# Appendix B

# Computation of required membrane surface

This appendix aims to explain the methodology for the computation of the membrane surface needed to reach a certain concentration in different cases.

## B.1 One unique module

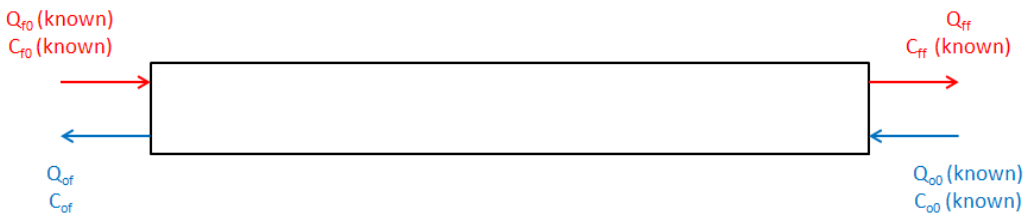


Figure B.1: Schema of one single module contactor. $Q$ is the flow, $C$ the concentration. The first index $f$ refers to the feed, $o$ to the osmotic. The second one 0 means it is the initial value, $f$ the final one.

Figure B.1 shows a representation of the system. Several parameters are known:

- $C_{f0}$ the inlet feed concentration
- $Q_{f0}$ the inlet feed flowrate
- $C_{ff}$ the final feed concentration (the objective to reach)
- $Q_{o0}$ the inlet osmotic flowrate
- $C_{o0}$ the inlet osmotic concentration
- $K$ the mass transfer coefficient (determined experimentally)

Final feed flowrate $Q_{ff}$ is given by a conservation of the $Na_2CO_3$ mass:

$$Q_{ff} = \frac{C_{f0} \cdot Q_{f0}}{C_{ff}} \tag{B.1}$$

The difference of inlet and outlet flow is only due to water removal by the membrane and is sent to the osmotic side. Therefore:

$$\Delta Q = Q_{f0} - Q_{ff} = Q_{of} - Q_{o0} = J_{avg} \cdot A \tag{B.2}$$

The final osmotic flowrate is given by:

$$Q_{of} = Q_{o0} + \Delta Q \tag{B.3}$$

The final osmotic concentration is obtained by a $NaCl$ mass balance:

$$C_{of} = \frac{C_{o0} \cdot Q_{o0}}{Q_{of}} \tag{B.4}$$

As the concentrations can vary a lot between the inlet and the outlet in both sides, the mean transmembrane flux $J_{avg}$ will be computed with the help of a logarithmic mean for the activities:

$$J_{avg} = K \cdot p^{vap}(T) \frac{(a_{f,in} - a_{p,out}) - (a_{f,out} - a_{p,in})}{\ln(a_{f,in} - a_{p,out}) - \ln(a_{f,out} - a_{p,in})} \tag{B.5}$$

where the activities $a_i$ and the partial vapor pressure $p^{vap}$ can be computed with the relationship in Appendix A. The area required in the module is then given by:

$$A = \frac{\Delta Q}{J_{avg}} \tag{B.6}$$

## B.2 Several modules in series

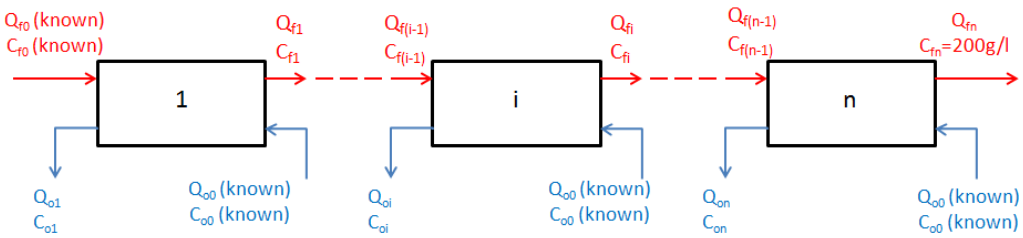


Figure B.2: Schema of several modules in series. $Q$ is the flow, $C$ the concentration. The first index $f$ refers to the feed, $o$ to the osmotic. The second index refers to the module the flow leaves, the value 0 referring to the initial value. Each module contains a membrane area equal to $A$.

Figure B.2 shows the representation of the system. Parameters that are known are:

- $C_{f0}$ the inlet feed concentration
- $Q_{f0}$ the inlet feed flowrate
- $C_{fn}$ the final feed concentration (the objective to reach)
- $Q_{o0}$ the inlet osmotic flowrate
- $C_{o0}$ the inlet osmotic concentration

The principle of calculation is based on iteration step by step. The flux removed at each step depends on the feed and osmotic concentrations. The dependence is determined by experiments (see Section 5.1):

$$J_i = 2.0015 \exp\left(-0.001 C_{f(i-1)}\right) \text{ if } C_{o0} \approx 300 \frac{g}{l} \tag{B.7}$$

$$J_i = 1.3748 \exp\left(-0.004 C_{f(i-1)}\right) \text{ if } C_{o0} \approx 200 \frac{g}{l} \tag{B.8}$$

with $C_{f(i-1)}$ in $\frac{g}{l}$, $J_i$ in $\frac{ml}{min \cdot m^2}$.

By volume conservation, the feed flow leaving the module $i$ $Q_{fi}$ is given by:

$$Q_{fi} = Q_{f(i-1)} - A \cdot J_i \tag{B.9}$$

The concentration of this flow is computed thanks to a mass conservation:

$$C_{fi} = \frac{C_{f(i-1)} Q_{f(i-1)}}{Q_{fi}} \tag{B.10}$$

These computations are repeated by iteration until the final concentration is reached thanks to a computation program such as Matlab. The total area used, $A_{tot}$, is given by:

$$A_{tot} = n \cdot A \tag{B.11}$$

where $n$ is the number of modules in series.