

# MUTUAL FUND FEE STRUCTURE WITH DECEPTIVE FEATURES

Author : *Matteo Sibilla*

Thesis Director : *Johannes Johnen*

Thesis Joint Director : *Laura Zanetti*

Reader : *Lorenzo Trimarchi*

Academic Year 2023-2024

## **In order to obtain the Joint Degree**

Master 120 en Sciences économiques, Orientation générale, Finalité  
approfondie (UCL/UNamur)

and

Laurea specialistica in Discipline Economiche e Sociali (Bocconi)



# Summary

- 1. **INTRODUCTION** ..... 3
- 2. **RELATED LITERATURE AND CONTRIBUTIONS** ..... 7
- 3. **BASELINE MODEL** ..... 13
  - 3.1. Capital Asset Pricing Model..... 13
  - 3.2. Extended CAPM Model with Fairness Concerns..... 16
- 4. **MUTUAL FUND FEE STRUCTURING** ..... 26
  - 4.1. Myopic Fund ..... 28
  - 4.2. Forward Looking Fund..... 34
- 5. **COMPARATIVE ANALYSIS** ..... 40
- 6. **IMPLICATIONS FOR FUND MANAGEMENT** ..... 45
- 7. **CONCLUSION** ..... 48
  
- REFERENCES**..... 50
- APPENDIX A**..... 58
- APPENDIX B** ..... 61
- APPENDIX C** ..... 66
- APPENDIX D**..... 68

## INTRODUCTION

The mutual funds sector plays a crucial role in the global financial system, providing an accessible and efficient means for individual and institutional investors to diversify their portfolios. As of the end of 2023, the global mutual fund industry managed over \$55 trillion in assets<sup>1</sup>. In the United States alone, mutual funds accounted for approximately \$25 trillion in assets under management, representing about 22% of U.S. household financial assets (Statista, 2024a). With the number of open-end funds worldwide increasing by more than 55 percent from 2011 to 2022 (Statista, 2024b), a mutual fund is typically described as a collective investment vehicle that aggregates capital from multiple investors to purchase a diversified portfolio of assets, including stocks, bonds, and short-term debt instruments (U.S. Securities and Exchange Commission, 2021). Most mutual funds charge annual fees consisting of two components: a fixed percentage of assets under management and an incentive fee that is a percentage of the fund's annual net profits, which is often defined as the fund's total earnings above and beyond some minimum threshold such as the LIBOR<sup>2</sup> return (Grinblatt and Titman, 1989). The U.S. Securities and Exchange Commission (SEC), has recently underscored the importance of transparency in fee disclosure to protect investors from unexpected costs that are not immediately apparent<sup>3</sup>. More generally, the SEC has expressed concerns regarding deceptive elements within the fee structures of investment funds, which can erode investor returns over time, particularly when compounded over long periods<sup>4</sup>. Despite the growing concerns of regulators worldwide and reports in the popular press, the economic literature offers limited insights into how the time horizon considered by mutual funds influences their decisions to shroud fees and the resulting impact on profitability. Precisely, how does the time horizon influence mutual funds' pricing strategies? What are the implications of the interaction between shrouded and transparent fees on a fund's profitability and long-term stability?

---

<sup>1</sup> See Investment Company Institute. (2024). "2024 Investment Company Fact Book."

<sup>2</sup> The London Interbank Offered Rate (LIBOR) is a benchmark interest rate used by major global banks to lend to one another in the international interbank market for short-term loans. It serve as a reference rate for a wide range of financial products, including mortgages, student loans, derivatives, and corporate loans. However, LIBOR faced significant scrutiny due to concerns about its reliability and vulnerability to manipulation. As a result, financial markets have increasingly transitioned to alternative reference rates, such as the Secured Overnight Financing Rate (SOFR) in the United States, as part of a global effort to adopt more transparent and robust benchmarks.

<sup>3</sup> See <https://www.sec.gov/investment/private-fund-advisers#:~:text=The%20quarterly%20statement%20rule%20requires,related%20persons%20by%20the%20fund.>

<sup>4</sup> See [https://www.law.com/nationallawjournal/2023/08/22/sec-set-to-require-fund-advisers-to-disclose-performance-fees-expenses/?slreturn=20240703053632.](https://www.law.com/nationallawjournal/2023/08/22/sec-set-to-require-fund-advisers-to-disclose-performance-fees-expenses/?slreturn=20240703053632)

While theories on optimal fund fees are rather scarce (see, e.g., Nanda, Narayanan, and Warther, 2000; Das and Sundaram, 2002; Berk and Green, 2004; Pastor and Stambaugh, 2012), the existing literature primarily addresses the relationship between fixed and incentive fees. This study seeks to fill this gap by developing a theoretical framework that combines an investor's optimal allocation model with a fund profit maximization model. For instance, Gabaix and Laibson (2006) illustrate why even competing firms might offer deceptive products, and Heidhues, Koszegi, and Murooka (2016) argue that the presence of superior products like index funds renders deception by managed mutual funds more profitable. This paper extends these concepts to a model of mutual fund pricing, demonstrating how the consideration of the time horizon influences pricing strategies. It examines whether a fee structure<sup>5</sup> that includes both transparent and hidden components affects the time-dependent profitability of a mutual fund. Additionally, it investigates how varying levels of transparent and hidden fees influence the fund's profitability under two scenarios: when the fund adopts a myopic perspective, focusing on short-term profit maximization, and when it takes a long-term approach, aiming to sustain profitability over an extended period. Moreover, despite innovative efforts within the industry, mutual funds face significant challenges in maintaining longevity. According to the Global Asset Management Report 2024 by Boston Consulting Group<sup>6</sup>, a mere 37% of mutual funds launched in 2013 were still active a decade later, in 2023. This marks a notable decline from 2010, when 60% of funds established ten years earlier remained in operation. How does fee structuring influence the survival rate of mutual funds? Should funds strategically shroud or unshroud fees to enhance their long-term viability? The analysis reveals that a higher proportion of hidden fees relative to transparent fees is associated with elevated short-term profitability; however, this approach gradually undermines the fund's long-term viability, eventually leading to non-profitability. Conversely, a greater reliance on transparent fees may result in lower initial profits, but it fosters more stable and consistent returns, ultimately enhancing the fund's longevity and financial sustainability.

I build on the work of Cooper et al. (2021), who identified persistent and pervasive fee dispersion in the mutual fund industry and demonstrated that these fees have significant economic effects on investors, fund performance and fund survival rate. Using a Capital Asset

---

<sup>5</sup> Both transparent and hidden fixed fees are calculated as a percentage of the total assets under management of the fund, consistent with findings from previous research (see Getmansky et al., 2015).

<sup>6</sup> See <https://web-assets.bcg.com/a3/8a/9a5b5365468d8e5db5993160fa2a/2024-gam-report-may-2024-r.pdf>.

Pricing Model (CAPM) alpha<sup>7</sup>, they assessed fee relevancy by estimating the average net alpha across a sample of U.S. and international equity funds from 1980 to 2017. Their findings indicate that funds with higher fees exhibit lower CAPM net alphas, suggesting that fees are indeed important from an investor's perspective. Building on their methodology, I develop a CAPM model that describes the capital allocated by investors to a fund, considering the presence of both transparent and hidden fees. Transparent fees are visible to investors, allowing them to accurately assess the risk premium<sup>8</sup> associated with the fund. In contrast, hidden fees are not visible to investors and therefore are not factored into their optimal allocation decisions. Previous research (Zahera and Bansal, 2018) indicates that investors' decisions are heavily influenced by recent events and past performances. For instance, when investors encounter unexpectedly low returns, they are likely to reduce their holdings in that asset. Conversely, when they experience returns that exceed their expectations, they tend to increase their exposure to that asset (Kuchler and Stroebel, 2021). Hidden fees introduce unexpected variations in returns, deviating from investors' expectations<sup>9</sup>. Holding other factors constant, hidden fees incurred in period  $t$  result in lower-than-expected returns in period  $t + 1$ . To incorporate this effect into the CAPM model, I draw on Su (2023) work on consumer fairness concerns. In his two-sided platform model, fairness concerns are defined by the difference between prices charged to loyal and non-loyal consumers, adjusted for consumers' aversion to perceived unfair treatment. Similarly, I characterize investors' fairness concerns as the discrepancy between expected returns and actual received returns, adjusted for investors' aversion to unexpected variations in returns.

The extended CAPM model, incorporating fees and fairness concerns, allows me to define and characterize the total capital that investors allocate to the fund, referred to as the total Assets Under Management (AUM). It is crucial to note that changes in the fee structure impact the fund's profits through two main channels: direct and indirect. The direct channel indicates that

---

<sup>7</sup> In the context of the Capital Asset Pricing Model (CAPM), alpha refers to the measure of an investment's performance relative to a market index or benchmark. Specifically, alpha indicates the excess return of an investment compared to the expected return predicted by the CAPM.

<sup>8</sup> The risk premium is the additional return an investor expects to receive from holding a risky asset, as opposed to a risk-free asset. It compensates the investor for taking on the higher risk associated with the investment.

<sup>9</sup> Investors generally accept that returns may vary due to market risk, which typically does not invoke fairness concerns as these fluctuations are inherent in the investment process. However, when actual returns fall short of expectations due to hidden fees—costs not anticipated in their initial expectations—investors may perceive this as unfair. To isolate the impact of hidden fees from market variability, I assume a constant expected return across all periods (see *Assumption 2* in *Section 3.2*). This approach specifically highlights fairness concerns linked to hidden fees, calculated as the difference between expected and actual returns, since these fees appear in the actual return equation but not in the expected return equation.

higher fees directly increase revenue. Conversely, the indirect channel shows that higher fees result in a lower AUM due to a reduced risk premium, leading to decreased optimal allocation by investors (Kumples and McCrae, 1999; Blake et al., 1999). Therefore, the fund must balance this trade-off, as excessively high fees can lead to a significant reduction in AUM, jeopardizing profitability. I then integrate this model with a profit maximization framework to analyze the effects of fee structure on the fund's profitability under two scenarios. First, I examine a myopic fund that maximizes its immediate next period profits without considering future periods, thereby focusing on short term profitability. Next, I conduct the analysis on a forward-looking fund that accounts for long-term profitability. I find that the interplay between transparent and hidden fees within a fund's fee structure significantly influences its profitability. Specifically, a myopic fund, focused solely on short-term gains, maximizes its immediate profits by increasing hidden fees and reducing transparent fees. Lower transparent fees attract more assets under management (AUM) in the short term, while the negative impact of hidden fees is deferred to future periods, which the myopic fund disregards. This strategy, although increasing short-term profits, ultimately leads to the fund's non-profitability and potential bankruptcy over time. Conversely, a forward-looking fund, which considers the long-term effects of its fee structure, tends to minimize hidden fees and increase transparent fees to maintain an overall balanced fee level. This approach accounts for the detrimental future impact of hidden fees. Additionally, the level of hidden fees is inversely proportional to the length of the forward-looking fund's time horizon. As a result, the forward-looking fund maintains stable profitability over the long run, avoiding bankruptcy.

The structure of the paper is as follows. *Section 2* reviews the related literature and outlines the main contributions of this study. *Section 3* introduces the baseline framework and extends the CAPM model to incorporate investors fairness concerns and fund fee structure. *Section 4* discusses the structuring of mutual fund fees and examines its implications under the scenarios of both myopic and forward-looking funds. *Section 5* provides an extended analysis and comparison between these two types of funds. *Section 6* explores implications for fund management, and *Section 7* concludes with a summary of findings and recommendations for future research as well as for policy regulators.

## 2 RELATED LITERATURE AND CONTRIBUTIONS

This paper builds upon two disjoint literatures, namely those on mutual fund's fee structure and shrouded fees, both of which are discussed below. Additionally, since this study employs the Capital Asset Pricing Model (CAPM) as a foundational framework for analyzing investor behavior, the relevant core literature on CAPM will be reviewed at the end of this section.

*Literature on Mutual Funds' Fee Structure.* The literature is broadly divided on whether mutual fund fees impact investors. Neoclassicists argue that in perfectly competitive markets, fees do not affect net returns (Khorana, Servaes and Tufano, 2009; Wahal and Wang, 2011; Cremers et al., 2016; and Hoberg, Kumar, and Prabhala, 2018), while others suggest that fees significantly influence investor outcomes, particularly in imperfect markets (Haslem, Baker, and Smith, 2006; Gil-Bazo and Ruiz-Verdu, 2009; and Barras, Scaillet, and Wermers, 2010). This paper aligns with the latter view, emphasizing the role of fees in affecting fund profitability and investor decisions. In the neoclassical view of mutual funds (Berk and Green, 2004; Berk and van Binsbergen, 2015; Zhu, 2018; Pastor, Stambaugh, and Taylor, 2020; Stambaugh, 2019) with competitive markets and rational investors, in an equilibrium state where fund size adjusts based on investor flows, the gross returns before fees (gross-of-fee alphas)<sup>10</sup> will, on average, equal the fees charged. Consequently, the net returns after fees (net alphas) will average out to zero<sup>11</sup>. This suggests that, in a perfectly competitive market with rational investors, mutual fund fees should not affect the overall performance for investors because any potential gains are offset by the fees (Ippolito and Turner, 1987; Pastor, Stambaugh, and Taylor, 2020). For instance, Berk and van Binsbergen (2015) demonstrate through a study of U.S. and international equity funds that while the average gross value added is positive—indicating fund manager skill—the net-of-fee alphas are zero. This finding aligns with the neoclassical perspective that suggests fund fees do not impact investors. In contrast to the perfectly competitive market model proposed by neoclassicists, several studies highlight the imperfections in the mutual fund market and suggest that fees do indeed matter for investors. Two important papers, Elton,

---

<sup>10</sup> "Gross-of-fee alphas" are the returns generated by a mutual fund before deducting fees, while "net alphas" are the returns after deducting fees.

<sup>11</sup> The idea here is that in a competitive market, mutual fund managers who can generate high gross returns will attract more investors, increasing the fund's size. However, as the fund grows, the manager's ability to maintain high returns diminishes due to factors like market impact and liquidity constraints. Consequently, the additional returns (alpha) generated by the manager are just enough to cover the fees charged to investors.

Gruber, and Busse (2004) and Hortacsu and Syverson (2004), provide evidence of a non-perfectly competitive mutual fund industry by documenting substantial price dispersion<sup>12</sup> among essentially identical S&P 500 index funds. Their findings suggest that mutual fund markets are not perfectly competitive and that fees do matter to investors<sup>13</sup>. To explain the substantial price dispersion found by these two papers, Elton, Gruber, and Busse (2004) argue that the failure of the law of one price<sup>14</sup> in the S&P 500 index fund market can be attributed to arbitrage combined with the presence of uninformed investors who fail to accurately account for market characteristics and fund cost structures. In a similar vein, Gil-Bazo and Ruiz-Verdú (2008) suggest that fund managers with varying levels of ability target different investor segments. They argue that high-ability managers tend to attract sophisticated investors, who are more likely to scrutinize fee structures, thereby driving down fees due to competitive pressures. Conversely, lower-ability managers target less sophisticated investors, allowing them to charge higher fees by obscuring additional costs and leveraging hidden components. These two papers suggest that shrouding practices and the targeting of investors based on their level of sophistication could be a significant factor behind the substantial fee dispersion observed in the mutual fund industry. This implies a concrete effect of shrouding on both fee levels and fund profitability. Anagol and Kim (2012), in their study of the Indian mutual fund sector, further support this observation by demonstrating that closed-end funds, which were permitted to charge higher shrouded fees, saw a corresponding short-term increase in revenue. However, the specific conditions under which these practices manifest and their precise impact on fund profitability remain unclear (Alda and Ferruz, 2012). This study seeks to address this gap by providing a framework that examines the implications of shrouding in mutual fund management. Furthermore, it is important to acknowledge that there are alternative explanations for the observed fee dispersion in mutual funds. For instance, Hortacsu and Syverson (2004) attribute fee dispersion to non-financial fund differentiation and search frictions, while Cremers, Ferreira, Matos, and Starks (2016) argue that varying competitive pressures across different countries also play a critical role in shaping mutual fund fees. Similarly, Nanda, Wang, and Zheng (2009) propose other contributing factors, indicating that

---

<sup>12</sup> Price dispersion refers to the variation in prices or fees charged for essentially similar products or services in a market. In this context, price dispersion refers to the variability in fees that different funds charge for managing similar portfolios or providing comparable investment services. This phenomenon suggests that despite offering similar products, different funds charge different fees.

<sup>13</sup> This is a direct consequence of their finding that investors do not always benefit equally from competitive pressures, which lead to significant variations in fees and, consequently, net returns.

<sup>14</sup> The "law of one price" is a financial theory that states that in an efficient market, identical goods should have only one price when there are no transaction costs and no barriers to trade.

fee dispersion is a multifaceted issue within the industry. These later papers do not consider shrouding as a factor contributing to fee dispersion and the variability in fund profitability. The literature on fee structuring in mutual funds, particularly when deceptive features such as shrouded fees are involved, is notably diverse and lacks consensus. This work contributes to this ongoing debate by shedding light on the impact and implications of the interplay between shrouded and transparent fees on fund profitability and long-term stability. The model developed in this study integrates the presence of these fees into the risk premium within the framework of the Capital Asset Pricing Model. This approach aims to provide a more nuanced understanding of how these fees influence fund performance and investor behavior.

*Literature on Shrouded Fee and Investors Naivete.* The largest area of study in behavioral industrial organization examines scenarios where profit-maximizing firms, who have a comprehensive understanding of their operational environment and can accurately predict consumer behavior, interact with consumers who consistently make systematic errors. Existing models of consumer mistakes suggest that, beyond misinterpreting prices or products, consumers also make strategic errors (Gabaix and Laibson, 2006). Specifically, they fail to recognize their own naivete when evaluating the products and contracts offered by firms (Heidhues and Kőszegi, 2018). The literature on pricing strategies and firm profitability is extensive within the two-sided market sector, both within cross-group price discrimination (Caillaud & Jullien, 2003; Rochet & Tirole, 2003; Armstrong, 2006; Belleflamme & Johnen, 2023) and within-group price discrimination (e.g., Liu & Serfes, 2013; Carroni, 2018; Shiller, 2020). However, it is not well-defined in the financial landscape of capital management and mutual funds, where it usually focuses on the relationship between fixed fee and incentive fee (Khorana, Servaes and Tufano, 2009)<sup>15</sup>. The impact of fee contracts and performance fees, in fact, has been extensively explored in both theoretical (e.g., Grinblatt and Titman (1987), Grinblatt and Titman (1989); Kritzman, 1987; Starks, 1987; Heinkel and Stoughton, 1994; and Lynch and Musto, 1997) and empirical research (e.g., Golec (1992), Baucus et al. (1993); Brown et al., 1996; Coles et al., 2000), while the impact of hidden fee on mutual fund's profitability is scarce (Agarwal et al., 2017; Heidhues et al., 2016). This study seeks to address this gap in the literature by investigating how the interplay between transparent and hidden fees impacts a fund's profitability. Specifically, it incorporates shrouded fees into the investor's optimal allocation problem, thereby advancing the understanding of how these deceptive fee

---

<sup>15</sup> For further discussion see Malhotra and McLeod (1997), Dobronogov and Murthi (2005).

structures influence fund performance and longevity. Gabaix and Laibson (2006) offer a foundational framework for competing firms deceptive pricing strategies adoption, while Johnen and Somogy (2024) further develop this by distinguishing between transparent and hidden fees. This paper builds upon and extends these concepts by applying them specifically to the pricing strategies of mutual funds. The findings contribute to a broader comprehension of the dynamics between fee structuring and capital allocation in the context of mutual funds. Both intuition and substantial empirical evidence indicate that firms in various markets often have a deeper understanding of consumer behavior than the consumers themselves (Heidhues and Kőszegi, 2018; Nair and Antony, 2015; Thaler, 1980). Indeed, as shown by Cruickshank (2000), who presents findings from a U.K. Treasury survey on high fees within the U.K. banking sector, the vast majority of people have no information about the fees for common financial services at their own bank<sup>16</sup>. This enables firms to take advantage of consumer misunderstandings by increasing prices, shrouding information and offering suboptimal products. Notable examples are credit markets (Ausubel, 1991; Agarwal et al., 2008; Stango and Zinman, 2009, 2014) and retail banking (Competition and Markets Authority, 2017; Cruickshank, 2000; Office of Fair Trading, 2008). Alan et al. (2018), argue that when consumers underestimate and are inattentive to overdraft costs, firms respond by shrouding overdraft prices and setting high hidden fees. This is consistent with Johnen (2019), who argues that consumer naivety, often due to overconfidence or lack of awareness, allows firms to exploit these tendencies by raising prices. Empirical evidence that consumers often fail to choose the best price, and the implications for firm behavior, are discussed in detail in Grubb (2015). As Englmaier et al. (2012) illustrate, the discovery of paying more than others due to shrouded attributes provokes fairness concerns among consumers. Similarly, Brown et al. (2018) indicate that fairness concerns arise in investment decisions when investors' expectations are unmet<sup>17</sup>. Building on the standard CAPM model, I follow Brown et al. (2018) and Su, J. (2023) to integrate investors' fairness concerns arising from lower-than-expected returns at  $t + 1$  due to hidden fees set by funds in period  $t$ . A key contribution of the model developed in this paper is applying deceptive pricing concepts to a model of mutual-fund pricing, showing how the time horizon taken into account by funds affects their pricing strategy. Moreover, the model

---

<sup>16</sup> Respondents were asked “*How accurately do you feel you know the charge for services on your account?*” Response categories included: “*Exactly,*” “*Roughly,*” “*Not at all,*” and “*Did not apply.*” About half of respondents choose “*Not at all.*”

<sup>17</sup> For an in-depth exploration of fairness concerns and their impact on economic behavior, refer to Fehr et al. (1993), who demonstrated the influence of fairness on market prices in auctions, and Rabin (1993), who extensively discusses fairness in economic theory.

explicitly accounts for how hidden fees generate fairness concerns among investors in future periods. This incorporation of fairness dynamics offers a novel perspective on how fee structures influence investor behavior and fund performance over time.

*Literature on Capital Asset Pricing Model.* Asset pricing theories have its origin in what is widely known as Capital Asset Pricing Model, a foundational theory in financial economics introduced independently by William Sharpe (1964) (Sharpe, William F. 1964) and John Lintner (Lintner, J. 1965; Lintner, J., 1975), with the contributions of Treynor (1961) and Mossin (1966) on the relationship between the systematic risk and expected return of an asset. The paramount importance of CAPM, which provides a framework for understanding the relationship between expected return and risk in financial markets, results in a Nobel Prize for Sharpe in 1990. The CAPM builds on Bernoulli's Expected utility theory on the trade-off between risk and return (Bernoulli, D., 2011; originally published in 1738; translated by Dr Louise Sommer, 1954) and on Harry Markowitz' mean-variance portfolio model (Markowitz, Harry, 1952; Markowitz, Harry, 1959). The model posits that investors are risk-averse and, when selecting portfolios, they focus solely on the mean and variance of their one-period investment returns. In Markowitz's framework, the risk associated with a portfolio is measured by the variance of its returns. Over the years, the model has been modified and extended to incorporate various features and assumptions. Merton's (1973) intertemporal capital asset pricing model (ICAPM) is an extension of the CAPM, in which investors are not only concerned about their end-of-period payoff, but also with the opportunities they will have to consume or invest this payoff. Breeden (1979) and Lucas (1978) extend the traditional CAPM and develop the Consumption Capital Asset Pricing Model (CCAPM) which links between consumption and stock returns, thus, it relies on the aggregate consumption in order to understand and predict future asset prices instead of the market portfolio's return in the traditional CAPM. Roll (1977), argues that the CAPM is not testable unless the market portfolio of all assets is used in the empirical test. The Arbitrage Pricing Theory (APT) of Ross, developed primarily by Ross (1976), has been proposed as a testable alternative to the CAPM. Roll and Ross (1980) explain that the APT demonstrates that since any market equilibrium must be consistent with no arbitrage profits, then every equilibrium will be characterized by a linear relationship between each asset's expected return and its return's response amplitude (Elbannan, 2015). Shefrin and Statman (2000) extended the model to develop a behavioral portfolio theory (BPT) and explore its implications for portfolio construction and security design, exploiting Friedman and Savage's (1948) observation on lotteries. Fama and French

(2003) introduced a crucial assumption that investors can borrow and lend at a uniform risk-free rate,  $r_f$ , which remains constant regardless of the amount borrowed or lent and is identical for all investors. I employ this assumption to construct a theoretical model that utilizes the CAPM to explain how investors allocate their capital to an investment vehicle when comparing it with a risk-free outside option. The model has undergone extensive empirical testing (Sharpe, W. F., & Cooper, G. M., 1972; Blume, 1970; Blume and Friend, 1973; Black et al., 1972; Fama and MacBeth, 1973), and is widely accepted for its validity and intuitive logic in explaining investor decision-making. In this work, I build upon the foundational model to investigate investor behavior. The model is expanded to include both shrouded and transparent fees, as well as considerations of fairness. This contribution further enhances CAPM-related models by integrating an intertemporal dimension where consumers respond to fees over time when making investment decisions.

### 3 BASELINE MODEL

I now introduce the baseline model. First, I describe the standard Capital Asset Pricing Model (CAPM), which serves as the foundation for the analysis. Next, I extend the model to include both transparent and hidden fees, as well as investors' fairness concerns. This extended model is then integrated in *Section 4* with a profit maximization framework. The analysis proceeds by examining two scenarios: first, a myopic fund that maximizes only immediate next-period profits without regard for future periods; and second, a forward-looking fund that aims to maximize its profits over a long-term horizon.

#### 3.1 CAPITAL ASSET PRICING MODEL

In the context of portfolio allocation, investors seek to optimize their returns by allocating their capital between risk-free assets and risky portfolios (Fama and French, 2003). This allocation strategy is traditionally guided by the principles of the Capital Asset Pricing Model, where the expected return is balanced against the associated risk. Investors choose the proportions allocated to the risk-free asset and the risky portfolio and allocate a percentage  $\alpha$  to the risky portfolio, indicated as  $p$ , and a percentage  $1 - \alpha$  to the risk-free asset, indicated as  $f$ . The return of the complete portfolio is therefore  $r_c = \alpha r_p + (1 - \alpha)r_f$ , and introducing the expectation operator  $E[r_c] = \alpha E[r_p] + (1 - \alpha)r_f$ , or re-arranging the terms  $E[r_c] = r_f + \alpha(E[r_p] - r_f)$ . The risk of the complete portfolio is identified by the standard deviation  $\sigma$ . Since, all the risk of the complete portfolio is due to the risky portfolio,  $\sigma_c = \alpha\sigma_p$ , and thus the variance of the complete portfolio is  $\sigma_c^2 = \alpha^2\sigma_p^2$ . From this, it can be written  $\alpha = \frac{\sigma_c}{\sigma_p}$ , and substituting this into the former equation yields:

$$E[r_c] = r_f + \sigma_c \frac{(E[r_p] - r_f)}{\sigma_p}$$

This equation represents the Capital Allocation Line (CAL), with  $E[r_p] - r_f$  being the Risk Premium and  $\frac{(E[r_p] - r_f)}{\sigma_p}$  the Sharpe Ratio or Risk-to-Reward Ratio. Since  $E[r_p] - r_f \geq 0$  and

$\sigma_p \geq 0$ , the Risk-to-Reward Ratio, which represents the slope of the equation, is a positive quantity. The CAL is represented as a line graph, with the y-axis being the expected return, and the x-axis the amount of risk, measured by the standard deviation of returns.

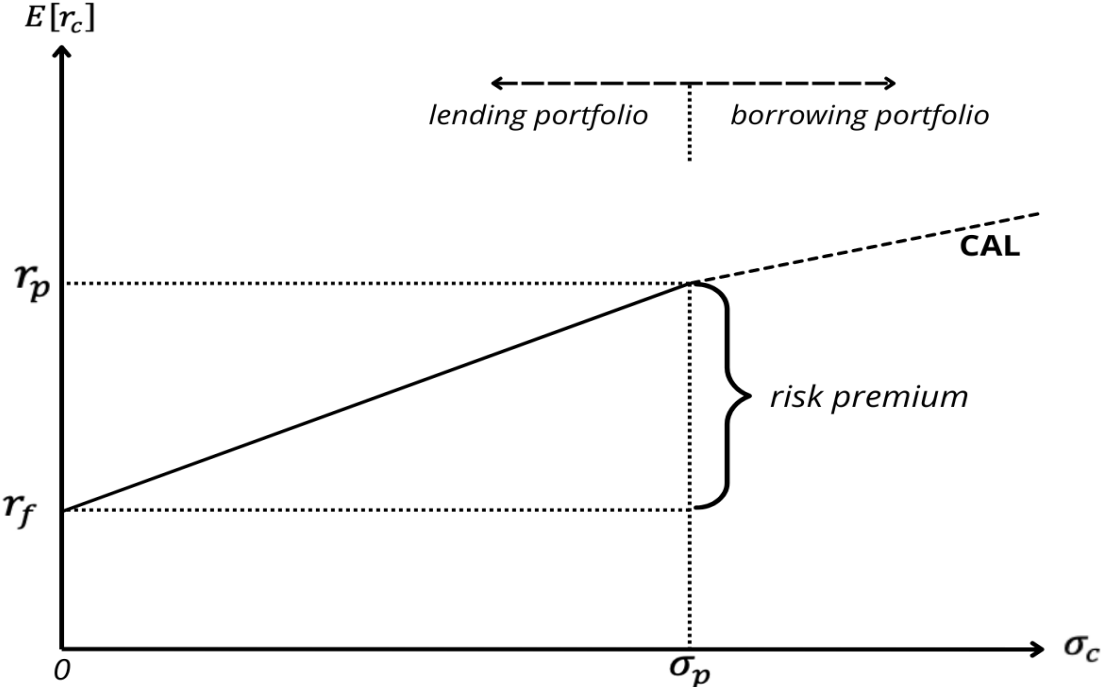


Figure 1: Capital Allocation Line.

The line displays the relationship between the risk and expected return of portfolios that combine a risk-free asset with a market portfolio of riskier assets. The starting point is the risk-free rate, where risk is zero, and the line extends upward to show increasing levels of risk and their corresponding expected returns. When investors allocate their entire capital to the risk-free asset ( $\alpha = 0$ ), they are positioned at the point  $(0, r_f)$  on the plane. Conversely, when investors allocate their full capital to the risky asset ( $\alpha = 1$ ), they are located at the point  $(\sigma_p, r_p)$  on the plane. Any allocation along the line connecting these two points represents a linear combination of the two assets. If  $\alpha > 1$ , it indicates that the investor is leveraging by borrowing additional funds to invest in the risky asset. Investors assign a certain utility to various investments. They derive a positive utility from the expected return of the complete portfolio, and a negative utility from the risk of the complete portfolio. The utility function is of the form  $U = E[r_c] - \frac{1}{2}\gamma\sigma_c^2$ , where  $\frac{1}{2}$  is an arbitrary scaling factor and  $\gamma$  is the coefficient of risk aversion. Investors maximize their utility by choosing the optimal allocation between the risky portfolio and the risk-free asset

$$\max_{\alpha} U = E[r_c] - \frac{1}{2}\gamma\sigma_c^2 = r_f + \alpha(E[r_p] - r_f) - \frac{1}{2}\gamma\alpha^2\sigma_p^2$$

Taking the partial derivative with respect to  $\alpha$ ,  $U' = (E[r_p] - r_f) - \gamma\alpha\sigma_p^2 = 0$ , and rearranging terms yields

$$\alpha^* = \frac{(E[r_p] - r_f)}{\gamma\sigma_p^2}$$

Investors thus allocate  $\alpha^*$  in the risky portfolio and  $1 - \alpha^*$  in the risk-free asset. The difference  $E[r_p] - r_f$  captures the excess return, or the risk premium, that investors anticipate receiving by investing in the risky asset instead of the risk-free asset. This risk premium is a key motivator for investors to consider risky assets, as it compensates for the additional risk undertaken. Thus, a reduced  $E[r_p]$  or an increased  $r_f$  reduces the risk premium, leading to a lower proportion of capital allocated to the risky portfolio (Sharpe, 1964; Lintner, 1975). In the denominator,  $\sigma_p^2$  is the variance of the return of the risky asset, which measure the asset's risk or volatility. A higher variance indicates greater risk associated with the asset's returns. The inclusion of  $\sigma_p^2$  in the denominator ensures that the allocation to the risky asset is inversely proportional to its risk; as the risk increases, the optimal allocation  $\alpha^*$  decreases. A higher  $\gamma$  indicates greater risk aversion, meaning the investor requires a higher risk premium to be willing to invest in the risky asset. Conversely, a lower  $\gamma$  suggest a higher tolerance for risk, resulting in a larger allocation to the risky asset.

I make the following assumption:

**Assumption 1.** The borrowing rate  $r^b$  and lending rate  $r^l$  are strictly positive quantities and are higher than the risk-free rate  $r_f$ .

1.  $r^b, r^l > 0$ .
2.  $r^b, r^l > r_f$ .

The main consequence is that,  $\alpha^*$  cannot be negative<sup>18</sup>, as this would suggest that investors are borrowing funds to invest in the risk-free asset. However, the risk-free asset offers the lowest possible return because it involves no risks, according to CAPM theory (Fama and French, 2003). Conversely, the borrowing rate is higher than the risk-free rate due to the borrower's solvency risk, which necessitates a risk premium. A negative alpha would imply that the investor is paying to invest in an asset that yields less, resulting in a certain loss. Even in the best-case scenario, where the borrowing rate equals the risk-free rate, investors would be borrowing at a rate equal to what they receive, leading to a guaranteed zero profit.

### 3.2 EXTENDED CAPM MODEL WITH FAIRNESS CONCERNS

I introduce the concept of the Mutual Fund as the vehicle for the risky portfolio. A Fund operates by pooling capital from multiple investors to create a larger, collective investment portfolio and strategically allocates the pooled capital across various asset classes and investment opportunities with the aim of maximizing returns while managing risk. From the investor's perspective, the risky asset considered in their optimal allocation problem coincides with the fund's portfolio. Investors then decide on the proportion of their portfolio to allocate between the fund and the risk-free asset. This inclusion allows to analyze how different fee structures can influence investors' optimal allocation strategies and the fund's profitability. The return generated by the fund is  $r_p$ , and the associated risk, measured in terms of standard deviation, is represented by  $\sigma_p$ . There are  $t$  periods, in which the fund, with marginal cost  $c \geq 0$ , invests the capital collected from investors. The fund sets transparent and hidden fees  $f_t$  and  $a_t \in [0, \bar{a}]$ , respectively. The assumption of a cap on the hidden fee follows the logic outlined by Morrison and Taubinsky (2023)<sup>19</sup>. To streamline the analysis, I assume through the main text a unit mass of naïve investors<sup>20</sup>, each investing a capital  $C = 1$ . Naïve investors take only transparent fees into account, but they end up paying also the hidden fee (Gabaix and Laibson, 2006; Armstrong and Vickers, 2012). The hidden fee captures unexpected lower returns (costs)

---

<sup>18</sup> The proof is presented in *Appendix A.1*.

<sup>19</sup> Their work on attention elasticities shows that some consumers systematically underreact to sales taxes whereas others systematically overreact, but higher stakes decrease both under- and overreaction. This is consistent with consumers using heterogeneous rules of thumb to compute the opaque tax when the stakes are low, but using costly mental effort at higher stakes. For further discussion see Gabaix and Laibson (2006), Heidhues and Köszegi (2018) or Johnen and Somogyi (2024).

<sup>20</sup> A discussion on the implications of integrating different types of investors into the analysis, along with recommendations for future research, is provided in *Section 7*.

of naïve investors. Investors maximize their perceived utility. First, consider the utility maximization problem of investors accounting for the transparent fee  $f$

$$\max_{\alpha} U = r_f + \alpha(E[r_p] - f - r_f) - \frac{1}{2}\gamma\alpha^2\sigma_p^2$$

Fairness concerns associated with shrouding practices are not included here but will be introduced in the subsequent analysis, where I extend the utility function to incorporate the impact of hidden fees on investors' utility. Here  $f > 0$ , indicates that a higher fee reduces the perceived utility, leading to a lower optimal allocation to the fund, as the optimal allocation  $\alpha^* = \frac{(E[r_p]-f-r_f)}{\gamma\sigma_p^2}$  is negatively correlated with  $f$ . According to the principles of the CAPM, when the expected return from the fund decreases, investors allocate less capital to the fund. This is consistent with the observations of Klumpes and McCrae (1999) and Blake et al. (1999), who noted that high fees, and thus a lower return from the fund, lead to a reduction in accumulated assets (AUM).

#### *Fairness Concerns.*

Christoffersen and Musto (2002) demonstrate that investors who are sensitive to performance often react to poor fund performance by either exiting the fund or adjusting their allocations accordingly. Recent research by Zahera and Bansal (2018) corroborate this finding by showing that indeed investors' decisions are significantly shaped by recent events and historical performance. Investors typically adjust their asset allocations based on the returns they experience. When returns are lower than expected, they tend to reduce their holdings, whereas higher-than-expected returns often lead to increased exposure (Kuchler and Stroebel, 2021). Since hidden fees are not apparent to naïve investors at time  $t$  but manifest their negative effects on returns in  $t + 1$ , they become a source of unexpectedly lower returns, thereby influencing investors' behavior through perceived unfairness (Fehr et al., 1993)<sup>21</sup>. I draw on Su J. (2023) work on consumer fairness concerns, to characterize investors' fairness concerns as a function of the difference between the expected return and the actual return. If investors receive a higher

---

<sup>21</sup> Investors who receive higher-than-expected returns from a fund experience an increased perception of fairness linked to higher perceived skills in active management (Stambaugh, 2019). This enhanced sense of fairness arises because the returns surpass their initial expectations, reinforcing their trust and satisfaction with the fund's performance. The positive discrepancy between expected and actual returns not only boosts their financial gains but also strengthens their belief in the fund's transparency and reliability, leading to a greater overall utility from their investment.

return than expected, they experience positive utility; conversely, if they receive a lower return, they experience negative utility, or disutility. Intuitively, when investors receive a higher-than-expected return, they are likely to increase their investment in the subsequent period, hoping for continued higher returns. Conversely, when they receive a lower-than-expected return, they tend to reduce their investment in the next period out of fear of further disappointment.

To streamline the analysis and focus on the effects of the fee structure on fund's profitability, I make the following assumption:

**Assumption 2.**  $E(r_{p,t}) = r_{p,t} = r_p$ .

This means that the expected return on the underlying assets of the fund  $E(r_{p,t})$  is constant and equal the actual return  $r_{p,t}$ , and both are equal to a constant value  $r_p$ . This assumption is made to neutralize the effects of varying market conditions and management's skill<sup>22</sup>, thereby allowing the analysis to focus solely on the impact of the fee structure. This simplification enables a clearer examination of how transparent and hidden fees influence investor behavior and fund profitability, without the added complexity of fluctuating asset performance.

Hidden fees, being unaccounted for by investors, effectively reduce the return they receive in the next period. Under *Assumption 2*, the expected return is consistently lower than the actual return, leading to the manifestation of fairness concerns as a disutility in the investor's utility maximization problem. The disutility, denoted as  $F_t(E_{t-1}(r_{c,t}), r_{c,t})$ <sup>23</sup>, arises from the unexpected lower returns caused by the hidden fees. This results in a consistent negative impact on the perceived utility, as the gap between expected and actual returns generates dissatisfaction and reduces overall utility. Intuitively, at time  $t$  investors expect to receive in  $t + 1$  a return  $E_t(r_{c,t+1})$ , but end up receiving  $r_{c,t+1} \leq E_t(r_{c,t+1})$ . This discrepancy arises from the investors'

---

<sup>22</sup> Previous research indicates that management skills are a primary factor influencing the returns generated by a fund's portfolio, significantly shaping investor behavior (Cuthbertson et al., 2016; Cremers et al., 2019; Berk and van Binsbergen, 2017). Stambaugh (2019) shows that greater skills allow managers to identify mispriced securities more accurately and thereby make better portfolio choices. A higher portfolio returns might induce higher fees to compensate for the excess risk premium over the benchmark. Greater skills also means, however, that active management corrects prices better and thus might reduce managers' return opportunities. *Assumption 2* removes these factors, isolating the determinants of variation in the expected returns of the complete portfolio  $E(r_c)$  to the fees set by the fund.

<sup>23</sup> In  $t = 0$  investors decide the optimal allocation based on available information and, since they do not take into account hidden fees, they do not experience negative utilities from unexpected lower returns. Thus,  $F_0() = 0$ . The first period in which they experience a positive disutility  $F_t() \geq 0$  is in  $t = 1$ .

failure to fully account for the entire fee structure, as they do not take into account the hidden fee (Gabaix and Laibson, 2006). At time  $t$ , investors expect to receive  $E_t[r_{c,t+1}] = \alpha_t^*(E_t[r_p] - f_t) + (1 - \alpha_t^*)r_f$ , but end up receiving  $r_{c,t+1} = \alpha_t^*(r_p - f_t - a_t) + (1 - \alpha_t^*)r_f$ . The difference between expected return and actual return is  $E_t[r_{c,t+1}] - r_{c,t+1} = \alpha_t^*a_t$ . Since  $\alpha_t^*a_t \geq 0$ ,  $r_{c,t+1} \leq E_t(r_{c,t+1})$ . *Lemma 1* summarizes these results<sup>24</sup>.

**Lemma 1.**

1.  $r_{c,t} = E_{t-1}(r_{c,t})$  for  $a_t = 0$ ;
2.  $r_{c,t} < E_{t-1}(r_{c,t})$  for  $a_t > 0$ .

The disutility persists over time but decreases at a diminishing rate, such that as time progresses, investors assign progressively lower weights to the disutility experienced in earlier periods<sup>25</sup>. Consequently, at time  $t = T$ , the disutility values follow the order  $F_1 < F_2 < \dots < F_T$ . I model the disutility equation in the following form:

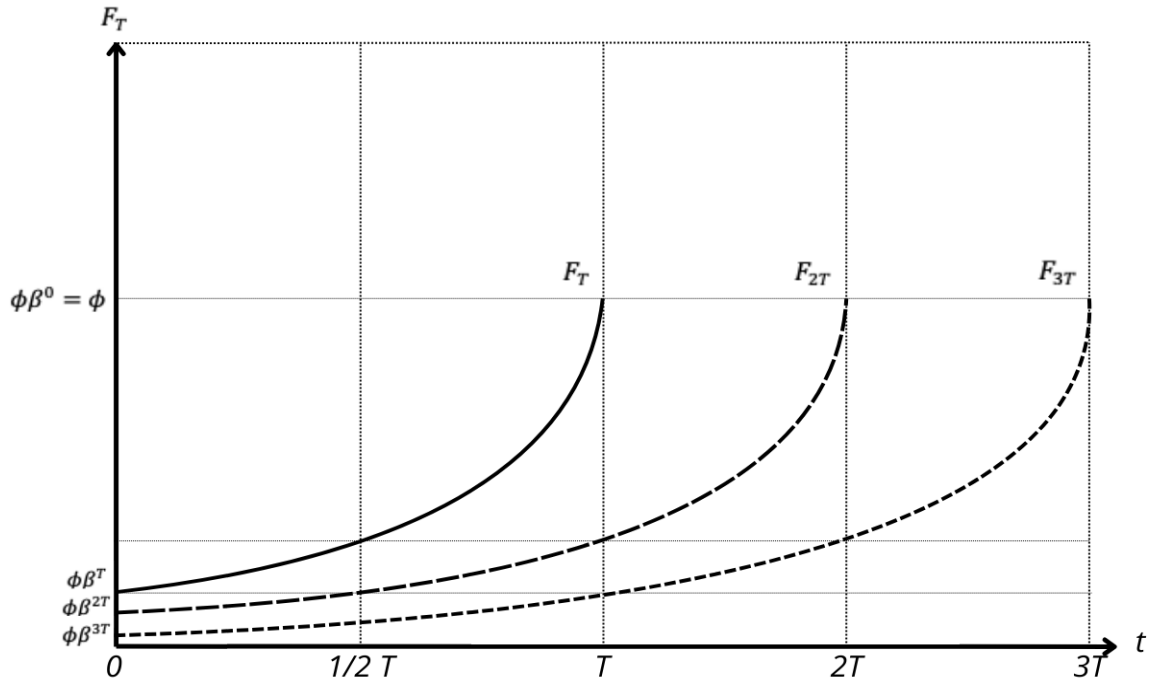
$$F_T(E_{t-1}(r_{c,t}), r_{c,t}) = \sum_{t=1}^T \phi \beta^{T-t} (E_{t-1}(r_{c,t}) - r_{c,t})$$

where  $\phi \geq 1$  is the fixed sensitivity to unexpected lower returns (i.e., investors' aversion to unexpected costs) and  $\beta \in [0,1]$  is the rate of time discount. Assuming a constant unit difference between the expected return and the actual return  $E_{t-1}(r_{c,t}) - r_{c,t}$ , the disutility at period  $T$ ,  $2T$  and  $3T$  follows the path illustrated in *Figure 2*.

---

<sup>24</sup> The proof is presented in *Appendix A.2*.

<sup>25</sup> Previous research indicates that investors' decisions are heavily influenced by recent events, often neglecting valuable information from events that occurred further in the past (Zahera and Bansal, 2018). This behavior is rooted in the '*recency effect*' literature, which suggests that the impact of events diminishes over time. Murdock (1962) and Tversky and Kahneman (1981) have shown that people tend to give more weight to recent events, leading to a decreased influence of older information on their decision-making.



**Figure 2:** Disutility Path for  $t = T, 2T, 3T$ .

Investors experience disutility due to returns that fall short of their expectations because the fund imposes hidden fees that reduce the actual gains. This unexpected shortfall in returns negatively impacts their satisfaction and influences their future investment behavior. As their dissatisfaction grows, investors gradually reduce their investment in the fund. When investors consistently receive lower returns than anticipated, they become cautious and adjust their future allocations to mitigate the risk of further disappointment (Englmaier et al., 2012). This ongoing process means that each period, investors face a disutility that shapes their subsequent investment decisions<sup>26</sup>. The disutility experienced by investors is cumulative, as each period adds to the total negative utility they perceive. Intuitively, the longer the periods of returns falling short of expectations, the greater the level of investor dissatisfaction. However, this accumulation occurs at a decreasing rate due to the diminishing impact of older disutility. Recent disutility has a more significant effect on investors' decision-making processes than older ones. For instance, the disutility from the previous period (most recent) is heavily weighted in their considerations, while disutility from much earlier periods (e.g., 20 periods

<sup>26</sup> It should be noted that the fairness factor developed here is symmetrically applied to both the positive and negative impacts of the difference between expected and actual returns. This is a simplifying assumption to facilitate computation and does not necessarily reflect real-life scenarios. Leibbrandt (2020) found that the negative effects caused by fairness concerns are more pronounced than the positive effects driven by self-interest psychology. However, since I assume a constant return by the investment vehicle to focus on the negative impact of hidden fees on investor behavior, it is unnecessary to differentiate the magnitude between positive and negative discrepancies in expected and actual returns in the fairness factor.

ago) have a much-reduced impact. This decreasing weight of older disutility can be explained by the investors' psychological inclination to prioritize more recent experiences over distant ones. The immediate past disutility is fresh in their memory and has a strong influence on their current perceptions and actions. In contrast, disutility from long ago are less relevant and therefore contribute less to the overall disutility. This phenomenon is known as the "*recency effect*," a cognitive bias where individuals tend to remember and give more importance to recent events than those that occurred in the distant past (Zahera and Bansal, 2018). This effect is well-documented in psychological literature (see e.g. Wulff et al., 2018), explaining why older disutilities are progressively discounted by investors.

From the equation, it can be observed that for high values of  $T$ , depending on the magnitudes of  $\beta$  and  $\phi$ , the effects of the oldest differences between expected and actual returns tend to approach zero. As a result, the disutility factor stabilizes over time. This means that as the time horizon extends, the impact of these parameters diminishes, leading to a steady state where the disutility no longer significantly changes. This stabilization implies that investors' negative reactions to deviations in expected returns become less pronounced over time, allowing for a more predictable and stable investment environment. If the cumulative impact of disutility from unexpected lower returns exceeds the risk premium associated with the fund, investors will shift all their capital to the risk-free asset. Conversely, if the cumulative disutility stabilizes at a level below the risk premium, investors will continue to allocate a portion of their capital to the fund, albeit a reduced percentage. In this latter case, in fact, despite facing consistently lower returns due to hidden fees, investors continue to achieve a positive return, which remains higher than the return offered by risk-free assets. This incentive to achieve a higher return, even if it involves perceived unfairness, motivates investors to maintain their allocations in the Fund<sup>27</sup>.

#### *Investors Intertemporal Behavior and Optimal Allocation.*

In the following analysis, I assume that investors exhibit myopic behavior. Specifically, drawing on the work of Herings, Mauleon and Vannetelbosch (2020), I posit that these investors focus solely on immediate returns and do not consider the future fee strategies or the long-term implications of the mutual fund's actions.

---

<sup>27</sup> See 'Assumption 3'.

**Period 0.** In  $t = 0$ , the Fund set the transparent and hidden fee levels at  $f_0$  and  $a_0$ , respectively. Investors maximize their perceived utility, taking only transparent fees into account, but end up paying also the hidden fee. Thus, their maximization problem in  $t = 0$  is

$$\max_{\alpha_0} U_0 = r_f + \alpha_0(E_0[r_p] - f_0 - r_f) - \frac{1}{2}\gamma\alpha_0^2\sigma_p^2$$

And the optimal allocation to the Mutual Fund is

$$\alpha_0^* = \frac{(E_0[r_p] - f_0 - r_f)}{\gamma\sigma_p^2}$$

This is represented as a point in the CAL. In  $t = 0$  investors expect to receive  $E_0[r_{c,1}] = \alpha_0^*(E_0[r_p] - f_0) + (1 - \alpha_0^*)r_f$ , but in  $t = 1$  they end up receiving  $r_{c,1} = \alpha_0^*(r_p - f_0 - a_0) + (1 - \alpha_0^*)r_f$ . Therefore in  $t = 1$ , they experience a disutility  $F_1(E_0[r_{c,1}], r_{c,1}) = \phi\{E_0[r_{c,1}] - r_{c,1}\}$  deriving from the unexpected lower return<sup>28</sup>

$$F_1(E_0[r_{c,1}], r_{c,1}) = \phi\alpha_0^*a_0$$

where  $\phi \geq 1$  is the sensitivity to unexpected lower return and the term  $\alpha_0^*a_0$  account for the fact that the disutility experienced is limited to the percentage actually invested in the Fund.

**Period 1.** In  $t = 1$ , investors maximize their new utility, taking into account the disutility  $F_1$ ,

$$\max_{\alpha_1} U_1 = r_f + \alpha_1(E_1[r_p] - f_1 - r_f) - \frac{1}{2}\gamma\alpha_1^2\sigma_p^2 - \alpha_1(\phi\alpha_0^*a_0)$$

The disutility from the unexpected lower return experienced at the end of  $t = 0$  enters in the above equation scaled by  $\alpha_1$  to account for the fact that investors' disutility is limited to the percentage they decide to invest in the fund after performing the maximization problem. The optimal allocation is

---

<sup>28</sup> In this model, as I assumed in *Assumption 2* that  $r_p$  is stable and investors correctly take into account  $r_p$  and its variability,  $E_0[r_p] = r_p$ .

$$\alpha_1^* = \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*a_0}{\gamma\sigma_p^2}$$

As before, in  $t = 1$  investors expect to receive  $E_1[r_{c,2}] = \alpha_1^*(E_1[r_p] - f_1) + (1 - \alpha_1^*)r_f$ , but in  $t = 2$  they end up receiving  $r_{c,2} = \alpha_1^*(r_p - f_1 - a_1) + (1 - \alpha_1^*)r_f$ . Therefore in  $t = 2$ , they experience a disutility  $F_2(E_1[r_{c,2}], r_{c,2})$  deriving from the unexpected lower return

$$F_2(E_1[r_{c,2}], r_{c,2}) = \sum_{t=1}^2 \phi\beta^{2-t}(E_{t-1}(r_{c,t}) - r_{c,t}) = \phi\beta\alpha_0^*a_0 + \phi\alpha_1^*a_1$$

In the above formula, the disutility experienced in the first period is now scaled by the time factor  $\beta \in (0,1)$ . As some time has passed, investors assign to the older disutility a lower weight than the one assigned to the most recent one<sup>29</sup>.

**Period 2.** Investor's new utility maximization problem in  $t = 2$  will therefore be

$$\max_{\alpha_2} U_2 = r_f + \alpha_2(E_2[r_p] - f_2 - r_f) - \frac{1}{2}\gamma\alpha_2^2\sigma_p^2 - \alpha_2(\phi\beta\alpha_0^*a_0) - \alpha_2(\phi\alpha_1^*a_1)$$

With the optimal allocation

$$\alpha_2^* = \frac{(E_2[r_p] - f_2 - r_f) - \phi\beta\alpha_0^*a_0 - \phi\alpha_1^*a_1}{\gamma\sigma_p^2}$$

In general, in every period  $T$ , investors maximize their perceived utility function

$$\max_{\alpha_T} U_T = r_f + \alpha_T(E_T[r_p] - f_T - r_f) - \frac{1}{2}\gamma\alpha_T^2\sigma_p^2 - \alpha_T\left(\sum_{t=1}^T \beta^{T-t}\phi\alpha_{t-1}^*a_{t-1}\right)$$

---

<sup>29</sup> See *Appendix A.3* for the detailed derivation process of the disutility factor.

Allocating<sup>30</sup>

$$\alpha_T^* = \frac{(E_T[r_p] - f_T - r_f) - \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1}}{\gamma \sigma_p^2}$$

The main intuition behind this formula aligns with the principles discussed earlier. However, in this context, the risk premium also accounts for the transparent fee  $f_T$ , which is visible to investors (Kouwenberg & Ziemba, 2007). The hidden fee is not explicitly considered, as it remains invisible to naïve investors. Consequently, a higher transparent fee will be recognized by investors<sup>31</sup>, resulting in a lower allocation to the fund. Additionally, the formula includes a negative term in the numerator that reflects the cumulative disutility arising from unexpected lower returns accumulated over time. This formula captures the behavior of investors when they consistently receive lower returns than expected. As they are unaware of the hidden fees, they calculate their portfolio allocations based on the information available to them. However, they ultimately face unexpected lower returns. These persistent unexpected lower returns accumulate over time, leading to a gradual reduction in the percentage of capital they decide to allocate to the fund in each period. This means that as their dissatisfaction grows, investors gradually reduce their investment in the fund. The disutility factor increases over time, albeit at a marginally decreasing rate. This implies that, after a sufficiently long period, the investor will reach a stable optimal allocation. If the cumulative disutility from unexpected lower returns reaches the fund's risk premium  $\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1} = (E_T[r_p] - f_T - r_f)$ , investors will transfer all their capital to the risk-free asset<sup>31</sup>, as  $\alpha_T^* = 0$ . However, if the cumulative disutility stabilizes at a level below the risk premium, investors will still allocate a portion of their capital to the fund, though at a reduced percentage. Intuitively, investors will recognize that the unexpected lower returns are constant and, since the risky asset still offers a higher return than the risk-free asset, they will derive positive utility by continuing to invest in the Fund rather than moving all their capital to the risk-free asset<sup>32</sup>.

---

<sup>30</sup> Note that when  $T = 0$ , the disutility term is zero. In the first period in fact, investors have not received their return yet, thus they do not experience any disutility or utility from the difference between expected and actual returns.

<sup>31</sup> As discussed in 'Section 3.1',  $\alpha_T^*$  cannot be negative, as this would imply that the investor is paying to invest in an asset that yields less, resulting in a certain loss. Therefore, as soon as the cumulative disutility  $\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1}$  surpasses the risk premium, investors have already moved all their capital to the risk free asset.

<sup>32</sup> A discussion on the implications of a stable level of fairness concern below the risk premium, among some recommendations for future research, is provided in Section 7.

To streamline the analysis, I make the following two assumptions throughout the main text

**Assumption 3.**  $\beta \approx 1$ .

This condition implies a high persistence of fairness concerns within the model. The main consequence is that upon reaching a certain period  $T_{lim}$ , the cumulative disutility equals the risk premium  $\sum_{t=1}^{T_{lim}} \beta^{T_{lim}-t} \phi \alpha_{t-1}^* a_{t-1} = (E_{T_{lim}}[r_p] - f_{T_{lim}} - r_f)$ .<sup>33</sup> Consequently, this induces investors to transfer all their capital to the outside option once period  $T_{lim}$  is reached. *Lemma 2* summarizes this result<sup>34</sup>.

**Lemma 2.** For a non-negative risk premium, investors optimal allocation is a continuous monotonic decreasing function in time, with

1.  $\alpha_T^* > 0$  for  $T < T_{lim}$ ;
2.  $\alpha_T^* = 0$  for  $T \geq T_{lim}$ .

**Assumption 4.**  $f_{TOT} \leq E(r_p) - r_f$ .<sup>35</sup>

This assumption summarize a natural restriction that arises within this framework: the total fee  $f_{TOT}$  charged by the fund must be lower than the risk premium,  $f_{TOT} \leq E(r_p) - r_f$ . If the total fee charged is equal or greater than the risk premium, investors will allocate their entire capital to the risk free asset, resulting in zero profits for the fund, as  $\alpha^* = \frac{(E[r_p] - f_{TOT} - r_f)}{\gamma \sigma_p^2} = \frac{(E[r_p] - E(r_p) + r_f - r_f)}{\gamma \sigma_p^2} = 0$ .

---

<sup>33</sup> Although this work follows Fama and French (2003) in using the risk-free rate as the outside option, it should be taken into consideration that investors might seek other investment vehicles or securities to allocate their capital if the investors' portfolio consistently underperforms  $r_{c,t} < E_{t-1}(r_{c,t})$ . In a competitive scenario, investors would indeed shift their capital to alternative options if the fund underperforms relative to its competitors, instead if it underperforms relative to expectations.

<sup>34</sup> The proof is presented in *Appendix A.4*.

<sup>35</sup> It should be noted that part of the total fee charged may be hidden and thus invisible to investors. However, as discussed earlier and following Morrison and Taubinsky (2023), if the hidden fee is too high, investors will incur costly screening processes, leading to the identification of the hidden fee.

## 4 MUTUAL FUND FEE STRUCTURING

In each period  $t$ , investors allocate a percentage  $\alpha_t^*$  to the Mutual Fund and a percentage  $1 - \alpha_t^*$  to the risk-free asset. The total Asset Under Management ( $AUM$ ) of the Fund in period  $t$  is calculated by aggregating the investments made by all investors  $i$  at time  $t$ ,  $AUM_t = \int \alpha_{i,t} C_{i,t}$ , with  $C_{i,t}$  being the capital of investor  $i$  at time  $t$ . For the sake of simplification, along with the assumption of a unit mass of investors, I also assume a homogenous and constant unit amount of capital invested by each investor, so as all investors invest an equal amount of capital  $C = 1$ . Thus

$$AUM_t = \alpha_t C = \alpha_t$$

As I restrict investors' allocation decisions to choosing between the mutual fund and the risk-free asset, a natural consequence arises. This restriction ensures the continuous reinvestment of capital between these two options. If investors decide to adjust their proportion of capital in the mutual fund, the remaining capital will automatically be allocated to the risk-free asset, the converse is also true. This assumption simplifies the model by eliminating the need to consider alternative investments, thereby focusing the analysis on the interaction between the mutual fund and the risk-free asset. The main implication of this restriction is that the total Assets Under Management of the mutual fund at any given period  $t$  is the cumulative sum of all the allocation adjustments made by investors up to that period

$$AUM_t = \alpha_0 + \sum_{s=1}^t \Delta\alpha_s$$

With  $\Delta\alpha_s = \alpha_s - \alpha_{s-1}$ . The fund with marginal costs  $c \geq 0$  sets transparent and hidden fees  $f_t$  and  $a_t \in [0, \bar{a}]$ , respectively. The profit function of the Fund at time  $t$  is thus defined as:

$$\pi_t = (f_t + a_t - c) \left( \alpha_0 + \sum_{s=1}^t \Delta\alpha_s \right)$$

or, in simpler terms

$$\pi_t = (f_t + a_t - c)AUM_t = (f_t + a_t - c)\alpha_t$$

The cumulative profit  $\Pi_T$  of the fund up to period  $T$  is intuitively the sum of all profits  $\pi_t$  up to period  $T$ :

$$\Pi_T = \sum_{t=0}^T \pi_t = \sum_{t=0}^T [(f_t + a_t - c)\alpha_t]$$

The fund's profit is therefore null in two cases: when the total assets under management are zero, which occurs when the percentage allocated by investors is zero,  $\alpha_t = 0$ , and when the total fee charged by the fund equals the fund's costs,  $f_t + a_t = c$ . This reflects the trade-off in the fund's fee structuring, as highlighted in the *Introduction*. The fee structure influences the fund's profit through both direct and indirect channels. A higher fee level directly increases the fund's profit through the factor  $(f_t + a_t)\alpha_t$ . However, a higher fee level reduces the risk premium, leading to a lower percentage of capital allocated to the fund by investors, which consequently reduces the assets under management.

**Fund's Type and Relative Fee Structure.** The mutual fund establishes both transparent and hidden fees with the objective of maximizing its profits. Adjustments to the fee levels and structure influence profitability through two primary channels: direct and indirect. Directly, an increase in fees generates higher revenue. Indirectly, however, higher fees result in a reduced AUM, as investors, accounting for the diminished returns due to the elevated fees, opt to allocate a smaller proportion of their capital to the fund. Consequently, the fund must strategically manage this trade-off to optimize overall profitability.

In the following section, I will apply and analyze the previous model to determine the optimal fee structure that maximizes the fund's profits. To thoroughly examine the implications of fee-setting strategies, I will consider two distinct scenarios. In the first case, the fund is assumed to be myopic, setting its fee structure to maximize profit in the immediate next period without regard for future periods. For example, at time  $t = 0$ , the fund will set its fees solely to optimize profit for that specific period, disregarding the potential impacts on subsequent periods. This approach reflects a short-term profit maximization strategy, where the fund's decisions are

driven by immediate returns. In the second scenario, the fund adopts a long-term strategy, structuring its fees to maximize profits over a pre-determined extended period. I refer to this type of fund as a "Forward-Looking Fund," characterized by its consideration of future periods and its ability to commit to a consistent fee strategy over time. This approach involves considering the intertemporal trade-offs and the cumulative impact of fee structures on investor behavior and AUM across multiple periods.

## 4.1 MYOPIC FUND

I now consider the case of a short-term oriented fund that aims to maximize its profit over the immediate term without regard for future periods. At the beginning of each period, the fund sets its fee structure for the forthcoming period by solving a profit maximization problem for that specific period. Thus, at time  $t = 0$ , the fund sets its transparent and hidden fees to maximize the profit  $\pi_0$ . Similarly, at time  $t = 1$ , it adjusts its fee structure to maximize the profit  $\pi_1$ , and this process continues for subsequent periods. As documented by previous researches (Stafylas et al., 2017; Brav et al., 2015), a myopic short-term profit maximization strategy may lead to several detrimental effects. Firstly, it can result in higher volatility in the fund's returns as frequent adjustments to the fee structure may create uncertainty among investors. Secondly, as I will show in the subsequent analysis, it can erode investor confidence and loyalty, leading to decreasing returns.

**Period 0.** In  $t = 0$  the fund performs the following maximization problem

$$\max_{f_0, a_0} \pi_0 = (f_0 + a_0 - c)\alpha_0^*$$

where  $\alpha_0^* = \frac{(E_0[r_p] - f_0 - r_f)}{\gamma\sigma_p^2}$ , recalling from *Section 3.2*, is the optimal allocation of investors in period  $t = 0$ . Performing the partial derivatives of  $\pi_0$  with respect to  $f_0$  and  $a_0$  yields the following system of equations<sup>36</sup>

---

<sup>36</sup> The detailed process is presented in *Appendix B.1*.

$$\begin{cases} f_0 = \frac{E_0[r_p] - r_f - \bar{a} + c}{2} \\ a_0 = \bar{a} \end{cases}$$

I discussed at the beginning of the section that, following Morrison and Taubinsky (2023),  $a \in [0, \bar{a}]$ . From *Assumption 4* the total fee level should be less than the risk premium,  $f + a < E[r_p] - r_f$ , as if it were equal, investor would allocate  $\alpha^* = 0$  to the fund. The level  $\bar{a}$  should be set by solving  $f_0 + a_0 < E[r_p] - r_f$ , which yield to  $a_0 < E[r_p] - r_f - c$ . Therefore, the upper bound of  $a$  is  $\bar{a} < E[r_p] - r_f - c$ .<sup>37</sup>The equation for setting the transparent fee positively depends on the risk premium factor and the marginal cost. This means that if the risk premium  $E_0[r_p] - r_f$  or the marginal costs  $c$  increase, the fund can set higher transparent fees. The rationale behind this is twofold. Firstly, a higher risk premium indicates a greater expected return from the risky asset compared to the risk-free asset. This increased return potential makes the investment more attractive to investors, who may be more willing to accept higher transparent fees in exchange for the higher anticipated returns. Consequently, the fund can leverage this increased attractiveness to justify higher fees without significantly deterring investors. Secondly, higher marginal costs necessitate an increase in fees to maintain profitability. Marginal costs represent the additional expenses incurred by the fund for managing each additional unit of investment. When these costs rise, the fund must adjust its fee structure to cover these expenses and ensure that it continues to operate profitably. By increasing the transparent fees, the fund can offset the higher marginal costs and sustain its financial health.

It is important to note that in  $t = 0$ , investors do not experience any disutility from unexpected lower returns due to hidden fees. Consequently, the disutility factor does not appear in the fund's fee structuring equation at this initial period. From this system, since  $\frac{\partial \pi_0}{\partial a_0} > 0$ , it becomes evident that the myopic fund will set the hidden fee at its maximum. This outcome is driven by the fact that investors are unaware of the hidden fees, and since the Fund does not consider future periods, it is more profitable to impose higher hidden fees, thereby gaining higher revenues through the direct channel without adversely impacting the indirect channel. The

---

<sup>37</sup> Note that for  $\bar{a} \rightarrow E[r_p] - r_f - c$  the transparent fee level tends to approach zero,  $f \rightarrow 0 < c$ . Consequently, the initial transparent fee is likely to be set lower than the marginal cost. This implies that the fund will primarily cover its costs through the hidden fees.

myopic fund, indeed, disregards the indirect channel's effects as these will manifest in the subsequent period, which lies beyond the fund's immediate focus. As a result, the transparent fee is set at its lowest possible value, since a higher fee would lead to a reduction in the AUM. By minimizing the transparent fee, the fund aims to maintain a higher level of investor capital allocation, thereby optimizing immediate profitability without considering future disutility impacts.

**Period 1.** In  $t = 1$ , the fund performs the following maximization problem:

$$\max_{f_1, a_1} \pi_1 = (f_1 + a_1 - c)\alpha_1^*$$

This time however, in the investor's optimal allocation  $\alpha_1^*$  compare a disutility factor  $F_1(E_0[r_{c,1}], r_{c,1}) = \phi\alpha_0^*\bar{a}$ , deriving from the unexpected lower returns experienced in the previous period. Investor's optimal allocation is thus set as  $\alpha_1^* = \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}$ , and performing the partial derivatives of  $\pi_1$  with respect to  $f_1$  and  $a_1$  yields the following system of equations<sup>38</sup>

$$\begin{cases} f_1 = \frac{E_1[r_p] - r_f - \phi\alpha_0^*\bar{a} - \bar{a} + c}{2} \\ a_1 = \bar{a} \end{cases}$$

At  $t = 1$ , the fund, similarly as before, sets the hidden fee at its maximum level. However, this time, the transparent fee is lower than the one set at  $t = 0$ ,  $f_1 < f_0$ . In the transparent fee-setting equation, there is now a negative term representing the disutility that investors experienced in the previous period from the unexpected lower returns due to the high hidden fee. Consequently, the transparent fee must be reduced to mitigate investor dissatisfaction and encourage continued investment. It is evident that at  $t = 1$ , the fund's profits are lower than at  $t = 0$ <sup>39</sup>. The total AUM is reduced, and while the hidden fee remains at its maximum value, the transparent fee is lower. Therefore, at  $t = 1$ , the Fund not only experiences the negative effects of the indirect channel but also suffers a reduction in revenue from the direct channel. Examining the equation for  $f_1$ , it can be decomposed into two terms:

<sup>38</sup> The detailed process is presented in *Appendix B.1*.

<sup>39</sup> The proof is presented in *Appendix B.2*.

$$f_1 = \frac{E_1[r_p] - r_f + c}{2} - \frac{\phi\alpha_0^*\bar{a} + \bar{a}}{2}$$

the first term represents a fixed quantity that accounts for the risk premium and marginal costs. This term reflects the positive influence of the expected return on the risky asset, the return on the risk-free asset, and the operational costs of managing the Fund. The second term represents the negative effect of the disutility factor experienced by investors. This term accounts for the disutility due to unexpected lower returns caused by hidden fees, which investors realized from the previous period. The presence of the disutility factor reduces the transparent fee that can be set without deterring investors. The transparent fee level remains positive as long as  $\frac{E_1[r_p] - r_f + c}{2} \geq \frac{\phi\alpha_0^*\bar{a} + \bar{a}}{2}$ . By *Assumption 3*, for the transparent fee to be viable, the positive impact of the risk premium and marginal costs must outweigh the negative effect of the disutility factor. If the disutility becomes too significant, the transparent fee may need to be set lower to maintain investor participation, highlighting the balance the fund must manage to optimize its fee structure while considering both revenue and investor satisfaction. Given that the disutility factor accumulate over time according to the following equation

$$F_t(E_{t-1}(r_{c,t}), r_{c,t}) = \sum_{t=1}^T \phi\beta^{T-t}(E_{t-1}(r_{c,t}) - r_{c,t})$$

the second term increases as time progresses. In  $t = 2$  in fact,  $f_2 = \frac{E_2[r_p] - r_f + c}{2} - \frac{\phi\beta\alpha_0^*\bar{a} + \phi\alpha_1^*\bar{a} + \bar{a}}{2}$

and clearly  $\frac{\phi\beta\alpha_0^*\bar{a} + \phi\alpha_1^*\bar{a} + \bar{a}}{2} \geq \frac{\phi\alpha_0^*\bar{a} + \bar{a}}{2}$ .<sup>40</sup> Generalizing, at  $t = T$

$$f_T = \frac{E_{T-1}[r_p] - r_f + c}{2} - \frac{\sum_{t=1}^T \beta^{T-t} \phi\alpha_{t-1}^* a_{t-1} + a_T}{2}$$

with  $f_T \leq f_{T-1}$ . Therefore, given that the first term remains constant under *Assumption 2* and constant marginal costs, and that the second term increases over time, there will eventually be a period  $t = N$  where  $\frac{E_{N-1}[r_p] - r_f + c}{2} = \frac{\sum_{t=1}^N \beta^{N-t} \phi\alpha_{t-1}^* a_{t-1} + a_N}{2}$  and  $f_N = 0$ .<sup>41</sup> Up to period  $N$ , the

<sup>40</sup> The detailed process is presented in *Appendix B.3*.

<sup>41</sup> The proof is presented in *Appendix B.4*.

fund set its hidden fee at the maximum level  $\bar{a}$  so that  $a_0 = a_1 = \dots = a_N = \bar{a}$ . Therefore, the hidden fee level remains constant until period  $N$ . Conversely, as shown above, the transparent fee level decreases over this same time horizon until it reaches zero at period  $N$ . Consequently, the profits of the fund decrease as time progresses. These results are summarized by *Lemma 3*<sup>42</sup>.

**Lemma 3.** Up to period  $t = N$ , a myopic fund set its hidden fee at its maximum value  $\bar{a}$ , while the transparent fee is a decreasing function of  $t$ , and reaches zero at  $t = N$ .

1.  $a_t = \bar{a} \quad \forall t \in [0, N], \quad \text{with } \bar{a} < E[r_p] - r_f - c.$
2.  $f_{t-1} > f_t \quad \forall t \in [1, N] \quad \text{and } f_N = 0.$

In period  $T = N + 1$  the transparent fee level is negative

$$f_{N+1} = \frac{E_N[r_p] - r_f + c}{2} - \frac{\sum_{t=1}^{N+1} \beta^{N+1-t} \phi \alpha_{t-1}^* a_{t-1} + a_{N+1}}{2} < 0$$

to keep  $f_{N+1} \geq 0$ , the fund needs to reduce the hidden fee  $a_{N+1}$ , so that  $a_{N+1} < a_N = \bar{a}$ . As time progresses, the negative factor in the transparent fee-setting equation, deriving from accumulating disutility, grows. To maintain a non-negative transparent fee level, the fund must progressively reduce the hidden fee level. Thus, after period  $N$ , where  $f_N = f_{N+1} = \dots = f_K = 0$ , with  $K$  being the terminal lifetime period of fund (i.e., the period after which the fund is no longer profitable), the hidden fee level must decrease as time progresses from  $N$  to  $K$ , such that  $a_N > a_{N+1} > \dots > a_K$ . *Lemma 4* summarizes this result.

**Lemma 4.**

1.  $f_{t-1} = f_t = 0 \quad \forall t \in [N, K], \quad \text{with } N < K.$
2.  $a_{t-1} > f_t \quad \forall t \in [N, K] \quad \text{with } N < K.$
3.  $f_t + a_t = c \quad \text{for } t = K.$

---

<sup>42</sup> The proof is presented in *Appendix B.5*.

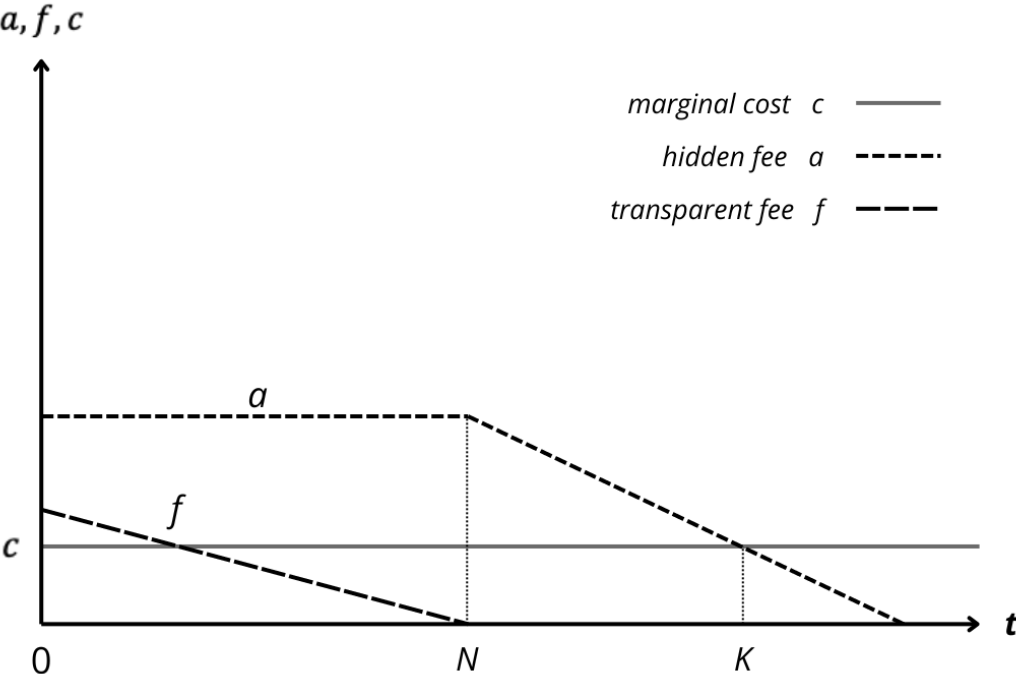
As the transparent fee level is constantly held at zero after period  $N$  and the hidden fee level decreases over time, there will eventually be a period  $K$  where the revenue from the fees equals the costs of the fund<sup>43</sup>

$$\pi_K = (f_K + a_K - c)\alpha_K = 0$$

or, equivalently, the total fees equal the marginal costs  $f_K + a_K = c$ . After period  $K$ , the fund is no longer profitable. From these results I derive the following proposition.

**Proposition 1.** *The myopic fund's fee structuring strategy sets the hidden fee at its maximum level, with the transparent fee adjusting downward, making the hidden fee weakly greater than the transparent fee. This results in the fund facing a terminal period, where its profitability cannot be sustained, that sets the stage for its inevitable bankruptcy.*

The following graph depicts the evolution in time of both the transparent and hidden fees from period 0 to the terminal period  $K$ , highlighting the dynamic adjustments made by the fund in response to investor's allocation.



**Figure 3:** Evolution of the Myopic Fund's Fee Structure Over Time.

<sup>43</sup> The proof is presented in Appendix B.6.

The graph above illustrates the evolution of the fund's fee structure over time. The hidden fee level is set at its maximum up to period  $N$ , while the transparent fee level decreases progressively. At time  $N$ , the transparent fee level reaches zero. To maintain a non-negative transparent fee level in subsequent periods, the fund must gradually reduce the hidden fee. After period  $K$ , however, revenues fall below costs, rendering the fund unprofitable. While the fund might survive for some additional periods due to the profits accumulated in earlier periods, it will eventually have to cease operations. It is crucial to note that since fee levels directly influence the investor's allocation decisions, excessively high fees diminish the fund's profits. This occurs because the reduction in capital allocated to the fund, resulting from higher fees, outweighs the revenue gains from the increased fees. The dotted lines in the graph represent the optimal maximum fee levels (both transparent and hidden) that the fund can set to maximize its profits. This optimal setting considers both the direct channel, which includes the revenue generated from the fees, and the indirect channel, which accounts for the reduced revenue due to lower capital allocation.

## 4.2 FORWARD LOOKING FUND

In contrast to the myopic fund approach, where fees are set period-by-period to maximize immediate profitability, the forward-looking fund adopts a strategy aimed at maximizing its long-term profits over a predefined horizon of time. The forward looking fund takes future periods into account and can commit to future fees. This strategic shift necessitates a comprehensive consideration of investor behavior and market dynamics across multiple periods within the horizon. Rather than focusing solely on short-term gains, the forward-looking fund evaluates the long-term implications of fee structures on investor allocations and overall profitability. The decision-making process of the forward-looking fund begins by setting transparent and hidden fee levels at the outset of the horizon, which remain constant throughout the entire period under consideration, reflecting the fund's commitment to optimizing its long-term profits. Unlike the myopic fund, which may overlook the compounding effects of fee structures over time, a forward-looking fund accounts for the persistence of the disutility experienced by investors due to unexpected lower returns resulting from hidden fees. While the myopic fund maximizes its profit  $\pi_t$  on a period-by-period basis, the forward-looking fund aims to maximize its cumulative profit  $\prod_T$  over a predefined period  $T$ .

$$\max_{f,a} \Pi_T = \sum_{t=0}^T \pi_t = \sum_{t=0}^T [(f + a - c)\alpha_t]$$

Given that the fund sets its fee structure at the beginning of the horizon and maintains these fees constant until period  $T$ , the transparent and hidden fee levels can be considered as fixed parameters. The cumulative profit is directly influenced by the accumulating capital allocated by investors each period, denoted as  $\sum_{t=0}^T \alpha_t$ . For  $T = 0$ , the total AUM are equal to the AUM in the initial period, as investors have not yet experienced any disutility from unexpected lower returns. However, for  $T \geq 1$ , investors begin to experience disutility. Consequently, the total AUM can be expressed as<sup>44</sup>

$$\sum_{t=0}^T \alpha_t = \begin{cases} \frac{E[r_p] - r_f - f}{\gamma\sigma_p^2}, & T = 0 \\ \alpha_0 + \left(\frac{E[r_p] - r_f - f}{\gamma\sigma_p^2}\right) (T) - \frac{a\phi}{\gamma\sigma_p^2} \left[ \sum_{t=1}^T \alpha_{t-1} \left( \sum_{s=t}^T \beta^{T-s} \right) \right], & T \geq 1 \end{cases}$$

Clearly, when  $T = 0$ , the fund's profit maximization problem is  $\max_{f,a} \Pi_0 = (f + a - c)\alpha^*$ , which is equal to the first period myopic fund's maximization problem. Therefore, the fee structure is

$$\begin{cases} f = \frac{E[r_p] - r_f - \bar{a} + c}{2} \\ a = \bar{a} \end{cases}$$

In this scenario, the myopic and forward-looking funds set the same level of fees. Consequently, the outcomes and implications discussed in *Section 4.1* also apply to this case. As the forward looking fund extends its time horizon to consider future periods, it must take into account the impact of hidden fees on investors' allocation decisions. When  $T = 1$ , for example, investors allocate  $\alpha_1^* = \frac{(E[r_p] - f - r_f) - \phi\alpha_0^*a}{\gamma\sigma_p^2}$ , thus the hidden fee reduce the AUM by a factor  $\phi\alpha_0^*$ .

---

<sup>44</sup> The detail procedure is presented in *Appendix C.1*.

In the forthcoming analysis, I will explore the fee structuring strategy of a forward-looking fund when it commits to a specific time horizon, denoted as  $T$ . It is crucial to understand that within each chosen time horizon, the fee levels remain constant; however, these fee levels vary depending on the length of the time horizon selected. For instance, if the fund opts for a time horizon of  $T = 5$  years, it will establish its fee levels at  $f_5$  and  $a_5$ , with these values remaining unchanged throughout the entire five-year period. Conversely, if the fund selects a longer time horizon, such as  $T = 10$  years, the fee structure will adjust to new levels,  $f_{10}$  and  $a_{10}$ , which will remain constant within that decade-long period. The analysis will demonstrate a correlation between the length of the time horizon and the weight of transparent fees in the fee structure. Specifically, longer time horizons are associated with a fee structure that increasingly favors transparent fees. For a sufficiently extended time horizon, the optimal fee structure for the fund may involve relying exclusively on transparent fees.

From the profit function  $\pi_t = (f + a - c)\alpha_t$ , the scaling factors of the direct channel (i.e., terms inside the brackets) to changes in fees is identical for both the transparent fee and the hidden fee, as a marginal change in the transparent fee  $\Delta f$  has the same impact as a marginal change in the hidden fee  $\Delta a$ <sup>45</sup>. However, the profit function is also influenced by the level of AUM, which depends on both the transparent and hidden fees. This interdependence suggests that while the direct impact of fee changes is equivalent, the overall effect on profitability is moderated by how these fees affect investor allocation and thus the AUM. The AUM in period  $T$  is

$$AUM_T = \frac{(E[r_p] - f - r_f) - \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a}{\gamma \sigma_p^2} = \frac{E[r_p] - r_f}{\gamma \sigma_p^2} - \frac{f}{\gamma \sigma_p^2} - \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a}{\gamma \sigma_p^2}$$

In this equation, the scaling factors of  $f$  and  $a$  are different. I define  $S_{f,T}$  as the scaling factor for  $f$  and  $S_{a,T}$  as the scaling factor for  $a$ . From the equation above  $S_{f,T} = \frac{1}{\gamma \sigma_p^2}$  and  $S_{a,T} = \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a}{\gamma \sigma_p^2}$ <sup>46</sup>. Note that  $S_{f,T}$  is a constant, thus  $S_{f,T} = S_f$ , while  $S_{a,T}$  is an increasing function in time<sup>47</sup>. The equation can thus be rewritten as

---

<sup>45</sup> See *Appendix C.2*.

<sup>46</sup> To streamline the analysis and concentrate on the magnitudes of the scaling factors, I exclude the negative sign from the term.

<sup>47</sup> See *Appendix B.4*.

$$AUM_T = \frac{E[r_p] - r_f}{\gamma\sigma_p^2} - S_{f,T}f - S_{a,T}a$$

Since  $f$  is scaled by  $\frac{1}{\gamma\sigma_p^2}$ , a marginal change in  $f$  has a direct and constant impact on  $AUM_T$ . On the contrary, the impact of a marginal change in  $a$  is scaled by the term  $\frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^*}{\gamma\sigma_p^2}$ , indicating that the effect of  $a$  on  $AUM_T$  is influenced by the cumulative disutility.

When  $T = 0$ , as investors do not account for the hidden fees,  $S_{a,T} = 0$  and thus  $S_f > S_{a,T}$ . As the disutility function accumulates over time, the scaling factor  $S_{a,T}$  increases with the passage of time. This implies that the negative impact of a marginal increase in the hidden fee on the AUM becomes progressively more significant as the time horizon advances. Consequently, since  $S_f$  is a constant, as time horizon grows, the difference  $S_f - S_{a,T}$  approaches 0 at some point in time  $T = T'$ . Therefore, for  $T < T'$ ,  $S_{a,T} < S_f$ , while for  $T > T'$ ,  $S_{a,T} > S_f$ . This result is summarized by *Lemma 5*<sup>48</sup>.

**Lemma 5.** Given functions  $S_f$  and  $S_{a,T}$ , there exists a point  $T' \in (0, T_{lim})$  such that  $S_{a,T'} = S_f$ , and:

1.  $S_{a,T} < S_f$  for  $T < T'$ ;
2.  $S_{a,T} > S_f$  for  $T > T'$ .

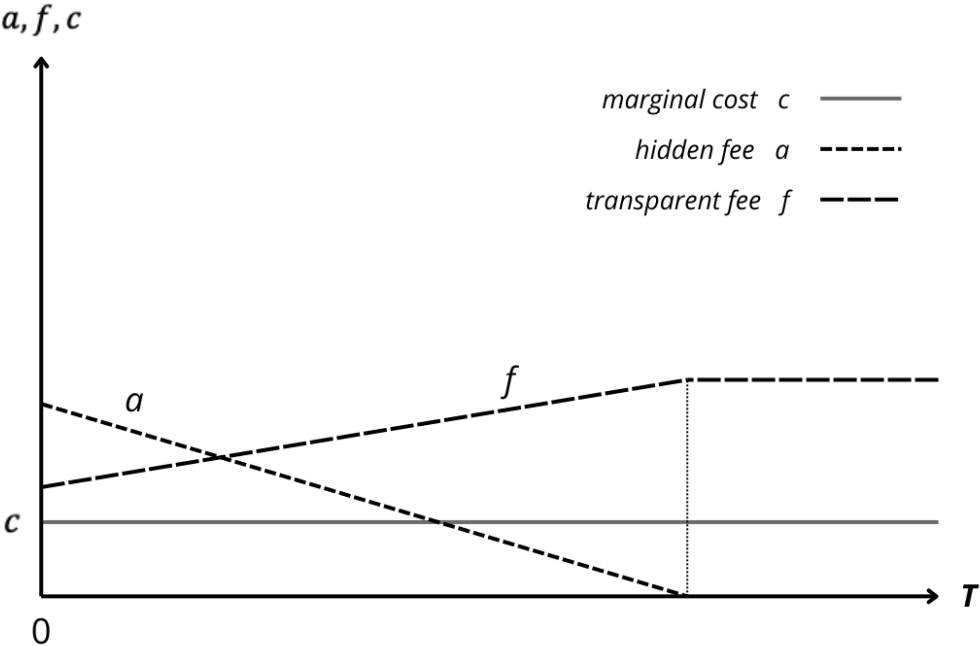
As long as the fund chooses a time horizon  $T < T'$ , it is more profitable to rely on hidden fees rather than transparent fees. This is because a marginal increase in the hidden fee  $a$  yields the same benefit as an increase in the transparent fee  $f$  through the direct channel, but it also causes a smaller reduction in AUM through the indirect channel. This outcome directly follows from *Lemma 5*. This implies that a forward-looking fund with a time horizon of  $T < T'$  will behave similarly to a myopic fund, setting a consistently high level of hidden fees throughout its entire horizon  $T$  and allowing the transparent fee to adjust accordingly. However, when the fund's time horizon is set at  $T \geq T'$ , where  $S_{a,T} \geq S_f$ , the fund will find it more profitable to rely on transparent fees rather than hidden fees.

---

<sup>48</sup> The proof is presented in *Appendix C.3*.

Consider a fund that sets its time horizon at  $T = T'$ , establishing an optimal fee structure with hidden fees  $a_{T'}$  and transparent fee  $f_{T'}$ . Now, imagine another scenario where a fund chooses a slightly longer time horizon,  $T = T' + 1$ , but maintains the same fee levels  $a_{T'}$  and  $f_{T'}$ . Although the fee levels remain constant over time, the effectiveness of the fee structure changes as the time horizon chosen by the fund changes. Given that for  $T \geq T'$ , the scaling factor  $S_{a,T}$  associated with the hidden fees becomes larger relative to the scaling factor  $S_f$  associated with transparent fees, the longer time horizon makes the hidden fee  $a_{T'}$  less profitable on a marginal basis compared to the transparent fee  $f_{T'}$ . Consequently, the optimal fee structure for a fund with a longer time horizon  $T = T' + 1$  would naturally involve a reduced reliance on hidden fees  $a_{T'+1}$  and an increased emphasis on transparent fees  $f_{T'+1}$ , compared to the fund with a shorter time horizon. As the time horizon extends, the pattern in fee structuring is clear: the longer the chosen time horizon, the more the fund relies on transparent fees rather than hidden fees. For a sufficiently extended time horizon, the optimal fee structure ultimately involves relying solely on transparent fees, with hidden fees minimized or entirely eliminated (Figure 4). From this discussion I derive the following proposition.

**Proposition 2.** *As a forward-looking fund selects longer time horizons, it optimally chooses a fee structure with a higher reliance on transparent fees and a lower reliance on hidden fees. For sufficiently long time horizons, the optimal fee structure involves exclusively transparent fees.*



**Figure 4:** Evolution of the Forward Looking Fund's Fee Structure on the Horizon.

As the time horizon of the fund increases, the reliance on the transparent fee becomes progressively more advantageous compared to the hidden fee, due to its superior profitability over time. This observation aligns with the findings of Kouwenberg and Ziemba (2007), who demonstrated that funds with higher incentive and fixed fees tend to exhibit significantly lower returns. While shrouding fees might initially appear to be an effective marketing strategy, the analysis in this work reveals that excessive shrouding can be detrimental to a fund's profitability. This conclusion supports the earlier work of Jovanovic (1982) and Milgrom (1981), who argued, in the context of the banking system, that shrouding could harm financial institutions. As outlined in the *Introduction* and *Section 2*, this research aligns with the literature emphasizing that fees significantly impact investor behavior. The results of this model suggest that investors are highly sensitive to the fee structure, which implies that fund managers should carefully consider the effects of fee structuring on various revenue channels<sup>49</sup>.

---

<sup>49</sup> See *Section 6* for a further discussion.

## 5 COMPARATIVE ANALYSIS

In this section I explore the implications of both a myopic approach and a forward-looking approach, comparing the two scenarios. Precisely, the analysis will focus on two types of forward looking funds, distinguished by the length of the time horizon they select and how it influences their fee structuring decisions. The first type of forward looking fund, referred to as ‘*Long-Term Fund*’, selects a sufficiently long time horizon, which results in a fee structure that relies exclusively on transparent fees, effectively setting hidden fees to zero. In contrast, the second type of forward-looking fund, referred to as ‘*Medium-Term Fund*’, opts for a shorter time horizon, leading to a fee structure that includes a positive level of hidden fees alongside transparent fees. The outcomes of these two forward-looking funds will then be compared with those of a myopic fund, which focuses solely on immediate gains without considering long-term implications.

*Long-Term Fund* ( $\mathbf{a} = \mathbf{0}$ ). This fund sets a sufficiently long time horizon, which leads to a fee structure exclusively reliant on transparent fees. For instance, *Proposition 2* in the previous section establishes that a forward-looking fund with a sufficiently long-term horizon will eventually eliminate the hidden fee, i.e.  $a_t = a = 0$ , depending solely on the transparent fee for profitability. When the hidden fee is set to zero, under *Assumption 2*, the disutility factor also becomes zero, as investors do not experience any disutility from unexpected lower returns caused by hidden fees. Consequently, a natural outcome is that investors will allocate a constant percentage of their portfolio to the fund. Thus, every period investors allocate  $\alpha_t^* = \alpha_{t+1}^* = \alpha^*$ . Since the fee level is fixed and the AUM remain constant, the fund's profits are stable over time,  $\pi_t = \pi_{t+1} = \pi$ . By *Assumption 4*,  $f_{TOT} \leq E(r_p) - r_f$ , and since  $a = 0$ ,  $f_{TOT} = f + a = f$ . Moreover, the marginal cost  $c$  is lower than the fees set by the fund<sup>50</sup>. These two conditions grants the fund profitability perpetually (*Figure 5*). This result is summarized by *Lemma 6*<sup>51</sup>.

---

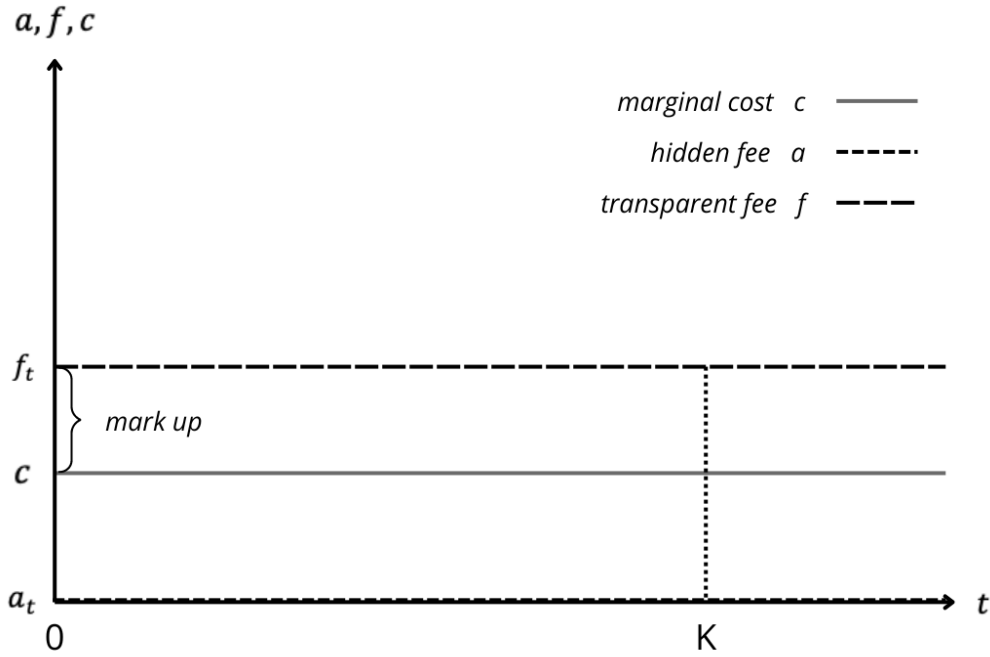
<sup>50</sup> The fund in this analysis operates as a monopolist with market power, allowing it to set a transparent fee above the cost.

<sup>51</sup> The proof is presented in *Appendix D.1*.

**Lemma 6.**  $\pi_t \geq 0 \quad \forall t \in [0, \infty)$  if and only if:

1.  $a_t = a = 0$ ;
2.  $f_{TOT} \leq E(r_p) - r_f$ ;
3.  $f_{TOT} > c$ .

A natural consequence of *Lemma 6* is that the forward looking fund, by relying solely on the transparent fee, is able to sustain profitability for a period  $T > K$ , with  $K$  being the terminal life-time period of the myopic fund.



**Figure 5:** Forward looking fund with  $a_t = 0$ .

The profit function of the fund is  $\pi_t = (f + a - c)\alpha_t$ , and since  $\alpha_t$  is constant and  $a = 0$ , it is clear that the fund is profitable as long as  $f > c$ . In this case, the marginal cost level coincide with the profitability level (i.e., the level at which the fund covers all its costs and generate profits). The forward-looking fund is able to maintain profitability for a longer duration compared to the myopic fund. It can be demonstrated that the myopic fund's profits are higher than those of the forward-looking fund in the initial periods<sup>52</sup>. This occurs because, at  $t = 0$ , the myopic fund sets the hidden fee at its maximum and the transparent fee at its minimum. Investors, who can only see the transparent fee, do not account for the hidden fee in their optimal allocation decisions, leading them to allocate a significantly higher percentage of their

<sup>52</sup> The proof is presented in *Appendix D.2*.

capital to the fund. In contrast, the forward-looking fund, which relies solely on the transparent fee, sets the hidden fee at zero and the transparent fee at a higher level. As investors take the transparent fee into consideration, they allocate a lower percentage of their capital to the forward-looking fund compared to the myopic fund. Consequently, the AUM of the myopic fund is initially higher than that of the forward-looking fund and given a constant total fee level  $f_{TOT} = f + a$ , the myopic fund's profits are temporarily higher as well. *Proposition 1* in *Section 4.1* asserts that under *Assumption 3*, the myopic fund will eventually reach a terminal period where it becomes unprofitable. This outcome is a direct consequence of excessive shrouding, which leads investors to experience cumulative disutility that eventually exceeds the risk premium. As a result, the myopic fund initially registers very high profits, but these profits steadily decline until the fund defaults. Conversely, as demonstrated by *Lemma 6*, the forward-looking fund, which relies solely on transparent fees, maintains consistent non-negative profits over time. This implies that while the myopic fund is more profitable in the initial periods, there will eventually come a point where the profits of the two funds converge. Beyond this point, the forward-looking fund becomes more profitable.

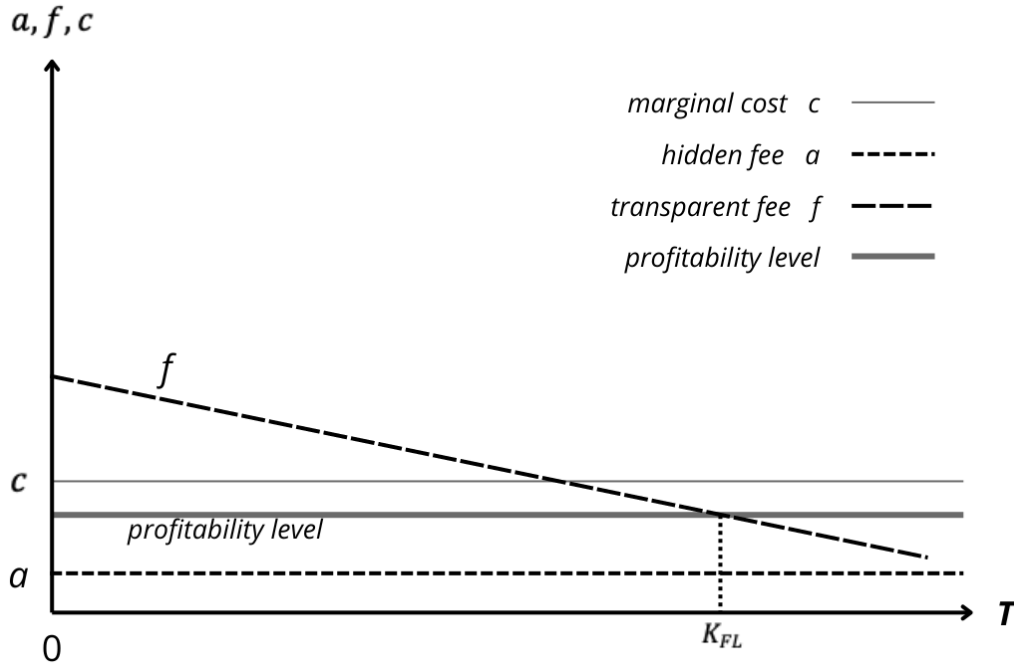
I now turn to the second scenario, where the forward-looking fund has a shorter time horizon, leading it to set a non-negative level of hidden fees.

*Medium-Term Fund ( $a > 0$ )*. In this scenario, the forward-looking fund operates with a shorter time horizon compared to the previous case, and I assume it sets a non-negative level of hidden fees within its time horizon. For simplicity and without loss of generality, I further assume that the hidden fee is set to cover part of the marginal costs  $c$ , such that  $a < c$ <sup>53</sup>. Under these conditions, the disutility factor becomes positive, leading to a cumulative reduction in investors' capital allocation due to the cumulative nature of the disutility function. As a result, the fund will need to account for the negative impact of this indirect channel on its overall profitability. Since now  $a > 0$ ,  $f = f_{TOT} - a < f_{TOT}$ . The fund, therefore, needs to adjust its transparent fee structure at the end of each chosen time horizon. Specifically, when  $a > 0$ , investors experience disutility, which necessitates a compensatory reduction in the transparent fee in the subsequent time period. This adjustment occurs as the fund reassesses its fee structure for the new time horizon to maintain profitability while addressing investor concerns. As the disutility factor

---

<sup>53</sup> The analysis and results holds also in the case where the hidden fee level is set higher than the marginal costs, i.e.  $a > c$ .

accumulates and grows over time, the required reduction in the transparent fee also increases correspondingly. By *Assumption 3*, the forward looking fund will eventually reach a terminal life-time period  $K_{FL}$  after which it is no longer profitable (*Figure 5*).



**Figure 6:** Forward looking fund's fees path.

In this scenario, the hidden fee is used to offset some of the fund's costs. Consequently, the profitability threshold for the transparent fee is lower than it would be if the hidden fee were set to zero, as the fund remains profitable as long as  $f > c - a$ . Although the hidden fee level remains constant, the disutility experienced by investors increases over time, causing a downward pressure on the transparent fee level in the subsequent time periods. Eventually, the transparent fee will reach the profitability threshold in period  $K_{FL}$ , where the fund's profits are zero, i.e.,  $f + a = c$ . It is possible to show that if  $0 < a < \bar{a}$ , the terminal lifetime period for the forward looking fund  $K_{FL}$  is higher than that of the myopic fund  $K$ <sup>54</sup>. Intuitively, when  $a < \bar{a}$ , the cumulative disutility is lower, which causes the transparent fee level to decrease at a slower rate. As a result, the total fee level takes longer to reach the profitability threshold, allowing the fund to remain profitable for a longer period of time. Similar to the previous case, the myopic fund experiences higher profits in the initial periods but faces a quicker path to default. In contrast, the forward-looking fund generates lower initial profits compared to the

<sup>54</sup> The proof is presented in *Appendix D.3*.

myopic fund but remains viable for a longer period. The underlying reason lies in the degree of shrouding employed by the funds. When the hidden fee is set higher (with a correspondingly lower transparent fee), investors initially allocate a higher percentage of capital, as they do not account for the hidden fee. Consequently, for a fixed level of total fees, the myopic fund enjoys higher initial profits. However, in later periods, the increasing negative pressure from investors' fairness concerns reduces the myopic fund's profits more rapidly than those of the forward-looking fund. From this discussion I derive the following proposition.

**Proposition 3.** *While a myopic fund achieves higher short-term profits, a forward-looking fund maintains greater longevity and stability.*

Previous research by Guasoni (2016) establishes a connection between myopic management and higher performance fees, which, although initially leading to greater portfolio risk exposure and short-term gains, ultimately result in a reduced survival rate for the fund. Proposition 3 in this study aligns with these findings, indicating that a myopic approach often shortens the fund's lifespan due to the unsustainable levels of total fees. This is further supported by the work of Gil-Bazo and Ruiz-Verdú (2008), who argue that less competent fund managers tend to target unsophisticated and naive investors by imposing higher fees, thereby diminishing long-term profitability and survival rates when compared to high-ability managers. These more skilled managers, who adopt a forward-looking strategy akin to the one proposed in this model, tend to set their fees at more reasonable levels, likely due to competitive pressures from sophisticated investors. Moreover, this study adds a new dimension to the existing literature by suggesting that forward-looking managers, who anticipate the long-term implications of their fee structures, may also be driven to lower fees as part of a strategic approach to enhance fund longevity and investor satisfaction. This aligns with the broader understanding that while high fees might generate short-term profits, they are detrimental to long-term fund stability, particularly as investors become increasingly sensitive to fee structures and their impact on net returns over time.

## 6 IMPLICATIONS FOR FUND MANAGEMENT

This section provides strategic recommendations for fund managers based on the findings of this study. The insights offered here aim to enhance fund profitability, investor satisfaction, and long-term stability by optimizing fee structures and adopting a strategic management approach. While hidden fees may offer the potential for higher short-term gains, excessive reliance on them can erode investor trust and long-term profitability. A balanced fee structure, where the weight of transparent fees increases over time, is more likely to ensure sustainable growth. This approach not only mitigates the risks associated with investor dissatisfaction but also capitalizes on the growing trend toward transparency in the financial markets<sup>55</sup>. For instance, regulatory frameworks such as the EU's Markets in Financial Instruments Directive (MiFID II)<sup>56</sup> in 2023 and the EU's Sustainable Finance Disclosure Regulation (SFDR)<sup>57</sup> in 2019 have both highlighted the critical importance of enhanced disclosure and transparency to bolster market stability and safeguard investors from concealed risks. If a fund's portfolio is predominantly composed of long-term investors with substantial capital commitments, it is advisable for the fund to prioritize a long-term perspective by relying more on transparent fees. Transparent fees build trust and align with the expectations of investors who are focused on stability and sustained returns over time. This approach mitigates the risk of investor dissatisfaction due to unexpected costs and fosters long-term relationships, which are crucial for maintaining a stable asset base. Conversely, if the portfolio is characterized by a high turnover rate, where investors frequently withdraw and deposit funds, the fund might benefit from implementing hidden fees. In such cases, the negative impact of cumulative disutility from hidden fees is less likely to materialize, as the transient nature of the investor base reduces the likelihood of long-term dissatisfaction. Therefore, for portfolios with a large number of small investors who frequently move in and out of the fund, relying on hidden fees can enhance profitability without significantly damaging investor relationships. In broad terms, fund managers should strategically differentiate between loyal, long-term investors and more transient, short-term investors. For the former group, transparent fees are preferable to maintain trust and

---

<sup>55</sup> The trend toward financial transparency gained significant momentum after the 2008 financial crisis, leading to several key legislations like the U.S. Dodd-Frank Wall Street Reform and Consumer Protection Act in 2010. This Act implemented comprehensive reforms to enhance transparency, reduce systemic risk, and protect consumers, marking a critical shift toward greater disclosure and oversight in financial markets. Consult <https://www.congress.gov/111/plaws/publ203/PLAW-111publ203.pdf>.

<sup>56</sup> See the official document <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32023R0960>.

<sup>57</sup> See the official document <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32019R2088>.

satisfaction. For the latter, a greater reliance on hidden fees could be more profitable, provided the fund carefully manages the balance to avoid excessive investor attrition.

Moreover, investment funds are often composed of various sub-units or specialized entities, each with distinct objectives, risk profiles, and time horizons. These sub-units, such as Special Purpose Vehicles (SPVs) and segregated portfolios, play a crucial role in the overall strategy of a fund by allowing for tailored investment approaches and risk management strategies. Given the diversity of these entities, it becomes imperative for fund managers to adopt a dynamic and context-specific fee structuring approach.

*Special Purpose Vehicles (SPVs).* A Special Purpose Vehicle (SPV) is a subsidiary entity established by a parent company to isolate and manage financial risk. These legal entities are created with a specific, narrow, and temporary objective, often related to investment or financing activities. SPVs are commonly utilized in the financial industry for asset securitization, joint ventures, or other financial transactions, providing a mechanism to limit the exposure to risks associated with these activities<sup>58</sup>. SPVs typically have a limited lifespan, tied to the completion of a specific project or the maturation of certain financial instruments. When fund managers establish SPVs with shorter lifespans, the incentive structures and fee management strategies must align with the unique objectives and constraints of these entities. Given the temporary nature of SPVs, the focus shifts towards maximizing returns within a constrained timeframe. In such scenarios, imposing higher hidden fees might be both justified and advantageous, as the immediate impact on investor disutility could be minimized due to the absence of a long-term relationship. Indeed, *Appendix D.2* shows that for the initial periods, the myopic fund profits are higher than that of the forward looking fund. However, it is crucial for fund managers to carefully calibrate the level of hidden fees, ensuring they do not entirely erode investor trust, even in the short term. The hidden fees should be strategically set high enough to maximize returns without provoking significant backlash or regulatory scrutiny.

*Segregated Portfolios.* Segregated portfolios are distinct sub-funds within a larger umbrella fund structure, each functioning as an independent legal entity. This structure enables the application of different investment strategies, asset pools, and fee structures within the same holding company<sup>59</sup>. Given that segregated portfolios may have varying lifespans, fund

---

<sup>58</sup> See <https://www.sec.gov/Archives/edgar/data/1055160/000105516022000009/R22.htm>.

<sup>59</sup> See <https://www.sec.gov/Archives/edgar/data/1839158/000149315221000394/partiandiii.htm>.

managers must tailor their fee strategies to align with the specific objectives and time horizons of each portfolio. For portfolios with shorter durations, it can be advantageous to implement higher hidden fees, similar to the approach used in SPVs. This strategy allows the fund to maximize returns within a limited timeframe, while ensuring that the immediate impact on investor disutility remains controlled. However, it is crucial for fund managers to calibrate these hidden fees carefully, balancing the potential for increased returns against the risk of eroding investor trust, even over the short term. This holds particularly true in the case of future flow of capital from a short-term segregated portfolio to a long-term segregated portfolio. Conversely, portfolios with longer durations should adopt a more balanced fee structure that gradually increases transparency over time. This approach not only fosters long-term investor satisfaction but also contributes to the overall stability of the fund. As the time horizon of the portfolio extends, increasing the transparent fee while reducing reliance on hidden fees becomes critical for sustaining investor confidence and optimizing long-term profitability. In managing these varying portfolios, fund managers should conduct regular evaluations of each portfolio's expected lifespan, risk profile, and investor base. By tailoring the fee structures to the specific characteristics of each portfolio, managers can optimize profitability across the entire holding, ensuring that both short-term gains and long-term stability are effectively balanced.

*Market Dynamics.* Fund managers should consider dynamically adjusting fee structures in response to evolving market conditions. During bearish market phases, increasing reliance on transparent fees may be prudent, as the combination of lower returns and heightened shrouding could significantly amplify investor disutility. Conversely, in stable or bullish markets, a moderate increase in hidden fees might be more acceptable, as the higher returns during these periods can offset the potential disutility associated with shrouded fees. Such dynamic fee structuring enables the fund to align investor satisfaction with revenue maximization across different market cycles. Additionally, it is crucial for fund managers to acknowledge investor preferences for stability within the fund's terms. In periods of high market volatility, emphasizing transparent fees can reassure investors of the fund's commitment to transparency and long-term stability, thereby fostering trust and loyalty.

## 7 CONCLUSION

This paper investigates the relationship between transparent and hidden fees within mutual funds, focusing on how these fee structures impact fund profitability and long-term stability. Transparent fees are factored into investors' optimal allocation decisions, whereas shrouded fees, being unaccounted for, result in unexpected lower returns in subsequent periods. This creates increasing disutility due to fairness concerns among investors. By developing a theoretical model rooted in the CAPM framework, this study integrates both types of fees with investors' fairness concerns. Furthermore, this model is combined with a profit maximization framework, wherein fees directly affect the fund's revenue and indirectly influence profits by impacting the capital allocated by investors. I then conduct a comparative analysis of two scenarios: a myopic, short-term-oriented fund and a forward-looking, long-term-oriented fund. The findings indicate that while shrouding boosts short-term profits, it is detrimental in the long run, ultimately leading to the fund's bankruptcy. Conversely, a greater reliance on transparent fees, though yielding lower short-term profits, enhances the fund's longevity, resulting in a higher survival rate for funds that minimize shrouding.

The analysis presented in this paper is based on several significant assumptions that should be acknowledged. First, I assume a high persistence of investors' time discount concerning fairness concerns, implying that investors remember past fund performances for an extended period. This assumption leads to a cumulative disutility that grows over time, ultimately causing the bankruptcy of funds that heavily rely on hidden fees. If persistence were lower, the disutility might stabilize below the threshold of profitability, potentially allowing even myopic funds to survive longer. This aspect warrants further investigation in future research. Additionally, the model assumes a homogeneous mass of naïve investors, simplifying the focus on the impact of shrouding within this market segment. However, in real-world scenarios, this assumption might not hold, as markets often include sophisticated investors as well. A more nuanced analysis should consider the presence of both naïve and sophisticated investors, each responding differently to fee structures. For instance, if naïve investors also exhibit a high turnover rate, the fund might benefit more from shrouding practices within this segment, while maintaining a transparent fee structure for sophisticated investors. This differentiated approach could optimize profitability across diverse investor bases.

The analysis indicates that short term oriented funds are more likely to engage in fee-shrouding practices, given their heightened incentives for short term profit maximization. This finding has significant policy implications. Regulators should prioritize oversight on funds that are inherently short-term in focus or operate within speculative segments of the market<sup>60</sup>, where the incentive to obscure fees is stronger. Specifically, investment vehicles with limited lifespans, such as those created for single-use operations or special purpose sub-funds, warrant particular attention. These entities are especially prone to shrouding due to their transient nature and the pressure to achieve immediate returns<sup>61</sup>. Additionally, to enhance transparency and reduce incentives for fee shrouding, regulators should prioritize fostering a long-term orientation within mutual funds. Incentives to align manager compensation with the long-term performance of the fund, as opposed to short-term gains, can mitigate the temptation to obscure fees for immediate profits. Moreover, regulators should incentivize the application of high-water marks<sup>62</sup> over extended periods, thereby encouraging a more long-term focus among fund managers<sup>63</sup>. By extending the timeframe for high-water marks, the appeal of shrouding practices diminishes, as the motivation to obscure fees for short-term gains decreases when performance is assessed over a longer duration. This approach fosters transparency and aligns the interests of the fund with sustained, long-term profitability rather than immediate returns. By aligning managerial incentives with the interests of long-term investors, such regulatory measures would likely encourage mutual funds to adopt more transparent practices, ultimately improving investor returns and increasing the survival rate of mutual funds.

---

<sup>60</sup> Previous research shows that speculative funds often face lower survival rates compared to those with longer time horizons. For example, Baba and Goko (2006) found that funds with lower volatilities tend to have higher survival probabilities. Similarly, Gregoriou (2002) suggested that monthly returns and volatilities significantly impact the mortality of hedge funds, which exhibit a median survival time of just 5.5 years—significantly shorter than longer-term oriented funds, such as Index Funds.

<sup>61</sup> See *Section 6*.

<sup>62</sup> A high-water mark is a provision in fund fee structures that ensures fund managers only earn performance fees if the fund's net asset value (NAV) exceeds its highest previous value.

<sup>63</sup> Aragon and Nanda (2012) demonstrate that high-water mark provisions, among other factors, tend to encourage fund managers to adopt a more conservative and long-term oriented approach in risk management.

## REFERENCES

- Agarwal, S., Chomsisengphet, S., & Lim, C.** (2017). What shapes consumer choice and financial products? A Review. *Annual Review of Financial Economics*, 9(1), 127-146.
- Agarwal, S., Driscoll, J. C., Gabaix, X., & Laibson, D.** (2008). Learning in the credit card market (No. w13822). *National Bureau of Economic Research*.
- Alan, S., Cemalcilar, M., Karlan, D., & Zinman, J.** (2018). Unshrouding: Evidence from bank overdrafts in Turkey. *The Journal of Finance*, 73(2), 481-522.
- Alda, M., & Ferruz, L. U. I. S.** (2012). The role of fees in pension fund performance. Evidence from Spain. *Finance a Uver*, 62(6), 518.
- Anagol, S., & Kim, H. H.** (2012). The impact of shrouded fees: Evidence from a natural experiment in the Indian mutual funds market. *American Economic Review*, 102(1), 576-593.
- Aragon, G. O., & Nanda, V.** (2012). Tournament behavior in hedge funds: High-water marks, fund liquidation, and managerial stake. *The Review of Financial Studies*, 25(3), 937-974.
- Armstrong, M., & Vickers, J.** (2012). Consumer protection and contingent charges. *Journal of Economic Literature*, 50(2), 477-493.
- Armstrong, M.** (2006). Competition in two-sided markets. *The RAND Journal of Economics*, 37(3), pp.668-691.
- Ausubel, L. M.** (1991). The failure of competition in the credit card market. *The American Economic Review*, 50-81.
- Baba, N., & Goko, H.** (2006). Survival analysis of hedge funds. *Institute for Monetary and Economic Studies and Financial Markets Department*, 6.
- Barras L. , Scaillet O. , Wermers R.** (2010). False discoveries in mutual fund performance: measuring luck in estimated alphas. *The Journal of Finance LXV*, 179–216.
- Baucus, D. A., Golec, J. H., & Cooper, J. R.** (1993). Estimating risk-return relationships: An analysis of measures. *Strategic Management Journal*, 14(5), 387-396.
- Belleflamme, P. and Johnen, J.** Non-price strategies of marketplaces: a survey. *LIDAM Discussion Paper*, 2023/15.
- Bernoulli, D.** (2011). Exposition of a new theory on the measurement of risk. *In The Kelly capital growth investment criterion: Theory and practice*, (pp. 11-24).

- Berk, J. B., & Green, R. C.** (2004). Mutual fund flows and performance in rational markets. *Journal of political economy*, 112(6), 1269-1295.
- Berk, J. B. & van Binsbergen, J. H.** (2015). Measuring skill in the mutual fund industry. *Journal of Financial Economics*, 118, 1–20.
- Berk, J. B., & van Binsbergen, J. H.** (2017). Mutual funds in equilibrium. *Annual Review of Financial Economics*, 9(1), 147-167.
- Black, Fischer, Michael C. Jensen and Myron Scholes.** (1972). “The Capital Asset Pricing Model: Some Empirical Tests,” in *Studies in the Theory of Capital Markets*. Michael C. Jensen, ed. New York: Praeger, pp. 79-121.
- Blake, D., Lehmann, B.N., Timmerman, A.** (1999). Asset allocation dynamics and pension fund performance. *Journal of Business*, 72:429-461.
- Blume, Marshall and Irwin Friend.** (1973). “A New Look at the Capital Asset Pricing Model.” *Journal of Finance*. 28:1, pp. 19-33.
- Blume, Marshall.** (1970). “Portfolio Theory: A Step Towards Its Practical Application.” *Journal of Business*. 43:2, pp. 152-174.
- Brav, A., Jiang, W., & Kim, H.** (2015). Recent advances in research on hedge fund activism: Value creation and identification. *Annual Review of Financial Economics*, 7(1), 579-595.
- Breeden, D. T.** (1979). An international asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economic*, 7, 265–296.
- Brown-Liburd, H., Cohen, J., & Zamora, V. L.** (2018). CSR disclosure items used as fairness heuristics in the investment decision. *Journal of Business Ethics*, 152, 275-289.
- Brown, K., Harlow, W.V., Starks, L.** (1996). Of tournaments and temptations: an analysis of managerial incentives in the mutual fund industry. *Journal of Finance*, 51, 85-110.
- Caillaud, B. and Jullien, B.** (2003). Chicken & egg: competition among intermediation service providers. *The RAND Journal of Economics*, 34(2), pp.309-328.
- Carroni, E.** (2018). Behaviour-based price discrimination with cross-group externalities. *Journal of Economics*, 125(2), pp.137-157.
- Christoffersen S., Musto, D.** (2002). Demand Curves and the Pricing of Money Management. *Review of Financial Studies*, 15(5):1499-1524.
- Coles, J., Suay, J., Woodbury, D.** (2000). Fund advisor compensation in closed-end funds. *Journal of Finance*, 55, 1385-1414.
- Competition and Markets Authority.** (2017). Retail banking market investigation Final report.

**Cooper, M. J., Halling, M., & Yang, W.** (2021). The persistence of fee dispersion among mutual funds. *Review of Finance*, 25(2), 365-402.

**Cremers M. , Ferreira M. , Matos P. , Starks L.** (2016). Indexing and active fund management: international evidence. *Journal of Financial Economics*, 120, 539–560.

**Cremers, K. M., Fulkerson, J. A., & Riley, T. B.** (2019). Challenging the conventional wisdom on active management: A review of the past 20 years of academic literature on actively managed mutual funds. *Financial Analysts Journal*, 75(4), 8-35.

**Cruickshank, D.** Review of Banking Services in the UK (London: HM Treasury, 2000).

**Cuthbertson, K., Nitzsche, D., & O'Sullivan, N.** (2016). A review of behavioural and management effects in mutual fund performance. *International Review of Financial Analysis*, 44, 162-176.

**Das S. R. , Sundaram R. K.** (2002). Fee speech: signaling, risk-sharing, and the impact of fee structures on investor welfare. *The Review of Financial Studies*, 15, 1465–1497.

**Dobronogov A., Murthi M.** (2005). Administrative fees and costs of mandatory private pensions in transition economies. *Journal of Pension Economics and Finance*, 4(1):31-56.

**Elbannan, M. A.** (2015). The capital asset pricing model: an overview of the theory. *International Journal of Economics and Finance*, 7(1), 216-228.

**Elton E. , Gruber M. , Busse J.** (2004): Are investors rational? Choices among index funds, *Journal of Finance* 59, 261–288.

**Englmaier, F., Gratz, L. and Reisinger, M.** (2012). Price discrimination and fairness concerns. *Munich Discussion Paper* No. 2012-7.

**Fama, E. F., & French, K. R.** (2003). The CAPM: Theory and evidence. *Center for Research in Security Prices (CRSP)*, University of Chicago, Working Paper, (550).

**Fama, Eugene F. and James D. MacBeth.** (1973). “Risk, Return, and Equilibrium: Empirical Tests.” *Journal of Political Economy*. 81:3, pp. 607-636.

**Fehr, E., Kirchsteiger, G. and Riedl, A.** (1993). Does fairness prevent market clearing? An experimental investigation. *The Quarterly Journal of Economics*, 108(2), pp.437-459.

**Friedman, M., & Savage, L. J.** (1948). The utility analysis of choices involving risk. *Journal of political Economy*, 56(4), 279-304.

**Gabaix, X., & Laibson, D.** (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, 121(2), 505-540.

**Getmansky, M., Lee, P. A., & Lo, A. W.** (2015). Hedge funds: A dynamic industry in transition. *Annual Review of Financial Economics*, 7(1), 483-577.

- Gil-Bazo J, Ruiz-Verdú P.** (2008): When cheaper is better: Fee determination in the market for equity mutual funds. *Journal of Economic Behavior and Organization*, 67(3-4):871-885.
- Gil-Bazo J. , Ruiz-Verdú P.** (2009): The relation between price and performance in the mutual fund industry, *Journal of Finance* 64, 2153–2183.
- Golec, J. H.** (1992). Empirical tests of a principal-agent model of the investor-investment advisor relationship. *Journal of Financial and Quantitative Analysis*, 27(1), 81-95.
- Gregoriou, G. N.** (2002). Hedge fund survival lifetimes. *Journal of Asset Management*, 3, 237-252.
- Grinblatt, M., & Titman, S.** (1987). The relation between mean-variance efficiency and arbitrage pricing. *Journal of Business*, 97-112.
- Grinblatt, M., & Titman, S.** (1989). Mutual fund performance: An analysis of quarterly portfolio holdings. *Journal of business*, 393-416.
- Grubb, M. D.** (2015). Failing to choose the best price: Theory, evidence, and policy. *Review of Industrial Organization*, 47, 303-340.
- Guasoni, P. and Oblój, J.** (2016), The incentives of hedge fund fees and high-water marks. *Mathematical Finance*, 26: 269-295.
- Haslem J. A. , Baker H. K. , Smith D. M.** (2006). Are retail S&P 500 index funds a financial commodity? Insights for investors. *Financial Services Review* 15, 99–116.
- Heidhues, P., & Kőszegi, B.** (2018). Behavioral industrial organization. *Handbook of Behavioral Economics: Applications and Foundations*, 1, 1, 517-612.
- Heidhues, P., Kőszegi, B., & Murooka, T.** (2016). Inferior products and profitable deception. *The Review of Economic Studies*, 84(1), 323-356.
- Heinkel, R., & Stoughton, N. M.** (1994). The dynamics of portfolio management contracts. *The Review of Financial Studies*, 7(2), 351-387.
- Herings, P. J. J., Mauleon, A., & Vannetelbosch, V.** (2020). Matching with myopic and farsighted players. *Journal of Economic Theory*, 190, 105125.
- Hoberg G. , Kumar N. , Prabhala N.** (2018). Mutual fund competition, managerial skill, and alpha persistence. *Review of Financial Studies* 31, 1896–1929.
- Hortacsu A. , Syverson C.** (2004). Product differentiation, search costs, and competition in the mutual fund industry: a case study of S&P 500 index funds. *Quarterly Journal of Economics* 119, 403–456.
- Ippolito R., Turner J.** (1987). Turnover, fees and pension plan performance. *Financial Analysts Journal*, 43(6):16-26.
- Jovanovic, B.** (1982). Truthful disclosure of information. *The Bell Journal of Economics*, 36-44.

- Johnen, J., & Somogyi, R.** (2024). Deceptive features on platforms. *The Economic Journal*, ueae016.
- Johnen, J.** (2019). Automatic-renewal contracts with heterogeneous consumer inertia. *Journal of Economics & Management Strategy*, 28(4), 765-786.
- Khorana A. , Servaes H. , Tufano P.** (2009). Mutual fund fees around the world. *Review of Financial Studies* 22, 1279–1310.
- Klumpes P. J. M., McCrae M.** (1999). Evaluating the financial performance of pension funds: An individual investor's perspective. *Journal of Business Finance and Accounting*, 26(3-4):261-281.
- Kouwenberg, R., & Ziemba, W. T.** (2007). Incentives and risk taking in hedge funds. *Journal of Banking & Finance*, 31(11), 3291-3310.
- Kritzman, M. P.** (1987). Incentive fees: Some problems and some solutions. *Financial Analysts Journal*, 43(1), 21-26.
- Kuchler, T., & Stroebel, J.** (2021). Social finance. *Annual Review of Financial Economics*, 13(1), 37-55.
- Leibbrandt, A.** (2020). Behavioral constraints on price discrimination: Experimental evidence on pricing and customer antagonism. *European Economic Review*, 121, 103303.
- Lintner, J.** (1965). “The Valuation of Risk Assets and the Selection of Risky Investments in Stock2”.
- Lintner, J.** (1975). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *In Stochastic optimization models in finance* (pp. 131-155). Academic Press.
- Liu, Q. and Serfes, K.** (2013). Price discrimination in two-sided markets. *Journal of Economics & Management Strategy*, 22(4), pp.768-786.
- Lucas, R. E.** (1978). Asset prices in an exchange economy. *Econometrica*, 46, 1429–1445.
- Lynch, A. W., & Musto, D. K.** (1997). Understanding fee structures in the asset management business. Markowitz, Harry. 1952. “Portfolio Selection.” *Journal of Finance*. 7:1, pp.77-91.
- Malhotra D., McLeod R.** (1997). An empirical analysis of mutual fund expenses. *Journal of Financial Research*, 20:175-190.
- Markowitz, H.** (1952). The utility of wealth. *Journal of political Economy*, 60(2), 151-158.
- Markowitz, H.** (1959). Portfolio Selection: Efficient Diversification of Investments. *Cowles Foundation Monograph* No. 16. New York: John Wiley & Sons, Inc.
- Merton, R. C.** (1973). An intertemporal capital asset pricing model. *Econometrica*, 41(5), 867–887.

- Milgrom, P. R.** (1981). Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics*, 380-391.
- Morrison, W., & Taubinsky, D.** (2023). Rules of thumb and attention elasticities: Evidence from under-and overreaction to taxes. *Review of Economics and Statistics*, 105(5), 1110-1127.
- Mossin, J.** (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the econometric society*, 768-783.
- Murdock Jr, B. B.** (1962). The serial position effect of free recall. *Journal of experimental psychology*, 64(5), 482.
- Nair, V. R., & Antony, A.** (2015). Evolutions and challenges of behavioral finance. *International Journal of Science and Research (IJSR)*, 4(3), 1055-1059.
- Nanda V. , Narayanan M. P. , Warther V. A.** (2000). Liquidity, investment ability, and mutual fund structure. *Journal of Financial Economics* 57, 417–443.
- Nanda V. , Wang Z. J. , Zheng L.** (2009): The ABCs of mutual funds: on the introduction of multiple share classes, *Journal of Financial Intermediation* 18, 329–361.
- Office of Fair Trading.** (2008). Personal current accounts in the UK. *Technical Rep.*
- Pastor L. , Stambaugh R. F. , Taylor L. A.** (2020). Fund tradeoffs, *Journal of Financial Economics*.
- Pastor L. , Stambaugh R. F.** (2012). On the size of the active management industry. *Journal of Political Economy* 120, 740–781.
- Portfolios and Capital Budgets.”** *Review of Economics and Statistics*. 47:1, 13-37.
- Rabin, M.** (1993). Incorporating fairness into game theory and economics. *American Economic Review*, 83(5), pp.1281-1302.
- Rochet, J.C. and Tirole, J.** (2003). Platform Competition in Two-Sided Markets. *Journal of the European Economic Association*, 1(4), pp.990-1029.
- Roll, R., & Ross, S. A.** (1980). An empirical investigation of the arbitrage pricing theory. *Journal of Finance*, 35(5), 1073–1103.
- Roll, R.** (1977). A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory. *Journal of financial economics*, 4(2), 129-176.
- Ross, S.** (1976). The arbitrage pricing theory. *Journal of Economic Theory*, 13(3), 341-360.
- Sharpe, W. F., & Cooper, G. M.** (1972). Risk-return classes of New York stock exchange common stocks, 1931–1967. *Financial Analysts Journal*, 28(2), 46-54.

- Sharpe, William F.** (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk". *Journal of Finance*. 19:3, pp. 425-442.
- Shefrin, H., & Statman, M.** (2000). Behavioral portfolio theory. *Journal of financial and quantitative analysis*, 35(2), 127-151.
- Shiller, B.R.** (2020). Approximating purchase propensities and reservation prices from broad consumer tracking. *International Economic Review*, 61(2), pp.847-870.
- Stafylas, D., Anderson, K., & Uddin, M.** (2017). Recent advances in explaining hedge fund returns: Implicit factors and exposures. *Global Finance Journal*, 33, 69-87.
- Stambaugh, R. F.** (2019). Skill and fees in active management (No. w26027). *National Bureau of Economic Research*.
- Stango, V., & Zinman, J.** (2009). What do consumers really pay on their checking and credit card accounts? Explicit, implicit, and avoidable costs. *American Economic Review*, 99(2), 424-429.
- Stango, V., & Zinman, J.** (2014). Limited and varying consumer attention: Evidence from shocks to the salience of bank overdraft fees. *The Review of Financial Studies*, 27(4), 990-1030.
- Starks, L. T.** (1987). Performance incentive fees: An agency theoretic approach. *Journal of Financial and Quantitative Analysis*, 22(1), 17-32.
- Statista.** (2024a). "Total net assets of mutual funds in the United States from 1997 to 2023."
- Statista.** (2024b). "Number of regulated open-end funds worldwide from 2011 to 2022."
- Su, J.** (2023). How Fairness Concerns Affect Online Platform' Discriminatory Pricing.
- Thaler, R.** (1980). Toward a positive theory of consumer choice. *Journal of economic behavior & organization*, 1(1), 39-60.
- Treynor, J. L.** (1961). Market value, time, and risk. *Time, and Risk* (August 8, 1961).
- Tversky, A., & Kahneman, D.** (1981). The framing of decisions and the psychology of choice. *science*, 211(4481), 453-458.
- U.S. Securities and Exchange Commission.** "Mutual Funds". *Archived from the original on 2021-10-08*. Retrieved 2021-10-03.
- Wahal S. , Wang A.** (2011). Competition among mutual funds. *Journal of Financial Economics* 99, 40–59.
- Wulff, D. U., Mergenthaler-Canseco, M., & Hertwig, R.** (2018). A meta-analytic review of two modes of learning and the description-experience gap. *Psychological bulletin*, 144(2), 140.

**Zahera, S. A., & Bansal, R.** (2018). Do investors exhibit behavioral biases in investment decision making? A systematic review. *Qualitative Research in Financial Markets*, 10(2), 210-251.

**Zhu M.** (2018). Informative fund size, managerial skill, and investor rationality. *Journal of Financial Economics* 130, 114–134.

## APPENDIX A

### A.1

If the risk premium is a positive quantity  $E[r_p] - r_f > 0$ , the investor will allocate a positive percentage  $\alpha^*$  of his portfolio to the fund and his total return will be  $E[r_c] = \alpha^* E[r_p] + (1 - \alpha^*) r_f$ . Since  $E[r_p] - r_f > 0$ ,  $E[r_c] > r_f$ .

If instead  $E[r_p] - r_f = 0$ , the investor allocate  $\alpha^* = 0$  to the fund, preferring the outside option  $r_f$ . His total return is therefore  $E[r_c] = 0E[r_p] + (1 - 0)r_f = r_f$ .

Suppose now the risk premium is a negative quantity, that is  $E[r_p] - r_f < 0$ . The optimal allocation  $\alpha^*$  is therefore  $\alpha^* = \frac{(E[r_p] - r_f)}{\gamma \sigma_p^2} < 0$ . This means that the investor is short selling the fund (i.e., is borrowing capital) to increase his exposure to the risk free asset. His total return is thus  $E[r_c] = \alpha^* r^b + (1 - \alpha^*) r_f = \alpha^* r^b + r_f - \alpha^* r_f$ , where  $r^b$  is the borrowing rate and  $\alpha^* r^b - \alpha^* r_f$  is the “arbitrage factor”, with  $\alpha^* r^b$  being the cost of borrowing and  $\alpha^* r_f$  the return earned from the investment. By *Assumption 1*,  $r^b > r_f$ , and since  $\alpha^* < 0$ , the arbitrage factor is a negative quantity,  $\alpha^* r^b - \alpha^* r_f < 0$ . Consequently  $E[r_c] = r_f + \alpha^* r^b - \alpha^* r_f < r_f$ . The investor is thus better off without arbitrage and an allocation of  $\alpha^* < 0$  is dominated by an allocation  $\alpha^* \geq 0$ .

### A.2

**Proof of Lemma 1.** At time  $t$ , investors expect to receive  $E_t[r_{c,t+1}] = \alpha_t^*(E_t[r_p] - f_t) + (1 - \alpha_t^*) r_f$ , but end up receiving  $r_{c,t+1} = \alpha_t^*(r_p - f_t - a_t) + (1 - \alpha_t^*) r_f$ . The difference between expected return and actual return is therefore

$$E_t[r_{c,t+1}] - r_{c,t+1} = [\alpha_t^*(E_t[r_p] - f_t) + (1 - \alpha_t^*) r_f] - [\alpha_t^*(r_p - f_t - a_t) + (1 - \alpha_t^*) r_f]$$

$$E_t[r_{c,t+1}] - r_{c,t+1} = \alpha_t^* E_t[r_p] - \alpha_t^* f_t + (1 - \alpha_t^*) r_f - \alpha_t^* r_p + \alpha_t^* f_t + \alpha_t^* a_t - (1 - \alpha_t^*) r_f$$

$$E_t[r_{c,t+1}] - r_{c,t+1} = \alpha_t^* E_t[r_p] - \alpha_t^* r_p + \alpha_t^* a_t$$

From *Assumption 2*  $E_t[r_p] = r_p$ , thus  $E_t[r_{c,t+1}] - r_{c,t+1} = \alpha_t^* a_t$ . Since by *Assumption 1*,  $\alpha_t^* \geq 0$ , and  $a_t \in [0, \bar{a}]$ ,  $E_t[r_{c,t+1}] - r_{c,t+1}$  is a positive quantity. Thus  $r_{c,t+1} \leq E_t(r_{c,t+1})$  for  $a_t \geq 0$ .

### A.3

**Derivation of disutility in  $t = 1$ .** In  $t = 0$ , investors expect to receive  $E_0[r_{c,1}] = \alpha_0^*(E_0[r_p] - f_0) + (1 - \alpha_0^*)r_f$ , but in  $t = 1$  they end up receiving  $r_{c,1} = \alpha_0^*(r_p - f_0 - a_0) + (1 - \alpha_0^*)r_f$ . The disutility experienced in  $t = 1$  can be derived as follow

$$F_1(E_0[r_{c,1}], r_{c,1}) = \phi\{[\alpha_0^*(E_0[r_p] - f_0) + (1 - \alpha_0^*)r_f] - [\alpha_0^*(r_p - f_0 - a_0) + (1 - \alpha_0^*)r_f]\}$$

$$F_1(E_0[r_{c,1}], r_{c,1}) = \phi\{\alpha_0^*E_0[r_p] - \alpha_0^*r_p + \alpha_0^*a_0\}$$

From *Assumption 2*  $E_0[r_p] = r_p$ , thus  $F_1(E_0[r_{c,1}], r_{c,1}) = \phi\alpha_0^*E_0[r_p]$ .

**Derivation of disutility in  $t = 2$ .** In  $t = 1$  investors expect to receive  $E_1[r_{c,2}] = \alpha_1^*(E_1[r_p] - f_1) + (1 - \alpha_1^*)r_f$ , but in  $t = 2$  they end up receiving  $r_{c,2} = \alpha_1^*(r_p - f_1 - a_1) + (1 - \alpha_1^*)r_f$ . Therefore in  $t = 2$ , they experience a disutility  $F_2(E_1[r_{c,2}], r_{c,2})$  deriving from the unexpected lower return

$$F_2(E_1[r_{c,2}], r_{c,2}) = \sum_{t=1}^2 \phi\beta^{2-t}(E_{t-1}(r_{c,t}) - r_{c,t})$$

$$F_2(E_1[r_{c,2}], r_{c,2}) = \phi\beta\alpha_0^*a_0 + \phi\{[\alpha_1^*(E_1[r_p] - f_1) + (1 - \alpha_1^*)r_f] - [\alpha_1^*(r_p - f_1 - a_1) + (1 - \alpha_1^*)r_f]\}$$

$$F_2(E_1[r_{c,2}], r_{c,2}) = \phi\beta\alpha_0^*a_0 + \phi\alpha_1^*a_1$$

### A.4

**Proof of Lemma 2.** Investors optimal allocation at time  $t = T$  is  $\alpha_T^* = \frac{(E_T[r_p] - f_T - r_f) - \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1}}{\gamma \sigma_p^2}$ . I decompose the function into its constituent terms  $\alpha_{T,1}^* =$

$\frac{(E_T[r_p] - f_T - r_f)}{\gamma\sigma_p^2}$  and  $\alpha_{T,2}^* = \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1}}{\gamma\sigma_p^2}$ . By *Assumption 2*,  $E_{T-1}[r_p] = E_T[r_p] = E[r_p]$ , therefore, given the constant nature of  $r_f$  and the cap on the total fee<sup>64</sup>  $f_{TOT} = f + a < E[r_p] - r_f$ , the term  $\alpha_{T,1}^*$  is a continuous function. When  $f$  is set at its maximum value  $f \rightarrow f_{TOT}$ , and thus  $a \rightarrow 0$ . Since  $f < E[r_p] - r_f$ , it can be written  $f = E[r_p] - r_f - \varepsilon$ , with  $\varepsilon$  an infinitely small factor. For a constant risk premium,  $\alpha_{T,1}^*$  ranges between  $\frac{(E_T[r_p] - r_f)}{\gamma\sigma_p^2}$ , when  $f_T = 0$ , and  $\frac{\varepsilon}{\gamma\sigma_p^2} \rightarrow 0$ , when  $f_T$  is at its maximum. *Appendix B.3* shows that  $f_T$  is a continuous monotonic decreasing function with  $f_0 > f_1 > \dots > f_T$ . Therefore, when  $T \rightarrow 0$ ,  $f_T \rightarrow 0$  and  $\alpha_{T,1}^* \rightarrow \frac{(E_T[r_p] - r_f)}{\gamma\sigma_p^2}$ . Conversely, when  $T \rightarrow \infty$ ,  $f_T \rightarrow E[r_p] - r_f - \varepsilon$  and  $\alpha_{T,1}^* \rightarrow 0$ . This proves that  $\alpha_{T,1}^* \in \left( \frac{(E_T[r_p] - r_f)}{\gamma\sigma_p^2}, 0 \right)$  is a continuous monotonic decreasing function. I will show now that  $\alpha_{T,2}^*$  is a continuous and strictly monotonic function. The factors  $\beta$  and  $\phi$  are constant factors and thus does not affect the continuity of  $\alpha_{T,2}^*$ . *Assumption 3* implies the continuity of  $\alpha_{t-1}^*$ , as it is defined  $\forall t$ . Since the summation of continuous functions (with respect to  $t$ ) preserves continuity and the division by  $\gamma\sigma_p^2$  is a scalar operation that does not affect continuity,  $\alpha_{T,2}^*$  is a continuous function. Given that  $\alpha_{T,2}^{*'} \geq 0$  for all  $T$ ,  $\alpha_{T,2}^*$  is monotonically increasing. When  $T = 0$ ,  $\alpha_{T,2}^* = 0$ , as investors do not experience any disutility. When  $T \rightarrow \infty$  however, given the high persistence assumed in *Assumption 3*,  $\lim_{T \rightarrow \infty} \alpha_{T,2}^* = \infty$ . Therefore, when  $T = 0$ ,  $\alpha_{T,1}^* > \alpha_{T,2}^*$ , while when  $T \rightarrow \infty$ ,  $\alpha_{T,1}^* < \alpha_{T,2}^*$ . Given the continuity of the two functions, the *Intermediate Value Theorem* implies that there exists at least one point  $T = T_{lim}$ , where  $\alpha_{T_{lim},1}^* = \alpha_{T_{lim},2}^*$ , and thus  $\alpha_{T_{lim}}^* = 0$ . Since  $\alpha_{T,1}^*$  is monotonic decreasing and  $\alpha_{T,2}^*$  is monotonic increasing, for  $T < T_{lim}$ ,  $\alpha_{T,1}^* > \alpha_{T,2}^*$ , and for  $T > T_{lim}$ ,  $\alpha_{T,1}^* < \alpha_{T,2}^*$ .

---

<sup>64</sup> See Appendix B.4 below for the transparent fee continuity proof.

## APPENDIX B

### B.1

**Myopic Fund Fee Derivation in  $t = 0$ .** The fund maximization problem in  $t = 0$  is  $\max_{f_0, a_0} \pi_0 = (f_0 + a_0 - c)\alpha_0^*$ , and can be rewritten as  $\max_{f_0, a_0} \pi_0 = (f_0 + a_0 - c) \frac{(E_0[r_p] - f_0 - r_f)}{\gamma\sigma_p^2}$ . The

partial derivative of  $\pi_0$  with respect to  $f_0$  is  $\frac{\partial \pi_0}{\partial f_0} = \frac{E_0[r_p] - 2f_0 - r_f}{\gamma\sigma_p^2} - \frac{a_0}{\gamma\sigma_p^2} + \frac{c}{\gamma\sigma_p^2}$ . Setting  $\frac{\partial \pi_0}{\partial f_0} = 0$

and solving for  $f_0$ , yields  $f_0 = \frac{E_0[r_p] - r_f - \bar{a} + c}{2}$ .

The partial derivative of  $\pi_0$  with respect to  $a_0$  is  $\frac{\partial \pi_0}{\partial a_0} = \frac{(E_0[r_p] - f_0 - r_f)}{\gamma\sigma_p^2}$ , which is not dependent on  $a_0$ . Therefore the higher profit  $\pi_0$  is reached when  $a_0$  is set to its maximum  $\bar{a}$ .

**Myopic Fund Fee Derivation in  $t = 1$ .** The fund maximization problem in  $t = 1$  is

$\max_{f_1, a_1} \pi_1 = (f_1 + a_1 - c)\alpha_1^*$ . Since  $\alpha_1^* = \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}$ , the maximization problem can be

written as  $\max_{f_1, a_1} \pi_1 = (f_1 + a_1 - c) \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}$ . The partial derivative of  $\pi_0$  with respect

to  $a_1$  is  $\frac{\partial \pi_1}{\partial a_1} = \frac{E_1[r_p] - 2f_1 - r_f - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2} - \frac{a_1}{\gamma\sigma_p^2} + \frac{c}{\gamma\sigma_p^2}$ . Setting  $\frac{\partial \pi_1}{\partial a_1} = 0$  and solving for  $f_1$ , yields  $f_1 =$

$$\frac{E_0[r_p] - r_f - \bar{a} + c}{2}.$$

The partial derivative of  $\pi_0$  with respect to  $a_1$  is  $\frac{\partial \pi_1}{\partial a_1} = \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}$ . By the same logic,

the higher profit  $\pi_1$  is reached when  $a_1$  is set to its maximum  $\bar{a}$ .

### B.2

The fund profit in  $t = 0$  is  $\pi_0 = (f_0 + \bar{a} - c)AUM_0 = (f_0 + \bar{a} - c) \frac{(E_0[r_p] - f_0 - r_f)}{\gamma\sigma_p^2}$ , with  $f_0 =$

$$\frac{E_0[r_p] - r_f - \bar{a} + c}{2}.$$

The fund profit in  $t = 1$  is  $\pi_1 = (f_1 + \bar{a} - c)AUM_1 = (f_1 + \bar{a} - c) \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}$ , with  $f_1 = \frac{E_1[r_p] - r_f - \phi\alpha_0^*\bar{a} - \bar{a} + c}{2}$ . I will show that  $\pi_0 > \pi_1$ , by showing first

that  $(f_0 + \bar{a} - c) > (f_1 + \bar{a} - c)$ , and then that  $AUM_0 > AUM_1$ .

Since  $\bar{a}$  and  $c$  are constant, the first relation can be written as  $f_0 > f_1$ . Suppose instead that  $f_0 < f_1$ , and substituting the values for these two terms, yields

$$\frac{E_0[r_p] - r_f - \bar{a} + c}{2} < \frac{E_1[r_p] - r_f - \phi\alpha_0^*\bar{a} - \bar{a} + c}{2}$$

$$E_0[r_p] - r_f - \bar{a} + c - E_1[r_p] + r_f + \phi\alpha_0^*\bar{a} + \bar{a} - c < 0$$

By Assumption 2,  $E_0[r_p] = E_1[r_p]$  and thus it remains  $\phi\alpha_0^*\bar{a} < 0$ . However, since  $\phi$ ,  $\alpha_0^*$  and  $\bar{a}$  are non-negative quantities,  $\phi\alpha_0^*\bar{a} > 0$ , which contrast with  $f_0 < f_1$ . Therefore  $f_0 > f_1$ .

$$AUM_0 = \frac{(E_0[r_p] - f_0 - r_f)}{\gamma\sigma_p^2} \text{ and substituting the value of } f_0 \text{ yields } AUM_0 = \frac{(E_0[r_p] - \left(\frac{E_0[r_p] - r_f - \bar{a} + c}{2}\right) - r_f)}{\gamma\sigma_p^2}. \text{ Rearranging terms, } AUM_0 = \frac{(E_0[r_p] - r_f + \bar{a} - c)}{2\gamma\sigma_p^2}. \text{ } AUM_1 = \frac{(E_1[r_p] - f_1 - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}, \text{ and substituting the value of } f_1 \text{ yields } AUM_1 = \frac{(E_1[r_p] - \left(\frac{E_1[r_p] - r_f - \phi\alpha_0^*\bar{a} - \bar{a} + c}{2}\right) - r_f) - \phi\alpha_0^*\bar{a}}{\gamma\sigma_p^2}. \text{ Rearranging terms yields } AUM_1 = \frac{E_1[r_p] - r_f - \phi\alpha_0^*\bar{a} + \bar{a} - c}{2\gamma\sigma_p^2}.$$

Suppose that  $AUM_0 < AUM_1$ .

$$\frac{(E_0[r_p] - r_f + \bar{a} - c)}{2\gamma\sigma_p^2} < \frac{E_1[r_p] - r_f - \phi\alpha_0^*\bar{a} + \bar{a} - c}{2\gamma\sigma_p^2}$$

$$E_0[r_p] - r_f + \bar{a} - c - E_1[r_p] + r_f + \phi\alpha_0^*\bar{a} - \bar{a} + c < 0$$

By Assumption 2,  $E_0[r_p] = E_1[r_p]$  and thus it remains  $\phi\alpha_0^*\bar{a} < 0$ . However, since  $\phi$ ,  $\alpha_0^*$  and  $\bar{a}$  are non-negative quantities,  $\phi\alpha_0^*\bar{a} > 0$ , which contrast with  $AUM_0 < AUM_1$ . Therefore  $AUM_0 > AUM_1$ . Since  $f_0 > f_1$  and  $AUM_0 > AUM_1$ ,  $\pi_0 > \pi_1$ .

### B.3

The transparent fee in  $t = 1$  is  $f_1 = \frac{E_1[r_p] - r_f + c}{2} - \frac{\phi\alpha_0^*\bar{a} + \bar{a}}{2}$ , while the transparent fee in  $t = 2$  is

$$f_2 = \frac{E_2[r_p] - r_f + c}{2} - \frac{\phi\beta\alpha_0^*\bar{a} + \phi\alpha_1^*\bar{a} + \bar{a}}{2}.$$

$$f_1 - f_2 = \frac{E_1[r_p] - r_f + c}{2} - \frac{\phi\alpha_0^*\bar{a} + \bar{a}}{2} - \frac{E_2[r_p] - r_f + c}{2} + \frac{\phi\beta\alpha_0^*\bar{a} + \phi\alpha_1^*\bar{a} + \bar{a}}{2}$$

By *Assumption 2*  $E_1[r_p] = E_2[r_p]$ , thus

$$f_1 - f_2 = -\frac{\phi\alpha_0^*\bar{a}}{2} + \frac{\phi\beta\alpha_0^*\bar{a} + \phi\alpha_1^*\bar{a}}{2} = \frac{\phi\bar{a}}{2}(-\alpha_0^* + \beta\alpha_0^* + \alpha_1^*)$$

$$f_1 - f_2 = \frac{\phi\bar{a}}{2}[\alpha_0^*(\beta - 1) + \alpha_1^*]$$

By *Assumption 3*  $\beta \approx 1$ , and so  $\beta - 1 \approx 0$ . Therefore,  $f_1 - f_2 = \frac{\phi\bar{a}}{2}[\alpha_1^*]$ . Since  $\frac{\phi\bar{a}}{2}$  is a non-negative quantity and, by *Assumption 1*,  $\alpha_1^* \geq 0$ ,  $f_1 - f_2 > 0$ . This means that  $f_1 > f_2$ .

#### B.4

The transparent fee at  $t = T$  is  $f_T = \frac{E_{T-1}[r_p] - r_f + c}{2} - \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1} + a_T}{2}$ . To analyze the asymptotic behavior of the function  $f_T$  as  $T \rightarrow \infty$ , I decompose the function into its constituent terms  $f_{T,1} = \frac{E_{T-1}[r_p] - r_f + c}{2}$  and  $f_{T,2} = \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1} + a_T}{2}$ . By *Assumption 2*,  $E_{T-1}[r_p] = E_T[r_p] = E[r_p]$ , therefore, given the constant nature of  $r_f$  and  $c$ , the term  $f_{T,1}$  is a time-independent constant function and thus continuous, with  $\lim_{T \rightarrow \infty} f_{T,1} = \frac{E[r_p] - r_f + c}{2}$ . I will show now that  $f_{T,2}$  is a continuous and strictly monotone function. The factors  $\beta$  and  $\phi$  are constant factors and thus does not affect the continuity of  $f_{T,2}$ . *Appendix B.1* shows that  $a_T$  is set at its maximum level  $\bar{a}$ , thus  $a_T$  is also a constant term that does not affect the continuity of  $f_{T,2}$ . The same process apply for  $a_{t-1}$ , since  $a_{t-1} = a_t = a_T$ . *Assumption 3* implies the continuity of  $\alpha_{t-1}^*$ , as it is defined  $\forall t$ . Since the summation of continuous functions (with respect to  $t$ ) preserves continuity and the division by 2 is a scalar operation that does not affect continuity,  $f_{T,2}$  is a continuous function. Given that  $f_{T,2}' \geq 0$  for all  $T$ ,  $f_{T,2}$  is monotonically increasing. When  $T = 0$ ,  $f_{0,2} = \frac{a_0}{2} = \frac{\bar{a}}{2}$ . From *Assumption 4*,  $\bar{a} < E[r_p] - r_f - c$ . This means that when  $T = 0$ ,  $f_{0,2} < f_{0,1}$ , as

$$\frac{\bar{a}}{2} - \frac{E_0[r_p] - r_f + c}{2} < 0$$

$$\frac{\bar{a} - (E_0[r_p] - r_f + c)}{2} < 0$$

By *Assumption 2*,  $E_0[r_p] = E[r_p]$ , thus  $\bar{a} - (E_0[r_p] - r_f + c) < 0$ . Given that  $f_{T,1}$  is a constant value and  $f_{T,2}$  is continuous and monotonically increasing with  $f_{T,2}$  initially taking a value lower than  $f_{T,1}$ , the *Intermediate Value Theorem* implies that there exists at least one point  $T = N$ , where  $f_{N,1} = f_{N,2}$ .

## B.5

**Proof of Lemma 3.1.** *Appendix B.1* shows the myopic fund fee derivation process at  $t = 0,1$ , where it sets its hidden fee level at the maximum level  $\bar{a}$ . Generalizing, at any time period  $T$ , the optimal investor allocation is

$$\alpha_T^* = \frac{(E_T[r_p] - f_T - r_f) - \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1}}{\gamma \sigma_p^2}$$

For  $T = 2$  thus the previous periods hidden fee levels, from *Appendix B.1*, are constant and

equal to  $\bar{a}$ . Therefore  $\alpha_2^* = \frac{(E_2[r_p] - f_2 - r_f) - \sum_{t=1}^2 \beta^{2-t} \phi \alpha_{t-1}^* \bar{a}}{\gamma \sigma_p^2}$ . The fund maximization problem in

$t = 2$  is  $\max_{f_2, a_2} \pi_2 = (f_2 + a_2 - c) \alpha_2^*$ , and can be rewritten as  $\max_{f_2, a_2} \pi_2 = (f_2 + a_2 -$

$c) \frac{(E_2[r_p] - f_2 - r_f) - \sum_{t=1}^2 \beta^{2-t} \phi \alpha_{t-1}^* \bar{a}}{\gamma \sigma_p^2}$ . Performing the partial derivative of  $\pi_2$  with respect to  $a_2$

yields  $\frac{\partial \pi_2}{\partial a_2} = \frac{(E_2[r_p] - f_2 - r_f) - \sum_{t=1}^2 \beta^{2-t} \phi \alpha_{t-1}^* \bar{a}}{\gamma \sigma_p^2}$ , thus the optimal level of hidden fee is again  $\bar{a}$ . Since

up to a discrete period  $T$ ,  $a_{t-1} = a_t = a_{t+1} = \dots = a_{T-1} = \bar{a}$ . The partial derivative of  $\pi_T$  with

respect to  $a_T$  yields  $\frac{\partial \pi_T}{\partial a_T} = \frac{(E_T[r_p] - f_T - r_f) - \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* \bar{a}}{\gamma \sigma_p^2}$ . Therefore the optimal level of hidden

fee is again  $\bar{a}$ .

**Proof of Lemma 3.2.** *Appendix B.4* shows that  $f_T$  can be decomposed into two ‘ad hoc’

functions  $f_{T,1} = \frac{E_{T-1}[r_p] - r_f + c}{2}$  and  $f_{T,2} = \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1} + a_T}{2}$ , with  $f_T = f_{T,1} - f_{T,2}$ . From

*Appendix B.4* both  $f_{T,1}$  and  $f_{T,2}$  are continuous, and since the summation of continuous

functions preserve continuity,  $f_T$  is a continuous function. As  $f_{T,2}$  is monotonically increasing,

$-f_{T,2}$  is monotonically decreasing. By symmetry, as the initial value of  $f_{T,2}$  is lower than the

initial value of  $f_{T,1}$ , the initial value of  $-f_{T,2}$  is higher than the initial value of  $f_{T,1}$ . The

*Intermediate Value Theorem* implies that there exists at least one point  $T = N$ , where  $f_{N,1} = f_{N,2}$ . Given that  $f_{T,1}$  is constant and  $-f_{T,2}$  monotonically decreasing,  $f_T$  is monotonically decreasing, implying that  $f_0 > f_1 > \dots > f_t > f_T$ . *Appendix B.4* demonstrate that at  $T = N$ ,  $f_{T,1} - f_{T,2} = 0$  and thus  $f_T \rightarrow 0$  for  $T \rightarrow N$ .

## B.6

**Proof of Lemma 4.** The profit of the fund is  $\pi_t = (f_t + a_t - c)\alpha_t$ . *Lemma 3* states that  $a_t$  is a continuous function equal to the constant value  $\bar{a} \forall t \in [0, N]$ , while  $f_t$  is a continuous and monotonically decreasing function  $\forall t \in [0, N]$ .

For  $t > N$ ,  $f_t$  is continuous and equal to a constant value  $f_{t-1} = f_t = 0$ , thus in the interval  $(N, \infty)$ ,  $f_t$  is constant and equal to 0. Therefore, after period  $N$ ,  $\pi_t = (a_t - c)\alpha_t$ .  $a_t$  is a continuous and monotonic decreasing function in the interval  $(N, \infty)$ , reaching  $a_t = 0$  as time grows. Given that  $c$  is a constant time-independent value, the *Intermediate Value Theorem* implies that there exist at least one period  $K$  where  $a_K = c$ , Therefore  $\pi_t = 0(\alpha_t) = 0$ .

## APPENDIX C

### C.1

The total cumulative AUM up to period  $T$  is  $\sum_{t=1}^T AUM_t = \sum_{t=1}^T \alpha_t$ , or  $\sum_{t=1}^T AUM_t = \alpha_0 + \alpha_1 + \dots + \alpha_T$ . Substituting the values of  $\alpha_0, \alpha_1, \alpha_T$

$$\sum_{t=1}^T AUM_t = \frac{(E[r_p] - f - r_f)}{\gamma\sigma_p^2} + \frac{(E[r_p] - f - r_f) - \phi\alpha_0^*a}{\gamma\sigma_p^2} + \dots + \frac{(E[r_p] - f - r_f) - \sum_{t=1}^T \beta^{T-t} \phi\alpha_{t-1}^*a}{\gamma\sigma_p^2}$$

$$\sum_{t=1}^T AUM_t = \alpha_0 + \left( \frac{(E[r_p] - f - r_f)}{\gamma\sigma_p^2} - \frac{\phi\alpha_0^*a}{\gamma\sigma_p^2} \right) + \dots + \left( \frac{(E[r_p] - f - r_f)}{\gamma\sigma_p^2} - \frac{\sum_{t=1}^T \beta^{T-t} \phi\alpha_{t-1}^*a}{\gamma\sigma_p^2} \right)$$

$$\sum_{t=1}^T AUM_t = \alpha_0 + \left( \frac{(E[r_p] - f - r_f)}{\gamma\sigma_p^2} \right) T - \frac{\phi\alpha_0^*a}{\gamma\sigma_p^2} - \dots - \frac{a\phi \sum_{t=1}^T \beta^{T-t} \alpha_{t-1}^*}{\gamma\sigma_p^2}$$

$$\begin{aligned} \sum_{t=1}^T AUM_t &= \alpha_0 + \left( \frac{(E[r_p] - f - r_f)}{\gamma\sigma_p^2} \right) T \\ &\quad - \frac{\phi a}{\gamma\sigma_p^2} \left( \alpha_0^* + \sum_{t=1}^1 \beta^{T-t} \alpha_{t-1}^* + \sum_{t=1}^2 \beta^{T-t} \alpha_{t-1}^* + \dots + \sum_{t=1}^T \beta^{T-t} \alpha_{t-1}^* \right) \end{aligned}$$

$$\sum_{t=1}^T AUM_t = \alpha_0 + \left( \frac{E[r_p] - r_f - f}{\gamma\sigma_p^2} \right) (T) - \frac{a\phi}{\gamma\sigma_p^2} \left[ \sum_{t=1}^T \alpha_{t-1} \left( \sum_{s=t}^T \beta^{T-s} \right) \right]$$

### C.2

The profit function of the fund  $\pi_t = (f + a - c)\alpha_t$  depends on  $(f + a - c)$  and  $\alpha_t$ . The first term represents the ‘*direct channel*’. Suppose for simplicity a unitary level of  $\alpha_t$  such that  $\alpha_t = 1$ , and that  $\alpha_t$  is not influenced by the fee level. The profit of the fund is now  $\pi_t = f + a - c$ . Suppose an increase in the transparent fee level of  $p$ . The profit of the fund is now  $\pi_t = (f + p) + a - c$ . The profit variation is therefore  $\Delta\pi_{t,p} = (f + p) + a - c - (f + a - c) = p$ . Suppose now the same increase  $p$  happens on the hidden fee. The profit of the fund is now  $\pi_t = f + (a + p) - c$ . The profit variation is therefore  $\Delta\pi_{t,a} = f + (a + p) - c - (f + a - c) = p$ . Clearly,  $\Delta\pi_{t,p} = \Delta\pi_{t,a}$ .

### C.3

**Proof of Lemma 5.** The transparent fee scaling factor  $S_f = \frac{1}{\gamma\sigma_p^2}$  is constant and time-independent, thus  $S_f$  is a continuous function with non-negative finite value. The hidden fee scaling factor  $S_{a,T} = \frac{\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^*}{\gamma\sigma_p^2}$  is on the other hand a time-dependent function. *Appendix B* showed that the cumulative term  $\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^*$  is a continuous and strictly monotonic function in time. The factor  $\gamma\sigma_p^2$  does not affect the continuity of a function and thus  $S_{a,T}$  is a continuous strictly monotonic increasing function.  $S_{a,T} = 0$  for  $T = 0$ , as investors do not experience any disutility at the initial period, while by *Assumption 3*, for  $\beta \approx 1$ ,  $\lim_T S_{a,T} \rightarrow \infty$ . Given that  $S_f$  is a constant finite non-negative value in the interval  $(0, \infty)$ , at the initial period  $S_{f,0} > S_{a,0}$ , while for  $T \rightarrow \infty$ ,  $S_f < S_{a,T}$ . Given the continuity of the functions, the *Intermediate Value Theorem* ensure the existence of at least one point  $T = T'$  where  $S_f < S_{a,T'}$ . As a consequence, given the strictly monotonic increasing nature of  $S_{a,T}$ , for  $T < T'$ ,  $S_f < S_{a,T}$ , while for  $T > T'$ ,  $S_f > S_{a,T}$ .

## APPENDIX D

### D.1

**Proof of Lemma 6.** Consider a fund that set the hidden fee  $a_t = a_{t+1} = 0$ . In  $t = 0$ , investors allocate  $\alpha_0^* = \frac{(E_0[r_p] - f - r_f)}{\gamma\sigma_p^2}$ . By *Assumption 4*,  $f_{TOT} \leq E(r_p) - r_f$  and since  $a_t = 0$ ,  $f_{TOT} = f$ .

Therefore the condition  $f \leq E(r_p) - r_f$  ensures a non-negative risk premium and consequently a non-negative asset allocation to the fund. Investors expect to receive in  $t = 1$ ,  $E_0[r_{c,1}] = \alpha_0^*(E_0[r_p] - f) + (1 - \alpha_0^*)r_f$ . They then receive  $r_{c,1} = \alpha_0^*(r_p - f) + (1 - \alpha_0^*)r_f$ . The disutility experienced in  $t = 1$  can be derived as follow

$$F_1(E_0[r_{c,1}], r_{c,1}) = \phi\{[\alpha_0^*(E_0[r_p] - f) + (1 - \alpha_0^*)r_f] - [\alpha_0^*(r_p - f) + (1 - \alpha_0^*)r_f]\}$$

$$F_1(E_0[r_{c,1}], r_{c,1}) = \phi\{\alpha_0^*E_0[r_p] - \alpha_0^*r_p\}$$

From *Assumption 2*  $E_0[r_p] = r_p$ , thus  $F_1(E_0[r_{c,1}], r_{c,1}) = 0$ . In  $t = 1$  investors expect to receive  $E_1[r_{c,2}] = \alpha_1^*(E_1[r_p] - f) + (1 - \alpha_1^*)r_f$ , and receive  $r_{c,2} = \alpha_1^*(r_p - f) + (1 - \alpha_1^*)r_f$ . Therefore in  $t = 2$ , they experience a disutility  $F_2(E_1[r_{c,2}], r_{c,2})$

$$F_2(E_1[r_{c,2}], r_{c,2}) = 0 + \phi\{[\alpha_1^*(E_1[r_p] - f) + (1 - \alpha_1^*)r_f] - [\alpha_1^*(r_p - f) + (1 - \alpha_1^*)r_f]\}$$

$$F_2(E_1[r_{c,2}], r_{c,2}) = 0$$

At some time  $T$  investors optimal allocation is  $\alpha_T^* = \frac{(E_T[r_p] - f - r_f) - \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1}}{\gamma\sigma_p^2}$ , and since

the disutility factor is constantly null, i.e.  $\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a_{t-1} = 0$ ,  $\alpha_T^* = \frac{(E_T[r_p] - f - r_f)}{\gamma\sigma_p^2}$ . By

*Assumption 2*,  $E_0[r_p] = E_1[r_p] = \dots = E_T[r_p] = E[r_p]$ , therefore  $\alpha_0^* = \alpha_1^* = \dots = \alpha_T^* > 0$ .

Consequently,  $AUM_t$  is a constant non-negative value. The profit of the fund at time  $t$  is  $\pi_t = (f - c)AUM_t$ , given that  $(f - c)$  is a positive scalar,  $(f - c)AUM_t$  is a constant non-negative value in time. Therefore  $\pi_0 = \pi_1 = \dots = \pi_t > 0$ .

## D.2

I use the indexing  $M$  for the myopic fund and  $F$  for the forward looking fund. At  $t = 0$  the myopic fund sets  $f_{0,M} = \frac{E_0[r_p] - r_f - \bar{a} + c}{2}$  and  $a_{0,M} = \bar{a}$ . Its profit function is  $\pi_{0,M} = (f_{0,M} + a_{0,M} - c)AUM_{0,M}$ , with  $AUM_{0,M} = \alpha_{0,M}^* = \frac{(E_0[r_p] - f_{0,M} - r_f)}{\gamma\sigma_p^2}$ . The forward looking fund, on the other hand, sets  $a_{0,F} = 0$  and  $f_{0,F} = \frac{(E_0[r_p] - r_f + c)}{2}$ . Its profit function is  $\pi_{0,F} = (f_{0,F} - c)AUM_{0,F}$ , with  $AUM_{0,F} = \alpha_{0,F}^* = \frac{(E_0[r_p] - f_{0,F} - r_f)}{\gamma\sigma_p^2}$ . I now compare the two profit functions to demonstrate that, at  $t = 0$ ,  $\pi_{0,M} > \pi_{0,F}$ . First,  $AUM_{0,M} = \frac{(E_0[r_p] - r_f)}{\gamma\sigma_p^2} - \frac{f_{0,M}}{\gamma\sigma_p^2}$ , while  $AUM_{0,F} = \frac{(E_0[r_p] - r_f)}{\gamma\sigma_p^2} - \frac{f_{0,F}}{\gamma\sigma_p^2}$ . Performing the difference of assets under management

$$AUM_{0,M} - AUM_{0,F} = \frac{(E_0[r_p] - r_f)}{\gamma\sigma_p^2} - \frac{f_{0,M}}{\gamma\sigma_p^2} - \frac{(E_0[r_p] - r_f)}{\gamma\sigma_p^2} + \frac{f_{0,F}}{\gamma\sigma_p^2}$$

$$AUM_{0,M} - AUM_{0,F} = \frac{1}{\gamma\sigma_p^2} (f_{0,F} - f_{0,M})$$

Given that  $f_{0,M} = \frac{E_0[r_p] - r_f - \bar{a} + c}{2}$  and  $f_{0,F} = \frac{(E_0[r_p] - r_f + c)}{2}$ , the difference  $f_{0,F} - f_{0,M} = \frac{\bar{a}}{2} > 0$ . Therefore,  $AUM_{0,M} - AUM_{0,F} = \frac{\bar{a}}{2\gamma\sigma_p^2} > 0$  and so  $AUM_{0,M} > AUM_{0,F}$ . Moving to the direct revenue channels (i.e., terms inside the brackets of the profit functions) it is clear to see that the direct revenue channel for the myopic fund ( $f_{0,M} + a_{0,M} - c$ ) is higher than that of the forward looking fund ( $f_{0,F} - c$ ). Performing the difference  $(f_{0,M} + a_{0,M} - c) - (f_{0,F} - c)$  and rearranging terms yields  $f_{0,M} - f_{0,F} + a_{0,M} > 0$ . It has just been proved that  $f_{0,F} - f_{0,M} = \frac{\bar{a}}{2}$ , therefore  $f_{0,M} - f_{0,F} = -\frac{\bar{a}}{2}$ , while  $a_{0,M} = \bar{a}$ . Thus,  $-\frac{\bar{a}}{2} + \bar{a} = \frac{\bar{a}}{2} > 0$ . Consequently, the direct revenue channel for the myopic fund is higher than that of the forward looking fund. Given that, not only the myopic fund has a higher initial AUM,  $AUM_{0,M} > AUM_{0,F}$ , but also has a higher direct revenue channel  $(f_{0,M} + a_{0,M} - c) > (f_{0,F} - c)$ , the profit of the myopic fund is initially higher than that of the forward looking fund,  $\pi_{0,M} > \pi_{0,F}$ .

### D.3

*Appendix B.3* shows that the profit of the myopic fund is a decreasing function in time, due to the cumulative disutility  $F_M = \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* \bar{a}$ . If the forward looking fund sets  $0 < a < \bar{a}$ , the cumulative disutility is  $F_F = \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a$ . Since  $\beta$  and  $\phi$  are fixed,  $F_F < F_M$ . *Proposition 1* states that the myopic fund will reach a terminal period, which, by *Lemma 3 and 4*, is reached at time  $k$ . This is a result of the *Intermediate Value Theorem* on the continuity nature of the fee and profitability level functions. The disutility functions  $F_F$  and  $F_M$  are, under *Assumption 2*, both continuous and strictly monotonic increasing. The function  $F_M - F_F$ , given that summation preserve continuity, is continuous and monotonic increasing, since  $\sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* \bar{a} > \sum_{t=1}^T \beta^{T-t} \phi \alpha_{t-1}^* a$ . For an assumed equal level of marginal costs  $c$  for both the myopic and forward looking fund, the profitability level is reached faster by the myopic fund. Intuitively, given that  $F_F < F_M$ , investors in the myopic fund scenario reduce the capital allocated faster, while the total fee level decreases sharper as well, as shown by *Lemma 3*. The profit function of the myopic fund has therefore, in absolute terms, a higher downwards slope than that of the forward looking fund. Consequently, from the *Intermediate Value Theorem*, it will cross the profitability level sooner than the forward looking fund.