

Optimal Transmission Expansion Planning

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I would like to dedicate this master's thesis to my loving grandfather, whom I have unfortunately seen going away during my studies.

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

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Abstract

This master's thesis investigates different methods to tackle a problem that power system designers, such as [Tractebel](#), face every day: finding the optimal expansion planning of a power system. This research presents a robust method able to handle a variety of power systems, in a reasonable amount of time, and with a respectable convergence.

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Notations

Abbreviations

TEP	Transmission Expansion Planning
OTEP	Optimal Transmission Expansion Planning
LP	Linear Programming
MILP	Mixer-Integer Linear Programming
MINLP	Mixer-Integer Non-Linear Programming
RO	Robust Optimization
ARO	Adaptive Robust Optimization
TSO	Transmission System Operator

Sets

$\mathcal{N}$	Set of buses (nodes).
$\mathcal{L}_e$	Set of existing transmission lines, where $l = (m, n) \in \mathcal{L}_e$ is the line with endpoints m and n .
$\mathcal{L}_c$	Set of prospective/candidate transmission lines.
Ω_n^-, Ω_n^+	Set of incoming and outgoing arcs at bus n .

<i>Parameters</i>	
I_l	Investment cost for candidate line l .
L_l	Length of candidate line l .
$\bar{L}$	Maximum total length of installed lines.
T_l	Maximum flow capacity on line l .
B_l	Susceptance of line l .
M_l	Big-M value for line l .
P_n	Power generation at bus n .
D_n	Load at bus n .
$\overline{\Delta\theta}$	Maximum difference in voltage angle between two adjacent buses.
<i>Decision variables</i>	
x_l	Investment decision on candidate line k (binary variable).
f_l	Algebraic power flow on line k .
θ_n	Phase angle of bus n .

The *Transmission Expansion Planning (TEP)* problem is a complex task whose main objective consists in finding ways to expand a transmission system while respecting predefined criteria. Generally, the objective of an expansion plan is to ensure that power systems are able to meet the forecasted demand without load interruption or damage to the physical integrity of the equipment while minimising investment and operational costs. To date, the problem is often solved by engineers trying to find the less expensive way to expand the transmission system, but without solving a formal optimization problem. This approach has two drawbacks: first it can be time-consuming and, second, it is unable to provide a guarantee on the quality of the solution obtained, i.e. is the solution close to the cheapest one?

In this context, we understand the need of the formalization of the TEP problem as an optimization problem in order to *automatically* find the best expansion plan. This formalization is referred to as *Optimal Transmission Expansion Planning (OTEP)* and consists in finding the optimal expansion of the transmission network subject to technical and economic constraints. This problem is a complex decision-making process to decide where, when and what reinforcements should be placed in the power network. The resulting optimization problem consists in minimizing an objective function (usually the total investment cost) subject to different sets of constraints. The constraints are introduced in the optimization problem with the objective of translating several planning criteria to respect. This optimization problem is usually formulated as a *large-scale mixed-integer linear programming (MILP)* problem considering a pre-defined list of candidate circuits. The definition of the list of candidate circuits has a major impact on the final expansion plan and is usually built only based on the experience of the planners.

The purpose of this master's thesis is twofold: **(i)** first, one is interested in the ways the OTEP problem has already been approached in literature and, **(ii)** second, one desires to obtain

a method that could provide good solutions to the OTEP problem within a reasonable amount of time. To tackle these challenges, the master's thesis is structured in three parts:

- First, in part [I](#), after a literature review on the different ways to formulate the transmission expansion planning (TEP) problem, a mathematical presentation (objective function, constraints, etc.) of the TEP problem and a formulation as an optimization problem are given .
- Second, in part [II](#), we review from literature the methods traditionally used to solve the TEP problem and then present and justify our own way to approach the problem.
- Finally, in part [III](#), the performances of our approach, on the one hand, and [Gurobi](#) (a commercial solver), on the other hand, are compared.

Part I

Model

Literature review on the transmission expansion planning (TEP) problem.

As mentioned in the introduction [1](#), the transmission expansion planning problem is a well known challenge that has a multitude of formulations depending on the criteria, selected by transmission system operators, that must be respected. The idea of this chapter is to present the most common formulations of the TEP problem encountered in the scientific literature. Generally, the difference between two TEP formulations lies in the planning horizon, the definition of the candidate transmission lines, the costs to minimize, the consideration of uncertainty, the way to consider uncertainty, etc.

Hence, hypotheses have to be made in order to choose the right TEP problem's formulation; they are presented in section [3.1](#) together with [Tractebel](#)'s requirements. Moreover, since certain formulations of the TEP problem are very dissimilar, some well-know methods to solve the TEP problem are performant for some formulations and useless for others. It is thereby important to be aware of the different formulations in order to consciously adopt the most appropriate solving methods or at least to be able to select the promising ones.

2.1 Planning horizon

The first distinction concerns the **planning horizon**; traditionally the TEP can be classified in either *static (single-stage)* or *dynamic (multi-stage)* planning. One talks about *static* planning if the planner seeks the optimal expansion plan for a single year on the planning horizon, i.e. the transmission system operator (TSO) seeks the answer to the questions: "what" transmission facilities^{[1](#)} should be added to the network and "where" should they be installed. Hence, in *static*

¹For instance, in the IEEE-24 buses system [A.1](#), one has transmission lines of different voltage as well as switches.

planning, only the final optimal network state is sought. By contrast, in *dynamic* planning, the operation costs are also considered during the construction of the expanded network. As a consequence, in addition to the questions "what" and "where", the TSO seeks also the answer to the question "when" a certain facility should be built, i.e. his objective is to find the best expansion strategy over all future years given a time horizon by elaborating a construction plan over the years of the lines to add to the current network [34, 56].

The *static* version of the TEP was first of interest to the scientific community and has been approached in literature using either mathematical programming [2, 25] or heuristics [26]. *Dynamic* planning models are tremendously more complex; finding the optimal construction plan over all years requires a large number of variables and constraints to ensure the coherence of the investments over time, resulting in an enormous computational effort to get the optimal expansion plan. Hence, few articles approach the problem from this point of view [18, 57]; instead, to achieve reasonable computation time, pseudo-dynamic procedures are considered, i.e. the successive resolution of static expansion problems. Pseudo-dynamic procedures often take two forms: (i) **a forward procedure**, i.e. solving static expansion problems sequentially for all years, starting from the first one [9]; (ii) **a backward procedure** which starts from the last year and goes back in time while trying to respect the decisions taken [48]. Since in the later periods the network is often more stressed than in the first ones, backward procedures generally achieve better performances than forward procedures [8].

In our case, we are interested in static planning; our objective is to find the best investment plan for a given network, i.e. obtaining the cheapest expanded power system while respecting all TSO's constraints, rather than knowing when the new transmission lines should be added to the existing power system.

2.2 AC/DC model

If we want to model the steady-state of a power transmission network in the most accurate way (the AC-formulation), one should consider a set of nonlinear algebraic equations known as the power flow. The AC formulation of the TEP problem is rarely discussed in literature since it is modelled as a mixed-integer nonlinear programming (MINLP) problem, which are problems amongst the most difficult to solve due to their intrinsic mathematical complexity [61]. Moreover, for non-convex MINLP problems, such as the AC-formulated TEP problem, solvers give either a heuristic solution or no solution at all [63]; a globally optimal solution is seldom obtained [62]. Hence, even though the AC network model represents the electric power network

accurately, one prefers modelling the TEP problem as a mixed-integer linear programming (MILP) problems via a DC approximation of the network [63]. To obtain linear equations, one makes four assumptions: **(i)** ignoring the reactive power balance equations, **(ii)** assuming all voltage magnitudes to be nominal, **(iii)** ignoring line losses, and, finally, **(iv)** ignoring the tap dependence in the transformer reactances. Moreover, it is a common assumption to consider that the difference in voltage phase angle of the adjacent buses in the network is small; hence one can use the following approximations:

$$\sin(\theta_i - \theta_j) \simeq (\theta_i - \theta_j), \quad (2.1)$$

$$\cos(\theta_i - \theta_j) \simeq 1. \quad (2.2)$$

Starting from equation 2.3, which represents the active power transferred on the transmission line between buses i and j when losses are neglected, one can derive the DC-power flow, i.e. the linearized power flow equations, by using the five assumptions previously mentioned:

$$f_{i,j} \stackrel{\text{i, iii and iv}}{=} B_{i,j} \cdot V_i \cdot V_j \cdot \sin(\theta_i - \theta_j), \quad (2.3)$$

$$\stackrel{ii}{=} B_{i,j} \cdot \sin(\theta_i - \theta_j), \quad (2.4)$$

$$\stackrel{2.1}{=} B_{i,j} \cdot (\theta_i - \theta_j), \quad (2.5)$$

where $B_{i,j}$ is the susceptance of the line between buses i and j , V_i and V_j are the voltage magnitudes in buses i and j , and θ_i and θ_j are the phase angles in buses i and j respectively. Based on approximation 2.5, real power injection in each bus is approximated by the unique solution of the following set of linear equations (full rank):

$$\sum_{n=1}^N P_n = 0, \quad (2.6)$$

$$\mathbf{P} = \mathbf{T} \cdot \boldsymbol{\theta}, \quad (2.7)$$

where N is the number of buses, $\mathbf{P} = (P_n)$, $n \in \{1, \dots, N\} - \{h\}$ is the vector of real power injections, $\boldsymbol{\theta} = (\theta_n)$, $n \in \{1, \dots, N\} - \{h\}$ is the vector of bus voltage angles, where h is the hub node ², and where $\mathbf{T}$ is defined as follows:

²A node arbitrarily chosen where we impose its voltage angle to 0.

$$T_{mn} = \begin{cases} -\frac{1}{X_{mn}}, & (m,n) \in A, \quad m \neq n \\ \sum_{n'=1, n' \neq m}^N \frac{1}{X_{mn'}}, & m = n \\ 0, & (m,n) \notin A \end{cases} \quad (2.8)$$

where A is the set of arcs in the network, and $X_{mn} = B_{mn}^{-1}$ is the reactance of the line linking buses m and n . Equations 2.6-2.7 are known as the node-based direct-current power flow equations. [44, 45]

For small power systems it is easy to find examples for which the results given by the DC-power flow are either exact or totally wrong; however, for larger systems, the errors resulting from such an approximation are hard to quantify analytically [63]. Article [33] provides a complete analysis of different existing approximations of the power flow, whereas article [62] presents a less relaxed model of the network compared to the DC-formulation that leads to better solutions to the TEP problem.

Finally, active power losses are generally neglected like in the DC-power flow; however considering losses can influence the optimal transmission plan. Article [1] shows how to obtain a MILP while considering line losses by using a piecewise linear approximation to approach the quadratic loss term.

2.3 Objective function

Generally, the objective function of the TEP problem is a combination of the following most common costs:

- **Investment costs**, representing the costs coming from the construction of the new lines composing the expansion plan.
- **Load shedding costs**, representing the cost of not serving demand.
- **Operation costs**, representing the expenses related to the transmission of the power along the expanded power network.

The different objective functions found in the scientific literature are usually a combination of these costs, such as: minimizing only the investments costs [3, 10, 55, 42], minimizing the investment and load shedding costs [26, 31], minimizing the investment and operation costs [1, 29] and, finally, minimizing all three costs [16, 12].

2.4 Candidate lines selection

In general, solving a transmission expansion planning problem means selecting the transmission lines to build among a set of candidate lines in order to satisfy given constraints while minimising a given objective function. Of course the set of candidate lines to consider has a major impact on the optimal expansion planning. As mentioned in the introduction, this set is traditionally manually defined by the transmission systems operators only based on their experience. Ideally, we would like to consider all possible candidate transmission lines in order to find the best expansion plan possible; however, if we consider the possibility to reinforce each existing transmission line, $(N - 1)!$ candidate lines should be considered, where N is the number of buses. Of course, considering all possible candidate lines is foolish since most of them can be eliminated based on geographical conditions, line length, etc. However, it is obvious that for large networks, considering even a part of all possible candidate lines is not an option since it leads to an intractable combinatorial optimization problem; hence a formalization in the selection of the candidate lines is requested. In article [59], a so-called *congestion buses* approach is proposed to select the candidate transmission lines automatically. Article [39] proposes a selection of the candidate lines in three steps: (i) first, a selection is performed based on sensitivities, (ii) then a filter is applied to keep to most promising lines in order to obtain a tractable problem without compromising global optimality and finally (iii) the relationships among investments is studied. The research in this domain remains limited and will probably be explored in the coming years.

2.5 Uncertainty

The TEP problem tries to find the best way to expand or reinforce an existing transmission network; the decisions of reinforcement have to be made under great uncertainty due to the uncertain nature of (i) the demand growth, (ii) the stochastic production of renewable generation facilities, such as wind turbines, (iii) the reliability of the equipment, such as the transmission lines, (iv) the construction of new generating facilities, (v) the political policies, etc. In the literature, depending on how we consider the uncertainty, the TEP problem is classified into two categories: **deterministic** and **non-deterministic approaches**. In non-deterministic approaches, the optimal expansion plan takes all possible future cases into account by according an occurrence probability to them [56]. Generally, a large number of scenarios are needed to represent the uncertainty accurately, which results in computationally complex problems that are often intractable; moreover, it is often impossible to determine the probability distribution function of the uncertain parameters and thus to obtain scenarios representing the reality

correctly. An alternative to stochastic programming is to consider confidence intervals for the uncertain parameters and use robust optimization (RO) approaches [52, 13, 24]. The advantage of this approach is that RO approaches do not require the generation of scenarios but robust sets instead; the problem complexity is thereby reduced compared to stochastic programming. However the drawback of robust optimization is that the results are often too conservative. More details about the optimization techniques to use in the TEP problems are presented in chapter 5, where we investigate the strategy to adopt in order to address the TEP problem described in chapter 3.

Presentation of the problem

As mentioned in the literature review [2](#), the purpose of a power transmission network is to transfer power from generation plants to load centers securely, efficiently, reliably and economically. Any practical transmission network is expanding and thus, the *transmission expansion planning (TEP)* problem is to identify where to construct new transmission lines in the future, so that the forecasted demand can be managed without load interruption or damage to the physical integrity of the equipment [\[60\]](#). These expansion's decisions have to be made under uncertainty due to (i) the power demand growth and (ii) the stochastic production of some generation facilities, such as renewable energies, etc. [\[52\]](#). These decisions need to be taken carefully and in a robust manner since they are in general irrevocable because of their high investment costs. It is therefore frequently considered that the expanded network must be able to manage the worst future situation.

The purpose of the *optimal transmission expansion planning (OTEP)* problem is to find the best expansion planing. When a superlative is given, such as *best*, one should always give a criterion for comparison. For example, *based on the [FIFA ranking](#)*, one can say that, in 2015, the greatest nation in football was Belgium. This assertion could be shocking for Brazilian or German people, however I made the hypothesis that the FIFA ranking was the only reference for comparison and therefore it is a non-arguable fact that Belgium was the best nation in football is 2015.

From this example, we see that rigorously defining the TEP problem is crucial; an optimal expansion plan for one TSO is not necessarily optimal for another. In this chapter, one defines an *OTEP* by mentioning all the constraints it has to meet and on which criterion two expansion plans are compared (the objective function). In a first step, constraints inherent to a power

system are presented; additional constraints will then be inserted in the model in order to obtain a more sophisticated and robust expanded transmission network.

3.1 Hypotheses

Finding the optimal transmission expansion plan is a difficult task; to simplify the model of the power system to expand and to facilitate the research of the optimal solution, we will make, under the supervision of [Tractebel](#), the following assumptions:

- A1** We consider DC-power flow (losses on transmission lines are thus not considered).
- A2** The problem is solved for a one-period horizon, i.e. one does not consider that lines can be built on different periods, i.e. we consider a static planning.
- A3** No uncertainty is considered. Instead, one assumes that the optimal expansion planning must be able to deal with different scenarios that are given, i.e. different instances of load and generator net injections. We are not in the case of stochastic programming since we only consider a limited number of scenarios; however they have been selected in order to represent the extreme cases that should be manageable by the power network.
- A4** The power system's structure can only be modified by adding candidate lines, i.e. no bus is built or removed and no existing line is decommissioned.
- A5** A list of candidate transmission lines is given in order to address the TEP problem. The main objective is thus to find the best combination of lines in a given set and not defining the lines to built.

3.2 Notations

In its simplest form, a power system is composed of generators, loads, buses and lines. The transmission network can be seen as a graph where the nodes correspond to the buses and the edges to the transmission lines, an example of a representation is given in figure [3.1](#).

In this section, we present the notations used in the problem's description. We will consider the following sets (the notations are similar to those used in [\[29\]](#)):

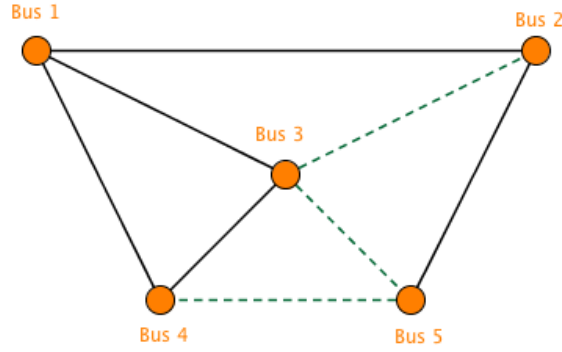


Fig. 3.1 Example of a representation of a power network as a graph. The plain lines are the existing transmission lines in the network and the dotted lines are the candidate ones.

Sets	
$\mathcal{N}$	Set of buses (nodes).
$\mathcal{L}_e$	Set of existing transmission lines, where $l = (m, n) \in \mathcal{L}_e$ is the line with endpoints m and n .
$\mathcal{L}_c$	Set of prospective/candidate transmission lines.
Ω_n^-, Ω_n^+	Set of incoming and outgoing arcs at bus n .

The following parameters will also be considered in order to characterize the components of the network :

Parameters	
I_l	Investment cost for candidate line l .
L_l	Length of candidate line l .
$\bar{L}$	Maximum total length of installed lines.
T_l	Maximum flow capacity on line l .
B_l	Susceptance of line l .
M_l	Big-M value for line l .
P_n	Power generation at bus n .
D_n	Load at bus n .
$\overline{\Delta\theta}$	Maximum difference in voltage angle between two adjacent buses.

Finally, the decision variables of the problem are presented in the table hereafter:

Decision variables

x_l	Investment decision on candidate line k (binary variable).
f_l	Algebraic power flow on line k , i.e. a line can be considered as an oriented edge in a graph and is thus defined by its endpoints. Hence the flow over a line $l = (m, n) \in \mathcal{L}_e$ is by definition positive if it goes from m to n and negative otherwise.
θ_n	Phase angle of bus n .

Before going any further, note that : **(i)** sets are written in round capital letters, **(ii)** parameters in capital letters and **(iii)** variables in lowercase letters.

3.3 Objective function

By assumption, we will focus here on minimising the investment costs of the expansion planning only. Hence, $\sum_{l \in \mathcal{L}_c} I_l x_l$ is the objective function we try to minimise:

$$\min_{x_l, f_l, \theta_n} \sum_{l \in \mathcal{L}_c} I_l x_l. \quad (3.1)$$

3.4 Constraints

Since a transmission network model is a representation of reality, some physical properties must be satisfied such as the Kirchhoff's laws or the maximum transmission capacity of the lines; these constraints are inherent to the problem. Besides, *transmission system operators (TSOs)* are often adding constraints in order to ensure the security, flexibility and robustness of the expanded transmission network. In the following section, we detail, for each constraint, **(i)** the purpose of adding it and **(ii)** how to derive its mathematical expression.

Capacity constraints

A generator has a maximum production capacity; in the same manner, a transmission line has a maximum transmission capacity, which cannot be exceeded in order to preserve the physical integrity of the lines. It is thus natural to consider the following constraints where parameter T_l is directly derived from physical properties:

$$-T_l \leq f_l \leq T_l, \quad \forall l \in \mathcal{L}_e, \quad (3.2)$$

$$-T_l x_l \leq f_l \leq T_l x_l, \quad \forall l \in \mathcal{L}_c, \quad (3.3)$$

where constraints 3.2 and 3.3 represent respectively the maximum and minimum capacity constraints of the existing and candidate lines. If a candidate line l is not constructed, i.e. $x_l = 0$, constraint 3.3 imposes that the flow on the line be zero: $f_l = 0$.

Balance constraints (i.e. deriving from Kirchhoff's current law)

Kirchhoff's current law states that: «At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.» (source: Wikipedia). Applied to power systems modelled with DC-power flow, this law can be stated as follows: the sum of the incoming power (generators and incoming transmission lines) equals the outgoing power (loads and outgoing transmission lines). Mathematically this means that the following equation must be satisfied at each bus n of the power system:

$$P_n + \sum_{l \in \Omega_n^-} f_l = D_n + \sum_{l \in \Omega_n^+} f_l, \quad \forall n \in \mathcal{N}. \quad (3.4)$$

Kirchhoff's voltage law

Kirchhoff's voltage law states that: «The directed sum of the electrical potential differences (voltage) around any closed network is zero.» (source: Wikipedia). Applied to power systems, this law implies that equations 3.5-3.7 must be respected¹:

$$f_l - B_l(\theta_m - \theta_n) = 0, \quad \forall l = (m, n) \in \mathcal{L}_e, \quad (3.5)$$

$$f_l - B_l(\theta_m - \theta_n) \leq M_l(1 - x_l), \quad \forall l = (m, n) \in \mathcal{L}_c, \quad (3.6)$$

$$-f_l + B_l(\theta_m - \theta_n) \leq M_l(1 - x_l), \quad \forall l = (m, n) \in \mathcal{L}_c, \quad (3.7)$$

$$-\overline{\Delta\theta} \leq \theta_m - \theta_n \leq \overline{\Delta\theta}, \quad \forall l = (m, n) \in \mathcal{L}_e, \quad (3.8)$$

$$-\overline{\Delta\theta} - (1 - x_l)M_\Delta \leq \theta_m - \theta_n \leq \overline{\Delta\theta} + (1 - x_l)M_\Delta, \quad \forall l = (m, n) \in \mathcal{L}_c, \quad (3.9)$$

where constraint 3.5 represents the voltage's law for existing lines, whereas constraints 3.6 and 3.7 represent the voltage's law for candidate lines, i.e. if candidate line l is built $f_l - B_l(\theta_m - \theta_n) = 0$ since $x_l = 1$. If $x_l = 0$, constraints 3.6 and 3.7 have no impact on the solution if M_l is large enough. However the value of M_l has an impact on the optimisation's performances; indeed it is used to define the convex hull of the problem, therefore a larger value of M_l results in a larger convex hull and thus a larger feasible set for the relaxed problem². As a consequence,

¹The textbook of the course [LINMA2415 - Quantitative Energy Economics](#) taught by [Professor Papavasiliou](#) at the Université catholique de Louvain gives the details of how to derive these equations. [45]

²In commercial solvers, MILP problems are often solved via a Branch and Bound (B&B) tree, which solves a relaxation of the problem at each node. We see here the interest of having a convex hull as small as possible, i.e. the smallest it is, the quicker the relaxed problem is solved and the quicker is the search in the B&B tree.

the way to assign M_l 's value is not random; the main reference in the literature is Binato's article [10]. We will not address this problem in the rest of this master's thesis because, as justified later on, we will not try to find the exact solution of the problem; interested readers can refer to Kathleen Hemmer's master's thesis [30] where the question has been addressed. Constraints 3.8 and 3.9 are not necessary for the formulation, however it could potentially help the problem in term of stability. As for M_l , M_Δ must be large enough in order to guarantee that, if $x_l = 0$, constraint 3.9 does not restrict the problem.

Limited budget/time

The expansion of a transmission network cannot be achieved in one day and can be expensive. Consequently, it makes sense to consider a limited length of candidate lines $\bar{L}$ to be built. Each candidate line l has a length L_l , thus the sum of the lengths of the lines built should not be greater than $\bar{L}$:

$$\sum_{l \in \mathcal{L}_c} x_l L_l \leq \bar{L}. \quad (3.10)$$

Robustness

So far, we have established the expressions of all constraints as if there was a unique production-consumption scenario, i.e. the production P_n and consumption D_n at each bus $n \in \mathcal{N}$ are known and do not vary. Nowadays, power consumption is increasing every year and, depending on the weather and other factors, power production and consumption can vary. It would therefore be natural for the expanded network to be able to withstand different instances, i.e. different combinations of parameters P_n and D_n , which would represent a range of possible worst-case outcomes. The objective behind these scenarios is to expand the network in an appropriate way: (i) if the worst-case scenarios are too conservative, the resulting expansion plan will be too expensive, (ii) conversely, if the worst-case scenarios are too optimistic, we could end up with a network that could easily collapse. This justifies the consideration of robust optimization approaches, i.e. approaches that guarantee worst case protection within an uncertainty set $\mathcal{K}$. Obviously, the definition of the uncertainty set $\mathcal{K}$ of net injections causes a tradeoff between security and cost [52].

Definition 1 ($\mathcal{K}$ -Robust) *A transmission expansion planning is $\mathcal{K}$ -robust if the expanded network can handle each element of set $\mathcal{K}$.*

Each element $k \in \mathcal{K}$ defines the values of parameters D_n and P_n for all $n \in \mathcal{N}$, i.e. it defines the net injection at each bus $n \in \mathcal{N}$. Hence, to deal with this constraint, one needs to add continuous variables and to repeat all previous constraints for each element k . For instance,

the balance constraint 3.4 is transformed as follows for all $k \in \mathcal{K}$, the other constraints being transformed in a similar way:

$$P_n^{(k)} + \sum_{l \in \Omega_n^-} f_l^{(k)} - \sum_{l \in \Omega_n^+} f_l^{(k)} = D_n^{(k)}, \quad \forall n \in \mathcal{N}.$$

Even though the number of continuous variables (variables f and θ) of the problem increases linearly with the number of scenarios considered, i.e. the number of elements in $\mathcal{K}$, the number of binary variables x remains the same; as a consequence, the combinatorial complexity of the model is not affected by the incorporation of the robustness constraint [31, 42].

N-1 security criterion

The widely used *N-1 security criterion* states that load supply should be ensured not only under base-case conditions, but also in the case of single circuit failures. This criterion is modeled by repeating all network's constraints for each circuit contingency $s \in \mathcal{S}$, where $\mathcal{S}$ is the finite set of all possible circuit contingencies. As previously mentioned, the way to assign the value of M_l depends on the network, therefore since any contingency s generates a change in the network, one needs to compute a value of M_l for each contingency s .

As for the robustness, considering contingency s implies considering a particular case for which the power system must be able to respect all previous constraints, hence one adds continuous variables indexed by s as well as constraints in the same way as we did for the scenarios. Here are some examples of modified constraints for a given $s \in \mathcal{S}$, the other constraints being transformed in a similar way:

$$\begin{aligned} P_n + \sum_{l \in \Omega_n^-} f_l^{(s)} - \sum_{l \in \Omega_n^+} f_l^{(s)} &= D_n, & \forall n \in \mathcal{N} \\ f_l^{(s)} - B_l \left(\theta_m^{(s)} - \theta_n^{(s)} \right) &\leq M_l^s (1 - x_l), & \forall l = (m, n) \in \mathcal{L}_c, \\ -f_l^{(s)} + B_l \left(\theta_m^{(s)} - \theta_n^{(s)} \right) &\leq M_l^s (1 - x_l), & \forall l = (m, n) \in \mathcal{L}_c. \end{aligned}$$

As for the *robustness constraint*, the consideration of contingencies generates a linear increase in the number of continuous variables and does not affect the combinatorial complexity of the problem [31, 42]; a more detailed complexity analysis is done in section 4.1.

Definition 2 ((N-1)-Secure) A transmission expansion planning is (N-1)-secure if the expanded network can handle any single circuit contingency $s \in \mathcal{S}$. An element s of set $\mathcal{S}$ corresponds to the case where a circuit $l \in \mathcal{L}$ fails, where $\mathcal{L}$ is the set of lines of the expanded network.

3.5 Optimal Transmission Expansion Planning (OTEP) problem formulation

We have now all the elements needed to define the optimization problem to solve in order to get a solution to the *Optimal Transmission Expansion Planning (OTEP)* problem. The optimization problem is formulated as follows:

OTEP problem formulation

$$\begin{aligned}
 & \min_{x_l, f_l^{(s,k)}, \theta_n^{(s,k)}} \sum_{l \in \mathcal{L}_c} I_l x_l & (3.11) \\
 \text{s.t.} \quad & P_n^{(k)} + \sum_{l \in \Omega_n^-} f_l^{(s,k)} - \sum_{l \in \Omega_n^+} f_l^{(s,k)} = D_n^{(k)}, & \forall n \in \mathcal{N} \\
 & f_l^{(s,k)} - B_l \left(\theta_m^{(s,k)} - \theta_n^{(s,k)} \right) = 0, & \forall l = (m, n) \in \mathcal{L}_e, \\
 & f_l^{(s,k)} - B_l \left(\theta_m^{(s,k)} - \theta_n^{(s,k)} \right) \leq M_l^s (1 - x_l), & \forall l = (m, n) \in \mathcal{L}_c, \\
 & -f_l^{(s,k)} + B_l \left(\theta_m^{(s,k)} - \theta_n^{(s,k)} \right) \leq M_l^s (1 - x_l), & \forall l = (m, n) \in \mathcal{L}_c, \\
 & -\overline{\Delta\theta} \leq \theta_m^{(s,k)} - \theta_n^{(s,k)} \leq \overline{\Delta\theta}, & \forall l = (m, n) \in \mathcal{L}_e, \\
 & -\overline{\Delta\theta} - (1 - x_l)M_\Delta^s \leq \theta_m^{(s,k)} - \theta_n^{(s,k)} \leq \overline{\Delta\theta} + (1 - x_l)M_\Delta^s, & \forall l = (m, n) \in \mathcal{L}_c, \\
 & -T_l \leq f_l^{(s,k)} \leq T_l, & \forall l \in \mathcal{L}_e, \\
 & -T_l x_l \leq f_l^{(s,k)} \leq T_l x_l, & \forall l \in \mathcal{L}_c, \\
 & x_l \in \{0, 1\}, & \forall l \in \mathcal{L}_c, \\
 & s \in \mathcal{S}, \\
 & k \in \mathcal{K}.
 \end{aligned}$$

Theoretical analysis

This chapter focuses on a theoretical analysis of the TEP problem (3.5). This optimization problem is categorized as a mixed-integer programming (MIP) problem since it includes both continuous and integer variables:

- *binary* variables, x_l , for the decision of building or not building each candidate line l in $\mathcal{L}_c$,
- *continuous* variables, $f_l^{(s,k)}$, representing the flows on each line $l \in \mathcal{L}_e \cup \mathcal{L}_c$ for each contingency s and scenario k ,
- *continuous* variables, $\theta_n^{(s,k)}$, representing the phase angles of each bus $n \in \mathcal{N}$ for each contingency s and scenario k .

Moreover, since we have considered a DC-power flow model, the TEP problem is a MILP.

4.1 Complexity analysis

Obviously, as already mentioned, the combinatorial complexity of the problem is independent of the number of contingencies and scenarios. As a consequence, one could think that the number of scenarios or contingencies should not significantly impact the computational time since it is the combinatorial complexity that impacts the most the global complexity. This assertion is valid theoretically but not in practice; indeed a commercial solver, such as [Gurobi](#), has memory issues when the number of constraints and continuous variables becomes too important; even solving the root node causes difficulties.

Example 1 *For a network consisting of 35 buses, 50 existing lines and 49 candidate lines (100 elements in set $\mathcal{S}$, i.e. $|\mathcal{S}| = 100$; one base state and 99 contingency cases) and for 20*

scenarios considered ($|\mathcal{K}| = 20$), the number of flow and phase angle variables amounts to $(100 + 35)(100 \cdot 20) = 2.7 \cdot 10^5$. Moreover, more than 10^5 constraints are needed to enforce each constraint presented in section 3.4: the balance constraint, the Kirchhoff voltage constraint, the maximum flow on the lines, etc. Finally the number of combinations of candidate lines, i.e. investment plans, amounts to $2^{|\mathcal{L}_c|} \simeq 5.63 \cdot 10^{14}$ possibilities.

This example clearly illustrates the rapid augmentation of complexity when the problem size increases:

- the number of investment plans is exponentially proportional to the number of candidate lines, i.e. $\mathcal{O}\left(e^{|\mathcal{L}_c|}\right)$.
- the number of continuous variables and constraints is directly proportional to the square of the number of contingencies, i.e. $\mathcal{O}\left(|\mathcal{S}|^2\right)$.

As a consequence, this problem is intractable for large instances; indeed the *decision version* of the TEP problem, i.e. the task of deciding, given a plan p , whether the network has any expansions plan p' cheaper than p , belongs to the class of *NP-complete* problems. Here is what is said in reference [19]: « *The electrical energy transmission system expansion planning problem is a mixed integer nonlinear programming problem, and is **NP-complete**, that is, a problem for which no method exists that solves it in polynomial time. This problem presents a large number of local optimal solutions and is not convex. The problem is not solving successfully using exact optimization techniques when system size becomes large, and the system has an appreciable amount of buses isolated, because the number of solutions grows exponentially.* »

Hence the TEP problem is **NP-Hard**, i.e. if somebody certifies that investment plan p^* is the optimal investment plan, there exists no efficient algorithm to verify the assertion.

Definition 3 (Efficient algorithm) *An algorithm $\mathcal{A}$ is said to be efficient if it runs in a polynomial amount of time w.r.t. the size of its entries.*

Until today, no efficient algorithm is known for solving NP-complete problems. There is thus little hope to find a method able to solve exactly the TEP problem in a reasonable amount of time as shown in the next section 4.2.

4.2 Expected performances

One method for finding the optimal expansion plan could be to investigate all possibilities, i.e. performing a brute-force search. However, this method can result non feasible in practice; for instance, a mid-size problem like the one in example 1, results in already more than 10^{14} possible expansion plans. If we consider that a computer can check the feasibility of 10^6 plans per second, more than three years would be necessary for a single computer to test all possibilities.

Definition 4 (Brute-force search) *In computer science, brute-force search or exhaustive search, is a very general problem-solving technique that consists in systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the problem's statement. (source: Wikipedia)*

One can also focus on the performances of the randomized enumeration algorithm.

Definition 5 (Randomized algorithm) *For a transmission expansion planning problem, the randomized algorithm generates randomly, uniformly and independently candidate plans and tests their feasibility until a stopping criterion is reached.*

4.2.1 Finding the optimal solution with an randomized algorithm

In this section, we are interested in the probability of finding the optimal expansion planning after having tested a fixed number m of plans generated randomly, uniformly and independently. Let n be the number of candidate lines and $N = 2^n$ the number of candidate expansion plans. The probability $\mathbb{P}(p^*|m)$ of finding the optimal solution after m samples¹ is given by the following formula:

$$\mathbb{P}(p^*|m) = 1 - \bar{\mathbb{P}}(p^*|m) \stackrel{i.i.d.}{=} 1 - (\bar{\mathbb{P}}(p^*|1))^m = 1 - \left(\frac{N-1}{N}\right)^m, \quad (4.1)$$

where $\bar{\mathbb{P}}(p^*|m)$ is the probability of not finding the optimal solution after m samples. Let us note, as mentioned previously, that there exists no efficient certificate of the optimality of an investment plan. This means that after m samples, one might have the optimal investment plan but verifying its optimality would need the call of a non-efficient algorithm.

Figure 4.1 represents how the probability of succeeding in finding the optimal investment plan evolves with the number of samples realized. We observe that no matter the number

¹We consider here that a sample is a candidate expansion plan generated randomly, uniformly and independently.

of candidate lines, the shapes of the cumulative distribution functions are identical. For 10 candidate lines (1024 possible investment plans), 3216 random plans have to be investigated in order to have a probability higher than 0.95 to get the optimal solution. A brute force approach would thus be wiser if such a guarantee of probability is required since there are less possibilities than the number of random plans to check. This approach has thus little hope to succeed in obtaining the optimal solution.

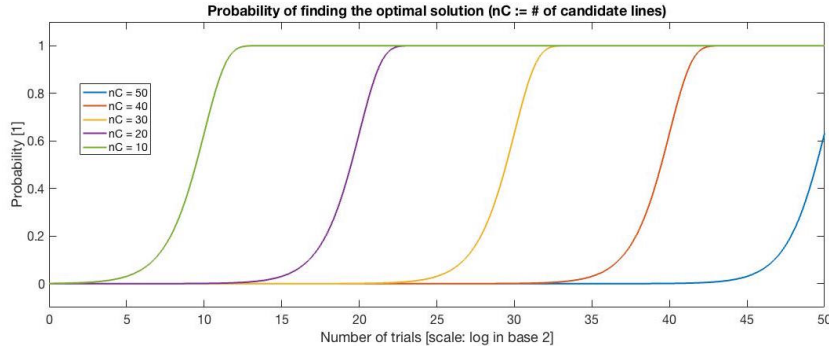


Fig. 4.1 Evolution of the probability of finding the optimal solution with the number of samples.

We could also focus on the probability of getting the optimal solution after having found a number m_f of feasible expansion plans. Of course an *a priori* knowledge of the total number of feasible plans N_f is required. For instance if we consider that 1% of the plans are feasible then $N_f = 0.01N$ and the probability $\mathbb{P}(p^*|m_f)$ of finding the optimal solution after finding m_f feasible plans is given by the following formula:

$$\mathbb{P}(p^*|m_f) = 1 - \bar{\mathbb{P}}(p^*|m_f) \stackrel{i.i.d.}{=} 1 - (\bar{\mathbb{P}}(p^*|1_f))^{m_f} = 1 - \left(\frac{N_f - 1}{N_f}\right)^{m_f}. \quad (4.2)$$

Figure 4.2, represents how the probability of succeeding in finding the optimal investment plan evolves with the number of random feasible plans tested. As expected, the behavior is similar to the previous situation; the graph is just shifted to the left by a value equal to $\log_2(100) = 6.64$. Hence, in this case one observes that the probability of finding the optimal expansion plan increases faster than previously, however a sample corresponds to a feasible plan, which happens, by hypothesis, once in a hundred random expansion plans. There is thus no interest in focusing on the number of feasible expansion plans instead of random ones if one cannot find a way to generate random feasible expansion plans with a larger probability than the proportion of feasible plans among all expansion plans.

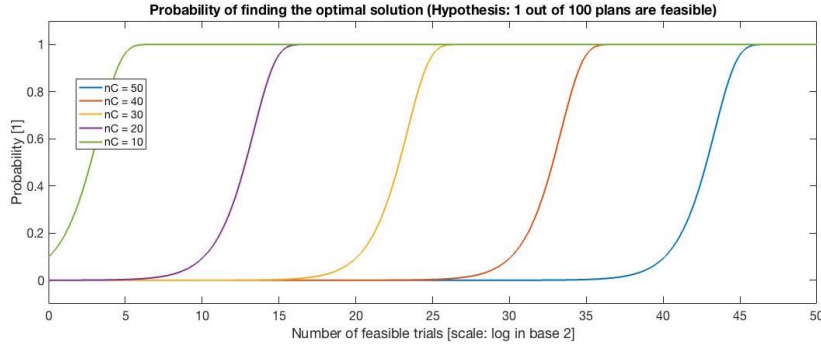


Fig. 4.2 Evolution of the probability of finding the optimal solution with the number of feasible plans found.

4.2.2 Finding a solution close to optimality with an randomized algorithm

As mentioned in section 4.1, the decision version of the TEP problem belongs to the class of NP-complete problems, which are problems that can, today, not be solved by efficient algorithms. Hence obtaining the optimal solution to the TEP problem can be illusory, whereas obtaining a solution of a relative good quality, i.e. close from optimality, is more accessible. In this section, we derive the probability of finding a *good* expansion plan with a randomized algorithm based on different hypotheses stated hereafter.

Hypotheses

To derive the probability of finding a feasible expansion plan near the optimal one, we make different hypotheses:

- H1** each candidate line has the same cost C .
- H2** each possible investment plan p has the same probability to be the optimal one p^* , i.e. $\mathbb{P}(p = p^*) = \frac{1}{2^n}$. This implies with **H1** that the number of candidate lines built in the optimal solution, n_{p^*} , follows a binomial distribution, i.e. $n_{p^*} \sim \mathcal{B}(n, 0.5)$.
- H3** the optimal solution is unique, i.e. only one plan p^* is feasible given the optimal cost C_{p^*} .
- H4** an investment plan p strictly more expensive than the optimal one p^* is feasible if and only if all the candidate lines built in p^* are also built in p . As a consequence, the

probability for a given investment plan p of cost C_p to be feasible amounts to:

$$\mathbb{P}(p \text{ is feasible} | C_p \text{ and } C_{p^*}) = \begin{cases} \frac{\frac{(n-n_{p^*})!}{(n_p-n_{p^*})! \cdot (n-(n_p-n_{p^*}))!}}{\frac{n!}{n_p!(n-n_p)!}}, & \text{if } C_p \geq C_{p^*}, \\ 0 & \text{otherwise,} \end{cases} \quad (4.3)$$

where

- n_p and n_{p^*} are the number of candidate lines in investment plan p and in the optimal investment plan p^* respectively; C_p is the cost of plan p . Let us note that, considering **H1**, the condition $C_p \geq C_{p^*}$ is identical to $n_p \geq n_{p^*}$,
- the *denominator* $= \frac{n!}{n_p!(n-n_p)!}$ represents the number of different investment plans that build n_p candidate lines among the n available,
- the *numerator* $= \frac{(n-n_{p^*})!}{(n_p-n_{p^*})! \cdot (n-(n_p-n_{p^*}))!}$ represents the number of different investment plans that build n_p candidate lines among the n available and where all the candidate lines of the optimal solution p^* are built.

Figure 4.3, represents an example of the behavior of expression 4.3 based on the *IEEE-24 buses system* for which all candidate lines' cost has been fixed to one, i.e. $C = 1$ ². This system considers 28 candidate lines and its optimal plan builds 14 of them, i.e. $n_{p^*} = C_{p^*} = 14$.

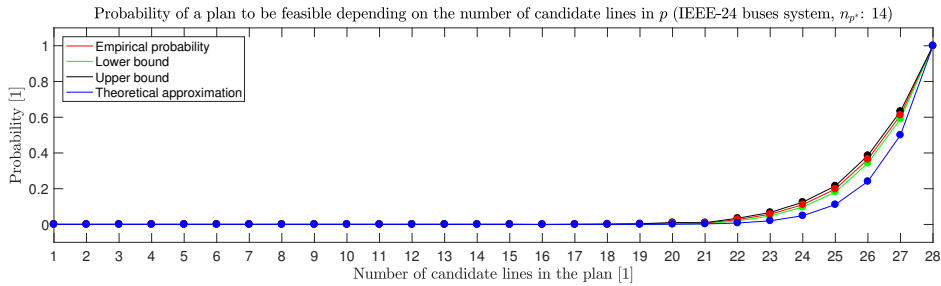


Fig. 4.3 Illustration of the probability for an investment plan p of being feasible. In red color, the effective probabilities obtained via the law of large numbers. The 95% confidence interval is represented by the space between the green and black curves. In blue color, the approximate probabilities (via equation 4.3)

Let us explain how to obtain the confidence interval. First, for a fixed number of candidate lines to insert, the probability for a plan of being feasible is equivalent to a Bernoulli

²A detailed presentation of the *IEEE-24 buses system* is given in Appendix A.1.

random variable of a certain parameter p , which corresponds to the proportion of feasible expansion plans among all possible ones. Moreover, for a Bernoulli distribution, if the estimate $\hat{p}$ of parameter p is not near 0 or 1, and if the sample size n is sufficiently large (i.e. $n\hat{p} > 5$ and $n(1 - \hat{p}) > 5$), then the $(1 - \alpha)^{th}$ -confidence interval can be estimated with the use of the Central Limit Theorem:

$$\left[\hat{p} - z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}, \hat{p} + z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \right]$$

where $z_{1-\alpha/2}$ is the $(1 - \alpha/2)^{th}$ quantile of the standard normal distribution. If $\hat{p} = 0$ and $n > 30$, the 95% confidence interval is approximately $[0, \frac{3}{n}]$ [32].

In figure 4.3, one observes that the approximation of the probability for a plan p to be feasible (in blue) is lower than its effective value (in red); this makes our approach wise since our interest is to find an upper bound on the number of random realizations to simulate in order to guarantee the proximity to optimality. Therefore, being pessimistic when considering the feasibility of an investment plan fits with the objective of finding an upper bound. One could have expected this observation for different reasons:

1. First, hypothesis **H1** assumes that there is only one optimal solution, which is not necessarily the case. Hence, for the approximation of the feasibility of an investment plan (equation 4.3), one should not verify the presence of one set of candidate lines but of several sets, i.e. one set of candidate lines per different optimal expansion planning.
2. Second, hypothesis **H4**'s statement: «*an investment plan p strictly more expensive than the optimal one p^* is feasible if and only if all the candidate lines built in p^* are also built in p .*» is not correct. Indeed, it could be possible that a plan is feasible even if it has not a subset of lines that corresponds to the set of lines of an optimal solution. Figure 4.4 shows an example where it is not the case; hypothesis **H4** is thus not a necessary condition to obtain feasible expansion plans.
3. Finally let also mention that adding a line to a feasible investment plan may result in an infeasible investment plan as illustrated in Figure 4.5; hence hypothesis **H4** is also not a sufficient condition to obtain feasible expansion plans.

To summarize, points 1 and 2 justify the under-estimation in the approximation of the probability for a plan to be feasible (equation 4.3), whereas point 3 could explain an over-estimation. However in practice, we observe that points 1 and 2 have more effect

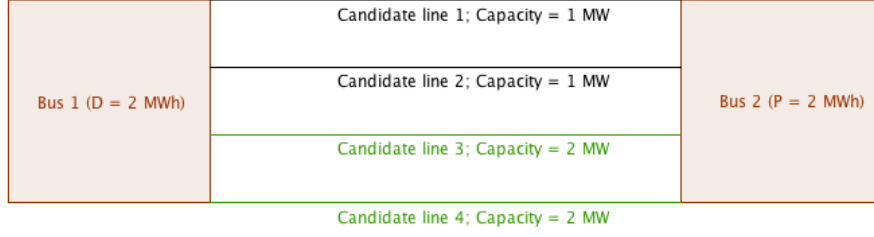


Fig. 4.4 Illustration of a counter-example of the statement mentioned in **H4**. Here, each line has a cost of 1; the optimal solution is to build lines 3 and 4 for a cost of 2. However building lines 1, 2 and 3 or 1, 2, and 4 are also feasible expansion plans even though they do not build both lines 3 and 4, i.e. the set of candidate lines of the optimal solution.

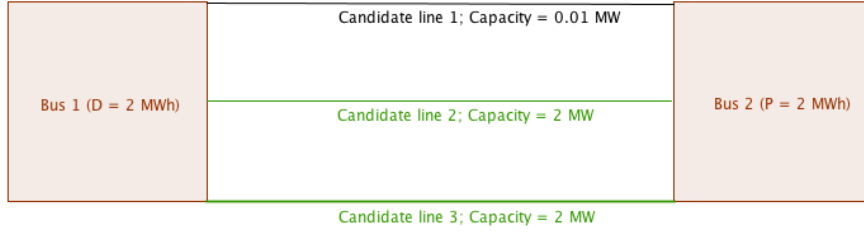


Fig. 4.5 Like in example 4.4, the optimal investment plan is to build lines 2 and 3. However building all the lines would lead to an infeasible expansion plan. Indeed, when there is no contingency, the flow is equally distributed on the three lines, which leads to an overloading of line 1 that has a capacity of 0.01 MW. We assume here that all lines have the same susceptance.

on the estimation than point 3, which lead to a global under-estimation as illustrated in figure 4.3.

Probability analysis

Based on hypotheses **H1-H4**, it is possible to derive the probability $\mathbb{P}(|C_{p^{r.a.}} - C_{p^*}| \leq k \cdot C | n_{iter} = m)$ of obtaining a feasible plan $p^{r.a.}$ with the randomized algorithm that is at most $k \cdot C$ more expensive than the optimal plan p^* after m samples. It is possible to express this probability in terms of number of candidate lines built in the plan instead of in terms of cost:

$$\mathbb{P}(|C_{p^{r.a.}} - C_{p^*}| \leq k \cdot C | n_{iter} = m) \stackrel{\mathbf{H1}}{=} \mathbb{P}(|n_{p^{r.a.}} - n_{p^*}| \leq k | n_{iter} = m), \quad (4.4)$$

where $n_{p^{r.a.}}$ is the number of candidate lines (of cost C) that are built in feasible plan $p^{r.a.}$. This probability can be computed as follows:

$$\begin{aligned}
& \mathbb{P}(|n_{p^{r.a.}} - n_{p^*}| \leq k | n_{iter} = m) \\
&= \sum_{i=0}^n \mathbb{P}(|n_{p^{r.a.}} - n_{p^*}| \leq k | n_{iter} = m \text{ and } n_{p^*} = i) \cdot \mathbb{P}(n_{p^*} = i), \\
&= \sum_{i=0}^n \mathbb{P}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = m \text{ and } n_{p^*} = i) \cdot \mathbb{P}(n_{p^*} = i), \\
&= \sum_{i=0}^n [1 - \bar{\mathbb{P}}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = m \text{ and } n_{p^*} = i)] \cdot \mathbb{P}(n_{p^*} = i), \\
&\stackrel{i.i.d.}{=} \sum_{i=0}^n (1 - [\bar{\mathbb{P}}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = 1 \text{ and } n_{p^*} = i)]^m) \cdot \mathbb{P}(n_{p^*} = i). \tag{4.5}
\end{aligned}$$

Deriving $\mathbb{P}(n_{p^*} = i)$ is easy since we have assumed that $n_{p^*} \sim \mathcal{B}(n, 0.5)$ in **H2**; deriving $\bar{\mathbb{P}}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = 1 \text{ and } n_{p^*} = i)$ needs more details: $n_{p^{r.a.}}$ is a random variable that represents the number of candidate lines of the best feasible investment plan given a certain number of realisations, hence after one sample, the probability that $n_{p^{r.a.}}$ equals $\bar{n}$ knowing the best investment plan p^* equals

$$\mathbb{P}(n_{p^{r.a.}} = \bar{n} | n_{iter} = 1 \text{ and } n_{p^*}) = \mathbb{P}(n_p = \bar{n}) \cdot \mathbb{P}(p \text{ is feasible} | n_{p^*} \text{ and } n_p = \bar{n}), \tag{4.6}$$

where $n_p \sim \mathcal{B}(n, 0.5)$. We can then derive the following probability:

$$\begin{aligned}
& \bar{\mathbb{P}}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = 1 \text{ and } n_{p^*} = i) \\
&= 1 - \mathbb{P}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = 1 \text{ and } n_{p^*} = i), \\
&= 1 - \sum_{j=0}^k \mathbb{P}(n_{p^{r.a.}} = i + j | n_{iter} = 1 \text{ and } n_{p^*} = i), \\
&\stackrel{4.6}{=} 1 - \sum_{j=0}^k \mathbb{P}(n_p = i + j) \cdot \mathbb{P}(p \text{ is feasible} | n_{p^*} = i \text{ and } n_p = i + j). \tag{4.7}
\end{aligned}$$

Finally, inserting expression 4.7 in equation 4.5 gives the following result:

$$\begin{aligned}
& \mathbb{P}(|C_{p^{r.a.}} - C_{p^*}| \leq k \cdot C | n_{iter} = m) \\
&= \sum_{i=0}^n (1 - [\bar{\mathbb{P}}(|n_{p^{r.a.}} - i| \leq k | n_{iter} = 1 \text{ and } n_{p^*} = i)]^m) \cdot \mathbb{P}(n_{p^*} = i), \\
&\stackrel{4.7}{=} \sum_{i=0}^n \left[1 - \left(1 - \sum_{j=0}^k \mathbb{P}(n_p = i + j) \cdot \mathbb{P}(p \text{ is feasible} | n_{p^*} = i \text{ and } n_p = i + j) \right)^m \right] \cdot \mathbb{P}(n_{p^*} = i).
\end{aligned}$$

Everything needed to compute this expression is now available since **(i)** both n_p and n_{p^*} follow a binomial distribution $\mathcal{B}(n, 0.5)$ and **(ii)** the probability for p to be feasible knowing the

cost of plan p and p^* is given by equation 4.3. Figure 4.6 represents this probability for different values of k ; as our intuition would have told us, the larger k , the more probable to find a feasible plan $p^{r.a.}$ that has a cost $C_{p^{r.a.}} \in [C_{p^*}, C_{p^*} + k \cdot C]$ for a given number of random realisations tested. Of course, if $k = 0$ is considered, one get the same graph as the one represented on figure 4.1. An arbitrary choice of k to guarantee the proximity of $p^{r.a.}$ to the optimal plan p^* , could be $k = 5$. From the graphs hereafter, one observes that the number of samples needed to reach a given probability confidence is already much lower than for finding the exact solution. However, for large systems, guaranteeing to be close to the optimal solution (for example $k = 5$) would need too large a number of samples to be computationally feasible; we see here the need of searching a plan in a more clever way.

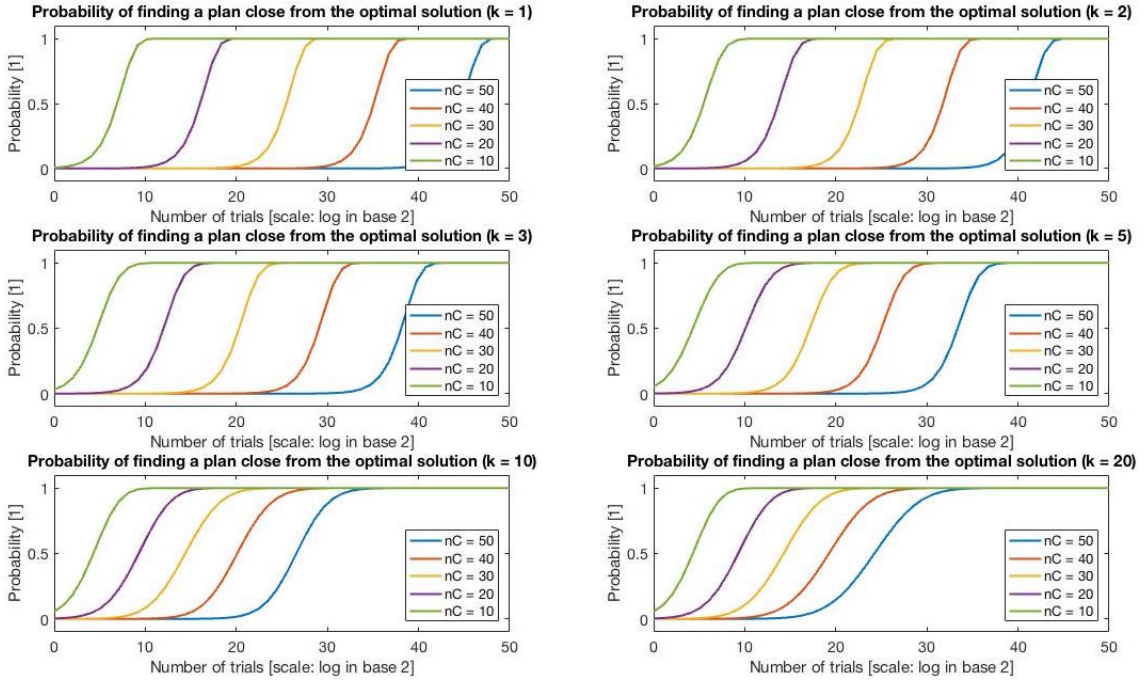


Fig. 4.6 Illustration of the probability to find a feasible plan close to optimal given a certain number of random realisations n_{iter} and given a maximum cost gap $k \cdot C$.

Part II

Methods

The transmission expansion planning (TEP) problem has extensively been addressed in the scientific literature. The way to model the problem has a significant impact on the efficiency of well-known methods traditionally used to solve MILP. Hence, before proposing methods to solve the TEP problem, one should first look at the existing methods and evaluate if they could be promising in our situation.

In this part, the objective is to get efficient methods that could tackle the optimization problem established in part I in a reasonable amount of time. To continue the literature review undertaken in chapter 2 where different TEP formulations have been presented, we present, in chapter 5, the most common methods used so far to solve the TEP problem depending on its formulation. In chapter 6, we investigate mathematical optimization models applicable to our formulation whereas chapter 7 focuses on heuristics and presents the method we have developed.

Literature review on the methods to solve the TEP problem.

In the literature, two main approaches are reported to solve the transmission expansion planning (TEP) problem: **(i)** mathematical optimization models and **(ii)** heuristics. Other approaches exist, referred to as meta-heuristics, that have characteristics of both types of approaches. [34, 29].

5.1 Mathematical Optimization Models

Mathematical optimization models solve a mathematical formulation of the TEP problem, such as the one derived in section 3.5, in order to obtain the optimal expansion plan. Obviously, the optimality trait of the obtained expansion plan depends on the TEP formulation; the solution of the optimization problem differs, for instance, as regards to considering or not the losses on the transmission lines. In these models, the optimal solution is the result of an optimization problem characterized by **(i)** an objective function allowing the comparison between expansion plans, and **(ii)** a set of constraints representing the requirements imposed by the TSOs to the expanded power system.

The TEP problem is modelled as an integer optimization problem that can be linear or not depending on the chosen way to represent the power flow. Hence, most of the methods proposed so far, solve the TEP problem using traditional optimization techniques such as linear programming [27], non-linear programming [35, 38], dynamic programming [18, 57] or mixed integer programming (such as *Branch and bound*) [4, 28, 31, 36]. However those analytical methods have some drawbacks [31]:

- *Linear programming* is only used for the transportation model, i.e. a relaxed version of the DC model where Kirchhoff's voltage law constraints are eliminated. As a consequence, the optimal expansion plan obtained must be considered with caution since it arises out of a (too?) relaxed model.
- *Non-linear programming* has still reliability and robustness difficulties when dealing with large complex power systems, especially for non-convex problems such as the TEP problem.
- *Branch and Bound* resorts to selective/partial enumerations, which can lead to memory issues for large problems. Moreover it has also high execution times.

Because of the drawbacks of the most common methods, decomposition strategies were investigated in order to treat the problem in several simpler pieces, such as Benders decomposition or hierarchical decomposition [51]. G.C. Oliveira and S. Binato were among the first to apply Benders decomposition to the TEP problem [9]. Benders decomposition was first proposed in 1962 [5] and has received much attention in the literature since then. The idea of this decomposition is to apply a *divide-and-conquer* strategy: the variables of the original problem are divided into two subsets, i.e. (i) first-stage variables whose values are determined by the so-called *master* problem and (ii) second stage variables whose values are obtained by solving the *slave* problem for a given first-stage solution¹. Benders decomposition is particularly advantageous when (i) the number of variables linking the two stages is small or when (ii) the nature of the master and slave problems are different; the TEP problem presents both aspects. The way to apply Benders decomposition to the TEP problem is further detailed in section 6.2.1.

In section 3.4, we have presented the $(N - 1)$ -security criterion, widely requested by transmission system operators (TSOs) in order to guarantee a certain level of reliability and security of the expanded power system. In a similar way as in section 3.4, article [42] shows how to formulate the problem by repeating all network constraints for each circuit contingency². In the literature, different ways have been proposed in order to solve the TEP problem under the $(N - 1)$ -security criterion. Article [31] explains how to apply Benders decomposition based on a stochastic model of the TEP problem. In contrast, in articles [14, 41] the authors decided to have recourse to adjustable robust optimization to circumvent the tractability issues experienced by conventional methods relying on explicitly modelling the whole contingency

¹For interested readers, the textbook of the course [LINMA2491 - Operations research](#) taught by [Professor Papavasiliou](#) at the Université catholique de Louvain gives more detail about Benders decomposition [46].

²Let us remind that imposing the robustness of a power system, i.e. considering different instances of net power injections (section 3.4), can be treated in the same way as contingencies.

set; this approach is similar to the way article [52] addresses the TEP problem while considering uncertainty of future demand and generation facilities production. The methods are presented in sections 6.2.1 and 6.2.2.

The main issue with mathematical optimization models is that they usually require a large computational effort and present difficulties to deal with the expansion of medium and large power systems [26]. In certain circumstances, they are even subject to convergence problems [25, 53]. This major drawback of mathematical optimization models explains why the scientific community decided to approach the TEP problem with heuristics as well.

5.2 Heuristics and Meta-Heuristics

Heuristics methods are the current alternative to mathematical optimization models. All techniques that are building solutions step-by-step instead of resolving an optimization problem are categorized as heuristics. These procedures perform local searches until they are not able to find any better solution; the search is then interrupted by invoking the satisfaction of a predefined stopping criteria. Hence, heuristics are simplified procedures used to identify feasible solutions of complex problems requiring little computational effort compared to mathematical optimization models. Even though heuristics return relatively quickly feasible solutions, they fail in providing, mathematically speaking, bounds or proofs on the quality/optimality of their solution; they rarely find the optimal expansion plan, especially when they are used on real power systems [50, 40].

Meta-heuristics are heuristic techniques enhanced by a search procedures inspired mainly by natural mechanisms. They are adequate to solve complex combinatorial problems and usually succeed in identifying optimal or suboptimal solutions even for large power systems. The main drawbacks of such algorithms is that they require large computational efforts.

Many different heuristics/meta-heuristics have been applied to the TEP problem; due to the abundance of heuristics applied to the TEP problem, we decided to only mention but not implement the most well-known ones hereafter; for more information on their performances we refer the interested readers to cited articles [34, 17]:

- A first class of heuristics are those iteratively selecting the candidate lines to add to an initial plan, based on *sensivities* [6, 21, 47]. The sensitivity analysis can focus exclusively an electric sensitivities such as in [6], or on sensitivities related to the power system behavior such as load curtailment [47].

- With the progress of computer performances, interest in methods appropriate to parallel processing has risen. The most well-known method being *genetic algorithms* GA, which are based on the mechanism of evolution (natural selection). The particularity of these methods is that, contrary to conventional optimization techniques performing point-to-point searches, it searches from population-to-population. The following articles explain how to approach the TEP problem using GA [23, 60].
- Ant Colony Search (ACS) system introduced in 1992 [20] is an algorithm inspired by the behavior of ant colonies and used to solve combinatorial problems. Article [37] proposes an adaptation of this algorithm to the TEP problem.
- The transmission expansion planning has been approached using many other ways: game theory [15, 43], simulated annealing [22], greedy randomized adaptive search procedure [7], etc.

We can see that a multitude of approaches have been proposed, however none of them has really taken the lead, i.e. the performances of each approach is dependent on the power system, the performances are thus variables depending on the network structure. In chapter 7, we propose a heuristic approach that does not require strong computational efforts and that does not require a particular power systems structure to perform correctly.

Mathematical optimization

In chapter 5, we have presented the most well-known methods used to solve the transmission expansion problem. In this chapter, we give more detail about the methods that can be applied to our formulation of the TEP problem (cfr. section 3.5). The common trait of the following methods is that they solve exactly the TEP problem 3.11.

We first explain how to implement the TEP problem in a commercial solver, such as Gurobi, then we detail two Benders decomposition approaches to solve our TEP problem and finally we have a word about the *Branch and Bound* method.

6.1 Solver method

The first reflex to have when dealing with the transmission expansion planning problem is to try to solve it in a commercial solver; indeed for small/medium power systems, it is probable that a commercial solver, such as *Gurobi*, is able to solve the TEP problem within a reasonable amount of time. As a consequence, in our case there would not be any interest in approaching the problem with any special mathematical approach.

Depending on the power system and the candidate lines, the TEP problem formulated as in section 3.5 might be infeasible, i.e. no combination of candidate lines makes the expansion planning feasible. However, by allowing load curtailment, the TEP problem is guaranteed to be feasible; this approach is for example used in [10, 31] to guarantee the feasibility of the slave problem in Benders decomposition, which justify the only presence of optimality cuts.

The consideration of load curtailment introduces new continuous variables, $r_n^{(s,k)}$, at each bus for each contingency and scenario; the complexity of the optimization problem to solve is

thus enhanced. Moreover, if load shedding has no cost, the optimal solution would be to shed all loads, which is obviously not the optimal solution researched. Hence, as suggested in [10], we should tolerate load shedding but at a high cost, C_{LS} , in order to ensure no load shedding occurs at final solution when the TEP problem is feasible. As a consequence, to load shedding corresponds the amount of network infeasibility. The easiest way to assign a value to C_{LS} is to consider a cost equal to the maximum investment cost possible, i.e. $C_{LS} = \sum_{l \in \mathcal{L}_c} I_l$. The adapted objective function of TEP problem while considering load curtailment writes as:

$$\min_{x_l, f_l, \theta_n} \sum_{l \in \mathcal{L}_c} I_l \cdot x_l + C_{LS} \cdot \left(\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}} r_n^{(s,k)} \right). \quad (6.1)$$

Formulated like this, the TEP problem stays infeasible due to the enforcement of the power balance constraint 3.4 since the production at each bus is fixed. A most natural way to render the TEP problem feasible when allowing load curtailment, is to consider that power generation at each bus and for all contingencies and scenarios, $g_n^{(s,k)}$, can vary between 0 and its maximum value P_n . Constraint 3.4 is then replaced by the following set of constraints:

$$g_n^{(s,k)} + \sum_{l \in \Omega_n^-} f_l^{(s,k)} - \sum_{l \in \Omega_n^+} f_l^{(s,k)} = D_n - r_n^{(s,k)}, \quad \forall n \in \mathcal{N}, \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \quad (6.2)$$

$$0 \leq r_n^{(s,k)} \leq D_n, \quad \forall n \in \mathcal{N}, \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \quad (6.3)$$

$$0 \leq g_n^{(s,k)} \leq P_n, \quad \forall n \in \mathcal{N}, \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \quad (6.4)$$

where constraints 6.3 and 6.4 impose that load and generator shedding should be non-negative and should not exceed their maximum value, D_n and P_n respectively.

Table 6.1 includes the time needed for both approaches (with or without load curtailment) to solve the transmission expansion planning problem for the IEEE-24 buses power system A.1. As expected from the design of both approaches, they find the same solution; no load shedding is used in the second approach. We also observe that, when the TEP problem is feasible, the time needed to find the optimal solution is less important in the case where no load shedding is inserted in the problem; this was of course expected since when load shedding is allowed, the problem complexity increases. One observes here the trade-off between (i) the computational time and (ii) the certainty of having a useful solution. Indeed, even for small power systems, such as the IEEE-24 buses power system, one sees the consequent increase in computation time needed when load shedding is considered.

IEEE-24 buses system	Gurobi without LS	Gurobi with LS
Time [s]	285.45	3465.11
Optimal cost [k€]	113600	113600
Quantity of LS [MWh]	-	0.0

Table 6.1 Time needed to solve the TEP problem for the IEEE-24 buses system with Gurobi.

6.2 Benders decomposition

In the following section, we explain how to apply Benders decomposition to our problem using two approaches: **(i)** first we consider a stochastic formulation and **(ii)** second we explain how to combine Benders decomposition with adjustable robust optimization.

6.2.1 Bender decomposition using a stochastic model

Binato *et al.* were among the first researchers to apply Benders decomposition to the TEP problem in [10]. As discussed when commenting equations 3.6 and 3.7, their major contribution was to propose a clever way to assign the value of the disjunctive parameters M_l . Their work has then been further developed in many ways, as explained in the introduction of article [31]. Article [31] is particularly relevant in our case since it explains how to apply Benders decomposition while considering the $(N - 1)$ -security criterion. Since imposing robustness to a power system is very similar to imposing the $(N - 1)$ -security criterion, applying Benders decomposition in our situation requires only few modifications.

The first step is to include load curtailment in our formulation 3.11 by using a stochastic optimization model. This is done by transforming the objective function as in equation 6.1 and by transforming the power balance constraint by the set of equations 6.2-6.4. Let us remark that normally, in stochastic optimization, we should insert probabilities $p_{s,k}$ representing the probability that contingency s occurs in scenario k as illustrated in equation 6.5. However, by hypothesis, we desire the power system to be able to manage each situation equivalently; we can thus, without loss of generality, remove $p_{s,k}$ from the objective function since it does not vary and can be integrated in C_{LS} .

$$\min_{x_l, f_l, \theta_n} \sum_{l \in \mathcal{L}_c} I_l \cdot x_l + C_{LS} \cdot \left(\sum_{n \in \mathcal{N}} p_{s,k} \cdot \left[\sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}} r_n^{(s,k)} \right] \right). \quad (6.5)$$

The extensive stochastic formulation can be formulated as in 6.6:

Stochastic optimization model	
$\min_{x_l, f_l^{(s,k)}, \theta_n^{(s,k)}, g_n^{(s,k)}, r_n^{(s,k)}} \quad (6.6)$	$\sum_{l \in \mathcal{L}_c} I_l \cdot x_l + C_{LS} \cdot \left(\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}} r_n^{(s,k)} \right)$
s.t.	$g_n^{(s,k)} + \sum_{l \in \Omega_n^-} f_l^{(s,k)} - \sum_{l \in \Omega_n^+} f_l^{(s,k)} = D_n - r_n^{(s,k)}, \quad \forall n \in \mathcal{N},$ $f_l^{(s,k)} - B_l \left(\theta_m^{(s,k)} - \theta_n^{(s,k)} \right) = 0, \quad \forall l = (m, n) \in \mathcal{L}_e,$ $f_l^{(s,k)} - B_l \left(\theta_m^{(s,k)} - \theta_n^{(s,k)} \right) \leq M_l^s (1 - x_l), \quad \forall l = (m, n) \in \mathcal{L}_c,$ $-f_l^{(s,k)} + B_l \left(\theta_m^{(s,k)} - \theta_n^{(s,k)} \right) \leq M_l^s (1 - x_l), \quad \forall l = (m, n) \in \mathcal{L}_c,$ $-T_l \leq f_l^{(s,k)} \leq T_l, \quad \forall l \in \mathcal{L}_e,$ $-T_l x_l \leq f_l^{(s,k)} \leq T_l x_l, \quad \forall l \in \mathcal{L}_c,$ $x_l \in \{0, 1\}, \quad \forall l \in \mathcal{L}_c,$ $0 \leq r_n^{(s,k)} \leq D_n, \quad \forall n \in \mathcal{N},$ $0 \leq g_n^{(s,k)} \leq P_n, \quad \forall n \in \mathcal{N},$ $s \in \mathcal{S}, k \in \mathcal{K}.$

To simplify the notations we will rather consider matrix notations 6.7, which are classically used in Benders decomposition; matrix notations help making the distinction between first and second stage decisions.

Stochastic optimization model (matrix notations)	
$\min_{x, y_{s,k}} \quad (6.7)$	$c^T \cdot x + \sum_{s \in \mathcal{S}, k \in \mathcal{K}} q_{s,k}^T \cdot y_{s,k}$
s.t.	$A \cdot x \leq b, \quad (6.8)$ $T_{s,k} \cdot x + W_{s,k} \cdot y_{s,k} \leq h_{s,k}, \quad (6.9)$ $x \in \mathcal{X}, y_{s,k} \geq 0. \quad (6.10)$

In formulation 6.7, x denotes the first stage decision variables restricted by an integer set $\mathcal{X}$; $y_{s,k}$ are the continuous second stage decisions for each scenario k and contingency s ; A and b are parameter matrix and vector independent of the scenarios and contingencies; $T_{s,k}$, $W_{s,k}$ and $h_{s,k}$ are parameter matrices and vector for each scenario $k \in \mathcal{K}$ and contingency $s \in \mathcal{S}$. In article [49] it is suggested to assign the value of the disjunctive parameters as follows: $M_l = \frac{2\pi}{B_l}$, where B_l is the susceptance of circuit l . We note that second stage variables $f_l^{(s,k)}$ and

$\theta_n^{(s,k)}$ may be negative, whereas we desire non-negative second stage variables as mentioned in equation 6.10; this issue is easily solved by adding non-negative variables to the problem, i.e. $f_l^{(s,k)} = f_l^{(s,k)+} - f_l^{(s,k)-}$.

We can now apply Benders decomposition which is interesting in our situation since, as explained before, (i) the number of variables linking first stage to second stage problems, i.e. investment decisions, is relatively small compared to the total number of variables, and (ii) the nature of both problems are different: the master problem is a MILP whereas the slave problem is a LP. The master problem 6.11 is formulated hereinafter where $Q = \sum_{s \in \mathcal{S}, k \in \mathcal{K}} Q_{s,k}$:

Benders decomposition: master problem (MP)		
$\min_x$	$c^T \cdot x + Q$	(6.11)
s.t.	$A \cdot x \leq b, x \in \mathcal{X}$	

Given the first stage decisions $\bar{x}$ obtained by solving the master problem, the slave problem must be solved. However, since there are no interactions between each scenario and contingency, it is more efficient to solve a slave problem 6.12 for each scenario k and contingency s ; the separation of these problems enables the use of parallel processing.

Benders decomposition: slave problem (SP)		
$\min_{y_{s,k}}$	$q_{s,k}^T \cdot y_{s,k}$	(6.12)
	$W_{s,k} \cdot y_{s,k} \leq h_{s,k} - T_{s,k} \cdot \bar{x},$	
	$y_{s,k} \geq 0.$	

The dual DP 6.13 of SP is formulated hereinafter:

Benders decomposition: dual of the slave problem (DP)		
$\max_{u_{s,k}}$	$(h_{s,k} - T_{s,k} \cdot \bar{x})^T \cdot u_{s,k}$	(6.13)
	$W_{s,k}^T \cdot u_{s,k} \leq q_{s,k},$	
	$u_{s,k} \leq 0.$	

Based on the argument developed in the previous section 6.1, the primal of the slave problem (SP) is always feasible since load curtailment is tolerated. As a consequence, feasibility cuts will never be generated. By duality, the dual problem DP is bounded and its objective function

value provides a lower bound on SP, which allows to generate an optimality cut as follows:

$$Q_{s,k} \geq (h_{s,k} - T_{s,k} \cdot x)^T \cdot \bar{u}_{s,k}, \quad (6.14)$$

where $\bar{u}_{s,k}$ is the optimal solution of DP 6.13. The value of Q is then constrained in the master problem by using these optimality cuts:

$$Q \geq \sum_{s \in \mathcal{S}, k \in \mathcal{K}} (h_{s,k} - T_{s,k} \cdot x)^T \cdot \bar{u}_{s,k}^i, \quad i = 1, \dots, N \quad (6.15)$$

where N is the number of iterations already realized. One can also split Q in the master problem as the sum of $Q_{s,k}$ and impose the following lower bounds on $Q_{s,k}$:

$$Q_{s,k} \geq (h_{s,k} - T_{s,k} \cdot x)^T \cdot \bar{u}_{s,k}^i, \quad i = 1, \dots, N. \quad (6.16)$$

This last version of the master problem is called the *multicut strategy*; it has been demonstrated that this version has a faster convergence rate than the classical version [11]. To increase the convergence, one can use the fact that no load curtailment is actually tolerated, which imposes that $Q = 0 \Leftrightarrow Q_{s,k} = 0, \forall k \in \mathcal{K}, s \in \mathcal{S}$. Hence the new multi-cut master problem can be transformed as follows:

Benders decomposition: master problem (MP), multi-cut version		
$\min_x$	$c^T \cdot x + Q$	(6.17)
s.t.	$A \cdot x \leq b, \quad x \in \mathcal{X},$ $0 \geq (h_{s,k} - T_{s,k} \cdot x)^T \cdot \bar{u}_{s,k}^i, \quad i = 1, \dots, N.$	

Algorithm 1 (Benders decomposition: multi-cut version) *The Benders decomposition algorithm applied to our situation can thus be formulated as follows [31]:*

1. **Step 1.** Set $i = 1$, $LB = -\infty$, $UB = +\infty$, and solve MP without cut. Update the lower bound as follows $LB = \max(LB, LB')$, where LB' is the objective function value of the master problem.
2. **Step 2.** Solve DP 6.13 for each scenario and contingency given the master first stage decision $\bar{x}$. Save the solution $\bar{u}_{s,k}$ and update the upper bound as follows $UB = \min(UB, UB')$, where $UB' = c^T \bar{x} + \sum_{s \in \mathcal{S}, k \in \mathcal{K}} (h_{s,k} - T_{s,k} \cdot \bar{x})^T \cdot \bar{u}_{s,k}$.

3. **Step 3.** Evaluate the stopping criteria, i.e. if $UB - LB \leq \varepsilon$ (e.g. $\varepsilon = 10^{-6}$) then STOP; otherwise we generate optimality cuts 6.16 and insert them in the master problem before going back to step 1 with $i = i + 1$.

6.2.2 Bender decomposition using adjustable robust optimization

In section 3.1, we made the hypothesis that the uncertainty of production generation facilities would be represented by several scenarios representing extreme cases; this hypothesis enables to approach the problem via the Benders decomposition strategy based on a stochastic model of the problem as presented in section 6.2.1. In practice, rather than having extreme scenarios, one represents more often the uncertain parameters, such as future production and demand, through robust sets. Indeed the probability distribution functions of those parameters, needed in the stochastic formulation, are often inaccessible. Moreover, considering extremes scenarios can lead to an enormous amount of variables and constraints; as a consequence, approaching the TEP problem via the previous approach 6.2.1 can rapidly lead to an intractable problem due to limited memory. Therefore, to circumvent the tractability issues associated with conventional contingency-constrained methods, articles [14, 54] propose a different approach to solve the TEP problem, which consists in formulating the problem as an adjustable robust optimization model and to apply the Benders decomposition strategy. Adjustable robust optimization (ARO) models, which are comparatively less complex than stochastic programming models, obey the following philosophy:

- The optimal expansion plan is sought by minimizing the same objective function as in the stochastic formulation 6.6.
- This optimal expansion plan is sought knowing that once the investment decisions are taken, the worst scenario occurs, i.e. given a transmission expansion plan, uncertain parameters take the values that maximize objective function 6.6.
- Once the uncertain parameters are fixed, the power system reacts in order to minimize the impact of those on the objective function by finding the best combination of the remaining variables.

This philosophy is translated in mathematics in an optimization problem at three levels:

- The upper level determines optimal non-adjustable decisions, i.e. decisions that must be feasible for every deviation of the uncertain parameters.
- The middle level determines the worst-case parameters values leading to maximum feasibility damage of the upper-level decisions.

- The lower level aims at finding the best reaction, by means of adjustable variables, that minimize the upper-level infeasibility.

The trilevel optimization model can be explained as follows: for a given upper-level decision, the middle-level searches in the contingencies set the most damaging one in terms of power imbalance, given the best redistribution provided by the lower level. In other words, the two lowermost optimizations are the identification of the contingency leading to the largest system power imbalance.

References [14, 54] prove the equivalence between the trilevel optimization model derived and the original contingency-dependent formulation. They explain also how to apply Benders decomposition once, by strong duality, the two lowermost optimization problems being merged together in a single-level equivalent problem.

In conclusion, the advantage of Benders decomposition is that **(i)** it helps to derive lower and upper bounds on the best expansion plan achievable and **(ii)** it allows parallel processing, which gains in success these years with the increase in computational capabilities. We decided not to implement this method since, as studied in a previous master's thesis [30], the results of this method are not convincing, i.e. the lower bound estimation behaves badly which leads to difficulties in the convergence.

6.3 The *Branch and Bound* method

Branch and Bound (B&B) trees are often used in commercial solvers in order to solve MILP optimization problems. The idea of this method is to use a decision tree and to update recursively the upper and lower bounds until a stopping criterion is reached, e.g. $\frac{UB-LB}{LB} < \varepsilon$, where ε is a chosen tolerance. This method is famous but has unfortunately bad performances on the TEP problem; to understand more in detail this bad behavior let us recall the philosophy of *Branch and Bound* (B&B) trees with the help of figure 6.1.

6.3.1 A quick reminder on how it works

Figure 6.1 represents a result that we could get when applying *Branch and Bound* (B&B) to a minimization optimization problem composed of four binary variables only. The main idea of *Branch and Bound* (B&B) trees is to iteratively split the search set into two complementary parts, e.g. by imposing for a binary variable a value of 0 in left part of the tree and of 1 in the right part. As a consequence, each node of the tree has some fixed decisions to respect; for

each of those nodes the linear relaxation of the integer problem is solved. Let denote Z the solution of this relaxed problem, which is a lower bound of the potential feasible solutions in its children nodes; for example, in figure 6.1, the first node of level 2 has a value $Z = 12$, which means that any feasible node in the subtree of this node will always have a value greater than or equal to $Z = 12$. Based on these values, it is possible to derive lower and upper bounds of the global optimum at each level l :

- $LB_l = \min_{n \in \{\text{Node of level } l\}} (Z_n)$; the global lower bound of the problem is the minimum of the lower bounds of all subtrees, which is given by the value Z_n for a node n of the level l .
- $UB_l = \min [UB_{l-1}; \min_{n \in \{\text{Feasible nodes of level } l\}} (Z_n)]$; the upper bound corresponds to the best solution found so far.

When the algorithm is processing, it can avoid exploring some parts of the tree in two frequent cases:

1. First, a branch issued from a node n can be pruned if the value of the relaxed problem, Z , is greater than the upper bound.
2. Second, the relaxation of a certain node might be infeasible. Hence any child of this node is also infeasible and does not need to be explored.

Example 2 illustrates these cutting strategies based on figure 6.1.

Example 2 (Branch and Bound tree) *On the right of the figure, we observe the updates of the lower and upper bounds. In the south-west corner is the legend of the nodes colors. The root node is unfeasible but has a feasible relaxation $Z = 10$, its children are both infeasible but have also feasible relaxations. In the third level, four different cases occur: (i) the green node is feasible and updates the upper bound, (ii) the red node is infeasible relaxation, which implies that it is not necessary to visit its children, (iii) the orange node has a relaxation value of 15, which is greater than the upper bound ($UB_3 = 14$); it can thus be pruned and finally (iv) the last has a relaxation value strictly smaller than UB_3 ; hence its children need to be investigated. Finally in level 4, a feasible node has a cost of 12, which corresponds to the lower bound; one can thus terminate the algorithm.*

Theoretically the *Branch and Bound* method seems attractive, however it is not appropriate to the transmission expansion planning problem for one major reason: the continuous relaxation of this mixed integer problem does not give useful/tight lower bounds. As a consequence, a solver, such as Gurobi, will lose time in evaluating, at each node, the relaxed problem for

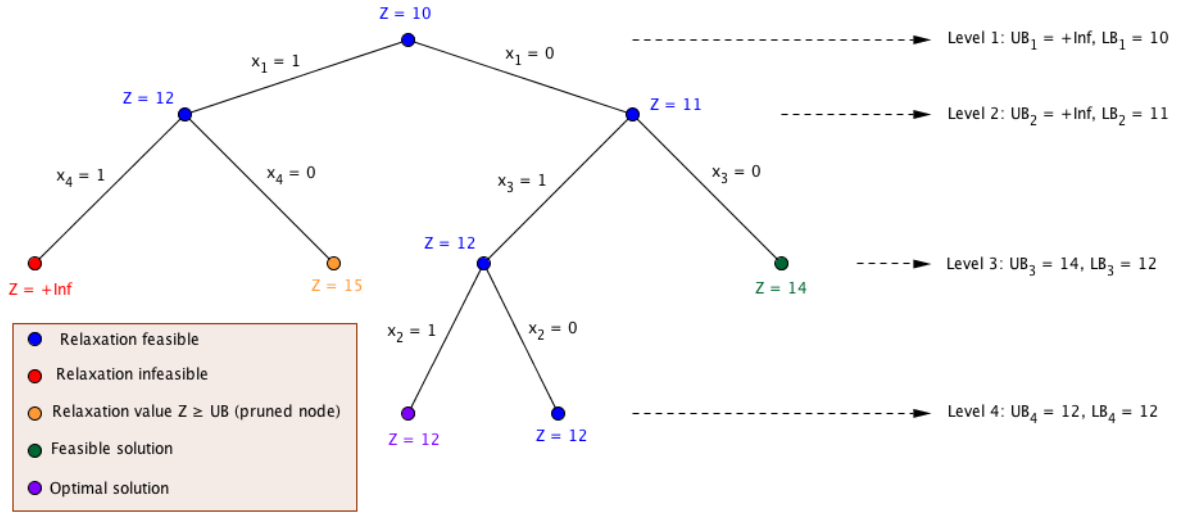


Fig. 6.1 Example of a B&B tree.

negligible benefits in the pruning strategy. Few attempts have been made in the perspective to improve the performance of the method: article [28] proposes a customization when applied to the transportation model, which is a very relaxed model of the TEP problem and in article [36] it is used only as a tool. Finally, one must mention that solving the relaxed problem or even checking the feasibility of an investment plan might cause memory and loading issues for commercial solvers when the power network size is large, e.g. the UK power system A.3.

6.3.2 How to apply the *Branch and Bound* method to the TEP problem?

From the previous section, we observe that solving the relaxed problem at each node of the tree is a waste of time. Hence, it could be interesting to implement a *Branch and Bound* tree, where the relaxed problems are not solved and where the feasibility of a plan is checked without calling a commercial solver.

Verifying the feasibility of an expansion plan requires that, for each scenario and contingency, (i) the flows respect the line capacities (ii) the net injections are respected. However, since one knows the net injections, it is easy to get the phase angles at each bus and as consequence the flows. Indeed, as described in section 2.2, the phase angles θ are obtained by solving the following linear system:

$$\mathbf{T} \cdot \boldsymbol{\theta} = \mathbf{P}, \quad (6.18)$$

where $\mathbf{P}$ is the vector of real power injections and $\mathbf{T}$ is a matrix whose elements are:

$$T_{mn} = \begin{cases} -\frac{1}{X_{mn}}, & \text{if } (m, n) \in A, m \neq n, \\ \sum_{n'=1, n' \neq m}^N \frac{1}{X_{mn'}}, & \text{if } m = n, \\ 0, & \text{if } (m, n) \notin A, \end{cases} \quad (6.19)$$

where A is the set of arcs in the network and where $X_{mn} = B_{mn}^{-1}$ is the reactance of arc (m, n) . Equation 6.18 is the matrix version of the balance constraints 3.4:

$$P_m = \sum_{n=1, n \neq m}^N \frac{1}{X_{mn}} \theta_m - \sum_{n=1, n \neq m}^N \frac{1}{X_{nm}} \theta_n, \quad \forall m \in \mathcal{N}.$$

For interested readers, we refer them to Appendix A.4. of the *Quantitative Energy Economics* course' textbook [45] for more detail on how to obtain those equations. With this approach, verifying the feasibility of an investment plan given a scenario and a contingency requires (i) the resolution of a linear system where the matrix is of size $|\mathcal{N}| - 1 \times |\mathcal{N}| - 1$, which remains relatively rapid to compute since the number of buses in a power system remains small enough and (ii) a *for* loop on the lines (number of iterations: $|\mathcal{L}_e \cup \mathcal{L}_c|$) to verify that each flow respects the line maximum capacity.

Table 6.2 compares the mean time needed to check the feasibility of an expansion plan when using either a commercial solver or the linear system approach, i.e. checking the feasibility of the power system for each scenario and contingency via the resolution of system 6.18.

System Method	Gurobi solver	Resolving $T \cdot \theta = P$
IEEE-24 buses (A.1)	1.20 s	0.02 s
UK system (A.3)	110.0 s	2.0 s

Table 6.2 Time needed to check the feasibility of an expansion plan [s].

From this table, one can conclude that calling a commercial solver to check the feasibility of an investment plan of large power systems is maybe not the best idea. However, even with this gain in efficiency, it is hopeless to obtain good performances with the *Branch and Bound* approach. Indeed, even if we assume that the optimal investment plan is quickly obtained, it might be computationally infeasible to prove its optimality in a reasonable amount of time as illustrated in example 3.

Example 3 *Let assume that the optimal investment plan uses 48 out of the 49 candidates lines. Then to prove optimality, all cheaper investment plans should be checked, which means that at least $2^{48} \simeq 2.81 \cdot 10^{14}$ plans must be considered; which leads to computational time issues.*

As a conclusion, approaching the transmission expansion planning with a *Branch and Bound* tree is hopeless since the number of plans to check is exponentially proportional to the number of candidate lines. However, we have shown here that checking the feasibility of an expansion plan is much more efficient via the linear system approach than calling a commercial solver.

Probabilistic method (Heuristic)

As mentioned in section 4, it is senseless to attempt to find, in a reasonable amount of time, the optimal solution of the transmission expansion planning problem when the number of candidate lines is large. In this chapter, we present and analyze the heuristic we have developed, called the *probabilistic method*. This heuristic has as objective to provide a feasible solution, of relatively good quality, to the TEP problem within a reasonable amount of time.

7.1 Ideas

The *probabilistic method* presented in the next section 7.2 arises as a result of two observations:

- First, as explained in section 6.3 and in [45], verifying the feasibility of an expansion plan in our formulation of the TEP problem 3.5 can be managed via the resolution of a linear system. This approach turned out to be more efficient than using an optimization commercial solver as illustrated in table 6.2. Hence, it could be relevant to design an algorithm that tests a multitude of expansion plans since one can check their verification at a greater pace than commercial solvers. As a consequence, we decided to reconsider the idea of randomized algorithms; indeed, if one could generate random investment plans with a higher probability to be feasible, we should be able to increase the chances of finding a *good* solution, which are low when generating completely random expansion plans as proved in section 4.2.2.
- Second, in section 4.2.2, we have derived an estimate of the probability of feasibility of an expansion plan p knowing its cost C_p and the optimal cost C_{p^*} (see figure 4.3). Based on figure 4.3, we can argue that this estimation is of relatively good quality; this partially validates the correctness of hypotheses **H1-H4** made in section 4.2.2. Hence the good fit of the estimation 4.3 with reality gave us two ideas detailed in the next paragraphs: (i)

first, based on empirical values, it is possible to derive a lower bound on the number of lines inserted in the optimal expansion plan and, (ii) second, the validity of hypotheses **H1-H4** suggests a strategy to generate random feasible expansion plans.

Deriving a lower bound on n_{p^*}

We have noticed in figure 4.3, that the estimate of the probability of feasibility of an expansion plan p was an under-estimation of the empiric probability, estimated using the law of large numbers (in red). To compute this estimation, one needs to know the number of candidate lines built in the optimal expansion plan n_{p^*} , which is unknown. However we can argue that if a value greater than or equal to n_{p^*} is used in equation 4.3, we should obtain an under-estimation of the empirical values. The contraposition of this statement is that, if, for a given estimate value $\bar{n}$ of n_{p^*} , one obtains an approximation that is greater than the empirical values, then $\bar{n} < n_{p^*}$. This assertion gives us a strategy (*confer* algorithm 2) to find a lower bound on the number of candidate lines built in the optimal expansion plan, n_{p^*} .

Algorithm 2 (Lower bound obtention)

1. Test different estimates $\bar{n}$ of n_{p^*} and check whether equation 4.3 gives an under or over-estimation of the empirical values.
2. Take as lower bound on n_{p^*} , the greatest value $\bar{n}$ where equation 4.3 returns an over-estimation of the empirical values.

Example 4 (A lower bound on n_{p^*} for the IEEE-24 buses system) Figure 7.1 represents the approximation of the feasibility probability computed via equation 4.3 for $\bar{n} = 10, 11$, and 12. When $\bar{n}$ equals 11 or 12, it is difficult to conclude that the obtained result is an over-estimation; in contrast, for $\bar{n} = 10$, one can say quite surely that the obtained result is an over-estimation. As a consequence, we can conclude that $n_{p^*} > 10$ which is verified since we know that $n_{p^*} = 14$.

Example 5 (A lower bound on n_{p^*} for the UK system) We can make the same analysis for the UK power system¹. Figure 7.2 represents the approximation of the feasibility probability computed via equation 4.3 for $\bar{n} = 63, 64$, and 65. For $\bar{n} = 63$, we can still say with confidence that the result obtained is an over-estimation. As a consequence, we can conclude that $n_{p^*} > 63$.

An idea to generate a solution to the TEP

From figures 7.1 and 7.2, we can observe that the shape of the empirical probability for a plan to be feasible is close from the one derived via equation 4.3. Hence we can suppose that, if

¹This power system is described in Appendix A.3

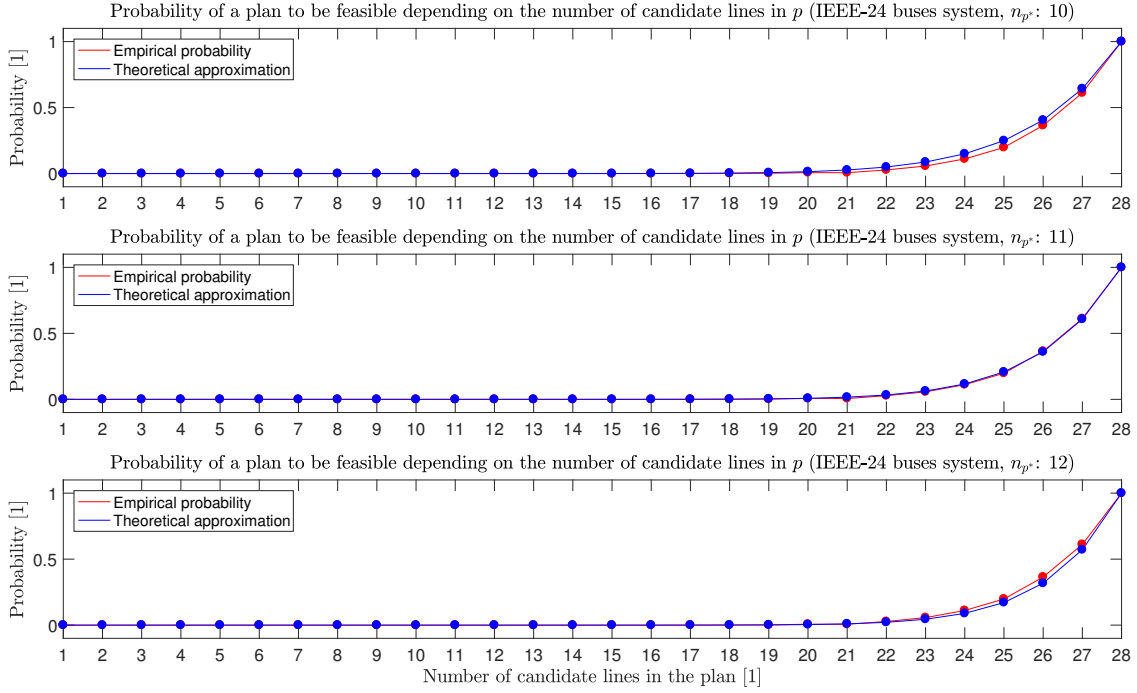


Fig. 7.1 Illustration of the probability for an investment plan p of being feasible for the IEEE-24 buses power system.

there is more than one solution to the TEP problem, the majority of the candidate lines built in these solutions are the same. As a consequence, if we generate a large number of random feasible expansion plans and if we observe that a candidate line is built in the majority of these plans, then it is highly probable that this candidate line (i) is needed to ensure the feasibility of an expansion plan and (ii) is built in the optimal expansion plan. From this observation, we have formulated algorithm 3 presented in the next section 7.2.1.

7.2 Probabilistic method

7.2.1 Algorithm

Algorithm 3 (Probabilistic method) *The probabilistic method is a recursive procedure based on the generation of random plans. This algorithm is composed of different steps:*

Step 1. Initialization.

- let $Cost_{min} = +\infty$ be the cheapest investment plan known so far.

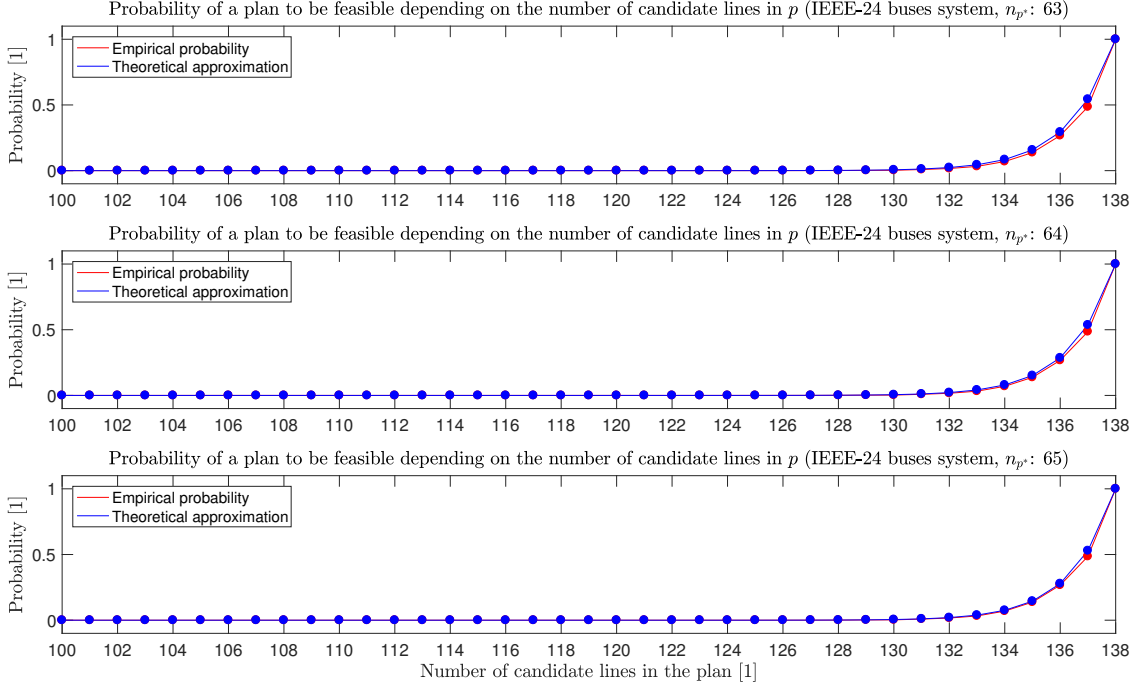


Fig. 7.2 Illustration of the probability for an investment plan p of being feasible for the UK power system.

- let $Cost_{min}^{Old} = +\infty$ be the cheapest investment plan known so far at the beginning of each iteration.
- let $proba := [p_1, p_2, \dots, p_{|\mathcal{L}_C|}]$ be a vector composed of the probabilities of selecting each candidate line while generating random expansion plans. Hence for $p_1 = \dots = p_{|\mathcal{L}_C|} = 0.99$, any candidate line has 99% chance of being included in any random expansion plan generated. At the beginning of the algorithm, there is a priori no reason to make any distinction between the candidate lines, hence we will set all probabilities equal to α , i.e. $p_1 = \dots = p_{|\mathcal{L}_C|} = \alpha$. The parameter α has an impact on how fast the algorithm will find feasible plans. Empirically the following assumption is verified: the more candidate lines are inserted in an expansion plan, the more probable it is for this expansion plan to be feasible. Hence, taking a value close to 1 for α increases the probability of generating feasible expansion plans.
- let $\bar{m}$ be the maximum number of random expansion plans tested per iteration.

Step 2. Recursion.

- (a) let $vec_{count} := [0, \dots, 0]$ be a vector with $|\mathcal{L}_c|$ entries and $m_c := 0$ be a scalar counting the number of feasible plans found.
- (b) **Generation.** Random expansion plans are generated until a number of m feasible plans cheaper than the best one at the beginning of the iteration (of cost $Cost_{min}^{Old}$) are found or until $\bar{m}$ expansion plans are tested. When a feasible plan cheaper than $Cost_{min}$ is found, the minimum cost $Cost_{min}$ is updated. Moreover, for any feasible plan p found, m_c is incremented and vector vec_{count} is updated as follows; component i is incremented by one if candidate line i is built in plan p .
- (c) **Probabilities update.** When m feasible expansion plans are found or when $\bar{m}$ plans have been tested, the vector of probabilities ($proba$) is updated as follows: $proba[i] = \max\left((1 - \beta), \min\left(\beta, \frac{vec_{count}[i]}{m_c}\right)\right)$. Parameter β is a safety parameter; indeed one desires to avoid making hasty decisions and one keeps considering building or not building all candidate lines.
- (d) **Evaluating the gap.** A relative gap can be computed at the end of each iteration. It is computed as follows:

$$gap_{rel} = \frac{Cost_{min} - Cost_{Built}}{Cost_{min}},$$

where $Cost_{Built}$ is the cost of building all the lines i for which $\frac{vec_{count}[i]}{m}$ equals 1, i.e. all the lines that are built in all feasible expansion plans found during the iteration.

- (e) **Evaluating the stopping criteria.** If during the iteration, one has not improved the best solution, i.e. $Cost_{min} = Cost_{min}^{Old}$, then the algorithm terminates.
- (f) **Update the best solution.** $Cost_{min}^{Old} = Cost_{min}$.
- (g) **Moving to the next iteration.** Go back to (a).

7.2.2 Description of the parameters

In the probabilistic algorithm 3, different parameters are used: α , β , m and $\bar{m}$; in this section we give an explanation on how to select these parameters.

- **Parameter α .** This parameter is used only for the first iteration. As mentioned above, taking a value close to 1 allows to generate random expansion plans that have a high probability to be feasible. However, this implies also that the random expansion plans obtained contain the majority of the candidate lines and are probably much too expensive compared to the optimal expansion plan. There is thus a trade-off between generating

plans with a high cost and a high probability to be feasible and generating plans with a lower cost but also with a lower probability to be feasible.

- **Parameter β .** The role of this parameter is to reconsider the decisions taken previously by limiting the probability to insert or not insert a candidate line in the random plans generated. For instance, if one has a pretty good idea on the decision to take for 100 candidate lines and if one takes a value of β equal to 0.99 then the probability of respecting the probable decisions is equal to $0.99^{100} \simeq 0.366$, which means that on average (i) one third of the random plans generated respects the advised decisions, and (ii) two thirds explore expansion plans "in the neighborhood" of the advised one and thus reconsider the advised decisions. Obviously, as presented in III, this parameter has an important impact on the convergence speed. For instance selecting a value of β slightly lower, e.g. of 0.95, results in an important difference with only 0.5% of the random plans generated respect the advised decisions; hence, if those decisions must be respected to ensure the feasibility of a plan, the algorithm will have to generate much more random plans in order to get the n feasible ones required to move to the next iteration.
- **Parameter m .** At the end of each iteration of algorithm 3, vector *proba* is updated. As mentioned previously, the i^{th} element of this vector is a probability estimation for candidate line i to be inserted in a feasible plan cheaper than $Cost_{min}^{Old}$; this estimation is then used in the next iteration as the best approximation of probability of line i to be inserted in a feasible plan cheaper than $Cost_{min}$. The probability for a line to be inserted in a random feasible plan cheaper than a specified cost can be seen as a Bernoulli random variable. Hence we should fix the value of m such that the size of the confidence interval of the estimation of each probability, i.e. $proba[i]$, is reasonable. From the Central Limit Theorem, one can derive the $(1 - \gamma)^{th}$ confidence interval as follows:

$$I_i = \left[\hat{p}_i - z_{1-\frac{\gamma}{2}} \sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{n}}, \hat{p}_i + z_{1-\frac{\gamma}{2}} \sqrt{\frac{\hat{p}_i(1-\hat{p}_i)}{n}} \right], \quad (7.1)$$

where $\hat{p}_i = proba[i]$ and $z_{1-\frac{\gamma}{2}}$ is the $(1 - \frac{\gamma}{2})^{th}$ quantile. Figure 7.3 represents the size of the confidence interval as a function of the estimated probability $\hat{p}_i$ for different values of n . Obviously, the larger n , the smaller the confidence interval. In algorithm 3, a high precision on the estimates $\hat{p}_i$ is not essential since these values are used to estimate the probability of insertion of line i in a feasible plan cheaper than $Cost_{min}$, whereas $\hat{p}_i$ is an estimate of this probability for plans cheaper than $Cost_{min}^{Old}$. As a consequence, considering a maximum size for the confidence interval of 0.1 seems already reasonable enough, i.e. $m = 400$.

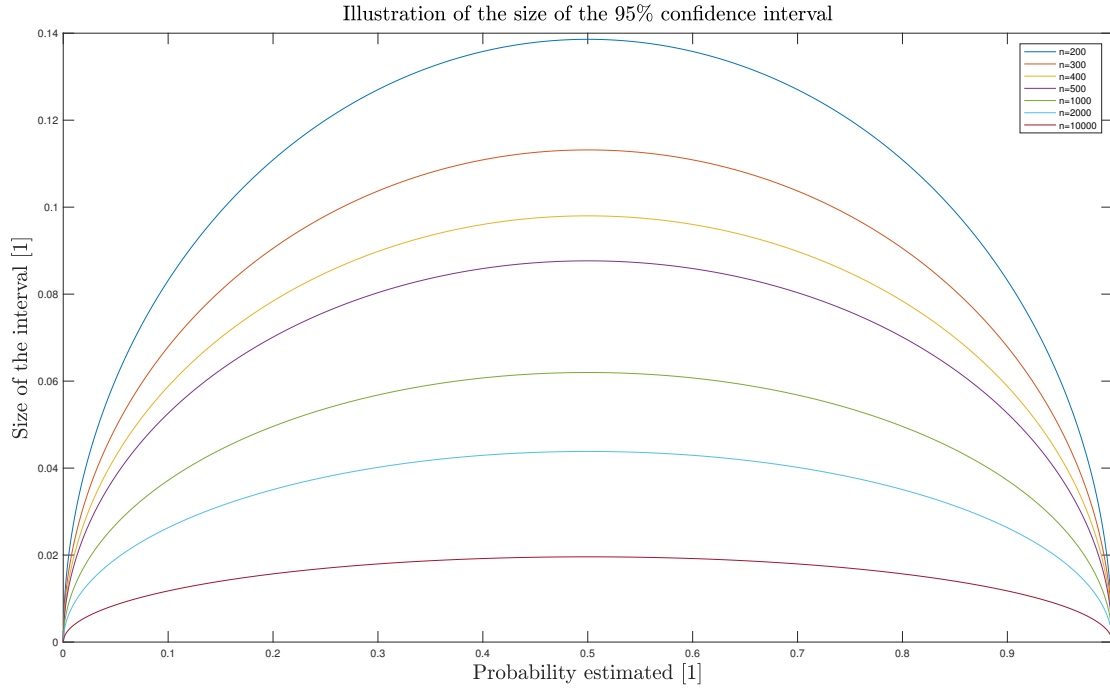


Fig. 7.3 Illustration of the size of the 95%-confidence interval for the approximation of Bernoulli's parameter, $\hat{p}$.

- **Parameter $\bar{m}$.** This parameter is critical for the stopping criteria of the algorithm. Indeed if, for an iteration, the algorithm cannot find a better solution than $Cost_{min}^{Old}$ when it generates $\bar{m}$ random expansion plans, it terminates. As a consequence, parameter $\bar{m}$ describes the proportion of the neighborhood to be investigated, near the optimal solution provided by the algorithm. Based on the hypotheses made in section 4.2.2, we can assume that if there exists a better solution than the one found so far, it should have the majority of their candidate lines built in common, which justifies the neighborhood search. An analysis of this parameter on the performance is carried out in part III.

7.3 Theoretical analysis

7.3.1 Convergence estimation

As mentioned in section 5.2, the major drawback of heuristic techniques is that they fail in providing a proof on the quality of the solution provided; moreover they rarely find the optimal expansion plan for real power systems [40, 50]. Unfortunately, this is also the case in the method we proposed in the previous section 7.2. However, if we consider that the hypotheses **H1-H4** made in section 4.2.2 are correct, we can still estimate the convergence. We will then comment what would be the expected convergence when the hypotheses are removed.

Convergence estimation based on hypotheses H1-H4.

The four hypotheses that were made are: **(H1)** each candidate line has the same cost C ; **(H2)** each possible investment plan p has the same probability to be the optimal one p^* ; **(H3)** the optimal solution is unique and **(H4)** an investment plan p strictly more expensive than the optimal one p^* is feasible if and only if all the candidate lines built in p^* are also built in p .

Based on these hypotheses, the behavior of the algorithm is illustrated in figure 7.4. The vertical axis represents the best cost achieved at the end of each iteration i , $Cost_{min}^i$; the length of the horizontal segment inside the polyhedral represents the maximum number of candidate lines that it is possible to insert in an expansion plan in order that its cost is smaller or equal to $Cost_{min}^i$. Of course, with **H1** the size of these segments shrinks at each iteration; the size of the green segment represents the number of candidate lines inserted in the optimal expansion plan.

To derive the probability of success of algorithm 3, we compute the probability of failure at each possible state of the algorithm, until we get the probability of reaching the final state, i.e. the optimal solution p^* . With **H2** we know that, given the number of candidate lines n , the number of lines built in the optimal solution follows a binomial distribution, $\mathcal{B}(n, 0.5)$. In a first step, we compute the probability of getting the right solution knowing the number of candidate lines inserted in the optimal plan, n_{p^*} ; we will then compute this probability for all possible n_{p^*} and merge all those probabilities, while respecting the binomial distribution, in order to get the final probability of success. To estimate the probability of success knowing n_{p^*} we follow the structure of algorithm 3:

- At the beginning of the algorithm, we know that the expansion plan that builds all the candidate lines is feasible. Hence, the first iteration of the algorithm has as objective to find a feasible solution strictly cheaper than the sum of all investment costs, i.e. $\sum_{l \in \mathcal{L}_c} I_l$.

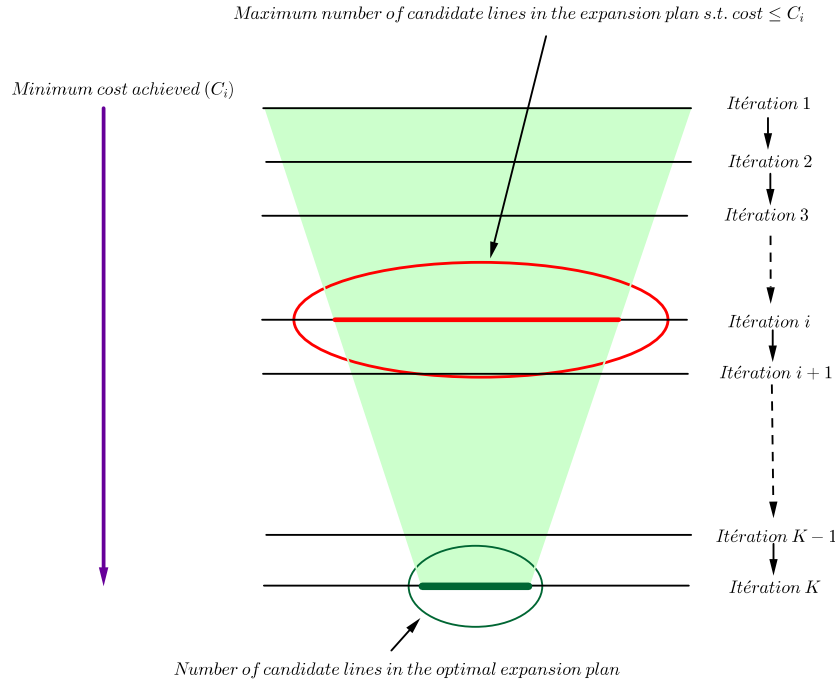


Fig. 7.4 Schematization of the convergence under hypotheses **H1-H4**.

Hypothesis **H4** stipulates that a random plan is feasible if and only if it builds the n_{p^*} optimal lines; as a consequence the probability for a plan to be feasible equals $\alpha^{n_{p^*}}$, and the probability that among the other lines at least one is not built, in order to get a cheaper plan, equals $p^+ = \mathbb{P}(X \leq n^\dagger - 1)$, where X is a random variable following a binomial distribution, $X \sim \mathcal{B}(n^\dagger, \alpha)$; α is the initial probability used to generate random plans and $n^\dagger$ is the number of candidate lines not inserted in the optimal solution, $n^\dagger = n - n_{p^*}$. In conclusion, the probability of success of iteration 1 equals $p^+ \cdot [1 - (1 - \alpha^{n_{p^*}})^{\bar{m}}]$, where $\bar{m}$ is the maximum random plans tested per iteration.

- At the end of the first iteration, we estimate the probability of transition to the other states, i.e. a state corresponding to the best cost achieved. Hence, in the case of a successful iteration, the probability to move from the initial state, state $n^\dagger$, to state i , i.e. the state where the minimum cost achieved equals $n_{p^*} + i \cdot C$ (cfr. **H1**) is computed as follows:

$$\mathbb{P}(\text{state}_{n^\dagger} \rightarrow \text{state}_i \mid \text{Iteration 1 is feasible}) \quad (7.2)$$

$$= \mathbb{P}(\min(D_j) = n_{p^*} + i \cdot C, j = 1, \dots, m \mid \text{Iteration 1 is feasible}), \quad (7.3)$$

$$= \mathbb{P}(\min(C_j) \leq n_{p^*} + i \cdot C, j = 1, \dots, m) \\ - \mathbb{P}(\min(C_j) \leq n_{p^*} + i \cdot C - 1, j = 1, \dots, m), \quad (7.4)$$

where D_j is a random variable representing the cost of a random feasible plan; C_j is a random variable representing the cost of a random feasible plan strictly cheaper than $(n_{p^*} + n^\dagger) \cdot C$. We have supposed that when an iteration is successful, the m feasible plans required to move to the next iteration are found. Probability 7.4 is computed with the help of the following result:

$$\begin{aligned} \mathbb{P}(\min(C_j) \leq n_{p^*} + i \cdot C, j = 1, \dots, m) &= 1 - \mathbb{P}(C_j > n_{p^*} + i \cdot C, j = 1, \dots, m), \\ &= 1 - \mathbb{P}(C_j > n_{p^*} + i \cdot C)^m. \end{aligned}$$

The probability to move from state $n^\dagger$ to i is thus equal to:

$$\mathbb{P}(\text{state}_{n^\dagger} \rightarrow \text{state}_i) = \mathbb{P}(\text{state}_{n^\dagger} \rightarrow \text{state}_i | \text{Iteration 1 is feasible}) \cdot p^+ \cdot [1 - (1 - \alpha^{n_{p^*}})^{\bar{m}}].$$

- At a state i , the random plans are generated based on vector *proba*, whose component i corresponds to the estimation of insertion of line i in feasible plans strictly cheaper than $n_{p^*} + i \cdot C$, the minimum cost achieved so far. Of course this estimation depends on the previous state, which explains why we have computed the transition probabilities between states in the previous point. Based on **H4** we can separate the candidate lines into two groups: a group of *useful* lines which have to be built to get a feasible expansion plan, and another group with the others lines categorized as *useless*. The components of vector *proba* corresponding to *useful* lines equals β , the maximum probability, no matter what the previous state was. For *useless* lines, this probability is dependent on the previous stage; for instance if the previous state is state i , i.e. the best optimal plan so far includes i useless lines, then the estimation of the probability of insertion of useless lines equals $\max((1 - \beta), \min(\beta, \frac{i-1}{i}))$. We assume that all lines have the same estimation since the situation is symmetric when considering the *useless* lines; the value of $\frac{i-1}{i}$ is chosen to be pessimistic, i.e. the greater the probability of inserting a *useless* line, the greater the chances that the algorithm is unable to find a cheaper solution than the current one.
- The algorithm will find the optimal solution if and only if it can reach the final state, i.e. state 0, where no useless lines are inserted. Hence the probability of success of the algorithm is obtained by summing all probabilities of transition from state i to state 0 for $i = 1, \dots, n^\dagger$.

Figure 7.5 represents the evolution of the probability of finding the optimal solution with the maximum number of random expansion plans allowed to be tested per iteration, $\bar{m}$, under

hypotheses **H1-H4**. Compared to the performances found in the theoretical analysis of the randomized algorithm 4.2.1 illustrated in figure 4.1, we observe that our approach requires significantly less random expansion plans. To correctly compare both graphs we must keep in mind that figure 7.5 illustrates the evolution of the probability with the maximum number of random expansions allowed to be tested *per iteration* and not the *total* number of random expansion plans tested like in figure 4.1. However, in the worst case, algorithm 3 requires $n^\dagger$ iterations to converge, which can be bounded by the total number of candidate lines, n ; as a consequence we should shift the curves of figure 7.5 to the right by a value of $\log_2(n)$, as illustrated in figure 7.6, to get comparable values with those of the randomized algorithm.

It is counterintuitive, as illustrated in figure 7.5, that the maximum number of random expansion plans requested at each iteration to obtain a high probability of success (e.g. > 0.8) decreases with the total number of candidate lines n . However this is justified in the next paragraph, where we consider the convergence of the algorithm when not respecting certain hypotheses among **H1-H4**. Figure 7.6 is consistent with our intuition, since it suggests that the greater the number of candidate lines, the greater the number of random expansion plans to generate in order to get a high probability of success.

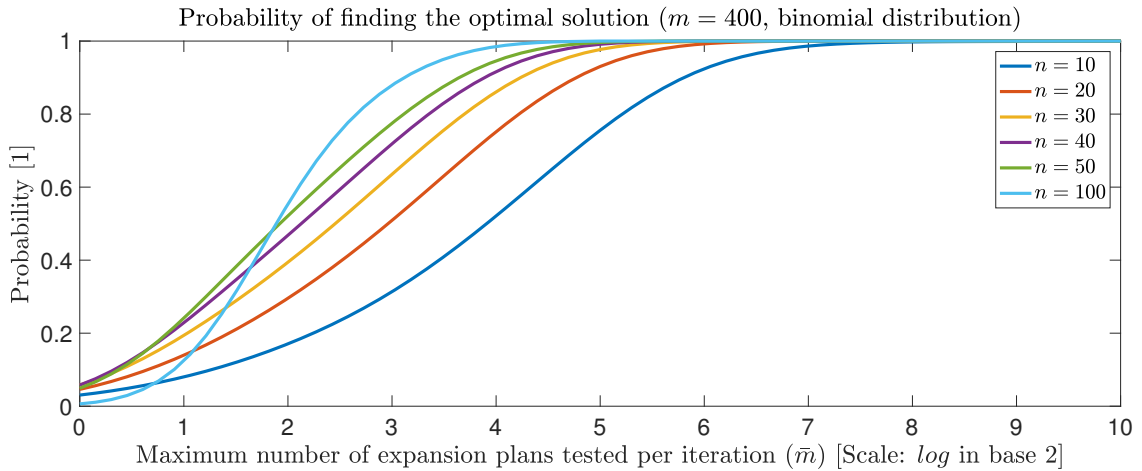


Fig. 7.5 Evolution of the probability of finding the optimal solution with the maximum number of random expansion plans allowed to be tested per iteration under hypotheses **H1-H4**.

Convergence estimation without hypotheses **H1-H4**.

Hypothesis H1. If we assume that the cost of the candidate lines is not uniform, the only impact on the algorithm's behavior is that, in the worst case situation, the difference in the

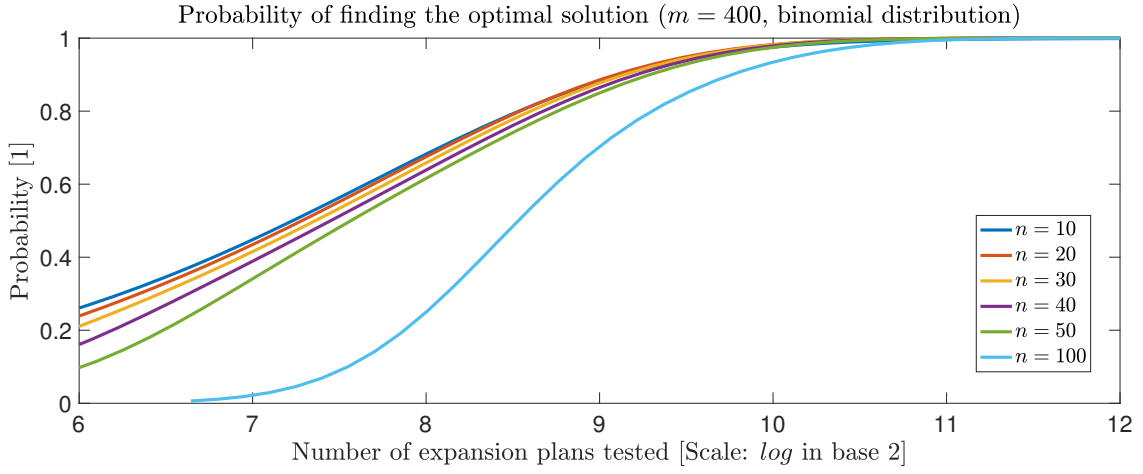


Fig. 7.6 Evolution of the probability of finding the optimal solution with the number of random expansion plans tested under hypotheses **H1-H4**.

investment costs of two successful iterations won't be constant anymore. As illustrated in figure 7.4, when we consider **H1**, the difference in the best cost achievable between two successive iterations equals C ; figure 7.7 schematizes the typical convergence when hypothesis **H1** is not taken into account. Both graphs consider the worst case situation where *useless* candidate lines are rejected one by one. In conclusion, dropping hypothesis **H1** has no influence on the convergence.

Hypothesis H2. This hypothesis stipulates that each possible investment plan p has the same probability to be the optimal one p^* , and thus that the number of candidate lines built in the optimal solution, n_{p^*} , follows a binomial distribution, i.e. $n_{p^*} \sim \mathcal{B}(n, 0.5)$. We could have made a different hypothesis on the distribution of n_{p^*} , for instance figures 7.8 represents the probability of finding the optimal solution with the maximum number of random expansion plans allowed to be tested when n_{p^*} follows a uniform distribution. We observe that in this case the maximum number of iterations $\bar{m}$ must be higher than in the previous situation in order to guarantee a high probability of success.

Figures 7.10 and 7.11 represent the probability of finding the optimal solution in the worst and best case respectively. Whatever the number of candidate lines n in the power system, the worst case scenario is when the number of lines in the optimal expansion plan equals $n - 1$. In the worst case, the probability for algorithm 3 to be successful equals $\alpha^{(n-1)} \cdot (1 - \alpha)$, which is a decreasing function in n . In contrast, the best case is when $n_{p^*} = 0$ and we observe that the

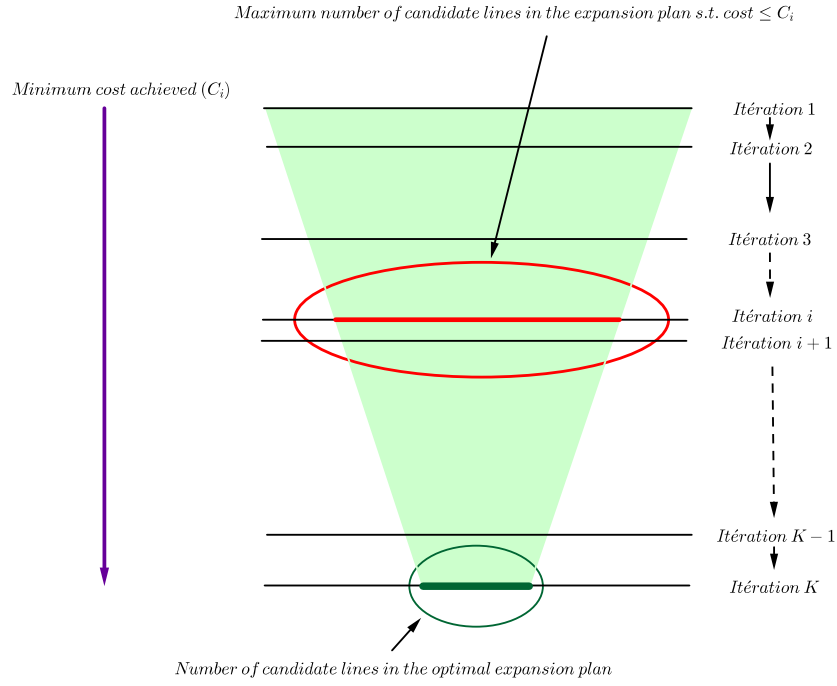


Fig. 7.7 Schematization of the convergence under hypotheses **H2-H4**.

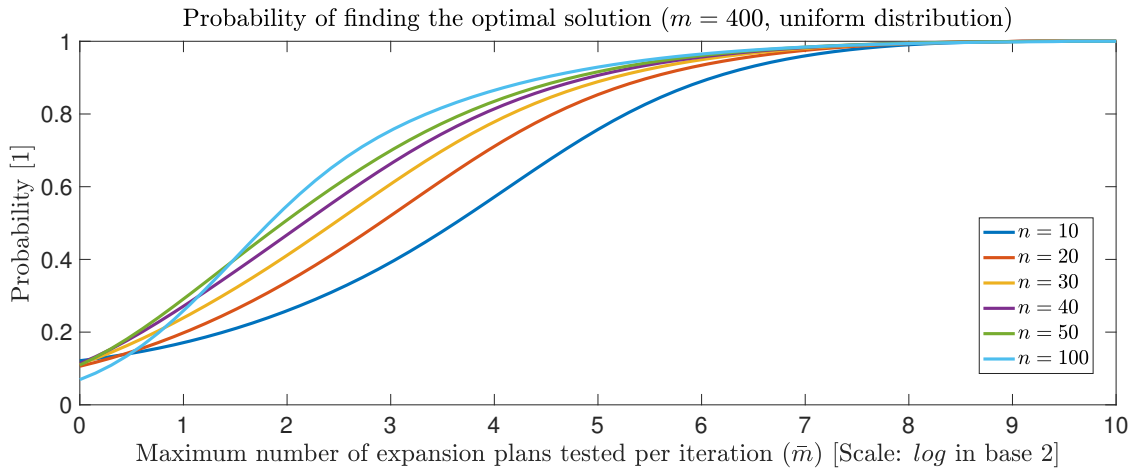


Fig. 7.8 Evolution of the probability of finding the optimal solution with the maximum number of random expansion plans allowed to be tested per iteration under hypotheses **H3-H4**.

greater n the greater the probability of success.

From figures 7.10 and 7.11, we observe that, the greater the number of candidate lines in the power system n , the greater the probability variation of finding the optimal solution

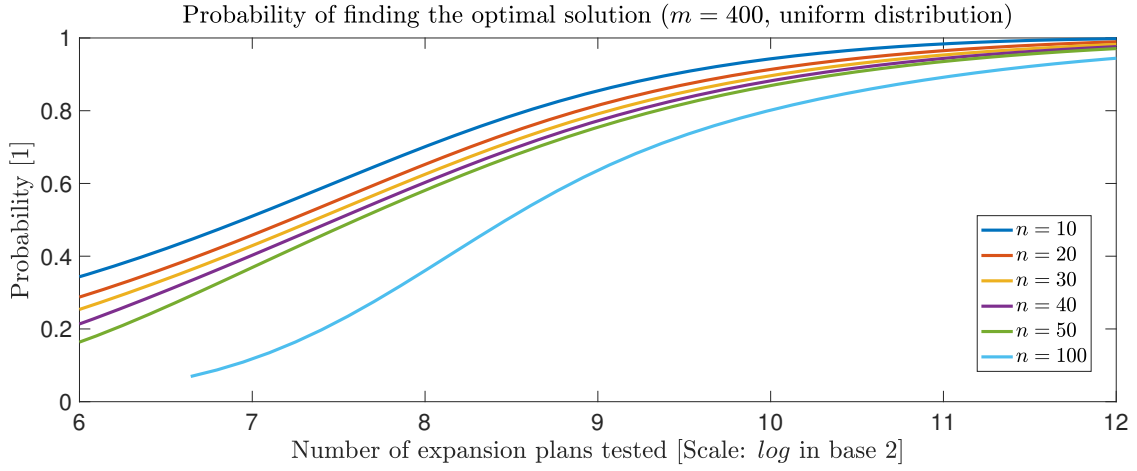


Fig. 7.9 Evolution of the probability of finding the optimal solution with the number of random expansion plans tested under hypotheses **H3-H4**.

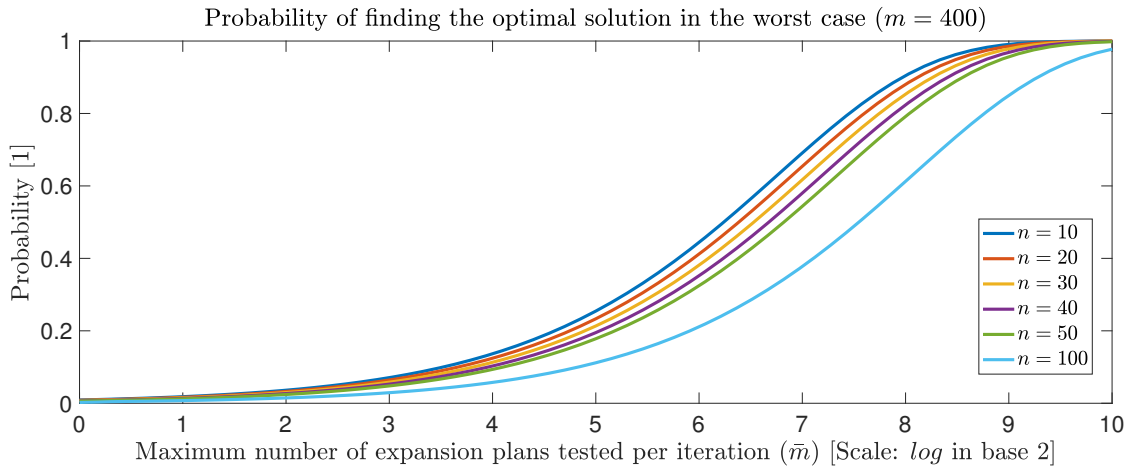


Fig. 7.10 Evolution of the probability of finding the optimal solution in the worst case with the maximum number of random expansion plans allowed to be tested per iteration under hypotheses **H3-H4**.

with n_{p^*} given a value of $\bar{m}$. However this observation does not justify why, if we consider that n_{p^*} follows a binomial or a uniform distribution, the maximum number of random plans allowed to be tested $\bar{m}$ needs to be greater for small values of n than for large ones in order to guarantee a high probability of success of the algorithm. To understand this fact we need one more information: figures 7.12 and 7.13 represent, for different values of $\bar{m}$, the evolution of the probability of success of algorithm 3 with the number of candidate lines in the optimal plan, n_{p^*} , when $n = 10$ and $n = 50$ respectively. As our intuition would have suggested, the greater $\bar{m}$, the greater the chances to find the optimal solution. The important fact is that the situation

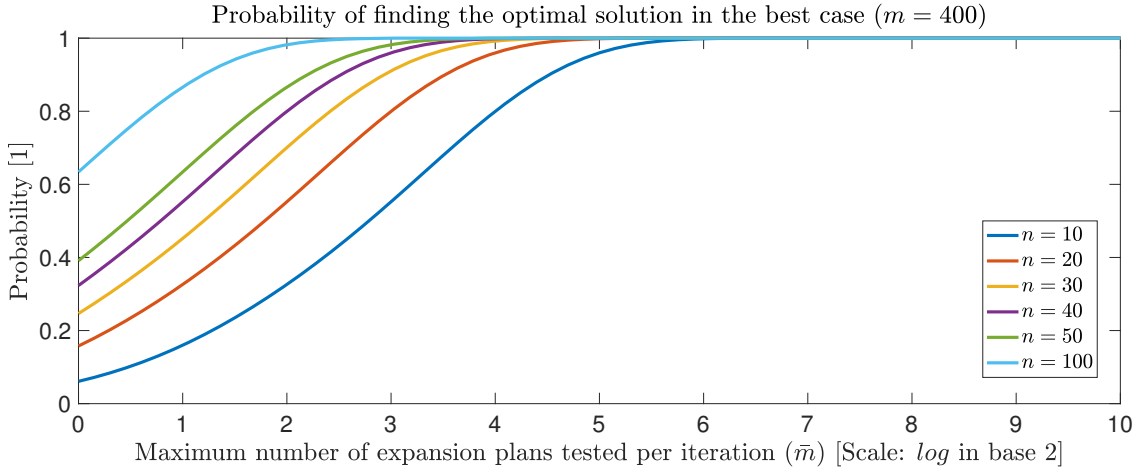


Fig. 7.11 Evolution of the probability of finding the optimal solution in the best case with the maximum number of random expansion plans allowed to be tested per iteration under hypotheses **H3-H4**.

is asymmetrical, i.e. the evolution of the probability between the best case, i.e. $n_{p^*} = 0$, and the worst case, i.e. $n_{p^*} = n - 1$, is not linear; indeed we observe that the probability of success decreases significantly only for values close to n when $\bar{m}$ is large. As a consequence, the facts that (i) the greater n , the greater the proportion of the cases when the probability of success is high, and that (ii) the uniform distribution puts the same weight on each case, justify the counterintuitive observation made previously, i.e. the maximum number of random expansion plans requested at each iteration to obtain a high probability of success (e.g. > 0.8) decreases with the total number of candidate lines n .

Hypothesis H3. Under this hypothesis it is assumed that the optimal solution is unique. If the optimal solution is not unique, different optimal sets of the same cost exist, where an optimal set is the set of all candidate lines built in an optimal solution. Considering the existence of different optimal solutions increases the probability of success of the algorithm; indeed the case of a unique solution is a restriction of this situation. If we want to schematize the convergence of the algorithm for the case where there are several optimal solutions, we should draw one convergence cone per solution, as illustrated in figure 7.14 in the case where there are 2 optimal solutions.

Hypothesis H4. From the previous sections, we have demonstrated that under hypothesis **H4** only, we could expect similar results as those presented under hypotheses **H1-H4**. Not

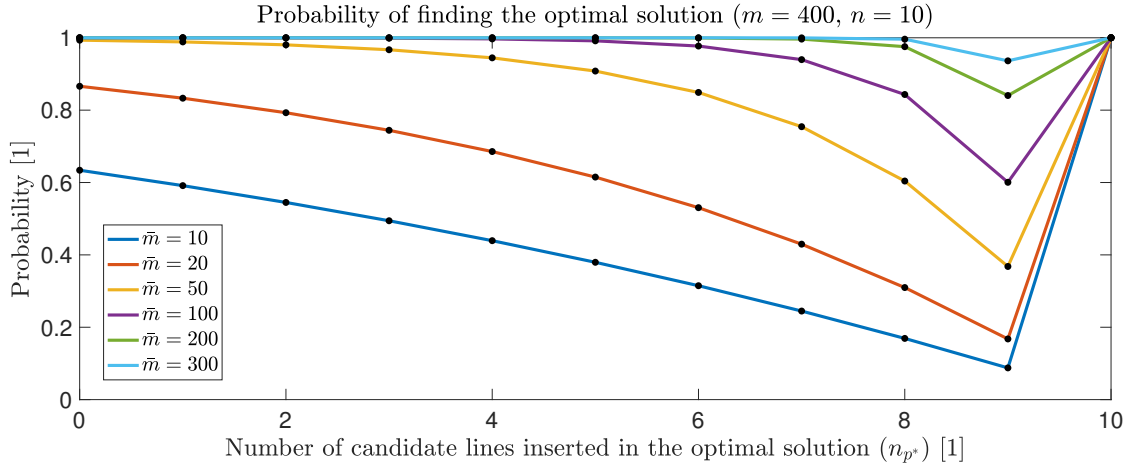


Fig. 7.12 Evolution of the probability of finding the optimal solution with the number of candidate lines built in the optimal solution under hypotheses **H3-H4** when $n = 10$.

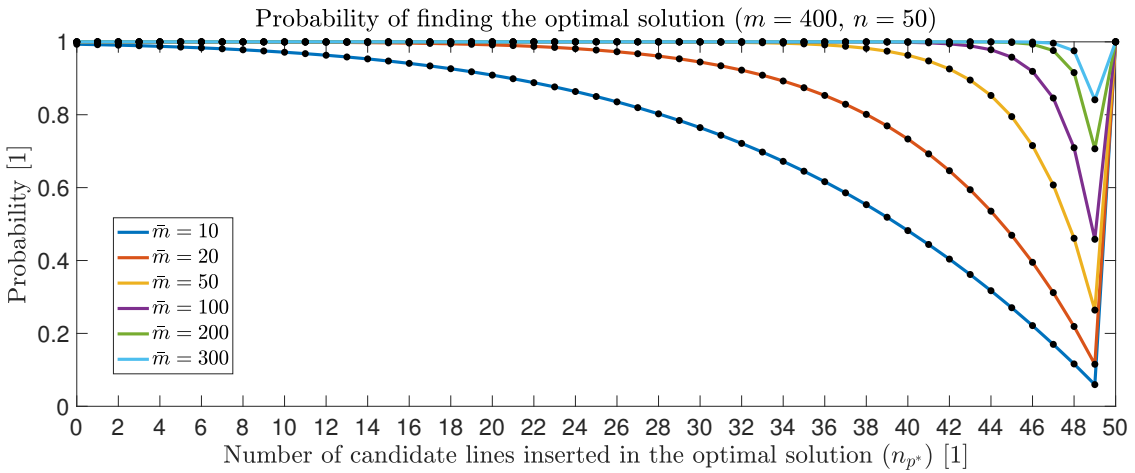


Fig. 7.13 Evolution of the probability of finding the optimal solution with the number of candidate lines built in the optimal solution under hypotheses **H3-H4** when $n = 50$.

considering hypothesis **H4** complicates tremendously the estimation of the probability of success; unfortunately we were not able to derive the probability of success.

7.3.2 Computational time estimation

Under hypothesis **H4**, we can estimate the worst case computational time using the results of table 6.2 and of figure 7.10. Table 6.2 gives the average time needed to validate the feasibility of a feasible expansion plan, which is always greater or equal than the time needed to check the potential feasibility of an infeasible expansion plan. Based on figure 7.10, we see that we

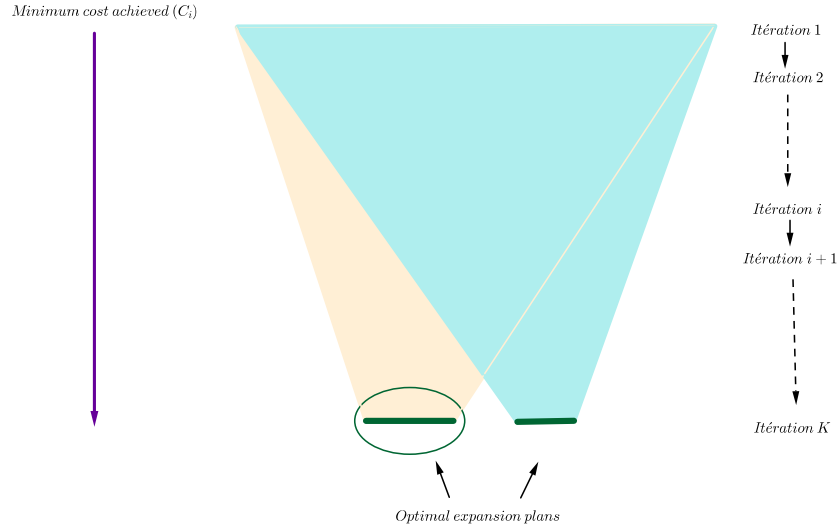


Fig. 7.14 Schematization of the convergence under hypothesis **H4** when there are 2 optimal solutions.

should take a value for $\bar{m}$ of about 1000 in order to guarantee a high probability of success ($1000 \simeq 2^{10}$). Hence we can derive upper bounds on the computational time for the IEEE-24 buses and UK power systems as presented in table 7.1.

System	UB on to time check feasibility [s]	UB on the computational time
IEEE-24 buses (A.1)	0.02	$0.02 \cdot \bar{m} \cdot n = 0.02 \cdot 1000 \cdot 28 = 560s$
UK system (A.3)	2.0	$2.0 \cdot 1000 \cdot 138 = 2.76 \cdot 10^5 s \simeq 77h$

Table 7.1 Upper bound estimate on the computational time of algorithm 2 under hypothesis **H4** given $\bar{m} = 1000$.

From table 7.1, we observe that there are three parameters that influence the upper bound estimate on the computational time of algorithm 3:

1. **The time needed to check the feasibility of an expansion plan.** In section 6.3.2, we presented the way to verify the feasibility of a plan via the resolution of a linear system and the verification of all lines's capacities and bus net-injections. The size of the matrix of the linear system is equal to $|\mathcal{N}| - 1 \times |\mathcal{N}| - 1$, $\mathcal{N}$ being the set of buses; hence the complexity of verifying the feasibility of a plan equals:

$$\mathcal{O}(|\mathcal{N}|^3) + \mathcal{O}(|\mathcal{L}_e| + |\mathcal{L}_c|) + \mathcal{O}(|\mathcal{N}|) \simeq \mathcal{O}(|\mathcal{N}|^3) + \mathcal{O}(|\mathcal{L}_e| + |\mathcal{L}_c|), \quad (7.5)$$

where $\mathcal{O}(|\mathcal{N}|^3)$, $\mathcal{O}(|\mathcal{L}_e| + |\mathcal{L}_c|)$, $\mathcal{O}(|\mathcal{N}|)$ are respectively the complexity of solving the linear system, of verifying the lines' capacities and of verifying the bus net-injections.

2. **The number of candidate lines, i.e. $|\mathcal{L}_c|$.**
3. **The maximum number of random plans allowed to be tested per iteration, $\bar{m}$.** As illustrated in figure 7.5, this value can be bounded by 2^{10} in order to guarantee a high probability of success of algorithm 3 whatever the number of candidate line n .

In conclusion, the global complexity of algorithm 3 is in

$$\mathcal{O}(|\mathcal{L}_c| \cdot |\mathcal{N}|^3) + \mathcal{O}(|\mathcal{L}_c| \cdot (|\mathcal{L}_e| + |\mathcal{L}_c|)), \quad (7.6)$$

which is a polynomial complexity. This justifies why our probabilistic approach performs better than the randomized algorithm (see section 4.2.1), which has an exponential complexity.

Finally, let us mention that the upper bounds derived in table 7.1 are obtained in the case of sequential computation; however parallel processing can be applied on algorithm 3, which empirically leads to a gain of a factor 3 on the computational time.

Part III

Analysis of the performances & conclusion

Analysis of the performances

In this chapter, we test the different methods presented in part II on three power systems of different size: (i) a *small* power system, the IEEE-24 buses power system A.1, which has 28 candidates lines; (ii) a *mid-size* power system, the RTS power system A.2, which has 60 candidates; and (iii) a large power system, the UK power system A.3, that has 138 candidates lines. We will compare the performances of the following methods:

- The solver method where load curtailment is not allowed (see section 6.1).
- The probabilistic method.
- The solver method where an upper bound on the optimal cost is given, e.g. the optimal cost returned by the probabilistic method.

All computational times reported in the next sections are obtained on a Mac Pro dating from mid-2010 which has two 2.66 GHz 6-Core Intel Xeon processors, 48 GB 1333 MHz DDR3 ECC of memory and which runs under OS X 10.9.5. Any simulation exceeding two days has been interrupted and is reported by a "-".

8.1 Solver method

Table 8.1 presents the time needed by Gurobi to solve the TEP problem when no load curtailment is tolerated. From this table, we can observe the rapid growth in the computational time required to solve the TEP problem with the number of candidate lines in the power system. Indeed the RTS power system has more or less twice the number of candidate lines as the IEEE-24 buses power system but requests around 11.25 times its computational time in order to reach the optimal solution. For the UK system, it is even worse: this power system is five times larger than the IEEE-24 buses power system in terms of candidate lines but Gurobi is not able to find the optimal solution (or even any feasible solution) in $2 \cdot 10^5$ seconds, which is already 712 times greater than the computational time required for the IEEE-24 buses power system. Finally, let us mention that Gurobi takes full advantage of parallel processing, so the results reported are comparable to those obtained via the parallel version of the probabilistic method reported in the next section 8.2.

System	Computational time
IEEE-24 buses	281.31 s
RTS system	3161.77 s
UK system	- s

Table 8.1 Computational time needed by Gurobi to solve the TEP problem [s].

8.2 Probabilistic method

In this section, we comment the results given by the probabilistic algorithm 3 in its parallelized version¹. An example of the output of this method is given in appendix B, where the probabilistic algorithm 3 is launched on the RTS power system. Since we obtained the optimal solution of only two power systems out of the three tested, we will split the convergence analysis of the probabilistic algorithm 3 in two: (i) first we focus on the two smaller power systems, the IEEE-24 buses and the RTS power systems, and make some correspondences with the theoretical analysis led in 7.3; (ii) we present the results obtained for the UK power system.

In the results presented hereafter in table 8.2, the mean values reported are computed based on 100 realizations in average, i.e. for each set of parameters, the probabilistic algorithm

¹Not being an expert in computer science, I used the parallel computing only while generating and verifying the feasibility of random plans with the help of the following architecture available in Julia *@parallel (a function) for*. More information on how to use parallel computing in Julia are available [here](#).

has been launched around 100 times. Unfortunately, it was not possible to make as much realizations on the larger power system since, for interesting sets of parameters, the algorithm 3 requires more than 19 hours to converge.

8.2.1 Analysis of the performances of the probabilistic method on the IEEE-24 buses and RTS power systems

IEEE-24 buses power system							
Parameters	C1	C2	C3	C4	C5	C6	C7
$m = 400, \beta = 0.99, \bar{m} = 16$	1.18 s	82.83%	939.34 k€	0.83%	5470 k€	4.82%	9.16
$m = 50, \beta = 0.99, \bar{m} = 1000$	1.84 s	98.99%	29.29 k€	0.00%	2900 k€	2.55%	8.18
$m = 400, \beta = 0.99, \bar{m} = 1000$	6.17 s	100.00%	0.00 k€	0.00%	0 k€	0.00%	7.06
$m = 400, \beta = 0.99, \bar{m} = 50000$	39.07 s	100.00%	0.00 k€	0.00%	0 k€	0.00%	7.00
$m = 400, \beta = 0.95, \bar{m} = 1000$	5.67 s	100.00%	0.00 k€	0.00%	0 k€	0.00%	6.40

RTS power system							
Parameters	C1	C2	C3	C4	C5	C6	C7
$m = 400, \beta = 0.99, \bar{m} = 16$	4.98 s	88.89%	0.51 M€	2.45%	4.58 M€	22.07%	16.73
$m = 50, \beta = 0.99, \bar{m} = 1000$	8.96 s	99.00%	0.05 M€	0.23%	4.74 M€	22.81%	15.23
$m = 400, \beta = 0.99, \bar{m} = 1000$	30.85 s	100.00%	0.00 M€	0.00%	0.00 M€	0.00%	12.51
$m = 400, \beta = 0.99, \bar{m} = 50000$	213.53 s	100.00%	0.00 M€	0.00%	0.00 M€	0.00%	12.62
$m = 400, \beta = 0.95, \bar{m} = 1000$	31.45 s	100.00%	0.00 M€	0.00%	0.00 M€	0.00%	11.32

Table 8.2 Performances of the probabilistic method on the IEEE-24 buses and RTS power systems. **C1:** computational time, **C2:** percentage of success, **C3:** mean gap, **C4:** mean relative gap, **C5:** mean gap when the optimal solution is not found, **C6:** mean relative gap when the optimal solution is not found, **C7:** mean number of iterations.

Table 8.2 presents the results of the probabilistic method for the IEEE-24 buses and RTS power systems. These results fit particularly well with the intuition we developed in the theoretical analysis:

- **Parameter $\bar{m}$.** As predicted in the theoretical analysis, the more random expansion plans allowed to be tested per iteration in order to get the m feasible ones required to move to the next iteration, the greater (i) the probability of finding the optimal solution and (ii) the computational time.
- **Parameter m .** This parameter is a key factor in the computation of vector *proba*, which contains the estimation of the probability of insertion of each candidate line in a feasible expansion plan cheaper than the best cost achieved so far by the algorithm. In figure 7.3, we had justified that taking a value of $m = 400$ was a good trade-off between (i) a good

estimation of the probability of insertion of each candidate line and **(ii)** the computational time of each iteration. As for the previous parameter, we observe from table 8.2 that the greater m , the greater the **(i)** computational time and **(ii)** the probability of success of the algorithm². Obviously, this can be justified by the fact that the greater m , the better the estimation of the probabilities of insertion, the easier for the algorithm to find feasible solutions, and thus the less probable for the algorithm to interrupt its search prematurely. Moreover, this argument justifies also the fact that the greater m , the fewer iterations required for the algorithm to converge. Indeed, at each iteration, the greater m , the more opportunities to improve the best cost achieved so far and thus the larger the difference between the best costs achieved between two iterations.

Finally, while comparing the results of the first two sets of parameters of table 8.2, we observe that the probability of success of the RTS power system is larger than for IEEE-24 buses power system. This could be surprising since the RTS power system has more candidate lines than the IEEE-24 buses power system, but this can be explained in two ways. First, these results validate the counterintuitive observation made in figure 7.5, which was that, given the maximum number of random expansion plans allowed to be tested per iteration $\bar{m}$, the probability of success increases with the total number of candidate lines n in the power system. Second, from Gurobi one knows that the optimal solution of the transmission expansion planning problem of both systems includes 14 candidate lines and, as observed in figures 7.12 and 7.13, the lower the percentage of candidate lines inserted in the optimal plan, the greater the probability of success of algorithm 3. Hence, since the IEEE-24 buses power system has only 28 candidate lines whereas the RTS power system has 60 candidate lines, the difference in the percentage of candidate lines built (50% versus 23.33%) could explain the difference in the success rate of algorithm 3.

- **Parameter β .** This parameter represents the maximum probability allowed in vector *proba*, which is the maximum probability for a candidate line to be inserted in a random expansion plan. In section 7.2.2, we had mentioned the following trade-off: the lower β , the lower the chances to find feasible solutions if vector *proba* is a good approximation of the probabilities of insertion, but the higher the probability of obtaining feasible solutions with low costs and thus the fewer iterations required. In table 8.2, the number of iterations required is indeed reduced for both power systems when we consider $\beta = 0.95$

²The first line of table 8.2 corresponds to the case where $m = 400$ and $\bar{m} = 16$. However, from the definition of the parameters, we know that in reality we have $m \leq \bar{m}$, since $\bar{m}$ is the maximum number of random plans allowed to be tested per iteration. As a consequence, we should look to the first line of table 8.2 as if $m = 16$.

instead of $\beta = 0.99$. However, we see that the impact on the computational time is more mitigated since for one power system it reduces the computational time but not for the other. This is explained by the fact that, although it reduces the total number of iterations, it complicates the obtaining of m feasible plans and thus it increases the computational time per iteration to an extent such that the benefit of lowering the number of iterations is abated.

In conclusion, table 8.2 clearly presents the benefits of approaching the TEP problem with the probabilistic approach instead of the solver method. Indeed, taking the following set of parameters $m = 400$, $\beta = 0.99$ and $\bar{m} = 1000$, leads to a very high probability of finding the optimal solution while requiring less than $\frac{1}{45}^{th}$ the computational effort of the solver method.

8.2.2 Probabilistic method on the UK power systems

As mentioned previously, it is though to comment the performances of the probabilistic algorithm on this power system since we do not know the optimal solution. Moreover, since one simulation requires a large computational time, we were not able to make a large number of tests in order to obtain meaningful statistical values.

Realization #	Parameters	C1	C2	C3
1	$m = 400, \beta = 0.99, \bar{m} = 1000$	1.32 h	159.18 M€	21
2	$m = 400, \beta = 0.99, \bar{m} = 50000$	19.19 h	149.88 M€	19
3	$m = 400, \beta = 0.99, \bar{m} = 50000$	23.63 h	149.70 M€	19
4	$m = 400, \beta = 0.99, \bar{m} = 50000$	28.16 h	148.19 M€	22
5	$m = 400, \beta = 0.99, \bar{m} = 100000$	42.27 h	149.70 M€	22
6	$m = 400, \beta = 0.95, \bar{m} = 100000$	36.52 h	149.88 M€	18

Table 8.3 Performances of the probabilistic method on the IEEE-24 buses and RTS power systems. **C1**: computational time, **C2**: best cost achieved, **C3**: number of iterations.

Using table 8.3 we verify also the few points explained in the previous section 8.2.1: (i) the greater $\bar{m}$, the greater the computational time, (ii) the lower β , the fewer iterations required by algorithm 3 to converge.

Even though we cannot estimate correctly the gap, we can observe that the values obtained with reasonable parameters ($\bar{m} \geq 50000$) are close from each other, i.e. the largest relative difference equals $\frac{149.88 - 148.19}{148.19} = 1.14\%$, which is relatively good since the first feasible solutions that the algorithm finds have a cost around 250 M€, which corresponds to a relative gap of 68.70%.

Finally, we should compare those performances with the solver method. Indeed after 58 hours, Gurobi has been unable to provide any feasible solution, whereas the probabilistic method is able to find a non-trivial solution in less than 2 hours and a more sophisticated one in 20 hours; there is thus a significant advantage in using the probabilistic method in order than the solver method if we wish to have *rapidly* an estimation on the cost of the optimal expansion plan.

8.3 Solver method with an upper bound

Table 8.4 presents the time needed by Gurobi to solve the TEP problem when no load curtailment is tolerated and when the investment cost of the optimal solution obtained with the probabilistic method is given as upper bound on the objective function. From this table, we can observe there is not a meaningful difference between obtaining the optimal solution as in 8.1 and certifying the optimality of an optimal solution.

System	Computational time
IEEE-24 buses	245.40 s
RTS system	3575.06 s
UK system	- s

Table 8.4 Computational time needed to certify the optimality of the optimal solution of the TEP problem [s].

8.4 Lower and upper bounds estimation on the optimal cost

In the description of algorithm 3, we mentioned that the upper bound at the end of each iteration was the best cost achieved, whereas the lower bound was estimated at the end of each iteration by $Cost_{Built}$, which is the total investment costs of building all candidate lines that are built in all feasible expansion plans found during the iteration. Figures 8.1, 8.2 and 8.3 show the evaluation of these bounds for a realization of algorithm 3 on the IEEE-24 buses, the RTS and the UK power systems respectively.

From a theoretical point of view, the upper bound estimation is always correct, whereas, under hypothesis **H1-H4** (see section 4.2.2), the lower bound is a probabilistic lower bound; the more the number of feasible expansion plans required per iteration (parameter m), the more confident we can be in the lower bound estimation. Of course, if hypotheses **H1-H4** are not

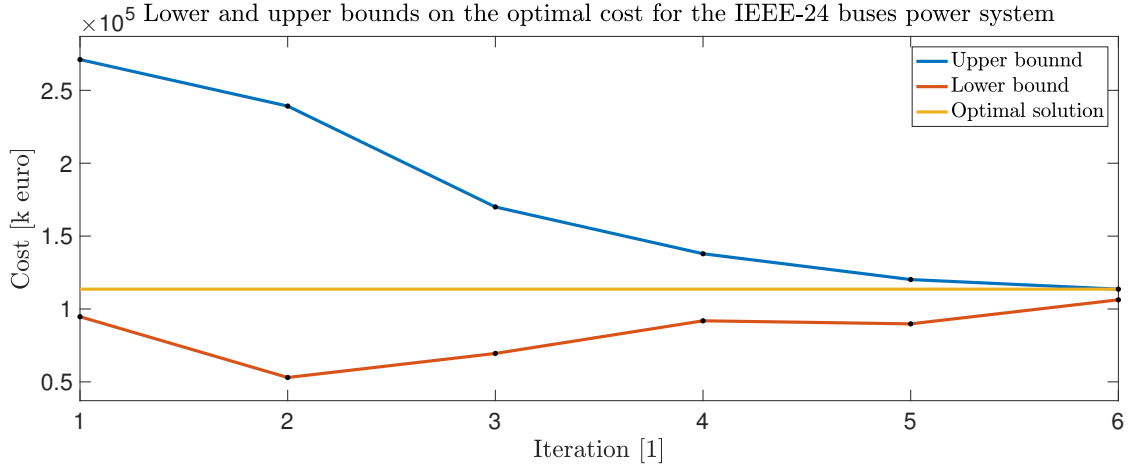


Fig. 8.1 Lower and upper bound estimation on the optimal cost for the IEEE-24 buses power system (Probabilistic algorithm).

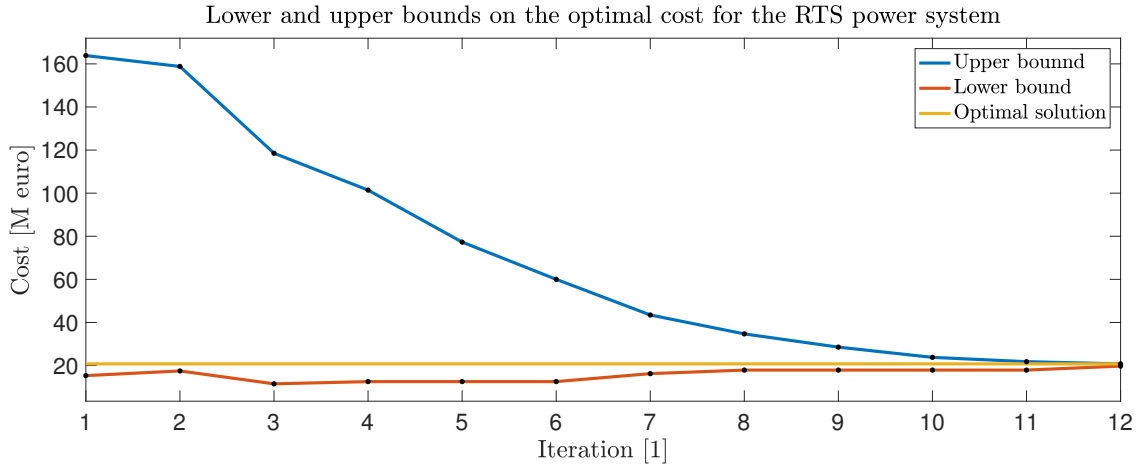


Fig. 8.2 Lower and upper bound estimation of the optimal cost for the RTS power system (Probabilistic algorithm).

considered, the lower bound should be taken with a grain of salt. For the two smaller system, we can assume, based on the results presented in table 8.2, that the systems respect quite well hypotheses **H1-H4**; as a consequence, the lower bound estimation are accurate for large values of m , and as we can observe on figures 8.1 and 8.2, we never get a lower bound greater than the optimal cost, which would be embarrassing. For the UK power system, the situation is not as good; indeed for certain iterations the given lower bound is erroneous since the algorithm finds solutions lower than those bounds in the last few iterations.

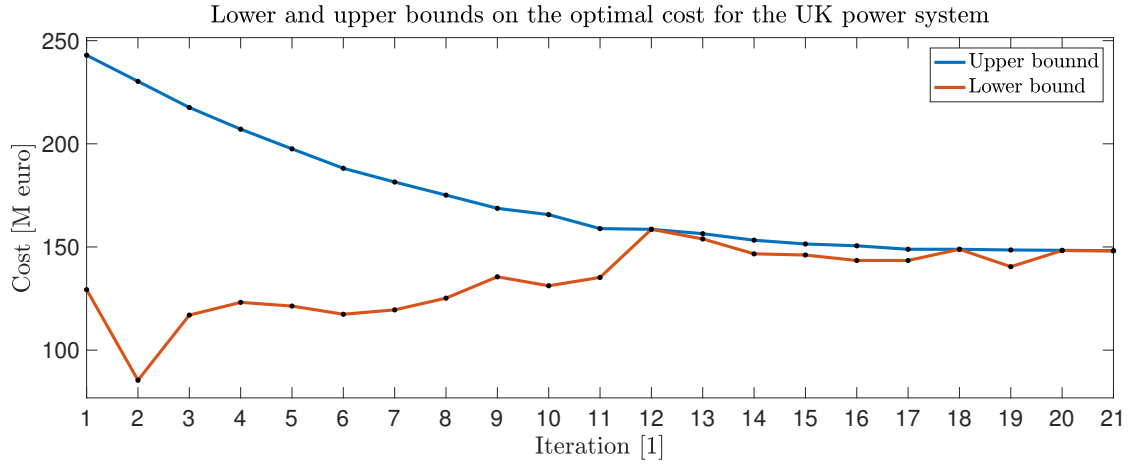


Fig. 8.3 Lower and upper bound estimation of the optimal cost for the UK power system (Probabilistic algorithm).

Figures 8.4 and 8.5 present the evolution of the lower and upper bound obtained with Gurobi. Gurobi does not return any upper bound for the first iterations, hence the values represented (all the constant part at the beginning until the first decrease) are values given by ourselves. From those figures, we can conclude that Gurobi concentrates its work on getting tighter lower bounds; however, compared with the probabilistic method, the convergence of the lower bound to the optimal solution is slower but smoother. The major drawback of using Gurobi is that it returns interesting feasible solutions only at the end of its computation; hence we have to wait a long time before getting any feasible solution, which, as for the UK power system, can lead to obtaining no solution at all in a reasonable amount of time, when the power system is too large.

Finally, in section 6.2, we have argued that we would not implement the Benders decomposition method since the convergence of the lower bound to the optimal solution behaves badly. To illustrate this, let us consider figure 8.6, issued from Ms. Hemmer master's thesis [30], which represents the convergence of the Benders decomposition method proposed by S. Binato in [10] and of other methods derived from this one and supposed to improve the convergence. In this figure, we observe two drawbacks of the methods: **(i)** first the upper bound remains more or less constant and **(ii)** the lower bound increases a lot in the first iterations and then stagnates in its evolution; there is thus little hope that the algorithm converges in a reasonable amount of time or that the algorithm provides interesting lower bounds.

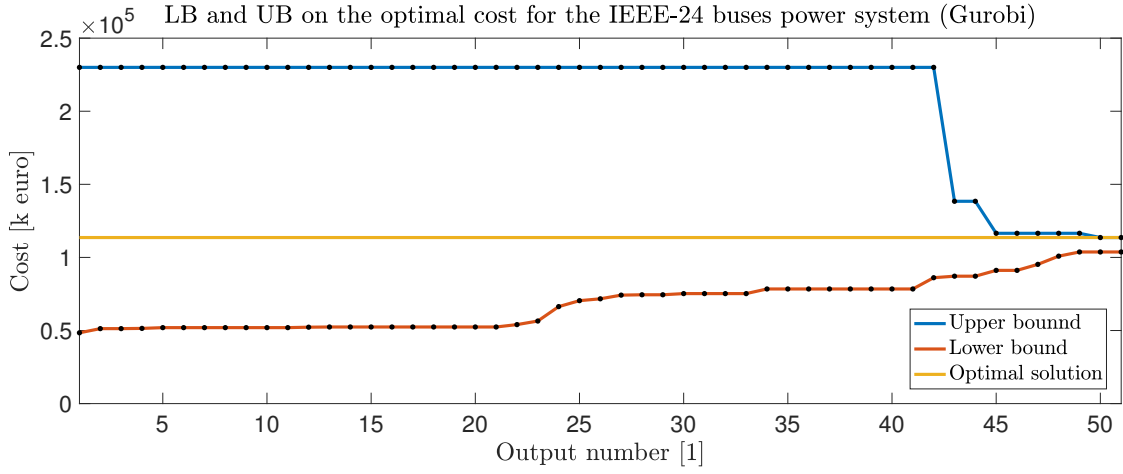


Fig. 8.4 Lower and upper bound estimation on the optimal cost for the IEEE-24 buses power system (Gurobi).

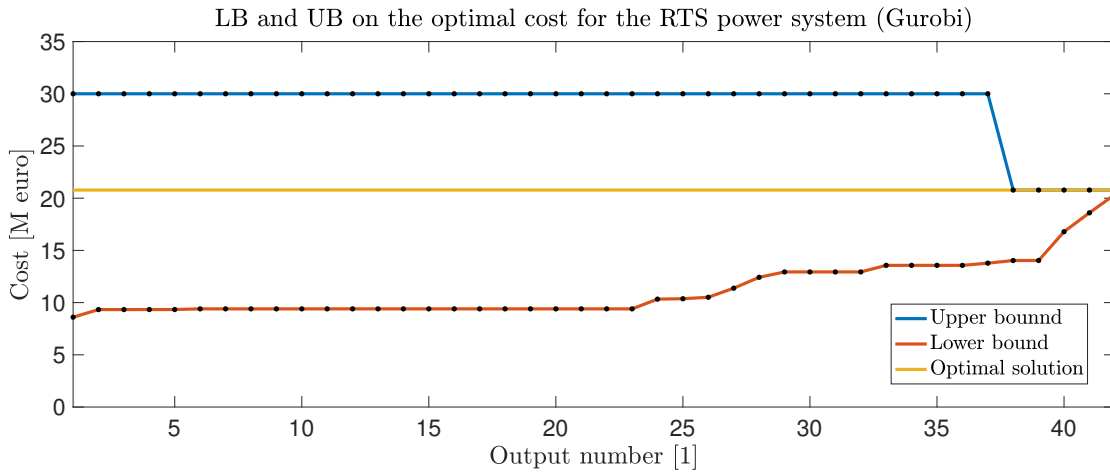


Fig. 8.5 Lower and upper bound estimation of the optimal cost for the RTS power system (Gurobi).

In conclusion, the validity of hypotheses **H1-H4** is requested in order to obtain meaningful lower bounds; since these hypotheses are not valid for all power systems, the lower bound returned by the probabilistic algorithm should be taken in consideration carefully. As mentioned at the end of the previous section, the huge advantage of the probabilistic algorithm is that it improves the upper bound at each iteration, whereas the solver method does it only at the end of its convergence; as a consequence using the probabilistic algorithm allows us to rapidly have an idea on the order of magnitude of the optimal cost. This is even more true regarding the

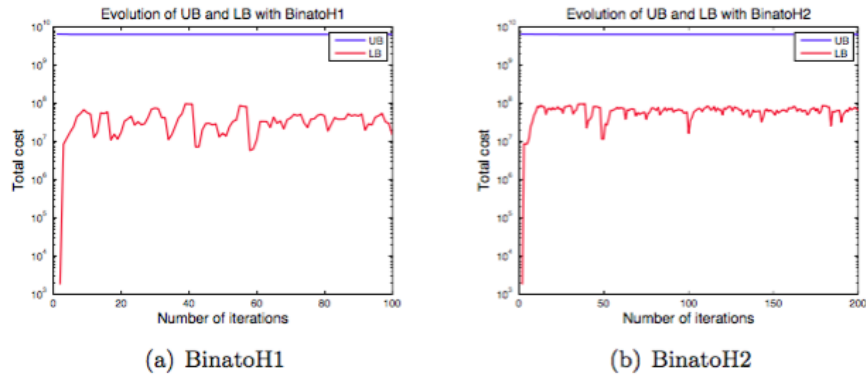


Figure 5.8: UB and LB with BinatoH1 and BinatoH2 for Irish network

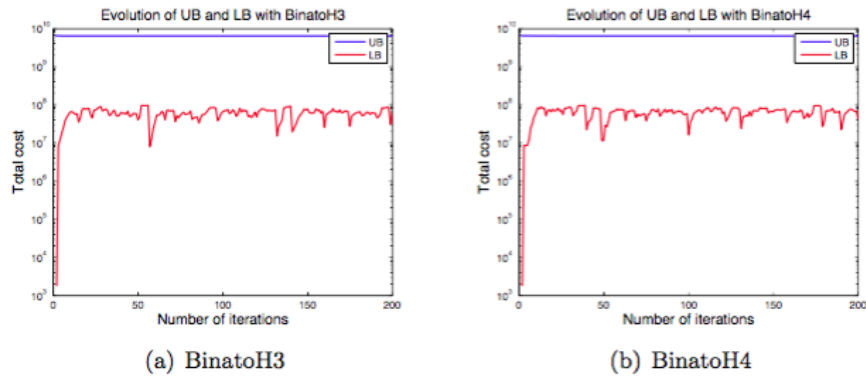


Fig. 8.6 Illustration of the convergence of the Benders decomposition method proposed by S. Binato [10] and other methods ameliorating the convergence.

shape of the upper bound: it first decreases quite steeply and then improves the best solution by little steps at the end of the convergence.

Conclusion

In this master's thesis, we have investigated different ways to formulate and solve the transmission expansion planning (TEP) problem. A literature review, as exhaustive as possible, was conducted and led us to the conclusion that mathematical programming methods were not the best option to solve the TEP problem when the number of candidate lines is large.

As a consequence, we have switched our approach and decided to look to heuristics. A multitude of heuristics have been proposed in literature to solve the TEP problems, such as genetic algorithms, simulated annealing, etc. Even though those heuristics behave well on certain types of power systems, they fail in being robust and in providing proofs of their convergence. Therefore, we decided to approach the TEP problem via randomization; based on intuitive hypotheses, we designed the *probabilistic algorithm*. The theoretical convergence of this algorithm was proven under the hypotheses considered; tests on real power systems have then revealed that the *probabilistic algorithm* has, for small and medium systems, a reasonable probability of finding the optimal expansion plan, while requiring a limited computational time.

Large power systems tend to respect to a lesser extent the hypotheses on which the *probabilistic algorithm* relies; the convergence results are thus more mitigated since the optimality of the returned solution is not guaranteed. However, we observe that the maximum difference in the solutions returned by the *probabilistic algorithm* is small enough, typically of 1.5%, which could lead us to think that we are close to the optimum. The major advantage of the method proposed is that the algorithm returns a relatively good solution, whereas a commercial solver does not return any solution at all, in this case of large power systems. Hence, the *probabilistic algorithm* could be a useful tool for transmission system operators in order to get a precise idea of the investment cost of the optimal expansion plan.

Different possibilities could be investigated in the future in order to improve the proposed method. First, one could investigate in what proportion the hypotheses made are respected in real power systems; based on this, one should be able to estimate the confidence in the best solution returned by the *probabilistic algorithm*. Second, one could examine whether it is possible to prove any convergence of the *probabilistic algorithm* when **H4** is not satisfied (cfr. section 7.3.1). Finally, once the *probabilistic algorithm* has converged, one could apply a local search, as proposed in [58], in order to diversify the search strategy and to improve the global convergence of the method.

Appendices

A.1 IEEE-24 buses system

The IEEE-24 buses power system we consider in this master's thesis is composed of 24 buses, 38 existing lines (table A.1) and 28 candidate lines (table A.2). The candidate lines were selected in order to have the possibility to double all single-circuit branches; to assign to cost of each candidate line, we made the following assumption: for a 138-kV line the cost amounts 300 k€ per km, for a 230-kV line the cost amounts 400 k€ per km and for a 138/230-kV transformer the cost amounts 2.1 M€¹.

As mentioned in the model part I, the optimal transmission network should be able to manage different scenarios representing worst cases. For this system, we consider 4 scenarios illustrated in tables A.3, which represent four extreme cases of the economic dispatch: peak load and high/low wind and off-peak load and high/low wind. The rationale is that, if the grid can withstand all extreme load conditions, it should withstand the intermediate load conditions.

The excel file containing the data is available in the following [link](#).

A.2 RTS system

The RTS power system is composed of 73 buses, 120 existing lines and 60 candidate lines. The excel file containing the data is available at the following [link](#). Note that the costs assigned to the candidate lines have no true meaning and have been arbitrarily chosen.

¹Lines length are available at the following [website](#).

A.3 UK system

The UK power system is composed of 548 buses, 899 existing lines and 138 candidate lines. The excel file containing the data is available at the following [link](#). Note that the costs assigned to the candidate lines have no true meaning and have been arbitrarily chosen.

Line name	Initial bus	End bus	Reactance [p.u.]	Capacity [MW]
L1	Bus 1	Bus 2	0,014	175
L2	Bus 1	Bus 3	0,211	175
L3	Bus 1	Bus 5	0,085	175
L4	Bus 2	Bus 4	0,127	175
L5	Bus 2	Bus 6	0,192	175
L6	Bus 3	Bus 9	0,119	175
L7	Bus 3	Bus 24	0,084	400
L8	Bus 4	Bus 9	0,104	175
L9	Bus 5	Bus 10	0,088	175
L10	Bus 6	Bus 10	0,061	175
L11	Bus 7	Bus 8	0,061	175
L12	Bus 8	Bus 9	0,165	175
L13	Bus 8	Bus 10	0,165	175
L14	Bus 9	Bus 11	0,084	400
L15	Bus 9	Bus 12	0,084	400
L16	Bus 10	Bus 11	0,084	400
L17	Bus 10	Bus 12	0,084	400
L18	Bus 11	Bus 13	0,048	500
L19	Bus 11	Bus 14	0,042	500
L20	Bus 12	Bus 13	0,048	500
L21	Bus 12	Bus 23	0,097	500
L22	Bus 13	Bus 23	0,087	500
L23	Bus 14	Bus 16	0,039	500
L24	Bus 15	Bus 16	0,017	500
L25	Bus 15	Bus 21	0,049	500
L26	Bus 15	Bus 21	0,049	500
L27	Bus 15	Bus 24	0,052	500
L28	Bus 16	Bus 17	0,026	500
L29	Bus 16	Bus 19	0,023	500
L30	Bus 17	Bus 18	0,014	500
L31	Bus 17	Bus 22	0,105	500
L32	Bus 18	Bus 21	0,026	500
L33	Bus 18	Bus 21	0,026	500
L34	Bus 19	Bus 20	0,040	500
L35	Bus 19	Bus 20	0,040	500
L36	Bus 20	Bus 23	0,022	500
L37	Bus 20	Bus 23	0,022	500
L38	Bus 21	Bus 22	0,068	500

Table A.1 Information about the existing lines in the IEEE-24 buses system.

Line name	Initial bus	End bus	Reactance [p.u.]	Capacity [MW]	Investment cost [k€]
C1	BUS1	BUS2	0,0139	175	900
C2	BUS1	BUS3	0,2112	175	16500
C3	BUS1	BUS5	0,0845	175	6600
C4	BUS2	BUS4	0,1267	175	9900
C5	BUS2	BUS6	0,192	175	15000
C6	BUS3	BUS9	0,119	175	9300
C7	BUS3	BUS24	0,0839	400	2100
C8	BUS4	BUS9	0,1037	175	8100
C9	BUS6	BUS10	0,0605	175	4800
C10	BUS7	BUS8	0,0614	175	4800
C11	BUS8	BUS9	0,1651	175	12900
C12	BUS8	BUS10	0,1651	175	12900
C13	BUS11	BUS13	0,0476	500	13200
C14	BUS11	BUS14	0,0418	500	11600
C15	BUS12	BUS13	0,0476	500	13200
C16	BUS12	BUS23	0,0966	500	26800
C17	BUS13	BUS23	0,0865	500	24000
C18	BUS14	BUS16	0,0389	500	10800
C19	BUS15	BUS16	0,0173	500	4800
C20	BUS15	BUS24	0,0519	500	14400
C21	BUS16	BUS17	0,0259	500	7200
C22	BUS16	BUS19	0,0231	500	6400
C23	BUS17	BUS18	0,0144	500	4000
C24	BUS17	BUS22	0,1053	500	29200
C25	BUS21	BUS22	0,0678	500	18800
C26	BUS14	BUS16	0,0389	500	10800
C27	BUS16	BUS17	0,0259	500	7200
C28	BUS17	BUS18	0,0144	500	4000

Table A.2 Information about the candidate lines in the IEEE-24 buses system.

Bus name	SC1: P [MW]	SC1: D [MW]	SC2: P [MW]	SC2: D [MW]	SC3: P [MW]	SC3: D [MW]	SC4: P [MW]	SC4: D [MW]
BUS1	108	0	108	0	54	0	54	0
BUS2	97	0	97	0	48,5	0	48,5	0
BUS3	180	0	180	150	90	0	90	150
BUS4	74	0	74	150	37	0	37	150
BUS5	71	0	71	150	35,5	0	35,5	150
BUS6	136	0	136	150	68	0	68	150
BUS7	125	0	125	0	62,5	0	62,5	0
BUS8	171	0	171	150	85,5	0	85,5	150
BUS9	175	0	175	150	87,5	0	87,5	150
BUS10	195	0	195	150	97,5	0	97,5	150
BUS11	0	0	0	0	0	0	0	0
BUS12	0	0	0	0	0	0	0	0
BUS13	265	0	265	0	132,5	0	132,5	0
BUS14	194	0	194	0	97	0	97	0
BUS15	317	155	317	0	158,5	0	158,5	0
BUS16	100	155	100	0	50	0	50	0
BUS17	0	0	0	0	0	0	0	0
BUS18	333	800	333	700	166,5	325	166,5	0
BUS19	181	0	181	0	90,5	0	90,5	0
BUS20	128	0	128	0	64	0	64	0
BUS21	0	800	0	800	0	800	0	75
BUS22	0	300	0	300	0	300	0	300
BUS23	0	640	0	0	0	0	0	0
BUS24	0	0	0	0	0	0	0	0

Table A.3 Information about the scenarios of the IEEE-24 buses system (SC $\Leftrightarrow$ Scenario, P $\Leftrightarrow$ Production, D $\Leftrightarrow$ Demand).



Example of outputs

```
=====
===== Begin algorithm =====
=====
This instance is generated with random number: 770526
-----
-> Iteration: 1
Simulation of random instances ...
Number of feasible realisations requested: 400
* Update: Best plan until now: 310200.0
* Update: Best plan until now: 288600.0
* Update: Best plan until now: 286200.0
* Update: Best plan until now: 281000.0
Iteration information
Time needed to complete iteration: 0.94 sec
Number of random expansion plans tested: 439
Number of feasible expansion plans obtained: 400/400.
** Absolute gap estimation: 186300.0
** Relative gap estimation: 66.299
Number of candidate lines without any idea about the decision: 6
Lines probably built: C2 C4 C6 C7 C9 C10 C11 C12 C13 C14 C16 C17 C18 C19 C20 C21
C23 C24 C25 C26 C27 C28
Lines probably not built:
-----
-> Iteration: 2
Simulation of random instances ...
```

Number of feasible realisations requested: 400

* Update: Best plan until now: 272900.0

* Update: Best plan until now: 274600.0

* Update: Best plan until now: 268100.0

* Update: Best plan until now: 262200.0

* Update: Best plan until now: 257000.0

* Update: Best plan until now: 254200.0

* Update: Best plan until now: 251400.0

* Update: Best plan until now: 244400.0

* Update: Best plan until now: 239200.0

Iteration information

Time needed to complete iteration: 1.02 sec

Number of random expansion plans tested: 835

Number of feasible expansion plans obtained: 400/400.

** Absolute gap estimation: 186200.0

** Relative gap estimation: 77.843

Number of candidate lines without any idea about the decision: 17

Lines probably built: C2 C7 C10 C11 C18 C20 C21 C23 C26 C27 C28

Lines probably not built:

-> Iteration: 3

Simulation of random instances ...

Number of feasible realisations requested: 400

* Update: Best plan until now: 192500.0

* Update: Best plan until now: 182000.0

Iteration information

Time needed to complete iteration: 0.93 sec

Number of random expansion plans tested: 576

Number of feasible expansion plans obtained: 400/400.

** Absolute gap estimation: 118200.0

** Relative gap estimation: 64.945

Number of candidate lines without any idea about the decision: 17

Lines probably built: C2 C7 C10 C11 C18 C20 C21 C23 C26 C27 C28

Lines probably not built:

-> Iteration: 4

Simulation of random instances ...

Number of feasible realisations requested: 400

* Update: Best plan until now: 168500.0

* Update: Best plan until now: 165200.0

* Update: Best plan until now: 164000.0

* Update: Best plan until now: 157400.0

* Update: Best plan until now: 154400.0

* Update: Best plan until now: 153500.0

* Update: Best plan until now: 145700.0

Iteration information

Time needed to complete iteration: 0.98 sec

Number of random expansion plans tested: 730

Number of feasible expansion plans obtained: 400/400.

** Absolute gap estimation: 39400.0

** Relative gap estimation: 27.042

Number of candidate lines without any idea about the decision: 15

Lines probably built: C2 C7 C10 C11 C14 C18 C20 C21 C23 C26 C27 C28

Lines probably not built: C24

→ Iteration: 5

Simulation of random instances ...

Number of feasible realisations requested: 400

* Update: Best plan until now: 142700.0

* Update: Best plan until now: 136400.0

* Update: Best plan until now: 134200.0

* Update: Best plan until now: 132500.0

* Update: Best plan until now: 131300.0

* Update: Best plan until now: 123200.0

Iteration information

Time needed to complete iteration: 0.79 sec

Number of random expansion plans tested: 663

Number of feasible expansion plans obtained: 400/400.

** Absolute gap estimation: 33400.0

** Relative gap estimation: 27.11

Number of candidate lines without any idea about the decision: 12

Lines probably built: C2 C7 C10 C11 C14 C18 C20 C21 C23 C26 C27 C28

Lines probably not built: C16 C17 C24 C25

-> Iteration: 6

Simulation of random instances ...

Number of feasible realisations requested: 400

* Update: Best plan until now: 122900.0

* Update: Best plan until now: 116500.0

* Update: Best plan until now: 113600.0

Iteration information

Time needed to complete iteration: 1.07 sec

Number of random expansion plans tested: 1105

Number of feasible expansion plans obtained: 371/400.

-> The limit on the number of random expansion plans allowed to simulate is reached.

Moving to the next iteration.

** Absolute gap estimation: 7300.0

** Relative gap estimation: 6.426

Number of candidate lines without any idea about the decision: 7

Lines probably built: C2 C7 C10 C11 C14 C18 C20 C21 C23 C26 C27 C28

Lines probably not built: C4 C5 C12 C13 C15 C16 C17 C24 C25

-> Iteration: 7

Simulation of random instances ...

Number of feasible realisations requested: 400

Iteration information

Time needed to complete iteration: 0.76 sec

Number of random expansion plans tested: 1000

Best cost achieved: 113600.0

This iteration has reached the maximum amount of expansion plans allowed to be tested and could not find a better solution.

Stopping criterion reached: could not get a better solution than the one found in the previous iteration.

```
=====
===== End algorithm =====
=====
```

Minimum investment cost found: 113600.0

Time needed to stop the algorithm: 6.484247569

Best transmission expansion plan: C1 C2 C7 C10 C11 C14 C18 C20 C21 C22 C23 C26
C27 C28



Implementation

The implementation of the methods studied in the performance part are implemented in Julia and can be downloaded on the following website: <https://julien.vaes.uk/english/acpro.html>.

The main file, *Implementation* contains 4 subfolders:

- ***Code*** contains the implementation of all methods.
- ***Data*** contains the data of the three power systems used (IEEE-24 buses, RTS and UK).
- ***Output*** saves all output files of the implemented methods.
- ***Script*** contains all scripts in order to launch the methods from the terminal.

To launch a method, one needs to move via the terminal in the *Script* folder and launch a command line of the type:

```
bash run.sh ARG1 ARG2 ARG3 ARG4 ARG5 ARG6,
```

where

◇ ***ARG1*** is the method we desire to launch. The user have the choice between the following methods:

1. ***proba*** corresponds to the probabilistic method.
2. ***probaParallel*** corresponds to the parallel version of the probabilistic method (using all processors of the computer).
3. ***gurobi*** corresponds to the solver method when load curtailment is forbidden.

4. ***gurobiWithLS*** corresponds to the solver method when load curtailment is tolerated.
 5. ***gurobiWithUpperBound*** corresponds to the solver method when load curtailment is forbidden and when one gives an upper bound on the optimal cost, for instance the optimal solution returned by the probabilistic method.
 6. ***probaGurobi*** corresponds to the probabilistic method and where the end of the algorithm is computed by Gurobi, i.e. the probabilistic method sends the problem to Gurobi once it has obtained a small relative gap, e.g. 5%.
 7. ***probaParallelGurobi*** corresponds to the parallel version of the probabilistic method and where the end of the algorithm is computed by Gurobi.
- ◇ ***ARG2*** correspond to the power system for which we desire to find the optimal expansion planning:
 1. ***IEEE-24*** corresponds to the IEEE-24 buses power system [A.1](#).
 2. ***RTS*** corresponds to the RTS power system [A.2](#).
 3. ***UK*** corresponds to the UK power system [A.3](#).
 - ◇ ***ARG3*** must be equal to ***y*** if one desires to create the data based on the excel file located in the ***Data*** folder; if one chooses ***n***, the algorithm will take the existing data files of the power system located in the ***Data*** folder.
 - ◇ ***ARG4*** must be equal to ***y*** if one desires to save the output of the method in folder ***Output***; ***n*** otherwise.
 - ◇ ***ARG5*** is the number of realizations one desires to launch. If this argument is not mentioned the method is launched only once.
 - ◇ ***ARG6*** corresponds to the upper bound one desires to send to Gurobi if method ***gurobi-WithUpperBound*** is chosen. This argument do not need to be completed for any other method.

Here are two examples of valid command lines:

```
bash run.sh probaParallel RTS y y 10
```

```
bash run.sh gurobiWithUpperBound IEEE-24 n n 2 113600.00
```

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