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The Performance of the Norwegian Sovereign Wealth Fund

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I. Executive Summary

In this Thesis, we decided to analyse one of the world's biggest mysteries: Sovereign Wealth Funds. Usually, those funds work very hard to keep their information private and ambiguous. This is the reason behind the analysis of the Norwegian Sovereign Wealth Fund. This Fund uses an opposite strategy than all its peers. This Fund decided that it would be very transparent in its investments and its strategies. This helped us understand its ascension between 2006 and 2021 and this is what this Thesis will be all about. Understanding how this Fund performs exceptionally while bearing similar risk than a US Bond. This is also the reason behind the inclusion of a Risk Parity Portfolio as a benchmark. We wanted to see how the Fund would perform against the safest portfolio created to this day. This portfolio is unique in the way it attributes risks to each asset class.

In this Thesis, we will analyse the performance of the Fund against the benchmarks by using a variety of performance measures. Those performance measures are a blend of Drawdown ratios, Downside Risk ratios and risk-adjusted returns measures.

We discovered that key changes in investments between 2007–2009 would be a precedent for future exceptional performances from the Fund. Our results also showed that low risk would be able to be maintained thanks to important investments from the Fund in Fixed-Income in the United States of America. We ended our Thesis by underlying that the Fund was always able, between 2006–2021, to change their strategies at key moments which helped them to sail through crises like no other benchmark was able to.

II. Acknowledgments

I would, firstly, like to thank my parents for their trust in my capabilities and their continued encouragements during this though year. They have proven to be essential for my academic success and their influence on my 6 years at the Université Catholique de Louvain should not be underestimated.

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1. Introduction

The goal of this Thesis was to resolve all the mystery surrounding the Norwegian Sovereign Wealth Fund. When talking with people around me, it became clear that people heard a lot of rumors about this subject but few really knew what this Fund was all about. We wanted to create a Thesis that would help inform people about this Fund and its goal. We tried to be very thorough in our research and divided our Thesis in distinct chapters to create a thread.

Our first goal was to explain what a Sovereign Wealth Fund was and the common misconceptions. After reaching this goal, we dived into the matter itself. Because our Thesis started in 2020, it was difficult to avoid talking about the change of CEO for the Fund. We continued our literature review by highlighting the main differences between this Fund and the other Sovereign Wealth Funds. It also seemed important to us to analyse the 2020 Report. It was important for us to inform the reader with as much information possible about the Fund before analyse its performance. We ended the first step of our literature review with a presentation of all the benchmarks used in this Thesis. We wanted to highlight the Risk Parity Portfolio, first and foremost, for its nature and for its purpose in this Thesis. This portfolio will really show us how good the Fund really is in terms of Risk Management.

The second step of our literature review was a presentation and explanation of all the performance measures used. It was really important for us to explain each parameter in each ratio because this helped the reader understand our analysis more thoroughly.

After informing the reader about the Sovereign Wealth Fund, the change of CEO, its 2020 report, the different benchmarks and the performance measures used, we were able to start our empirical analysis. The main goal of this analysis was understanding “how” the Fund is outperforming all the benchmarks. To understand this, we needed to dig through key reports between 2006–2021. We needed to learn the different strategies used from the Fund to present such exceptional results. The second step in learning the “how” was looking at the different performance measure. Those performances measures would inform us in which areas the fund was outperforming the benchmarks. By gathering that information, we were able to figure out “how” the Fund was able to outperform its peers.

We are ending our introduction by highlighting some limits to our results. The Fund has proven to be very transparent with its results but there is still a lot of information we don’t know about the Fund. We have seen that the Fund doesn’t inform the public about its cash and derivatives.

We don’t know how much profit the Fund gathers thank to its derivatives because we have not been able to find information about this subject. Moreover, we have seen in portfolio management that cash is very important. It helps us keep high liquidity and it helps us invest in opportunities when markets are down.

2. Literature review

2.1 The Norwegian Wealth Fund

Let's start this paper by defining and understanding the returning concepts. Investopedia defines a Sovereign Wealth Fund as "*a state-owned investment fund composed of money generated by the government, often derived from a country's surplus reserves. SWFs provide a benefit for a country's economy and its citizens*.[1]" (Investopedia, 2020). This definition is very interesting because criticism about SwF has been growing rapidly in the last couple of years. The criticism, submitted by companies/countries, is about the goal of those funds. They accuse those funds of stealing intellectual properties, threatening national securities and their actions to be politically funded. Those criticisms are fed by the great mystery surrounding a multitude of Sovereign Wealth Fund's. Let's take the Abu Dhabi Investment Authority which is created by the United Arab Emirates. This Fund is very private about their allocation, investments and goals. The general public doesn't even know the total value of all the assets of this SwF (Between 250–875 Billion dollars). This is why we have chosen the Norwegian one. This Fund is unlike its peers. It's very open about their investments, about their goals, about the Fund altogether.

2.1.1 Nicolai Tangen's appointment as new CEO

Let's take a deeper look at this Fund. The official Fund's name is the Government Pension Fund Global. It was created in 1990 after the country decided that their Oil Revenue (discovered in 1969) should be used and invested with care. The main goal of this Fund is to ensure that Revenues are not wasted in the short-term and are used/invested to the gain of the Future Generations. After the first deposit of money, in 1996, the Fund decided to always invest abroad to avoid conflict of interests. This is a very recurring theme with the Fund. Since the appointment of the new CEO of the Wealth Fund (March 2020) there have been criticism, uproar and questions about his conflict of interests with its own company "AKO Capital". Immediately after the announcement it became abundantly clear that there need to be a *"necessary distance established between the Government Pension Fund Global, the AKO system and Nicolai Tangen's personal wealth.[2]"* (Norges Bank, 2020). The details,

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[1] Investopedia. (2020, November 26) *Sovereign Wealth Fund (SWF.)* https://www.investopedia.com/terms/s/sovereign_wealth_fund.asp

[2] Norges Bank. (2020, May 11). *The Governor's comments on letter from Norges Bank's Supervisory Council.*

arrangements of his contract would be discussed on 27 May 2020. During this meeting, it would be discussed that Nicolai Tangen would not exercise ownership or control over the AKO management. Fast forward to August 2020 and we see that there are still problems about this appointment. The head of the supervisory Council has explained that conflict of interests have not been eliminated between Mr Tangen personal wealth, AKO Capital and the wealth Fund. We also discovered that the Wealth Fund broke the following rules with his appointment: Ethical principles and Transparency Laws. It also “forgot” to inform the Prime Minister that Mr Tangen was allowed to keep his stake in his company. After tiring negotiations in August, Mr Tangen agreed the 24th to liquidate his entire stake in AKO Capital and transfer them to the AKO Foundation, he also agreed to deposit all his personal investments in multiple banks. Eventually, on 3 September 2020, Mr Tangen took over the job of CEO of the Norwegian Sovereign Wealth Fund.

This “drama” shows two things about the Fund. It shows first and foremost the transparency about it. The fact that everything was displayed and explained in the press shows the Fund’s goal to be as transparent as possible and it’s objective to avoid any ethical wrongdoing. It also shows that the common goal is more important than the individual. Mr Tangen explained multiple times he wasn’t ready to sell off his assets. By showing this balance of power, the Fund shows that it will never risk or put under pressure the future of their country or their fund. The Fund shows right now how they are different from another big corporation or wealth fund. How another big corporation would have hidden or accepted Mr Tangen demands, the Fund shows its resilience to follow the path laid down for them when the Fund was created in 1990. It’s very promising for the Fund to handle what is described as the biggest crisis in their history, in such a good way.

After explaining how the Fund avoids conflicts of interests between the people involved and the fund, we will talk a little bit about how the Fund works.

2.1.2 How the Fund is different than its peers

Like we explained before, the Fund has been created thanks to the oil/gas revenue generated by Norway. Nowadays, those revenues account only for half of the value of the Fund. The value of the fund is a blend between government funding, oil/gas revenue and income from equity

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https://www.norges-bank.no/en/news-events/news-publications/News-items/2020/2020-05-11-governor-comments/

(investments in 9000 companies), fixed income (lending to other countries) and real estate investments (rental income from hundreds of buildings). It’s important to note that only a small part of the government budget (1/5) is dedicated to the Fund. The government also agreed on a rule of thumb, known as a fiscal rule, around their spending. It is advised to spend no more than the equivalent of the real return on the fund. This ensures the government to spend the “surplus” and leave the capital alone. By using this fiscal rule, they ensure that the future generations will benefit from the resting capital (which will only grow in the future).

We will finish this chapter by explaining how this Fund, again, distances themselves from the average wealth fund. Nowadays, Hedge funds/Wealth funds/Mutual funds try to create a portfolio of investments searching for the highest return possible and accepting a certain risk rate on their portfolio. Mostly they are looking for a Max return/minima risk. This is not the case for this Fund. This Fund has the same idea but they created a list of companies in which they aren’t willing to invest in or work with. How does this process work? Firstly, the Fund needs to respect the rules that are presented by the parliament and the Norwegian Prime Minister. One of those rules is that the Fund's Executive Board needs to listen to the recommendations offered by the Council of Ethics (established by the Prime Minister) about their ethical evaluation of some companies. After listening to the recommendations, the Executive board will choose (since 1 January 2015) if they want to exclude, observe, invest or divest in those companies. Previously, Exclusions were made by the Prime Minister. When looking at the list, we understood exclusions can be due to multiple reasons.
Below we will cite a couple of the most recurring reasons of exclusions:

1) Production of coal or coal-based energy

2) Production of nuclear weapons

3) Production of tobacco

4) Severe environmental damage

5) Serious violations of human rights

6) Unacceptable greenhouse gas emissions

7) Other particularly serious violations of fundamental ethical norms

8) Gross corruption

9) Production of cluster munitions

10) Serious violations of individuals' rights in situations of war or conflict

From the list of companies excluded from investment, we can find a couple of big names in the corporation's world: Airbus SE, Berkshire Hathaway Energy (Owned 90% by Warren Buffett's Berkshire Hathaway), Boeing Co, British American Tobacco, Philip Morris Co, …Those companies are very powerful and have a huge influence in the financial world. Their stocks are very stable and could help a portfolio lower their risk by a great deal. By choosing to exclude those companies, the Fund takes a stand against bad behavior. Compagnies in the list are repetitive offenders that the council of Ethics have under observation before exclusion. The goal is to invest in companies that will help the future generations. This is underlined by the fact they decide to exclude tobacco companies from their portfolio. This is extremely interesting because those companies are not "actively" participating in the destruction of the world today like companies excluded for "severe environmental damage", "gross corruption" or even "production of nuclear weapons". Of course, the production of tobacco is demanding in terms of environmental damage but they decided to disinvest in those companies because they thought tobacco has a bad influence on health of the future generations and also, it's active lobbying to council the negative effects of their products on health and environment. The fund made a conscious choice to fund good influences and avoid bad influences. They decided that this world needs to be better and they want to help in the change. We aren't talking about active change, but a change in investment is a start.

We will now look deeper how the Fund invests. We are going to look where they invest the most, what the companies targeted, what are the countries/cities with the most influence on the Fund, where the Fund soared in the last years, …

Like we already explained, the Fund has invested in more than 73 countries and in 9123 companies. They invest in equities, real estate and fixed income.. We explained above that all this information is given by the Fund itself because they are prone to transparency and openness about their practices. Below we will look at the Fund's 2020 report to understand their performance and their investments. This report is particularly interesting because we will see the influence of the Corona Pandemic on their yearly return. In the report they will explain what changes they made to their strategies to counter the consequences of this pandemic.

2.2 The 2020 Report of the Norwegian Sovereign Wealth Fund

At the end of the year, the Fund had 71.5% invested in Equities, 3.8% in unlisted Real Estate, 24.7% in Fixed Income. It's well known that a portfolio needs frequent rebalancing and the Fund's policy is when Equity proportion rises above 72% or under 68%. The Fund also understood that this Corona pandemic damaged Norwegian households and that more money is needed for the citizens. That's why they withdrew from the Fund 347 billion Kroner in 2020. They also decided to augment their investments in the 4th quarter of the year in US Techs firms because they saw that those firms were handling the Corona Pandemic better than other companies.

Firstly, we will start with some key points of the year 2020:

- Repeated lockdowns and measures to contain the virus resulted in negative growth in the global economy
- Growing optimism for the Year 2021 thanks to mass immunisation and the development of effective vaccines.
- Different governments influenced the real economy with support packages to companies/citizens and Central Banks made support purchases in financial markets while cutting their interest rates.
- USA launched the CARES Act that brought *"direct payments to households, higher unemployment benefits, and forgivable loans and grants for small businesses[3]"* (Government Pension Fund Global, 2021).
- The EU countries created a recovery fund of 750 billion euros to support economies damaged the most by this pandemic.
- The pandemic caused a sharp and unusual decline in equity prices in short-term but the end of 2020 would show us that that decline was only short-lived.

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[3] Government Pension Fund Global. (2021). *2020 annual report of the Government Pension Fund Global*. Retrieved from

https://www.nbim.no/contentassets/fd871d2a4e2d4c1ab9d3d66c98fa6ba1/annual-report_2020_government-pension-fund-global_web

- Results were very different between sectors. While the Technological sector surged, the Financial sector wasn't able to fully recover from the Q1 crash.

The Fund invested in 9123 companies in 2020. Those investments are predominantly made in North America (45.6% of their total investments) and more precisely in the United States of America (41.6% of their total investments). Europe stays their second most important investment with 32% and Asia-Pacific closes the top 3 with 20.1%. We see below their favourite investment destinations. We can immediately see that their return is heavily reliant on the performance of their USA investments. It is also important to note that their investments in Emerging Markets drastically declined from 8% to 2.8% of their total investments in 2020. This is a sign that the Fund was unsure of how those markets would handle the pandemic and decided to transfer that money in "safer" investments like North America, Europe.

Table 1: Ten largest holdings by country for the Fund as on 31 December 2020.

Country	Total	Equity	Fixed income	Unlisted real estate
US	41.6	30.0	10.5	1.1
Japan	8.9	5.9	3.0	0.0
UK	7.0	5.1	1.5	0.4
Germany	5.6	3.2	2.3	0.1
France	5.4	3.4	1.5	0.5
China	3.8	3.8	0.0	-
Switzerland	3.7	3.2	0.4	0.1
Canada	2.5	1.4	1.1	-
Australia	2.1	1.4	0.7	-
South Korea	1.8	1.4	0.4	-

Source: Government Pension Fund Global Annual Report 2020

2.2.1 Equities

Equity investments returned 12.1% in 2020. This was thanks to their investments in US Stocks. Those investments account for 42% of their total equity investments. Those investments returned 21.3% in 2020 and this was mainly thanks to their investments in US Technology firms. The Fund wouldn't have returned 12.1% if they had such large investments in Europe. European equity investments only returned 3.9% while it accounts for 30.9% of their total equity investments. Moreover, Asia-Pacific continues to prove itself as an interesting investment opportunity with an 18.3% return in 2020 while accounting for 23.5% of the Fund's total equity investments. This is underlined when we look at the table below.

Table 2: The Fund's return on their largest equity investments by country

Country	Return in international currency	Return in local currency	Share of equity investments
US	17.6	21.3	42.0
Japan	10.5	8.3	8.2
UK	-12.2	-12.3	7.2
China	33.9	29.7	5.3
France	2.1	-3.3	4.8
Switzerland	10.1	3.6	4.5
Germany	12.3	6.3	4.5
Taiwan	34.8	30.3	2.1
Australia	7.1	0.7	2.0
South Korea	41.1	36.7	2.0

Source: Government Pension Fund Global Annual Report 2020

The table shows us that the three best-performing countries for the Fund are South Korea, Taiwan and China. While China doesn't have the same importance for the Fund as the USA has, we see that the Fund is starting to grow their investments there. With a return of 33.9% in international currency, China has proven to be a 5.3% of equity investments well used. It's interesting to see that the worst-performing "country" is the UK. This is probably due to Brexit and 2020 being the year that the UK finally leaves Europe completely. Negotiations and uncertainty about their future have not proven to be in their favour for the last 2 years.

What does the second table tell us? We cannot forget that this table has been made after different lockdowns in a lot of countries. This means those numbers are likely to be different in normal circumstances. We see that investments are diversified in a lot of industries with a favour for Technology corporations (18.5% of total equity investments) and financial corporations (20.4% of total equity investments). This table also tells us exactly why the CEO of the Fund decided to increase investments in the US-based Technology Firms. We see that, after such a tough year, only two sectors record negative returns. Their favourite sector of investments, the financial sector, recorded a negative return of 6.3% and the Oil and gas sector with a negative return of 25.3%. The table above has a mistake in it. The Basic materials sector returned 21% and Oil and Gas returned -25.3%.

Table 3: The Fund's return on their equity investments by sector. International currency.

Sector	Return	Share of equity investments[1]
Financials	-6.3	20.4
Technology	41.9	18.5
Industrials	17.2	13.8
Consumer goods	16.7	12.0
Health care	13.7	11.7
Consumer services	17.3	11.5
Oil and gas	21.0	4.4
Basic materials	-25.3	3.0
Utilities	10.3	2.6
Telecommunications	1.7	2.4

Source: Government Pension Fund Global Annual Report 2020

The Fund explained that the financial sector didn't know how to cope with lockdown measures which translated in *"recession, lower interest rates and higher expected loan losses*[4]*"* (Government Pension Fund Global, 2020). The Technology sector has done particularly well, because of the lockdown people were searching for solutions to home-school, teaching through streaming, online shopping, entertainment due to lockdown boredom. The second-best sector was the Basic materials one. This was due to higher commodities prices caused by supply interruptions and expansionary fiscal and monetary policy. The third-best sector was Consumer services. This is completely normal because people were forced to buy their things on the internet. This led to online retailers profiting from this pandemic. Healthcare did well because people were going in masses to hospitals and some pharmaceutical companies made the decision to increase prices of some medication rumored to help with covid-19. Last sector that did very bad was the oil/gas sector. This was due to an increase of supply from Saudi Arabia and a weakened demand due to the pandemic (people were staying home meaning they weren't consuming petrol). There was also weak demand in markets for natural gas, petrochemicals and refined products.

[4] Government Pension Fund Global. (2020). *Half year report*. https://www.nbim.no/contentassets/ae6bdcba846e430bb07e81577639dd1d/government-pension-fund-global---half-year-report-1h-2020.pdf

The Fund's 3 largest investments in equity holdings are Apple Inc, Microsoft Corp and Amazon.com Inc. Those investments together account for 5.8% of the Fund's equity portfolio.

2.2.2 Real Estate

This chapter is pretty short because the Fund only invested 3.8% of their total investments in Real Estate. The Fund reported a negative return of 5% for the Year 2020. This was largely due to their listed real estate investments that returned -14.9% in 2020. We will explain below what the difference is between listed real estate investments and unlisted real estate investments. We cannot forget that the Fund limits their real estate investment to 7% of their total investments. The Fund unlisted Real Estate's return is created thanks to rental income, operating costs, movements in exchange rates, changes in the value of properties and debt and transaction costs for property purchases and sales.

Like we explained above, the Fund's real estate investments are split in two subcategories: listed real estate (34.9%) and unlisted real estate (65.1%).

Table 4: Value in Millions of Kroner of the different real estate investments as on 31 December 2020

	Value[1]
Unlisted real estate investments	273,109
Listed real estate investments	146,677
Aggregated real estate investments	419,786

[1] Including bank deposits and other receivables

Source: Government Pension Fund Global Annual Report 2020

What is the difference between those 2 real estate investments? A paper written in 2019 by AmpCapital explained that *"Listed real estate, also known as Real Estate Investment Trusts (REITs), generally offer daily liquidity, a stable yield, access to a wide range of asset types, and the potential for capital growth. Because they are traded on the stock market, in the very short term they are highly correlated to the broader equities market but behave like real estate over the longer term.*[5]*"* (Talbot & Maydew, 2019). They also explain that, in comparison with

[5] Talbot, C., & Maydew, J. (2019, February). *Investing in listed vs unlisted real estate*. AMP Capital. Retrieved from https://www.ampcapital.com/au/en/insights-hub/articles/2019/february/investing-in-listed-vs-unlisted-real-estate

unlisted real estate investments, listed real estate needs a low capital outlay and that this will help a portfolio manager increase their diversification.

This explanation has proven to be true because the Fund reported a negative return of 14.9 percent for their 2020 listed real estate investments. We have seen at the end of 2020's Q2 that the listed real estate investments were highly correlated with the financial equities of the Fund. Both categories were returning negative returns between 15–20%. Like AmpCapital explained, this correlation is temporary because those listed real estate investments will behave like real estates in the long term. The Fund's listed real estate investments are mostly in residential, office, retail sectors and are equally distributed between Europe and the United States. Their biggest listed ownership is in Shaftesbury PLC with the ownership of 25.8%.

This paper also outlays the benefits of investing in Unlisted Real Estate. *"That said, investing in unlisted real estate also has its attractions for investors including generally offering a stable, secure income stream, low volatility of returns, a low correlation to other asset classes and a natural hedge against inflation.*[6]*"* (Talbot & Maydew, 2019). This explanation is also justified by the Fund's numbers.

The Fund also explains that the pandemic impacted the real estate investments of the Fund. The Fund explained that an important number of retail tenants filed for bankruptcy. This resulted in a negative return of 12.6% for their retail properties which are part of their unlisted real estate investments. The Fund has received 93% of their due rent for their unlisted real estate investments. This is due to their retail tenants where only 64% of them have paid their due rent in 2020. This is further evidence that their retail properties have returned underwhelmingly in 2020. This is not the only category in their unlisted real estate investments that were impacted by the pandemic. We have seen that people were forced to use the internet for shopping and grocery shopping due to lockdown measures. This lowered the need for physical shops and developed the need for warehouses, depots and storehouses. This has helped the unlisted logistic properties category for the Fund and returned 9.2% in 2020.

[6] Talbot, C., & Maydew, J. (2019, February). *Investing in listed vs unlisted real estate*. AMP Capital. Retrieved from https://www.ampcapital.com/au/en/insights-hub/articles/2019/february/investing-in-listed-vs-unlisted-real-estate

The return is a low negative which shows the stability of those investments regarding the times we are currently living in. Like the explanation above, rental income is stable and positive. What created this negative return is the different changes in value due to the crisis we are currently living in.

2.2.3 Fixed Income

In this Chapter, we will look at the Fund's Fixed Income Investments. Like we acknowledged above, the Fund has 24.7 percent of their Total investments in Fixed Income Investments. Those investments returned 7.5% in 2020. Thanks to the tables below, we will see what the type and quality of bonds are favored, which countries are favored for their bonds and why, and we will explain how those investments panned out.

The table below shows us that more than half of their Fixed Income investments are in Government bonds (56.5%). Those bonds are risk-free and mostly pay coupon payments every few months (depends on terms of contract). We will see below that 23.2 percent of Fixed Income investments are US Treasuries. We will also explain the differences between the bonds. Government bonds are bonds issued from a country's government in their local currency, while government-related bonds are bonds issued from a country's government in another currency. Inflation-linked bonds are bonds issued by a government where the principal's value adjusts with inflation meaning they are designed to hedge the inflation risk of a bond. Corporate bonds are bonds issued by companies and not governments. The Securitised bonds are usually covered bonds, like explained in the report. *"Covered bonds are debt obligations issued by credit institutions which offer a so-called double-recourse protection to bondholders: if the issuer fails, the bondholder has a direct and preferential claim against certain earmarked assets and an ordinary claim against the issuer's remaining assets*[7]*."* In Securitised bonds we can also find Asset-Backed Securities and Mortgage-Backed Securities.

[7]European Comission. (2015, October 2015). *Covered bonds*. https://ec.europa.eu/info/business-economy-euro/banking-and-finance/financial-supervision-and-risk-management/managing-risks-banks-and-financial-institutions/covered-bonds_en

Table 5: The Fund's fixed income investments return, in percent, by sector in 2020

Sector	Return	Share of fixed-income investments[1]
Government bonds[2]	6.4	56.5
Government-related bonds[2]	7.2	11.6
Inflation-linked bonds[2]	9.1	6.3
Corporate bonds	7.7	26.1
Securitised bonds	6.7	5.8

[1] Does not sum up to 100 percent because cash and derivatives are not included.

Source: Government Pension Fund Global Annual Report 2020

After explaining and understanding the different bonds, it's not a surprise that returns have proven to be positive. Bond returns are stable and fixed and they are in direct exposure to interest rate risk. During this pandemic, governments have cut interest rates and issued new bond purchase programmes because they were in dire need of money. It is also no surprise that the return of corporate bonds has been one of the highest of all bonds-related investments. Corporate bonds are riskier than government bonds and like we have seen in the past, a higher risk generates higher returns. The Fund hasn't explained their strategy behind this allocation of their Fixed Income investments but it is important to have a nice blend of different bonds to ensure a good return and a very low risk. Corporate bonds bring higher returns but higher risks, government bonds bring low risk but "normally" low returns that can fall behind rising inflation. Government-related bonds bring the same advantages and disadvantages as a government bond but we add currency change to the mix. The Inflation-linked bonds advantage is also its disadvantage because when inflation is high they are yielding high returns but when deflation is happening, those bonds don't offer a lot of security.

After trying to understand the Fund's strategy regarding their Fixed-Income investments allocation, we are now looking at the countries favored by the Fund and why? It's absolutely logical that the fund decided to invest almost 1/5 of their fixed-income investments in US Bonds. This is because US Treasuries has proven in the past to be risk-free and always on point with their payments. The US also boasts that the bonds are backed by "the full faith and credit" of the US Government. It is also well known that the US has never failed to pay back people with their bonds since their creation in 1776.

Table 6: The largest bond holdings in millions of Kroner for the Fund as on 31 December 2020

Issuer	Country	Holding
United States of America	US	717,935
Japanese government	Japan	294,195
Federal Republic of Germany	Germany	138,176
UK government	UK	87,746
French Republic	France	72,442
Spanish government	Spain	51,345
Australian government	Australia	41,065
South Korean government	South Korea	36,045
Canada Mortgage & Housing Corp	Canada	31,675
Italian Republic	Italy	31,261
Government of Canada	Canada	31,242
Mexican government	Mexico	24,444
Bank of America Corp	US	17,648
Government of Indonesia	Indonesia	16,785
Government of Denmark	Denmark	13,805
Province of Ontario Canada	Canada	13,643
Goldman Sachs Group Inc/ The	US	13,575
Government of Ireland	Ireland	13,127
Morgan Stanley	US	12,519
Government of South Africa	South Africa	12,475

Source: Government Pension Fund Global Annual Report 2020

The Fund has invested 717,935 million Kroner in US Bonds as on 30 June 2020. The second-largest bond holding is Japan and the third is Germany. This is no surprise because you need to be present in every Market. By investing big in those countries, they ensure investments on the bond market in Asia, America and Europe. This is again a way to diversify their portfolio and lower their risk. It is surprising that they didn't invest in Chinese bonds because, per 2017, they are the third country with the most Total Amount of Debt Securities Outstanding. This is due to a couple of facts. First and foremost, China does not have the seniority of other countries like the United States. Countries like the United States, Japan have shown in the past to always pay back lenders. Moreover, the Chinese market is very illiquid. In Asia, China is the second most illiquid market behind Malaysia. Other reasons why investors could avoid the Chinese Bond market are: the unpredictability of food inflation due to different diseases, uncertainty about

Chinese law enforcement (state-owned firms could be protected in case of wrongdoing), hedging tools being less sophisticated than those in more developed Bond markets, … Those reasons could be why the Fund decided to avoid investing too much on the Chinese Bond Market.

Before analyse the table below, we will explain what the different credit ratings mean. Firstly, the Investment grade designated below is from Standard & Poor's because Moody's ratings go from Aaa to Baa3. Like Investopedia defines, those ratings "*evaluate a bond issuer's financial strength, or its ability to pay a bond's principal and interest, in a timely fashion.*[8]" (Investopedia, 2020). By financial strength, Investopedia means a government or corporation's ability to be liquid, pay back lenders and maintain their future goals and expectations. The higher the rating, the safest investment but the lowest yields.

Table 7: The bond holdings in percent for the Fund as on 30 June 2020.

Table 7 The fund's bond holdings as at 30 June 2020 based on credit ratings. Percent.

	AAA	AA	A	BBB	Lower rating	Total
Government bonds	25.6	7.9	9.6	4.7	1.0	48.8
Government-related bonds	4.7	5.0	1.9	0.5	0.0	12.2
Inflation-linked bonds	4.6	1.3	0.3	0.4	0.0	6.6
Corporate bonds	0.2	1.7	11.2	12.9	0.4	26.4
Securitised bonds	5.0	0.9	0.1	0.0	0.0	6.0
Total	**40.1**	**16.8**	**23.0**	**18.6**	**1.5**	**100.0**

Source: Government Pension Fund Global Annual Report 2020

It is no surprise to see the Fund invest greatly in AAA government bonds (25.6%). Those bonds are safe investments with a constant return and a guarantee of returned principal. The table below also shows us that investments in A and BBB corporate bonds are almost as large as in AAA government bonds. This is probably due to the fact that those bonds guarantee interesting yields to keep these Fixed-Income investments with a sufficient return. If the Fund decided to invest everything of their Fixed Income investments in AAA government bonds, they would assume zero risk but also very little reward. It is important to find a good balance, like with equities, with the different Fixed Income investments.

[8] Investopedia. (2020, March 25). *Bond Rating.* https://www.investopedia.com/terms/b/bondrating.asp#:~:text=A%20bond%20rating%20is%20a%20letter%2Dbased%20credit%20scoring%20scheme,Junk%20bonds%20have%20lower%20ratings.

We are going to use different performance measurements to analyse properly how the Norwegian Sovereign Wealth Fund developed over time. We are also going to use those measurements to make comparisons between the SwF and the different indexes.

2.3 Different Benchmarks Used Against the Fund

In this chapter, we will look at the different benchmarks used for comparison with the Fund. We will also look at the bond used for comparison with the Fund. We will also explain how we created the Risk Parity Portfolio and its performance against the Fund. The main goal of this chapter is understanding what each index, portfolio and bond represents and how it punches up against the Fund.

S&P500's full name is the Standard and Poor's 500. It is a stock market index of the 500 largest listed companies in the United States. This index is a capitalization-weighted index. This means its components, the companies, are weighted according to their market value of their outstanding shares. The more a company is worth, the more its weight on the index, in this case the S&P 500. The index is not only cap-weighted but also float-adjusted. To understand float-adjusted, we need to understand that every stock receives a float factor. This factor calculates the proportion of outstanding shares owned by the general public. It's important to understand also that closely-held shares are outstanding shares owned by governments, royalties and company insiders. When calculating Float-adjusted weighting, we multiply the Float Factor by a company Market capitalization. The result will be used to weigh its value against the other companies of the index. To conclude, the S&P 500 is a free-float, capitalization-weighted index. We will look at the numbers of shares owned by the general public and we will use that to value the weight of a company in the S&P 500. The 10 biggest companies in the S&P 500 are Apple Inc, Microsoft, Amazon.com, Facebook, Alphabet Inc, Tesla Inc, Berkshire Hathaway, JPMorgan & Chase and Johnson & Johnson. We see that a lot of those companies have been invested in by the Fund.

To become part of the S&P 500, a company needs to be selected by a committee and needs to fill a couple of requirements.

1. Market capitalization needs to be equal or bigger than 11.3 billion dollars.
2. Annual dollar traded to float-adjusted market capitalization> 1.0
3. In each of the 6 months leading to the evaluation date for a company's eligibility, there should be a minimum monthly trading of 250,000 shares.

4. Listed publicly on the NYSE or Nasdaq
5. Based in the US
6. *"Securities that are ineligible for inclusion in the index are limited partnerships, master limited partnerships and their investment trust units, OTC Bulletin Board issues, closed-end funds, exchange-traded funds, exchange-traded notes, royalty trusts, tracking stocks, preferred stock, unit trusts, equity warrants, convertible bonds, investment trusts, American depositary receipts, and American depositary shares.[9]"* (S&P Global, 2021)

The index is rebalanced and modified every quarter but the Fund tries to minimize turnover.

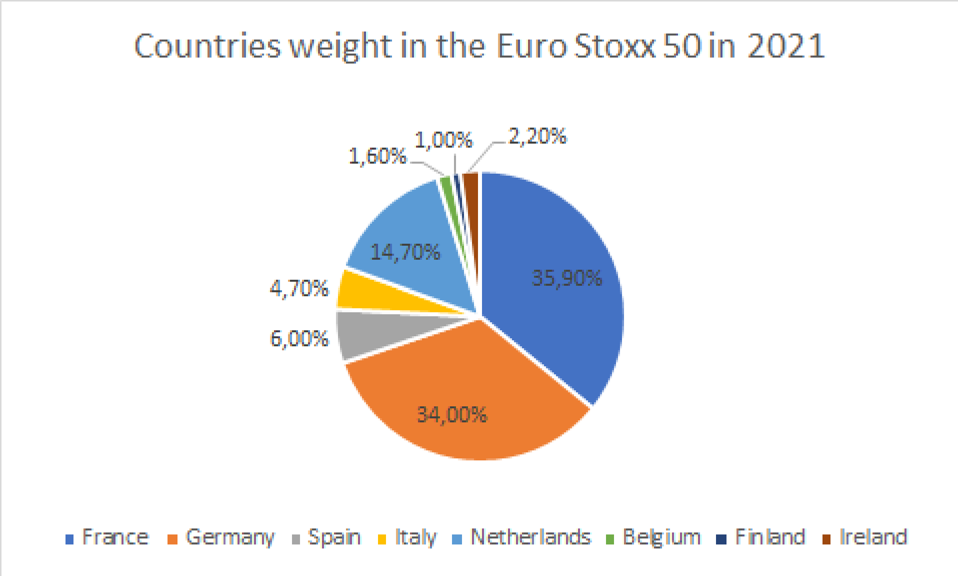
Euro Stoxx 50 is a stock market index that incorporates the 50 biggest companies from the Eurozone. The market index is owned by the Deutsche Börse Group. Those companies are judged by their market capitalization and the index is, like the S&P 500, a free-float capitalization-weighted index. Every month the index is restructured and new companies are added. To become part of the Euro Stoxx 50, a company needs to fill a couple of requirements:

1. Needs to be in the Top 40 of biggest market capitalization in the Eurozone
2. Needs to stay in the Top 60 of biggest Market capitalization every month or the company is deleted from the index

This shows it's tougher to be added to the Euro Stoxx 50 than to be deleted. Even if you are not in the Top 40 you can stay in it because a company is only deleted when its market capitalization is lower than the 60ste biggest market capitalization in the Eurozone. The ten biggest companies in the index are Sap AG, ASML, Total, Sanofi, LVMH, Linde AG, Allianz, Siemens AG, Schneider Electric, L'Oréal. We see that the top 10 is dominated by French and German companies and this trend is also seen when looking at the weight of each country in the index.

[9] S&P Global. (2021, June). *S&P U.S. Indices Methodology*. https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-us-indices.pdf

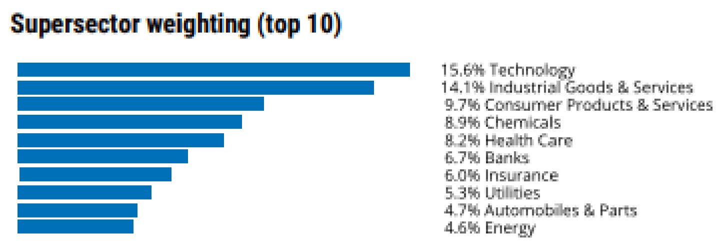
Figure 1: Countries weight in the Euro Stoxx 50 in 2021



Source: Bjorn Pieters (2021)

This index is very diversified, but in recent years technology companies are starting to break into the Top 50 of biggest market capitalization in the Eurozone. We see this trend when looking at the top 10 sectors in the index.

Figure 2: Top 10 Supersector weighting in the Euro Stoxx 50 in 2021



Source: Blue-Chip indices Euro Stoxx 50 Index

Nikkei 225 is a stock market index representing the Tokyo Stock Exchange. The Nikkei is made of the 225 biggest companies in Japan. It is one of the oldest market indexes created in September 1950. It is a price-weighted index and operates in the Japanese Yen. The Dow Jones Industrial Average is also a price-weighted stock market index. As the CFI explains, "*A price-weighted index is a type of stock market index in which each component of the index is weighted according to its current share price. In price-weighted indices, companies with a high share price have a greater weight than those with a low share price. Therefore, the price movements of companies with the highest share price have the largest impact on the value of the* *index.*[10]*"*(Corporate Finance Institute, n.d.). It is also known that, between 2013 and 2017, the Bank of Japan has purchased a significant number of stocks from companies in the Nikkei 225. This creates fear and investors fear that the Nikkei is boosted artificially by the Bank of Japan. This bank is a top 10 shareholder in 90% of the companies in the Nikkei and it also owns around 75% of all Japanese ETFs. The top 10 largest companies by market capitalization are Toyota, SoftBank, Sony, NTT Docomo, Keyence, Nippon Telegraph & Telephone, Fast Retailing, Recruit, Tokyo Electron, Shin-Etsu Chemical. At the end of 2020, the heaviest sector in the Nikkei 225 stock index is Consumer Discretionary with 20.56% followed closely by the Industrial sector (18.37%) and Information Technology sector (17.18%).

US 10-Year Treasuries is a bond issued by the US government. The most well-known US Treasuries are 3-month, 6-month, 12-month, 2 Year, 5 Year, 10 Year and 30 Year. Investors usually use the 10-Year US Treasuries rates as a risk-free rate. We are going to use the 10-Year US Gov total return. This will help us understand how the bond fluctuates with time and how it affects their rate. The main goal is to use this bond as if it was a stock and that is why we need to look at the monthly returns and how the rates fluctuate between each month rather than looking at the 10-Year rate and how much we would earn in 10 Years.

2.4 Risk Parity Portfolio

We will start our explanation with a definition found in a book called Global Asset Allocation from Meb Faber: *"Risk Parity is a term that focuses on building a portfolio based on allocating weights based on risk rather than dollar weights in the portfolio*[11]*"* (Meb Faber, 2015). Risk Parity is based on the Modern Portfolio Theory but it modifies some key aspects: It has a different approach of investing through the use of leverage and it allows short selling. Usually with Modern Portfolio Theory strategies, the investors concentrate their investments in stocks and bonds whereas with Risk Parity strategies the investor can invest in a bigger range of assets. This strategy also changes the way of investment. Investopedia explains that" instead *of generating allocations to different asset classes to arrive at an optimal risk target, risk parity*

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[10] Corporate Finance Institute. (n.d.) *Price-Weighted Index.* https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/price-weighted-index/#:~:text=A%20price-weighted%20index%20is%20a%20type%20of%20stock,weight%20than%20those%20with%20a%20low%20share%20price.

[11] Faber, M. T. (2015). *Global asset allocation : A survey of the world's top investment strategies*, p49

strategies use the optimal risk target level as their basis for investing[12] *"* (Investopedia, 2021). Usually when investors wanted higher returns they needed to bear additional risks. Risk Parity equalizes the amount of volatility and risk through the different assets by using leverage.

Ray Dalio was likely the first investor to create a Risk Parity Portfolio. He called it the All Weather portfolio and had a Strategic Allocation that was slightly different to the Risk Parity Portfolio used in this Thesis.

Our Risk Parity Portfolio is divided into 7 types of investments. Our strategic allocation is: 10.6% of total investments in US Large Cap, 7.9% of total investments in Foreign Developed Equities, 17.9% of total investments in Corporate Bonds, 37.3% of total investments in 10 Year Bonds, 9.8% of total investments in Commodities, 10.5% of total investments in Gold and 5.9% of total investments in REITs. We will explain below the different indexes used for the different categories of investments and how we calculated the different allocations for the different investments.

[12] Investopedia. (2021, February 1). *Risk Parity.* https://www.investopedia.com/terms/r/risk-parity.asp

2.5 Performance measures

2.5.1 Modern Portfolio Theory

To understand the different performance measures, we need to understand its history. The Modern Portfolio Theory, Mean-Variance analysis, was created in 1952 by Nobel Prize winner Harry Markowitz.

Before explaining the Modern Portfolio Theory, we need to explain the different assumptions this theory is based on:

1. The market is efficient and all investors are equal. All information needed by investors to make returns are available. Returns by investors are possible either by predicting future movements by looking at historical data or by technical analysis of a company's fundamentals.
2. All investors are risk averse. Their main goal is avoiding risk.
3. All investors have a goal to earn a maximum rate of return on all of their investments.
4. Decisions by investors are based on expected returns and variance or standard deviation of those returns. The expected return is calculated by dividing the security's purchase price by the income per year and adding annual capital gains.
5. Investors are considered price takers and they are trading without transaction costs.
6. Securities are liquid and highly divisible
7. Investors borrow or lend money at the risk-free rate of interest

What Markowitz tried to explain is that the more types of assets we own, the less risk our portfolio is subjected to. Risk is assessed by looking at the variances of different asset prices. The higher the variance of an asset, the higher the volatility. This means an asset is very risky and very unstable which could lead to high rewards but also high losses. This theory is a more extensive research about diversification.

The theory develops the idea that an investor will always look at the highest possible expected return with the lowest possible risk. It is entirely possible to create a low-risk portfolio with high-risk assets. This theory is the best represented by a single phrase: The whole is greater than the sum of the parts.

For example, in a two-asset portfolio:

- Portfolio return: $\mathrm{E}(R_p) = w_A \, \mathrm{E}(R_A) + w_B \, \mathrm{E}(R_B) = w_A \, \mathrm{E}(R_A) + (1 - w_A) \, \mathrm{E}(R_B)$.
- Portfolio variance: $\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB}$

The Portfolio's return is the sum of two assets, proportionally-weighted, Expected returns. Whereas the Portfolio's volatility is the correlation between the two assets standard deviation.

The main criticism of this theory is that portfolios are evaluated by their variances rather than their downside risk. This means that we are only looking at variances and returns and we are missing other variables. It is possible that two portfolios have the same variance and return but have completely different line charts. One variance can be due to multiple small losses in the last months/years whereas one variance can be due to spectacular declines/rises.

2.5.2 Capital Asset Pricing Model

This pricing model is based on the Modern Portfolio Theory. There are a couple of well-known economists that worked on this model between 1961–1966. The first was Jack Treynor, followed by William F. Sharpe, John Lintner, Jan Mossin. The last person working on this model was Fischer Black (1972), but he created a model without the presence of an asset without risk.

In this model, we assume the same assumptions as in the Modern Portfolio Theory seen above. To understand this model, we need to understand the formula representing it.

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f)$$

E (Ri) = Expected Return of the portfolio

Rf = Risk-free rate of interest (ex: 10-year Us-Treasury bond)

β_i = Sensitivity measure of how much risk an investment will add to a portfolio similar to the market

E (Rm) = Expected return of the market E

(Rm) - Rf = Market risk premium

E (Ri) - Rf = Risk Premium

Before explaining this formula, we need to understand better what the Beta means in this formula.

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)} = \rho_{i,m}\frac{\sigma_i}{\sigma_m}$$

The beta is the covariance between the investments return and the market return divided by the variance of the market return. We can also calculate the beta by dividing the standard deviation of the investment by the standard deviation of the market and multiplying it by the correlation coefficient between them. When the beta is above one, the stock is riskier than the market. When the beta is under one, the stock will reduce the portfolio's risk.

The beta is also known as the systematic risk and explains the probability that certain events will affect market returns, resource production and aggregate income. Systematic risk in portfolio management is risk that hasn't been eliminated by diversification also known as returns in excess of the risk-free rate.

There are 4 types of Beta. The first type is when the Beta value is equal to 1. This means that a portfolio's price is highly correlated with the market used as a benchmark. When the Beta value is 1, there is a presence of systematic risk. This also means that the portfolio is bearing the same risk as the market because it moves in similar ways. The second type is when the Beta value is less than one. This happens when a portfolio is less volatile than the market used as a benchmark. The third type is when the Beta value is greater than one. This means the portfolio is more volatile than the market used as a benchmark. Usually portfolios with large investments in Technology stocks and small caps are more likely to have larger betas than one. Usually those portfolios have larger excess returns than its peers but they are also more likely bearing additional risks. The last type is when the beta is negative. This is when the portfolio is inversely correlated to the market used as a benchmark. This would mean that when the portfolio goes down, the markets go up and inversely. Usually Put options, inverse ETFs and industry groups like gold miners have negative betas.

After understanding the beta, we can explain what CAPM means and what the problem is with this model. We know money loses its value over time, that's why investors feel the need to be rewarded for the risk they take. The risk-free rate in the formula equals the value of money overtime and the rest of the formula is the additional risk taken by the investor. After explaining the Risk-free rate in the formula and the beta above, we only need to explain the market risk premium. The market risk premium is the expected market return minus the risk-free rate.

Thanks to the return found, we will evaluate if a stock is evaluated at his right price considering its risk, it's expected return and the time value of money.

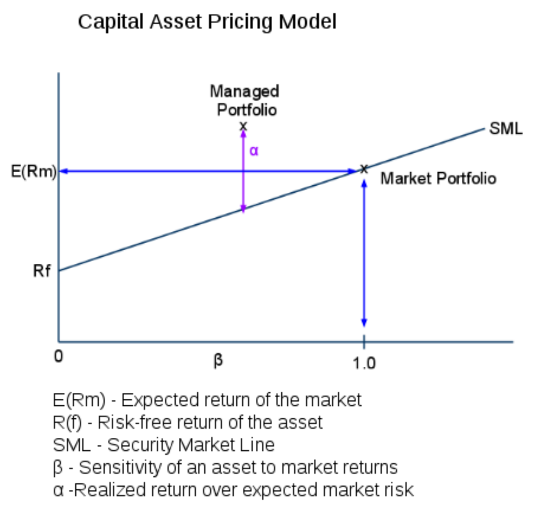
After all this explanation, we also need to assess this model's limitations. Firstly, the assumptions, on which this model is based, are flawed. There are two assumptions that doesn't reflect reality:

1. "*Securities markets are very competitive and efficient (that is, relevant information about the companies is quickly and universally distributed and absorbed)*
2. *These markets are dominated by rational, risk-averse investors, who seek to maximize satisfaction from returns on their investments*[13]" (Kenton, 2021).

Secondly, the model underlines that the risk-free rate will remain constant over time. This is almost impossible because it is constantly moving and a change in the risk-free rate impacts the cost of capital and could lead to an overvaluation of the stock. Thirdly, the expected market return used in the formula will probably be a well-known index (S&500, Nikkei, etc.…), this could lead to imperfect results. Lastly, risk in the formula is measured by a stock's variance. This means we put the same weight on upward price movements and downwards movements, although they do not bear the same risk.

This is an example of the capital asset pricing model in a graph.

Figure 3: Capital Asset Pricing Model



Source: Boglehead (2019)

[13] Kenton, W. (2021, April 1). *Capital Asset Pricing Model (CAPM)*. Investopedia https://www.investopedia.com/terms/c/capm.asp

2.5.3 Jensen's Alpha

We have seen above that a security, if fairly priced, should never be under or above the Security Market Line. The formula calculates that the alpha is positive when the portfolio is outperforming the market. We will explain how this formula is calculated. Firstly, the excess return is calculated by subtracting the portfolio's return by the risk-free rate. After finding the excess return, we will subtract it by multiplying the excess market return by the Beta. The excess market return is calculated by subtracting the market return by the risk-free rate. We have seen above how a Beta is calculated. In this formula, Beta is used to calculate the systematic risk relative to the market. Thus, Jensen's alpha calculates the excess return adjusted for systematic risk.

$$\alpha_J = (R_i - R_f) - \beta_{iM} \cdot (R_M - R_f)$$

where $(R_i - R_f)$ is the Realized Risk Premium

and where $\beta_{iM} \cdot (R_M - R_f)$ is the Estimated Risk Premium

2.5.4 Sharpe ratio

The Sharpe ratio has been created by William F. Sharpe, one of the Capital Asset Pricing model's predecessors. This ratio will help us understand better the risk-adjusted return of a portfolio. By deducting the portfolio's return by the risk-free rate of interest, we only have the excess of return that's left over. This excess of return needs to be divided by the overall risk/volatility that the portfolio is bearing. The answer of this formula is going to help us understand precisely if our risky investments are giving us a decent return or not. A high Sharpe ratio means that our investments are yielding a great risk-adjusted return.

A Sharpe ratio can give us a multitude of information. An increase of the Sharpe ratio can be due to the fact that we added some investments to our portfolio. Like we have seen above, when adding extra investments with a low correlation to a portfolio, the portfolio's volatility is decreasing. This is due to diversification. We can also calculate an expected Sharpe ratio by using expected portfolio returns and expected risk-free rates. A Sharpe ratio also shows us if our portfolio is subjected to too much risk in terms of our return. If we have a high return but we are also subjected to high risk, our Sharpe ratio will not be very high because our excess of return is canceled by our additional risk. Lastly, a negative Sharpe ratio doesn't tell us anything. This is probably due to a low portfolio return or a high risk-free rate.

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

This Sharpe ratio formula has his limitations. Sharpe ratio's main criticism is its denominator. In this formula, we are using the standard deviation to calculate the total risk of a portfolio. The standard deviation expects returns to be normally distributed. This is not on hold with reality because movements can be very surprising and volatile during short periods. We also explained this limitation with the Capital Asset pricing model, giving the same weight on upward movements and downwards movements is an incorrect way of interpreting risk. Finally, Sharpe ratio's can be adjusted to show "false" information about our portfolio. By using a long-time interval for our portfolio, we lower the portfolio's standard deviation. This will lead to a higher Sharpe ratio and, thus, a better risk-adjusted return than reality. Lower we will look at a ratio (Sortino's) that will cancel Sharpe's limitations by only considering negative deviations.

2.5.5 M Squared

Msquared also noted M^2 was created by Leah Modigliani and Franco Modigliani. It is a risk-adjusted return that is an addition between the risk-free rate and the multiplication of the Sharpe ratio and the market risk. This addition creates Msquared. The market risk is calculated by subtracting the benchmark's standard deviation by the portfolio's standard deviation.

$$M^2 = \widetilde{r} + SR \times (\widetilde{\sigma}_b - \widetilde{\sigma})$$

There are two ways of calculating Msquared. Certain investors prefer the first way because of the subtraction between the benchmarks standard deviation and the portfolio's standard deviation. This subtraction helps them understand the rewards behind a portfolio with low risk. The second method doesn't capture the rewards that easily.

$$M^2 = (\widetilde{r} - \widetilde{r}_F) \times \frac{\widetilde{\sigma}_b}{\widetilde{\sigma}} + \widetilde{r}_F$$

2.5.6 Treynor ratio

The Treynor ratio has been created by one of Capital Asset Pricing model's predecessors, Jack Treynor. The Treynor ratio is pretty similar to the Sharpe ratio. We are going to calculate the portfolio's excess return by subtracting his return by the risk-free rate. After finding the excess return, we are going to divide it by the portfolio's beta, also known as his systematic risk. Like we have seen above, systematic risk is risk that cannot be eliminated by diversification. That's where this ratio is different from the Sharpe ratio. One is calculated by looking at the overall risk of a portfolio and one is calculated by using the risk that cannot be eliminated from the portfolio. Like the Sharpe ratio, when the calculated result is negative, this ratio has no meaning whatsoever. This ratio shows us how investors are remunerated while bearing risk that cannot be eliminated.

$$\text{Treynor Ratio} = \frac{r_p - r_f}{\beta_p}$$

Like with every ratio, Treynor ratio has his limitations and weaknesses. Firstly, this ratio is based on historical performances. We have seen the last couple of years that the market cannot be predicted and that historical performances can or cannot be a predicament of the future. Secondly, we have seen above that to create a beta, we need to use a good Benchmark to make good comparisons or to yield precise calculations. When searching for a market variance, we need to first understand which market is the most similar to my portfolio. When my portfolio is mostly made of US Large stocks, we are going to use the S&P500 as a benchmark. Lastly, it is very difficult to compare Treynor ratios between them because all those ratios depend on their portfolio's beta. The higher the ratio the better, but we don't know how better this portfolio is compared to the other one.

2.5.7 Information ratio

The information ratio was created in 1994. The goal of this formula was to compare a portfolio against a chosen benchmark. It is very important to use a benchmark that is similar to the portfolio's main investments. This ratio has no meaning if the benchmark used is completely different to the portfolio's main interests. In this ratio, we will subtract the portfolio's return by a benchmark return. After finding the return of a zero-cost strategy, we need to divide that return by the tracking error, also called residual risk. Before continuing the explanation of this ratio, we will need to explain clearly what a tracking error is. The tracking error measures how consistent a portfolio follows an index movement. When the tracking error is low, this means the portfolio is consistently presenting better results than the benchmark. A high-tracking error means that the portfolio is unstable and consistently presents worse results than the index. To calculate a tracking error, we take the standard deviation of the difference in returns between the portfolio and the benchmark.

When the information ratio is negative, this means the portfolio is outperformed by its benchmark. This usually means the portfolio has a higher risk and a lower return than the benchmark. The Information ratio can also be between 0.40-0.60. This is usually a good result for the portfolio manager. This shows that the portfolio is outperforming the benchmark by an important margin. It is very rare to have an information ratio of 1 during an important Time Horizon. This would mean that the portfolio has never once been beaten during the whole Time Horizon.

There is also a different information ratio called the Model-based Information ratio. This ratio replaces the benchmark's return by the portfolio's expected return. This model is interesting when we are dealing with an actively managed portfolio and when:

1. No benchmark can be identified
2. The portfolio is too diverse and unique to be using a single benchmark index
3. The core investment needs the adding of a satellite portfolio
4. The benchmark is flawed/misfit/misspecified

$$IR = \frac{E[R_p - R_b]}{\sigma} = \frac{\alpha}{\omega} = \frac{E[R_p - R_b]}{\sqrt{\mathrm{var}[R_p - R_b]}},$$

Before we finish this chapter, we need to assess this ratio's limitations. It's very difficult to compare Information ratios between each other because portfolios are very different between each other and everybody is a benchmark that resembles the most of their portfolio. This means we have lots of information ratios with different benchmarks and different portfolios what would make comparisons very difficult. Also, the information ratio needs an important time horizon to make great results. Whatsoever, this ratio stays important and interesting because it's using a benchmark to compare our portfolio. This makes it easy to understand if our portfolio is beating the market or not.

2.5.8 Roy measure

The Roy measure created in 1952 is also called Roy's Safety-First Criterion. This formula explains how a portfolio performs against a minimum acceptable return. Actually, Roy's idea was to compare the portfolio's return against a minimum required return while subjected to a certain level of risk. To find the answer to this formula, we need to subtract the portfolio's expected return by the investors reservation rate (Minimum required return) while dividing the total by the portfolio's standard deviation. The Roy measure helps an investor understand if his portfolio is outperforming or underperforming his expectations because he decides what his minimum required return is.

$$Roy_{p,i} = \frac{\bar{r}_p - \bar{r}_{r,i}}{\sigma_p}$$

2.5.9 Sortino ratio

The Sortino ratio is an extension of the Sharpe Ratio like we acknowledged above. This ratio has the same numerator as the Sharpe ratio but it differs by its denominator. When subtracting the portfolio's return by the risk-free rate, we are not going to divide it by the portfolio's standard deviation but we are going to divide it by the portfolio's negative returns standard deviation, also called standard deviation of the downside risk.

The downside deviation is a risk measure created in 2006 by Estrada. This measure has the ability to only consider results below a minimum that investors call reservation rate. In this formula, we are also giving different weights to movements. The higher the fluctuation, the larger the weight submitted.

$$DD_P = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\max(r_r - r_{p,t}, 0))^2}$$

This ratio is important because we neglect the positive volatility of the portfolio. This is because positive volatility influences the Sharpe ratio and can give a "biased" risk-adjusted performance result.

$$\text{Sortino Ratio} = \frac{R_p - r_f}{\sigma_d}$$

2.5.10 Upside-Downside ratio

The Upside-Downside ratio is using the downside deviation as his denominator like the Sortino ratio. This ratio is different from his peer through his numerator. The numerator of this ratio represents the total number of traded securities closing above the price they open with each day. This ratio helps us understand in which direction the market is moving at this very moment. If there are more securities closing above their opening price than securities closing under their opening price, we are saying that the market is in an upward slope.

$$Upside/Downside\ Ratio = \frac{Advancing\ Issues}{Declining\ Issues}$$

To make calculations easier, we mostly take a moving average to avoid capturing insignificant movements. This ratio can help us make conclusions about a market. When this ratio is high, this could mean that the market is becoming overbought whereas a low ratio tells us that a market is becoming oversold. Oversold means that a security is traded at a lower price than normally and that logically there is a potential that there will be an upward slope in the future with this security. Overbought means that a security is traded at a high price than its “normal” price, we could imagine that this security in the future will decline and be in a downside movement.

2.5.11 Omega Ratio

The omega ratio was created by Shadwick and Keating in their article “A Universal Performance measure”. To calculate Omega, we need to divide the upward potential by the downside potential. Those are two calculations that we haven’t seen before.

$$\Omega = \frac{Up_P}{Down_P}$$

The upside potential is calculated by measuring a portfolio or a security’s ability to generate returns above a minimum. We also need to acknowledge how many times a security or portfolio had an upside movement and the amplitude of this upside.

$$Up_P = \frac{1}{T}\sum_{t=1}^{T} \left(R_{P,t} - R_L, 0\right)$$

The downside potential is calculated by measuring a portfolio or security's ability to generate returns under a minimum. We also need to acknowledge, like above, the number of downside movements this security/portfolio had and its amplitude.

$$Down_P = \frac{1}{T}\sum_{t=1}^{T} \left(R_L - R_{P,t}, 0\right)$$

The omega ratio's big advantage is that we consider the whole range of returns. This is important because it is entirely possible that some returns are asymmetric due to a certain investment strategy. By looking at an entire distribution of returns, we are able to consider its skewness and kurtosis.

2.5.12 Drawdown and It's Variables

When we are talking about drawdown in finance, we are talking about a period where a security or portfolio is subjected to a continuous losing return. There are different drawdowns. There is the average drawdown, the maximum drawdown, the largest individual drawdown, the drawdown duration and the drawdown deviation.

The drawdowns help us make a couple of conclusions about a security or a portfolio because it is a measure of downside volatility. The drawdowns help us understand how much a security or a portfolio is down from its peak. This is different from a loss because a loss only looks at the purchase price and the exit price. A Drawdown only stops being a drawdown when it has passed its previous peak. If an investment goes from 100 to 90, we are talking about a 10% drawdown and this drawdown continue until the moment an investment hits its new peak.

The average drawdown helps us understand if an investment is having frequent negative movements and if the investment is stable or not. We mostly use a 3-year period for calculating an average drawdown.

If a drawdown is

$$D(T) = \max\left[\max_{t\in(0,T)} X(t) - X(T), 0\right] \equiv \left[\max_{t\in(0,T)} X(t) - X(T)\right]_+$$

Where Max X (t) represents its peak and X (T) represents its position right now.

Then the Average Drawdown is

$$\mathrm{AvDD}(T) = \frac{1}{T}\int_0^T D(t)\,dt$$

Where we look at the time average of a drawdown during a certain period of time (3-year mostly). It is very difficult to make comparisons between average drawdowns because investors mostly use different periods of time or number of drawdowns during a certain period.

The maximum drawdown, also known as peak to valley drawdown, is the largest difference between its peak and its lowest point during a certain period. It is the maximum potential loss someone could record when looking at past data. The maximum drawdown can be calculated like this.

$$MDD = \frac{Trough\ Value - Peak\ Value}{Peak\ Value}$$

The maximum Drawdown also has its limitations. The maximum drawdown doesn't show how long it took to find its previous peak or if the previous peak has been touched. The maximum drawdown also doesn't show us if those negative movements are frequent or not.

The drawdown duration, also known as Recovery time, is the time it took an investment to go from its peak to the lowest point and back to its peak. The Recovery Time can be calculated from its lowest point or it can be calculated from its highest point.

The drawdown deviation is a standard deviation calculated by using individual drawdowns from a certain period. This is used to calculate the risk of drawdowns happening.

2.5.13 Sterling Ratio

The sterling ratio was created by a company called Deane Sterling Jones. This ratio is similar to the Maximum Drawdown. When looking at the Maximum Drawdown, like we explained above, we take the worst point an investment had during a certain period and we wait until it touches its peak again. In the Calmar ratio, we replace the average largest Drawdown in the formula by the maximum drawdown.

There are 2 widely used formulas when calculating the Sterling ratio. The original sterling ratio divides a portfolio's return by the Average largest drawdown added by 10%.

$$\text{Original Sterling ratio } OSR = \frac{\tilde{r}}{\overline{D}_{Lar} + 10\%}$$

We add 10% because the average Largest Drawdown is smaller than the Maximum Drawdown. This is due to the fact that we take a couple of drawdowns to calculate the average. Thus, those smaller drawdowns impact the largest drawdowns creating a smaller average than the maximum drawdown.

$$SR = \frac{Annual\ Portfolio\ Return - Annual\ Risk\text{-}Free\ Rate}{Average\ Largest\ Drawdown}$$

The Sterling ratio that we are currently using is the one we see above. The 10% is deleted from the equation because it has no important purpose for comparison. When Jones invented the Sterling ratio T-bills were yielding 10%, which is something that doesn't happen anymore.

2.5.14 Calmar Ratio

The Calmar ratio was created in 1991 by fund manager Terry Young. He wanted to create a ratio that was "less sensitive" than the Sterling ratio. This ratio is also called drawdown ratio. Calmar is an acronym for California Managed Annual Reports. When the ratio was initially created, there was no risk-free rate in the numerator.

The Calmar ratio is exactly the same as the Sterling ratio apart from the fact that the denominator is the Maximum Drawdown and not the average Largest Drawdown. Mostly the Maximum Drawdown is calculated on a 3-year period. It is also important that we use the same period and the same frequency of data when analyse the performance of different portfolios

$$\text{Calmar ratio } CR = \frac{\tilde{r} - \tilde{r}_F}{D_{Max}}$$

The Calmar ratio has a couple of weaknesses and strengths. His main weakness is also its strength: The use of the Maximum Drawdown as a measure of risk. The Calmar ratio is updated monthly and its 3-Year Time Horizon makes the ratio more reliable than its peers using shorter Time Horizons and more subject to natural market volatility. On the other hand, using the Maximum Drawdown as a risk measure makes us ignore volatility. This makes the ratio less statistically significant and useful.

There is also a variation of the Calmar ratio called Sterling-Calmar ratio. This ratio is the same as the Calmar ratio but we use the average annual maximum drawdown. This ratio also uses a Time Horizon of 3 years.

$$\text{Sterling-Calmar ratio } SCR = \frac{\tilde{r} - \tilde{r}_F}{\overline{D}_{max}}$$

2.5.15 Pain index

The pain index was created in 2006 by Thomas Becker and Aaron Moore of Zephyr Associates. This risk measure is very similar to its peers but it differs in its definition. The pain index uses losses as a measure of risk. To be precise, it measures the depth, the frequency and the duration of losses.

The pain index is calculated by synerging the depth, frequency and duration of losses. The longer a loss takes to hit its peak again, the larger the pain index will be. We also need to take into account how much the investment went down but also if this is a recurring trend. After looking at those 3 sides of a loss, we can measure if the Pain Index is big or small. The smaller the Pain index, the better an investment opportunity an asset is. Mostly, when searching for portfolios or investment strategies with low pain indexes, we find saving accounts, money markets or certificates of deposit. Money is never lost but we are not yielding impressive returns.

$$\text{Pain index } PI = \sum_{i=1}^{i=n} \frac{|D_i'|}{n}$$

This risk measure is mostly used for investors that are risk averse and scared of losing money when investing, prioritising capital preservation. Below we will see the pain ratio which helps us but a portfolio's return against the index. This will help us understand if the risk taken, or not taken, is worth its return. The higher the pain ratio, the better. This would mean return is important against the risk taken by the investor.

$$\text{Pain ratio } PR = \frac{\tilde{r} - \tilde{r}_F}{\sum_{i=1}^{i=n} \frac{D_i'}{n}}$$

In the Pain ratio, we use the same numerator as the Sharpe ratio. We can clearly see that this a recurring trend with the ratios seen above. We will subtract the portfolio's return by the risk-free rate to find how much this portfolio is yielding above the risk-free rate. After finding this answer, we will divide it by the pain index that we developed above.

2.5.16 Ulcer Index

The Ulcer index was created in 1987 by Peter G Martin and Byron McCann. This index is similar to the drawdown deviation because it mainly measures the downside risk, but it differs in the fact that we take into account the time recovering from its peak and the depth of the drawdown. This index will perform badly if a portfolio has long and deep drawdowns because of the weight this ratio put on those losses. The index is mostly calculated on a 14-day period. The higher the value of the index, the longer it takes an investment to recover from recent highs.

$$\text{Ulcer index } UI = \sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}$$

Where D' represents Drawdown since the previous peak in period i.

2.5.17 Martin ratio

The Martin ratio, also known as the Ulcer performance index, was created by Peter G Martin. Like we have seen above, the numerator is the same as the Sharpe, Pain, etc. ratio. The denominator is the Ulcer index. In this case, the higher the Martin ratio, the better the portfolio is performing adjusted to the risk.

$$\text{Martin ratio } MR = \frac{\tilde{r} - \tilde{r}_F}{\sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}}$$

2.5.18 Burke ratio

This ratio was first presented in 1994 by Burke in a paper called "A sharper Sharpe ratio". Like we have seen above, the Burke ratio's numerator is the same as Sharpe, etc.We are going to subtract the portfolio's return by the risk-free rate. When calculating the denominator, we take the square root of each drawdown. Taking the square root of drawdown impacts the ratio greatly because we are putting extra weight on larger drawdowns than smaller ones. This means, the larger the drawdown a portfolio is subjected to, the lower the Burke ratio, the worse the Fund is performing with this ratio.

$$\text{Burke ratio } BR_d = \frac{\tilde{r} - \tilde{r}_F}{\sqrt{\sum_{j=1}^{j=d} D_j^2}}$$

Some investors use the modified Burke ratio. This ratio is the same as the Burke ratio apart from the fact that we divide the square root of the largest drawdowns by n (the amount of data points). This doesn't change anything at the result we are getting and doesn't change a portfolio's ranking. The modified Burke ratio just takes into account the number of drawdowns that happened during a certain period.

$$\text{Modified Burke ratio } MBR_d = \frac{\tilde{r} - \tilde{r}_F}{\sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}}}$$

2.5.19 Omega-Sharpe ratio

The Omega-Sharpe ratio is the average portfolio return subtracted by the target return and the total divided by the sum of the upside and downside potential that we have seen above. Mostly, this ratio will rank portfolio's in a similar fashion than the rank created by the Omega ratio.

$$OmegaSharpeRatio(R, MAR) = \frac{r_p - r_t}{\sum_{t=1}^{n} \frac{max(r_t - r_t, 0)}{n}}$$

2.5.20 Value-at-Risk

The Value-at-Risk, also called VaR, was developed in 1990 when the CEO of JP Morgan asked for a one-page report 15 minutes before the market close where the firm's risk was assessed. *"VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence level[14]"* (Bacon, 2013). For example, when the annual VaR is 10 million dollars at a 95% confidence level, this means there is a 5% chance that the loss will exceed 10 million dollars. The VaR is mainly calculated by using historical data. This has its weaknesses because, for example, the one-day VaR is predicting at 99% of confidence that the investment will never lose more than its second-worst historical loss. In this Thesis, we will use the VaR in 3 different ways.

[14] CARL BACON, Practical risk-adjusted performance measurement, p120

The first method is the Parametric VaR, also called Variance-Covariance method, also called Gaussian method. This method only uses the standard deviation and the mean return to calculate the VaR because it assumes that the returns are normally distributed.

The VaR with 95% confidence is: $\mathrm{Va}R_{95\%} = \bar{r} - 1.65 \times \sigma$

The second method used to calculate a VaR is the modified VaR. Thanks to the Cornish-Fisher expansion, a Var is influenced by the kurtosis and skewness of the return distribution. The skewness is usually used to measure the symmetry of a distribution whereas the kurtosis is usually used to measure the weight of the distribution tails. In conclusion, kurtosis analyses the tail shape and skewness analyses the overall shape.

The last method used is the Historical Simulation method. This method will rank historical returns from best to worst using the current portfolio's holdings. *"The value at risk is determined at the 95th percentile for a 95% confidence level and the gain at risk in effect is the 5th percentile[15]"* (Bacon, 2013). This method predicts that future returns will move exactly to their past.

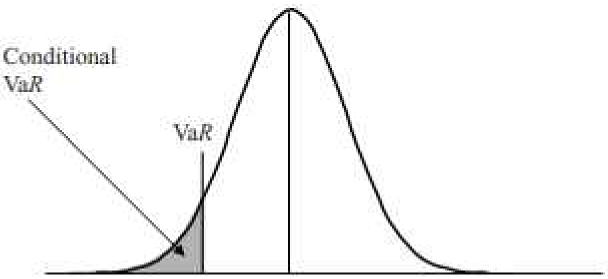
2.5.21 Expected Shortfall

The Expected Shortfall is also called the conditional value-at-risk. It is a measure that complements the Var's weaknesses. The Var will calculate how much will be the expected loss at a certain confidence level but the Var will not calculate how much will be lost when this threshold is surpassed at that confidence level. That is the Expected Shortfall's job. In other words, the Var will focus on the probability of the biggest expected loss, whereas the Expected Shortfall will focus on the probability of that loss and its size.

The ES is mainly used to evaluate credit or market risk of a portfolio. The ES will take a time horizon and will make an average of all the returns that are below the Var of a portfolio at a certain confidence level. If in 95% of the cases, the loss will never exceed a certain amount, the ES will make an average of all the amounts that surpassed that certain amount. A lot of investors favor the Expected Shortfall because it is a more coherent risk measure than the VaR.

[15] Bacon, C. R. (2013). *Practical risk-adjusted performance measurement*. Wiley. P125

Figure 4: Graphical representation of the conditional VaR on a normal distribution



Source: Practical Risk-Adjusted Performance Measurement, p132, BACON

When calculating the ES, we will use the same methods explained above: The Historical Simulation method, the Modified method, and the Variance-covariance method.

2.5.22 Value-at-Risk and Expected Shortfall ratios

We have seen above that the Sharpe ratio has inspired a lot of different risk measures. In the ratios presented below, we will see that the different methods of Var and the different methods of ES will be used as risk measures instead of the standard deviation usually used when calculating the Sharpe ratio. The numerator doesn't change and is still the portfolio's return subtracted by the risk-free rate.

Figure 5: The 3 different ratios when using the VaR as a risk measure

$$Hist\ VaR\ Sharpe_p = \frac{\bar{r}_p - \bar{r}_f}{Hist\ VaR_p}$$

$$Gauss\ VaR\ Sharpe_p = \frac{\bar{r}_p - \bar{r}_f}{Gauss\ VaR_p}$$

$$Modified\ VaR\ Sharpe_p = \frac{\bar{r}_p - \bar{r}_l}{Modified\ VaR_p}$$

Source: "Testing Marc Faber's asset allocation", Guillaume Genin, p40 (2017)

Figure 6: The 3 different ratios when using the Expected Shortfall as a risk measure

$$Hist\ ES\ Sharpe_p = \frac{\bar{r}_p - \bar{r}_f}{Hist\ ES_p}$$

$$Gauss\ ES\ Sharpe_p = \frac{\bar{r}_p - \bar{r}_f}{Gauss\ ES_p}$$

$$Modified\ ES\ Sharpe_p = \frac{\bar{r}_p - \bar{r}_f}{Modified\ ES_p}$$

Source: "Testing Marc Faber's asset allocation", Guillaume Genin, p40 (2017)

3. Empirical Analysis

After thoroughly explaining all the different ratios, formulas and technical terms, we are going to analyse the results of those measurements. We used professional software to ensure that our data is used properly and the ratios don't bear any mistakes. This software, called Rstudio, is an open-source programming tool that helps us make sense of data. The data used in the programme is the monthly return of the 10-Year Us government Treasury (US10GOV), the Norwegian Wealth Fund (NOBSGLIA, the S&P 500 (SPX), the Nikkei (NKYTR) and the Euro Stoxx 50 (SX5E Index) between 31 September 2006 and 31 January 2021. We also decided that we would create a Risk Parity Portfolio and we would analyse its monthly return in Rstudio against the benchmarks and the Fund. Nonetheless, we decided to use the monthly returns between January 2010 and January 2021 because our Risk Parity portfolio's allocation is based on data between 2005 and 2010. The data was calculated in excel and imported from the Bloomberg platform available on the Louvain-la-Neuve Campus. Mikael Petitjean helped us import the data in Rstudio to ensure no mistakes were made when launching the programme and creating the different results (Rstudio code can be found Appendix 3-4).

We will now explain the different stages in our Empirical Analysis. The first stage will be a quick explanation and reminder about the strategic allocation of the Norwegian Sovereign Wealth Fund and the Risk Parity Portfolio. This will help us understand how the investments are made and the differences between each other and the different indexes. In the second stage, we will analyse the results presented by Rstudio. We will start with a little glimpse at the median, mean, minima, maxima, skew, kurtosis, etc. After this little glimpse, we will look at a graph created by Rstudio. This graph represents the Cumulative returns of each fund, index, government bond between January 2006 and January 2021. The second cumulative returns graph will show how the Fund and the benchmarks performed between 2010 and 2021 against our self-created Risk Parity Portfolio. Once the Cumulative results are analysed, we will analyse other graphs created by Rstudio called Returnsplot where we see the different positive and negative variations over the years of each participant. We will end this "return" stage with a look at the monthly and annualised returns of each participant.

The next result approached is the standard deviation and the correlation between each participant. This will be presented in a table where the correlation between participants is weighted in stars. Correlation coefficients above 0.3 are considered significant enough to receive 3 stars. We will explain this more thoroughly below. Before looking at the ratios, we finish this returns and volatility paragraph with the global return, the annualised return and the volatility of each participant.

The third stage in our empirical analysis is about ratios. In this stage we will compare the different participants' CAPM against a benchmark's CAPM of our choice. This benchmark can be the S&P 500 but also another participant. We follow our analysis with a monthly and annualised Sharpe ratio. We will sometimes use the 10-Year government bond as a risk-free rate, and sometimes we will use 0 as a risk-free rate. Moreover, we will analyse the results of a variety of ratios or formulas like the Treynor ratio, Msquared, different downside risk ratios (omega, upside/downside potential, Omega-Sharpe, Sortino, …), the 5 worst drawdowns of each participant, and the different drawdown ratios (Sterling, Calmar, Ulcer, Pain, Martin, Burke ratios).
The fourth stage of our empirical analysis will be a presentation of the different VaR and Expected Shortfall results. We will also take a glimpse at the Sharpe ratio, with the VaR or ES as denominator, results. The fifth stage will be a roundabout of everything we have learned and a conclusion about our Fund and its results.

3.1 The Norwegian Wealth Fund's Strategic Asset Allocation

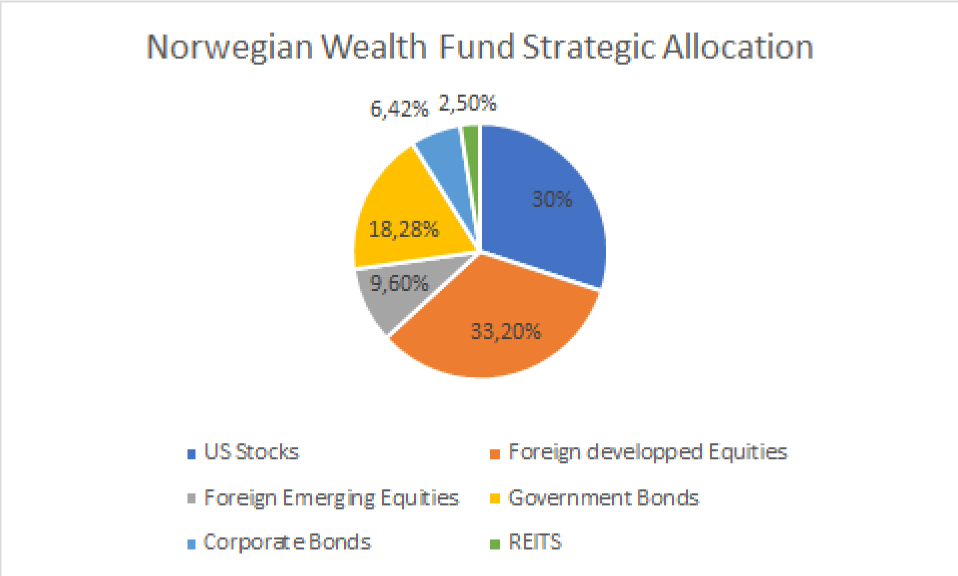
The Fund has a fairly simple strategic allocation. They invest 72.8% in equities, 24.7% in Fixed-Income and 2.5% in Real Estate. They try to never invest more than 7% of all their investments in Real Estate. The Fund also has a rule that they should never own more than 10% of a single company. The equity investments are distributed between North America (43.9%), Europe (30.9%), Asia-Pacific (23.6%), and others (1.9%). The Fund doesn't provide information about their investments in foreign developed markets, foreign emerging markets. Thus, we made this allocation by using the FTSE list of developed, emerging countries. The whole portfolio is made of 30% of US stocks, 33.2% of Foreign Developed stocks and 9.6% in Foreign Emerging stocks.

Their Fixed-Income investments, which account for 24.7% of the Fund's total investments, are distributed between Government bonds (56.5%), Government-related bonds (11.6%), Inflation-linked bonds (6.3%), Corporate bonds (26.1%) and Securitised bonds (5.8%). It's normal, according to the Funds 2020 report, that the different percentages above don't add up to 100% because cash and derivatives are not included. To make it easier to understand and to make the graph as clear as possible, we will divide the Fixed-Income investments in two divisions: Government bonds (73.9%) and Corporate bonds (26.1%).

Their real estate investments, which account for 2.5% of the Fund's total investments, are distributed between unlisted real estate investments (65.1%) and listed real estate investments (34.9%). We explained above what each real estate investment means.

Below the Norwegian Sovereign Wealth Fund Strategic Asset Allocation.

Figure 7: Constituents of the Norwegian Sovereign Wealth Fund asset allocation



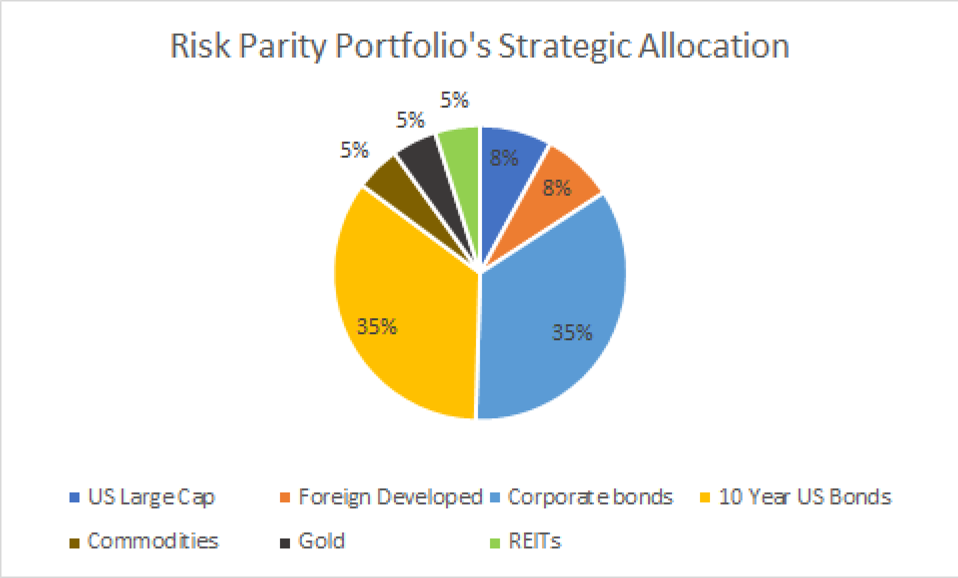
Source: Bjorn Pieters (2021)

3.2 Risk Parity Portfolio's Strategic Allocation

We used the Global Asset Allocation book from Meb Faber to find the 7 most used categories of investment when creating a Risk Parity portfolio. We saw that they used an allocation for the Risk Parity portfolio that is slightly different from the allocation found when creating our own Risk Parity Portfolio. We will explain below how we calculated the allocation.

This graph shows us the "perfect" Risk Parity portfolio based on historical performances between 1973 and 2013. This is what allocation Meb Faber advised us to have when creating a risk Parity Portfolio. We decided to create our own allocation because we deemed it important to understand the different allocations behind our portfolio.

Figure 8: The Risk Parity Portfolio's Optimal Strategic allocation



Source: Meb Faber Global Asset Allocation (2015)

3.2.1 The Methodology behind the creation of our Risk Parity Portfolio

After deciding the 7 categories of investments, based on the book above, we needed to find how much share each investment would have in the portfolio. Our first step was finding the different representations for our categories. We explained below the thought process behind those choices. We, then, needed to calculate the returns for each month for each category. This would help us assess the different correlations and covariances between the investments. We used the Formula Covariance. S in excel between the 7 investments to help us find the correlations between them. The Formula used in excel to create such a table is the one mentioned below.

```
=COVARIANCE.STANDARD(INDEX($J$2:$P$2;$R5):INDEX($J$50:$P$50;$R5);INDEX($J$2:$P$2;T$3):INDEX($J$50:$P$50;T$3))*252
```

Important note: The J and P columns are the different returns for the different asset classes whereas the R and S columns help us define between which assets the correlation is happening. We multiplied the formula by 252 because there are 252 trading days each year. We used Index in the formula because it helps us return a value to another value in a table. It is also important to know that we only used the data between 31/12/2005 and 31/12/2009 included to create the portfolio. This is why, when calculating the covariance, we only used 50 lines. This created the table seen below.

Table 8: Covariance-Correlation between the asset classes from 31/12/2005 to 31/12/2009

		1	2	3	4	5	6	7
	Covariance	MSCI EAFE	S&P 500 CC	US BENCHI	ISHARES IB	FTSE Nareit	S&P GSCI G	Dow Jones Commodity Index TR - RETURN IND. (OFCL)
1	MSCI	0,956891	0,711836	-0,040535	0,234778	1,148218	0,226008	0,605302
2	S&P 500	0,711836	0,633728	-0,039153	0,158662	1,036635	0,081504	0,383982
3	US Bench	-0,040535	-0,039153	0,157225	0,089996	-0,072863	0,118489	-0,056485
4	ISHARE	0,234778	0,158662	0,089996	0,234395	0,298982	0,08557	0,078633
5	FTSE	1,148218	1,036635	-0,072863	0,298982	2,530574	0,107964	0,52272
6	Gold	0,226008	0,081504	0,118489	0,08557	0,107964	0,985788	0,531456
7	Dow Jones	0,605302	0,383982	-0,056485	0,078633	0,52272	0,531456	0,983443

Source: Bjorn Pieters (2021)

Our third step was finding the variance, and thus the volatility with the Racine formula in Excel. We calculated the variance based on the covariance table seen above and the supposition that each asset class would represent the same share of total investments in the portfolio. Because we divided 100% of total investments by 7 (number of asset classes), we found that each share of investments would represent 14,2857%. Below this is the formula used to calculate the variance.

```
=PRODUITMAT(U16:AA16;PRODUITMAT($T$5:$Z$11;TRANSPOSE(U16:AA16)))
```

Important note: U16:AA16 are the different shares of investments for the different asset classes and $T$5:$Z$11 represents the values in the correlation table seen above. Produitmat helped us multiply two arrays against each other whereas Transpose helped us arrange the horizontal values from the asset classes in vertical values to allow them to be used in the calculation of the variance.

This created a variance of 0.385825 and a volatility of 0.621149.

Table 9: The portfolio's volatility and weights between asset classes

				Asset Class weights				
Portfolio	Volatility	Variance	MSCI	S&P500	US Bench	ISHARE	FTSE	Gold	Dow Jones
Equal-weig	0,621148	0,385825	0,142857	0,142857	0,142857	0,142857	0,142857	0,142857	0,142857
Risk Parity									

Source: Bjorn Pieters (2021)

The next step was a multiplication of the covariance against the different weights. We used the formula seen below to calculate the table.

```
{=PRODUITMAT(U16:AA16;$T$5:$Z$11)}
```

We multiplied the covariance table with the asset class weights table to create such results.

Table 10: The portfolio's covariance multiplied by the different weights

		Covariance*Weights				
MSCI	S&P500	US Bench	ISHARE	FTSE	Gold	Dow Jones
0,548928	0,423885	0,022382	0,168717	0,796033	0,305254	0,435579

Source: Bjorn Pieters (2021)

We needed to accomplish one last step before we were able to find the optimal allocation for the Risk Parity portfolio. Our final step was finding the different risk contributions for each asset class. Because we assumed since the beginning that each asset class would represent 14.28%, we will multiply the Covariance*weights table above by 14.28% for each asset class. This showed us which asset classes were the riskiest and, thus, which asset classes needed a decrease in their share of investments. We saw that the FTSE and the MSCI were the riskiest assets.

Table 11: The asset's risk contributions to the portfolio

	Risk Contributions					
MSCI	S&P500	US Bench	ISHARE	FTSE	Gold	Dow Jones
0,203248	0,156949	0,008287	0,06247	0,294742	0,113025	0,161279

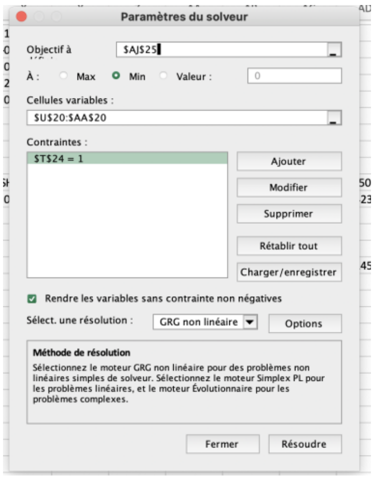
Source: Bjorn Pieters (2021)

After finding the different risk contributions, we needed to know the different deviations between the asset classes. It is important to know the different deviations between the asset classes because the sum of the deviations between them will be used to calculate the allocations for the portfolio. We calculated the different deviations between the assets by subtracting the different risk contributions by 1/7 and using ABS (absolute) in the formula allows us to change any negative values into positive values. The formula used can be seen below.

```
=ABS(AK20-1/$T$22)
```

This created a sum of deviation of 0.48958. We used solver to find the optimal allocation for the Risk Parity Portfolio. Our "cellules variables" were the different weights possible for our Risk Parity Portfolio. It was important to establish that $T$24 was equal to 1 because the sum of weights cannot go beyond 100. Lastly, the “Objectif à ” was the sum of the deviation from the Risk Parity portfolio if we gave each asset class an equal share of investment.

Table 12: Utilisation of Solver to find the optimal allocation for the portfolio



Source: Bjorn Pieters (2021)

This created the results that we will explain below with a volatility of 42.72%.

Table 13: Optimal Asset Allocation for the Risk Parity portfolio

Solver									
Risk Parity	0,427203	0,182502	7,9%	10,6%	37,3%	17,9%	5,9%	10,5%	9,8%

Source: Bjorn Pieters (2021)

Because we already explained how we created the strategic allocation above, we will concentrate ourselves on the indexes used for each category. Our risk parity portfolio is divided in 7 types of investment and we used indexes that represent, the best they can, those categories.

In this portfolio, we invested 10.6% of our total investments in US Large Cap equities. We decided that the S&P 500 Composite Total Return would be the best representation of US Large Cap equities because it represents the 500 largest companies in the US and it’s total return on a month-to-month basis.

In the Global Asset Allocation book, that we already talked about above, a balanced Risk Parity portfolio has 7.9% of their Total investments invested in Foreign Developed equities. We used the MSCI EAFE Total Return USD Index to represent the Foreign Developed equities. We used this index because it *"captures large and mid-cap representation across 21 Developed Markets countries around the world, excluding the US and Canada*[16]*"* (MSCI EAFE Index, 2021).

We are now going to explain the two representations used for the two largest investments in this portfolio. Firstly, for this Risk Parity portfolio, we invested 37.3% of the total investments in 10 Year US Bonds. We used the US Benchmark 10 Year DS Government Total Return index. This Total return index includes Income gains and Capital gains or losses depending on investment rate fluctuations. This index helps us capture the real returns for the 10-Year US bond by looking, month-to-month, how its return is influenced by income gains (coupons) and Interest rate variation.

Secondly, in this Risk Parity portfolio, we invested 17.9% of our total investments in Corporate bonds. We thought the best representation for this investment was the Ishares IBOXX Investment grade Corporate Bond ETF Total Return Index. We decided that this index represented well the Corporate Bonds category because it is a blend of 1000+ high-quality corporate bonds and there is an exposure to a whole range of Investment grade U.S. Corporate bonds (Microsoft, Goldman Sachs, AT&T, …).

Firstly, the portfolio has a 10.5% share of their Total investments in Gold. We used the S&P GSCI Gold Total Return as representation for those investments. *"The S&P GSCI Gold Index, a sub-index of the S&P GSCI, provides investors with a reliable and publicly available benchmark tracking the COMEX gold future*[17]*"* (S&P Global, 2021). We used the Total Return because this helps us capture the actual rate of return with interest, dividends, capital gains and distributions.

Secondly, in our Risk Parity portfolio, we invested 5.9% of our total investments in REITs. We used the FTSE NAREIT All Equity REITS Total Return Index for this category because the "*constituents of the index include all tax-qualified REITs with more than 50 percent of total assets in qualifying real estate assets other than mortgages secured by real property*[18]*"* (Nareit, 2021).

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[16]MSCI EAFE Index. (2021). *MSCI EAFE Index (USD).* Retrieved from https://www.msci.com/documents/10199/822e3d18-16fb-4d23-9295-11bc9e07b8ba

[17] S&P Global. (2021). *S&P GSCI Gold.* https://www.spglobal.com/spdji/en/indices/commodities/sp-gsci-gold/#overview

[18] Nareit. (2021). *FTSE NAREIT All Equity REITs (FNER).* https://www.reit.com/data-research/reit-indexes/real-time-index-returns/fner-ftx

Lastly, we invested 9.8% of our total investments in Commodities. We used the Dow Jones Commodity Index Total Return because it represents 28 different commodities from agricultural to precious metals to energy products and represents US commodities.

3.3 Return and Volatility

In this chapter, we are going to analyse different graphs. The goal is to understand how the different funds/indexes have moved over the years and if past movements prove future movements. Like we already explained before, the 4 participants are the Nikkei, the S&P500, the Norwegian Sovereign Wealth Fund and the 10-Year US treasuries. We calculated the first rate of return of each participant starting 31 January 2006. This makes the graph very telling and shows us which investment would have been the more profitable.

3.3.1 Cumulative Return between 2006–2021 for the Fund and the 4 benchmarks excluding the Risk Parity Portfolio

We will start our analysis with one of the "worst" performers in this graph. The Nikkei has underperformed the 10 Year US-Treasuries between 2006 and 2018. There have only been two times where the Nikkei was overperforming the 10-Year US Treasuries: Between January 2018 and January 2019, and since April 2020. It is very surprising to see the Nikkei, which represents the 225 largest Japanese companies, being outperformed by a bond. Moreover, the US Treasuries are bearing absolutely zero risk whereas the Nikkei has a much larger risk because it is primordially made of stocks. Nikkei's bad cumulative return is due to their slow recovery from the 2008 Financial crisis. We can see in this graph that the index only recovered fully and hit their 2006 levels at the end of 2013 whereas the US treasuries hit their initial level in January 2012 already. Moreover, the S&P 500 didn't suffer as much as the Nikkei because their cumulative return didn't crash during the financial crisis and afterwards. We will explain this below. We gathered a couple of reasons that could explain why the Nikkei had such difficulties recovering from the 2008 Financial crisis. First and foremost, Japan lost one of the country's biggest companies during the crisis. This company, called Yamato Life Insurance, was labelled "established life insurer". After the crisis, the country started their path to growth. It became abundantly clear that this path will not be without slumps. In 2011, the Nikkei suffered again after Fukushima's accident that led to investor panic. This was the first nuclear drama since Chernobyl and this created tensions on the Japanese stock market. The Nikkei would suffer greatly during that time. The Japanese stock market would not be the only market to suffer. The downgrading of the United States credit rating from AAA to AA+ by Standard & Poor's and Europe's worries of a sovereign debt crisis would lead to bigger slumps in the stock market. The Nikkei would suffer again early 2013 after investors became worried about *"weak economic data from China and indications that the US Federal Reserve may start dialing down* *its bond-buying programme as early as June (the following month)*[19]*"* (Petroff, 2013). After 2015, the Nikkei would move very similarly to the S&P 500 and would suffer only two more setbacks that the S&P 500, although lesser, would also endure.

The 10-Year US treasuries have been really stable through the years. This bond is really liked because it will always yield a decent return, no matter the times. We see that during and after the 2008 financial crisis, when stocks were yielding bad returns, that the bond returned better than the Nikkei and the S&P 500.

Even more surprising, it wasn't until the end of 2016 that the S&P 500 cumulative return was higher than the US treasuries. It is not too surprising to see that the bond was doing greatly during the 2008 financial crisis because people search for safe ways to place their money when everything is crashing. Stocks and bonds have an extensive history together. We will explain the different possibilities in movements between stocks and bonds. Usually when stocks are doing badly, bonds go up. This is because people are searching for a safe investment while still searching for ways to win money. The bonds go up because demand is rising for bonds and declining for stocks because people are scared to lose money. Bonds have a lower return than stocks but they are also much safer because they got a government's assurance behind. There is also a possibility where stocks and bonds go up at the same time, this is due to the fact that there is too much liquidity or too much money chasing too few investments. There is also a possibility when bonds and stocks go down, this happens when everyone is panicking and investors are selling everything. This is why portfolio managers always have a certain percentage of their portfolio invested in gold because gold goes up when everything is going down. This is why in this graph we see that the Treasuries cumulative return always slows down when indexes are yielding higher returns. We see that, in 2020, stocks are going up greatly while the bonds are slowly going down in cumulative returns and even being surpassed by the Nikkei.

The S&P 500 has been doing better than the Nikkei since 2006. They are also outperforming the US Treasuries since the end of 2016. We can see that its movement is mirrored by the Nikkei but the S&P always stays above them. This is probably due to the fact that US Stocks handled and recovered better from the 2008 financial crisis. The S&P 500 recovered completely from the crisis in 2013. It has gone from trading at 666.8 in March 2009 to 3386.2 in February 2020

[19] Petroff, A. (2013, May 23). *Japan plunge spooks global markets.* CNN. https://money.cnn.com/2013/05/23/investing/japan-nikkei-fall/

which is a return of almost 400%. Investors are also talking about a 10-Year Bull market. This rise is probably due to stable economic growth and low interest rates guaranteed by the FED. When interest rates are low, people tend to invest more because loans are cheap. This also means bonds are not yielding high returns, which forces opportunistic people on the stock market. Thus, people are preferring dividend payments than coupon payments. Nevertheless, the S&P 500 suffered a small blow when the corona virus arrived. The stock market suffered after an important number of countries issued quarantines. Those quarantines forced an important number of companies and small businesses to close their shops. We talked about it earlier, but the financial market, retail market and Horeca market were the biggest sufferers of those widely spread quarantines. The S&P500 went from 3386.2 in February 2020 to 2237.4 end March 2020. We see now that this slump was temporary because in 2021 the S&P 500 has never been higher in their history. This is due to a couple of factors. First, the US government issued trillions of dollars of fiscal stimulus for companies and individuals. Secondly, the US government issued extensive loan programme for struggling small businesses. Thirdly, the pharmaceutical industry was really optimistic with their vaccine research and vaccine production. Lastly, the Fed was constantly working hard to keep interest rates low. Like we explained above, this would help make Americans and companies invest in the stock market and stay away from bonds or saving accounts. This blend between giving Americans stimulus checks and keeping interest rates low which makes loaning very cheap, creates a perfect environment for the stock market. It has never been this easy to invest in a stock than nowadays. We could also make a parallel with the development of cryptocurrency but that is not the theme of this thesis.

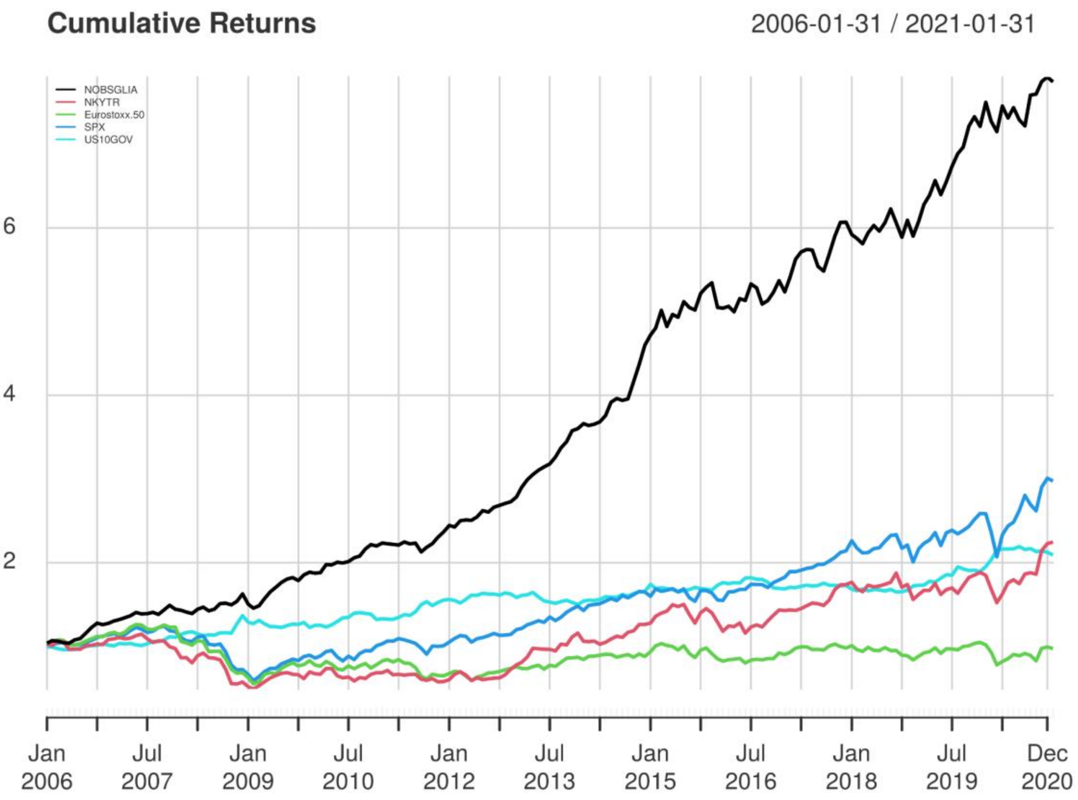
Without a doubt we can clearly state that the Euro Stoxx 50 is the worst performing index in this graph. Surprisingly, the index was moving similarly to the Nikkei and the S&P 500 between 2006 and early 2010. We will see that, like with the Nikkei, the aftermath of the 2008 financial crisis will have a huge impact on this index. While the S&P 500 parted ways with the consequences of this crisis early 2010, the Euro Stoxx and the Nikkei needed more time. It has now become clear that the Euro Stoxx never really recovered fully from this crisis because while the Nikkei started their resurgence early 2013, the Euro Stoxx 50 missed the boat. Moreover, the Euro Stoxx never found their pre-crisis levels. This index is also returning far less than the US bonds which is the safest investment possible. Another problem with this index is its volatility. Between 1998 and 2019, this index returned 2.9% a year and boasted a volatility of 20%. This means your investment is submitted to important risk while having a lower return than its peers. We will analyse these numbers more profoundly below. There are a couple of reasons on why the Euro Stoxx had such difficulties during and after this financial crisis. Firstly, European countries tend to overweight their positions in banks. This has proven to be a bad decision because Banks in Europe have struggled during the aftermath of the crisis. Like Jeremy Schwartz, CFA Graduate explains in his Seeking Alpha article: *"European banks have been under enormous pressure. Structural forces are weighing on net interest margins, a key measure of bank profitability and lending activity, and there is a lot of excess capacity in European banks, but low valuations make consolidation very hard.[20]"* (Schwartz, 2019). Secondly, Europe doesn't have huge technological firms like the USA. Europe's weight, over the last decade, in the global Information Technology sector is 4%. We have seen that the S&P 500 has their performance boosted thanks to Tech firms. Those Technology firms also helped the S&P 500 in their Resurgence after their March 2020 crash. We will also see below that the Norwegian Sovereign Wealth Fund has their performance boosted greatly thanks to their investments in Technology firms. Thirdly, Germany, Europe's driving force, has been struggling with a manufacturing slowdown due to soaring global auto sales, global capital goods expenditures and their tough relationship and weakness with China. Lastly, huge uncertainties around Brexit and its aftermath didn't help this index hit its full potential.

The Norwegian Sovereign Wealth Fund's chart is very impressive. There hasn't been a moment where one of the participants or bonds have been close to yielding the same returns. This is also the only participant that hasn't been impacted by the 2008 Financial crisis. We see that they didn't have a couple of years of "recovery" like its peers. We are going to try to find the different reasons on how they managed to survive the 2008 crisis that easily and how they managed to explode their cumulative returns between 2006 and 2021. We see that the Fund had two patches where their return was exponential. First, between 2009 and 2015, the Fund went from boosting similar results than the S&P 500 in January 2009 to boosting more than 3 times the S&p 500's cumulative return in January 2015. We will find the reason for this success. Firstly, their portfolio, like we have seen above, their portfolio is not made 100% of stocks. They got some diversification going thanks to their investment in Fixed-Income (24.7% in 2020). This is very important in times of crisis because it helps diminish the impact of negative stock returns. Like we have seen above, when stocks are going down, there is a great probability that bonds will go up. This is not the only reason for this exponential growth. Another reason for this growth

[20] Schwartz, J. (2019, October 23). *Can Europe's Dismal Decade Turn Around?.* Seeking Alpha. https://seekingalpha.com/article/4298100-can-europes-dismal-decade-turn-around

is their diversification in their equities investments. They have invested in European equities, North-American equities and Asia-Pacific equities. Their Asian investments have proven to be very profitable because China's development between 2006 and 2020 cannot be underestimated. This can be proven by the fact that two of their 10 biggest equity investments are in Alibaba and Tencent. We also know that the Fund has invested more than 290 billion NOK in Japanese bonds (Additional information can be found Appendix 1).

Figure 9: Cumulative Returns between 2006–2021 for the Fund, S&P 500, Euro Stoxx 50, Nikkei and 10-Year US Treasuries



Source: Rstudio (2021)

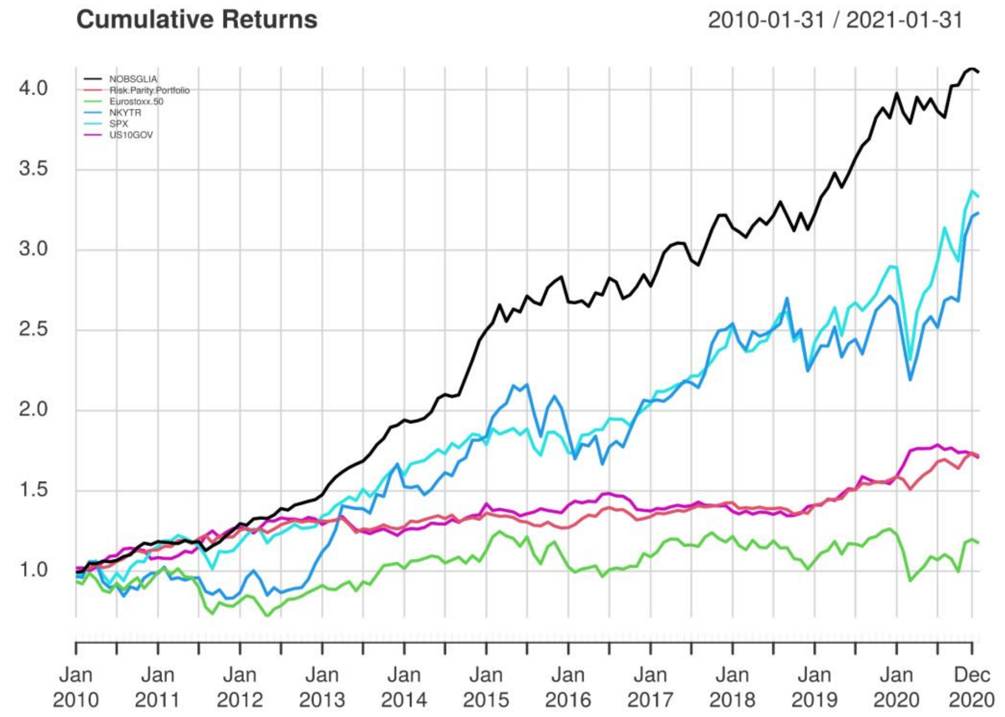
3.3.2 Regarding the Risk Parity Portfolio

Below we will analyse the Cumulative returns graph between 2010 and December 2020. We see that the Risk Parity portfolio has a very similar cumulative graph than the 10-Year US Bond. This is normal because in the Risk Parity Portfolio, 37.3% of the total investments are in 10-Year US Bonds. Nonetheless, it is surprising to see that the Risk Parity portfolio has been outperformed a couple of times between 2010 and 2020. We see that between January 2015 and January 2017, the bond performed better than the portfolio. This was not due to extraordinary performance from the bond, but this was more due to a bad end of the year 2015 for US Stocks, Gold and Commodities.

This happened again between January 2020 and December 2020. It is logical that in 2020, the 10-Year US bond outperformed the portfolio because, early 2020, stocks were suffering due to lockdown measurements whereas bonds were surging. We cannot forget that 19.5% of total investments in the portfolio are in equities. We also saw in the portfolio that Real Estate and commodities were suffering greatly early 2020. This led to the 10-Year US bond outperforming the Portfolio.

We will finish the analysis of this graph with a little word about the Fund and its benchmarks. We see that Nikkei performs in a very similar way to the S&P 500. This is something we talked about previously and will talk about later. We see that, if the Nikkei wouldn’t have suffered so much from the 2008 Financial crisis, that we would be looking at the same kinds of returns that the S&P 500. Moreover, we see that the Euro Stoxx 50 is still the worst performing index and the fund is still boosting extraordinary results. Nonetheless, their results on a 10-Year basis are less extraordinary than on a 15-Year basis. This is largely due to their handling of the 2008 Financial crisis and their increase in investments in US Large cap. This makes the Fund follow a similar pattern to the S&P 500 (Additional information can be found Appendix 2).

Figure 10: Cumulative Returns between 2010 and 2020 for the Fund and all the benchmarks including the Risk Parity Portfolio



Source: Rstudio (2021)

3.3.3 Key changes and investments for the Sovereign Fund between 2009–2019

While looking at the reports between 2009–2015, we noticed a couple key investments that helped the Fund yield such impressive returns. We will look at each year and list the different key investments that helped them become such a powerhouse nowadays.

In 2007, the Fund decided to change their portfolio allocation. It took 2 years to go from a 40% equity share of the Fund's portfolio to 60% of the fund's portfolio. They invested 1.1 Trillion Kroner between 2007 and 2009 to boost their equity investments. The Fund was smart because they took advantage of the low equity prices due to the 2008 financial crisis and they sold government bonds that were at historically high prices at that time. Their main equity investments were still in European equities at the time. This proved to be very successful because the fund had a record return of 25.6%. Their regional breakdown of the equity portfolio in 2009 was 50.3% in Europe, 35.3% in America and Africa and 14.4% in Asia Oceania. Their regional breakdown of the fixed-income portfolio in 2009 was 58.6% in Europe, 36% in

America and 5.4% in Asia and Oceania. Also, their biggest equity holdings were different: HSBC Holdings, Royal Dutch Shell and BP. All those companies were based in the UK.

In 2010, the Fund decided to make another key change to their portfolio. The Ministry of Finance decided that 5% maximum of the portfolio should be invested in Real Estate. This means they lowered the Fixed income share in the Fund’s portfolio allocation. The Fund returned 9.6% thanks to their equity investments that returned 13.3% in 2010. We can see that 3 sectors had been particularly profitable in that year. Their equity investments in Basic Materials returned 25.2%, in Industrials returned 22.2%, and in Consumer Goods returned 20.4%. Those investments count for 34.5% of the equity investments. The worst-performing equity sector was the Financial sector with a 4% return. Those investments count for 21.4% in their equity investments. The Fund also noted that US and Spanish equities influenced the most of their excess return. Their regional breakdown of the Fund’s equity and fixed-Income investments didn’t change that much. The Fund’s main bond issuer is still the USA. The Fund decided to lower their investments in BP and reinforce their positions in Nestlé, which is now in their top 3 equity investments with Shell and HSBC.

In 2012, the Fund returned 13.4%. That return was impressive because 2012 was a year with slow economic growth and market turmoil. Like we have seen in 2010, the return was mostly thanks to their equity investments that returned 18.1%. Their Fixed-Income investments returned 6.7% which is pretty high for fixed-income investments. That year, the Fund decided to lower their investments in Europe from 53% to 48% of their total portfolio. This was deemed surprising because their European equity investments returned 20.8% in 2010 thanks to European Financial stocks yielding 34% return. The Fund’s 2012 goal was to start investing more in emerging equity investments. The Fund’s allocation that year was 61.2% in equities, 0.7% in Real Estate and 38.1% in Fixed-Income. The main sources of return in their equity investments were the Financial sector with 29.7%, the Consumer Goods sector with 24.5%, the Industrials sector with 20.5% and the Consumer Services sector with 22.4% which represent 58.7% of their equity investments. Every continent was yielding important returns with America yielding 16% and Asia yielding 14%. Asia’s reported return was mainly due to their equity investments in China after Chinese authorities raised the total quota of foreign investors from 30 Billion dollars to 80 billion dollars. The Fund was also quietly increasing their investments in US shares over the years which accounted for 28% of their total equity investments. The top 3 performing companies were HSBC, Apple and Nestlé. Moreover, the

Fund decided to increase their holdings in Government bonds and government-related bonds from 66% to 73%.

In 2013, the Fund returned 15.9%, their second-best return in history at that moment. This was primarily due to their equity investments that returned 26.3% because their Fixed-Income investments returned 0.1%. This is particularly surprising considering their portfolio allocation didn't change a whole lot. That year we see that European investments went down again from 48% to 45.2%. This is an interesting trend after seeing the Eurostoxx 50's "bad" performance during those years. Investment in America and Asia remained stable with 32.8% and 14.8%. US shares investments went up again from 28% to 31.7% of equity investments. This helped the Fund a lot because North America was the best-performing region in 2013 with 33.9% return in equity investments. This was mainly due to low borrowing costs for corporations thanks to the FED. Every continent was performing really well in 2013 with their European equities returning 28.8%, their Japanese equities returning 29.6% and their Chinese equities returning 17.4%. We also see below that returns were booming in different sectors. We see that Consumer Services (34.8%), Healthcare (35%) and Telecommunications (37.5%) were the Fund's most profitable sectors. This is an interesting table because we see the Fund's main investments and their most profitable sectors.

Table 14: The Fund's 2013 Return by sector for their equity investments in International currency

Sector	Return in international currency	Share of equity investments*
Financials	27.1	23.8
Industrials	29.4	14.4
Consumer goods	26.1	14.0
Consumer services	34.8	10.2
Healthcare	35.0	8.7
Oil and gas	16.1	8.4
Technology	30.6	7.5
Basic materials	5.1	6.4
Telecommunications	37.5	3.9
Utilities	16.4	3.5

*Does not sum up to 100 because cash and derivatives are not included.

Source: Government Pension Fund Global Annual Report 2013

Their 3 largest equity investments are Nestlé, Royal Dutch Shell and Novartis. While their most profitable investments were Telecom Company Vodafone, Us Asset Manager BlackRock and Swiss Drugmaker Roche.

In 2019, the Fund returned 19.9%. This was thanks to their equity investments that returned 26%. Their fixed-income investments returned 7.6% and their real estate investments returned 6.8%. We see that in 2019, the Fund continued their divestment in European stocks. Previously, in 2013, their European investments represented 45.2% whereas, in 2019, their European investments represented 33.7%. Moreover, the Fund grew their investments in North America from 32.8% in 2013 to 43.9% in 2019. They talked about this change of strategy in the early 2010s and they continued to follow through. We see that their investments in Asia-Pacific also increased because it represented, in 2019, 19.2% of their total investments whereas, in 2013, it represented 14.8%. This explains the pattern that we are seeing with the different indexes. We see that the S&P 500 is performing much better than the Nikkei that is, at his turn, performing better than the Euro Stoxx 50. This is why it is a logical explanation that the Fund is disinvesting in Europe and increase their investments in Asia and North America. This is shown by the Fund reporting in 2019 that their investments in US Stocks increased from 31.7% in 2013 to 39.8%. US stocks were also their primary source of return with 31.4% whereas European stocks returned 25.2% and Asia-Pacific equities returned 19.5%. Us stocks return was boosted thanks to US Tech firms. We see that the technology sector returned 42.3% in 2019 with a 14.6% share in equity investments. The Fund reported that this was "*driven by software and semiconductor producers. Software stocks were buoyed by the transition to cloud solutions, and semiconductor stocks by expectations of an upswing in that market*[21]" (Government Pension Fund Global, 2020). We see that the Industrials sector returned 30.3% thanks to changes in US Monetary policy and to a possible resolution of the 2018 US-China trade war. Moreover, Utilities returned 26.9% thanks to low interest rates in financial markets and initiatives from power companies to cut their carbon emissions. Lastly, we see that the Fund's 3 largest investments aren't Nestlé, Shell and Novartis anymore. The Funds' 3 largest equity holdings are Apple, Microsoft, Alphabet Inc. Apple and Microsoft contributed the most to the Fund's return with Nestlé. Another notable change that we see in 2019 is their increased investments in equities and their smaller investments in Fixed-Income. The 2019 portfolio allocation of the fund is 4.1% in Real Estate, 26.5% in Fixed-Income and 69.4% in Equities. The Fund lowered their Fixed-Income

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[21] Government Pension Fund Global. (2020). 2019 annual report of the Government Pension Fund Global. P41. Retrieved from

https://www.nbim.no/contentassets/3d447c795db84a18b54df8dd87d3b60e/spu_annual_report_2019_en_web.pdf investments by almost 12% in 6 years. Moreover, their biggest return in Fixed-Income was thanks to their Mexican Peso and Indonesian Rupiah investments with 26.3% and 19.3%. Again, their largest Fixed-Income investment is with the US government (45.6%). They returned 9.6% in local currency which is a big indicator on how the Fixed-Income investments returned 7.6% in 2019. This was because the FED decided to change their strategy of raising its policy rate and decided to cut rates by 25 basis points 3 times.

Table 15: The Fund's return in 2019 for their equity investments in international currency

Table 11 Return on the fund's equity investments in 2019 by sector. International currency. Percent

Sector	Return	Share of equity investments [1]
Financials	23.7	23.6
Technology	42.3	14.6
Industrials	30.3	13.4
Consumer goods	23.6	11.5
Health care	24.3	11.3
Consumer services	25.1	10.7
Oil and gas	12.9	5.0
Basic materials	18.3	4.4
Utilities	26.9	2.8
Telecommunications	13.9	2.7

[1] Does not sum up to 100 percent because cash and derivatives are not included.

Source: Government Pension Fund Global Annual Report 2019

3.4 Correlation and Volatility

In this chapter, we are going to look at the data of each benchmark and we are going to see if there is any correlation between a benchmark, the Risk Parity Portfolio and the Norwegian SwF. We are also going to look for correlations between benchmarks. The goal is to see which benchmark is similar/different to the Fund and if they are similar between them. Before starting our analysis, we need to define what a correlation is, what the significance of correlation means and the different scales of Skewness and Kurtosis.

3.4.1 Types of Correlations and it's Different Degrees

Correlation helps us understand to what extent two variables are related and in what direction this relationship goes. There can be a positive correlation and a negative correlation. A correlation is positive when the two variables are moving in the same direction. This means when one index goes up, the other index also goes up. A negative correlation is when the variables move in the opposite direction. This means when one index goes up, the other one goes down. This is why correlation ranges between -1 and +1. The closer the correlation is to 1, negative or positive, the higher the significance of this correlation is. There are 7 degrees of correlation between two variables possible:

1. Perfect Positive Correlation: This is when the correlation coefficient is 1 between two variables. The points are all on the same line and upward sloping because the correlation is positive.
2. High Degree Positive Correlation: This is when the correlation coefficient is between 0.7 and 1. The points are trying to create an upward sloping line. The closer the points are to a straight line, the closer the correlation coefficient is to 1.
3. Strong (Positive or Negative) Correlation: This when the correlation coefficient is above 0.4. The points are starting to be closer and closer to create a straight line.
4. Moderate (Positive or Negative) Correlation: This is when the correlation coefficient is between 0.2 and 0.4. The dots are pretty far from each other and have difficulties creating an upward sloping or downward sloping straight line.
5. Weak (Positive or Negative) Correlation: This is when the correlation coefficient is below 0.2. Those correlations are insignificant. This means there is no connection between the dots and the points aren't creating an upward or downward sloping straight line.

6. High Degree Negative Correlation: This is when the correlation coefficient is between -0.7 and -1. The points are trying to create a straight downward sloping line.
7. No correlation: When the correlation coefficient is 0.

3.4.2 Kurtosis and Skewness

We already explained above what Kurtosis and Skewness means but we will explain it again quickly. The skewness measures the lack of symmetry. It measures if the distribution looks the same on the right and on the left. Kurtosis, on the other hand, measures if data is heavy-tailed or light-tailed regarding a normal distribution. When data has high kurtosis, we can notice heavy tails. When data has low kurtosis, we can notice light tails. In other words, the Kurtosis looks at the peak of the distribution curve and how pointy it is.

The Skewness is calculated by multiplying by 3 the subtraction between the Mean and the Median. After this total, we divide the result by the standard deviation.

$$\textbf{Skewness} = \frac{\mathbf{3(Mean - Median)}}{\textbf{Standard Deviation}}$$

When the distribution is skewed, it either tends to the left or to the right. When the distribution tends to the right, this means the mean is bigger than the median and that there is positive skewness. When the distribution tends to the left, the mean is lower than the median and the distribution has negative skewness. There are also different scales of skewness. When skewness is above 1 or under -1, we say that the distribution is highly skewed. When skewness is between 0.5 and 1 or -0.5 and -1, we say that the distribution is moderately skewed. Lastly, when skewness is between -0.5 and 0,5, we say that the distribution is approximately symmetric.

Kurtosis measures tailedness of the distribution and measures the outliers or tails of the distribution. When the distribution is more peaked than a normal distribution, the kurtosis is positive. When the distribution is flatter than a normal distribution, the kurtosis is negative. There are 3 types of distributions based on Kurtosis. The first type is Leptokurtic. This means that the Kurtosis is above 0. It is easily recognizable thanks to the distribution's fat tails and sharp peaks. It also has less variable. The second type is Mesokurtic. This is when Kurtosis is equal to 0 or close to zero. The distribution is then medium peaked. The last type is Platykurtic. This is when Kurtosis is negative. This distribution is also easily recognizable because it has a flat peak and it's highly dispersed.

3.4.3 Results for the Years 2006–2021 for the Fund and the benchmarks excluding the Risk Parity portfolio

After carefully explaining the different types of correlation, the different degrees of correlation and the different scales of Kurtosis and Skewness, we can now analyse our results. We will first look at the correlation between the benchmarks before turning to the Norwegian SwF and its correlation with the different benchmarks.

In this table, we see that every benchmark has a negative correlation with the 10-Year US Treasuries. This is completely normal and proves our precedent point developed above. We explained that mostly when stocks go up, bonds go down and vice-versa. We also stated that there are scenarios where stocks and bonds go up together or go down together. With this table, we see that our first thought was right. The 10-Year US Treasuries is negatively correlated to the S&P 500 (-0.34), to the Nikkei (-0.40) and to the Euro Stoxx 50 (-0.33). Those stats are very similar. If we are going to use the different degrees of correlation presented above, we can say that there is a moderate negative correlation between the 10-Year US Treasuries and the other benchmarks. We also see if we are looking at the different dots that the correlation between the Treasuries and the Nikkei is the strongest because the dots create a much steeper downward sloping line than with the other benchmarks. Rstudio also calculated that the 10-Year US Treasuries graph has a 2.26 Kurtosis. This is the highest Kurtosis of all the benchmarks. This means the distribution is Leptokurtic. Moreover, we see that the graph has a sharp peak and heavy tails. This shows that the Kurtosis is positive. High kurtosis usually means that the investment will yield extreme values (negative and positive). Rstudio also shows us that the Treasuries skewness is 0.65. This means the distribution is moderately skewed. Moreover, the Treasuries is the only distribution that has a positive skewness of all the participants. We can see on the graph that the distribution tends to the left with its tail on the right. This is because the median is bigger than the mean. A positive skewness usually means, in finance, that an investment will guarantee frequent small losses and a few large gains.

The S&P 500 is very correlated with the Nikkei (0.72) and the Euro Stoxx 50 (0.82). Like we have seen above, between those three benchmarks there is a High Degree Positive correlation. This is due to the fact that those markets are very similar. There are a lot of companies present in the United States that are also present in Europe and in Japan. We also see that those markets suffer at the same time and find growth at the same time. Even if the growth or slump are not of the same degree, it mostly happens during the same time horizon. There are two striking examples where the S&P 500, Nikkei and Euro Stoxx 50 mirror their moves. The first example is the 2008 Financial crisis. We see that the three indexes suffered and bottommost at the same time. At the end of the day, Euro Stoxx and Nikkei needed more time to find their initial growth but they touched rock bottom at the same time as the S&P 500. This is also due to technological advancements. Before it took a couple of days before everybody was aware there was a financial crisis happening. Nowadays, when a financial crisis happens, the whole world is aware and everybody acts to protect themselves. Customers, Compagnies in the whole world act instantly to avoid big losses. The second example, where we see that the three indexes are highly correlated, is when the corona pandemic started. Every index recorded huge losses due to market uncertainty and lockdowns ordered by governments. We see this little slump on the cumulative returns graph at the beginning of 2020. Those are two examples where the indexes moved similarly and showed us their correlation between them. It is also normal that the Euro Stoxx 50 is more correlated with the S&P 500 than with the Nikkei. This is due to the close post-war relationship between the USA and Europe. The S&P 500's kurtosis is 1.61. This means the distribution is Leptokurtic because the kurtosis is positive. The three indexes have Leptokurtic distributions because they all have sharp peaks and heavy tails. Investments with Leptokurtic distributions are considered risky because their returns are prone to extreme values on either side. The S&P 500's skewness is -0.65. This means the distribution is moderately skewed like the 10-Year US Treasuries. The distribution also tends to the right with its tail on the left side which means that the skewness is negative. Distributions with negative skewness mostly return many small wins and a few large losses.

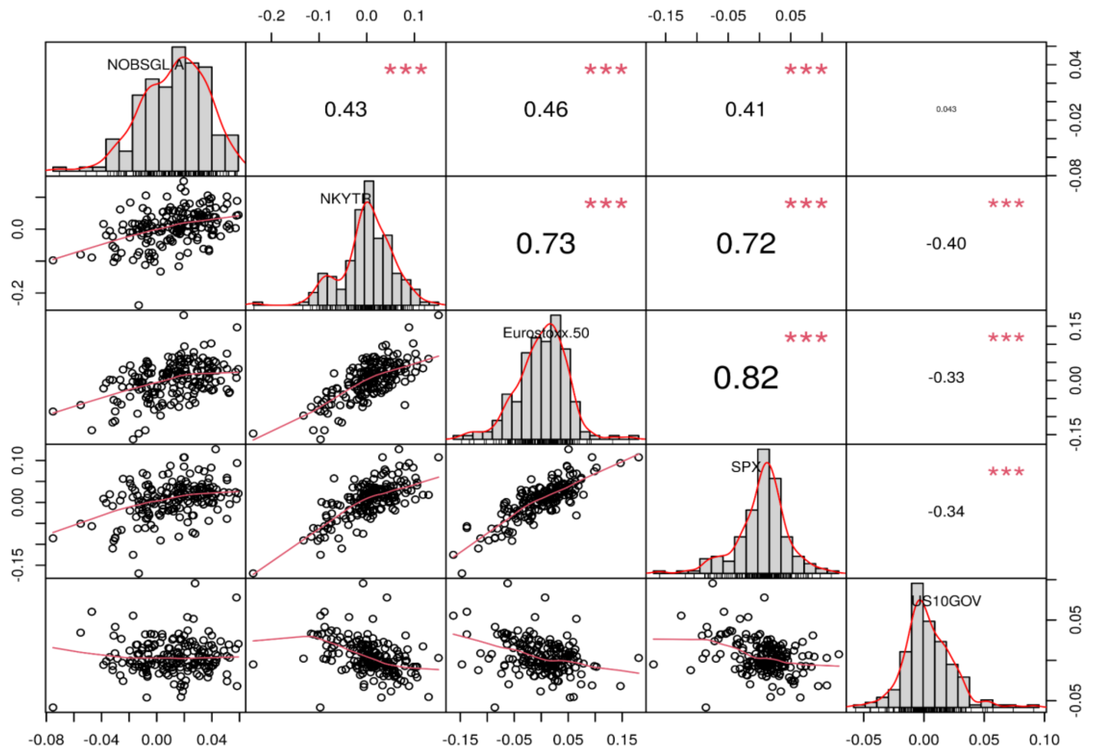
The Euro Stoxx 50 is also very correlated with the Nikkei (0.73) and the S&P 500 (0.82). Those correlations can be named "High Degree Positive correlations". We already established above the different reasons for this high correlation between those markets. We see that the dots, in the table below, create a nice pattern that makes it easy to identify a straight line between the S&P 500 and the Euro Stoxx 50. The same can be said for the relationship between the Nikkei and the Euro Stoxx 50. We see that the correlations are positive because the dots start from the low left side and develop to the high right side. The Euro Stoxx 50's kurtosis is 1.11. This means its distribution is Leptokurtic. This is a returning trend with indexes. It has become clear that indexes are volatile and can generate important returns to compensate for the risk. We also see again that the distribution has heavy tails and a sharp peak. The Euro Stoxx 50's skewness is -0.31. Because the skewness is between -0.5 and 0,5, we consider its distribution as Approximately Symmetric. Like we have seen above, investments with negative skewness usually report many small wins and huge losses. The graph also shows that the distribution tends to the right with its tail on the left side.

The Nikkei has a "High Degree Positive Correlation" with the Euro Stoxx 50 (0.73) and the S&P 500 (0.72). This correlation is due, like we have seen before, to the different similarities in culture and markets between them. We see that the two correlations are positive because the dots start at the low left of the graph and end at the top right of the graph. This creates an upward sloping straight line. The dots also show us that the correlation between the Nikkei and the Euro Stoxx 50 is lower than the correlation between the Euro Stoxx 50 and the S&P 500 due to higher distance and disparity between dots. Nikkei's kurtosis is 1.42. Like we announced before, this is another index where its distribution is Leptokurtic. Again, the recurring trend is a sharp peak and heavy tails. This shows us the kurtosis is positive. Like we have seen above, a leptokurtic distribution means that the investment bears an important volatility with a possibility of high returns. Moreover, Nikkei's skewness is -0.60. Like its peers, the skewness is negative. This means its distribution tends to the right with its tail on the left side. Negative skewness usually means the investment reports many small wins and few huge losses. The distribution is also considered moderately skewed because its skewness is between -0.5 and 0.5.

The Norwegian Sovereign Wealth Fund has a strong positive correlation with the 3 indexes: Nikkei (0.43), Euro Stoxx 50 (0.46), S&P 500 (0.41). We see that the correlations are positive because there is an upward sloping pattern with the dots of the 3 graphs. The correlation is considered significant but is smaller than the correlation between indexes. This is probably due to the fact that the SwF has a portfolio made of bonds, real estate and stocks whereas the three indexes have a portfolio made of 100% stocks. This means that sometimes the Fund is yielding positive returns when stocks are slumping thanks to their different investments in bonds. We also see a more important correlation between the Euro Stoxx 50 and the Fund than with the other indexes. This is logical because they are based in Norway and they have made substantial investments in European markets. Lastly, the correlation between the Fund and the 10-year US Treasuries is 0.045. This means the correlation between the two participants is considered very weak. This weak correlation is shown in their graph. The different dots have difficulties creating an upward or downward sloping line. We also see that the dots aren't even able to create a distinctive line. This weak correlation is due to their investments in US Bonds. Firstly, the Fund has invested in thousands of different bonds and stocks. Secondly, they have invested in different types of US bonds which makes the correlation between the 10-Year US treasuries and the whole portfolio of the Fund negligible. Let's now talk about the Fund's distribution.

The Fund has the lowest kurtosis of all participants (0.11). When the distribution has kurtosis close to zero or equal to zero, we say that the distribution is Mesokurtic. A Mesokurtic distribution is easily recognizable because the distribution is medium peaked. Such distributions are usually a sign that those investments are bearing moderate risk levels. It also shows us that high levels of return only happen on a few occasions and big losses are scarce. Moreover, the Fund's skewness is -0.45. This means the distribution is moderately skewed because its skewness is between -0.5 and 0.5. Again, like with the 3 indexes, the distribution tends to the right with a heavy tail on the left. Negative skewness usually means frequent small wins and a few huge losses. The Fund is performing very well and this is shown. With low kurtosis, meaning moderate risk levels and decent return, and negative skewness, meaning many small wins and few huge losses, the Fund is proving to handle difficult times better than the three indexes. This has also been shown because they handled the 2008 Financial crisis very well and very quickly while the different indexes needed years before finding their pre-crisis levels. This has also been shown with the corona pandemic and stock crash between February 2020 and March 2020. The Fund suffered like the three indexes but they recovered quickly to ensure growth.

Table 16: Correlations between the Fund and the benchmarks for the Years 2006–2021.



Source: Rstudio (2021)

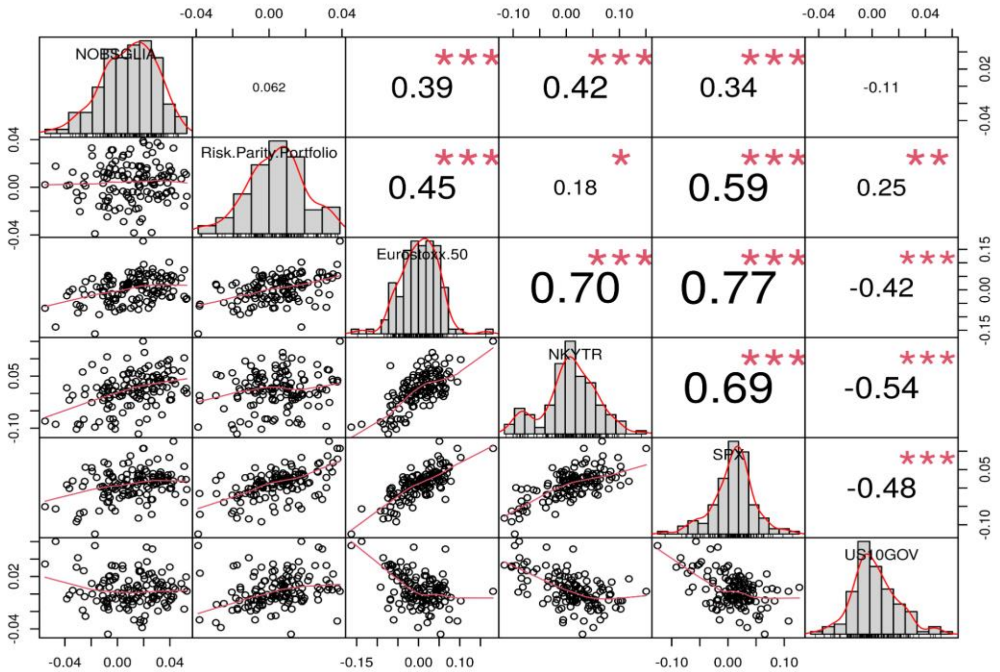
3.4.4 Results for the Fund and the benchmarks including the Risk Parity portfolio for the Years 2010–2020

The Risk Parity Portfolio has a Kurtosis of -0.09 and a skewness of -0.18. Like we explained above, when the skewness is between -0.5 and 0,5, we say that the distribution is approximately symmetric. Like with the other benchmarks with an approximately symmetric distribution, we see that the distribution tends to the right with a heavy tail to the left. Moreover, the distribution is Mesokurtic because the distribution has kurtosis close to zero and it is medium peaked like the Sovereign Wealth Fund. Usually Mesokurtic investments are less susceptible to extreme returns or extreme losses and are very stable investments. When distributions have fatter tails, like with Leptokurtic distributions, extreme losses and gains are more likely to happen.

The Risk Parity Portfolio has a strong positive correlation with the Euro Stoxx 50 (0.45) and the S&P 500 (0.59). This is normal because in the Risk Parity portfolio the S&P 500 represents 10.6% of the portfolio. Moreover, it is also logical that the Euro Stoxx 50 has a significant correlation with the portfolio because 7.9% of the portfolio is invested in Foreign Developed equities (largely European Equities). It is also very interesting to see that the portfolio has a moderate positive correlation with the 10-Year US bond. This is largely due to their high share of investments in 10-year US bonds (37.3%). It is also important to note that, whereas usually benchmarks or Equities centered portfolios have negative correlations with bonds, this portfolio has a positive correlation with a bond. The portfolio has its lowest correlations with the Fund (0.062) and the Nikkei (0.18). The weak positive correlation is probably because in the portfolio there is a low presence of Japanese Equities in the Foreign Developed Equities. Moreover, we have seen that the Fund has invested in 10-Year US bonds, but this portion of investment is so small that this created no statistically significant correlation between the portfolio and the Fund.

We will finish by noting that, on a 10-year period, the correlations are still significant between the benchmarks and the Fund. The relationships between each other haven’t changed much. We could note that the change of investments for the Fund in the last 10 years created a negative correlation with the 10-Year US bond whereas it was non-existent on a 15-year basis. We could also note that negative correlation with the bond became stronger on a 10-year basis.

Table 17: Correlations between the Fund and the benchmarks including the Risk Parity portfolio for the Years 2010–2020.



Source: Rstudio (2021)

3.5 Volatility

3.5.1 Volatility for the Fund and the benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021

Risk is usually measured by looking at an index, stock or bond's standard deviation. It can help us understand the market volatility and the usual different movements. When standard deviation is high, it leads us to believe that the stock moves frequently and is not stable. This means the investment is considered to be "risky". When standard deviation is low, movement is scarce and an investment can be considered low risk. Usually stocks have a much more important standard deviation than bonds because their prices are much more susceptible to movement whereas bonds are government issued and mostly very stable and low risk over time. The standard deviation also shows us the range of movement of a price. The wider the range of an investment with its mean, the riskier the investment is. This is normal because this means an investment can vary a lot in a year.

The table below shows us that the SwF has an incredible low standard deviation considering their return. Their standard deviation is higher than the 10-Year US Treasuries, which is normal, but far lower than the S&P 500, Euro Stoxx 50 and Nikkei. It is normal that the bond has the lowest standard deviation because that investment is very stable and boosts consistent returns. When investing in the 10-Year US Treasuries in 2021, you will receive for 10-Year straight a return of 1.7% each year without the possibility of default. Moreover, this is why it is so surprising that the Fund has such a low standard deviation while returning frequently more than 10%. This is probably thanks to their Fixed-Income investments. By investing between 25–30% of their portfolio in Fixed-Income, they ensure that they have consistent return and a high diversification each year. Moreover, we cannot forget that their biggest Fixed-Income investment is in United States issued bonds. They have invested 717,935 million Kroner in US Bonds as on 30 June 2020. Such a large investment helps the Fund lower their standard deviation, thus, their risk. It would be very interesting to see how the Fund's risk developed overtime because they have invested more in equities the last few years and they lowered their investments in Fixed-Income. A reminder that in 2006, the Fund had invested 60% in Fixed-Income and 40% in Equities. Since 2007, they have gradually increased their investments in equities to 71.5% of their total investments in 2020. Moreover, they have shifted their main investments in European Equities to US Tech equities in the last few years. We see that the

S&P 500 has a lower standard deviation than the Euro Stoxx 50 which could lead us to believe that those increased US Investments helped lower the Fund's standard deviation over the years.

Lastly, the Nikkei is the index with the highest standard deviation. This is normal because, like we mentioned above, they had difficulties finding consistent returns after the 2008–2009 financial crisis. It is interesting to see that the worst performing index, the Nikkei, is also boosting the highest standard deviation. This means the investment is very risky and has a low return. To conclude, the Fund is not only beating everyone with their cumulative return, but we also see that they have the lowest risk of all the indexes and their standard deviation is slightly higher than that of a US bond. This Nikkei's high standard deviation can also be due to their high dependency to the United States. Japan is highly dependent on their exports to the United States. This means when something happens to the United States or in the United States, the Japanese Economy and the Nikkei are directly impacted. Moreover, they are more impacted than the S&P 500 because they are always trailing behind the United States movements. Also, because the Nikkei is price-weighted, the index is highly influenced by the movement of the big companies in the index. This would lead to high dependency and could lead to a high standard deviation.

An important note is that the Annualised Deviation is the Monthly standard deviation times the square root of 12.

Table 18: Mean, Monthly and Annualised standard deviation for the Fund and its benchmarks excluding the Risk Parity portfolio for the years 2006–2021.

	NOBSGLIA	NKYTR	Eurostoxx.50	SPX	US10GOV
Mean Absolute deviation	0.0196	0.0416	0.0393	0.0317	0.0157
monthly Std Dev	0.0242	0.0557	0.0512	0.0436	0.0210
Annualized Std Dev	0.0840	0.1930	0.1775	0.1509	0.0728

Source: Rstudio (2021)

3.5.2 Volatility for the Fund and the benchmarks including the Risk Parity Portfolio for the Years 2010–2020

It is very reassuring looking at this data, because this means we created our Risk Parity portfolio correctly. The Portfolio is less volatile/risky than the Fund and the 10-Year Us bond. This is because of the way we created our portfolio. The portfolio is created in such a way where the riskiest assets have less weight than the "safest" investments. The main goal was to create a portfolio where each asset bears the same amount of risk than the other. That is why it is important to diminish the weight of certain assets because they are more volatile than the others.

We see that, on a 10-Year period, the Fund and the benchmarks are considered less volatile than on a 15-Year period. This is absolutely normal because we haven’t considered the 2008–2009 financial crisis in the standard deviations below. When the 2008 financial crisis was happening, stocks were crashing and this increased their volatility on a 15-Year basis.

Table 19: Mean, Monthly and Annualised standard deviation for the Fund and its benchmarks including the Risk Parity portfolio for the years 2010–2020.

	NOBSGLIA	Risk.Parity.Portfolio	Eurostoxx.50	NKYTR	SPX	US10GOV
Mean Absolute deviation	0.0183	0.0130	0.0388	0.0395	0.0300	0.0146
monthly Std Dev	0.0225	0.0165	0.0492	0.0517	0.0405	0.0189
Annualized Std Dev	0.0779	0.0570	0.1704	0.1792	0.1402	0.0653

Source: Rstudio (2021)

3.6 Performance Measurements

3.6.1 Performance Measures for the Fund and its benchmarks excluding the Risk Parity portfolio for the Years 2006–2021

We will now look at the different performance measures. We will start this chapter by looking how the Fund performs with its measures against the three indexes used as a benchmark. We will talk mostly about the Tracking error, R squared, Information Ratio, Treynor ratio, Alpha and Beta.

3.6.1.1 Tracking error

We explained it earlier, the Tracking Error is the standard deviation of the difference between the Fund's return and the return of a benchmark of our choosing. If the Fund's return is significantly higher than the benchmark's return, the tracking error will be directly impacted in a positive way. It is also normal that an actively managed portfolio has a higher tracking error when calculated against a benchmark. We cannot forget that the tracking error calculates the deviation between the Fund and one of the three indexes. When the tracking error is low, this means the Fund is consistently boosting better results than the benchmarks.

To find the three Tracking Errors, we used each index as a benchmark. We see that the highest Tracking Error is between the Fund and the Nikkei (0.1742). This shows that the Fund is outperforming the Nikkei the most. Altogether, the Fund is outperforming each index. It has a Tracking Error of 0.1391 when the S&P 500 is used as a benchmark and a Tracking Error of 0.1579 with the Euro Stoxx 50. The higher the Tracking Error, the more the Fund outperforms the benchmark. We see again the recurring trend that the S&P 500 is the strongest benchmark.

3.6.1.2 Information Ratio

Explaining the Tracking Error leads us to the Information Ratio. Like seen above, the Information ratio is the difference between the Fund's return and the benchmark's return divided by the Tracking Error. We explained the different limitations of the Information Ratio and we made sure that those limitations didn't impact our results. Firstly, we used the same portfolio to be compared against and we used an important enough Time Horizon to make our results interesting.

The results are rather impressive. After assessing that the S&P 500 is the strongest benchmark with the lowest Tracking Error, we must say that there is not a huge difference between the

Tracking Errors. This makes it rather surprising that there is such a difference between the three Information Ratios. The Information ratio that sticks out is the one where the Euro Stoxx 50 is used as a benchmark. Whereas the other Information Ratios are 0.5061 and 0.5182 when the S&P 500 and the Nikkei are used as benchmarks, when the Euro Stoxx 50 is used as a benchmark the Information ratio is 0.9325. While by no means the Information ratios are low, such a difference should be noted.

Like with the Tracking Error, the Information Ratio shows us if the Fund is outperforming the benchmark. The higher the Information ratio, the larger the outperformance. This is normal because we use the difference between returns divided by the standard deviation of the difference between the return of the portfolio and the return of the benchmark. The Information ratio also shows us how much and how often the Fund is trading in excess of the benchmark. To conclude, the Fund is consistently outperforming the three indexes but it has boosted remarquable results against the Euro Stoxx 50. This is probably due to the fact that the Euro Stoxx 50 has a higher standard deviation with far worse results than the Fund and the rest of the indexes. We cannot forget that the Euro Stoxx 50 is the worst performing index in cumulative returns by a margin and the second most volatile benchmark behind the Nikkei. The Information ratio also shows us the remarquable skill of the managers overseeing the Fund. Maintaining a portfolio with a low standard deviation while boosting remarquable returns is an incredibly difficult task to complete. We see that there are a couple of negative Information ratios, this is when the benchmark outperforms the “portfolio”. When the S&P 500 is used as a benchmark and the Euro Stoxx as the “portfolio”, we see that there is a large negative Information ratio.

3.6.1.3 R-Squared

R-squared can also be called the coefficient of determination. Usually, R-squared explains, in percentage, how much the Fund’s movement is influenced by the benchmark’s movement. In other words, R-squared uses the Fund’s return and compares it against the three benchmarks and this comparison is expressed in a percentage between 0–100. For example, we see that 46.48% of the Fund’s movement is explained by movement from the S&P 500. This is a pretty low R-squared and means that the Fund doesn’t generally follow the movements of the S&P 500. We can divide the R-squared values in three tiers.

1. If R-squared is between 1 and 40%, we say that the Fund has a low correlation to the benchmark

2. If R-squared is between 40 and 70%, we say that the Fund has an average correlation to the benchmark
3. If R-squared is between 70–100%, we say that the Fund has a high correlation to the benchmark

If the R-squared would have been between 85 and 100% then we would have considered that the Fund is following the benchmark. We will see in the tables below that the Fund also doesn't follow movements from the Nikkei (48.69%) and the Euro Stoxx 50 (48.62%). Thus we can say that the Fund has an average correlation to the three benchmarks when using R-squared as a measure. It is important to note that when we used the correlation table above, the Fund also reported an average correlation with the three benchmarks and shows us the results are consistently correct.

The R-squared value is important in considering the importance of the Beta. When R-squared is close to 100%, the beta is much more useful. Whereas R-squared usually measures how much change of prices are correlated with a benchmark, Beta measures the range of those price changes against the benchmark. R-squared can also be used for diversification purposes. It helps us choose which investments in our portfolio are worth keeping by looking at their performances against the Fund.

3.6.1.4 Beta

Like we explained above, the Beta measures how volatile the Fund is against the market as a whole. In this case, the betas are meaningful because the Fund is related to the three benchmarks used. We are going to remind ourselves what the 4 types of Beta values are.

1. Beta value = 1
2. Beta value <1
3. Beta value> 1
4. Negative Beta value

We will see that the three Betas found with the Fund are less than one. The Fund's beta against the S&P 500 is 0,3937, against the Nikkei is 0,3275, against the Euro Stoxx 50 is 0.3558. When the Beta is below 1, this means the portfolio is less volatile than the benchmark used. In this case, the Fund is less volatile than the three benchmarks. This is something we noticed in the beginning of our results and this is a recurring trend. Something interesting that we noticed is the different betas between the benchmarks. The beta value between the Nikkei and the S&P is

1,0024. This shows that the price is highly correlated to the S&P 500. We could even say that the Nikkei is slightly more volatile than the S&P 500. This is something we have talked about in the beginning of our thesis and that is now shown in the results. Similar results are seen between the Euro Stoxx 50 and the S&P 500 with a beta of 0.9941. Again, this is something we have explained before, the Euro Stoxx 50 follows the S&P 500 a lot and is highly influenced by their actions.

Nevertheless, Beta has its limitations. The fact that the Beta is created by historical data makes the results less meaningful when predicting future movements for the Fund. A Beta is also a less useful long-term measure because companies and indexes can change their volatility and dependencies over the years.

3.6.1.5 Alpha

Because we have already explained how Jensen's alpha is calculated, we are going to concentrate ourselves on the analysis of the results. Like we already know, alpha is a measure that explains how much a portfolio or stock is outperforming the market used as a benchmark. In this case we have used the Fund against the three same benchmarks we have used previously. We see that the Fund is outperforming the S&P 500 with 0,0063, the Nikkei with 0.0068 and the Fund outperforms the Euro Stoxx 50 with 0.0085. Again, we are seeing that the S&P 500 is the best performing index against the Fund and the Euro Stoxx 50 is the worst performing index against the Fund. While Beta is a measurement of risk, alpha is a measure of returns. After seeing the cumulative returns between 2006 and 2021 using the three indexes and the Fund, it was already clear that there was a clear winner and what the best performing index and the worst performing index were.

If alpha would have been negative, like when we are using the Euro Stoxx 50 as our portfolio and the S&P 500 as the benchmark (-0.0058), this would mean that the benchmark is outperforming the portfolio. The S&P 500 has been outperforming the Euro Stoxx 50 on every metric since the beginning of this thesis.

3.6.1.6 Treynor Ratio

The Treynor ratio divides the excess return by the beta between a portfolio and a benchmark. To find the excess return, we need to calculate the difference between the Fund's return and the risk-free rate. In this case, we will use the 10-Year US Treasuries rate as the risk-free rate.

The higher the Treynor ratio, the better the investment because this would lead us to believe that the excess return is high or the Beta is low. We see that the highest Treynor ratio is 0.2615 when the Nikkei is used as a benchmark. This is due to the low Beta found between the Fund and the Nikkei. This is also due to the big difference in return between the Fund and the 10-year US Treasuries. The recurring trend is that the worst performing index is the Euro Stoxx 50 but the most volatile index is the Nikkei. This is why the highest alpha is when the Euro Stoxx 50 is used as a benchmark and the lowest beta is when the Nikkei is used as a benchmark.

The lowest Treynor ratio is when the S&P 500 is used as a benchmark. This is because they have the highest Beta of the three benchmarks and because the Fund doesn't outperform this benchmark like the others.

We also see that when we don't use the Fund as our portfolio and we use one of our benchmarks, we have consistently negative Treynor ratios. This is because the 10-Year US Treasuries has outperformed the Nikkei and the Euro Stoxx 50 between 2006 and 2021. Thus, the excess return is negative between those two benchmarks and the US Treasuries used as the risk-free rate. If we would have used the S&P 500 as our portfolio and weighted it against the risk-free rate, we would have had a positive excess return because the S&P 500 has outperformed the bond.

Table 20: The different performance measures for the Years 2006–2021 when using the S&P 500 as our benchmark

	NOBSGLIA to SPX	NKYTR to SPX	Eurostoxx.50 to SPX
Alpha	0.0063	-0.0009	-0.0058
Beta	0.3937	1.0024	0.9941
Beta+	0.2879	0.7287	0.9782
Beta-	0.4152	0.9793	0.9756
R-squared	0.4648	0.6634	0.7713
Annualized Alpha	0.0783	-0.0113	-0.0676
Correlation	0.6817	0.8145	0.8782
Correlation p-value	0.0000	0.0000	0.0000
Tracking Error	0.1391	0.1344	0.1019
Active Premium	0.0704	-0.0199	-0.0768
Information Ratio	0.5061	-0.1478	-0.7536
Treynor Ratio	0.2175	-0.0063	-0.0595

Source: Rstudio (2021)

Table 21: The different performance measures for the Years 2006–2021 when using the Nikkei as our benchmark

	NOBSGLIA to NKYTR	NKYTR to NKYTR	Eurostoxx.50 to NKYTR
Alpha	0.0068	0.0000	-0.0044
Beta	0.3275	1.0000	0.7455
Beta+	0.2571	1.0000	0.6497
Beta-	0.3191	1.0000	0.8900
R-squared	0.4869	1.0000	0.6569
Annualized Alpha	0.0846	0.0000	-0.0520
Correlation	0.6978	1.0000	0.8105
Correlation p-value	0.0000	0.0000	0.0000
Tracking Error	0.1742	0.0000	0.1380
Active Premium	0.0903	0.0000	-0.0569
Information Ratio	0.5182	NaN	-0.4124
Treynor Ratio	0.2615	-0.0063	-0.0794

Source: Rstudio (2021)

Table 22: The different performance measures for the Years 2006–2021 when using the Euro Stoxx 50 as our benchmark

	NOBSGLIA to Eurostoxx.50	NKYTR to Eurostoxx.50	Eurostoxx.50 to Eurostoxx.50
Alpha	0.0085	0.0045	0.0000
Beta	0.3558	0.8812	1.0000
Beta+	0.3160	0.7749	1.0000
Beta-	0.4085	0.9948	1.0000
R-squared	0.4862	0.6569	1.0000
Annualized Alpha	0.1066	0.0556	0.0000
Correlation	0.6973	0.8105	1.0000
Correlation p-value	0.0000	0.0000	0.0000
Tracking Error	0.1579	0.1380	0.0000
Active Premium	0.1472	0.0569	0.0000
Information Ratio	0.9325	0.4124	NaN
Treynor Ratio	0.2407	-0.0071	-0.0592

Source: Rstudio (2021)

3.6.2 Performance measures for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

3.6.2.1 Tracking error

Because we already established above how we calculate the Tracking error, we are going to concentrate on the results for the Risk Parity portfolio. We see that the Risk Parity portfolio has its lowest Tracking error when calculated against the Fund (0.0937). This is absolutely normal because both of them have very low standard deviations. The portfolio has its worst Tracking error when calculated with the Nikkei (0.1779). Like we already said above, this is normal because the Nikkei has the highest standard deviation of all the benchmarks and its return is much better than that of the Risk Parity portfolio.

3.6.2.2 Information ratio

Like with the Tracking error, we will not repeat the formula of the Information ratio and we will only concentrate on the results for the Risk Parity Portfolio. We see that the portfolio has a negative Information ratio with the Euro Stoxx 50. This is normal because the portfolio has a better return than the Euro Stoxx 50. Again, we are seeing that the Nikkei has a much better Information ratio than on a 15-Year basis. Their Information ratio is slightly worse than that of the S&P 500 and this is mainly due to the small difference in return in favor of the S&P 500 and the bigger standard deviation for the Nikkei. Lastly, we see that the Fund has an outstanding performance against the Risk Parity portfolio when calculating the Information ratio. This is mainly due to a low Tracking error and a high excess return between the two.

3.6.2.3 R-squared

When looking at R squared, we see that 83.61% of the portfolio's movement is influenced by the S&P 500 and vice-versa. This shows that there is a high correlation between both of them. We also see that the portfolio's movement is the least influenced by the Fund (26.82%). This can be considered a low correlation between the two participants. We already talked about the reasons for those high or low correlations when analysing the volatility and correlations between the benchmarks.

3.6.2.4 Beta

When the fund is used as a benchmark, we see that the Beta is very low against the portfolio (0.3648). This is because the portfolio is less volatile than the Fund. This beta implies that the portfolio's movement will be 36.48% of the Fund's movement. We also immediately see that the Betas are very big when using the Risk Parity portfolio as our benchmark against the indexes (all bigger than 2). This is because they are all twice more volatile than the portfolio created. We see the same pattern when looking at the Betas for the Fund. We see that all the benchmarks are more volatile than the Fund by 12% for the S&P 500, 28.26% for the Euro Stoxx 50 and 49.19% for the Nikkei. Thus, it is also normal that, when the Nikkei is used as a benchmark, the portfolio has its lowest Beta because its movement is the least influenced by the Nikkei's movement.

3.6.2.5 Alpha

We see that every benchmark is outperforming the Risk Parity portfolio besides the Euro Stoxx 50 with a negative Alpha. We also note that the Fund outperforms every other benchmark and that the S&P 500 outperforms the Nikkei (0.0017) more greatly than the Nikkei outperforms them (0.0003).

3.6.2.6 Treynor ratio

We see that the Risk Parity portfolio has a negative Treynor ratio. This is logical because its return is lower than the 10-Year US bond's return. This leads to negative Treynor ratios. The same can be said about the Euro Stoxx 50. The Fund has a decent Treynor ratio, but its highest is when the benchmark used is the Euro Stoxx 50 (0.2260). This is because the Beta used in the calculations is the lowest (0.3436). The Risk Parity portfolio has its worst Treynor ratio when the benchmark used is the Nikkei (-0.0117). This is because the Beta is the biggest when the Nikkei is used as Benchmark against the portfolio.

Table 23: The different performance measures for the Years 2010–2020 when using the Fund as our benchmark

```
> table.CAPM(mydata[,1:5], mydata[,1,drop=FALSE], Rf = mydata[,6,drop=FALSE]) # Report a lot of information on the CAPM (alpha, beta, R
ared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)
```

	NOBSGLIA to NOBSGLIA	Risk.Parity.Portfolio to NOBSGLIA	Eurostoxx.50 to NOBSGLIA	NKYTR to NOBSGLIA	SPX to NOBSGLIA
Alpha	0.0000	-0.0024	-0.0104	-0.0040	-0.0019
Beta	1.0000	0.3648	1.2826	1.4919	1.1276
Beta+	1.0000	0.2656	0.7938	1.0382	0.6669
Beta-	1.0000	0.5893	1.8252	1.7087	1.5099
R-squared	1.0000	0.2682	0.4406	0.5186	0.4444
Annualized Alpha	0.0000	-0.0290	-0.1180	-0.0474	-0.0224
Correlation	1.0000	0.5178	0.6638	0.7201	0.6666
Correlation p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Tracking Error	0.0000	0.0937	0.1575	0.1625	0.1356
Active Premium	0.0000	-0.0860	-0.1213	-0.0242	-0.0213
Information Ratio	NaN	-0.9178	-0.7703	-0.1489	-0.1568
Treynor Ratio	0.0776	-0.0076	-0.0330	0.0323	0.0471

Source: Rstudio (2021)

Table 24: The different performance measures for the Years 2010–2020 when using the Risk Parity Portfolio as our benchmark

```
> table.CAPM(mydata[,1:5], mydata[,2,drop=FALSE], Rf = mydata[,6,drop=FALSE]) # Report a lot of information on the CAPM
ared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)
```

	NOBSGLIA to Risk.Parity.Portfolio	Risk.Parity.Portfolio to Risk.Parity.Portfolio
Alpha	0.0067	0.0000
Beta	0.7351	1.0000
Beta+	0.3459	1.0000
Beta-	1.0944	1.0000
R-squared	0.2682	1.0000
Annualized Alpha	0.0837	0.0000
Correlation	0.5178	1.0000
Correlation p-value	0.0000	0.0000
Tracking Error	0.0937	0.0000
Active Premium	0.0860	0.0000
Information Ratio	0.9178	NaN
Treynor Ratio	0.1056	-0.0028

	Eurostoxx.50 to Risk.Parity.Portfolio	NKYTR to Risk.Parity.Portfolio	SPX to Risk.Parity.Portfolio
Alpha	-0.0018	0.0060	0.0057
Beta	2.1950	2.0382	2.1957
Beta+	1.9975	1.1724	2.0333
Beta-	2.2973	2.3934	2.2668
R-squared	0.6404	0.4803	0.8361
Annualized Alpha	-0.0213	0.0742	0.0705
Correlation	0.8002	0.6930	0.9144
Correlation p-value	0.0000	0.0000	0.0000
Tracking Error	0.1532	0.1779	0.1161
Active Premium	-0.0354	0.0618	0.0647
Information Ratio	-0.2308	0.3472	0.5573
Treynor Ratio	-0.0193	0.0236	0.0242

Source: Rstudio (2021)

Table 25: The different performance measures for the Years 2010–2020 when using the Euro Stoxx 50 as our benchmark

```
> table.CAPM(mydata[,1:5], mydata[,3,drop=FALSE], Rf = mydata[,6,drop=FALSE]) # Report a lot of information on the CAPM (alpha, beta, R
ared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)
```

	NOBSGLIA to Eurostoxx.50	Risk.Parity.Portfolio to Eurostoxx.50	Eurostoxx.50 to Eurostoxx.50	NKYTR to Eurostoxx.50	SPX to Eurostoxx.50
Alpha	0.0073	0.0005	0.0000	0.0075	0.0070
Beta	0.3436	0.2917	1.0000	0.8590	0.7432
Beta+	0.2440	0.3006	1.0000	0.6116	0.7520
Beta-	0.4697	0.3658	1.0000	1.0179	0.9070
R-squared	0.4406	0.6404	1.0000	0.6419	0.7208
Annualized Alpha	0.0917	0.0063	0.0000	0.0942	0.0877
Correlation	0.6638	0.8002	1.0000	0.8012	0.8490
Correlation p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Tracking Error	0.1575	0.1532	0.0000	0.1358	0.1094
Active Premium	0.1213	0.0354	0.0000	0.0971	0.1001
Information Ratio	0.7703	0.2308	NaN	0.7154	0.9152
Treynor Ratio	0.2260	-0.0095	-0.0424	0.0560	0.0714

Source: Rstudio (2021)

Table 26: The different performance measures for the Years 2010–2020 when using the Nikkei as our benchmark

```
> table.CAPM(mydata[,1:5], mydata[,4,drop=FALSE], Rf = mydata[,6,drop=FALSE]) # Report a lot of information on the CAPM
ared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)
```

	NOBSGLIA to NKYTR	Risk.Parity.Portfolio to NKYTR	Eurostoxx.50 to NKYTR	NKYTR to NKYTR	SPX to NKYTR
Alpha	0.0046	-0.0014	-0.0063	0.0000	0.0017
Beta	0.3476	0.2356	0.7472	1.0000	0.6617
Beta+	0.1890	0.1855	0.6321	1.0000	0.5026
Beta-	0.4463	0.3486	0.9582	1.0000	0.8885
R-squared	0.5186	0.4803	0.6419	1.0000	0.6567
Annualized Alpha	0.0572	-0.0168	-0.0726	0.0000	0.0210
Correlation	0.7201	0.6930	0.8012	1.0000	0.8104
Correlation p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Tracking Error	0.1625	0.1779	0.1358	0.0000	0.1299
Active Premium	0.0242	-0.0618	-0.0971	0.0000	0.0029
Information Ratio	0.1489	-0.3472	-0.7154	NaN	0.0227
Treynor Ratio	0.2233	-0.0117	-0.0567	0.0481	0.0803

Source: Rstudio (2021)

Table 27: The different performance measures for the Years 2010–2020 when using the S&P 500 as our benchmark

```
> table.CAPM(mydata[,1:5], mydata[,5,drop=FALSE], Rf = mydata[,6,drop=FALSE]) # Report a lot of information or
ared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)
```

	NOBSGLIA to SPX	Risk.Parity.Portfolio to SPX	Eurostoxx.50 to SPX	NKYTR to SPX	SPX to SPX
Alpha	0.0045	-0.0022	-0.0073	0.0003	0.0000
Beta	0.3941	0.3808	0.9698	0.9925	1.0000
Beta+	0.2278	0.4133	0.9077	0.5952	1.0000
Beta-	0.5278	0.3976	0.9759	1.0499	1.0000
R-squared	0.4444	0.8361	0.7208	0.6567	1.0000
Annualized Alpha	0.0551	-0.0257	-0.0843	0.0041	0.0000
Correlation	0.6666	0.9144	0.8490	0.8104	1.0000
Correlation p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Tracking Error	0.1356	0.1161	0.1094	0.1299	0.0000
Active Premium	0.0213	-0.0647	-0.1001	-0.0029	0.0000
Information Ratio	0.1568	-0.5573	-0.9152	-0.0227	NaN
Treynor Ratio	0.1970	-0.0073	-0.0437	0.0485	0.0531

Source: Rstudio (2021)

3.7 Sharpe ratio

3.7.1 Sharpe ratio for the Fund and its benchmarks excluding the Risk Parity portfolio for the Years 2006–2021

The Sharpe ratio is calculated by dividing the excess return by the standard deviation. In this chapter we will calculate the Sharpe ratio in two different ways. Firstly, we will calculate the excess return of the Sharpe ratio by using the 10-Year US Treasuries rate as a risk-free rate. After finding the monthly Sharpe ratio and the annualised Sharpe ratio using this way, we will use our second way of calculating the Sharpe ratio. In this case, we will calculate the Excess return by using 0 as our risk-free rate rather than the 10-Year US Treasuries.

The goal is to have the highest Sharpe ratio possible because that would show us that the Fund or index has an important excess return regarding their standard deviation which is used as a risk measure.

Firstly, let's remind ourselves of the different standard deviations of each participant.

Table 18: Mean, Monthly and Annualised standard deviation for the Fund and its benchmarks excluding the Risk Parity portfolio for the years 2006–2021.

	NOBSGLIA	NKYTR	Eurostoxx.50	SPX	US10GOV
Mean Absolute deviation	0.0196	0.0416	0.0393	0.0317	0.0157
monthly Std Dev	0.0242	0.0557	0.0512	0.0436	0.0210
Annualized Std Dev	0.0840	0.1930	0.1775	0.1509	0.0728

Source: Rstudio (2021)

3.7.1.1 Monthly Sharpe ratio when the monthly risk-free rate is 0.4%

The best performing participant is the Fund. The Fund is performing 5 times better than the S&P 500 and 10 times better than the Nikkei. The Fund has not only a much better cumulative return than those two benchmarks, it also has a much better standard deviation. We can also notice that the Euro Stoxx 50's Monthly Sharpe ratio is negative. This would mean that the Euro Stoxx 50 is performing worse monthly than the 10-Year US Treasuries because the excess return is negative. The S&P 500 is performing almost 2 times better than the Nikkei, this is due to their lower monthly standard deviation and higher monthly returns.

3.7.1.2 Annualised Sharpe ratio when the annualised risk-free rate is 5.2%

Firstly, we are going to explain how we calculate the Annualised Sharpe ratio from the Monthly Sharpe ratio. *"The annualized Sharpe Ratio is computed by dividing the annualized mean* *monthly excess return by the annualized monthly standard deviation of excess return. Equivalently, the annualized Sharpe Ratio equals the monthly Sharpe Ratio times the square root of 12[22]"* (Stanford, n.d.).

The results show an incredible difference between the Fund and the three benchmarks. While the best performing index (S&P 500) is struggling to outperform the 10-Year US Treasuries, the Fund has vastly outperformed the bond. This is due to their very close standard deviation with the bond (0.0840 against 0.0728) and their excess return against the bond each year.

This time we are seeing two negative Sharpe ratios. The Nikkei has a very small negative Sharpe ratio while the Euro Stoxx 50 has a big negative Sharpe ratio. This would mean that, annually, the Nikkei has been slightly outperformed by the 10-Year US Treasuries while the Euro Stoxx 50 has been vastly outperformed.

3.7.1.3 Daily Sharpe ratio when the risk-free rate is 0

In this calculation, we are going to elaborate the hypothesis that the risk-free rate is 0. This would lead to excess returns being equal to the participants' returns. This is going to show us what the risk-adjusted return is for each participant.

Firstly, we see that the Fund is, again, performing better than the benchmarks. This time we are also going to calculate the 10-Year Us Treasuries Sharpe ratio. Now that we are using 0 as the risk-free rate, we are enabled to calculate the Sharpe ratio for the 10-Year US Treasuries. We see that the 10-Year US has a better daily Sharpe ratio than the S&P 500. This is surprising because the S&P 500 is usually the best performing benchmark. This high Daily Sharpe ratio is fueled by the lowest standard deviation of all participants and a better return than the Nikkei and the Euro Stoxx 50.

We cannot forget that those stats are "tainted" by the 2008 Financial crisis and its aftermath. It would be interesting to see the Nikkei's Sharpe ratio since 2013–2014 because the Japanese economy really started to hit their pre-crisis levels around those years. We could argue that their Sharpe ratio would be higher than that of the 10-Year US if we started calculating from those years. The same could be said about the S&P 500. This would also be because, like we said before, bond returns are decreasing when economies are thriving due to low rates. It would be

[22] Stanford. (n.d.). *Instructions for Performance Measurement Worksheet https://web.stanford.edu/~wfsharpe/ws/wi_perf.htm#:~:text=The%20annualized%20Sharpe%20Ratio%20is,the%20square%20root%20of%2012.*

very surprising to see a higher Sharpe ratio for a bond than for a stock when economies are thriving. Surprisingly when watching the Euro Stoxx 50's cumulative return since 2013–2014, it could be argued that they would still have a worse Sharpe ratio than the 10-Year US.

3.7.1.4 Annualised Sharpe ratio when the risk-free rate is 0

Surprisingly, we see that the Euro Stoxx 50 has a negative Annualised Sharpe ratio. This would mean that the Euro Stoxx 50 has a negative annualised return. This is normal because they haven't reached their pre-2008 levels today. This would mean, if you would have invested in 2006 in those stocks, that you are still losing on your investments. We see that the S&P 500 is still being beaten by the 10-Year US. Lastly, the Fund is still outperforming each benchmark with ease. It is normal that the Fund has such a large Sharpe ratio because they are constantly boosting annual returns of 10–15% while having the standard deviation of a bond.

Table 28: The different Sharpe ratios for the Fund and its benchmarks excluding the Risk Parity portfolio for the Years 2006–2021.

```
> SharpeRatio(mydata[,1:4], Rf = mydata[,5,drop=FALSE], FUN="StdDev") #Report the traditional monthly Sharpe ratio
                                 NOBSGLIA        NKYTR
StdDev Sharpe (Rf=0.4%, p=95%): 0.3036442366 0.03149100962
                               Eurostoxx.50
StdDev Sharpe (Rf=0.4%, p=95%): -0.06116935813
                                       SPX
StdDev Sharpe (Rf=0.4%, p=95%): 0.06191048253
> SharpeRatio(mydata[,1:5], Rf = 0, FUN="StdDev") #Report the traditional daily Sharpe ratio
                               NOBSGLIA       NKYTR
StdDev Sharpe (Rf=0%, p=95%): 0.4811886608 0.1087480396
                            Eurostoxx.50          SPX
StdDev Sharpe (Rf=0%, p=95%): 0.02285719793 0.1607332581
                                US10GOV
StdDev Sharpe (Rf=0%, p=95%): 0.2048486026
>
> SharpeRatio.annualized(mydata[,1:4], Rf = mydata[,5,drop=FALSE])
                                    NOBSGLIA
Annualized Sharpe Ratio (Rf=5.2%) 1.019512112
                                          NKYTR
Annualized Sharpe Ratio (Rf=5.2%) -0.03258556924
                                  Eurostoxx.50
Annualized Sharpe Ratio (Rf=5.2%) -0.3334027322
                                          SPX
Annualized Sharpe Ratio (Rf=5.2%) 0.09506936555
> SharpeRatio.annualized(mydata[,1:5], Rf = 0)
                                  NOBSGLIA        NKYTR
Annualized Sharpe Ratio (Rf=0%) 1.730910522 0.2855012278
                                Eurostoxx.50          SPX
Annualized Sharpe Ratio (Rf=0%) -0.01030896736 0.4968680983
                                    US10GOV
Annualized Sharpe Ratio (Rf=0%) 0.6892258916
```

Source: Rstudio (2021)

3.7.2 Sharpe ratio for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

We already know that the Risk Parity portfolio has the lowest standard deviation of all the participants. We also know that the order (from best to worst participants) is the same when looking at a 10-Year period than when we are looking at a 15-Year period.

Firstly, we see that the risk-free rate is similar to a 10-Year period than on a 15-Year period when looking at the monthly returns. Secondly, we already established previously how a Sharpe ratio is calculated. Thirdly, like we have seen above, the Euro Stoxx 50 has a negative Sharpe ratio because its return is lower than that of the 10-Year US Bond.

3.7.2.1 Sharpe ratio when using the US bond as our Risk-free rate

It became immediately clear that the Risk Parity portfolio would not have an incredibly high Sharpe ratio. This is due to their very similar return to the 10-Year US bond used as a risk-free rate. We see that on a monthly basis the portfolio beats the US bond slightly but when we are calculating the annualised Sharpe ratio, the US bond beats the portfolio and this creates a negative Sharpe ratio. On a 10-Year period, it is interesting to see that the Nikkei has a much better Annualised Sharpe ratio with a risk-free rate (0.26849) than on a 15-Year basis (-0.03258). It is still worse than the S&P 500 due to their higher standard deviation and slightly lower return. Lastly, we see that the Fund performed worse on a 10-year basis than on a 15-Year basis, this is due to the better performance from the US bond.

Table 29: Sharpe ratios when using the monthly 10-Year US Treasury rate as our monthly risk-free rate

```
> SharpeRatio(mydata[,1:6], Rf = mydata[,6,drop=FALSE], FUN="StdDev") #Report the traditional monthly Sharpe ratio
                                   NOBSGLIA
StdDev Sharpe (Rf=0.4%, p=95%): 0.2991727747
                                Risk.Parity.Portfolio
StdDev Sharpe (Rf=0.4%, p=95%):       0.0003862595932
                                  Eurostoxx.50       NKYTR
StdDev Sharpe (Rf=0.4%, p=95%): -0.03623693426 0.115957581
                                         SPX US10GOV
StdDev Sharpe (Rf=0.4%, p=95%): 0.1408797207       0
```

Source: Rstudio (2021)

Table 30: Sharpe ratios when using the Annualised 10-Year US Treasury rate as our Annualised risk-free rate

```
> SharpeRatio.annualized(mydata[,1:6], Rf = mydata[,6,drop=FALSE])
                                    NOBSGLIA
Annualized Sharpe Ratio (Rf=5%) 0.9964545649
                                Risk.Parity.Portfolio
Annualized Sharpe Ratio (Rf=5%)        -0.04855879345
                                Eurostoxx.50        NKYTR
Annualized Sharpe Ratio (Rf=5%)  -0.24877341 0.2684914103
                                         SPX US10GOV
Annualized Sharpe Ratio (Rf=5%) 0.3786449039       0
```

Source: Rstudio (2021)

3.7.2.2 Sharpe ratio when using 0 as our Risk-free rate

It is very surprising to see that the Risk Parity portfolio has the best Sharpe ratio from all the benchmarks. This is all thanks to their incredibly low standard deviation. We have seen above that their return, on an annualised basis, is slightly less than that of the US bond but its standard deviation is that much better, that the portfolio has a better Sharpe ratio than its peers. Moreover, we see that the bond performed better on a 10-year basis than on a 15-year basis. This is because the bond also suffered greatly, less than stocks, between 2008–2010.

Important note about the calculations regarding the Annualised risk-free returns: (0.004+1)^(12-1)

Table 31: Sharpe ratios when using 0 as our daily risk-free rate

```
> SharpeRatio(mydata[,1:6], Rf = 0, FUN="StdDev") #Report the traditional daily Sharpe ratio
                                  NOBSGLIA
StdDev Sharpe (Rf=0%, p=95%): 0.4858543538
                              Risk.Parity.Portfolio
StdDev Sharpe (Rf=0%, p=95%):           0.255484852
                               Eurostoxx.50        NKYTR
StdDev Sharpe (Rf=0%, p=95%): 0.04913943206 0.1971187815
                                       SPX      US10GOV
StdDev Sharpe (Rf=0%, p=95%): 0.2445907485 0.2226934495
```

Source: Rstudio (2021)

Table 32: Sharpe ratios when using 0 as our Annualised risk-free rate

```
> SharpeRatio.annualized(mydata[,1:6], Rf = 0)
                                  NOBSGLIA
Annualized Sharpe Ratio (Rf=0%) 1.744644447
                                Risk.Parity.Portfolio
Annualized Sharpe Ratio (Rf=0%)          0.8762325781
                                 Eurostoxx.50        NKYTR
Annualized Sharpe Ratio (Rf=0%) 0.08566623232 0.6234707391
                                         SPX      US10GOV
Annualized Sharpe Ratio (Rf=0%) 0.8176825407 0.7558777704
>
```

Source: Rstudio (2021)

3.8 Modified Treynor ratio

3.8.1 Modified Treynor ratio for the Fund and its benchmarks excluding the Risk Parity portfolio for the Years 2006–2021

The Modified Treynor ratio is calculated by dividing the excess return by the Systematic risk. The modified Treynor ratio is different from the Treynor ratio by its denominator. In the Treynor ratio we are using the portfolio's Beta as the denominator whereas in the modified Treynor ratio we are using the Systematic risk. Carl R. Bacon explains that the Systematic risk is calculated by multiplying the Beta by the market risk. Again, in this calculation, we are going to use the 10-year US as the risk-free rate. We are going to calculate the modified Treynor ratio using 4 different Systematic Risk. Each systematic risk will be influenced by the Beta of one of the 4 participants.

3.8.1.1 Modified Treynor ratio using the S&P 500 as our benchmark when calculating the Systematic Risk

We see that the Treynor ratio is much higher for the Fund when using the Systematic risk as our denominator (Modified Treynor ratio: 1,1294 vs Treynor ratio: 0.2175). This is probably because the market risk is very low for the Fund. Their market risk is so diversified thanks to their multiple investments in hundreds of markets.

We are seeing that the Nikkei's modified Treynor ratio is worse than the normal one. (Modified Treynor ratio: -0.03349 vs Treynor ratio: -0.0063). This is probably due to their high market risk which is increasing its systematic risk. We have seen before that the Japanese economy is seen as one of the most volatile markets of all the developed markets.

The same pattern is seen with the Euro Stoxx 50 (Modified Treynor ratio: -0.3161 vs Treynor ratio: -0.0595). We have already seen that their Beta and excess return are lower than the Nikkei's. We also think that their worse Treynor ratio is due to their high market risk. Even if the European market is less volatile than the Japanese market, the market is considered riskier than the American one.

To conclude, both markets have negative Modified Treynor ratios and this is probably due to their high market risk and high betas while still boosting negative or very low excess returns.

Table 33: Modified Treynor ratio when using the S&P 500 as our benchmark when calculating the Systematic Risk

```
> #CAPM useful when portfolios or individual stocks must be compared to a benchmark
> TreynorRatio(mydata[,1:3], mydata[,4], Rf = mydata[,5,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                  NOBSGLIA          NKYTR  Eurostoxx.50
Treynor Ratio:  1.155153097 -0.03332997074 -0.3161557888
```

Source: Rstudio (2021)

3.8.1.2 Modified Treynor ratio using the Euro Stoxx 50 as our benchmark when calculating the Systematic risk

We see that, again, the Nikkei is having negative modified Treynor ratios and that is also a pattern that we are going to observe below. This time we see the modified Treynor ratio for the S&P 500 (0.08676). We will see below that the S&P 500's modified Treynor ratio will always be positive and around those numbers. This is because the excess return doesn't change when different benchmarks are used to calculate the Systematic risk. The only variables that change each calcul are the market risk and the Beta using another benchmark. The S&P 500 ratio is positive because we have seen before that the benchmark has a positive excess return whereas the Nikkei and the Euro Stoxx 50 have mixed results in that field.

Table 34: Modified Treynor ratio when using the Euro Stoxx 50 as our benchmark when calculating the Systematic Risk

```
> #CAPM useful when portfolios or individual stocks must be compared to a benchmark
> TreynorRatio(mydata[,1:4], mydata[,3], Rf = mydata[,5,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                  NOBSGLIA          NKYTR  Eurostoxx.50           SPX
Treynor Ratio:  1.129437986 -0.03349366859 -0.2776591369 0.08676503784
```

Source: Rstudio (2021)

3.8.1.3 Modified Treynor ratio using the Nikkei as our benchmark when calculating the Systematic risk

In this paragraph, we see that each ratio is higher than when the Euro Stoxx 50 is used to calculate the Systematic risk. This is because we noticed that the Beta is lower when the Nikkei is the benchmark (0.7455 vs 0.8812 when the benchmark is Euro Stoxx 50) than when the Euro Stoxx was the benchmark. This would mean that the Systematic risk is lower when the Nikkei is used as our benchmark to calculate such risk.

This creates a lower modified Treynor ratio when the excess return is negative and a higher positive modified Treynor ratio when the excess return is positive. This is why the Fund and the S&P 500 are presenting better ratios and the Euro Stoxx 50 is presenting a worse ratio.

Table 35: Modified Treynor ratio when using the Nikkei as our benchmark when calculating the Systematic Risk

```
> #CAPM useful when portfolios or individual stocks must be compared to a benchmark
> TreynorRatio(mydata[,1:4], mydata[,2], Rf = mydata[,5,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                   NOBSGLIA           NKYTR  Eurostoxx.50           SPX
Treynor Ratio:  1.128573117 -0.02714676311 -0.3425757639 0.09355617788
```

Source: Rstudio (2021)

3.8.1.4 Modified Treynor ratio using the Fund as our benchmark when calculating the Systematic risk

When the Fund is used to calculate systematic risk, we see that the S&P 500 has its highest modified Treynor ratio since the beginning. This is due to the low systematic risk used to calculate the ratio. The low systematic risk is because the beta is very low when the Fund is used as our benchmark. Moreover, the market risk is lower than when the other benchmarks are used. This is, like we already said before, because the Fund's investments are in a multitude of markets which makes the effect of one market negligible. Like above, a low beta means that positive excess returns boast better modified Treynor ratios whereas negative excess returns boast worse modified Treynor ratios.

Table 36: Modified Treynor ratio when using the Fund as our benchmark when calculating the Systematic Risk

```
> #CAPM useful when portfolios or individual stocks must be compared to a benchmark
> TreynorRatio(mydata[,1:4], mydata[,1], Rf = mydata[,5,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                    NOBSGLIA           NKYTR  Eurostoxx.50          SPX
Treynor Ratio:  0.7875055962 -0.03890398648 -0.3982178386 0.1117741794
```

Source: Rstudio (2021)

3.8.2 Modified Treynor ratio for the Fund and its benchmarks including the Risk Parity portfolio for the Years 2010–2020

We immediately see that the Risk Parity portfolio has a negative modified Treynor ratio. This is because their annualised return is smaller than the annualised return from the 10-Year US bond. This creates a negative excess return and, thus, a negative modified Treynor ratio. We knew that when we created this Risk Parity portfolio that they would perform badly when looking at performance measurements involving returns. We cannot forget that the main goal was to create a portfolio with the least risk possible, even less riskier than a bond.

Again, we are seeing that the Nikkei is performing much better than on a 15-year basis. Again, this is due to their exceptional recovery since 2012 and their very similar return to the S&P 500.

Nonetheless, they are still one of the more volatile indexes. The Fund's performance is still very good but the slight drop-off is due to the better performance from US bonds on a 10-Year basis.

Table 37: Modified Treynor ratio when using the Fund (1), Risk Parity Portfolio (2), Euro Stoxx 50 (3), Nikkei (4), S&P 500 (5) as our benchmark when calculating the Systematic Risk

```
> TreynorRatio(mydata[,1:5], mydata[,1], Rf = mydata[,6,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                    NOBSGLIA Risk.Parity.Portfolio Eurostoxx.50        NKYTR          SPX
Treynor Ratio:  0.7257492233        -0.07095269709 -0.308894007 0.3014909926 0.4402472386
> TreynorRatio(mydata[,1:5], mydata[,2], Rf = mydata[,6,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                   NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50        NKYTR          SPX
Treynor Ratio:  1.401498798        -0.03674199711 -0.2562286839 0.3132836252 0.3209525102
> TreynorRatio(mydata[,1:5], mydata[,3], Rf = mydata[,6,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                   NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50        NKYTR          SPX
Treynor Ratio:  1.093318872        -0.04591359372 -0.2050450161 0.2709957946 0.3456697851
> TreynorRatio(mydata[,1:5], mydata[,4], Rf = mydata[,6,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                  NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50        NKYTR          SPX
Treynor Ratio:  1.00779255        -0.05301645399 -0.2559304683 0.2171149744 0.3621436586
> TreynorRatio(mydata[,1:5], mydata[,5], Rf = mydata[,6,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio
                  NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50        NKYTR          SPX
Treynor Ratio:  1.08872749        -0.04018273598 -0.2415163885 0.2679209232 0.2934702159
```

Source: Rstudio (2021)

3.9 M Squared

3.9.1 M Squared for the Fund and its benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021

In our chapter about the different performance measurements, we already talked about M Squared. M squared is calculated by multiplying the Sharpe ratio by the market risk and adding the risk-free rate after that multiplication. We are going to use 0 as our risk-free rate. Moreover, we are going to calculate M squared using 4 different benchmarks to calculate the different market risks to have more accurate results.

After having a quick look at the results, it isn't difficult to find a pattern. We see that the Euro Stoxx 50 is the only benchmark having a negative M Squared. This is because it is constantly performing worse than the Nikkei, S&P 500 and the Swf. The Euro Stoxx 50's Monthly Sharpe ratio is not only negative, it is also the worst of all the benchmarks. The Euro Stoxx 50 has a slightly better standard deviation than the Nikkei but the difference is so negligible that this doesn't help the Euro Stoxx have a better M squared.

Another pattern seen is that M squared results are constantly increasing with the standard deviation. We see that the Fund sees its M squared increased when riskier benchmarks are added to calculate it. To remind ourselves, the riskiest benchmark is the Nikkei and the least volatile benchmark is the S&P 500. We see that M squared results are consistent with our assumptions.

We also concluded that it wasn't really necessary to use the USGov10 as our risk-free rate when calculating M squared because it doesn't change anything in our assumptions. If we would have used the USGov10, the Euro Stoxx 50 would still be the worst performing benchmark but it would have been positive thanks to the USGov10.

Table 38: M Squared when using the S&P 500 as our benchmark

```
> MSquared(mydata[,1:4], mydata[,4],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                          NOBSGLIA        NKYTR     Eurostoxx.50            SPX
MSquared (Risk free = 0) 0.2611646 0.04307722034 -0.001555445705 0.07496884239
```

Source: Rstudio (2021)

Table 39: M Squared when using the Euro Stoxx 50 as our benchmark

```
> MSquared(mydata[,1:4], mydata[,3],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                             NOBSGLIA         NKYTR     Eurostoxx.50            SPX
MSquared (Risk free = 0) 0.3071530224 0.0506626795 -0.001829343829 0.08817009092
```

Source: Rstudio (2021)

Table 40: M Squared when using the Nikkei as our benchmark

```
> MSquared(mydata[,1:4], mydata[,2],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                             NOBSGLIA          NKYTR     Eurostoxx.50           SPX
MSquared (Risk free = 0) 0.3340668243 0.05510191734 -0.001989637213 0.09589585686
```

Source: Rstudio (2021)

Table 41: M Squared when using the Fund as our benchmark

```
> MSquared(mydata[,1:4], mydata[,1],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                             NOBSGLIA        NKYTR      Eurostoxx.50           SPX
MSquared (Risk free = 0) 0.1453664951 0.0239771567 -0.0008657746508 0.04172831179
```

Source: Rstudio (2021)

3.9.2 M Squared for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

The Risk Parity Portfolio has the second-best M squared of all the participants. This is no surprise because, without using the US Bond as our risk-free rate, the portfolio has the second-best Sharpe ratio. Moreover, we see that M Squared is significantly smaller when using the Risk Parity portfolio as our market risk. This is because the portfolio is very diversified and, thus, has a lot of different markets that influence the market risk. This creates a very low market risk. The market risk is even lower than with the Fund. The highest M squared recorded is when the Nikkei is used to calculate the market risk.

Table 42: M Squared when using the Fund (1), Risk Parity Portfolio (2), Euro Stoxx 50 (3), Nikkei (4), S&P 500 (5) as our benchmark

```
> MSquared(mydata[,1:5], mydata[,1],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                             NOBSGLIA Risk.Parity.Portfolio   Eurostoxx.50          NKYTR           SPX
MSquared (Risk free = 0) 0.1359286944          0.06826901062 0.006674425341 0.04857583657 0.06370726157
> MSquared(mydata[,1:5], mydata[,2],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                              NOBSGLIA Risk.Parity.Portfolio   Eurostoxx.50          NKYTR           SPX
MSquared (Risk free = 0) 0.09947284722          0.04995937683 0.004884355694 0.03554787893 0.04662108121
> MSquared(mydata[,1:5], mydata[,3],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                             NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50         NKYTR          SPX
MSquared (Risk free = 0) 0.2972178883           0.1492751127 0.01459411212 0.1062145681 0.1393005196
> MSquared(mydata[,1:5], mydata[,4],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                             NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50         NKYTR          SPX
MSquared (Risk free = 0) 0.3126541151           0.1570278241 0.01535206793 0.1117308988 0.1465351932
> MSquared(mydata[,1:5], mydata[,5],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate
                            NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50          NKYTR          SPX
MSquared (Risk free = 0) 0.244673916           0.1228853573 0.01201407689 0.08743731566 0.1146741329
```

Source: Rstudio (2021)

3.10 Analysis of the different Downside Risk ratios

In the performance measurement chapter above, we have already explained all the ratios below. Nonetheless, we will, in one or two phrases, remind ourselves how to calculate each ratio. This will help us in our analysis and will also help us understand better the meaning behind each result. We also have already established, before starting the calculations, that the MAR is equal to 0. The MAR is an abbreviation for Minimum Acceptable Return. After fixing a threshold, with the MAR, we are going to be able to calculate and analyse the different Downside risks.

3.10.1 The different Downside Risk ratios for the Fund and its benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021.

3.10.1.1 Monthly and Annualised Downside risk

The downside risk measures the variability of the underperformance of an investment below a certain threshold or minimum. Like we said above, in this case, the threshold is 0. We will look at each investment's negative return because positive returns are excluded from the equation.

The downside risk will help us understand how investments fare when markets are declining. We are only looking at the potential losses and not the potential gains. Usually investments with high downside risks have high potential gains. This is something we have explained before. Usually high risks can lead to high rewards. To calculate the Downside risk, we are looking at all the historical data of each investment since 2006. This will help us have better and more precise results.

$$\text{Downside risk } \sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\min[(r_i - r_T), 0]^2}{n}}$$

The Nikkei is the benchmark with the highest annual and monthly downside risk. We have already seen that this benchmark has the highest standard deviation. The Nikkei has also shown that it was more affected by the 2008 crisis than the worst performing benchmark, the Euro Stoxx 50. We can see on the graph "cumulative returns" above, that the Nikkei suffered more important losses than every other benchmark.

We see that the Fund has a very close downside risk to the 10-Year US Treasuries.

Whatsoever, the Fund has proven to still be presenting exceptional results. Having such a close downside ratio with the 10-Year US gov is remarquable. We have seen, by watching the monthly returns of each benchmark, that the Fund is always outperforming the markets. Let's look at the beginning of the 2008 Financial crisis, where the 31/08/08 monthly return for the Fund is -1% whereas the S&P 500 is -6%, the Euro Stoxx 50 is -14% and the Nikkei is -11%. This shows that, even when the Fund is doing bad, they are outperforming the benchmarks by a big margin. Another example, 31/03/20, we see that the monthly return for the Fund is -2% whereas the Euro Stoxx 50 has -16% monthly return, the Nikkei has -10% monthly return and the S&P 500 has -13% monthly return. The Fund has proven to be performing better in good and tough times. We will take a closer look at those negative returns when analyse the drawdown chapter.

3.10.1.2 Omega ratio

The Omega ratio is calculated by dividing the Upside Potential by the Downside Potential. This ratio is entirely based on past results and could (or not) be a telling for the future. The Omega ratio is driven by the Upward Potential and, logically, a high Upward Potential creates a High Omega ratio. A high Omega ratio could also be driven by a low Downside Potential.

The Fund has an Omega ratio of 3,1831. This is not thanks to their incredibly high Upside potential. We have seen before that the Upside potential of the Fund is influenced by low returns when other benchmarks are suffering negative returns. We now see that this "theory" is proven true thanks to the Omega ratio. The Fund has a high Omega ratio because the Downside potential is so low. They are very rarely suffering negative returns and when they are suffering negative returns, they are "in average" smaller than that of the 10-Year US Treasuries.

The Euro Stoxx 50 has a 3 times lower Omega ratio than the Norwegian Sovereign Wealth Fund. They are presenting the worst Omega ratio of all the benchmarks. This is because they have the highest Downside potential of all the benchmarks. When their high Downside Potential is paired with their average Upside potential, we get a very low Omega ratio. The Nikkei has a better Omega ratio, not thanks to their Downside potential which is fairly similar to that of the Euro Stoxx 50, but because they have the highest Upside Potential of all the participants.

3.10.1.3 Sortino ratio

The Sortino ratio is calculated like the Sharpe ratio but we are dividing the excess return by the downside risk.

It is entirely logical that the Fund is presenting the best Sortino ratio (0.9384) because it boosts the highest excess return and the second-lowest Downside risk of all the participants. The second-best performer is the 10-Year US Gov (0.3794). This is mainly thanks to their incredibly low Downside risk. This downside risk paired with a "respectable" excess return creates a better Sortino ratio than that of all the other benchmarks.

Again, the Euro Stoxx 50 is the worst performing benchmark with a Sortino ratio of 0.0313. This is mainly due to their very low Excess return. The cumulative returns chart above has shown us that the Euro Stoxx 50 hasn't hit its pre-2008 levels. The Euro Stoxx 50 pairs this abysmal excess return with the second-highest Downside risk. This creates a Sortino ratio close to 0. Such a low Sortino ratio is usually a sign that such investments should not be pursued because their excess returns are smaller than their Downside risks. This means that you are penalised for your investment and you are bearing additional risks without reaping the rewards.

The Sortino ratio is sometimes preferred to the Sharpe ratio because it only takes the downside risk into account whereas the Sharpe ratio takes the whole standard deviation into account. When taking the whole standard deviation into account we are penalising risks that result in positive returns.

3.10.1.4 Upside potential

The upside potential is calculated by averaging the sum of all positive returns.

$$\text{Upside potential } \mu_U \sum_{i=1}^{n} \frac{\max[(r_i - r_T), 0]}{n}$$

Surprisingly, the Fund does not have the highest upside potential. A logical theory would be that the Fund doesn't suffer a lot of negative returns, which would mean that, when calculating the upside potential, the Fund's average return is watered down but low returns when the benchmarks are suffering negative returns.

Moreover, it is no surprise that the Nikkei has the highest upside potential. This is due to their high volatility. The Nikkei suffers negative returns one month and the second month the market is trying to recalibrate. This creates an important downside potential and important upside potential.

We see that the 10-Year US Treasuries has the lowest Upside potential of all the participants. This is normal because the bond isn't well known for its high returns, it is well-known for its safety and low standard deviation. The Fund is always boosting stable positive returns whereas the Nikkei suffers negative returns one month and positive returns another month.

To conclude, the Nikkei has the highest average sum of positive returns whereas the 10-Year US Treasuries has the lowest average sum of positive returns.

3.10.1.5 Downside potential

The downside potential is calculated by averaging the sum of all negative returns.

$$\text{Downside potential } \mu_D = \sum_{i=1}^{n} \frac{\min[(r_i - r_T), 0]}{n}$$

We see that, surprisingly, the Fund has a better Downside potential than the bond. This would mean its average negative return is slightly better than that of the 10-Year US Gov This is probably because the Fund never suffers big monthly negative results like we have seen above with the other benchmarks. Again, the two results are very similar but this doesn't change the fact that a Fund with such high returns should not have a lower downside potential than a bond.

Looking at the results below, we see that the Euro Stoxx 50 has the highest downside potential. We already have seen in the cumulative returns chart that this is due to their slow recovery since the 2008 Financial crisis. The Nikkei is boosting smaller negative results the last couple of years than the Euro Stoxx 50.

To conclude, you would average the highest losses when investing in the Euro Stoxx 50 whereas you would average the lowest losses when investing in/like the Fund.

3.10.1.6 Upside potential ratio

The Upside potential ratio is calculated by dividing the Upside potential by the Downside risk. This ratio is interesting because we are pairing the average positive return against the variability of negative returns. We are going to look at which participant is outperforming the others.

$$\text{Upside potential ratio } UPR = \frac{\text{Upside Potential}}{\text{Downside Risk}}$$

Firstly, we are seeing that the Fund and the 10-Year US have a very similar Upside Potential ratio (1,1397 vs 1,1346). The Fund is hedging it out by 0.0051. This is because the Fund has a better Upside Potential than the 10-Year US Gov This advantage is more important than the lower Downside risk for the 10-Year US Gov It has been seen with the cumulative returns that the Fund is boosting more important positive returns than bonds even if the bond is facing fewer negative returns than the Fund.

Surprisingly, the worst performing benchmark is the S&P 500 with an upside potential ratio of 0.5940. Firstly, they have a lower Upside potential than the Nikkei and the Euro Stoxx 50. Secondly, they have a lower Downside risk than the Nikkei and the Euro Stoxx 50. This is not logical because their Upside Potential ratio should be higher than that of the Euro Stoxx 50. When dividing the Upside Potential (0.201) from the S&P 500 by its Downside deviation (0.1044), we get 0.6677. Whereas, when calculating the Euro Stoxx 50 Upside Potential ratio, we get 0.5427 when using the monthly downside risk (0,0374). The Fund still has the best Upside Potential ratio with 1.37 when dividing the Upside Potential by the monthly downside risk.

3.10.1.7 Omega-Sharpe ratio

The Omega-Sharpe ratio can be calculated in two ways. First, the way we have expressed above. We are going to subtract the average portfolio return by the target return (in this case 0) and we are going to divide it by the negative Downside Potential. The second way we can calculate the Omega-Sharpe ratio is by subtracting 1 from the Omega ratio.

$$\textit{Omega}\text{-Sharpe ratio} \approx \Omega - 1$$

We see that in this case, we have used the second way. This calculation doesn't change anything in our analysis of the Omega ratio results and our analysis above can be used for this ratio also.

Table 43: The different Downside Risk ratios for the Years 2006–2021

```
> #table.DownsideRiskRatio(mydata[,1:19],MAR=0.01/12) #Report various Downside Risk Ratios (Sortino, UDR, Omega, et
c.)
> table.DownsideRiskRatio(mydata[,1:5],MAR=0) #Report various Downside Risk Ratios (Sortino, UDR, Omega, etc.)
                         NOBSGLIA  NKYTR Eurostoxx.50    SPX US10GOV
monthly downside risk      0.0124 0.0392       0.0374 0.0301  0.0113
Annualised downside risk   0.0431 0.1356       0.1297 0.1044  0.0393
Downside potential         0.0053 0.0181       0.0191 0.0131  0.0057
Omega                      3.1831 1.3339       1.0612 1.5339  1.7607
Sortino ratio              0.9384 0.1547       0.0313 0.2322  0.3794
Upside potential           0.0170 0.0242       0.0203 0.0201  0.0100
Upside potential ratio     1.1397 0.6686       0.6779 0.5940  1.1346
Omega-sharpe ratio         2.1831 0.3339       0.0612 0.5339  0.7607
```

Source: Rstudio (2021)

3.10.2 The different Downside Risk ratios for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

We immediately see that every downside risk is lower on a 10-Year period than on a 15-Year period. This is normal because we haven't taken into account the 2008–2009 financial crisis in the downside risks below. This crisis boosted the standard deviation and the downside risk of the Fund and its benchmarks. The Risk Parity portfolio also has a lower Downside risk (0.0335) than the Fund (0.0395) and the bond (0.0352). This means we created a less volatile portfolio than the 10-Year US bond because its standard deviation is lower and also its downside risk is lower than those two. Thus, the portfolio is doing what it was created to do.

We also see that the portfolio only has a worse Omega ratio than the Fund (3,2170) and the S&P 500 (1,8959). This is because the portfolio has the best downside potential of all the participants. This means that this portfolio, when it is suffering negative returns, is suffering the lowest negative returns of all the participants. Its low Downside Potential outweighs the higher upside potential from the 10-Year US bond when calculating the Omega ratio. This also means that the S&P 500 higher Upside potential outweighs the portfolio's lower Downside potential.

The Risk Parity portfolio has the second-best Sortino ratio behind the Fund. This is because the Fund has a better excess return than the Risk Parity portfolio and slightly worse downside risk. It is also important to note that every benchmark has a better Sortino ratio thanks to lower Downside risk and higher Excess return.

We see that the portfolio has the lowest Upside potential (0.0089) of all the participants. This is because, when it is presenting positive returns, those returns are usually low. We even see that the bond is outperforming them (0.0094) in that instance. When creating this portfolio, the main goal was to create a portfolio with the lowest risk possible using a batch of different assets. The main goal was never to have a high return but a low risk.

Again, like we explained above without using the Risk Parity portfolio, the Upside potential ratio is not calculated according to the formula seen with Bacon. We see that the S&P 500 has the worst Upside potential ratio but its upside potential is higher than that of the Euro Stoxx and its downside risk is lower than that of the Euro Stoxx. This makes the results of the Upside Potential ratio seen in Rstudio non-utilizable. When using the formula seen with Bacon, the Risk Parity Portfolio has the third-best Upside Potential ratio with 0.9175 while the 10-Year US Gov has a Upside potential ratio of 0.9215.

The Omega-Sharpe ratio isn't of use because it is the Omega ratio minus 1. So our analysis is no different from the Omega ratio.

Table 44: The different Downside Risk ratios for the Years 2010–2020

```
> table.DownsideRiskRatio(mydata[,1:6],MAR=0) #Report various Downside Risk Ratios (Sortino, UDR, Omega, etc.)
```

	NOBSGLIA	Risk.Parity.Portfolio	Eurostoxx.50	NKYTR	SPX	US10GOV
monthly downside risk	0.0114	0.0097	0.0344	0.0332	0.0253	0.0102
Annualised downside risk	0.0395	0.0335	0.1191	0.1151	0.0875	0.0352
Downside potential	0.0049	0.0047	0.0182	0.0154	0.0111	0.0052
Omega	3.2170	1.8946	1.1325	1.6603	1.8959	1.8125
Sortino ratio	0.9574	0.4344	0.0703	0.3070	0.3920	0.4134
Upside potential	0.0159	0.0089	0.0207	0.0256	0.0210	0.0094
Upside potential ratio	1.1673	0.9038	0.7688	0.7585	0.7293	1.1796
Omega-sharpe ratio	2.2170	0.8946	0.1325	0.6603	0.8959	0.8125

Source: Rstudio (2021)

3.11 Drawdowns

We are now going to be looking at the 5 most severe drawdowns for each participant. We are going to look at the severity, the length and the time of recovery of the 5 most severe drawdowns for each participant. We will also see when those drawdowns occurred. We will start this analysis with the Fund and we will analyse each benchmark accordingly.

To remind ourselves, we are going to explain every term in a couple of statements. The first term that we are going to explain is the Depth of the Drawdown. This looks at how much a price changed when it went from its peak to its lowest point. The second term that we are going to explain is the Length of the Drawdown. The Length of the drawdown is equal to the sum of the “To through” and “Recovery”. The Length of a drawdown means how long it took the investment to go from its peak to its lowest point and back to its initial peak. The third term that we are going to explain is the “To trough” term. This term explains how long it took, in months, to hit its lowest point and go through it. The last erm we are going to explain is the Recovery term. This term explains how much time it took an investment to hit its initial price from its lowest point.

3.11.1 The 5 most severe Drawdowns for the Fund and its benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021

3.11.1.1 The 5 most severe Drawdowns for the Norwegian Sovereign Wealth Fund

We will start our analysis with the Fund. We will see a recurring trend with every participant when looking at their 5 most severe Drawdowns. Every participant suffered their largest and longest Drawdown because of the 2008 Financial crisis. Also, we noticed that the indexes or funds reached their lowest point on 28 February 2009. It is absolutely normal that the 10-Year US Treasuries didn’t suffer their lowest point at that moment because bonds usually go up when stocks are crashing. We will usually see below that when the bond is reaching its lowest point, stocks are in full recovery.

Let’s go back to the Fund. We see that the Fund suffered its largest drawdown at the end of the 2008–2009 Financial crisis. The Fund lost 10% of its value from 01/01/09 to 28/02/2009. This is not surprising because we explained above that between 2007 and 2009 the Fund invested an additional 1.1 Trillion Kroner in Equities. This means their share of Equity investments was increasing at the start of 2009 and would suffer important losses for 2 months. We cannot forget that the Fund increased their Equity share of Total investments from 40% to 60% during that time. The Fund then needed 3 months of recovery until it reached its initial peak again. This is a very fast recovery and highlights the exceptional performances of the Fund. Our Theory is that the Fund increased their Equity investments right after the lowest point at the end of February 2009 and continued increasing its Equity share during the Year 2009 until it reached 60% at the end of the Year.

This is also a reason why the Fund didn’t suffer large and deep drawdowns during the 2008 Financial crisis like the 3 indexes seen below. The Fund had invested 60% of their portfolio in bonds and we have seen that bonds thrive when stocks are suffering. We know that they chose a change of strategy at the end of 2007 and this change of strategy ended at the end of 2009. This is why the two largest drawdowns of the Fund are quick (5 months and 9 months) and not deep (-10% and -7%). We could argue that their change of strategy wasn’t arbitrary.

Table 45: The 5 most severe Drawdowns for the Norwegian Sovereign Wealth Fund

```
> table.Drawdowns(mydata[,1,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
        From     Trough         To Depth Length To Trough Recovery
1 2009-01-31 2009-02-28 2009-05-31 -0.10      5         2        3
2 2007-12-31 2008-03-31 2008-08-31 -0.07      9         4        5
3 2016-01-31 2016-04-30 2016-12-31 -0.06     12         4        8
4 2018-09-30 2018-10-31 2019-02-28 -0.05      6         2        4
5 2011-06-30 2011-08-31 2011-11-30 -0.05      6         3        3
```

Source: Rstudio (2021)

3.11.1.2 The 5 most severe Drawdowns for the Nikkei

This drawdown table will help to support a lot of arguments we have presented during our thesis. Firstly, we see that the Nikkei suffered its largest drawdown for 78 months between July 2007 and December 2013. They lost 57% of their value between its peak in 2007 and its lowest point in February 2009. They lost 57% in 20 months and it took 58 months before they returned to their initial peak. This supports our argument that the Nikkei suffered a lot from the 2008 Financial crisis and suffered from the aftermath for a long time (58 months to be precise).

The second-largest drawdown the Nikkei suffered was due to the 2015–2016 stock market selloff. This was an extremely turbulent period where investors were selling shares globally due to a variety of reasons we will list below:

- Slow Growth in China’s GDP
- Crash in Petroleum prices
- The end of quantitative easing from the United States in October 2014
- June 2015’s Greek debt default
- Rise in Bonds rates in the beginning of 2016

- The Brexit vote after the 2016 UK EU membership referendum

We will see below that the S&P 500 also suffered some difficulties between June 2015 and July 2016 and this will support a theory we have presented before. Like we explained before, the Nikkei usually follows the movement of the American stock market and the Aftermath of those slumps are usually damaging the Nikkei more importantly than the S&P 500. We see that the second-largest drawdown happened between August 2015 and June 2017. This is a year later than the end of the drawdown that the American stock market suffered. The Nikkei's drawdown lasted 23 months and the Nikkei's price fell from 23%. In parallel, the S&P 500 only lost 9% of its value between 2015 and 2016.

Table 46: The 5 most severe Drawdowns for the Nikkei

```
> table.Drawdowns(mydata[,2,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
        From     Trough         To Depth Length To Trough Recovery
1 2007-07-31 2009-02-28 2013-12-31 -0.57     78        20       58
2 2015-08-31 2016-06-30 2017-06-30 -0.23     23        11       12
3 2020-01-31 2020-03-31 2020-11-30 -0.19     11         3        8
4 2018-10-31 2018-12-31 2019-12-31 -0.17     15         3       12
5 2014-01-31 2014-04-30 2014-09-30 -0.12      9         4        5
```

Source: Rstudio (2021)

3.11.1.3 The 5 most severe Drawdowns for the Euro Stoxx 50

In the table below, we see something we have talked about before. The Euro Stoxx 50 hasn't hit its pre-2008 levels due to the important aftermath of the 2008 Financial crisis. We see that the drawdown started in June 2007 and hasn't ended. They lost 56% of its value between June 2007 and February 2009. The index suffered its lowest point, like all the other indexes, at the end of February 2009. It took them 21 months, since the start of the drawdown, to reach their lowest point. We see that their recovery is NA because they haven't been able to add 56% to its value since February 2009. This means the Euro Stoxx 50 has been in Drawdown for 165 weeks between June 2007 and January 2021.

This slow recovery and underperformance is due to a couple factors. Morgan Stanley explained it well in its equity strategy note:" *A significant driver of European equity underperformance over the last decade has been its sector mix, with the region's stock market skewed away from Technology and towards more "legacy" industries where the underlying growth trends are more muted*[23]*"* (Ponthus & Masoni, 2020). This explains perfectly how the American stock

[23] Ponthus, J., Masoni, D. (2020, 2 September). *Tech sector tightens its grip over Europe's stock markets.* Reuters. https://www.reuters.com/article/us-markets-stocks-tech-idUSKBN25T1NN

market always found ways to outperform their peers. US Tech firms boosted the S&P 500 and helped it reach record levels in 2020–2021. Those investments from the Norwegian SwF in Us Tech have also proven to be fruitful because they also reached record levels in 2020–2021.

Table 47: The 5 most severe Drawdowns for the Euro Stoxx 50

```
> table.Drawdowns(mydata[,3,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
        From     Trough         To Depth Length To Trough Recovery
1 2007-06-30 2009-02-28       <NA> -0.56    165        21       NA
2 2006-04-30 2006-05-31 2006-09-30 -0.06      6         2        4
3 2007-02-28 2007-02-28 2007-03-31 -0.02      2         1        1
4 2006-11-30 2006-11-30 2006-12-31  0.00      2         1        1
Warning message:
In table.Drawdowns(mydata[, 3, drop = FALSE], 5, 2) :
  Only 4 available in the data.
```

Source: Rstudio (2021)

3.11.1.4 The 5 most severe Drawdowns for the S&P 500

The S&P 500 suffered a large drawdown between November 2007 and March 2013. This drawdown lasted 65 months and it took 16 months for the index to hit its lowest point in February 2009. We see that the index needed a long time (49 months) to hit its initial peak after reaching its lowest point. This recovery is still shorter than the recovery needed by the Nikkei (58 months) before they were able to hit their initial peak again. The S&P 500 also lost smaller value from its peak than the Nikkei (53% and 57%) when they reached their respective lows.

Their second-largest drawdown happened in January 2020. This drawdown would prove to be the start of a huge bull market where American stocks would not stop their growth. This drawdown lasted 7 months and would lead to a 20% loss of value from its initial peak in January 2020. They would bottom through in March 2020 and reach its initial peak in July 2020 when covid-19 regulations settled down. This bull market was and is fueled "*by the largest federal government stimulus ever, historic support from the Federal Reserve and optimism about how quickly the economy is likely to bounce back next year as coronavirus vaccines become widely distributed*[24]"(Shaban & Long, 2020).

[24] Shaban, H., Long, H. (2020, 31 December). *The stock market is ending 2020 at record highs, even as the virus surges and millions go hungry*. The Washington Post. https://www.washingtonpost.com/gdpr-consent/?next_url=https%3a%2f%2fwww.washingtonpost.com%2fbusiness%2f2020%2f12%2f31%2fstock-market-record-2020%2f

Table 48: The 5 most severe Drawdowns for the S&P 500

```
> table.Drawdowns(mydata[,4,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
        From     Trough         To Depth Length To Trough Recovery
1 2007-11-30 2009-02-28 2013-03-31 -0.53     65        16       49
2 2020-01-31 2020-03-31 2020-07-31 -0.20      7         3        4
3 2018-10-31 2018-12-31 2019-04-30 -0.14      7         3        4
4 2015-06-30 2015-09-30 2016-07-31 -0.09     14         4       10
5 2020-09-30 2020-10-31 2020-11-30 -0.07      3         2        1
```

Source: Rstudio (2021)

3.11.1.5 The 5 most severe Drawdowns for the 10-Year US Treasuries

The information that we are going to see below is particular because we are going to watch the different drawdowns a bond suffered. Usually their markets move in opposite ways of stock markets. We immediately see that each drawdown is substantial in its length but not in its size. This is normal because bond rates are very stable and losses are scarce. We see that their largest drawdown was 10% between January 2009 and August 2010. This is because stocks started to go up from February 2009 and, thus, bond rates started to go down at the same time. Their drawdown length was 20 months and they needed 13 months to recover from its lowest point.

Another interesting information we see in the table below, is the second-largest drawdown. We see that the bond suffered a drawdown that lasted 35 months and ended in June 2019. This drawdown was very slow because the bond only bottomed after 26 months and only needed 7 months to recover from all its 9% of losses. It's interesting to see that the bond reached its initial peak in June 2019 because, little did we know that the bond would need another 2 years before hitting that June 2019 peak and that the Coronavirus would plummet the bond rates for the ¾ of 2020 at around 0.60%.

Table 49: The 5 most severe Drawdowns for the 10-Year US Treasuries

```
> table.Drawdowns(mydata[,5,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
        From     Trough         To Depth Length To Trough Recovery
1 2009-01-31 2009-07-31 2010-08-31 -0.10     20         7       13
2 2016-08-31 2018-09-30 2019-06-30 -0.09     35        26        9
3 2013-05-31 2013-12-31 2014-11-30 -0.09     19         8       11
4 2010-10-31 2010-12-31 2011-07-31 -0.06     10         3        7
5 2015-02-28 2015-06-30 2016-02-29 -0.05     13         5        8
```

Source: Rstudio (2021)

3.11.2 The 5 most severe Drawdowns for the Risk Parity portfolio for the Years 2010–2020

We need to bear in mind that the drawdowns below aren't influenced by the 2008–2009 Financial crisis. We have seen that the crisis created long and large drawdowns for all the participants above between 2008 and 2010.

The Risk Parity portfolio suffered its longest and largest drawdown between 28 February 2015 and 30 June 2016. This drawdown lasted 17 months and the portfolio lost 7% of its value in 11 months. The portfolio, then, needed a 6-month recovery to hit its pre-drawdown peak from 28 February 2015. Surprisingly, between 2010–2020, the Fund suffered lower and shorter drawdowns than the portfolio. This shows, again, how the Fund is incredibly resilient and that high losses are non-existent for the Fund. Its largest drawdown between 2010 and 2020 is a 6% loss in value and a drawdown length of 12 months. The Fund only needed 8 months to recover from its lowest point to its initial peak. This drawdown occurred between January 2016 and December 2016. We explained above, why the years 2015–2016 were difficult for the different benchmarks.

Table 50: The 5 most severe Drawdowns for the Risk Parity Portfolio

```
> table.Drawdowns(mydata[,2,drop=FALSE], 5, 2) #Identify the 5 most sev
utiny
```

	From	Trough	To	Depth	Length	To Trough	Recovery
1	2015-02-28	2015-12-31	2016-06-30	-0.07	17	11	6
2	2013-05-31	2013-06-30	2014-05-31	-0.07	13	2	11
3	2016-08-31	2016-11-30	2017-08-31	-0.05	13	4	9
4	2018-02-28	2018-10-31	2019-03-31	-0.05	14	9	5
5	2020-02-29	2020-03-31	2020-05-31	-0.05	4	2	2

Source: Rstudio (2021)

3.12 Drawdowns ratios

After analyse the 5 most severe drawdowns for each benchmark, we are going to see if the different drawdown ratios are in line or in contradiction with our theories. We will, again, explain quickly how each ratio is calculated. We also used 0 as our risk-free rate when calculating each ratio. This will help us understand better all the variables in each ratio and how they impacted the results. We will see recurring trends when watching the results of the different benchmarks. Firstly, the Fund has consistently the best results when analysing the different drawdown ratios. Secondly, the Euro Stoxx 50 will mostly present negative Drawdown ratios due to their cumulative negative return.

3.12.1 The different Drawdown ratios for the Fund and its benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021

3.12.1.1 Sterling ratio

The Sterling ratio is calculated by dividing the Annual Portfolio Return by the Average Largest Drawdown. We explained in the performance measurements chapter how the Average Largest Drawdown is calculated.

Without a surprise, the Fund has the best Sterling ratio with 0.7103. This is because they have a big annual return and their 3 or 5 largest drawdowns create an average smaller than all the other benchmarks. For example, we take the three largest drawdowns of the Fund. This creates an average of 7.66% per year whereas when calculating the Average Largest Drawdown for the 10-Year US Gov we get 9.33% annually. Even with a low return annually, the 10-Year US Gov manages to present the second-best Sterling ratio by a margin (2x times better than the S&P 500 SR). The fact that the 10-Year US Gov is outperforming the indexes by such a margin is due to the high Average Largest Drawdown that the indexes are presenting.

For example, the S&P 500 has an Average Largest Drawdown of 29%. This average can be considered high when looking at the Fund or the 10-Year US Gov The Nikkei has an average largest drawdown of 33% when looking at their last 3 largest drawdowns. We see that the S&P 500 has a better Sterling ratio than the Nikkei (0.1198 vs 0.0820). This is largely due to their far better annual return because the difference between both Average Largest Drawdowns is negligible.

It’s difficult to make an Average Largest Drawdown with the Euro Stoxx 50 because they have been in drawdown since the 2008 financial crisis. Moreover, we see that their Sterling ratio is negative (0.0028). This is because they have had a negative cumulative return since 2006 which means that their average annual return is negative.

3.12.1.2 Calmar ratio

The Calmar ratio is calculated by dividing the average annual portfolio return by the Maximum Drawdown.

Again, the Fund has the best Calmar ratio with 1,3888. This is because of their large average return and their very low Maximum Drawdown. They have the same Maximum Drawdown as the 10-Year Us Gov with 10%. Because they have the same denominator, we can see how big the difference is between both numerators. The Fund presents a Calmar ratio of 1,3888 whereas the 10-Year US Gov presents a Calmar ratio of 0.4844.

The other benchmarks have very high Maximum Drawdowns. The Nikkei has a Maximum Drawdown of 57%, the Euro Stoxx 50 has one of 56% and the S&P 500 has a Maximum Drawdown of 53%. This is shown in the Calmar ratio because the S&P 500 outperforms the Nikkei more with this ratio than with the Sterling ratio.

Again, the Euro Stoxx 50 has a negative ratio because the index has a cumulative negative return.

3.12.1.3 Burke ratio

The Burke ratio is very similar to the Sterling ratio. The big difference between both ratios is that the Burke ratio square roots the largest drawdowns to increase the impact of those drawdowns.

This creates very similar results to the Sterling ratio results. We also noticed that every result is slightly smaller than the Sterling ratio results. This is because the denominator is square rooted in the Burke ratio.

The S&P 500 and Nikkei are impacted more greatly than the other benchmarks because they have such a Large Average Largest Drawdown. We know that the Nikkei has a larger average than the S&P 500 (33% vs 29%). This is probably why, when square rooting this average, the S&P 500 presents even better results than with the Sterling ratio, against the Nikkei.

Again, the Euro Stoxx 50 is presenting a negative result due to their average negative return.

3.12.1.4 Pain index

The Pain index merges 3 parameters of loss when calculated. We explained it above, the pain index is calculated by synerging the depth, frequency and duration of losses.

In this case, the lower the Pain Index, the better the result for the benchmark. We see that the Fund has the best Pain index (0.0116). When thinking about it, the Fund has the lowest, shortest and fewest drawdowns of all participants. This makes it logical that it has the lowest Pain Index.

The S&P 500 is closer, with its Pain index, to the Fund and 10-Year US Gov than to the two other benchmarks. This is because the S&P 500 has deep, but quick, drawdowns in comparison with other indexes.

The Euro Stoxx 50 has probably the highest Pain index (0.2368) because it has the second-most drawdowns, the longest drawdown by far and slightly the second deepest drawdown of all participants. This never-ending drawdown since 2008 has probably been instrumental in creating the highest Pain index of all participants.

3.12.1.5 Ulcer index

Like Carl R.Bacon explained, the Ulcer index *"is similar to the drawdown deviation with the exception that the impact of time 'under water' is combined with the depth of drawdown by selecting the negative return for each period below the previous peak or high water mark.*[25]*"* (Bacon, 2013). This means that indexes with long and deep drawdowns are going to be penalised more because those drawdowns will be squared.

This is also why the participant with the best Ulcer index is the Fund (0.0224) and the participant with the worst Ulcer index is the Euro Stoxx 50 (0.3002). This is because the Fund has relatively shallow drawdowns. The Fund combines those shallow drawdowns with quick recoveries which makes them the best performing participant when calculating the Ulcer Index. The 10-Year US Gov has a slightly worse Ulcer index than the Fund (0.0442). This is not due to the depth of their drawdowns (they have the same depth as the Fund). This is due to the duration of its drawdowns. We have seen that the Fund averages, of its 3 largest drawdowns in depth, a duration of 8.67 weeks for each drawdown to go from peak to valley and back to peak.

[25] Bacon, C. R. (2013). *Practical risk-adjusted performance measurement*. Wiley. P99

Whereas the 10-Year US Gov averages a duration of 24.67 weeks to go from peak to valley and to its peak again. This is where the Fund vastly outperforms the US Bond.

We know that the worst performing index, when calculating the drawdown ratios, is the Euro Stoxx 50. They have the highest Ulcer index of all participants (0.3002). This is, without a doubt, due to their never ending drawdown and recovery since the 2008 Financial crisis. We know the Nikkei suffered a slightly worse depth of drawdown than the Euro Stoxx 50 during the 2008 Financial crisis but the Nikkei recovered from it whereas the Euro Stoxx 50 hasn't recovered from the initial drawdown.

3.12.1.6 Pain ratio

The pain ratio is calculated by dividing the average annual portfolio return by the Pain index. This helps us see the impact of the return on the Pain Index. We will see that the Euro Stoxx 50 will have a negative Pain ratio because its return is negative. We will also see that the Fund has the highest Pain ratio with a very large margin because their average annual return is so big and their Pain index is so small.

This is also a big difference between the 10-Year US Gov and the Fund. The Fund is presenting exponential returns whereas the bond is presenting very low returns. The Fund isn't combining low returns with long drawdowns like the US Bond. The Fund is combining high returns with short and shallow drawdowns which leads to very good drawdown ratios. We should not forget that the US Bond is presenting good Drawdown ratios, we are just witnessing exceptional Drawdown ratios from the Fund.

Our analysis about the three other benchmarks hasn't changed much. The S&P 500 has a better return than the US bond but its Pain ratio is impacted by the much bigger Pain index in comparison to the Bonds Pain Index. This is because the index suffered much deeper drawdowns and slightly longer drawdowns than the bond.

3.12.1.7 Martin ratio

The Martin ratio is calculated by dividing the average annual portfolio return by the Ulcer index. Like with the Pain ratio, we are going to be able to see the impact of the return on a risk measurement. Again, the Euro Stoxx 50 has a negative Martin ratio because the index has a cumulative negative return.

The Fund has the highest Martin ratio of all participants. This is because they have the highest return of all participants and have the lowest Ulcer index. Our analysis of the Martin ratio will be very similar to our analysis of the Pain ratio and the Ulcer index.

Our main takeaways are that the Fund is the big winner and that the Euro Stoxx 50 is the big loser when calculating the different Drawdown ratios. Most of those ratios are based on the participants' return and the Euro Stoxx 50 has a negative return which makes it the worst performing benchmark of all the participants. Moreover, we have seen that even when the return wasn't a needed parameter, the index managed the best worst performing benchmark. This was largely due to their unrecovered drawdown since the 2008 financial crisis. Lastly, we have seen that the US bond has been outperformed by the Fund when looking at all the drawdown ratios. This was very surprising because bonds are widely considered to be the safest investment possible and losses are scarce. This highlights the exceptional standards the Fund has reached since its creation. The Fund is returning better than indexes while being a safer investment than the most well-known bond.

Table 51: Drawdown ratios for the Years 2006–2021

```
> table.DrawdownsRatio(mydata[,1:5], Rf= 0) #Report various Drawdowns Ratios (Sterling, Burke, Calmar, etc.)
               NOBSGLIA  NKYTR Eurostoxx.50    SPX US10GOV
Sterling ratio   0.7103 0.0820      -0.0028 0.1198  0.2464
Calmar ratio     1.3888 0.0963      -0.0033 0.1426  0.4844
Burke ratio      0.7070 0.0742      -0.0025 0.1290  0.2449
Pain index       0.0116 0.1868       0.2368 0.0837  0.0321
Ulcer index      0.0224 0.2785       0.3002 0.1616  0.0442
Pain ratio      12.5011 0.2949      -0.0077 0.8952  1.5652
Martin ratio     6.4783 0.1978      -0.0061 0.4639  1.1351
> #Average of Daily 10YR YTM = 0
```

Source: Rstudio (2021)

3.12.2 The different Drawdown ratios for the Risk Parity Portfolio for the Years 2010–2020

We immediately see that all the ratios are better for all the participants. This is something we have seen over and over again. This is because the 2008–2009 Financial crisis doesn't figure in the ratios below. We are only seeing the recovery from this crisis for the Fund and the different benchmarks.

We are going to start our analysis of the Drawdown ratios for the Risk Parity portfolio, with the Sterling ratio. The portfolio has a worse Sterling ratio (0.2962) than the Fund (0.8244), Nikkei (0.3396) and S&P 500 (0.3822). This comes without a surprise because the portfolio has a worse annual return than those 3 participants. Moreover, its largest Average Drawdown is much lower than that of the indexes but the indexes annual return outweighs the difference in Average Drawdowns. This is because the drawdowns are much smaller for all the participants. This creates a higher reward for higher returns.

For the Calmar ratio, it absolutely makes sense that the Risk Parity portfolio is the second-best performing participant. The portfolio has much lower Maximum Drawdown than that of his peers. The portfolio has a Maximum Drawdown of 0.07 whereas the US gov has a Maximum Drawdown of 0,09, the S&P 500 of 0,20, the Nikkei of 0.23 and the Euro Stoxx 50 of 0.30. The Fund is vastly outperforming its peers (2,0949) because it has a lower Maximum Drawdown (0.06) than the portfolio and a higher return than all the benchmarks.

With the Burke ratio, the Fund is again outperforming its peers (0.8930). This is because, in this ratio, we put additional weight on large drawdowns. This is also the reason why the 10-Year US bond is outperforming the S&P 500 for this ratio. The S&P 500 has a much larger drawdown than the bond. The Risk Parity portfolio is the second-best performing participant with 0.3572 thanks to its low drawdowns.

The Fund is, like usual, the best performing participant when looking at the Pain index. It is interesting to see that the Pain index is much better for the Fund (0.0101) and the portfolio (0.0158) than for the other benchmarks. This is because, like we explained above, the Pain index is a blend of 3 parameters of loss and the Euro Stoxx 50 (0.0897) and the Nikkei (0.0658) have proven to be performing badly with the 3 parameters. Both suffer long and big drawdowns between 2010–2020.

The Ulcer index shows us similar results than the Pain index. This is because it is calculated on 2 similar parameters than the Pain index (depth and longevity of losses). The portfolio is again the second-best performing participant with 0.0246.

The Pain ratio is interesting because it blends return and the Pain index. This helps us understand which investment is worth the risk. We see that the S&P 500 has the second-best Pain ratio behind the Fund. This shows that, when looking at the depth, frequency and longevity of losses and the returns of the respective benchmarks, the S&P 500 is the best investment behind the Fund.

This point of view is confirmed when calculating the Martin ratio, we see that the S&P 500 is the best investment behind the Fund over a 10-Year period. The Risk Parity portfolio is the third-best investment on a risk-adjusted basis. We know that this portfolio has very low risk of high losses but it also has a very low probability of high returns.

Table 52: Drawdown ratios with the Risk Parity Portfolio for the Years 2010–2020

```
> table.DrawdownsRatio(mydata[,1:6], Rf= 0) #Report various Drawdowns Ratios (Sterling, Burke, Calmar, etc.)
```

	NOBSGLIA	Risk.Parity.Portfolio	Eurostoxx.50	NKYTR	SPX	US10GOV
Sterling ratio	0.8244	0.2962	0.0368	0.3396	0.3822	0.2548
Calmar ratio	2.0949	0.7274	0.0492	0.4879	0.5733	0.5265
Burke ratio	0.8930	0.3572	0.0253	0.2306	0.2972	0.3037
Pain index	0.0101	0.0158	0.0897	0.0658	0.0238	0.0302
Ulcer index	0.0191	0.0246	0.1195	0.0936	0.0448	0.0416
Pain ratio	13.4422	3.1719	0.1627	1.6979	4.8264	1.6350
Martin ratio	7.1116	2.0284	0.1221	1.1941	2.5572	1.1865

Source: Rstudio (2021)

3.13 Value at risk

We will keep our explanations quick and brief because we already explained thoroughly the VaR and its 3 methods in the performance measurement chapter. In this chapter we will see how well the Fund and its benchmarks perform when using the VaR as a risk measure. The VaR calculates, for a certain level of confidence and when the markets are trading under normal conditions, what the maximum expected losses can be during a certain Time Horizon. We are going to use 3 different methods to calculate the VaR to see how our results change depending on the method used. This risk measure will help us see how much risk some investments are bearing and if certain investments are as safe as they look like. We will always use a level of confidence of 99%. This means there is only a 1% chance that the actual loss is higher than the expected loss.

3.13.1 Value at risk for the Fund and its benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021

3.13.1.1 Var with the Gaussian method

The Gaussian method, also called variance-covariance method, assumes that equity returns are normally distributed. This method estimates the expected return and the standard deviation to create a normal distribution graph. We will compare the normal distribution against actual returns to find the worst losses and highest gains.

The US Bond is presenting the lowest VaR from all the participants by a negligible margin when using the Gaussian method (-0.04444). The Fund has a very close VaR when using the Gaussian method (-0.04457). This method estimates that the Fund and the bond have very similar standard deviations and expected returns. This method uses probability, based on past price movements between 2006 and 2021, to calculate the maximum expected losses the participants will suffer at a 99% level of confidence. The method judged that the Fund and the bond would suffer very similar maximum expected losses under normal conditions because they have very similar standard deviations and because their price movements are similar when suffering negative returns.

This method also judged that the Nikkei would be the worst performing benchmark (-0.12319). This is normal because the Nikkei has the highest standard deviation and it suffered the largest loss of all the benchmarks during the 2008 financial crisis. Furthermore, its price movements in the past have been very volatile which could have led the Gaussian method to think that this index would suffer the largest expected loss of all benchmarks.

The S&P 500 is performing much better under this method than under the historical method. This is because the S&P 500 has a much lower standard deviation than the Nikkei and the Euro Stoxx 50. Furthermore, its past price movements have been more upward scaling than downwards scaling. This is why under normal conditions, the Gaussian method calculated that the S&P 500 would suffer much lower expected losses than the Nikkei and the Euro Stoxx 50.

3.13.1.2 Var with the Historical method

The Historical method is calculated by using past returns and ranking them from worst to best. It assumes future risks can be calculated by looking at past results. We assume that, for a level of confidence of 99%, that the bond will have the lowest expected losses from all the participants. This agrees with what we have seen in the Drawdown chapter. The US bond had a maximum actual loss of 10% between 2006 and 2021 when looking at the historical results. This was very similar to the Fund and this is probably a reason why the Fund is presenting a similar VaR to the US Bond (-0.04867 vs -0.04550). The bond is probably having a slightly better VaR because the Fund suffered negative returns more times than the bond according to the Downside risk seen in the Drawdown chapter.

We see that the Euro Stoxx 50 is the worst performing benchmark when we use the historical method (-0.139744). They expect the index to suffer the highest loss of all the benchmarks with 99% confidence. This is probably because this index never recovered from their initial loss from 2008 whereas the Nikkei found a way to recover and even boasts a higher cumulative return than the US Bond. The VaR probably also deemed that, when the market is trading under normal conditions, the Euro Stoxx 50 is the most susceptible to make larger losses than its peers when looking at past data.

Surprisingly, the Nikkei and the S&P 500 have very similar VaRs when using the Historical method (-0.11972 vs -0.11296). This is probably because the Nikkei is always following the American stock market. This means that, under normal conditions, when the market is suffering losses, the Nikkei will probably suffer the same losses and, mostly, those losses will be slightly higher. This is something we see between those two VaRs because the Nikkei's VaR is slightly worse than the S&P 500's.

3.13.1.3 Var with the Modified method

The modified method will subject the normal distribution to skewness and kurtosis corrections. Beware, this modified method can only be used if the confidence level is above 95.84%? Otherwise, the results can be deemed “unusable”.

The modified method deems that the bond would suffer the lowest expected losses of all the participants (-0.04242). The modified method generated bigger margins between the participants and also expected larger losses for each participant. We are seeing similar results than the Gaussian method. This is because the modified method is based on the normal distribution. The big difference is that we add skewness and kurtosis corrections to boost the expected losses. This means big losses are highlighted and boosted and, thus, higher expected losses are calculated.

Table 53: Value at risk calculated using the 3 different methods for the Years 2006–2021

```
> VaR(mydata[,1:5], p=.99,method="gaussian")
          NOBSGLIA        NKYTR   Eurostoxx.50
VaR -0.04457748968 -0.123193891 -0.1176687179
               SPX       US10GOV
VaR -0.09404544901 -0.0444423098
> VaR(mydata[,1:5], p=.99,method="historical")
          NOBSGLIA         NKYTR   Eurostoxx.50
VaR -0.04867594966 -0.1197268052 -0.1397445673
              SPX        US10GOV
VaR -0.1129688441 -0.04550058463
> VaR(mydata[,1:5], p=.99,method="modified")
          NOBSGLIA         NKYTR   Eurostoxx.50
VaR -0.05159633071 -0.1593520402 -0.1413387408
              SPX        US10GOV
VaR -0.1248739222 -0.04242369577
>
```

Source: Rstudio (2021)

3.13.2 Value-at-Risk for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

The Risk Parity portfolio has, for a given level of confidence of 99%, the lowest VaR of all the participants when using the Gaussian method. Whereas, when using the Historical and Modified method, the bond has the lowest VaR of all the participants for a given level of confidence of 99%.

In the Gaussian method, the Risk Parity portfolio has the lowest VaR because a normal distribution is created based on its standard deviation and expected return. This method uses probability based on past price movements to calculate the maximum expected loss. This method judged, after calculating the standard deviation of the different price movements, what the expected maximum loss was.

In the Historical method and Modified method, the bond has a lower VaR than the Risk Parity portfolio. The historical method, in contrast to the Gaussian method, is a non-parametric method. This means no assumptions are made about the past returns whereas assumptions are made when calculating the Gaussian method. The VaR is directly calculated from past returns in this instance. We have seen that the bond has more positive returns than the portfolio and that their maximum drawdowns are very similar. This is probably the reason why the bond has a better VaR than the portfolio.

Because the modified method uses Kurtosis and skewness in the normal distribution, we understand why the bond has a lower VaR. We know for a matter of fact that greater negative kurtosis and skewness increase the Value-at-risk. The bond has a positive kurtosis whereas the portfolio has a negative kurtosis. Moreover, the bond has positive skewness whereas the portfolio has negative skewness.

Table 54: Value at risk calculated using the 3 different methods for the Years 2010–2020

```
> VaR(mydata[,1:6], p=.99,method="gaussian")
          NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50        NKYTR            SPX       US10GOV
VaR -0.04119792743         -0.03394039522 -0.1115593271 -0.109697845 -0.08392442272 -0.0394974724
> VaR(mydata[,1:6], p=.99,method="historical")
         NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50       NKYTR         SPX        US10GOV
VaR -0.0443298544         -0.03812325596 -0.1232056242 -0.10279006 -0.08932368 -0.03666078893
> VaR(mydata[,1:6], p=.99,method="modified")
          NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50         NKYTR            SPX        US10GOV
VaR -0.04549543155         -0.03572072515 -0.1293572875 -0.1192664538 -0.1011540247 -0.03547371137
```

Source: Rstudio (2021)

3.14 Expected Shortfall

The Expected Shortfall, also known as the conditional Value at Risk, is a risk measure that helps us complement the VaR's shortcomings. Like we explained before, the Expected Shortfall helps us calculate the expected loss when the maximum expected loss threshold from the VaR is crossed. The Expected Shortfall is particularly interesting for volatile investments because the VaR Threshold is usually exceeded whereas, with safer investments, the VaR threshold is rarely exceeded. We already explained enough about the Expected Shortfall in the performance measurement chapter.

Like with the VaR, we are going to calculate the Expected Shortfall, with a 99% level of confidence, using 3 different methods: the Gaussian method, Historical method and Modified method. The closer the Expected Shortfall to 0, the less likely the investment will suffer high losses after crossing the VaR threshold. We will not explain again how those methods are calculated because we already did that above and in the performance measurement chapter.

3.14.1 Expected Shortfall for the Fund and its benchmarks excluding the Risk Parity Portfolio for the Years 2006–2021

3.14.1.1 Expected Shortfall with the Gaussian method

We see that the results are very similar to the results seen above when calculating the VaR using the Gaussian method. Again, the Fund and the 10-Year US Gov are very similar in their Expected Shortfall. Because the Gaussian method is based on expected standard deviations and expected returns, the results will be very similar to the results seen above. The Nikkei is expected to have the highest losses when the VaR threshold is crossed because it has the highest expected standard deviation and the second lowest expected return.

3.14.1.2 Expected Shortfall with the Historical method

The results are slightly different between the VaR and the Expected Shortfall when using the Historical method. The Nikkei is the investment that is the most susceptible to suffer the highest losses once the VaR threshold is crossed whereas, when looking at the VaR results using the Historical method, the Euro Stoxx is the most susceptible to suffer the highest expected loss from all benchmarks. This means that, for a level of confidence of 99%, the Eurostoxx would suffer the highest expected loss but, when those expected losses threshold crossed, the Nikkei would suffer the highest expected losses using the Historical method. Those results are based on past results and ranked from worst to best.

Again, the bond slightly outperforms the Fund with its Expected Shortfall. Based on past results and with a 99% level of confidence, the bond will have a lower maximum expected loss than the Fund and, even if its VaR threshold is crossed, it will have a lower maximum expected loss than the Fund.

3.14.1.3 Expected Shortfall with the Modified method

The modified method will have similar results than the Gaussian method because it is based on a normal distribution. The only difference is that we add skewness and kurtosis to the distribution. This will generate higher expected losses for the different participants but this will not change the results significantly in their order.

Table 55: Expected Shortfall calculated using the 3 different methods for the Years 2006–2021

```
> ES(mydata[,1:5], p=.99,method="gaussian")
        NOBSGLIA        NKYTR Eurostoxx.50          SPX
ES -0.05277014122 -0.1420214281 -0.134979431 -0.1087643191
         US10GOV
ES -0.05154296648
> ES(mydata[,1:5], p=.99,method="historical")
        NOBSGLIA         NKYTR  Eurostoxx.50          SPX
ES -0.0651365114 -0.1854671591 -0.1549535835 -0.1472719276
         US10GOV
ES -0.05217515919
> ES(mydata[,1:5], p=.99,method="modified")
        NOBSGLIA         NKYTR  Eurostoxx.50          SPX
ES -0.0652831027 -0.1969919964 -0.1756973431 -0.1525684675
         US10GOV
ES -0.05293113108
```

Source: Rstudio (2021)

3.14.2 Expected Shortfall for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

In this case, the Expected Shortfall is lower for the Risk Parity portfolio than for the bond when using the Gaussian method and the Historical method. This means that when using those two methods, for a level of confidence of 99%, the portfolio has the lowest maximum expected loss when the VaR threshold is crossed.

This result is logical because the portfolio has never suffered larger drawdowns than the bond. Thus, it is normal that, when looking at past results with assumptions or not, the portfolio will never have higher maximum expected losses, when the VaR threshold is crossed, than the bond. Unsurprisingly, we see that the Nikkei is performing better on a 10-Year period than on a 15-

Year period. We have seen before that the 2008–2009 financial crisis influenced a lot the standard deviations of the different benchmarks and the Nikkei in particular. Whereas, in a 15-year period, the Nikkei had the largest negative ES. In a 10-Year period, the Nikkei has a lower ES than the Euro Stoxx 50 thanks to its resurgence from that said crisis.

Lastly, the modified method still considers the Bond as the participant with the lowest ES. Like we said above, this is due to the Kurtosis and skewness being positive and the portfolio having negative skewness and kurtosis. We have seen above, that negative skewness and kurtosis increase VaR and, thus, Expected Shortfall.

Table 56: Expected Shortfall calculated using the 3 different methods for the Years 2010–2020

```
> ES(mydata[,1:6], p=.99,method="gaussian")
         NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50         NKYTR            SPX        US10GOV
ES -0.04879074867        -0.03949683457 -0.1281615781 -0.1271623459 -0.09759162753 -0.04586245467
> ES(mydata[,1:6], p=.99,method="historical")
         NOBSGLIA Risk.Parity.Portfolio Eurostoxx.50         NKYTR           SPX        US10GOV
ES -0.05109404226        -0.03856367118 -0.150441058 -0.1096657897 -0.1084481383 -0.04222227046
> ES(mydata[,1:6], p=.99,method="modified")
         NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50         NKYTR           SPX        US10GOV
ES -0.05524346851        -0.04184175875 -0.1652567462 -0.1414806791 -0.1280989286 -0.03769266381
```

Source: Rstudio (2021)

3.15 Sharpe ratio vs Var and Expected Shortfall

3.15.1 Sharpe ratio vs VaR and Expected Shortfall for the fund and its benchmarks for the Years 2006–2021

For the last chapter, before closing our results, we are going to calculate the Sharpe ratio using 3 different denominators: The standard deviation, the Var or the Expected Shortfall. We will use a risk-free rate equal to 0 and a given level of confidence of 99%. Those results are interesting because we are going to blend the different participant returns with different risk measures. There will be no surprises when calculating those Sharpe ratios because the bond is the best performing participant with its lowest standard deviation, VaR and Expected Shortfall. The bond will have a lower Sharpe ratio than the fund because the Fund has such an exceptional return. The bond will still have a higher Sharpe ratio than all the other benchmarks because their risk measures are so high in comparison to him.

Moreover, the Nikkei is outperforming the Euro Stoxx 50 with all its Sharpe ratios. This is not thanks to their risk measures because they have a higher standard deviation, VaR and modified and gaussian Expected Shortfall than the Euro Stoxx 50. This is because they have a much better return than the Euro Stoxx 50. The S&P 500 is outperforming the two other indexes because they have superior returns and lower standard deviation, VaR and ES than the two other indexes.

Table 57: Sharpe ratios vs Var and Expected Shortfall for the Years 2006–2021

```
> SharpeRatio(mydata[,1:5], Rf = 0,p=0.99, method="gaussian")
                                 NOBSGLIA        NKYTR
StdDev Sharpe (Rf=0%, p=99%): 0.4811886608 0.10874803958
VaR Sharpe (Rf=0%, p=99%):    0.2616970667 0.04918134165
ES Sharpe (Rf=0%, p=99%):     0.2210681650 0.04266145554
                               Eurostoxx.50           SPX
StdDev Sharpe (Rf=0%, p=99%): 0.022857197929 0.16073325811
VaR Sharpe (Rf=0%, p=99%):    0.009950651573 0.07444182850
ES Sharpe (Rf=0%, p=99%):     0.008674509921 0.06436775628
                                   US10GOV
StdDev Sharpe (Rf=0%, p=99%): 0.20484860261
VaR Sharpe (Rf=0%, p=99%):    0.09685220072
ES Sharpe (Rf=0%, p=99%):     0.08350965811
> SharpeRatio(mydata[,1:5], Rf = 0,p=0.99, method="historical")
                                 NOBSGLIA        NKYTR
StdDev Sharpe (Rf=0%, p=99%): 0.4811886608 0.10874803958
VaR Sharpe (Rf=0%, p=99%):    0.2396624692 0.05060555009
ES Sharpe (Rf=0%, p=99%):     0.1790976833 0.03266799831
                               Eurostoxx.50           SPX
StdDev Sharpe (Rf=0%, p=99%): 0.022857197929 0.16073325811
VaR Sharpe (Rf=0%, p=99%):    0.008378718660 0.06197208836
ES Sharpe (Rf=0%, p=99%):     0.007556330012 0.04753733654
                                  US10GOV
StdDev Sharpe (Rf=0%, p=99%): 0.2048486026
VaR Sharpe (Rf=0%, p=99%):    0.0945995649
ES Sharpe (Rf=0%, p=99%):     0.0824977935
> SharpeRatio(mydata[,1:5], Rf = 0,p=0.99, method="modified")
                                 NOBSGLIA        NKYTR
StdDev Sharpe (Rf=0%, p=99%): 0.4811886608 0.10874803958
VaR Sharpe (Rf=0%, p=99%):    0.2260974400 0.03802173373
ES Sharpe (Rf=0%, p=99%):     0.1786955247 0.03075678683
                               Eurostoxx.50           SPX
StdDev Sharpe (Rf=0%, p=99%): 0.022857197929 0.16073325811
VaR Sharpe (Rf=0%, p=99%):    0.008284214268 0.05606386875
ES Sharpe (Rf=0%, p=99%):     0.006664189639 0.04588703878
                                   US10GOV
StdDev Sharpe (Rf=0%, p=99%): 0.20484860261
VaR Sharpe (Rf=0%, p=99%):    0.10146064437
ES Sharpe (Rf=0%, p=99%):     0.08131954524
>
```

Source: Rstudio (2021)

3.15.2 Sharpe ratio vs Var and ES for the Fund and its benchmarks including the Risk Parity Portfolio for the Years 2010–2020

The table below calculated Sharpe ratios using VaR and ES as our denominator. Moreover, we are going to use the three different methods seen above to create more precise results. It is interesting to analyse the results below because we blend returns and risk together to find the best investment possible between all the participants.

Because we have already analysed the Sharpe ratio using the standard deviation, we are going to concentrate ourselves with the VaR and the Expected shortfall.

Firstly, we see that the Risk Parity portfolio has the best Sharpe ratio behind the Fund when using the VaR from Gaussian method. We have seen above that the portfolio had the lowest VaR from all the participants and this is the reason for its second place behind the Fund. We have also seen that the portfolio has the second-best Sharpe ratio when using the ES from the Gaussian method. We have also seen above that the Risk Parity portfolio had the lowest ES from all the participants when using the Gaussian method.

Secondly, we have seen that, when using the historical method to calculate the VaR used as the denominator to calculate the VaR, the bond (0.11452) and the S&P 500 (0.11085) have better Sharpe ratios. This is because the difference in return between the Risk Parity portfolio and the S&P 500 is so important that it outweighs the difference in VaR. The bond beats the portfolio because it has a lower VaR than the portfolio. This creates their better Sharpe ratios. We also see that when using the ES has our denominator using the historical method that the Risk Parity portfolio has the second-best Sharpe ratio of all the participants. This is normal because it has the lowest ES of all the participants.

Thirdly, the bond has a better Sharpe ratio than the Risk Parity portfolio when using the modified method to calculate the denominator. This is logical because we have seen above that the bond has the lowest VaR and ES from all the participants. It is still surprising to see that the Fund is outperforming the benchmarks with such differences.

Table 58: Sharpe ratios vs Var and Expected Shortfall for the Years 2010–2020

```
> SharpeRatio(mydata[,1:6], Rf = 0,p=0.99, method="gaussian")
                              NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50      NKYTR         SPX
StdDev Sharpe (Rf=0%, p=99%): 0.4858543538          0.2554848520 0.04913943206 0.19711878149 0.2445907485
VaR Sharpe (Rf=0%, p=99%):    0.2652432376          0.1238954204 0.02166215381 0.09296008318 0.1179890708
ES Sharpe (Rf=0%, p=99%):     0.2239660582          0.1064657353 0.01885600458 0.08019292758 0.1014653091
                                  US10GOV
StdDev Sharpe (Rf=0%, p=99%): 0.22269344946
VaR Sharpe (Rf=0%, p=99%):    0.10630305639
ES Sharpe (Rf=0%, p=99%):     0.09154987594
> SharpeRatio(mydata[,1:6], Rf = 0,p=0.99, method="historical")
                              NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50      NKYTR          SPX
StdDev Sharpe (Rf=0%, p=99%): 0.4858543538          0.2554848520 0.04913943206 0.19711878149 0.24459074849
VaR Sharpe (Rf=0%, p=99%):    0.2465036668          0.1103016893 0.01961448853 0.09920726564 0.11085710587
ES Sharpe (Rf=0%, p=99%):     0.2138697815          0.1090419923 0.01606353569 0.09298725539 0.09130783438
                                  US10GOV
StdDev Sharpe (Rf=0%, p=99%): 0.22269344946
VaR Sharpe (Rf=0%, p=99%):    0.11452841466
ES Sharpe (Rf=0%, p=99%):     0.09944282934
> SharpeRatio(mydata[,1:6], Rf = 0,p=0.99, method="modified")
                              NOBSGLIA Risk.Parity.Portfolio  Eurostoxx.50      NKYTR          SPX
StdDev Sharpe (Rf=0%, p=99%): 0.4858543538          0.2554848520 0.04913943206 0.19711878149 0.24459074849
VaR Sharpe (Rf=0%, p=99%):    0.2401883284          0.1177204415 0.01868170979 0.08550200387 0.09789194924
ES Sharpe (Rf=0%, p=99%):     0.1978056764          0.1004991105 0.01462339879 0.07207712641 0.07730091704
                                 US10GOV
StdDev Sharpe (Rf=0%, p=99%): 0.2226934495
VaR Sharpe (Rf=0%, p=99%):    0.1183609460
ES Sharpe (Rf=0%, p=99%):     0.1113930832
```

Source: Rstudio (2021)

4. Conclusion

In this thesis, our goal was to understand the complexity behind the Fund. We heard that the Fund was known for being the largest Sovereign Wealth Fund. We heard that the Fund is very transparent with its data and that the Fund has an exhaustive list of companies banned for investment. The Fund was also known for their important development in the last 10–15 years. This was all our preconceived information before starting this thesis. We discovered much more information after writing this thesis. This SwF was dealing with an important internal crisis at the end of 2020. This was more about the new CEO's personal conflict of interest than the Fund's performance. We learned that the new CEO, Nicolai Tangen, was allowed initially to keep its stake in his company AKO Capital and that this was ignored by the Prime Minister of Norway. This created uproar and would lead to Nicolai Tangen liquidating its stake in AKO Capital while also depositing its money in multiple banks to avoid conflict of interest with its personal wealth. During the introduction of this thesis, we explained the different reasons why the Fund would not invest in certain companies. We saw that those reasons ranged from production of nuclear weapons to environmental damage or even corruption. The Fund decided that some well-known companies like Berkshire Hathaway Energy and Airbus SE were on the exclusion list because of their dark handling. Our introduction also leads us to have a quick look at the 2020 report of the Fund. This report helped us immediately assess how the Fund invests. We saw how the portfolio was allocated and their attraction to US Equities and Bonds (41.6% of their total investments). We were surprised by the fact that the Fund had invested such a large part of their portfolio in Equities (71.5%) while his other investments were so small (24.7% in Fixed Income and 3.8% in Real Estate). The report helped us understand how the Fund worked from the inside and the different strategies behind their investments. We ended our introduction with an introduction and quick explanation of the different benchmarks used against the Fund. Those benchmarks were the Risk Parity Portfolio, the S&P 500, the Nikkei, the Euro Stoxx 50 and the 10-Year US Treasury. We developed which are the most important companies in each index and how companies are added and excluded from said indexes. We also explained how each index was weighted and we were surprised that the Nikkei was a price-weighted index while the others were float-adjusted capitalization-weighted indexes. We also explained quickly the history of the Risk Parity portfolio and its creator.

After ending our introduction, we started the second step in our thesis. This chapter is important because it helps us understand the results analysed in the third chapter. We started off by explaining the Modern Portfolio Theory. This theory, developed by Harry Markowitz, is pivotal in understanding how a portfolio works. It establishes that investors will always search for the lowest risk while also searching for the highest expected return. We followed this theory with the Capital Asset Pricing model. This model is based on the Modern Portfolio Theory and explains how the Expected return of a portfolio is calculated. This model will introduce us to Beta and we explained how Beta is calculated and its importance in Portfolio management. After elaborating those theories, we started our introduction of the different ratios used in this thesis. We made sure to always explain the limitations of each ratio while also validating the importance of those ratios. We introduced ratios like the Treynor ratio, the Sharpe ratio, the Information ratio, etc. We also didn't forget to present the drawdown ratios in this chapter. Some of those Drawdown ratios are the Sterling ratio, the Pain index, Ulcer index, etc. We also didn't forget to explain what drawdown means and the different types of drawdown. We ended our performance measurement chapter with a quick look at the value-at-risk and the Expected Shortfall. It was important to elaborate those risk measures because they assess a risk that was not calculated with the ratios or the drawdowns. We saw that the VaR and the ES also used different calculation methods like the Gaussian method, the Historical method and the Modified method.

After finishing our performance measurement chapter, we were able to correctly and precisely analyse the results presented to us in Rstudio. This programme enabled us to calculate the different returns, drawdowns and ratios without the risk of making mistakes in those calculations. It is also important to note that our analysis is divided in two periods. We are going to analyse the performance of the Fund against the S&P 500, the Euro Stoxx 50 and the Nikkei on a 15-Year Time horizon. We are, then, going to analyse the performance of the Risk Parity portfolio against the Fund and the 3 benchmarks seen above on a 10-Year Time Horizon. We explained in our thesis that the reason behind this split analysis is that we wanted to create our own Risk Parity portfolio based on results from the different investment categories used between 2005–2010. We explained thoroughly how we created this portfolio in excel and what the optimal allocation was.

We started our analysis with a chapter about the different returns and volatility. We looked for abnormalities in the cumulative returns graph and tried to understand why some returns were underwhelming. We quickly understood the impact of the 2008–2009 Financial crisis and how the aftermath impacted the Nikkei and the Euro Stoxx 50. We were immediately surprised by the exceptional cumulative return of the Fund and the low impact of the financial crisis on the Fund. We even argued that the financial crisis was beneficial for the Fund because they announced in 2007 that they would change their portfolio allocation in the 2 years to come. They increased their Equity investments from 40% of their total investments to 60% of their total investments. This increase in equity investments happened when equities were trading at all-time lows. We also highlighted the key changes made by the Fund between 2009 and 2019. This helped us understand in depth how the Fund changed strategies over the years and how their success was years in the waiting. We were also surprised to see that the Euro Stoxx 50 hadn't recovered from the 2008 Financial crisis whereas the Nikkei only recovered 4–5 years after the initial crash. We also analysed the cumulative returns graph on a 10-Year period with the Risk Parity portfolio. We discovered that the portfolio had a very similar return to the 10-Year US bond and that that wasn't a surprise because of the importance of the bond in the portfolio. We were also surprised to see that the Nikkei was slightly outperformed by the S&P 500 whereas in the 15-Year period the difference between both of them was much bigger. We, thus, discovered the importance of the 2008–2009 financial crisis on the Nikkei.

After analyse the cumulative returns graph, we started analyse the different correlations between the Fund and its benchmarks. We quickly explained the different types of correlation and the difference between the skewness or the kurtosis of the distribution. We quickly saw that there were high correlations between the 3 indexes. We explained that those markets were very similar and moved accordingly. We used the example of the 2008 financial crisis to explain how those 3 markets are interconnected. It also became abundantly clear that the American markets regenerated quicker than the two other markets. This is because the European markets and Japanese markets are following the movements from the American markets. This leads us to believe that those two markets suffer more when crises happen. We ended this chapter by highlighting the Fund's kurtosis. The Fund had the lowest kurtosis of all the participants (0.11). When an investment has a low kurtosis, we say that this investment bears a moderate level of risk. Such investments are also known for their rare big losses and few high levels of return. After ending the 15-year analysis, we started our analysis on a 10-Year period with the portfolio. We discovered that the portfolio had a Mesokurtic distribution like the Fund and that extreme losses and gains were rare. The portfolio also had important correlations with the S&P 500 (0.59) and the Euro Stoxx 50. We understood that this was largely due to the portfolio's investments in the S&P 500 and the MSCI EAFE Total Return USD Index. We ended this analysis by highlighting the small positive correlation with the bond. Usually bonds are negatively correlated with indexes or Funds, but in this case the portfolio is positively correlated with the bond.

We, then, started to look at the volatility of each participant. The Fund immediately distanced themselves from the 3 other indexes. The Fund presented a similar standard deviation to the 10-Year US Treasury (0.0840 vs 0.0728). This was very surprising because the Fund only invested 25% of their total investments in Fixed-Income. We also noticed that the Nikkei was the most volatile of the 3 markets. This is something we will see frequently when analyse the different risk measures. We will see that Nikkei's return is the only reason it is not performing worse than the Euro Stoxx 50. In a 10-Year period, the Risk Parity portfolio had the lowest standard deviation of all the participants. This means we did a good job in creating the Risk Parity portfolio and that our portfolio was indeed the safest investment.

After analyse the different returns and volatility, we could start to analyse the different performance measurements explained before. We looked at the Treynor ratio, Information ratio, R-squared, Beta, Alpha and the Tracking error. Those ratios showed us that the Fund outperformed every index. The tracking error showed us that the Fund outperforms the three indexes with its return's standard deviation and that an actively managed portfolio has a better Tracking error against the three benchmarks. We see that the Nikkei is the most outperformed by the Fund when looking at the standard deviation. The Information ratio would show us the incredible underperformance, in terms of returns, by the Euro Stoxx 50. R-squared showed us that the Fund's movement is averagely correlated to the three benchmarks. This means that only 45–50% of the time, the Fund's movement is explained by the movements of one of the benchmarks. We see that the Beta from the Fund is below one when calculated against the three benchmarks. This means the Fund is less volatile than the benchmarks. We see that the Fund is less volatile against the benchmarks by a margin. Again, the S&P 500 is the strongest of the three benchmarks. The Treynor ratio was calculated using the 10-Year US Treasuries returns as the risk-free rate. The Fund outperformed the Nikkei the most when the Treynor ratio was calculated. This was due to the low beta between the Fund and the Nikkei. We cannot forget that the Beta is based on the volatility of the market. We have constantly seen that the Nikkei is the most volatile market but the Euro Stoxx 50 is the market with the lowest growth. This pattern will also be highlighted in a 10-Year period. We saw that the Nikkei remained the riskiest asset but this time the index presented very similar returns to the S&P 500. Whereas the Risk Parity portfolio proved to be the safest investment. We saw that the portfolio had the lowest Tracking error when calculated with the Fund and the highest when calculated with the Nikkei. The portfolio also proved to have consistently negative Information ratios when calculated against the S&P 500, the Fund and the Nikkei. It only had a positive Information ratio when calculated against the Euro Stoxx 50. This is normal because it has a negative excess return against those 3. R-squared showed us that the portfolio’s movement is highly correlated with that of the S&P 500 and that it is the least correlated with the Fund. The Beta showed us that the benchmarks are at least twice more volatile than the Risk Parity portfolio. It also showed us the difference in volatility between the Fund and the different benchmarks. Alpha showed us that, in terms of returns, the portfolio is outperformed by all the participants beside the Euro Stoxx 50. We ended this chapter with the Treynor ratio. The Risk Parity portfolio had a negative Treynor ratio because it has a negative return against the 10-Year US Treasury.

Once those metrics are analysed, we could concentrate on the Sharpe ratio and the modified Treynor ratio. We calculated the Sharpe ratio in two ways. In one case we used 0 as our risk-free rate and, in the other case, we used the 10-Year US Gov as our risk-free rate. This created very similar results beside the fact that the Euro Stoxx 50 had a negative Sharpe ratio when we used the 10-Year US Gov as the risk-free rate. This was because the Euro Stoxx 50 has outperformed the bond between 2006–2021. The Fund’s excess return was vastly bigger than that of the other benchmarks. Moreover, it has a lower standard deviation than the other benchmarks. This creates a Sharpe ratio bigger than all the other benchmarks. The modified Treynor ratio was negative for the Euro Stoxx 50 again. This is, again, because they have a lower return than the risk-free rate. We also analysed M-squared. Those results showed us that the Euro Stoxx 50 is the only benchmark with a negative M-squared. This is due to their negative Sharpe ratio. The Fund had the best M-square because it has the best Sharpe ratio and the lowest Market risk of the benchmarks. When looking at a 10-Year period, our main takeaways are that the Nikkei presented a much better Sharpe ratio than on a 15-Year period due to better returns on that period and that the Risk Parity portfolio has the second-best Sharpe ratio when 0 is used as the Risk-free rate. This high Sharpe ratio is mainly due to its incredibly low standard deviation because we saw that when the 10-year US was used as the risk-free rate, the portfolio had a negative Sharpe ratio. We will see the same pattern with the modified Treynor ratio. The portfolio has a negative modified Treynor ratio due to its lower return than the bond. We also noted that the bond performed better on a 10-Year period than on a 15-Year period. We ended our chapter by highlighting the low market risk from the portfolio. This resulted in the second-best M-squared.

After seeing the different calculations showing the exceptional return of the Fund and its low standard deviation, we decided that we should also calculate some risk measures. We started with some Downside risk ratios and, after analyse them, we ended this chapter with the different

Drawdown ratios. The downside risks showed us that the 10-Year US Gov is the safest investment from all the participants. It surprised us that the Downside potential was smaller for the Fund than for the bond. This meant that the Fund losses, on average, less than the bond when they were losing. The other noticeable information we found was that the Nikkei had the highest Upside potential of all investments. This is probably because the Nikkei is very volatile and can suffer large downsides but also large upsides. We ended this chapter by concluding that the Fund had the best downside risk ratios and that it largely outperformed its peers. In a 10-Year period, we immediately saw that each ratio was better for all the participants. This was mainly because the 2008–2009 financial crisis didn't figure in those ratios. The Risk Parity portfolio had the lowest Downside risk, lowest Upside potential, lowest Downside potential. This largely meant that when the portfolio suffers losses, usually those losses are very small. This also meant that the portfolio rarely presents high returns. We also concluded when blending returns and downside risk ratios, that the Fund is the best performing participant. We also saw that the portfolio and the S&P 500 are fighting for second best. The S&P 500 has the better Omega ratio, but the portfolio has the better Sortino ratio.

After analyse the Downside risks, we decided to analyse the 5 most severe drawdowns of each investment and to analyse the Drawdown ratios for each participant. We concluded that the Nikkei suffered the largest drawdown but the Euro Stoxx 50 suffered the longest drawdown. (That hasn't ended since 2008). The Fund also showed us that its losses are infrequent and small. They have a very similar loss pattern than the 10-Year US Gov but their returns pattern is much better. The Drawdown ratios helped us to come to such conclusions. Again, we saw that when the Euro Stoxx 50 returns are punched against the risk-free rate, the ratios are negative due to their low return. In a 10-year period, the Risk Parity portfolio suffered the second-lowest drawdown from all the participants. The Fund proved to have shorter and smaller drawdowns in a 10-Year period than the portfolio. This was surprising to us when looking at the downside risk and the downside potential from the portfolio. When looking at the different Drawdown ratios, we discovered that the portfolio is the second-best performing participant behind the Fund. The portfolio has the second-best Calmar ratio, Burke ratio, Pain index, Ulcer index. Again, like we have seen consistently in this thesis, when blending returns and risk measures, the S&P 500 outperforms the portfolio with a better Sterling ratio, Pain ratio and Martin ratio.

We ended our thesis with an analysis of the Value-at-risk and the Expected Shortfall for each participant. We used three different methods of calculations to help us present more precise results: the Gaussian method, the Modified method and the Historical method. We concluded that, against a level of confidence of 99%, the 10-Year US Gov would suffer the lowest maximum expected loss whereas the Nikkei (in the Gaussian method and the modified method) would suffer the largest maximum expected loss. The Expected Shortfall showed us, with a 99% level of confidence, also that the 10-Year US Gov would suffer the lowest loss if the VaR threshold was crossed whereas the Nikkei would be the benchmark that would suffer the largest loss if the VaR threshold was crossed. In a 10-year period, we concluded that the lowest VaR or Expected Shortfall depended on the method of calculation. We saw that when using the Gaussian method, the portfolio would present the lowest VaR or ES. We also saw that when using the modified method, the bond would have the lowest VaR or ES. The historical method would prove to present mixed results, with a lower ES for the bond and a lower VaR for the portfolio than its peers. We ended our analysis with some Sharpe ratios using the different VaR and ES calculated by the different methods.

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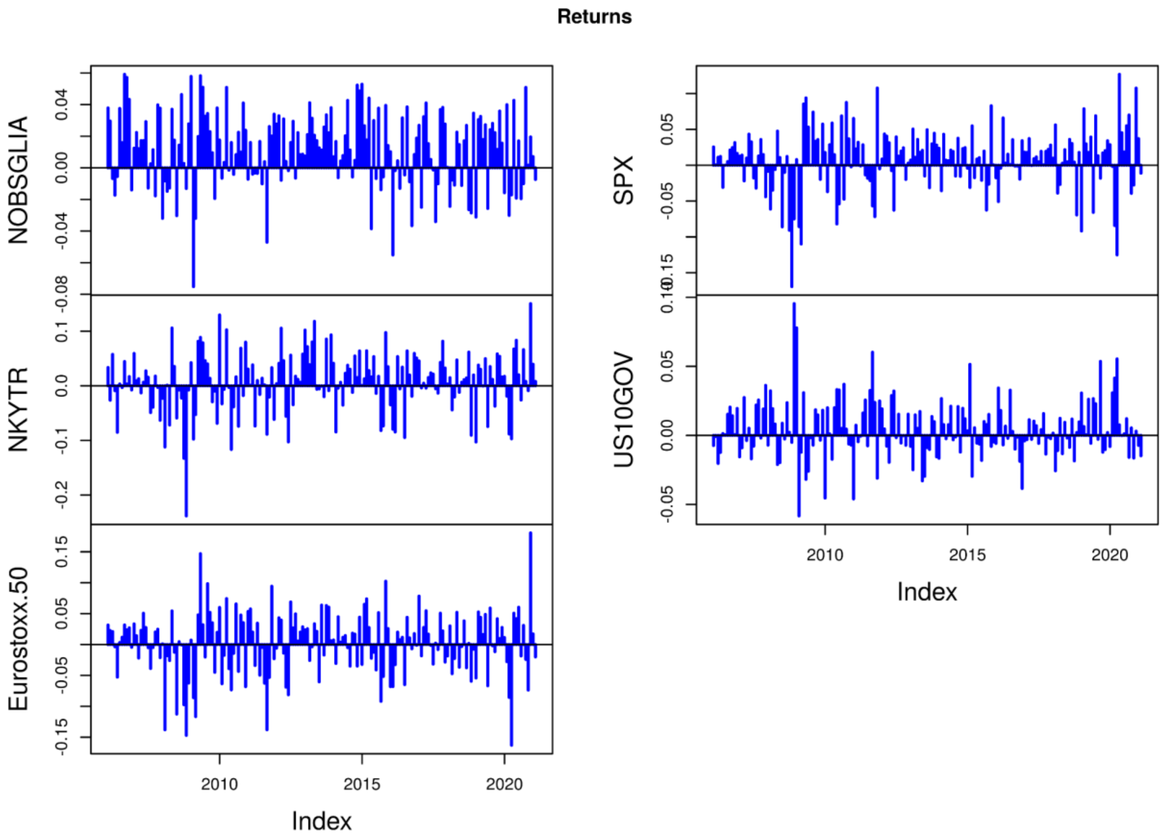
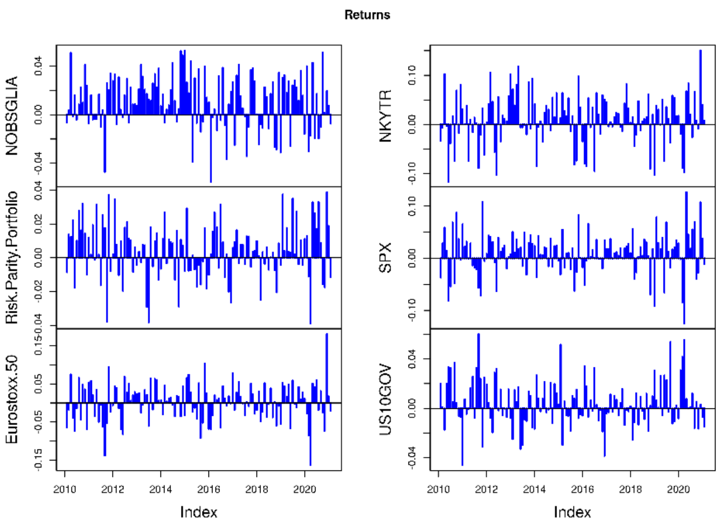
List of Appendices

Appendix 1: Year-on-Year returns for the Fund and the benchmarks excluding the Risk Parity Portfolio between 2006-2021



Appendix 2: Year-on-Year returns for the Fund and the benchmarks including the Risk Parity Portfolio between 2010-2020



Appendix 3: Rstudio Code for the Fund and 4 benchmarks results for the Years 2006-2021

```
R.version
if (!require("PerformanceAnalytics")) {
  install.packages("PerformanceAnalytics", dependencies = TRUE)
  library(dplyr)
}
if (!require("readr")) {
  install.packages("readr", dependencies = TRUE)
  library(dplyr)
}
if (!require("psych")) {
  install.packages("psych", dependencies = TRUE)
  library(dplyr)
}
if (!require("zoo")) {
  install.packages("zoo", dependencies = TRUE)
  library(dplyr)
}
if (!require("zoo")) {
  install.packages("zoo", dependencies = TRUE)
  library(dplyr)
}
if (!require("openxlsx")) {
  install.packages("openxlsx", dependencies = TRUE)
  library(dplyr)
}
if (!require("zoo")) {
  install.packages("zoo", dependencies = TRUE)
  library(dplyr)
}

options("scipen"=50, "digits"=10)

library("openxlsx")  # To read xlsx



db <- read.xlsx("BJORN.xlsx", sheet = 1, detectDates = TRUE)
library("readr")   # Fast csv write
write_csv(db, path="FR.csv")
db <- read.table("FR.csv", header=T,sep=",")
library(zoo)
mydata <-read.zoo(db)

View(mydata)


# mean,median,14th and 75th quartiles,min,max
summary(mydata)
```

```
library(psych)
# item name ,item number, nvalid, mean, sd,
# median, mad, min, max, skew, kurtosis, se
describe(mydata)

library(PerformanceAnalytics)

start(as.zoo(mydata))
end(as.zoo(mydata))
colnames(mydata)

# graphical analysis of manager data

png("CReturnsPlot.png",width=7,height=5,units="in",res=1000)
#pdf("CumulativeReturns.pdf",width=7,height=5)
chart.CumReturns(mydata[,c(1:5)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(1:3,13)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(4:6,13)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(7:9,13)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(10:12,13)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(1:5,13)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(7:12,13)], main="Cumulative Returns", wealth.index=TRUE,
legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
dev.off()


# panel function to put horizontal lines at zero in each panel
my.panel <- function(...) {
  lines(...)
  abline(h=0)
}
# use plot.zoo() to create a multiple panel time series plot
# plot returns over time to illustrate monotone missing data
plot.zoo(mydata[,1:5], main="Returns",plot.type="multiple", type="h", lwd=2, col="blue",
panel=my.panel)
#plot.zoo(mydata[,1:18], main="Returns",plot.type="multiple", type="h", lwd=2,
col="blue",         panel=my.panel)
dev.copy(png,'ReturnsPlot.png',width=8,height=6,units="in",res=1000)
dev.off()

#mydata = mydata["2001-09-30::2006"] # remove data prior to 2001-09-30 b/c some data are
not observed
#head(mydata) # give a quick of the data
```

#table.CalendarReturns works only for monthly returns
table.CalendarReturns(mydata[,1:5], digits = 1, as.perc = TRUE,geometric = TRUE) # display calendar (monthly and annualized) returns for the first time series (in % and geom mean)

table.Stats(mydata[,1:5]) # give stats for all the time series ? including monthly arith and geom mean returns

chart.Correlation(mydata[,1:5], histogram=TRUE, pch="+")
#chart.Correlation(mydata[,1:18], histogram=TRUE, pch="+")
dev.copy(png,'CorrelationPlot.png',width=8,height=6,units="in",res=1000)
dev.off()

tail(cumprod(1+mydata[,1:5])-1,1) #Global Return

Return.annualized(mydata[,1:5]) #Annualised Return: Geometric average (%), identical to annualised log returns

table.Variability(mydata[,1:5]) #Give volatility among others (i.e. annual std dev)

#CAPM useful when portfolios or individual stocks must be compared to a benchmark
table.CAPM(mydata[,1:3], mydata[,4,drop=FALSE], Rf = mydata[,5,drop=FALSE]) # Report a lot of information on the CAPM (alpha, beta, R squared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)

SharpeRatio(mydata[,1:4], Rf = mydata[,5,drop=FALSE], FUN="StdDev") #Report the traditional monthly Sharpe ratio
SharpeRatio(mydata[,1:5], Rf = 0, FUN="StdDev") #Report the traditional daily Sharpe ratio

SharpeRatio.annualized(mydata[,1:4], Rf = mydata[,5,drop=FALSE])
SharpeRatio.annualized(mydata[,1:5], Rf = 0)

#CAPM useful when portfolios or individual stocks must be compared to a benchmark
TreynorRatio(mydata[,1:3], mydata[,4], Rf = mydata[,5,drop=FALSE],modified = TRUE)
#Report the modified Treynor ratio
MSquared(mydata[,1:3], mydata[,4],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate

#table.DownsideRiskRatio(mydata[,1:19],MAR=0.01/12) #Report various Downside Risk Ratios (Sortino, UDR, Omega, etc.)
table.DownsideRiskRatio(mydata[,1:5],MAR=0) #Report various Downside Risk Ratios (Sortino, UDR, Omega, etc.)

table.Drawdowns(mydata[,1,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
table.Drawdowns(mydata[,2,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
table.Drawdowns(mydata[,3,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
table.Drawdowns(mydata[,4,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny
table.Drawdowns(mydata[,5,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for

```
the asset under special scrutiny
#table.Drawdowns(mydata[,6,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

table.DrawdownsRatio(mydata[,1:5], Rf= 0) #Report various Drawdowns Ratios (Sterling, Burke, Calmar, etc.)
#Average of Daily 10YR YTM = 0

VaR(mydata[,1:5], p=.99,method="gaussian")
VaR(mydata[,1:5], p=.99,method="historical")
VaR(mydata[,1:5], p=.99,method="modified")

ES(mydata[,1:5], p=.99,method="gaussian")
ES(mydata[,1:5], p=.99,method="historical")
ES(mydata[,1:5], p=.99,method="modified")

SharpeRatio(mydata[,1:5], Rf = 0,p=0.99, method="gaussian")
SharpeRatio(mydata[,1:5], Rf = 0,p=0.99, method="historical")
SharpeRatio(mydata[,1:5], Rf = 0,p=0.99, method="modified")
```

Appendix 4: Rstudio Code for the Fund and 5 benchmarks results for the Years 2010-2020

```
R.version

if (!require("PerformanceAnalytics")) {

  install.packages("PerformanceAnalytics", dependencies = TRUE)

  library(dplyr)

}

if (!require("readr")) {

  install.packages("readr", dependencies = TRUE)

  library(dplyr)

}

if (!require("psych")) {

  install.packages("psych", dependencies = TRUE)

  library(dplyr)

}

if (!require("zoo")) {

  install.packages("zoo", dependencies = TRUE)

  library(dplyr)

}
```

```
if (!require("zoo")) {
  install.packages("zoo", dependencies = TRUE)
  library(dplyr)
}
if (!require("openxlsx")) {
  install.packages("openxlsx", dependencies = TRUE)
  library(dplyr)
}
if (!require("zoo")) {
  install.packages("zoo", dependencies = TRUE)
  library(dplyr)
}

options("scipen"=50, "digits"=10)

library("openxlsx")  # To read xlsx

db <- read.xlsx("Copie de Test pour R (version 2)-2 Bjorn.xlsx", sheet = 4, detectDates = TRUE)
library("readr")   # Fast csv write
write_csv(db, path="FR.csv")
db <- read.table("FR.csv", header=T,sep=",")
library(zoo)
mydata <-read.zoo(db)

View(mydata)

# mean,median,14th and 75th quartiles,min,max
summary(mydata)

library(psych)
# item name ,item number, nvalid, mean, sd,
```

```
# median, mad, min, max, skew, kurtosis, se
describe(mydata)

library(PerformanceAnalytics)

start(as.zoo(mydata))
end(as.zoo(mydata))
colnames(mydata)

# graphical analysis of manager data

png("CReturnsPlot.png",width=7,height=5,units="in",res=1000)
#pdf("CumulativeReturns.pdf",width=7,height=5)
chart.CumReturns(mydata[,c(1:6)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(1:3,13)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(4:6,13)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(7:9,13)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(10:12,13)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(1:5,13)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
#chart.CumReturns(mydata[,c(7:12,13)], main="Cumulative Returns", wealth.index=TRUE, legend.loc="topleft", cex.legend = 0.4) # plot cumulative returns using PerformanceAnalytics
dev.off()

# panel function to put horizontal lines at zero in each panel
my.panel <- function(...) {
  lines(...)
  abline(h=0)
```

```
}

# use plot.zoo() to create a multiple panel time series plot

# plot returns over time to illustrate monotone missing data

plot.zoo(mydata[,1:6], main="Returns",plot.type="multiple", type="h", lwd=2, col="blue", panel=my.panel)

#plot.zoo(mydata[,1:18], main="Returns",plot.type="multiple", type="h", lwd=2, col="blue", panel=my.panel)

dev.copy(png,'ReturnsPlot.png',width=8,height=6,units="in",res=1000)

dev.off()


#mydata = mydata["2001-09-30::2006"] # remove data prior to 2001-09-30 b/c some data are not observed

#head(mydata) # give a quick of the data


#table.CalendarReturns works only for monthly returns

table.CalendarReturns(mydata[,1:6], digits = 1, as.perc = TRUE,geometric = TRUE) # display calendar (monthly and annualized) returns for the first time series (in % and geom mean)


table.Stats(mydata[,1:6]) # give stats for all the time series ? including monthly arith and geom mean returns


chart.Correlation(mydata[,1:6], histogram=TRUE, pch="+")

#chart.Correlation(mydata[,1:18], histogram=TRUE, pch="+")

dev.copy(png,'CorrelationPlot.png',width=8,height=6,units="in",res=1000)

dev.off()


tail(cumprod(1+mydata[,1:6])-1,1) #Global Return


Return.annualized(mydata[,1:6]) #Annualised Return: Geometric average (%), identical to annualised log returns


table.Variability(mydata[,1:6]) #Give volatility among others (i.e. annual std dev)
```

#CAPM useful when portfolios or individual stocks must be compared to a benchmark

table.CAPM(mydata[,1:5], mydata[,1,drop=FALSE], Rf = mydata[,6,drop=FALSE]) # Report a lot of information on the CAPM (alpha, beta, R squared, correlation, tracking ? error ?, Information ratio, Treynor ratio, etc.)

SharpeRatio(mydata[,1:6], Rf = mydata[,5,drop=FALSE], FUN="StdDev") #Report the traditional monthly Sharpe ratio

SharpeRatio(mydata[,1:6], Rf = 0, FUN="StdDev") #Report the traditional daily Sharpe ratio

SharpeRatio.annualized(mydata[,1:6], Rf = mydata[,5,drop=FALSE])

SharpeRatio.annualized(mydata[,1:6], Rf = 0)

#CAPM useful when portfolios or individual stocks must be compared to a benchmark

TreynorRatio(mydata[,1:6], mydata[,5], Rf = mydata[,6,drop=FALSE],modified = TRUE) #Report the modified Treynor ratio

MSquared(mydata[,1:5], mydata[,1],Rf = 0) #Report the M2 with annualized geom. mean risk-free rate

#table.DownsideRiskRatio(mydata[,1:19],MAR=0.01/12) #Report various Downside Risk Ratios (Sortino, UDR, Omega, etc.)

table.DownsideRiskRatio(mydata[,1:6],MAR=0) #Report various Downside Risk Ratios (Sortino, UDR, Omega, etc.)

table.Drawdowns(mydata[,1,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

table.Drawdowns(mydata[,2,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

table.Drawdowns(mydata[,3,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

table.Drawdowns(mydata[,4,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

table.Drawdowns(mydata[,5,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

table.Drawdowns(mydata[,6,drop=FALSE], 5, 2) #Identify the 5 most severe drawdowns for the asset under special scrutiny

```
table.DrawdownsRatio(mydata[,1:6], Rf= 0) #Report various Drawdowns Ratios (Sterling, Burke, Calmar, etc.)

#Average of Daily 10YR YTM = 0


VaR(mydata[,1:6], p=.99,method="gaussian")

VaR(mydata[,1:6], p=.99,method="historical")

VaR(mydata[,1:6], p=.99,method="modified")


ES(mydata[,1:6], p=.99,method="gaussian")

ES(mydata[,1:6], p=.99,method="historical")

ES(mydata[,1:6], p=.99,method="modified")


SharpeRatio(mydata[,1:6], Rf = 0,p=0.99, method="gaussian")

SharpeRatio(mydata[,1:6], Rf = 0,p=0.99, method="historical")

SharpeRatio(mydata[,1:6], Rf = 0,p=0.99, method="modified")
```

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