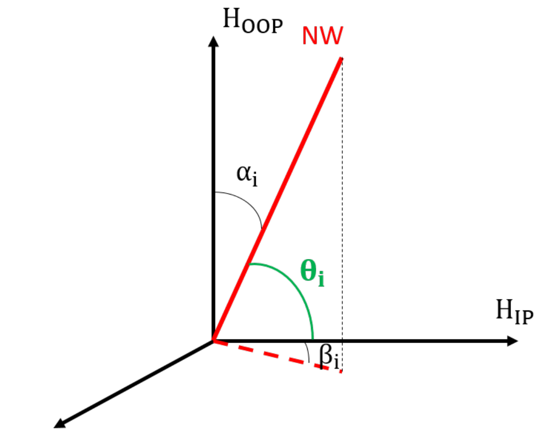
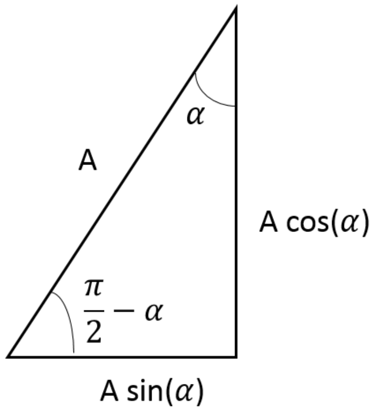
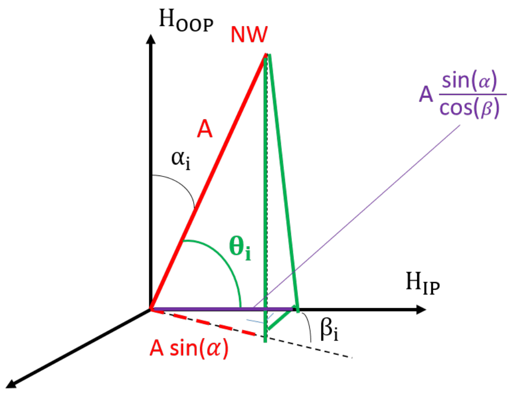
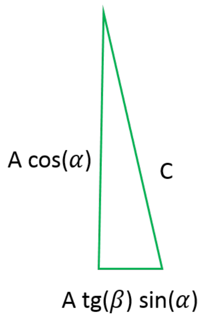
Appendix D: Angular relation in the three-dimensional system

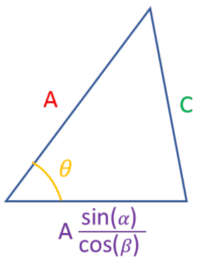








$$C^2 = A^2\cos^2(\alpha) + A^2\sin^2(\alpha)tg^2(\beta)$$



$$C^2 = A^2 + A^2\frac{\sin^2(\alpha)}{\cos^2(\beta)} - 2A^2\frac{\sin(\alpha)}{\cos(\beta)}\cos(\theta)$$

$$A^2 + A^2\frac{\sin^2(\alpha)}{\cos^2(\beta)} - 2A^2\frac{\sin(\alpha)}{\cos(\beta)}\cos(\theta) = A^2\cos^2(\alpha) + A^2\sin^2(\alpha)\mathrm{tg}^2(\beta)$$

$$1 + \frac{\sin^2(\alpha)}{\cos^2(\beta)} - 2\frac{\sin(\alpha)}{\cos(\beta)}\cos(\theta) = \cos^2(\alpha) + \sin^2(\alpha)\mathrm{tg}^2(\beta)$$

$$1 + \frac{\sin^2(\alpha)}{\cos^2(\beta)} - \cos^2(\alpha) - \sin^2(\alpha)\mathrm{tg}^2(\beta) = 2\frac{\sin(\alpha)}{\cos(\beta)}\cos(\theta)$$

$$\cos(\theta) = \frac{\cos(\beta)}{2\sin(\alpha)} + \frac{\sin(\alpha)}{2\cos(\beta)} - \frac{\cos^2(\alpha)\cos(\beta)}{2\sin(\alpha)} - \frac{\sin(\alpha)\sin^2(\beta)}{2\cos(\beta)}$$

$$\cos(\theta) = \frac{\cos(\beta)}{2\sin(\alpha)}(1 - \cos^2(\alpha)) + \frac{\sin(\alpha)}{2\cos(\beta)}(1 - \sin^2(\beta))$$

$$\cos(\theta) = \frac{\cos(\beta)}{2}\sin(\alpha) + \frac{\sin(\alpha)}{2}\cos(\beta) = \cos(\beta)\sin(\alpha)$$