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Quadratic Funding & Project Choice

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1. Introduction

In economics, public goods are goods that are both non-excludable and non-rivalrous. Individuals cannot be excluded from use, and the use from one individual does not reduce availability to others. Also, a public good can be used simultaneously by more than one individual.[i]

Figure 1 : Public goods categorisation

Let's take the example of a Youtube channel that shares videos to popularize economic concepts. The videos of the channel would be considered as public goods as we don't have to compete to access the videos, and we cannot be excluded from them either by being priced out or by belonging to a specific group of people. The aim of these videos is to bring free education for every person that has access to Youtube.

Now, creating these Youtube videos represents a certain fix cost, affected by several factors.[ii] Assuming that the creator wants to do everything in his power to create a real community, he will have to create better quality videos than the others.

First, quality video design requires quality tools. By quality tools, I mean a good camera, a good microphone to record audio commentary, a suitable set (and therefore good lighting) and editing software, which can sometimes be quite expensive.

A professional staff may also be needed, particularly for video editing, which sometimes requires a certain amount of expertise, but also if the video shoot requires particular shots or complex staging. There is also another form of cost, which is a non-monetary cost : time. Preparing, shooting and editing a video generally takes a lot of time.

Creating a video therefore represents a certain amount of fixed costs. In the context of financing Youtube videos, and more broadly public goods in general, the Quadratic Funding mechanism could be an interesting alternative to traditional financing methods such as 1p1v democratic systems, private exclusion-based efforts, charitable organizations,... All of these institutions inevitably face certain limitations. Even when functioning as intended, one-person-one-vote democratic systems respond to the preferences of the majority, which does not always align with what would maximize overall value. Such systems can suppress minority voices or, conversely, be manipulated by minorities to avoid oppression. Additionally, they tend to be expensive to establish, inflexible, and slow to adapt to emerging needs or new organizational structures. On the other hand, private, and thus exclusion-based initiatives, although more adaptable, are often difficult and expensive to put in place. They also lack effective feedback loops to ensure they meet the needs of their members, and inefficiently exclude potential users. Charitable organizations, while often more adaptable and responsive than the other two forms, depend on motivations that are difficult to consistently align with the common good, especially beyond the small groups where they tend to be most effective (Ostrom, 1990) [iii]. Beyond these small groups, they are often swayed by status-driven motives and narrow, even exclusionary, interests.

After a quick introduction about the concept of Quadratic Voting in Section 2, an extended literature review of the Quadratic Funding mechanism is presented.

Quadratic Funding (QF) is an innovative crowdfunding mechanism designed to optimize the distribution of funds for public goods. The core idea of QF is to match individual contributions to a project with a larger pool of funds in a way that better reflects the collective interest of a community. The more contributors there are to a project, the more matching funds it receives, but the matching is not linear, it follows a quadratic formula. The total funding for a project under the Quadratic Funding mechanism is determined by the square of the sum of the square roots of individual contributions. QF is designed to encourage widespread participation by amplifying the impact of small contributions. This contrasts with traditional linear matching mechanisms where the size of the contribution directly determines the matching amount. In QF, small contributions from a large number of people can have a significant effect on the total funding received. The mechanism inherently prioritizes projects that have broad support across a community, rather than those that receive large contributions from a few individuals. This ensures that public goods which are valued by many, even if they cannot afford to contribute large amounts, are adequately funded.

As Quadratic Funding is an innovative and recent concept, there is not much literature on the subject. Through sections 3 and 4, this thesis gives some insights about what has been contributed to the subject of Quadratic Funding to this day. Primary sources include academic papers on mechanism design, public goods theory, and crowdfunding, particularly the foundational work by Buterin, Hitzig, and Weyl on Quadratic Funding and several papers of R. Pasquini, criticising and supplementing the work proposed by Buterin, Hitzig and Weyl.

Section 3 starts with some basic models of Quadratic Funding to lay out the principles. Then, some more realistic assumptions are added to the model, such as the incorporation of the deficit by individual or the possibility to make negative contributions.

After this, I present some limitations and failures of the model in Section 4. Some alternative versions of the basic QF mechanism are presented, these are adapted to take into account limited matching funds. Despite its advantages, QF is not immune to potential challenges like collusion or strategic behavior, where groups might try to manipulate the system to favor their preferred projects. That is why I also discuss the problem of collusion and fraud. Finally, I end the literature review by discussing the failures of concavity and a solution to these failures.

Through this thesis, I contribute to the subject of Quadratic Funding by exploring how funding decisions are made when contributors have non-concave utility functions, as it is the case for a YouTube channel with a certain amount of fixed costs below which the channel cannot produce its content. More precisely, I analyse the special cases in which two projects are perfect substitutes or perfect complements.

First, in section 5, I implement a theoretical model with the simplest assumptions (two projects to fund that are completely independent, and only two contributors) to be able to analyse the optimal levels of funding of each project according to the behaviour of the individual contributors. The analysis reveals that optimal funding levels can be found by equating the marginal social welfare gains to the marginal costs. The solution suggests that, depending on individual contributor preferences and the utility functions, the social planner might choose to allocate funds equally between projects or favor one project over another.

Finally, in Section 6, I make two theoretical extensions of the basic model. First, the model is extended to consider cases where the two projects are perfect substitutes, meaning that they produce identical content. Therefore, funding both does not increase utility beyond funding just one. The model demonstrates that the social planner should ideally fund only one project to avoid redundancy and maximize social welfare. On the other hand, the model shows that a standard QF mechanism leads to an all-or-nothing outcome, where one project secures all the funding, while the other receives none. This is scenario is rather desirable considering the case of perfect substitutes projects, as it avoids duplication of efforts and ensures that resources are used effectively. The second extension of the model explores the scenario where the two projects are perfect complements, each project's utility thus depends on the success of the other. An example given is two YouTube channels where one teaches personal finance basics, and the other applies these principles to investing. The model indicates that in such cases, balanced funding is critical; underfunding one project significantly reduces the overall utility derived from both. The optimal strategy for the social planner is to ensure that both projects are funded sufficiently and equally. On the other hand, the model shows that under a standard Quadratic Funding mechanism with perfect complements, asymmetric contributions lead to a suboptimal outcome where the utility derived from the projects is limited by the underfunded project.

2. The Quadratic Voting mechanism

To better understand the Quadratic Funding mechanism, it is important to introduce the principles of Quadratic Voting (QV) proposed by Glen Weyl and Eric Posner (2017) [iv], on which is built the concept of Quadratic Funding.

2.1. Quadratic Voting : principles

Quadratic Voting allows individuals to express not only their preference for or against an issue but also the intensity of that preference. QV works as follows : each voter is allocated a budget of "voice credits" every year, which they can distribute across various referenda (public votes on specific issues) during the year or save for future use. To cast votes on an issue, individuals use their voice credits. The Quadratic cost function is as follows:

$\text{Cost}(n) = n^2$

Where:

- n is the number of votes cast for a particular option.
- $\text{Cost}(n)$ is the total cost of casting those n votes.

The cost of casting multiple votes on a single issue increases quadratically. If a voter casts n votes, the cost is n^2. This quadratic cost function means that casting one vote costs 1 credit, two votes cost 4 credits, three votes cost 9 credits, and so on. The cost of buying votes increases with the number of votes you purchase. This structure ensures that while individuals can express strong preferences, doing so becomes increasingly expensive, discouraging vote monopolization by a single individual.

In the Quadratic Voting system, a referendum is approved if the votes in favor exceed the votes against. By structuring the cost of votes in this way, QV aims to capture the intensity of voters' preferences. This contrasts with traditional one-person-one-vote systems, where the strength of preference is not captured. In a standard pricing model, the influence (or voting power) individuals get is directly proportional to the amount of money they spend. This means those who care a lot about an issue can buy many votes because the cost per vote remains constant. Conversely, those who care little might not buy any votes because even the first vote seems too expensive. This results in individuals who are very passionate about an issue potentially buying all the votes, giving them disproportionate influence, while those who are indifferent or only mildly interested might not participate at all, leading to a lack of representation for moderate viewpoints. In this situation thus, votes are too cheap for those who care a lot, but too expensive for those who care little. Quadratic Voting addresses these issues by making the cost of votes increase quadratically rather than linearly, as it is shown with the marginal cost column in the following table. The marginal cost of casting the next vote grows proportionally to the number of votes cast.

Number of votes	Total Cost	Marginal Cost
1	1	1
2	4	3
3	9	5
4	16	7

Figure 2 : Number of votes, their cost and marginal cost under QV

By making each additional vote more expensive, those who care a lot will be less likely to buy many votes, which prevents them from dominating the decision-making process. At the same time, those who care little can still buy a few votes at a relatively low cost, ensuring their voices are also heard. Therefore, Quadratic Voting aims to create a more balanced and representative decision-making process by adjusting the cost structure of voting to account for the intensity of individuals' preferences.

2.2. Vickrey-Clarke-Groves (VCG) Mechanism

To fully appreciate the innovation of QV, it is essential to compare it with the Vickrey-Clarke-Groves (VCG) mechanism, which has been a foundational approach in mechanism design for achieving efficient outcomes in collective decisions. The VCG mechanism, named after its creators, works by asking individuals to declare how much they would pay to achieve a certain outcome, and then selecting the outcome with the highest declared value. Importantly, individuals who are pivotal in determining the outcome (those whose payment tips the balance) are required to pay an amount that reflects the cost their decision imposes on others. This payment system is intended to align individual incentives with social welfare, ensuring that the outcome maximizes collective value.

However, the VCG mechanism is not without its flaws. It is highly sensitive to collusion, where a small group of individuals can manipulate the system to achieve their desired outcome without bearing the corresponding costs. Additionally, VCG often requires individuals to make very large payments, which can be problematic if participants are risk-averse or have budget constraints. These issues have limited the practical application of the VCG mechanism, despite its theoretical appeal.

QV addresses some of these limitations by allowing individuals to buy votes incrementally. The quadratic cost function of QV makes it less susceptible to the extreme payments that characterize VCG. This cost structure also reduces the incentive for collusion, as the benefits of manipulating the system are outweighed by the rapidly increasing costs of additional votes.

2.3. Groves-Ledyard (GL) Mechanism

The Groves-Ledyard (GL) mechanism, another influential approach in collective decision-making, is designed for continuous social choices, such as determining the optimal level of a public good. In this mechanism, individuals announce their expectations for the amount of a public good that should be produced and propose increments to this amount. The mechanism then aggregates these proposals, adjusts the provision of the public good, and penalizes any individual whose proposal deviates significantly from the group consensus. The penalties are designed to encourage truthful reporting and to achieve an efficient allocation of resources.

While the GL mechanism offers an elegant solution to continuous public good problems, it is complex and difficult to implement in practice. It requires extensive information about individual preferences and relies on iterative processes to converge on an optimal solution. The mechanism’s complexity and the need for common knowledge among participants about the expected level of public goods make it less practical for large-scale applications.

QV simplifies the core idea of the GL mechanism by focusing on binary decisions (e.g., yes or no votes) rather than continuous choices. In QV, individuals buy votes on an issue, and the decision is made based on the aggregate votes. This simplification avoids the need for iterative processes and reduces the informational burden on participants, making QV more suitable for practical applications.

2.4. Challenges and Concerns with QV

Despite its potential advantages, QV is not without challenges. These challenges can be broadly categorized into issues related to collusion, strategic behavior, efficiency, and the robustness of QV in real-world implementations.

2.4.1. Collusion

Collusion is a significant concern in any voting system where individuals can influence outcomes through payments or vote purchases. In the VCG mechanism, collusion is particularly problematic because a small group of individuals can declare exorbitant payments to manipulate the outcome without actually having to pay, provided their collective influence is large enough to be pivotal. This makes VCG highly vulnerable to strategic manipulation.

QV, while not immune to collusion, is more resilient due to its quadratic cost structure. In QV, if a group of individuals attempts to collude by purchasing a large number of votes, the cost increases quadratically, making it financially prohibitive for small groups to exert disproportionate influence.

For instance, if one person buys 10 votes, it costs them 100 units, but if 10 people each buy 1 vote, the total cost is only 10 units. This makes large-scale collusion more difficult, as the cost to achieve the same level of influence is much higher for smaller groups. However, if collusion does occur, it requires a significant number of participants or high individual expenditures to have a meaningful impact, making it less likely than in VCG.

2.4.2. Strategic Behavior

Another challenge for QV is strategic behavior by participants, especially in the context of income effects and risk aversion. In VCG, the potential for large, unpredictable payments can deter participation or lead individuals to understate their true preferences to avoid financial risk. QV mitigates this by providing a more predictable cost structure, where individuals know in advance how much they will need to pay based on the number of votes they wish to purchase.

However, strategic behavior can still arise in QV, particularly in close elections where the outcome is uncertain. In such scenarios, individuals might attempt to buy enough votes to sway the outcome, but the quadratic cost function makes it costly to do so, especially if the election is not close. When the election is not close, individuals are less likely to engage in extreme vote-buying behavior, as the probability of being pivotal is low.

2.4.3. Efficiency and Practical Implementation

QV has been shown to achieve near-optimal outcomes in large populations, where the law of large numbers helps average out individual strategic behaviors. This means that as the number of participants increases, the aggregate outcome of QV tends to be more efficient and closer to the social optimum. However, achieving this efficiency requires careful design and implementation, particularly in terms of managing income effects and ensuring that participants have accurate information about the stakes involved.

One of the theoretical advantages of QV is its ability to accommodate budget constraints and large income effects. Unlike VCG, where individuals may face extreme financial burdens, QV allows participants to manage their expenditures more effectively, as they can decide in advance how much they are willing to spend on votes. This predictability makes QV more stable and less prone to the volatility that can plague other voting mechanisms.

2.4.4. Voter Behavior and Information

The success of QV also depends on the behavior and motivations of voters. QV assumes that participants are rational and will vote in a way that maximizes their utility. However, in reality, voters may not always act rationally. They might overestimate their influence, vote to express their identity rather than their true preferences, or be motivated by factors unrelated to the outcome of the vote.

In situations where elections are not close, QV is relatively robust to deviations from rational behavior. The quadratic cost structure naturally limits the impact of irrational voting, as the cost of excessive voting becomes prohibitive. However, in close elections, non-rational behavior can introduce noise into the voting process, potentially leading to inefficiencies. The extent to which QV can handle such noise depends on whether the heterogeneity of preferences outweighs the irrationality of voter behavior.

2.4.5. Comparative Performance

Finally, the performance of QV is often compared to traditional voting systems, particularly the one-person-one-vote (1p1v) model. The analogy to a market economy is frequently drawn, where QV, like a market, allows individuals to trade influence based on their willingness to pay. In theory, this should lead to more efficient outcomes, as those with stronger preferences have a greater say in the decision.

However, as with markets, QV is not immune to failures. If voters are significantly irrational, the traditional 1p1v system might perform better because it avoids the complexities introduced by vote-buying. In cases where voter irrationality is high, the simplicity and egalitarianism of 1p1v may be preferable to the more complex and potentially unequal outcomes of QV.

2.5. Quadratic Voting : criticisms

The innovative Quadratic Voting system has obviously been criticised. In this section, I will refer to the criticisms made by David K. Levine in his review essay [v] of "Radical Markets" by Eric Posner and Glen Weyl [vi].

The author provides a detailed critique of the principles of the Quadratic Voting mechanism, particularly focusing on its practical implications and potential drawbacks. Levine expresses skepticism about the efficacy of QV in large-scale elections. He argues that in real voting systems, the cost of turning out to vote already reflects the intensity of preferences to some extent. For example, more motivated and passionate voters are more likely to turn out and vote. Levine questions whether QV would significantly improve upon this existing dynamic.

Levine is also concerned that QV could disproportionately empower special interest groups. In traditional voting systems, each voter has an equal say, but QV allows those with more resources to have a greater influence. This could lead to outcomes where well-funded interest groups exert undue influence over policy decisions, potentially at the expense of broader public interests. That being said, the author also raises ethical and moral considerations regarding QV. He argues that there is a demand for making every vote equal, which is based on principles of equity and fairness. By allowing individuals to purchase more influence through additional votes, QV could undermine these principles and lead to perceptions of inequality and unfairness in the democratic process.

Levine also points out that real policies are not determined solely by voting but also by lobbying and political advocacy. He argues that minority views are often well-represented through lobbying efforts and that changing the voting system alone may not address the broader issues of minority representation and influence in politics. Levine is particularly concerned that empowering minorities through QV might inadvertently strengthen the influence of special interests and bigots, rather than achieving more equitable representation.

Another point of concern from the author is the potential coordination problems in multi-candidate elections with QV. In such settings, the probability of being pivotal for candidates who are not among the top two is significantly lower. This could lead to inefficient allocation of votes and suboptimal outcomes. He suggests that existing systems like runoff elections, where the top two candidates advance to a second round, may be more effective in addressing coordination issues.

Finally, Levine advocates for more research on QV and related voting mechanisms before considering their widespread implementation. He emphasizes that while there is enthusiasm for the potential of these systems, the evidence on their effectiveness and impact is not yet conclusive. Levine calls for careful study and experimentation to understand how these mechanisms work in practice and to ensure that they achieve the desired outcomes without unintended negative consequences.

3. The Quadratic Funding mechanism

Now that we've seen what the Quadratic Voting mechanism is, let's delve into the principles of the Quadratic Funding mechanism, as proposed by Buterin, Hitzig and Weyl[vii]. All the content of sections 3 and 4 will be based on their paper, with some more in-depth reflections at times, and evidence from other papers to complete the analysis of BHW.

3.1. Quadratic Funding : principles

Quadratic Funding (QF) is a crowdfunding mechanism that enhances available resources by allowing community members to make donations of any size, which serve as votes for fund allocation. The more widespread the support, the larger the matching funds provided. QF pools small donations from many people and matches them with larger amounts from a central fund, using a unique formula that rewards projects based on the number of contributors rather than the size of donations. This approach better captures the public's wants and needs, fosters community involvement and shared ownership, and creates incentives for smaller contributors, thereby multiplying the impact of existing funds.

According to this mechanism, there are basically two sources of funds for a project. Let's take the example of a Youtube channel that shares videos to popularize economic concepts, as mentioned in the introduction.

The first source of funds is the contributions from individuals. Individual contributions in the context of QF are the amounts donated by members of the community. These contributions are collected voluntarily, with individuals donating money to the projects (in the example we took, this could be Youtube channels) or causes they support. The contributions come from a diverse donor base, with each person contributing according to their capacity and interest.

On top of the contributions received from individuals, there is another source of funds obtained through the Quadratic Funding mechanism, which is the funds from the matching pool. A matching pool is a pool of money provided by matching partners used to "magnify" the individual contributions to the fund projects. The funds of the matching pool can be obtained through public institutions and governments, large philanthropic organizations and foundations, corporate contributions (companies), crowdfunding/community contributions or even cryptocurrency and blockchain projects.

It may be hard to imagine that public institutions, companies or even foundations would want to fund a YouTube channel, but there are several examples that can be mentioned. Indeed, some foundations that promote educational content have already provided funds to Youtube channels. For example, Khan Academy, a 501(c)(3) nonprofit organization, supports initiatives that promote education and make learning accessible to a broad audience.[viii] Also, corporations like Google often support educational projects as part of their Corporate Social Responsibility (CSR) efforts, aiming to improve community engagement and educational outcomes.[ix] The community that benefits from the educational content might also contribute directly to a matching pool through crowdfunding campaigns, on platforms like Kickstarter, Patreon or GoFundMe. This is the case of the Youtube Crashcourse, that creates free, high-quality educational videos, which is supported by crowdfunding on the Patreon platform[x].

Through all the financing methods mentioned above, the funds collected could be stored/pooled into a matching pool. Now, let's see mathematically how a project such as a Youtube channel would be funded through the theoretical principles of the Quadratic Funding mechanism.

3.2. Quadratic Funding : simple setup

In this section, we will focus on establishing the basic principles and benefits of the Quadratic Finance (QF) mechanism through a simple model. Indeed, for simplicity, we will initially make several assumptions to clearly illustrate how QF works under ideal conditions.

For the moment, we'll make the assumption that QF is financed by the deficit through a per-capita tax on each individual, and that individuals ignore their impact on the budget and costs imposed by it. Under this assumption, each individual perceives their contribution as a drop in the ocean, not significantly affecting the overall budget or creating additional costs. This perception simplifies decision-making for contributors, focusing on the direct benefits they receive from the public good rather than the indirect budgetary impact. In fact, the actual impact of individual contributions on the total budget is minimal, justifying the assumption for analytical purposes. This allows us to isolate and highlight the direct effects of the QF mechanism on public good funding.

3.2.1. QF mechanism : formula

The Quadratic Finance mechanism is defined by the following formula:

$$F^p = \left(\sum_i \sqrt{c_i^p}\right)^2$$

Here, F^p is the total funding received by a project p through the matching pool of funds, and c_i^p represents the contribution made by individual i towards project p. The QF mechanism translates individual contributions into the total funding level F^p for each project. The key property of QF is that the funding received by the project is proportional to the square of the sum of the square roots of individual contributions to this project.

In the following subsections, I aim to prove that the QF mechanism leads to the optimal provision of public goods by ensuring that the total marginal value of the public good equals the cost of providing it.

3.2.2. Utility Function

Each individual i derives utility from the funding level F^p of a project p. This utility is denoted by $V_i^p(F^p)$. The total utility of an individual is given by:

$$\sum_p V_i^p(F^p) - \sum_p c_i^p - t_i$$

where t_i is a tax imposed on individual i, as we consider for the moment that QF is funded by the deficit through a per-capita tax on each individual.

3.2.3. Optimality Condition

Through derivation, it's possible to show that the QF mechanism satisfies the optimality condition for public goods provision, which is $V'^p(F^p) = 1$. This condition means that the marginal value derived from the public good should equal the cost of providing it.

Let's set up the maximization problem of QF.

Individual i chooses their contribution c_i^p to maximize their total utility. The maximization problem is:

$$max_{c_i^p} \sum_p V_i^p(F^p) - \sum_p c_i^p - t_i$$

To find the optimal contribution, we derive the first-order condition. Differentiating the utility function with respect to c_i^p and setting it to zero gives:

$$2V_i'^p(F^p)\left(\sum_j \sqrt{c_i^p}\right)\frac{1}{2\sqrt{c_i^p}} = 1$$

This simplifies to:

$$V_i'^p(F^p) = \frac{\sqrt{c_i^p}}{\sum_j \sqrt{c_i^p}}$$

Summing this equation over all individuals i leads to:

$$\sum_i V_i'^p(F^p) = 1$$

Since $V_i'^p(F^p)$ is the marginal value each individual derives from the project, summing these values should equal 1 for optimal provision.

The above derivation shows that under the QF mechanism, the marginal value derived from the project by all individuals equals the cost of providing it. Thus, in its simplest setup, the QF mechanism ensures the optimal provision of the chosen project.

3.2.4. Simple model : some insights about QF

This baseline analysis of the Quadratic Funding mechanism gives us some insights about the concept.

First, the QF mechanism modifies the traditional linear funding principle by introducing a non-linear relationship between contributions and funding. This non-linearity helps to address the free-rider problem and encourages broader participation. The free-rider problem occurs when individuals benefit from resources, goods, or services without paying for them, relying on others to bear the costs. This often leads to underfunding or depletion of those resources because everyone has an incentive to avoid contributing, assuming that others will cover the cost. Public goods, like clean air, national defense, open-source software, or even Youtube channels as we took for example, are particularly vulnerable to this issue because they are non-excludable (everyone can use them) and non-rivalrous (one person's use doesn't diminish another's) [xi].

By magnifying the effect of small contributions, as small contributions are heavily subsidized under QF because the square root function grows more slowly than the linear function, QF reduces the incentive to free-ride. Individuals understand that even a modest donation can substantially increase the funding a project receives due to the matching mechanism. This encourages more people to contribute, as their donations are effectively leveraged to have a larger impact. Furthermore, the quadratic formula ensures that projects with widespread support receive more funding, reflecting the collective interest of the community rather than just the preferences of a few wealthy individuals. Conversely, larger contributions receive less subsidy, aligning more closely with private goods. This helps to balance the influence of large and small contributors.

Also, as we have seen with the last step of the maximization problem, by derivation, the QF mechanism leads to the utilitarian optimal provision of public goods by ensuring that the total marginal value equals the cost, which is the core condition for optimal public goods provision.

3.3. Intuitive Derivation of the Quadratic Funding Mechanism

Another way to define the Quadratic Funding function is to start from the individuals' degree of contribution, and think about the fact that an appealing normative feature to counter the free-rider problem might be that individuals should not act in accordance with solely selfish motives but rather consider the collective benefit. Based on that and using a differential equation approach, we can obtain a more intuitive understanding of the Quadratic Funding mechanism.

Let's assume that each individual i has a "degree of contribution" $h(c_i)$, which is a function of their contribution c_i. The total funding g is a function of the sum of these contributions:

$$g\left(\sum_i h(c_i)\right)$$

Here, the goal is to find the functions h and g that lead to optimal public goods provision. The prevailing rationale for free-riding is the assumption that each individual is willing to contribute to a public good, provided that others are similarly inclined. Thus theoretically, this implies that if an individual increases their contribution by 1%, the total funding should increase by 1% of the total contributions:

$$\frac{\partial g(\sum_i h(c_i))}{\partial c_j} = \frac{g(\sum_i h(c_i))}{h(c_j)}$$

This differential equation can be simplified and solved to find the forms of h and g. First, let's express the partial derivative in terms of h and g:

$$\frac{\partial g(\sum_i h(c_i))}{\partial c_j} = g'\left(\sum_i h(c_i)\right)h'(c_j)$$

The equation becomes:

$$g'\left(\sum_i h(c_i)\right)h'(c_j) = \frac{g(\sum_i h(c_i))}{h(c_j)}$$

To satisfy this equation, g' and h' must have specific forms:

$$g'\left(\sum_i h(c_i)\right) = k\sum_i h(c_i)$$

$$h'(c_j) = \frac{1}{kh(c_j)}$$

where k is a constant.

Let's integrate g' and h' to find the forms of g and h :

$$g(x) = \frac{k}{2}x^2 + m$$

$$h(x) = 2\sqrt{\frac{x}{k}} + n$$

If we want to ensure that the funding of a project with no contributions is zero, the constants m and n should also be zero:

$$g(x) = \frac{k}{2}x^2$$

$$h(x) = 2\sqrt{\frac{x}{k}}$$

In a traditional private contributory scheme, the funding received by a public good is simply the sum of individual contributions. For a single contributor, this should directly translate into the funding amount without any distortion. By choosing $k = 2$, we ensure that the QF mechanism respects this property. Therefore, we obtain:

$$g(x) = x^2$$

$$h(x) = \sqrt{x}$$

The outcome of this derivation gives us the core definition of QF, as $g(x)$ is the total funding received by the project, and $h(x)$ is the degree of contribution of each individual. If we replace the expressions obtained just above in the basic formula we started with, we have :

$$g = F = \left(\sum_i \sqrt{c_i}\right)^2$$

Which is the formula of the Quadratic Funding mechanism we started with in subsection 3.2.1.

By translating the normative property we mentioned into a differential equation and solving it, we showed that the QF mechanism naturally emerges as the solution that aligns individual incentives with the collective goal of optimal public goods funding.

3.4. Key properties of the QF mechanism

In subsection 4.3 of their paper, the authors delve into the specific properties that make the Quadratic Finance (QF) mechanism both unique and effective for the funding of public goods. These properties ensure that the mechanism is not only theoretically sound but also practical and robust in various real-world scenarios.

The first property discussed is the homogeneity of degree one. This means that if all contributions are scaled by a common factor, the total funding is scaled by the same factor. Mathematically, if every individual's contribution is doubled, the total funding for the public good is also doubled. This property is crucial because it ensures that the mechanism is proportional and consistent regardless of the scale of contributions. It implies that changing the currency or the unit of measurement does not affect the mechanism's outcome, making it universally applicable.

The second key property is that the funding received by a public good grows quadratically with the number of contributors, assuming each contributes an equal amount. If each member of a community contributes one unit, the total funding is proportional to the square of the number of contributors. For instance, if ten people each contribute one unit, the funding is \(10^2 = 100\) units. This property effectively addresses the underfunding problem typical in linear systems where each additional contributor only adds a linear amount to the total funding. By contrast, QF ensures that the collective contribution of a larger group has a significantly amplified impact, encouraging broader participation.

QF inherently discourages fragmentation of contributions. If a community splits into smaller groups, each contributing separately, the total funding for the public good decreases. For example, if a community of 100 people splits into two groups of 50, the funding for each group is $50^2 = 2500$ units, totaling 5000 units, which is less than $100^2 = 10.000$ units if they had contributed as a single group. This property promotes cohesion and collaboration within communities, ensuring that the public good receives maximum funding when contributions are unified rather than divided.

Another important property is the reversion to a standard private good mechanism in extreme cases. When a single individual contributes a large portion of the total funds, the impact of other small contributions diminishes. The mechanism effectively treats this situation as a private good, where the funding is primarily driven by the large contributor. This ensures that the mechanism remains fair and efficient even when contributions are highly skewed towards a few large donors.

QF heavily subsidizes small contributions. Since the square root function grows more slowly than the linear function, small contributions have a disproportionately larger impact on the total funding. This property is particularly beneficial in encouraging participation from individuals who can only afford to contribute small amounts. It democratizes the funding process by ensuring that everyone's contributions, regardless of size, are meaningful and impactful.

The mechanism ensures that public goods are funded equitably and efficiently. The quadratic nature of the mechanism aligns the funding levels with the collective valuation of the public good, leading to a utilitarian optimal provision. This means that the total societal benefit derived from the public good is maximized relative to the cost.

Finally, QF also shows resilience against collusion. In a colluding group where members try to exploit the mechanism by artificially inflating their contributions, the system's design naturally limits the benefits of such actions. If a group of 100 people contributes €1000 each, they collectively receive €1,000,000. However, if one member defects and contributes nothing, the total funding only slightly decreases, leading to a minimal impact on the group but saving the defector's contribution. This disincentivizes collusion as individual members have strong incentives to act independently rather than as part of a colluding group. We will delve into deeper details later...

3.5. User interface

In their paper, Buterin, Hitzig & Weyl [vi] discuss the practical implementation of the Quadratic Finance (QF) mechanism. They provide a conceptual overview of how the QF mechanism could be realized in a real-world setting, how individuals would interact with the system, and the administrative considerations necessary for effective deployment. In this section, I try to develop these possible practical implementations with some concrete examples.

The QF mechanism's success hinges on a user-friendly interface that facilitates easy participation and transparent funding processes. The authors envision a system where any individual can propose new public goods projects at any time. This open-ended approach ensures that the set of public goods is dynamic and can adapt to the community's evolving needs and preferences. To maintain order and prevent spam, there may be an approval process administered by a trusted body, especially in cases where funding comes from philanthropic sources. This approval process ensures that only legitimate and valuable projects are listed. Practically, this might include filling out forms detailing the project's goals, benefits, required funding, and other pertinent information. Then, an administrative body would review proposals to ensure they meet certain criteria. This could involve checks for feasibility, community benefit, and alignment with the platform's goals. Approved projects would then be listed for public contributions.

Once a project is approved and listed, individuals could contribute funds at regular intervals, such as monthly. Regular contributions help maintain a steady flow of funding and allow for ongoing support of public goods. The system would provide contributors with real-time or near-real-time updates on the funding status of various projects. However, to prevent strategic manipulation or fraud, some delay or noise might be introduced into these updates.

Transparency is a critical component of the QF mechanism. Contributors need to understand the impact of their donations on the total funding of a project. The interface would include tools like visualizations and calculators to help individuals see how their contributions, combined with others, influence the total funding. For instance, a contributor could input a potential donation amount and see an estimate of the resulting total funding, accounting for the QF formula. These types of visualization tools already exist, for example you can try on the page wtfisqf.com[xii], on which there is a simulation tool where you can enter an amount for the matching pool and amounts from various contributors, so that you can see how much a project would be funded through the QF mechanism.

The design of the interface should also help avoid fragmentation of contributions. Given the QF mechanism's property that larger, unified groups receive more funding, the system should encourage consolidation of similar projects or contributions to a common goal rather than dispersal across many smaller, overlapping initiatives. To this end, the interface might include features that highlight popular or highly-funded projects, suggest related projects, and facilitate collaboration among project proposers and contributors. Graphs and charts displaying funding levels, contribution history, and other metrics could also help users understand the broader impact of their contributions and the overall health of funded projects. For instance, and not least, the platform Gitcoin Grants is a quarterly initiative designed to support early-stage projects and builders in the web3 ecosystem through a combination of crowdfunding and grants. The program uses, by the way, the Quadratic Funding mechanism to leverage funds for these projects. The grants are aimed at a wide range of projects, including open source software, web3 infrastructure, decentralized applications (dApps), and climate solutions. By pooling funds and distributing them based on community support, Gitcoin Grants helps innovative projects gain the resources they need to grow and succeed. Gitcoin Grants operates in funding rounds, typically running for a set period (for example a few weeks) with a matching pool for each round. During these rounds, projects compete for funding from the community and matching pools.[xiii]

Security and coordination features are essential for the QF mechanism. The authors mention the need for security measures to protect against fraud and ensure that contributions are genuine. This could involve identity verification processes (for example through processes like email confirmation, social media verification, or government-issued ID checks) and regular audits of contributions and funding allocations. Additionally, coordination tools might include forums or discussion boards where contributors and project proposers can communicate, plan, and align their efforts. Also, we will come back to this later, but it is important to consider the risks of collusion and fraud within such a funding mechanism. We will analyse the theoretical implications of collusion on the model in one of the next sections.

Also, but more anecdotally, the overall look and feel of the QF system require careful thought and user-centered design. The interface should be intuitive, making it easy for individuals to navigate, understand, and participate in the funding process. This might involve user-friendly dashboards, clear instructions, and accessible support resources. The system's design could also incorporate elements of gamification to encourage participation and engagement, such as badges for frequent contributors or milestones for projects reaching certain funding levels.

3.6. Deficit incorporation

As I said before, to see how the mechanism works under ideal conditions, the baseline analysis of Quadratic Funding we made in the previous sections was based on a simple model including assumptions that are not necessarily realistic in practice. Now, let’s include more realistic assumptions and see how this influences the behavior of contributors and the overall effectiveness of the QF system.

In the previous sections thus, the QF mechanism was analyzed under the assumption that contributors ignore their impact on the deficit. A budget deficit occurs when the total funding provided to public goods exceeds the total contributions made by individuals. The basic formula of QF (see subsection 3.2.1. QF mechanism : formula) often results in funding levels that are higher than the sum of individual contributions because of the quadratic nature of the mechanism. This difference between the funding provided and the contributions collected creates a deficit that needs to be financed. It's really important to think about the deficit when it comes to making sure the QF mechanism stays sustainable and practical. If we don't manage the deficit, the system could end up facing financial instability or needing an unrealistic amount of external funding. By including the deficit in the decision-making process, we can make adjustments to account for this financial gap, which will lead to more realistic and efficient funding outcomes.

3.6.1. Impact of Deficit Considerations

When contributors consider the impact of their contributions on the overall budget deficit, their behavior changes. Each contributor has a shadow value, λ_i, which represents their concern about reducing the deficit. This concern could stem from personal taxes they might face or from reduced funding of other public goods. If the government decides to cover the deficit by increasing taxes, λ_i represents how much more tax individual i would have to pay. If the government decides to cover the deficit by reduced funding of other public goods, λ_i measures the impact of this reduction on the individual's utility.

3.6.2. Definition of Aggregate Cost of Deficit

The aggregate cost of an increased deficit is denoted by:

$$\Lambda = \sum_i \lambda_i$$

This represents the total burden or cost that the deficit imposes on all individuals in the society. Assuming that λ_i is on the order of $\frac{1}{N}$ (where N is the number of contributors), the aggregate cost, Λ, is around 1. This assumption implies that in a large society, the impact of the deficit on any single individual is very small. Each person's share of the total deficit is tiny because the cost is spread out across many people.

3.6.3. Adjusted Utility Maximization

With the deficit consideration, each individual i aims to maximize their utility, taking into account their contribution's impact on the deficit. The adjusted utility function is:

$$U_i = V_i^p(F^p) - c_i^p - \lambda_i\left(\left(\sum_j \sqrt{c_j^p}\right)^2 - \sum_j c_j^p\right)$$

Where:

- $\left(\sum_j \sqrt{c_j^p}\right)^2$ is the total funding through matching pool for the project p using the QF mechanism.
- $\sum_j c_j^p$ is the sum of individual contributions.
- $\lambda_i\left(\left(\sum_j \sqrt{c_j^p}\right)^2 - \sum_j c_j^p\right)$ represents the cost of the deficit imposed on individual i.

To maximize their utility, each individual i adjusts their contribution c_i^p based on the marginal utility they derive from the public good and the marginal cost of their contribution, considering the deficit. The first-order condition for this maximization is derived by taking the derivative of the adjusted utility function with respect to c_i^p and setting it to zero:

$$2\left[V_i^p(F^p) - \lambda_i\right]\left(\sum_j \sqrt{c_j^p}\right)\frac{1}{2\sqrt{c_i^p}} = 1 - \lambda_i$$

This simplifies to:

$$V_i^{p\prime}(F^p) - \lambda_i = \frac{\sqrt{c_i^p}}{\sum_j \sqrt{c_j^p}}(1 - \lambda_i)$$

This condition ensures that the marginal benefit of contributing an additional unit to the public good equals the marginal cost, adjusted for the deficit impact.

By aggregating the first-order conditions across all individuals, we derive an overall condition for the public good:

$$V_i^p(F^p) - \wedge = 1 - \sum_j \lambda_j \frac{\sqrt{c_j^p}}{\sum_j \sqrt{c_j^p}}$$

Let us remind that here, $\wedge$ represents the aggregate cost of the deficit across all individuals:

$$\wedge = \sum_i \lambda_i$$

As we said, in a large population, it is assumed that each individual's share of the deficit (λ_i) is very small, on the order of $\frac{1}{N}$ (where N is the number of contributors). Therefore, the ratio $\frac{\sqrt{c_j^p}}{\sum_j \sqrt{c_j^p}}$ is much smaller than $\wedge$. This allows us to simplify the equation to:

$$V_i^{p\prime}(F^p) - \wedge \approx 1$$

Therefore, the first-order condition simplifies to:

$$V_i^{p\prime}(F^p) \approx 1 + \wedge$$

3.6.4. Implications of Underfunding

The above result suggests that incorporating the deficit leads to a slight underfunding of public goods compared to the ideal situation where the deficit is not considered. The term $1 + \wedge$ indicates that the marginal value of the public good is slightly higher than 1, reflecting the additional cost imposed by the deficit.

However, as Buterin, Hitzig & Weyl [vi] note in their paper, this underfunding is bounded and manageable. They explore various scenarios to illustrate that incorporating the deficit does not fundamentally alter the QF mechanism's effectiveness.

In the scenario where most project funded through QF benefit only a small fraction of individuals, the impact of the deficit on individual contributors is minimal. This is because the deficit's cost is spread over a large population, making each person's share of the deficit relatively small. As long as the assumption under which negative contributions are not allowed, the slight underfunding caused by the deficit remains minor, and the conclusions drawn from the baseline analysis we made (which ignores the deficit) largely hold true.

Suppose a certain Youtube channel primarily benefits a specific type of users. For instance, a Youtube channel that shares economic concepts will favour students in economics, while other students won't have any interest in it. The contributions to this channel might be substantial within the entire Youtube users community, but the overall community ensures that the deficit's cost is distributed thinly. Consequently, the financial burden on each individual is minor, leading to a situation where the QF mechanism still provides close-to-optimal funding levels, with minimal underfunding due to the deficit.

Now, if negative contributions are allowed (further analysis of the impact of negative contributions on the model will come in the next section), this would enable individuals to defund public goods that they perceive as having negative externalities. Therefore, this mechanism could still help moderate the underfunding issue by providing a way to reduce funding for projects that are not universally beneficial.

Consider a Youtube channel that causes significant inconvenience or harm to a subset of the community. For example, a Youtube channel on economic that shares explanations that are false, based on unverified sources, or that shares content plagiarised from another channel. This could distort the learning of economics students, teaching them false facts about economics, or the plagiarised YouTube channel might not get the credit it deserves. Allowing negative contributions lets these affected students reduce the funding for such projects. However, this approach introduces the risk of negative contributions being abused. If not carefully managed, individuals or groups could use negative contributions to undermine projects based on personal biases or vendettas, rather than genuine negative externalities. Thus, while negative contributions can help address underfunding, they require stringent safeguards to prevent misuse. We will come back to this later.

A particularly favorable scenario for the QF mechanism involves funding from external sources that we mentioned earlier (public institutions and governments, large philanthropic organizations and foundations, companies, crowdfunding/community contributions or even cryptocurrency and blockchain projects) rather than relying on tax revenue from contributors. When the deficit is covered by philanthropic donations, contributors are less concerned about the financial implications of their contributions, as they are not directly bearing the cost of the deficit. In this case, the contributors' primary focus is on the utility derived from the funded project, leading to more accurate and optimal funding levels.

3.7. Negative Contributions

An interesting alternative of the model to explore is the concept of allowing negative contributions within the Quadratic Finance (QF) mechanism. Negative contributions enable individuals to effectively "defund" public goods that they perceive as harmful or undesirable. In this section, we will discuss the benefits, potential issues, and theoretical basis of incorporating negative contributions.

The basic idea behind negative contributions is that not all public projects yield purely positive outcomes. Some projects might generate negative externalities that affect certain segments of the population. For instance, a public park might lead to noise pollution, or a new road might cause environmental damage. Allowing negative contributions provides a mechanism for those negatively impacted to express their preferences and reduce the funding for such projects.

3.7.1. Definition of ± Quadratic Finance (±QF)

The natural extension of QF to incorporate negative contributions is termed the ± Quadratic Finance Mechanism (±QF). In this variant, citizens can choose to either fund or defund a public good. The formal definition of the mechanism is:

$$F^p = \Phi^{\pm QF}\left(\sum_i \pm\sqrt{c_i^p}\right)^2$$

where:

- F^p is the total funding for public good p .

- c_i^p is the contribution made by individual i to public good p .

- $\pm\sqrt{c_i^p}$ indicates that contributions can be either positive or negative, based on the individual's valuation of the public good.

In this mechanism, individuals with positive valuations ($V_i^{p\prime} \geq 0$) for a public good will make positive contributions, while those with negative valuations ($V_i^{p\prime} \leq 0$) will make negative contributions. This setup allows for a more nuanced expression of public preferences, accommodating both supporters and detractors of a public project.

3.7.2. Utility Maximization with Negative Contributions

The introduction of negative contributions modifies the utility maximization framework for individuals. For positive contributors, the utility function remains the same as in the original QF mechanism:

$$U_i = V_i^p(F^p) - c_i^p$$

Therefore, the first-order condition for utility maximization remains similar to the original QF mechanism:

$$2V_i^{p\prime}(F^p)\left(\sum_j \sqrt{c_j^p}\right)\frac{1}{2\sqrt{c_i^p}} = 1$$

Simplifying, we get:

$$V_i^{p\prime}(F^p) = \frac{\sqrt{c_i^p}}{\sum_j \sqrt{c_j^p}}$$

However, for negative contributors, the utility function includes a term for the disutility caused by the public good:

$$U_i = -V_i^p(F^p) - c_i^p$$

Therefore, the first-order condition for maximizing utility with negative contributions is:

$$-2V_i^{p'}(F^p)\left(\sum_j \pm\sqrt{c_j^p}\right)\frac{1}{2\sqrt{c_i^p}} = 1$$

This simplifies to :

$$V_i^{p'}(F^p) = -\frac{\sqrt{c_j^p}}{\sum_j \pm\sqrt{c_j^p}}$$

Summing these conditions across all contributors, we obtain the aggregate first-order condition:

$$V^{p\prime}(F^p) = 1$$

This result shows that allowing negative contributions can theoretically lead to optimal funding levels, where the total marginal value derived from a public good equals the cost of providing it.

3.7.3. Practical Considerations and Challenges

This variation of the model implies challenges we need to take into account. Indeed, allowing negative contributions introduces complexity in implementation, as the system must handle both funding and defunding simultaneously. Also, the platform needs to ensure that the total funding remains non-negative and that projects are not arbitrarily defunded due to malicious activities.

However, negative contributions empower individuals to act as "vigilante enforcers" against projects they perceive as harmful. While this can help prevent the funding of detrimental projects, it also risks misuse, where individuals might defund projects for personal or ideological reasons rather than genuine harm.

Therefore, it is important to balance the influence of positive and negative contributions to ensure fair and equitable outcomes. This might involve setting limits on the extent of negative contributions or implementing safeguards to prevent abuse.

4. Limitations and failures of QF

4.1. The Capital-constrained Quadratic Funding mechanism

As I said in the section about the principles of the QF mechanism, the funds collected into the matching pool are less likely to come from tax revenue from individuals, but rather from external sources. We just mentioned the fact that in the scenario where funding comes from external sources, individuals tend to not consider their impact on the deficit. However, this does not imply that all these public institutions, companies, philanthropic organizations,... have unlimited funds to allocate to projects funded by QF. This is a limitation in the model.

Therefore, this first subsection will address an alternative form of the Quadratic Funding mechanism, proposed by Buterin, Hitzig & Weyl in their paper [vi] : the Capital-constrained Quadratic Funding mechanism (CQF).

4.1.1. CQF mechanism : principles

Indeed, this budget limitation necessitates a mechanism that can work within these financial constraints while still achieving the optimal distribution of funds.

To address this issue, the authors define the Capital-constrained Quadratic Finance mechanism (CQF) as a weighted combination of QF and private contributions. The formula for CQF is given by:

$$F^p = \alpha \left(\sum_i \sqrt{c_i^p} \right)^2 + (1 - \alpha) \sum_i c_i^p$$

where:

- F^p is the total funding for public good p .
- c_i^p is the contribution made by individual i to public good p .
- α is a parameter between 0 and 1 that adjusts the weight of QF versus private contributions.

The key feature of CQF is that α can be adjusted to ensure that the budget is not exceeded. When $\alpha \to 0$, the mechanism relies more on private contributions, which are self-financing. As α increases, the influence of QF increases, providing more subsidies for contributions. This means that when α is low, the mechanism is closer to private contributions, which require no external funding. Conversely, when α is high, the mechanism provides substantial subsidies but requires more external funding.

By setting α appropriately, the mechanism can ensure that the total funding does not exceed the available budget (by setting α low enough, the deficit can be eliminated). This flexibility allows philanthropists or governments to manage their financial commitments effectively while still promoting public goods.

4.1.2. CQF mechanism : individual rationality

A key point of CQF is that it maintains individual rationality, meaning that contributors always benefit from using the mechanism compared to contributing through a purely private channel.

The marginal impact of an individual's contribution under CQF is always greater than under a purely private contribution system. Mathematically, this translates to:

$$\frac{\partial F^p}{\partial c_i^p} = \alpha \frac{\sum_j \sqrt{c_j^p}}{\sqrt{c_j^p}} + 1 - \alpha$$

This derivative shows that the marginal value of a contribution under CQF is always greater than or equal to 1, exceeding the marginal impact of a contribution made through purely private channel. This ensures that contributors have a strong incentive to participate in the CQF system.

4.1.3. Underfunding and Efficiency

In CQF, individuals choose their contributions to maximize their utility, taking into account the weighted combination of QF and private contributions. The first-order condition for maximizing utility is:

$$V_i^{p\prime}\left(\alpha \frac{\sum_j \sqrt{c_j^p}}{\sqrt{c_j^p}} + 1 - \alpha\right) = 1$$

By simplification, we obtain that:

$$V_i^{p\prime} \approx \left(\frac{\sqrt{c_j^p}}{\alpha \sum_j \sqrt{c_j^p}}\right)$$

Which simplifies to:

$$V^{p\prime} = \frac{1}{\alpha}$$

This indicates that, for large communities where individual contributions are small relative to the total, the marginal value of public goods funding is approximately $\frac{1}{\alpha}$. In large populations thus, this underfunding is less severe than in purely private contributions, making CQF a significant improvement. Additionally, CQF ensures that funding is allocated approximately optimally across different public goods, adhering to principles of efficient taxation and public goods provision.

4.2. Variant of the CQF mechanism

In this section, I aim to provide another view of the QF problem in the context of limited matching funds. To this end, I'll be taking up Ricardo A. Pasquini's paper "Optimal Allocation of Limited Funds in Quadratic Funding"[xiv], in which the author investigates the allocation of limited matching funds to public good projects using a variation of the Quadratic Funding (QF) mechanism, initially proposed by Buterin, Hitzig, and Weyl (2019).

4.2.1. Proposed variation

The variation of the model Pasquini proposes relies on two main assumptions. First, in his model, projects do not receive the funds directly from contributors but receive a percentage of the total funds according to the QF rule. This means that instead of receiving funds from the matching pool and individual contributions, only the funds the matching pool are distributed. Second, the individual contributions are retained and could be added to the matching pool in future funding rounds.

This mechanism aims to equalize the marginal social benefits across projects, driving towards a socially optimal allocation of limited resources. Let's now delve into the analysis of the model.

4.2.2. Basic setup

The mechanism begins by defining two sets:

- $p \in P$ represents public good projects that are competing to receive funding.
- $i \in I$ represents individual contributors who choose to support these projects by committing specific amounts of money, denoted by c_i^p for each project p.

In addition to these individual contributions, there is a pool of matching funds provided by donors, denoted by D . These funds are used to supplement the contributions made by individuals, in accordance with the rules of Quadratic Funding.

4.2.3. Funding Formula

The mechanism promises to fund each project p an amount F^p . The amount F^p that a project receives is determined by the following two scenarios:

First, the unconstrained case. If the total amount required to fulfill the Quadratic Funding rule for all projects $\sum_p F^{p,QF}$ is less than or equal to the available matching funds D, then each project receives exactly the amount calculated by the Quadratic Funding rule, i.e., $F^p = F^{p,QF}$.

On the other hand, there is the constrained case. If the total amount required by the Quadratic Funding rule exceeds the available matching funds (i.e., $\sum_p F^{p,QF} > D$), then the funds are distributed proportionally. Specifically, each project p receives a fraction of the total funds available in the pool D, proportional to what it would have received under the unconstrained Quadratic Funding rule. This is mathematically expressed as:

$$F^p = \frac{F^{p,QF}}{\sum_p F^{p,QF}} D$$

This formula ensures that the sum of all project funding does not exceed the total available matching funds D.

The mechanism operates under the assumption that the matching pool D is fixed. As contributors invest in a project p , they alter the share of the matching pool that project p will receive. However, their contributions do not increase the total pool D, instead, they change the distribution of this fixed amount among all projects.

A notable feature of this mechanism is that the contributions c_i^p made by individual contributors are not directly allocated to the projects they support. Instead, these contributions are retained by the mechanism, and they can be added to the matching pool D in subsequent rounds of funding. This approach differs from traditional models where individual contributions are immediately allocated to the projects.

In the constrained case, projects compete for a fixed amount of funds. The proportional allocation ensures that all projects receive some funding, but the amount is scaled based on their relative size under the Quadratic Funding rule.

4.2.4. Individual contributor problem

This section will delve into the decision-making process of individual contributors, still in the case of the variant proposed by Pasquini. We will explore how contributors determine the amount of money they commit to different public good projects, given the constraints of limited matching funds, providing a mathematical and conceptual analysis of the contributors' optimization problem and the resulting dynamics of fund allocation.

Setup

Each individual contributor i derives a certain level of utility from the public goods funded by the QF mechanism. This utility Is denoted as V_i^p, which represents the currency-equivalent utility that contributor i gains from project p. The utility functions V_i^p are assumed to be independent across different public goods, meaning that the utility derived from one project does not affect the utility derived from another.

Contributors face the challenge of how much to contribute to each project p , balancing their desire to maximize their overall utility against the reality of limited matching funds.

Optimization Problem

The individual contributor's optimization problem is defined as choosing the amount of money to commit to each project p, denoted by c_i^p, in a way that maximizes their overall utility. The total utility that contributor i seeks to maximize can be expressed as the sum of the utilities they derive from all the public goods projects, minus the amount they contribute. Mathematically, this is represented as:

$$max_{\{c_i^p\}p\in P} \sum_{p'\in P} V_i^{p'}\left(\frac{D}{\sum_{p'\in P} F^{p',QF}}\ F^{p',QF}\right) - c_i^{p'}$$

This expression reflects the contributor's decision-making process:

The first term inside the sum represents the utility derived from each public good p', which is a function of the matching funds D and the Quadratic Funding rule $F^{p',QF}$.

The second term, $c_i^{p'}$, represents the cost of contributing to the project, which is subtracted from the total utility.

First Order Condition (FOC)

To find the optimal contribution c_i^p, the paper derives the First Order Condition (FOC) for the optimization problem. This condition characterizes the point at which a contributor's utility is maximized, meaning any small change in the contribution amount c_i^p would not increase the total utility.

The FOC is expressed as:

$$V_i'^p(.)\left(\frac{\sum_i\sqrt{c_i^p}}{\sqrt{c_i^p}}\frac{D}{\sum_{p'\in P}F^{p',QF}}-\frac{D}{\left(\sum_{p'\in P}F^{p',QF}\right)^2}F^{p',QF}\frac{\sum_i\sqrt{c_i^p}}{\sqrt{c_i^p}}\right)-1$$
$$+\sum_{p'\in P,\,p'\neq p}V_i'^{p'}(.)\left(-\frac{D}{\left(\sum_{p'\in P}F^{p',QF}\right)^2}F^{p',QF}\frac{\sum_i\sqrt{c_i^p}}{\sqrt{c_i^p}}\right)=0$$

Breaking this down:

$V_i'^p(.)$ represents the marginal utility that contributor i derives from public good p.

The first term inside the parentheses reflects the direct effect of contributing to project p, scaled by the ratio of total matching funds D to the sum of the Quadratic Funding allocations across all projects.

The second term inside the parentheses represents the indirect effect of the contribution, where increasing c_i^p raises the matching requirements across all projects $\sum_{p'\in P}F^{p',QF}$.

The second part of the equation sums the marginal utility effects across all other projects p' that are not project p. It reflects how contributions to project p affect the distribution of matching funds among the other projects.

Simplification and Interpretation

To simplify the complex FOC, the paper groups terms and factors to provide a clearer interpretation of the contributor's decision-making process. The key conclusion of this analysis can be expressed in the following rearranged equation:

$$V_i'^p(.)-\sum_{p'\in P}V_i'^{p'}(.)\frac{F^{p',QF}}{\sum_{p'\in P}F^{p',QF}}=\frac{\sqrt{c_i^p}}{\sum_i\sqrt{c_i^p}}\left(\frac{\sum_{p'\in P}F^{p',QF}}{D}\right)$$

The left side of the equation represents the difference between the sum of marginal utilities from contributing to project p and a weighted average of marginal utilities across all projects. If this difference is positive, it means that contributor iii has an incentive to invest more in project p .

The right side of the equation represents the cost associated with reallocating funds by contributing to project p. This cost increases as more funds are contributed because the denominator D (available matching funds) is fixed.

A key insight from this analysis is that contributors are incentivized to contribute to projects that offer the highest marginal utility relative to the overall funding environment. As contributors make decisions based on their marginal utility, the system naturally tends to equalize the marginal benefits across all projects. This process drives the allocation of funds towards a socially optimal distribution, where the marginal utility derived from each project is balanced.

4.3. Collusion and Fraud

The QF mechanism, while innovative, is not immune to strategic manipulations. Two main vulnerabilities identified by Buterin, Hitzig, and Weyl (BHW) are collusion and identity fraud. In this section, we will analyse the challenges of these vulnerabilities, their potential impact on the model, and propose measures to mitigate them.

4.3.1. Collusion and Fraud: Definitions and Risks

Fraud involves a single agent misrepresenting themselves as multiple agents to inflate contributions and receive more matching funds.. This could lead to a scenario where one person disproportionately benefits from the funding mechanism.

Collusion, on the other hand, occurs when multiple agents cooperate to manipulate the mechanism for mutual benefit, often at the expense of other participants. There are basically two types of collusion : horizontal and vertical collusion. Horizontal Collusion involves multiple participants with similar goals collaborating to increase their combined benefits, while vertical collusion involves cooperation between different levels or sides of the mechanism, such as between contributors and project organizers. This manipulation of the distribution of funds can distort the intended equitable distribution of funds.

4.3.2. Specific threats and simple examples of fraud and collusion

Let's consider a scenario of fraud, where a single individual fraudulently represents themselves as 20 different contributors. If each of these fake identities contributes €1, the system calculates the funding as:

$$\alpha\left(\sum_{i=1}^{20}\sqrt{1}\right)^2$$

If $\alpha = 0.1$, the total funding becomes

$$0.1 \times 20^2 = 40 \text{ euros.}$$

The fraudster has effectively turned a €20 contribution into €40 of funding, doubling their money through manipulation.

Similarly, in a collusion scenario, a group of 100 people each contributing €10 could coordinate to appear as separate contributors. The system would calculate the funding based on:

$$\alpha\left(\sum_{i=1}^{100}\sqrt{10}\right)^2$$

If $\alpha = 0.1$, the total funding becomes

$$0.1 \times \left(100 \times \sqrt{10}\right)^2 = 10.000 \text{ euros.}$$

If one member of this group decides to defect and contribute nothing, the total funding only slightly decreases, but the defecting member saves their contribution while still benefiting from the collective funding, creating a strong incentive to cheat within the group. Let's specify the analysis of this latter example, using the concept of reciprocal backing.

4.3.3. Reciprocal Backing

In his paper *"Quadratic Funding and Matching Funds Requirements"* [xv], Pasquini proposes a detailed analysis of the reciprocal backing concept in the context of Quadratic Funding.

Reciprocal backing refers to a situation where contributors with their own projects invest in each other's projects, expecting the favor to be returned. This behavior can be explicit (through an agreement) or implicit (through unspoken understanding). By strategically spreading their contributions, project team members can maximize the total funding their projects receive.

Reciprocal Backing in a one-period model

To formalize this, consider two contributors, each with a project in the mechanism. Assume each contributor has an amount c to invest. If both contributors decide to invest half of their funds in each other's projects, expecting reciprocal investments, they can increase their overall funding. This situation can be modeled as a 2x2 simultaneous game with the following payoff matrix:

	Invest	Do Not Invest
Invest	c, c	$-\frac{c}{2}, \frac{c(1+2\sqrt{2})}{2}$
Do Not Invest	$\frac{c(1+2\sqrt{2})}{2}, -\frac{c}{2}$	$0, 0$

In this matrix, the "Invest-Invest" payoff of c for both contributors arises because the quadratic funding rule amplifies their contributions. For example, if both contributors invest $\frac{c}{2}$ in each other's projects, the payoff for each would be:

$$\left(2\sqrt{\frac{c}{2}}\right)^2 - c = c$$

If one contributor invests but the other does not reciprocate, the investing contributor suffers a loss due to the reduced matching funds, resulting in a negative payoff. This creates a classic Prisoner's Dilemma where the Nash equilibrium is for both contributors not to invest. In a one-period model thus, collusion is mitigated by the unilateral incentives to deviate from the collusive agreement.

However in practice, as Pasquini points out, Quadratic Funding might involve several rounds of funding, as proposed by the Gitcoin Grants project (section 3.5 on User Interface).

Reciprocal Backing in the case of repeated rounds of funding

In a scenario of repeated rounds of funding, reciprocal backing can become a sustained strategy. If contributors expect future rounds, they may use "trigger strategies" or threats to enforce reciprocal behavior. This leads to a Nash equilibrium where reciprocal backing is maintained over multiple rounds.

The proof relies on the repeated game framework, where the fear of losing future reciprocal investments enforces cooperation among contributors. When the number of colluding participants exceeds two, the strategy can still be profitable even if some participants deviate.

For instance, in a collusion with n participants, the strategy remains beneficial if a certain percentage α^* of participants continue to invest reciprocally. This percentage is derived from the condition that the sum of the square roots of investments must exceed the cost of contributions:

$$\alpha^* > \frac{1}{\sqrt{n}}$$

This implies that as the number of colluding participants increases, the required percentage of reciprocal investors decreases. For example, with 25 participants, the strategy remains profitable if at least 20% of them reciprocate.

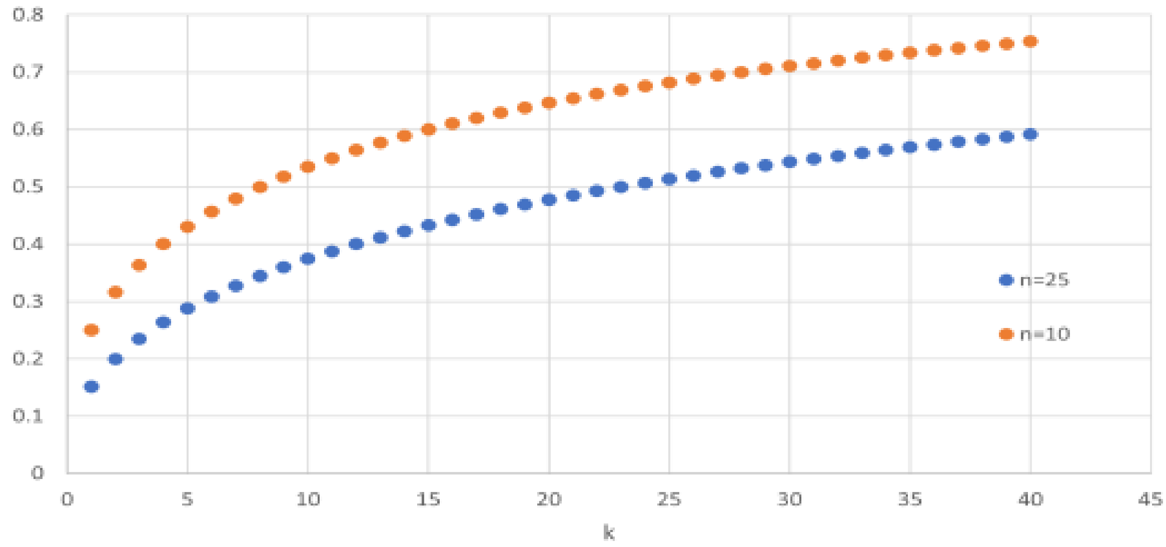
Collusion with Limited Funds

Under the assumption limited matching funds, as we analysed in section 4.1 with the CQF model, collusion incentives are reduced but not eliminated.

Indeed, under the assumption of restrictive funds, a collusion strategy with n participants is profitable under deviation if a percentage α^{**} still colludes where:

$$\alpha^* > \frac{k\left(\left(1-\frac{1}{k}\right)+\sqrt{\left(1-\frac{1}{k}\right)^2+4\left(\frac{n}{k}\right)}\right)}{2n}$$

For example, with 25 participants and a k value of 20 (indicating significant funding constraints), the collusion strategy is profitable if at least 60% of contributors reciprocate. The graph below illustrates this relationship, showing how the required percentage of reciprocators changes with different levels of k:



It's important to note that in practice, not all participants face the same budget constraints. For example, Gitcoin Grants runs multiple categories of projects with separate matching pools. The assumption is thus that colluding projects in different categories do not compete for the same funds, increasing the incentive for reciprocal backing across categories of projects rather than within categories.

Empirical Evidence of reciprocal backing

To support the assumption we just made, let's take some elements of section 4 of Pasquini's paper, in which the author proposes empirical evidence of reciprocal backing through the example of the Gitcoin Grants project.

The empirical analysis covers contributions made during the 7th and 8th Gitcoin Grants rounds. The dataset includes information on individual contributions, the projects supported, and the identities of project team members. The author aims to show that reciprocal backing is stronger across project categories because projects in different categories do not compete for the same matching funds pool.

The figure below shows the relationship between the number of reciprocal contributions and the total number of projects contributed to.

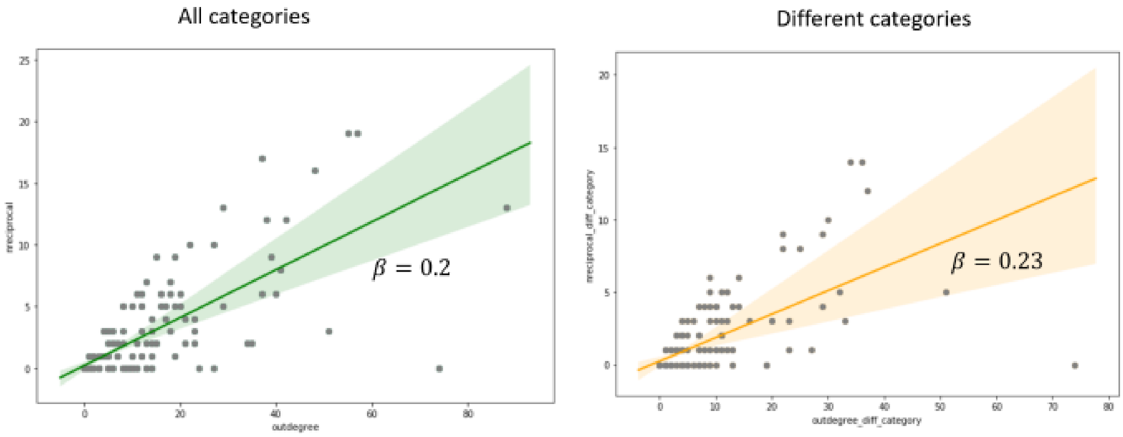


Figure 3 : Reciprocal contributions as function of total contributions

The linear approximation shows a slope of 0.2, confirming that about 20% of contributions are reciprocated, which suggests that reciprocal backing is a prevalent strategy. This means that for every ten contributions made by project team members, two are invested back into their projects by the projects they supported.

The right panel of Figure 3 shows that contributions in additional projects are reciprocated with a probability of 23% when considering cross-category investments. This is slightly higher than the overall reciprocal backing rate, suggesting that cross-category reciprocal backing is indeed a significant behavior.

Additionally, the author provides a detailed analysis of reciprocal backing across different categories. Through the following table, Pasquini compares the percentage of reciprocal investments received from projects in other categories with the percentage of projects not in the same category.

Table 3: Reciprocal backing.

Category	Number of projects %	Number of projects-complement %	Reciprocal investments in other categories %
Community	33.49	66.51	71.30
Dapp Tech	33.49	66.51	55.55
Infra Tech	22.64	77.36	76.00
Matic	8.96	91.04	20.51

For the category of projects "Community", 71% of reciprocal contributions come from projects in other categories, whereas only 66% of projects are outside the Community category. This indicates a higher propensity for cross-category reciprocal backing in this category.

In contrast, the "Dapp Technology" category shows 55% of reciprocal contributions from other categories, compared to 66% of projects outside the Dapp category. This suggests a lower rate of cross-category reciprocal backing in the Dapp category.

Finally, the “Infrastructure Technology” category has a percentage of 76% for cross-category reciprocal contributions, closely matching the 77% of projects not in the same category, indicating a balanced reciprocal backing behavior.

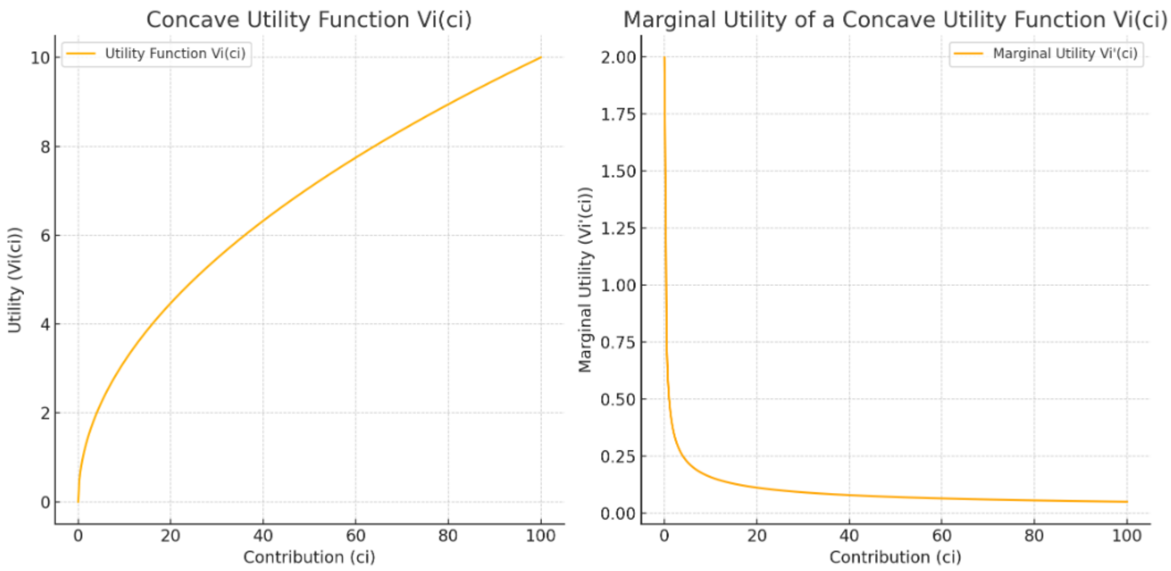
These findings indicate that while reciprocal backing occurs both within and across categories, the incentives for cross-category reciprocal backing are generally stronger.

4.4. Failures of concavity and solutions

In the previous sections, we assumed that the utility functions of the QF mechanism were concave. However, in real-world scenarios, utility functions may not always be concave. This section aims to analyse this failure of the model and provide solutions to address this issue, as Buterin, Hitzig and Weyl made in their paper.

4.4.1. Failure of concavity

The QF mechanism relies on the assumption that individuals' utility functions are smooth and concave. Concave utility functions are characterized by diminishing marginal returns, meaning that as the quantity of a good increases, the additional utility gained from consuming an additional unit decreases. This property ensures that individuals allocate their resources in a way that balances their marginal benefits and costs, leading to efficient outcomes. Graphically, this is how concave utility functions and its corresponding marginal utility would be represented:

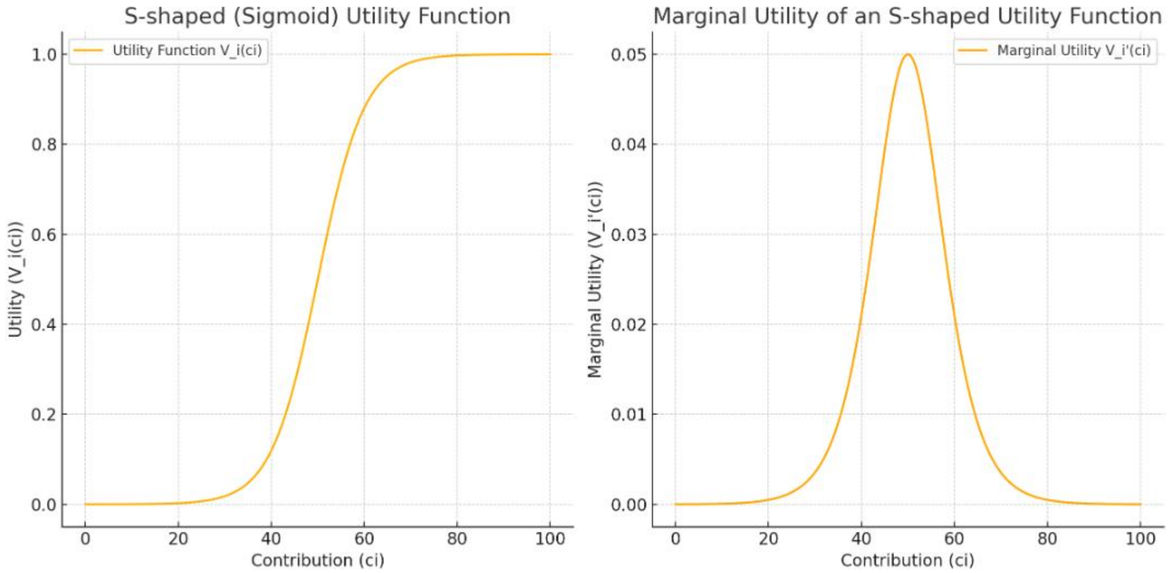


The first graph shows the relationship between the contribution amount c_i and the utility $V_i(c_i)$ derived from that contribution. As the contribution amount increases, the utility increases, but at a decreasing rate. This is characteristic of a concave function, where the slope (rate of increase) diminishes as c_i grows.

The second graph shows the marginal utility, which is the derivative of the utility function with respect to the contribution amount. The marginal utility decreases as the contribution amount increases, reflecting diminishing returns on additional contributions. This decrease in marginal utility is what defines the concavity of the utility function.

However, in real-world scenarios, utility functions may not always be concave. Indeed, some public goods may have threshold effects. A threshold public good is a public good that is provided if and only if contributions reach some critical threshold [xvi]. Under this threshold, no individual contributor derives utility from the project. This is often the case for public infrastructure projects or more broadly for projects with fixed costs.

Graphically, this implies that the utility function of individual contributors is not concave anymore, but rather sigmoidal (S-shaped):



The graph on the left shows the relationship between the contribution amount c_i and the utility $V_i(c_i)$, illustrating the threshold effect and the S-shaped curve.

The graph on the right shows the marginal utility of such a function, or the rate of change of the utility with respect to contributions. It highlights the peak around the threshold, where contributions are most impactful.

4.4.2. Potential Solution

To address the issues of concavity, the authors propose a dynamic solution called "assurance contracts. Assurance contracts are agreements in which individuals commit to contributing to a public good only if a critical mass of contributions is reached. This approach can help overcome the coordination problem that arises when individuals are unwilling to contribute unless they are assured that others will also contribute.

Assurance contracts work as follows: Individuals pledge to contribute a certain amount to a public good, but their contributions are only collected if the total pledges meet or exceed a predetermined threshold. If the threshold is not met, the contributions are not collected, and the public good is not funded. This mechanism ensures that individuals only contribute when their contributions are likely to result in the provision of the public good, thereby addressing the failures of concavity.

5. Quadratic Funding and Project Choice

5.1. The model

From a pure economical point of view, let’s set up a theoretical model of Quadratic Funding. Let’s take the initial example of a Youtube channel. As we said in the introduction, creating Youtube videos involves a certain amount of fixed costs : good equipment to record, editing software, professional staff, etc. This means that individual contributors will only derive utility/value from the videos if there is enough funding for the Youtube channel to create and post its videos. Under this threshold of fixed costs, videos can’t be produced and no contributor derives utility from them. This implies non-concave utility functions.

The basic form of our model will analyse the case in which there are two projects (Youtube channels A and B) with no interaction between them (the two channels are independent), and we will consider for simplicity that there are only two individual contributors involved in the funding mechanism.

5.1.1. Utility Functions and Social Welfare

Assume that each contributor's utility function for a given project follows a concave, Stone-Geary form due to the parameter $\gamma < 1$:

$$V_i^p(F^p) = A_i^p \frac{1}{\gamma}(F^p - x^p)^\gamma$$

$V_i^p(F^p)$ represents the utility that contributor i derives from funding project p, minus the individual contributions c_i.

F^p is the funding level of project p through matching pool of funds, with x^p being some minimal fixed costs that project p has to produce its content. If $F^p < x$, meaning that the funding received is lower than the fixed costs, individual contributors derive negative utility because their total utility would be minus the contributions. Because of this, nobody would want to fund it. Therefore, for the above mentioned utility function to remain non-negative, the condition is that $F^p \geq x$.

A_i is a constant reflecting the contributor's valuation of the project, in other words the willingness of individual i to pay for project p.

γ is a parameter that shapes the utility function, with $\gamma < 1$ indicating decreasing marginal utility as funding increases.

This Stone-Geary utility function suggests that contributors derive satisfaction at a lower rate as a project receives more funding, especially after reaching the critical threshold of fixed costs.

The total social welfare, $W(F_A, F_B)$, from funding both projects (A and B) is the sum of the utilities of all contributors:

$$W(F_A, F_B) = \sum_i \left(V_i^A(F^A) + V_i^B(F^B) - c_i - t\right)$$

Which means:

$$W(F_A, F_B) = \sum_i \left(A_i^A \frac{1}{\gamma} (F^A - x^A)^\gamma + A_i^B \frac{1}{\gamma} (F^B - x^B)^\gamma - c_i^A - c_i^B - t \right)$$

Each contributor gains utility from the funding of Projects A and B, modeled by the concave, Stone-Geary utility functions $A_i^A \frac{1}{\gamma} (F^A - x^A)^\gamma$ and $A_i^B \frac{1}{\gamma} (F^B - x^B)^\gamma$, where $\gamma < 1$. This utility increases as the funding levels F_A and F_B increase, especially after surpassing the critical threshold x.

The terms $-c_i^A$ and $-c_i^B$ reflect the costs that contributors cause by making their contributions. These costs directly reduce the net welfare of each contributor, as they represent a loss of resources that could have been used elsewhere.

The tax level t is a fixed amount subtracted from each contributor's welfare, further reducing the overall social welfare. This could represent a general tax imposed to fund the matching pool or other public goods, reducing the disposable income of the contributors.

5.1.2. Optimization Problem

Let's first consider that the social planner's objective is to maximize the total social welfare by choosing the optimal levels of funding F^A and F^B for the two projects:

$$max_{F^A, F^B} W(F_A, F_B) = \sum_i \left(A_i^A \frac{1}{\gamma} (F^A - x^A)^\gamma + A_i^B \frac{1}{\gamma} (F^B - x^B)^\gamma - c_i^A - c_i^B - t \right)$$

As we said in the previous section, there is a non-negative funding constraint. The funding for each project must be non-negative and must meet or exceed the fixed cost x^p to ensure that utility is non-negative, such as:

$$F^A \geq x^A , \;\; F^B \geq x^B$$

Also, there is a budget-constraint. Indeed, let's consider that the combined funding for both projects is financed through lump-sum taxes. Therefore, $F^A + F^B$ cannot exceed t:

$$F^A + F^B \leq t_1 + t_2 = t$$

Also, let us remind the basic assumption according to which there is no interaction between the two projects. It is important to start by setting out the optimisation problem in a fairly simple framework, so that we can later analyse the case in which the two projects are perfect substitutes or perfect complements.

5.1.3. First-Order Conditions

To find the optimal funding levels F^A and F^B, we derive the first-order conditions (FOCs) with respect to F^A and F^B. Given that the marginal cost of increasing F^A or F^B is one (due to the fact that it is financed through one-for-one taxation), the first-order conditions for the maximization problem are derived as follows:

For optimal funding of project A (F^A) we have:

$$\frac{\partial W}{\partial F^A} = \sum_i \left(A_i^A \frac{1}{\gamma} (F^A - x^A)^{\gamma - 1} \right) = 1$$

For optimal funding of project B (F^B), we have :

$$\frac{\partial W}{\partial F^B} = \sum_i \left(A_i^B \frac{1}{\gamma} (F^B - x^B)^{\gamma - 1} \right) = 1$$

These first-order conditions equate the sum of the marginal utilities gained from increasing funding for each project to the marginal cost of doing so, which is one due to the direct tax burden.

Since $\gamma < 1$, the utility functions exhibit concavity, meaning that as the funding for a project surpasses its fixed cost x^p, the marginal utility decreases. The planner will seek to fund each project to the point where the sum of the marginal utilities of additional funding justifies the cost of the tax burden.

The planner will allocate resources between the projects to balance these marginal utilities. Specifically, the planner will increase funding for Project A until the marginal utility equals the marginal cost of funding (which is 1). The same applies to Project B. If one project offers higher marginal utility at a lower level of funding, the planner will prioritize funding that project until the first-order condition is met.

5.1.4. Applications

Let's set up a first example of the optimization problem, replacing the different parameters with specific values to verify the potential optimality of our model.

- Let's assume that there are only 2 individual contributors, which means $i \in (1\,;2)$.

- We keep the assumption according to which there are only two projects (A and B) to fund, and we assume that the fixed costs these projects have to produce their content is equal to 1 :

$$x^A = x^B = 1$$

- To keep the assumption of concave utility functions, we set the parameter γ as equal to $\frac{1}{2}$.

- For now, let's keep the parameter reflecting the contributors' valuation of the projects, A_i^p in its general form.

Utility Function

The utility function for contributor i from funding project p (where p is either A or B) is:

$$V_i^p(F^p) = A_i^p \frac{1}{\gamma} (F^p - x^p)^\gamma = 2A_i^p \sqrt{F^p - 1}$$

Where F^p is the funding level of project p, and $F^p \geq 1$ to ensure non-negativity utility.

Now, we want to solve the optimization problem to maximize total social welfare $W(F_A\,, F_B)$ by determining the optimal funding levels F_A and F_B for projects A and B, respectively.

Total Social Welfare Function

The total social welfare function $W(F_A, F_B)$ is the sum of the utilities of the two contributors minus the total tax burden:

$$W(F_A, F_B) = \sum_{i=1}^{2} \left(2A_i^A\sqrt{F_A - 1} + 2A_i^B\sqrt{F_B - 1} - F_A - F_B\right)$$

Simplifying, we get:

$$W(F_A, F_B) = 2\left(2A_1^A\sqrt{F_A - 1} + A_2^A\sqrt{F_A - 1} + A_1^B\sqrt{F_B - 1} + A_2^B\sqrt{F_B - 1}\right)$$

Optimization Problem

The aim is to maximize $W(F_A, F_B)$ with respect to F_A and F_B, subject to the constraint $F_A \geq 1$ and $F_B \geq 1$. Therefore, we get:

$$max_{F_A, F_B} W(F_A, F_B) = 2\left(\left(A_1^A + A_2^A\right)\sqrt{F_A - 1} + (A_1^B + A_2^B)\sqrt{F_B - 1}\right) - F_A - F_B$$

First-Order Conditions

To find the optimal values of F_A and F_B, we take the partial derivatives of $W(F_A, F_B)$ with respect to F_A and F_B and set them equal to 1, reflecting the marginal cost of funding we have:

For F_A:

$$\frac{\partial W}{\partial F_A} = \frac{A_1^A + A_2^A}{\sqrt{F_A - 1}} = 1$$

For F_B:

$$\frac{\partial W}{\partial F_B} = \frac{A_1^B + A_2^B}{\sqrt{F_B - 1}} = 1$$

Solving the First-Order Conditions

Solving these equations for F_A and F_B, we get:

For F_A :

$$\sqrt{F_A - 1} = A_1^A + A_2^A$$

$$F_A - 1 = \left(A_1^A + A_2^A\right)^2$$

$$F_A = 1 + \left(A_1^A + A_2^A\right)^2$$

For F_B :

$$\sqrt{F_B - 1} = A_1^B + A_2^B$$

$$F_B - 1 = (A_1^B + A_2^B)^2$$

$$F_B = 1 + (A_1^B + A_2^B)^2$$

Corner Solution and Optimal levels of funding

When the society is better off not funding one of the projects at all, we call this a corner solution.

For instance, let's suppose that $A_1^A + A_2^A$ is very large compared to $A_1^B + A_2^B$, making F_A significantly more valuable.

If F_A provides much higher utility, it may be optimal to set $F_B = 0$. That means the value derived by individuals from project B is so low that it may be optimal not to fund this project at all, and allocate all the budget to F_A.

Example

Let's assume specific values:

$A_1^A = 2, A_2^A = 3, A_1^B = 0{,}5, A_2^B = 0{,}5$

Then:

$A_1^A + A_2^A = 5$, so $F_A = 1 + 25 = 26$

$A_1^B + A_2^B = 1$, so $F_B = 1 + 1 = 2$

Given these values, it is optimal to allocate all the resources to Project A because it provides higher marginal utility, while Project B is not funded at all.

5.1.5. Standard Quadratic Funding

When applying a standard Quadratic Funding (QF) mechanism to this scenario, the funding decisions are determined by the contributions made by individual contributors rather than by a central social planner. Let's recall that Quadratic Funding is a mechanism where the total funding a project receives is the sum of individual contributions plus an additional amount derived from a matching pool, which amplifies the impact of small contributions.

Let us remind the basic principles.

Quadratic Funding Mechanism Overview

In a standard Quadratic Funding mechanism:

Each contributor i contributes an amount c_i^p to project p (either A or B in our example).

The total contributions to project p from all contributors are denoted as $C^p = \sum_i c_i^p$.

The total funding that project p receives under QF is given by:

$$F^p = \left(\sum_i \sqrt{c_i^p}\right)^2$$

The QF mechanism amplifies the contributions by squaring the sum of the square roots of individual contributions.

Contributor's Utility Function

Each contributor i derives utility from the funding level of project p, as specified by the Stone-Geary utility function:

$$V_i^p(F^p) = A_i \frac{1}{\gamma}(F^p - x)^\gamma$$

Here, we assume that

A_i , the valuation parameter for contributor i, is the same across both projects but can differ between contributors.

$x = 1$, the fixed costs are the same for both projects.

$\gamma = \frac{1}{2}$, giving the utility function a concave, Stone-Geary shape.

Individual Contribution Decision

Each contributor i decides how much to contribute to each project based on their utility from the project's funding and the cost of their contribution. The goal of each contributor is to maximize their net utility:

$$max\ V_i^p(F^p) - c_i^p$$

Given that $F^p = \left(\sum_i \sqrt{c_i^p}\right)^2$ under QF, the utility derived by contributor i from project p is:

$$V_i^p(F^p) = 2A_i(F^p - 1)^{\frac{1}{2}} = 2A_i\left(\left(\sum_i \sqrt{c_i^p}\right)^2 - 1\right)^{\frac{1}{2}}$$

Solving for Contributions

For simplicity, we will make the assumption that both contributors don't make zero contributions. The first-order condition for each contributor i to maximize their utility with respect to their contribution c_i^p to project p is given by:

$$\frac{\partial V_i^p}{\partial c_i^p} = \frac{\partial}{\partial c_i^p}\left[2A_i\left(\left(\sum_i \sqrt{c_i^p}\right)^2 - 1\right)^{\frac{1}{2}}\right] - 1 = 0$$

The derivative of the utility function with respect to cipc_i^pcip involves applying the chain rule:

$$\frac{\partial V_i^p}{\partial c_i^p} = 2A_i \frac{\left(\sum_i \sqrt{c_i^p}\right)}{\sqrt{\left(\left(\sum_i \sqrt{c_i^p}\right)^2 - 1\right)}} \cdot \frac{1}{2\sqrt{c_i^p}} - 1 = 0$$

Simplifying, we get:

$$A_i \frac{\left(\sum_i \sqrt{c_i^p}\right)}{\sqrt{\left(\left(\sum_i \sqrt{c_i^p}\right)^2 - 1\right)}} \cdot \frac{1}{\sqrt{c_i^p}} = 1$$

This reflects the balance between the marginal utility derived from an additional unit of contribution and the marginal cost (which is 1 per unit of contribution). The solution depends on the values of A_i , but also the aggregate contributions $\sum_i \sqrt{c_i^p}$ and the specific project p.

Funding Outcomes Under Quadratic Funding

Under QF, the total funding received by each project depends on the contributions from both contributors. Since each contributor's utility is increasing in F^p (especially after the funding surpasses the fixed cost $x = 1$), they have an incentive to contribute. However, the distribution of contributions will depend on the relative valuations A_i of the contributors.

We can now analyse the different scenarios:

If $A_1 > A_2$, Contributor 1 values the projects more highly and will contribute more. This will lead to a higher F_A and F_B, with the project that both contributors prefer receiving more total funding.

If both A_1 and A_2 are equal , the contributions may be more balanced, leading to similar funding levels for both projects, assuming equal initial preferences.

Potential for Corner Solutions

In this scenario, a corner solution may occur if the contributions to one project dominate those to the other, driven by differences in the contributors' valuations A_i. For example:

If A_1 is significantly larger than A_2 and both contributors prefer project A , most of the contributions will flow to project A , leaving project B underfunded.

The other way round, if A_2 strongly prefers project B, the contributions might be concentrated in project B, leading to a corner solution where project B receives the majority of the funding.

6. Extensions

6.1. The case of perfect substitutes projects

In the previous sections, we considered for simplicity that there was no interaction between the two projects (the two Youtube channels were independent). However, in some more realistic scenarios, this is not the case. If you're searching for a specific content on Youtube, it's not unusual to see two channels producing very similar content, or even plagiarism in some cases. Therefore, this section aims to analyse the case in which the two projects are perfect substitutes and observe the implications of this assumption on the model and its conditions for optimality of funding.

6.1.1. Perfect substitutes : The model

When dealing with two YouTube channels (or projects) that are perfect substitutes, meaning they aim to produce the exact same content, the optimal funding strategy becomes more straightforward. In this scenario, the utility derived from funding both projects is not additive instead, it mirrors the utility of funding just one of them. Consequently, it doesn't make sense to allocate resources to both channels because doing so would result in redundant content with no additional utility benefit.

6.1.2. Perfect substitutes : Utility function

As we said, the utility function for each contributor i when funding either of the two projects, Project A and B, is based on the idea that the utility of having both projects funded is the same as having just one project funded. Therefore, it becomes:

$$V_i(F_A, F_B) = A_i \frac{1}{\gamma}\left(max(F_A - x^A, F_B - x^B)\right)^{\gamma}$$

The max function captures the notion that the utility is determined by the better-funded project. Since the projects are perfect substitutes, the utility depends on the highest funding level relative to the fixed cost.

6.1.3. Perfect substitutes : Total social welfare function

The total social welfare $W(F_A, F_B)$ for this case is then:

$$W(F_A, F_B) = \sum_i \left(A_i \frac{1}{\gamma}\left(max(F_A - x^A, F_B - x^B)\right)^{\gamma} - c_i^A - c_i^B - t\right)$$

6.1.4. Perfect substitutes : Optimization Problem

Given that it doesn't make sense to fund both projects, the social planner's objective is to maximize the total social welfare by choosing the optimal level of funding for only one of the projects, either F_A or F_B, while setting the other to zero:

$$max_{F_A, F_B} W(F_A, F_B) = \sum_i \left(A_i \frac{1}{\gamma} \left(max(F_A - x^A, F_B - x^B) \right)^\gamma - c_i^A - c_i^B - t \right)$$

Still subject to the budget-constraint such as:

$$F_A + F_B \leq t$$

However, the key point here is that F_A or F_B should be zero for the optimal allocation. So, the problem simplifies to choosing the project that will be funded and ensuring it receives the optimal funding.

6.1.5. Perfect substitutes : Simplified Problem

To decide whether to fund Project A or Project B in the simplified problem, we need to compare the potential social welfare from funding each project individually and choose the one that maximizes total social welfare. The decision process involves calculating the social welfare for both projects when funded separately and then selecting the project that provides the highest social welfare.

If you decide to fund Project A and not Project B ($F_B = 0$), the total social welfare $W(F_A, F_B)$ is:

$$W_A(F_A) = \sum_i \left(A_i \frac{1}{\gamma} (F_A - x^A)^\gamma - c_i^A - t \right)$$

Conversely, if you decide to fund Project B and not Project A, the total social welfare is:

$$W_B(F_B) = \sum_i \left(A_i \frac{1}{\gamma} (F_B - x^B)^\gamma - c_i^B - t \right)$$

Maximize Social Welfare for each project

The social planner should determine the optimal funding level F_A for project A and F_B for project B by maximizing their respective social welfare functions. This involves solving the first-order conditions for each project.

For project A:

$$\frac{\partial W}{\partial F_A} = \sum_i A_i \, \gamma \, (F_A - x^A)^{\gamma-1} - \frac{\partial \left(c_i^A \right)}{\partial (F^A)} = 0$$

This equation determines the optimal funding level F_A for project A.

For project B:

$$\frac{\partial W}{\partial F_B} = \sum_i A_i \, \gamma \, (F_B - x^B)^{\gamma-1} - \frac{\partial \left(c_i^B \right)}{\partial (F^B)} = 0$$

This equation determines the optimal funding level F_B for project B.

Compare the Social Welfare outcomes

After solving the first-order conditions, we have the optimal social welfare values $W_A({F_A}^*)$and $W_B({F_B}^*)$ corresponding to the optimal funding levels ${F_A}^*$ and ${F_B}^*$ for projects A and B.

Decision rule

For example, if $W_A({F_A}^*) > W_B({F_B}^*)$, meaning that the optimal social welfare is larger for project A than for project B, then the social planner will decide to fund project A and not to fund project B $(F_B = 0)$

This decision rule ensures that the project that maximizes overall social welfare is funded.

Simplified problem

Here, let's assume we choose to fund Project A, the optimization problem becomes:

$$max_{F_A} W(F_A) = \sum_i \left(A_i \frac{1}{\gamma} (F_A - x^A)^\gamma - c_i^A - t \right)$$

subject to:

$$F_A \le t$$

And set $F_B = 0$.

The first-order condition for this optimization problem becomes:

$$\frac{\partial W}{\partial F_A} = \sum_i A_i \, \gamma \, (F_A - x^A)^{\gamma - 1} - 1 = 0$$

6.1.6. Implications

The solution indicates that the social planner should fully fund only one project (either F_A or F_B), while setting the funding for the other project to zero. This approach avoids redundancy and ensures that resources are not wasted on duplicate content.

The chosen project should be the one that can achieve the highest utility given the available matching funds and its fixed costs. If Project A is funded, F_A should be set at the optimal level that maximizes $W(F_A)$, while F_B receives no funding.

Also, funding both projects would not increase social welfare because the utility derived from the second project is identical to that from the first. Therefore, the social planner must recognize that doubling the funding across two identical projects does not yield twice the utility, it yields the same utility as funding just one project.

6.1.7. Quadratic Funding in the case of perfect substitutes projects

Now, let's analyse the allocation of resources between two projects in a scenario where they are perfect substitutes and a standard Quadratic Funding (QF) mechanism is in place. Again, we'll consider that contributors have the same valuation for both projects but that this valuation differs across individuals.

Utility function under Quadratic Funding in the case of perfect substitutes projects

Again, let's take the scenario in which each individual contributor i has the same parameter A_i for both projects, but this parameter A_i can differ across individuals, the utility functions remain as:

$$V_i(F_A, F_B) = A_i \frac{1}{\gamma} \left(max(F_A - x^A, F_B - x^B) \right)^\gamma$$

Again, A_i represents contributor i's valuation of the projects, and F_A and F_B are the funding levels of Projects A and B, respectively.

x^A and x^B are the fixed costs for each project. The utility function depends on the maximum of the two funding levels since the projects are perfect substitutes.

Quadratic Funding mechanism

In a standard Quadratic Funding mechanism, the matching funds provided by the QF mechanism are proportional to the square of the sum of the square roots of individual contributions, such as:

$$M^A = \left(\sum_i \sqrt{c_i^A} \right)^2$$

$$M^B = \left(\sum_i \sqrt{c_i^B} \right)^2$$

M^A and M^B being the matching funds provided through the QF mechanism.

Therefore, the total funding for each project, including matching funds, is:

$$F^A = M^A$$

$$F^B = M^B$$

Interaction between the two projects

Since the two projects are perfect substitutes, the contributors and the QF mechanism must distribute funds between them. However, because the utility derived from funding both projects is identical to the utility from funding just one, the QF mechanism will naturally drive the allocation toward favoring one project over the other, depending on the distribution of contributions.

Allocations at equilibrium

At equilibrium, the QF mechanism will likely result in one project being fully funded while the other is essentially neglected. The project with the higher initial contributions will attract more matching funds, eventually receiving the majority of the available resources.

We can think of this equilibrium as a self-reinforcing cycle because as more contributions are directed toward one project, the matching mechanism amplifies these contributions, further increasing the funding gap between the two projects. Contributors then rationally decide to contribute only to the better-funded project, further widening the gap.

Implications

This has several implications. First, the QF mechanism's principle of amplifying the more popular project leads to a concentration of resources on one project. This is consistent with the idea that funding both projects is redundant since they are perfect substitutes.

If we think from the point of view of efficiency, the QF mechanism effectively prevents resources from being wasted on redundant projects because by concentrating funding on one project, QF ensures that the matching funds are used in a way that maximizes collective utility, since the utility function is non-additive.

Thus, in this context, QF leads to an all-or-nothing outcome, where one project secures all the funding, while the other receives none. This is scenario is rather desirable considering the case of perfect substitutes projects, as it avoids duplication of efforts and ensures that resources are used effectively. Let's also keep in mind that the equilibrium project that gets funded is likely to be determined by initial contributions.

6.2. The case of perfect complements

6.2.1. The model

Now, another possible scenario is two Youtube channels that are perfect complements. For example, channel A teaches the basics of managing personal finances, including budgeting, saving, debt management, and financial planning. The other, channel B, focuses on applying personal finance principles to investing, covering topics like stock market investing, retirement planning, and portfolio management. In this case, the foundational knowledge from channel A is necessary to make informed investment decisions as taught in channel B, meaning that the utility derived from one project is contingent on the success of the other. If only one project is funded, the utility derived from it is minimal or zero because the full value is only realized when both projects are funded together.

6.2.2. Perfect complements : Utility function

Given that the two projects are perfect complements, the utility that each contributor i derives depends on the combined funding of both projects. The utility function for a contributor i when both projects A and B are funded can be modeled as:

$$V_i(F_A, F_B) = A_i \frac{1}{\gamma}\left(min(F_A - x^A, F_B - x^B)\right)^{\gamma}$$

The min function captures the idea that the utility is solely determined by the lower of the two funding levels (relative to the fixed costs), which reflects the fact that both projects are complements. If one project is underfunded, the overall utility is limited by the underfunded project.

6.2.3. Perfect complements : Total social welfare function

The total social welfare W(FA,FB)W(F_A, F_B)W(FA,FB) derived from funding both projects is:

$$W(F_A, F_B) = \sum_i \left(A_i \frac{1}{\gamma} \left(min(F_A - x^A, F_B - x^B)\right)^{\gamma} - c_i^A - c_i^B - t \right)$$

6.2.4. Perfect complements : Optimal funding strategy

Given that the projects are perfect complements, the social planner's objective is to maximize the total social welfare by ensuring that both projects are funded at similar levels, ideally ensuring that both projects meet or exceed their fixed costs:

$$max_{F_A, F_B} W(F_A, F_B) = \sum_i \left(A_i \frac{1}{\gamma} \left(min(F_A - x^A, F_B - x^B)\right)^{\gamma} - c_i^A - c_i^B - t \right)$$

Still subject to budget-constraint and non-negative funding constraint, such as:

$$F_A + F_B \leq t$$

$$F_A \geq x^A, \quad F_B \geq x^B$$

6.2.5. Perfect complements : First-Order Conditions and Equilibrium

The first-order conditions now reflect the need to balance funding between the two projects because their utility is interdependent. The optimal funding levels F_A and F_B should be equal, ensuring that neither project is underfunded, as underfunding one would severely limit the overall utility.

Therefore, for F_A , we have:

$$\frac{\partial W}{\partial F_A} = \sum_i \left(A_i \, \gamma \left(min(F_A - x^A, F_B - x^B)\right)^{\gamma - 1}. \; I(F_A \leq F_B) - \frac{\partial (c_i^A)}{\partial (F_A)} \right) = \lambda$$

For F_B, we also have:

$$\frac{\partial W}{\partial F_B} = \sum_i \left(A_i \, \gamma \left(min(F_A - x^A, F_B - x^B)\right)^{\gamma - 1}. \; I(F_B \leq F_A) - \frac{\partial (c_i^B)}{\partial (F_B)} \right) = \lambda$$

The functions $I(F_A \leq F_B)$ and $I(F_B \leq F_A)$ are such that:

$$I(F_A \leq F_B) = \begin{cases} 1 & if \; F_A \leq F_B \\ 0 & if \; F_A > F_B \end{cases}$$

$$I(F_B \leq F_A) = \begin{cases} 1 & if \; F_B \leq F_A \\ 0 & if \; F_B > F_A \end{cases}$$

For example, if $F_A \leq F_B$, then $I(F_A \leq F_B) = 1$ and $I(F_B > F_A) = 0$, which means that the utility will primarily depend on F_A, the lower-funded project and that the planner should allocate additional resources to project A until it matches the funding of project B. This ensures that the utility derived from the two projects depends on the project that is underfunded relative to the other.

6.2.6. Implications

Given that the projects are perfect complements, the optimal strategy for contributors is to contribute equally to both projects. This ensures that both projects are funded sufficiently to deliver the combined utility. To achieve this optimal outcome, a central authority (such as the government) may need to intervene by bundling the projects together and ensuring that they are funded equally. This intervention prevents the scenario where one project receives disproportionate contributions, which would lead to suboptimal utility.

If one project receives significantly more funding than the other, the overall utility will be constrained by the underfunded project. Thus, in equilibrium, the optimal contribution from each contributor would be the same for both projects to ensure balanced funding. If one project starts to receive more funding, contributors should shift their contributions to the underfunded project to maximize overall utility. If contributors fail to coordinate and one project receives less funding, the equilibrium could be suboptimal, with reduced overall social welfare. This possibility shows the importance of mechanisms that encourage balanced funding.

6.2.7. Application

Let's use a simple numeric example to better illustrate the need for balanced funding. Let's assume that the fixed costs for projects A and B are the same and equal 20: $x^A = x^B = 20$. Also, let's assume that $A_i = 10$. Same as before, to keep the assumption of convex utility functions, we set the parameter γ as equal to $\frac{1}{2}$.

Given these values, the utility function for each contributor is:

$$V_i(F_A, F_B) = 20 \times \sqrt{min(F_A - 20, F_B - 20)}$$

Let's evaluate the total social welfare for different funding scenarios:

First, let's consider he scenario of balanced funding, which means that the total the total amount of matching funds in the pool $D = 100$ is equally split between the two projects A and B, such as $F_A = F_B = 50$. In this scenario the total social welfare is:

$$W(F_A, F_B) = 20 \times \sqrt{min(50 - 20)} = 20 \times \sqrt{30} \approx 109.4$$

Net utility, which means minus costs and tax, for each contributor would then be: $109.4 - 10 - 2 = 97.4$

Now, let's consider a scenario of imbalanced funding, for example $F_A = 70$ and $F_B = 30$. In this case, the total social welfare becomes:

$$W(F_A, F_B) = 20 \times \sqrt{min(70 - 20, 30 - 20)} = 20 \times \sqrt{10} \approx 63.2$$

Net utility, as well as the total social welfare, would be much lower in this case: $63.2 - 10 - 2 = 51.2$

Add some little discussion on how this could be implemented. Some agency that would pay subsidy for content providers, or if some billionaires want to make donations to charities for content providers, what would you recommend to them ? To avoid that two people are doing exactly the same thing, duplicative spending.

6.2.8. Quadratic Funding in the case of perfect complements projects

Again, let's analyse the allocation of resources between two projects in a scenario where they are perfect complements and a standard Quadratic Funding (QF) mechanism is in place. We'll focus on how contributions and matching funds would be allocated by the QF mechanism, given that the utility derived from one project depends on the funding of the other.

Utility function under Quadratic Funding in the case of perfect substitutes projects

As before, the utility for each contributor i for the two projects A and B is given by:

$$V_i(F_A, F_B) = A_i \frac{1}{\gamma}\left(min(F_A - x^A, F_B - x^B)\right)^\gamma$$

Utility with QF and Perfect Complements

Given the QF mechanism, the utility that each contributor i derives from the two projects considering that they are perfect complements is:

$$V_i(F_A, F_B) = A_i \frac{1}{\gamma}\left(min\left(\left(\sqrt{c_1^A} + \sqrt{c_2^A}\right)^2 - x^A, \left(\sqrt{c_1^B} + \sqrt{c_2^B}\right)^2 - x^B\right)\right)^\gamma$$

Equilibrium contributions under QF

Let's make the assumption that contributors do not contribute symmetrically. Suppose for example that contributor 1 prefers project A and contributes more to it, this means:

$$c_1^A > c_2^A$$

$$c_1^B < c_2^B$$

With this, we have the total funding for each project such as:

$$F_A = \left(\sqrt{c_1^A} + \sqrt{c_2^A}\right)^2$$

$$F_B = \left(\sqrt{c_1^B} + \sqrt{c_2^B}\right)^2$$

Given this asymmetry, the utility for each contributor is now limited by the lower of the two funding levels. Let's assume that $F_A > F_B$:

$$V_i(F_A, F_B) = A_i \frac{1}{\gamma}\left(\left(\left(\sqrt{c_1^B} + \sqrt{c_2^B}\right)^2 - x^B\right)\right)^\gamma$$

Here, the utility is limited by the lower funded project F_B, which reflects the fact that the utility of the complementary projects is constrained by the weaker one.

Implications

Since the utility is determined by the minimum of the two funding levels, contributors have an incentive to shift their contributions to the underfunded project to maximize their utility, as follows:

If contributor 1 sees that project B is underfunded relative to project A, he might reduce his contribution to project A and increase his contribution to project B to boost overall utility.

Similarly, contributor 2 might adjust his contributions if he notices an inequality, aiming to ensure that both projects are adequately funded.

If contributors do not adjust their contributions and the imbalance persists, the utility for all contributors remains suboptimal because the lower funded project limits the overall utility. This creates an incentive toward the equilibrium situation, where contributors progressively shift their contributions to balance the funding levels.

Therefore, we can conclude that under a standard Quadratic Funding mechanism with perfect complements, asymmetric contributions lead to a suboptimal outcome where the utility derived from the projects is limited by the underfunded project. However, the QF mechanism, by amplifying contributions, encourages contributors to recognize this complementarity and adjust their contributions to balance the funding levels between the projects. In equilibrium, contributors will balance their contributions to ensure that both projects are sufficiently funded to maximize the combined utility. This balanced funding is critical because with perfect complements the overall utility is determined by the project which receives less funding.

7. Conclusion

This thesis has provided an in-depth analysis of the Quadratic Funding (QF) mechanism, exploring its theoretical foundations, practical applications, and limitations. By examining both the basic principles of QF and extending the model to incorporate more complex scenarios, such as budget constraints, collusion, and strategic behavior, this work offers a comprehensive understanding of how QF can be used to fund public goods in a more efficient and equitable manner.

Quadratic Funding stands out for its ability to amplify the impact of small contributions, thus addressing the free-rider problem that often plagues traditional public goods funding mechanisms. By matching individual contributions according to a quadratic formula, QF ensures that the total funding for a project better reflects the collective interest of the community, thereby encouraging widespread participation. Under ideal conditions, QF can achieve an optimal provision of public goods, where the total marginal value derived from the good equals the cost of providing it.

However, I have also highlighted the challenges of implementing QF in real-world settings. One significant issue is the potential for underfunding when contributors take into account their impact on a budget deficit. The analysis shows that when contributors consider the financial implications of their contributions, slight underfunding occurs, as individuals adjust their behavior to minimize their own financial burden. This issue is further complicated by the introduction of negative contributions, which allow individuals to defund projects they perceive as harmful. While negative contributions can theoretically lead to optimal funding levels, they also risk strategic defunding and a reduction in the overall effectiveness of the QF mechanism.

To address these challenges, I have introduced the Capital-constrained Quadratic Funding (CQF) mechanism, proposed by BHW, which adjusts the QF formula to operate within a fixed budget by combining QF with private contributions. This variant ensures that the total funding does not exceed the available budget while maintaining the key advantages of QF, such as encouraging broad participation and optimizing public goods provision. In large populations, where individual contributions are small relative to the total, CQF minimizes the impact of budget constraints, thereby reducing the extent of underfunding.

Moreover, practical considerations are central to the successful implementation of QF, particularly in digital and decentralized contexts. A user-friendly interface that promotes transparency and encourages collaboration among contributors is essential for fostering widespread participation. Additionally, the risks of collusion and strategic manipulation must be mitigated to preserve the fairness and efficiency of the QF mechanism. While the quadratic cost structure of QF is more resilient than traditional mechanisms like the Vickrey-Clarke-Groves (VCG) mechanism, it still requires robust safeguards to prevent abuse.

I also have introduced the concept of ± Quadratic Funding (±QF), which allows for negative contributions. This extension theoretically enables optimal funding levels by accommodating both supporters and detractors of a public good. However, it also introduces significant practical challenges, including the potential for strategic defunding and the complexity of implementing such a system in practice. Ensuring that projects are not arbitrarily defunded due to malicious activities will be crucial to maintaining the integrity of the funding process.

I developed a theoretical model that analyses the optimal allocation of funds between two projects, with the simplest assumptions: two completely independent projects and two contributors. This model revealed that the optimal funding levels could be determined by equating the marginal social welfare gains with the marginal costs. The analysis suggested that depending on individual contributors' preferences and utility functions, a social planner might choose to allocate funds equally between projects or favor one project over another. This finding is significant as it highlights the flexibility of the QF mechanism in adapting to different funding environments and the potential for optimizing resource allocation based on contributors' preferences.

I then explored scenarios where the two projects are either perfect substitutes or perfect complements. When projects are perfect substitutes, meaning they produce identical content, the model demonstrates that the QF mechanism naturally leads to an all-or-nothing outcome. In this scenario, one project receives all the funding, while the other receives none. This outcome is particularly desirable as it prevents redundancy and ensures that resources are used most effectively. The result underscores the efficiency of QF in scenarios where multiple projects provide similar benefits, allowing for a streamlined allocation of resources that avoids waste.

Conversely, in the case of perfect complements, where the utility of one project is dependent on the success of the other, the analysis shows that balanced funding is crucial. The model indicates that under a standard QF mechanism, if contributions are asymmetric, the resulting funding allocation is suboptimal, with the utility derived from both projects being limited by the underfunded one. This finding is particularly important as it highlights a potential weakness in the

QF mechanism when dealing with interdependent projects. In such cases, the social planner must ensure that both projects are sufficiently and equally funded to maximize overall utility.

The insights from Sections 5 and 6 show that while QF is highly effective in many scenarios, its application must be carefully managed in cases involving interdependent projects. The findings suggest that the standard QF mechanism may need to be adjusted or supplemented with additional rules or guidelines to ensure optimal outcomes in these more complex cases.

Furthermore, while QF is inherently designed to encourage broad participation and amplify the impact of small contributions, its effectiveness can vary significantly depending on the nature of the projects being funded. For instance, in the case of perfect substitutes, the all-or-nothing outcome is efficient and desirable. However, in the case of perfect complements, the standard QF mechanism might fall short, requiring a more nuanced approach to ensure that resources are allocated in a way that maximizes collective utility.

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Disclaimer : No content inside this paper was solely produced by any AI language model tool, all the content is either based on literature, or based on my intuition. However, AI has been used from time to time to rephrase sentences when it felt needed.

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