

École polytechnique de Louvain

Metasurface antennas for radar applications

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Abstract

Metasurface (MTS) antennas consist of a dielectric slab on which periodically sub-wavelength metal patterns of varying size and orientation are printed. A feeder is located inside the substrate, and generates a surface wave (SW) that is progressively perturbed by the slow spatial variation in the shapes of the patches. That perturbation generates leaky waves (LW), potentially producing a well controlled radiation pattern. MTS antennas have the advantage to be flat and easy to manufacture, which makes them a very attractive technology for many wireless applications. In particular, this master thesis is devoted to the analysis and design of metasurface-based sparse-array antennas with a limited number of feeds. The goal is to propose an alternative and more efficient technology to phased arrays, for beam scanning applications. The proposed antenna would have a wider feed spacing, resulting in a lighter and less power-consuming antenna. This would be an innovative technology for many radar applications, such as traffic radars or synthetic aperture radars (SAR) for Earth observation.

This master thesis first shows how the Method of Moments (MoM) can be used to solve the electric field integral equation (EFIE) for the current distribution on the antenna. To that aim, the MTS can accurately be modeled by a surface sheet impedance. The analysis is first carried out in one dimension for finite antennas, and is then extended to infinite periodic antennas. Then, the design of one-dimensional antennas is performed. The goal is to optimize the surface impedance of the antenna such that it radiates a desired radiation pattern. Finally, the previous methodology for the analysis of one-dimensional antennas is extended to two-dimensional antennas. It is an essential step towards the design of more realistic antennas that can be manufactured.

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Mathematical notations and Acronyms

Mathematical notations

$j = \sqrt{-1}$	imaginary unit
x	a scalar
$\mathbf{x}$	a vector
$\mathbf{X}$	a matrix
$\hat{\mathbf{x}}$	a unit vector
$f(x, y, z)$	a quantity in the spatial domain
$\tilde{f}(k_x, k_y, z)$	a quantity in the 2D spectral domain
$\text{Re}(x)$	the real part of x
$\text{Im}(x)$	the imaginary part of x
$ x $	the absolute value of x
$\delta(x)$	the Dirac delta function of x
$\mathbb{N}_0$	the set of natural numbers, excluding zero
$\mathbb{Z}$	the set of integers
$\times$	the cross product operator
$\otimes$	the convolution operator

Acronyms

1D	One Dimension
2D	Two Dimensions
AF	Array Factor
BW	Bandwidth
EEP	Embedded Element Pattern
EFIE	Electric Field Integral Equation
EM	Electromagnetic
FS	Fourier Series
FT	Fourier Transform
IBC	Impedance Boundary Condition
IFT	Inverse Fourier Transform
ImPACT	Impulsing PARagidm Change through disruptive Technologies
LHCP	Left Handed Circular Polarization
LW	Leaky Wave
MIMO	Multiple Input Multiple Output
MoM	Method of Moments
MTM	Metamaterial
MTS	Metasurface
PEC	Perfect Electric Conductor
RHCP	Right Handed Circular Polarization
RIS	Reconfigurable Intelligent Surfaces
RP	Radiation Pattern
SAR	Synthetic Aperture Radar
SLL	Sidelobe level
SW	Surface Wave
TE	Transverse Electric
TM	Transverse Magnetic
VED	Vertical Elementary Dipole

Chapter 1

Introduction

1.1 Historical context

Antennas are the "eyes and ears" of wireless communications. The history of antennas dates back to the end of the 19th Century and is based on the famous work of the physicist James Clerk Maxwell (1831-1879) who proposed the well-known Maxwell's equations. Those equations show that the time-varying electric fields and magnetic fields can sustain each other and form electromagnetic waves that propagate through space [1]. Then, Guglielmo Marconi (1874-1937) was the first to achieve a long distance wireless communication in 1901 by transferring a signal across the Atlantic, from England to Canada. Since then, antenna technology continued to evolve and steady progress was made until today. Among the breakthroughs, we can cite the introduction of the microwave technologies (1940s), microstrip antennas (1970s) and adaptive arrays (1970s) [2].

Electromagnetic (EM) based wireless systems play an essential role in our daily life, not only for communication purposes, but also in remote sensing, radar, medical imaging and radio astronomy. With the rapid increase in complexity and demand for new technologies, it has become a challenge to propose efficient and reliable wireless systems. For instance, there is an interest in smart antennas, including Multiple Input Multiple Output (MIMO) configurations to improve the performance and data capacity of the system. Another famous technology is phased array antennas that can offer high gain and beam shaping or beam scanning capabilities to meet the required specifications of many applications. However, they require a lot of electronics to feed and reconfigure all the active elements. This results in very dense systems consuming a considerable amount of power. Over the past twenty years, metamaterials (MTM) have been proposed. Engineers focused on the integration of those materials in antenna systems because their interaction with electromagnetic waves exhibits unique properties that can be exploited to optimize the performance of antennas and other electromagnetic devices. It makes it a very attractive

candidate to design more efficient, smaller, cheaper and less power-consuming antenna systems.

1.2 Metasurface antennas

Metamaterials (MTMs) are artificially structured materials, resulting from an engineering process. A MTM consists of a periodic structure of dense sub-wavelength elements, as shown in Figure 1.1. Their composition, position, shape and orientation can be smartly adjusted to offer properties that are not found in the nature when coupled with EM waves, such as negative permittivity or permeability, which opens the door to new perspectives for the design of antennas. However, their fabrication being complex, more interest has been given to their two-dimensional (2D) counterpart, commonly referred to as metasurfaces (MTS). Metasurfaces are thin surfaces composed of periodically sub-wavelength metal patterns of varying size and orientation printed on a (sometimes grounded) dielectric slab, as shown in Figure 1.2.

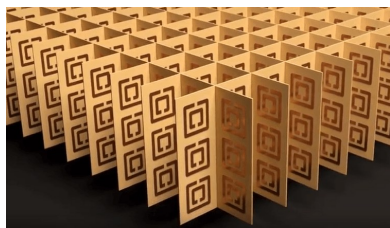


Figure 1.1: Example of a metamaterial [3].

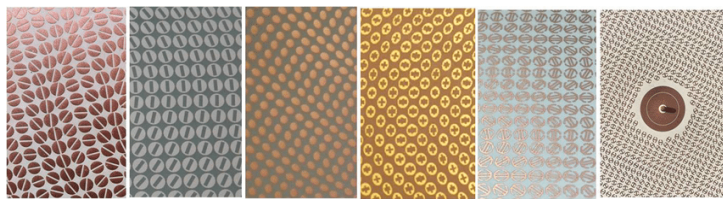


Figure 1.2: Examples of textured layout for implementing the MTS [4].

MTS antennas consist of a grounded dielectric slab on which those varying sub-wavelength conductive patches are printed [5]. The basic idea behind MTS antennas is to play with the arrangement of the sub-wavelength patches to control the aperture fields in phase, amplitude and polarization. It makes them a powerful technology to design antennas that transform a known incident field into an arbitrarily desired radiation pattern. Moreover, MTS antennas are flat, light, cheap and easy to manufacture, making them a very good candidate for a variety of applications. An example of a MTS antenna is shown in Figure 1.3a and zooms at different scales of this antenna are summarized in Figure 1.3b.

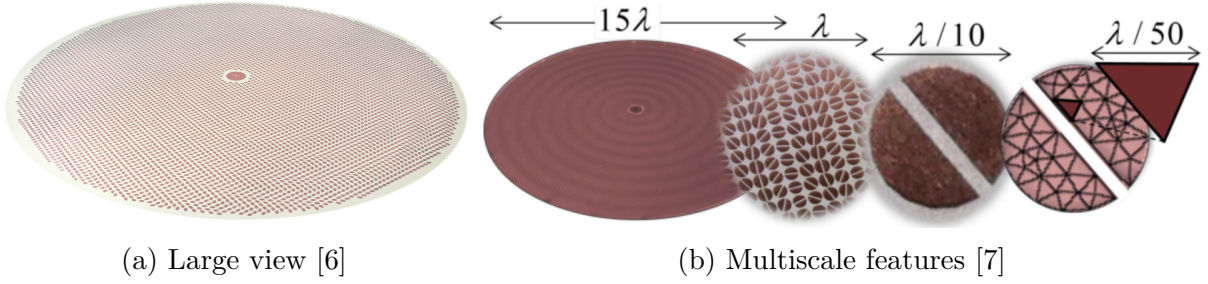


Figure 1.3: Example of a circular MTS antenna at multiple scales.

1.2.1 Modulated metasurface antennas

Modulated metasurface antennas are a class of leaky-wave (LW) antennas. A feeder generates a transverse magnetic (TM) surface wave (SW) that propagates within the metasurface, in the dielectric slab. This surface wave is progressively perturbed by the slow spatial variation in the shapes of the patches. That perturbation generates leaky waves, potentially producing a well controlled radiation pattern [8] [9], as illustrated in Figure 1.4. Leaky-wave radiation from modulated impedance surfaces was first demonstrated by Oliner and Hessel [10]. MTS antennas can cover many wavelengths and include sub-wavelength details. Therefore, their numerical analysis is very complex. In this respect, the method of moments (MoM) has been exploited. This method includes a large part of the physical process through the use of Green's functions [11]. In particular, the metallization layer can reliably be modeled as an electric spatially modulated sheet impedance lying on a grounded slab [12], creating impedance boundary conditions (IBC) which dictate the link between the tangential electric and magnetic fields on the antenna. The Method of Moments (MoM) can be used to solve Maxwell's equations formulated in their surface-integral form. This analysis procedure will be detailed in Chapter 2.

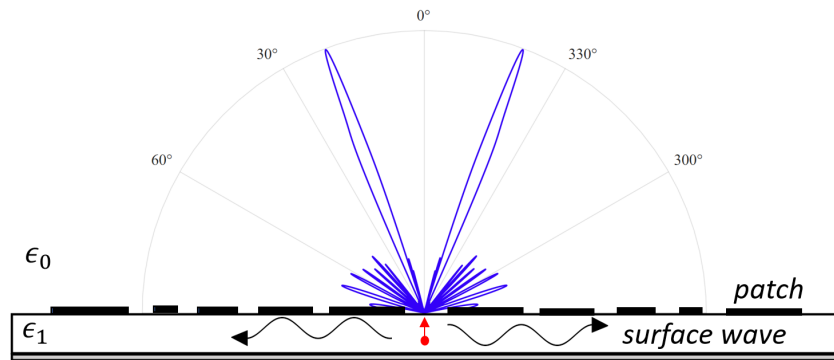


Figure 1.4: MTS antenna principle.

To understand and analyze MTS antennas, it is often more convenient to switch from

the spatial domain (x, y, z) to the 2D spectral domain (k_x, k_y, z) thanks to a 2D Fourier transform (FT) defined as follows in two dimensions:

$$\tilde{E}(k_x, k_y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x, y, z) e^{j(k_x x + k_y y)} dx dy \quad (1.1)$$

Similarly, the inverse Fourier transform (IFT) allows to revert to the spatial domain:

$$E(x, y, z) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{E}(k_x, k_y, z) e^{-j(k_x x + k_y y)} dk_x dk_y \quad (1.2)$$

Basically, the Fourier transform allows the decomposition of the fields into a continuum of plane waves, where the following relation holds for the wavenumbers in free-space:

$$k_x^2 + k_y^2 + k_z^2 = k_0^2 = \omega^2 \mu \epsilon \quad (1.3)$$

where μ and ϵ are the permeability and the permittivity of the material and $\omega = 2\pi f$ is the pulsation. $\lambda = 2\pi/k_0 = c/f$ is the homogeneous-medium wavelength, c is the speed of light in free-space, f is the frequency and k_0 is the free-space wavenumber. Note that $\text{Im}(k_z) \leq 0$ must be imposed to satisfy the radiation condition. Finally, it is important to introduce the concept of visible and invisible region in the spectral domain. Indeed, Section 2.2.2 will show that the radiation pattern (RP) of the antenna is directly linked to the spectral aperture electric field. However, only the region for which $k_x^2 + k_y^2 \leq k_0^2$ contributes to the far-field radiation. This spectral domain is called the visible region. In the same way, the invisible region is defined to be the region where $k_x^2 + k_y^2 > k_0^2$.

The basic principle behind modulated MTS is explained here for a one-dimensional (1D) antenna (invariant along y (space) and $k_y = 0$ (spectral)) but the same reasoning applies for two-dimensional (2D) antennas. First, we consider an unmodulated (constant) capacitive surface impedance. In this case, the TM surface wave generated by the feeder will propagate in the MTS. However, this SW will not radiate because its wavenumber k_{sw} is outside the visible region, as shown in Figure 1.6. If the surface is modulated (with a periodicity p) at the wavelength scale, several harmonics of the fundamental wavenumber are created and some of them could fall inside the visible region, causing radiation. Those harmonics are called Floquet modes and are at the following wavenumbers:

$$k_x^{(n)} = k_{sw} + n \frac{2\pi}{p} \quad \forall n \in \mathbb{Z} \quad (1.4)$$

In Figure 1.6, we observe that in the modulated case, the -1 harmonic causes radiation because it is located in the visible region. If the antenna must look at a certain angle, the MTS is designed such that only the -1 harmonic falls in the visible region, and its location determines in which direction the antenna radiates:

$$\sin \theta_n = \frac{k_x^{(n)}}{k_0} \quad (1.5)$$

This equation comes from the polar coordinate system transformation reminded in Figure 1.5.

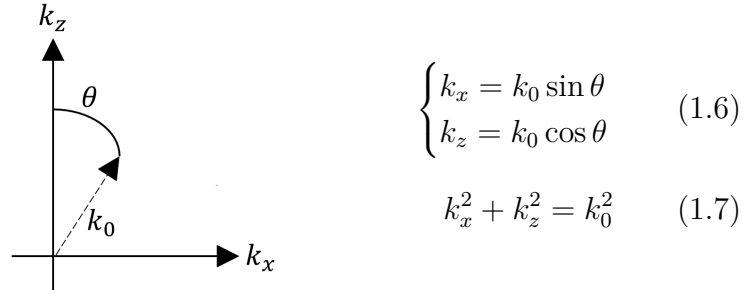


Figure 1.5: Polar coordinate system.

And it confirms the intuition behind the visible region through the following relation:

$$|\sin \theta_n| \leq 1 \implies |k_x^{(n)}| \leq k_0 \quad (1.8)$$

This notion of harmonics is essential to understand the link between the impedance and the direction of radiation of a modulated MTS antenna. However, it supposes a very simple impedance variation and results are modified if the impedance behavior is more complex.

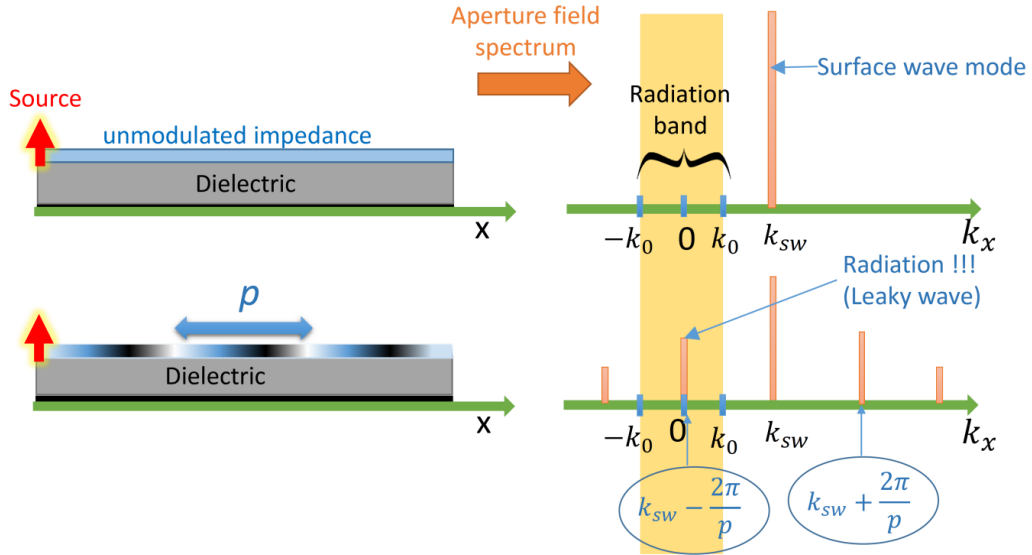


Figure 1.6: Basic principle of a MTS antenna in the spectral domain [13].

1.3 Applications and challenges

Metasurface antennas have the advantage to be flat, compact, directive, reconfigurable, light, cheap and easy to fabricate. They are also usually easily fed and easily matched antennas. This makes them a very good candidate for many applications in a variety of domains, such as wireless communications, remote sensing, radar, medical imaging and radio astronomy. As a first example, research is conducted to use metasurface technology to design reconfigurable intelligent surfaces (RIS) [14] whose goal is to manipulate and control the transmission paths of radio waves in a given environment. The idea is to place reflectors in cities to redirect the beams in a given direction. This can be used for 5G communications in order to increase the directivity and quality of the transmission or to reach areas that are obstructed by obstacles to increase the coverage area. In the same way, it can be used to avoid illuminating areas that should stay out of radiation. The potential applications of RIS in cities are illustrated in Figure 1.7. The rapid and efficient reconfigurability of those surfaces is a complex problem and is still under research [15].

Another important field of application is radar in general, such as traffic radars and imaging radars. Radars often require beam scanning to observe a given region in space. A famous and mature way to scan is to use phased arrays to enable electronic scanning, offering excellent beam controllability. However, it is well-known that phased arrays require a maximum spacing of half-wavelength (up to one wavelength in case of limited scan range) between active elements to avoid grating lobes, resulting in a dense electronics. So, the drawback of this technology is its large volume, weight, power consumption and cost. As explained in this master thesis, metasurface antennas may be used as an alternative to dense phased arrays, to design sparse arrays with a wider spacing between feed points (larger than the wavelength), while still avoiding grating lobes. The reduced number of feeds, and hence of electronic channels, will result in a lighter antenna that consumes less power. The result will combine the advantages of phased arrays (good directivity and reconfigurability) with the advantages of metasurfaces (low losses, low profile, simplicity of fabrication and lighter electronic front-end), making MTS antennas a very good candidate for radar applications. Furthermore, the flatness of those antennas as well as their ability to shape the radiation pattern make them a very suitable and innovative technology for space applications [16]. For example, it can be used for synthetic aperture radars (SAR) which is a type of radar that has been widely used for Earth observation over the past 30 years. SAR instruments take advantage of their motion to improve the resolution of the image in the azimuth direction (i.e. along track). In the spotlight mode, the radiation pattern is continuously tilted such that the beam always points towards the same zone and the resolution can be improved by combining observations made at consecutive positions through array processing [17] (Figure 1.8). There can also be a need for scanning in the other direction (i.e. across track) to increase the observable swath. An example of a SAR satellite scanning in both directions is the

strix- α SAR, which was developed under a Japanese government ImPACT (Impulsing PARadigm Change through disruptive Technologies) program and successfully launched in 2020 [18]. However, space applications come also with some additional constraints, such as heat dissipation, deployment and vibrations in the structure that should be taken into account in the design of the antenna. Finally, a last challenge regards the bandwidth (BW) of the antenna, which is limited due to the frequency dispersion of the propagating waves. But techniques exist to enlarge the bandwidth like considering a non-uniform modulation period, namely by matching the local radial period of the modulation at different positions with the SW wavenumber at different frequencies. This allows to widen the BW up to 25%, at the price of a significantly lower aperture efficiency [19].

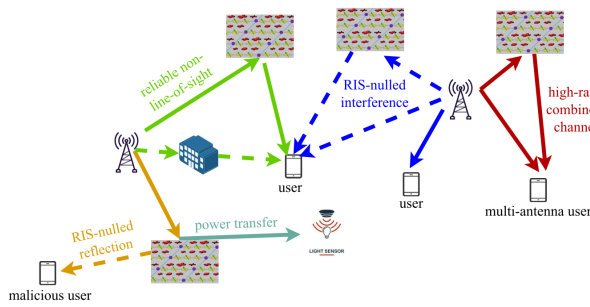


Figure 1.7: Potential applications of RIS in smart radio environments [15].

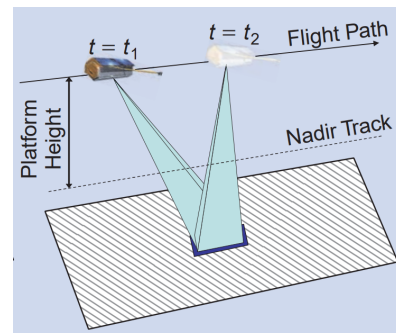


Figure 1.8: Spotlight mode of a SAR [17].

Finally, it is important to tell a few words about the scanning techniques used with MTS, which are still in their infancy. The first and probably most popular approach consists of tuning or switching each individual printed patch [20], but this requires an extremely dense controlling circuit, such that the advantage over phased arrays becomes less significant. A second method makes use of tunable dielectric materials, such as liquid crystals [21], but those are in general quite lossy and are difficult to apply to 2D scanning. A third approach resorts to a large number of feed points, each creating an independent beam [22], but full coverage of the field of view with discrete beams remains a huge challenge. A limited continuous scanning, over just one beamwidth, has been proposed in [23] by combining two patterns with opposite phase slopes, created by widely separated feeds. A much wider scan range is necessary for SAR, with scan angles of the order of 30 degrees, if we want to strongly enhance the capabilities of spaceborne SAR. In this master thesis, we propose a new idea, which consists of designing the MTS antenna such that the embedded element pattern is very sharp and filters out the grating lobes of the array factor, creating a directive radiation pattern in one given direction. Moreover, this direction can be electronically modified to offer scanning. More details about this idea for scanning with MTS antennas will be provided in the next chapters.

1.4 Goal and outline

To summarize, the goal of this master thesis is to analyze and design metasurface antennas such that they can be used to design sparse arrays with a wide spacing between feed points. The result looked for is a smooth aperture with a limited number of feeds [23], rather than an array of discrete elements. The goal is to propose MTS antennas that have scanning capabilities and to apply them to radar applications. Moreover, metasurface antennas have other advantages, like their flatness and their reduced power consumption that make them an attractive technology for space applications.

This master thesis is organized in several chapters that first deal with the analysis and then with the design of MTS antennas. As MTS antennas are very complex structures, the analysis is first carried out for one-dimensional finite antennas in Chapter 2. In particular, Chapter 2 demonstrates how the Method of Moments can be implemented to determine the current distribution and electric field propagating in such antennas, for a known excitation. Then, different simulations are carried out to understand the behavior of MTS antennas. Similarly to the finite case, the same analysis are conducted for infinitely periodic one-dimensional MTS antennas in Chapter 3. Those antennas involve an infinite array of feeding points and the main goal of this chapter is to determine the embedded element pattern associated with each excitation. This allows one to build the total directivity, as the array factor can be found analytically. It is an essential step towards the final goal of this master thesis which is to offer scanning with MTS antennas. Note that if the spacing between feeding points is much larger than half-wavelength, the field of view will be limited. Once the coding implementations are validated in Chapters 2 and 3 for finite and infinite antennas respectively, Chapter 4 explains the design procedure towards the desired embedded element pattern. The design methodology amounts to determining the needed surface impedance to obtain a desired radiation pattern for a given excitation in a sparse periodic structure. The design is done for infinitely periodic MTS antennas, and then the impact of array truncation is observed. Then, Chapter 5 introduces how the coding implementation can be extended to the analysis of two-dimensional finite MTS antennas. This is the first step towards the design of two-dimensional antennas, which can then be manufactured and measured. Finally, Chapter 6 concludes this work and tells a few words about further research directions.

Chapter 2

Analysis of 1D finite metasurface antennas

This chapter presents the necessary tools for the analysis of one-dimensional finite metasurface antennas. In particular, it shows how the Method of Moments can be used to determine the current distribution along the antenna. Of course, 1D antennas cannot be fabricated through an industrial process but it is of good practice to start by a simplified version of the problem (1D antennas) to tackle the main issues before designing a more realistic version (2D antennas). It will be a powerful tool to validate some results and understand basic principles behind MTS antennas.

2.1 Problem statement

In this chapter, we consider a 1D metasurface antenna oriented along the x direction, as shown in Figure 2.1 below. It consists of a finite MTS of length a printed on an infinite dielectric grounded slab along x with thickness d . The length of the MTS is also the antenna length. For all the simulations, we will use the substrate ROGERS4350B [24] of relative permittivity $\epsilon_r = 3.66$, and thickness $d = 1.524$ mm. The working frequency is $f = 24$ GHz and the impedance of each medium is $\eta = \sqrt{\mu/\epsilon}$, with μ the permeability and ϵ the permittivity of the medium. The metasurface is implemented with patches and modeled by a surface sheet impedance denoted as $Z_s(x)$. The antenna is excited at its center by an infinite array along y of vertical elementary dipoles (VED) along z , represented in red in Figure 2.1. The excitation will be modeled more precisely in Section 2.1.2. Except for the excitation modeling, the structure is invariant along y and the whole analysis can be carried in one dimension only. Before diving into complete explanations regarding how the MoM is used to analyze 1D antennas, a short reminder on Green's function is provided in Section 2.1.1.

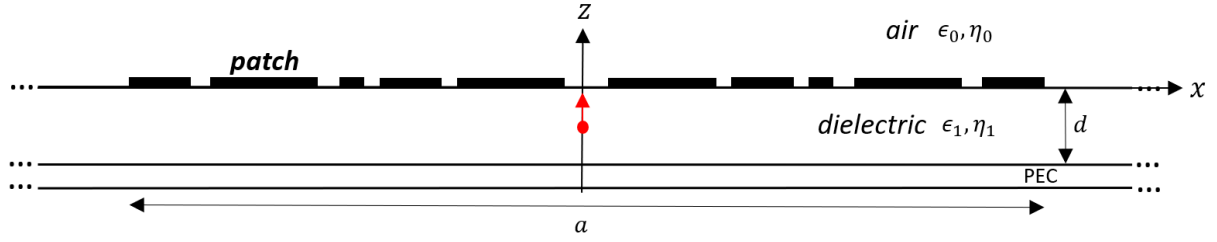


Figure 2.1: 1D metasurface antenna.

2.1.1 Green's function

The Green's function is introduced here as it will be useful in the following sections for the MoM formulation. The Green's function corresponds to the field distribution associated with a given point source, in a given material environment. It can be viewed as the spatial impulse response of the radiation operator [25]. In free-space, it is given by:

$$\mathbf{G}(\mathbf{r}, \mathbf{r}') = \mathbf{G}(R = |\mathbf{r} - \mathbf{r}'|) = \frac{e^{-jk_0 R}}{4\pi R} \quad (2.1)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are respectively the points at which the field is evaluated and the source is located. So, the Green's function ($\mathbf{G}$) allows one to determine the electric field ($\mathbf{E}$) radiated by a known current distribution ($\mathbf{J}$) [26]:

- In the spatial domain, with a convolution:

$$\mathbf{E}(\mathbf{r}) = \mathbf{G}(\mathbf{r}, \mathbf{r}') \circledast \mathbf{J}(\mathbf{r}') \quad (2.2)$$

- In the spectral domain, with a product:

$$\tilde{\mathbf{E}}(k_x, k_y) = \tilde{\mathbf{G}}(k_x, k_y) \tilde{\mathbf{J}}(k_x, k_y) \quad (2.3)$$

In its more general form, the Green's function is dyadic and its tensor notation comes from the vector nature of currents and fields. The Green's function for horizontal currents (in the xy plane) can be computed analytically in the spectral domain for a layered medium, which consists of several layers along z that are infinite in the x and y directions. Note that a MTS antenna corresponds to the specific case of a single-layer substrate. A rigorous proof is detailed in [1] and the final expression is given below.

$$\tilde{\mathbf{G}}(k_x, k_y) = \begin{bmatrix} \tilde{G}^{xx} & \tilde{G}^{xy} \\ \tilde{G}^{yx} & \tilde{G}^{yy} \end{bmatrix} = \tilde{G}_A(k_\rho) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \tilde{G}_V(k_\rho) \begin{bmatrix} k_x^2 & k_x k_y \\ k_x k_y & k_y^2 \end{bmatrix} \quad (2.4)$$

Note that this expression depends on $\tilde{G}_A(k_\rho)$ and $\tilde{G}_V(k_\rho)$, which are two scalar potentials that depend on $k_\rho = \sqrt{k_x^2 + k_y^2}$ only. Their analytical expressions are delineated in [1] for completeness.

2.1.2 The excitation

This short section aims at telling a few words about the way the antenna is excited. We consider that the metasurface antenna is excited by a wall infinite along y of vertical elementary dipoles (VED) along z . Each VED is located at $(x', y', z') = (0, y, -d/2)$, as shown in Figure 2.1. The current distribution $\mathbf{J}_s(x, y, z)$ associated with the excitation can be written as follows:

$$\mathbf{J}_s(x, y, z) = J_{s0} \delta(x) \delta(z + \frac{d}{2}) \hat{\mathbf{z}} \quad (2.5)$$

where J_{s0} is the amplitude of the current distribution and $\delta(x)$ is the Dirac delta function. The infinite wall of excited elements allows one to create a 1D problem because the total field radiated by this wall will only depend on k_x , while k_y is always zero, given the invariance of sources and materials in the direction of the wall. Indeed, it gives:

$$\tilde{E}_{wall}(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{xz}(x, y) e^{jk_x x} e^{jk_y y} dx dy \quad (2.6)$$

$$= \int_{-\infty}^{+\infty} E_{xz}(x) e^{jk_x x} dx \int_{-\infty}^{+\infty} e^{jk_y y} dy \quad (2.7)$$

$$= \tilde{E}_{xz}(k_x) 2\pi\delta(k_y) \quad (2.8)$$

where $\tilde{E}_{xz}(k_x)$ refers to the x component of the spectral electric field generated by a VED oriented along z . The spectral electric field on the metasurface ($z = 0$) generated by each VED can be computed analytically and it should be noted that it only contains a TM component [1][13]. The result is reminded below in Cartesian coordinates:

$$\tilde{E}_{xz}(k_x) = \frac{k_x}{\sqrt{k_x^2}} \tilde{f}(k_x) e^{jk_x x'} \quad (2.9)$$

with

$$\tilde{f}(k_x) = \frac{\eta_0 k_x}{2\epsilon_r k_0} \left(e^{-jk_{z1} \frac{d}{2}} - \frac{-\Gamma e^{-jk_{z1} \frac{d}{2}} - e^{-jk_{z1} \frac{3d}{2}} + \Gamma e^{-jk_{z1} \frac{5d}{2}} - \Gamma e^{-jk_{z1} \frac{3d}{2}}}{1 + \Gamma e^{-jk_{z1} 2d}} \right) \quad (2.10)$$

where

$$\Gamma = \frac{\epsilon_r k_{z0} - k_{z1}}{\epsilon_r k_{z0} + k_{z1}} \quad \text{and} \quad k_{z1}^2 + k_x^2 = \omega^2 \mu \epsilon_1 \quad (2.11)$$

where index 1 refers to the dielectric medium and Γ is the reflection coefficient at the air-dielectric interface.

2.2 Method of Moments

MTS antennas are composed of plenty of small different patches with details at the sub-wavelength scale, making them very complex structures. Therefore, efficient numerical methods are necessary to compute the electromagnetic fields generated by a given excitation within the antenna. Those methods are generally based on the solution of Maxwell's equations in integral form to compute the current distribution on the antenna. From those currents, fields and radiation patterns can be deduced in a second phase. This chapter reminds the main equations needed to build and solve the MoM system of equations in order to find the current distribution. More complete and rigorous mathematical developments can be found in [27].

2.2.1 Method of Moments formulation

We consider that the metasurface antenna is modeled as an impedance sheet distribution on an infinite grounded dielectric slab. The impedance sheet $\mathbf{Z}_s$ imposes the ratio between the total electric field and the surface current $\mathbf{J}$ on the metasurface. This leads to the following electric field integral equation (EFIE), which provides the link between a point source and its radiated fields [28]:

$$\hat{\mathbf{n}} \times \left[\iint_{S'} \mathbf{G}(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') dS' - \mathbf{Z}_s(\mathbf{r}) \mathbf{J}(\mathbf{r}) \right] = -\hat{\mathbf{n}} \times \mathbf{E}_i(\mathbf{r}) \quad (2.12)$$

where $\hat{\mathbf{n}}$ is the unit vector perpendicular to the surface of the antenna S' , $\mathbf{G}$ is the spatial Green's function of the grounded slab, $\mathbf{E}_i$ is the incident electric field (excitation) and $\mathbf{r}$ and $\mathbf{r}'$ refer to the position of observation and of the source, respectively.

In one dimension, equation (2.12) becomes:

$$\int_{x'} G^{xx}(x, x') J^x(x') dx' - Z_s^{xx}(x) J^x(x) = -E_i^x(x) \quad (2.13)$$

where the x exponent refers to the x component of each vector. This exponent will be kept for the Green's function but will be omitted for the other variables to simplify the notations for the upcoming explanations. The Method of Moments is used to solve equation (2.13) to determine the current distribution on the metasurface. To do so, the desired current distribution is approximated by a linear combination of N well-chosen basis functions:

$$\mathbf{J}(x) = \sum_{n=1}^N i_n F_b^n(x) \hat{\mathbf{x}} \quad (2.14)$$

where i_n is the weight associated with the n^{th} basis function F_b^n . The choice of the basis functions is important because it will impact the quality of the approximated current distribution. There exist two types of basis functions: sub-domain functions only

defined on a sub-domain region (e.g. rooftop functions) and full-domain functions which are defined on the overall domain (e.g. polynomial and trigonometric functions). The advantage of full-domain basis functions is that they allow a reduction of the number of unknowns and a good conditioning of the system of equations [29] [9]. In this master thesis, we will use Gaussian basis functions which are defined on the entire domain but are still very located due to their particular shape. Those functions are defined as:

$$F_b^n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_n)^2}{2\sigma^2}} \quad (2.15)$$

where σ and μ are the standard deviation and the mean of the Gaussian function, respectively. Those parameters are chosen such that there are enough basis functions per unit length to represent the current (typically 10 to 20 basis functions per wavelength), and such that they cross each other around -1 dB from their top (σ is typically the distance between the maxima of two consecutive basis functions). It should look like in Figure 2.2.

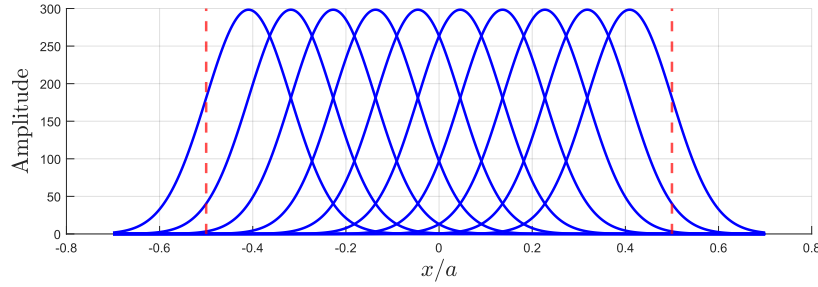


Figure 2.2: Basis functions along the antenna.

Note that there is no basis function centered at the edges of the antenna to limit the current outside the MTS. A nice property of those basis functions is that their Fourier transform (FT) remains Gaussian:

$$\tilde{F}_b^n(k_x) = e^{jk_x\mu_n} e^{-\sigma^2 \frac{k_x^2}{2}} \quad (2.16)$$

After inserting (2.14) into (2.13) and testing the fields with the same basis functions ($F_t^n(x) = F_b^n(x)$: Galerkin testing), the EFIE can be written in a matrix form where a linear system of equations allows one to find the current distribution:

$$(\mathbf{Z}_G - \mathbf{Z}_{IBC}) \mathbf{i} = \mathbf{v} \quad (2.17)$$

where $\mathbf{Z}_G$ is named the substrate matrix, $\mathbf{Z}_{IBC}$ is the impedance boundary condition (IBC) matrix, $\mathbf{i}$ is the vector containing the unknown current coefficients and $\mathbf{v}$ is the excitation vector. It can be shown that their mathematical expressions are:

$$Z_G(m, n) = \int_x \int_{x'} F_t^m(x) G^{xx}(x - x') F_b^n(x') dx' dx \quad (2.18)$$

$$Z_{IBC}(m, n) = \int_x F_b^n(x) Z_s(x) F_t^m(x) dx \quad (2.19)$$

$$v(n) = - \int_x E_i(x) F_t^n(x) dx \quad (2.20)$$

Note that the Green's function and the excitation field have an analytical expression in the spectral domain, which has been developed in Sections 2.1.1 and 2.1.2, respectively. This pleads in favor of computing $\mathbf{Z}_G$ and $\mathbf{v}$ in spectral domain. Their expressions are written below and the mathematical proofs are given in Appendix A.

$$Z_G(m, n) = \frac{1}{2\pi} \int_{k_x} \tilde{F}_t^m(-k_x) \tilde{G}^{xx}(k_x) \tilde{F}_b^n(k_x) dk_x \quad (2.21)$$

$$v(n) = \frac{-1}{2\pi} \int_{k_x} \tilde{E}_i(k_x) \tilde{F}_t^n(-k_x) dk_x \quad (2.22)$$

Remarks for the software implementation

The analysis is performed using the MATLAB® language [30]. The coding implementation mainly consists of building the matrices $\mathbf{Z}_G$ and $\mathbf{Z}_{IBC}$ and the vector $\mathbf{v}$ to solve the linear system of equations (2.17). It seems simple, but it is interesting to take a closer look at the structure and particularities of the matrices and vectors to make the implementation efficient. First, it is obvious that $\mathbf{Z}_{IBC}$ is a symmetric matrix, meaning that only the upper triangular matrix needs to be computed with integrals. Secondly, $\mathbf{Z}_G$ is a Toeplitz and symmetric matrix. Indeed, a small mathematical development can show that each entry of the matrix only depends on the difference between the maxima of the basis and testing functions: $Z_G(m, n) = f(\mu_n - \mu_m)$. This allows a lot of computation savings as only the first line of the matrix needs to be computed. The rest of the matrix can be deduced from the first line of the previously computed matrix. Finally, care must be taken when computing $\mathbf{Z}_G$ and $\mathbf{v}$ as they both contain poles around $k_x = k_{sw, substrate} = \pm 1.26 k_0$ due to the Green's function of the substrate and the VED excitation. Thus, a parabolic complex contour is used for the computation of those integrals. As we know the approximate location of those poles, the contour is only used for $|k_x| < 2 k_0$. The contour is represented in the complex plane of k_x in Figure 2.3, where $k_{x,i}$ and $k_{x,r}$ are the imaginary and real parts of k_x , respectively. More details about its mathematical expression are provided in Appendix B.

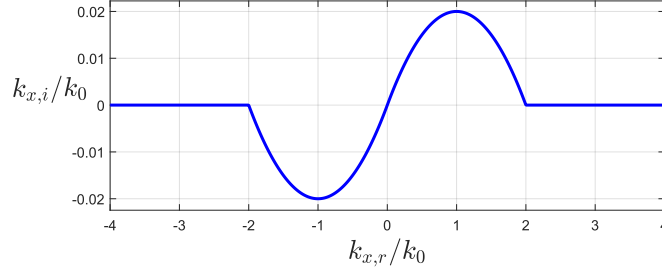


Figure 2.3: Complex contour.

2.2.2 Radiation pattern and directivity

Once the linear system of equations (2.17) is solved, the current distribution can be approximated using (2.14). Note that its Fourier transform can be computed similarly as the Fourier transform of the basis functions is known analytically. Then, the electric field can be directly computed in the spectral domain using the Green's function:

$$\tilde{\mathbf{E}}(k_x) = \tilde{\mathbf{J}}(k_x) \tilde{G}^{xx}(k_x) + \tilde{\mathbf{E}}_{wall}(k_x) \quad (2.23)$$

It corresponds to the electric field propagating in the whole structure, which corresponds to the finite MTS printed on an infinite grounded slab. One could be tempted to compute the electric field as follows:

$$\mathbf{E}(x) = \mathbf{J}(x) Z_s(x) \quad (2.24)$$

However, this computes the electric field on the MTS only, so it is not complete and one must use equation (2.23) followed by an inverse Fourier transform if the spatial electric field is of interest. Then, the radiation pattern can be obtained as follows [26]:

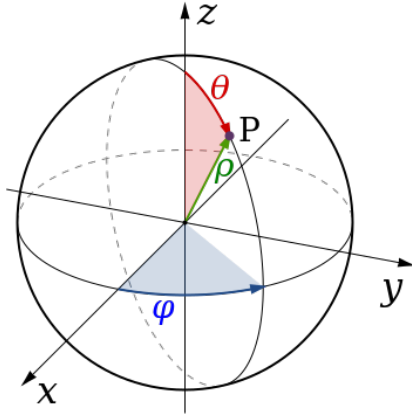
$$\tilde{\mathbf{F}}(k_x, k_y) = \frac{-j}{\lambda} \left[\hat{\mathbf{u}} \times (\hat{\mathbf{n}} \times \tilde{\mathbf{E}}(k_x, k_y)) \right] \quad (2.25)$$

where $\hat{\mathbf{u}}$ is the unit vector in the direction of observation and $\hat{\mathbf{n}}$ is the unit vector perpendicular to the surface of the antenna. In one dimension, the amplitude of the pattern and electric field can be expressed as:

$$\tilde{F}(k_x, k_y) = 2\pi \tilde{F}(k_x) \delta(k_y) \quad (2.26)$$

$$\tilde{E}(k_x, k_y) = 2\pi \tilde{E}_x(k_x) \delta(k_y) \quad (2.27)$$

A spherical system of coordinates can be considered to ease the computations:



$$\begin{cases} x = \rho \sin \theta \cos \phi \\ y = \rho \sin \theta \sin \phi \\ z = \rho \cos \theta \end{cases} \quad (2.28)$$

$$\begin{cases} k_x = k_0 \sin \theta \cos \phi \\ k_y = k_0 \sin \theta \sin \phi \\ k_z = k_0 \cos \theta \end{cases} \quad (2.29)$$

$$k_0^2 = k_x^2 + k_y^2 + k_z^2 = k_\rho^2 + k_z^2 \quad (2.30)$$

Figure 2.4: Spherical coordinate system [31].

Note that for a 1D MTS antenna, the expression of the radiation pattern simplifies to:

$$\mathbf{F}_a(\theta) = \tilde{\mathbf{F}}(k_x) = \frac{-j}{\lambda} \tilde{E}_x(k_x) (-\cos \theta \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{z}}) \quad (2.31)$$

and its norm is given by:

$$|\tilde{\mathbf{F}}(k_x)| = \frac{|\tilde{E}_x(k_x)|}{\lambda} \quad (2.32)$$

Then, the total power radiated by the MTS antenna can be computed from the radiation pattern. As we consider a 1D antenna, it corresponds to the power per unit length along y and is given by:

$$P_{rad} = \frac{1}{2\eta} \int_{-\pi/2}^{\pi/2} |\mathbf{F}_a(\theta)|^2 d\theta \quad [\text{W/m}] \quad (2.33)$$

Finally, the directivity of the antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions [32]. It corresponds to the ratio between the radiation pattern of the MTS antenna and the radiation pattern of an isotropic antenna (F is constant and $\theta \in [-\pi, \pi]$) radiating the same total power:

$$D(\theta) = \frac{|\mathbf{F}_a(\theta)|^2}{|\mathbf{F}_a(\theta)|_{isotropic}^2} = \frac{|\mathbf{F}_a(\theta)|^2}{(2\eta P_{rad})/(2\pi)} = \frac{2\pi |\mathbf{F}_a(\theta)|^2}{2\eta P_{rad}} \quad (2.34)$$

Note that for the more general case of a 2D MTS antenna, the expressions of the radiated power and the directivity are:

$$P_{rad} = \frac{1}{2\eta} \int_0^{2\pi} \int_0^{\pi/2} |\mathbf{F}_a(\theta, \phi)|^2 \sin \theta d\theta d\phi \quad [\text{W}] \quad (2.35)$$

$$D(\theta, \phi) = \frac{|\mathbf{F}_a(\theta, \phi)|^2}{|\mathbf{F}_a(\theta, \phi)|_{isotropic}^2} = \frac{|\mathbf{F}_a(\theta, \phi)|^2}{(2\eta P_{rad})/(4\pi)} = \frac{4\pi |\mathbf{F}_a(\theta, \phi)|^2}{2\eta P_{rad}} \quad (2.36)$$

2.3 Results

This section provides the results obtained for a 1D MTS antenna simulated using the methodology described in this chapter. The goal is to analyze the simulated results for different cases to validate the methodology presented. To that aim, a sinusoidal sheet impedance is considered, as proposed in [13] and [26]:

$$Z_s(x) = jX_0(1 + M \cos(k_{sw}x - k_0x \sin \theta_p)) \quad (2.37)$$

with X_0 the average surface impedance, M the modulation depth, k_{sw} the main SW wavenumber propagating in the MTS antenna, k_0 the free-space wavenumber and θ_p the main direction of radiation. This impedance has been designed such that the -1 harmonic of the electric field falling in the visible spectrum will radiate a beam in the θ_p direction. Indeed, the periodicity of the impedance is:

$$p = \frac{2\pi}{k_{sw} - k_0 \sin \theta_p} \quad (2.38)$$

so, the location of the -1 harmonic can be predicted by equation (1.4) and it will be located in:

$$k_x^{(-1)} = k_{sw} - \frac{2\pi}{p} = k_0 \sin \theta_p \quad (2.39)$$

as expected.

Unmodulated metasurface antenna

First, an unmodulated MTS antenna ($M = 0$) of length 20λ is considered:

$$Z_s(x) = jX_0 \quad (2.40)$$

with $X_0 = -772.50 \, \Omega$. The simulated spatial and spectral current distributions are reported in Figures 2.5a and 2.5b, respectively. The spectral electric field is shown in Figure 2.6.

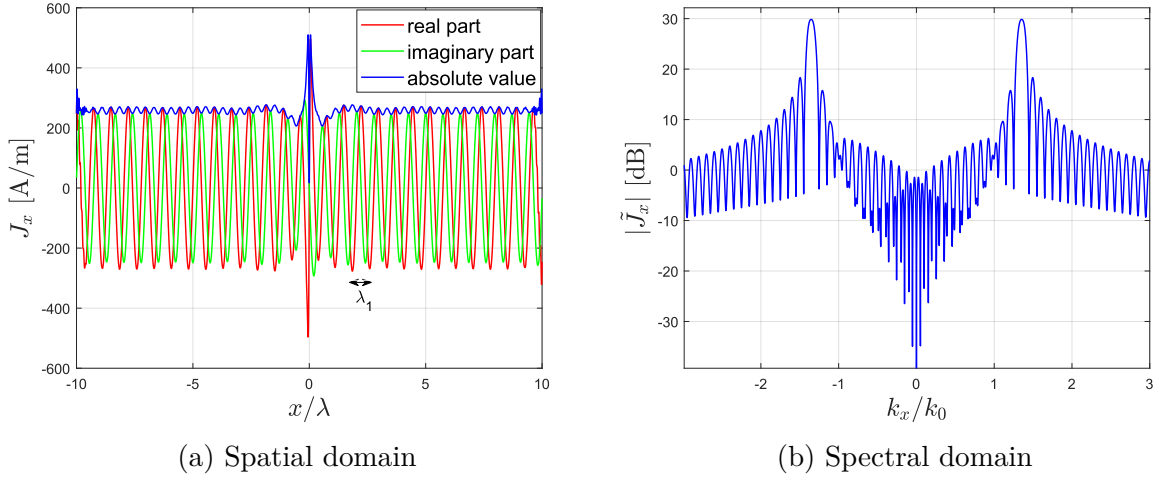


Figure 2.5: Current distribution for an unmodulated MTS antenna.

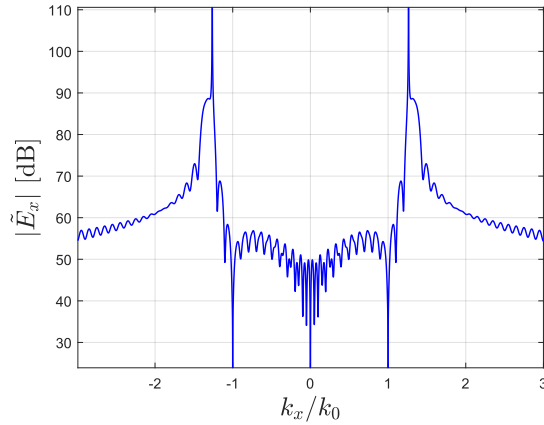


Figure 2.6: Spectral electric field for an unmodulated MTS antenna.

Firstly, we observe that the real and imaginary parts of the current distribution are strictly odd and quasi-sinusoidal, except close to the feeding point at the center of the antenna. This is confirmed by its spectral distribution that contains two main peaks around $|k_x| = 1.35 k_0$. Indeed, close inspection of Figure 2.5a reveals that the main wavelength of the current distribution is $\lambda_1 = 0.74 \lambda$, meaning that its main wavenumber is $k_x = k_0/0.74 = 1.35 k_0$ as expected. Moreover, we observe that the spectral electric field is similar to the spectral current distribution because they are linked through equation (2.23). In Figure 2.6, we observe that the spectral electric field has a main peak around $k_x = 1.314 k_0 = k_{sw,obs}$ and a pole in $k_x = 1.26 k_0 = k_{sw,substrate}$. This is a very important observation that characterizes the main wavenumbers of the electric field propagating in

the structure. To understand it, it is important to remind that the considered antenna is composed of a finite impedance sheet printed on an infinite dielectric slab. In fact, $k_{sw,obs}$ refers to the main mode of the SW electric field propagating in the structure where the MTS is present. And there is a pole in $k_{sw,substrate}$ because it is the main mode of the SW electric field propagating in the substrate beyond the MTS, towards infinity. This pole comes from the Green's function that considers an infinitely extended substrate. This is illustrated in Figure 2.7 below.

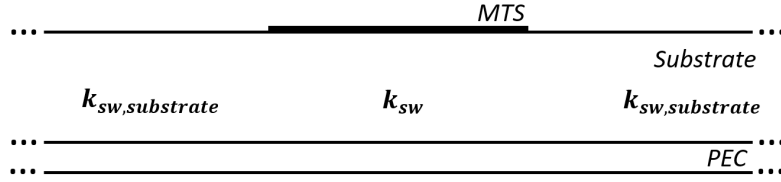


Figure 2.7: Main wavenumbers present in the structure.

It can also be noted that the spectral electric field has no peak in the visible spectrum. This confirms the theoretical approach presented in Figure 1.6 which states that for an unmodulated MTS antenna there is only one peak in $k_x = k_{sw}$ and no harmonics generated, and thus no radiation. To be precise, there is a little bit of radiation as the spectral electric field is not strictly zero in the visible region but it is negligible and we cannot say that the antenna really radiates power in a certain direction.

Modulated metasurface antenna

Then, the modulated surface impedance introduced in equation (2.37) is considered, with the following set of parameters:

Average surface impedance	X_0	$-772.50 [\Omega]$
Modulation depth	M	$0.4 [-]$
SW wavenumber	k_{sw}	$1.35 k_0 [m^{-1}]$
Direction of radiation	θ_p	$20.49 [^\circ]$

Table 2.1: Sheet impedance parameters

The antenna length is still 20λ and the impedance has a periodicity of λ . The simulated spatial and spectral current distributions are shown in Figures 2.8a and 2.8b. The spectral electric field and directivity are shown in Figures 2.9a and 2.9b, respectively.

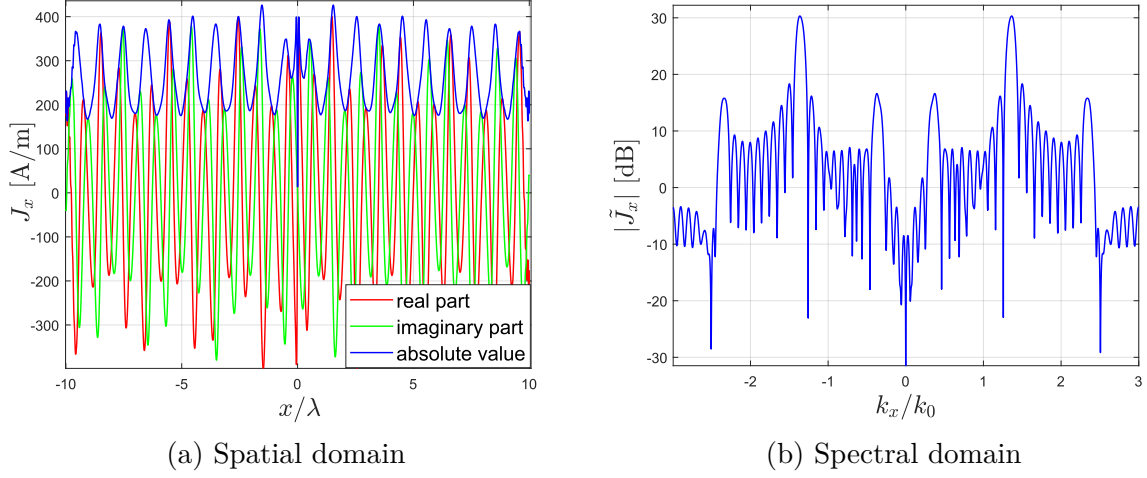


Figure 2.8: Current distribution for a modulated MTS antenna.

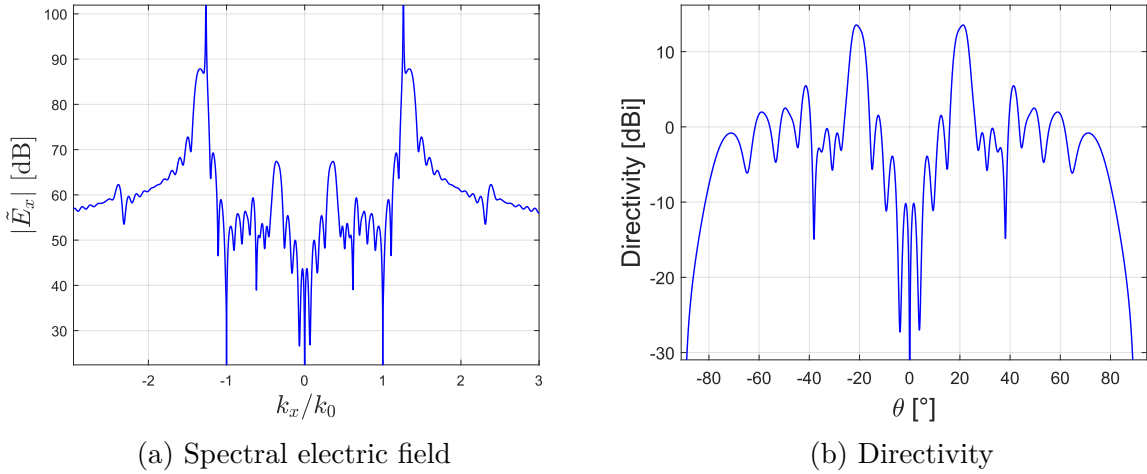


Figure 2.9: Spectral electric field and directivity for a modulated MTS antenna.

As predicted by theory in Figure 1.6, we observe that the modulation of the impedance sheet generated various harmonics around $k_x = 1.339 k_0 = k_{sw,obs}$ for the spectral electric field. This wavenumber is slightly shifted compared to the unmodulated case and close to $1.35 k_0$, which is the approximate value used to build the sheet impedance. There is a peak in $k_x = 0.362 k_0$ (-1 harmonic) and a peak in $k_x = 2.384 k_0$ ($+1$ harmonic). The position of those peaks is confirmed by equation (2.39), with $p = \lambda$:

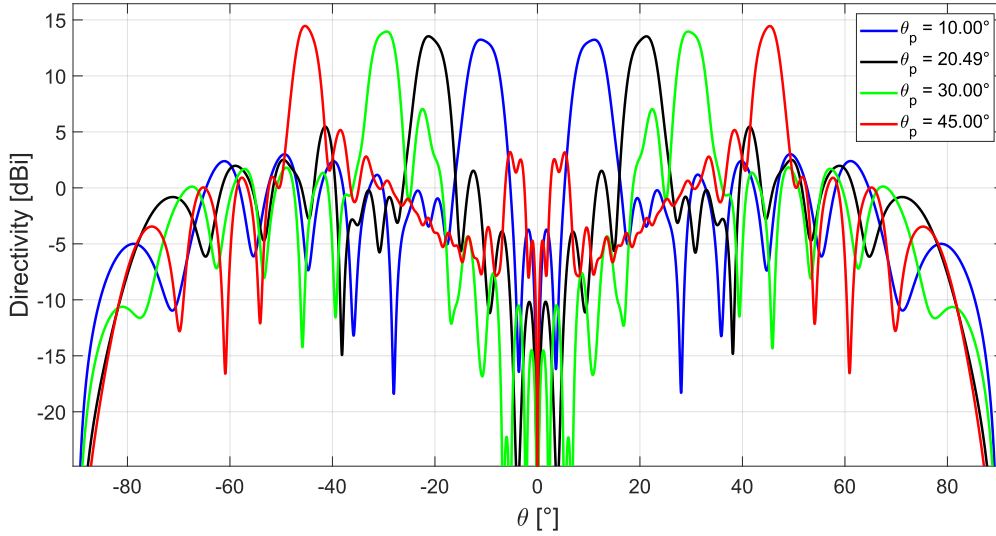
$$k_x^{(-1)} = 1.339 k_0 - k_0 = 0.339 k_0 \approx 0.362 k_0 \quad (2.41)$$

$$k_x^{(+1)} = 1.339 k_0 + k_0 = 2.339 k_0 \approx 2.384 k_0 \quad (2.42)$$

As expected, the -1 harmonic falls into the visible region, causing radiation of the structure at $\theta = \arcsin(0.362) = 21.22^\circ$. The same reasoning applies for the harmonics generated by the main negative wavenumber. Indeed, we observe in Figure 2.9b that the main directions of radiation are $\theta_{beam} = \pm 21.22^\circ$. It should be noted that there is no radiation at broadside ($\theta = 0^\circ$) because the spectral electric field is odd as the Green's function is even and the spectral current and excitation are odd. This is due to the even character of the impedance sheet.

Regarding the current distribution (Figure 2.8a), there are some differences with the unmodulated case (Figure 2.5a). Firstly, we clearly see the impact of the presence of several harmonics on the real and imaginary parts of the current distribution, which do not look like a pure sinus anymore. This is what allows radiation and it is caused by the modulation of the impedance as previously explained. It is also observable through the different peaks present in Figure 2.8b, some of which clearly are in the visible domain. Secondly, we clearly observe a decreasing average current in the modulated case. This is due to the power radiated by the antenna, which is of course much bigger for the modulated case.

Then, the impact of the value of θ_p is analyzed. The aim is to verify that the main beam of the directivity moves around this value. Simulations are done for $\theta_p = \{10.00^\circ, 20.29^\circ, 30.00^\circ, 45.00^\circ\}$ in Figure 2.10 and the directions of maximum directivity are summarized in Table 2.2. For each simulation, the antenna length is 20 times the periodicity of the impedance sheet. As the periodicity depends on θ_p , the antenna length is different for each simulation.


 Figure 2.10: Directivity for different beam directions θ_p .

θ_p	10.00°	20.49°	30.00°	45.00°
θ_{beam}	10.92°	21.22°	29.37°	45.37°

Table 2.2: Directions of maximum directivity.

We observe that the main beam is located very close to the theoretical radiation direction for each case, as expected. Furthermore, we observe a second beam located around $\theta = 4.59^\circ$ for $\theta_p = 45^\circ$. This is explained by the fact that the -2 harmonic falls into the visible spectrum as well in this case: $k_x^{(-1)} = 0.71 k_0$ and $k_x^{(-2)} = 0.08 k_0$.

Finally, we are interested in observing how a very long antenna will behave. To that aim, the initial set of parameters for the impedance sheet presented in Table 2.1 is considered. The length of the antenna is 250λ , which is 3.125 m. This is an extremely long antenna and the goal is to check how the results change when the length of the antenna is increased. The goal is to have a first idea of how an infinite antenna would behave. A specific code for infinite antennas will be developed and results will be compared in Chapter 3. The simulated current distribution is shown in Figure 2.11. The current is only shown for $|x| < 60\lambda$ for clarity but the same behavior is observable until the end of the antenna. The spectral electric field and the directivity are shown in Figures 2.12a and 2.12b, respectively.

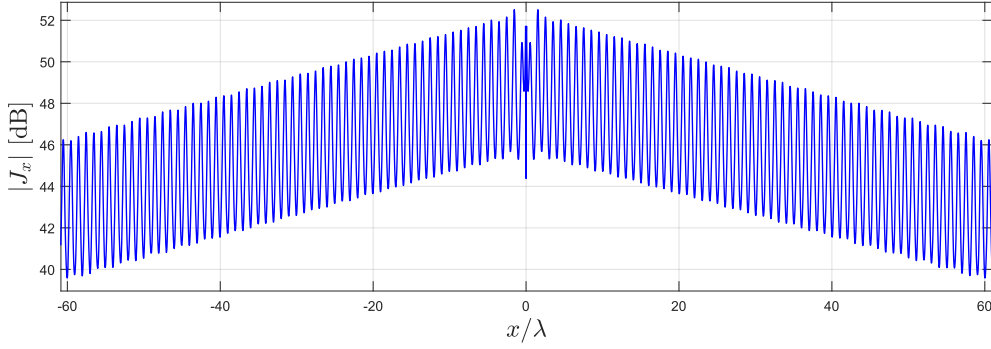


Figure 2.11: Spatial current distribution for a modulated MTS antenna of length $a = 250\lambda$.

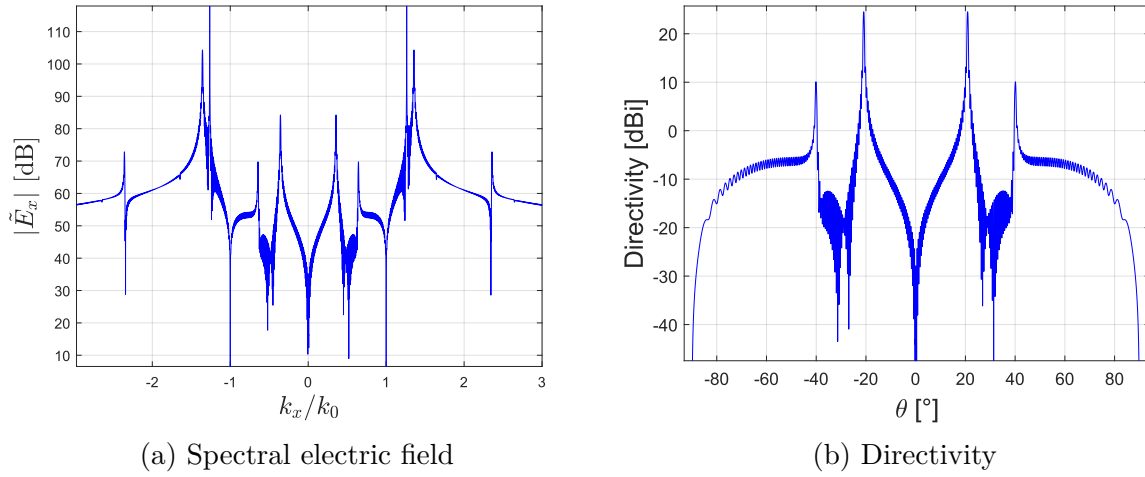


Figure 2.12: Spectral electric field and directivity for a modulated MTS antenna of length $a = 250\lambda$.

We observe a similar behavior for the current distribution which is oscillating and decreasing along the antenna as the distance to the feed increases. The spectral electric field and directivity are very similar to the case of a shorter antenna, except that the beams are much more narrow for a larger antenna. As the length of the antenna increases, the fields propagate along a wider domain, allowing sharper radiation patterns. In particular, we have a maximum directivity of 13.53 dB for $a = 20\lambda$ and 25.91 dB for $a = 250\lambda$, making the antenna more directive when its length increases.

Chapter 3

Analysis of 1D infinite periodic metasurface antennas

The goal of this chapter is to extend the work of the previous chapter about 1D finite antennas with one excitation to analyze periodic infinite 1D antennas. Such antennas are composed of an infinite array of phase shifted feeding points. It will be shown that it allows the antenna to scan over a certain region, if the appropriate phase shift is applied to each excitation and if the embedded element pattern of each excitation has an ideal rectangular shape. Of course, infinite antennas do not exist but it allows us to analyze and design antennas without considering parasitic effects at the edges of the antenna. Then, infinite antennas can be approximated by sufficiently long periodic antennas in practice. This chapter presents the MoM formulation, while some simulated results are shown and analyzed.

3.1 Problem statement

We consider a 1D infinite antenna along the x direction, translationally invariant along the y direction. The antenna has a periodicity of a because it is composed of an infinite repetition of unit cells of length a with the same surface impedance and excited with VED of same amplitudes, but with a phase shift of $-\psi$ between consecutive cells. Note that it means that the periodicity of the surface impedance is also a . The antenna is illustrated in Figure 3.1, where the reference cell is highlighted in red. The reference cell is the center cell going from $x = -a/2$ to $x = a/2$ with the reference phase shift of 0° .

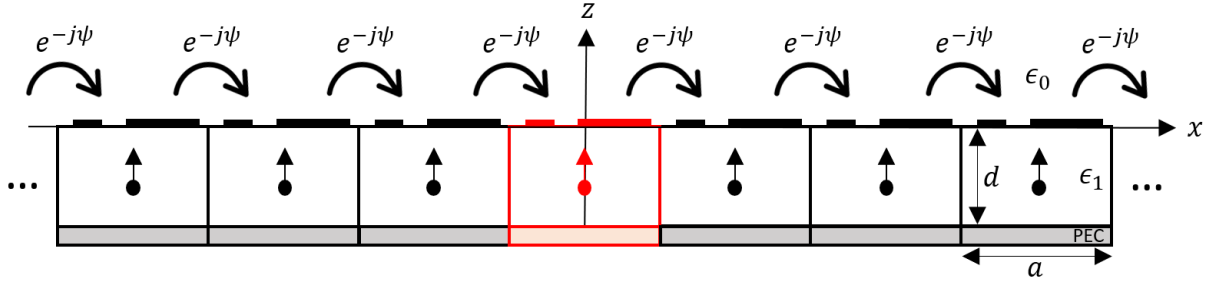


Figure 3.1: 1D periodic metasurface antenna.

Note that in this case both the MTS and the substrate are infinitely long along the x direction. We consider the same values for the frequency and the substrate parameters as for the finite case: $f = 24$ GHz, $d = 1.524$ mm and $\epsilon_r = 3.66$. The phase shift ψ and the unit cell length a are left as parameters for the moment.

3.1.1 Motivation for the study of infinite antennas

Firstly, it is important to explain why it is relevant to study infinite metasurface antennas. In fact, this type of antenna is well suited for the design of very directive radiation patterns that radiate a main beam in a given direction. Then, the phase shift between consecutive cells can be tuned to change the main beam direction and offer scanning capabilities. A reminder about the total radiation pattern of a MTS antenna excited by a phased array is provided below to mathematically prove the scanning ability of the antenna.

The total radiation pattern of a MTS antenna excited by an array of feeding points is the sum of the pattern of each VED multiplied by its corresponding phase shift. As the antenna is infinite and all the VED are identical up to a phase shift, the total pattern is the product between the embedded element pattern (EEP) of each vertical elementary dipole in the metasurface and the array factor (AF), taking into account the effect of the array of excited elements [33]:

$$\mathbf{F}(\hat{\mathbf{u}}) = \sum_{n=-\infty}^{+\infty} \mathbf{F}_{n,0}(\hat{\mathbf{u}}) e^{jk_0 \rho_n \cdot \hat{\mathbf{u}}} \quad (3.1)$$

$$= \sum_{n=-\infty}^{+\infty} \mathbf{F}_0(\hat{\mathbf{u}}) e^{-jn\psi} e^{jk_0 na \sin \theta} \quad (3.2)$$

$$= \mathbf{F}_0(\hat{\mathbf{u}}) \sum_{n=-\infty}^{+\infty} e^{jn(-\psi + k_0 a \sin \theta)} \quad (3.3)$$

$$= \underbrace{\mathbf{F}_0(\theta)}_{EEP} \underbrace{R(\theta)}_{AF} \quad (3.4)$$

where $\boldsymbol{\rho}_n = na \hat{\mathbf{x}}$ is the vector from the reference cell to the n^{th} cell and $\hat{\mathbf{u}} = \sin \theta \hat{\mathbf{x}} + \cos \theta \hat{\mathbf{z}}$ is the vector for the direction of observation. In the case of an infinite array of excitations, the array factor simplifies to a Dirac comb (see Appendix C.1 for more details):

$$R(\theta) = \sum_{n=-\infty}^{+\infty} e^{jn\Psi} \quad \text{with } \Psi = -\psi + ak_0 \sin \theta \quad (3.5)$$

$$= 2\pi \sum_{m=-\infty}^{+\infty} \delta(\Psi - 2\pi m) \quad (3.6)$$

where Ψ is the total phase shift which is the sum of the imposed phase shift ($-\psi$) and the phase shift due to the difference of propagation distance between consecutive cells ($ak_0 \sin \theta$). We observe that the array factor is only non zero for specific values of θ . The main lobe is located at $\Psi = 0$, while the grating lobes are located at non zero multiples of 2π . The idea is to use the metasurface to produce sharp embedded element patterns (EEP) that have a nearly rectangular shape to only keep one Dirac delta in the final pattern (main lobe), meaning that the antenna only looks in a specific direction depending on the imposed phase shift ψ . In other words, those sharp patterns will naturally filter out the grating lobes of the array factor (AF) that are in the visible region, and allow for a larger inter-element spacing, at the expense of a smaller field of view. This is exactly what is needed for scanning applications such as radars. Note that such sharp radiation patterns correspond to very wide spatial electric field (or equivalent currents) distributions whose visible part should ideally have a sinc function shape [34]. It means that the current distributions generated by the different feeds should widely overlap, which is very practical (and seems possible) with metasurface antennas, since they allow currents to smoothly spread over the surface, see Figure 3.2 below for an illustration.

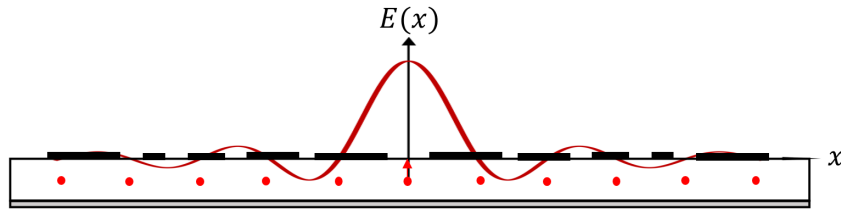


Figure 3.2: Sinc shape of the electric field contributing to radiation.

In practice, the antenna will not be infinite but we hope that if it is sufficiently long, its behavior will be similar to the one of an infinite antenna. It can be verified beforehand with a finite antenna simulation. In the case of an array with N excitation ports, the expression of the array factor is not a Dirac comb anymore and is given by:

$$R(\theta) = e^{j(N-1)\frac{\Psi}{2}} \frac{\sin\left(\frac{N\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)} \quad \text{with } \Psi = -\psi + ak_0 \sin \theta \quad (3.7)$$

which is a function with a periodicity of 2π along Ψ and with $N-2$ sidelobes over each period. The whole idea of filtering the grating lobes with a sharp EEP is illustrated in Figure 3.3 below for an antenna with a finite array of excitations. Note that the width of the rectangular EEP must be equal to the periodicity of the array factor ($2\pi/a$ along k_x), such that the grating lobes filtering works up to the limit scanning angles. The EEP is centered in $k_x = 0$ to allow a symmetric scanning. So, the range of scanning angles is limited by the unit cell size a and applying a phase shift of $-\psi$ between consecutive cells allows one to radiate a beam towards $\theta = \arcsin(\psi/(ak_0))$.

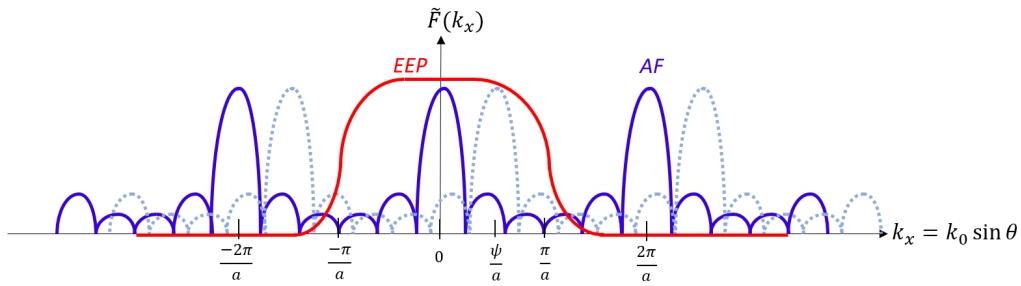


Figure 3.3: Sharp filtering of the grating lobes of the array factor (AF, dashed with scanning) by the embedded element pattern (EEP).

In the following sections, we focus on the computation of the EEP, keeping in mind that a sharp rectangular pattern is desired.

3.1.2 Embedded element pattern computation

The embedded element pattern (EEP) is the pattern radiated by the antenna when one single port is excited in the metasurface and all the other ports are switched off. A first technique is to compute it exactly as in Chapter 2 with an extremely long (nearly infinite) antenna. If the antenna is sufficiently long, the current at the edges of the antenna will be very small due to the losses by radiation across the antenna and this current will not contribute to the radiation pattern anymore. However, this technique requires a very long antenna, which is computationally not effective and supposes that the current is negligible from a certain distance from the feed.

There exists another strategy to determine the embedded element pattern of an infinite antenna, which has been demonstrated in [35]. That paper shows that the EEP in direction θ computed on the infinite MTS can be obtained from the pattern computed only on the unit cell when all the elements are excited with a phase shift $\psi = ak_0 \sin \theta$ between two consecutive elements. In other words, the integral over an infinite antenna of the current caused by one excited element is replaced by the integral over a finite cell

of the current generated by an infinite array of active phase-shifted elements. In one dimension, it gives:

$$\tilde{\mathbf{F}}_0(k_x = k_0 \sin \theta) = \int_{-\infty}^{+\infty} \mathbf{J}_0(x) e^{jxk_0 \sin \theta} dx \quad (3.8)$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} \mathbf{J}_\infty(x, \psi = ak_0 \sin \theta) e^{jxk_0 \sin \theta} dx \quad (3.9)$$

where $\mathbf{J}_0(x)$ is the current density generated by one single element and $\mathbf{J}_\infty(x, \psi)$ is the current density generated when all the elements are excited with a phase shift of ψ between consecutive elements. Thus, the EEP can be computed when all elements are excited with a specific phase shift for each direction of interest, meaning that the MoM system of equations should be built and solved again for each direction. This is nevertheless more efficient because it drastically reduces the domain of integration and we will show that the MoM equations are greatly simplified for a periodic antenna.

3.2 Method of Moments

This section describes the Method of Moments formulation for a periodic and infinite MTS antenna. It starts from the formulation for a finite antenna developed in Section 2.2.1 and shows how it can be simplified for a periodic antenna.

3.2.1 Method of Moments formulation

First, a quick remark regarding the location of basis and testing functions is necessary. Indeed, as the antenna has a periodicity (up to a phase shift) of a , the basis and testing functions are implicitly repeated with a period a . This allows us to only use basis and testing functions on the reference cell, from $x = -a/2$ to $x = a/2$. Those functions should be placed such that the periodicity is respected between consecutive cells meaning that there is no Gaussian function centered in $x = a/2$ because it is the same function (up to a phase shift) as the one centered in $x = -a/2$. The goal is that the entire antenna is covered after periodizing the basis functions with their corresponding phase shift. This is illustrated in Figure 3.4.

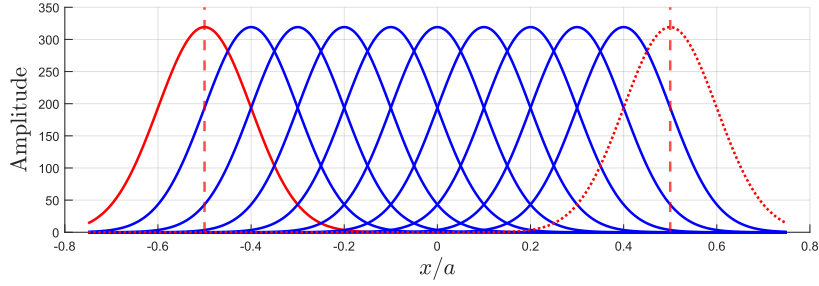


Figure 3.4: Basis functions on the reference cell (solid lines). They are implicitly repeated in the other cells, as shown with the red dashed function which is the repetition of the first red function.

The MoM system of equations can be built and solved with those basis and testing functions to find the coefficients associated with each basis function. It allows to approximate the reference phase shift current distribution:

$$J_0(x) = \sum_{n=1}^N i_n F_b^n(x) \quad (3.10)$$

The total current can be obtained on the whole antenna by periodizing the reference current and by multiplying it each time by the corresponding phase shift:

$$J_{tot}(x) = J_0(x) \circledast \sum_{m=-\infty}^{+\infty} \delta(x - ma) e^{-jm\psi} \quad (3.11)$$

$$= J_0(x) \circledast \sum_{m=-\infty}^{+\infty} \delta(x - ma) e^{-j\frac{x}{a}\psi} \quad (3.12)$$

Then, the Fourier transform of this periodic current function can be computed. Note that the convolution becomes a product in the spectral domain and that the Fourier transform of a Dirac comb remains a Dirac comb (see Appendix C.2 for details):

$$\tilde{J}_{tot}(k_x) = \tilde{J}_0(k_x) \frac{2\pi}{a} \sum_{p=-\infty}^{+\infty} \delta(k_x - p\frac{2\pi}{a} - \frac{\psi}{a}) \quad (3.13)$$

$$= \frac{2\pi}{a} \sum_{p=-\infty}^{+\infty} \tilde{J}_0(k_x^p) \delta(k_x - k_x^p) \quad (3.14)$$

with k_x^p the Floquet wavenumbers:

$$k_x^p = \frac{\psi}{a} + p\frac{2\pi}{a} \quad (3.15)$$

We observe that, since the current is a periodic function in the spatial domain, it is a sampled function in the spectral domain. This is a very important observation as it will simplify the integrals for the computation of $\mathbf{Z}_G$ and $\mathbf{v}$ into summations. For the substrate matrix, it yields:

$$Z_G(m, n) = \frac{1}{2\pi} \int_{k_x} \tilde{F}_t^m(-k_x) \tilde{G}_{tot}^{xx}(k_x) \tilde{F}_b^n(k_x) dk_x \quad (3.16)$$

$$= \frac{1}{2\pi} \int_{k_x} \tilde{F}_t^m(-k_x) \frac{2\pi}{a} \sum_{p=-\infty}^{+\infty} \tilde{G}_0^{xx}(k_x^p) \delta(k_x - k_x^p) \tilde{F}_b^n(k_x) dk_x \quad (3.17)$$

$$= \frac{1}{a} \sum_{p=-\infty}^{+\infty} \tilde{F}_t^m(-k_x^p) \tilde{G}_0^{xx}(k_x^p) \tilde{F}_b^n(k_x^p) \quad (3.18)$$

Similarly, we have the following simplifications for the excitation vector:

$$v(n) = \frac{-1}{2\pi} \int_{k_x} \tilde{E}_{i,tot}(k_x) \tilde{F}_t^n(-k_x) dk_x \quad (3.19)$$

$$= \frac{-1}{2\pi} \int_{k_x} \frac{2\pi}{a} \sum_{p=-\infty}^{+\infty} \tilde{E}_{i,0}(k_x^p) \delta(k_x - k_x^p) \tilde{F}_t^n(-k_x) dk_x \quad (3.20)$$

$$= \frac{-1}{a} \sum_{p=-\infty}^{+\infty} \tilde{E}_{i,0}(k_x^p) \tilde{F}_t^n(-k_x^p) \quad (3.21)$$

$$(3.22)$$

The simplification from integrals to summations allows us to accelerate the implementation. Moreover, the complex contour is not necessary anymore if we make sure that no sample is located too close to the poles. The substrate matrix $\mathbf{Z}_G$ is still a Toeplitz matrix but it is not symmetric anymore because the sampling on the Floquet wavenumbers is not symmetric around $k_x = 0$. The integral for the computation of $\mathbf{Z}_{IBC}$ is unchanged as it is computed in the spatial domain. However, one should keep in mind that the basis and testing functions are periodic up to a phase shift such that there is for example a strong interaction between the first basis function and the last testing function (see Figure 3.4) because the latter is also located just before the first one, with a different phase.

3.2.2 Radiation pattern and directivity

The exact same technique as for finite antennas is used in this case to compute the radiation pattern and the directivity, refer to Section 2.2.2 for details. However, note that in this case both the spatial and spectral approaches are possible for the computation of the spectral electric field in the structure, as we consider an infinite MTS printed on an infinite dielectric layer. Both approaches start from the current distribution $J(x)$ and are detailed below.

Spatial-domain approach

The electric field is first computed in the spatial domain thanks to the impedance sheet:

$$E(x) = J(x)Z_s(x) \quad (3.23)$$

Then, its Fourier transform is computed numerically on the reference cell:

$$\tilde{E}(k_x) = \int_{-\frac{a}{2}}^{\frac{a}{2}} E(x) e^{jk_x x} dx \quad (3.24)$$

Spectral-domain approach

The Fourier transform of the current is first computed numerically on the reference cell:

$$\tilde{J}(k_x) = \int_{-\frac{a}{2}}^{\frac{a}{2}} J(x) e^{jk_x x} dx \quad (3.25)$$

Then, the spectral electric field is obtained through the Green's function:

$$\tilde{E}(k_x) = \tilde{J}(k_x) \tilde{G}^{xx}(k_x) + \tilde{E}_{wall}(k_x) \quad (3.26)$$

3.3 Results

Similarly to the case of finite antennas in Section 2.3, simulation results are given in this section for infinite 1D MTS antennas. The code implements the methodology described in this chapter. The MTS antenna is modeled with the same impedance sheet as in equation (2.37), which is reminded here for convenience:

$$Z_s(x) = jX_0(1 + M \cos(k_{sw}x - k_0x \sin \theta_p)) \quad (3.27)$$

with X_0 the average surface impedance, M the modulation depth, k_{sw} the main SW wavenumber propagating in the MTS antenna, k_0 the free-space wavenumber and θ_p the main direction of radiation. Care must be taken for the periodicity of this impedance which is:

$$p = \frac{2\pi}{k_{sw} - k_0 \sin \theta_p} \quad (3.28)$$

Indeed, the antenna is infinite in the x direction but has a periodicity of a which is the length of each cell. So, the length of each cell must be a positive multiple of the periodicity of the impedance sheet:

$$a = n p \quad n \in \mathbb{N}_0 \quad (3.29)$$

Unmodulated metasurface antenna

Firstly, we consider the unmodulated case with $X_0 = -772.50 \, \Omega$. The size of each cell is arbitrarily chosen to $a = \lambda$ but it does not impact the results and any cell size can be chosen as the impedance is constant. The spectral electric field is shown in Figure 3.5 for the spatial and the spectral approaches introduced in equations (3.23) and (3.26), respectively.

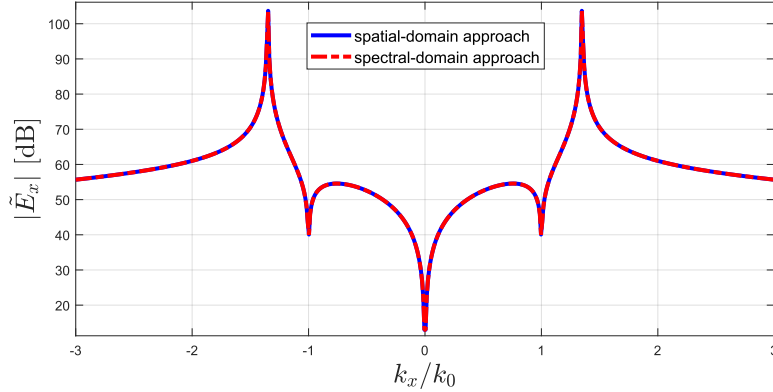


Figure 3.5: Spectral electric field for both approaches.

Firstly, we observe that both approaches lead to same results, as expected. The shape is very similar to the one in Figure 2.6 for a finite antenna except that it looks smoother for an infinite antenna. This is because there is no reflection or undesired effects at the edges of the antenna, as the structure has an infinite length. Moreover, the current can propagate until infinity so it is theoretically totally attenuated (or negligible) beyond a certain distance from the feed. Again, we observe a main peak around $k_x = 1.35 k_0$, which corresponds to the main wavenumber of the electric field propagating in the antenna. But it does not lead to radiation, as it is located outside the visible region. One major difference with the finite case is that we do not observe the substrate pole in $k_x = 1.26 k_0 = k_{sw,substrate}$. This is logical, as that pole was linked to the wave propagating in the infinite substrate beyond the finite MTS, but both the MTS and the substrate are infinite in this case.

Modulated metasurface antenna

Then, the modulated surface impedance reminded in equation (3.27) is considered, with the same set of parameters as in Table 2.1. Note that this impedance has a periodicity of λ . The simulated spectral electric field is shown in Figures 3.6 for different cell sizes. We observe as expected that the results are independent from the cell size if the size is a positive multiple of the impedance periodicity. Then, we observe, as it was the case for

the finite MTS, that harmonics are generated around the main surface wave wavenumber. The -1 harmonic falls in the visible region and it is more clear in this case that the -2 harmonic is also located in the visible region, but has a lower magnitude:

$$k_x^{(-1)} = 1.36 k_0 - k_0 = 0.36 k_0 \quad (3.30)$$

$$k_x^{(-2)} = 1.36 k_0 - 2 k_0 = -0.64 k_0 \quad (3.31)$$

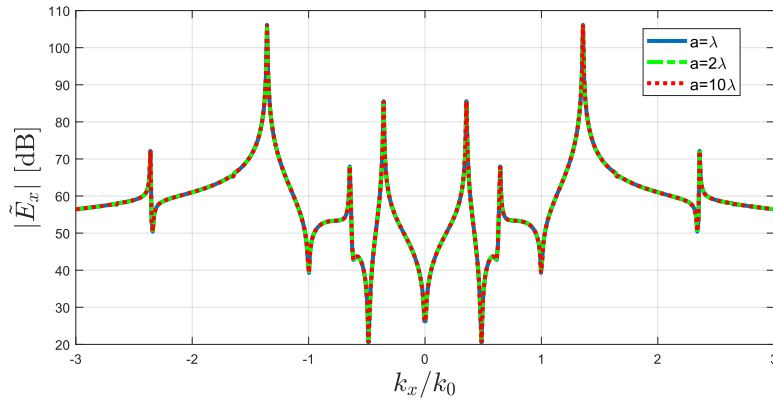


Figure 3.6: Spectral electric field for different cell sizes.

Figure 3.7 compares the results obtained for different basis functions. The implementation with real Gaussian basis and testing functions used in the framework of this master thesis is compared with the code of Dr. Modeste Bodehou¹ who used the same methodology but with complex exponential basis and testing functions. This comparison is a solid validation to observe the nearly perfect agreement between both results in Figure 3.7.

¹He is a researcher at UCLouvain working on MTS antennas. He is collaborating with Prof. Christophe Craeye and myself for my master thesis.

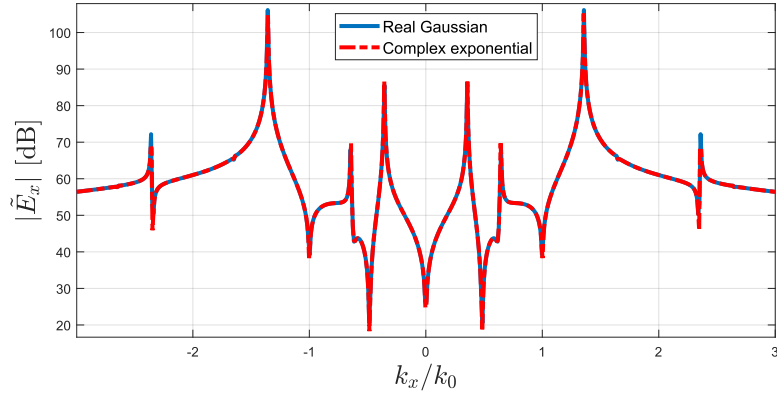


Figure 3.7: Spectral electric field for different basis and testing functions.

Finally, we would like to compare the results of an infinite antenna with a finite but very long antenna. To that aim, we compare the results obtained in Figures 2.12a and 2.12b for a modulated finite antenna of length 250λ with the results in Figure 3.6 and the resulting directivity for an infinite antenna. Both antennas are modulated with the same impedance sheet and have exactly the same parameters, except for the antenna length. The comparison is shown in Figures 3.8a and 3.8b below for the spectral electric field and the directivity, respectively.

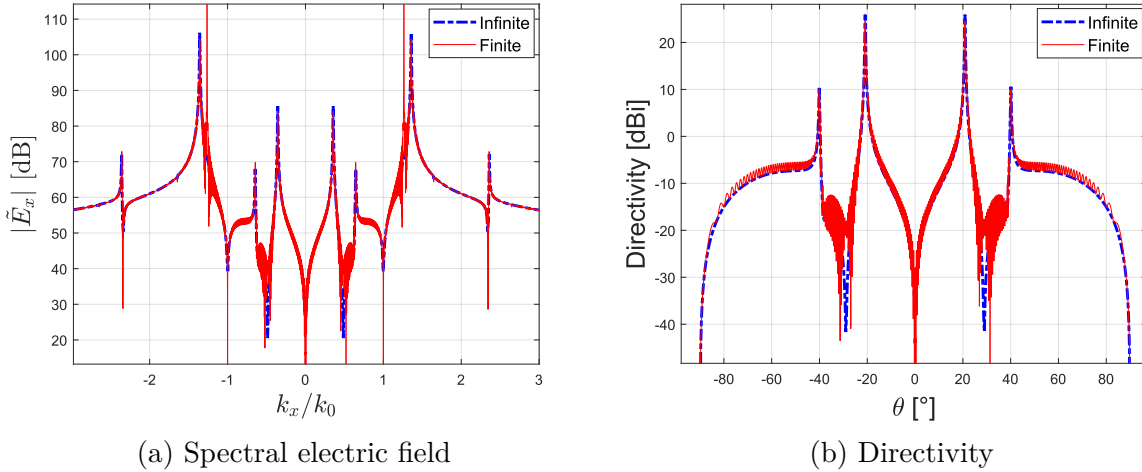


Figure 3.8: Comparison of the spectral electric field and directivity between a finite ($a = 250\lambda$) and infinite modulated MTS antenna.

We observe a strong correlation between both cases. Intuitively, it seems logical that a very long antenna should tend towards an infinite antenna. In particular, the results

are influenced by the losses in the structure and more precisely by the rate at which the current decays in the MTS. In theory, both results should be very close if the finite antenna is sufficiently long, such that the current becomes negligible at its edges. Note that there would still be a difference regarding the substrate poles, that would only be present in the finite case, as they are associated with the fields propagating in the structure beyond the MTS. As the current decays very slowly in a 1D antenna, the antenna length should be very long to obtain results similar than with an infinite antenna. However, this slow decay of the current is specific to one-dimensional antennas and we expect more current losses for two-dimensional antennas.

Chapter 4

Design of 1D metasurface antennas

This chapter is devoted to the design of 1D MTS antennas. In particular, we are interested in optimizing the surface impedance of the antenna, such that the embedded element pattern (EEP) of each feed is ideally very sharp and rectangular, as shown in Figure 4.1. The idea is to suppress the grating lobes of the array factor with the EEP, as illustrated in Figure 3.3 of Section 3.1.1. The design procedure is carried out for infinite periodic antennas. Then, the impact of array truncation (finite number of cells) is analyzed.

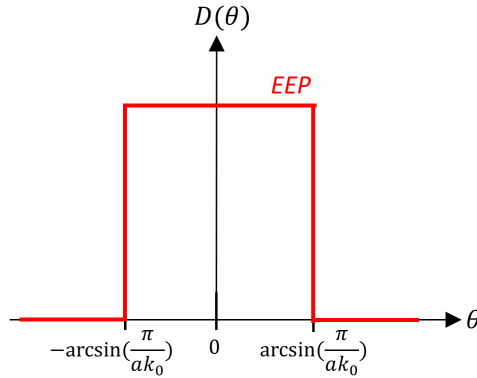


Figure 4.1: Directivity of the ideal embedded element pattern (EEP).

Note that the size of the unit cell a is an important parameter, that should be judiciously chosen. In fact, we are interested in a cell size bigger than $\lambda/2$ in order to increase the spacing between feeding points, compared to a classical phased array. The goal is to reduce the number of electronic channels needed to configure the antenna for scanning. However, the maximum range of scanning angles is limited by the cell size and is given by (cf. Section 3.1.1 for more details):

$$|\theta_{scan}| \leq \arcsin\left(\frac{\pi}{ak_0}\right) \quad (4.1)$$

We clearly observe the trade-off between the feeding point spacing and the scanning range of the antenna. In practice, the cell size is fixed by the specifications of the application. For the design methodology presented in this chapter, we propose to consider a cell size of $a = 2\lambda$. It is an interesting cell size as it increases the feed spacing by a factor 4 (1D antenna), and still allows a scanning range of the order of 28.96° :

$$|\theta_{scan}| \leq \arcsin\left(\frac{\pi}{2\lambda\frac{2\pi}{\lambda}}\right) = \arcsin\left(\frac{1}{4}\right) = 14.48^\circ \quad (4.2)$$

This type of antenna would meet the specifications of many radar applications. For the optimization process, we consider an operating frequency of $f = 24$ GHz and a substrate relative permittivity and thickness of $\epsilon_r = 3.66$ and $d = 1.524$ mm, respectively. The performance of the optimized antenna, as well as the impact of the cell size on the design methodology will be evaluated in this chapter.

4.1 Design of 1D infinite periodic metasurface antennas

4.1.1 Design methodology

The design methodology consists of optimizing the impedance sheet, such that the obtained embedded element pattern is close to the ideal one. The optimization starts from an initial impedance and minimizes a well chosen cost function at each iteration to reach the ideal pattern. Note that the cost function must be properly defined and will gather three essential criteria: the sharpness of the transitions around $|\theta| = \arcsin(\pi/(ak_0))$, the flatness of the pattern for $|\theta| < \arcsin(\pi/(ak_0))$, and the maximum level of the grating lobes for $|\theta| > \arcsin(\pi/(ak_0))$. The function to be optimized is the surface impedance sheet $Z_s(x)$. Note that this impedance must have a periodicity of $a = 2\lambda$, and must be purely imaginary (no resistive part). Hence, a practical way to impose those constraints in the optimization loop is to decompose the impedance into a Fourier series and to optimize its Fourier series coefficients, denoted as $X[k]$:

$$Z_s(x) = \sum_{k=-\infty}^{+\infty} X[k]e^{jk\omega_0 x} \quad \text{with } \omega_0 = \frac{2\pi}{a} \quad (4.3)$$

If we impose the impedance to be purely imaginary (link between the positive and negative coefficients), and if the series is truncated to $2N + 1$ coefficients, its expression simplifies to (see Appendix C.3 for details):

$$Z_s(x) = j \left(\text{Im}(X[0]) + 2 \sum_{k=1}^N \text{Im}(X[k]) \cos(k\omega_0 x) + \text{Re}(X[k]) \sin(k\omega_0 x) \right) \quad (4.4)$$

which contains $2N + 1$ real parameters for the optimization.

Everything is set for the optimization, except for the initial impedance that must be well chosen, as it will strongly affect the optimization result. We have shown through equation (2.39) that the periodicity of the impedance, and thus its frequency, imposes the main direction of radiation. Based on this principle, we propose an impedance whose spatial frequency varies linearly, such that its continuous range of frequencies matches a continuous range of radiation directions. The goal is to design the impedance such that its radiation direction range is $|\theta| < \arcsin(\pi/(ak_0))$. This can theoretically be achieved with a chirp impedance whose frequency varies linearly on the reference cell. Its mathematical expression on the reference cell is given by:

$$Z_s(x) = \begin{cases} jX_0(1 + M_0 \sin(2\pi(\frac{f_2-f_1}{a}x^2 + f_2x) + \phi)) & \text{if } \frac{-a}{2} \leq x < 0 \\ jX_0(1 + M_0 \sin(2\pi(\frac{f_1-f_2}{a}x^2 + f_2x) + \phi)) & \text{if } 0 \leq x \leq \frac{a}{2} \end{cases} \quad (4.5)$$

where X_0 is the average impedance, M_0 is the modulation depth, ϕ is the phase shift which is an additional parameter and f_1 and f_2 are the frequencies at the edges and the center of the cell, respectively. Depending on the parameters, the frequency variation is not necessarily visible to the naked eye but the trend over one unit cell is highlighted in Figure 4.2 for convenience.

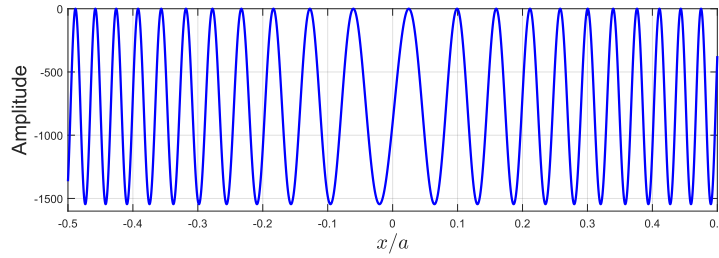


Figure 4.2: Example of a chirp impedance in one cell.

According to equation (2.38), the frequency range of the impedance must be:

$$f_1 = \frac{k_{sw} - k_0 \sin(\theta_{p1})}{2\pi} \quad (4.6)$$

$$f_2 = \frac{k_{sw} - k_0 \sin(\theta_{p2})}{2\pi} \quad (4.7)$$

where θ_{p1} and θ_{p2} are the edges of the radiating area, and are chosen to meet the specifications. For example, we can choose $\theta_{p1} = -\arcsin(\pi/(ak_0))$ and $\theta_{p2} = 0$ such that the harmonics of the positive and negative main SW wavenumber contribute each to one side of the radiation pattern. The simulated directivity for $\theta_{p1} = -\arcsin(\pi/(ak_0))$,

$\theta_{p2} = 0$, $X_0 = -772.50 \, \Omega$, $M_0 = 1$, $\phi = 0$ and $a = 2\lambda$ is shown in Figure 4.3 below. Note that it has been obtained by decomposing $Z_s(x)$ into a Fourier series (with $N = 15$) to ensure the periodicity of the impedance.

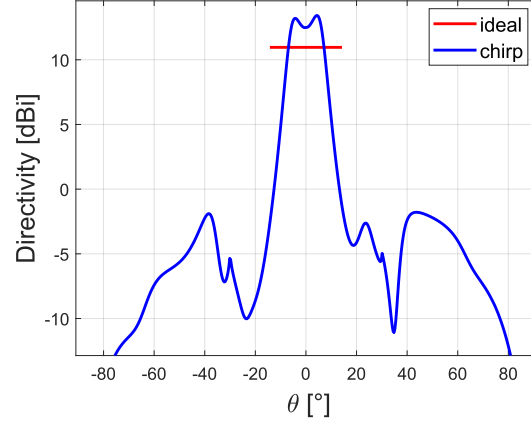


Figure 4.3: Directivity of the EEP obtained with the chirp impedance, for $\theta_{p1} = -14.48^\circ$, $\theta_{p2} = 0^\circ$, $X_0 = -772.50 \, \Omega$, $M_0 = 1$, $\phi = 0$ and $a = 2\lambda$.

We observe that the global behavior of the embedded element pattern is as expected. Of course, it is not perfect but it appears as an appropriate start to launch the optimization loop.

4.1.2 Results

The optimization is conducted with *fminsearch*, a routine available in the MATLAB library, based on the Nelder-Mead simplex algorithm [36]. The optimization starts from the initial chirp impedance proposed in Section 4.1.1, with $N = 15$. After several tests, we can observe that among the $2N + 1$ parameters to optimize, only 2 parameters have a major impact on the resulting EEP. So, only those two parameters are kept for the optimization and the optimized impedance sheet is given by:

$$Z_s(x) = jX_0 (1 + M \sin(3\omega_0 x)) \quad (4.8)$$

with $X_0 = -933.63 \, \Omega$ and $M = 1.294$. In fact, only the average impedance and the third odd harmonic seem to be useful to shape the EEP. The obtained EEP is shown in Figure 4.4 below.

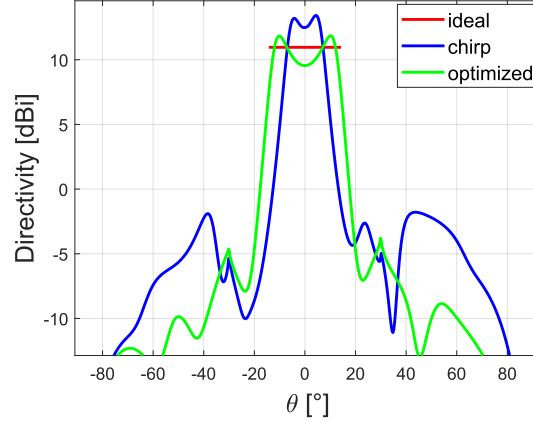
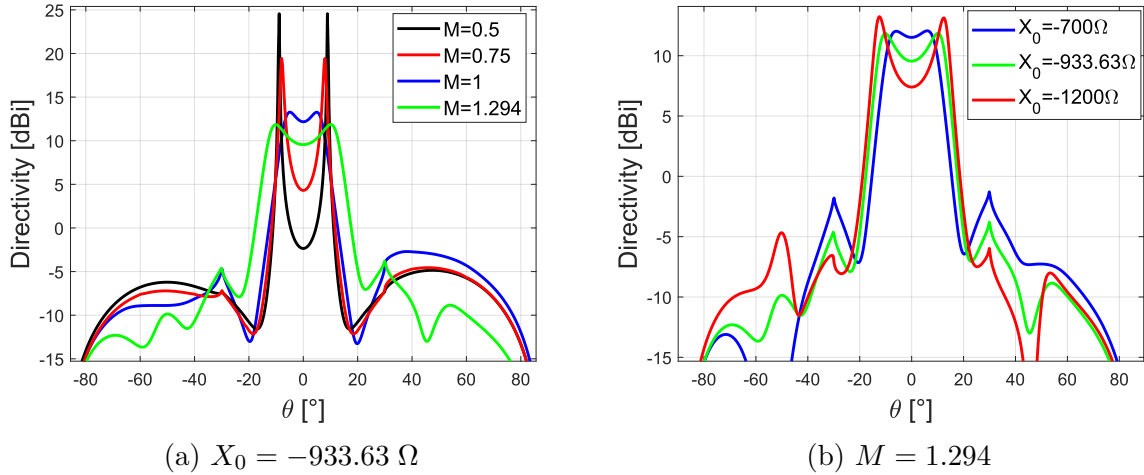


Figure 4.4: Directivity of the optimized EEP, with $X_0 = -933.63 \, \Omega$ and $M = 1.294$.

We clearly see the improvement in terms of sidelobe level (SLL) and width of the pattern in the radiating band. This is a very interesting result as it demonstrates that a simple sinusoidal impedance can produce an EEP close to the ideal one. To understand how it works, the impact of the modulation depth and the average impedance are analyzed in Figures 4.5a and 4.5b, respectively.



(a) $X_0 = -933.63 \, \Omega$

(b) $M = 1.294$

Figure 4.5: Impact of X_0 and M on the EEP directivity.

Firstly, we clearly observe in Figure 4.5a that both -1 harmonic peaks fall into the visible region, around $\theta = \pm \arcsin(\pi/(ak_0))$. Then, we observe that the modulation depth plays an important role in the flatness of the pattern in the radiating band. Indeed, a larger modulation depth produces more leakage and, in turn, radiation from a smaller

portion of the MTS. That finally leads to broader patterns. We can conclude that increasing M is a powerful tool to flatten the pattern in the radiating band, as two peaks on either sides of the rectangular window may merge. Secondly, we observe in Figure 4.5b that the average impedance allows one to modify the main SW wavenumber propagating in the antenna k_{sw} , and consequently to change the location of the -1 harmonic falling into the visible region. This parameter has also an impact on the flatness of the pattern as the hollow at the center of the pattern increases if the visible peaks are farther apart. Note that the impedance is purely anti-symmetric, allowing radiation at broadside, which would be impossible with a purely symmetric impedance. This allows us to simplify the design methodology by only selecting the odd harmonic of the impedance that will generate -1 harmonics near the edges of the radiating band. Then, its average and modulation depth are optimized to reach a better performance.

Regarding the quality of the obtained EEP in Figure 4.4, we observe that the transitions around $|\theta| = \arcsin(\pi/(ak_0))$ are very sharp, as expected. Furthermore, the maximum directivity is 11.84 dB and the directivity at $\theta = 0^\circ$ is 9.55 dB, which is 2.29 dB below the maximum directivity. We can conclude that the pattern is sufficiently flat in the radiating band, if we authorize a 3 dB difference in this band. Based on this constraint, we can find the maximum scanning angle which is defined as the angle at which the directivity is 3 dB below the maximum directivity. It gives a maximum scanning angle of $|\theta_{max}| = 13.54^\circ$, which is close to the theoretical maximum angle of $|\theta_{scan}| = 14.48^\circ$.

The level of the highest grating lobe is -3.79 dB (highest value outside the rectangular shape), which gives a sidelobe level (SLL) of -15.63 dB below the maximum directivity. This provides attenuation for the grating lobes outside the scanning region, but might not be sufficient for many applications. So, we are interested in investigating solutions to improve the SLL up to -20 dB or -25 dB. To that aim, the cost function is modified to give more importance to the SLL criterion. Also, other harmonics are reintroduced and different initial impedances are used for the optimization loop. However, no improvement on the result was observed. Note that we observed in Figure 3.5 that there is a little bit of radiation, even with an unmodulated impedance, which might explain why it is difficult to obtain lower grating lobes. The abrupt character of the generated current close to the feed might also be responsible for the significant radiation outside the rectangular domain.

4.1.3 Feed design

This section shows how a smart design of the feed can help to improve the antenna performance. The key is to take advantage of the infinite and periodic aspect of the impedance. As the actual periodicity of the impedance is $2\lambda/3$, the feed can be translated by any multiple of $\lambda/3$, without affecting the amplitude of its embedded element pattern. The potential locations of the feed are illustrated in Figure 4.6.

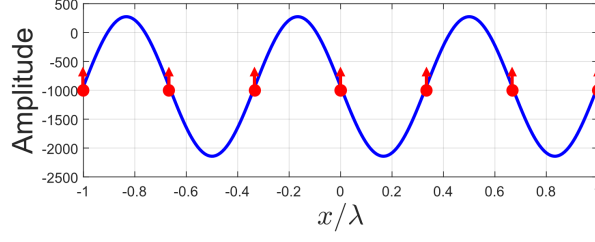


Figure 4.6: Impedance sheet in one unit cell, with the location of feeds that would have the same EEP amplitude.

The idea is to use a small array of feeds within each cell such that its array factor increases the gain of the embedded element pattern at broadside, and possibly also reduces the SLL. As we are interested in increasing the gain at broadside, the feeds within a same cell are all in phase. For a first evaluation of the performance, we consider the same amplitude for all feeds. The impact of the feed design is shown in Figure 4.7 for different configurations: one feed in $x = [0]$, two feeds in $x = [-\lambda/3; \lambda/3]$ and four feeds in $x = [-\lambda; -\lambda/3; \lambda/3; \lambda]$.

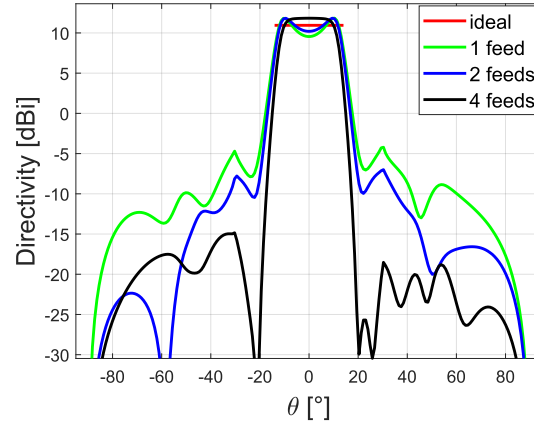


Figure 4.7: EEP directivity for different feed designs, with $X_0 = -933.63 \Omega$ and $M = 1.294$.

For an array of four feeds, we clearly observe a huge improvement in terms of SLL and flatness of the EEP in the radiating band. The SLL is reduced to -26.67 dB, but it comes with the reduction of the field of view to $|\theta_{max}| = 11.78^\circ$. It shows that the feed can have a real impact on the performance of the antenna in terms of SLL, flatness and scanning range. In particular, the feed can be designed such that the antenna meets the required specifications of a given application.

A last optimization is carried out to optimize simultaneously the amplitude of each feeding point, as well as the average and modulation depth of the impedance. The positions of the feeds are fixed to $x = [-\lambda; -\lambda/3; \lambda/3; \lambda]$. As we know that the main effect of the array is to increase the gain at broadside, we expect to improve the flatness and scanning range of the EEP with smaller X_0 and M . Indeed, the optimization gives the following parameters: $X_0 = -1379.40 \Omega$, $M = 1.282$ and $w = [1.117; 0.727; 0.646; 1.270]$ where w contains the amplitude associated with each feed. The obtained EEP is shown in purple in Figure 4.8 for different zoom ranges, where the green and black curves are the same as in Figure 4.7 ($X_0 = -933.63 \Omega$ and $M = 1.294$).

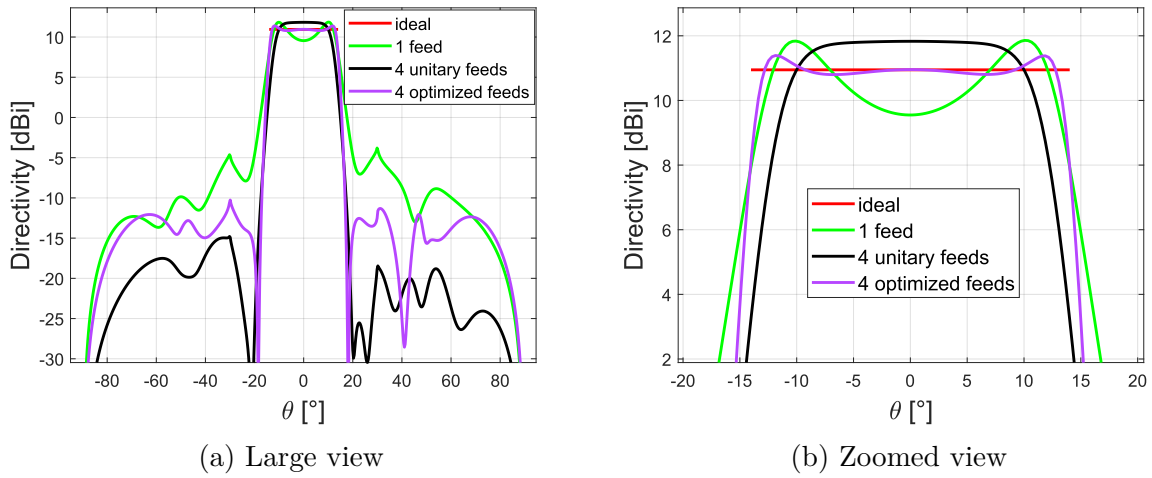


Figure 4.8: Optimized EEP directivity with four feeds.

We observe on those Figures that the optimized EEP is larger and sharper but has higher grating lobes. Nevertheless, the SLL is -21.65 dB which is still a great result. The EEP is extremely flat in the radiating band and the scanning range is $|\theta_{max}| = 13.95^\circ$, which is excellent for many applications.

Note that the final design has a feed spacing of $2\lambda/3$. However, as feeds within the same cell are in phase while scanning, the electronics required to apply the phase shifts is only used between consecutive cells, and thus every 2λ . So, there is still a considerable reduction of the density of electronic components, as compared with classical phased arrays. Of course, the number of feeding points per cell can be limited at the cost of reduced performance, if necessary.

4.1.4 Impact of the cell size

According to equation (4.1), the cell size can be decreased if a wider scanning range is necessary, at the cost of a higher density of electronic components. This section aims at evaluating the performance of the design methodology proposed in this chapter, for a smaller cell size. We consider a cell size $a = 1.5\lambda$, which allows one to design an antenna with a theoretical maximum scanning range of 38.94° :

$$|\theta_{scan}| \leq \arcsin\left(\frac{\pi}{ak_0}\right) = 19.47^\circ \quad (4.9)$$

Again, only the third odd harmonic is used to describe the impedance sheet. The optimized average impedance and modulation depth are $X_0 = -83.90 \, \Omega$ and $M = 3.560$, respectively. The obtained EEP is shown in Figure 4.9.

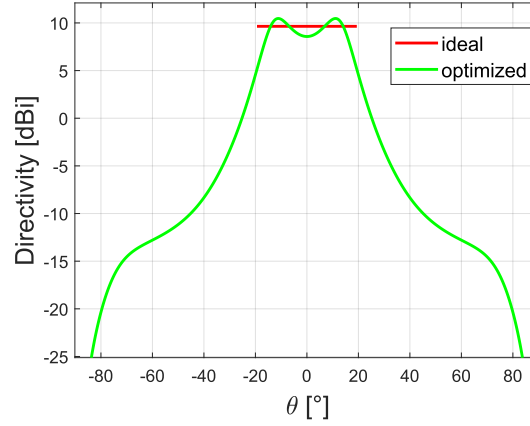


Figure 4.9: Optimized EEP directivity for $a = 1.5\lambda$.

We observe that the scanning range can be increased, at the cost of a less flat and sharp pattern. Intuitively, we understand that both -1 harmonic peaks should be further apart when the cell size decreases, causing a trough at the center of the EEP and less sharp transitions at the edges of the radiating band. It clearly shows the limitation of the design methodology for smaller cell sizes. Similarly to Section 4.1.3, we investigate the ability to improve the radiation pattern by considering an array of four feeds per cell. Firstly, we consider the same impedance sheet with four unit amplitude feeds at $x = [-3\lambda/4; -\lambda/4; \lambda/4; 3\lambda/4]$. Then, the amplitude of each feed, as well as the average impedance and modulation depth, are re-optimized to improve the performance of the antenna. It gives $X_0 = -106.98 \, \Omega$, $M = 1.751$ and $w = [2.011; 0.439; 0.439; 2.011]$. Results are shown in Figure 4.10.

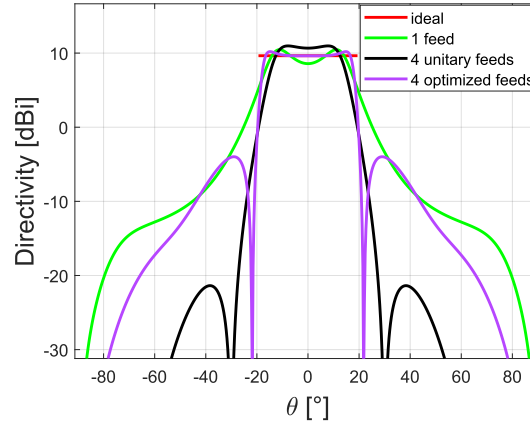


Figure 4.10: Optimized EEP directivity for $a = 1.5\lambda$ and different feed designs.

We observe that an appropriate feed design allows one to sharpen the radiation pattern. However, we clearly observe a trade-off between the reduction of the SLL and the width of the main beam. Compared to an antenna with a cell size $a = 2\lambda$, it is more difficult to satisfy all the criteria regarding the desired EEP (flat, sharp and wide). We can conclude that the methodology seems very well suited for a cell size of 2λ but limited for smaller cell sizes. Note that it has not been fully optimized and that a deeper analysis would be necessary to confirm the observed trend. One should also investigate if adding more degrees of freedom for the optimization, by allowing a more complex impedance, could improve the performance.

4.2 Impact of array truncation

Section 4.1 has shown how to target a desired EEP for an infinite and periodic antenna. It is an essential step, but we are interested in obtaining similar results for finite antennas of reasonable size, that can then be manufactured for practical applications. This section aims at observing the impact of array truncation on the performance of the antenna. We come back to a cell size of $a = 2\lambda$.

Firstly, we consider a sufficiently long antenna (31 cells) such that the EEP of the center cell is not impacted anymore if the antenna length is further increased. The EEP of the three designs in Figure 4.8 are simulated and shown in Figure 4.11, where they are compared with the infinite case previously simulated.

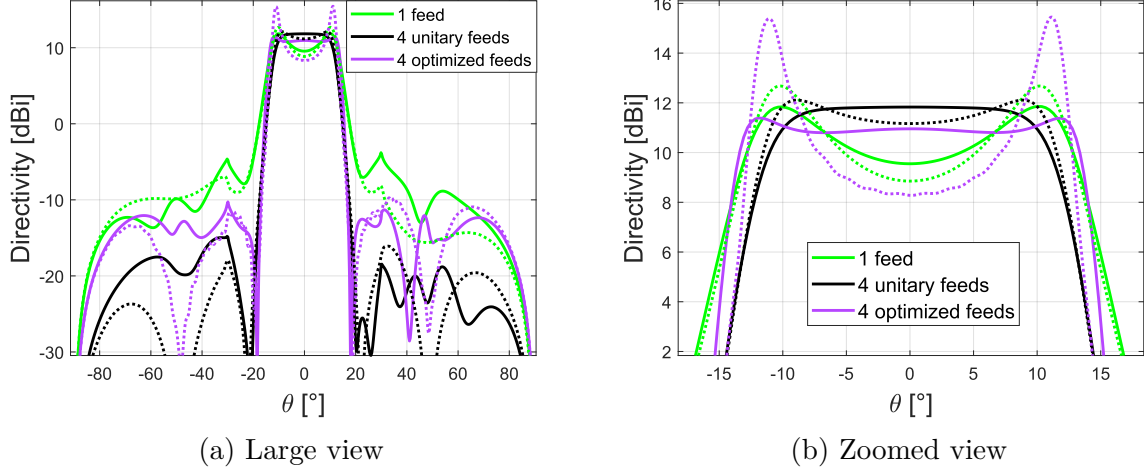


Figure 4.11: Effect of array truncation (31 cells) on the EEP of the center cell (dotted line), compared with the infinite case previously simulated (solid line).

There is a difference between the EEP simulated in the finite and infinite cases, but we observe a convergence between both cases, for all feed designs. The SLL and sharpness of the EEP remain approximately identical for a finite antenna. However, Figure 4.11b highlights the impact of the truncation on the flatness of the EEP in the radiating band. All EEPs are less flat, but especially the last design with 4 optimized feeds. This is probably linked to the location of the -1 harmonics and the modulation depth.

Secondly, we are interested in observing how the EEP differs from one cell to another, and especially how it behaves for cells close to the edges of the array. To that aim, a more realistic antenna of length 22λ (11 cells) is simulated. The design with four unitary feeds is kept ($X_0 = -933.63 \Omega$ and $M = 1.294$) and the EEP of each cell is simulated to observe the behavior. The results are shown in Figure 4.12.

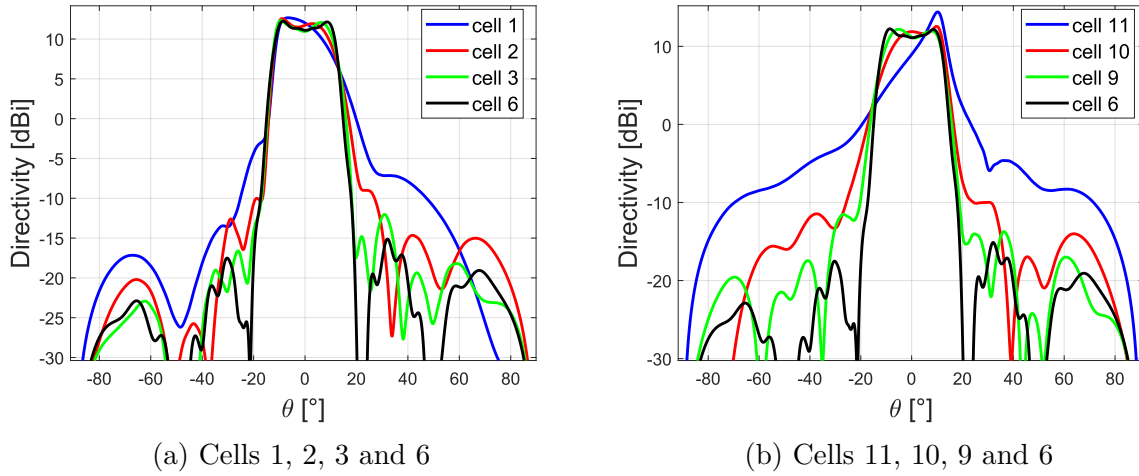
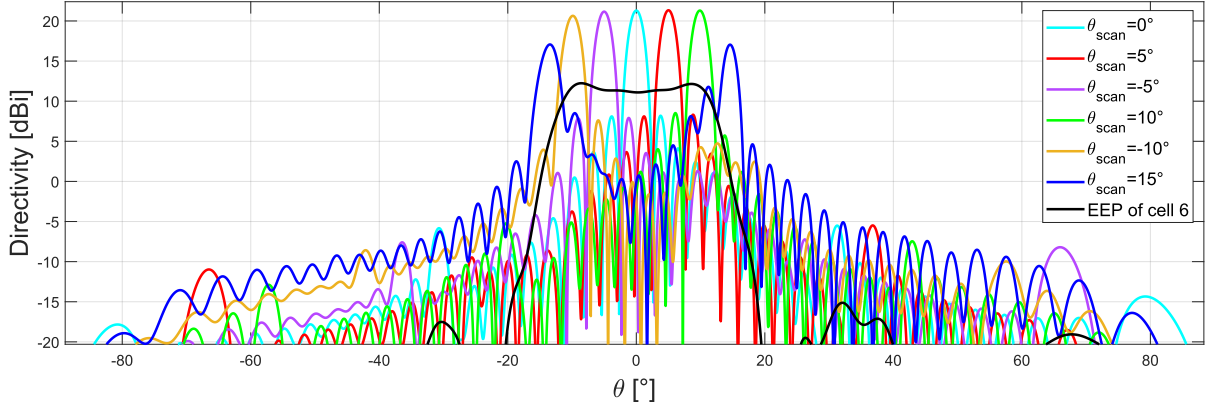


Figure 4.12: EEP directivity of different cells, for an antenna of 11 cells (cell 6 is the center cell).

We observe that the EEP varies from one cell to another, and is especially different from the desired EEP close to the edges of the antenna. This is an expected result as the cell edges are much more impacted by the reflections and other effects at the ends of the array, where the MTS is brutally interrupted. In practice, the MTS can be re-designed close to the edges to account for those differences.

4.2.1 Scanning demonstration

Finally, the ability of a finite antenna to perform scanning is evaluated. We consider the antenna of length 22λ (11 cells) analyzed in Section 4.2, with four unitary feeds per cell, and the impedance sheet described in equation (4.8). To perform scanning, a phase shift of $-\psi$ between consecutive cells is applied to impose radiation at an angle $\theta_{scan} = \arcsin(\psi/(ak_0))$, as explained in Section 3.1.1. Results are shown in Figure 4.13 for various directions θ_{scan} . The EEP of the center cell is superimposed on the graph to ease the understanding.

Figure 4.13: Antenna scanning for various directions θ_{scan} .

We clearly observe the ability of the MTS array to perform scanning over a given region. This region is limited by the cell size and by the EEP, if the latter is narrower than the ideal EEP. Figure 4.13 clearly shows that the EEP shape directly impacts the scanning performance of the antenna. We showed in Section 4.1.3 that the scanning range of this antenna (for the infinite case) was $|\theta_{max}| = 11.78^\circ$, which is confirmed here. For $|\theta_{scan}| = 15^\circ$, we observe an undesired high grating lobe at $\theta = -13.45^\circ$, but the patterns are excellent for $|\theta_{scan}| \leq 10^\circ$.

It is interesting to compare the performance of the proposed MTS antenna with the specifications of typical radar applications, such as synthetic aperture radars (SAR) introduced in Chapter 1. Recent SAR missions (from 2018) such as the Argentinean SAOCOM mission, the ICEYE constellation, and the Strix- α mini-satellite consider an incident angle scanning range of the order of $20\text{-}30^\circ$ for their satellite [37] [18]. This scanning range is comparable to the one shown in Figure 4.13 which is around 23.56° , and a wider scanning range could be achieved by improving the design methodology for lower cell sizes. We can conclude that the design of MTS antennas seems well suited to synthetic aperture radars, in terms of scanning capabilities. However, the impact of other parameters and constraints such as the frequency, heat dissipation and deployment of the structure must be deeply analyzed to ensure the compatibility of the antenna with the application. Many tests and validations would be needed before the satellite could be launched, but this chapter confirms the potential of metasurface-based sparse-array antennas for space applications.

Finally, note that we considered a 1D antenna and that those results should be applied to 2D antennas to evaluate the accuracy of the methodology. Then, the design methodology should be extended to 2D antennas (cf. simulation technique in Chapter 5) to allow one to design more realistic antennas. Finally, the impact of other parameters

such as the relative permittivity and the thickness of the substrate could be investigated to optimize the performance.

Chapter 5

Analysis of 2D finite metasurface antennas

This chapter aims at extending the MoM formulation detailed and validated in Chapter 2 to analyze finite two-dimensional MTS antennas. It is an essential step towards the analysis and design of more realistic antennas.

5.1 Problem statement

The analyzed structure corresponds to the same structure presented in Figure 2.1 but with an additional dimension along y and one single feeding point instead of an infinite wall. It is a rectangular antenna of length a , width b and thickness d that is fed by a single VED located in $(x', y', -d/2)$. An illustration of the considered antenna is shown in Figure 5.1.

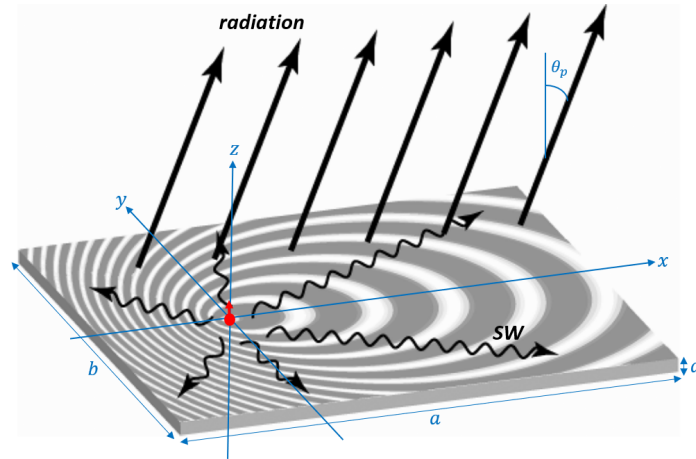


Figure 5.1: 2D MTS antenna [5].

5.2 Method of Moments formulation

The Method of Moments formulation is very similar to the 1D case except that the Gaussian basis and testing functions depend on x and y and are located all over the antenna (Gaussian ring functions) [29]:

$$F_b^n(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{(x-\mu_{x,n})^2}{2\sigma_x^2} - \frac{(y-\mu_{y,n})^2}{2\sigma_y^2}} \quad (5.1)$$

$$\tilde{F}_b^n(k_x, k_y) = e^{j(k_x\mu_{x,n} + k_y\mu_{y,n})} e^{-\sigma_x^2 \frac{k_x^2}{2} - \sigma_y^2 \frac{k_y^2}{2}} \quad (5.2)$$

Each 2D basis function is the product of two 1D Gaussian functions along x and y where μ and σ refer to the mean and the standard deviation of the function, respectively. There are N different locations for those functions on the antenna ($n \in \{1, \dots, N\}$). At each location, there is one function oriented along x ($F_b^{n,x}(x, y)$) and another one oriented along y ($F_b^{n,y}(x, y)$), so there are $2N$ basis and testing functions in total. This allows one to build the MoM linear system of equations by blocks:

$$\left(\begin{bmatrix} \mathbf{Z}_G^{xx} & \mathbf{Z}_G^{xy} \\ \mathbf{Z}_G^{yx} & \mathbf{Z}_G^{yy} \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_{IBC}^{xx} & \mathbf{Z}_{IBC}^{xy} \\ \mathbf{Z}_{IBC}^{yx} & \mathbf{Z}_{IBC}^{yy} \end{bmatrix} \right) \begin{bmatrix} \mathbf{i}^x \\ \mathbf{i}^y \end{bmatrix} = \begin{bmatrix} \mathbf{v}^x \\ \mathbf{v}^y \end{bmatrix} \quad (5.3)$$

where the first index in the exponent (x or y) refers to the orientation of the testing function and the second one to the orientation of the basis function. The expressions of $Z_{IBC}^{xy}(m, n)$, $Z_G^{xy}(m, n)$ and $v^x(n)$ are summarized below for basis and testing functions along y and x , respectively. The expressions of the other blocks of the linear system of equations can be easily deduced from those ones.

$$Z_{IBC}^{xy}(m, n) = \int_x \int_y F_b^{n,y}(x, y) Z_s^{xy}(x, y) F_t^{m,x}(x, y) dx dy \quad (5.4)$$

$$Z_G^{xy}(m, n) = \frac{1}{4\pi^2} \int_{k_x} \int_{k_y} \tilde{F}_t^{m,x}(-k_x, -k_y) \tilde{G}^{xy}(k_x, k_y) \tilde{F}_b^{n,y}(k_x, k_y) dk_x dk_y \quad (5.5)$$

$$v^x(n) = \frac{-1}{4\pi^2} \int_{k_x} \int_{k_y} \tilde{E}_i^{xz}(k_x, k_y) \tilde{F}_t^{n,x}(-k_x, -k_y) dk_x dk_y \quad (5.6)$$

Note that $\tilde{E}_i^{xz}(k_x, k_y)$ and $\tilde{E}_i^{yz}(k_x, k_y)$ are the x and y components of the spectral electric field generated by a VED along z , which is the excitation. Their expressions are detailed in [1] and are a two-dimensional extension of the results given in Section 2.1.2. Their expressions are reminded below in polar coordinates (k_ρ, α):

$$\begin{cases} \tilde{E}_{xz}(k_\rho, \alpha) = \tilde{f}(k_\rho) \cos \alpha e^{j(k_\rho \cos \alpha x' + k_\rho \sin \alpha y')} \\ \tilde{E}_{yz}(k_\rho, \alpha) = \tilde{f}(k_\rho) \sin \alpha e^{j(k_\rho \cos \alpha x' + k_\rho \sin \alpha y')} \end{cases} \quad (5.7)$$

with

$$\tilde{f}(k_\rho) = \frac{\eta_0 k_\rho}{2\epsilon_r k_0} \left(e^{-jk_{z1}\frac{d}{2}} - \frac{-\Gamma e^{-jk_{z1}\frac{d}{2}} - e^{-jk_{z1}\frac{3d}{2}} + \Gamma e^{-jk_{z1}\frac{5d}{2}} - \Gamma e^{-jk_{z1}\frac{3d}{2}}}{1 + \Gamma e^{-jk_{z1}2d}} \right) \quad (5.8)$$

The notations and definitions of the variables are similar to those in Section 2.1.2, refer to this section for more details.

Once (5.3) is solved for the current distribution, the electric field and directivity can be computed as explained in Section 2.2.2, which was treating 1D finite antennas.

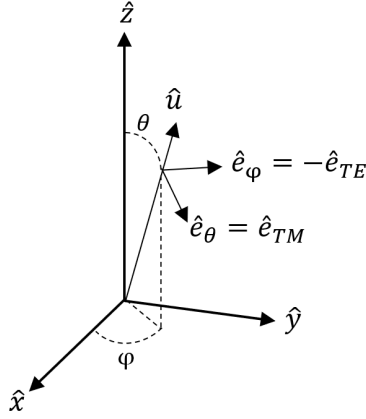
5.3 Results

This section provides the results of simulated 2D MTS antennas. We used the same substrate with a relative permittivity $\epsilon_r = 3.66$ and thickness $d = 1.524$ mm and a frequency of 24 GHz. The antenna is modulated with a spiral-type MTS impedance sheet, which is a 2×2 matrix in the two-dimensional case [13] [38]. In polar coordinates (ρ, ϕ) , it is defined as:

$$\begin{cases} Z^{\rho\rho}(\rho, \phi) = jX_0(1 + M \cos(k_{sw}\rho - k_0\rho \cos \phi \sin \theta_p - \phi)) \\ Z^{\rho\phi}(\rho, \phi) = Z^{\phi\rho}(\rho, \phi) = jX_0M \sin(k_{sw}\rho - k_0\rho \cos \phi \sin \theta_p - \phi) \\ Z^{\phi\phi}(\rho, \phi) = jX_0(1 - M \cos(k_{sw}\rho - k_0\rho \cos \phi \sin \theta_p - \phi)) \end{cases} \quad (5.9)$$

with X_0 the average surface impedance, M the modulation depth, k_0 the free-space wavenumber and k_{sw} the main SW wavenumber propagating in the MTS antenna. This impedance is anisotropic and designed such that it radiates a squinted broadside right-handed circularly polarized (RHCP) pencil beam at $\theta = \theta_p$.

The resulting directivity depends on the polarity of the antenna. Firstly, a reminder on the transverse electric (TE) and transverse magnetic (TM) polarization vectors for the propagating field is shown in Figure 5.2.



$$\hat{e}_{TE} = \frac{1}{k_\rho} \begin{bmatrix} k_y \\ -k_x \\ 0 \end{bmatrix} \quad (5.10)$$

$$\hat{e}_{TM} = \frac{1}{k_\rho k_0} \begin{bmatrix} k_x k_z \\ k_y k_z \\ -k_\rho^2 \end{bmatrix} \quad (5.11)$$

Figure 5.2: Systems of coordinates used to define polarizations.

The radiation patterns in both polarizations can be obtained by extracting the TM and TE components of the total radiation pattern (projection on those polarization vectors). It allows the definition of the co-polar and cross-polar patterns, which correspond to the right-handed circular polarization (RHCP) and the left-handed circular polarization (LHCP), respectively:

$$F_{co-pol} = \frac{1}{\sqrt{2}}(F_{TM} - jF_{TE}) \quad (5.12)$$

$$F_{cross-pol} = \frac{1}{\sqrt{2}}(F_{TM} + jF_{TE}) \quad (5.13)$$

Then, the co-polar and cross-polar directivities can be obtained as explained in Section 2.2.2.

Analysis of a $4\lambda \times 4\lambda$ square antenna

Firstly, we consider a square antenna of side 4λ with the set of parameters summarized in Table 5.1 for the sheet impedance. This antenna is supposed to radiate a RHCP main beam at $\theta = 0^\circ$ in the plane of $\phi = 0^\circ$.

Average surface impedance	X_0	$-772.49 [\Omega]$
Modulation depth	M	$0.6 [-]$
SW wavenumber	k_{sw}	$1.35 k_0 [m^{-1}]$
Direction of radiation	θ_p	$0 [^\circ]$

Table 5.1: 2D square sheet impedance parameters.

The simulated current is shown in Figure 5.3a below. We clearly observe a spiral shape caused by the impedance sheet. Note that for this simulation, 20 Gaussian basis

functions per wavelength were used. Increasing the number of basis functions would probably smoothen the current distribution graph, but it would require too much memory and computation time for a standard laptop. The co-polar and cross-polar directivities in the plane $\phi = 0^\circ$ are shown in Figure 5.3b. We observe that the main direction of radiation for the RHCP case is located at $\theta = 0^\circ$, as expected.

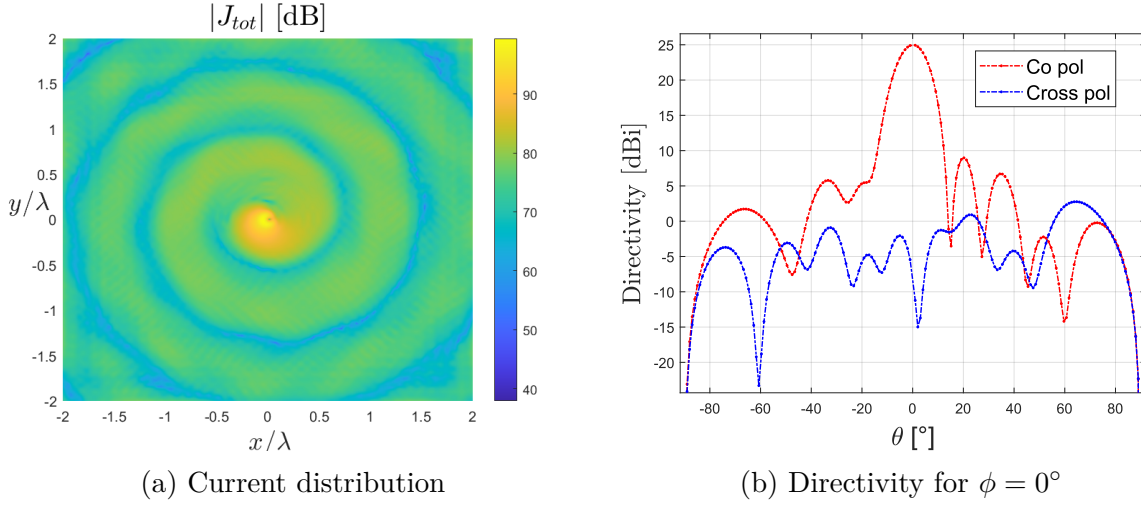


Figure 5.3: Simulation of a $4\lambda \times 4\lambda$ square antenna.

Analysis of a $7\lambda \times 4\lambda$ rectangular antenna

Secondly, we are interested in simulating a known antenna from the literature to compare the results and validate the coding implementation. To that aim, the rectangular antenna of length 7λ and width 4λ proposed in [39] is simulated. Note that paper [39] uses the same MoM formulation but with rooftop basis functions. The antenna works at the same frequency with the same substrate parameters and it is modeled with the same sheet impedance as in equation (5.9), but with the set of parameters in Table 5.2.

Average surface impedance	X_0	$-772 [\Omega]$
Modulation depth	M	$0.4 [-]$
SW wavenumber	k_{sw}	$1.35 k_0 [m^{-1}]$
Direction of radiation	θ_p	$30 [^\circ]$

Table 5.2: 2D rectangular sheet impedance parameters.

The results from the paper are given in Figure 5.5a where the current distribution and the directivity are shown. Note that in the notations of the paper, the *IBC* curves refer to the simulation of the antenna with the surface impedance and the *MTS* curves refer to the

physical MTS structure implementing that surface impedance (with coffee-bean patches). Both curves are very similar but, as we simulate the antenna modeled by the impedance sheet, we are interested in comparing our results with the *IBC* curves in Figure 5.5a. Finally, the surface impedance has been excluded on a centered disk with 0.3λ radius to account for the region without MTS where a realistic feeder is located, but this effect is neglected in our simulations. The obtained current distribution is shown in Figure 5.4. This current looks very similar to the one in the top-left corner of Figure 5.5a, which is a first positive point.

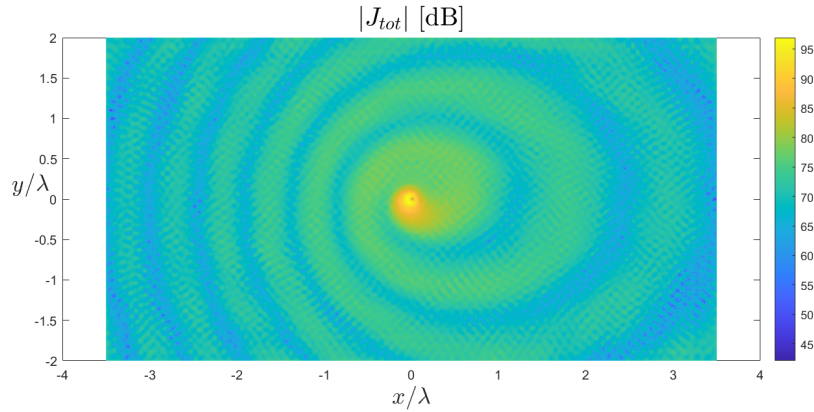


Figure 5.4: Simulated current distribution for a $7\lambda \times 4\lambda$ rectangular antenna.

Then, the co-polar and cross-polar directivities are computed and shown in Figure 5.5b. Again, we observe a main RHCP beam direction at $\theta_p = 30^\circ$, as expected. Moreover, we observe a good agreement with the curves from the paper in Figure 5.5a. Again, we limited the number of basis functions to 20 per wavelength to limit the needed computational resources. This may explain the small differences between both Figures. The area without MTS around the feeder and the choice of different basis and testing functions are also responsible for differences between both Figures.

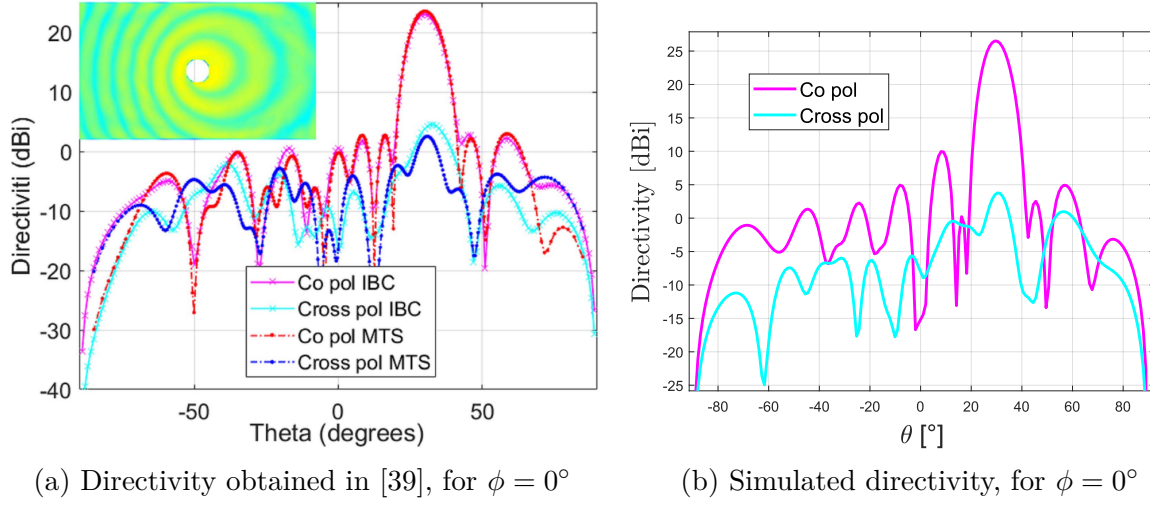


Figure 5.5: Comparison between the directivity obtained in [39] and the simulation, for a $7\lambda \times 4\lambda$ rectangular antenna.

This confirms the correct implementation of the code in the two-dimensional case. However, improvements regarding the computation time and memory allocation are necessary to simulate larger antennas with a higher accuracy. Then, the code could be extended to simulate antennas that are infinite and periodic in the x and/or y direction. This can easily be implemented following the same methodology presented in Chapter 3 for one-dimensional antennas, i.e. through a regular sampling of the spectral-domain Green's function. Finally, the design of MTS antennas to achieve a specific radiation pattern could be carried out following the lines developed in Chapter 4. This will open the door to many applications and offer a powerful tool to design periodic 2D MTS antennas, which can then be synthesized and measured.

Chapter 6

Conclusion and future work

6.1 Conclusion

This master thesis was devoted to the analysis and design of modulated metasurface antennas. Firstly, the Method of Moments was used to solve the electric field integral equation for the current distribution on the antenna. It has been shown that the dense surface of varying patches can be accurately modeled by a surface impedance sheet. In particular, the impedance boundary condition dictates the link between a given excitation and the resulting currents and fields propagating in the structure. This allows one to simulate one-dimensional antennas and to observe the impact of the excitation and impedance on the obtained radiation pattern. This analysis was carried out for both finite and infinite (periodic) antennas. Consistent results were observed in both cases. Note that the simulations of sufficiently long antennas converged adequately to the simulations of infinite antennas.

Then, a design methodology for one-dimensional antennas was proposed. The goal was to optimize the surface impedance sheet of the antenna such that it radiates a desired pattern. In particular, we are targeting an ideal rectangular embedded element pattern for each feed, such that it filters out the grating lobes of the array factor. The idea is to allow beam scanning with a wider feed spacing compared with conventional phased arrays, and hence a reduction of the density of electronic components. The optimization process has shown that a purely sinusoidal sheet impedance can provide excellent results. The simplicity of this result is very convenient. It was also shown that an appropriate design of the feed can improve the performance. The proposed design methodology is simple and effective, but sensitive to the cell size and the effect of array truncation.

The range of scanning angles of the designed metasurface antenna is around 23.56° , with a feed spacing of 2λ , which is very convenient for many applications requiring beam

scanning. Note that there is a trade-off between the feed spacing and the field of view, that should be tuned to meet the specifications of a given application. In addition, metasurface antennas are flat and easy to manufacture, making them a suitable technology for space applications such as synthetic aperture radars.

Finally, the developments for the analysis of one-dimensional antennas were extended to the more general case of two-dimensional antennas. Agreements with literature results confirmed the accuracy of the methodology. Similarly to the one-dimensional case, it could be used to design two-dimensional antennas. Those antennas could then be manufactured and measured to assess the effectiveness of the methodology.

6.2 Future work

Firstly, efforts could be made to improve the efficiency of the coding implementation for the analysis of MTS antennas. There exist more advanced methods for computing numerical integrals, that could be investigated to improve the computation time and memory allocation. This would allow one to analyze larger two-dimensional antennas more accurately on a conventional laptop. Then, the same methodology can be used to design two-dimensional antennas and evaluate their performance. If the results are not satisfactory, further research might be needed to improve the output of the optimization loop. These antennas could then be manufactured and measured in an anechoic chamber to quantify the efficiency of the design process.

Regarding the design of one-dimensional antennas, strategies must be investigated to obtain similar performance for any cell size, such that it can be fixed by the specifications of the application. It could start by considering a more complex impedance for the optimization. Note that the optimization converged to a purely anti-symmetric impedance but that it has not (yet) been proven that the impedance must be strictly anti-symmetric. The impact of the symmetric and anti-symmetric parts of the impedance should be more deeply studied, and it could pave the way to a more efficient design methodology. Furthermore, it has been shown that the feed has a real impact on the performance. A smart design of the feed was proposed to improve the performance but further improvements could be considered. The trade-off between feed complexity and performance should be assessed to design realistic feeding systems. Finally, the impact of array truncation must be counteracted to limit the deterioration of the performance. One could study the possibility of designing the MTS differently near the edges of the array.

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Appendices

A Derivation of $\mathbf{Z}_G$ and $\mathbf{v}$ in the spectral domain

This appendix shows how $\mathbf{Z}_G$ and $\mathbf{v}$ can be computed in the spectral domain, based on their expression in the spatial domain. For the substrate matrix, it gives:

$$Z_G(m, n) = \int_x \int_{x'} F_t^m(x) G^{xx}(x - x') F_b^n(x') dx' dx \quad (1)$$

$$= \int_x \int_{x'} F_t^m(x) \frac{1}{2\pi} \int_{k_x} \tilde{G}^{xx}(k_x) e^{-jk_x(x-x')} dk_x F_b^n(x') dx' dx \quad (2)$$

$$= \frac{1}{2\pi} \int_{k_x} \tilde{G}^{xx}(k_x) \int_x F_t^m(x) e^{-jk_x x} dx \int_{x'} F_b^n(x') e^{jk_x x'} dx' dk_x \quad (3)$$

$$= \frac{1}{2\pi} \int_{k_x} \tilde{G}^{xx}(k_x) \tilde{F}_t^m(-k_x) \tilde{F}_b^n(k_x) dk_x \quad (4)$$

For the excitation vector, we have:

$$v(n) = - \int_x E_i(x) F_t^n(x) dx \quad (5)$$

$$= - \int_x \frac{1}{2\pi} \int_{k_x} \tilde{E}_i(k_x) e^{-jk_x x} dk_x F_t^n(x) dx \quad (6)$$

$$= \frac{-1}{2\pi} \int_{k_x} \tilde{E}_i(k_x) \int_x F_t^n(x) e^{-jk_x x} dx dk_x \quad (7)$$

$$= \frac{-1}{2\pi} \int_{k_x} \tilde{E}_i(k_x) \tilde{F}_t^n(-k_x) dk_x \quad (8)$$

B Complex contour for the integral computation of $\mathbf{Z}_G$ and $\mathbf{v}$

As introduced in Section 2.2.1, a complex contour is necessary for the computation of $\mathbf{Z}_G$ and $\mathbf{v}$ to avoid the poles present in their integral expressions. As we know that the poles are located around $k_x = k_{sw, substrate} = \pm 1.26 k_0$, the contour is only used for $|k_x| < 2 k_0$.

The considered contour is parabolic and is shown in Figure 1 in the complex plane of k_x , where $k_{x,i}$ and $k_{x,r}$ are the imaginary and real parts of k_x , respectively.

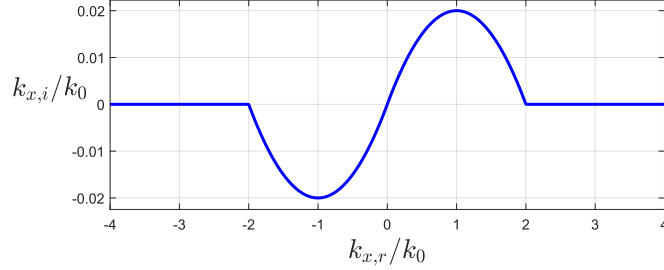


Figure 1: Complex contour.

The mathematical expression of the contour is:

$$k_{x,i}(k_{x,r}) = \begin{cases} \frac{-k_0}{50} \left(1 - \frac{(k_{x,r} + k_0)^2}{k_0^2} \right) & \text{if } -2k_0 < k_{x,r} < 0 \\ \frac{k_0}{50} \left(1 - \frac{(k_{x,r} - k_0)^2}{k_0^2} \right) & \text{if } 0 \leq k_{x,r} < 2k_0 \\ k_{x,r} & \text{otherwise} \end{cases} \quad (9)$$

and its derivative is:

$$\frac{dk_{x,i}}{dk_{x,r}}(k_{x,r}) = \begin{cases} \frac{2}{50k_0}(k_{x,r} + k_0) & \text{if } -2k_0 < k_{x,r} < 0 \\ \frac{-2}{50k_0}(k_{x,r} - k_0) & \text{if } 0 \leq k_{x,r} < 2k_0 \\ 1 & \text{otherwise} \end{cases} \quad (10)$$

This allows one to compute the integrals for $\mathbf{Z}_G$ and $\mathbf{v}$ on this complex path by using the above equations for k_x and dk_x :

$$k_x = k_{x,r} + jk_{x,i} = k_{x,r} + jk_{x,i}(k_{x,r}) \quad (11)$$

$$dk_x = dk_{x,r} + jdk_{x,i} = dk_{x,r} \left(1 + j \frac{dk_{x,i}}{dk_{x,r}}(k_{x,r}) \right) \quad (12)$$

C Useful notes on Fourier series

C.1 Array factor simplification

This Appendix demonstrates equality (13) below, which is used to simplify the array factor expression of an infinite array of elements.

$$\sum_{n=-\infty}^{+\infty} e^{jn\Psi} = 2\pi \sum_{m=-\infty}^{+\infty} \delta(\Psi - 2\pi m) \quad (13)$$

Let's consider the following signal, which has a periodicity of a :

$$p(x) = \sum_{m=-\infty}^{+\infty} \delta(x - ma) \quad (14)$$

As it is periodic, it can be decomposed into a Fourier series (FS):

$$p(x) = \sum_{k=-\infty}^{+\infty} P[k] e^{jk\frac{2\pi}{a}x} \quad (15)$$

where the Fourier series coefficients $P[k]$ are given by:

$$P[k] = \frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} p(x) e^{jk\frac{2\pi}{a}x} dx \quad (16)$$

$$= \frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} \sum_{m=-\infty}^{+\infty} \delta(x - ma) e^{jk\frac{2\pi}{a}x} dx \quad (17)$$

$$= \frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} \delta(x) e^{jk\frac{2\pi}{a}x} dx \quad (18)$$

$$= \frac{1}{a} \quad (19)$$

So, we have:

$$p(x) = \frac{1}{a} \sum_{k=-\infty}^{+\infty} e^{jk\frac{2\pi}{a}x} = \sum_{m=-\infty}^{+\infty} \delta(x - ma) \quad (20)$$

In our case, the periodicity is $a = 2\pi$, which gives:

$$R(\Psi) = \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} e^{jk\Psi} = \sum_{m=-\infty}^{+\infty} \delta(\Psi - m2\pi) \quad (21)$$

C.2 Fourier transform of a Dirac comb

This Appendix demonstrates that the Fourier transform of a Dirac comb is still a Dirac comb and provides its expression [40]. One can start from the result demonstrated in Appendix C.1:

$$p(x) = \sum_{m=-\infty}^{+\infty} \delta(x - ma) \quad (22)$$

$$= \frac{1}{a} \sum_{k=-\infty}^{+\infty} e^{jk \frac{2\pi}{a} x} \quad (23)$$

This result shows that an infinite Dirac comb is equal to an infinite summation of complex exponentials. The Fourier transform of $p(x)$ can then be easily computed and it gives:

$$\tilde{P}(k_x) = \frac{2\pi}{a} \sum_{p=-\infty}^{+\infty} \delta(k_x - p \frac{2\pi}{a}) \quad (24)$$

C.3 Fourier series decomposition of a purely imaginary function

This appendix shows how the Fourier series decomposition of a purely imaginary function can be simplified. We consider the decomposition of the impedance sheet $Z_s(x)$ which has a periodicity of a and is purely imaginary (no resistive part), with $\omega_0 = \frac{2\pi}{a}$:

$$Z_s(x) = \sum_{k=-\infty}^{+\infty} X[k] e^{jk\omega_0 x} \quad (25)$$

$$= \sum_{k=-\infty}^{-1} X[k] e^{jk\omega_0 x} + X[0] + \sum_{k=1}^{+\infty} X[k] e^{jk\omega_0 x} \quad (26)$$

$$= \sum_{k=1}^{+\infty} X[-k] e^{-jk\omega_0 x} + X[0] + \sum_{k=1}^{+\infty} X[k] e^{jk\omega_0 x} \quad (27)$$

$$= X[0] + \sum_{k=1}^{+\infty} (X[-k] + X[k]) \cos(k\omega_0 x) + (X[k] - X[-k])j \sin(k\omega_0 x) \quad (28)$$

The impedance is purely imaginary if $X[0]$ is purely imaginary, and if the real and imaginary parts of the Fourier series coefficients are linked as follows:

$$-\text{Re}(X[-k]) = \text{Re}(X[k]) \quad (29)$$

$$\text{Im}(X[-k]) = \text{Im}(X[k]) \quad (30)$$

The impedance simplifies to:

$$Z_s(x) = j \left(\text{Im}(X[0]) + 2 \sum_{k=1}^{+\infty} \text{Im}(X[k]) \cos(k\omega_0 x) + \text{Re}(X[k]) \sin(k\omega_0 x) \right) \quad (31)$$

where $\mathbb{I}\mathfrak{m}(X[0])$, $\mathbb{I}\mathfrak{m}(X[k])$ and $\mathbb{R}\mathfrak{e}(X[k])$ are purely real coefficients. If only a finite number of coefficients is kept, the impedance is:

$$Z_s(x) = j \left(\mathbb{I}\mathfrak{m}(X[0]) + 2 \sum_{k=1}^N \mathbb{I}\mathfrak{m}(X[k]) \cos(k\omega_0 x) + \mathbb{R}\mathfrak{e}(X[k]) \sin(k\omega_0 x) \right) \quad (32)$$

which contains $2N + 1$ real parameters.

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