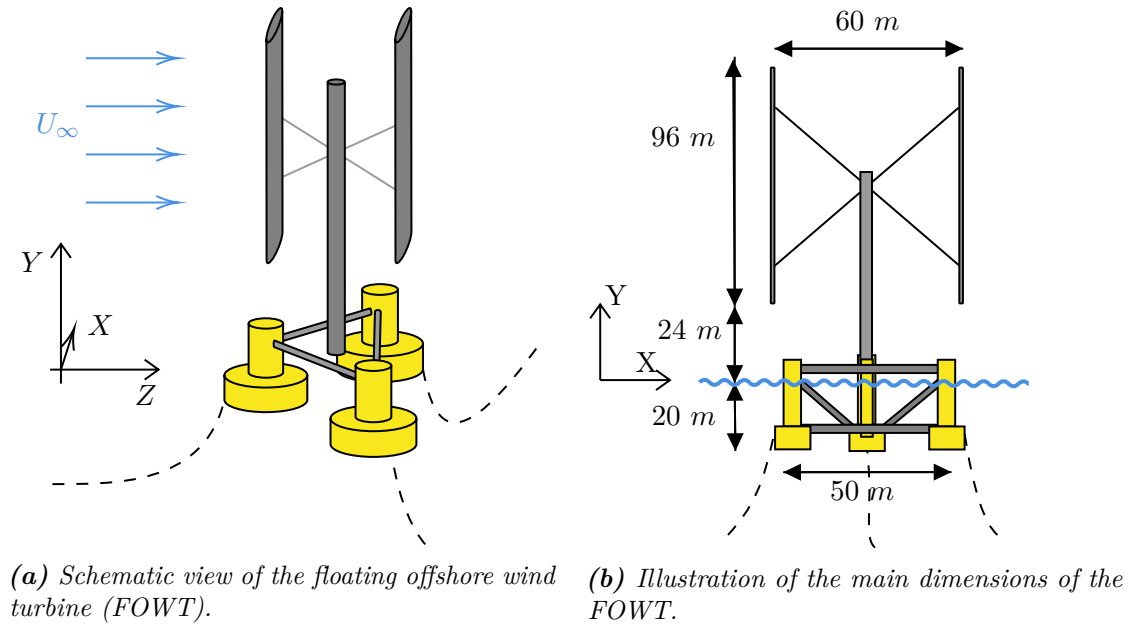


Figure 4.3: Illustrations of the implemented VAWT

4.2.1 VAWT properties

The selected wind turbine is a H-type wind turbine. Its NACA0018 blades are 96 m long with a chord of 6 m. They are located 24 m above the SWL. Blades weight was calculated thanks to values given in [61]. The specific mass per unit length, referenced for 5 meter chord profile, was corrected by a factor equal to the ratio of the profiles area. For the inertia mass matrix computation, blades and tower were considered as cylinders. Eventually the VAWT diameter is 60m, which gives an aspect ratio (AR) of 1.6. Those primary properties are listed in table 4.1 while main VAWT dimensions are depicted on sketch 4.3b.

Elevation of the tower base (above SWL)	10 m
Length of the tower	80 m
Length of blades	96 m
Blades chord	6 m
VAWT diameter	60 m
Tower mass	249 719 kg
Blade mass (each blade)	30 302 kg

Table 4.1: VAWT main properties

4.2.2 Semi-submersible platform properties

Based on the DeepCwind semi-submersible, this triangle-shaped platform is composed by three primary columns, ensuring buoyancy, and by a main column carrying the wind turbine mast. For the sake of clarity, schematic view of this subsystem is given in

figure 4.4. Primary columns, also called floaters, are divided into two cylinders of different diameter: the upper column and the base one also referred to as the heave disc. The latter plays an important role in the vertical dynamic response of the platform under waves loads.

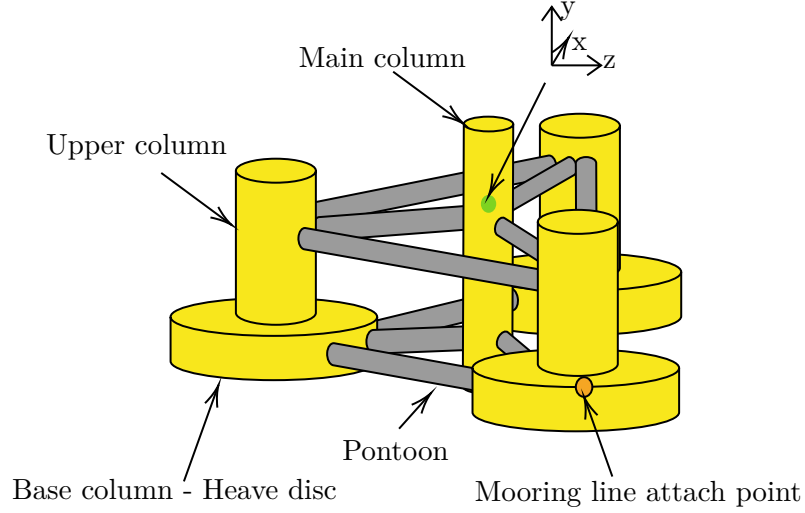


Figure 4.4: Sketch of the platform.

An attractive aspect of this system is the wide range of reachable water depth it offers. Able to go down to shallow water (down to 25 m), they offer flexibility in the wind farm site choice. Contrary to instinctive impressions, it is harder to design catenary mooring for shallower water, since a small horizontal motion will exert high tensions in the line [41].

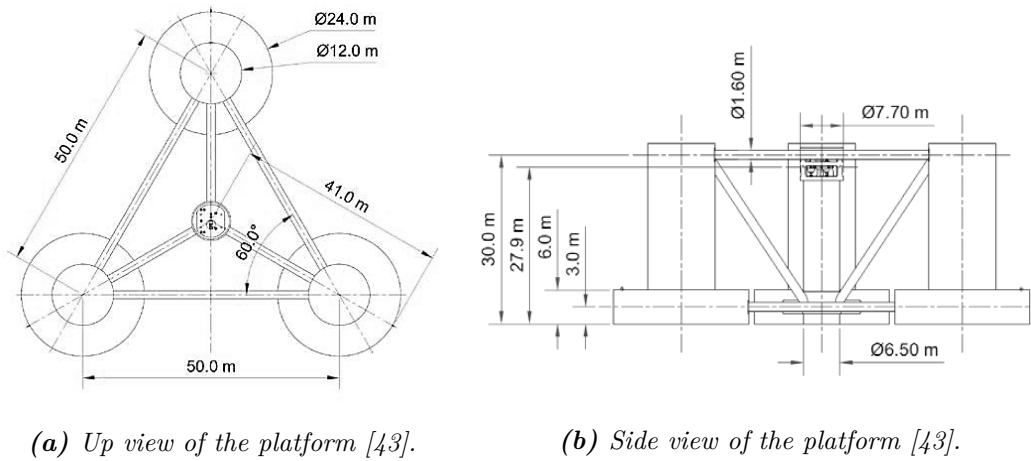


Figure 4.5: Detailed drawings of the platform.

4.2. Floating platform properties

At static equilibrium, main column retains the tower 10 m above the SWL. Each column, connected together with pontoons, starts above the SWL and continues beneath water surface. The inertial frame of the global system is taken on the main column, at the SWL. Plans of the platform are provided in figures 4.5a and 4.5b.

Upper and base column are filled with water, mainly for reaching the optimal depth needed for the heave disc operation. The total weight of the platform, including ballast water, is equal to $1.32 \cdot 10^7$ kg. The center of mass is located 13.46 m underneath the SWL, lying on the negative y -axis.

4.2.3 Mooring lines properties

Anchor lines eventually prevent the system from drifting under the influence of waves and current. Therefore, three catenary mooring lines were added to ensure sea-keeping function. Fairleads, ends of the cable positioned on the platform side, are fixed on the top of the base column. Those cable ends are thus located 14 m below the SWL. On the sea plane, they are attached at a radius of 40.8 m from the center of the structure. On the other end, anchor points are fixed to the sea floor 200 m under the SWL at radius of 837.6 m from the semi-submersible center line. This configuration is illustrated in figure 4.6. The 3 lines are separated by an angle of 120 deg. All their properties

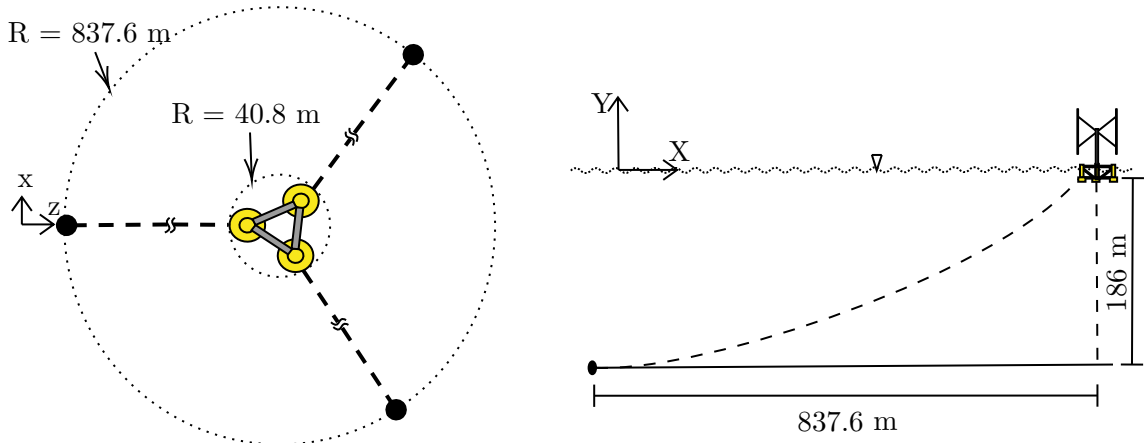


Figure 4.6: Configuration of the mooring cables.

4.3. Dynamic forces on a floating platform

Number of mooring lines	3
Angle between adjacent line	120 deg
Depth to anchor below SWL	200 m
Depth to fairleads below SWL	14 m
Radius to anchors from platform center lines	837.6 m
Radius to fairleads from platform center lines	40.868 m
Unstretched mooring line length	835.5 m
Mooring line nominal diameter	0.0779 m
Equivalent mooring line mass density	116.6 kg/m
Equivalent mooring line extensional stiffness	753.6 MN

Table 4.2: Mooring system main properties from [43] and [55].

4.3 Dynamic forces on a floating platform

Now that platform properties were described, we can go deeper in the description of dynamic underlying floating technologies. Prediction of a FOWT behavior implies a multidisciplinary analysis. Soft substructures restoring forces lead to rotor motions and impacts its aerodynamics. It results in complex coupling effects between the submodels, namely aerodynamics, hydrodynamics, servodynamics and structural dynamics [56]. In this Master's thesis, great attention is paid to the combination of aerodynamics, hydrodynamics and structural submodels. The following paragraph offers a broad view of all the forces involved in this system.

Sketch 4.7 shows the different loads acting on the floating platform alone. For the sake of clarity, only one ballast is illustrated. Three types of forces are taken into account: weight (in red), hydrostatic and hydrodynamic loads (in blue) and finally the mooring lines reaction (in yellow). The gravitational forces is constant and always pointing in the negative y -direction. It is easily implemented thanks to an inherent feature of ROBOTRAN. Fluid-structure interaction can be decomposed into two components: hydrostatic and hydrodynamic forces. Hydrostatic forces, responsible for the platform buoyancy, are constant when the system is at equilibrium, yet acting like a spring in case of perturbation. Hydrodynamic forces result mainly in waves and current loading. Finally, mooring lines prevent the platform from being carried away by waves. Their intensity depends on their tension but also on their displacement.

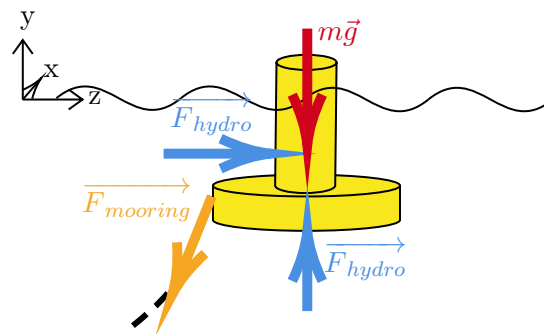


Figure 4.7: Dynamic forces on a floating platform.

As reproducing floating bodies dynamics requires ability to generate waves, the following sections first explain our wave generator implementation, based on the JONSWAP spectrum. Then it goes throughout the description of each force individually.

4.3.1 Ocean waves generation

Waves are the most evident and observable phenomenon that naturally occurs at the ocean or sea surface. It is therefore crucial to develop an accurate model of the kinematic and dynamic behaviour of waves in order to include them as an external loading on our platform.

Ocean and sea waves are mainly produced by two phenomena. On the one hand, waves are produced by tides. Tides are caused by the small changes in the gravitational attraction due to the motion of the earth through the solar system, and particularly due to the motion of the moon around the earth.

On the other hand, ocean waves are also produced by wind through a three step mechanism. Starting from a smooth free of wind sea surface, when wind suddenly blows :

1. Small waves are formed by the pressure differences caused by the turbulence in the wind.
2. Those waves are also hit by the same wind, and gain in height and energy as the pressure difference across their profile increases. This process is unstable and hence the formed waves end up collapsing at some point.
3. The constant forming and collapsing begins to interact with the rest of the ocean. Waves interact with each other and distribute their energy.

The irregular nature of wind causes the sea surface, and more specifically the sea elevation, *i.e.* the height of the waves above or below a mean value, to be continually varying. A deterministic approach, where at any time and any location the properties of a wave can be described is an unfeasible task. Hence a statistical approach is preferred for describing wave properties, as average wave height, wave periods and directions. These properties appear to vary slowly in time and space. This approach tends to find a certain probability distribution giving an accurate description of the sea elevation $\xi(\mathbf{x}, t)$ over space and time.

Spectral description of ocean surface waves

Statistics are based on records of the sea elevation. These records are mainly provided by floating buoys, giving the actual sea elevation over time. As these buoys are fixed, the measures give no information about the direction of the waves, but provide $\xi(t)$ at a specific location. An example of record over a time period T is given in Figure 4.8.

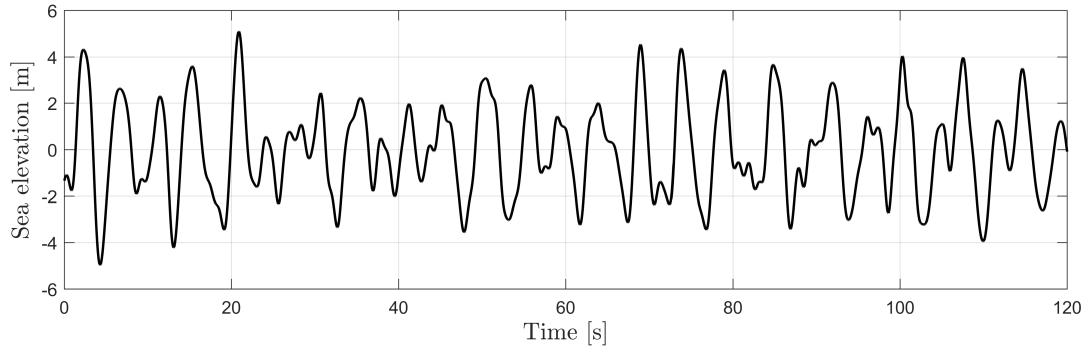


Figure 4.8: Record of wave elevation by a floating buoy.

The most common method to statistically describe ocean waves is based on the spectral analysis. For a continuous function, Joseph Fourier showed that almost every function can be obtained as a linear combination of an infinite number of harmonic oscillations. Describing the above elevation $\xi(t)$ would give

$$\xi(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(2\pi n f t) + b_n \sin(2\pi n f t)) \quad (4.1)$$

where $n f$ is the n^{th} harmonic of the fundamental frequency $f = \frac{1}{T}$, a_n and b_n are the coefficients of the Fourier series, provided by

$$a_n = \frac{2}{T} \int_T \xi(t) \cos(2\pi n f t) dt \quad (4.2)$$

$$b_n = \frac{2}{T} \int_T \xi(t) \sin(2\pi n f t) dt \quad (4.3)$$

Using a zero-mean level, implying $a_0 = 0$, and trigonometric relations, one can rewrite the surface elevation as :

$$\xi(t) = \sum_{n=1}^{\infty} c_n \cos(2\pi n f t + \alpha_n) \quad (4.4)$$

where α_n is the phase of the n^{th} harmonic.

Using the definition of a complex exponential, the Fourier series can be rewritten as

$$\xi(t) = \sum_{n=-\infty}^{\infty} Z_n e^{i2\pi n f t} \quad (4.5)$$

$$Z_n = \frac{1}{T} \int_T \xi(t) e^{-i2\pi n f t} dt \quad (4.6)$$

Z_n being the complex amplitude of the n^{th} harmonic. The latter form is convenient to compute the wave spectrum $S_{(w_n)}$ or $S_{(nf)} \equiv Z_n Z_n^*$.

4.3. Dynamic forces on a floating platform

In practice, floating buoys do not record the elevation continuously, but instead sample the records at fixed intervals Δt . The elevation is hence recorded at discrete moments $\xi_j = \xi(j\Delta t)$ in time and the Fourier integrals become sums, N being the number of samples recorded and $T = N\Delta t$ the total length of the record. The sampling process gave up an infinite amount of information about the surface, and this has a consequence on the bandwidth that is well defined by the samples. According to the sampling theorem, the wave record contains information about the frequencies being in the range

$$\frac{1}{T} < f < \frac{1}{2\Delta t} \equiv N_y \text{ the Nyquist frequency.} \quad (4.7)$$

One hence has to chose Δt small enough to lose little useful information.

Using one of these sampled sea elevations, one can compute the discrete Fourier transform of the record :

$$Z_n = \frac{1}{N} \sum_{j=0}^{N-1} \xi_j e^{(-i2\pi jn/N)} \quad (4.8)$$

$$\xi_j = \sum_{n=0}^{N-1} Z_n e^{(i2\pi jn/N)} \quad (4.9)$$

$$(4.10)$$

The spectrum S_n of the associated record is then given by

$$S_n = \frac{1}{N^2} (|Z_n|^2 - |Z_{N-n}|^2) \quad n = 1, 2, \dots, \left(\frac{N}{2} - 1\right) \quad (4.11)$$

$$S_0 = \frac{1}{N^2} |Z_0|^2 \quad (4.12)$$

$$S_{N/2} = \frac{1}{N^2} |Z_{N/2}|^2 \quad (4.13)$$

One can observe that the variance of the elevation $\langle \xi_j \rangle^2$ is given by the sum of the spectrum values.

Linear theory

On can expand the idea of Fourier series to include spatial dependency of the elevation $\xi(\mathbf{x}, t)$. Waves would then not only be described by a wave frequency, but also by wavelenghts λ , the distance between two successive wave crests at a given time. The wave frequency is related to the wave number $k = \frac{2\pi}{\lambda}$ by the dispersion relation:

$$\omega^2 = gk \tanh(kh) \quad (4.14)$$

where h is the water depth, and g the acceleration of gravity. Linear theory is a useful approximation of the dispersion relation when dealing with great water depths. In this

case, $h \gg \lambda$ and the relation becomes $\omega^2 = gk$. The local sea surface can hence be represented by a linear superposition of cosine waves having many different wave lengths or frequencies. In this approximation, the Fourier series cease to represent a mathematical tool, but really represent that the sea surface is composed of harmonic oscillations. The phase is an uniformly distributed random variable between 0 and 2π , and the amplitude is considered as a random variable too.

From different records and the associated simple spectrum S_n , one can attend to statistically build an amplitude distribution over the frequencies. But rather than the amplitude, the variance density is chosen as a statistical parameter. Indeed, linear theory implies that the variance of the sea surface displacement is related to the wave energy density E by :

$$E = \frac{1}{2} \rho_w g < \xi^2 > \quad [J/m^2] \quad (4.15)$$

For now on, ρ_w will refer to sea water density while ρ stands for air density.

The spectrum S_n is directly related to the variance, which is proportional to the energy of the waves. The simple spectrum may be a discrete function, but in fact all frequencies are present in real waves. The *Wave Density Spectrum* or *Energy Density Spectrum*, is the continuous counterpart of the simple spectrum S_n for one record, and is denoted $S(f)$ or $S(\omega)$. It gives the distribution of the variance of the sea surface, but also represents how the total energy of the waves is distributed among the different frequencies they are composed of.

Some predefined wave spectra

Having a mathematical description of the wave spectrum provides a powerful tool to rapidly compute a randomly generated sea surface. In designing ships and offshore structures, a predefined spectrum depending on the wind velocity and other parameters could be of use to numerically test preliminary design prototypes. The commonly used ocean wave spectra are the Pierson-Moskowitz and the JONSWAP ones for deep water areas. One has to keep in mind that these are attempts to describe the ocean wave spectrum and are highly idealized.

a) *Pierson-Moskowitz Spectrum*

This spectrum was proposed by Pierson and Moskowitz in 1964. They elaborated the concept of a fully developed sea: waves in equilibrium with the wind, blowing steadily over a large area for a long time. They fitted many measurements of waves, selected when the wind had blown steadily long enough, and found a specific spectrum $S(\omega)$ with $\omega = 2\pi f$ the angular velocity.

b) *JONSWAP Spectrum*

The JONSWAP Spectrum, for *Joint North Sea Wave Observation Project* (1973) is particularly well suited for North-Sea waves, and adds to the Pierson-Moskowitz

4.3. Dynamic forces on a floating platform

Spectrum a peak enhancement factor γ^r . This factor was added to improve the fit to their measurements.

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[\frac{-5}{4} \left(\frac{\omega_p}{\omega} \right)^4 \right] \gamma^r \quad [m^2/Hz] \quad (4.16)$$

$$r = \exp \left[\frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2} \right] \quad (4.17)$$

where g is the acceleration of gravity, α the spectrum intensity and ω_p the peak angular velocity. The values of the constants were experimentally determined as :

- $\alpha = 0.076 \left(\frac{U_{10}^2}{Fg} \right)^{0.22} \quad [/]$
- $\omega_p = 22 \left(\frac{g^2}{U_{10}F} \right)^{1/3} \quad [rad/s]$
- $\gamma = 3.3 \quad [/]$ for the North-Sea
- $\sigma = \begin{cases} 0.07 & \omega \leq \omega_p \\ 0.09 & \omega > \omega_p \end{cases}$

F being the fetch, *i.e.* the distance over which the wind is assumed to blow with constant velocity, and U_{10} the wind velocity 10 meters above the sea level. The added peak enhancement factor γ^r leads to better fitting of the waves that are inherently non-linear.

The corresponding frequency spectrum is given by $S(f) = S(\omega \frac{\partial \omega}{\partial f}) = 2\pi S(2\pi f)$. The factor 2π ensures that the area under the curve remains the same, implying that no additional energy is being produced or destroyed.

As in the case for simple wave spectra, the variance is given by the integral of $S(w)$, and is related to the total energy density:

$$\langle \xi \rangle^2 = \int_0^\infty S(\omega) d\omega = \sum_{n=0}^\infty \frac{1}{2} c_n^2 \quad [m^2] \quad (4.18)$$

$$E_{tot} = \frac{1}{2} \rho_w g \langle \xi^2 \rangle \quad [J/m^2] \quad (4.19)$$

In this spectrum model, the energy contained in the waves is directly proportional to the fetch distance F .

Another important parameter is the significant wave height $H_{1/3}$, sometimes referred to as H_s . Initially, it was defined as the mean of the height of the highest third of the waves. It is now related to the standard deviation of the sea-surface based on measures of the sea displacement. In this case :

$$H_{1/3} \simeq 4 \langle \xi^2 \rangle^{1/2} = 4 \left(\int_0^\infty S(\omega) d\omega \right)^{1/2} \quad [m] \quad (4.20)$$

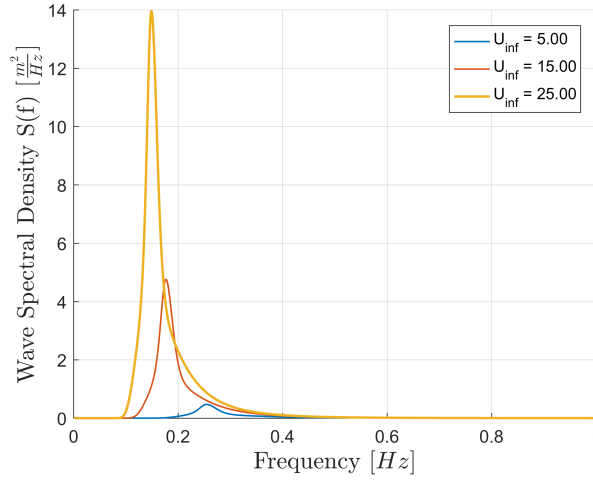


Figure 4.9: Spectral density obtained with the JONSWAP model for different wind velocities. The fetch distance is $F = 50 \text{ km}$.

Figure 4.9 shows the obtained spectrum with a fetch length of $F = 50 \text{ km}$ and for three different wind velocities $U_{10} = U_{\infty}$. The fetch length was chosen as a mean value of existing European offshore wind turbines [64].

One observes that the JONSWAP spectrum density has a narrow frequency bandwidth, going near a zero value for frequencies above 1 Hz .

Randomly generated waves with JONSWAP spectrum

Using the JONSWAP spectrum, and assuming that the direction of propagation is aligned with the wind direction, one can generate, for a given wind velocity and fetch distance, N terms of the Fourier series composing the sea elevation. Those N terms define N frequencies between $f \in]0; 1] \text{ [Hz]}$ with increments of Δf . The resulting sea elevation is composed of cosine waves with different amplitudes c_n , frequencies f_n , angular velocities $\omega_n = 2\pi f_n$ and uniformly distributed initial phases α_n .

Having the discretization $\Delta\omega = 2\pi\Delta f$ and $\omega_n = n\Delta\omega$, one can use equation (4.18) to obtain approximations of the amplitudes of each cosine wave :

$$c_n \simeq \sqrt{2S(\omega_n)\Delta\omega} = \sqrt{2S(f_n)\Delta f} \quad (4.21)$$

The elevation $\xi(t)$ can thus be written as the sum of the N defined cosine waves :

$$\xi(t) = \sum_{n=1}^N \sqrt{2S(\omega_n)\Delta\omega} \cos(\omega_n t + \alpha_n) \quad (4.22)$$

According to Figure 4.9, the bandwidth with a significant amplitude is narrow, hence N should be sufficiently large to

4.3. Dynamic forces on a floating platform

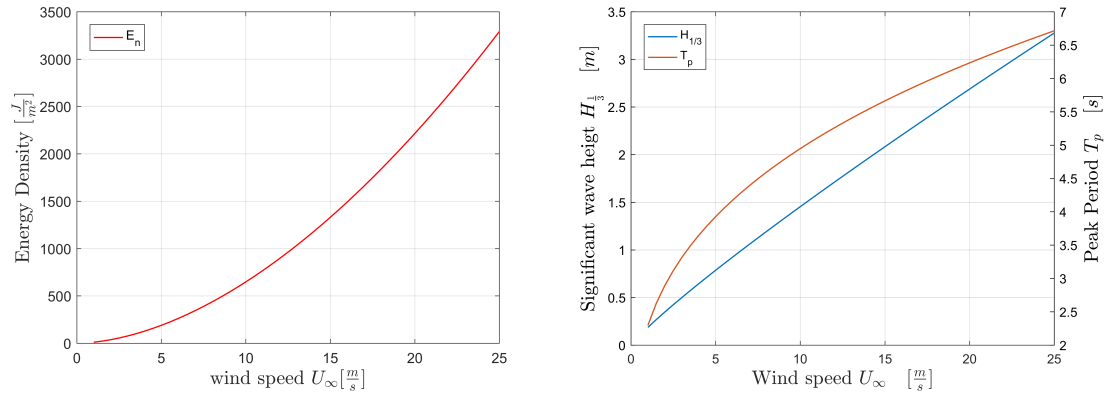


Figure 4.10: Significant wave height $H_{1/3}$, peak period T_p and Energy density as a function of the wind velocity $U_{10} = U_\infty$, according to the JONSWAP model. The fetch length is $F = 50$ km.

1. Cover the whole spectrum between $f \in]0; 1]$ $[Hz]$;
2. Be representative enough of the total energy density contained in the real waves.

Those two considerations lead to choose a strict minimum value of $N = 75$. The resulting sampling is shown on Figure 4.11. One can observe that only few frequencies having a significant amplitude are represented by this sampling, due to the narrowness of the bandwidth.

According to Figure 4.10, the resulting elevation should have a significant wave height of about 2 m, and a peak period of approximately 5.5 s.

The elevation function as described by equation (4.22) for $N = 75$ is plotted over time in Figure 4.12. A representative time interval being 40 s. One can effectively observe that the main crests occur approximately at every peak period $T_p = 5.5$ s. However the maximal amplitude attained is slightly smaller than $H_{1/3}$, the significant wave height, as the sampling process discards an infinite number of frequencies that should be present in real waves.

The obtained elevation function $\xi(t)$ over time was used in the analysis of a single VAWT on offshore platform, where waves motion are used in ROBOTRAN to compute hydrodynamic loads. This is discussed in subsection 4.4.

4.3.2 Hydrostatic loads

Hydrostatic plays an important role in studying fluid structure interactions. Here, it allows to ensure static properties of the system. It is performed by means of

$$\mathbf{F}^{Hydrostatic}(\mathbf{q}) = \rho_w \mathbf{g} V_0 - \mathbb{C}^{Hydrostatic} \mathbf{q} \quad (4.23)$$

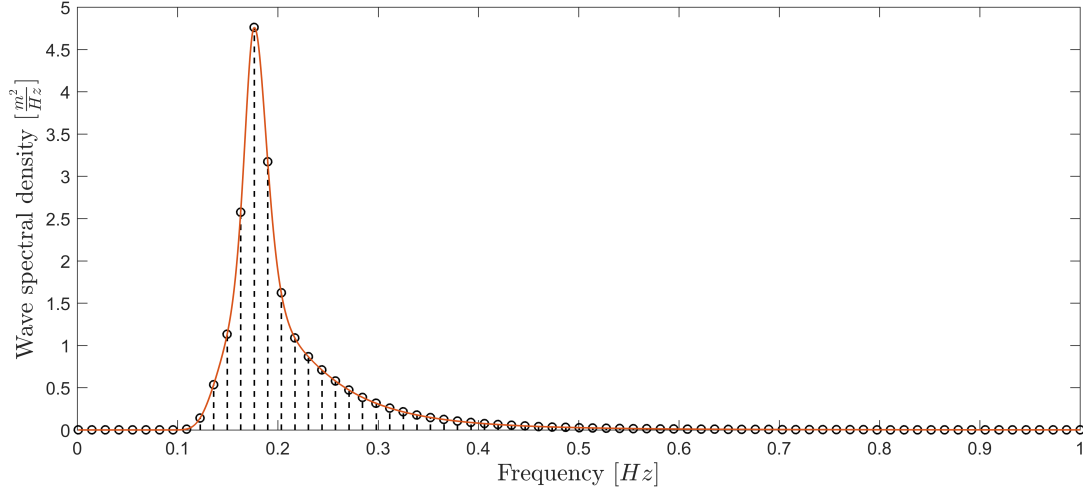


Figure 4.11: Final sampling of the JONSWAP wave spectrum model, with $N = 75$, wind velocity $U_\infty = 15 \frac{m}{s}$ and fetch length $F = 50 \text{ km}$.

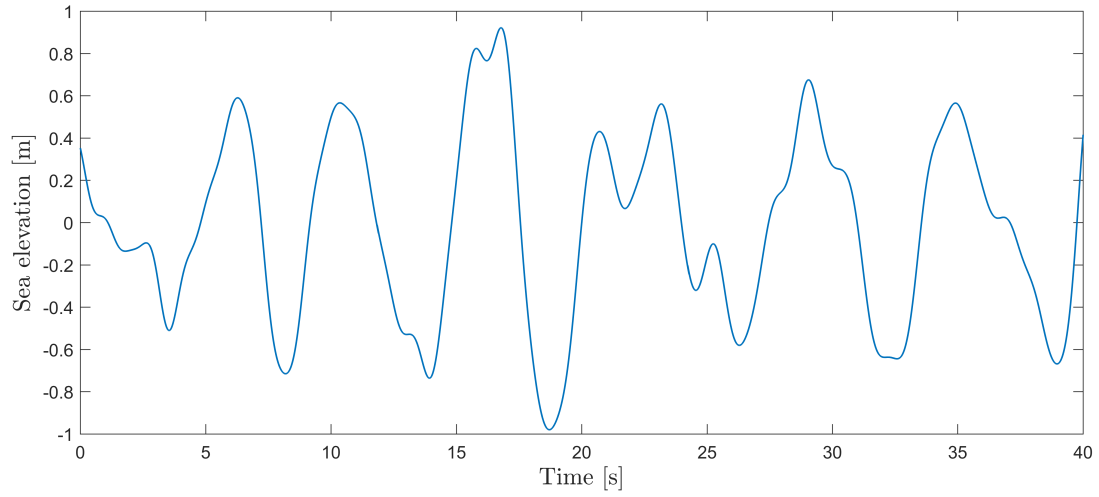


Figure 4.12: Resulting elevation function $\xi(t)$ for $N = 75$ representative frequencies of the wave spectral density. The wind velocity is $U_\infty = 15 \frac{m}{s}$ and the fetch distance is $F = 50 \text{ km}$.

where $\mathbf{g}$ is the gravitational acceleration, pointing towards the positive y -direction, V_0 is the displaced volume of fluid at rest position, $\mathbb{C}$ is the linear hydrostatic restoring matrix, resulting from effects of water-plane at the center of buoyancy, and $\mathbf{q}$ represents the set of platform DOF. Loads are positive in the positive motion of the DOF.

In equation 4.23, the first term of the right-hand side (RHS) models the buoyancy force from the well known Archimede's principle whilst the second term express the change in hydrostatic force and moment in response to platform displacement. Note that the constant numerical values are obtained from [43] by means of a code-to-code verification and can be found in the appendices A.1.

4.3.3 Hydrodynamic loads

Hydrodynamic loads on offshore wind systems are composed of different contributions: excitation from incident waves, radiation of outgoing waves from platform motion and viscous forces. There are several ways to treat those components. Among those, the most common ones are the Potential-flow theory-based method and Morison's equation.

The potential flow theory-based method is considered as the more viable one for the present considered structure. However, it implies the use of three-dimensional panel methods able to solve the linearized potential-flow hydrodynamic radiation and diffraction problems for arbitrary geometries [43]. Even though this model could give a better insight on loads applied to the platform, a simplified hydrodynamic model is sufficient to study qualitatively the impact of sea-induced motion on the VAWTs aerodynamics and wakes. Hence, a simplified hydrodynamic model using Morison's formulation is implemented [43].

Hydrodynamic loads are accounted for thanks to the Morison's formulation. This model is quite common when analyzing fixed-bottom structures of offshore wind turbine. It can be adapted to floating rotor at the expense of some assumptions:

- the effect of diffraction can be simplified with the long wavelength approximation;
- radiation damping is negligible;
- flow separation may occur.

Physically, those approximations are admissible in the case of moderate to extreme sea-state, which means waves with a 10 s period and 3 m height. This model takes the main components of waves forces into account, namely incident-wave-induced excitation, radiation-induced added mass and flow-separation viscous drag. Since immersed components of the platform are cylindrical, those forces are decomposed according to two axes: fluids moving perpendicularly to the cylinder generate so-called transverse flow forces while the vertical displacements of the cylinder will generate a heave force.

Transverse flow loads

For a cylinder in steady, transverse flow, the wave forces per unit length on the cylinder moving with velocity $\dot{\mathbf{q}}$ can be expressed as:

$$\mathbf{F}_t = \frac{1}{2}C_d\rho_w D(\mathbf{u}_t - \dot{\mathbf{q}}_t)|\mathbf{u}_t - \dot{\mathbf{q}}_t| + (1 + C_a)\rho_w \frac{\pi D^2}{4}\ddot{\mathbf{u}}_t - C_a\rho_w \frac{\pi D^2}{4}\ddot{\mathbf{q}}_t \quad (4.24)$$

where D is the diameter of the cylinder, $\mathbf{u}_t$ is the transverse velocity of a water particle, $\dot{\mathbf{u}}$ refers to its time derivative, C_d represents the drag coefficient and C_a is the added mass coefficient. The added mass expresses the inertia added to the system due to the fact that an accelerating body needs to move the fluid around it. Subscript t refers to transverse direction.

Heave plate loads

In the axial direction, the hydrodynamic force is computed with a modified Morison's equation, in order to take into account dynamic pressure effect. The formula is given below.

$$F_y = \frac{1}{2}\rho_w C_{dz} A_c (v - \dot{q}_2)|v - \dot{q}_2| + \rho_w C_{az} V_R (\dot{v} - \ddot{q}_2) + \frac{\pi}{4} D_h p_b - \frac{\pi}{4} (D_h^2 - D_c^2) p_t \quad (4.25)$$

C_{dz} is the drag coefficient in the heave direction, A_c the cross sectional area of the heave-plate in the y -direction, v the vertical velocity of a water particle and $\dot{q}_2$ the heave velocity of the platform. D_h refers to the heave-plate diameter while D_c is the diameter of the upper column.

The two first terms of the RHS refer to the drag and added mass loads, as in the previous case. Coefficients of drag and added mass are obviously different in the heave direction than in the transverse direction. The other terms correspond to the Froude-Krylov force.

The Froude-Krylov forces, along with diffraction force neglected in the present model, represent the non-viscous forces acting on floating bodies. The floater's shape was studied to take benefit of those forces, mainly due to the unsteady pressure field generated by undisturbed waves. Indeed, the base cylinder offers a free under water surface, $\frac{\pi}{4}(D_h^2 - D_c^2)$, pointing to the y direction where dynamic pressure, proportional to the square of the relative velocity, is applied on. It then reduces the positive vertical motions of the FOWT. When the platform moves downward, a reaction force opposed to the velocity appears. Heave motion fluctuations are consequently damped. This configuration is illustrated in figure 4.13.

Vertical wave velocity is generated thanks to the previously described JONSWAP model. Transverse waves group velocity is supposed constant. Hydrodynamic forces acting on the pontoons are neglected. The main assumptions underlying the loads decomposition into transverse and axial direction ignores the possible inclination of the columns.

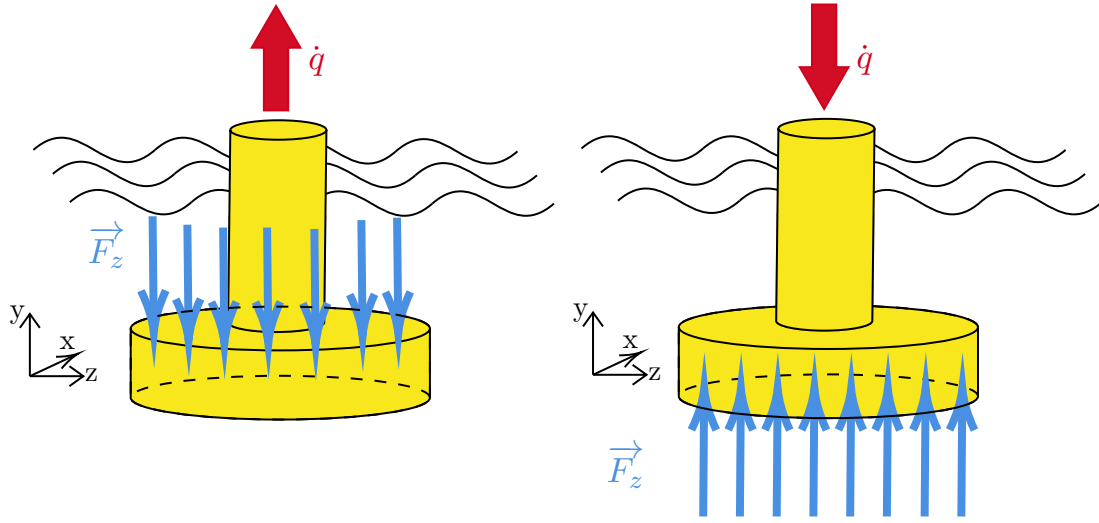


Figure 4.13: Illustration of heave plate functioning.

4.3.4 Mooring lines model

As said previously, the platform is subject to transverse loads due to wind and waves forces, one must thus ensure that the structure will be maintained within a certain position: station-keeping function is then assumed by the mooring system. Mooring lines play a important role in predicting the global response of the platform. As mooring lines are sized on basis of extreme fatigue loading over their life-time, accurate prediction of loads they undergo is of obvious interest. Different ways to analyze mooring system exists, the most common ones are: static, quasi-static and dynamic models. A small review about the models and their discrepancies is offered, followed by a description of the dynamic model used in the cuurent work.

The static model only considers constant loads, such as gravity, buoyancy and mean drift forces. It is generally used to determine the equilibrium between the mean environmental loads and the restoring force of the mooring loads [57].

To enhance the accuracy of the model, the quasi-static approach considers the motion of the system to be uniform and linear between two successive positions. During this change of position, loads are set to be constant. Such models provide satisfactory results and are popular for efficient modeling of FOWT motions. However, they are neglecting hydrodynamic and inertial forces on the line, which can affect the structure's response. Furthermore, Hall *et al.* compared quasi-static and finite-element analysis mooring lines models and concluded that mooring dynamics have an influence on platform motions only when those motions are large. Inaccuracies arise not only on loads predictions but also impact the response on the rotor's blades [58]. In sake for consistency with moderate to severe assumptions, the present work implements a dynamic mooring system model.

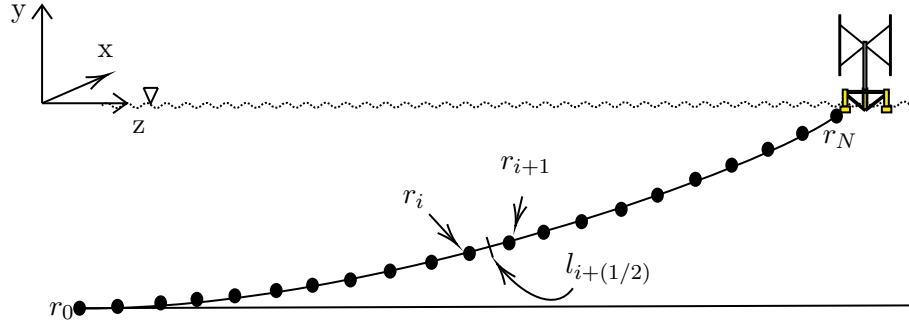


Figure 4.14: Mooring line discretization - Inspired by [59].

The aerodynamic solver coupled with the MBS software already requiring a high computational complexity, the mooring lines model should have a minimum impact on the computational time. The current computation method is based on the researches led by Hall *et al.* [59]. It is a simple lumped-mass mooring line model that captures the dynamic line characteristic necessary to predict tensions whereas higher-fidelity features are dropped to spare on computational cost. In this approach, the cable is discretized into N evenly-sized line segments connecting $N + 1$ node points. Indexing starts at the anchor node, and cable segment located between index i and $i + 1$ gets the index $i + (1/2)$. Line discretization and indexing is sketched in figure 4.14.

The inertial frame is defined as in ROBOTRAN: the z -axis is in the flow direction, the x -axis in the streamwise direction while the y -axis is aligned with the VAWTs mast. The position of each node is defined by the vector $\mathbf{r}_i$ containing its (x, y, z) coordinates. Each unstretched segment $i + 1/2$ has identical properties: diameter d , density ρ_{moor} , Young's modulus E and internal damping coefficient C_{int} .

Forces acting on the lines, depicted in figure 4.15, are the following: internal axial stiffness and damping forces, weight and buoyancy forces, hydrodynamic forces from Morison's equation, and forces from contact with the seabed. Throughout all computations, tangent direction at each node is approximated by the average tangent directions of the two adjacent elements. The resulting direction vectors are essential to calculate hydrodynamic forces, *i.e.* transverse and axial drag as well as added mass. Next subsections present the implementation of each force component.

Internal forces

Axial stiffness, axial damping and weight compose the internal force of the mooring system. For convenience, buoyancy and weight are accounted together for each segment $i + (1/2)$, according to equation 4.26.

$$W_{i+(1/2)} = \frac{\pi}{4} d^2 l (\rho_w - \rho_{moor}) g \quad (4.26)$$

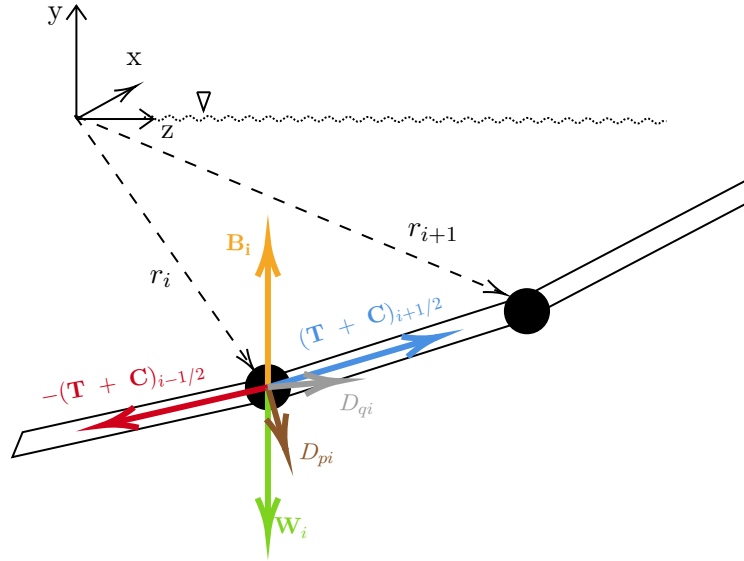


Figure 4.15: Forces on each element - Inspired by [59].

As this force is equally spread among two neighbouring nodes, the net buoyancy/weight vector at node i is

$$\mathbf{W}_i = \frac{1}{2}(W_{i+(1/2)} + W_{i-(1/2)})\hat{\mathbf{e}}_y, \quad (4.27)$$

where $\hat{\mathbf{e}}_y$ is a unit vector in the positive y -axis direction.

The net tension in a cable segment is formulated in the vector form as:

$$\mathbf{T}_{i+(1/2)} = E \frac{\pi}{4} d^2 \epsilon_{i+(1/2)} \left(\frac{\mathbf{r}_{i+1} - \mathbf{r}_i}{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|} \right)$$

$\epsilon_{i+(1/2)}$ is the strain factor, which can be expressed by means of $\mathbf{r}$ and l , $\left(\frac{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|}{l} - 1 \right)$, and simplified to give a more compact form of segment $i + (1/2)$ tension, (equation 4.28). The tension force direction is defined as pointing from node i to node $i + (1/2)$, indicating the force on node i . The opposite of this vector is the tension applied on node $i + 1$.

$$\mathbf{T}_{i+(1/2)} = E \frac{\pi}{4} d^2 \left(\frac{1}{l} - \frac{1}{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|} \right) (\mathbf{r}_{i+1} - \mathbf{r}_i) \quad (4.28)$$

The tension force is applied solely if $\|\mathbf{r}_{i+1} - \mathbf{r}_i\| > l$, *i.e.* only if there is a positive tension in the cable. Compression is not taken into account in the model.

Internal damping, also quite important to ensure numerical stability, is considered by means of:

$$\mathbf{C}_{i+(1/2)} = C_{int} \frac{\pi}{4} d^2 \dot{\epsilon}_{i+(1/2)} \left(\frac{\mathbf{r}_{i+1} - \mathbf{r}_i}{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|} \right) \quad (4.29)$$

C_{int} is the internal damping coefficient, and has the units of stress per strain rate (Pa-s).

The strain rate of the segment, $\dot{\epsilon}_{i+(1/2)}$, is computed thanks to

$$\begin{aligned}\dot{\epsilon}_{i+(1/2)} &= \frac{\partial \epsilon}{\partial t} = \frac{\partial}{\partial t} \left(\frac{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|}{l} - 1 \right) \\ &= \frac{1}{2l} \frac{1}{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|} \frac{\partial}{\partial t} \left[(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2 \right]\end{aligned}\quad (4.30)$$

which finally gives, in terms of x , y and z :

$$\dot{\epsilon}_{i+(1/2)} = \frac{1}{l} \frac{1}{\|\mathbf{r}_{i+1} - \mathbf{r}_i\|} \left[(x_{i+1} - x_i)(\dot{x}_{i+1} - \dot{x}_i) + (y_{i+1} - y_i)(\dot{y}_{i+1} - \dot{y}_i) + (z_{i+1} - z_i)(\dot{z}_{i+1} - \dot{z}_i) \right] \quad (4.31)$$

External forces

Concerning the external forces, Hall *et al.* showed the small influence of waves dynamics in the mooring line model on the final fairlead tension. They have compared model with and without waves forces on the cable against experimental data. Discrepancies between the two models are just over 3%. To simplify the implementation, it has been decided to ignore wave kinematics, modeling forces caused by the line motion in still water only. As previously explained, one must express the relative fluid velocities and accelerations in terms of transverse and tangent components before resolving forces applied on the cable. The latter direction, $\hat{\mathbf{q}}_i$, is defined by

$$\hat{\mathbf{q}}_i = \frac{\mathbf{r}_{i+1} - \mathbf{r}_{i-1}}{\|\mathbf{r}_{i+1} - \mathbf{r}_{i-1}\|} \quad (4.32)$$

In quiescent water, the relative velocity between fluid particles and segment is equal to $-\dot{\mathbf{r}}_i$. The projection of this vector onto $\hat{\mathbf{q}}_i$ gives the axial component of this flow, $(-\dot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i)\hat{\mathbf{q}}_i$. The transverse component, on the other hand, is formulated as $(\dot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i)\hat{\mathbf{q}}_i - \dot{\mathbf{r}}_i$.

As for the hydrodynamic loads on the floater, hydrodynamic loads on the cable are computed using Morison's equation. Mooring line segment are rightly considered as a cylinder. The transverse drag on the segment, denoted by the subscript p, is calculated thanks to

$$\mathbf{D}_{pi} = \frac{1}{2} \rho_w C_{dn} dl \|(\dot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i)\hat{\mathbf{q}}_i - \dot{\mathbf{r}}_i\| [(\dot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i)\hat{\mathbf{q}}_i - \dot{\mathbf{r}}_i] \quad (4.33)$$

where C_{dn} is the transverse drag coefficient. A tangent drag force is also expressed to take into account skin friction and form drag (equation 4.34).

$$\mathbf{D}_{qi} = \frac{1}{2} \rho_w C_{dt} \pi dl \|(-\dot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i)\hat{\mathbf{q}}_i\| (-\dot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i)\hat{\mathbf{q}}_i \quad (4.34)$$

where C_{dt} is the tangential drag coefficient.

Similarly, added masses in both direction are described in formula 4.35 and 4.36, where $\mathbb{A}_{pi}$ and $\mathbb{A}_{qi}$ correspond to added mass matrix in transverse and tangential direction. Unlike for the floater case, Froude-Krylov forces are not modeled since waves dynamics are neglected.

4.3. Dynamic forces on a floating platform

$$\mathbb{A}_{p_i} \ddot{\mathbf{r}}_i = \rho_w C_{an} \frac{\pi}{4} d^2 l [(\ddot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i) \hat{\mathbf{q}}_i - \ddot{\mathbf{r}}_i] \quad (4.35)$$

$$\mathbb{A}_{q_i} \ddot{\mathbf{r}}_i = \rho_w C_{at} \frac{\pi}{4} d^2 l (-\ddot{\mathbf{r}}_i \cdot \hat{\mathbf{q}}_i) \hat{\mathbf{q}}_i \quad (4.36)$$

Lastly, cable-seabed interactions have to be implemented. A common approach consists in replacing the ground into a simple spring-damper system. This force is only applied in the vertical direction whenever a segment touches the sea floor, otherwise the ground reaction is equal to zero. Horizontal friction from contact with the seabed are not modeled. In expression 4.37, $\mathbf{B}_i$ represents the ground force vector, c_b returns to the viscous damping per unit area whereas k_b represents the stiffness per unit area.

$$\mathbf{B}_i = dl [(z_{bot} - z_i)k_b - \dot{z}_i c_b] \hat{\mathbf{e}}_y \quad (4.37)$$

Mass and integration

Since the model accounts for added mass, it is required to express the mass of each segment in the matrix form. Then, the 3-by-3 mass matrix of node i can be rewritten as:

$$\mathbb{M}_i = \frac{\pi}{4} d^2 l \rho_{moor} \mathbb{I} \quad (4.38)$$

where $\mathbb{I}$ is the identity matrix.

Added mass terms, also proportional to $\ddot{\mathbf{r}}$, are rearranged and included in the mass matrix to obtain the semi-explicit equation of motions of the system.

$$\mathbb{A}_i = \mathbb{A}_{p_i} + \mathbb{A}_{q_i} = \rho_w \frac{\pi}{4} d^2 l [C_{an} (\mathbf{I} - \hat{\mathbf{q}}_i \hat{\mathbf{q}}_i^\top) + C_{at} (\hat{\mathbf{q}}_i \hat{\mathbf{q}}_i^\top)] \quad (4.39)$$

Then, the complete equation of motion to solve for each node i is:

$$\underbrace{[\mathbb{M}_i + \mathbb{A}_i]}_{\text{mass and added mass}} \ddot{\mathbf{r}}_i = \underbrace{\mathbf{T}_{i+(1/2)} - \mathbf{T}_{i-(1/2)} + \mathbf{C}_{i+(1/2)} - \mathbf{C}_{i-(1/2)}}_{\text{int. stiffness and damping}} + \underbrace{\mathbf{W}_i + \mathbf{B}_i}_{\text{weight and contact}} + \underbrace{\mathbf{D}_{\mathbf{p}_i} + \mathbf{D}_{\mathbf{p}_i}}_{\text{drag}} \quad (4.40)$$

Hall *et al.* advise to solve these equation by means of a Runge-Kutta 2 scheme. However, looking at the stability region of such a scheme, one can remark that the eigenvalues located on the complex axis of the $\lambda \Delta t$ plane are not included in this region. This can lead to numerical instability. For the linear problem, that kind of situation is likely to happen. Since this model is dedicated to coupling resolution methods, selection of the time step of the integration scheme will be constrained by the other simulation tools. Consequently, we proceed to the time integration by means of a Runge-Kutta 4 scheme, which is more robust thanks to a stability region including the imaginary axis of the $\lambda \Delta t$ plane. A comparison between the two stability regions are provided in figure 4.16.

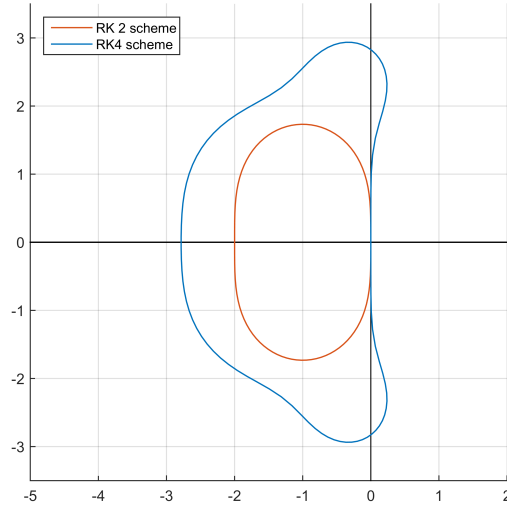


Figure 4.16: *Stability region of Runge-kutta schemes*

4.4 Numerical implementation

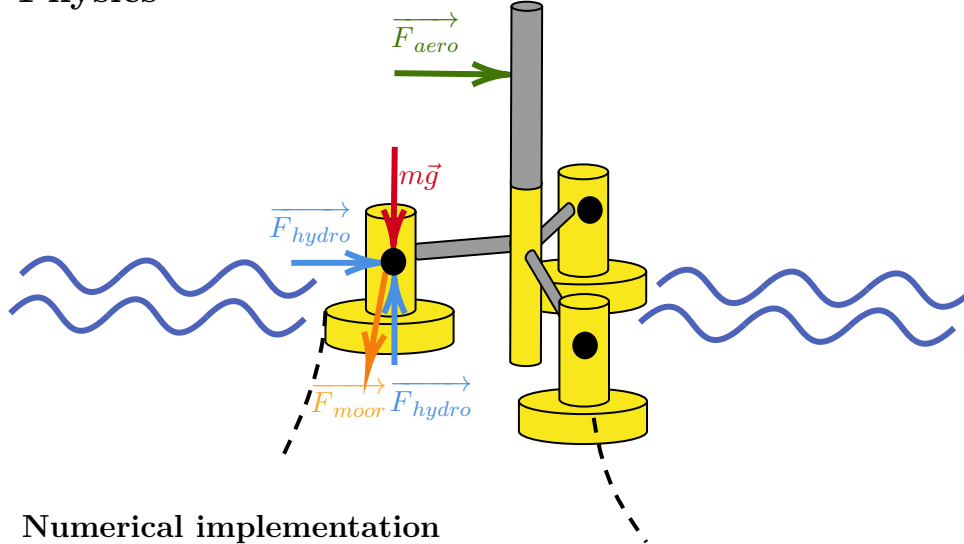
This section provides a schematic review of the interactions between all the previously described models.

The objective of the developed tools is to provide a first insight on FOWT aerodynamics and wakes. For the first time, a full 3D simulation tool will be used to analyze the dynamics of FOWT, but above all it will give a really precise overview on the FOWT wake characteristics. To this end, it has been coupled to a really efficient MBS solver, communicating simultaneously with a dynamical mooring lines lumped-mass model. Furthermore, a simplified hydrodynamic solver has been implemented, making it possible to excite the FOWT under regular and irregular waves.

In order to obtain results as realistic as possible, the semi-submersible platform is modelled by means of three floaters, where hydrodynamics as well as mooring lines reactions are applied. Forces are thus imposed on three separate points of the system (represented by black points in fig. 4.17). Not to mention, the 6 DOF are unconstrained.

The final simulation tool thus includes *vpm*, *ROBOTRAN*, the mooring line lumped-mass model, as well as the hydrodynamic model coupled with a waves generation tool. Even though this may seem tricky in a first sight, synchronization between those models can be easily understood by looking at the figure 4.17. This schematic view was simplified to ease the reading. Forces are applied on each floater. Recall that the inertial frame is located on the main column, at the sea water level.

Physics



Numerical implementation

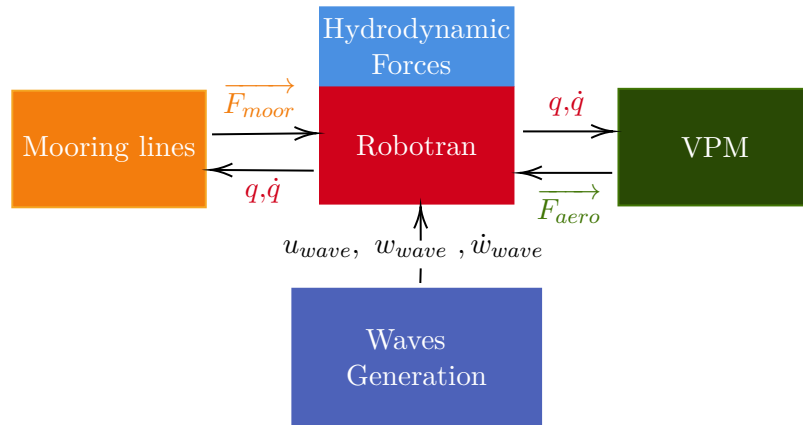


Figure 4.17: Schematic view of the coupled simulation tool

Starting from the bottom of the flow chart, one can observe that waves, either JON-SWAP spectrum or regular ones, are generated only once. They are stored in a function that ROBOTRAN uses every time it needs to compute the hydrodynamic forces. The latter are directly implemented in the external forces feature of the MBS solver, undoubtedly reducing computational cost. For each time step of ROBOTRAN, mooring lines dynamics is resolved and fairleads tensions are send back to the MBS software. At the end of the cycle, the latter computes position and velocity of the whole system and sends them to the aero-solver. Then, each time VPM perform a substep, this closed-loop is called. The coupling between ROBOTRAN and VPM has already been presented in chapter 1.

4.4.1 Model verification

Before going any further with the investigation on wake's reaction to rotor sea-induced motions, we must ensure that our model outcomes are in accordance with the reality. Numerous ways of coding the complex coupling dynamics makes it hard to find a reference on which we could base our entire validation. Different kind of platforms, hydrodynamic softwares and aerodynamic solvers create a wide range of potential implementations. For the hydrodynamic part, most of papers use proven software's accounting for waves-radiations and mooring systems in many different ways, while we implemented a lumped-mass mooring model along with Morison's equations. On the other hand, it is the first time that a vortex particle-mesh method is used to study offshore wind turbine's aerodynamic while most of the research uses Double Multiple Streamtube method [47, 48]. Therefore, our result's verification is mainly based on a qualitative comparison against the proven software's results. Achieving same order of magnitude is sufficient as our main focus is towards the analysis of the aerodynamics and wakes of wind turbines. Nevertheless and to the authors knowledge it is the first time aerodynamic and hydrodynamic loads are simultaneously studied within this context.

As a preliminary verification, the lumped-mass mooring model outcomes were compared to those provided by Hall *et al.* [55] under static conditions. A difference of 1% in pretension values (initial tension in the mooring lines) suggests our implementation has been properly performed. Static loading of the mooring lines are sketched in figure A.1, available in the appendices.

Afterwards, we proceeded in a free decay test, whereby the platform is released from a specified initial position. Still water and no wind conditions are applied. The resulting frequency response of the system is referred to as the natural frequency. Each DOF is tested separately. Those tests are quite relevant to examine the stiffness and the damping of the system, which are mainly due to hydrostatic, hydrodynamic and mooring lines loads. Valid solely in moderate to high sea states conditions, discrepancies are expected to emerge as compared to [62]. Differences between both outcomes are within a range of 40%, yet they are not taking into account their mooring model. Nevertheless, the global

DOF	Natural period [s]
Heave displacement	13.33
Surge displacement	66.6
Sway displacement	100.0
Pitch	18.18
Yaw	100.0
Roll	18.18

Table 4.3: System natural periods.

behavior of the structure follows the same pattern. Table 4.3 summarizes the obtained results. We can already observe interaction between the DOF. Sway and yaw motions affect each other, while symmetries of the platform give the same period for pitch and roll decaying character.

Last verification asserts consistency of our model subject to dynamic excitations. Hence, the platform response is evaluated under regular waves loads. We endeavour to replicate the results obtained in [55], where the structure undergoes regular waves characterized by a height H of 10.304 m and a period T of 12.1 s . Heave motions are of the same order of magnitude than those from Hall *et al.* Both models results in oscillations between -1.75 m and 1.75 m .

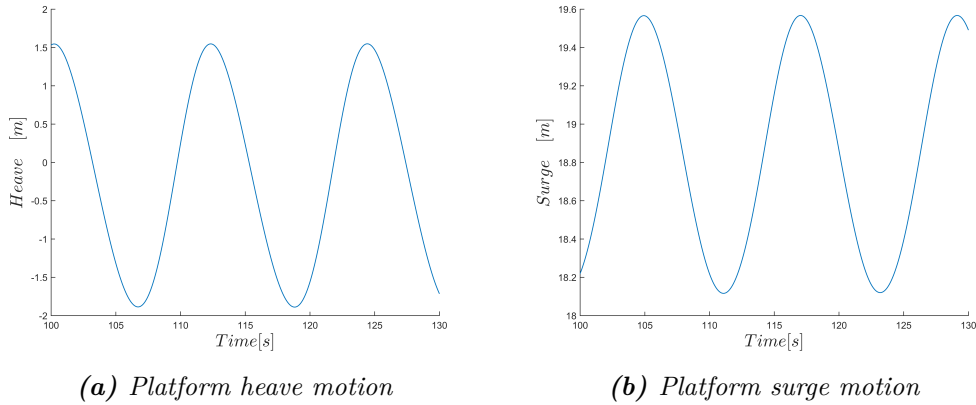


Figure 4.18: Platform motions under waves loading characterized by a height H of 10.304 m and a period T of 12.1 s , given after the transient response.

However, some discrepancies appear for the surge and pitch dynamics. Indeed, in our model, we consider a wave's velocity of 6 m/s aligned with the flow. This results in transverse loads pushing our FOWT up to 18 m away from its original position, leading to high mooring line tensions. The surge motion amplitude is consequently kept only within 1 m . In the reference paper, wave velocity is not precised. Still, amplitudes of surge oscillations are approximately of 6 m , while the platform is only 3 m away from its starting point. Their forces in mooring cables are then well below our tension values.

4.5. Results

The same argue applies for pitch motions. Indeed, the higher tension in the mooring lines, the greater the inclination of the resultant force pointing towards the seabed. Since only the first cable undergoes higher stress, this causes higher pitch oscillations, which are in the range of -8.5 to 2.5 degrees in our case compared to solely -1.5 to 1.5 degrees in [55]. A smaller velocity input should decrease those differences. Our results in heave and surge can be found in figure 4.18.

In conclusion, even though small discrepancies between the current and some references implementations appear, they are explainable and within a reasonable range. The general behavior of a floating structure has been obtained in three different configurations: under static conditions, during a free decay test or under regular waves loading. Since results are within an appropriate enough order of magnitude, a deeper analysis of the coupled dynamics has been performed.

4.5 Results

In this section, results of three different simulations are compared, namely a static wind turbine, a FOWT subjected to regular waves loading and another one exposed to a JONSWAP based wave description. We expect that the interactions between the motions and orientations of the platform and the aerodynamics loads will behave differently depending on the sea conditions.

The simulations were all performed with a constant wind velocity of 15 m/s , while imposing a TSR of 3 and a solidity of 0.2. The chord based Reynolds is equal to $6 \cdot 10^6$ while the diameter based one is $6 \cdot 10^7$. Regular waves loading considers a crest of 6 m , a period of 10 s as well as a constant wave group velocity of 6.13 m/s whereas the irregular wave loading presents a significant wave height H_s of 2.08 [m] , a significant period T_s of 5.66 s , $\gamma = 3.3$ and a waves velocity equal to 4.23 m/s . This corresponds to a distance to shore of 50 km . The water depth considered is 200 m . All three simulations are run over 75 non-dimensional times, within a domain of $[4D \times 4D \times 8D]$ discretized with 8 million particles.

	Re_c	σ	λ	AR	$H_s \text{ [m]}$	$T_s \text{ [s]}$	$u_{\text{waves}} \left[\frac{\text{m}}{\text{s}}\right]$
Static	$6 \cdot 10^6$	0.2	3.0	1.6	0	0	0
Regular	$6 \cdot 10^6$	0.2	3.0	1.6	6	10	6.13
JONSWAP	$6 \cdot 10^6$	0.2	3.0	1.6	2.08	5.6624	4.42

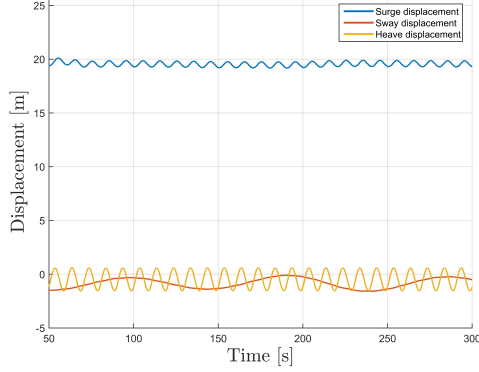
Table 4.4: Non-dimensional numbers and wave characteristics of the simulations.

4.5.1 Motion analysis

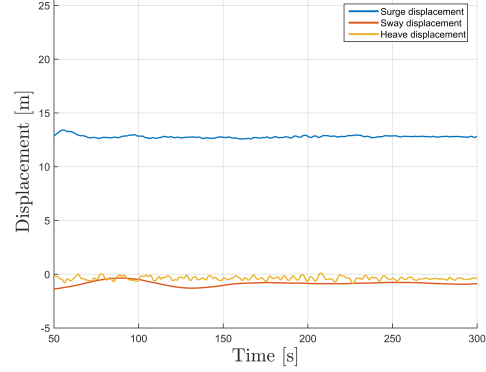
Motion time series of the platform under regular and JONSWAP excitations are provided in figures 4.19. During the first 30 seconds, the system reaches its dynamic equi-

4.5. Results

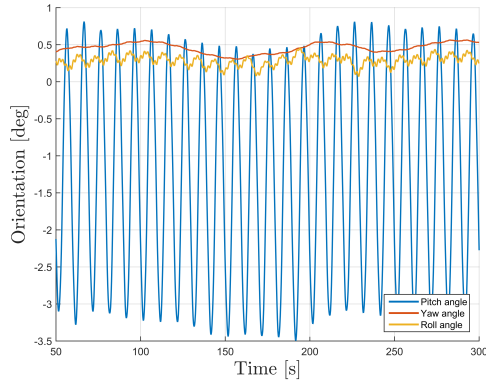
librium. Therefore, the platform undergoes high surge, pitch, yaw and roll displacements. The other DOF are less sensitive to the transient response.



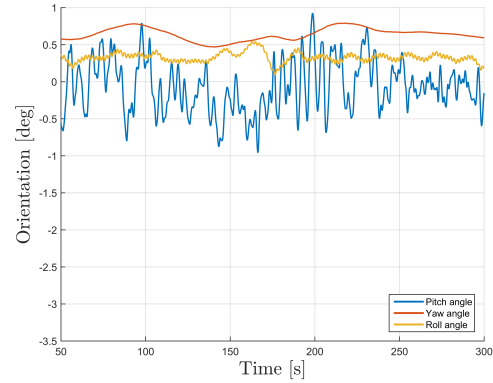
(a) Platform heave, surge and sway time-series under regular waves



(b) Platform heave, surge and sway time-series under JONSWAP waves spectrum



(c) Platform pitch, yaw and roll time-series under regular waves



(d) Platform pitch, yaw and roll time-series under JONSWAP waves spectrum

Figure 4.19: Time series of the floating platform response under regular and JONSWAP waves and constant wind loads. The complete time series including transient response is available in the appendices A.2.

This new equilibrium is characterized by oscillations of all the DOF around a mean position (whose values are listed in table 4.5). Translational motions amplitudes lies within 3 m. One can already underline the benefit of the heave plates mechanism. Amplitude excitations of 6 m only lead to an amplitude response of 2.5 m, for the regular waves case. Surge and heave displacements follows the waves period whilst sway response is subjected to a different behaviour. This excitation results from the Magnus effect, which applies a constant force in the negative x -direction. Therefore, the structure oscillates in the transverse direction and ultimately reach a final x position. Such a behavior is observable for the JONSWAP load case. However, the sway oscillations do

not disappears for the regular waves case. Waves frequency is actually equal to 10 times the sway fundamental frequency, leading to resonance effects. Fortunately, transverse hydraulic forces are strong enough to partially damp those excitations, preventing the system to exceeds its elastic range. To avoid excessive fatigue loads, one must ensure that the waves frequencies are not likely to excite the natural frequency of the design.

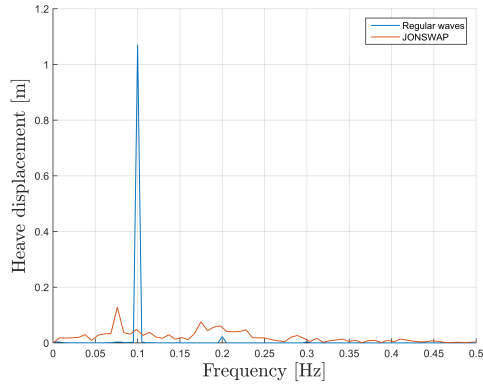
Concerning the rotational DOF, the pitch orientation is highly affected by the waves swell whereas yaw oscillations present strong similarity with the transverse displacement. The latter is caused by a higher tension in the mooring line located in the north-east side of the structure. The Magnus force breaks the system symmetry along the x -axis and stretches the mooring line: the resulting moment on the platform induces a rotation around the vertical axis.

Finally, looking at the time-series in the frequency domain (figure 4.20) confirms that motions are affected by several sources. Waves excitation (the peak at 0.1 Hz for the regular waves shown in figure 4.20a) influences all the DOF, except for the transverse displacement. Translational and rotational motions strongly interact due to the mooring line system. As an example, the sway oscillation frequency (peak at 0.016 Hz), is also significant in the roll and yaw frequency response. Finally, the blade passing frequency (peak at 0.47 Hz), also known as the 2P frequency, shows another impact of the aerodynamic forces on the system's motion. Those excitations caused by oscillations of the aerodynamic load are mainly present for change of the FOWT rotation axis (pitch and roll).

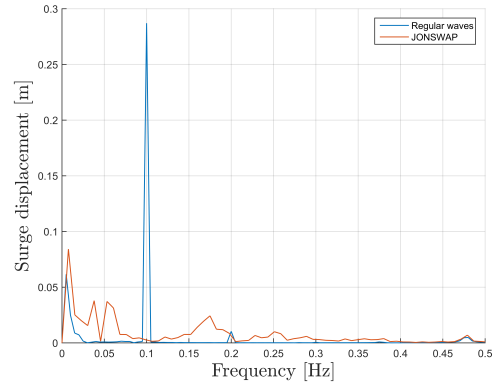
Mean values	Regular waves spectrum	JONSWAP waves spectrum
Heave position	-0.4640 [m]	-0.3775 [m]
Surge position	19.5349 [m]	12.7667 [m]
Sway position	-0.8088 [m]	-0.8940 [m]
Pitch angle	-1.3135 [deg]	-0.0965 [deg]
Yaw angle	0.4531 [deg]	0.6334 [deg]
Roll angle	0.2832 [deg]	0.3279 [deg]

Table 4.5: System mean position under regular and JONSWAP waves spectrum

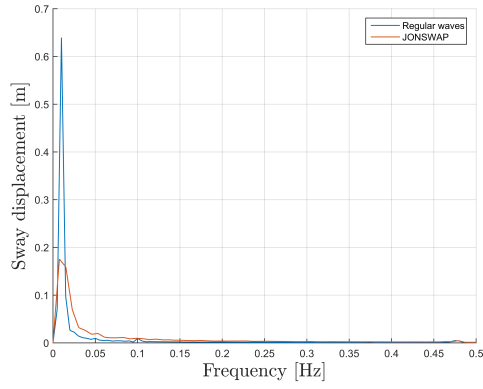
4.5. Results



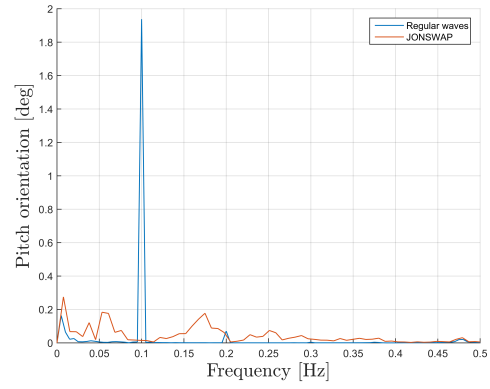
(a) Heave motions



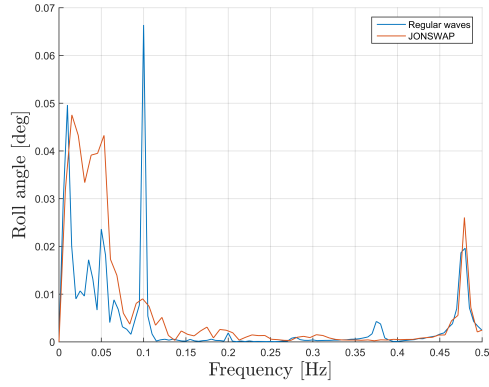
(b) Surge motions



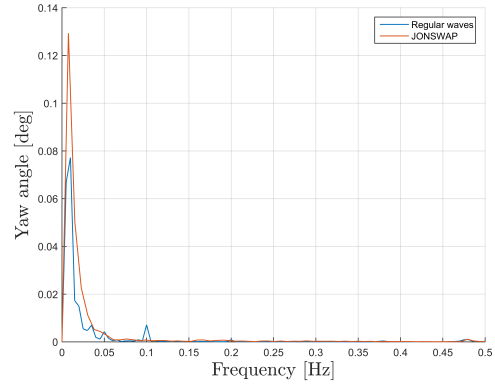
(c) Sway motions



(d) Pitch motions



(e) Roll motions



(f) Yaw motions

Figure 4.20: Floating platform response in the frequency domain. Simulation parameters are those of table 4.4

4.5.2 Aerodynamic analysis

The former section presented the motions and orientations of the platform caused by the anchor system and the hydrodynamic and aerodynamic loads on the setup. These motions certainly affect the aerodynamic quantities of the wind turbine. Indeed, any displacement or different orientation of the platform and turbine cause a variation in relative velocities as seen by the blades, and hence in angles of attack and resulting performances.

Table 4.6 contains the global power coefficients for the same setup in different sea conditions, as well as the transverse C_x and streamwise C_z load coefficients defined in section 3.3. The definitions of the latter two coefficients imply that values are given in the reference frame of the computational domain. All values are averaged over the 5 last rotor revolutions. Additionally, the maximum standard deviation in this averaging process, and hence associated with an azimuth angle, is given for C_p by $\sigma_{C_p, \max}$.

We can observe that the output power of the setups in any wave conditions loses in performances compared to the same setup remaining static throughout the computation. A relative loss of 13% is encountered for the regular waves, being the most severe sea conditions in terms of significant wave height, platform displacements and orientations. The latter also induces the greatest standard deviations associated with displacements of large periods. The statistically described sea condition, however, does not significantly affect the performances of the setup. The power loss encountered is only about 3% compared to the motionless platform, and yields comparable standard deviations. In terms of loadings, the streamwise and transverse loading coefficients exhibit different behaviours, and are mainly influenced by the averaged values of yaw and roll angles of the dynamic equilibrium, listed in table 4.5. Static, JONSWAP and finally regular configurations present absolute averaged pitch orientations in an ascending order, while the associated streamwise loadings C_z are declining. The pitch angle reduces the effective area on which the free stream velocity impinges, *i.e.* the free stream velocity component normal to the blades is reduced with pitch angle. The averaged negative pitch angle encountered is due to a positive average surge in the streamwise direction, stretching the mooring cable aligned with the z -axis. The average surge is influenced by the wave group velocity, being higher for the regular sea condition. For the JONSWAP sea condition,

	Static	Regular	Jonswap
C_p	0.2317	0.2010	0.2244
$\sigma_{C_p, \max}$	0.0514	0.1337	0.0654
$C_{x, \text{mean}}$	-0.1094	-0.1184	-0.1332
$C_{z, \text{mean}}$	0.7614	0.7133	0.7471

Table 4.6: H-type VAWT's global performances for different sea behaviours. The output power coefficient C_p , and the transverse and streamwise loadings C_x and C_z are given for each setup. The maximum standard deviation is denoted by $\sigma_{C_p, \max}$. Setup parameters are those of table 4.4.

the surge displacement is less severe, enabling a higher average sway displacement due to the Magnus effect, oriented in the negative x -direction. This implies a higher stretching of the mooring line in the positive $x - z$ quarter (see figure 4.6) and results in a positive averaged roll angle. The ascending order of averaged roll angles corresponds with the values of the transverse loading coefficients C_x on the turbine. This relation can be attributed to the gyroscopic effect, being significant for fluctuations of high frequencies as it is the case for the roll angle compared to the pitch angle.

Figures 4.21a–4.21f present the evolution of the angle of attack α , the power coefficient C_p , the tangential and normal force coefficients C_t and C_n and the streamwise and transverse loading coefficients C_z and C_x with the azimuth position. The values are averaged over 5 rotor revolutions. Additionally, the standard deviation envelopes of the aerodynamic quantities are given for each sea condition. Regarding the evolution of the angle of attack α , we observe that the highest standard deviation envelopes are obtained for the regular waves. We suggest the following mechanism: longer periods are associated with higher wave group velocities, increasing the hydrodynamic loads in the streamwise direction and hence the average surge in dynamic equilibrium. The amplitude of fluctuations around this equilibrium depends mainly on the significant height of the waves. Both induce higher pitch angle fluctuations and a more severe mean value of the latter. The higher the average absolute value of the pitch angle, the lower the normal component of the free stream velocity is, with respect to the blades. This results in lower angles of attack both in the upstream and downstream region of the turbine and finally in lower power extractions.

Even though few diameters downstream were included in the computational domain, the time-averaged streamwise velocity $\frac{\bar{w}}{U_\infty}$ at mid blade and the corresponding time-averaged velocity deficit $1 - \frac{\bar{w}}{U_\infty}$ at each diameter give insights of the near wake behaviours in the different sea conditions. These are shown in figures 4.22a–4.22c.

First of all, we notice that the turbines are moved downstream by the average value of surge for the regular and JONSWAP model of waves, the black circle representing the cylinder swept by the blades for a static turbine. The near wakes of the turbines for the different sea conditions exhibit very different behaviours. The static turbine presents a wider wake, but also a significant recirculation region compared to the other models. Especially, the turbine's wake for the regular sea configuration appears very narrow with a lower velocity deficit. This may be due to the pitch angle evolution, alternatively deflecting the wake towards the positive and negative spanwise directions, resulting in lower velocity deficits in the mid plane of the turbine.

Finally, figures 4.23a–4.23d present the displacement and momentum widths L_1 and L_2 in vis-à-vis of the displacement and momentum surfaces S_1 and S_2 , as defined in section 3.2. We recall that the difference in their definitions lies in the number of dimensions being numerically integrated at each diameter downstream.

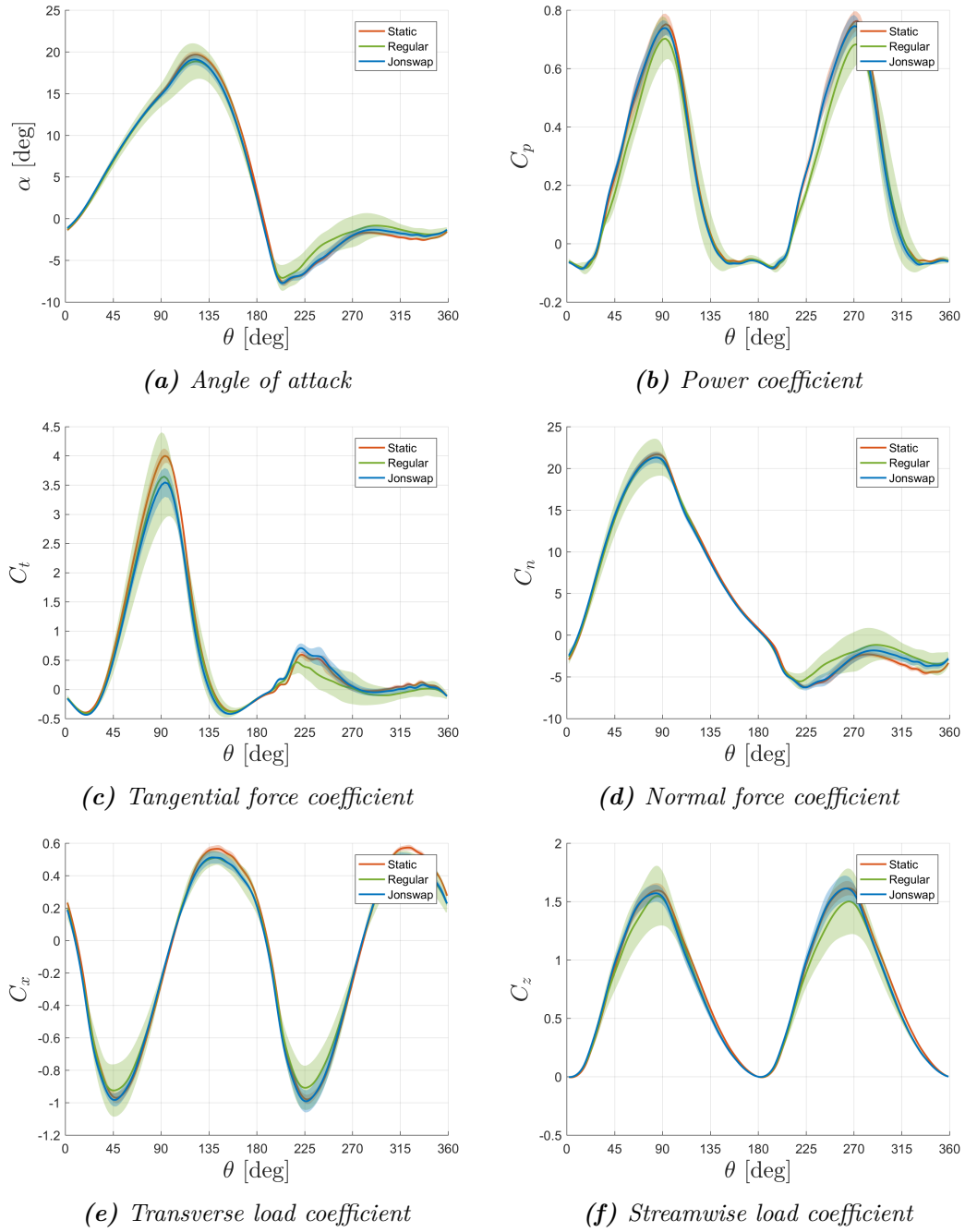


Figure 4.21: Diagnostics of the H-type VAWT for different sea behaviours. Simulation parameters are those of table 4.4. Values are averaged over 5 rotor revolutions. The standard deviation envelopes are given as shaded areas.

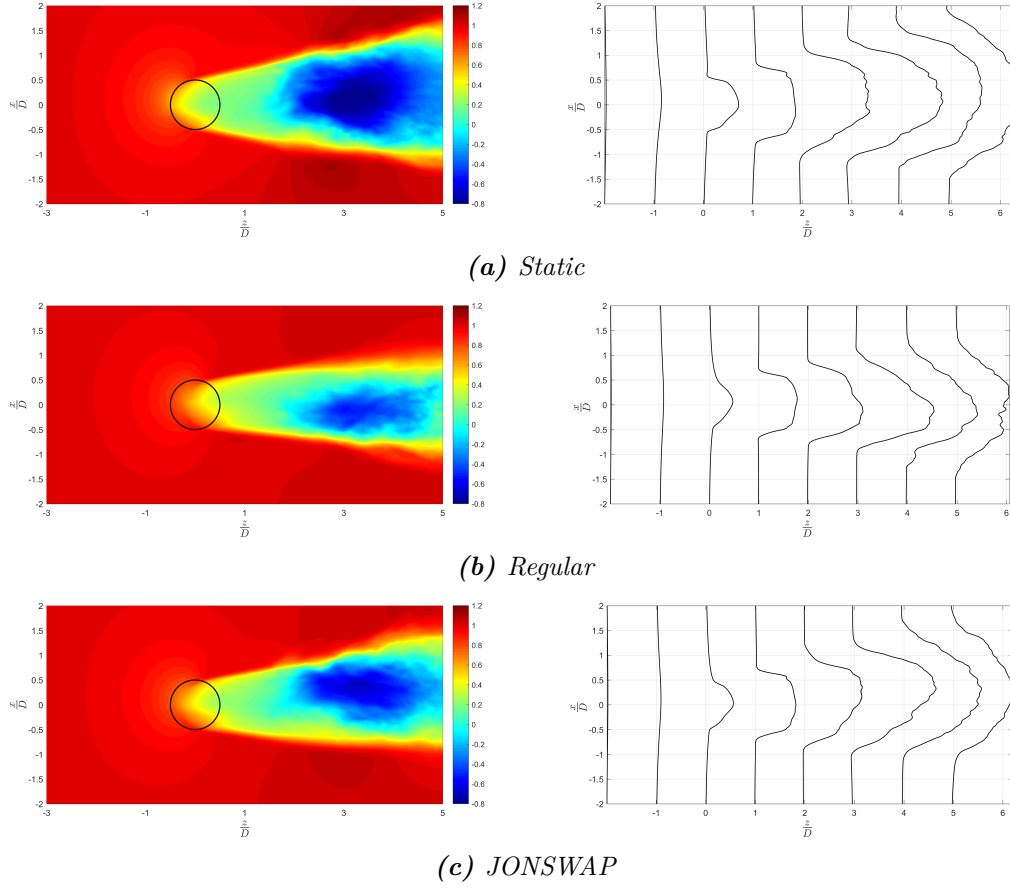


Figure 4.22: Time-averaged streamwise velocity $\frac{\bar{u}}{U_\infty}$ and velocity deficit $(1 - \frac{\bar{u}}{U_\infty})$ at each diameter for different sea conditions. Values are averaged over $t = 10$ non-dimensional times.

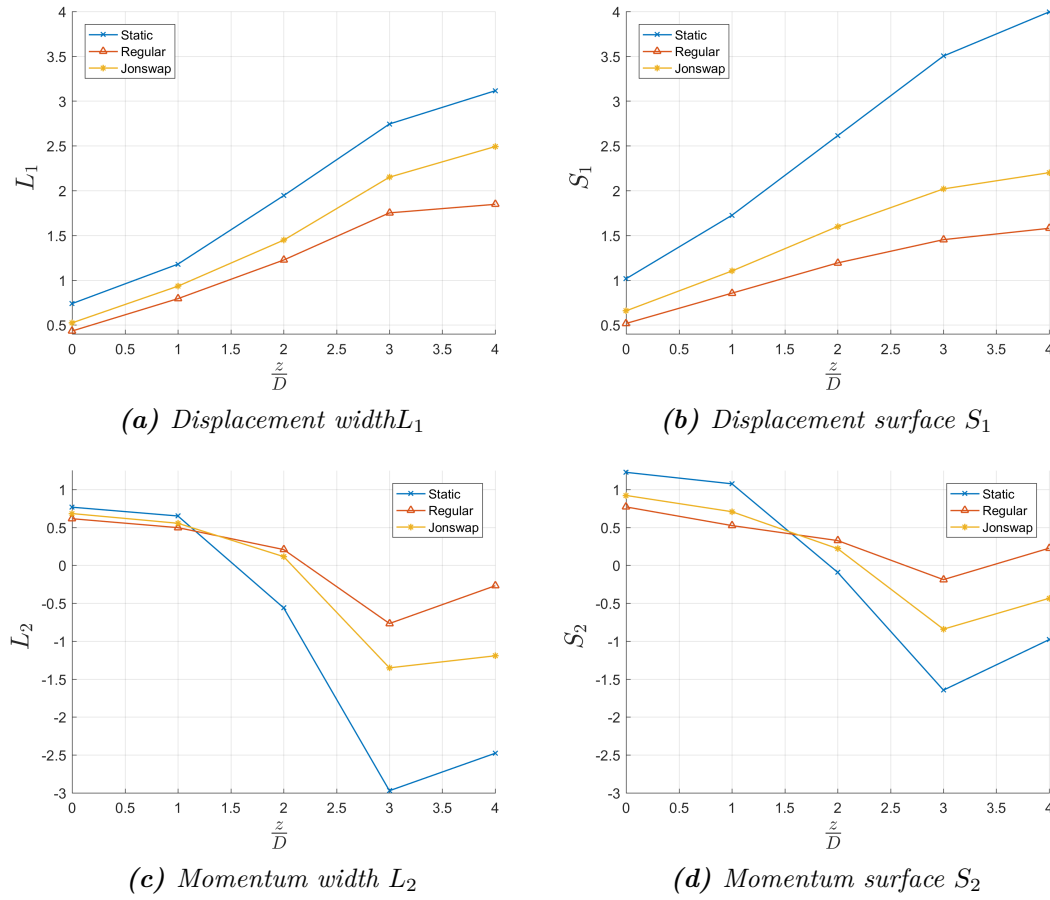


Figure 4.23: Displacement and momentum widths L_1 and L_2 and surfaces S_1 and S_2 based on the statistical streamwise velocity deficit at several diameters downstream. The parameters of the simulation are those of table 4.4.

Regarding the displacement width and surface, the static layout presents a massive overshoot compared to the turbines subjected to dynamic wave loadings, indicating the strength of the recirculation region. The obtained width of the wake for the static platform and turbine also induces greater leakage effects, as the unbounded condition on the sides allows for the fluid to *leave* the computational domain. Nevertheless the slope of the regular waves hydrodynamic loading appears to flatten faster than for the JONSWAP model, and will probably decay faster in the far wake. The unceasing movement of the platform forces the shed vortical structures to interact, leading to intenser stretching of the vortices and bringing an overwhelming contribution to the transition to turbulence.

This only constitutes a preliminary analysis of the wakes of floating offshore vertical-axis wind turbines, as we are limited by the availability of the velocity and vorticity fields further downstream, and a relatively small computational domain. Future research should be performed to detect the wake decay mechanisms from the near to the very far wake.

4.6 Conclusions

Greatly inspired by the TwinFloat industrial project, a floating wind turbine mounted on a semi-submersible platform was conceived along with a numerical tool to understand the complex dynamic underlying such a structure. The entire floating wind turbine system consists in a full-scale H-type wind turbine mounted on the DeepCwind semi-submersible platform maintained by mooring lines. The simulation tool joins forces of an aero-solver (VPM), a multibody software (ROBOTRAN), an irregular waves generator and a lumped-mass mooring line model proposed by [55]. After a qualitative verification in regards to references results ([48], [55] and [62]), simulations were run to compare performances of a vertical-axis wind turbine in static conditions against one under regular and irregular waves loads. Results show that:

- The hydrostatic and hydrodynamic forces damp the waves-induced motions
- The floating wind turbine motions are strongly affected not only by the waves swell but also by the aerodynamic forces. Influences of the latter are mainly due to the Magnus effect and the inherent blade passing frequency.
- The mooring lines system is highly responsible for coupling between translational and rotational degrees of freedom of the platform.
- The performances of the wind turbine are inversely proportional to the waves significant height and period.
- The induced motions of the rotor led to higher vortex interactions, which are expected to shorten the transition to turbulence.

Strong coupling effects underlying a floating offshore wind turbine under waves loading are of various origins and modeling of such a setup requires efficient and accurate tools. The individual wind turbine performances are expected to decrease on a floating platform, but a resulting shorter wake might be a benefit for an entire offshore wind turbine farm.

Conclusions and perspectives

The two main objectives of this Master's thesis were to elucidate the gain of performances associated to double-rotor configurations and to clarify the influence of hydrodynamic loads on a floating offshore vertical-axis wind turbine.

In order to achieve the aforementioned goals, a thorough comprehension of the used numerical tools was needed. Chapter 1 presented the underlying equations and methodology of the aerodynamic solver *VPM* and the multibody system solver *ROBOTRAN*. The coupling methodology between the two software codes, *VIR*, was presented to the reader.

Chapter 2 introduced the different double-rotor layouts -namely counter-up (CU), counter-down (CD) and co-rotating (CO)- and offered a methodical two-dimensional analysis of their aerodynamics and wakes. Their results were systematically compared to the single rotor in the flow. Then, the most promising configuration, *i.e.* the counter-up layout, was used as a base case to study the effects of turbine inter-distance and wind inflow direction on the performances and wakes of the setup. Unfortunately, an ill-calibrated tip-speed ratio implied large stalled regions over a blade's revolution, disabling to highlight the positive interactions between the turbines. Even though the gains were not prominent, the setup behaved as expected with turbine inter-distance and wind inflow directions. It confirmed that the former parameters are closely related and considerably influence the performances of the turbines.

While Chapter 2 provided a glimpse of the key physics behind the double-rotor concept, Chapter 3 took into account three-dimensional effects. Indeed, the tip-induced velocities, not present in a two-dimensional analysis, were found to significantly impact the aerodynamics and wakes of each different double-rotor configuration. Based on the counter-up configuration, the number of blades and the effect of aspect ratio were assessed. The analysis resulted in relative gains between 6 % and 8 % depending on the layout, compared to the single turbine. The physics behind these mutual benefits between rotors were elucidated. In terms of wake behaviour, we showed that wake bending effect is a new parameter that has to be taken into account. With an emphasis on the counter-up configuration, only very small aspect ratios benefit from faster recovery rates due to the double-rotor arrangement.

Finally, Chapter 4 provided a qualitative and quantitative comparison of a full-scale vertical-axis wind turbine mounted on a semi-submersible platform in different sea conditions. First, a hydrodynamic model, based on Morison's equations, was implemented in ROBOTRAN, along with a lumped-mass mooring line model. The resulting platform, having 6 degrees of freedom, was validated against verified existing software codes. A single rotor was thereafter added to the model, using the aforementioned coupling of VPM and ROBOTRAN. The obtained setup was numerically tested in three sea conditions -namely static, regular waves and a statistical description of wave spectrum- and their aerodynamics and wakes were investigated. The simulations showed that the global performances of the turbine were slightly reduced by the wave model, while their wakes benefited from an enhanced mixing leading to faster recovery rates.

Research on VAWTs and especially on their floating implementation is in its early stages. Meeting the objectives of this Master's Thesis, by explaining the key physics of the double-rotor layouts and highlighting the hydro- and aerodynamic behaviors of a floating vertical-axis wind turbine using vortex particle-mesh methods, provides a solid basis in the field. It actually paves the way for further research, as it opens a certain number of perspectives. Similar investigations on the other rotor-layouts could be done to objectively list their respective drawbacks and strengths. One can also think about optimizing vertical-axis wind farms made of closely spaced-rotors by combining the different layouts. The ultimate goal could consist in merging the two main parts of this work in order to provide an analysis of offshore wind farms, composed of double-rotor layouts mounted on a single semi-submersible platform, as proposed by *Nenuphar*. In any event, the future appears to be challenging and seems to foreshadow significant changes in Europe's energetic landscape.

Code availability

The immersed lifting-line enabled *VPM* code and its Fourier-based solver library are proprietary of the UCL-TFL. The Parallel Particle-Mesh (PPM) library is an open-source library (ETHZ/CSE Lab,2011).

The symbolic software to analyze multibody systems (MBS) is freely obtainable online. Install wikis and tutorials are available on the same website.

Competing interests

The authors declare having no conflicts of interests.

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Appendix A

Floating offshore wind turbine

Water density (ρ)	1025 kg/m ³
Water depth (h)	200 m
Center of buoyancy below SWL	- 13.15 m
Hydrostatic restoring in heave ($C_{33}^{\text{Hydrostatic}}$)	3.836 1e6 N/m
Hydrostatic restoring in roll ($C_{44}^{\text{Hydrostatic}}$)	-3.776 1e8 Nm/rad
Hydrostatic restoring in pitch ($C_{55}^{\text{Hydrostatic}}$)	-3.776 1e8 Nm/rad
Added mass coefficient (C_a) for all members	0.63
Added mass coefficient (C_{az}) for base columns in z-axis direction	0.33
Drag coefficient (C_d) for upper columns	0.61
Drag coefficient (C_d) for base columns	0.63
Drag coefficient (C_{dz}) for base columns in z-axis direction	1.6
Platform roll inertia about CM	6.827 10 ⁹ kg/m ²
Platform pitch inertia about CM	6.827 10 ⁹ kg/m ²
Platform yaw inertia about CM	1.226 10 ¹⁰ kg/m ²

Table A.1: Floating platform properties - from [43]

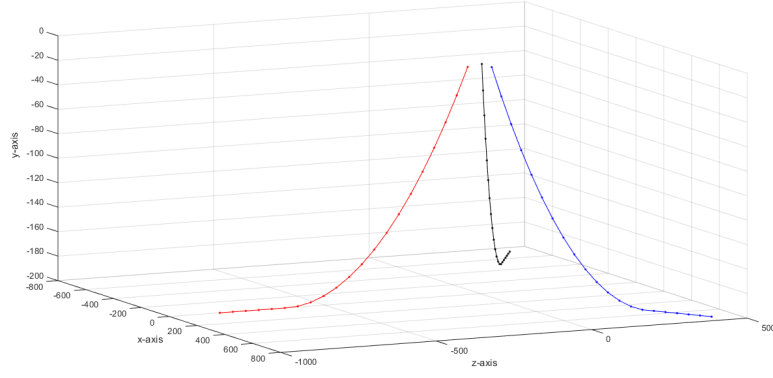
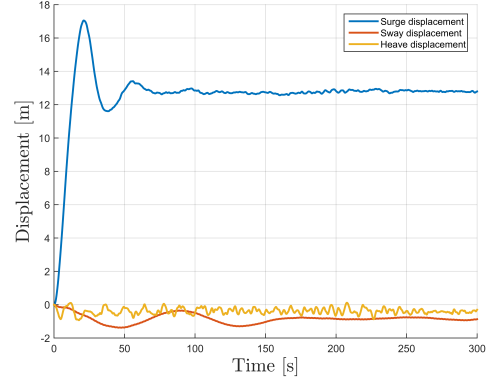
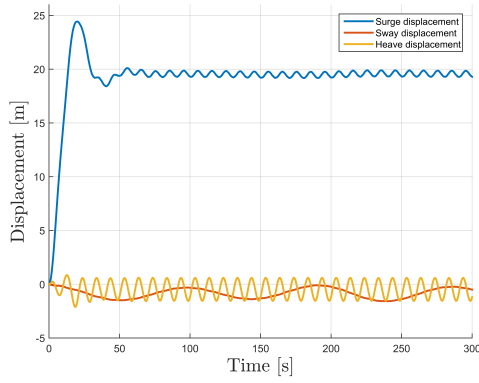
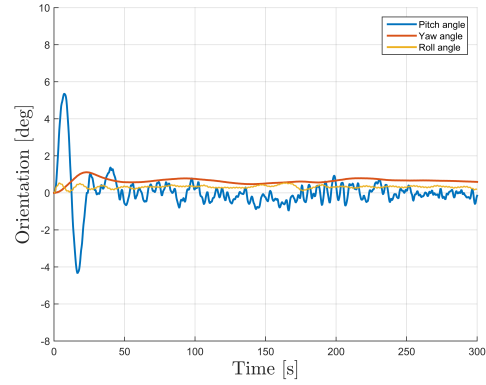
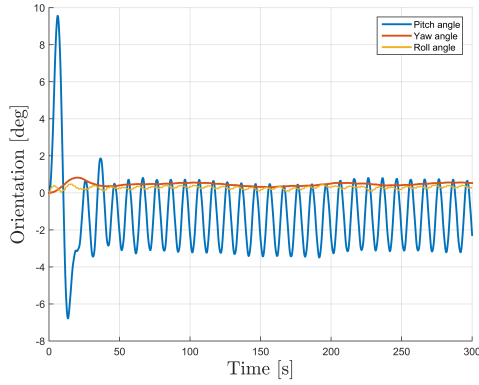


Figure A.1: Mooring lines equilibrium under static conditions



(a) Platform heave, surge and sway time-series **(b)** Platform heave, surge and sway time-series under JONSWAP waves spectrum



(c) Platform pitch, yaw and roll time-series under regular waves **(d)** Platform pitch, yaw and roll time-series under JONSWAP waves spectrum

Figure A.2: Time series of the floating platform response under regular and JONSWAP waves and constant wind loads - Complete time series.

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