

École polytechnique de Louvain

Discrete element modelling of blocky masonry structures under static and dynamic large displacements: validation against experimental benchmarks

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Abstract

This thesis has been dedicated to the modelling of masonry seismic behaviour using the Discrete Element Method (DEM). In response to the recent evolution of numerical methods, particularly DEM, which has shown promising results in this field, this study aimed to address the need for assessing these methods, focusing on the LMGC90 software implemented with the Non Smooth Contact Dynamics (NSCD) method.

The presented numerical results are obtained by varying several parameters and are compared against experimental data to explore and evaluate the software's capabilities. In total, five experimental benchmarks, spanning 2D and 3D scenarios in static and dynamic settings, are analysed and simulated throughout this thesis. The first benchmark study will consist of a thorough parametric study, aiming to explore the modelling options offered by the software.

The thesis specifically steers the research towards 3D and dynamic benchmarks. Ultimately, the study addresses the complications encountered while simulating 3D structures and discusses potential solutions.

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CHAPTER 1

Introduction

1.1 Motivation

Masonry, one of the oldest and most widely employed construction techniques, continues to be prevalent in both contemporary new buildings and significant number of historical structures. The prevalence of masonry in the built environment demands a comprehensive understanding of its structural behaviour, particularly under seismic loading conditions. [1, 2, 3]

Belgium has been identified as having significant exposure and vulnerability to earthquakes, with a large portion of their building stock consisting of Unreinforced Masonry (URM) structures. The weak seismic performance of masonry structures was highlighted by recent and past earthquakes and poses significant economic and human risks. For instance, the city of Liege (Belgium), which faced substantial damage during an earthquake in 1983, emphasizing the need to understand and predict the behaviour of these structures under seismic loading. [1, 2, 4, 5, 6]

In response to the challenges posed by seismic vulnerability, advanced numerical simulation methods have emerged as valuable tools for investigating the seismic behaviour of masonry structures. Among these methods, the Discrete Element Method (DEM) has demonstrated remarkable success in various engineering applications, including masonry modelling, and its potential to accurately capture the complex behaviour of materials and interactions between bodies is recognized [1, 4].

In recent years, the LMGC90 software, implemented with the Non Smooth Contact Dynamics (NSCD) method (employing the DEM approach), has undergone significant developments, incorporating specific features tailored to the modelling of Unreinforced Masonry (URM) buildings [5, 7, 8]. The primary motivation behind this thesis is to evaluate the capabilities of the LMGC90 software in accurately simulating the seismic behaviour of masonry structures. By thoroughly assessing the software's capabilities and limitations in this domain, this research aims to contribute to the advancement of numerical methods and enhance the seismic engineering knowledge related to masonry modelling.

1.2 Objectives

The main objective of this research is to comprehensively evaluate the capabilities of the LMGC90 software, implemented with the DEM, for simulating the seismic behaviour of masonry structures. This study aims to establish new validations based on experimental results.

To achieve these objectives, the thesis will begin by conducting a literature review, aiming to understand the challenges involved in modelling masonry and explore the various modelling options offered by the LMGC90 program. This review will provide insights into existing studies and identify areas that require further investigation and validation through experimental data.

As the primary focus is on evaluating the LMGC90 software's performance in modelling masonry structures under seismic conditions, parametric studies will be carried out, aiming to reproduce the experimental masonry behaviour. The present work aims to simulate various benchmarks, experimenting with the static and dynamic behaviour of masonry in 2D and 3D settings, using experimental data for validation.

An extensive study will be conducted on an first 2D static experimental benchmark to explore the software's modelling capabilities. Subsequent benchmarks will be chosen based on the findings of the literature review and existing studies. The thesis is focused on shifting towards the examination of 3D and/or dynamic benchmarks due to the limited availability of validation work in these areas within the literature.

1.3 Research novelty

Unlike some other DEM approaches, the existing literature lacks comprehensive studies aiming to validate the LMGC90 software with experimental benchmarks. While a few comparisons between numerical and experimental results have been found, they are primarily limited to static and 2D scenarios and are not exhaustive.

This research addresses these gaps by providing new and extensive validations of the NSCD method through the LMGC90 software. By enhancing the reliability and applicability of the software for simulating the seismic behaviour of masonry structures, this work contributes to the advancement of numerical methods in this field.

The novelty of this research lies in its parametric study, which aims to evaluate the influence of various parameters on the simulation results, thereby bringing them closer to experimental data. This allows for a more comprehensive exploration of the LMGC90 software's capabilities and limitations in capturing the intricate behaviour of masonry materials and interactions between blocks. Furthermore, this thesis aims to extend the evaluations to more advanced benchmarks, specifically focusing on 3D and dynamic simulations.

1.4 Thesis structure

Following this introductory chapter, Chapter 2 will delve into a literature review aiming to investigate the seismic behaviour of masonry structures and their modelling. This chapter will also seek to understand the principles and features of the method used within the framework of this thesis. Additionally, it will present other studies conducted on this subject.

The validation of the software through experimental benchmarks will be distributed across three separate chapters. In Chapter 3, two static 2D benchmarks will be addressed, with the first one involving a more in-depth parametric study. Chapter 4 will focus on 3D static benchmarks. Finally, Chapter 5 will explore dynamic benchmarks, encompassing both 2D and 3D simulations. Each of the five validations will be structured as follows: presentation of the experimental benchmark, followed by the presentation of the numerical model established with LMGC90 and the parameters studied, and finally, an overview of the obtained numerical results.

Ultimately, Chapter 6 will conclude this thesis, summarizing the essential points and conclusions drawn from the analyses conducted throughout this research.

CHAPTER 2

Literature review

2.1 Introduction

This chapter serves as an initial literature review to explore the subject of masonry modelling and the existing studies conducted on this topic. This literature review aims to understand the challenges involved in numerical modelling and to identify the specific features and functionalities of the LMGC90 software and the NSCD method. The review will provide insights into selecting appropriate benchmarks, justifying assumptions, and exploring simulation parameters throughout the thesis.

2.2 Masonry: definition and behaviour

2.2.1 Definition

Masonry construction has been used since ancient times and remains prevalent in modern buildings. It is a construction technique that involves assembling individual natural or manufactured units (blocks) [3, 9]. These elements are typically bound together by mortar joints, forming cohesive walls. In cases where the blocks are not bound by mortar, the masonry is referred to as dry-jointed.

Masonry can be classified based on various factors, such as the type of element used (bricks, stones, concrete blocks, etc.), joint type (dry or mortared), unit assembly, or construction technique [3, 10]. The wide range of construction techniques, assembly

methods, unit size, and materials employed results in diverse characteristics and mechanical behaviours of masonry structures [3, 9].

2.2.2 Mechanical properties and behaviour

Mechanical properties of masonry

Masonry as a structure can be characterized by its strength properties. Mechanical tests can be performed to determine the masonry's characteristics [3]. However, the mechanics of masonry are complex and highly nonlinear. Masonry is a heterogeneous and anisotropic material, consisting of two distinct parts: units (blocks) and joints. As such, its mechanical properties depend on both the material properties and the bond between the blocks [3, 11, 12].

Mechanical properties of blocks

Masonry properties are influenced by the nature of the blocks, which generally exhibit similar properties. They typically have good compressive strength, ranging from 20 MPa for hollow bricks to as high as 200 MPa for certain types of stones. However, their tensile strength is often negligible, with a maximum value of approximately 30 MPa for specific stones, exhibiting quasibrittle tensile behaviour [1]. Masonry elements generally have densities between 2000 and 3000 kg/m³, and their Poisson's ratio usually ranges from 0.2 to 0.3. The elasticity modulus varies significantly. Table 2.1 presents usual values for mechanical properties of different block materials [3, 10].

Material	Density [kg/m ³]	Elastic modulus [GPa]	Poisson coefficient [ν]	Tensile strength [MPa]	Compressive strength [MPa]
Brick	3000	5-15	0.2	3-8	10-30
Concrete	2300	30	0.3	3	30
Stone	2000-2900	20-100	0.2-0.3	1-30	10-200

Table 2.1: Usual mechanical properties for different block materials [3, 10].

Mechanical properties of joints

Mortar joints provide cohesion, shear resistance, and load transfer within the structure [1]. The mechanical behaviour of mortar depends directly on its components (cement, water, aggregates, etc.) [3]. Like blocks, mortar has good compressive strength but low tensile strength, exhibiting quasibrittle behaviour. However, its strength is generally lower than that of blocks, often making it the weakest link in masonry structures and controlling failure modes in many cases [10, 1].

In the case of dry-joint masonry (which is the case for all the benchmarks studied in the following chapters of this thesis), the contact occurs directly between the units. Consequently, the tensile strength is completely absent [3]. The most commonly used law to describe the horizontal interaction between two blocks is Coulomb's law, which directly depends on the material's friction coefficient (more detailed in section 2.5.2.3). The friction coefficient μ depends on the material roughness and the axial stress, a typical value for masonry is around 0.6 (value found for cut soft stone) [13].

2.3 Seismic action and evaluation methods

2.3.1 Seismic action

An earthquake is a natural ground motion resulting from the movement of the Earth's crust, generating seismic waves due to the release of energy during rock ruptures [3]. The earthquake's scope is characterized by its intensity, quantifying the effects at a specific point on the Earth's surface, and its magnitude, characterizing the energy released at the epicenter [3]. Seismic signals can be characterized by parameters such as peak ground acceleration (PGA) or duration [3, 14].

2.3.2 Masonry seismic behaviour and failure

During earthquakes, masonry structures are subjected to dynamic forces, causing significant damage due to the horizontal and vertical loads [14, 3]. The behaviour of a masonry wall differs based on the direction of loading [3]:

- Within its plane, a masonry wall experiences substantial horizontal loads, leading to significant in-plane shear forces, especially when supporting vertical loads.

- The out-of-plane behaviour of a wall depends on its connections with bracing walls, requiring a 3D analysis. Failure can involve either the braced wall or the bracing wall, experiencing out-of-plane bending. Several out-of-plane failure mechanisms have been identified by D'Ayala and Speranza (2003) [15].

Taforel (2012) [3] notes that the seismic response of masonry begins with deformations within the linear range, followed by a non-linear phase characterized by crack development, potential separation of masonry units, and failure of individual elements. Five local mechanisms can lead to failure, where (a), (b) and (d) are the most frequent [3, 16]:

- **(a)** joint separation (tension)
- **(b)** block sliding along joints
- **(c)** unit cracking due to shear and compression stress
- **(d)** diagonal unit cracking due to tensile stress
- **(e)** brick unit splitting

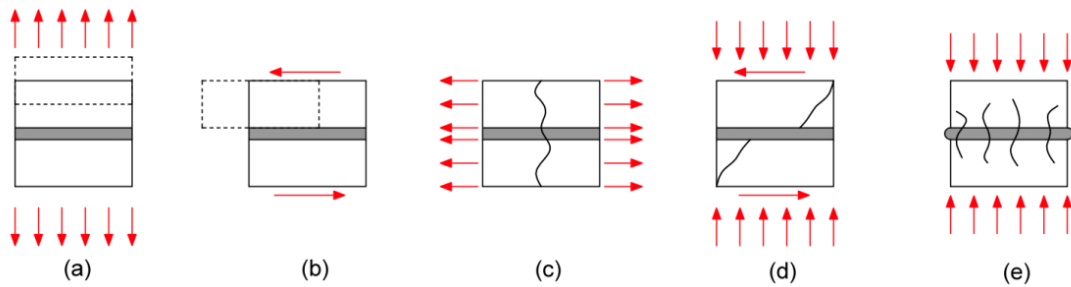


Figure 2.1: Masonry failure mechanisms [16].

2.3.3 Seismic design method

Due to the vulnerability of masonry and the risk of damage during seismic events, it is crucial to consider this aspect in the design of buildings and assess the seismic risk of existing (historical) structures. Standards like EUROCODE 8 propose seismic design methods for masonry constructions, considering peak ground acceleration (PGA) levels with a 10% probability of exceedance over 50 years, specific to each region [14].

Consequently, there is a growing interest in the development of assessment and design methods. Both experimental and numerical studies play a key role in this regard [1]. Over the years, numerical methods especially have been developed and, as the focus of this study aims to achieve, the advancement of these methods relies on their validation, which includes comparison with experimental results.

2.3.4 Experimental tests for masonry assessment

As the validation of the present numerical method used is based on experimental tests, this section provides an overview of existing methods. Approaches typically begin with experimental tests, which provide the most realistic way to study masonry behaviour [11]. Calvi et al. (1996) [11] summarize the different types of experimental tests for unreinforced masonry:

- *Material tests:* Material tests are primarily focused on obtaining mechanical properties of masonry at a local scale, such as compression strength, elastic modulus, and shear strength. Since masonry is a composite material, the characterization of its properties should not solely rely on individual material tests (blocks or mortar) but also on interaction tests that consider brick-mortar interface properties.

While these tests offer simplicity and relatively low cost, their results are particularly useful for analytical and numerical models set-up. However, they will not be the subject of study in this research.

- *Structural component tests:* Simple masonry wall panels are loaded most of the time axially to study their in-plane resistance, which is crucial for buildings to withstand seismic actions. Structural component tests can be conducted either statically or dynamically, providing information about masonry wall properties.

Although these tests are more complex and expensive as they consider entire walls, they are more comprehensive than material tests. One of these experimental tests will be studied in Section 4.2.3.2.

- *Structural system tests:* This category of tests involves studying 2D or 3D masonry structures and can be further divided into two main types:
 - *Quasistatic tests:* Quasistatic tests apply equivalent static forces to represent the seismic response of structures. One frequently used example is tilting tests, two of which will be studied in Sections 4.2 and 5.1.

- *Dynamic tests*: Dynamic tests involve subjecting masonry structures to dynamic loads using shaking tables. These tests provide a more in-depth understanding of masonry behaviour, as they analyse the structural response history over time and consider the accurate distribution of inertia forces. Chapter 5 is dedicated to two dynamic benchmarks, allowing for a comprehensive exploration of the dynamic response of masonry structures.

Both types of structural system tests are essential for characterizing masonry behaviour, as they yield different valuable information. The choice of the equivalent load or the applied ground motion must be carefully considered in both cases. These tests are generally more complex and expensive but offer a more realistic representation of masonry behaviour. They can be conducted at either reduced scale or full scale.

Experimental tests can be carried out in laboratories or on-site. They can be either destructive or non-destructive [11, 17]. These tests provide valuable insights into the local and global mechanisms of masonry.

2.4 Modelling of masonry structures

2.4.1 Global overview of existing methods

Numerical modelling of masonry structures poses significant challenges due to the material's composite nature and non-linear behaviour. Over the years, various methods have been developed to address these complexities, aiming to capture different failure modes with accuracy and simplicity [10, 18]. The choice of the most suitable method depends on factors such as desired accuracy, computational time, and available problem data [10, 19]. In this section, different methods for modelling masonry structures are summarized, highlighting their characteristics and applications.

Modelling strategy

As discussed in Section 2.2.2, masonry exhibits a composite nature, which results in various approaches to modelling it. Among all numerical methods for masonry modelling, D'Altri et al. [1, 9] identify four main categories of modelling strategies:

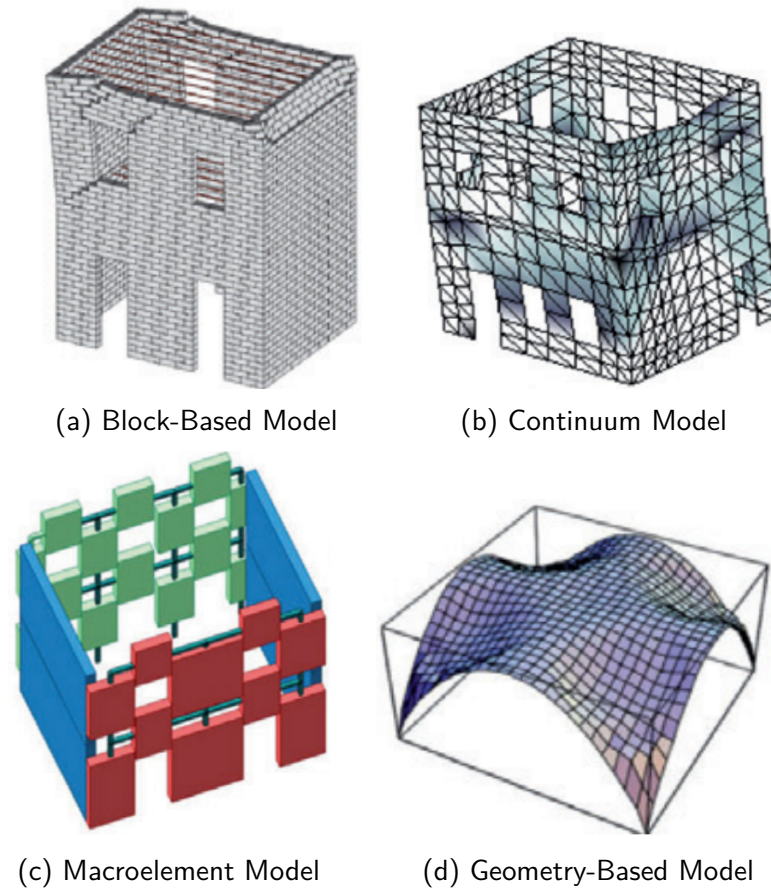


Figure 2.2: Example for the different presented modelling strategies [1].

- **BBM (Block-Based Model - Figure 2.2a):** This method defines the structure block-by-block, using a discontinuum idealization. It distinguishes between individual blocks and joints, making it suitable for capturing the composite nature of masonry. The interaction between blocks can be modelled through various formulations. DEMs follow this strategy, including the NSCD method implemented in LMGC90. BBM is often used in research to explore specific characteristics or behaviours of masonry, as this thesis focusses on this approach. However, these models may require significant computational effort.
- **CM (Continuum Model - Figure 2.2b):** CM represents masonry as a continuous deformable body and reproduces masonry's mechanical behaviour using a continuum constitutive law, which can be challenging to formulate due to the composite nature of masonry. For instance, Finite Element Methods (FEMs) follow this strategy. CM is more commonly used for masonry building studies and is preferred in engineering design [20].

- MM (Macroelement Model - Figure 2.2c): MM treats structures as arrangements of panel-scale structural components (macroelements, representing piers and spandrels) with mechanical-based behaviour. The overall seismic behaviour is closely linked to the in-plane panel response and load redistribution from the diaphragms. For instance, most of these methods model the structure as an arrangement of nonlinear beams. However, local failure cannot be captured.
- GBM (Geometry-Based Model - Figure 2.2d): GBM represents the masonry structure as a rigid body, requiring only geometric and loading input. Structural equilibrium and collapse are studied through solutions based on limit analysis, which can be based on static or kinematic theorems. Although GBM may not be suitable for displacement-based seismic assessment methods, it offers efficient and intuitive tools for assessing the safety and behaviour of masonry arrangements.

DEM and FEM

Most current methods for assessing masonry behaviour in literature fall into two categories: DEM and FEM, belonging to the two first modelling strategies discussed above (BBM and CM) and leading respectively to discontinuum and continuum assumptions, also known as micro- and macro-modelling [9, 19, 20].

Formulations in masonry structure simulations for DEM and FEM are varied. Both methods have evolved, in some cases borrowing features from each other. For example, DEMs may include internal meshes for blocks to account for their elasticity [20]. In general, DEM can be differentiated from FEM based on certain features [20]:

- DEM assumes that blocks are rigid, and deformations can be considered at joints.
- Interaction between blocks must be defined (contact discretization, contact law).
- DEM allows full separation of the blocks.
- DEM uses time-stepping algorithms.

2.4.2 Discrete Element Method (DEM)

The Discrete Element Method (DEM) is a numerical simulation technique used to model the mechanical static or dynamic behaviour of systems composed of multiple discrete bodies, blocks, or particles [20, 21]. Originally developed during the 1960s for analysing slopes in hard rock, DEM has found widespread applications, particularly in

geotechnical engineering and materials science, encompassing various geo-materials like rock, soil, concrete, and masonry [20].

In the context of masonry, DEM represents the structure as a collection of discrete blocks interacting along their boundaries [20, 21]. The fundamental concept behind all DEM approaches is the idealization of masonry as a "discontinuum," which separates the mechanical behaviour of individual units and the interactions between them. This approach treats joints as surfaces of contact between distinct bodies, allowing DEM to model strongly nonlinear behaviours such as joint sliding and total blocks separation [20]. These behaviours can lead to significant relative movements between individual bricks, resulting in changes in the structural geometry and connectivity [20].

Three general features of DEM for masonry simulation, as defined by Sarhosis et al. [19], are:

- The method incorporates finite-sized discrete bodies that can move (and, in some cases, deform) independently of each other.
- Finite displacements and rotations of the discrete elements are permitted.
- As the calculation proceeds, the method automatically identifies and updates new contacts between discrete elements while accounting for the loss of existing contacts.

2.4.2.1 Modelling assumptions

When employing DEM for masonry simulation, certain assumptions about the model need to be made [20]:

- **Block representation:** There are two assumptions regarding the mechanical behaviour of a block:
 - The block behaves like a **rigid body**. In this case, deformations are assumed to be negligible (in rigid contact models) or concentrated in the elastic contacts (in deformable contact models).
 - The model accounts for **block deformability**. The most general approach is to discretize each block into a Finite Element (FE) mesh.

Both block representations can be modelled in LMGC90. For the purpose of this study, all the simulations presented of this study assume rigid blocks.

- **Contact representation:** The interaction between blocks can be represented in two main ways:
 - **Contact points** (Figure 2.3a): Each contact force is a function of the relative block displacement at that point. This assumption is the easiest and most common, and it is the one implemented in the LMGC90 software.
 - **Joint element** (Figure 2.3b): The contact can be modelled using edge-edge formulations, where the contact is discretized based on interacting line segments of each block.

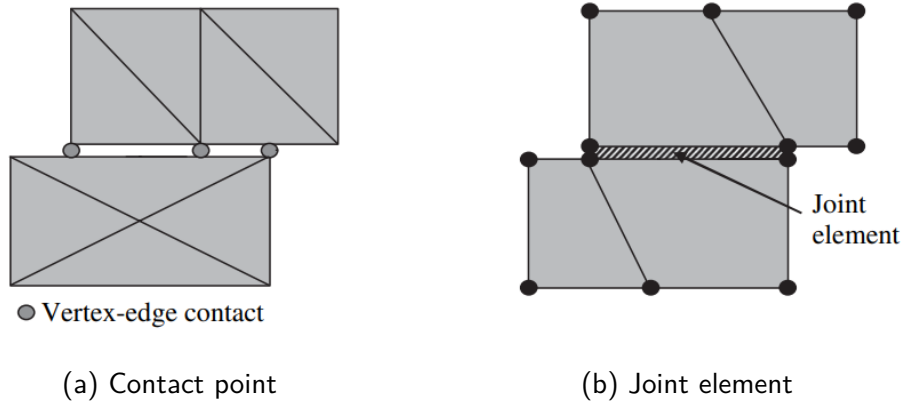


Figure 2.3: Different contact representation [20].

- **Contact behaviour (law):** Two main ways to define the contact behaviour are:
 - **Rigid contact** or hard contact (Figure 2.4a).
 - **Deformable contact** or soft contact (Figure 2.4b).

Both contact behaviours can be defined in LMGC90 and will be compared in most of the numerical studies of this thesis. The contact law corresponding to these assumptions are detailed in Section 2.5.2.3.
- **Solution methods:** DEM mostly uses explicit time-stepping algorithms, which are common for solving its equations. However, some methods use an implicit algorithm, as is the case for the NSCD method.
- **Contact detection and update:** Various methods exist to determine and update contacts between blocks as their positions evolve. The choice of method significantly impacts simulation time.

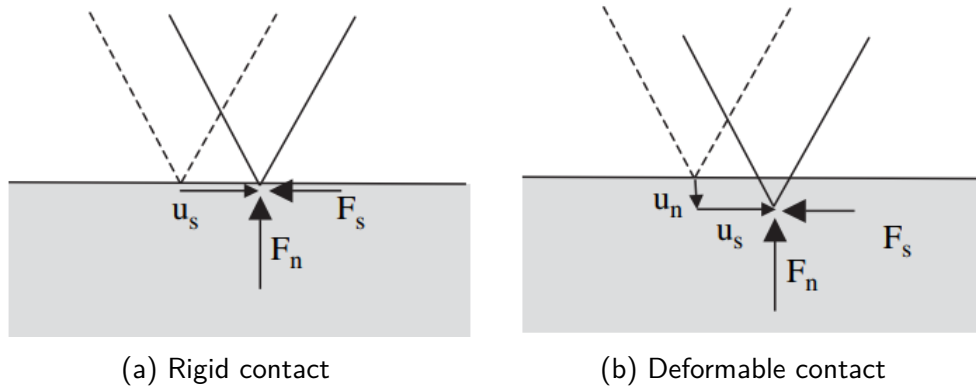


Figure 2.4: Different contact behaviour [20].

2.5 Description of the LMGC90 software and the NSCD method

2.5.1 The LMGC90 software

All the simulations carried out for the study of benchmarks during this thesis were done with LMGC90. This open-source software platform was developed at the LMGC (Laboratoire de Mécanique et Génie Civil, unit of the University of Montpellier), with the aim of developing an advanced simulation tool and making it accessible for engineering purposes [7, 8]. The program LMGC90 finds applications in various fields such as granular physics, rock mechanics or masonry buildings studies [7, 8].

LMGC90 is implemented with the Non-Smooth Contact Dynamics (NSCD) method, which is developed in section 2.5.2. A special feature is that it allows the modelling of large objects collections in interaction. The software can model bodies with various geometries and behaviours in 2D or in 3D and define their interactions with a wide range of contact laws (rigid, elastic, elasto-plastic, etc) [7, 8].

2.5.2 The NSCD method

The Non-Smooth Contact Dynamic (NSCD) method, developed by Moreau and Jean in the 1980s and implemented in the LMGC90 software [3, 18, 22]. It is part of the Discrete Element Methods (DEMs) and therefore shares the characteristics mentioned in the previous section 2.4.2.

This approach is specifically designed to solve dynamic frictional contact problems [18], using non-smooth interaction laws and incorporating the dynamic equation implicitly, in contrast to conventional methods based on smooth interaction laws (continuous and differentiable functions) with explicit integration schemes [3].

2.5.2.1 Strategy and implemented method

The NSCD method addresses the challenge of obtaining the overall behaviour of a collection of objects by considering the dynamics of each element individually and accounting for interactions between them [3]. The problem is at two levels [3, 18, 22, 4]:

- At a global level: The movement of the particles is described by the equations of motion based on global variables (Section 2.5.2.2).
- At a local level: The interactions between the bodies are described by the contact laws based on local variables (Section 2.5.2.3).

The passage from the local level to the global level is accomplished by employing matrices $\mathbb{H}(q)$ and ${}^T\mathbb{H}(q)$ (Figure 2.5), with q , the discrete displacement. These allow to make the connection between the relative velocities $\mathcal{V}^\alpha$ at a given contact α and the set of bodies velocities $\mathbf{V}$, and between the relative contact forces $\mathcal{R}^\alpha$ at a given contact α and the set of resulting contact forces $\mathbf{R}$ (Figure 2.5).

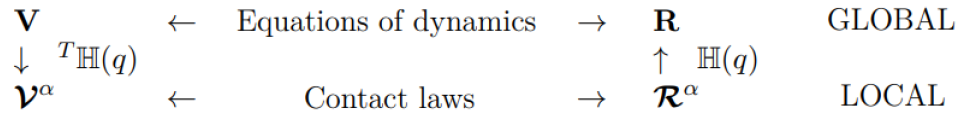


Figure 2.5: Implemented LMGC90 strategy [3].

2.5.2.2 Dynamic equation

The implemented equation governing a body kinematics in the NSCD method is developed in equation 2.1 [3, 4, 22, 23]:

$$\mathbb{M}\dot{\mathbf{V}} = \mathbf{F}_{int}(q, v, t) + \mathbf{F}_{ext}(t) + \mathbf{R} \quad (2.1)$$

where $\mathbb{M}$ is the inertia matrix made up of mass and inertia components,
 $\dot{V}$ is the derivative of the velocity function,
 $\mathbf{F}_{int}(q, v, t)$ represents the internal forces,
 $\mathbf{F}_{ext}(t)$ represents the external forces,
 $\mathbf{R}$ represents the contact forces (accounting for regular contact forces and those due to body shocks as well).

The equation can also be rewritten in differential form to take account of the time step. This is the equation that the program considers for its solution strategy.

2.5.2.3 Description of the used contact laws

The software LMGC90 offers various contact laws that can take into account numerous parameters (friction, cohesion, etc.). The simulation results presented in the following chapters will only use the two following contact laws: rigid law (IQS_CLB) and elastic law (ELASTIC_REPELL_CLB).

Rigid contact law (IQS_CLB)

Using a rigid contact law, contact interactions are defined by the two following laws [3, 4, 22, 23]:

- In the **axial** direction: the Signorini law or inelastic shock law (Figure 2.6):

$$\begin{cases} \text{if } g > 0 & \mathcal{R}_n = 0 \text{ and } \mathcal{V}_n \text{ arbitrary} \\ \text{if } g \leq 0 & \mathcal{V}_n \geq 0 \text{ and } \mathcal{V}_n \cdot \mathcal{R}_n = 0 \end{cases} \quad (2.2)$$

where g is the gap between two bodies,
 $\mathcal{R}_n$ is the normal reaction,
 $\mathcal{V}_n$ is the normal relative velocity.

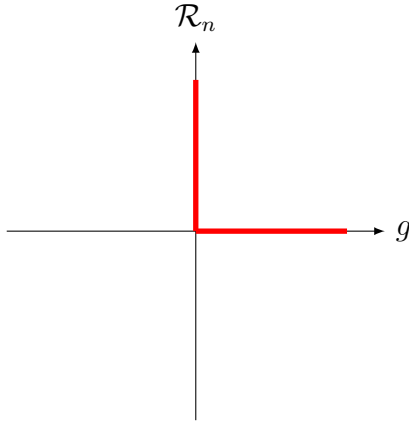


Figure 2.6: Signorini law

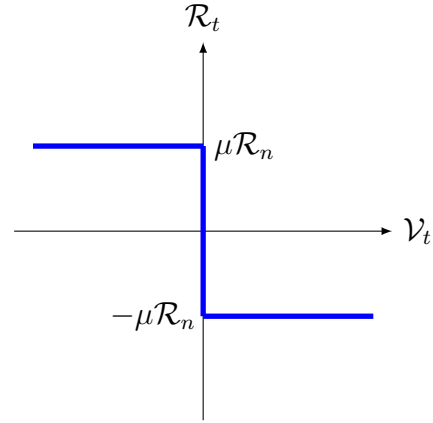


Figure 2.7: Coulomb law

- In the **shear** direction: the Coulomb law (Figure 2.7):

$$\mathcal{R}_n \geq 0 \text{ and } \|\mathcal{R}_t\| \leq \mu \cdot \mathcal{R}_n \begin{cases} \|\mathcal{R}_t\| < \mu \cdot \mathcal{R}_n \Rightarrow \mathcal{V}_n = 0 \\ \|\mathcal{R}_t\| = \mu \cdot \mathcal{R}_n \Rightarrow \mathcal{V}_n = -\lambda \cdot \frac{\mathcal{R}_t}{\|\mathcal{R}_t\|} \end{cases} \quad (2.3)$$

where μ is the friction coefficient,
 $\mathcal{R}_t$ is the tangential reaction,
 $\mathcal{V}_t$ is the tangential relative velocity,
 $\lambda > 0$.

Elastic contact law (ELASTIC_REPELL_CLB)

Using an elastic contact law, contact interactions are defined by the two following laws [23, 24]:

- In the **axial** direction: the elastic law (Figure 2.8):

$$\begin{cases} \text{if } g > 0 & \mathcal{R}_n = 0 \text{ and } \mathcal{V}_n \text{ arbitrary} \\ \text{if } g \leq 0 & \mathcal{V}_n \geq 0 \text{ and } \mathcal{R}_n = -k \cdot \mathcal{V}_n \end{cases} \quad (2.4)$$

where k is the constant axial stiffness of the contact.

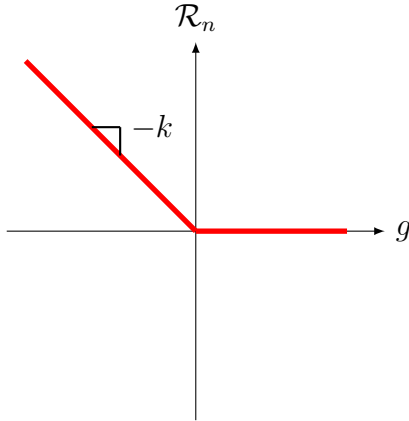


Figure 2.8: Elastic Law

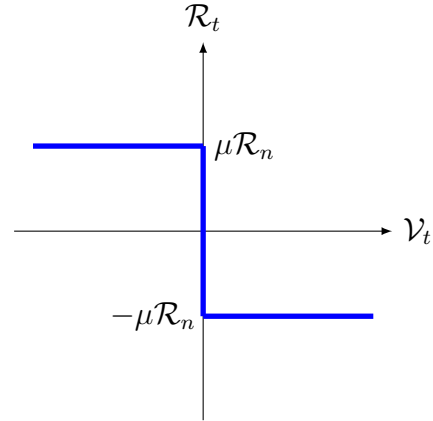


Figure 2.9: Coulomb law

- In the **shear** direction: the Coulomb law (Figure 2.9):

$$\mathcal{R}_n \geq 0 \text{ and } ||\mathcal{R}_t|| \leq \mu \cdot \mathcal{R}_n \begin{cases} ||\mathcal{R}_t|| < \mu \cdot \mathcal{R}_n \Rightarrow \mathcal{V}_n = 0 \\ ||\mathcal{R}_t|| = \mu \cdot \mathcal{R}_n \Rightarrow \mathcal{V}_n = -\lambda \cdot \frac{\mathcal{R}_t}{||\mathcal{R}_t||} \end{cases} \quad (2.5)$$

2.5.3 Use of the program in the context of the thesis

The software is used through an interface that utilizes the Python programming language. A first Python file is used to define the geometry of the different elements (bricks) and the parameters of the simulated problem. One notable feature of the program is its ability to define different geometries, properties, and contact laws for each element in a single simulation. A second Python file is used to specify the simulation parameters, such as time step, simulation time, and optionally a condition for stopping the simulation. The various functions and details of the contact laws can be found in the program's documentation [23]. The 2D and 3D models have entirely distinct functions.

The software can provide a visual representation of the structure at specified moments during the simulation, along with the relative values of displacements and forces for each degree of freedom (DoF). The images presented in this document are generated using the scientific visualization software Paraview. The graphs showing numerical data from the simulations are created using a Jupyter Notebook (Python).

Before proceeding with the validation studies, a preliminary familiarization with the software was conducted by following the tutorials provided with the software and reproducing two problems that were previously simulated with LMGC90 by Bouckaert et al. (2021) [5].

2.6 Previous studies

2.6.1 Other studies realised with LMGC90

A preliminary review was conducted to search for studies already carried out using the LMGC90 software (for masonry simulations). Several practical cases have been studied with the software: for instance, seismic resistance of a bridge [25] or the behaviour of a medieval church [26]. Other studies have aimed to examine the behaviour of specific types of masonry structures, such as the seismic behaviour of domes [27].

However, few works focusing on comprehensive validation of numerical models established with LMGC90 using experimental results are found in the scientific literature. Some studies include comparisons between numerical and experimental results. For instance, an examination of failure modes of an in-plane loaded masonry panel [28], a study on the numerical stability of the software based on results from a tilting test (static) [24] or a brief presentation of several numerical and experimental failure mechanisms with rigid and deformable blocks [29]. Apart from a few more examples, the comparisons between numerical and experimental results are mostly limited to those just mentioned. They generally involve relatively simple geometry masonry structures (often in 2D) subjected to static loading. These studies generally demonstrate that LMGC90 generally captures masonry behaviour, but they tend to be relatively concise and do not explore the influence of various parameters on the results.

Other validation works comparing LMGC90 results with numerical outcomes can also be found in the literature. For example, Bouckaert et al. (2021) [5] compared LMGC90 results with those from UDEC for static 2D benchmarks (walls subjected to horizontal loading) and dynamic benchmarks (columns and frames). Studies involving larger-scale structures like towers have compared seismic responses using other numerical methods (DEM) as well [30, 31].

2.6.2 UDEC

Among the software that uses the DEM, UDEC and its 3D equivalent 3DEC are the most widely used programs in studies on the seismic behaviour of masonry. The main difference with LMGC90 is that UDEC represents its contacts are with joint elements (Figure 2.3b) and assumes that elements have round corners [32].

In recent years, the development of DEM has enabled numerous numerical studies on masonry structures. The method finds extensive applications in understanding the behaviour of masonry in various scenarios, including masonry towers, ancient columns or colonnades, masonry walls, arches, vaults, and even more complex or larger masonry structures [19]. The literature contains numerous examples of its applications, such as the study of the collapse mechanism of a 3D C-shaped masonry structure with the software 3DEC (by Lemos and Campos (2017) [33]) or the study of seismic behaviour of ancient multidrum columns with the software UDEC (by Papastamatiou and Psycharis (1993) [34]). Additionally, the literature provides validations of the UDEC program using experimental benchmarks (for instance, Bui et al. (2017) [35] compare numerical results with experimental ones based on static and dynamic, 2D and 3D benchmarks) or analytical models (for instance, De Lorenzis et al. (2007) [36] compare numerical and analytical response of masonry arches under impulse base motion). Many other examples are compiled and explained in the review conducted by Sarhosis et al. (2019) [19].

2.7 Conclusion

In conclusion, this literature review has allowed the study of masonry behaviour and supplied insights into existing numerical methods and modelling strategies for assessing masonry seismic responses. It has also contextualised the NSCD method implemented in LMGC90 among other numerical methods. This method is part of the DEM, considering a collection of rigid bodies interacting with each other, enabling the simulation of masonry's composite and non-linear behaviour. A distinguishing attribute of this software lies in its representation of block interactions via contact points. Within this thesis, the contact behaviour is characterized by either rigid or elastic contact laws. Moreover, the review of the literature has highlighted the paucity of validation work involving the LMGC90 software against experimental benchmarks.

CHAPTER 3

Validation with 2D static benchmarks

This part of the thesis aims to evaluate the ability of the program to reproduce masonry behaviour in 2D static cases. The first problem, presented in Section 3.1, will be thoroughly analysed, testing a large number of parameters to draw relevant conclusions and apply these analyses to subsequent problems. The choice of the first benchmark was motivated by its comprehensive procedures and results, making it suitable for a detailed parametric study. The first benchmark involves an experiment that assesses the impact of foundation settlements on a masonry wall.

In the second part of this chapter (Section 3.2), a second benchmark will be analysed more briefly to validate the previous observations. This benchmark focuses on the behaviour of a dry-jointed wall subjected to in-plane loading.

3.1 Masonry wall subjected to foundation settlements

3.1.1 Description of the experimental benchmark

This study was designed and realised by Portioli and Cascini (2016) [37]. Two experiments considering two full-scaled walls of different heights reproduced the impact of foundation settlements by lowering a part of the wall foundation. These aim to study the wall behaviour and the base reaction throughout the experiment.

3.1.1.1 Experimental setup and parameters

Two experiments were conducted on two simple brick panels of different heights, labeled 8C and 12C. Both models consist of the same dry-jointed bricks with dimensions of $200 \times 100 \times 50$ mm [37]. As shown in Figure 3.1, the 8C and the 12C walls are composed of respectively eight and twelve rows of seven-and-a-half bricks (for total wall dimensions of $1100 \times 400 \times 50$ mm and $1100 \times 600 \times 50$ mm). The specified value of masonry density was 12 kN/m^3 and the friction coefficient was 0.4 [37]. However, the document does not state how these two values were determined.

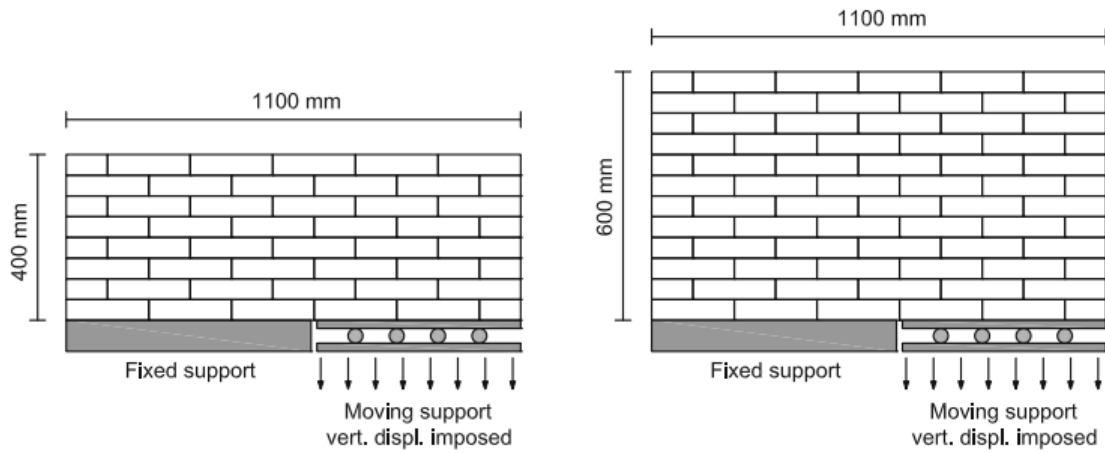


Figure 3.1: Geometry and boundary conditions (8C and 12C wall) for the experimental panels tested by Portioli and Cascini (2016) [37], image from [38].

The support of the walls (foundation) was divided into two parts. The experimental setup allowed the right-hand side of the foundation to be moved vertically (Figure 3.1). For both experiments, a vertical incremental downwards displacement of up to 50 mm (steps of 1 mm) was imposed on this support to reproduce the settlement of the foundation. Rollers have also been placed under the foundation to allow it to move freely horizontally.

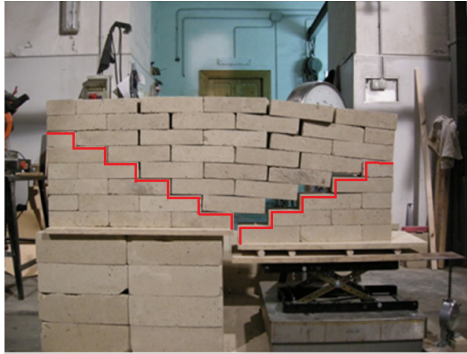
The experimental setup also allowed measuring the vertical displacement of the right-hand side of the foundation and its base reaction throughout the experiment (for every millimeter).

3.1.1.2 Experimental Results

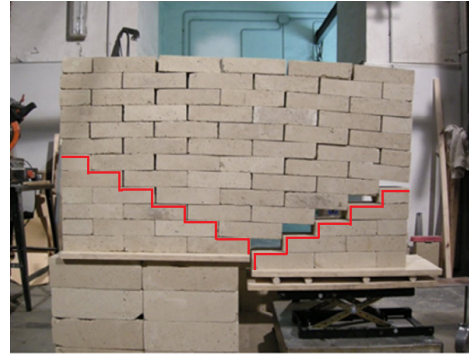
The results of the experiment are characterised by two elements:

3 – Validation with 2D static benchmarks

- The **failure mechanism** of the walls (pictures) for displacement values of 50 mm. These differ for both walls. For the 8C wall (Figure 3.2a), three parts of the wall can be distinguished: two triangular-shaped sets of bricks at the bottom and a middle set where cracks are visible. The same three zones can be observed for the 12C wall, but it splits into two distinct parts as the foundation goes down (Figure 3.2b). The left-hand part of the wall deforms slightly and cracks appear.



(a) 8C wall



(b) 12C wall

Figure 3.2: Experimental failure mechanism obtained by Portioli and Cascini (2016) [37].

- The **force-displacement relation** (Figure 3.3) shows the relationship between the computed displacement and the base reaction values of the right-hand foundation, for displacement values between 0 and 50 mm. This relationship can be likened to a classic force-displacement relationship of a material, but reversed as the value of the base reaction decreases. Initially, there is a linear zone at the beginning of the curve, where its slope remains relatively constant, followed by a plateau.

Several characteristic values can be drawn from this relationship and are detailed for both walls in Table 3.1:

Wall	Initial slope [N/m]	Initial base reaction [N]	Final base reaction [N]
8C	32	254	104
12C	42	379	87

Table 3.1: Characteristic values of the experimental force-displacement relation

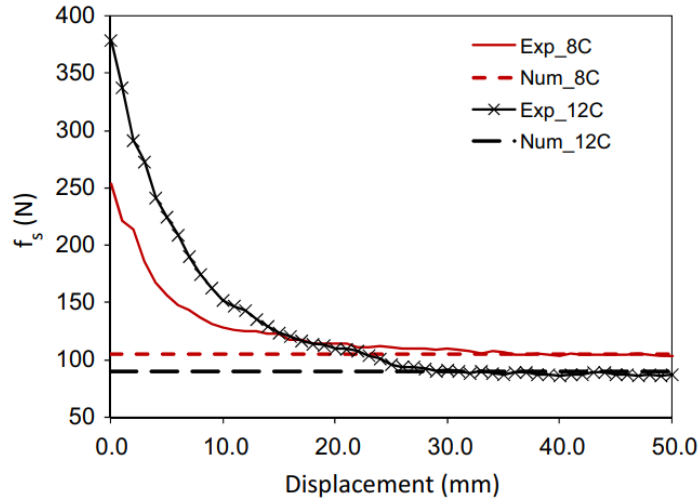


Figure 3.3: Base reaction vs vertical displacement, comparison between experimental and numerical results, obtained by Portioli and Cascini (2016) [37].

3.1.1.3 Numerical results from other studies

The numerical simulation of this problem has been previously studied in two other papers with the aim of reproducing the experimental results:

- The first study was conducted by Portioli and Cascini (2016) [37], in the same paper as the experiment. The numerical results closely match the experimental ones, with final base reactions of 105 N for the 8C wall and 90 N for the 12C wall (for a base displacement of 50 mm), as represented by the horizontal dashed lines in Figure 3.3. The failure mechanisms (Figures 3.4a and 3.4b) obtained in the numerical simulation are also similar to the experimental ones, although the

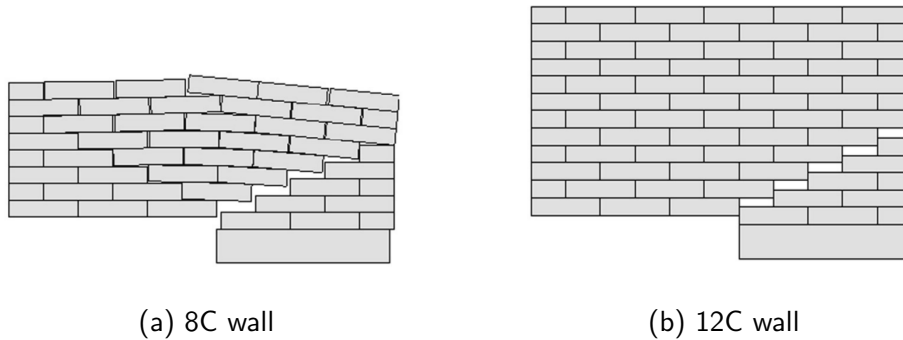


Figure 3.4: Numerical failure mechanism obtained by Portioli and Cascini (2016) [37].

3 – Validation with 2D static benchmarks

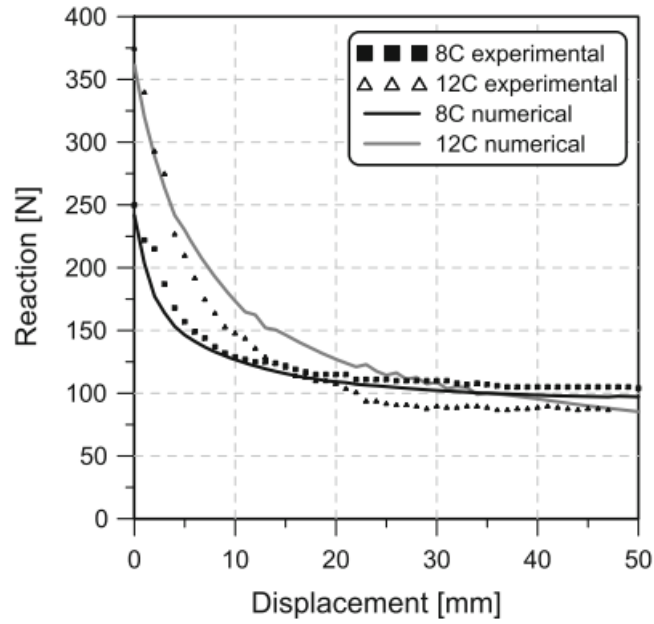


Figure 3.5: Base reaction vs vertical displacement, comparison between experimental and numerical results obtained by Spada (2019) [38].

left-hand part of the 12C wall remains completely upright and stiff, which differs from the experimental observation.

- The second study was conducted by Spada (2019) [38], using an interphase model that accounts for the elasticity of the bricks (FEM) and discretised bricks. The numerical force-displacement relations closely match the experimental ones, as shown in Figure 3.5. However, the failure mechanisms of the walls in the

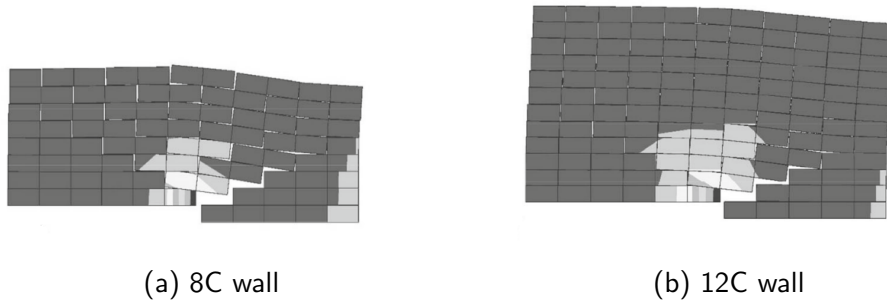


Figure 3.6: Numerical failure mechanism obtained by Spada (2019) [38].

numerical simulation (Figures 3.6a and 3.6b) show a too flexible behaviour, which differs from the experimental observations. In particular, the 12C wall does not completely split, and its left part still touches and rests on the right-hand bottom triangle of bricks.

3.1.2 Description of the numerical model

To simulate the experimental behaviour of the walls, numerical models for both the 8C and 12C configurations were created using the LMGC90 software. The models incorporate the geometric data, friction coefficient (0.4), and masonry density (12 kN/m^3) values from the experiment, as detailed in the previous section 3.1.1.1.

Each simulation duration was 210 seconds, with a time step of 10^{-4} s . The vertical displacement imposed to the right-hand side foundation is detailed in Figure 3.7. No move was imposed during the 10 first seconds, to allow the model to stabilise. Indeed, very slight movements of the blocks at the start of the simulation due to blocks adjustment can induce non-zero forces even before the base is moved, so this stabilisation period is necessary to dissipate these effects and observe only the effects of the base movement. Afterwards, the support was moved down at a constant speed up to 100 mm of foundation displacement. Throughout the simulation, its horizontal displacement was also set free, to match with the impact of the rollers beneath the foundation (Figure 3.1).

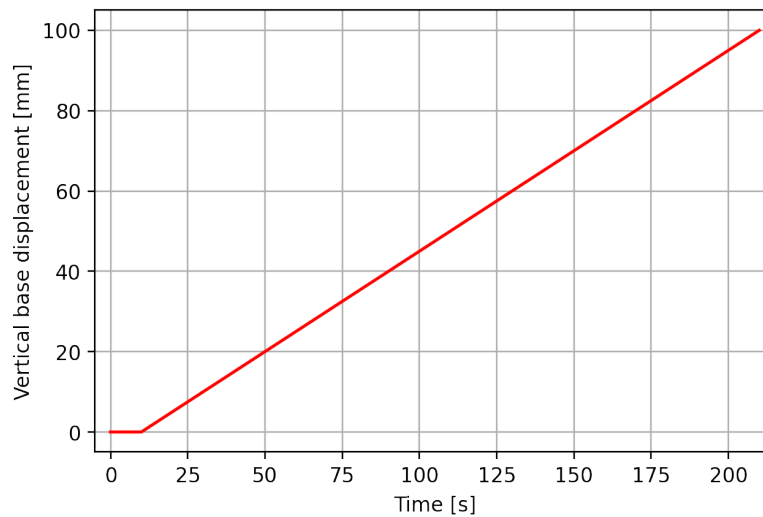


Figure 3.7: Vertical downwards displacement imposed to the right-hand side foundation during the simulation.

3.1.2.1 Studied parameters

The objective of this initial benchmark study was to investigate the influence of various parameters and thoroughly analyse the problem to obtain results that closely align with the experimental data. The models were developed by incorporating the characteristics mentioned in Section 3.1.2. Two categories of parameters were examined: contact laws and element (brick) types. The models were established step by step, starting with simpler parameters, and evolved based on the previous simulation results. Only the models that yielded significant conclusions are presented in this section, and Table 3.2 summarises all fourteen models along with their characteristics in Section 3.1.2.2.

Rigid contact law (Models 1-2)

The initial modelling of both 8C and 12C walls (Models 1 and 2) assumed simple rectangular bricks, each composed of four contact points at their corners (Figure 3.8a). The contact law chosen for these models was a simple rigid law (IQS_CLB), developed in Section 2.5.2.3.

Elastic contact law (Models 3-8)

Subsequently, additional simulations were conducted with the same four-point bricks (Figure 3.8a), but this time, an elastic contact law (ELASTIC_REPELL_CLB, developed in Section 2.5.2.3) was used. Among the two wall configurations, the 12C case was chosen as the reference due to its experimental results with distinct features, facilitating better comparisons than with the 8C model (e.g., clear separation between the 2 sets of bricks in the failure mechanism, resulting in a fixed value of the final base reaction).

As the experimental documentation lacked information on the elastic modulus of the masonry, several contact stiffness values were empirically tested. Initially, a stiffness of 5 MN/m was assumed to be realistic for the masonry, and the first simulation was initiated with this value. The contact stiffness was then adjusted to align the initial force-displacement curve slope with the experimental one (Figure 3.3). Four simulations (Models 3-6) were presented with contact stiffness values of 5, 2.5, 2, and 1.25 MN/m. As lower stiffness values led to a masonry behaviour that was too flexible in the 12C case, next simulations with an elastic contact law only considered stiffness values of 5 and/or 2.5 MN/m.

3 – Validation with 2D static benchmarks

Finally, the same elastic contact law was applied to the model of the 8C wall to confirm whether the same conclusions could be drawn for this configuration. This time, the simulations only considered stiffness values of 5 and 2.5 MN/m (Models 7-8). Other contact laws implemented in LMGC90 were tested in the 12C case but did not yield conclusive results.

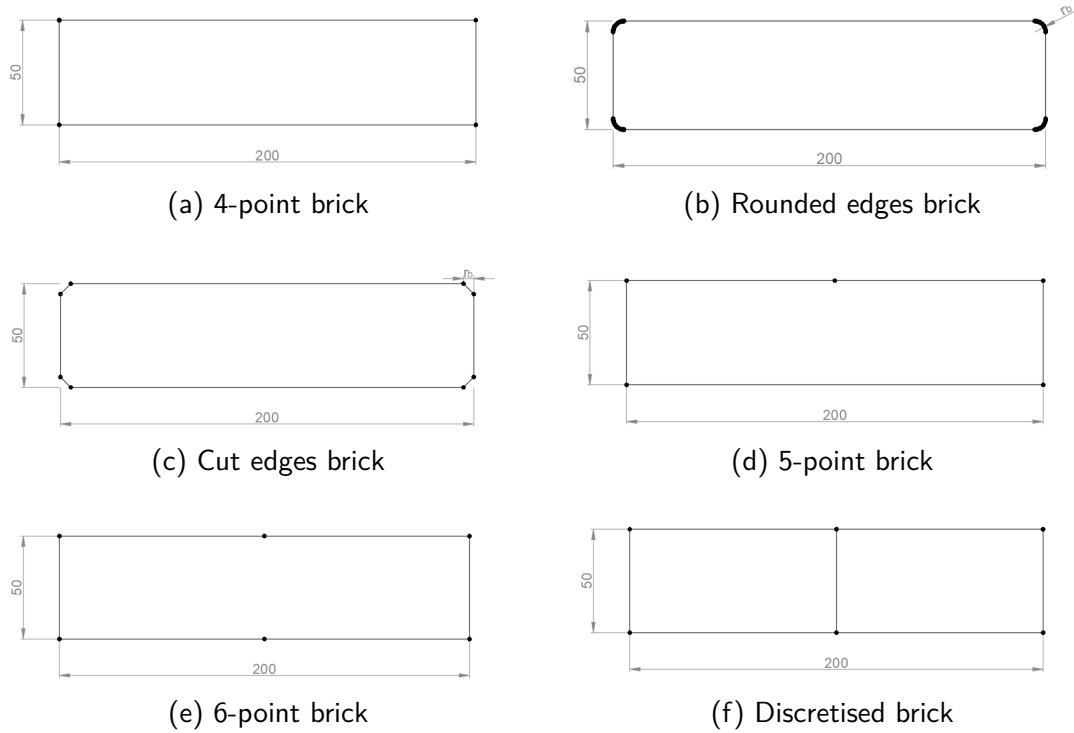


Figure 3.8: Different types of brick used.

Influence of the brick edges (Models 9-10)

As bricks in experimental conditions do not have completely sharp edges, the next simulation aimed to analyse the influence of rounded edges, inspired by Bouckaert et al. (2021) [5]. Once again, these simulations were performed on the 12C wall case to facilitate comparisons with the results.

The parameter was first evaluated through a simulation considering a rigid contact law. To create rounded edges, additional contact points in the shape of an arc of a circle were added to each corner of the brick, resulting in 40 contact points (10 in every

corner) instead of 4 for the initial rectangular brick (Figure 3.8b). Five different values of the edges curvature radius r_b were tested, ranging from 2 to 5 mm. However, the results are discussed but not presented as they were inconclusive.

The influence of the type of edges was also examined in other simulations, using an elastic contact law. Adding multiple contact points to create a rounded edge could have increased the wall's elasticity. To avoid this effect, the edges of the brick were simply cut to keep only two contact points per face (Figure 3.8c). Models 9 and 10 experiment values of radius of truncature r_b of 2 and 5 mm. Both simulations again considered a stiffness value of 5 MN/m, as considering a 2.5 MN/m led to inconclusive results.

Influence of the number of contact points (Models 11-12)

This section focuses on the influence of adding contact points to the bricks, which is assumed to provide more accurate results by allowing the software to detect contacts between blocks at more locations. Several types of bricks were tested on the example of the 12C wall, starting with a six-point brick (Figure 3.8e). However, most of these simulations resulted in numerical errors and random horizontal brick motion (possibly related to frictional force calculations).

The only successful results obtained were using five-point bricks (keeping only the top middle additional contact point, Figure 3.8d) with an elastic law and stiffness values of 5 and 2.5 MN/m (Models 9 and 10). Ideally, the stiffness should have been adapted to account for the new fifth point (representing an additional spring), but for comparison purposes, it was kept constant. As the program sets the same stiffness for every contact point, establishing a completely realistic model would be challenging anyway.

Influence of the bricks discretisation (Models 13-14)

To overcome the limitation of adding more contact points to a single brick, the impact of bricks discretisation (Figure 3.8f) on the simulation results was analysed, inspired by the study conducted by Spada (2019) [38], developed in Section 3.1.1.3. Again, this approach aims to provide more accurate results by introducing more contact points.

In this case, every brick was discretised into two half bricks (with the same height and half the length of the initial four-point brick, as shown in Figure 3.8f). The two half

elements of one single brick are joined and no displacement between them is allowed, forming one single body in LMGC90. Two simulations were performed on the example of the 12C wall, using the elastic contact law with stiffness values of 5 and 2.5 MN/m (Models 13 and 14).

3.1.2.2 Summary of studied models and compared parameters

Below is the summary of all the studied and presented models, along with their parameters and the corresponding section where the results are presented and discussed (Table 3.2):

N°	Wall	Contact law	Brick type	Section
1	12C	Rigid	4-point	3.1.3.1
2	8C			
3	12C	Elastic (5 MN/m)		3.1.3.2
4		Elastic (2.5 MN/m)		
5		Elastic (2 MN/m)		
6		Elastic (1.25 MN/m)		
7	8C	Elastic (5 MN/m)		
8		Elastic (2.5 MN/m)		
9	12C	Elastic (5 MN/m)	Cut edges ($r_b = 2 \text{ mm}$)	3.1.3.3
10			Cut edges ($r_b = 5 \text{ mm}$)	
11		Elastic (5 MN/m)	5-point	3.1.3.4
12		Elastic (2.5 MN/m)		
13		Elastic (5 MN/m)	Discretised	3.1.3.5
14		Elastic (2.5 MN/m)		

Table 3.2: Summary of all the studied and presented models.

3.1.3 Numerical results

In this section, the results obtained for each of the fourteen models will be presented and discussed. The comparison with the experimental results will be based on the following aspects:

- The **failure mechanism**, for a displacement of the foundation of 50mm.
- The **general shape** of the force-displacement curve.
- The **initial base reaction** of the foundation, just before the foundation starts moving down (after 10 seconds of simulation).
- The **initial slope**, when the wall still behaves elastically. To compute the initial slope, the first point is taken for a base displacement of 0 mm (after 10 s of simulation). The second point is the one which gives the slope with the correlation coefficient closest to 1, using the least squares method.
- The **final base reaction**. If the curve shows a peak, the final base reaction is characterised by its lowest value (peak). If there is no peak (or if the peak occurs after 50 mm of displacement), the final base reaction is taken as the value for a displacement of 50 mm.

All the quantified values are summarised in Table 3.3 and 3.4.

3.1.3.1 Rigid contact law (Models 1-2)

For the 12C wall simulation (Model 1), the failure mechanism shows that the wall splits into two distinct parts as the foundation goes down (Figure 3.9a), which is also observed in the experimental failure mechanism (Figure 3.2b). However, the left-hand part of the wall remains completely upright and stiff, whereas this part of the wall deforms in the experiment. The left triangular-shaped set of bricks observed experimentally is not visible on Figure 3.9a.

The obtained failure mechanism of the 8C wall (Model 2) (Figure 3.9b) is similar to the experimental one (Figure 3.2a). Again, three parts of the wall can be distinguished: two triangular-shaped sets of bricks at the bottom and a middle set where cracks are visible. However, the failure lines and the exact brick position slightly differ from reality for some bricks.

3 – Validation with 2D static benchmarks

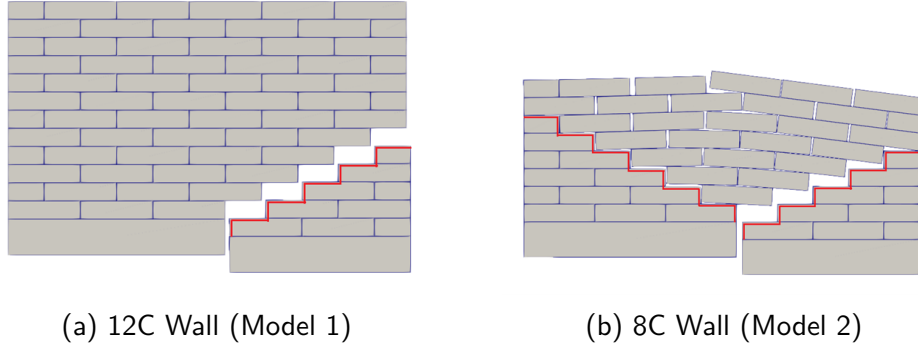


Figure 3.9: Numerical failure mechanisms for both 8C and 12C wall, obtained with LMGC90 using a rigid contact law (IQS_CLB).

The force-displacement relations (Figure 3.10) for both walls drop almost vertically. The initial slopes of the curves are 459 and 790 kN/mm (respectively for the 12C and 8C walls), which are more than 10,000 times higher than the experimental values (41 and 32 kN/mm). This indicates a very stiff behaviour of the wall, consistent with the observations on the failure mechanisms.

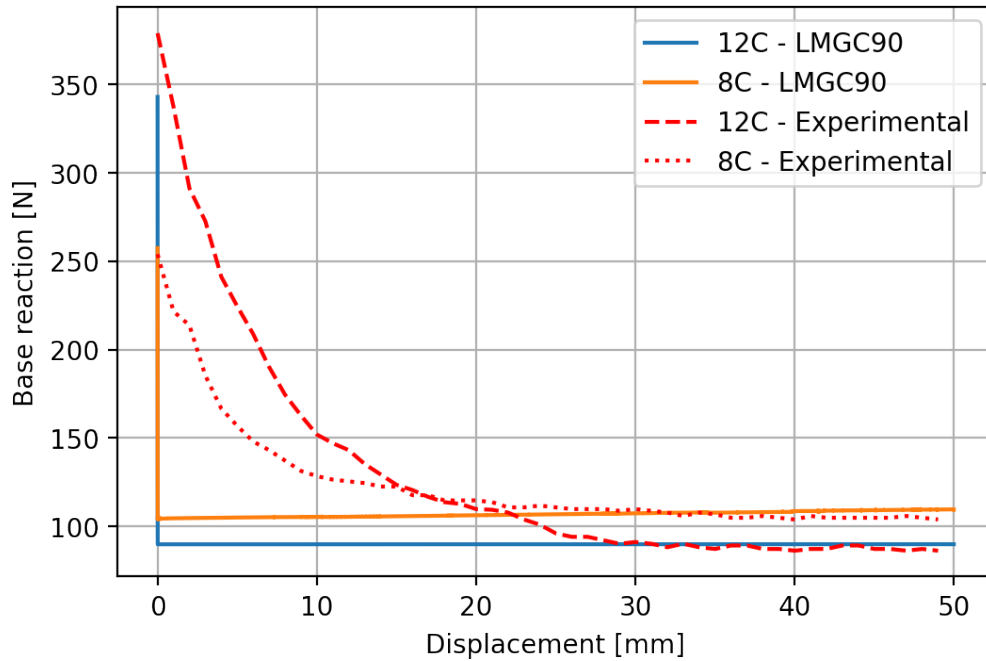


Figure 3.10: Base reaction vs vertical displacement, using a rigid contact law (for 8C and 12C walls, Models 1 and 2).

3 – Validation with 2D static benchmarks

The initial base reaction for the 12C wall (Model 1) is 343 N, slightly lower than the experimental value (379 N). The curve reaches its final value (90 N) almost immediately and remains constant as the weight distribution of bricks does not change after the wall splits into two parts. This value corresponds to the theoretical weight of the right-hand bottom triangle of bricks.

For the 8C wall (Model 2), the initial base reaction is 257 N, slightly higher than the experimental value (254 N). The force-displacement curve shows a peak at 104 N, close to the experimental peak value (103 N). Subsequently, the curve slightly rises, indicating a shift in the weight distribution as the right-hand side foundation continues to move down.

To conclude, the model shows a general masonry behaviour in terms of initial and final base reaction and failure mechanism similar to the one observed experimentally. However, by defining a rigid law, the simulation does not account for the joint stiffness and results in a too stiff behaviour of the masonry. The results for both walls are very similar to the ones obtained by Portioli and Cascini (2016) [37] in terms of failure mechanism (Figures 3.4b and 3.4a) and final base reaction. This confirms the expectations as both models and their contact laws are very similar.

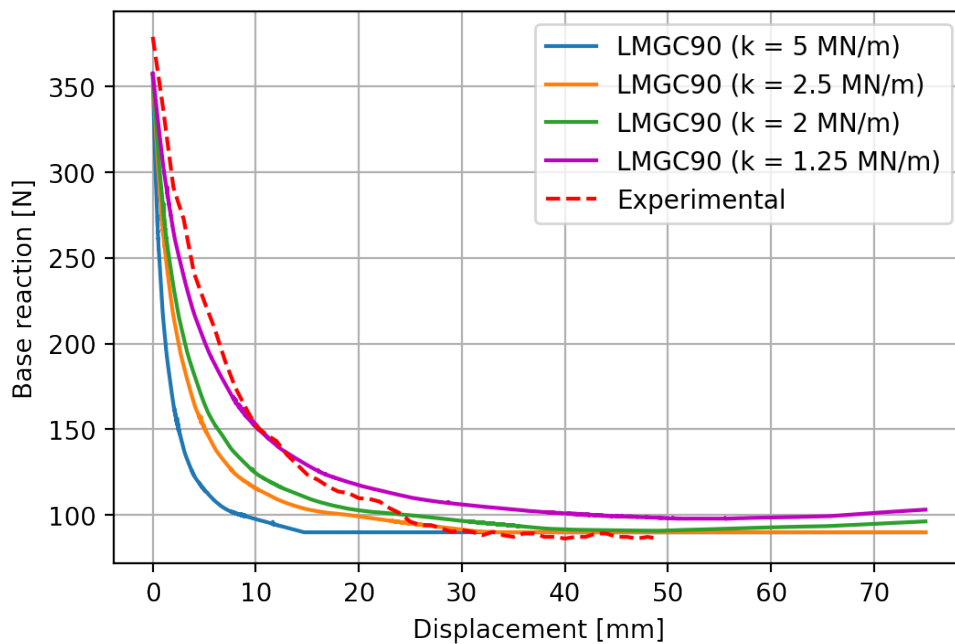
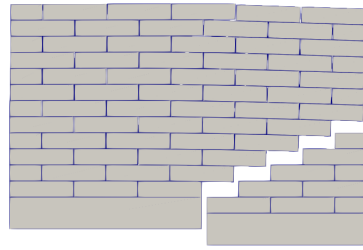


Figure 3.11: Base reaction vs vertical displacement (12C wall) for various contact stiffnesses (Models 3-6).

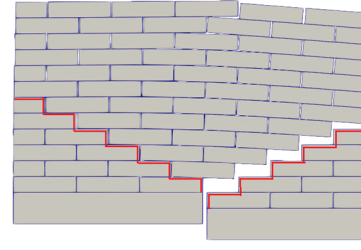
3.1.3.2 Elastic contact law (Models 3-8)

12C wall case (Models 3-6)

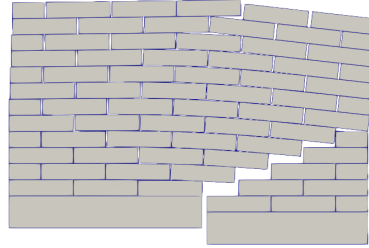
For the 12C wall case, the simulations were conducted with various joint stiffness values using an elastic contact law. Model 3, with a stiffness of 5 MN/m, already shows a failure mechanism (Figure 3.12a) and a force-displacement relation (Figure 3.11) closer to the experimental ones compared to the rigid law model. As the joint stiffness decreases, the simulation results align more closely with the experimental data (Figure 3.11). Model 4, with a stiffness of 2.5 MN/m, provides the closest results to the experiment, showing the left-hand set of bricks deforming similarly to the experiment (Figure 3.12b).



(a) $k = 5 \text{ MN/m}$ (Model 3)



(b) $k = 2.5 \text{ MN/m}$ (Model 4)



(c) $k = 2 \text{ MN/m}$ (Model 5)

Figure 3.12: Numerical failure mechanisms obtained with LMGC90 for the 12C wall, comparing different contact stiffnesses (ELASTIC_REPELL_CLB).

However, for stiffness values below 2 MN/m (Models 5 and 6), the failure mechanism starts to deviate from the experiments, with the left part of the wall touching and resting on the right-hand bottom triangle of bricks (Figure 3.12c). On Figure 3.11, the magenta and green curves go up after reaching a peak around the 50 mm of displacement, meaning that the weight of the wall begins to shift to the right-hand side foundation while this one keeps moving down.

The initial base reaction remains nearly unchanged for all stiffness values, around 358 N, and is closer to the experimental value than the rigid law results. For stiffness values above 2 MN/m, the final base reaction is equal to the weight of the triangle of bricks (90 N). However, for stiffness values below 2 MN/m, the curve peaks at more than 90 N, indicating that the moving foundation supports the weight of additional bricks. The curve then slightly rises, reminding the softening behaviour observed for the 8C case. The initial slope of the curves almost varies linearly with the imposed joint elasticity, with the closest value to the experiment obtained with the 2.5 MN/m stiffness (Model 4), having an initial slope of 111 N/mm, which is still more than 2 times the experimental initial slope (42 N/mm).

8C wall case (Models 7-8)

For the 8C wall case, similar observations were made. The force-displacement relation (Figure 3.14) also improves as the joint stiffness decreases. Model 7 and 8, with stiffnesses of 5 and 2.5 MN/m respectively, provide very similar failure mechanism (Figures 3.13a and 3.13b), that are both closer to the experimental one. This can be observed thanks to the top middle brick which rests on the left part of the wall (as on the experiment, Figure 3.2a) where it tilted to the right on the simulation carried out with the rigid law (Figure 3.9a).

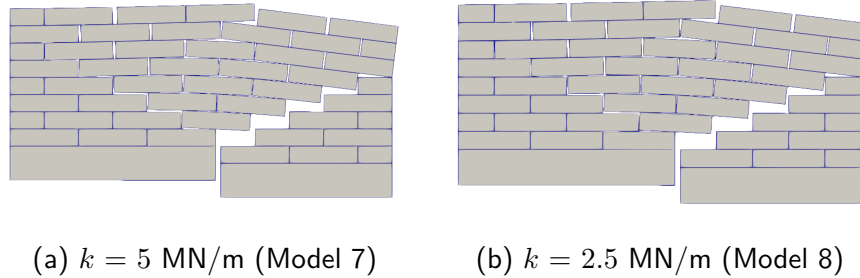


Figure 3.13: Numerical failure mechanisms obtained with LMGC90 for the 8C wall, comparing different contact stiffnesses (ELASTIC_REPELL_CLB).

The initial base reaction and initial slope follow a similar trend to the 12C case, with Model 8 (stiffness of 2.5 MN/m) providing the closest results. The final base reaction occurs between 10 and 20 mm of displacement, before the experimental curve reaches its minimum value. The numerical (105 N and 106 N for Models 7 and 8, respectively) and experimental (103 N) values are still close.

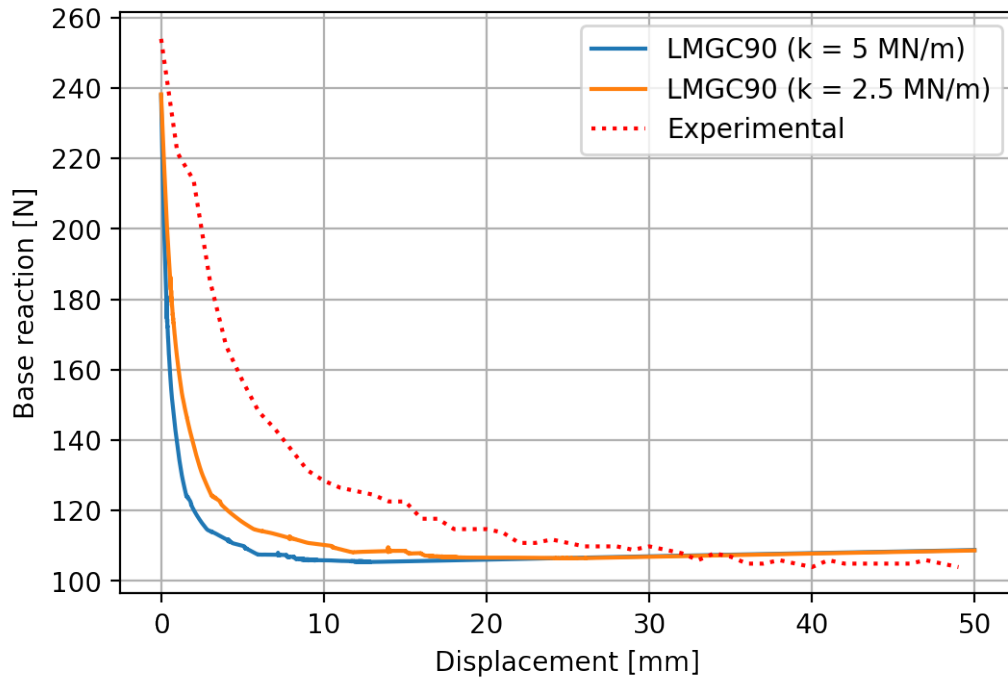


Figure 3.14: Base reaction vs vertical displacement (8C wall) for various contact stiffnesses (Models 7 and 8).

Conclusion

Based on the results obtained, Model 4 for the 12C wall (with a joint stiffness of 2.5 MN/m) and Model 8 for the 8C wall (also with a stiffness of 2.5 MN/m) are considered the closest to the experimental data for both walls. By accounting for the elasticity of the joints, LMGC90 better reproduces the general behaviour of the masonry. However, the elastic law has its limitations and works up to a certain point (minimum contact stiffness of 2.5 MN/m in the particular studied case).

3.1.3.3 Influence of the brick edges

Rigid contact law

When considering a radius of curvature value of 4 mm or smaller, the simulation results remain identical to those with sharp edges (Section 3.1.3.1). However, for a curvature greater than 4 mm, the failure mechanism starts to differ significantly, with the left part of the wall touching and resting on the right-hand side foundation (similar

to the failure mechanism in Figure 3.12c). Therefore, using a rigid contact law, this parameter does not contribute to bringing us closer to the experimental results.

Elastic contact law (Models 9-10)

Under an elastic contact law with a stiffness of 5 MN/m, the results vary depending on the defined radius of truncature (r_b). For a radius of truncature of 2 mm (Model 9), the failure mechanism (Figure 3.15) slightly better resembles the experimental one, with the inclination of the top middle brick being closer to the experiment. However, the overall trajectory of the force-displacement curve (Figure 3.16) remains largely unchanged, except for a slight delay in the moment when the wall splits (around 20 mm of displacement instead of 15 mm), corresponding on Figure 3.16 to the point where the curve reaches the final base reaction value.

On the other hand, increasing the radius of truncature (up to 5 mm, Model 10) leads to a failure mechanism where the left-hand side wall touches and rests on the right-hand bottom triangle of bricks (similar to the failure mechanism in Figure 3.12c). The force-displacement relation confirms this, with the trajectory of the curve deviating further from the theoretical 90 N peak and rising after reaching a peak. The same values of radius of truncature were experimented with a contact stiffness of 2.5 MN/m, but the simulation showed the same failure mechanism.

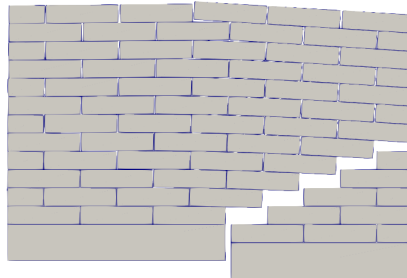


Figure 3.15: Numerical failure mechanism obtained with LMGC90 for the 12C wall, using cut edges bricks (Model 9).

Changing the curvature does not significantly impact the initial value of the base reaction (around 357 N, similar to the sharp edges case). However, cutting the brick edges changes the initial slope of the curve (234 N/m instead of 214 N/m), while changing the value of the radius of truncature r_b does not influence the initial slope.

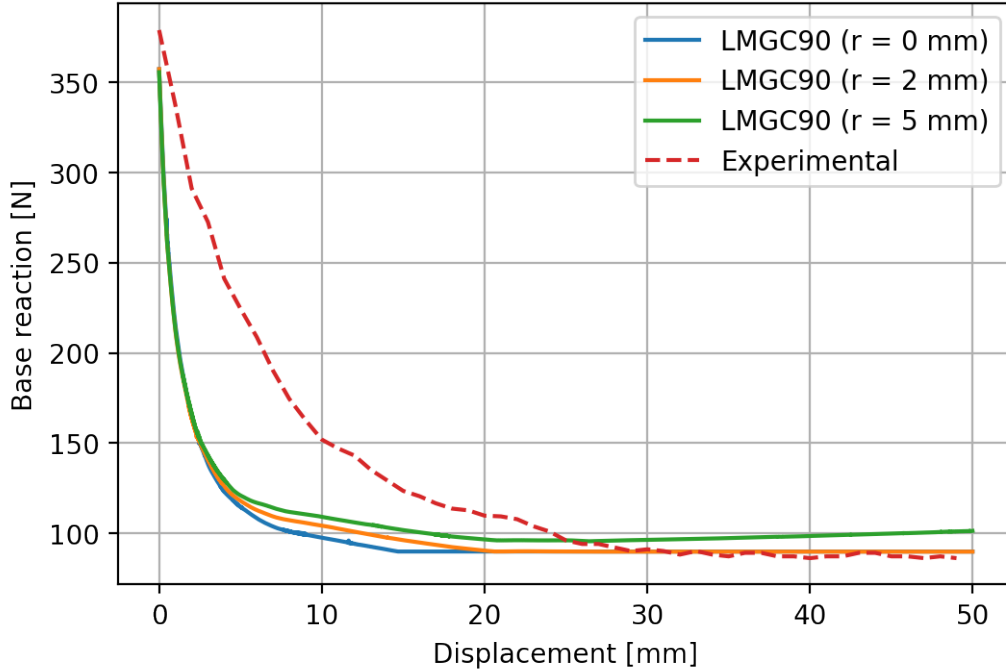


Figure 3.16: Base reaction vs vertical displacement (12C wall), comparing different values of radius of truncature r_b (Models 9 and 10).

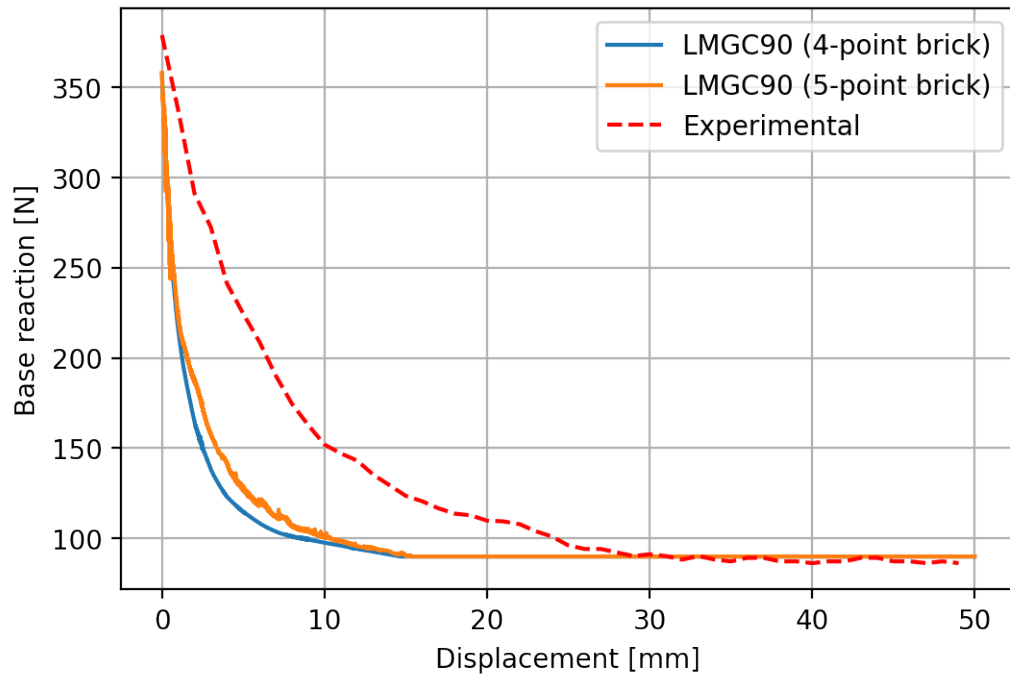
Conclusion

In summary, simulations using rounded edges bricks and a rigid contact law tend to deviate further from the experimental results. Under an elastic contact law, the influence of the radius of truncature is more noticeable, particularly in the behaviour of the wall just before it splits. However, as the initial curve (with sharp edges) still does not sufficiently match the experimental one, this parameter alone does not bring the simulations significantly closer to the experimental behaviour.

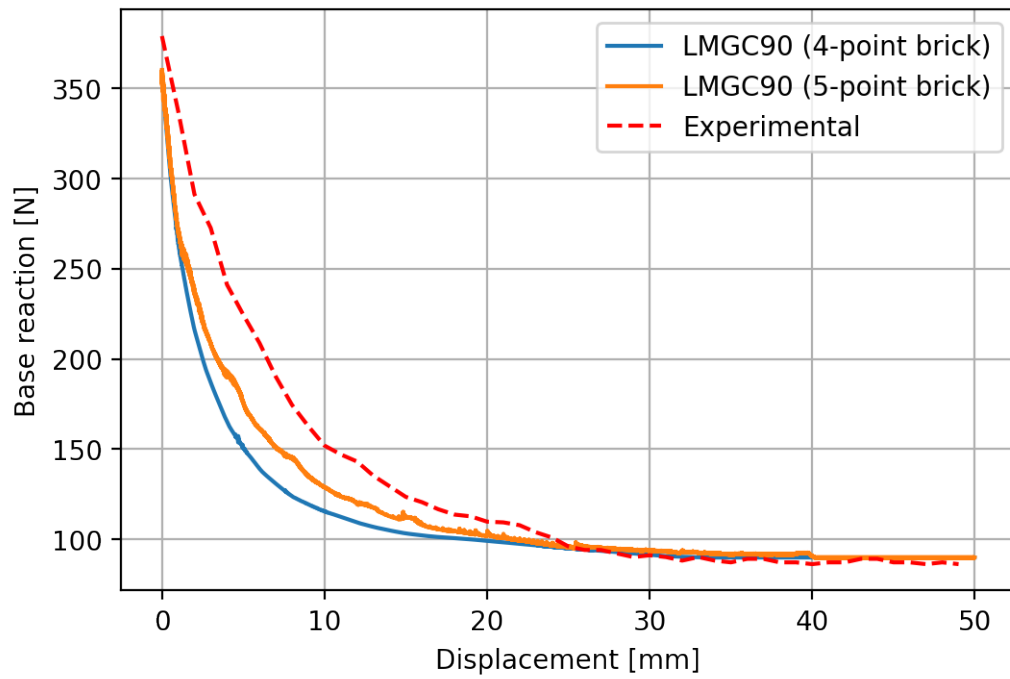
3.1.3.4 Influence of the number of contact points (Models 11-12)

Using the elastic contact law with stiffness values of 5 and 2.5 MN/m, the force-displacement relations (Figures 3.17a and 3.17b) show a closer resemblance to the experimental data, particularly in the case of 2.5 MN/m where the orange curve closely matches the experimental one. However, the curves exhibit a more unstable behaviour, becoming non-smooth and showing jumps. Despite these irregularities, their initial values (360 N and 358 N respectively for 5 and 2.5 MN/m) are slightly higher and closer to the experimental values.

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(a) $k = 5 \text{ MN/m}$ (Model 11)



(b) $k = 2.5 \text{ MN/m}$ (Model 12)

Figure 3.17: Base reaction vs vertical displacement (12C wall), comparing the 4-point and the 5-point bricks models.

Moreover, the initial slope of the curve is lower compared to the four-point bricks model, resulting in behaviour that is closer to the experimental observations. Surprisingly, adding a fifth contact point was expected to increase the elasticity of the wall. The base reaction for both cases reaches the value of 90 N, but at a higher displacement (40 mm), which is even later than observed in the experiment.

The failure mechanisms for both cases (Figures 3.18a and 3.18b) are quite similar to the ones obtained with the four-points brick model. However, the 5 MN/m case remains stiff (Figure 3.18a), whereas the 2.5 MN/m case exhibits a greater and slightly excessive deformation of the left part of the wall (Figure 3.18b). This is likely due to the delayed moment when the wall splits (corresponding to the curve reaching 90 N).

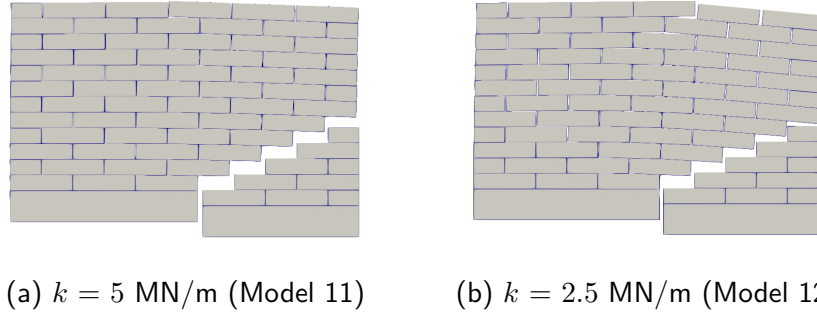


Figure 3.18: Numerical failure mechanisms obtained with LMGC90 for the 12C wall, using five-point bricks.

In summary, while using five contact points leads to more interesting force-displacement relations (especially with the 2.5 MN/m case being closer to the experimental data), the less realistic failure mechanism raises doubts about whether the model is truly an improvement. Moreover, the simulations display non-smooth force-displacement curves, probably due to computational instabilities. As a result, simulations with more than five points per brick lead to numerical errors.

3.1.3.5 Influence of the bricks discretisation (Models 13-14)

In the case of the 5 MN/m stiffness model (Model 13), the use of discretised bricks yields a slightly higher initial slope of the force-displacement curve (262 N/m) compared to the four-point bricks model. This difference can be attributed to the presence of two additional contact points on each brick, effectively introducing two extra springs. Consequently, the trajectory of the force-displacement curve (Figure 3.20) slightly deviates just before reaching its final value of 90 N. Figure 3.19 shows

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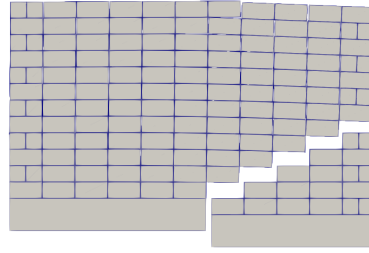


Figure 3.19: Numerical failure mechanisms obtained with LMGC90 for the 12C wall, using discretised bricks (Model 13).

that the numerical failure mechanism is similar to the one already obtain with the four-point bricks (Figure 3.12a).

However, Model 14 (using a 2.5 MN/m contact stiffness) with discretised bricks led to a failure mechanism where both parts of the wall still touch and rest on each other (Figure 3.12c), which is not in accordance with the experimental observations. Moreover, the computational time increases due to the doubled number of elements involved in the simulation, making this approach less practical for this particular study. Overall, the results suggest that the discretisation of bricks does not significantly impact the simulation outcomes.

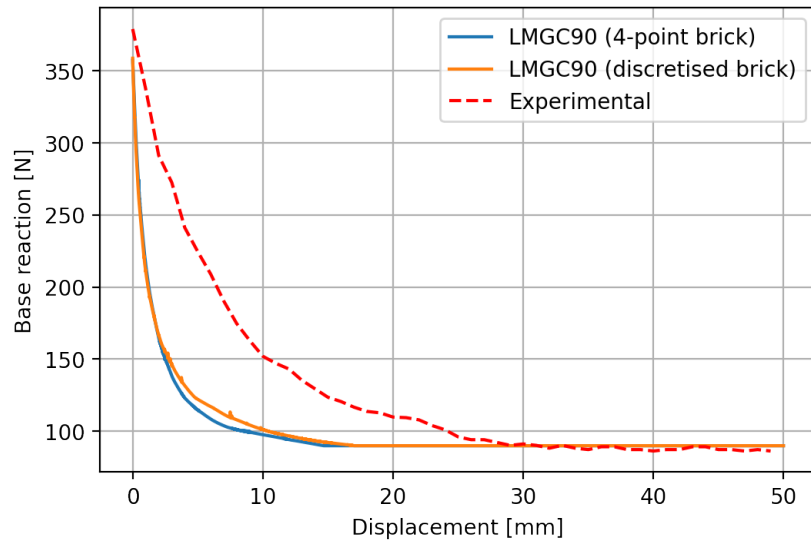


Figure 3.20: Base reaction vs vertical displacement (12C wall), using discretised bricks (Model 13).

3.1.4 Conclusion

In conclusion, this study aimed to compare the influence of different contact laws and brick types through simulations of a 2D static problem to approach the experimental results. The elastic contact law provided failure mechanisms and force-displacement relations that closely resembled the experimental ones (Models 4 and 8). However, the model still did not perfectly represent the elasticity of the wall, as it deviated from the experimental results below a joint stiffness of 2.5 MN/m. Other factors, such as changing the type of brick, adding additional contact points, or discretizing the elements, did not significantly improve the simulation results, except for the case using five-points bricks, which showed an improved force-displacement curve. All the quantified values are summarised in Tables 3.3 and 3.4.

In addition, comparisons can be made with other numerical studies that deal with this experiment. The first study by Portioli and Cascini (2016) [37] utilised rigid bricks and obtained failure mechanisms (Figures 3.4b and 3.4a) very similar to those obtained with LMGC90 using a rigid contact law, as discussed in section 3.1.3.1. The second study by Spada (2019) [38] employed an interphase model (DEM) with discretised bricks. While their force-displacement relation matched the experimental data better than the one obtained with LMGC90, the failure mechanism (Figures 3.6a and 3.6b) exhibited an overly flexible behaviour of the wall. Their model for the 12C wall did not exhibit the splitting of the wall into two parts as observed in the experiment.

However, some uncertainties exist in the simulation results due to limited information on parameter determination in the experimental data. Additionally, the experimental base reaction value was only measured at every millimeter, which makes it difficult to compare accurately the beginning (initial slope) and end of the graph (final base reaction). Moreover, other experimental observations, such as the horizontal displacement of the right-hand side of the foundation, could have provided additional comparison criteria, which are essential for establishing and evaluating numerical models.

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N°	Brick type	Contact law	Initial slope [N/m]	Initial base reaction [N]	Final base reaction [N]
Experimental			42	379	87
1	4-point	Rigid	459.332	343	90
3		Elastic (5 MN/m)	219	357	90
4		Elastic (2.5 MN/m)	111	358	90
5		Elastic (2 MN/m)	89	358	91
6		Elastic (1.3 MN/m)	55	358	98
9	Cut edges ($r_b = 2 \text{ mm}$)	Elastic (5 MN/m)	55	358	98
10	Cut edges ($r_b = 5 \text{ mm}$)	Elastic (5 MN/m)	55	358	98
11	5-point	Elastic (5 MN/m)	196	358	90
12		Elastic (2.5 MN/m)	101	360	90
13	Discretised	Elastic (5 MN/m)	262	359	90
14		Elastic (2.5 MN/m)	141	360	91

Table 3.3: Comparison of the initial slope, initial base reaction and final base reaction for all the presented simulations obtained for the 12C wall case.

N°	Brick type	Contact law	Initial slope [N/m]	Initial base reaction [N]	Final base reaction [N]
Experimental			32	254	103
2	4-point	Rigid	790.472	257	104
7		Elastic (5 MN/m)	215	238	105
8		Elastic (2.5 MN/m)	109	338	106

Table 3.4: Comparison of the initial slope, initial base reaction and final base reaction for all the presented simulations obtained for the 8C wall case.

3.2 Dry-jointed masonry wall with in-plane loading

3.2.1 Description of the experimental benchmark

A series of experiments were conducted by Lourenço et al. (2005) [39] to study the mechanical behaviour of dry-jointed stone walls subjected to in-plane loading. The experimental setup involved applying a vertical constant load, and then subjecting the wall to an increasing top horizontal load until failure (Figure 3.21). Four cases were established considering four different values of vertical load, but always using the same wall. For the purpose of this section, we will focus on analysing the SW30 case, which represents the scenario with the lowest vertical load (30 kN).

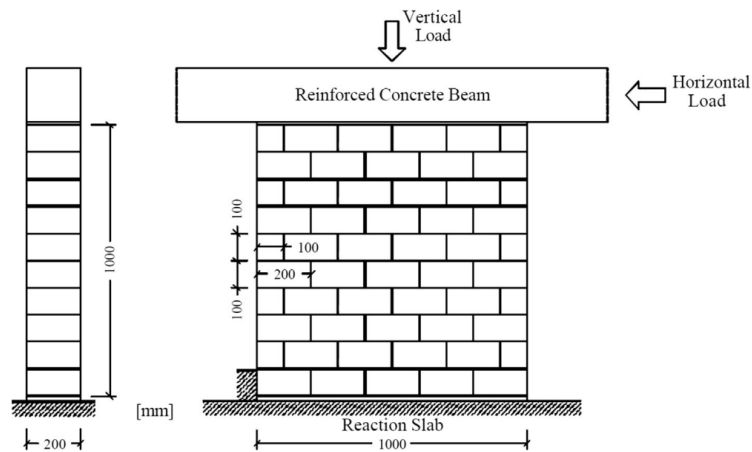


Figure 3.21: Experimental set-up and wall geometry by Lourenço et al. (2005) [39]

3.2.1.1 Experimental setup and parameters

The wall was composed of ten courses of five stone blocks (Figure 3.21). Blocks with dimensions 100 x 200 x 200 mm (height x span x width) were used, resulting in total wall dimensions of 1000 x 1000 x 200 mm. A concrete beam with dimensions 200 x 1000 x 200 mm (height x span x width) and another steel beam, weighing together 2.6 kN, were placed at the top of the wall.

Tests carried out on stone blocks allowed determining their density (25 kN/m³) and elastic modulus (15 GPa). The friction coefficient was determined by Lourenço and Ramos (2004) [40] in a previous study and was found to be 0.62.

The wall was loaded with a 30 kN constant vertical force and an increasing horizontal load. Both loads were applied to the concrete beam and then transferred to the wall. Horizontal displacement of the concrete beam and relative displacement between stone pieces were measured throughout the experiment. The horizontal displacement was computed as the difference between the horizontal displacement at the top of the beam and the displacement measured at the bottom of the wall. The experiment was replicated two times on two identical walls labeled SW30.1 and SW30.2.

3.2.1.2 Experimental results

Results of both SW30.1 and SW30.2 tests are characterised by two elements:

- The **horizontal force-displacement relation** (Figure 3.22): Both experiments exhibit a similar linear first-phase curve but display a non-linear and more unstable behaviour after 2 mm of displacement. Figure 3.22 also shows results from two other numerical studies carried out with DEM and FEM, presented by Orduña (2017) [41], that closely match both experimental curves for the linear first part.

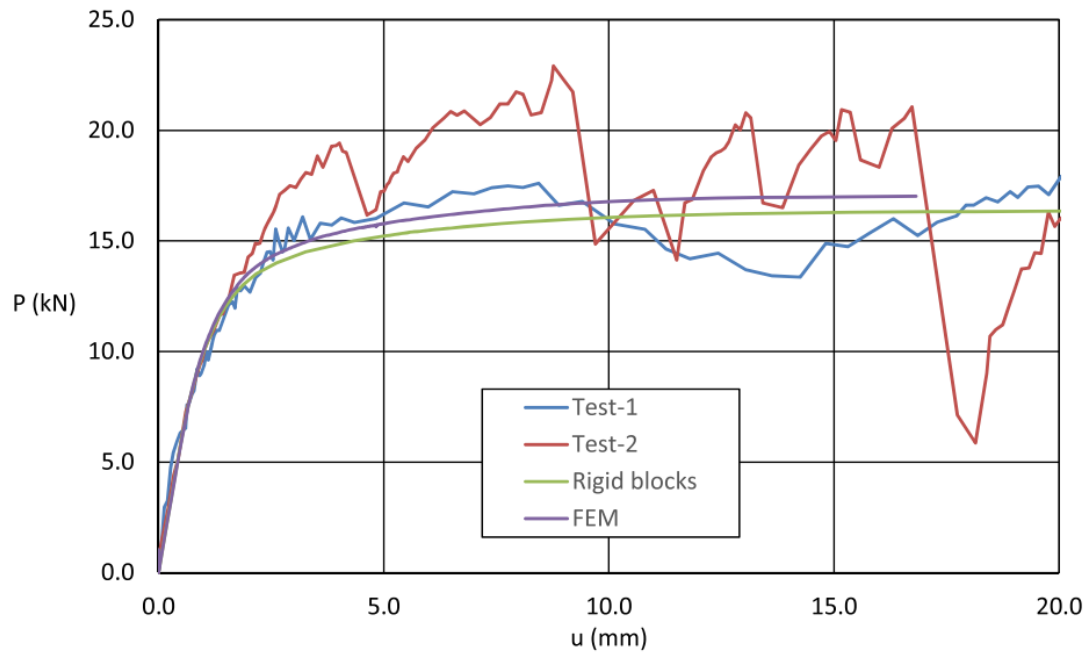


Figure 3.22: Horizontal force-displacement relation for SW30 experiments, compared with numerical results from other studies [41].

- The **failure mechanism**: The document provides two pictures shown in Figures 3.23a and 3.23b. The failure is formed by a simple rotation of the upper part of the wall and horizontal sliding of the blocks, resulting in a stepped diagonal crack. No sliding between the wall and the base was observed.

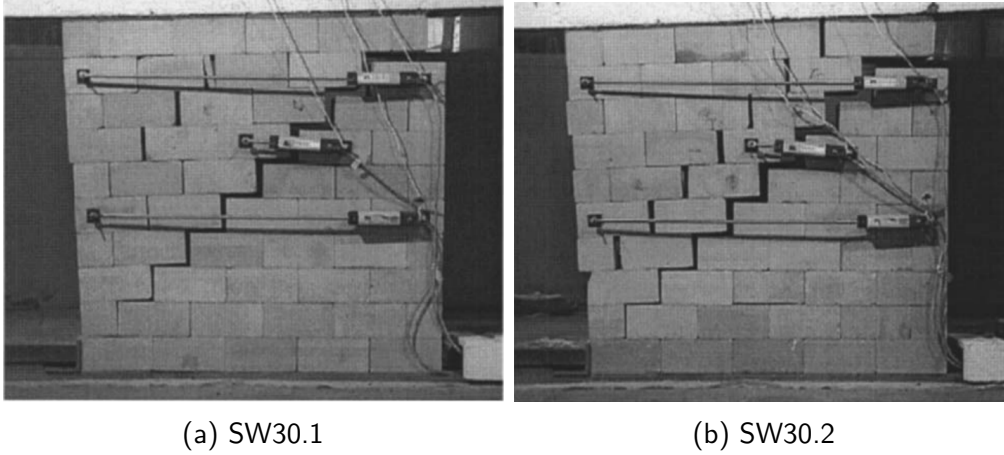


Figure 3.23: failure mechanisms obtained from SW30 experiments [39].

3.2.2 Description of the numerical model

The numerical simulation of the in-plane loading problem was performed in a 3D model. The model was based on the geometrical data, friction coefficient (0.62), and density (25 kN/m^3) from the experimental setup described above. To account for the additional weight of the beams, the density of the concrete was set to 24 kN/m^3 . As the experiment did not show any sliding between the wall and its base, a block, for which the displacement in every DoF was blocked, was placed to the left of the wall to avoid sliding.

The simulations were conducted for a total duration of 210 seconds, with a time step of 10^{-4} s . The applied loads are detailed in Figure 3.24. Only the vertical constant force of 30 kN was applied in one step to the concrete beam only after 5 seconds, to observe the impact of both loads separately and to avoid effect of initial block displacement. After the first 10 seconds, a horizontal force was linearly increased on the concrete beam, reaching a value of 25 kN at the end of the simulation. The simulations stopped when the horizontal displacement of the beam exceeded 50 cm, prior to the end of the simulation.

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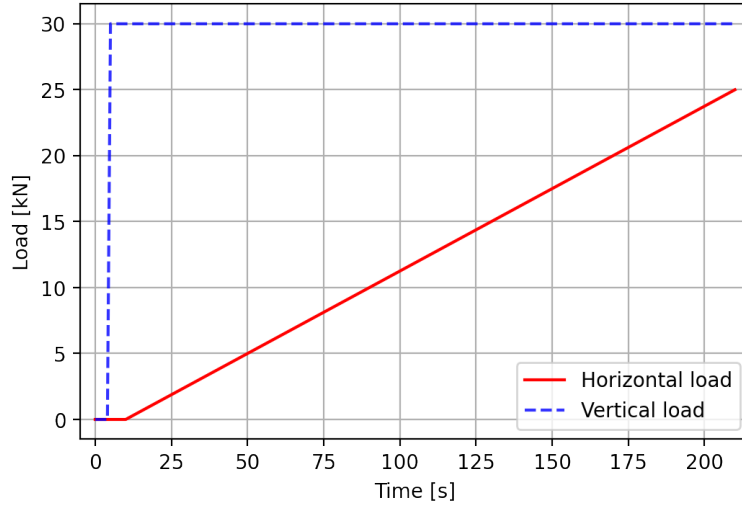


Figure 3.24: Vertical and horizontal loads applied to the concrete top beam during the simulations.

Two models were established, comparing the results using a rigid and an elastic law, as this comparison showed conclusive results in the previous study (Section 3.1). The model with the elastic law initially considered contact stiffness parameters of $k_{bed} = 3000$ MN/m for bed joints and $k_{head} = 750$ MN/m for head joints. These values were computed from the experimentally determined elastic modulus (15 GPa), will be explained in Section 3.2.2.1. Then, several simulations were carried out, increasing the values of stiffness to make the force-displacement curve align with the experimental one. In Section 3.2.3, only the simulation that was closest to the experiment is presented, for which the stiffnesses were 150 and 37.7 MN/m.

3.2.2.1 Calculation of contact stiffness k from the brick elastic modulus E

This section explains the calculation used to obtain the contact stiffnesses k_{head} (head joint) and k_{bed} (bed joint) from the experimentally computed elastic modulus E .

As mentioned in Section 2.5, the contact between bricks is modeled with contact points in LMGC90, where contacts behave like springs in compression (equation 2.4) while defining an elastic law. These contact points exhibit the deformability of the blocks, and an equivalent stiffness k of the joints must be found to model their elastic behaviour accurately.

To compute the contact stiffness to set in the program, a simple rectangular block with dimensions $b \times l \times h$ and bounded by eight contact points is assumed. There are

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two different contact types: bed joints (vertical contact, transferring a vertical load - Figure 3.25) and head joints (horizontal contact, transferring a horizontal load - Figure 3.26). For both cases, the calculation assumes that a particular elastic contact (blue spring) accounts for the axial stiffness of a rectangular-shaped part of the block in its corner, with dimensions $b/2 \times l/2 \times h/2$ (blue rectangle).

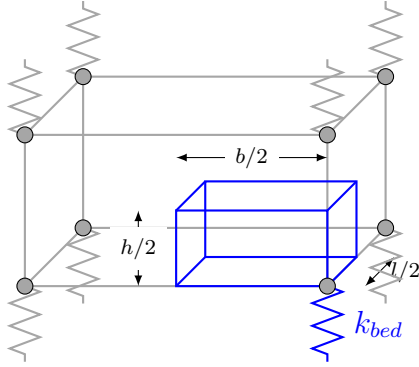


Figure 3.25: Block diagram considered for bed joints.

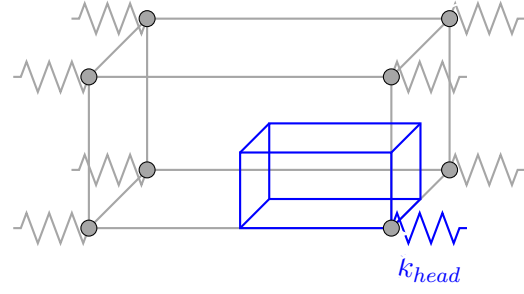


Figure 3.26: Block diagram considered for head joints.

Using Hooke's law ($\epsilon = \frac{\sigma}{E} = \frac{F/A}{E}$, $F = k \cdot \Delta L$), the axial stiffness of a block can be computed as follows:

$$k = \frac{E \cdot A}{L} \quad (3.1)$$

where E is the elastic modulus of the block, A is its cross-sectional area, and L is the length of the element in the axe of the force. Stiffnesses for both bed and head joints contacts are then calculated:

$$k_{bed} = E \cdot \frac{\frac{b}{2} \cdot \frac{l}{2}}{\frac{h}{2}} = 3000 \text{ MN/m} \quad (3.2)$$

$$k_{head} = E \cdot \frac{\frac{h}{2} \cdot \frac{l}{2}}{\frac{b}{2}} = 750 \text{ MN/m} \quad (3.3)$$

This method has been considered the most representative for reproducing the elastic behaviour of masonry in this case and will also be retained for subsequent benchmark studies. However, it should be noted that this computation assumes that a load is transmitted across the entire surface of the joint, which may not always be the case when wall begins to deform and bricks rest on and interact with each other at their corners.

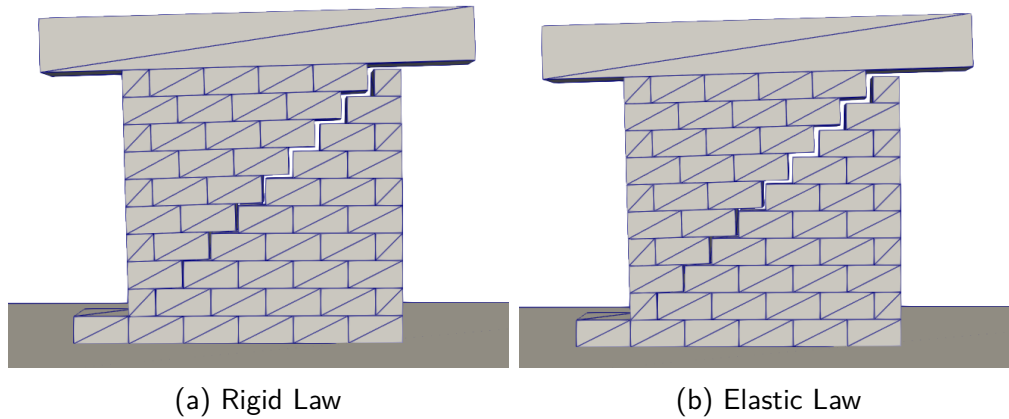


Figure 3.27: Numerical failure mechanism for a horizontal displacement of 27 mm

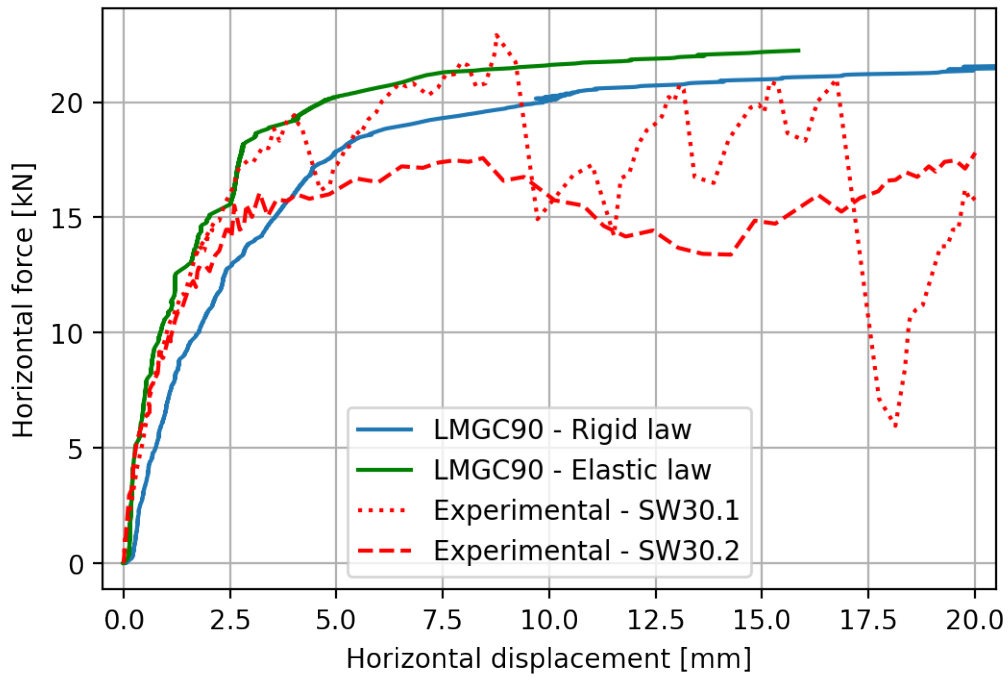
3.2.3 Numerical results

Rigid law

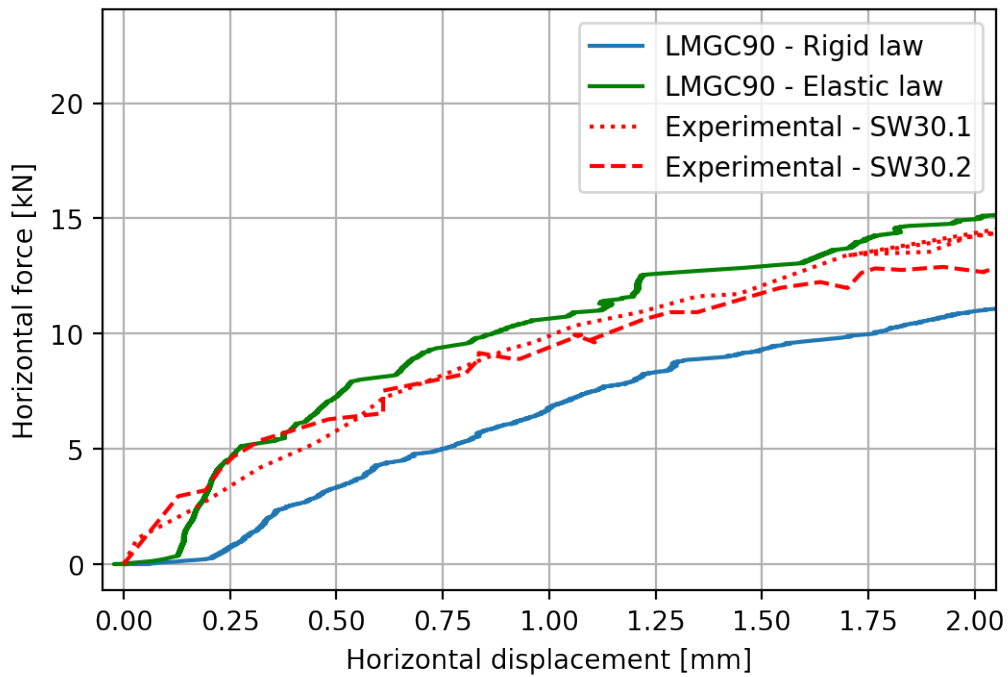
The simulation using a rigid contact law exhibits a similar overall behaviour of the wall in terms of force-displacement relationship (Figure 3.28a) and failure mechanism (Figure 3.27a) when compared to the experimental observations. However, the slope of the numerical curve (blue) is noticeably lower than the experimental one. Figure 3.28b shows that the beginning of the relation is characterised by a significant displacement (0.21 mm) when the horizontal force starts to be applied (after the initial 10 seconds of simulation, i.e. not due to the vertical load), causing a slight shift from the experimental curve. This offset has proved difficult to explain, even though several simulations have been carried out, experimenting with different parameters (blocking the displacement for the DoFs of bricks linked with an out-of-plane displacement of the wall, modifying the conditions on the DoFs of the additional block, etc.) in an attempt to eliminate it.

The simulated failure mechanism (Figure 3.27a) for a horizontal displacement of

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(a) Force-displacement relations for horizontal displacement between 0 and 20 mm



(b) Zoom on the first 2 mm of horizontal displacement

Figure 3.28: Comparison of the force-displacement relations obtained experimentally and numerically with rigid and elastic contact laws.

27 mm is similar to the experimental observations (Figure 3.23b). In both cases, failure is formed by rotation of the upper part of the wall, resulting in a stepped diagonal crack. However, the sliding of blocks, which was observed during the experiments, is only slightly exhibited in the simulation and is not clearly visible in Figure 3.27a.

Elastic law

Among the simulations using an elastic contact law, the one with contact stiffness values of 150 MN/m and 37.5 MN/m for bed joints and head joints, respectively, provided results closest to the experimental ones. The force-displacement curve (Figure 3.27) also presents a significant displacement (0.17 mm) when the horizontal force is applied, but is lower than with the rigid law. The force-displacement curve (green) closely follows the experimental curves (red) and then suddenly rises at around 2.5 mm of displacement. As both experimental results display scattered masonry behaviour in their non-linear phase, the part after about 2 mm of horizontal displacement is not relevant for direct comparison. Additionally, the failure mechanism (Figure 3.27b) remains similar to the one obtained using a rigid law.

Conclusion

Using an elastic contact law allows again for a better reproduction of the behaviour of masonry, as it considers the deformability of masonry through elastic contact points. However, the shift in both curves makes it difficult to directly assess and compare, for instance, the initial slope of the curves. Furthermore, the document [39] only provides one picture of the mechanism, which may present difficulties in assessing the model, as the simulation shows a slight evolution in the crack position (similar to that obtained with other studies).

Unlike the simulation of the problem already carried out by Orduña (2017) [41] with DEM, Figure 3.22), both LMGC90 simulations resulted in non-smooth curves. This illustrates the impact of the implemented method (NSCD, developed in Section 2.5.2) and can also explain the shift observed in the curves.

CHAPTER 4

Validation with 3D static benchmarks

As discussed in Section 2.3.2, masonry structures exhibit different behaviours in-plane and out-of-plane. Adopting a 3D approach to the problem allows for a more in-depth study and captures certain structural behaviours that involve responses in both planes.

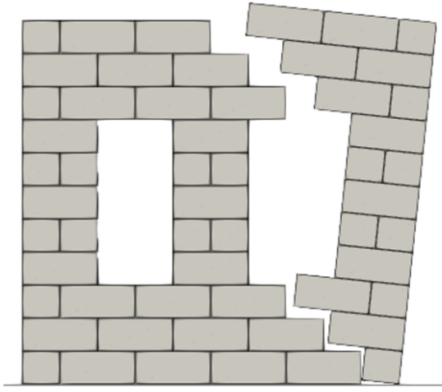
However, this 3D approach has significant implications from a computational standpoint. In 2D models, the elements have 3 DoFs with at least 4 contact points for rectangular bricks, and these contact points are assigned a length. 3D blocks have 6 DoF and are composed of at least 8 contact points, where contact points are assigned an area, and blocks are delimited by their faces and vertices [20]. The possible combinations of interactions between blocks are more complex in 3D [20, 4]. Consequently, the computational time increases significantly, making it less convenient, especially as 3D structures are generally composed of more elements. 3D modelling often leads to the need to reduce the time step, which may result in a less stable and accurate solution.

Due to these differences, 3D models are implemented differently in the LMGC90 interface. In this chapter, we first present a few simple tests (Section 4.1) that were carried out to get started with 3D models and to verify the software's ability to produce consistent results in very simple cases. Then, in Section 4.2, we aim to validate the capabilities of the software using more complex 3D static benchmarks, subjecting several masonry structures to tilting tests.

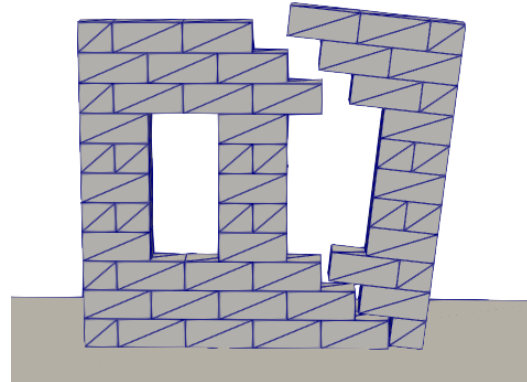
4.1 Validation with basic tests

To validate the capabilities of the software for simple cases, three tests were conducted. While these tests are not the central focus of this work and are intended for a preliminary validation, they are briefly mentioned here without further details.

- **Test 1:** A simple 3D block with a friction coefficient μ and its self-weight W is subjected to an increasing horizontal load. As expected, the software accurately reproduces the frictional forces as the block starts to move for a horizontal force equal to $\mu \cdot W$.
- **Test 2:** The same simple block, with length b , is subjected to an increasing moment, applied at its centre. As expected, the block starts to tilt around its corner when the applied moment is equal to $W \cdot b/2$.
- **Test 3:** The simulation in a 3D model of a 2D static benchmark that was previously analysed by Bouckaert et al. (2021) [5] using a 2D model in LMGC90. All blocks of a 2D wall (configuration shown in Figure 4.1a) were subjected to an increasing horizontal load until collapse. The same parameters and geometry as Bouckaert (2021) were used for the simulation with the 3D model in LMGC90. The results of the 3D simulation resulted in a collapse multiplier very close to the one obtained with the 2D model (0.272 instead of 0.266). Additionally, the failure mechanisms in both cases (Figures 4.1a and 4.1b) are also very similar.



(a) with a 2D model by Bouckaert (2021) [5]



(b) with a 3D model

Figure 4.1: Failure mechanisms of the wall obtained numerically with LMGC90

4.2 Dry stone masonry walls in bending

4.2.1 Description of the experimental benchmark (S13 wall)

The experimental benchmark conducted by Restrepo-Velez et al. (2013) [42] aimed to analyse the resistance of dry-jointed masonry structures to horizontal in-plane and out-of-plane loading. The test involved 1:5-scaled masonry walls of different configurations, each subjected to a progressive inclination of its base until collapse using a tilting table device.

A total of 42 different structure configurations were tested to observe the influence of various factors, such as wall length, presence and position of openings, connection quality between walls, existence of vertical overburden loads, and the number of stories. The experiments allowed to classify the failure mechanisms into different types of out-of-plane failure [15].

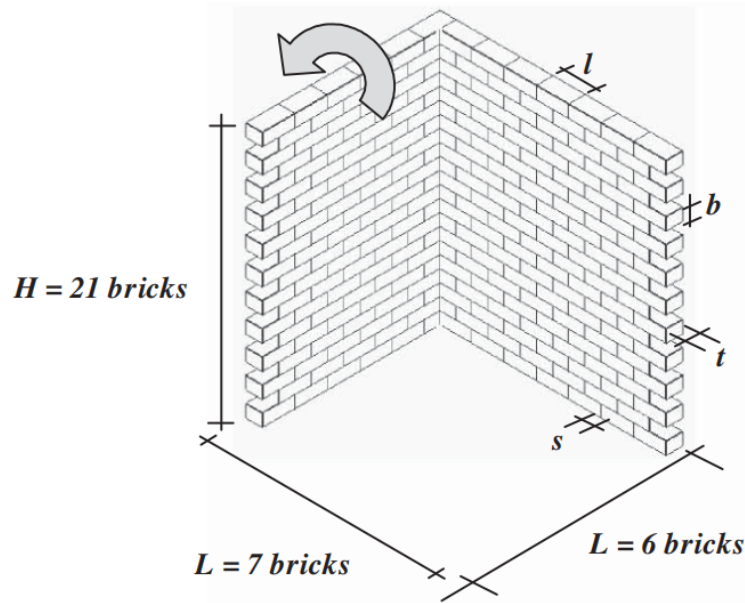


Figure 4.2: Geometry of the S13 wall [42]

In this section, we focus on the S13 wall configuration (Figure 4.2), chosen as the study case due to its lower number of elements (for computational time reasons) and its distinct failure mechanism shape. During collapse, the S13 wall forms three different planes, making it easier to compare the results. Other walls were then studied to validate the results.

4.2.1.1 Experimental setup and parameters

The S13 wall is an L-shaped structure comprising 21 layers of bricks (Figure 4.2). It consists of two planes: a six-brick-long front wall and a seven-brick-long side wall, with a total of 263 bricks. The dimensions of the wall are 478.7 mm (length) $\times$ 598.1 mm (height) $\times$ 593.0 mm (width).

All walls were constructed using marble bricks with dimensions 79.8 mm (length) $\times$ 39.7 mm (height) $\times$ 28.2 mm (width). The marble's density was 26.8 kN/m³. The friction coefficient was determined for various vertical stresses, and a value of 0.77 was selected, approximating the stress resulting from the self-weight of the wall at its midpoint. The brick properties were computed from a sample of 50 bricks.

The experiment involved gradually tilting the base of the wall using a jack until the wall collapsed. A board was employed to prevent the structure from sliding on its base during the test.

4.2.1.2 Experimental results

Results of the experiment are characterised by two elements:

- The **failure mechanism** of the collapsed wall (Figure 4.3). The wall clearly forms three distinct planes: one remaining straight, the largest one falling forward, and a small top section falling backward.

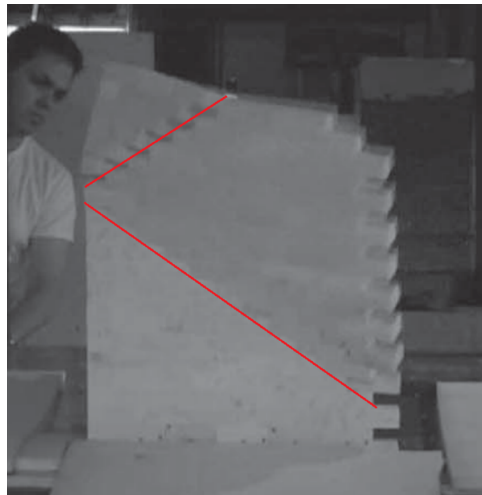


Figure 4.3: Failure mechanism of the S13 wall [42]

- The **collapse multiplier** λ , computed from the inclination angle of the platform that caused collapse, represents the ratio of horizontal to vertical components of the bricks' self-weight:

$$\lambda = \tan(\alpha_{\text{collapse}}) \quad (4.1)$$

The obtained collapse multiplier λ value for the S13 wall was 0.181.

4.2.2 Description of the numerical model

To simulate the experiment described in the previous section, a 3D model was developed in the LMGC90 program. This model incorporates the geometry of the structure, friction coefficient (0.62), and masonry density (26.8 kN/m³) obtained from the experiment. Except for the last model, all simulations adopt a rigid contact law. Various approaches for modelling the action on the bricks were explored.

4.2.2.1 Modelling of the action and studied parameters

Horizontal loading path (1)

During the simulation, all elements are subjected to their self-weight W . To model the effect of base tilting on the wall in the experiment, it was decided to subject all blocks of the wall to a horizontal load equal to λW , where λ is called the load factor and increases linearly from 0 to collapse (Figure 4.4a).

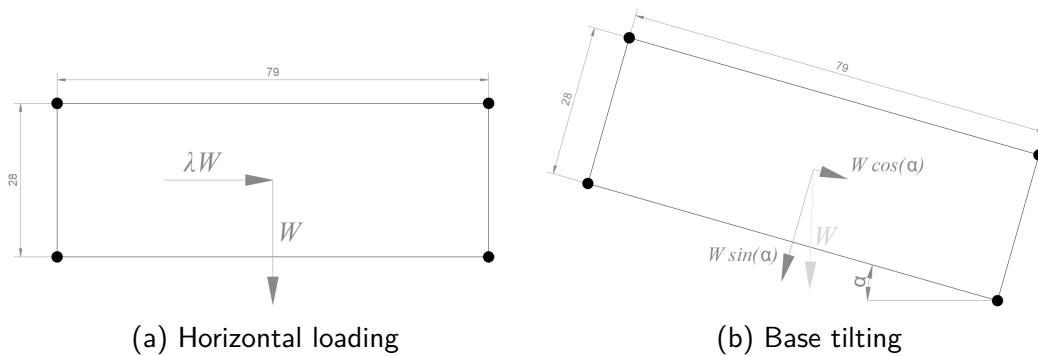


Figure 4.4: Comparison between forces applied on blocks during the simulations

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This first simulation duration was 210 seconds, with a time step of 10^{-3} s. No force was applied during the first 10 seconds to allow the model to stabilize. Afterward, the load factor λ linearly increases from 0 to 1 over 200 seconds. As other loading ways are experimented, this first loading path is labeled Loading path **(1)** (Figure 4.5). The simulation stops once the wall top exceed 10 cm of horizontal displacement, when the wall collapses, at a collapse multiplier λ_{max} .

Comparison with numerical tilting of the base

Another simulation was carried out to verify that loading the blocks horizontally was similar to impose a tilt of the wall base. To do so, the same problem was simulated (same structure and same parameters), but this time, imposing a linearly increasing α angle rotation of the base until the collapse of the wall. In this case, blocks are only subjected to their self-weight, which can be decomposed as show in Figure 4.4b. The load factor is then computed with the following equation:

$$\lambda = \frac{W \cdot \sin(\alpha)}{W \cdot \cos(\alpha)} = \tan(\alpha) \quad (4.2)$$

For the results of the two simulations to be compared, the velocity of the tilting is imposed so that the load factor reaches the value of 1 both after 210 seconds. Only very slight differences are expected since both loading paths lead to similar but not identical blocks loadings in time: the loading path **(1)** is linear while the loading follows a tangent function when tilting the base.

Comparison between different loading paths

As the problem is now three-dimensional and involves a significant number of elements, the computational time has increased considerably (around 50 hours for the two previously described simulations). To address this issue, different loading paths are proposed to shorten the computational time. Several simulations were conducted on the same S13 structure (with identical parameters) by subjecting the blocks to horizontal loading (Figure 4.4a). The aim was to observe the influence of the following factors:

1. Reducing the initial stabilizing duration.
2. Increasing the loading velocity.

3. Starting the simulation at a value closer to the experimental collapse multiplier λ_{max} (starting load factor of 0.15 for this S13 wall case).

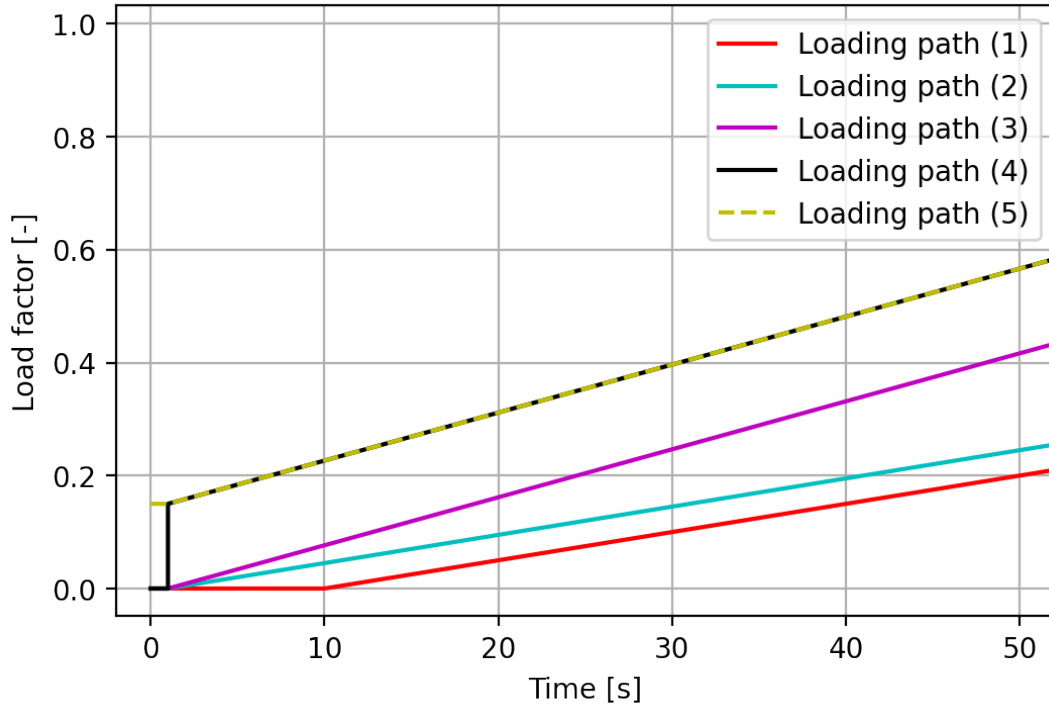


Figure 4.5: Load factor for loading paths **(1-5)**

In total, five loading paths were experimented (details shown in Figure 4.5):

- Loading path **(1)**: The load factor λ increases linearly from 0 to 1 over 200 seconds after an initial stabilization period of 10 seconds.
- Loading path **(2)**: The load factor λ increases linearly from 0 to 1 over 200 seconds after an initial stabilization period of 1 second.
- Loading path **(3)**: The load factor λ increases linearly from 0 to 1 over a shorter duration of $\frac{100}{0.85}$ seconds after an initial stabilization period of 1 second.
- Loading path **(4)**: The load factor λ increases linearly from 0.15 to 1 over a duration of $\frac{100}{0.85}$ seconds after an initial stabilization period of 1 second at 0.
- Loading path **(5)**: The load factor λ increases linearly from 0.15 to 1 over a duration of $\frac{100}{0.85}$ seconds after an initial stabilization period of 1 second at 0.15.

Indeed, looking at Figure 4.5 and assuming a collapse multiplier of 0.181 (experimental), the change in the loading path from (1) to (5) would decrease the simulation duration from approximately 50 to 6 seconds. This reduction in computational time would be significant, bringing it down from several days to just a few hours. This is a time-saving strategy for future simulations if the changes in the loading paths do not significantly affect the results.

Elastic law

In the last set of simulations, the focus was on studying the influence of adopting an elastic contact law. However, the original paper detailing the experiment [42] did not provide the elasticity modulus value for the blocks. Therefore, a value of 50.5 MPa, computed for marble in another study [43], was decided.

To obtain the contact stiffness parameters, a similar calculation as the one developed in Section 3.2.2.1 for the previous benchmark was employed. This led to the determination of the values k^{bed} and k^{head} as 11.2 and 1.4 GN/m, respectively. Since the simulation with these parameters resulted in a failure mechanism that became closer the experimental one, a second simulation was conducted, in which these stiffness parameters were divided by two, resulting in k^{bed} and k^{head} values of 5.6 and 0.7 GN/m, respectively. Both simulations are presented and compared. For simplicity, the models are characterised by the parameter k^{bed} . As the models resulted in smaller collapse multiplier, the wall was loaded horizontally following the loading path (3).

4.2.2.2 Compared parameters

The results of each simulation are compared with the experimental ones in terms of collapse multiplier and the failure mechanism. Table 4.1 provides a summary of the collapse multipliers for all the presented simulations.

The load factor-displacement relation obtained numerically does not exhibit a distinct point indicating failure (as shown in the example for the Loading path (1) simulation in Figure 4.6). This is because the horizontal loading keeps increasing during the simulation, even while the wall is collapsing. To address this issue, a *reaction ratio* was computed to help determine more rigorously at which displacement value to consider the collapse multiplier. The reaction ratio represents the ratio between the horizontal base reaction and the total weight of the bricks, as depicted by the black dashed curve in Figure 4.6. Ideally, both red and black curves should follow the same trajectory as long as the base takes up the wall loading. The moment where

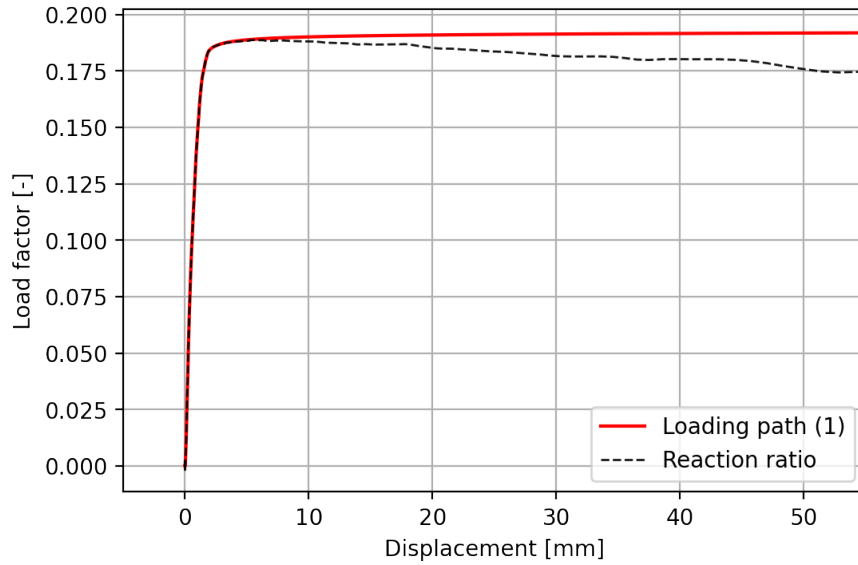


Figure 4.6: Load factor-displacement and reaction ratio relations for the simulation following Loading path **(1)**.

the wall collapses can be determined when the reaction ratio deviates from the load factor-displacement relationship, indicating that not all loads are being transferred to the base and the wall is collapsing.

For all simulations, the collapse point was considered when both the red and black curves deviated by more than 1%. Due to the highly unstable behaviour (Figure 4.11) of the simulation using an elastic law, the collapse multiplier value was arbitrarily taken for a structure displacement of 20 mm. All relevant relationships are plotted in Annex ??.

4.2.3 Numerical results

4.2.3.1 Horizontal loading path (1)

The factor-displacement relation plotted in Figure 4.6 already indicates a collapse multiplier of 0.190, which is quite close to the experimental collapse multiplier of 0.181 (5.0 % error). Despite this similarity in collapse multipliers, the failure mechanism obtained with this simulation (Figure 4.7b) differs from the experimental one (Figure 4.7a). In the numerical simulation, the wall only forms two different planes instead of three, as observed in the experiment. Additionally, the visible crack between these two planes occurs two bricks higher than it does in the experiment.

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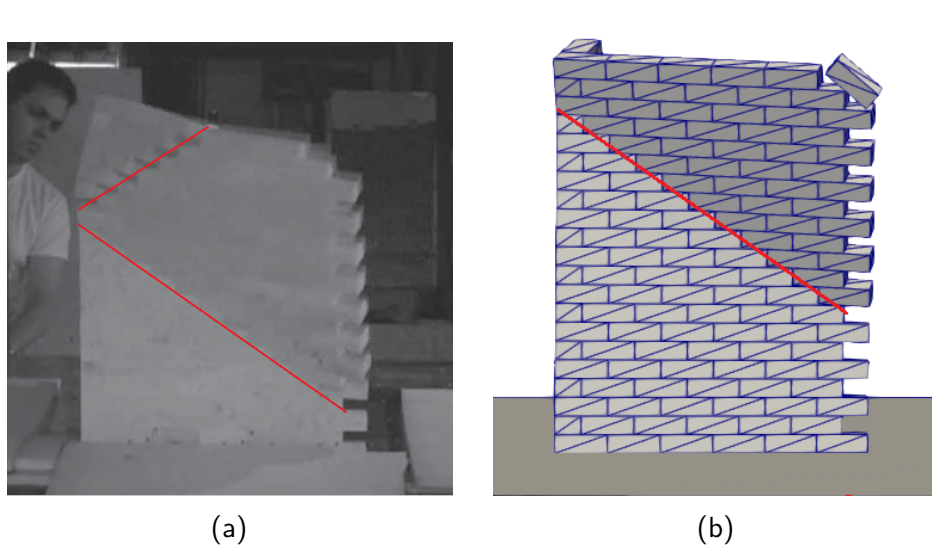


Figure 4.7: Failure mechanisms obtained **(a)** experimentally by Restrepo-Velez (2013) [42] **(b)** numerically with LMGC90 using a rigid law

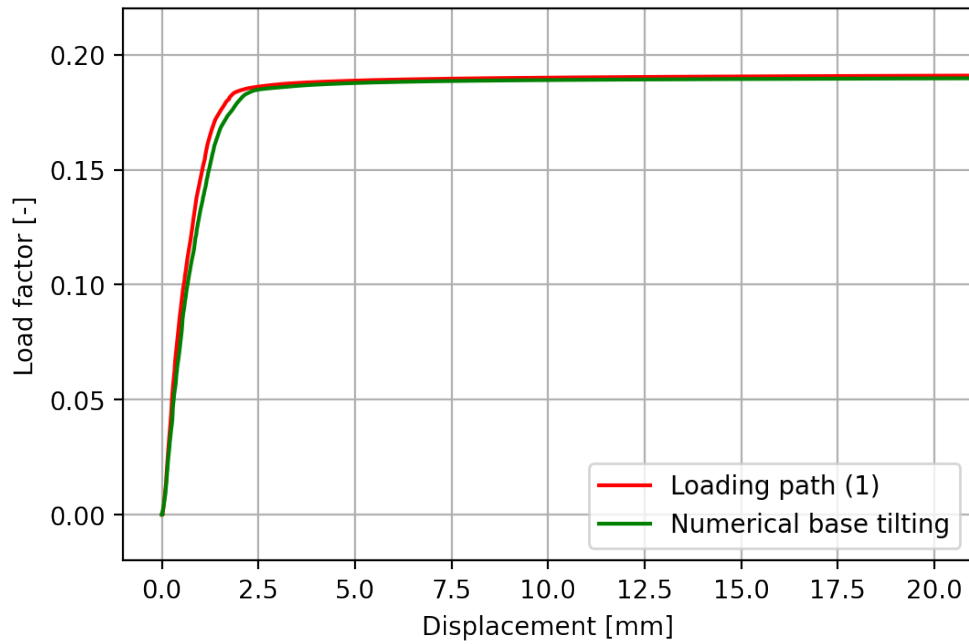


Figure 4.8: Comparison of load factor-displacement relations for simulations **(i)** loading the blocks horizontally (Loading path **(1)**) and **(ii)** tilting the base.

4.2.3.2 Comparison with numerical tilting of the base

The comparison of load factor-displacement relations for both loading methods, i.e., loading the blocks horizontally and tilting the base, is shown in Figure 4.8. Despite slight differences in the loading paths, the two curves exhibit similar trajectories, resulting in comparable collapse multipliers (0.189 instead of 0.190 for the loading path **(1)**, 4.4 % error). Additionally, the failure mechanism remains unchanged (Figure 4.7b), still composed of two failure planes. The similarity in results between the two different methods allow to conclude that the horizontal loading approach provides an effective way to simulate the experimental behaviour.

4.2.3.3 Comparison between different loading paths

The comparison of load factor-displacement relations for all five loading paths is shown in Figure 4.9. It can be observed that they generally result in the same collapse multiplier λ_{max} . The collapse multiplier is determined using the reaction ratio, as explained in Section 4.2.2.2. Every simulation shows the same failure mechanism (similar to Figure 4.7b), which is logical as the structure is loaded similarly.

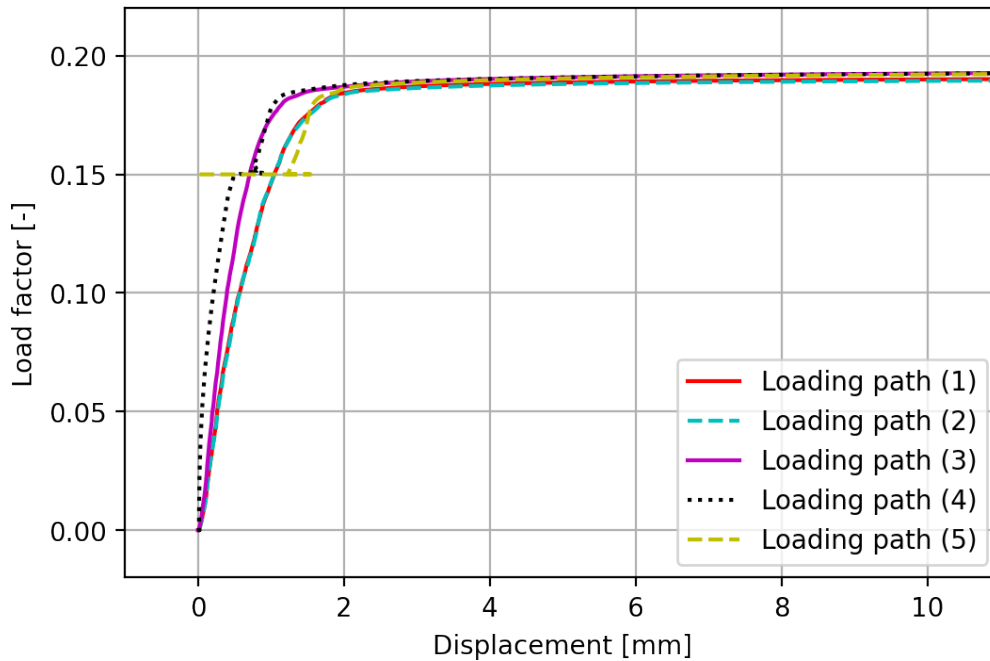


Figure 4.9: Comparisons between load factor-displacement relations for simulations following Loading paths **(1-5)**.

The influence of each loading path is analysed here:

- Loading path **(2)**: The red and blue curves in Figure 4.9 are almost identical, yielding the same collapse multiplier (0.190). This demonstrates that decreasing the initial stabilization duration to 1 second does not significantly influence the results.
- Loading path **(3)**: The load factor-displacement relation (magenta curve) rises more quickly and reaches a slightly higher collapse multiplier. This is due to the fact that the wall has less time to deform while the loading keeps increasing. However, this influence is not significant, and the simulation still results in a very close collapse multiplier (0.192 instead of 0.190, corresponding to a 6.1 % error).
- Loading path **(4)**: As the loading suddenly rises instantaneously from 0 to 0.15, the curve increases even faster until reaching a plateau at a load factor of 0.15. It then oscillates and closely follows the loading path **(3)** curve, resulting in the same collapse multiplier (0.192). Here, the collapse occurs after about 6 seconds.
- Loading path **(5)**: The load factor directly starts at 0.15 and then closely follows the loading path **(4)** curve. Although this does not reduce the computational time, it still yields similar results (0.192).

In summary, these simulations demonstrate that similar results can be obtained by changing the loading path. The primary difference is observed when changing the loading velocity, but this influence is negligible. By selecting the loading path **(5)**, the simulation duration can be reduced from 50 to 6 seconds, significantly saving computational time. These conclusions will be retained for further computations.

4.2.3.4 Elastic contact law

The first simulation, using a k_{bed} value of 11.2 GN/m, results in a failure mechanism (Figures 4.10a) that is closer to the experimental one (Figure 4.7a) compared to the rigid law simulation. Three planes can be distinguished, but the main crack is still one brick too high. The second simulation, using a k_{bed} value of 5.6 GN/m, leads to a failure mechanism (Figures 4.10b) that is identical to the experimental one (Figure 4.7a). The three planes and the cracks are exactly the same as observed in the experiment.

However, both simulations result in a collapse multiplier much smaller (0.139) than the experimental one (0.181), corresponding to a much higher error percentage of 23.2 %. The load factor-displacement relations (Figure 4.11) show a very unstable

behaviour of the computation. A decrease in the time step could have led to more stable and accurate results, but it was not considered due to computational time reasons.

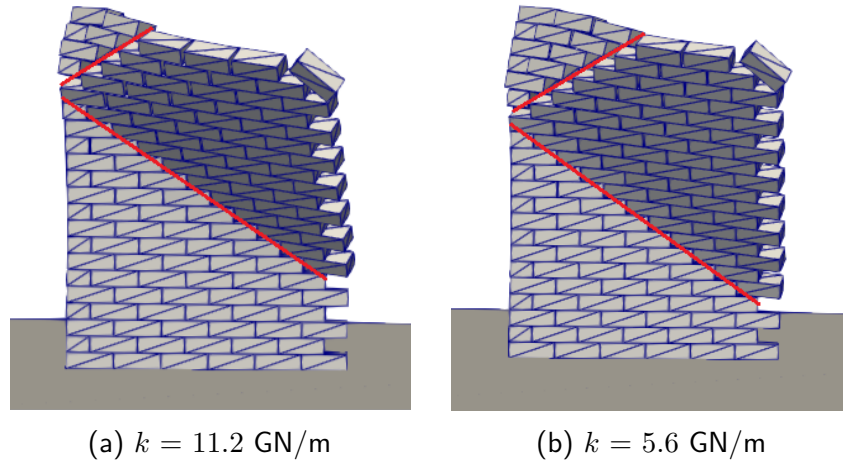


Figure 4.10: Failure mechanisms obtained from simulations using an elastic contact law, for different contact stiffness values.

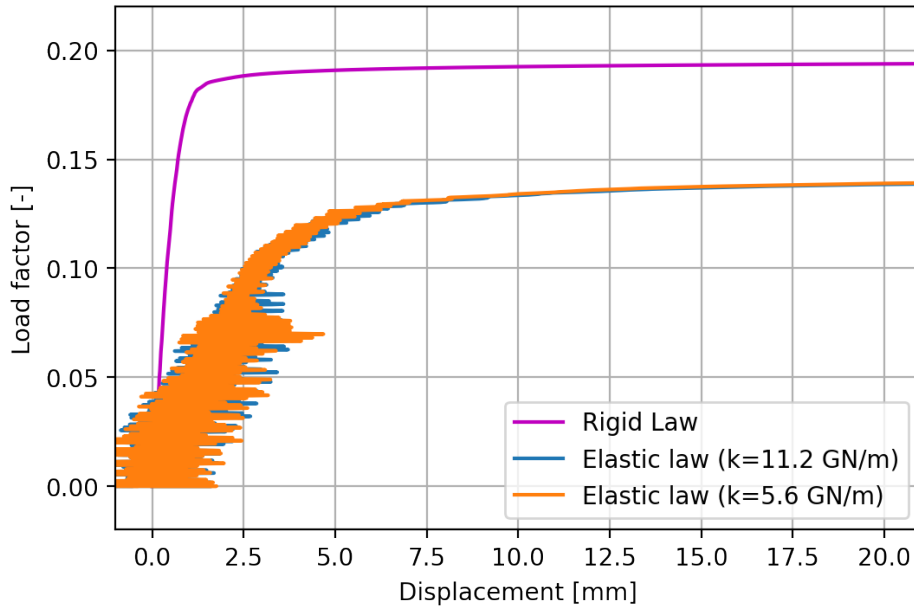


Figure 4.11: Load factor-displacement relations for simulations using elastic and rigid contact laws, following Loading path **(3)**.

4.2.4 Validation with other wall configurations

To validate the results obtained with the S13 wall simulations, additional simulations were performed for other benchmarks from the same set of experiments conducted by Restrepo-Velez et al. (2013) [42]. Various walls, including S10, S15, S17, and S20 (Figure 4.12) were experimented with, each having different features.

However, due to their large number of element or the specific geometry of some blocks, simulations presented difficulties, with errors in contact detection, leading to unusable results. Efforts have been made to address these issues. Finally, only two wall configurations yielded conclusive results: the S10 and S15 walls. The S17 and S20 wall cases are therefore not presented.

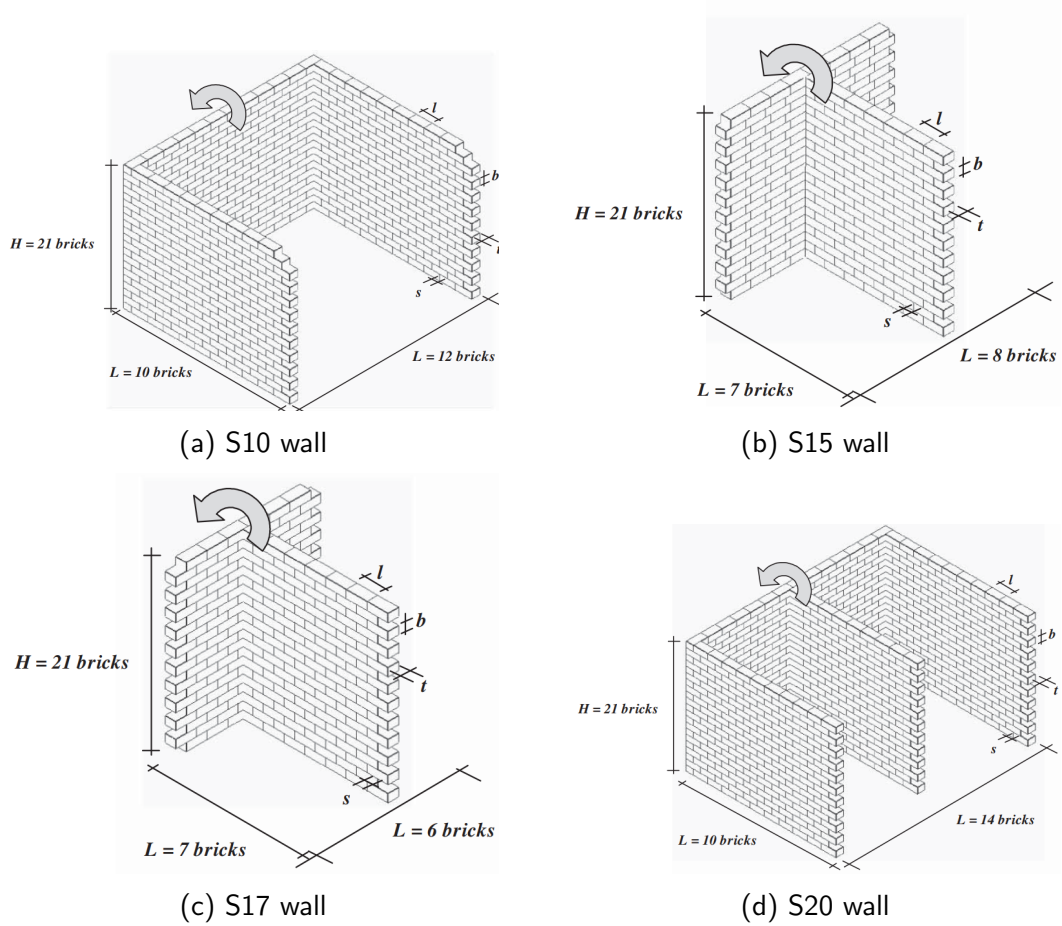


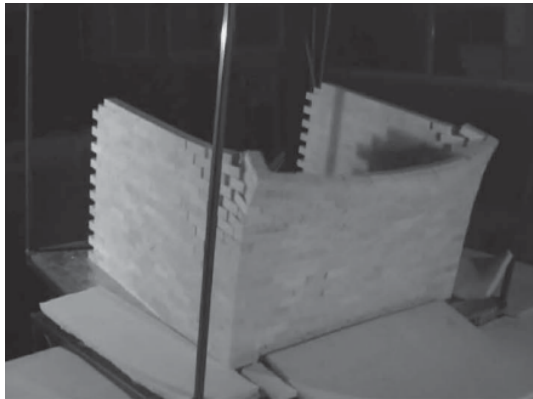
Figure 4.12: Geometry of the experimented wall configurations for validation results.

4.2.4.1 Description of the S10 and S15 walls (experimental)

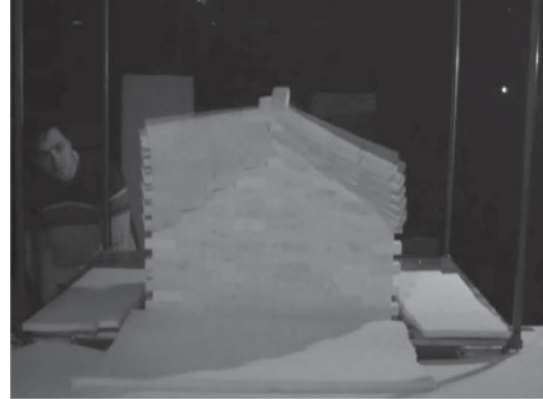
The S10 wall is a C-shaped structure, 21 bricks high, consisting of a twelve bricks long front wall and ten bricks wide side walls, composed of a total of 660 bricks (Figure 4.12c). Its dimensions are 945.5 x 906.4 x 593.0 mm (length x width x height). The S15 wall is a T-shaped structure, also 21 bricks high, composed of an eight bricks long front wall and a seven bricks long side wall, totalling 315 bricks (Figure 4.12d). Its dimensions are 591.2 x 670.0 x 593.0 mm. All parameters and experimental setup remain the same as described for the S13 wall in Section 4.2.1 (brick dimensions, density, friction coefficient, etc.).

4.2.4.2 Experimental results

The experimental collapse multipliers λ_{max} obtained for the S10 and S15 walls are respectively 0.213 and 0.199. Their failure mechanisms are shown in Figures 4.13a and 4.13b.



(a) S10 wall



(b) S15 wall

Figure 4.13: Experimental failure mechanisms obtained by Restrepo-Velez et al. (2013) [42] for both S10 and S15 wall cases.

4.2.4.3 Description of the numerical model

Two numerical models, similar to the S13 wall model, were established for both S10 and S15 walls, incorporating the aforementioned parameters and data. For both wall configurations, two simulations were conducted, one using a rigid contact law and the other using an elastic contact law ($k_{bed} = 11.2 \text{ GN/m}$). Unfortunately, using an elastic

law with the S15 model only led to numerical errors, and therefore, only the model using a rigid law is presented for this wall case. Horizontal loading was performed by following loading path (5) while using the rigid law and loading path (3) while using the elastic law.

4.2.4.4 Numerical results for the S10 wall case

S10 wall case

For the S10 wall case, the simulation using a rigid contact law resulted in a collapse multiplier of 0.25, slightly above the experimental value of 0.213, as for the S13 wall case, but the error percentage is higher (17.4 %). The simulation using an elastic law gave a collapse multiplier of 0.173, slightly below the experimental value, but the error (18.8 %) is not much higher than obtained using a rigid contact law. Both failure mechanisms closely resemble the experimental one (Figure 4.13a). The picture shown by the paper [42] does not allow to clearly differentiate the numerically obtained failure mechanisms.

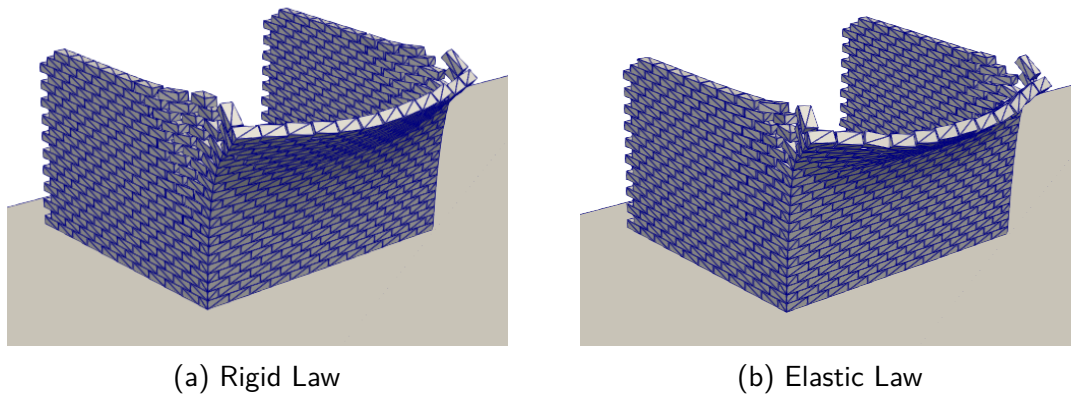


Figure 4.14: Numerical failure mechanisms obtained in the S10 wall case.

S15 wall case

For the S15 wall case, the obtained collapse multiplier was 0.228, which, like the S13 and S15 wall cases, was slightly above the experimental value of 0.199, corresponding to a 14.6 % error percentage. The simulation captured the global failure mechanism (Figure 4.15), but the picture provided by the paper does not allow to assess the obtained failure mechanism more accurately.

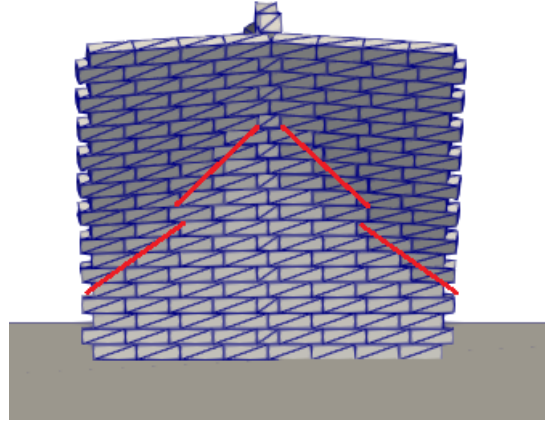


Figure 4.15: Numerical failure mechanism obtained in the S15 wall case.

4.2.5 Conclusion

The simulations of the three experiments demonstrate the capability of the LMGC90 software to capture the general behaviour of masonry structures in this 3D benchmark. Similar to the findings in the previous 2D benchmarks, the use of an elastic contact law results in failure mechanisms that closely resemble the experimental ones, particularly in the case of the S13 wall.

All obtained collapse multipliers and error percentages are summarised in Table 4.1 and are quite close to the experimental ones. The simulation using a rigid contact law are closer to the experimental values in the S13 case. For the S10 wall simulation, the two contact laws gave collapse multiplier as far apart as the experimental.

However, the unstable behaviour of the curve makes it difficult to assess the software's capabilities based on the collapse multiplier. Furthermore, the assessment can only be based on the final experimental collapse multiplier and not the complete load factor-displacement relation. All collapse multiplier λ_{max} are summarised in Table 4.1.

An important observation is the significant increase in computational time compared to the previously studied 2D benchmarks. This necessitates larger time steps, which in turn compromise the stability and accuracy of the numerical solutions, particularly when employing an elastic contact law. Additionally, the five presented wall configurations posed difficulties in obtaining results due to numerical errors, indicating limitations when dealing with 3D simulations. The contacts between bricks are not always detected, leading to bricks falling into each other or through the base. For

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the three presented wall configurations, S10, S13, and S15, numerical errors were resolved through modifications that are difficult to explain, such as shifting the structure horizontally (changing the coordinates of all blocks) or modifying the order in which the bricks are defined. Unfortunately, using more complex models or increasing the number of elements (as in the S17 and S20 models) did not yield satisfactory results.

Wall model	Loading path	Contact law	Collapse multiplier [-]	Error [%]
S13	Experimental		0.181	-
	(1)	Rigid	0.190	5.0
	(2)		0.190	5.0
	(3)		0.192	6.1
	(4)		0.192	6.1
	(5)		0.192	6.1
	Numerical tilting		0.189	4.4
	(3)	Elastic ($k = 11.2$ GN/m)	0.139	23.2
	(3)	Elastic ($k = 5.6$ GN/m)	0.139	23.2
S10	Experimental		0.213	-
	(5)	Rigid	0.250	17.4
	(3)	Elastic ($k = 11.2$ GN/m)	0.173	18.8
S15	Experimental		0.199	-
	(5)	Rigid	0.228	14.6

Table 4.1: Summary of the collapse multipliers λ_{max} obtained for all simulations of the benchmark.

CHAPTER 5

Validation with dynamic benchmarks

This chapter focuses on the dynamic behaviour of masonry structures and aims to validate LMGC90's capabilities in this context using dynamic benchmarks. Unlike the previous static analyses, dynamic simulations consider inertia forces and time-dependent loadings, providing a more comprehensive understanding of structural responses.

The chapter begins by addressing a 2D problem: the rocking stability of masonry arches. Subsequently, a 3D perspective is taken, where a rectangular masonry structure is subjected to harmonic loading. Given the challenges highlighted in the preceding chapter, which showed complexities in modelling 3D structures, the addition of dynamic loads is expected to introduce further intricacies.

5.1 Rocking stability of masonry arches

5.1.1 Description of the experimental benchmark

This section presents an experimental investigation conducted by DeJong et al. (2008) [2, 44] to explore the seismic behaviour of masonry arches. The experiment aimed to gain a fundamental understanding of arch dynamics and to evaluate the assumptions of an analytical model previously established. The experimental setup involved subjecting two small-scale arches of different geometries and seismic base motion.

A shake table was utilized to expose the arches to sinusoidal impulses characterised by different periods and acceleration amplitudes. This study aimed to validate the analytical model established by De Lorenzis et al. (2007) [36] using experimental data. In order to calibrate the analytical model, tilting tests were performed for both arches.

5.1.1.1 Arches geometry and experimental setup

The experimental setup involved two arches with distinct geometries (Figure 5.1), as outlined in Table 5.1. The variations between the arches pertain to the number of voussoirs n_A and the angle β_A .

Arch	R_A [cm]	t_A [cm]	t_A/R_A [-]	β_A [degrees]	n_A [-]
Arch 1	20	3	0.15	162	16
Arch 2	20	3	0.15	152	15

Table 5.1: Characteristics of the tested arches [44].

The voussoirs of the arches were cut from autoclaved aerated concrete (AAC) blocks to the dimensions mentioned above (Table 5.1), with an additional out-of-plane thickness of 10 cm. The base supports, also constructed from AAC, were tailored to ensure no sliding between the blocks and the base. It should be noted that due to the arches collapse and reconstruction multiple times during testing, minor variations in the final block geometry were observed. Density and coefficient of friction values were not computed for this experiment. It will be demonstrated that these parameters do not exert a significant influence on the numerical outcomes.

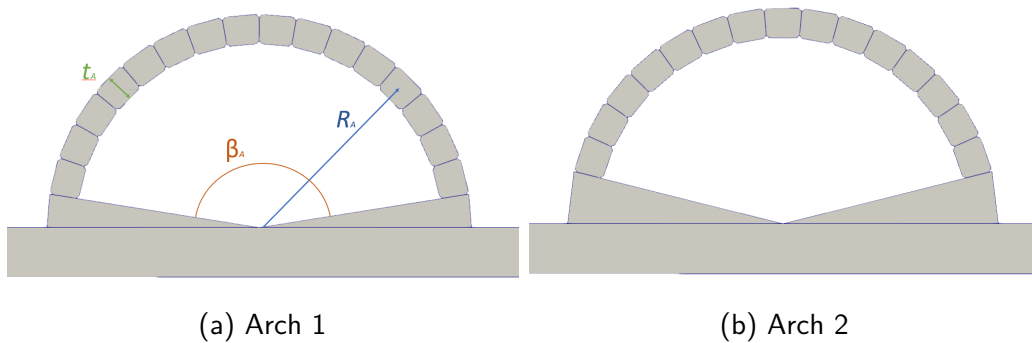


Figure 5.1: Geometries of both arches (LMGC90)

5.1.1.2 Tilting test

While efforts were undertaken to minimize damage to the voussoirs during the seismic tests, minor chipping was observed at the sharp corners of the blocks during initial testing. These experimental block measurements revealed that corner rounding caused a 10-15% reduction in thickness. This rounding of corners consequently decreased the contact thickness between the blocks, thereby affecting the t_A/R_A ratio.

Tilting tests were first conducted on both arches in order to quantify this thickness reduction and facilitate comparison with the analytical model for the seismic test results. The base platform was incrementally rotated until failure occurred, following a similar procedure to the previous benchmark studied (Section 4.2). The resulting adjusted thickness was then incorporated into the analytical model for a comprehensive comparison with the experimental seismic test outcomes.

Results

The tilting test outcomes for both arches are presented in Table 5.2. The horizontal acceleration γ_{exp} can be derived from the collapse inclination angle α_{exp} using the equation:

$$\gamma_{exp} = \tan(\alpha_{exp}) \quad (5.1)$$

For both arches, the theoretical horizontal acceleration $(\gamma_{100})^1$ considering 100% of arch thickness ($t_A = 3$ cm) was higher than the experimental horizontal acceleration γ_{exp} obtained from the tilting tests, likely attributed to corner damage. However, when considering 80% of the arch thickness (γ_{80}) as a theoretical value, a close alignment with the experimentally measured values for both arches was observed. To sum up, adopting a theoretical 20% reduction in thickness resulted in alignment between experimental and analytical results for the tilting tests.

Beyond the measured 10-15% thickness reduction, a further 5-10% thickness reduction could be attributed to slight block size variations and construction geometry imperfections. This would align with the theoretically estimated 20% thickness reduction acquired from the tilting tests.

¹The experiment documentation [44, 2] does not explicitly state the method used to compute these theoretical values.

5 – Validation with dynamic benchmarks

Arch	α_{exp} [degrees]	γ_{exp} [g]	γ_{100} [g]	γ_{80} [g]
Arch 1	11.1	0.20	0.31	0.21
Arch 2	17.5	0.31	0.41	0.30

Table 5.2: Test arch characteristics [2, 44]

5.1.1.3 Seismic test

A shake table was employed to subject both arches to primary sinusoidal impulses characterised by different periods T and acceleration amplitude A_p . For several impulse periods, several seismic tests were carried out, increasing and decreasing the acceleration amplitude to find the failure point.

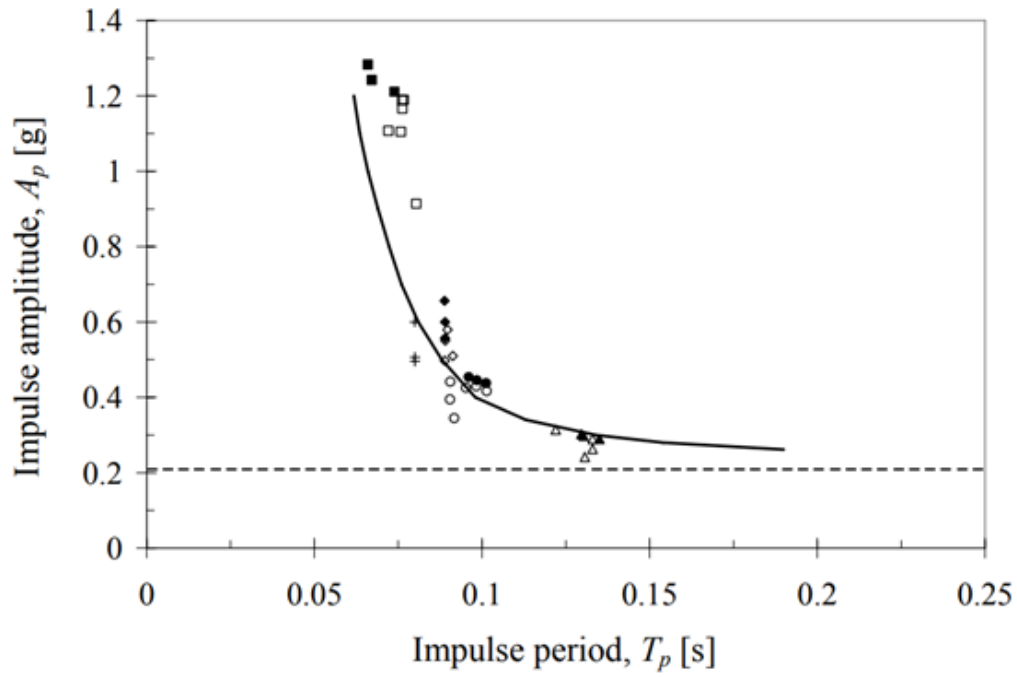
Results

Seismic test outcomes for both arches are depicted in Figures 5.2a and 5.2b. Each point on the graph represents the result of a single test, characterized by the period T and the amplitude A_p of the applied sinusoidal impulse. Filled and open data points represent collapse and recovery points, respectively.

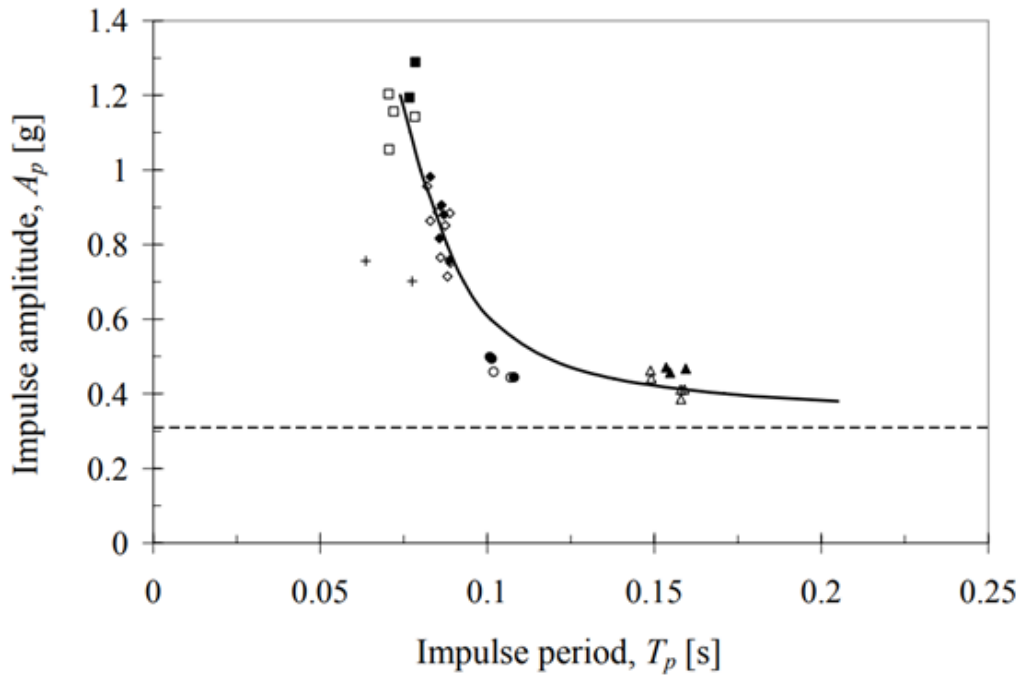
The black curves represent analytical results assuming an 80% arch thickness, as determined from the tilting tests. Ideally, these curves should lie between collapse and recovery points. Figures 5.2a and 5.2b indicate a good alignment between both analytical and experimental results. As the period increases, the curves approach the horizontal acceleration (computed using the tilting tests), represented by the dashed lines. No images of the arches during the experiment are provided in the document. However, it is mentioned that only minor sliding was occasionally observed during testing.

5.1.1.4 Comparison with UDEC

The analytical model outlined by De Lorenzis et al. (2007) [36] is also compared with numerical results (UDEC). The document indicates that both numerical and analytical results aligned well, among others for sinusoidal impulses. This suggests that comparable results can be anticipated in our study of the actual problem.



(a) Arch 1



(b) Arch 2

Figure 5.2: Comparison between the analytical model [36] and the experimental results of the seismic test for both arches [44].

5.1.2 Description of the numerical model

Both arches were simulated using a 2D model created with the LMGC90 program, incorporating the experimental characteristics of the arches described in Table 5.1 (Figures 5.1a and 5.1b). By default, the blocks have sharp edges, but the code also accommodates the specification of a radius of truncature r that can be applied to truncate the block edges, effectively reducing the length of each brick edge by two times r (i.e., r on each corner).

As the document does not state the material density, an arbitrary value of 2600 kg/m^3 was set. Extensive simulations involving different density values indicated that the parameter's influence was negligible. Given the limited sliding observed between blocks in the study, a friction coefficient of 1 (the maximum value allowable in the program) was chosen. All the presented results were obtained using a rigid contact law, as the use of an elastic law did not yield significantly divergent outcomes.

5.1.2.1 Tilting test simulations

Tilting test simulations were executed for both arches, involving a linearly increasing tilting of the base until collapse. Numerous simulations were conducted, adjusting the parameter r to obtain a simulated collapse inclination angle closely matching the experimental value α_{exp} .

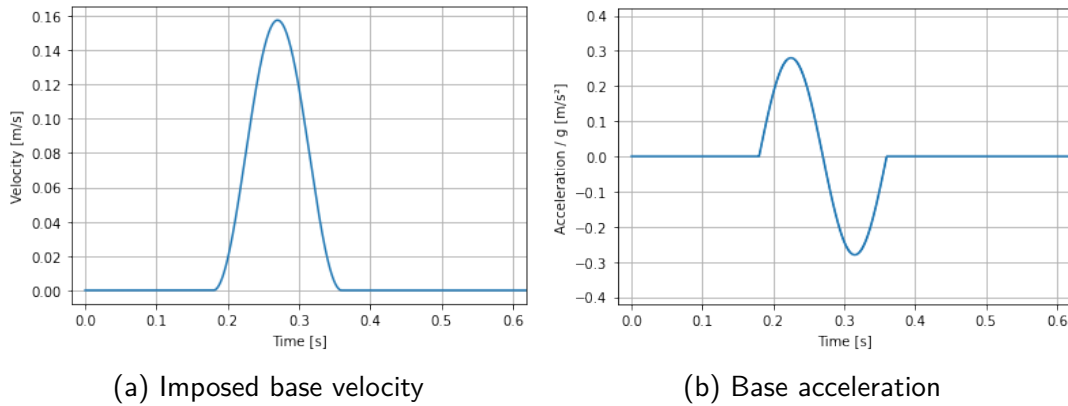


Figure 5.3: Imposed base velocity and base acceleration for $A_{max} = 0.28 \text{ g}$ and $T = 0.18 \text{ s}$

5.1.2.2 Seismic test simulations

Simulations subjected both arches to a sinusoidal acceleration impulse with varying values of period T and acceleration amplitude A_p (example in Figure 5.3b). These simulations will consider truncated bricks with the r value determined by the tilting test. To obtain failure points for several periods and compare the experimental points obtained by DeJong et al. (2008) [44], a loop was used to run the simulations for each step of the period (steps of 0.01 s), increasing the acceleration amplitude value incrementally (steps of 0.1 g) until failure of the arch occurred.

As shown in Figure 5.3, the impulse was only applied after a period of time T to allow the model to stabilise. As the LMGC90 interface only allows the imposition of a certain velocity to elements, a cosine-impulse of velocity (as shown in Figure 5.3a) detailed in equation 5.2 was imposed on the base to achieve the desired sinusoidal acceleration impulse (Figure 5.3)

$$v(t) = \frac{A_{max}}{\omega} \cos(\omega \cdot t) + \frac{A_{max}}{\omega} \quad (5.2)$$

where the angular frequency $\omega = \frac{2\pi}{T}$ and the term $\frac{A_{max}}{\omega}$ is included to account for initial conditions.

5.1.3 Numerical results

5.1.3.1 Tilting test results

Table 5.3 presents the collapse inclination angles α_{num} obtained for different r values. For arch 1, the numerical results closely align with the experimental results for an r value of 3.5 mm, while for arch 2, the value is slightly lower at around 3 mm. Despite the slightly different values obtained for each arch, it was decided to use a common r value of 3 mm for both arches, as it still provides results close to the experimental ones for both arches. This corresponds to a 20% reduction in thickness, which matches the results obtained by DeJong et al. (2008) [44] and could allow for easier comparison with their analytical results as well. Once again, these results confirm the program's ability to accurately reproduce the static behaviour of masonry.

	Arch 1	Arch 2
r [mm]	α_{num} [°]	α_{num} [°]
Exp.	11.1	17.5
0	17.6	22.6
1	15.9	20.9
2	14.0	19.0
3	12.1	17.1
3.5	11.1	16.1

Table 5.3: Collapse inclination angle α_{exp} and horizontal acceleration γ obtained from the ting test simulations for different r values.

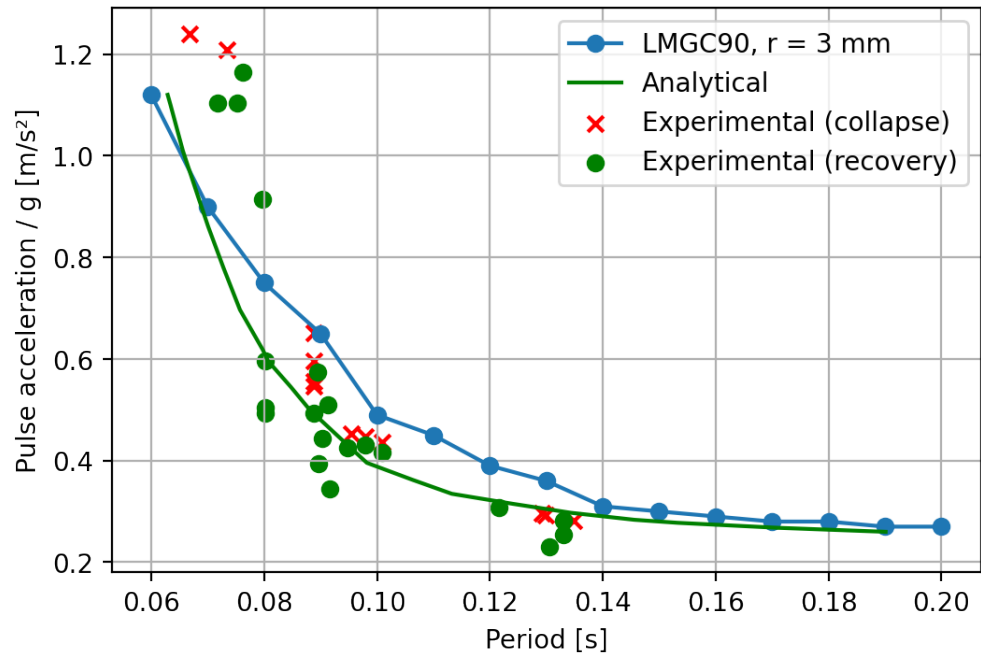
5.1.3.2 Seismic test results

Figures 5.4a and 5.4b present a comparison between the numerically obtained failure curves for both arches and the experimental data points. The desired outcome is for these curves to lie between the experimental points representing collapse (red crosses) and those representing recovery (green points).

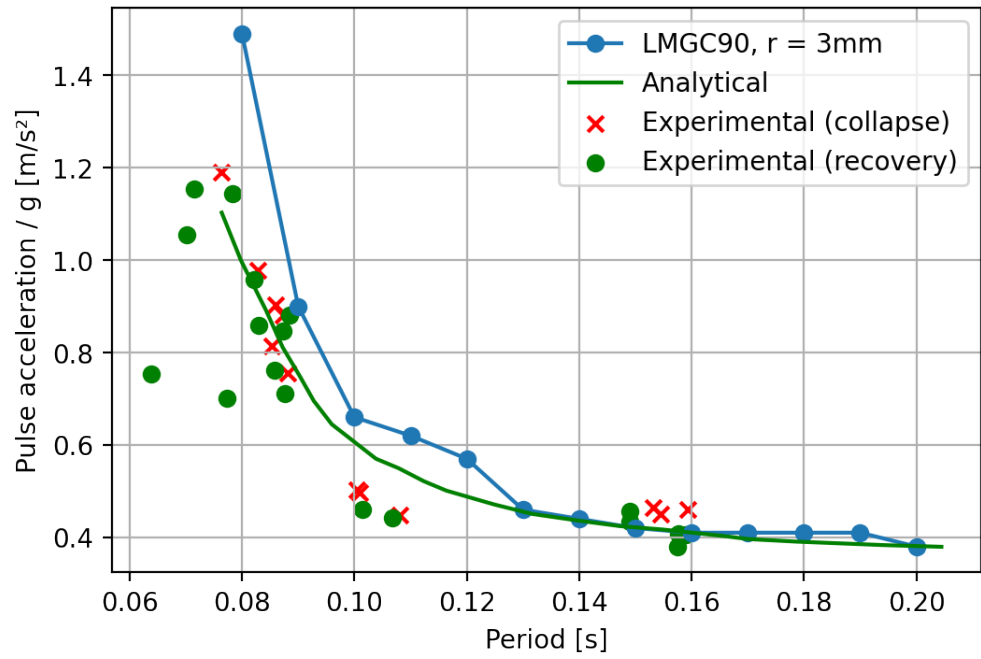
In general, the numerical curves closely align with the experimental results, particularly for period values exceeding 0.8 s. Below a period of 0.8 s, both curves slightly deviate from the experimental results, but in different ways. While the arch 1 case tends to overestimate the failure value, the arch 2 case tends to underestimate it. Additionally, the numerical curves show a good correspondence with the analytical curves, although the analytical curve appears slightly closer to the experimental one.

The software demonstrates a general ability to replicate the experimental behaviour of masonry structures in this dynamic benchmark. The dispersion observed in the data points makes it challenging to assess the model more accurately. Furthermore, no conclusion can be drawn from the failure mechanism of the arches as the document of the experiment [44] does not share any picture of the experiment. However, the simulations align with the experimental observation of minimal sliding, similar to the behaviour during the experiments. It is worth noting that a value of $r = 3$ was chosen for simplification, although more accurate results could have been achieved by considering individualized values for each arch.

5 – Validation with dynamic benchmarks



(a) Arch 1



(b) Arch 2

Figure 5.4: Comparison between experimental, analytical and numerical results of the seismic tests.

5.2 Harmonic seismic loading of a rectangular 3D masonry structure

5.2.1 Description of the experimental benchmark

This set of experiment described by Bui et al. (2017) [35] aimed to assess the "box action" effect of a 4-walls rectangular masonry structure (Figure 5.5). To achieve this, a 1/10-scaled wall made of sugar blocks was subjected to a harmonic loading by mean of a shake table.

5.2.1.1 Experimental set-up and parameters

The considered 10 blocks high rectangular structure was composed of 4 identical 9 blocks long walls (Figure 5.5). The structure is then composed of 360 sugar blocks of dimensions 27 x 12 x 17 mm (length x height x thickness), for total structure dimensions of 260 x 260 x 120 mm. The blocks were dry-jointed and the measured friction angle was between 28.5 ° and 32 °. The density of the sugar blocks was 1081 kg/m³. No elastic modulus is given.

An harmonic loading was applied to the structure, by imposing a horizontal displacement of the shake table defined by the following equation:

$$y(t) = A \sin(\omega \cdot t) \quad (5.3)$$

where A is the amplitude of the displacement and ω is the angular frequency. The maximum acceleration A_{max} can then be computed based on these two parameters:

$$A_{max} = -\omega^2 \cdot A \quad (5.4)$$

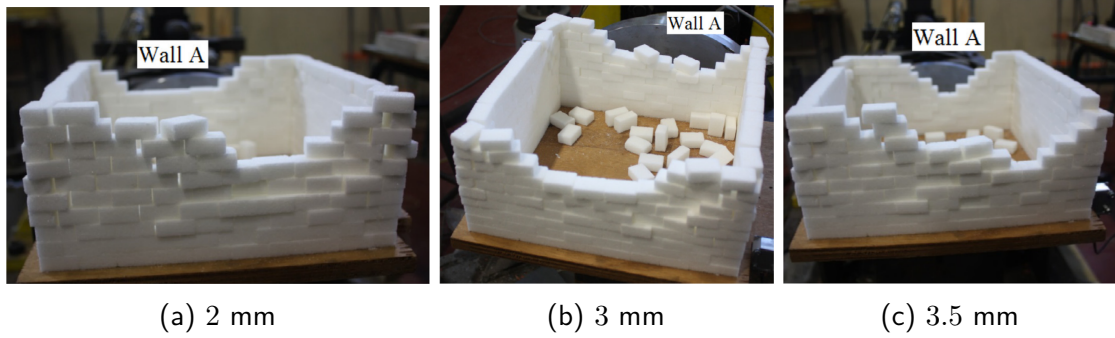


Figure 5.5: Experimental failure mechanisms for different values of displacement amplitude A [35].

5.2.1.2 Experimental results

Results of the experiment are characterised by two elements:

- The maximal acceleration A_{max} obtained for the three values of amplitude A form the red curve in Figure 5.6. The red curve tends through the static maximum acceleration equal to 15.8 g (blue curve).

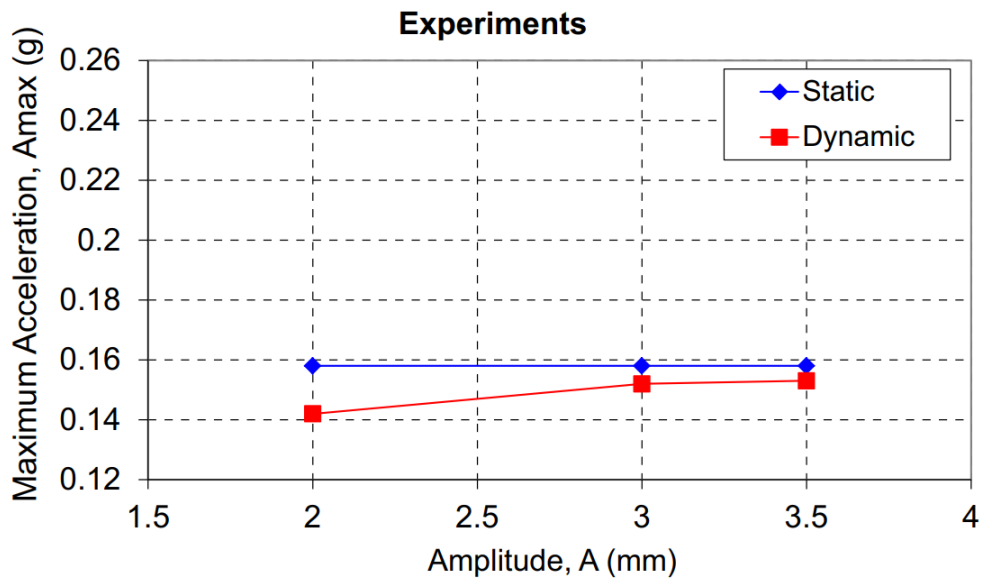


Figure 5.6: Maximum acceleration vs amplitude obtained experimentally [35].

- The three wall failure mechanisms (pictures) shown in Figures 5.5, corresponding to the three points displayed on Figure 5.5. In the three cases, the picture shows

that both side walls remained almost intact where both front and back toppled out of their plane. This confirms the masonry's resistance behaviour within its plane and its weaker behaviour out-of-plane. Slight sliding between blocks can also be observed.

5.2.2 Description of the numerical model

The rectangular masonry wall was modelled in LMGC90 in a 3D model, incorporating the geometry of the structure and the density (1081 kg/m^3) described in the document. The friction coefficient was set to $0.583 (= \tan(30.3^\circ))$. A rigid contact law is defined for every simulation.

Every displacement amplitude case presented in the experiment (2, 3 and 3.5 mm) was considered. An example of the applied horizontal base velocity and corresponding horizontal displacement are shown in Figure 5.7 for a 2 mm amplitude. For every simulation, the displacement was only applied after five periods ($= 5 \cdot \frac{2\pi}{\omega}$) to allow the model to stabilise. Then, the base was subjected to a horizontal harmonic velocity described by equation 5.5, to obtained the desired harmonic base displacement. A simulation duration of 50 s was considered to be sufficiently long to observe the impact of the harmonic loading.

$$v(t) = A \cdot \omega \sin(\omega \cdot t) \quad (5.5)$$

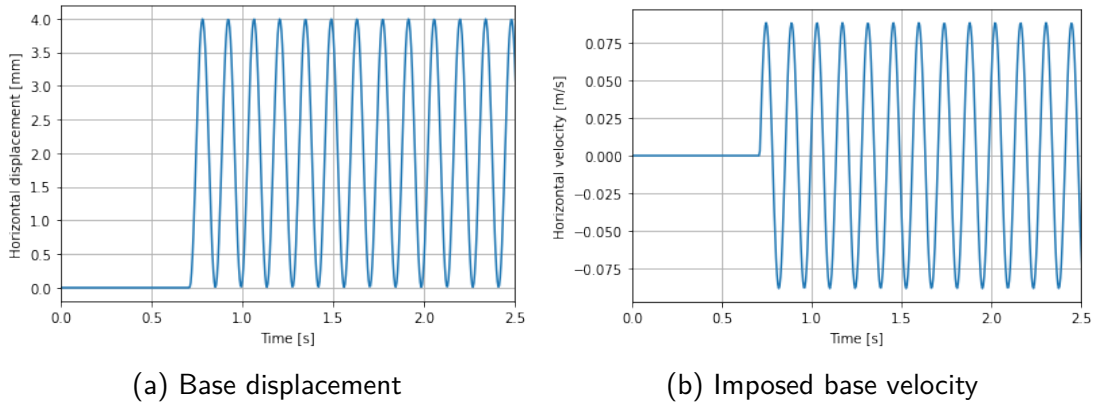


Figure 5.7: Imposed base velocity and displacement for $A = 2 \text{ mm}$ and $A_{max} = 0.4 \text{ g}$

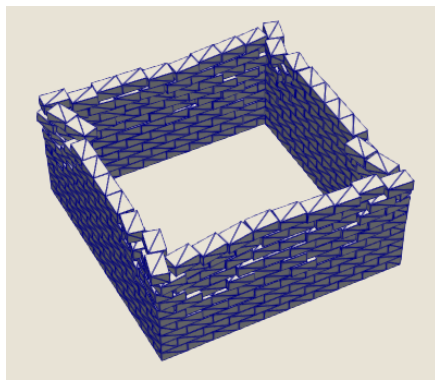
For every displacement amplitude case, various simulations were conducted, modifying the angular frequency ω to increase or decrease the acceleration amplitude A_{max} until finding the collapse point. In this case, collapse of the structure is defined as being reached when one or more bricks detach and fall from the wall. Collapse is expected to be reached by overturning the front and back walls, as observed for the experiment.

5.2.3 Numerical results

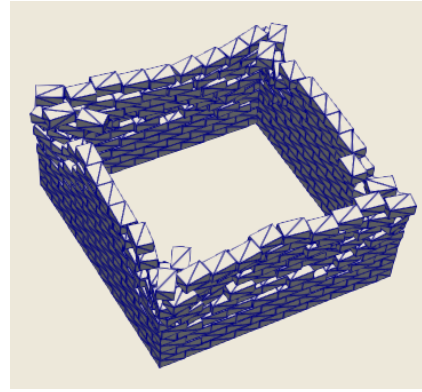
As already observed for 3D static benchmarks in Chapter 4, the first simulation of this problem were characterised by numerical errors, where bricks passed through each other. Numerous simulations were carried out to avoid these. Again, the problem was solved by shifting the structure and defining the blocks in a different order, which is not supposed to change the results.

However, all simulations still led to unexpected observations: the friction forces between blocks (in the out-of-plane direction of the wall) seem not to be properly exhibited, blocks (mainly at the four corners) start sliding in different ways when the harmonic loading starts to be applied, which is difficult to explain. Figure 5.8 shows the evolution of the structure block sliding for a displacement amplitude of 2 mm and an acceleration amplitude of 0.5 g. As the harmonic base displacement keeps going on, the bricks keep sliding progressively, forming a discontinuous wall, until collapse of one front wall (reached after 32 s of simulation in this case). Similar results were observed for lower acceleration amplitudes (up to 0.1 g) and for the two other displacement amplitudes cases.

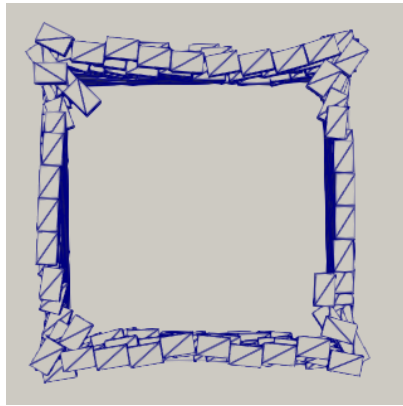
These failure mechanisms are very far from the ones obtained experimentally. With LMGC90, failure starts from the corners (Figure 5.8), where blocks fall from the middle of the front and back walls for the experiment. As the failure is obtained by the progressive sliding of the blocks, the simulations obviously do not reproduce the experimental behaviour of masonry, making it difficult to define an minimum acceleration amplitude for which the structure collapses. It was then not possible to obtain failure points for the three displacement amplitude values and compare them with the ones shown in Figure 5.6. The use of an elastic contact law led to similar results.



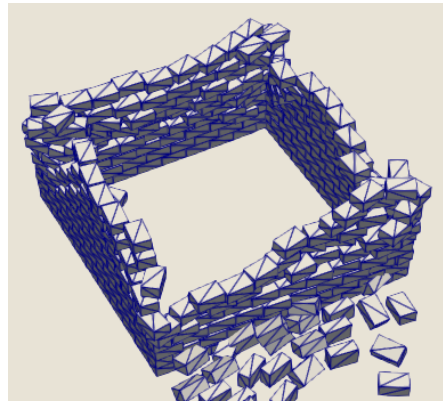
(a) After 7 s of simulation



(b) After 15 s of simulation



(c) After 15 s of simulation



(d) After 32 s of simulation

Figure 5.8: Deformed shape of the rectangular masonry structure at different moments of the simulation for a displacement amplitude of 2 mm and an acceleration amplitude of 0.5 g.

5.3 Conclusion

Two dynamic benchmarks were experimented in this chapter. The first one showed a good alignment between experimental and numerical results, when considering truncated blocks to account for the block edge damage. However, when dealing with a 3D masonry structure, the software did not allow to obtain usable results, due to the excessive sliding of the bricks. As for the study of 3D static benchmarks, some simulations showed numerical errors, which were also expected due to the large number of elements of the structure.

CHAPTER 6

Conclusions and future developments

This thesis has been dedicated to the modelling of masonry seismic behaviour using the DEM. In response to the recent evolution of numerical methods, the study aimed to address the need for assessing these methods, with a particular focus on evaluating the LMGC90 software implemented with the NSCD method. The primary goal was to explore the software's ability to simulate masonry behaviour across various scenarios.

The initial research into the NSCD method contextualised it among other numerical methods for assessing masonry's seismic behaviour. LMGC90, as a DEM software, discretizes masonry blocks into rigid bodies. Its special feature is that it defines blocks interaction through contact points, resulting in a non-smooth contact approach. The contact behaviour is defined by specified contact law.

Throughout the last three chapters, five benchmarks were investigated, involving both 2D and 3D static and dynamic problems. The results were obtained by varying various parameters, enabling comparisons between numerical and experimental outcomes.

The first benchmark, involving the settlement of a wall foundation (2D static), underwent a comprehensive parametric study. It revealed that **(i)** using an elastic contact law, which considers the elasticity of masonry through elastic block contacts, yielded results closer to experimental data **(ii)** aspects such as brick truncation, additional contact points, or brick discretization didn't significantly enhance the results.

Similar conclusions were drawn from the study of 3D structures subjected to tilting

tests (static). The use of an elastic contact law was once again validated. However, simulations involving 3D models drastically increased the computation time and introduces numerical errors. Efforts were made to mitigate this by adjusting the time step or the loading type, but numerical errors still persisted in some cases, making some results unusable.

The study's final focus was on dynamic problems. The examination of the seismic response of arches (2D problem) showed good agreement between numerical and experimental results. However, the simulations for the 3D dynamic benchmark, involving a rectangular structure subjected to harmonic motion, led to numerical errors. Despite attempts to address these issues, the simulations remained unsuccessful.

Generally speaking, this thesis has enabled to work on certain issues relating to numerical modelling. When setting up a numerical model, it is essential to consider the choices and assumptions to be made in order to get closer to the experimental conditions without compromising the computation time and the accuracy of the results.

The influence of the assumptions made on the numerical results needs to be understood, hence the interest in parametric studies, which enable the influence of one single parameter on the results to be studied. However, a parametric study can be time-consuming. In the context of this thesis, taking time to reflect on the parameters studied and, in particular, the interest of experimenting with them would probably have saved many more time-consuming simulations.

A good understanding of the numerical method is also essential. The LMGC90 software offers a wide range of simulation options. Even after working on it for a long time, certain parameters or features remain unclear, especially as the documentation and tutorials provided by the program are not always very exhaustive. As an area for improvement, it would have been better to take an even more in-depth look at it, which could have provided explanations or solutions to the numerical errors obtained for the 3D models.

The selection of experimental benchmarks for validation must also be carefully considered. Most of the time, the choice of parameters is based on experimental data, hence the importance of the information and details provided by the scientific literature. The completeness of the first benchmark allowed a in-depth study of the problem. In some cases where experimental information is lacking, the choice of certain parameters must be justified and thought through, but constitutes an additional uncertainty. Moreover, it should not be forgotten that the comparisons made in the studies of the five benchmarks are based on experimental data, which themselves have their uncertainties

6 – Conclusions and future developments

and disparities, as demonstrated by some of the benchmarks studied. In the context of this work, more reflection and research could have been carried out on the selection of benchmarks, prior to their investigation. Indeed, realising the lack of information or irrelevance of a benchmark study could have been a waste of time in the context of this work.

In general, with the exception of the last problem which failed to produce results, the results obtained with LMGC90 demonstrated alignment with the experimental results. However, the major problem encountered while dealing with 3D problems especially dynamic ones, is the increase in computational time and in some cases, the inability to provide consistent results due to numerical errors. The impossibility of simulating the latter problem could highlight a limitation of the software. As an area for improvement, it could have been interesting to investigate the sources of these numerical errors more closely. Especially as studies involving a large number of elements have already been carried out. It might have been beneficial to begin with simpler 3D dynamic benchmarks to comprehend error sources before progressing to more complex problems.

As another potential direction for future research, every additional validation work adds to the comparative pool. Particularly in the realm of 3D dynamic benchmarks, which are relatively scarce in literature, investigations could help unveil and address the observed errors. Moreover, research could expand into specific types of masonry or structures not covered in this study, like mortared-joint masonry. LMGC90 can provide contact laws that consider cohesion which may be suitable for simulating this kind of masonry.

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