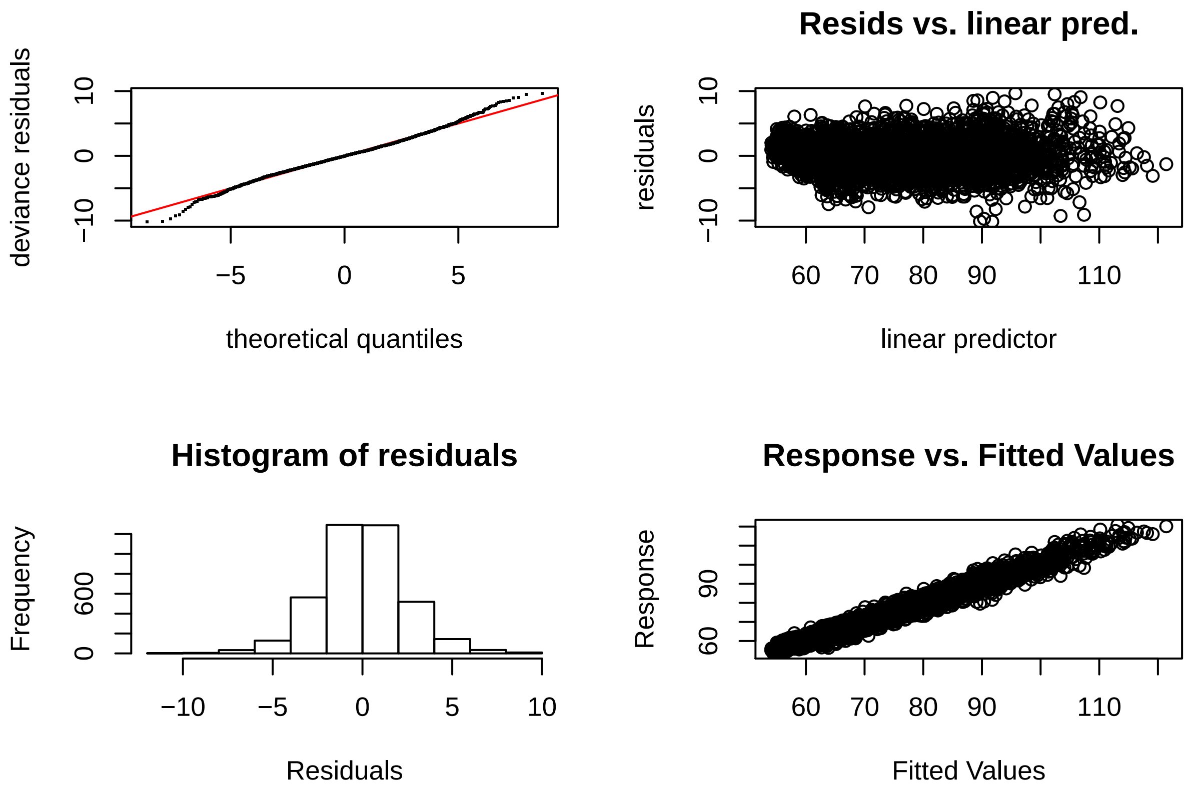
External appendix A

Elisa Ribesse

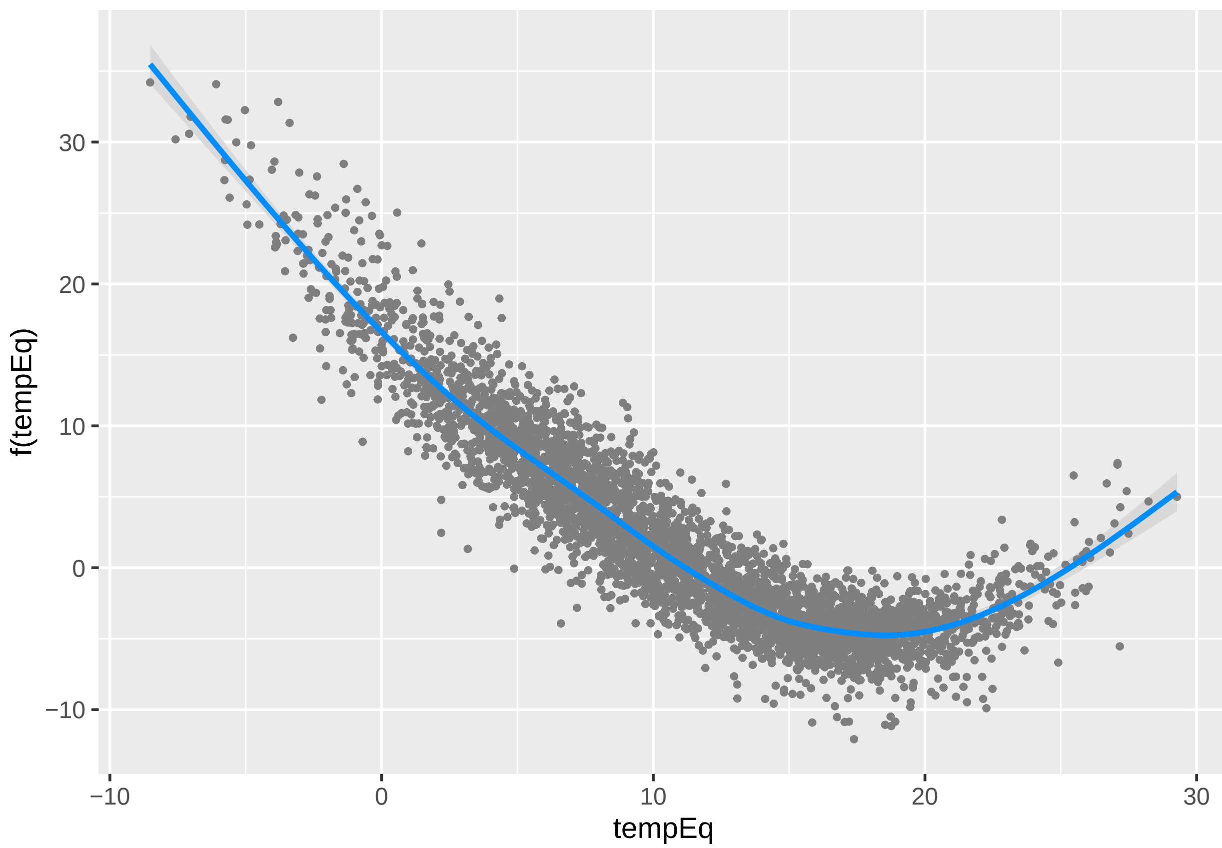
Looking for a good model for electricity:

Main effects

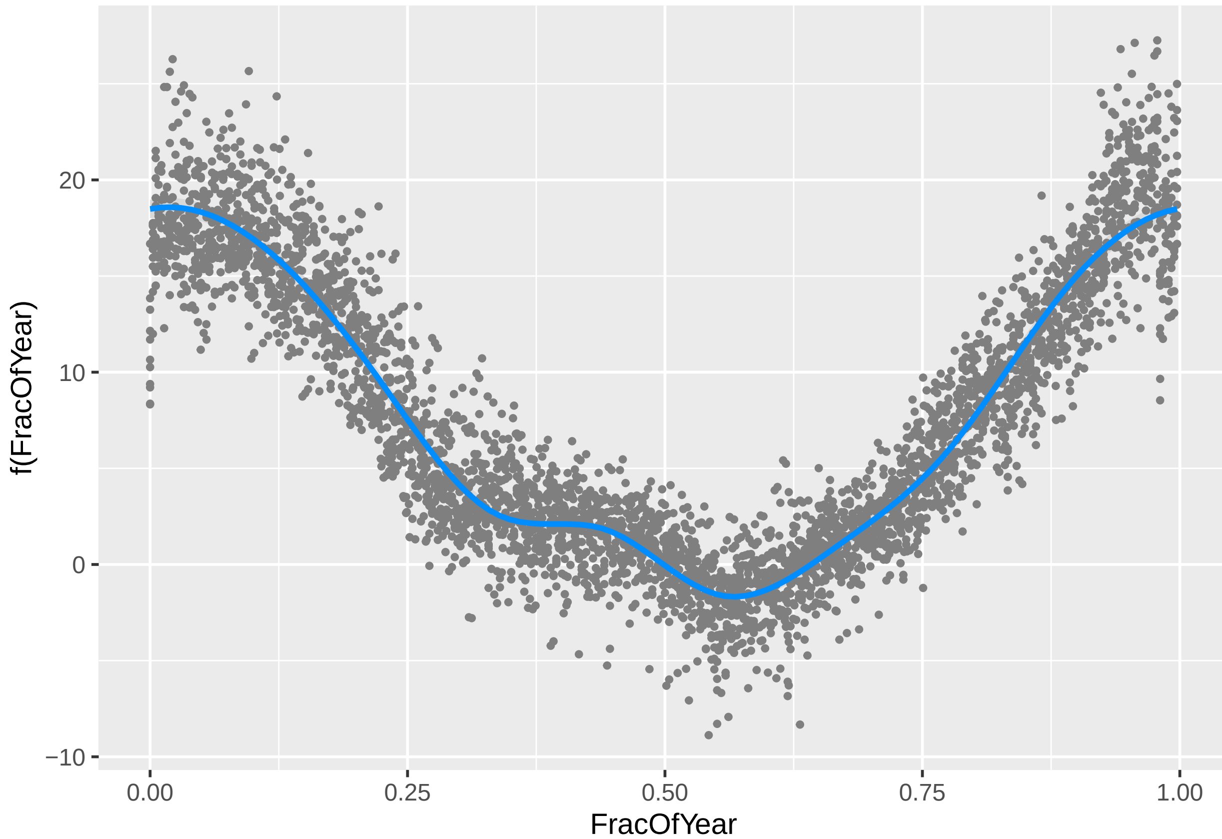
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc") + s(trend,
##     bs = "cr")
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 75.06196    0.03737    2009   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                 edf Ref.df      F p-value
## s(tempEq)     8.061  8.710 1196.7  <2e-16 ***
## s(FracOfYear) 7.925  8.000 1070.9  <2e-16 ***
## s(trend)      8.295  8.862  793.2  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =   0.97   Deviance explained = 97.1%
## -REML = 9218.3  Scale est. = 5.6085    n = 4017
```



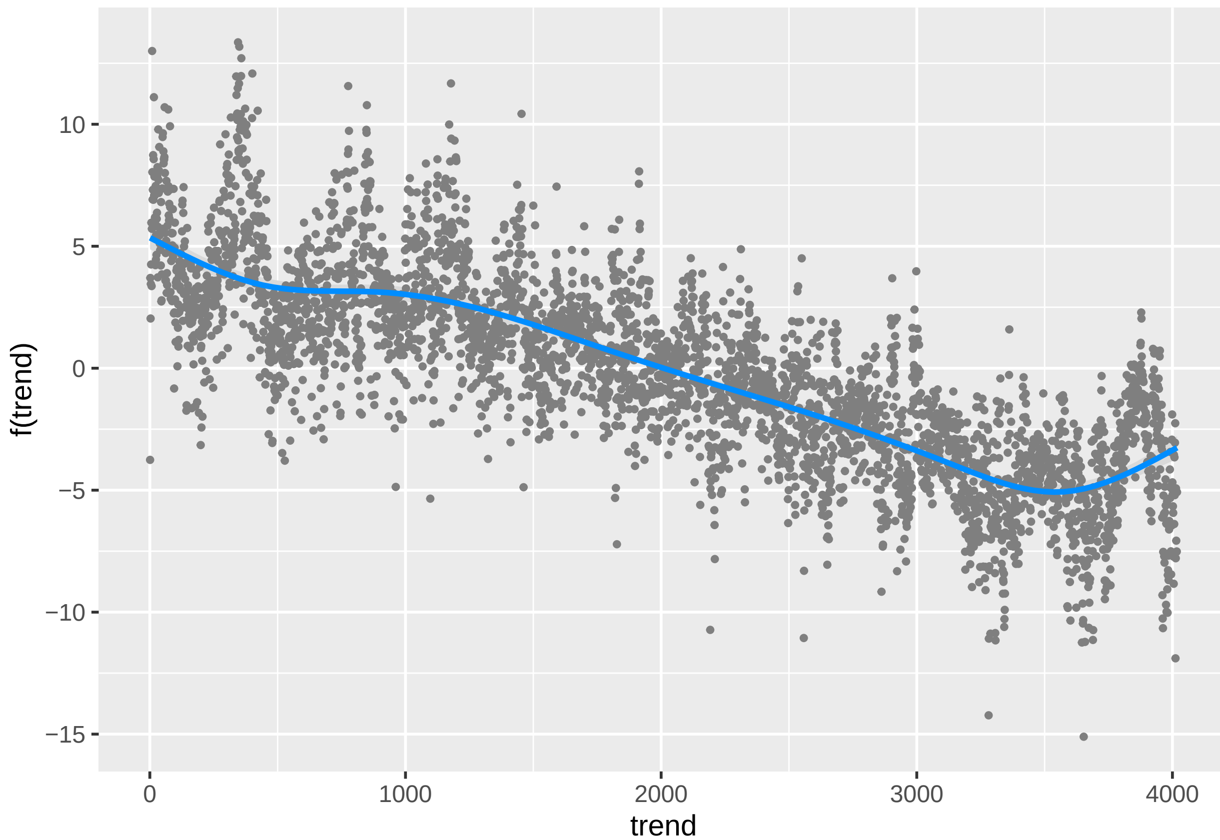

```
##
## Method: REML   Optimizer: outer newton
## full convergence after 6 iterations.
## Gradient range [-0.0001971743,0.0001897036]
## (score 9218.275 & scale 5.608459).
## Hessian positive definite, eigenvalue range [3.208156,2007.021].
## Model rank =  27 / 27
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                  k'  edf k-index p-value
## s(tempEq)      9.00 8.06    0.99    0.31
## s(FracOfYear) 8.00 7.93    0.95  <2e-16 ***
## s(trend)       9.00 8.30    0.37  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## [[1]]
```




```
##
## [[2]]
```

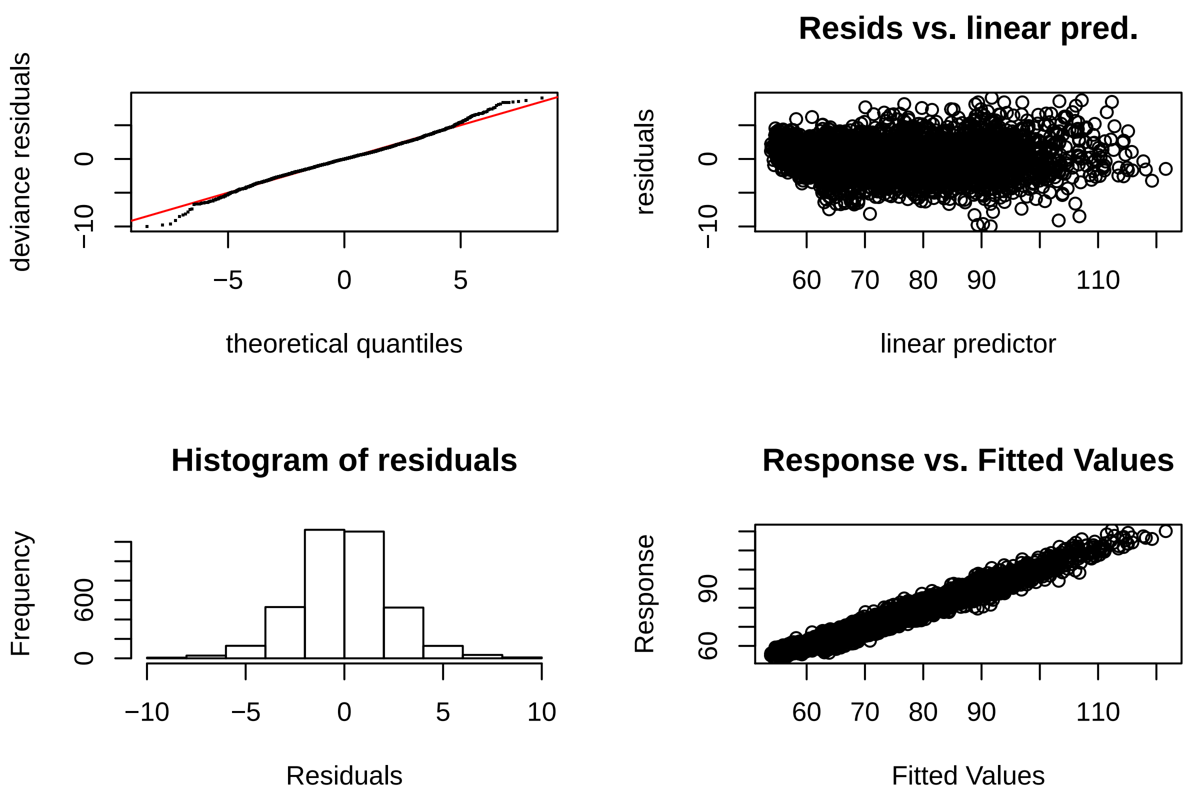



```
##
## [[3]]
```

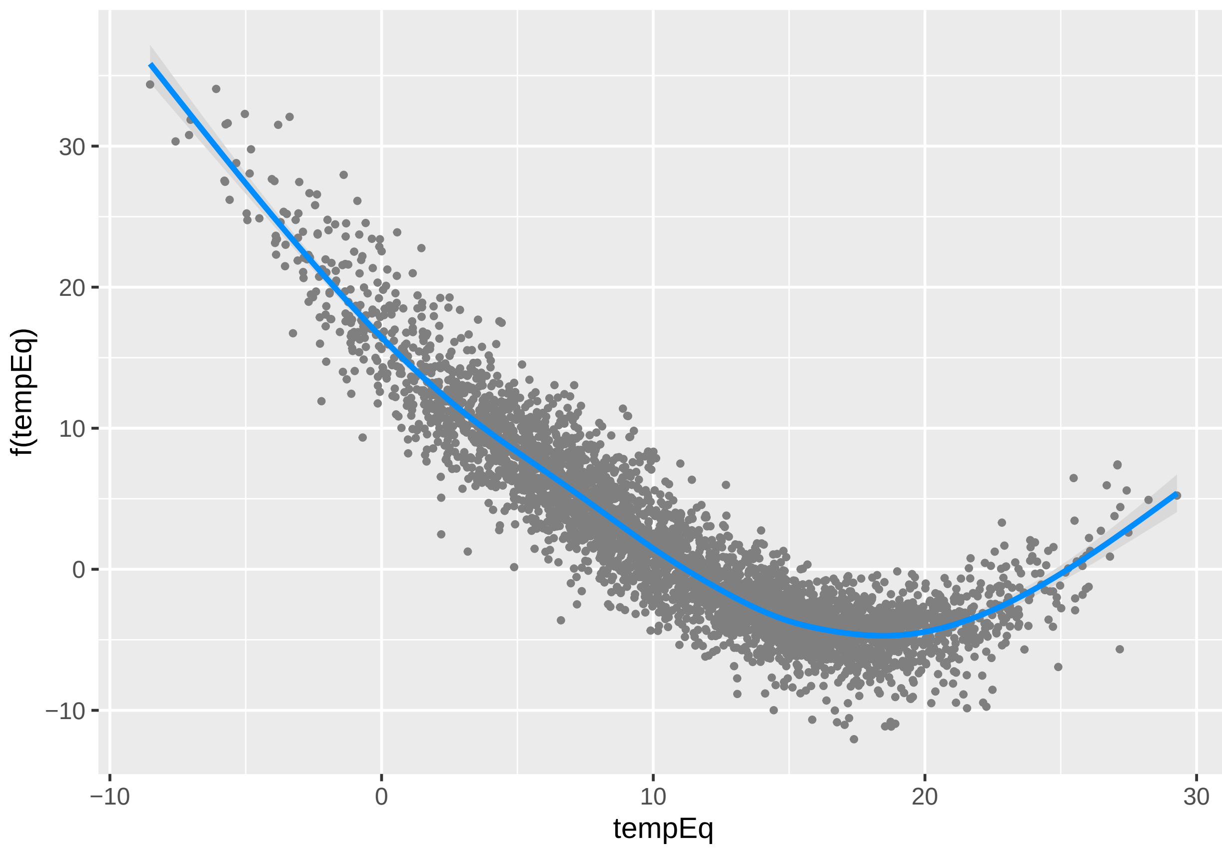


Gam check not relevant for "trend": we want a smooth curve not depending of the fracOfYear Pattern ###Test with higher k for fracOfYeaar

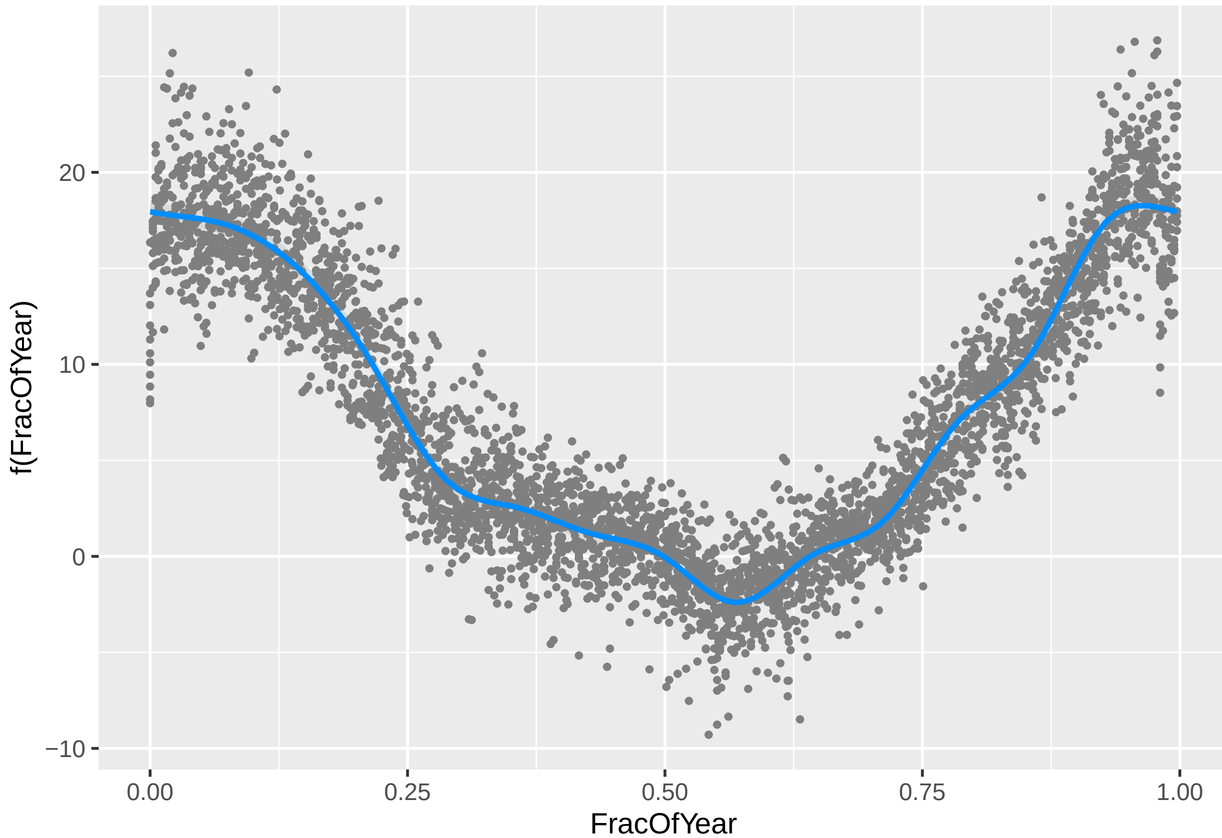
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 75.06196    0.03655    2054   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                   edf Ref.df      F p-value
## s(tempEq)       8.088  8.725 1228.3  <2e-16 ***
## s(FracOfYear) 12.716 13.000  709.7  <2e-16 ***
## s(trend)        5.893  5.995 1229.0  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.972   Deviance explained = 97.2%
## -REML = 9138.8  Scale est. = 5.3667    n = 4017
```



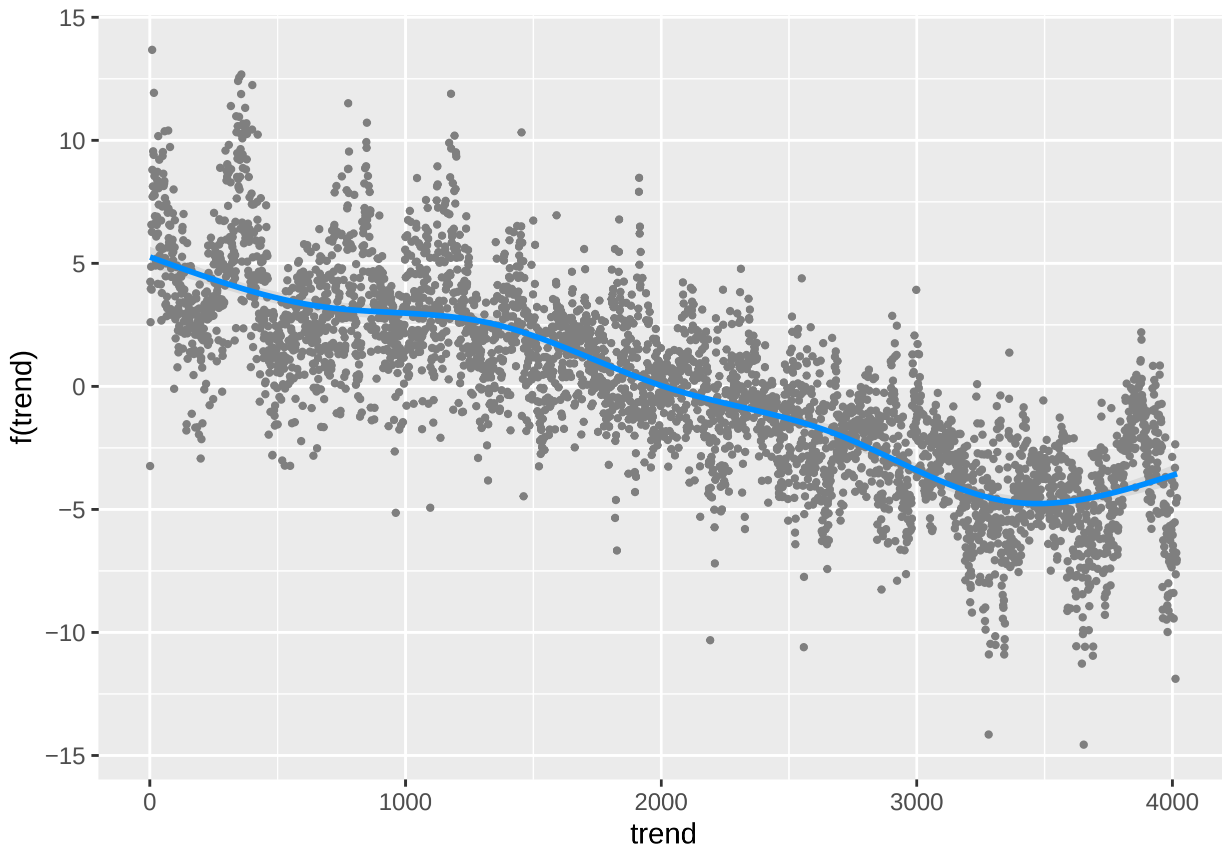

```
##
## Method: REML   Optimizer: outer newton
## full convergence after 5 iterations.
## Gradient range [-0.0002556855,0.0002072619]
## (score 9138.821 & scale 5.366747).
## Hessian positive definite, eigenvalue range [2.247641,2007.03].
## Model rank =  29 / 29
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                  k'   edf k-index p-value
## s(tempEq)       9.00  8.09    0.99    0.28
## s(FracOfYear) 13.00 12.72    1.00    0.52
## s(trend)        6.00  5.89    0.38  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## [[1]]
```




```
##
## [[2]]
```




```
##
## [[3]]
```

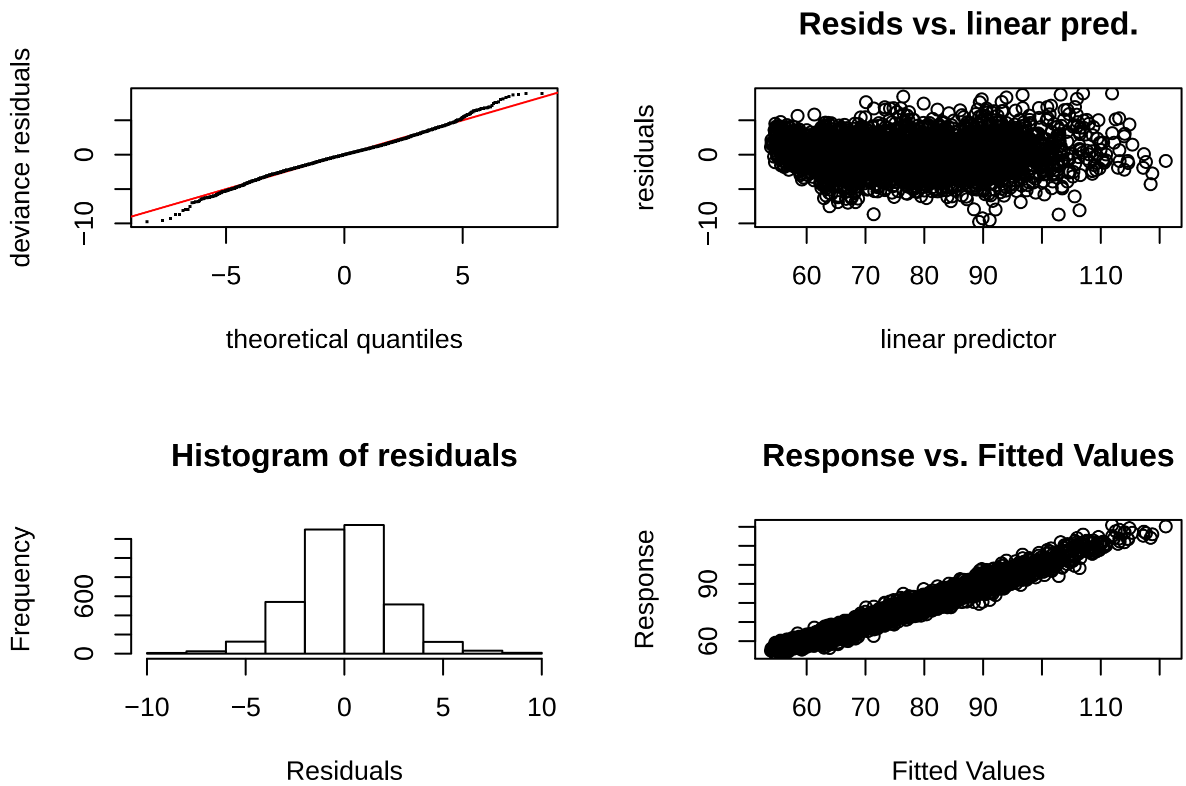


Interactions

Add (tempEq,FracOfYear)

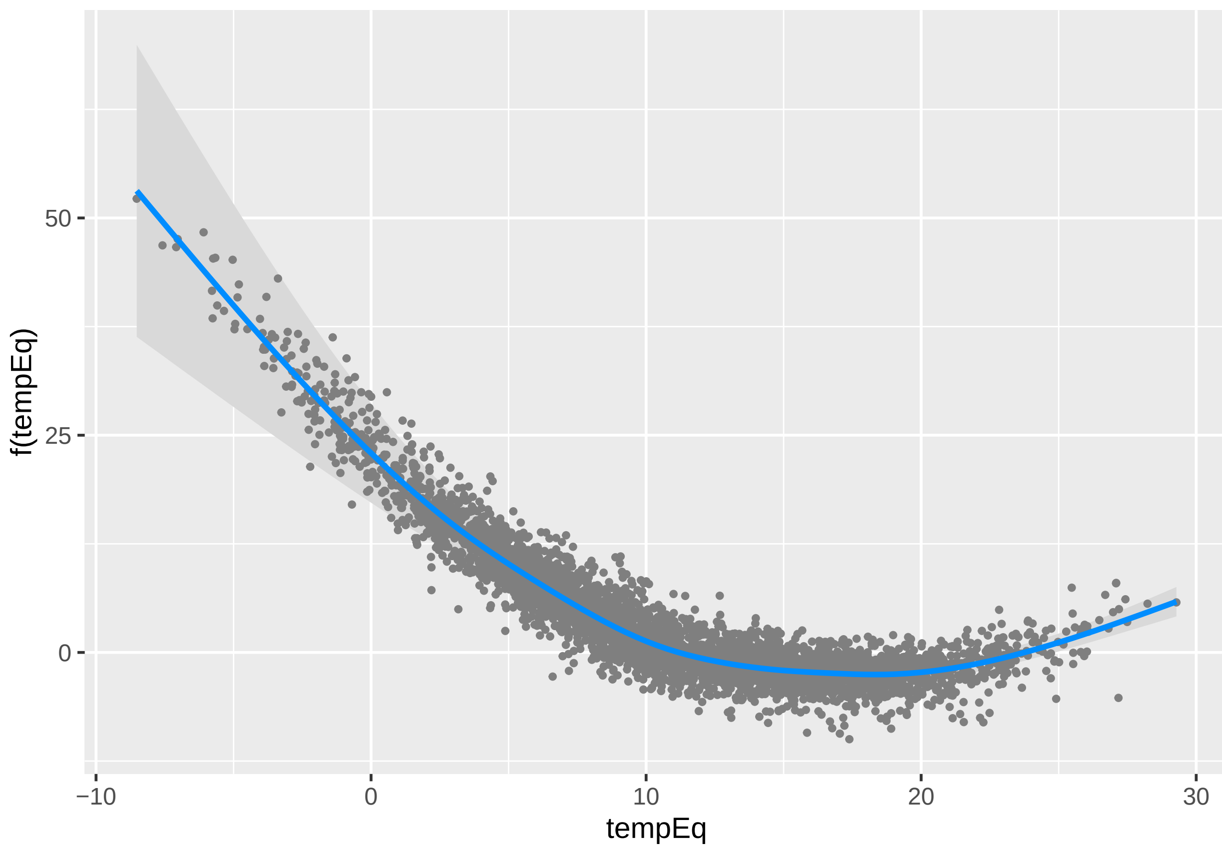
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.4653     0.2811   264.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                          edf Ref.df       F p-value
## s(tempEq)              7.639  8.464   99.29  <2e-16 ***
## s(FracOfYear)         12.693 13.000  136.61  <2e-16 ***
## s(trend)               5.893  5.995 1273.48  <2e-16 ***
## ti(tempEq,FracOfYear)  9.477 12.000   11.71  <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.973   Deviance explained = 97.3%
## -REML = 9086.1  Scale est. = 5.1859    n = 4017
```

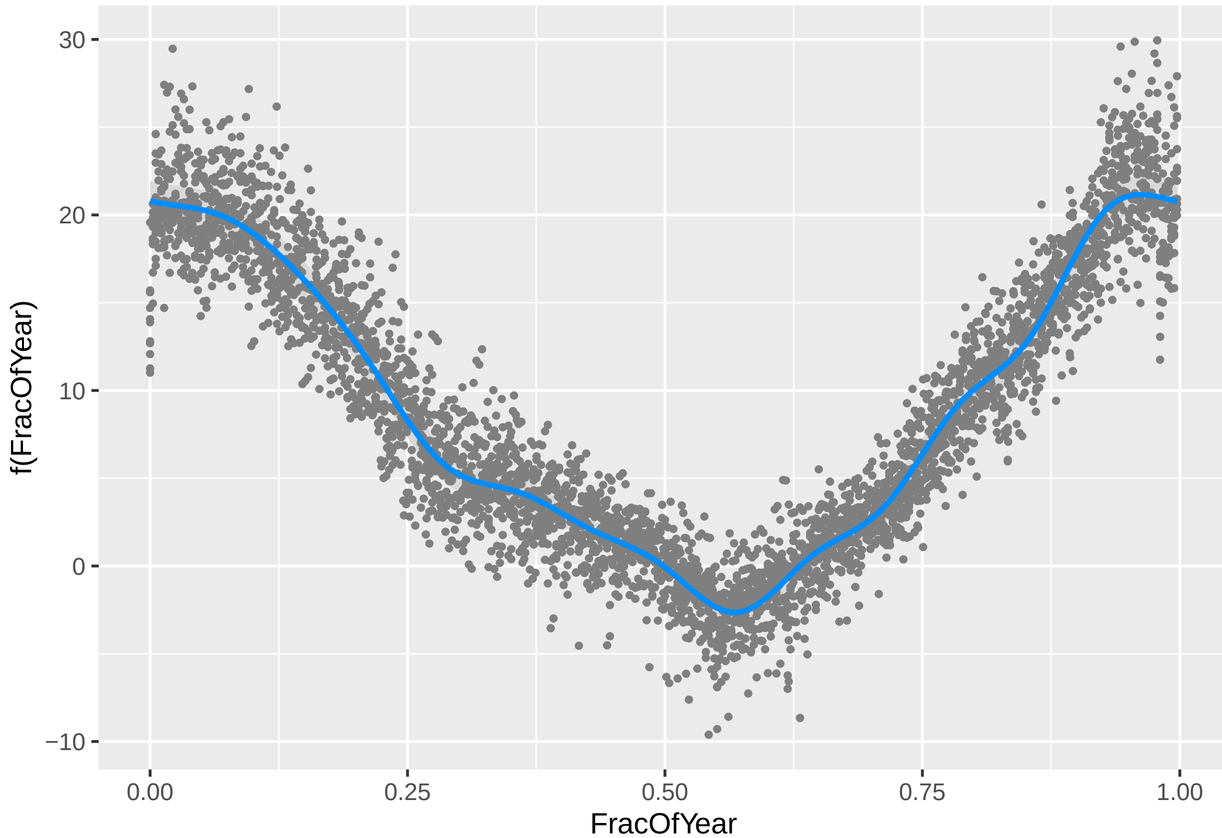



```
##
## Method: REML   Optimizer: outer newton
## full convergence after 7 iterations.
## Gradient range [-0.0002124876,9.23419e-05]
## (score 9086.057 & scale 5.185866).
## Hessian positive definite, eigenvalue range [0.6353694,2007.035].
## Model rank =  41 / 41
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                          k'   edf k-index p-value
## s(tempEq)              9.00  7.64    0.99    0.34
## s(FracOfYear)         13.00 12.69    1.00    0.34
## s(trend)               6.00  5.89    0.39  <2e-16 ***
## ti(tempEq,FracOfYear) 12.00  9.48    0.93  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

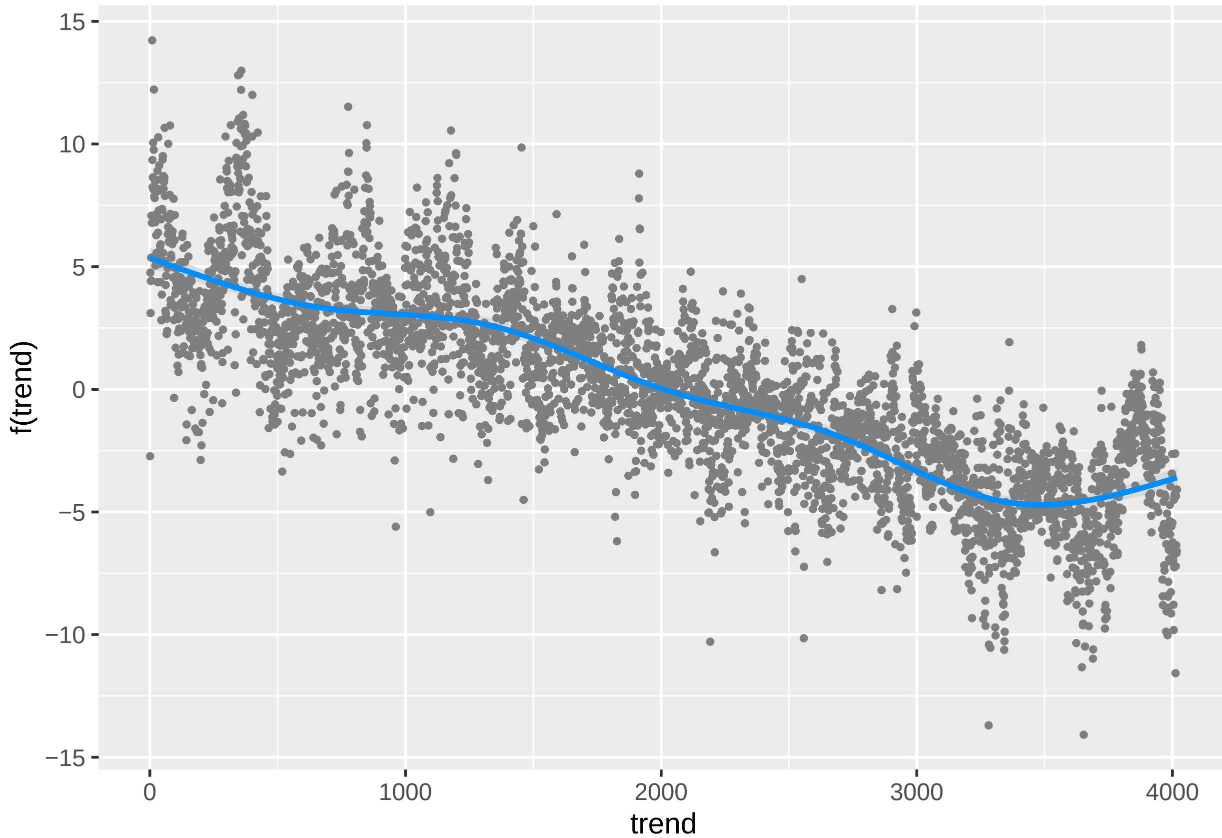
```
## [[1]]
```

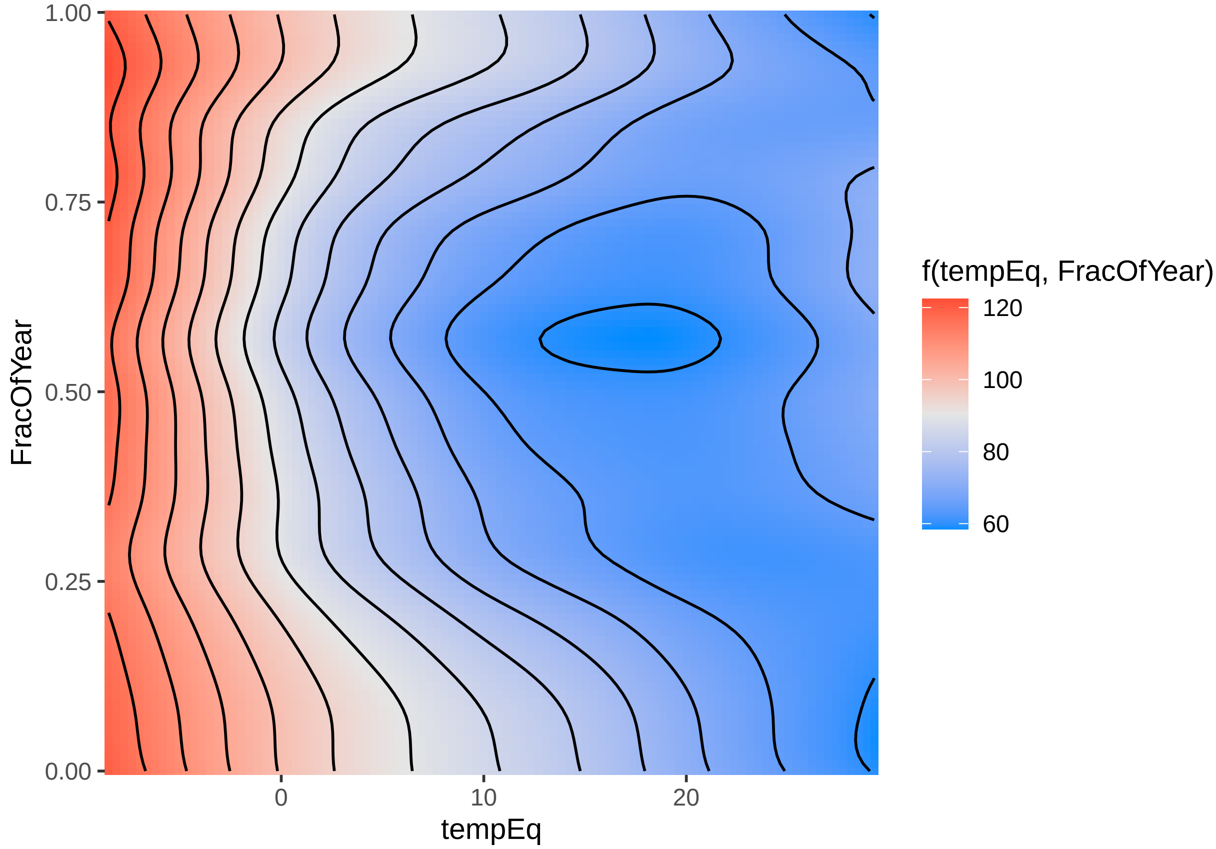



```
##
## [[2]]
```




```
##
## [[3]]
```

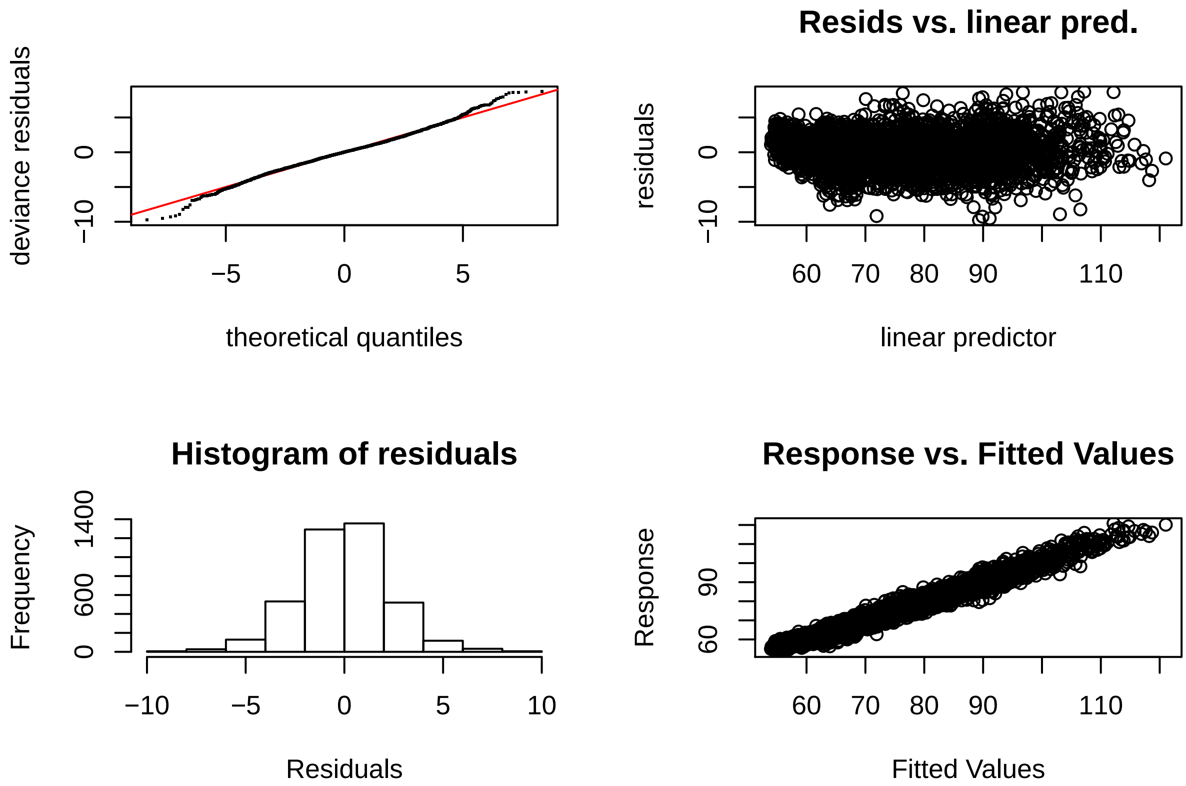

15
10
5
0
−5
−10
−15
f(trend)
0
1000
2000
3000
4000
trend



Higher k for the interaction

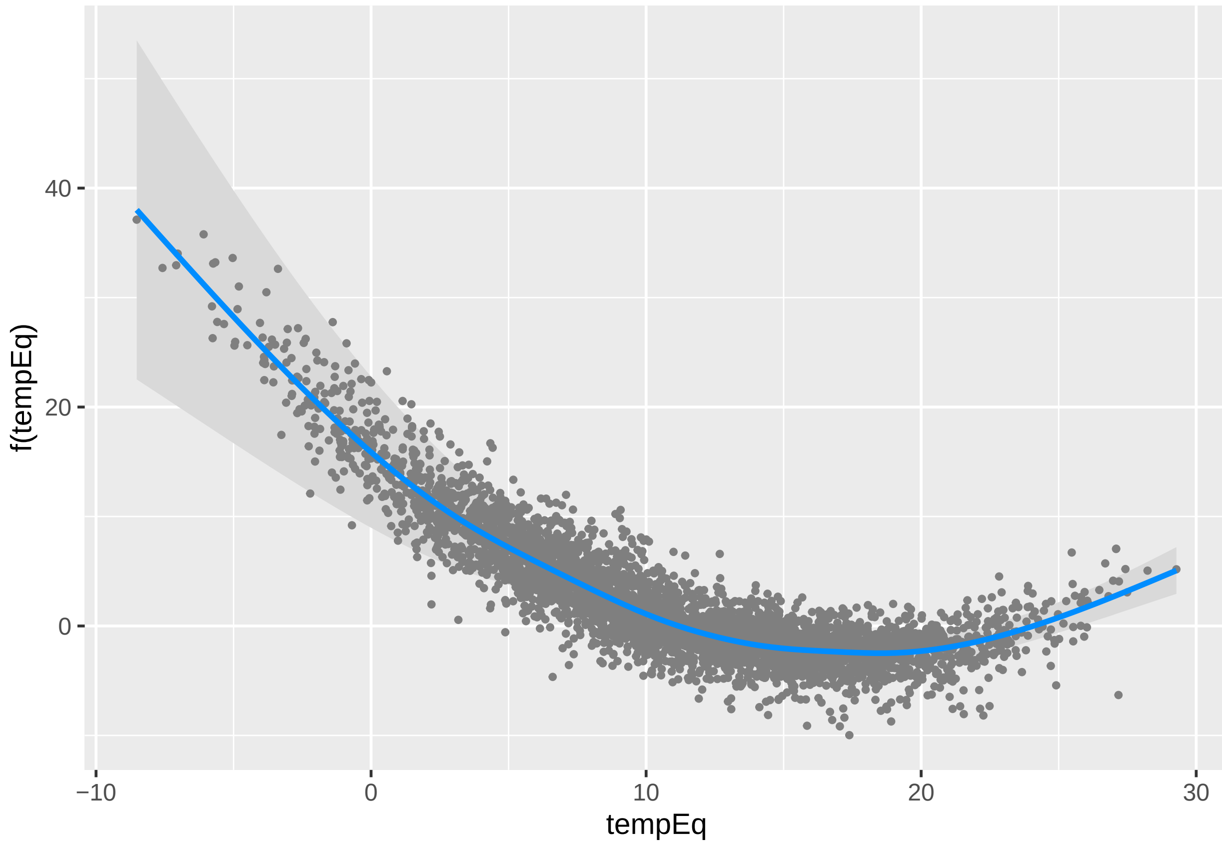
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.4426     0.3373   220.7   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                          edf Ref.df        F p-value
## s(tempEq)              7.619  8.370   47.126  <2e-16 ***
## s(FracOfYear)         12.626 13.000   78.473  <2e-16 ***
## s(trend)               5.889  5.994 1271.131  <2e-16 ***
## ti(tempEq,FracOfYear) 14.122 30.000    5.206  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## R-sq.(adj) =  0.973   Deviance explained = 97.3%
## -REML =   9082  Scale est. = 5.1656    n = 4017
```

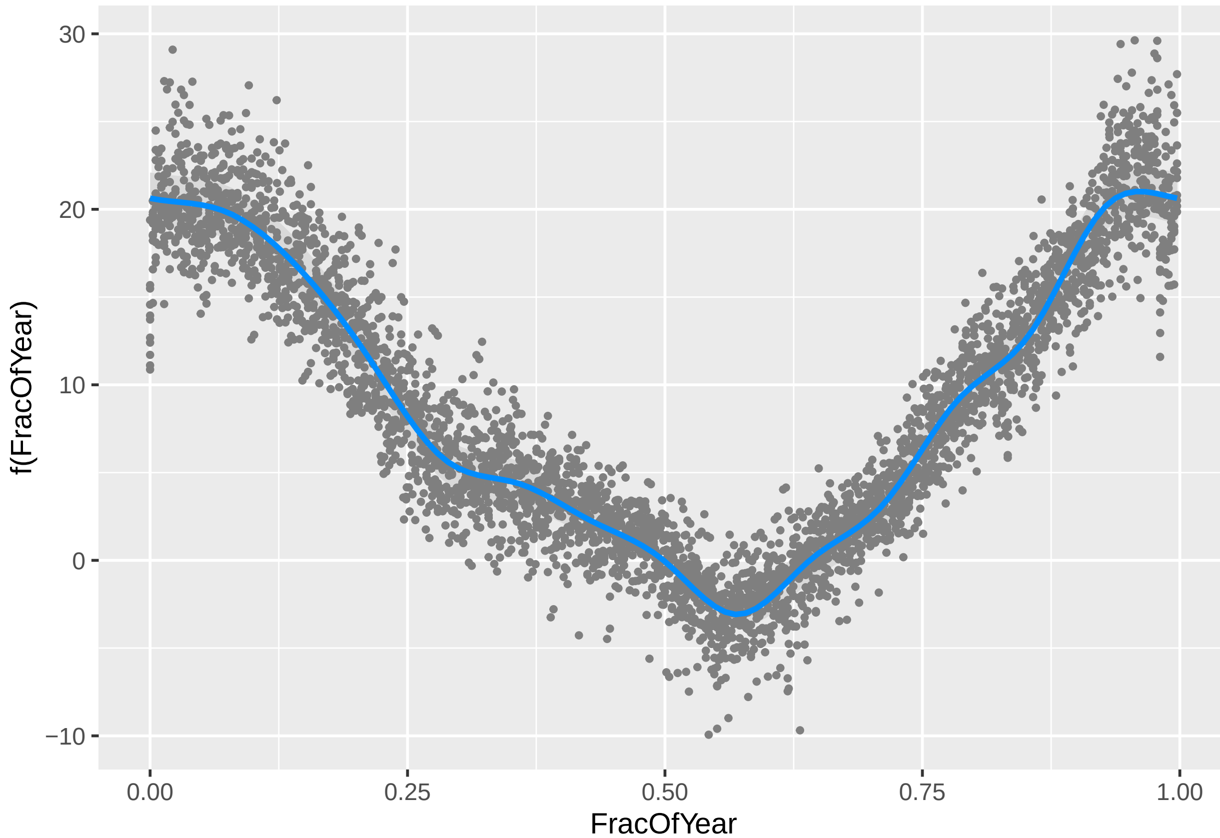



```
##
## Method: REML   Optimizer: outer newton
## full convergence after 6 iterations.
## Gradient range [-0.001683413,0.0006260171]
## (score 9081.96 & scale 5.165596).
## Hessian positive definite, eigenvalue range [0.7798269,2007.043].
## Model rank =  59 / 59
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                          k'   edf k-index p-value
## s(tempEq)              9.00  7.62    0.99    0.32
## s(FracOfYear)         13.00 12.63    1.00    0.40
## s(trend)               6.00  5.89    0.39  <2e-16 ***
## ti(tempEq,FracOfYear) 30.00 14.12    0.96  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

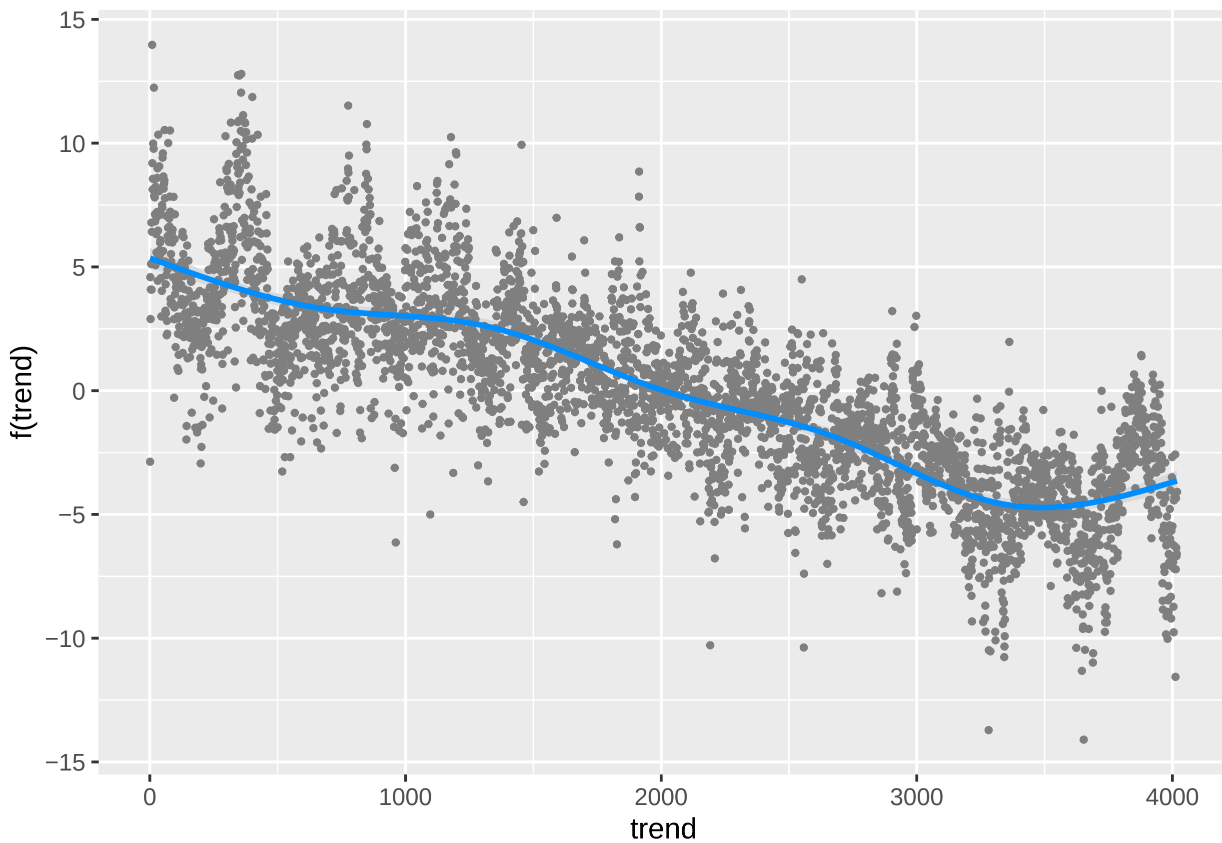
```
## [[1]]
```

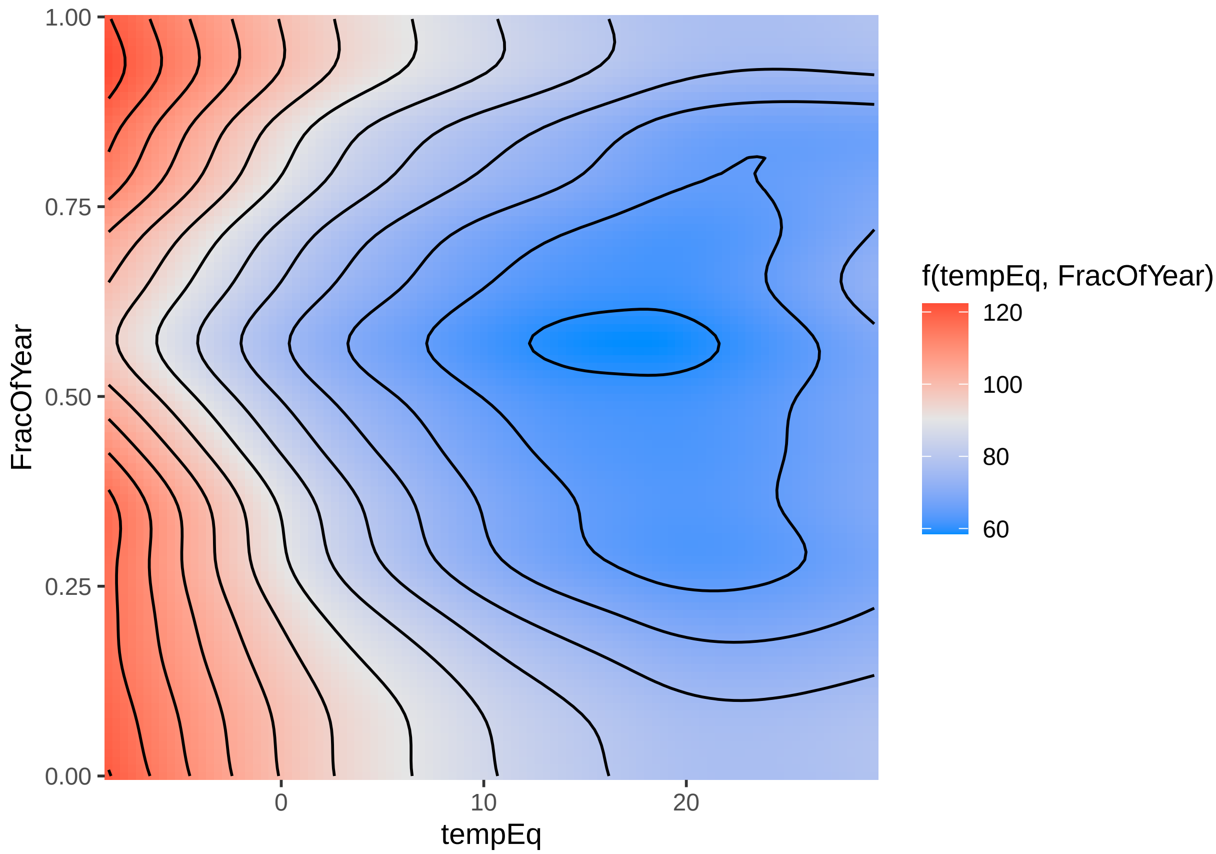



```
##
## [[2]]
```




```
##
## [[3]]
```


15
10
5
0
−5
−10
−15
f(trend)
0
1000
2000
3000
4000
trend

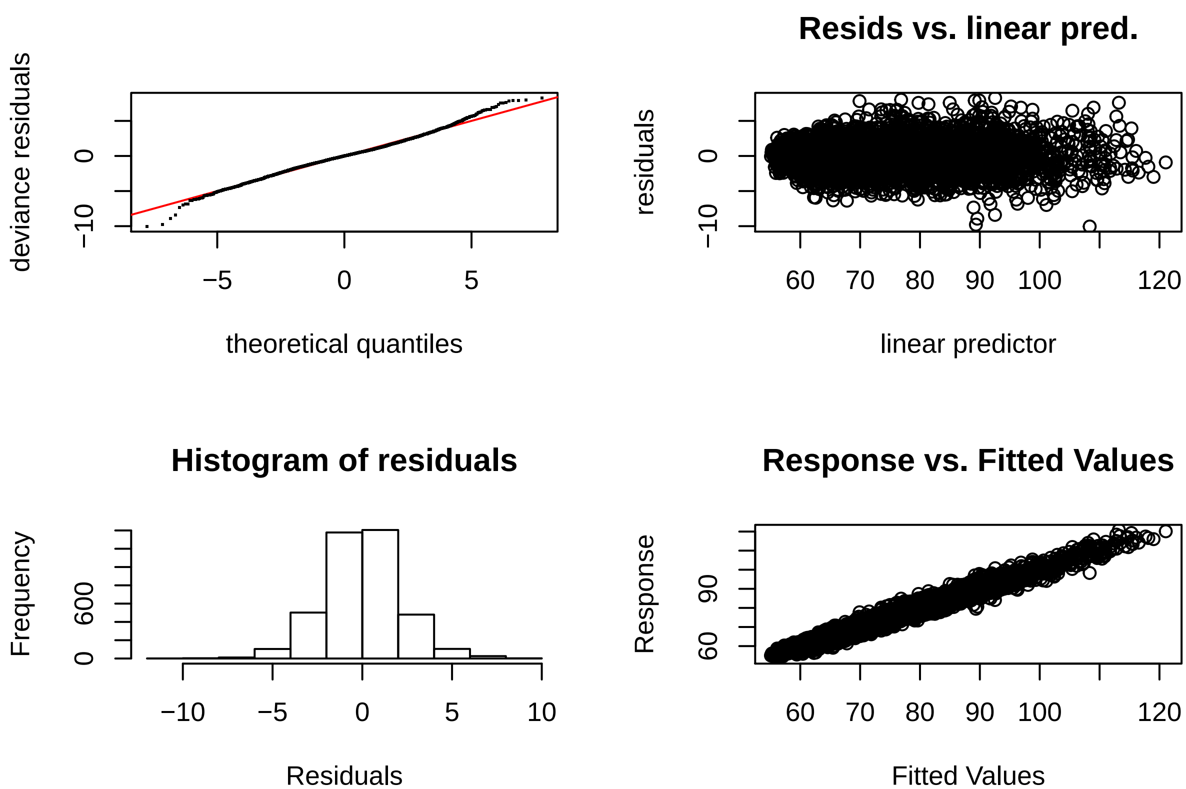


gam.check: edf =12.44 < k' =30. K large enough

Add (tempEq,trend)

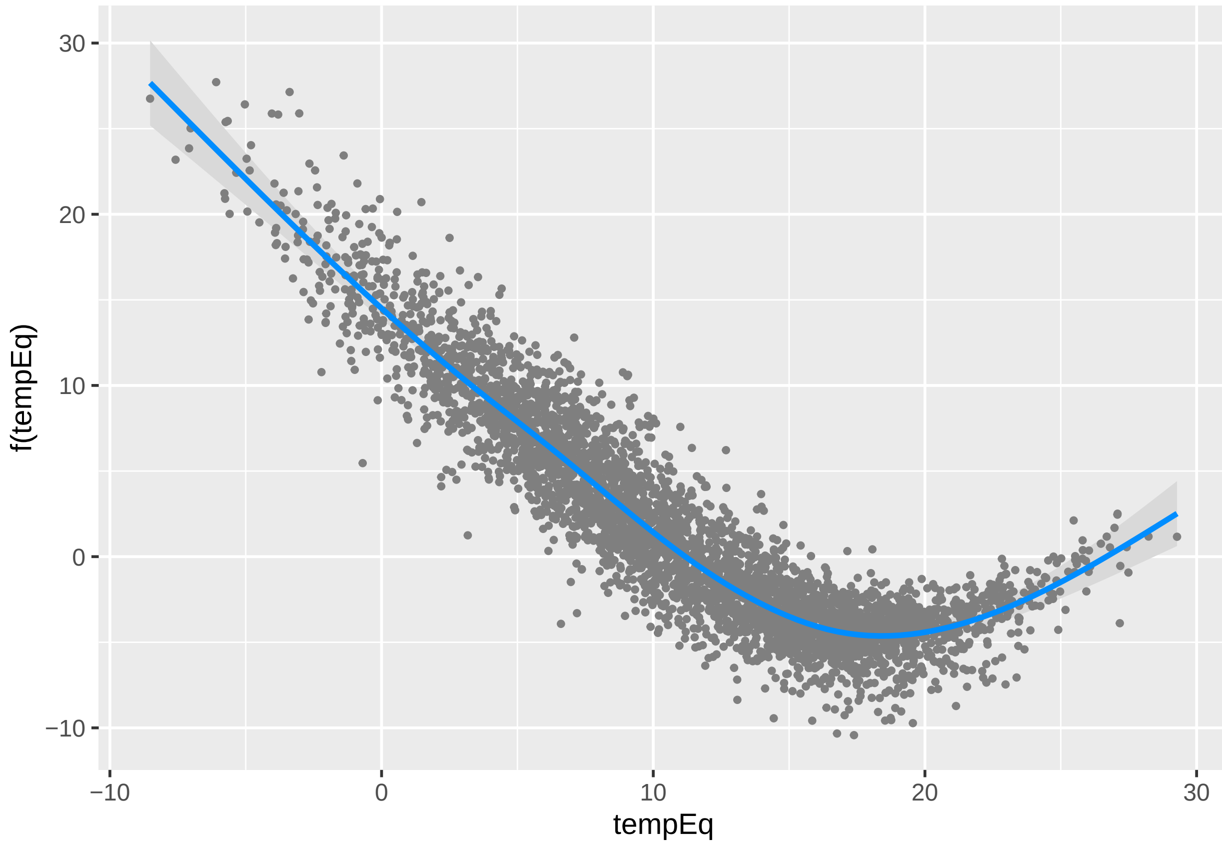
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, trend, bs = c("cr",
##     "cr"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 74.95230    0.03408    2199   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                     edf Ref.df       F p-value
## s(tempEq)         7.900  8.615 1125.81  <2e-16 ***
## s(FracOfYear)    12.785 13.000  854.19  <2e-16 ***
## s(trend)          5.882  5.994 1454.29  <2e-16 ***
## ti(tempEq,trend)  9.721 11.826   65.67  <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.976   Deviance explained = 97.6%
## -REML = 8794.7  Scale est. = 4.4913    n = 4017
```

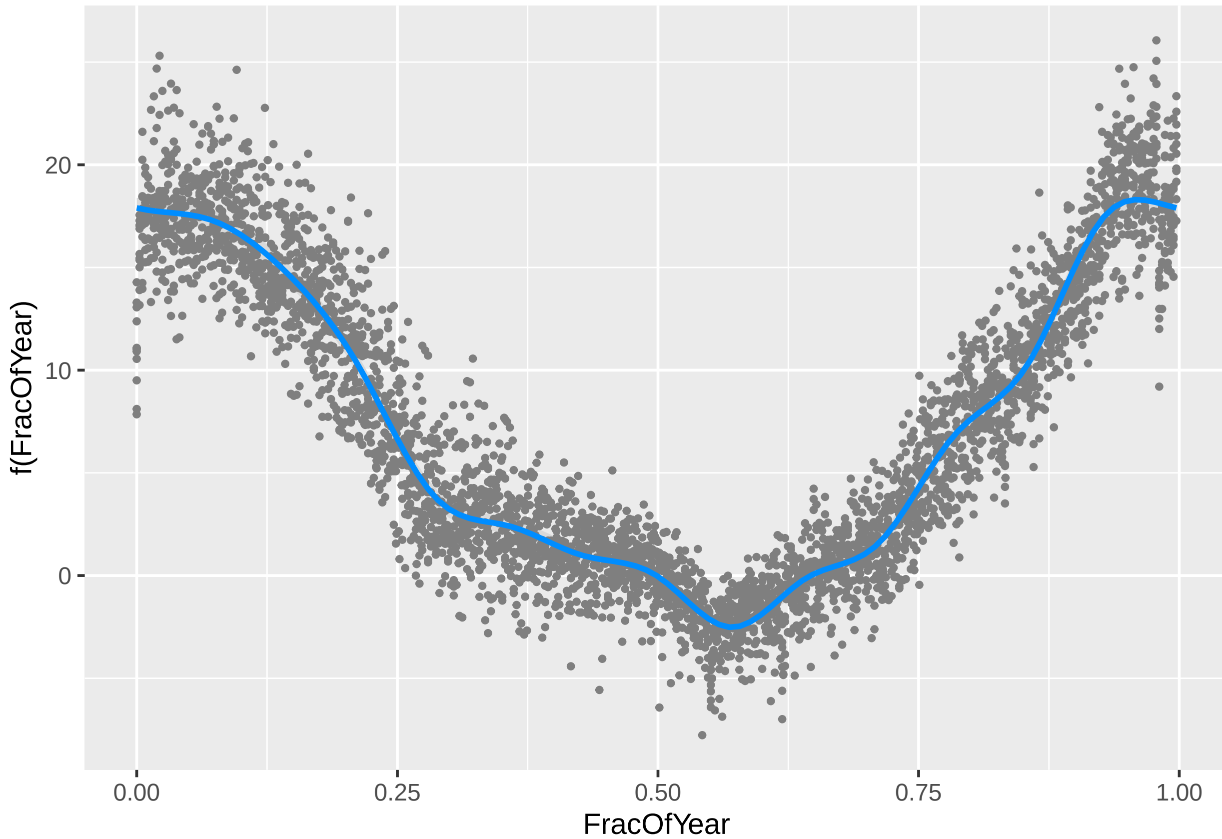



```
##
## Method: REML   Optimizer: outer newton
## full convergence after 9 iterations.
## Gradient range [-0.0001777434,0.0001405041]
## (score 8794.66 & scale 4.491307).
## Hessian positive definite, eigenvalue range [0.9182005,2006.535].
## Model rank =  45 / 45
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                       k'   edf k-index p-value
## s(tempEq)           9.00  7.90    0.99   0.360
## s(FracOfYear)      13.00 12.78    1.00   0.555
## s(trend)            6.00  5.88    0.45  <2e-16 ***
## ti(tempEq,trend) 16.00  9.72    0.98   0.025 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

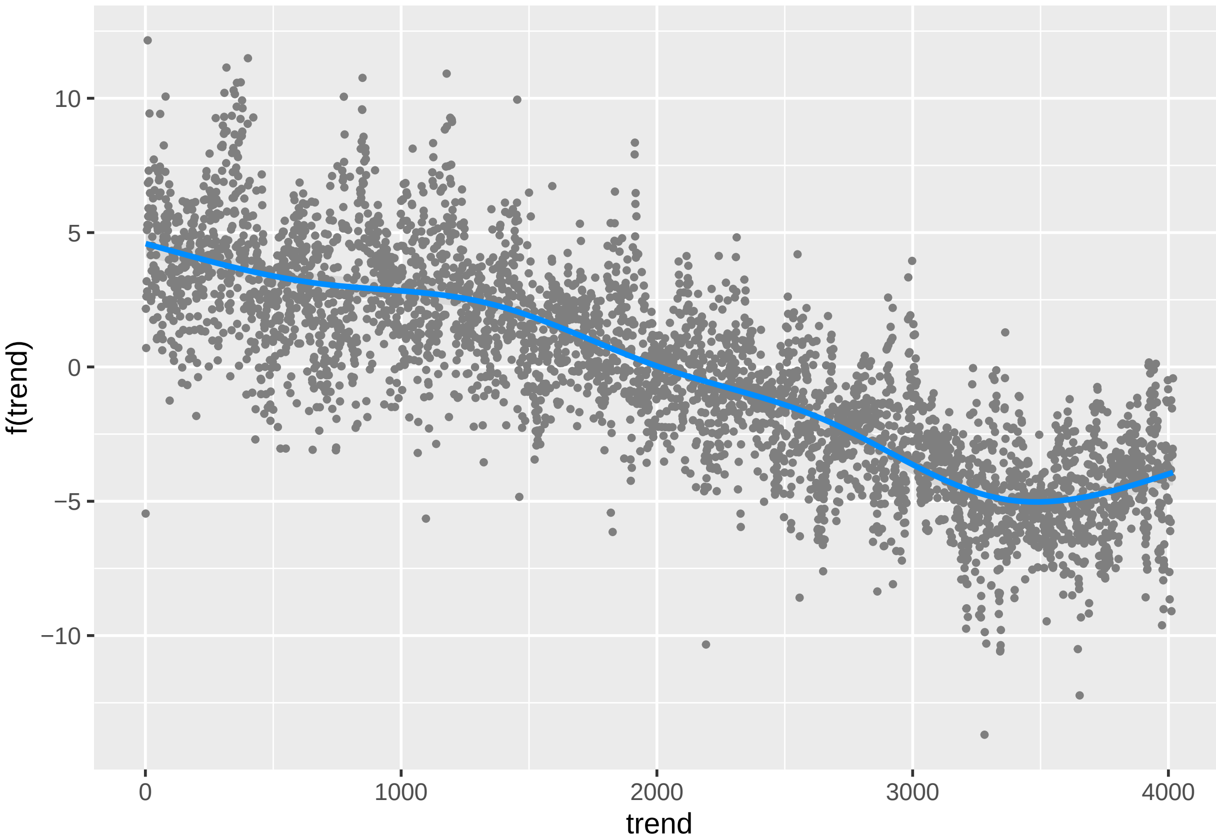
```
## [[1]]
```



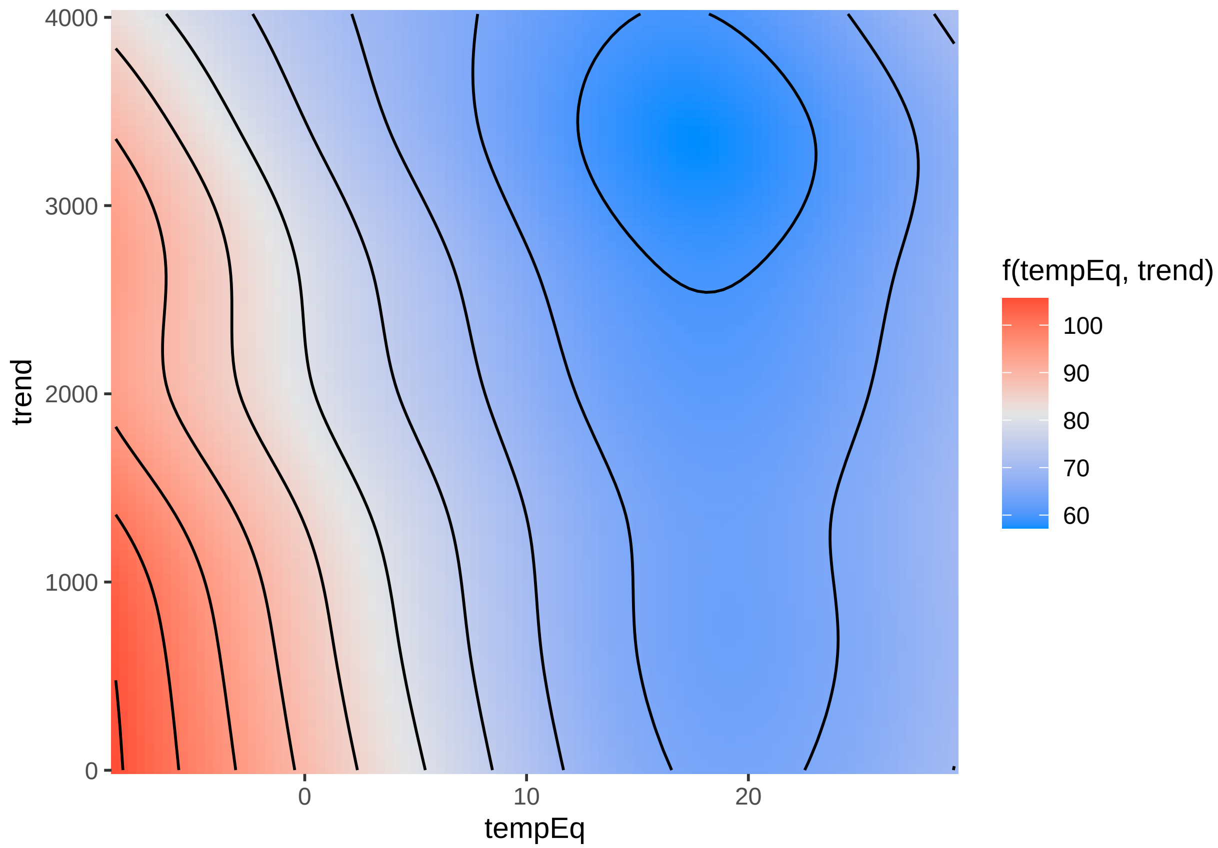

```
##
## [[2]]
```




```
##
## [[3]]
```



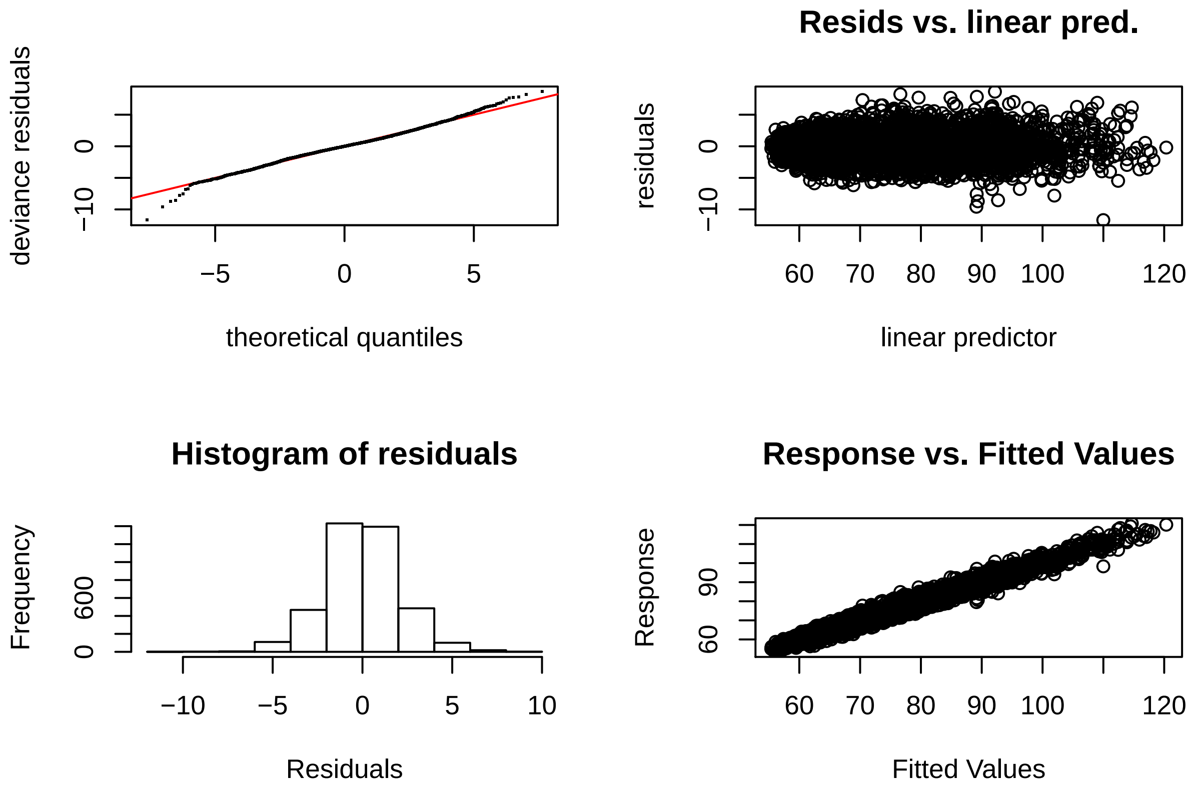
10
5
0
−5
−10
f(trend)
0
1000
2000
3000
4000
trend



Add (trend,FracOfYear)

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(trend, FracOfYear, bs = c("cr",
##     "cc"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 75.05581    0.03313    2265   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                        edf Ref.df       F p-value
## s(tempEq)            8.108  8.735 1292.74  <2e-16 ***
## s(FracOfYear)       12.798 13.000  889.05  <2e-16 ***
## s(trend)             5.908  5.996 1534.46  <2e-16 ***
## ti(trend,FracOfYear) 11.423 12.000   78.33  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## R-sq.(adj) =  0.977   Deviance explained = 97.7%
## -REML = 8739.4  Scale est. = 4.343     n = 4017
```

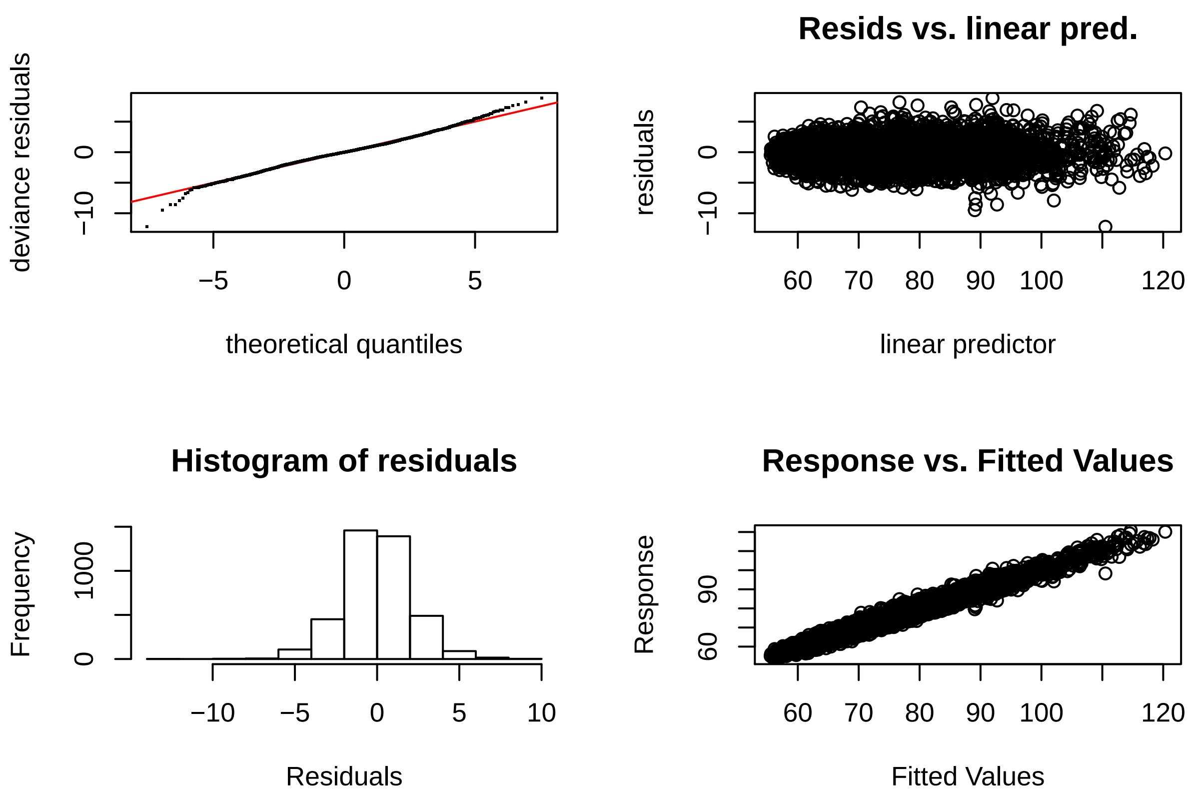



```
##
## Method: REML   Optimizer: outer newton
## full convergence after 7 iterations.
## Gradient range [-0.001097639,0.0007395871]
## (score 8739.425 & scale 4.342957).
## Hessian positive definite, eigenvalue range [1.100156,2007.04].
## Model rank =  41 / 41
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                          k'   edf k-index p-value
## s(tempEq)              9.00  8.11    0.99    0.25
## s(FracOfYear)         13.00 12.80    1.01    0.69
## s(trend)               6.00  5.91    0.47  <2e-16 ***
## ti(trend,FracOfYear) 12.00 11.42    0.96  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Higher k

```
##
```

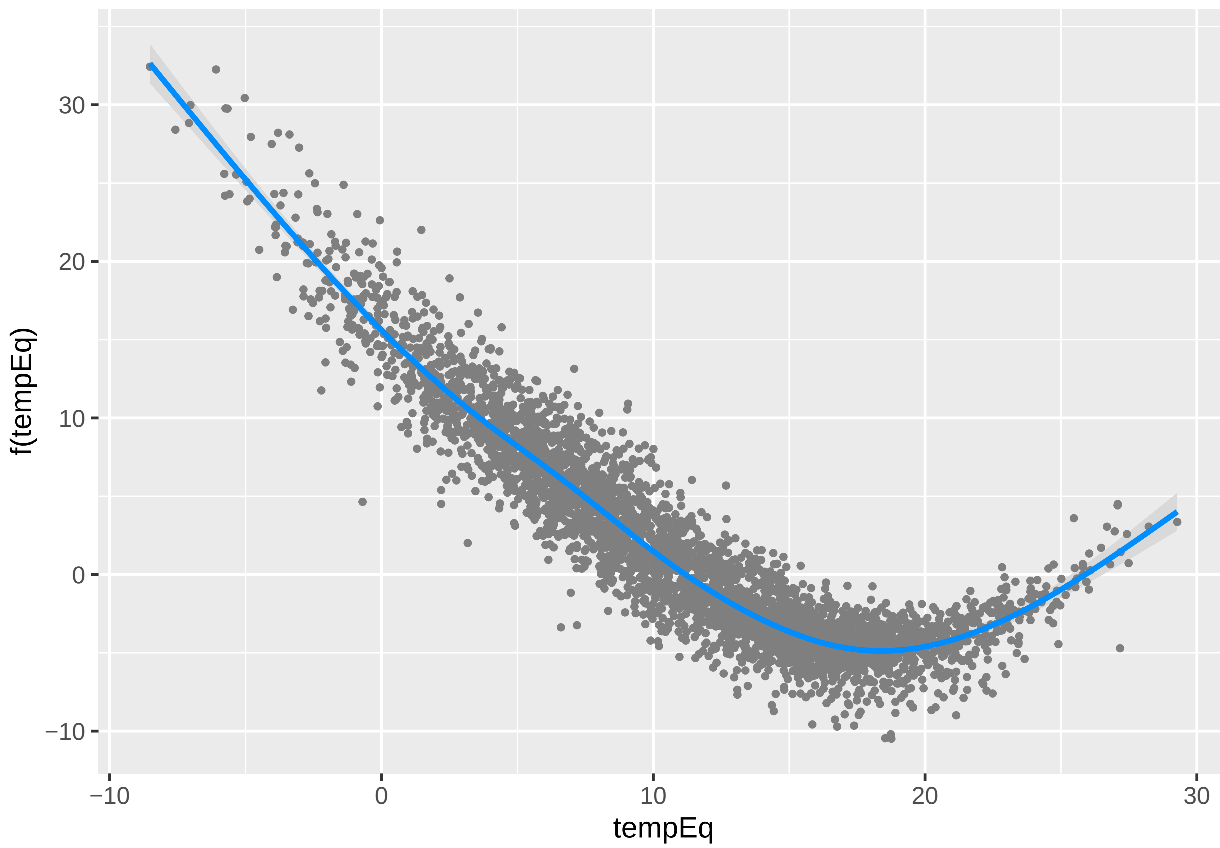
```
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(trend, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 12))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 75.06883    0.03307    2270   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                         edf Ref.df       F p-value
## s(tempEq)             8.093  8.725 1276.29  <2e-16 ***
## s(FracOfYear)        12.806 13.000  896.73  <2e-16 ***
## s(trend)              5.916  5.996 1535.73  <2e-16 ***
## ti(trend,FracOfYear) 40.934 60.000   17.72  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.978   Deviance explained = 97.8%
## -REML = 8721.2  Scale est. = 4.2367    n = 4017
```



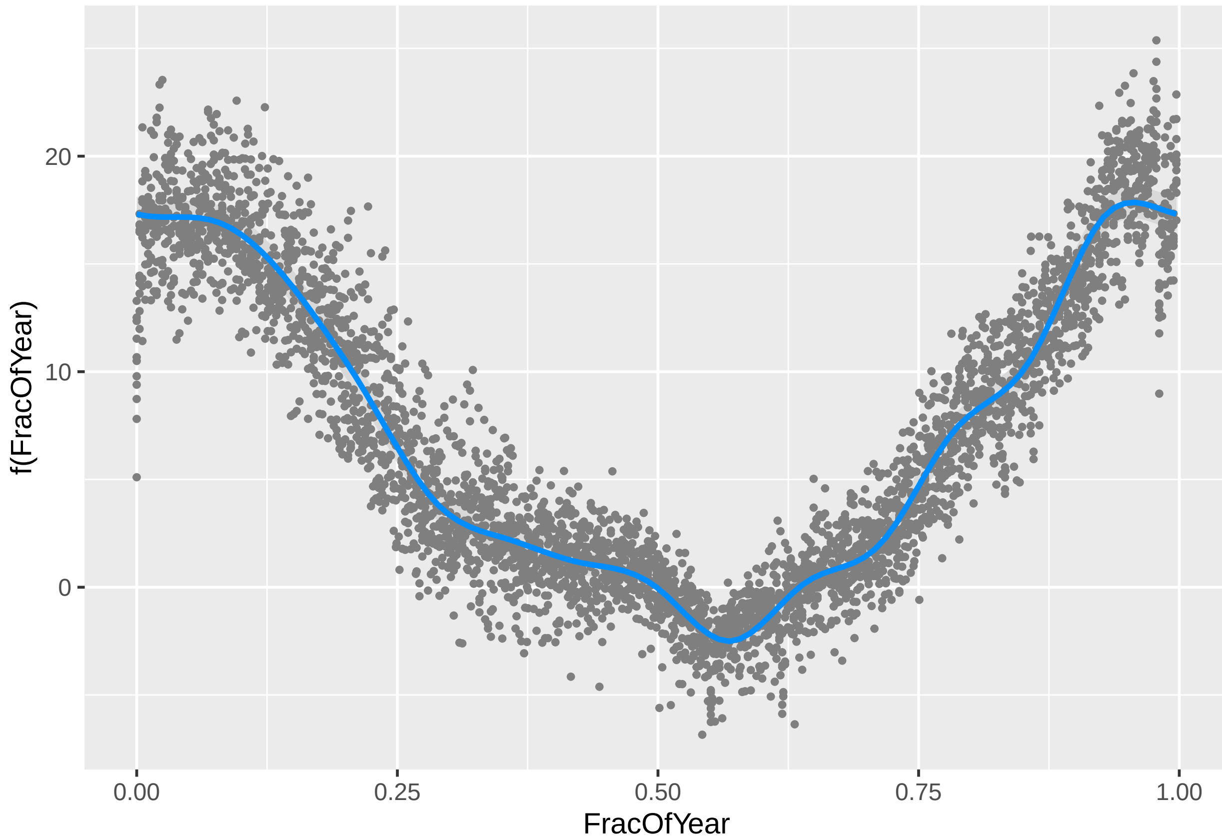

```
##
```

```
## Method: REML   Optimizer: outer newton
## full convergence after 4 iterations.
## Gradient range [-0.005217523,0.0006480696]
## (score 8721.167 & scale 4.236693).
## Hessian positive definite, eigenvalue range [2.275574,2007.148].
## Model rank =  89 / 89
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                         k'   edf k-index p-value
## s(tempEq)             9.00  8.09    0.99    0.38
## s(FracOfYear)        13.00 12.81    0.99    0.23
## s(trend)              6.00  5.92    0.48  <2e-16 ***
## ti(trend,FracOfYear) 60.00 40.93    0.96  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

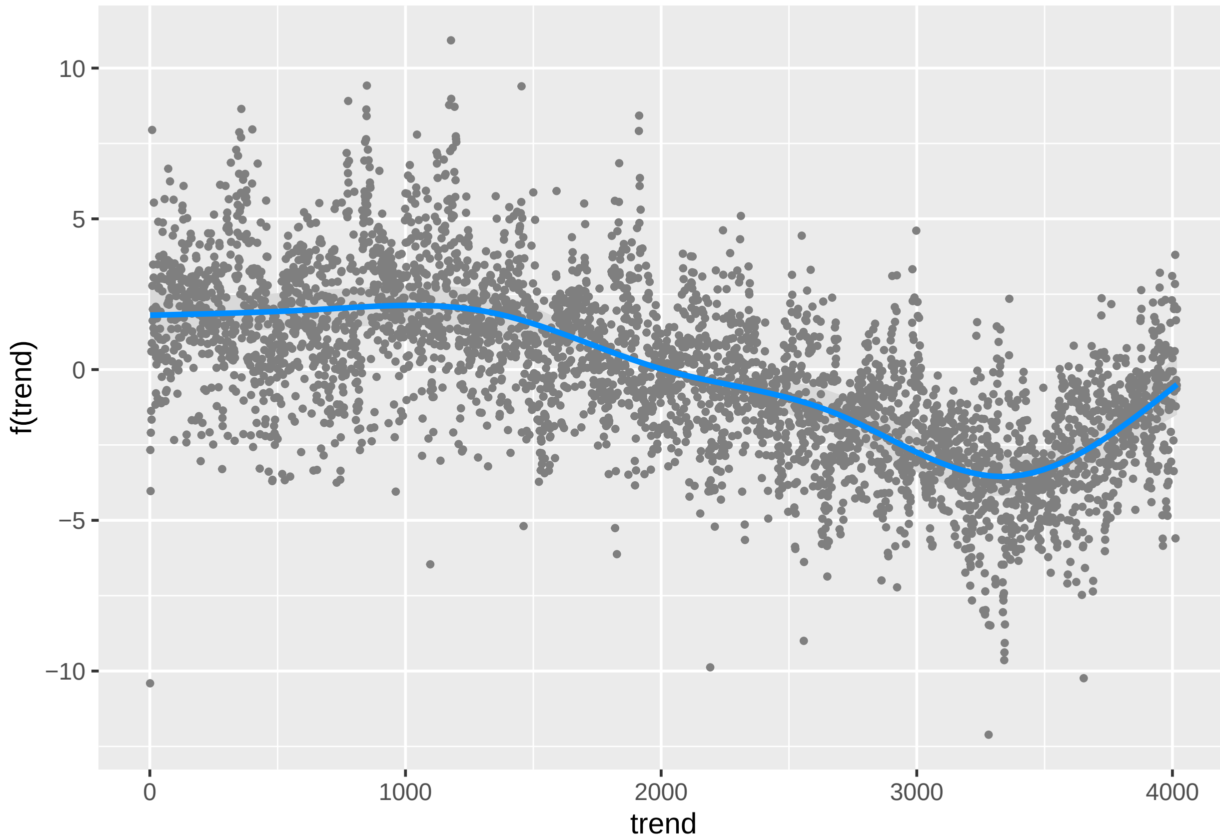
```
## [[1]]
```

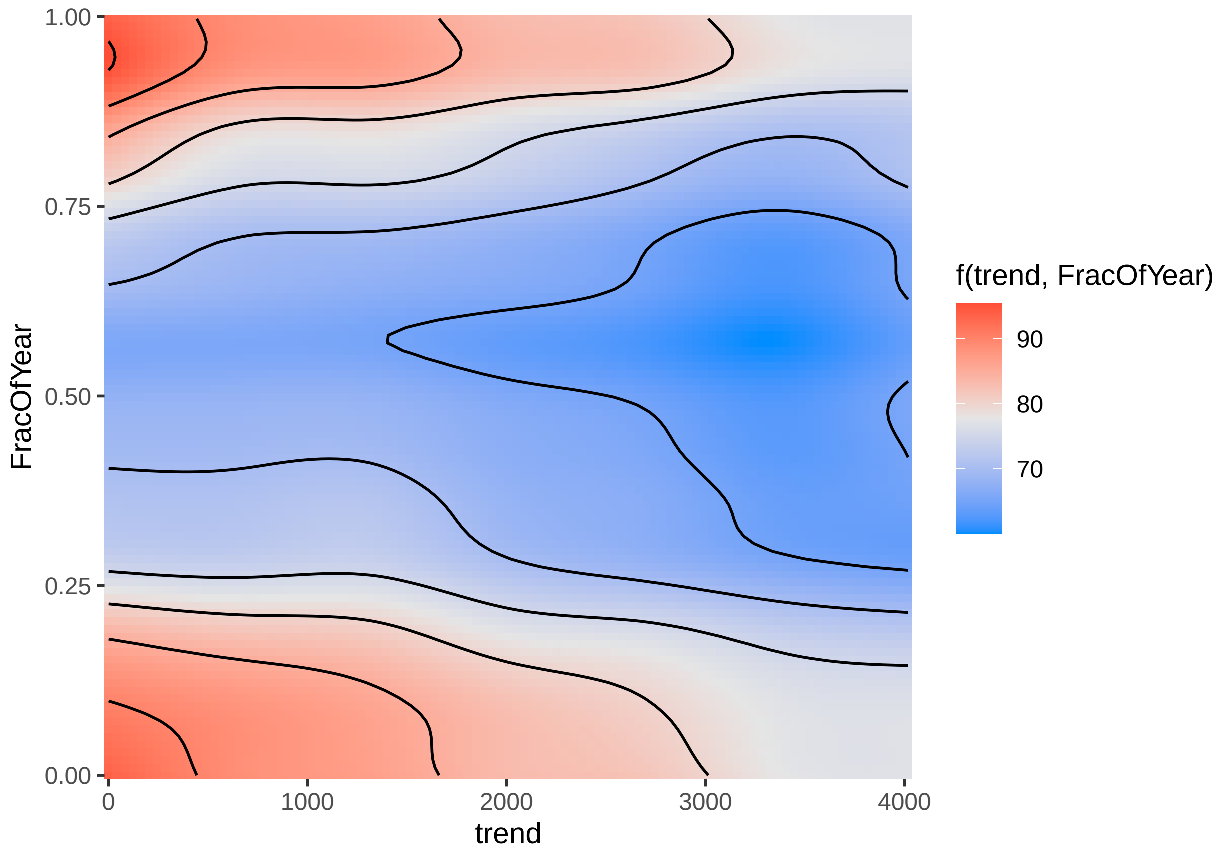



```
##
## [[2]]
```




```
##
## [[3]]
```

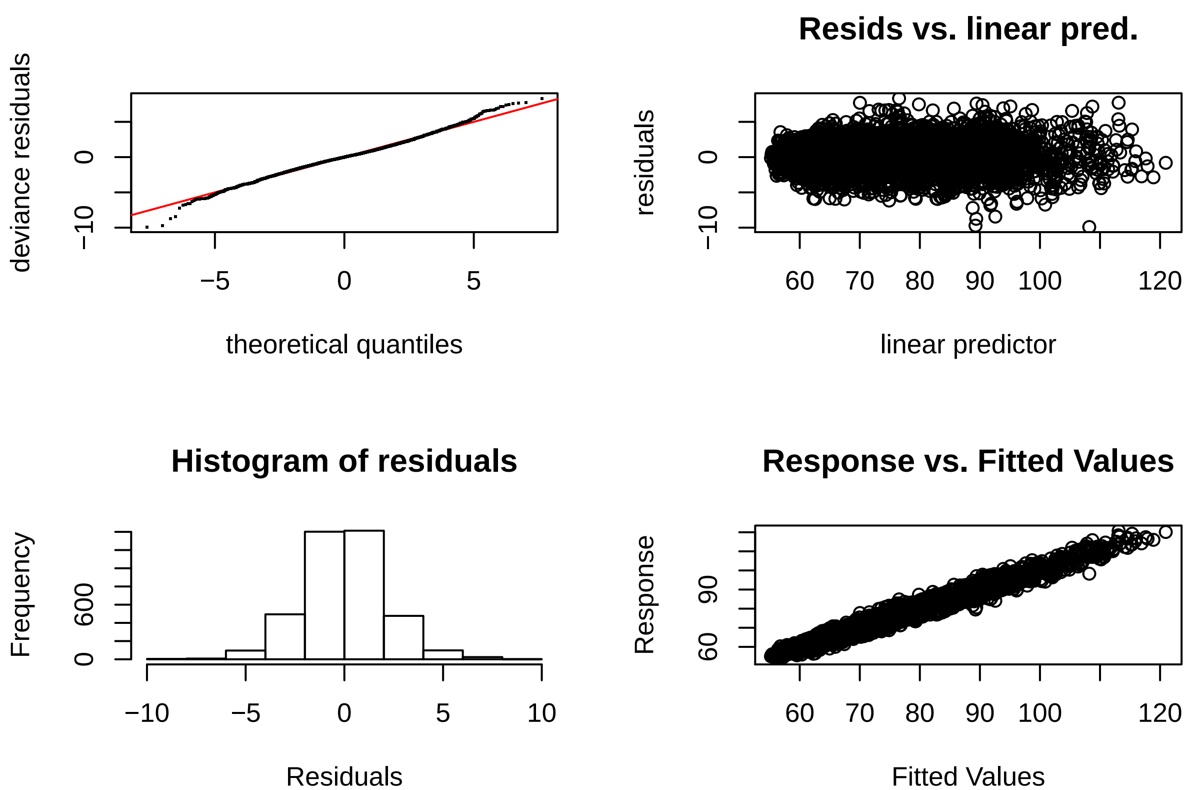

10
5
0
−5
−10
f(trend)
0
1000
2000
3000
4000
trend



gam.check: edf =12.44 < k' =30. K large enough ###Combination of interactions

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(tempEq, trend, bs = c("cr", "cr"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.3524     0.2877   258.5   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                          edf Ref.df        F p-value
## s(tempEq)              6.910  7.813   59.920  <2e-16 ***
## s(FracOfYear)         12.721 13.000   90.914  <2e-16 ***
## s(trend)               5.880  5.993 1511.558  <2e-16 ***
## ti(tempEq,FracOfYear) 15.149 30.000    5.145  <2e-16 ***
## ti(tempEq,trend)       8.243 10.220   74.906  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

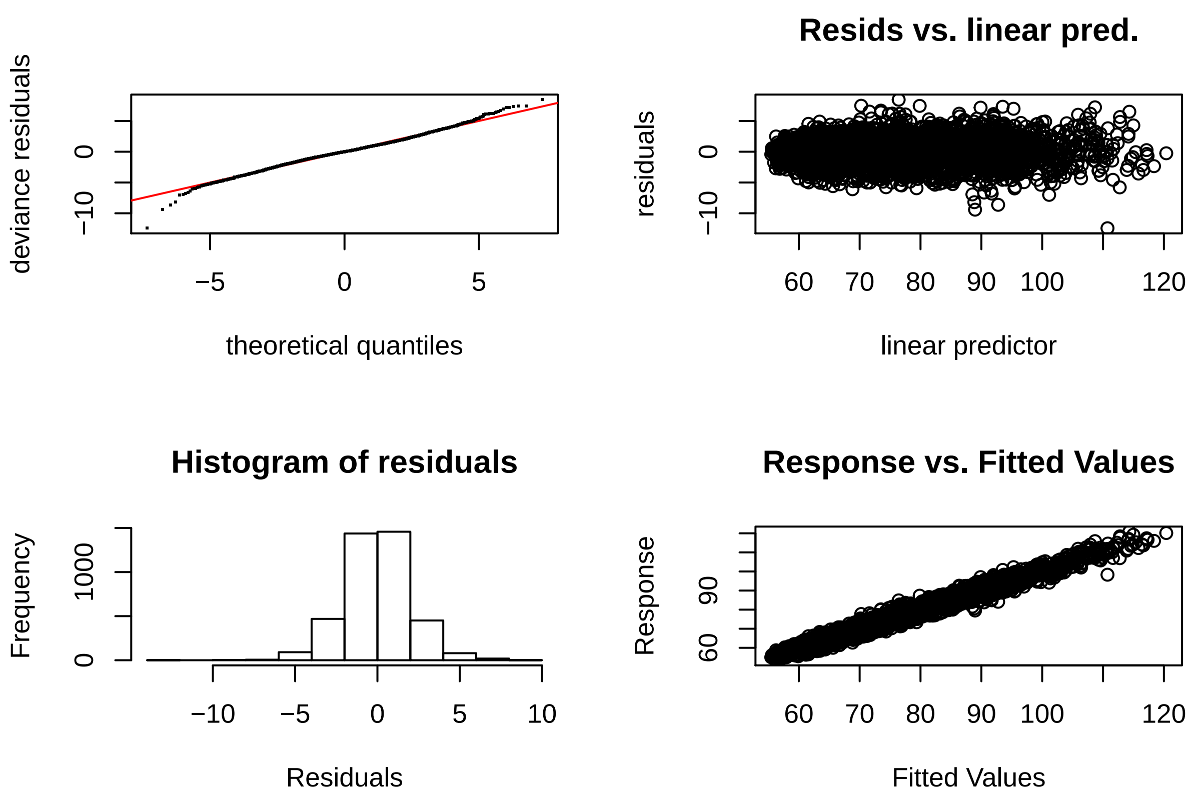
```
## R-sq.(adj) =  0.977   Deviance explained = 97.7%
## -REML = 8738.9  Scale est. = 4.333     n = 4017
```




```
##
## Method: REML   Optimizer: outer newton
## full convergence after 8 iterations.
## Gradient range [-1.304639e-05,2.076147e-06]
## (score 8738.893 & scale 4.332972).
## Hessian positive definite, eigenvalue range [0.5584506,2006.546].
## Model rank =  75 / 75
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                         k'   edf k-index p-value
## s(tempEq)             9.00  6.91    0.99    0.40
## s(FracOfYear)        13.00 12.72    1.00    0.58
## s(trend)              6.00  5.88    0.47  <2e-16 ***
## ti(tempEq,FracOfYear) 30.00 15.15    0.93  <2e-16 ***
## ti(tempEq,trend)      16.00  8.24    0.97    0.04 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

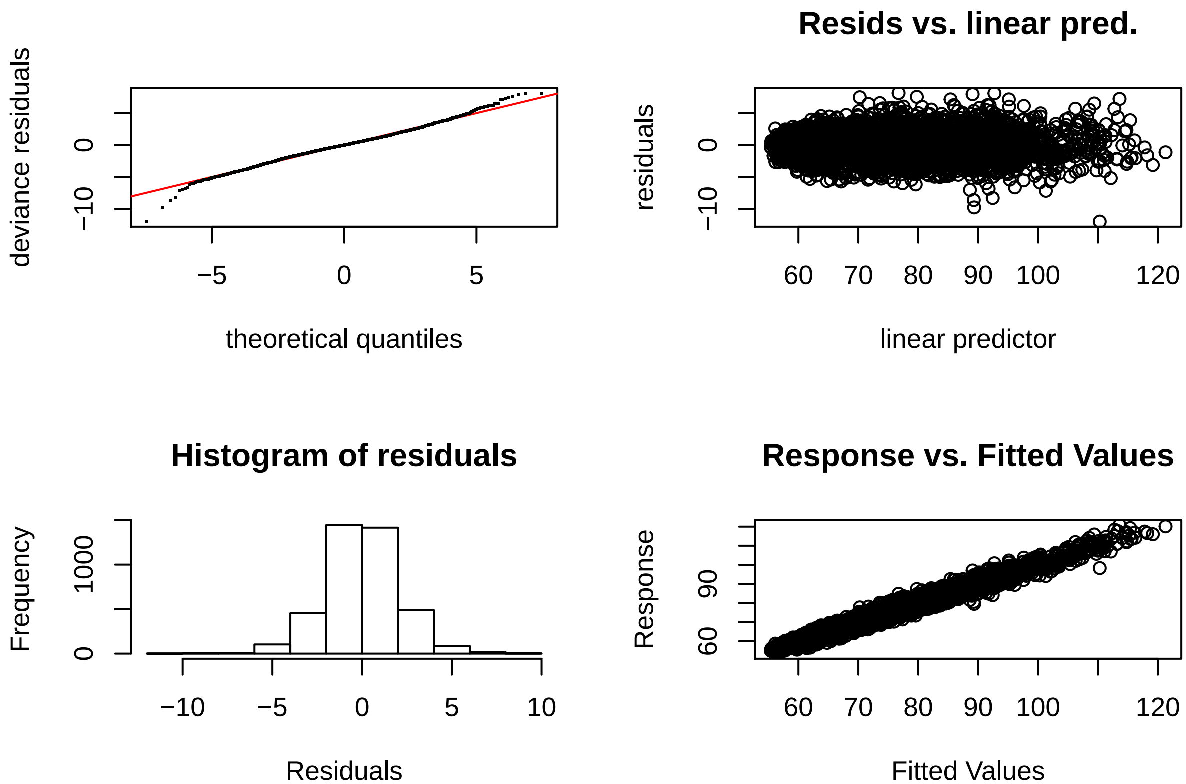
```
##
## Family: gaussian
## Link function: identity
##
```

```
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(trend, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 12))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.8581     0.2806   266.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                           edf Ref.df        F p-value
## s(tempEq)               7.346  8.154   66.321  <2e-16 ***
## s(FracOfYear)          12.763 13.000  107.480  <2e-16 ***
## s(trend)                5.919  5.996 1610.836  <2e-16 ***
## ti(tempEq,FracOfYear) 14.493 30.000    6.689  <2e-16 ***
## ti(trend,FracOfYear)  43.976 60.000   19.010  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.979   Deviance explained = 97.9%
## -REML = 8642.6  Scale est. = 4.0206    n = 4017
```




```
##
## Method: REML   Optimizer: outer newton
```

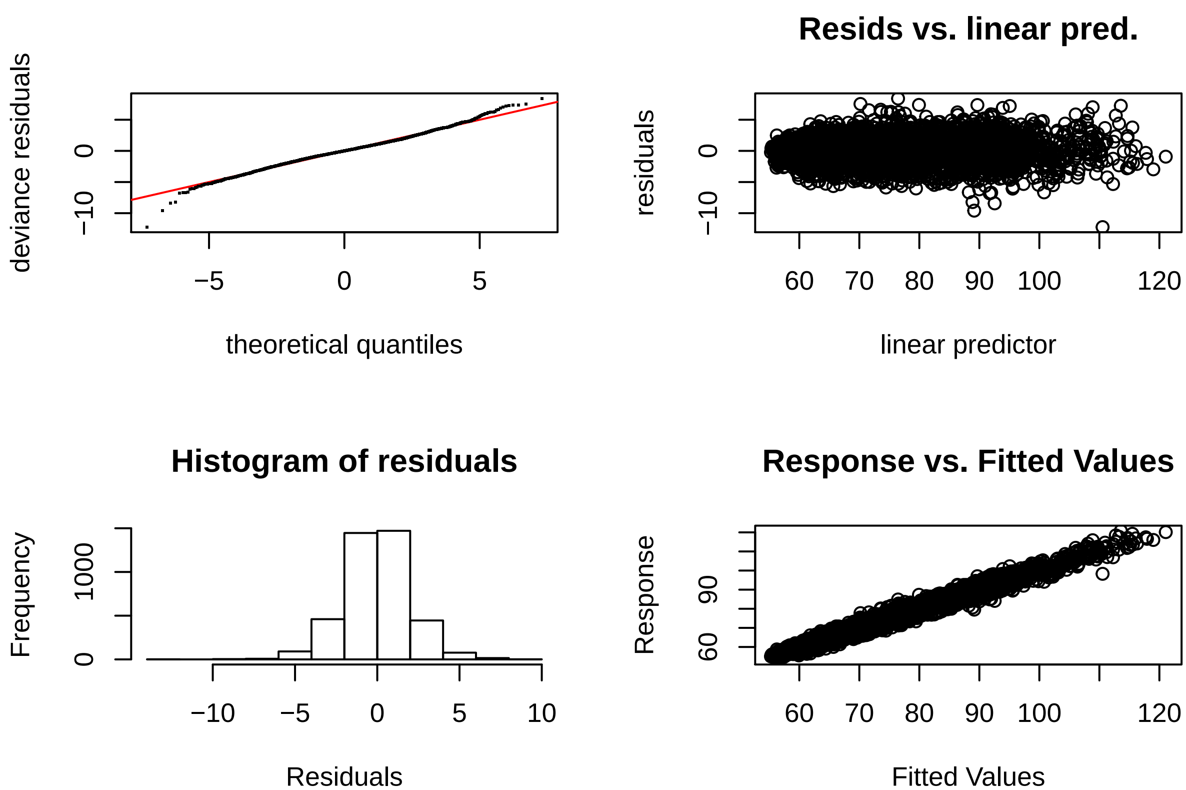
```
## full convergence after 8 iterations.
## Gradient range [-0.003473388,0.001794092]
## (score 8642.607 & scale 4.020625).
## Hessian positive definite, eigenvalue range [0.9504122,2007.18].
## Model rank =  119 / 119
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                          k'   edf k-index p-value
## s(tempEq)              9.00  7.35    0.99    0.23
## s(FracOfYear)         13.00 12.76    0.98    0.10 .
## s(trend)               6.00  5.92    0.50  <2e-16 ***
## ti(tempEq,FracOfYear) 30.00 14.49    0.91  <2e-16 ***
## ti(trend,FracOfYear)  60.00 43.98    0.95  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, trend, bs = c("cr",
##     "cr")) + ti(trend, FracOfYear, bs = c("cr", "cc"), k = c(7,
##     12))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 75.03351    0.03391    2213   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                         edf Ref.df        F  p-value
## s(tempEq)             7.998  8.673 1108.270  < 2e-16 ***
## s(FracOfYear)        12.806 13.000  905.990  < 2e-16 ***
## s(trend)              5.911  5.995 1518.372  < 2e-16 ***
## ti(tempEq,trend)     11.361 13.425    6.620 3.68e-12 ***
## ti(trend,FracOfYear) 40.061 60.000    5.363  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.978   Deviance explained = 97.9%
## -REML = 8689.8  Scale est. = 4.1459    n = 4017
```




```
##
## Method: REML   Optimizer: outer newton
## full convergence after 6 iterations.
## Gradient range [-0.003530779,0.002869096]
## (score 8689.846 & scale 4.145858).
## Hessian positive definite, eigenvalue range [1.099703,2006.651].
## Model rank =  105 / 105
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                         k'   edf k-index p-value
## s(tempEq)             9.00  8.00    0.99    0.16
## s(FracOfYear)        13.00 12.81    0.99    0.21
## s(trend)              6.00  5.91    0.49  <2e-16 ***
## ti(tempEq,trend)     16.00 11.36    0.98    0.04 *
## ti(trend,FracOfYear) 60.00 40.06    0.96  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

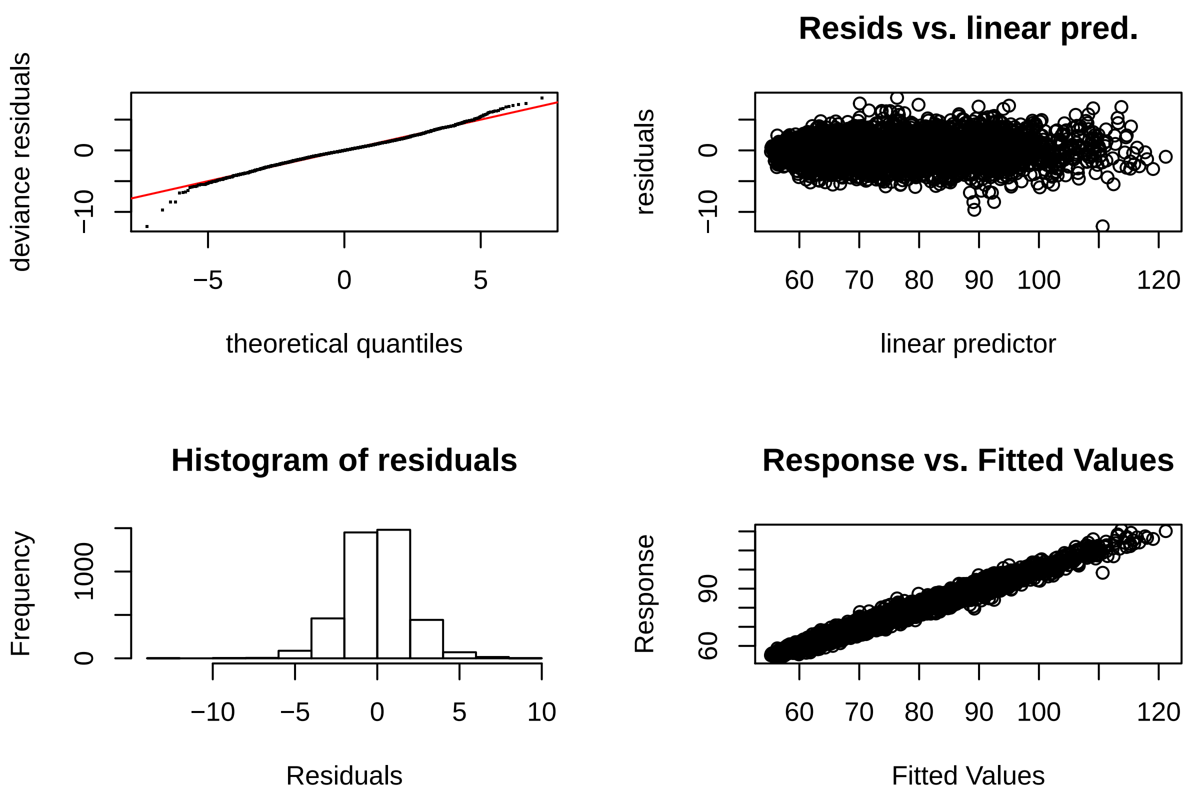
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
```

```
##     "cc"), k = c(7, 7)) + ti(tempEq, trend, bs = c("cr", "cr")) +
##     ti(trend, FracOfYear, bs = c("cr", "cc"), k = c(7, 12))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.7053     0.2786   268.2   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                          edf Ref.df        F  p-value
## s(tempEq)              7.148  8.008   66.863  < 2e-16 ***
## s(FracOfYear)         12.762 13.000  110.850  < 2e-16 ***
## s(trend)               5.917  5.995 1591.748  < 2e-16 ***
## ti(tempEq,FracOfYear) 14.223 30.000    6.033  < 2e-16 ***
## ti(tempEq,trend)       9.384 11.606    4.942 6.46e-08 ***
## ti(trend,FracOfYear)  43.070 60.000    6.102  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.979   Deviance explained =   98%
## -REML = 8620.9  Scale est. = 3.9638    n = 4017
```




```
##
## Method: REML   Optimizer: outer newton
## full convergence after 6 iterations.
## Gradient range [-0.001284614,0.00107839]
```

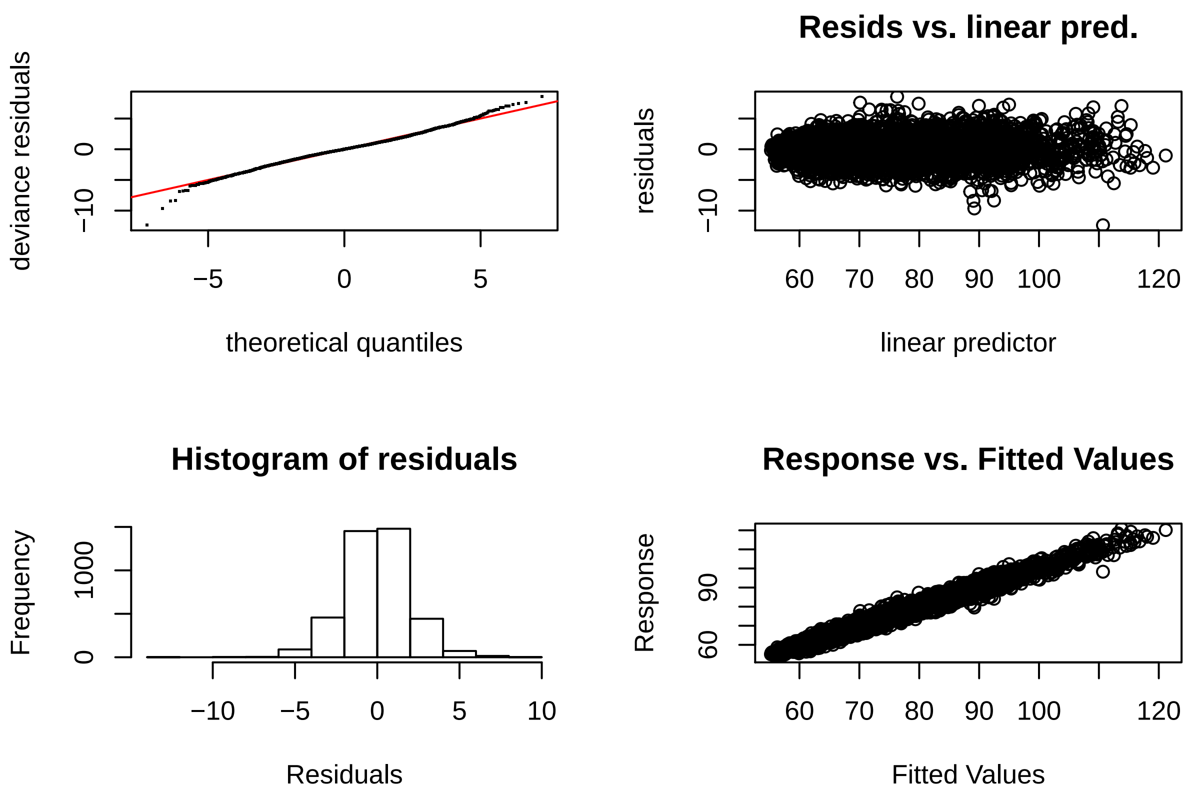
```
## (score 8620.926 & scale 3.963833).
## Hessian positive definite, eigenvalue range [0.7353986,2006.678].
## Model rank =  135 / 135
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                          k'   edf k-index p-value
## s(tempEq)              9.00  7.15    0.99   0.215
## s(FracOfYear)         13.00 12.76    0.98   0.095 .
## s(trend)               6.00  5.92    0.51  <2e-16 ***
## ti(tempEq,FracOfYear) 30.00 14.22    0.91  <2e-16 ***
## ti(tempEq,trend)      16.00  9.38    0.98   0.075 .
## ti(trend,FracOfYear)  60.00 43.07    0.95  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(tempEq, trend, bs = c("cr", "cr")) +
##     ti(trend, FracOfYear, bs = c("cr", "cc"), k = c(7, 12)) +
##     ti(tempEq, FracOfYear, trend, bs = c("cr", "cc", "cr"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.6762     0.2808   265.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                                 edf Ref.df       F  p-value
## s(tempEq)                     6.990  7.876  64.229  < 2e-16 ***
## s(FracOfYear)                12.758 13.000 109.813  < 2e-16 ***
## s(trend)                      5.880  5.975 223.952  < 2e-16 ***
## ti(tempEq,FracOfYear)        14.151 30.000   5.962  < 2e-16 ***
## ti(tempEq,trend)              2.196  2.434   4.421  0.00599 **
## ti(trend,FracOfYear)         44.429 60.000   4.727  < 2e-16 ***
## ti(tempEq,FracOfYear,trend)  22.406 48.000   1.557 3.29e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.979   Deviance explained =   98%
## -REML = 8610.3  Scale est. = 3.9042    n = 4017
```




```
##
## Method: REML   Optimizer: outer newton
## full convergence after 7 iterations.
## Gradient range [-0.01051786,0.004389617]
## (score 8610.269 & scale 3.904233).
## Hessian positive definite, eigenvalue range [0.0003261739,2006.717].
## Model rank =  183 / 183
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                               k'   edf k-index p-value
## s(tempEq)                   9.00  6.99    0.99    0.24
## s(FracOfYear)              13.00 12.76    0.97    0.06 .
## s(trend)                    6.00  5.88    0.52  <2e-16 ***
## ti(tempEq,FracOfYear)      30.00 14.15    0.91  <2e-16 ***
## ti(tempEq,trend)           16.00  2.20    0.99    0.12
## ti(trend,FracOfYear)       60.00 44.43    0.95  <2e-16 ***
## ti(tempEq,FracOfYear,trend) 48.00 22.41    0.91  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##
## Family: gaussian
## Link function: identity
##
## Formula:
```

```
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(tempEq, trend, bs = c("cr", "cr")) +
##     ti(trend, FracOfYear, bs = c("cr", "cc"), k = c(7, 12)) +
##     ti(tempEq, FracOfYear, trend, bs = c("cr", "cc", "cr"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  74.6762     0.2808   265.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                                edf Ref.df       F  p-value
## s(tempEq)                    6.990  7.876  64.229  < 2e-16 ***
## s(FracOfYear)               12.758 13.000 109.813  < 2e-16 ***
## s(trend)                     5.880  5.975 223.952  < 2e-16 ***
## ti(tempEq,FracOfYear)       14.151 30.000   5.962  < 2e-16 ***
## ti(tempEq,trend)             2.196  2.434   4.421  0.00599 **
## ti(trend,FracOfYear)        44.429 60.000   4.727  < 2e-16 ***
## ti(tempEq,FracOfYear,trend) 22.406 48.000   1.557 3.29e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.979   Deviance explained =   98%
## -REML = 8610.3  Scale est. = 3.9042    n = 4017

##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(trend, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 12)) + ti(tempEq, FracOfYear, trend, bs = c("cr",
##     "cc", "cr"))
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   74.767      0.277     270   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                                edf Ref.df       F  p-value
## s(tempEq)                    7.021  7.897  66.032  < 2e-16 ***
## s(FracOfYear)               12.759 13.000 109.346  < 2e-16 ***
## s(trend)                     5.887  5.976 223.291  < 2e-16 ***
## ti(tempEq,FracOfYear)       14.303 30.000   6.164  < 2e-16 ***
## ti(trend,FracOfYear)        45.083 60.000  13.135  < 2e-16 ***
## ti(tempEq,FracOfYear,trend) 23.718 48.000   2.121 2.76e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## R-sq.(adj) =  0.979   Deviance explained =   98%
## -REML = 8616.6  Scale est. = 3.9106    n = 4017
```




```
##
## Method: REML   Optimizer: outer newton
## full convergence after 8 iterations.
## Gradient range [-0.001512917,0.0003177952]
## (score 8616.58 & scale 3.910565).
## Hessian positive definite, eigenvalue range [0.1051232,2007.209].
## Model rank =  167 / 167
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                                 k'   edf k-index p-value
## s(tempEq)                     9.00  7.02    0.99   0.260
## s(FracOfYear)                13.00 12.76    0.97   0.045 *
## s(trend)                      6.00  5.89    0.52  <2e-16 ***
## ti(tempEq,FracOfYear)        30.00 14.30    0.91  <2e-16 ***
## ti(trend,FracOfYear)         60.00 45.08    0.96  <2e-16 ***
## ti(tempEq,FracOfYear,trend)  48.00 23.72    0.92  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comparison AIC

```
##              df      AIC
## elec2   28.97309 18179.36
## elec3   43.93974 18042.16
## elec4   40.02382 17476.45
## elec5   71.73512 17273.56
## elec6   54.68087 17348.84
## elec7   89.73624 17082.21
## elec8   83.71562 17199.87
## elec9   99.82033 17037.04
## elec10 116.81992 16993.45
## elec11 116.15808 16998.68
```

Add Holiday and WeekDay

Only WeekDay

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(tempEq, trend, bs = c("cr", "cr")) +
##     ti(trend, FracOfYear, bs = c("cr", "cc"), k = c(7, 12)) +
##     ti(tempEq, FracOfYear, trend, bs = c("cr", "cc", "cr")) +
##     as.factor(WeekDay)
##
## Parametric coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           74.81147    0.27630 270.758  < 2e-16 ***
## as.factor(WeekDay)Mon -0.58538    0.10960  -5.341 9.76e-08 ***
## as.factor(WeekDay)Sat  1.24906    0.10939  11.419  < 2e-16 ***
## as.factor(WeekDay)Sun -1.07345    0.10955  -9.799  < 2e-16 ***
## as.factor(WeekDay)Thu  0.02552    0.10937   0.233  0.81549
## as.factor(WeekDay)Tue -0.34045    0.10960  -3.106  0.00191 **
## as.factor(WeekDay)Wed  0.30458    0.10953   2.781  0.00545 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                                edf Ref.df       F p-value
## s(tempEq)                    7.070  7.934  68.936 < 2e-16 ***
## s(FracOfYear)               12.787 13.000 120.934 < 2e-16 ***
## s(trend)                     5.892  5.979 234.787 < 2e-16 ***
## ti(tempEq,FracOfYear)       14.564 30.000   6.592 < 2e-16 ***
## ti(tempEq,trend)             2.039  2.264   5.007 0.00381 **
## ti(trend,FracOfYear)        46.828 60.000   5.322 < 2e-16 ***
## ti(tempEq,FracOfYear,trend) 24.610 48.000   1.985 1.8e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## R-sq.(adj) =  0.982   Deviance explained = 98.2%
## -REML =   8366  Scale est. = 3.4247    n = 4017
```

Only Holiday

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, trend, bs = c("cr",
##     "cr")) + ti(tempEq, FracOfYear, bs = c("cr", "cc"), k = c(7,
##     7)) + ti(trend, FracOfYear, bs = c("cr", "cc"), k = c(7,
##     12)) + ti(tempEq, FracOfYear, trend, bs = c("cr", "cc", "cr")) +
##     as.factor(Holiday)
##
## Parametric coefficients:
##                                Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     73.7675     0.3582 205.929  < 2e-16 ***
## as.factor(Holiday)No             0.8388     0.2226   3.769 0.000167 ***
## as.factor(Holiday)SummerHoliday   0.5453     0.3437   1.587 0.112668
## as.factor(Holiday)WeekHoliday    -1.7473     0.3629  -4.814 1.53e-06 ***
## as.factor(Holiday)WinterHoliday  -0.4154     0.3302  -1.258 0.208483
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                               edf Ref.df       F  p-value
## s(tempEq)                   6.738  7.675  62.148  < 2e-16 ***
## s(FracOfYear)              12.666 13.000  95.027  < 2e-16 ***
## s(trend)                    5.913  5.989 137.572  < 2e-16 ***
## ti(tempEq,trend)            7.935  9.601   4.508 5.18e-06 ***
## ti(tempEq,FracOfYear)      14.538 30.000   6.105  < 2e-16 ***
## ti(trend,FracOfYear)       43.995 60.000   4.180  < 2e-16 ***
## ti(tempEq,FracOfYear,trend) 14.901 48.000   1.454 2.08e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =   0.98   Deviance explained = 98.1%
## -REML = 8550.1  Scale est. = 3.7886    n = 4017
```

Both

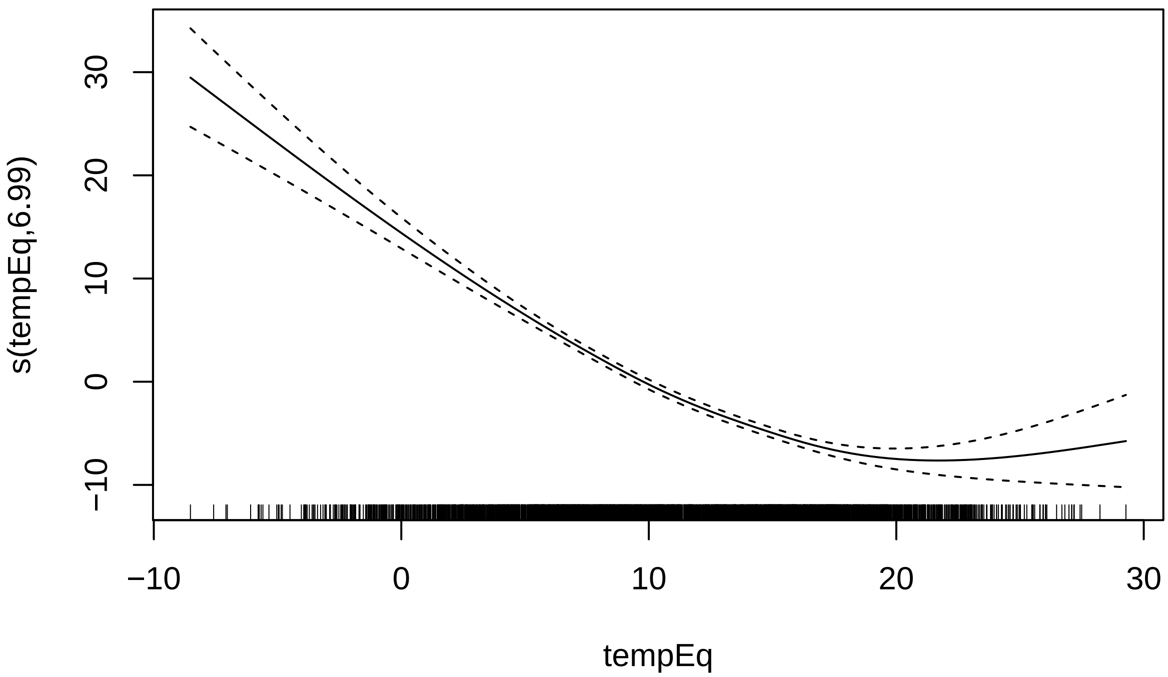
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## GWhElec ~ s(tempEq, bs = "cr") + s(FracOfYear, bs = "cc", k = 15) +
##     s(trend, bs = "cr", k = 7) + ti(tempEq, FracOfYear, bs = c("cr",
##     "cc"), k = c(7, 7)) + ti(trend, FracOfYear, bs = c("cr",
```

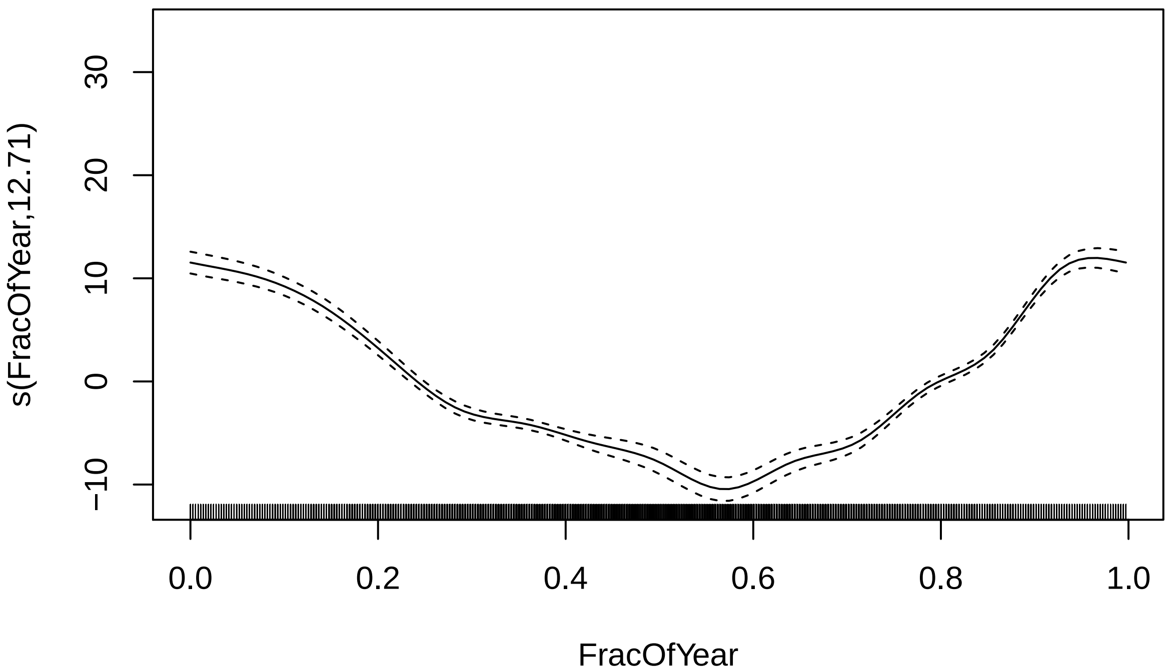
```
##     "cc"), k = c(7, 12)) + ti(tempEq, trend, bs = c("cr", "cr")) +
##     ti(tempEq, FracOfYear, trend, bs = c("cr", "cc", "cr"), k = c(7,
##         7, 7)) + as.factor(WeekDay) + as.factor(Holiday)
##
## Parametric coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     73.99858    0.34555 214.148  < 2e-16 ***
## as.factor(WeekDay)Mon           -0.55722    0.10718  -5.199 2.11e-07 ***
## as.factor(WeekDay)Sat            1.18605    0.10728  11.055  < 2e-16 ***
## as.factor(WeekDay)Sun           -1.14034    0.10755 -10.603  < 2e-16 ***
## as.factor(WeekDay)Thu            0.05993    0.10687   0.561 0.574973
## as.factor(WeekDay)Tue           -0.35286    0.10708  -3.295 0.000992 ***
## as.factor(WeekDay)Wed            0.29008    0.10703   2.710 0.006754 **
## as.factor(Holiday)No             0.81678    0.21013   3.887 0.000103 ***
## as.factor(Holiday)SummerHoliday  0.53371    0.32382   1.648 0.099402 .
## as.factor(Holiday)WeekHoliday   -1.64249    0.34128  -4.813 1.55e-06 ***
## as.factor(Holiday)WinterHoliday -0.38463    0.31245  -1.231 0.218399
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                                edf  Ref.df       F p-value
## s(tempEq)                    6.993   7.860  70.954  <2e-16 ***
## s(FracOfYear)               12.710  13.000 108.870  <2e-16 ***
## s(trend)                     5.797   5.880 106.446  <2e-16 ***
## ti(tempEq,FracOfYear)       14.080  30.000   5.595  <2e-16 ***
## ti(trend,FracOfYear)        42.076  60.000   4.105  <2e-16 ***
## ti(tempEq,trend)             5.163   5.922   1.376   0.199
## ti(tempEq,FracOfYear,trend) 53.950 180.000   1.077  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.983   Deviance explained = 98.3%
## -REML = 8289.5  Scale est. = 3.2584    n = 4017
```

AIC in order to select the model

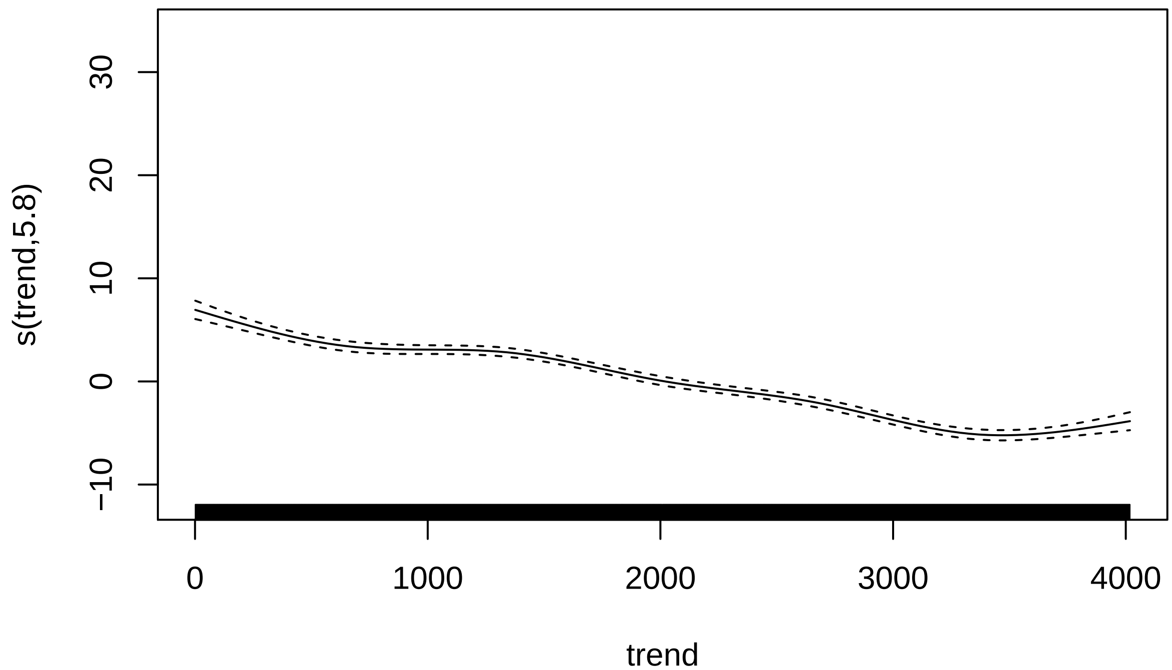
```
##              df      AIC
## elec10 116.8199 16993.45
## elec12 127.4093 16476.91
## elec13 119.5895 16876.28
## elec14 162.0631 16314.25
```

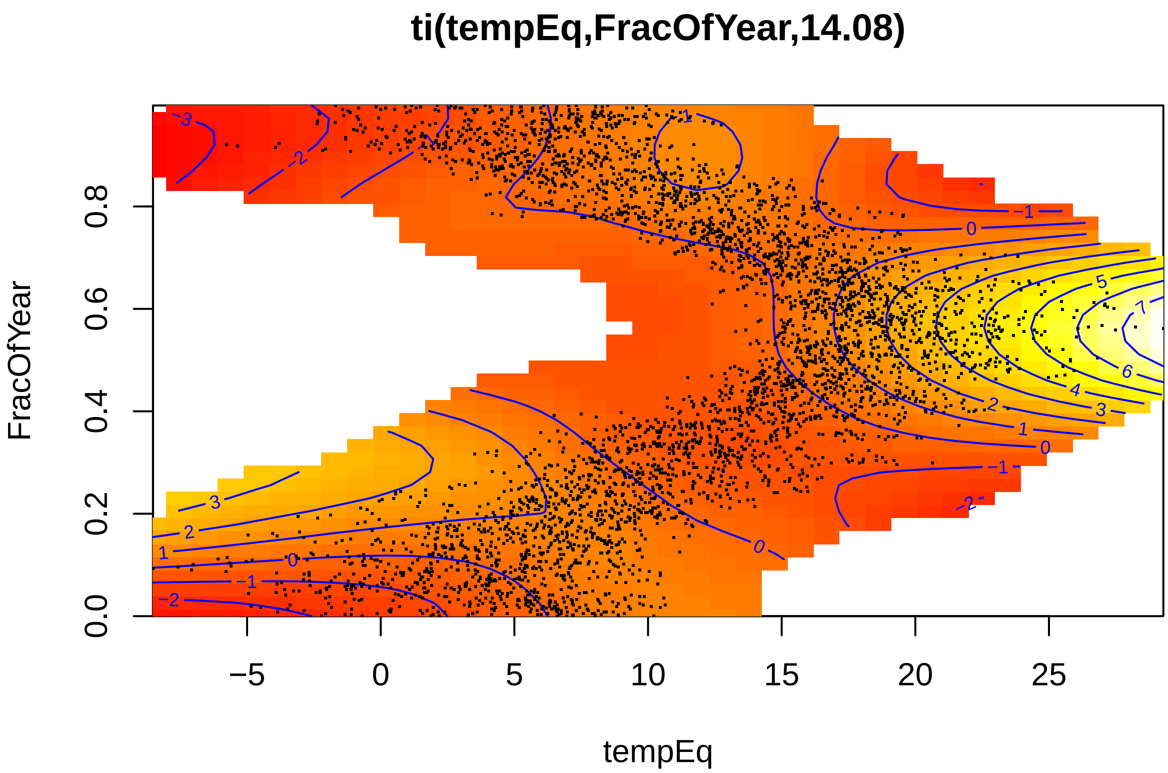
One choose elec14: weekday and holiday variable are kept ##Visualisation

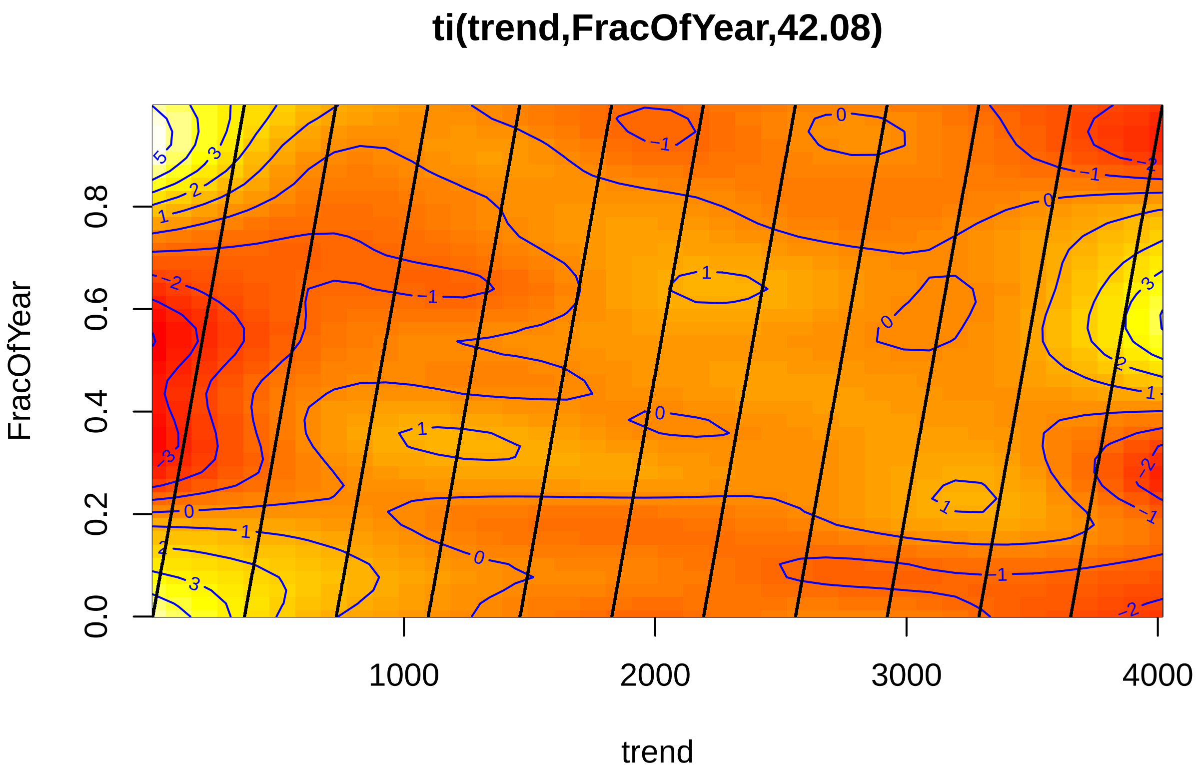

s(tempEq,6.99)
30
20
10
0
−10
−10
0
10
20
30
tempEq

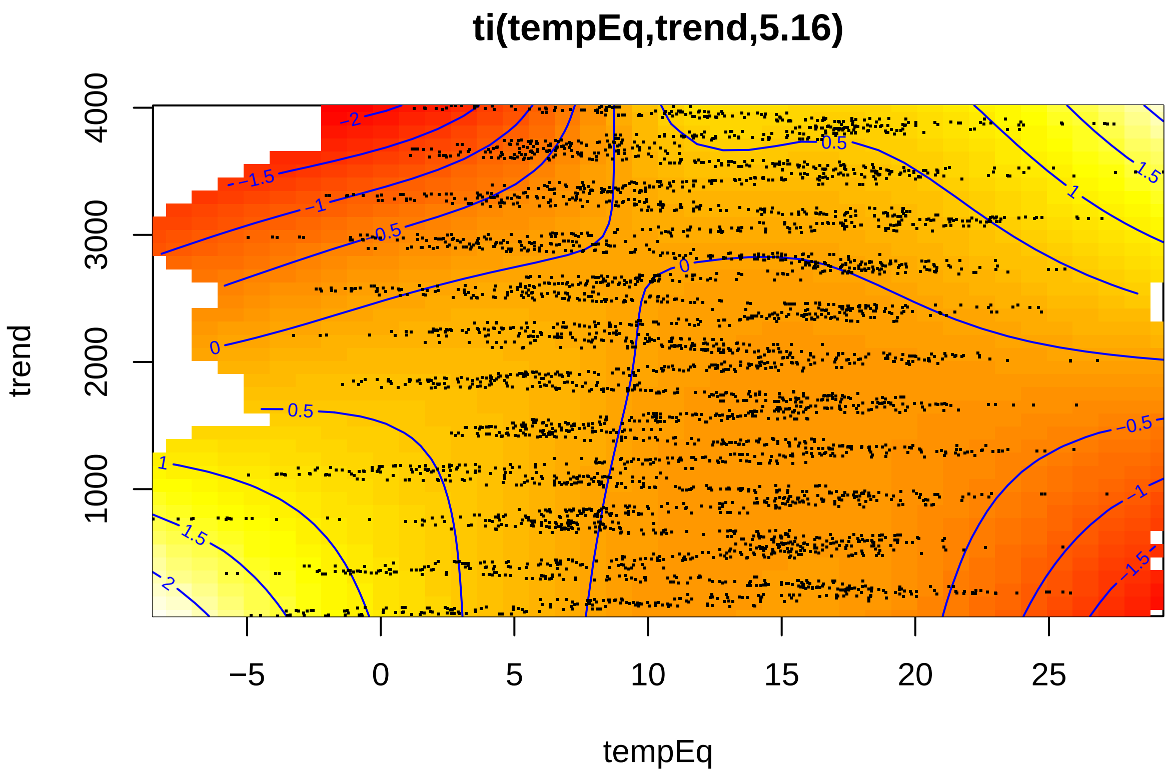


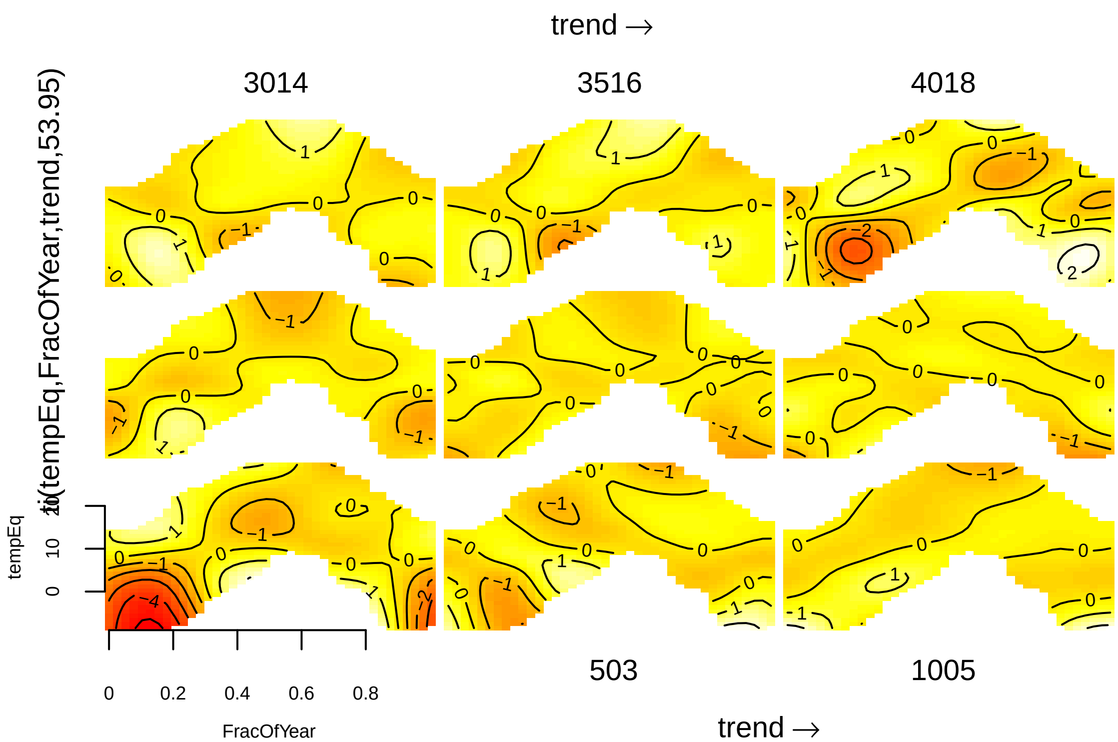
s(FracOfYear,12.71)
30
20
10
0
−10
0.0
0.2
0.4
0.6
0.8
1.0
FracOfYear


s(trend,5.8)
−10
0
10
20
30
0
1000
2000
3000
4000
trend

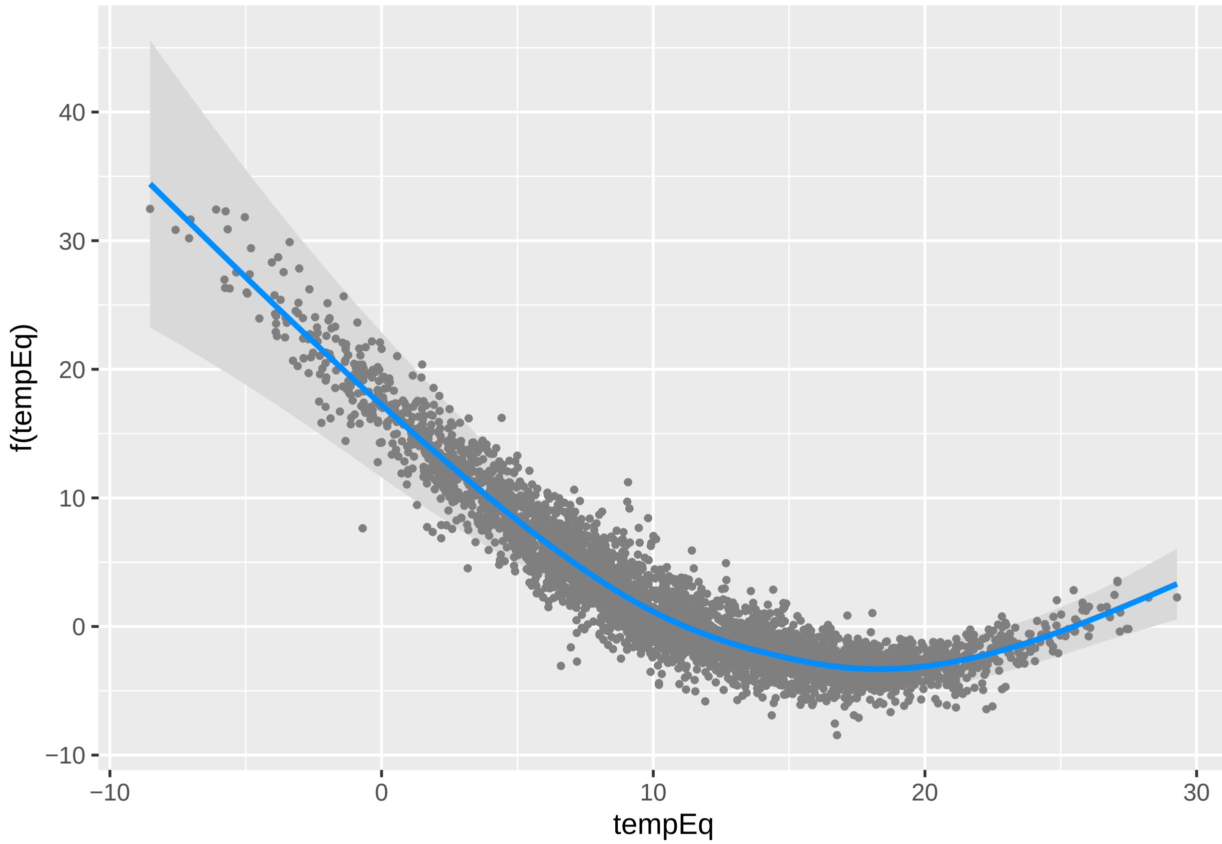

ti(tempEq,FracOfYear,14.08)
FracOfYear
0.0
0.2
0.4
0.6
0.8
−5
0
5
10
15
20
25
tempEq


ti(trend,FracOfYear,42.08)
FracOfYear
0.0
0.2
0.4
0.6
0.8
1000
2000
3000
4000
trend

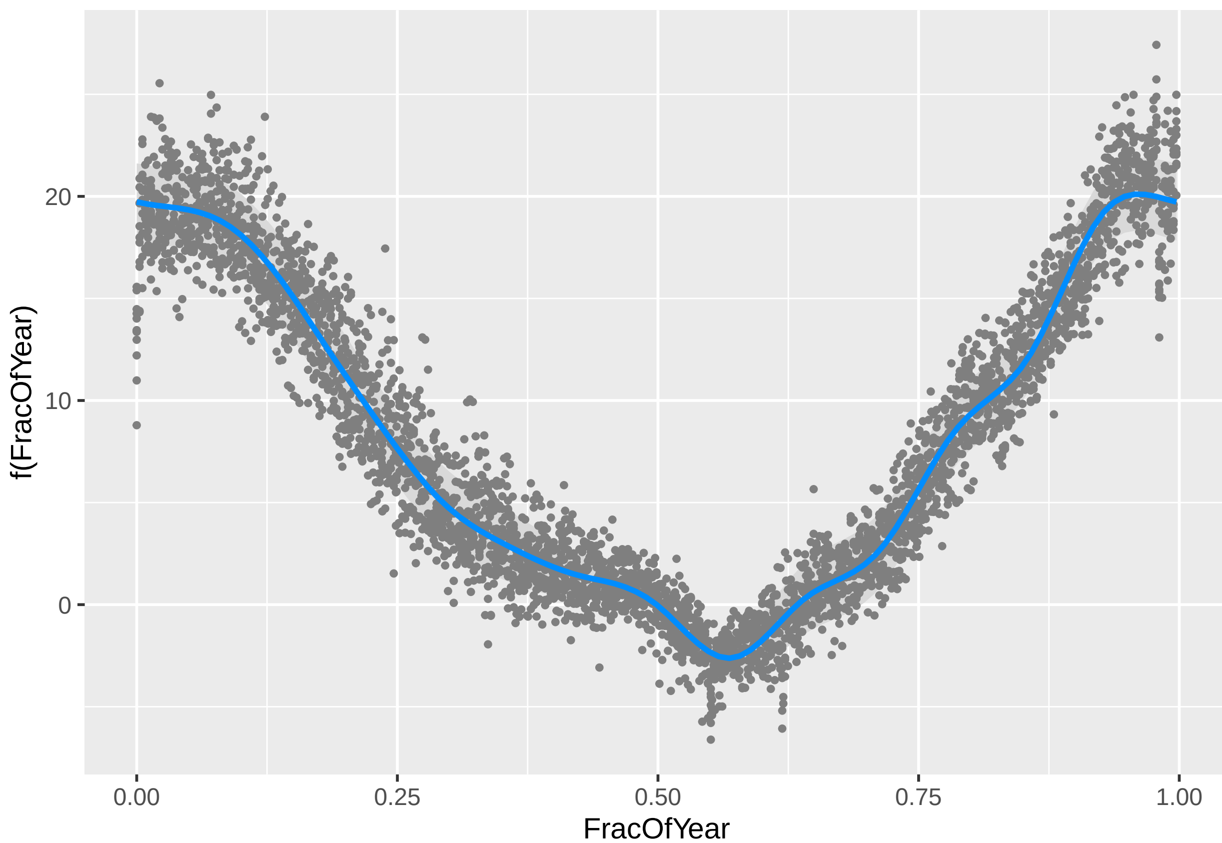

ti(tempEq,trend,5.16)
trend
4000
3000
2000
1000
−5
0
5
10
15
20
25
tempEq


trend →
3014
3516
4018
ti(tempEq,FracOfYear,trend,53.95)
tempEq
0
10
20
0
0.2
0.4
0.6
0.8
FracOfYear
503
1005
trend →

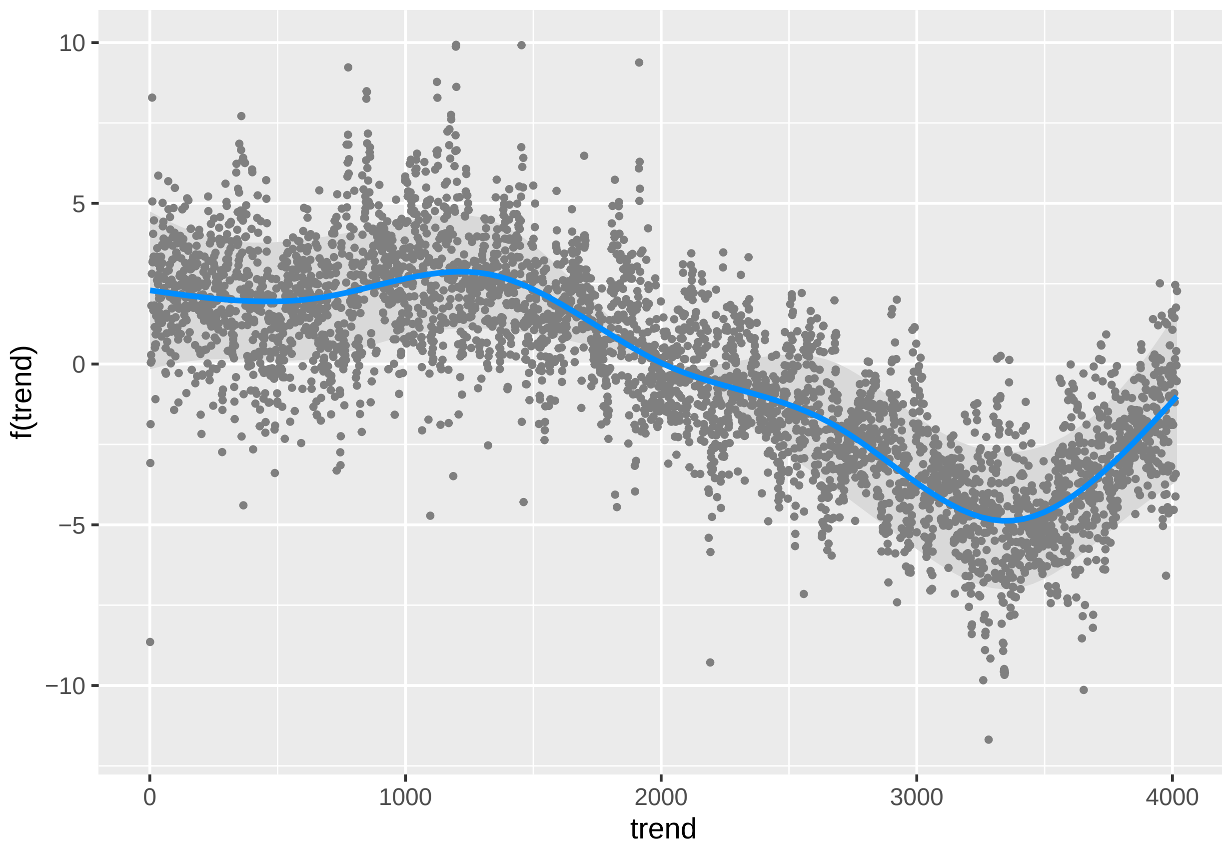
```
## [[1]]
```



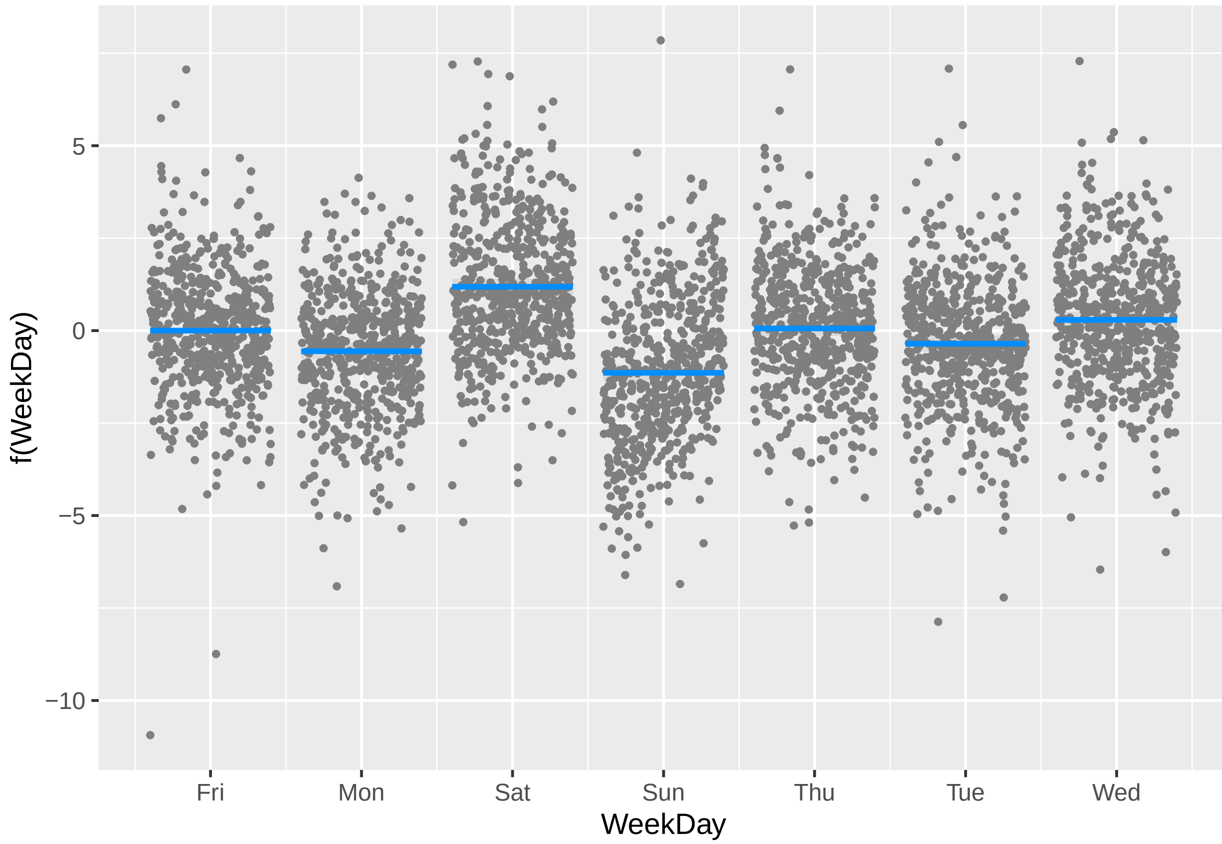

```
##
## [[2]]
```



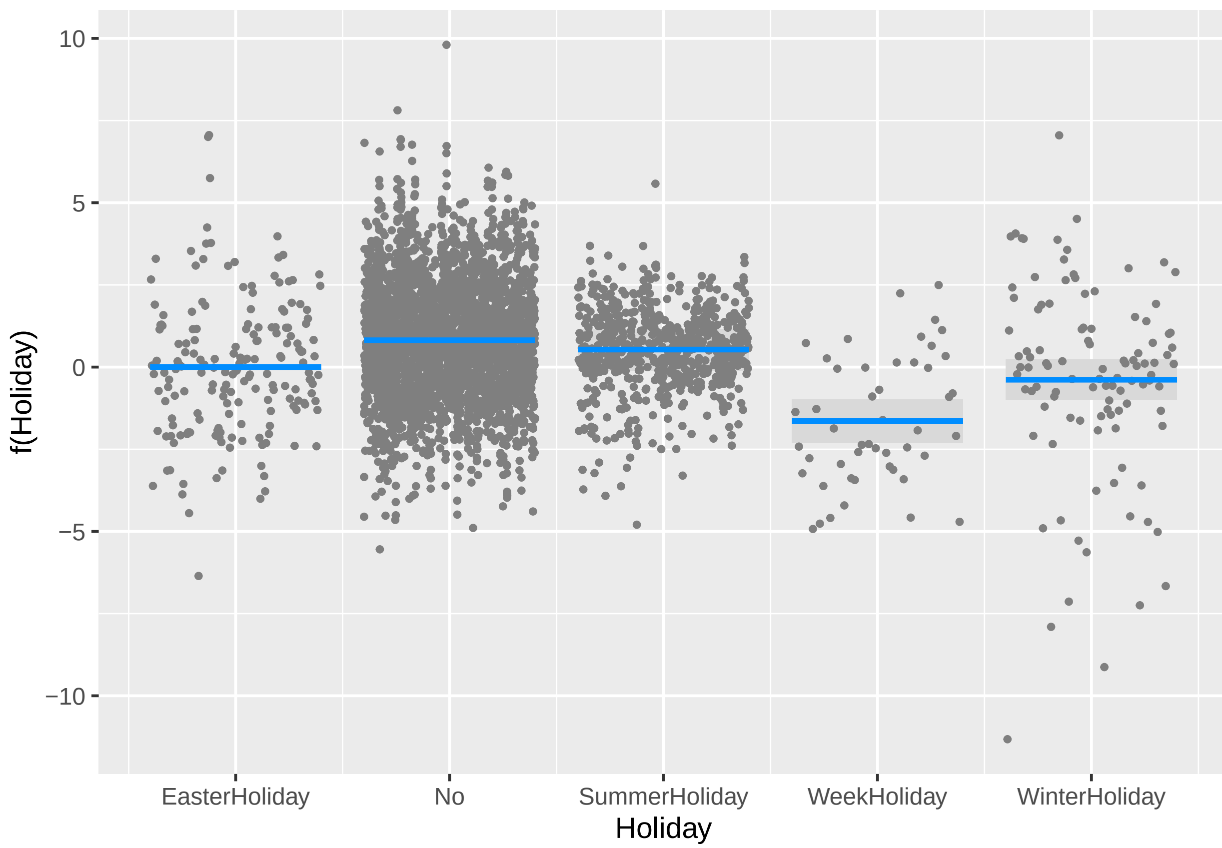

```
##
## [[3]]
```

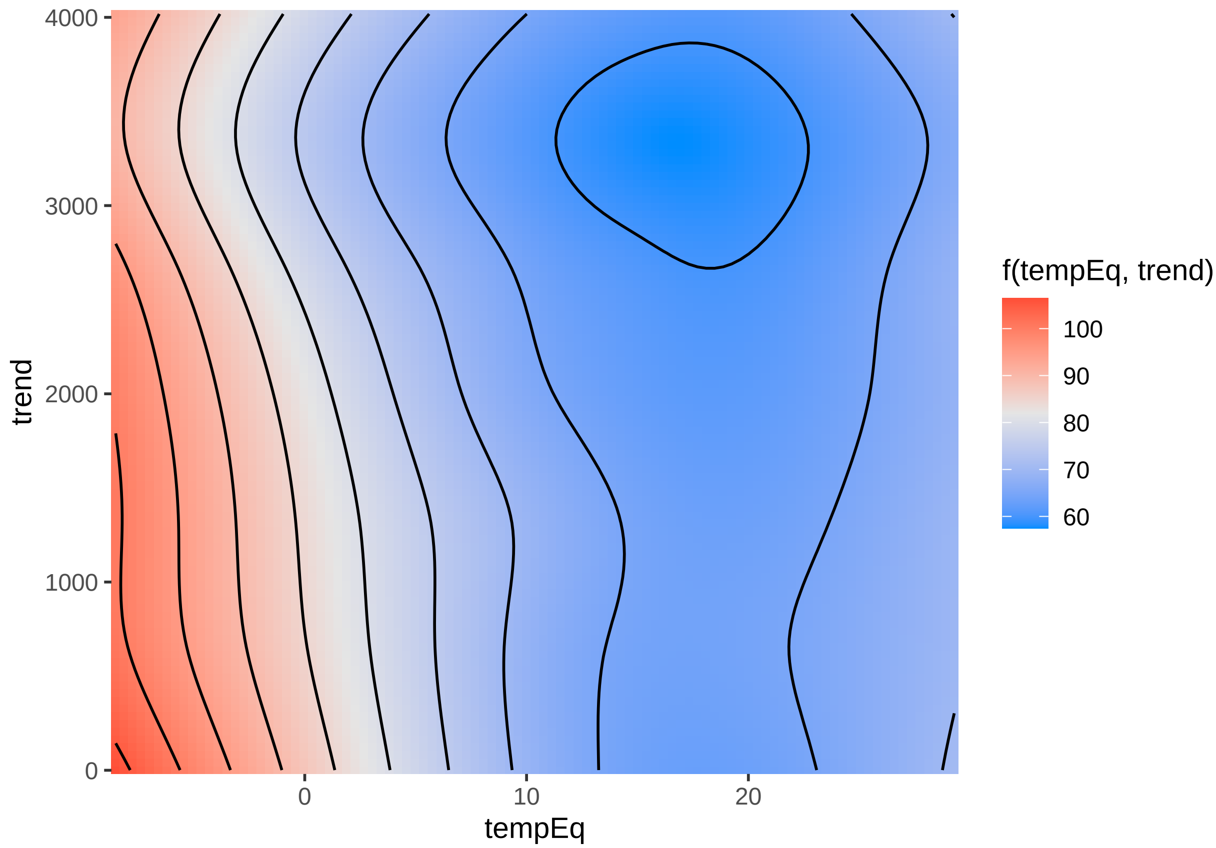


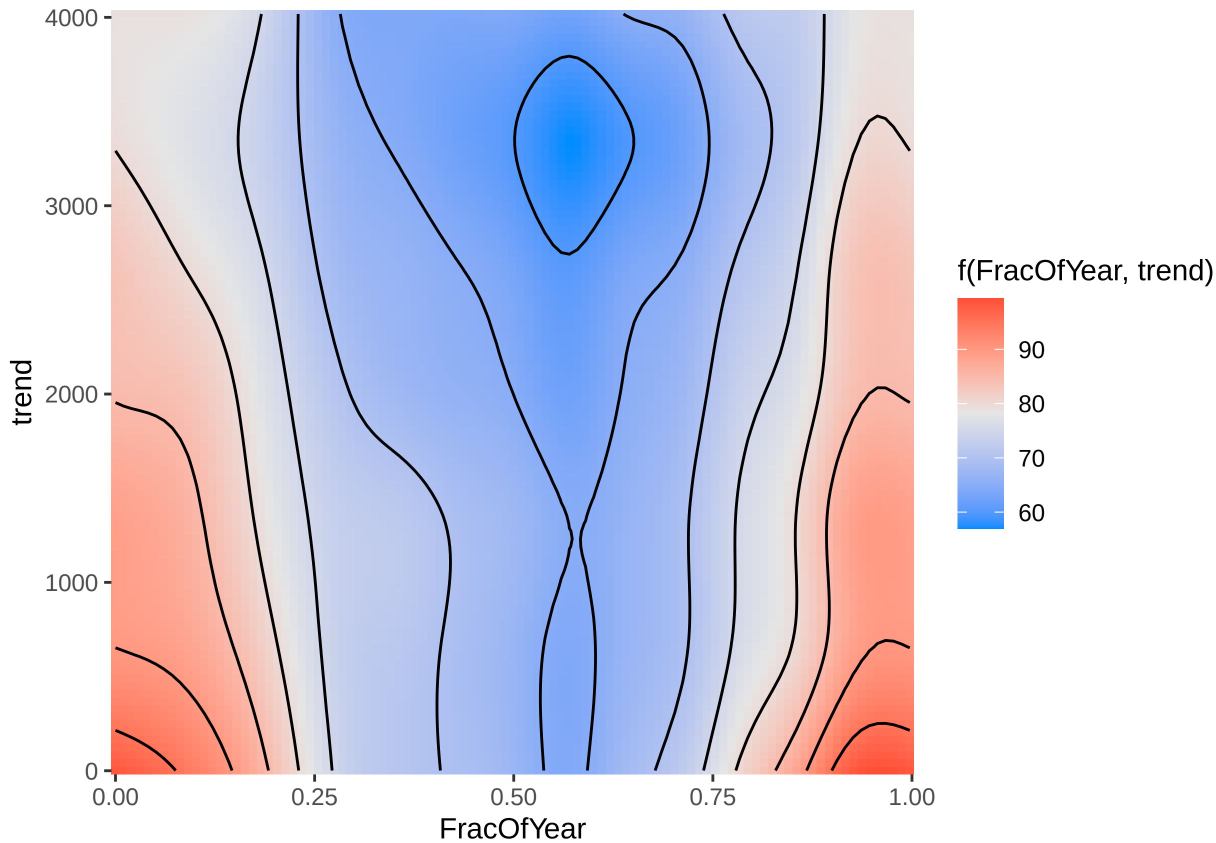

```
##
## [[4]]
```



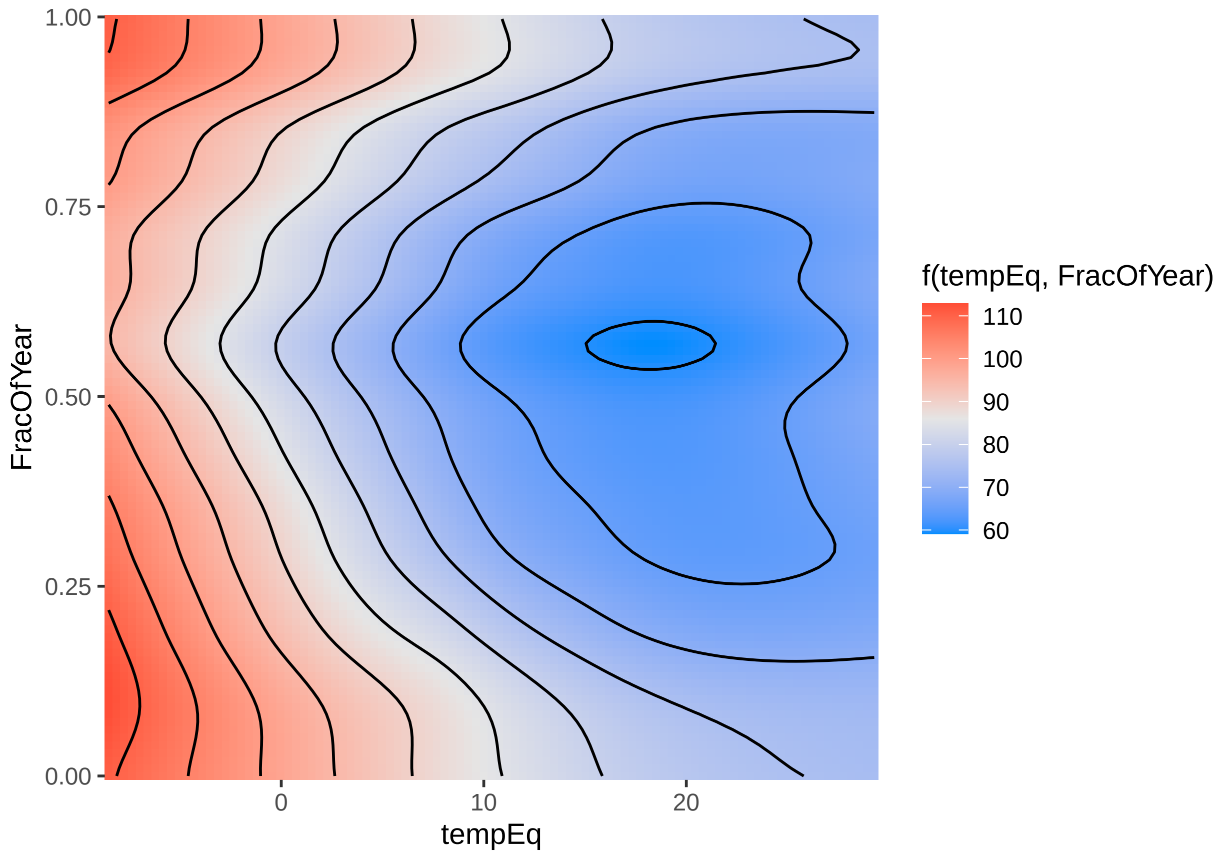

```
##
## [[5]]
```


10
5
0
−5
−10
f(Holiday)
EasterHoliday
No
SummerHoliday
WeekHoliday
WinterHoliday
Holiday


4000
3000
2000
1000
0
trend
0
10
20
tempEq
f(tempEq, trend)
100
90
80
70
60



4000
3000
2000
1000
0
trend
0.00
0.25
0.50
0.75
1.00
FracOfYear
f(FracOfYear, trend)
90
80
70
60


1.00
0.75
0.50
0.25
0.00
FracOfYear
0
10
20
tempEq
f(tempEq, FracOfYear)
110
100
90
80
70
60