

École polytechnique de Louvain

Effects of corrosion on the capacity of the nib of reinforced concrete dapped-end beams

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1 | Introduction

Since the twentieth century, reinforced concrete has been used extensively in the construction industry. Concrete itself has demonstrated impressive performances in time, but, after the introduction of steel reinforcement into the concrete matrix, serious degradation issues have surfaced. In the past, inadequate construction practices were responsible for most of deterioration problems. But today, even with quality control on construction sites, structures made in reinforced concrete are degrading at an alarming rate.

This issue arises due to the corrosion of this newly introduced reinforcement, triggered by aggressive substances penetrating the concrete matrix and entering in contact with the surface of the steel. The two major phenomena leading to corrosion are the presence of an excessive amount of chloride ions in the pore solution and/or carbonation caused by carbon dioxide [1].

Surface cracking dramatically increases the speed at which these substances penetrate the matrix. The use of fast hardening concrete is believed to have raised the cracking potential of structures [2]. The improvement in early age performance may fit the tight schedules on the construction site, but this propensity to cracking shortens the lifespan of the structure by drastically reducing the initiation time of corrosion.

Corrosion of the embedded reinforcement lowers the load carrying capacity of the structure by reducing the cross-sectional area of steel present in the concrete. If an element reaches a limit state due to this deterioration, which often goes undetected until the damage is already advanced, this could lead to dramatic outcomes for the overall structural integrity and thus safety of users, as corrosion has been shown to reduce the ductility of affected elements [3], which could then fail without adequate prior warnings.

Considering that reinforced concrete is being used worldwide for a wide range of constructions, it is crucial to evaluate the extent of deterioration of existing structures and to design future infrastructure for proper durability and structural stability in the face of environmental degradation.

The present document is a follow-up of *Influence of degradation phenomena on the failure of structural elements in normal and prestressed reinforced concrete* by Pier Giorgio MALERBA, Luca SGAMBI, Giordano GOTTI and Diego IELMINI [4, 5]. The authors will be referred in this paper using "MSG&I" for short.

They created a non-linear finite-element based computer program, which can analyse the behaviour of reinforced and prestressed concrete beams subjected to environmental degradation. This program couples the mechanical behaviour of the element to its deterioration, allowing for the study of crack propagation and its consequences on the ingress of aggressive substances. Using this program, they analysed the influence of localised corrosion on the failure mode of prestressed beams.

The same program will be used for the present paper, to analyse the extremities of dapped-end beams, whose cross-section is reduced at the supports. When used to build bridges, on

which de-icing salts can be used, they are subjected to a fairly aggressive environment. It is thus important to be able to evaluate how degradation affects those beams throughout the years, for assessing existing bridges and design future ones, to avoid sudden and catastrophic collapses.

This paper observes the following layout.

Chapter 2 : the process of degradation of reinforced concrete is reminded, namely carbonation, the penetration of chloride ions and the propagation of corrosion.

Chapter 3 : the method used to model the reinforced concrete material is described, essentially based on the Modified Compression Field Theory of Collins and Vecchio [6].

Chapter 4 : the dapped-end beam as well as a method to calculate its ultimate load, namely the strut-and-tie method, are presented. The chosen strut-and-tie model is that of Mata-Falcón et al. [7], and the equations characterising it are provided.

Chapter 5 : the non-linear finite-element program written by MSG&I is presented and its main features are outlined. The formulation of the problem to solve and the way they arrived to a solution are described.

Chapter 6 : the equations that were chosen to characterise the degradation are graphed to examine the influence of the parameters that they contain.

Then, the degradation analysis is conducted and its results are submitted. The limitations of the model are discussed, before interpreting the results and comparing them to previous studies. Conclusions are drawn and ensuing propositions on durability are presented.

2 | Degradation of reinforced concrete

Throughout its working life, reinforced concrete is subjected to various degradation phenomena, on either the concrete matrix or the steel reinforcement. Degradation of the matrix suppresses the natural protection it offers the reinforcement, which then deteriorates and affects the matrix further.

This chapter starts by briefly summarising the phenomena leading to the deterioration of the matrix and its major consequences. The rest of the chapter is dedicated to the degradation of reinforcement through carbonation and penetration of chlorides, before describing the ensuing corrosion propagation. The exposure classes and durability recommendations of the Eurocode are presented at the end.

2.1 Degradation of the concrete matrix

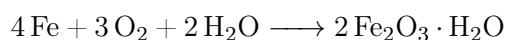
The processes leading to concrete deterioration can be divided into several categories : physical, chemical, mechanical, biological and structural. They include phenomena such as freeze-thaw cycles, attacks due to acids and sulphates, reactions between alkali and aggregates or abrasion, erosion and washout. This list is however non-exhaustive and more details can be found in papers such as Basheer [8].

The major consequence of concrete degradation is the initiation and propagation of cracking unrelated to the tensile stress introduced by the applied loading. For example, sulphates penetrating the matrix react to form expansive crystals that induce pressure inside the concrete, leading to cracking and possible detachment of the cover. This spalling exposes the steel reinforcement to the atmosphere, allowing the aggressive substances to reach it much more easily, and providing an ideal environment for the electrochemical reaction characterising corrosion, described in the next section.

2.2 Corrosion of the reinforcement

2.2.1 Description

Corrosion is an electrochemical process leading to the degradation of steel by stripping the original material and replacing it with an oxide. The chemical reactions describing corrosion can be condensed as :



The nature of this reaction thus requires the combination of oxygen and of an electrolyte to occur. Concrete is a porous material containing a variety of dissolved salts, generally a mix

of calcium, sodium and potassium hydroxides, consequently supplying the required electrolyte. This pore solution creates an environment with a very high pH value, of over 13, necessary for the protection of steel reinforcement. This protection comes in the form of a thin oxide film on the surface of the steel, difficult to dissolve if the pH is above 9 [9]. While this condition is met, the steel is called passivated, as it cannot actively participate in the electrochemical reaction due to the protective film preventing the oxygen from entering in contact with the material.

In reinforced concrete structures, the destruction of this natural protection has two major causes : carbonation due to the carbon dioxide present in the atmosphere and penetration of chloride ions. Other phenomena can cause the initiation of corrosion, but only these two will be considered in the numerical model, as they are most often predominant.

To characterise corrosion, two distinct periods in the life of the structure have to be considered, due to the fact that the leading parameters during these periods are different [1]. Figure 2.1 visually describes the corrosion process.

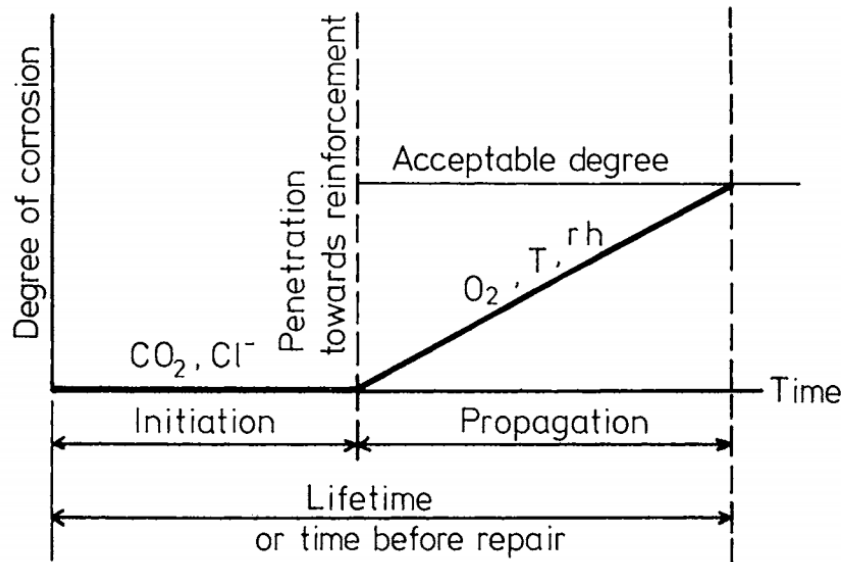


Figure 2.1: Representation of the corrosion process of steel in concrete [1]

Initiation time : period of time between the introduction of the degrading substances and the destruction of the protective oxide film.

The rate of penetration of carbon dioxide and chloride ions into the concrete matrix is studied during this stage.

Propagation time : once the steel becomes depassivated, corrosion is triggered if all the components required for the reaction are present, namely moisture and oxygen. The propagation period lasts until the structure is deemed to have reached one of its limit states due to the reduction of its ductility, load-carrying capacity or any other criterion.

The factors determining the rate of corrosion are now of interest.

Around a real structure, the humidity, temperature, etc. of the environment vary in time, and corrosion will not progress uniformly throughout the years and in every part of the structure. However, a constant rate of corrosion in time is assumed for the simplicity of the model. Using a constant rate for the design of structures is in any way advised [2].

2.2.2 Effects of corrosion

The rust product resulting from the corrosion of steel, made of various ferric oxides and hydroxides, is several times more voluminous than the initial material. The product accumulation on the surface of the reinforcement creates tensile stresses on the concrete matrix. As stated earlier in section 2.1, once the concrete cannot sustain these stresses anymore, the cover¹ cracks and detaches, exposing the steel further and accelerating the degradation process.

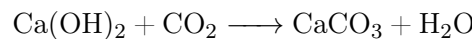
It has been shown by Vidal et al. [11] that cracks start to significantly influence the ingress of aggressive substances when their width reaches 0.3 mm. Cracking accelerates the corrosion process, and corrosion causes cracking of the cover. This implies that the damage development and the mechanical behaviour are coupled, which is taken into account in the model implemented by MSG&I.

The consumption of steel for the reaction entails a reduction over time of the cross sectional area of the reinforcement. As corrosion has been found to have a negligible effect on the yield and ultimate strength of the steel [3], the reduction in strength of the reinforced concrete element is considered as only resulting from the diminishing resisting area. The tensile capacity of the element decreases and can possibly lead to a brittle failure, as the elongation capacity of the reinforcement is lowered.

Corrosion affects the bond strength between the reinforcement and the matrix encasing it, this adhesion influencing its rate [12]. However, perfect adherence between steel and concrete is assumed during the analysis, for simplicity.

2.2.3 Carbonation**2.2.3.1 Description**

A majority of structures is exposed to the atmosphere, which contains carbon dioxide. This CO₂ dissolves into the pore solution, reacts with the calcium hydroxide present in the cement paste and creates calcium carbonate. The pH of the concrete matrix decreases greatly, and the protective film becomes damaged once the carbonation front reaches the cover depth. The general reaction can be summarised as :



Carbonation is not a completely harmful phenomenon. It has been indeed shown to improve the compression strength of the concrete by reducing its porosity, but at the cost of a diminished ductility [13]. However, this is not considered for this study.

2.2.3.2 Evolution - intact concrete

Since carbon dioxide penetrates the entire exposed surface of the concrete, the depassivation can be considered quite uniform along the reinforcement, with a speed depending on the exposure conditions ; for example, the frequency of wetting-drying cycles. The general penetration trend inside intact² concrete is generally accepted as :

$$x(t) = K t^{1/p} \tag{2.1}$$

¹Definition of the EN1992-1-1 [10] : "The concrete cover is the distance between the surface of the reinforcement closest to the nearest concrete surface (including links and stirrups and surface reinforcement where relevant) and the nearest concrete surface."

²"Intact" referring to a concrete that does not have any cracks on its surface.

with

x [mm] the thickness of the carbonated layer ;

K [mm/year^{1/p}] the carbonation coefficient ;

t [years] the time at which carbonation is evaluated ;

p [/] a parameter depending on the porosity of the concrete matrix.

For a porous concrete, p has been determined to be about 2, increasing in value as it becomes more compact, which then provides the following expression :

$$x(t) = K \sqrt{t} \quad (2.2)$$

The predictions given using this equation are fairly accurate for porous concretes but tend to overestimate the penetration inside compact concretes over long periods of time.

The carbonation coefficient K depends on parameters related to the environment and the concrete itself. The main influencing factors are :

Relative humidity : the rate of diffusion of carbon dioxide increases as the relative humidity of the matrix decreases, progressing most easily in air-filled pores. The only case for which carbonation cannot affect a structure is when it is completely and permanently submerged.

However, carbonation can only occur in the presence of water. The critical range of humidity was determined to be between 50 and 80% [1], which is the typical range found inside buildings.

CO₂ concentration : evidently, the speed of penetration increases with the quantity of carbon dioxide in contact with the surface of the concrete.

Temperature : an increase of temperature can lead to an acceleration of the penetration rate, as it improves the general mobility of substances.

Quantity and alkalinity of cement : the ability to fix carbon dioxide is proportional to the alkalinity of the cement paste and grows linearly with the amount of cement used.

Water/cement ratio and curing conditions : a decrease in w/c ratio reduces the capillarity of concrete, which slows carbonation down, but this can only be achieved with proper curing.

2.2.3.3 Evolution - cracked concrete

Cracks allow a faster penetration of carbon dioxide inside the structure, decreasing dramatically the time required for the initiation of corrosion. In this case, this simplified equation based on the crack width is used to determine the depth of the carbonation front over time [14] :

$$x(t) = 50 \sqrt{w} \sqrt[4]{t} \quad (2.3)$$

with w [mm] the width of the cracks. The carbonation coefficient K then no longer plays a role.

2.2.4 Penetration of chloride ions

2.2.4.1 Description

Studies concerned with the harmful effects of chloride ions on structures have led to their prohibition in concrete mixes. The external environment however remains the main provider of chloride-based salts, such as marine environments or through the use of de-icing salts during winter.

The chloride ions linked to the constituents of the cement matrix are not dangerous, unlike the ones present in pores. It may be complicated to measure the chloride content only inside the pores, meaning that the thresholds used in research often contain both.

The protective oxide film surrounding the reinforcement is destroyed when chloride ions reach a certain concentration threshold at the cover depth. The progression of those ions inside the matrix depends on the concrete properties, the transport mechanisms considered, the moisture content, the crack pattern, the cover thickness and the concentration of chloride ions on the surface of the element. It is considerably more complicated to study the propagation of these salts than the evolution of the carbonation front.

The interface quality between the concrete and its reinforcement influences the chloride concentration threshold required for the initiation of corrosion [15], but, as stated earlier, the bond is considered intact throughout the degradation process.

2.2.4.2 Transport mechanisms

The potential transport mechanisms characterising the propagation of chloride ions are varied: diffusion, capillary absorption, permeation and migration. The importance of each mechanism depends on the exposure conditions.

Modelling the penetration of chlorides into a concrete element can be performed by selecting the right penetration mechanism with an appropriate parameter to describe its rate. The evolution of the concentration inside the matrix can then be calculated over time at a certain depth. The major difficulty in doing so is not mathematical, as the equations are quite simple, but arises from the fact that the parameters are many and often unknown. Also, in real cases, a combination of several transport mechanisms is often present.

Only the diffusion mechanism will be taken into account in the numerical model of MSG&I. Diffusion takes place in the presence of a concentration gradient : when the surface of a saturated concrete comes in contact with a solution, the ions enter the water-filled pores. Fick's Second Law of Diffusion is usually taken as suitable to describe this process [16] :

$$\frac{\partial C}{\partial t} = -D(x, t) \frac{\partial^2 C}{\partial x^2} \quad (2.4)$$

with

C [%] the total chloride content by cement weight ;

D [mm²/s] the parameter characterising the diffusion rate ;

x [mm] the distance from the concrete surface ;

t [s] the time at which the penetration is evaluated.

2.2.4.3 Evolution - intact concrete

Even in the presence of transport mechanisms other than diffusion, it is possible to predict with sufficient precision the concentration profile of chlorides over time inside an homogeneous concrete matrix with this solution of equation (2.4) :

$$C(x, t) = C_s \left(1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_{app} t}} \right) \right) \quad (2.5)$$

with

C_s [%] the chloride content at the surface of the element ;

D_{app} [mm²/s] the apparent diffusion coefficient (obtained experimentally).

To find this solution, both of these coefficients are assumed to be constant in space and time. This is in reality not the case, as they happen to depend on many different parameters. Indeed, C_s depends on the concrete composition, structure placement, position and orientation of the surface, micro-climate around the structure, concentration of chlorides in the environment and exposure conditions (wind and rain), while D_{app} varies with the w/c ratio, compaction and curing of the concrete.

2.2.4.4 Evolution - cracked concrete

To take into account the presence of cracks, the Simplified Smeared Approach [17] is chosen, for which diffusivity and permeability increase with the cracking. This approach assumes that the ingress of chlorides into cracked concrete can also be approximated through a solution of Fick's Second Law, using an adapted diffusion coefficient :

$$C(x, t) = C_s \left(1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_{av} t}} \right) \right) \quad \text{with} \quad D_{av} = D_{app} + \frac{w}{s} D_{cr} \quad (2.6)$$

with

D_{av} [mm²/s] the average diffusion coefficient ;

D_{cr} [mm²/s] the diffusion coefficient inside the crack ;

w [mm] the width of the cracks ;

s [mm] the distance between cracks.

2.2.5 Propagation of corrosion

Once the carbonation front has reached a depth equal to the concrete cover, or the chloride concentration threshold has been reached at the reinforcement, corrosion is initiated if the required amounts of oxygen and moisture are present.

The corrosion rate of carbonated structures is linked to the resistivity of concrete and therefore its moisture content. The presence of chlorides creates an even more aggressive environment by increasing the range of humidity for which corrosion is significant [18], see figure 2.2. This can lead to unacceptable corrosion levels in a relatively short time.

The diameter reduction $d'(t)$ of a reinforcement bar once corrosion has been triggered can be estimated at time t by Faraday's law [19] :

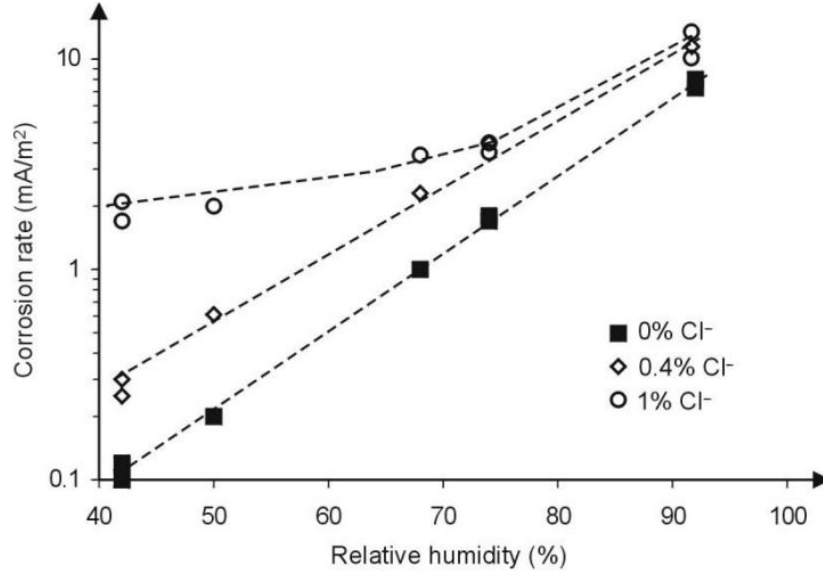


Figure 2.2: Corrosion rate depending on the relative humidity for several chloride surface concentrations [18]

$$d'(t) = i_{corr}(t - t_0) \quad (2.7)$$

with

i_{corr} [mm/year] the rate of corrosion ;

t [year] the time at which the diameter reduction is evaluated ;

t_0 [year] the time at which corrosion started.

The evolution of $d'(t)$ is displayed on figure 2.3. i_{corr} depends on the environmental conditions, the type of ongoing degradation (carbonation or chloride penetration) as well as the characteristics of concrete and steel and the crack width.

For an initial diameter D_0 , the variation of the cross sectional area through time can then be found as :

$$d(t) = D_0 - nd'(t) \quad (2.8)$$

$$A_s(t) = \begin{cases} \frac{N_s \pi D_0^2}{4} & t \leq t_0 \\ \frac{N_s \pi d(t)^2}{4} & t > t_0 \end{cases} \quad (2.9)$$

with

$d(t)$ [mm] the residual diameter at time t ;

n the number of attacked sides (1 for pitting, 2 for uniform), see figure 2.4 ;

N_s the number of rebars.

Pitting is the name given to a local corrosive attack on a rebar or prestressing cable. For the finite-element analysis, the corrosion is assumed uniform over the surface.

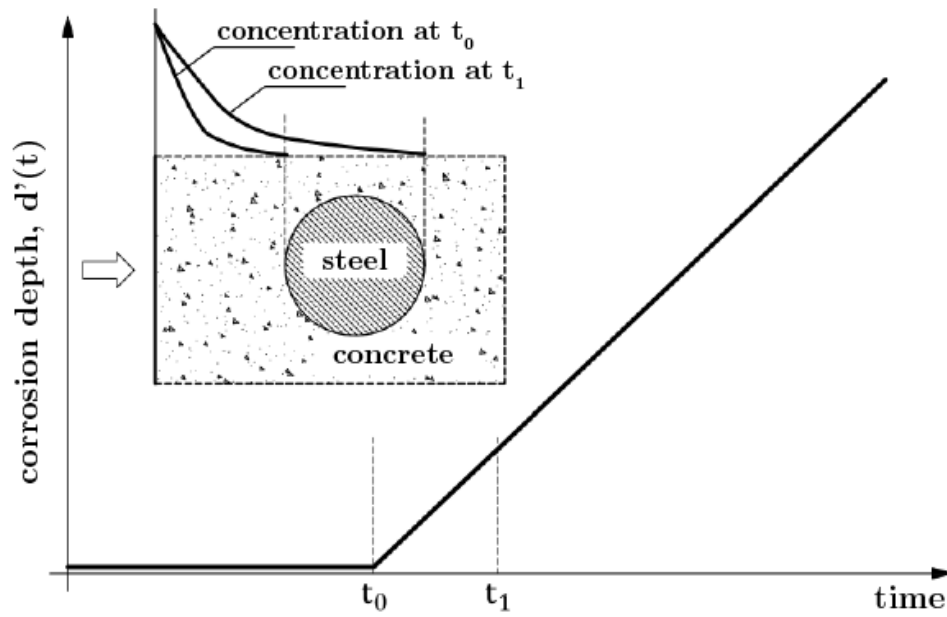


Figure 2.3: Diameter reduction in time for a constant corrosion rate i_{corr} [4, 5]

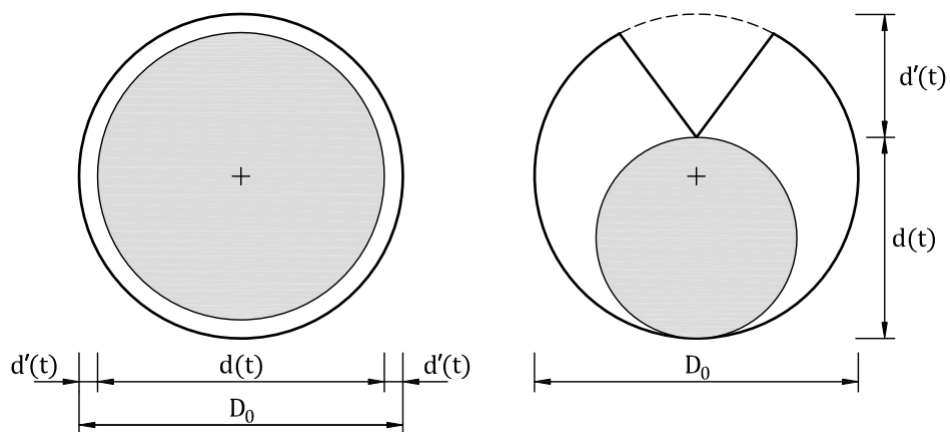


Figure 2.4: Geometry of corrosion of the reinforcement : (a) uniform attack ; (b) pitting [4, 5]

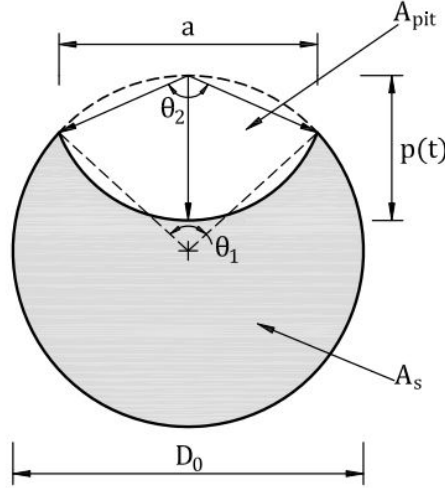


Figure 2.5: Representation of a localised corrosive attack on a reinforcement bar [20]

2.2.6 Corrosion of prestressed cables

MSG&I oriented their attention towards the influence of the breakage of cables due to localised corrosion on the failure mode of prestressed beams. In this case, as the cable area is small compared to the size of the element, a modest reduction in diameter can drastically lower the capacity of the cable. Indeed, unlike passive reinforcement, cables are subjected to a high tensile force and are likely to snap and suddenly lose all of their load-carrying capacity once a corrosion threshold has been reached.

The reduction in diameter is localised and takes a shorter time to create substantial damage. The pitting factor is described as :

$$R = \frac{P_{\max}}{P_{av}} > 1 \quad (2.10)$$

with

$P_{\max}$ [mm] the depth of corrosion in the case of pitting ;

P_{av} [mm] the depth of corrosion in the case of uniform corrosion.

The depth of the crater, see figure 2.4, is evaluated at time t using :

$$p(t) = d'(t) R \quad (2.11)$$

with

p [mm] the depth of the crater ;

$d'(t)$ [mm] the depth of corrosion ;

R [/] the pitting factor.

Since prestressing is not be applied on the dapped-end beams in this paper, the propagation of pitting corrosion is not reminded, but can be found in detail in [4, 5].

2.2.7 Typical values for the corrosion parameters

The following ranges of parameters for the study of degradation are used in [4, 5].

2.2.7.1 Carbonation coefficient

It is possible to determine K for existing structures by measuring the thickness of the carbonation front, but estimating K for new structures is a complex operation, as the parameter depends on all the factors listed in section 2.2.3.2 and more. The values found for real structures protected from rain vary from 2 to 15 mm/year^{1/2} :

High compacity, cement content > 350 kg/m ³	$K = 2$ to 6 mm/year ^{1/2}
Medium compacity	$K = 6$ to 9 mm/year ^{1/2}
Low compacity, cement content < 250 kg/m ³	$K > 9$ mm/year ^{1/2}

2.2.7.2 Chloride penetration

In the case of chloride penetration, the aggressiveness of the environment is described with the chloride surface concentration :

Low	$C_s = 0.00$ to 0.10 %
Medium	$C_s = 0.10$ to 0.20 %
High	$C_s = 0.20$ to 0.25 %
Very high	$C_s = 0.25$ to 0.37 %

2.2.7.3 Rate of corrosion

The rate of corrosion of a structure is highly influenced by its environment, especially the relative humidity of the air. Structures subjected to regular rain are usually exposed to an average relative humidity of 95%. For protected buildings, 90% is assumed.

As stated before, cracking increases the rate at which aggressive substances, water and oxygen reach the reinforcement, thus influencing the corrosion rate. However, a rate for intact concrete is taken due to lack of precise experimentation and data on this phenomenon [19]. The following range of values is used, proposed by Andrade et al. [21] :

- only carbonation and RH from 90 to 98% : 0.005 to 0.01 mm/year ;
- only carbonation and RH lower than 85% : ≤ 0.002 mm/year ;
- contamination by chlorides and RH = 100% : ≤ 0.01 mm/year ;
- contamination by chlorides and RH between 80 and 95% : 0.05 to 0.1 mm/year ;
- contamination by chlorides and RH below 70% : ≤ 0.002 mm/year.

2.3 Durability recommendations of the Eurocode

The exposition classes related to corrosion defined by the EN1992-1-1 [10] are presented on figure 2.6. The minimum values of the concrete cover depth required to satisfy durability requirements, depending on the structural class, are shown on figure 2.7.

Figure 2.6: Table 4.1 of EN1992-1-1 (partial) : Exposure classes related to environmental conditions in accordance with EN 206-1 [10]

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs
4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures

Figure 2.7: Table 4.4N of EN1992-1-1 : Values of minimum cover, $c_{\min,dur}$, requirements with regard to durability for reinforcement steel in accordance with EN 10080 [10]

Environmental Requirement for $c_{\min,dur}$ (mm)							
Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

The dapped-end beams described later in chapter 5 are often used to build bridges, where they are both exposed to the carbon dioxide present in the atmosphere and are under risk of water stagnation if the drainage system is inadequate, in addition to being exposed to rain. This water may contain a large amount of chloride ions if de-icing salts are used on the road that the bridge supports. Thus, the exposition class considered for the construction of these beams for bridges should be of either XC4 or XD3, depending on which phenomenon is deemed dominant

at this location.

A structural class of S4 is recommended by the EN1992-1-1 for a working life of 50 years. As bridges should be designed for twice that amount [22], this structural class should be increased by two, reaching S6. The strength class of the concrete also has an influence : for XC4 and XD3, the respective use of a strength over C40/50 and C45/55 allows the structural class to be decreased by 1, reducing to S5. The cover thickness chosen for an element thus depends on three major parameters : the working life, exposition class and concrete strength.

In addition to the minimum cover requirements, limitations are put on the crack width w . For reinforced concrete subjected to an exposure class of XD4 and XC3, this width should be limited to 0.3 mm.

Figure 2.8: Table 7.1N of EN1992-1-1 : Recommended values of $w_{\max}$ [mm] [10]

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4 ¹	0,2
XC2, XC3, XC4	0,3	0,2 ²
AC₂ XD1, XD2, XD3, XS1, XS2, XS3 AC₂		Decompression
Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and AC₁ this limit is set to give generally acceptable appearance. In the absence AC₁ of appearance conditions this limit may be relaxed. Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.		

3 | Evolution of the degradation

An analysis of the progression of degradation using the equations presented in the previous chapter is conducted, to determine the general influence of parameters such as the carbonation coefficient, chloride surface concentration and cracking state on the initiation time and rate of corrosion.

3.1 Degradation model

3.1.1 Required parameters

The parameters required for this analysis are :

- geometry : the cover thickness [mm], the crack width w [mm], the crack spacing s [mm] ;
- carbonation : the carbonation coefficient K [mm/year^{1/2}] ;
- chlorides : the surface content C_s [%], the critical content C_{crit} [%], the diffusion coefficients D_{app} and D_{cr} [mm²/s] ;
- corrosion : the rate of corrosion i_{corr} [mm/year], depending on whether or not the chloride concentration threshold has been reached at the reinforcement.

3.1.2 Reminder of the equations

A simple Python program was written, containing the following equations :

- equations (2.2) and (2.3) for the carbonation front propagation, respectively in an intact and cracked state ;
$$x(t) = K\sqrt{t} \quad x_{cr}(t) = 50 \sqrt{w} \sqrt[4]{t}$$
- equations (2.5) and (2.6) for the chloride ingress into the concrete matrix, with respectively $D_i = D_{app}$ and D_{cr} for an intact or cracked state ;

$$C(x, t) = C_s \left(1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_i t}} \right) \right)$$

- equation (2.7) for the diameter reduction at time t ;

$$d'(t) = i_{corr}(t - t_0)$$

3.1.3 Parameters

The diffusion coefficients and rates of corrosion are selected :

$$D_{app} = 5 \times 10^{-7} \text{ [mm}^2\text{/s]} ;$$

$$D_{cr} = 10^{-3} \text{ [mm}^2\text{/s]} ;$$

$$i_{corr} = 0.055 \text{ (carbonation only) or } 0.122 \text{ (chloride threshold reached) [mm/year].}$$

Several fixed cracking situations are studied, having the following naming system :

1. no cracking : $w = 0$ mm and $s = 0$ mm
2. light cracking : $w = 0.1$ mm and $s = 200$ mm
3. medium cracking : $w = 0.3$ mm and $s = 100$ mm
4. heavy cracking : $w = 0.5$ mm and $s = 50$ mm

3.2 Carbonation

This section focuses on the influence of carbonation alone.

Figures 3.1 and 3.3 respectively depict equations (2.2) and (2.3) for several carbonation coefficients K and crack widths w . The ensuing reductions of the diameter due to corrosion through carbonation only are shown on figures 3.2 and 3.4 for reinforcement protected by a cover of 30 mm.

The compacity of the concrete, which influences K , seems to be of great importance for the initiation of corrosion in intact concrete. Indeed, while a well compacted concrete will protect the reinforcement at a cover depth of 30 mm for more than a hundred years, poorly compacted concrete would only offer this protection for a decade.

However, concrete is inherently cracked, due to phenomena such as shrinkage. In this case, the evolution of the carbonation front is taken as only dependent on the crack width, and it can be seen that corrosion begins very early during the design working life even for widths below the durability requirement of 0.3 mm (section 2.3).

To visualise the influence of the cover depth on the initiation time of corrosion, the graphs of figures 3.5 and 3.6 are produced. It is observed that an increase of cover thickness plays a marginal effect on the onset of corrosion for a crack width of 0.3 mm. The difference is only marked for intact concrete.

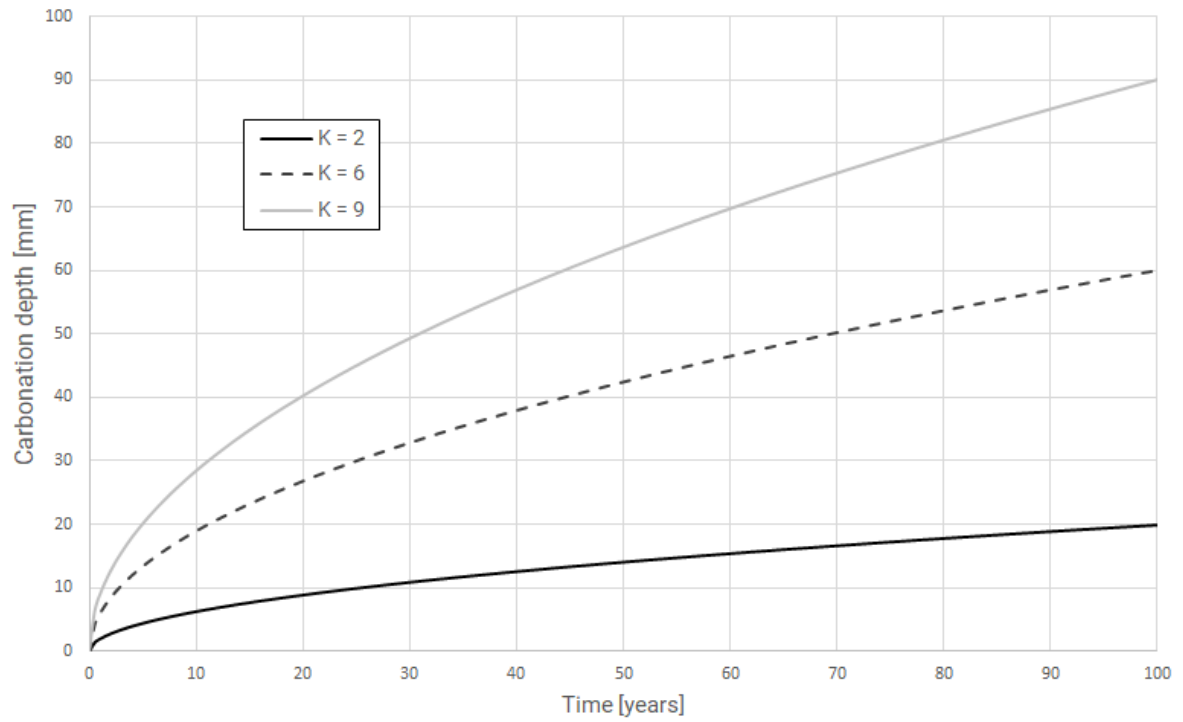


Figure 3.1: Evolution of the carbonation depth in intact concrete for several carbonation coefficients ; $K = 2, 6$ and $9 \text{ mm/year}^{1/2}$

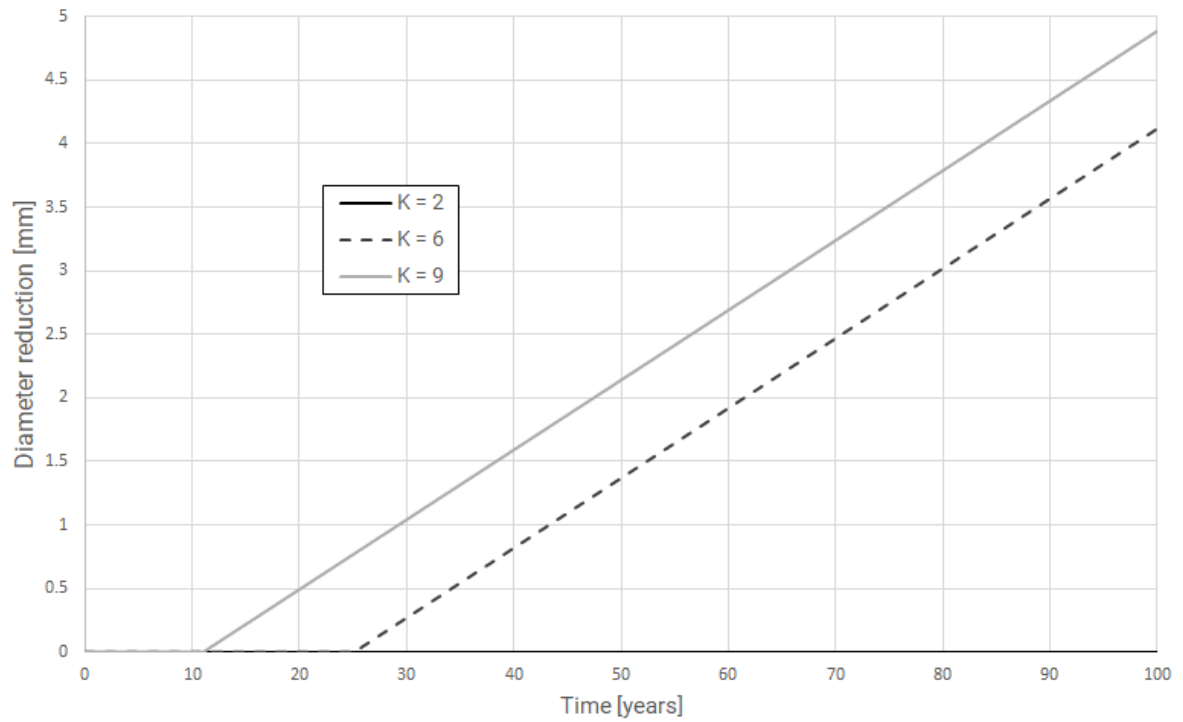


Figure 3.2: Diameter reduction due to carbonation only in intact concrete for different carbonation coefficients ; $K = 2, 6$ and $9 \text{ mm/year}^{1/2}$

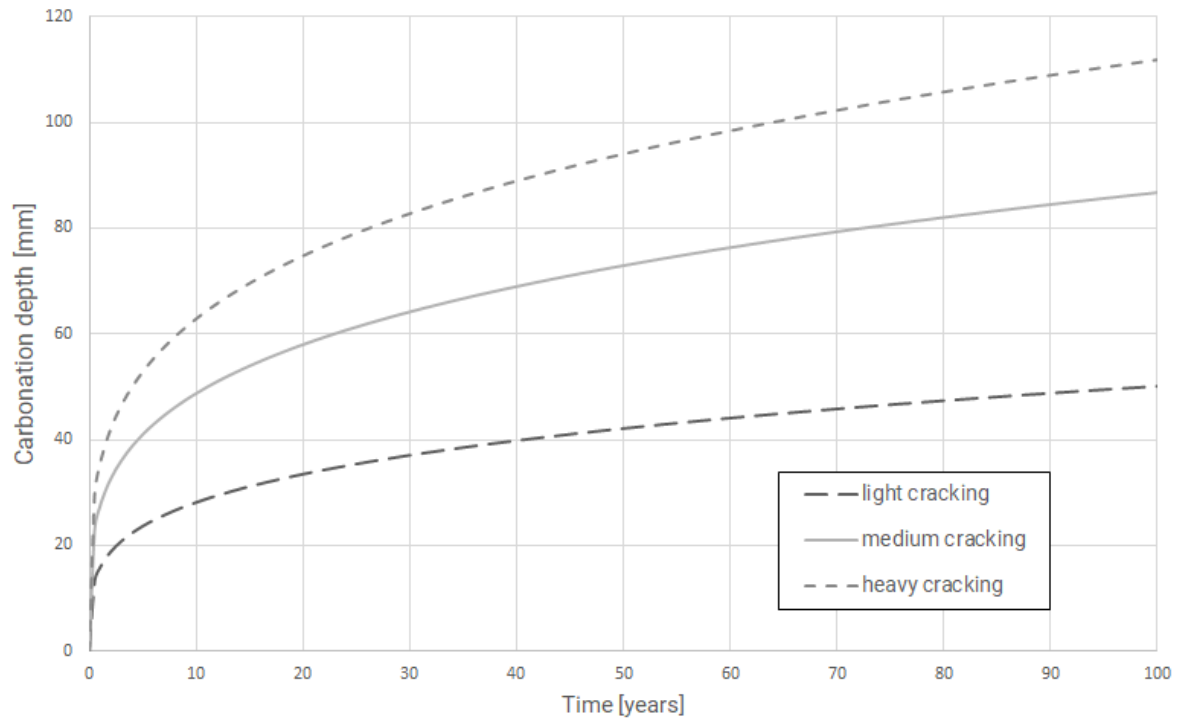


Figure 3.3: Evolution of the carbonation depth in cracked concrete for several crack widths ; $w = 0.1, 0.3$ and 0.5 mm

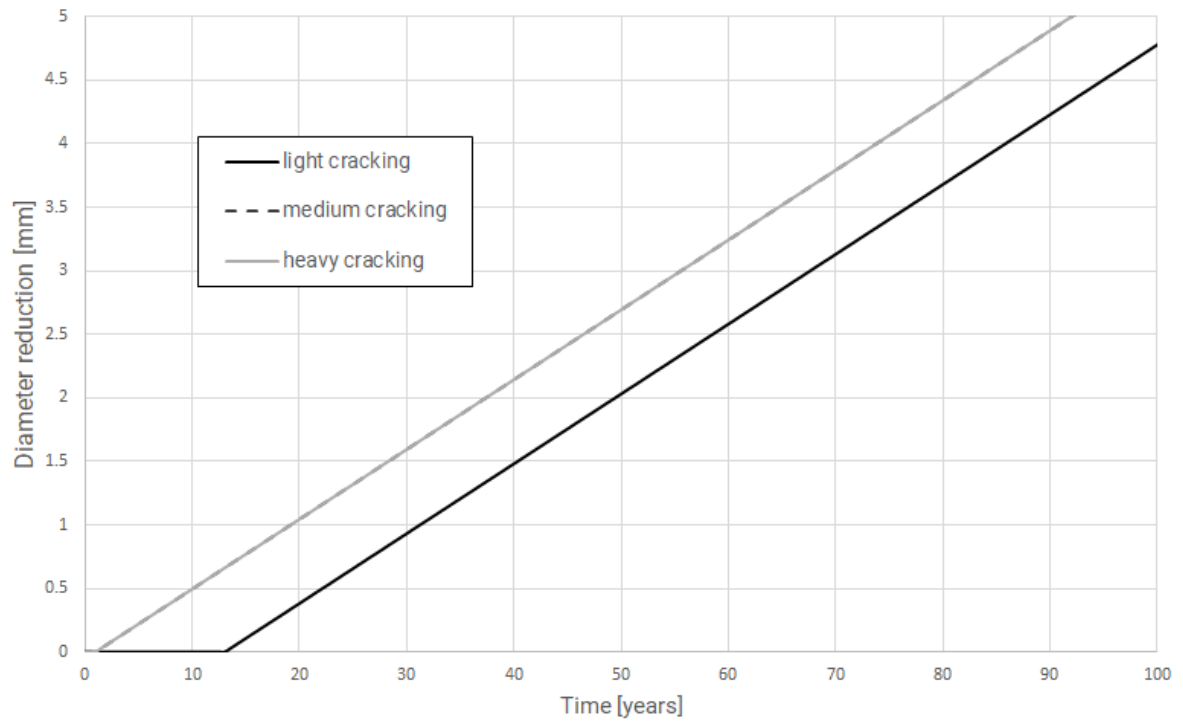


Figure 3.4: Diameter reduction due to carbonation only in cracked concrete for several crack widths ; $w = 0.1, 0.3$ and 0.5 mm

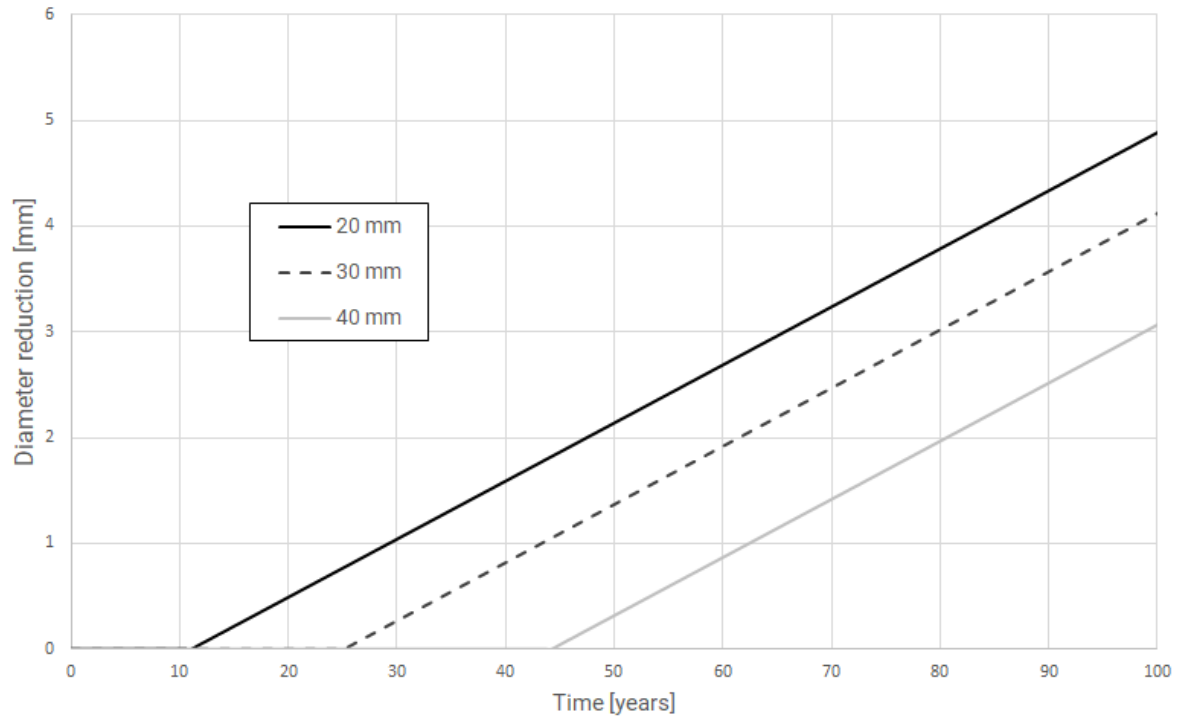


Figure 3.5: Influence of cover thickness on the initiation time of corrosion through carbonation only for intact concrete ; $K = 6 \text{ mm/year}^{1/2}$, covers of 20, 30 and 40 mm

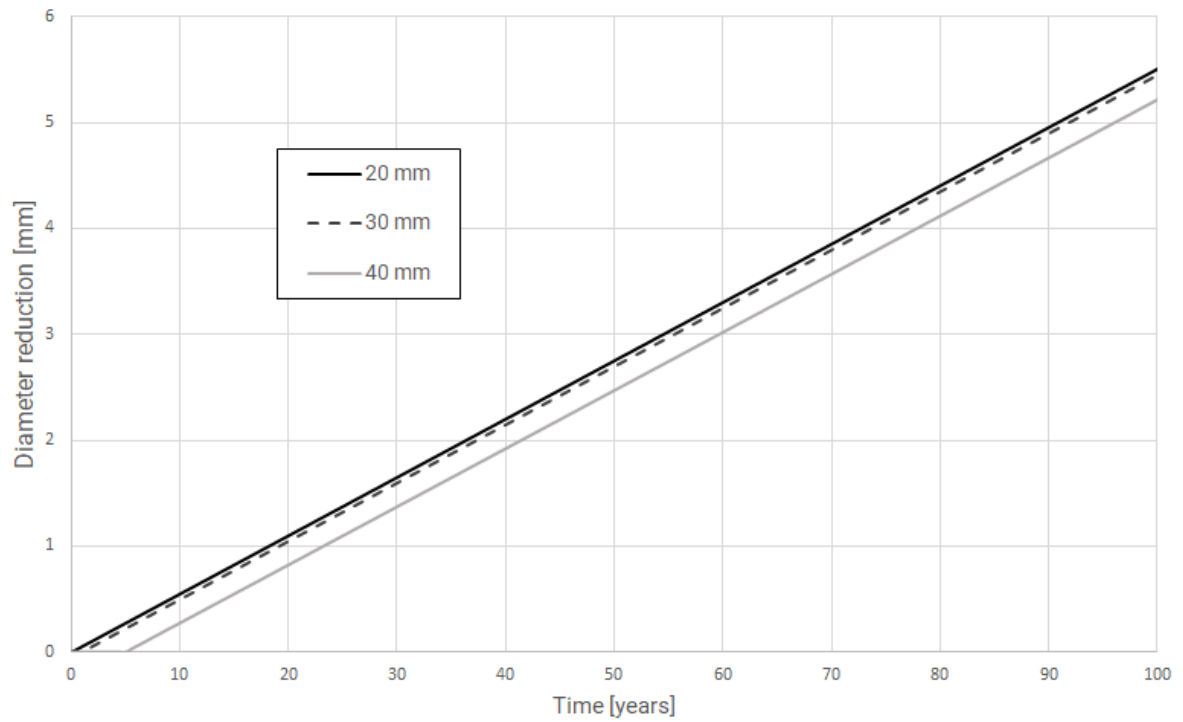


Figure 3.6: Influence of cover thickness on the initiation time of corrosion through carbonation only for cracked concrete ; $w = 0.3 \text{ mm}$, covers of 20, 30 and 40 mm

3.3 Penetration of chlorides

In this section, the attention is turned towards corrosion due to the sole penetration of chloride ions.

The influence of the surface chloride concentration on the concentration at cover depth and the reduction in diameter due to corrosion is presented in figures 3.7 to 3.10.

Unless the environment is extremely aggressive (C_s higher than 0.5%), chlorides alone do not induce corrosion during the lifetime of the structure, intact or cracked.

Cracking leads to a steep increase in the concentration at cover depth in the first 20 years but the progression become slower as time advances, almost reaching a plateau.

The influence of the cover thickness on the chloride concentration at the reinforcement is shown on figures 3.11 and 3.12.

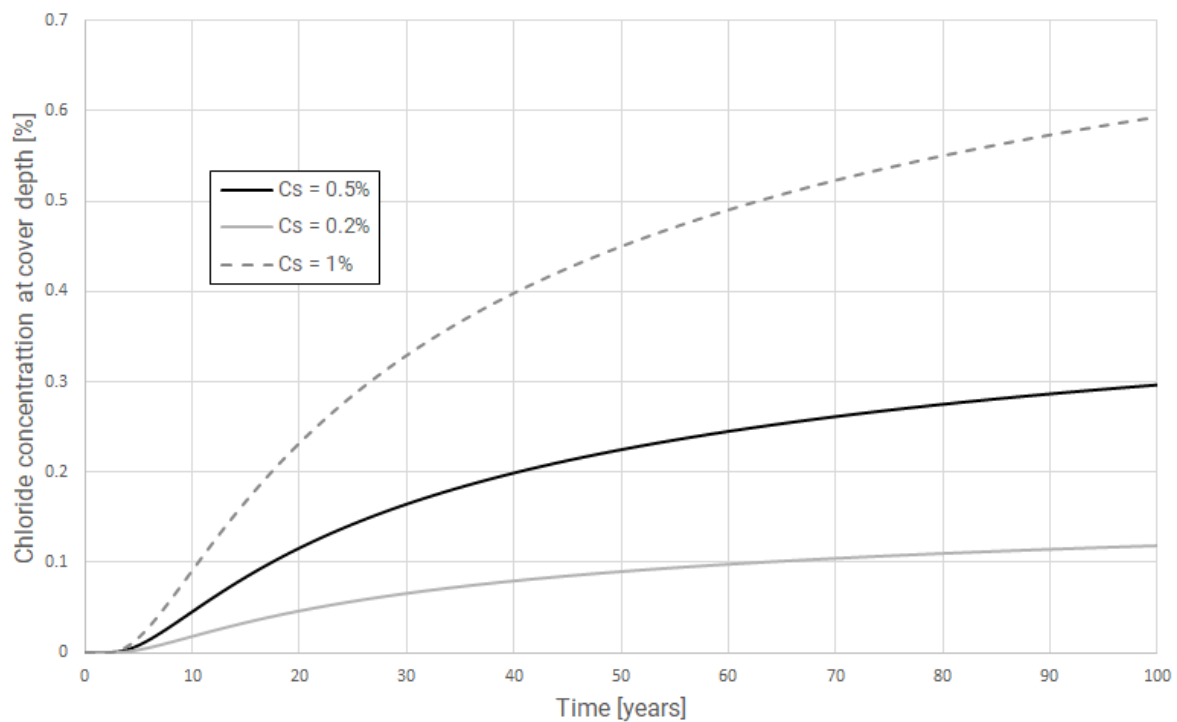


Figure 3.7: Influence of surface chloride concentration C_s on the concentration at cover depth ; $C_s = 0.2, 0.5$ and 1% , cover of 30 mm

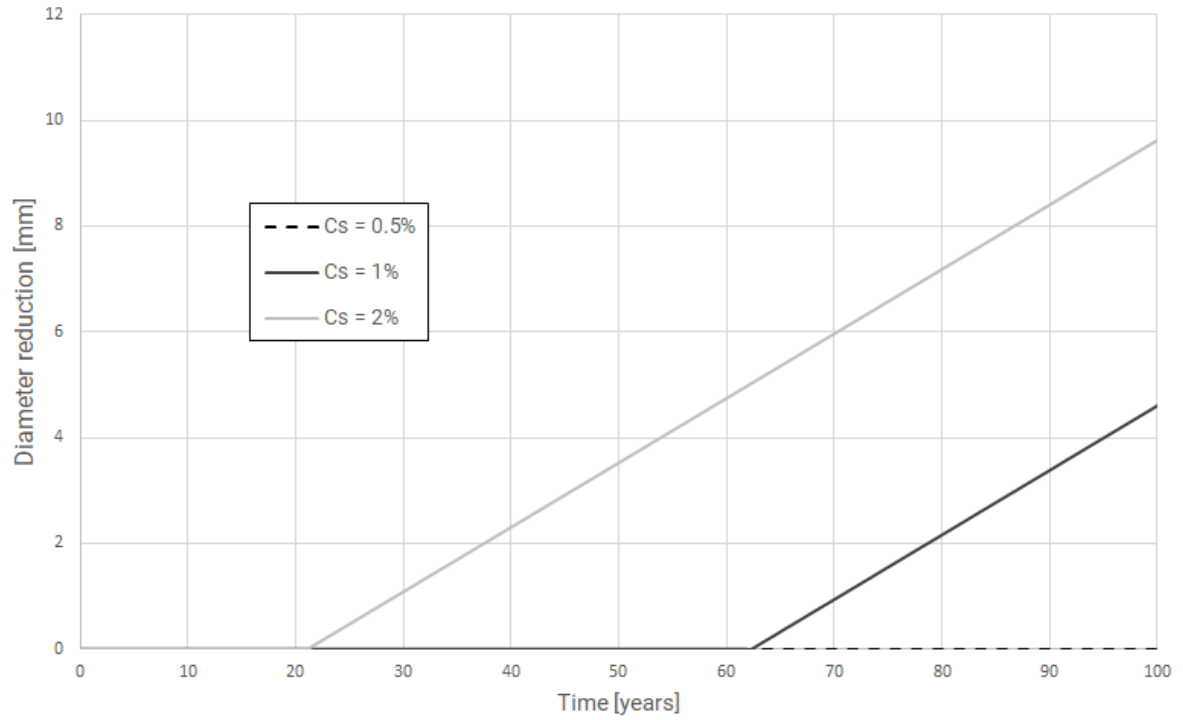


Figure 3.8: Diameter reduction due to chlorides only in intact concrete for different surface chloride concentrations ; $C_s = 0.5, 1$ and 2% , cover of 30 mm

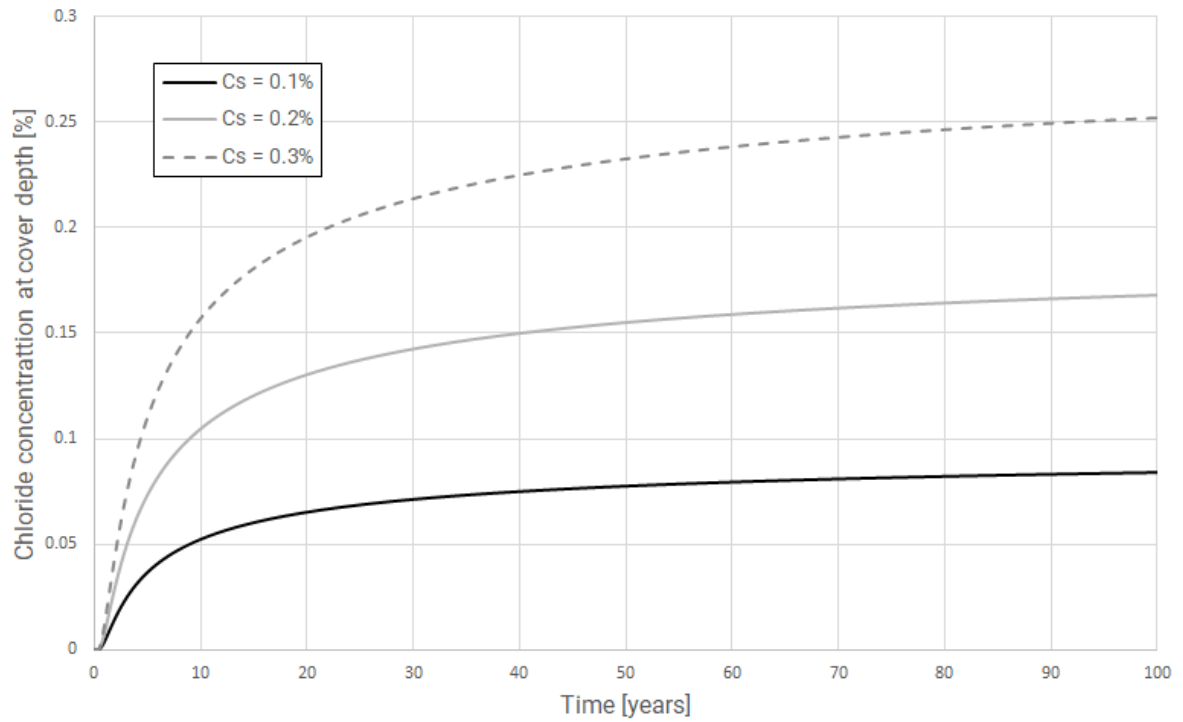


Figure 3.9: Influence of surface chloride concentration C_s on the concentration at cover depth ; $C_s = 0.1, 0.2$ and 0.3% , $w = 0.3$ mm, $s = 100$ mm, cover of 30 mm

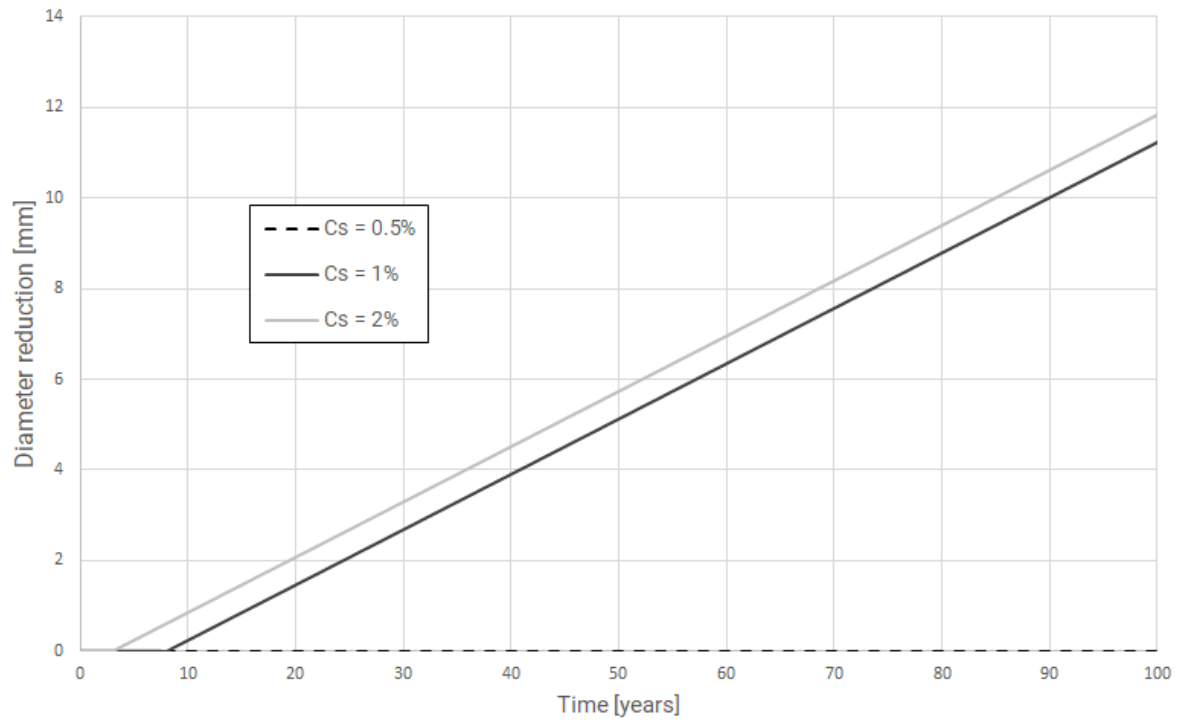


Figure 3.10: Diameter reduction due to chlorides only in cracked concrete for different surface chloride concentrations ; $C_s = 0.5, 1$ and 2% , $w = 0.3$ mm, $s = 100$ mm, cover of 30 mm

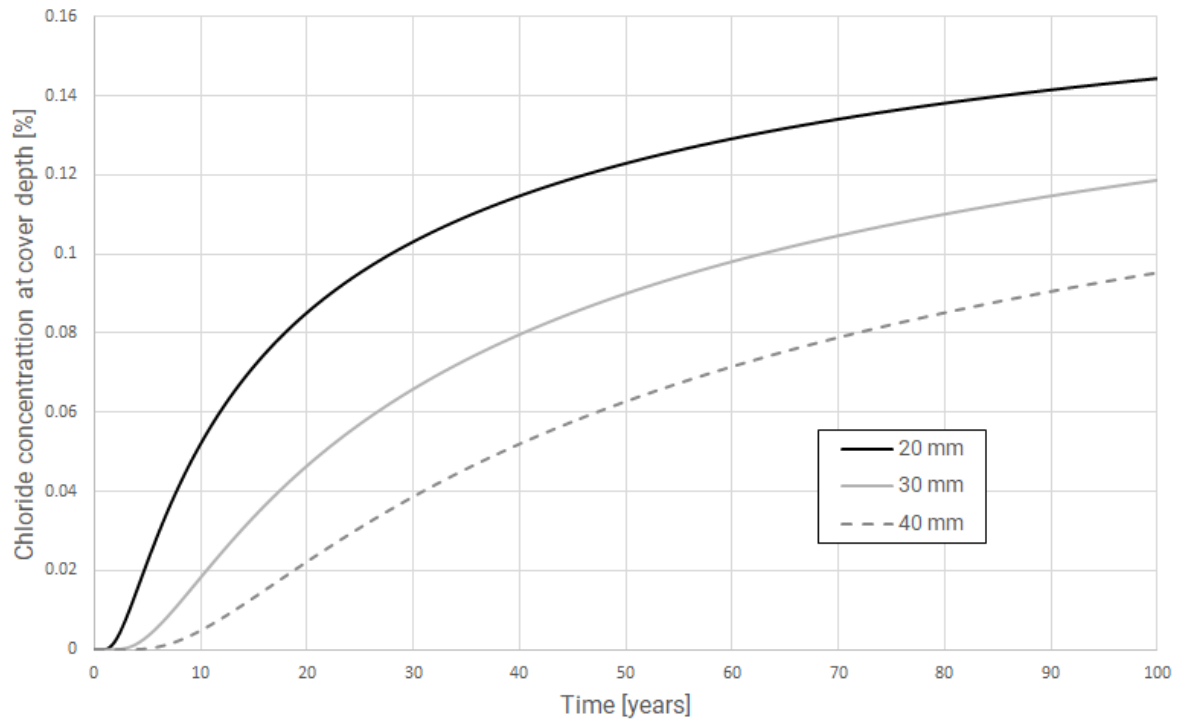


Figure 3.11: Influence of the cover thickness for intact concrete on the chloride concentration at the reinforcement ; $C_s = 0.2\%$, covers of 20, 30 and 40 mm

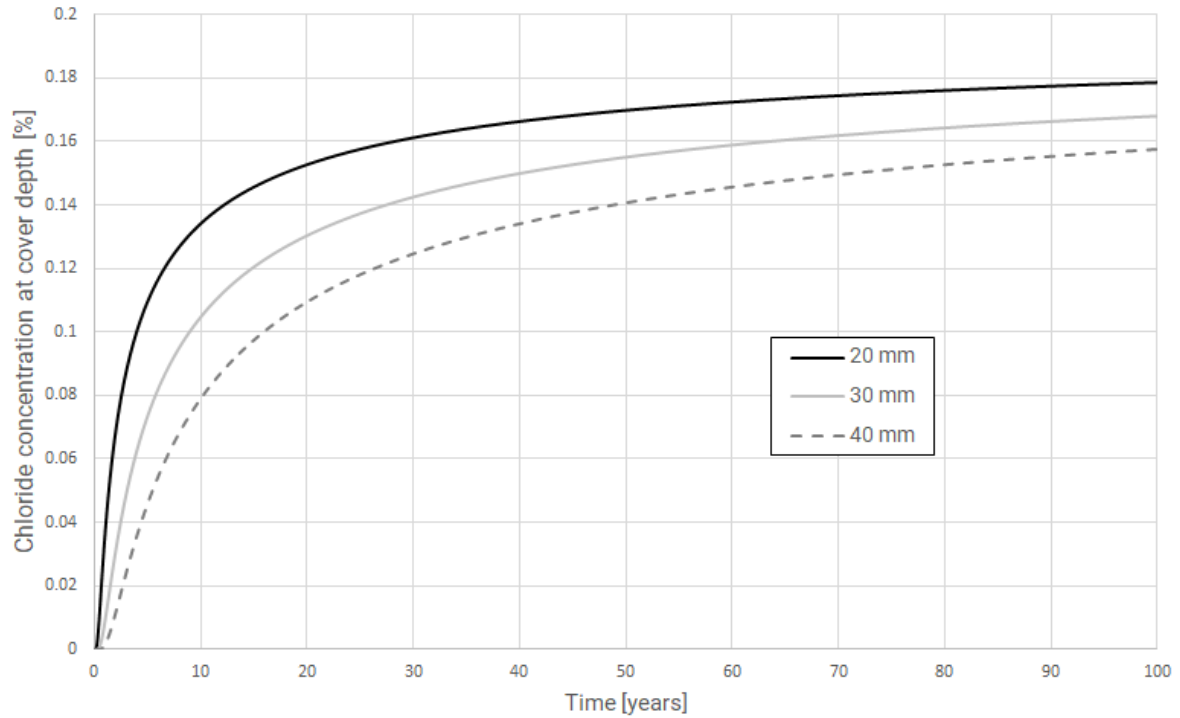


Figure 3.12: Influence of the cover thickness for cracked concrete on the chloride concentration at the reinforcement ; $C_s = 0.2\%$, $w = 0.3$ mm, $s = 100$ mm, covers of 20, 30 and 40 mm

3.4 Carbonation and chloride penetration

The coupled action of carbonation and chloride penetration is now studied. Figures 3.13 and 3.14 present the influence of the cover depth on the onset of corrosion for a surface chloride concentration of 0.2%. However, since the chlorides didn't participate in the corrosion for such a value, figure 3.15 was added with $C_s = 1\%$.

The reduction of diameter is more than 5 mm for $C_s = 0.2\%$ and shoots up to 12 mm for $C_s = 1\%$, with the cover thickness having little influence in both cases.

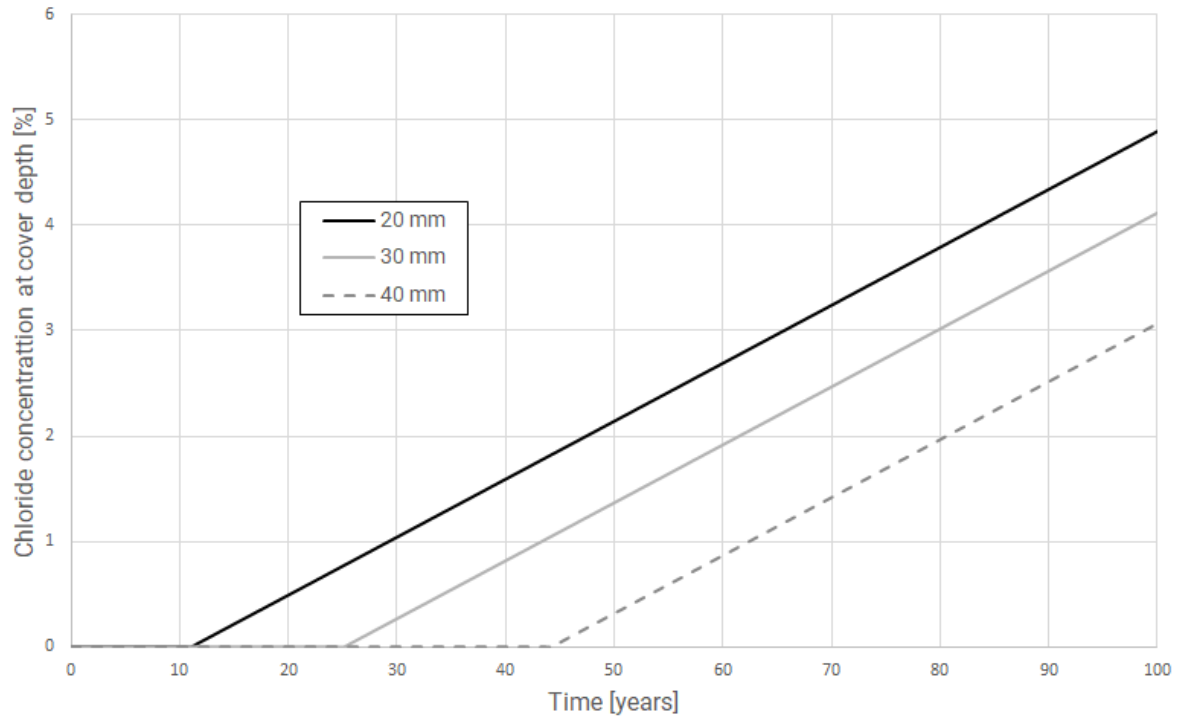


Figure 3.13: Diameter reduction under combined action of carbonation and chlorides on intact concrete ; $C_s = 0.2\%$, $K = 6 \text{ mm/year}^{1/2}$

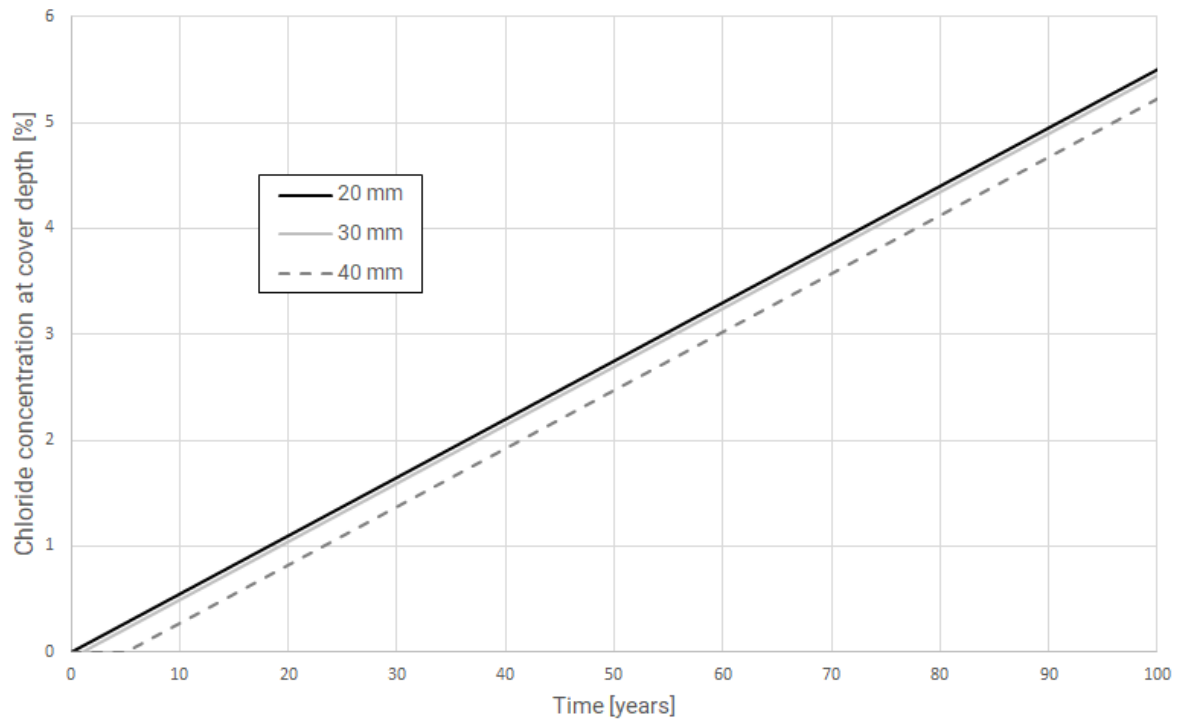


Figure 3.14: Diameter reduction under combined action of carbonation and chlorides on cracked concrete ; $C_s = 0.2\%$, $K = 6 \text{ mm/year}^{1/2}$, $w = 0.3 \text{ mm}$, $s = 100 \text{ mm}$

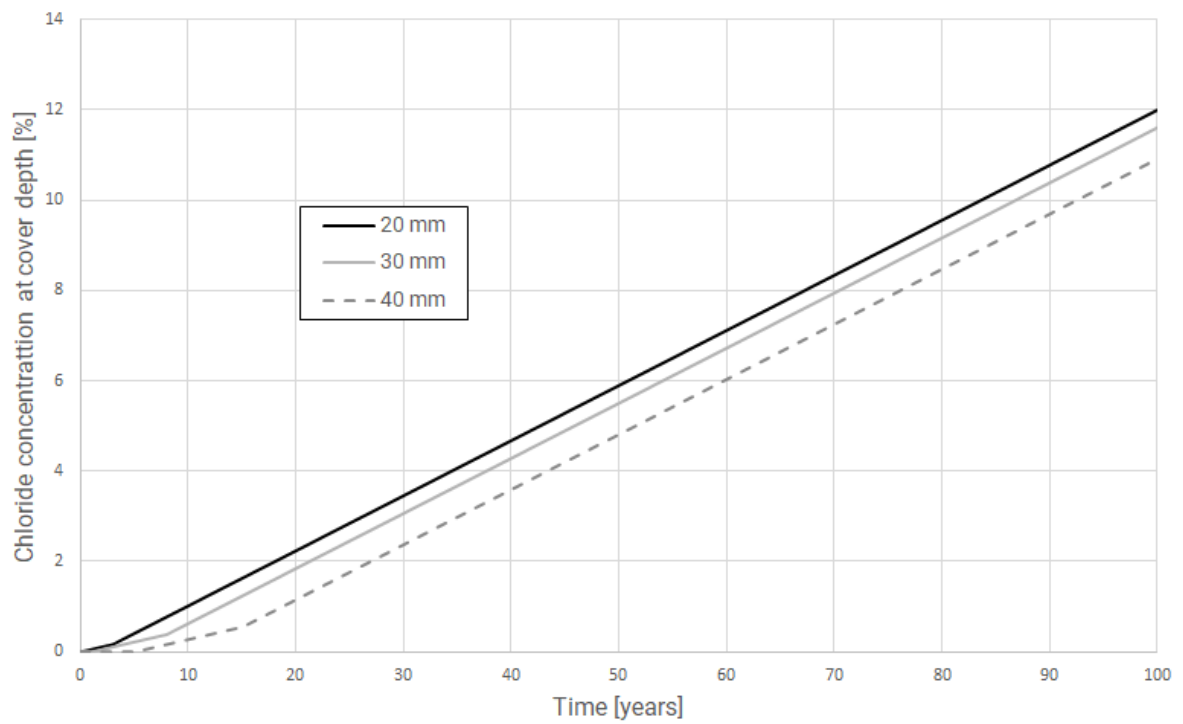


Figure 3.15: Diameter reduction under combined action of carbonation and chlorides on cracked concrete ; $C_s = 1\%$, $K = 6 \text{ mm/year}^{1/2}$, $w = 0.3 \text{ mm}$, $s = 100 \text{ mm}$

4 | Modelling of the reinforced concrete element

Since the elastic analysis is not capable of identifying collapse mechanisms depending on the interaction between the concrete matrix and its reinforcement, such as tension stiffening, there is need for a non-linear formulation. The formulation proposed and numerically implemented by MSG&I pays particular attention to conditions close to failure, namely the post-cracking phase. This structural analysis is then coupled with an adequate model of the previously described environmental degradation to which the reinforcement is subjected.

The theories used to construct the behavioural part of this non-linear formulation will be summarised in this chapter. Loading histories are not considered, which signifies that to one stress state corresponds a unique strain state and vice-versa. For simplicity, the constitutive relationships of concrete and steel are considered uncoupled, which is in reality not the case. For more in-depth descriptions of the theories, see [4, 5] and its bibliography.

4.1 The Compression Field Theory

4.1.1 Description and assumptions

The (Diagonal) Compression Field Theory (CFT), a two-dimensional model introduced by Collins and Mitchell in 1974 [23], is a fundamental contribution to the theory of elements behaving in membrane regime and to the shear theory. It treats reinforced concrete in terms of average stresses and strains - a valid assumption on an area sufficiently large to allow the determination of a defined crack pattern. The concrete material is taken as composite, anisotropic and homogeneous on average.

An element in membrane regime is an element only subjected to stresses in its lying plane. This slab element of constant thickness is made of a concrete matrix reinforced by two orthogonal sets of reinforcing steel bars uniformly distributed over the element. The stresses and strains acting on the element are represented on figure 4.1. It is assumed that the edges of the element remain straight and parallel after deformation.

The difference between the CFT and the later theories lies in the assumptions used. In the case of the CFT,

1. the principal directions of main stresses and main strains are taken as coincident ;
2. the reinforcement and concrete are perfectly bonded together ;
3. the angle of inclination of cracks rotates with the intensity of the load ;

4. after cracking, concrete is unable to sustain tension in the principal tensile stress direction.

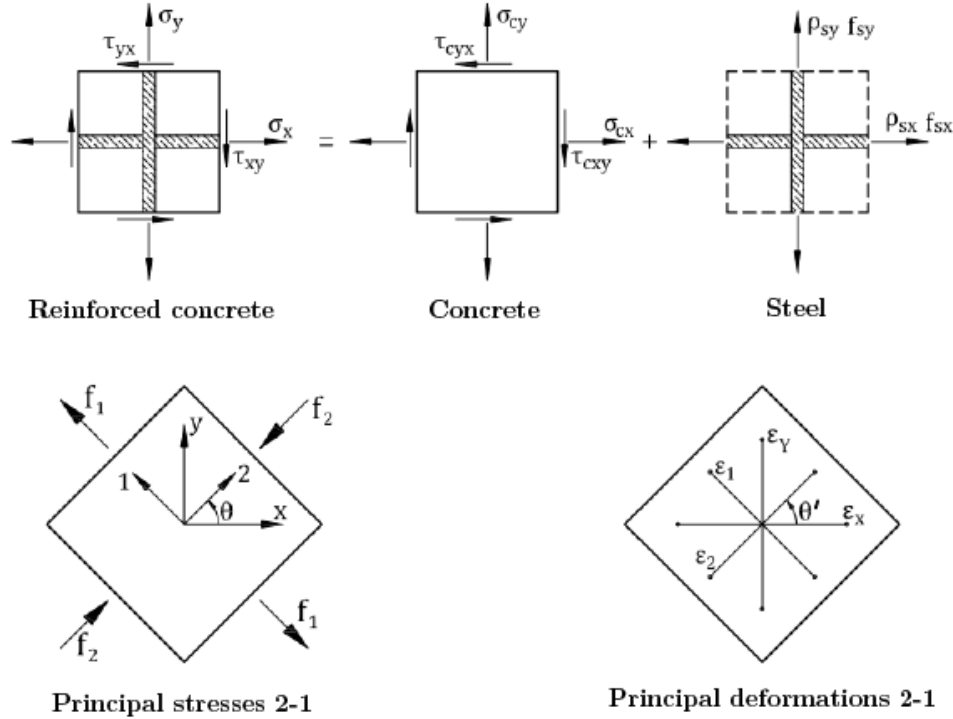


Figure 4.1: Representation of stress and strain of the membrane element [4, 5]

4.1.2 Equilibrium and compatibility

For cracked concrete, the principal tensile and compression stresses are denoted by f_{c1} and f_{c2} respectively (sometimes simply written as f_1 and f_2).

Applying a tangential stress τ_{xy} to the cracked membrane element creates stresses in the concrete matrix and reinforcement, inducing this compression stress f_{c2} of inclination θ with respect to the longitudinal axis. Per the first assumption, with θ_0 being the principal strain direction, the inclination of f_{c2} is then :

$$\theta = \theta_0 \quad (4.1)$$

The stresses in the longitudinal and transverse ordinary reinforcements are respectively f_{sx} and f_{sy} , with respective cross sections of A_{sx} and A_{sy} . The eventual prestressing applying an axial effort N on the element is represented by f_{px} and A_{px} . The various reinforcement ratios are :

$$\rho_{sx} = \frac{A_{sx}}{b_w j d} \quad \rho_{sy} = \frac{A_{sy}}{b_w s} \quad \rho_{px} = \frac{A_{px}}{b_w j d} \quad (4.2)$$

with the geometrical entities b_w , s and $j d$ represented on figure 4.2.

The three equilibrium equations inside the concrete are found using the diagrams and Mohr circles of figure 4.2 :

$$\begin{aligned} \sigma_{cx} &= \rho_{sx} f_{sx} + \rho_{px} f_{px} + \frac{N}{b_w j d} = \frac{\tau_{xy}}{\tan \theta} = f_{c2} \cos^2 \theta \\ \sigma_{cy} &= \rho_{sy} f_{sy} = \tau_{xy} \tan \theta = f_{c2} \sin^2 \theta \\ \text{with } f_{c2} &= \tau_{xy} \left(\tan \theta + \frac{1}{\tan \theta} \right) \end{aligned} \quad (4.3)$$

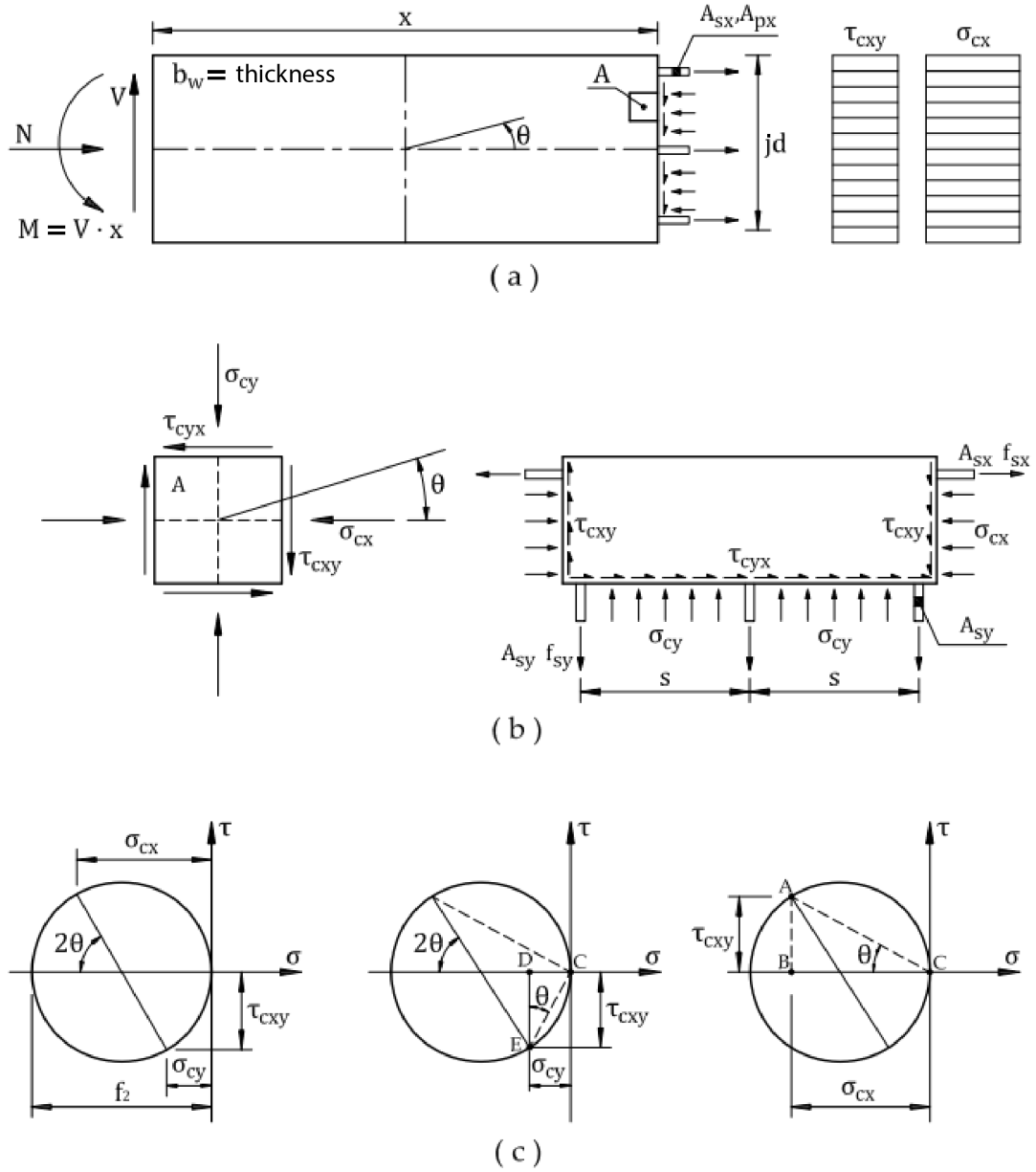


Figure 4.2: (a) and (b) Forces and efforts involved in the equilibrium, (c) Mohr circles representative of the average stress in the concrete [24]

The second assumption, of a perfect bond between steel and concrete, is written as :

$$\varepsilon_{sx} = \varepsilon_{cx} = \varepsilon_x \quad \varepsilon_{sy} = \varepsilon_{cy} = \varepsilon_y \quad (4.4)$$

allowing to determine θ through :

$$\tan^2 \theta = \frac{\varepsilon_x + \varepsilon_2}{\varepsilon_y + \varepsilon_2} \quad (4.5)$$

4.1.3 Constitutive equations

4.1.3.1 Steel

An elastoplastic relationship was chosen to represent the behaviour of the ordinary steel reinforcement, with a respective yield stress of f_{yx} and f_{yy} for the x - and y -direction. It is assumed that the axial stress only depends on the axial strain, and that the average shear stress that the reinforcement resists in its normal plane is zero.

Before yielding

$$\begin{aligned} f_{sx} &= E_s \varepsilon_x \\ f_{sy} &= E_s \varepsilon_y \end{aligned} \quad (4.6)$$

with E_s the Young modulus of the steel.

After yielding

$$\begin{aligned} f_{sx} &= f_{yx} \\ f_{sy} &= f_{yy} \end{aligned} \quad (4.7)$$

4.1.3.2 Concrete

For cracked concrete, the main compressive stress f_{c2} and its corresponding strain ε_2 are different than the ones obtained during a uniaxial test on a cylindrical specimen. Indeed, a thin reinforced slab has a lower strength and stiffness in its cracked state than a specimen subjected to a simple compression test [6], see figure 4.3.

Several stress-strain relationships have been formulated to take into account this decrease in resistance due to tensile deformations in the transverse direction of the compressed struts. The most common one is Hognestad's parabola :

$$\begin{aligned} f_{c2} &= f_{c2,\max} \left(2 \left(\frac{\varepsilon_2}{\varepsilon'_c} \right) - \left(\frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right) \\ \text{with } \frac{f_{c2,\max}}{f'_c} &= \frac{1}{0.8 + 170 \varepsilon_1} \leq 1 \end{aligned} \quad (4.8)$$

with

f'_c the resistance to uniaxial compression of a standard cylindrical specimen ;

ε'_c the peak compressive deformation of the concrete, usually assumed to be equal to -0.002 .

Concerning tension, the fourth assumption from section 4.1.1 leads to :

$$f_1 = 0 \quad \text{for} \quad \varepsilon_1 > \varepsilon_{cr} \quad (4.9)$$

ε_{cr} being the concrete strain at cracking.

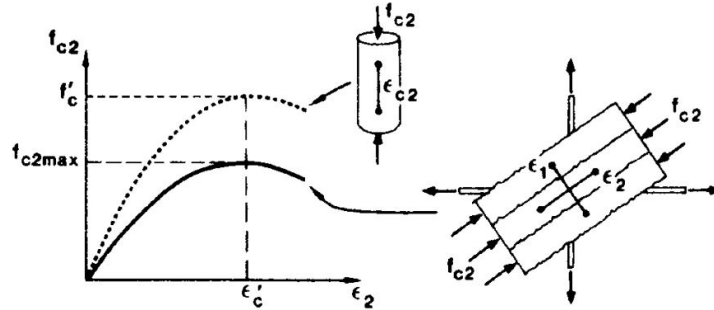


Figure 4.3: Comparison of the compression stress-strain relationship of a cylinder and a thin slab [6]

4.1.4 Shortcomings

The original formulation of the CFT has two major limitations. Firstly, ignoring the contribution in tension of cracked concrete, in particular the tension stiffening resulting from its interaction with the reinforcement, overestimates the deformation state. Secondly, assuming that the crack angle coincides with the principal compression direction does not allow to take into account the shear contribution of the concrete matrix.

The Modified Compression Field Theory (MCFT) has been introduced by Collins and Vecchio in 1986 [6] to tackle part of these shortcomings.

4.2 The Modified Compression Field Theory

The MCFT is currently the most widely used method for the study of the behaviour of a cracked reinforced concrete element subjected to in-plane stresses, depicted earlier on figure 4.1.

In this theory, cracked concrete is treated as a new material possessing its own stress-strain characteristics, which have been determined by Collins and Vecchio through a set of experiments. The equilibrium, compatibility and stress-strain relationships of steel and concrete are still formulated in terms of average stress and strain, but are an adaptation of the equations obtained in the CFT, accounting for the new set of assumptions, which is :

1. (CFT) the principal directions for stresses and strains are still taken as coincident ;
2. (CFT) the concrete and its reinforcement are perfectly bonded together ;
3. after cracking, the concrete matrix can sustain tensile stresses in-between cracks.

Figure 4.4 summarises the behaviour of the different materials.

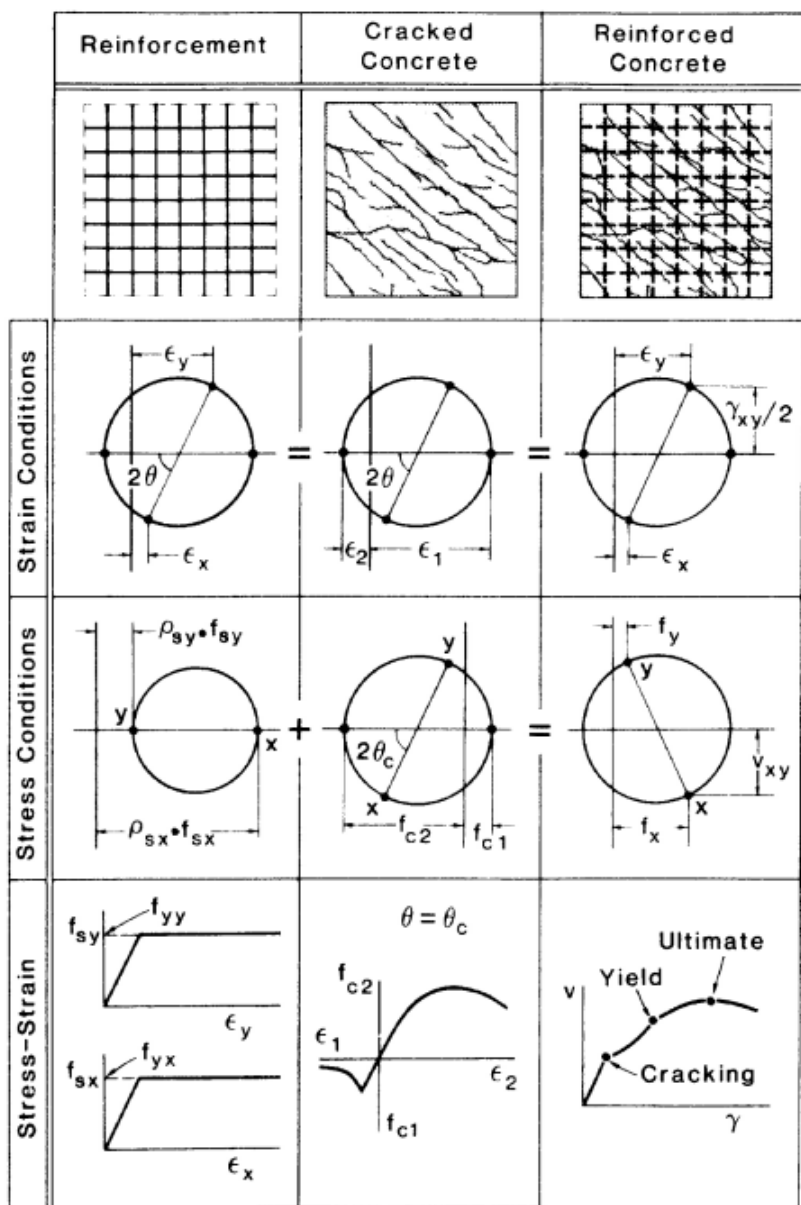


Figure 4.4: Summary of the MCFT for membrane elements [6]

4.2.1 Equilibrium and compatibility

Assuming that $\tau_{cx} = \tau_{cy} = \tau_{cxy}$, the stress conditions are fully known if f_{cx} , f_{cy} and τ_{cxy} are known. If θ_c is the inclination of the principal stresses in the concrete with respect to the x -axis, equilibrium is defined with the following equations, now taking into account the contribution of f_{c1} :

$$\begin{aligned}\sigma_{cx} &= f_{c1} - \frac{\tau_{cxy}}{\tan \theta_c} \\ \sigma_{cy} &= f_{c1} - \tau_{cxy} \tan \theta_c \\ f_{c2} &= f_{c1} - \tau_{cxy} \left(\tan \theta_c + \frac{1}{\tan \theta_c} \right)\end{aligned}\tag{4.10}$$

The new assumptions do not affect the compatibility equations obtained for the CFT, the equations presented in section 4.1.2 are thus still valid for this model.

4.2.2 Constitutive equations

4.2.2.1 Steel

The elastoplastic behaviour used for the reinforcement in section 4.1.3.1 has not been modified.

4.2.2.2 Concrete

The relationships obtained during the experiments describe the behaviour of concrete in tension as being (figure 4.5) :

$$f_{c1} = E_c \varepsilon_1 \quad \text{for } \varepsilon_1 \leq \varepsilon_{cr} \tag{4.11}$$

$$f_{c1} = \frac{f_{cr}}{1 + \sqrt{200} \varepsilon_1} \quad \text{for } \varepsilon_1 > \varepsilon_{cr} \tag{4.12}$$

with

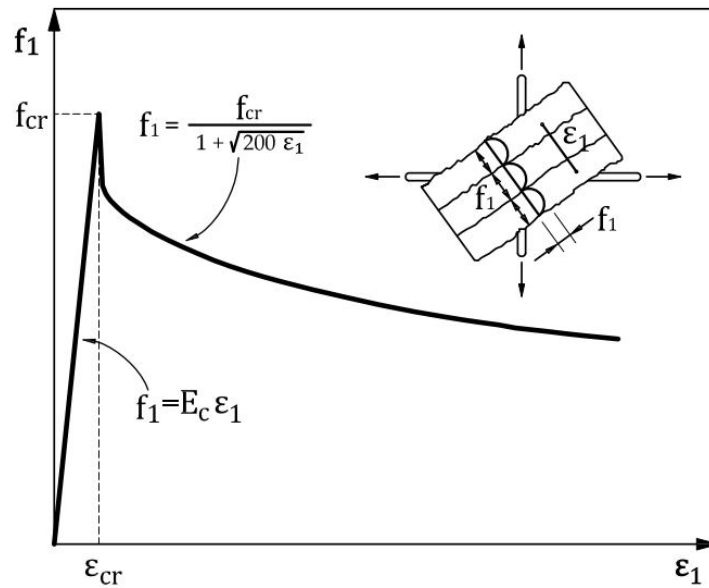


Figure 4.5: Behavior of cracked concrete in tension [4, 5]

f_{cr} the tensile stress at cracking ;

ε_{cr} the corresponding cracking strain ;

E_c the Young modulus of intact concrete.

In compression, equation 4.8 keeps the same shape but with a modified $f_{c2,max}$, now depending on the negative quantity ε'_c :

$$\frac{f_{c2,max}}{f'_c} = \frac{1}{0.8 - 0.34 \varepsilon_1/\varepsilon'_c} \leq 1 \quad (4.13)$$

4.2.3 Transmission of loads at cracks

The previous formulations deal with average stress and strain fields, which means they are not capable of giving information on the variations present at cracks. The MCFT addresses the transmission of stresses at cracks, as it influences the load pattern inside the element, thus allowing a more precise analysis.

4.2.3.1 Crack check control

For the majority of concretes, cracking occurs at the interface between the cement paste and the aggregates. This creates a roughness that can transfer shear through aggregate interlock. This phenomenon is illustrated on figure 4.6.

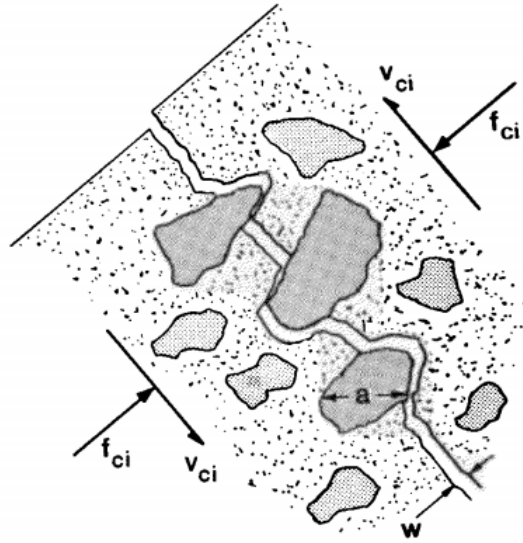


Figure 4.6: Transmission of shear stresses across cracks through aggregate interlock [6]

At cracks, the stress and strain fields differ greatly from the mean values used so far : the tensile stress in the steel reinforcement is higher than average while it is zero for the concrete. Concrete is however subjected to its highest tensile stress midway between the cracks, where the stress is lower than average in the reinforcement. These disruptions are represented on figure 4.7.

The ultimate capacity of the element may depend on its ability to transmit the tensile loads across the cracks. Thus, a crack check control is required, with the critical direction assumed perpendicular to the principal tensile strain direction.

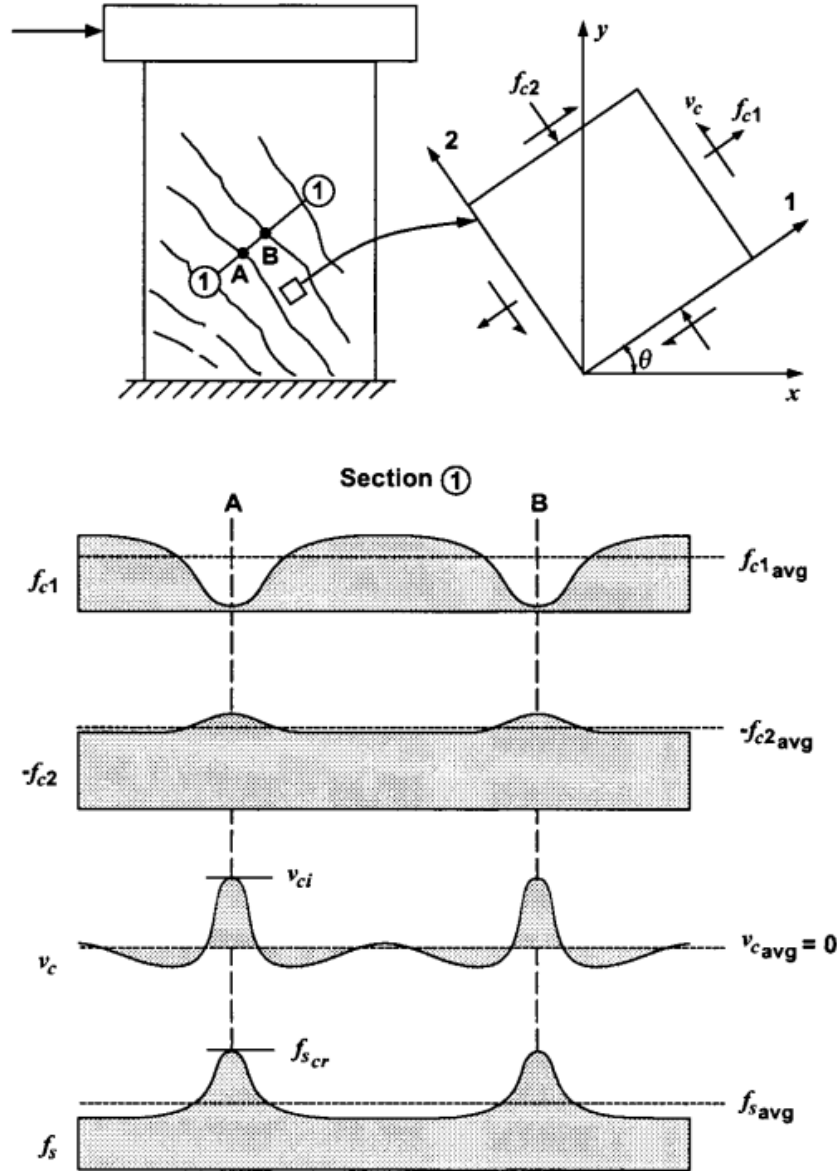


Figure 4.7: Disruptions of the stress fields at cracks [25]

The presence of local shear stresses v_{ci} may be required to transmit the load through the crack, possibly accompanied by local compression stresses f_{ci} . The equilibrium inside the element can be satisfied without the presence of these shear stresses only if the average stress in the reinforcement does not reach a certain threshold. This condition can be written as :

$$\rho_{sx}(f_{sx,cr} - f_{sx}) = \rho_{sy}(f_{sy,cr} - f_{sy}) = f_{c1} \quad (4.14)$$

but only if $f_{sx,cr} \leq f_{yx}$ and $f_{sy,cr} \leq f_{yy}$ (cr referring to the crack location).

The relationship between v_{ci} , the average crack width w and f_{ci} used in the MCFT is that of Walraven [26] :

$$v_{ci} = 0.18 v_{ci,max} + 1.64 f_{ci} - 0.82 \frac{f_{ci}^2}{v_{ci,max}} \quad (4.15)$$

with

$$v_{ci,max} = \frac{0.18\sqrt{-f'_c}}{0.3 + 24 w/(a + 16)} \quad (4.16)$$

with a the maximum aggregate size in [mm] and the stresses in [MPa].

If a solution is not possible due to yielding of the reinforcement at the crack, f_{c1} has to be reduced until one is found.

4.2.3.2 Evaluation of the crack width

The average crack width w can be estimated from the tensile strain and the average crack spacing $S_{m\theta}$ using

$$w = \varepsilon_1 S_{m\theta} = \varepsilon_1 \frac{1}{\frac{\sin \theta}{S_{mx}} + \frac{\cos \theta}{S_{my}}} \quad (4.17)$$

with the S_{mi} estimated using the expression of the CEB code (1985) :

$$S_{mx} = 2 \left(c_x + \frac{s_x}{10} \right) + k_1 k_2 \frac{d_{bx}}{\rho_{sx}} \quad (4.18)$$

$$S_{my} = 2 \left(c_y + \frac{s_y}{10} \right) + k_1 k_2 \frac{d_{by}}{\rho_{sy}} \quad (4.19)$$

with

ρ_{sx} the percentage of horizontal reinforcement (including the prestressing) ;

$$\rho_{sx} = \frac{A_{sx} + A_{px}}{A_c} \quad (4.20)$$

A_c the area of the concrete cross section ;

d_{bx} and d_{by} respectively the diameters of the longitudinal and transverse reinforcements ;

c_x the maximum distance between the longitudinal reinforcement and the beam axis ;

c_y the maximum distance between the transverse reinforcement and the beam axis ;

s_x the maximum spacing of the horizontal bars, which cannot exceed 15 d_{bx} ;

s_y the average spacing between the stirrups ;

k_1 a value depending on the smoothness of the bar (0.8 for ribbed bars and 0.4 for smooth ones) ;

k_2 a value depending on the type of loading (0.25 for pure traction and 0.125 in flexion).

4.2.4 Shortcomings

The MCFT is a method that has proven its efficiency over the years and is widely used for the determination of the strain states of reinforced concrete elements. However, it has deficiencies in several cases, leading to an under- or overestimation of shear strength and stiffness. The reader can discover them in details in [25].

These deviations arise due the observed tendency for the principal stress direction to lag behind the principal strain direction, those two directions having been assumed equal for the model. When the element starts to crack, the principal strain direction increases abruptly while there is little change in the stress direction. Afterwards, they both gradually increase, keeping that constant lag between them. Figure 4.8 shows the obtained stress and strain inclinations obtained during the tests conducted for the MCFT [6].

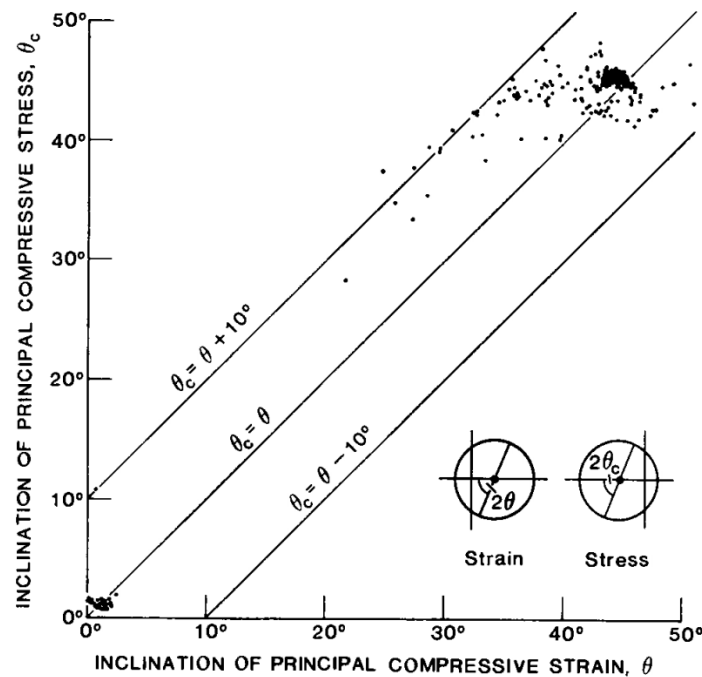


Figure 4.8: Comparison of principal compressive stress direction with principal compressive strain direction [6]

The cracks in concrete were assumed to align with the average principal stress directions, giving an average shear stress of zero in directions orthogonal to cracks. However, local conditions differ on the crack surfaces, giving rise to non-zero shear stresses. For the MCFT, as described before, the magnitude of the tensile stress that can be sustained by the element is reduced if the local stresses exceed a certain limit value. In reality, the relationship between tensile and shear stresses is not so direct, something that the Disturbed Stress Field Model (DSFM) wishes to capture [25].

4.3 The Disturbed Stress Field Model

The DSFM brings several improvements to the MCFT :

- the removal of the constraint coming from the assumptions of coinciding inclinations of principal strains and stresses ;
- a better representation of the behaviour and failure conditions influenced by the shear slip inside the cracks ;
- the shear stresses on the surfaces of cracks do not require a check anymore, which was computationally complicated ;
- the degree of softening of the concrete's response is reduced and more consistent with reality.

4.3.1 Influence of cracking

Two models exist to represent the evolution of cracking inside a concrete element :

Rotating crack model : gradual reorientation of the direction of the cracks, dictated by the loading and material response, as well as the principal stresses and strain directions.

Fixed crack model : the crack direction remains fixed in the direction of first cracking. Discrete new cracks can form at alternate inclinations if the stress conditions dictate it. An important aspect of this method is that it is possible to determine the shear stresses on the crack surfaces and the resulting shear slip.

The DSFM is a hybrid formulation of those two models, while the MCFT lays in the rotating crack model category. The DSFM has a new set of equilibrium, compatibility and constitutive equations to better take into account the disturbances in stress fields arising from the shear stresses on crack surfaces and the resulting slip, with a particular attention to compression softening and tension stiffening. An area containing several cracks is again considered, on which sectional forces can be considered relatively constant, but while keeping in mind the disturbances caused by these cracks.

f_{c1} , similarly than for the MCFT, approaches zero at crack locations and be greater than average between cracks due to tension stiffening. To compensate for this reduced tensile stress at cracks, the stress in the reinforcement f_s locally increases to maintain equilibrium, possibly coupled with shear stresses if the crack is not orthogonal to the reinforcement, as presented earlier in section 4.2.3 and represented on figure 4.7.

Inversely, f_{c2} is increased near cracks, due to aggregate interlock and equilibrium requirements. If there are non-zero local shear stresses v_{ci} at the crack, this creates a rigid body shear slip along the surfaces of that crack. This must be added to the average strains in the concrete, which leads to the deviation of principal stress and strain directions depicted on figure 4.8.

4.3.2 Equilibrium and compatibility

A reinforced concrete element experiencing deformation is subjected to both a continuum straining, resulting from mechanical compliance to stress and the smearing of crack width over the area, as well as a discontinuous slip along the cracks, due to the rigid body movement along their interface. These deformations are presented on figure 4.9.

The measured strains would contain both, leading to what is called the apparent strain state, denoted by $[\varepsilon]$. The actual net strains within the concrete, used to determine the average

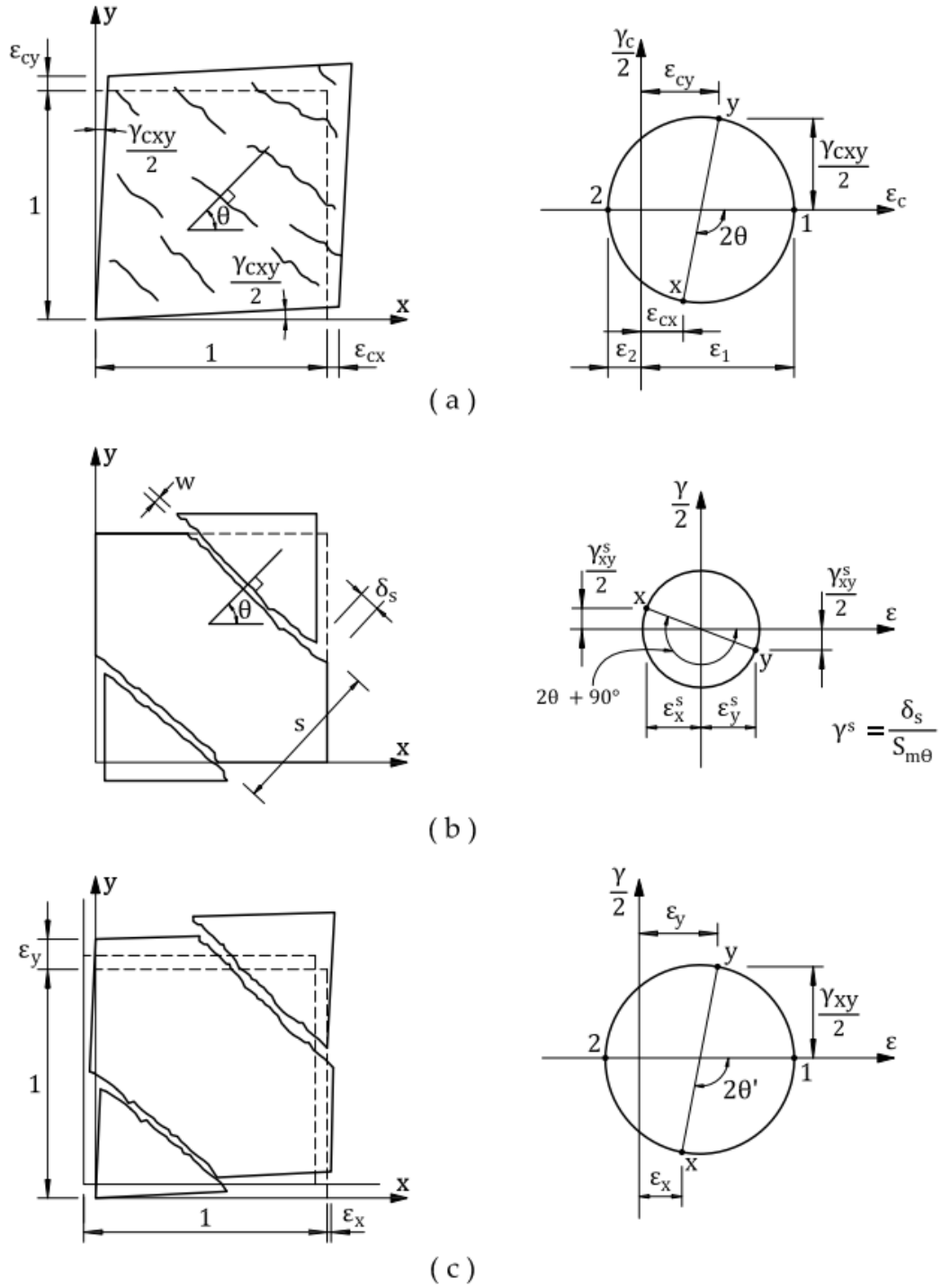


Figure 4.9: Compatibility conditions: (a) deformations due to average (smeared) constitutive response ; (b) deformations due to local rigid body slip along the crack; (c) combined deformations [25]

stresses inside, are referred to as $[\varepsilon_c]$. The corresponding principal strains are calculated using the following equation :

$$\varepsilon_{c1}, \varepsilon_{c2} = \frac{\varepsilon_{cx} + \varepsilon_{cy}}{2} \pm \frac{1}{2} \sqrt{(\varepsilon_{cx} - \varepsilon_{cy})^2 + \gamma_{cxy}^2} \quad (4.21)$$

and the inclination of cracks found using :

$$\theta = \frac{1}{2} \arctan \left(\frac{\gamma_{cxy}}{\varepsilon_{cx} - \varepsilon_{cy}} \right) \quad (4.22)$$

As for the MCFT, the reinforcement and concrete are taken as perfectly bonded together. The average strain in the i -th reinforcement component can thus be found with :

$$\varepsilon_{s_i} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos(2\alpha_i) + \frac{\gamma_{xy}}{2} \sin(2\alpha_i) \quad (4.23)$$

An initial prestressing $\varepsilon_{s_i}^0$ can be added to this formula.

In reality, other strains arise from elastic and plastic offsets (thermal expansion, cyclic loading, etc.). However, they are ignored for this model and the total strain will be determined as $[\varepsilon] = [\varepsilon_c] + [\varepsilon_s]$.

4.3.3 Constitutive equations

4.3.3.1 Concrete

The principal compression stress f_{c2} is no longer only dependent on ε_2 but now also on ε_1 . The following reduction factor was thus introduced :

$$\beta_d = \frac{1}{1 + C_s C_d} \leq 1.0 \quad (4.24)$$

with

C_d a correlation factor obtained through experimentation ;

$$C_d = 0.35 \left(-\frac{\varepsilon_{c1}}{\varepsilon_{c2}} - 0.28 \right)^{0.8} \quad (4.25)$$

C_s a coefficient taking into account the slip on the crack surfaces : $C_s = 1.0$ (MCFT) or $C_s = 0.55$ (DSFM).

The peak stress is then defined as

$$f_p = -\beta_d f'_c \quad (4.26)$$

with its corresponding strain

$$\varepsilon_p = -\beta_d \varepsilon_0 \quad (4.27)$$

The new compression response curve assumes the following shape (figure 4.10(a)) [27] :

$$f_{c2} = f_p \frac{n(\varepsilon_{c2}/\varepsilon_p)}{(n-1) + (\varepsilon_{c2}/\varepsilon_p)^{nk}} \quad (4.28)$$

with

n defined as

$$n = 0.8 - f_p/17 \quad (4.29)$$

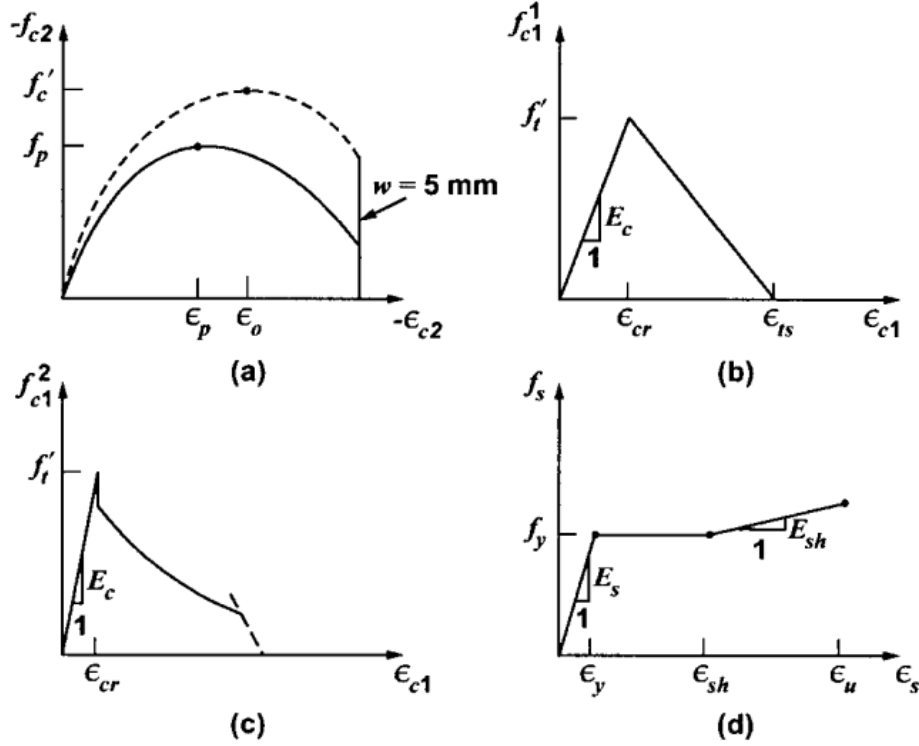


Figure 4.10: Constitutive relationships : (a) compressive softening model ; (b) tension softening model ; (c) tension stiffening model ; (d) reinforcing steel response [25]

k defined as

$$\begin{aligned} k &= 1.0 & \text{for } \epsilon_p < \epsilon_{c2} < 0 \\ k &= 0.67 - f_p/62 & \text{for } \epsilon_{c2} < \epsilon_p \end{aligned} \quad (4.30)$$

The part of equation (4.11) dealing with the pre-cracking state for the MCFT is kept to describe the tensile behaviour of the element. Experience suggests an estimation of the concrete tensile strength as :

$$f'_t = 0.65 (f'_c)^{0.33} \quad (4.31)$$

A representation of the various constitutive relationships of the DSFM is found on figure 4.10. The response goes to zero as the crack width reaches 5 mm.

Once the element begins to crack, the tensile stresses are carried through two independent mechanisms ; tension softening and stiffening.

Tension softening (figure 4.10(b)) : particularly significant in elements with little to no reinforcement, associated with fracture mechanisms ;

$$f_{c1}^{(1)} = f'_t \left(1 - \frac{\epsilon_{c1} - \epsilon_{cr}}{\epsilon_{ts} - \epsilon_{cr}} \right) \quad (4.32)$$

with ϵ_{ts} the terminal strain, calculated from

$$\epsilon_{ts} = \frac{2 G_f}{f'_t L_r} \quad (4.33)$$

G_f , the fracture energy parameter, is usually taken as equal to 75 N/m. L_r represents the characteristic length of the element.

Tension stiffening (figure 4.10(c)) : arises from the interaction between concrete and steel between cracks, where the load is transferred to the concrete by bond stresses, inducing a significant amount of tension inside the material. The degree of stiffening depends on several factors, including the reinforcement ratio and bar diameter ;

$$f_{c1}^{(2)} = \frac{f_t'}{1 + \sqrt{c_t \varepsilon_{c1}}} \quad (4.34)$$

$f_{c1}^{(2)}$ is limited by the stress which can be transmitted across cracks. Thus, the resulting average principal tensile stress in the concrete is taken as

$$f_{c1} = \max \left(f_{c1}^{(1)}, f_{c1}^{(2)} \right) \quad (4.35)$$

4.3.3.2 Steel

The stress-strain relationship for the reinforcement is now taken as trilinear in both tension and compression. This formulation can be used for both local and average strains ($f_{s,cr}$ and f_s).

$$f_s = \begin{cases} E_s \varepsilon_s & \text{for } 0 < \varepsilon_s < \varepsilon_y \\ f_y & \text{for } \varepsilon_y < \varepsilon_s < \varepsilon_{sh} \\ f_y + E_{sh}(\varepsilon_s - \varepsilon_{sh}) & \text{for } \varepsilon_{sh} < \varepsilon_s < \varepsilon_u \\ 0 & \text{for } \varepsilon_s > \varepsilon_u \end{cases} \quad (4.36)$$

with

E_{sh} the strain hardening modulus ;

ε_{sh} the strain at the beginning of strain hardening ;

ε_u the ultimate strain.

4.3.4 Modelling of the slip

It should be calculated using two procedures : one based on the calculation of the rotation lag of figure 4.8 and one based on the shear stresses v_{ci} transmitted through the crack. The maximum value obtained will be the one used in the constitutive equations, but the latter is generally used as it is simpler [28].

The following relationship originating from is used to determine the slip displacement along the crack surface δ_s for a concrete of cube strength f_{cc} [26] :

$$\delta_s = \frac{v_{ci}}{1.8 w^{-0.8} + (0.234 w^{-0.707} - 0.2) f_{cc}} \quad (4.37)$$

giving the average slip strain of

$$\gamma_s = \frac{\delta_s}{S_{m\theta}} \quad (4.38)$$

4.4 Model used for the analysis

Comparisons between the MCFT and DSFM have been carried out for the validation of the latter [29]. It has been determined through experimentation that the DSFM gives more precise

results for panels under constant shear, but, for more complex structures, the DSFM and MCFT generally give comparable results. In some cases, the MCFT can model the behaviour better in the sense that the load and strain at failure are evaluated more correctly.

Thus, the finite-element program implemented by MSG&I was based on the MCFT, with additional methods to better model the behaviour of elements subjected to more complex stress fields. The model is discussed in more detail in chapter 6.

5 | Dapped-end beams and the strut-and-tie method

This chapter presents the point of interest of this paper, namely the extremity of dapped-end beams. To determine the ultimate load that this type of element can sustain, equations based on the strut-and-tie method are presented, with the chosen model being the one presented by Mata-Falcón et al in 2019 [7].

5.1 Dapped-end beams

A dapped-end beam (DEB), or half-joint beam, is a type of beam frequently used in precast construction, whose section has been reduced at the extremities to create a support, see figure 5.1. While this geometry allows for a reduced floor height, easier assembly and a greater lateral stability compared to beams supported at their bottom face [30], it creates a locally disturbed region which relies heavily on its reinforcement to transfer the stress into the full depth of the beam.

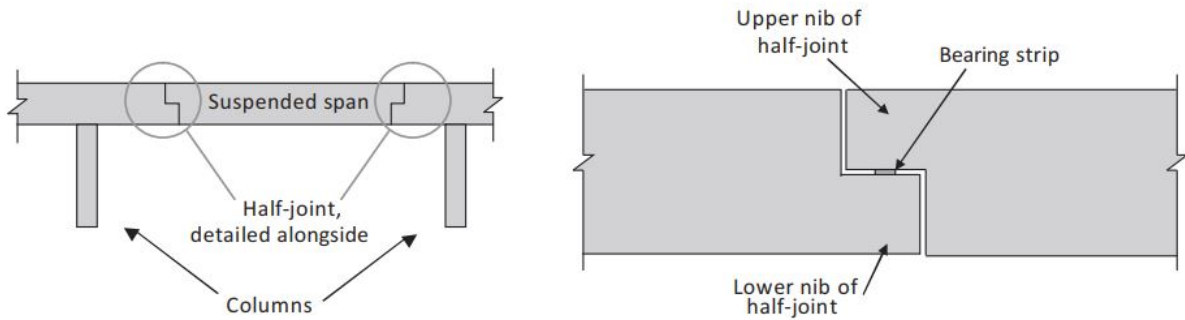


Figure 5.1: Half-joint principle for reinforced concrete bridges [31]

As regular full-depth beams have already been studied by MSG&I, the point of interest of this paper is solely the extremities of the DEB, called nibs, which resemble corbels in many aspects.

The major difference between the stress flow inside the nib of a dapped-end beam and a typical corbel is that while the inclined compression force is transmitted to the column in the case of the corbel, for a DEB this force must be resisted by a tensile force in the vertical reinforcement close to the full-depth face of the beam, as illustrated on figure 5.2. To satisfy equilibrium, this tensile force must be equal to the vertical shear force V acting on the nib. A group of stirrups should be placed close to that interface to resist this component, and be correctly anchored to the longitudinal reinforcement. Two typical reinforcement layouts are presented on figure 5.3.

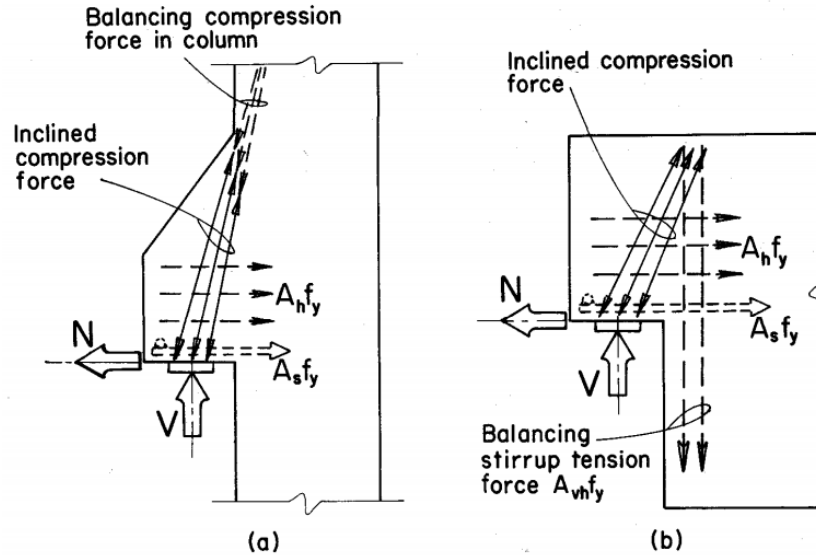


Figure 5.2: Internal forces for (a) a corbel on a column ; (b) the nib of a dapped-end beam [30]

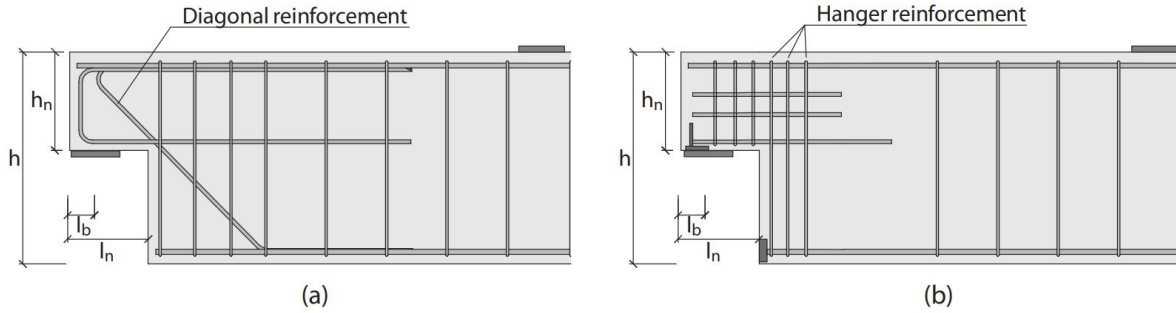


Figure 5.3: Typical reinforcement layouts for dapped-end beams : (a) with diagonal reinforcement ; (b) without [31]

The experiments of Mattock and Chan [30] on DEBs revealed that the nib can be designed as a corbel if the shear span is taken equal to the distance between the centre of application of the vertical load V and the centre of gravity of the vertical stirrup (also called hanger) reinforcement.

The cracking process was similar for all the tested specimens, with the first crack initiating at the re-entrant corner A (see figure 5.4) at about 20% of the ultimate strength. It propagated at an angle of 45° and extended to two-thirds of the height of the nib at service load. Increasing the load further created additional cracks and lengthened existing ones.

As the reinforcement began to yield, the cracks widened significantly and spalling occurred on top of the vertical hanger reinforcement depicted on figure 5.5. Right before failure, the same region was subjected to the crushing of the diagonal concrete strut.

The major diagonal cracks which can affect the element are the ones originating from corners A and B, thus proper reinforcement has to cross them to prevent a premature collapse. A more detailed cracking pattern is shown on figure 5.6.

The spalling phenomenon is illustrated on figure 5.7, which represents the reinforcement layout and failure of the reference specimen of an experimental campaign conducted by Desnerck et al. [31].

Most of the failure modes of DEBs are brittle in nature. Three of them have been identified [30, 32] :

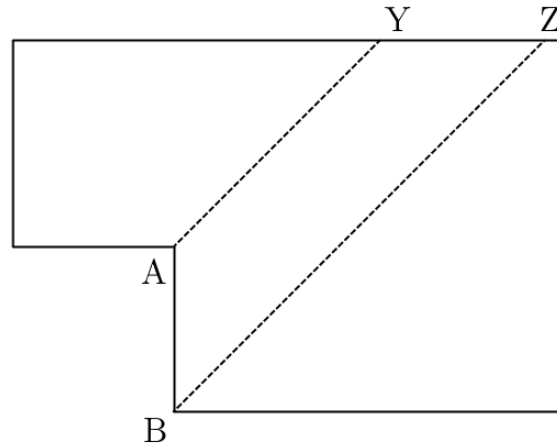


Figure 5.4: Location of potential diagonal cracks

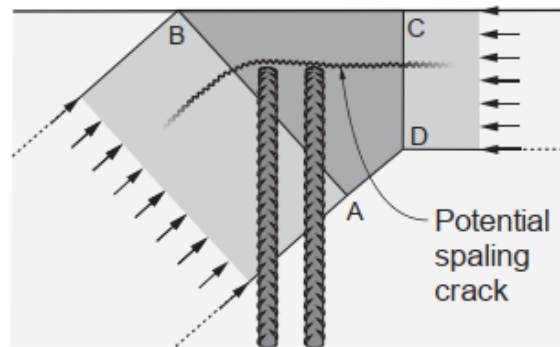


Figure 5.5: Nodal area on top of the hanger reinforcement [7]

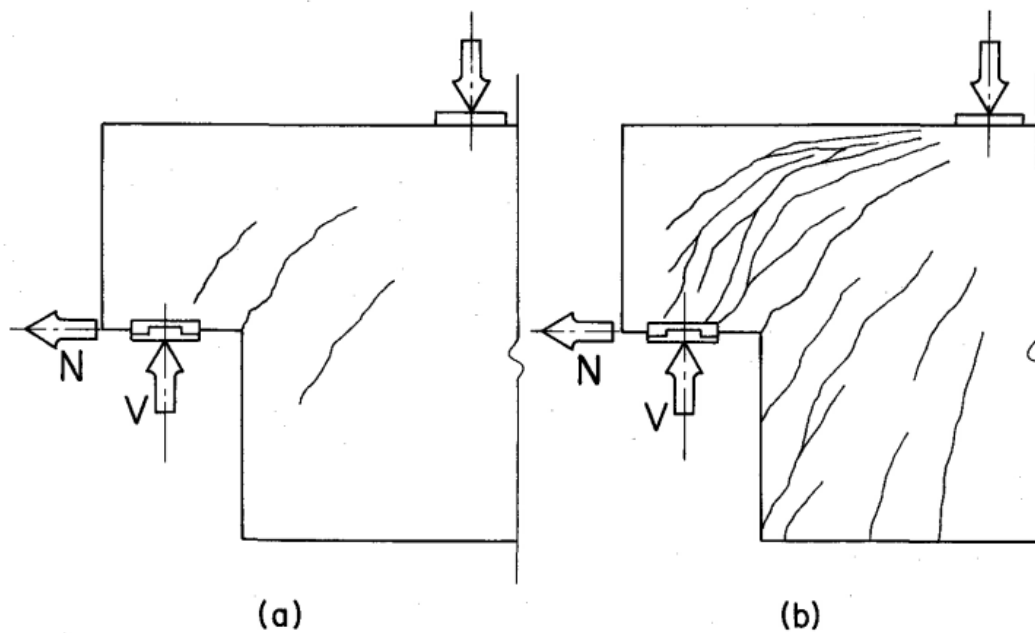


Figure 5.6: Typical cracking pattern at the extremity of a dapped-end beam (a) at service load ; (b) just before failure [30]

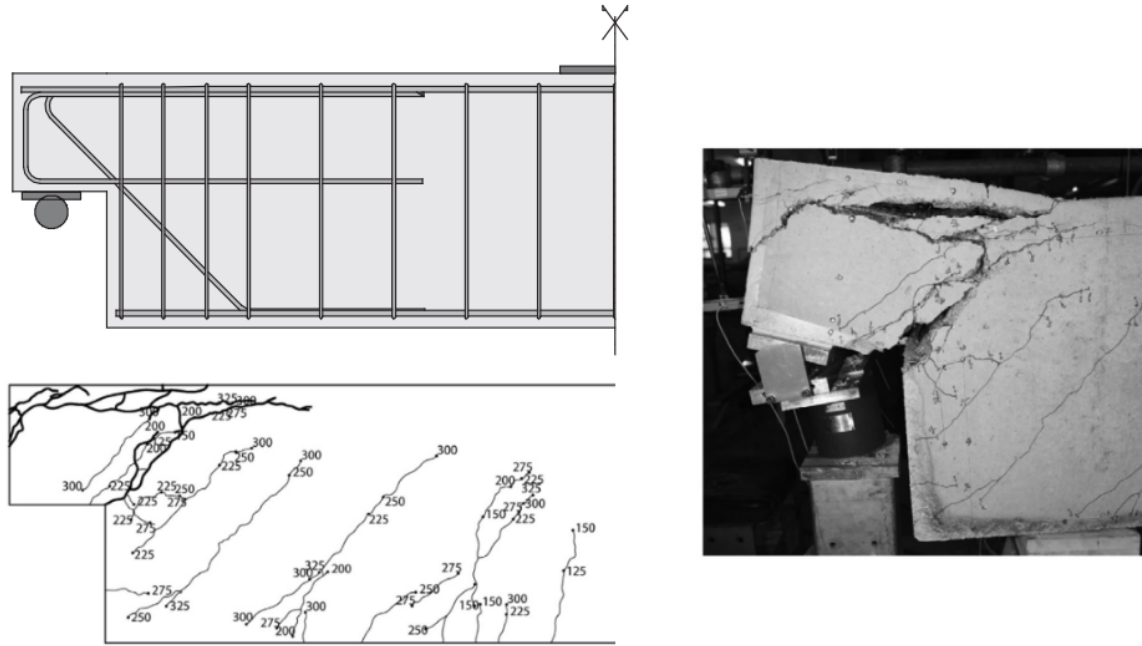


Figure 5.7: Spalling failure of a dapped-end beam [31]

- nib failure with inclined cracking from the re-entrant corner A ;
- flexural failure in the full-depth section ;
- failure plane originating from corner B.

5.2 Detailing

The cross-section of the element has an important impact on the diagonal cracking. The nib should have a minimum height of 45% of the full-section depth of the beam, as it plays a significant role in the shear capacity of the element [33]. Reducing the span-to-depth ratio of the nib increases its ultimate load and stiffness, leading to a higher shear strength [34]. Usually, the ratio between the length of the nib to its height is 90% on average.

It is essential to ensure the transmission of forces from the application point on the nib to the full-depth section of the beam by using proper reinforcement detailing. Two general layouts for the reinforcement of the nib of a DEB were represented on figure 5.3. Layout (a) is often used in Europe and contains a diagonal set of rebars. Compared to layout (b), this additional type of reinforcement influences greatly the behaviour at service load by reducing the width of the cracks (up to six times according to Clark et al. [35]) as well as the load at which they appear, and by increasing the stiffness of the element [31]. This reduction of the re-entrant corner crack arises from the fact that this diagonal reinforcement is oriented in the same manner as the principal strain direction [36].

For further crack control, the first vertical stirrup should be placed as close as possible to the edge of the full-depth interface while satisfying the cover thickness requirement. A maximum distance of 40 mm is suggested [33], but this may not fit with the cover durability requirements of the Eurocode (section 2.3). Increasing the spacing of the hangers can create significant

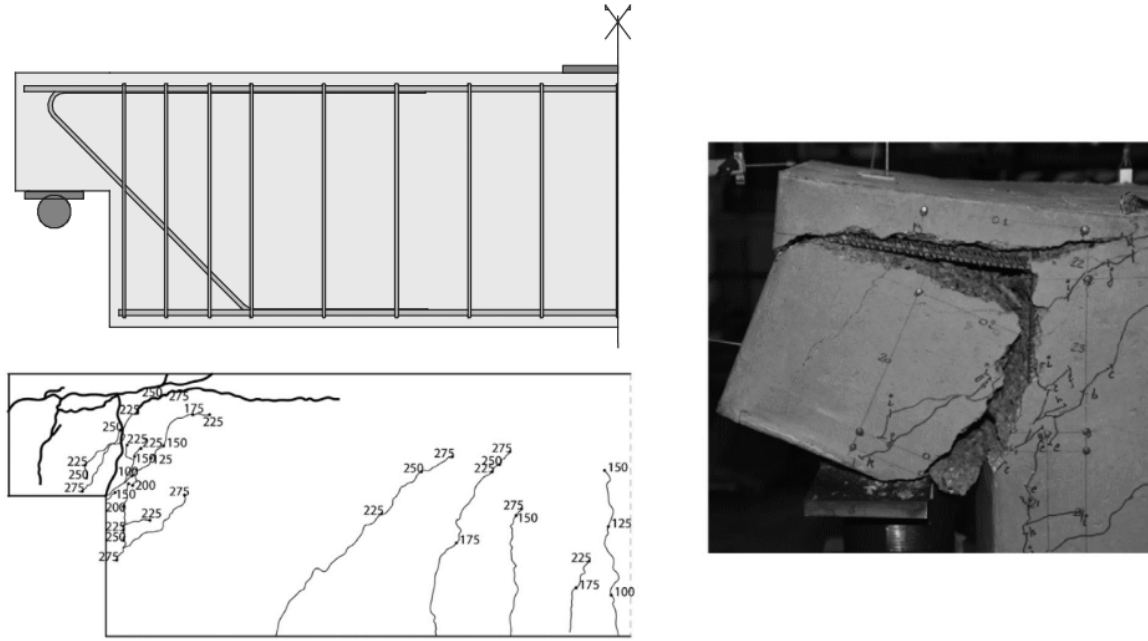


Figure 5.8: Failure of a dapped-end beam in the absence of U-bars [31]

shear cracking in the full-depth section and lowers the load that the nib can sustain. However, this is partially compensated by a positive effect on the spalling on top of the hangers (figure 5.5).

An appropriate anchorage of the longitudinal reinforcement in the nib has shown its importance, as specimens with improper detailing in the 2016 experimental campaign of Desnerck got their strength diminished by up to a third [31]. Anchorage is typically achieved by using angles or welded plates, which are also at risk of corrosion. Herzinger and Elbadry [37] conducted a study on the use of single or double-headed studs embedded inside the concrete element to reduce the risk of improper anchorage. These elements also facilitate the installation and compaction of concrete by reducing congestion.

Desnerck et al. also found that reducing the hanger reinforcement shifts the failure mode from the nib to the full depth of the beam (from A-Y to B-Z on figure 5.6) but does not considerably affect the overall capacity and amount of cracking. However, removing the horizontal U-bars or the diagonal reinforcement greatly weakened the element.

Removing these U-bars increases the angle of the crack at the re-entrant corner, which becomes almost vertical, as shown on figure 5.8.

Overall, the experiments of Desnerck on the influence of detailing drew the following conclusions [31] :

- proper detailing has a tremendous influence on the ultimate load that the nib can reach and any reduction of a certain type of reinforcement inside layout (a) leads to a clear reduction of the load-carrying capacity ;
- in all of the cases where a certain type of reinforcement was missing, cracks formed at the re-entrant corner once the load reached 20 to 33% of the failure load, with an angle of 40 to 45° in the presence of U-bars (70 to 75° in their absence) ;
- this crack at the re-entrant corner caused a brittle failure, unless the amount of shear

reinforcement was low, which then caused the failure to occur in the full depth of the beam ;

- the greatest decrease of the ultimate load came from the removal of the diagonal reinforcement ;
- in the case of insufficient reinforcement, the greatest amount of redistribution of forces between the bars occurred at the re-entrant corner.

Due to the advantages introduced by using diagonal reinforcement, layout (a) was chosen for this analysis.

5.3 Strut-and-tie model of the dapped-end beam

5.3.1 Description

In regions close to discontinuities, such as near a change in geometry or close to the point of application of a force - both being true in the case of the nib of a DEB - the Navier-Bernoulli hypothesis that plane sections remain plane after deformation cannot be applied, due to the non-linear distribution of strains inside the element. The strut-and-tie (ST) is one of the methods that have been put together to analyse and design these disturbed regions without resorting to complex and time-consuming non-linear calculations.

Since the ST is based on a lower-bound plasticity theorem, the calculated ultimate load has to be inferior to the real capacity of the element, putting the method on the side of safety. This also means that the structure possesses enough ductility to redistribute the applied loading.

The ST method is applied by choosing a distribution of internal stresses which satisfies equilibrium and support conditions while never violating the plasticity conditions of the concrete matrix and the steel reinforcement. This signifies that the dimensions of the element need to be sufficient to sustain its internal stresses [38].

The stresses inside the element are idealised as struts, ties and nodes. A truss representing the flow of stresses from the applied forces to the supports is drawn, leading to a continuous and balanced system of forces under the ultimate load applied. The geometry of this truss is determined by the designer, following experience, experiments and good practice. The different members of a ST model are described as such :

Struts : members representing uniaxial compression fields, often made of concrete, but sometimes containing reinforcement working in compression.

Ties : they represent the zones subjected to tensile stress, indicating where reinforcement has to be present.

Nodes : nodes are where ties, struts, forces and/or application points meet. They are thus volumes of concrete where different types of forces meet and balance each other. Nodes are named following which members meet at this point : CCC for struts, CCT if a tie is joining, CTT if more.

The reinforcement has to be sufficient to sustain the stress inside the ties, and has to extend beyond nodes to ensure proper anchorage. The axis of the reinforcement has to geometrically coincide with the axis of the corresponding tie.

Nodes have to be able to transmit the forces applied on them, with a reduced concrete strength of

$$f_{cu} = k_c f_c \quad (5.1)$$

with $k_c \leq 1$ an efficiency coefficient found in construction codes. Ideally, the axis of the forces meeting at the node should cross at one single point, but it's not always the case.

The following section will show an application of the ST method on a DEB, to find the reinforcement required to sustain a certain load and vice-versa.

5.3.2 Model implemented

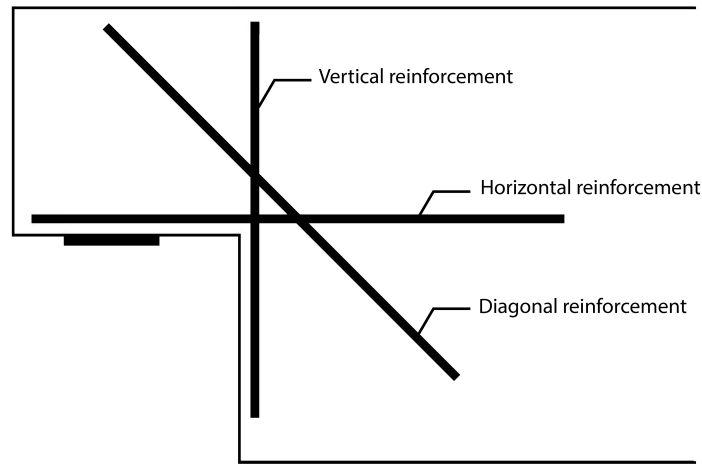


Figure 5.9: Reinforcement determined by the strut-and-tie method

The ST models presented on figures 5.10 (model A) and 5.11 (model B) were introduced by Mata-Falcón et al. after a set of experiments [7]. Their goal was to develop a simplified procedure to determine the load capacity of a DEB without neglecting the top concrete cover of figure 5.5. Indeed, neglecting it tends to be too conservative, but the CCT nodal regions checks found in standards could lead to unsafe results when considering the concrete cover, as the reinforcement in the DEB does not completely cross the nodal region and is usually yielded when reaching the ultimate state, which leads to more restrictive conditions.

The conditions for the application of this method are (1) the DEB possesses a clearly defined hanger reinforcement and (2) its model has no other reinforcement supporting the load other than represented on models A and B. The additional reinforcement of figure 5.3 does not need to be acknowledged for the ST analysis of the nib. Two other considerations are that the concrete does not fail at the CCT nodal region described earlier, and that the reinforcement in the horizontal and diagonal directions always reach their plastic capacity. However, for the hanger reinforcement, there are two possibilities, which is what differentiates the two models :

Model A : the vertical reaction V is equilibrated by the diagonal reinforcement (when present) and by the hanger reinforcement, which does not reach its maximum capacity.

Model B : in this model, the hanger reinforcement does reach its plastic capacity. The vertical reaction is now equilibrated by the diagonal reinforcement (when present), the hanger reinforcement, and also the shear stirrups in the length of the beam.

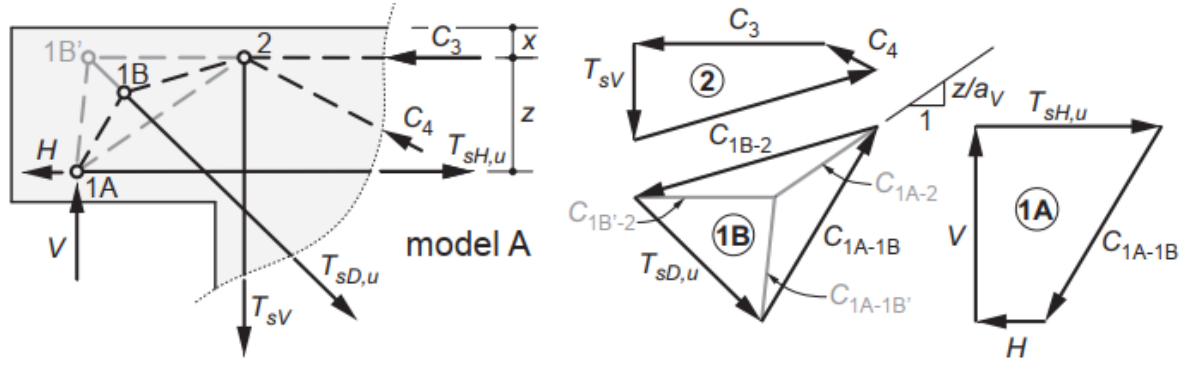


Figure 5.10: Strut-and-tie model A [7]

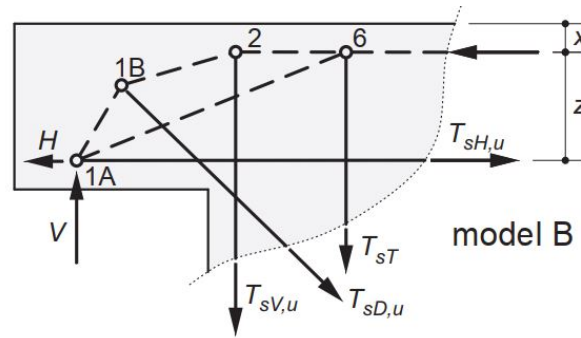


Figure 5.11: Strut-and-tie model B [7]

The major difference between the two models is thus whether or not the hanger reinforcement reaches its maximum capacity $T_{sV,u}$. The authors recommend the use of model A for design purposes, as it does not rely on the contribution of the shear reinforcement of the beam.

Model A allows the ultimate strength V_u to be computed without finding the strut forces C_3 and C_4 . The latter is produced by a phenomenon called fan anchorage mechanism, which is caused by the horizontal reinforcement, and is not present in model B.

In the following paragraphs, the equations characterising the two strut-and-tie models are presented, allowing the determination of the reinforcement required to sustain a certain loading and vice-versa.

5.3.3 Determination of the ultimate load (section-to-load)

The failure of the nodal top region limits the ultimate load that can be sustained by the DEB, unless it possesses a low amount of reinforcement. Thus, to develop the maximal strength capacity of the nib, this nodal region is required to fail. To easily determine the position of node 2 (N2), the assumption used by Mata-Falc3n et al. is that it depends on the failure condition of strut 1B-2.

5.3.3.1 Model A

The height of N2 above the reinforcement, z , can be determined using the geometrical and material data. The slope between N1A and N2 is (figure 5.10) :

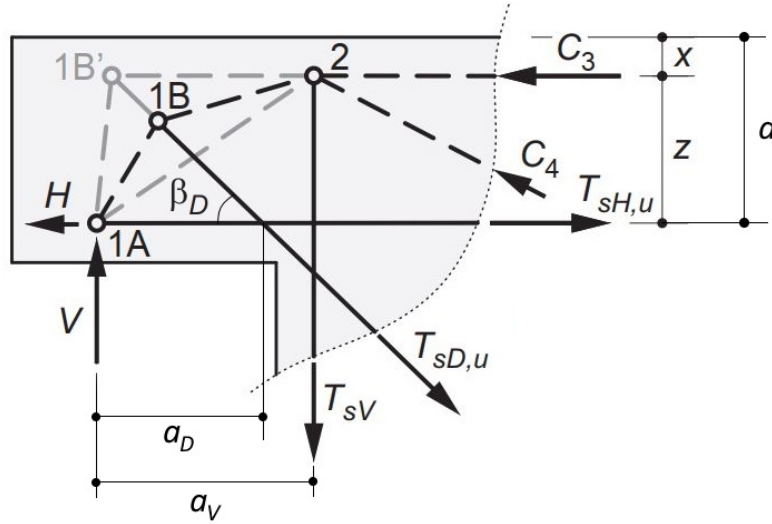


Figure 5.12: Geometrical parameters (adaptation of [7])

$$\frac{z}{a_V} = \lambda_d - \lambda_c + \sqrt{\lambda_c^2 + 2\lambda_c \left(\frac{d}{a_V} - \lambda_d \right) - 1} \quad (5.2)$$

with λ_c and λ_d being the following dimensionless coefficients, related to the capacities of the diagonal reinforcement and the concrete :

$$\lambda_c = \frac{k_c f_c b a_V}{T_{sH,u} + T_{sD,u} \cos \beta_D - H} \quad (5.3)$$

$$\lambda_d = \frac{a_D}{a_V} \frac{T_{sD,u} \sin \beta_D}{T_{sH,u} + T_{sD,u} \cos \beta_D - H} \quad (5.4)$$

with the chosen design values (the geometrical entities are depicted on figure 5.12) :

a_V [mm] the distance between N1A and the vertical hangers ;

a_D [mm] the horizontal distance between N1A and where the diagonal and horizontal reinforcement cross ;

d [mm] the distance between the horizontal reinforcement and the top of the beam ;

β_D [rad] the angle of the diagonal reinforcement with respect to the horizontal ;

$T_{sH,u}$ and $T_{sD,u}$ [kN] the tensile capacities of the horizontal and diagonal reinforcement, respectively ;

H [kN] the horizontal load applied on the nib (positive direction indicated on figure 5.12).

The ultimate vertical loading that can be applied to a nib with reinforcement strengths $T_{sH,u}$ and $T_{sD,u}$ and subjected to an horizontal tensile load of H is found as being :

$$V_u = (T_{sH,u} + T_{sD,u} \cos \beta_D - H) \frac{z}{a_V} + T_{sD,u} \sin \beta_D \left(1 - \frac{a_D}{a_V} \right) \quad (5.5)$$

Since model A requires the hanger reinforcement to reach its plastic capacity, its contribution can be assessed with :

$$T_{sV} = (T_{sH,u} + T_{sD,u} \cos \beta_D - H) \tan \theta_{1B-2} + C_4 \sin \theta_4 \quad (5.6)$$

with θ_4 the inclination of C_4 with respect to the horizontal. The angle of strut 1B-2 is :

$$\tan \theta_{1B-2} = \frac{z}{a_V} - \lambda_d \quad (5.7)$$

In the absence of diagonal reinforcement, θ_{1B-2} becomes θ_{1A-2} .

Equation (5.5) is only valid for model A. To determine if the model is applicable, the following check is made, arising from the fact that C_4 degenerates when going from model A to B :

$$(T_{sH,u} + T_{sD,u} \cos \beta_D - H) \tan \theta_{1B-2} \leq T_{sV,u} \quad (5.8)$$

5.3.3.2 Model B

Usually, the majority of the loading is taken by the vertical and diagonal (if present) reinforcements. However, if condition (5.8) is not fulfilled, model B has to be applied to find V_u , transforming equation (5.5) into :

$$V_u = T_{sV,u} + T_{sD,u} \sin \beta_D + T_{sT} \quad (5.9)$$

with T_{sT} the contribution of the shear reinforcement inside the full depth of the beam. Equilibrium at N1A leads to

$$T_{sT} = \frac{z}{a_T} (T_{sH,u} + T_{sD,u} \cos \beta_D - T_{sV,u} \cot \theta_{1B-2} - H) \leq T_{sT,u} \quad (5.10)$$

with

z [mm] determined using equation (5.2) ;

$T_{sT,u}$ [kN] the plastic tensile capacity of the tie representing the shear reinforcement ;

a_T [mm] the horizontal distance between N1A and N6 (see figure 5.11).

The horizontal tie does not yield if the shear reinforcement reaches maximum capacity. In this case, the force inside can be found by rearranging equation (5.10).

A conservative simplification is the assumption that the position of N2 is identical for both models. An iterative process could refine its position, but is not recommended by the authors.

5.3.4 Design approach (load-to-section)

5.3.4.1 Determination of the required reinforcement

As stated earlier, the use of model A is recommended to design the nib, as it does not rely on the shear reinforcement of the beam. To determine the reinforcement required to sustain the loading (V , H) applied on the nib, it is first necessary to impose α_D , the fraction of the vertical load V that is sustained by the diagonal reinforcement. Once this is done, the required capacities of the remaining ties can be determined with :

$$T_{sH,u} = V(1 - \alpha_D) \frac{a_V}{z} + H \quad (5.11)$$

$$T_{sV,u} = V \left(1 - \lambda_d \frac{a_V}{z} \right) \quad (5.12)$$

$$T_{sD,u} = \frac{V \alpha_D}{\cos \beta_D \frac{z}{a_V} + \sin \beta_D \left(1 - \frac{a_D}{a_V} \right)} \quad (5.13)$$

These equations require the vertical position of N2, previously determined through equation (5.2), which is now found using :

$$\frac{z}{a_V} = \frac{1 + \mu_c \lambda_d + \sqrt{1 - 2\mu_c \left(\frac{a_V}{d} - \lambda_d + \frac{a_V}{d} \lambda_d^2 \right) - \mu_c^2 (1 + \lambda_d^2)}}{\mu_c + 2 \frac{a_D}{d}} \quad (5.14)$$

with the dimensionless coefficient μ_c defined as :

$$\mu_c = \frac{V}{k_c f_c b d} \quad (5.15)$$

As λ_d of equation (5.4) depends on the horizontal and diagonal reinforcements, this procedure becomes iterative. An initial value for λ_d has to be chosen to launch the analysis, which converges however quite rapidly.

The forces in the ties are then obtained and the required amount of steel is determined depending on the chosen grade.

5.3.4.2 Strength of the concrete struts

Based on experimentation, the capacity of diagonal struts is expressed by a reduction factor k_c taking into account the presence of transverse strains as well as the cover spalling and that does not depend on the mode of failure. The maximum compression load that the strut 1B-2 can sustain is thus :

$$C_{1B-2} = k_c f_c b (2x \cos \theta_{1B-2}) \quad (5.16)$$

with

f_c [MPa] the compression strength of the concrete ;

b [mm] the width of the DEB ;

x [mm] the vertical distance between node 2 and the top edge of the beam ;

θ_{1B-2} [rad] the inclination of strut 1B-2 with respect to the horizontal.

Various values of the reduction coefficient k_c are provided by codes, such as the *fib* Model Code 2010 [39], EN 1992-1-1 [10] and ACI 318-14 [40]. The values provided in the case of CCT nodes on supports are deemed too unsafe unless the top concrete cover is neglected, following the experiments of [7]. This is due to the fact that the potential cracks across the diagonal strut have to be considered, as they can lead to delamination of the cover. Thus, the coefficients provided for oblique struts are more suitable and range from 0.55 to 0.46 depending on the concrete strength, see table 5.1.

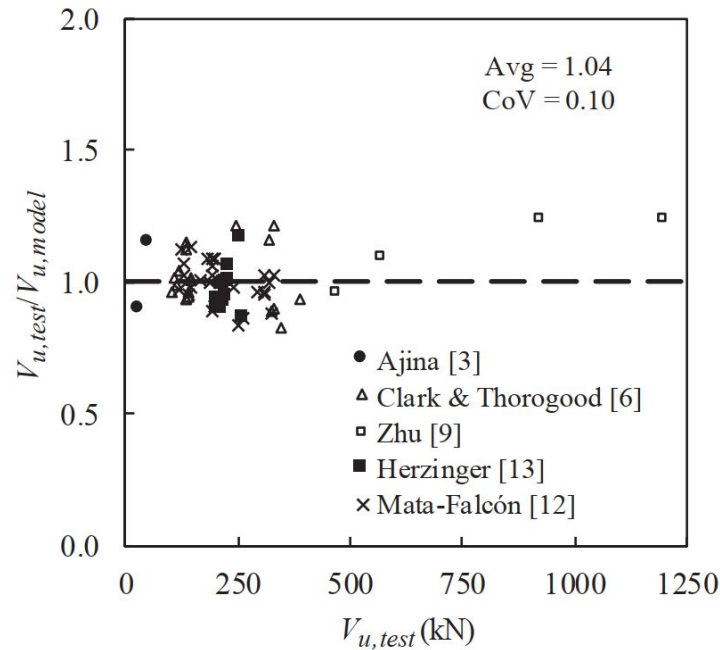
Table 5.1: Reduction factor of the compressive strength specified in *fib* Model Code 2010, EN 1992-1-1 and ACI 318-14 for struts with oblique tension

f_{ck} [MPa]	ACI 318-14 $k_c = 0.85 \times 0.6$	EN 1992 $k_c = 0.6 \eta'$	<i>fib</i> MC 2010 $k_c = 0.5 \eta_{fc}$
20	0.51	0.55	0.55
30	0.51	0.53	0.55
40	0.51	0.50	0.50
50	0.51	0.48	0.46

5.3.5 Accuracy of the model

Using the *fib* coefficient $k_c = 0.5 \eta_{fc}$, the model developed by Mata-Falc3n et al. was applied on DEBs studied by Ajina¹, Clark and Thorogood [35], Zhu [36] and Herzinger², as well as their own.

The ratio between experimental and predicted loads is plotted as a function of the experimental load, presenting the prediction accuracy of the two strut-and-tie models using the *fib* coefficients :

**Figure 5.13:** Accuracy of the strut-and-tie models A and B, by application on various experimental campaigns [7]

For the reinforcement used in practice, the authors state that the model gave accurate estimations for the ultimate load, regardless on whether or not the vertical load was accompanied by an horizontal force. The tests ran by Zhu, of lightly reinforced DEBs, yielded somewhat conservative results. However, these beams were below reinforcement standards.

¹Ajina JM., *Effect of steel fibers on precast dapped-end beam connections*, MSc thesis Brookings (South Dakota, EEUU): South Dakota State University; 1986, p. 271.

²Herzinger R., *Stud reinforcement in dapped ends of concrete beams*, PhD thesis Calgary (Alberta, Canada): University of Calgary; 2008, p. 324.

6 | Non-linear finite-element analysis

A numerical model was developed by MSG&I based on the membrane regime and degradation phenomena described in the previous chapters.

They decided to implement a two-dimensional non-linear finite-element analysis (NLFEA) of secant type formulated in displacements, based on the properties of materials proposed by the Modified Compression Field Theory [6] with certain additions. A summary of the way this NLFEA was created is presented in this chapter. For more details, see [4, 5].

6.1 Reference system

Each membrane element for which the secant stiffness matrix is formulated is composed of a concrete matrix containing a uniformly distributed reinforcement mesh of orthogonal sides, possibly embedded with prestressing cables [41, 42, 43].

The reference systems for the orientation of the various components have to be defined. They are (figure 6.1) :

- (X, Y) the general reference system of the entire structure ;
- (x, y) the local reference system for the concrete element, with an inclination of β with respect to (X, Y) ;
- $(1, 2)$ the reference system defined by the main stress directions, tension (1) and compression (2) ;

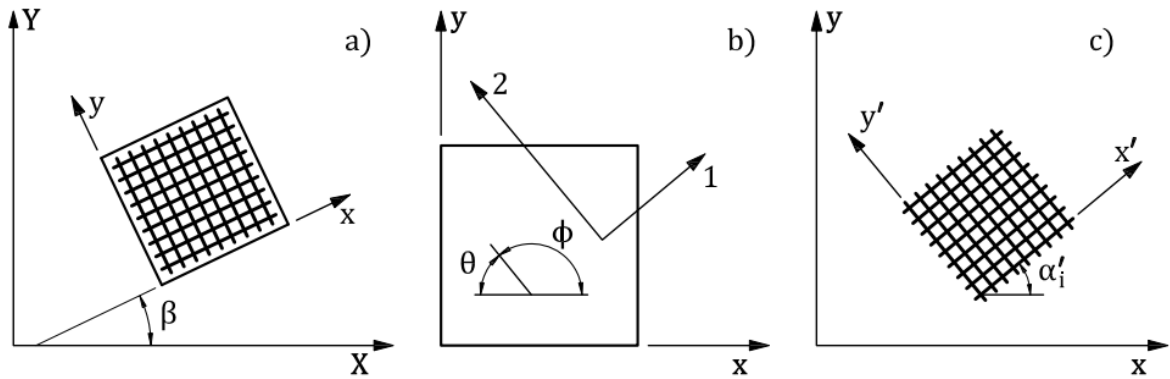


Figure 6.1: Orientation of a) the concrete element with respect to (X, Y) ; b) the main directions of tension and compression with respect to (x, y) with axes parallel to the reinforcement ; c) a generic smeared reinforcement layer with respect to (x, y) [4, 5]

- (x'_i, y'_i) the reference system for the i -th reinforcement layer, inclined by α_i with respect to (x, y) .

The system $(1, 2)$ is considered to be rotated by an angle $\pi/2 - \theta + \beta$ with respect to (X, Y) . The angle $\pi - \theta + \beta$ represents the main compression direction, assumed coincident with the cracking direction.

6.2 Characterisation through finite elements

Analysing the structural element as a whole would be too complex, which is why a finite-element based method was implemented. The structure is divided into a mesh of triangular elements, each possessing various geometrical and material characteristics depending on the part of the structure they represent. They are defined by three nodes possessing two degrees of freedom each (horizontal and vertical), giving the following displacement vector per triangle (figure 6.2) :

$$\mathbf{Q} = \{u_i \quad v_i \quad u_j \quad v_j \quad u_k \quad v_k\}^T \quad (6.1)$$

If $\mathbf{N}$ is the matrix containing the shape functions for this type of triangular element, then the displacement $\mathbf{q}$ at any designated point P of the element is determined using :

$$\mathbf{q} = \{u \quad v\}^T = \mathbf{N} \mathbf{Q} \quad (6.2)$$

With $\mathbf{L}$ the linear compatibility operator, the deformation field $\boldsymbol{\varepsilon}$ is found with :

$$\boldsymbol{\varepsilon} = \mathbf{L} \mathbf{q} = \mathbf{L} \mathbf{N} \mathbf{Q} = \mathbf{B} \mathbf{Q} \quad (6.3)$$

In the case of linear elasticity and in the absence of distortions, the stresses can be related to the displacements in the following way :

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} = \mathbf{D} \mathbf{B} \mathbf{Q} \quad (6.4)$$

with $\mathbf{D}$ the stiffness matrix of the material. For an isotropic material in a plane stress state, this matrix is formulated as (with E being the Young modulus of the material and ν its Poisson's ratio) :

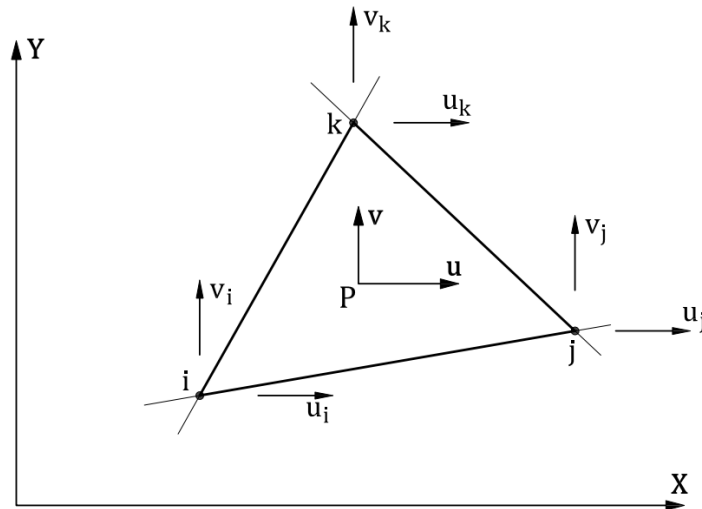


Figure 6.2: Representation of a triangular finite element [4, 5]

$$\mathbf{D} = \frac{E}{1 - \nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \quad (6.5)$$

The displacement field of an element is related to the adjacent elements by its nodal displacements $\mathbf{Q}$, which are the unknowns of the problem. The goal of the NLFEA is thus to determine the global vector $\mathbf{Q}$ for the whole structure. Once these displacements have been determined, other results can be computed through them, such as the stress field or cracking state of the structure.

In order to achieve this, the steps followed by the NLFEA program written by MSG&I is presented in the next section.

6.3 Overview of the method

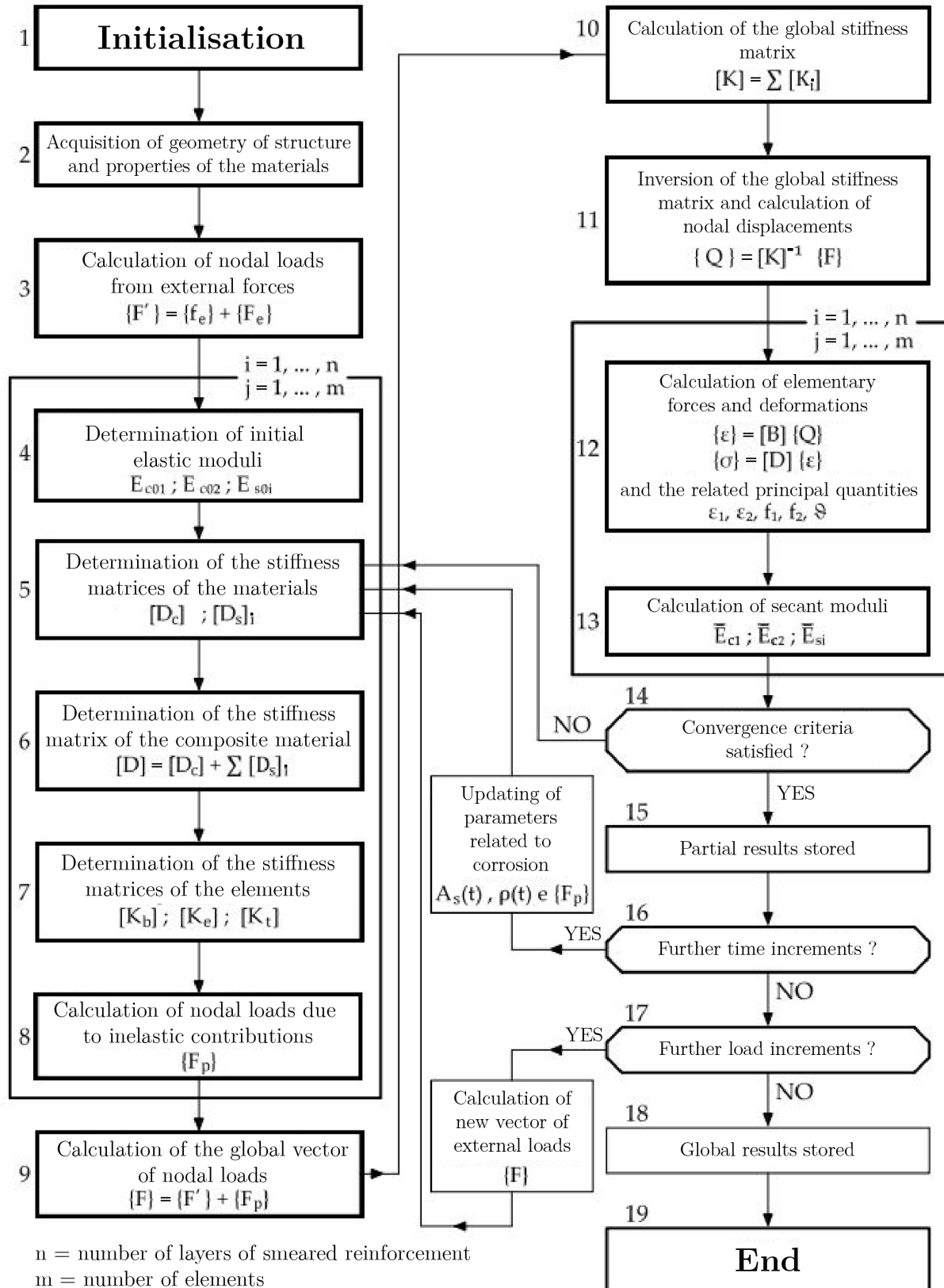


Figure 6.3: Visual illustration of the NLFEA [4, 5]

These steps are shortly summarised below.

1. **Initialisation** : the required input for the program is written following a specific template. It contains the coordinates of the mesh, the geometrical, material and degradation data, the loads applied on the structure, etc.
2. **Acquisition of geometry of structure and properties of the materials** : the information is extracted from the input and stored into variables, vectors and matrices to be used by the program.

Concentrated loads on a node could create localised high stresses leading to a punching failure. To avoid this, a strip of elastic elements is placed at the point of application, and anywhere this sort of phenomenon can occur. Additionally, the loading should be distributed on several nodes to further reduce the risk of instability in that region.
3. **Calculation of the nodal loads from the external forces** : concentrated loads are directly added to their corresponding degrees of freedom. Distributed loads, applied on the edge of elements, first have to be converted to nodal equivalent concentrated loads before being inserted in the load vector. The weight of each finite element is calculated and divided between the three nodes that define it.
4. **Determination of the initial elastic moduli** : the first step, namely equation (6.48), is calculated as if the structure was completely elastic by using the tangent Young moduli provided in the input. This allows the iterative process to begin.
5. **Determination of the stiffness matrices of the materials** (sections 6.4.1 and 6.4.2): the stiffness matrices for concrete and steel are calculated using equations (6.15) and (6.26).
6. **Determination of the stiffness matrices of the composite material** (section 6.4.3): the elements made of concrete and smeared reinforcement are considered as being made of a distinct material. Their stiffness matrix is determined with equation (6.28).
7. **Determination of the stiffness matrices of the elements** (sections 6.4.4 and 6.4.5): the stiffness matrices are also calculated for embedded elements and isolated bars, representing respectively the prestressing and localised reinforcement, equations (6.34) and (6.41).
8. **Calculation of nodal loads due to inelastic contributions** (section 6.4.4): the prestressing applied on the structure is converted to nodal equivalent forces, the equivalent force of the cable determined using equation (6.36).
9. **Calculation of the global vector of nodal loads** : the final load vector is found by adding the prestressing to the vector containing the weight, concentrated and distributed loads.
10. **Calculation of the global stiffness matrix** : the global stiffness matrix is found by combining those found for each element using a numerical assembly method.
11. **Inversion of the global stiffness matrix and calculation of the nodal displacements** (section 6.5.1) : once the global stiffness matrix has been computed, it can be inverted using the skyline method. Then, the nodal displacements are calculated using equation (6.47).
12. **Calculation of the elementary forces and deformations and their related principal quantities** : now, for each element, the stress and strain fields can be calculated. The principal strains and stresses and their inclinations are computed.

13. **Calculation of the secant moduli** : the new moduli can be determined using the new set of stresses and strains. They are found for each type of material or element with equations (6.13), (6.25), (6.37) and (6.45).
14. **Convergence criteria satisfied ?** (section 6.5.3) : if the convergence criteria are not fulfilled, the method iterates back to step 5 to find the material matrices using the new moduli and calculates the nodal displacements again.
15. **Partial results stored** : if the convergence criteria are attained, the results obtained so far are stored.
16. **Further time increments ?** : each iteration is done on several time increments to analyse the degradation process (section 6.5.4). If the iteration limit has not been reached, the program updates the parameters depending on the corrosion propagation, such as the reinforcement areas, and goes back to step 5.
17. **Further load increments ?** : the load is not applied at once to avoid divergence in the results (section 6.5.2). Thus, the total load needs to have been applied before ending the iterations. If that's not the case, a new load vector is calculated and the program goes back to step 5.
18. **Global results stored** : once the convergence is satisfied, the load has been fully applied and the propagation of corrosion has been calculated over the chosen period of time, the global results can be stored.
19. **End** : the results are returned. A graphical output was written in MATLAB by MSG&I to visualise the parameters over the mesh, such as the strain directions, principal stresses or the cracking pattern of the element.

6.4 Behavioural relationships

The equations that were chosen to characterise the behaviour of concrete, steel and the composite material are presented below. The way the prestressing and localised reinforcement are introduced in the structure is also described.

6.4.1 Concrete

6.4.1.1 Concrete in compression

Taking into account the softening behaviour of the material, the formulation for concrete under compression given in the MCFT is used (figure 6.4) :

$$f_{c2} = f_{c2,\max} \left(2 \left(\frac{\varepsilon_2}{\varepsilon'_c} \right) - \left(\frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right) \quad (6.6)$$

with

$$\frac{f_{c2,\max}}{f'_c} = \frac{1}{0.8 - 0.34 \varepsilon_1/\varepsilon'_c} \leq 1 \quad (6.7)$$

with

f'_c the resistance to uniaxial compression of a standard cylindrical specimen ;

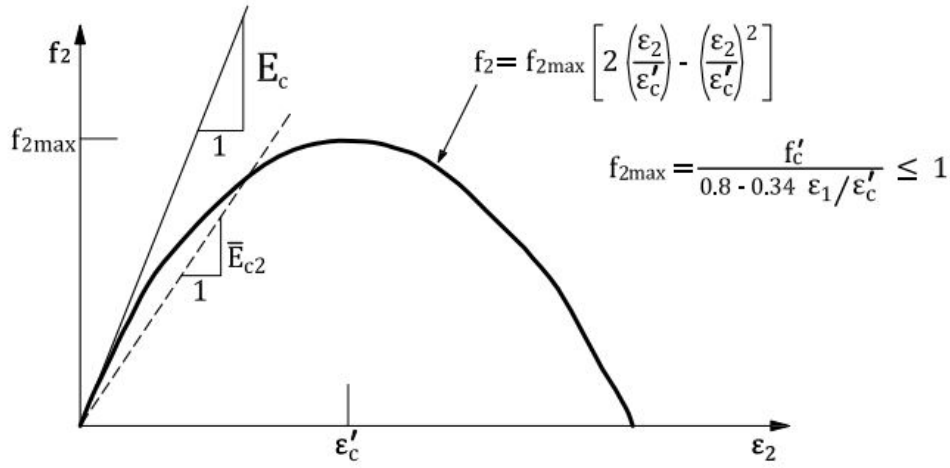


Figure 6.4: Behaviour of concrete in compression [4, 5]

ε'_c the peak compressive deformation of that standard specimen ;

ε_1 and ε_2 respectively the tension and compression principal strains.

6.4.1.2 Concrete in tension

The tension stiffening phenomenon is taken into account using the following relationships (figure 6.5) :

$$f_{c1} = E_c \varepsilon_1 \quad \text{for } \varepsilon_1 < \varepsilon_{cr} \quad (6.8)$$

$$f_{c1} = \frac{f_{cr}}{1 + \sqrt{c_t \varepsilon_1}} \quad \text{for } \varepsilon_1 \geq \varepsilon_{cr} \quad (6.9)$$

with

c_t the tension stiffening coefficient, respectively 200 and 500 for small and large elements, although the influence is minimal as demonstrated on figure 6.6 ;

E_c the Young modulus ;

$$E_c = \frac{2f'_c}{\varepsilon'_c} \quad (6.10)$$

ε_{cr} the strain at cracking ;

$$\varepsilon_{cr} = \frac{f_{cr}}{E_c} \quad (6.11)$$

f_{cr} the stress at cracking, which, in the absence of experimental data, is assumed to be :

$$f_{cr} = 0.33\sqrt{f'_c} \quad (6.12)$$

The stiffness matrix $\mathbf{D}$ has to be modified to represent the non-linear behaviour, using the following secant moduli :

$$\bar{E}_{c,1} = \frac{f_1(\varepsilon_1)}{\varepsilon_1} \quad \bar{E}_{c,2} = \frac{f_2(\varepsilon_1, \varepsilon_2)}{\varepsilon_2} \quad (6.13)$$

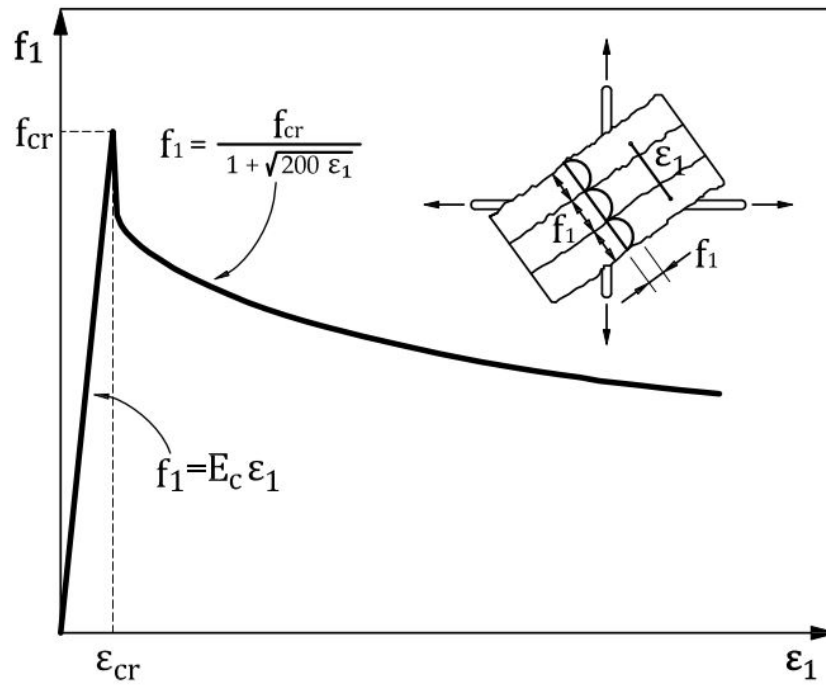


Figure 6.5: Behaviour of concrete in traction [4, 5]

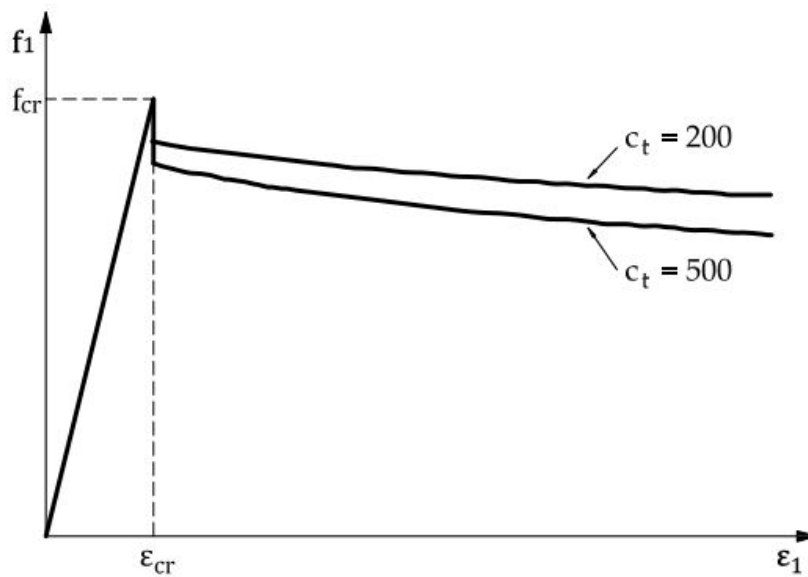


Figure 6.6: Influence of the tension stiffening coefficient c_t [4, 5]

$$\bar{G}_c = \frac{\bar{E}_{c,1} \bar{E}_{c,2}}{\bar{E}_{c,1} + \bar{E}_{c,2}} \quad (6.14)$$

After cracking, Poisson effects are assumed to be negligible. Thus, the stress inside the concrete matrix can be determined using this formulation :

$$\boldsymbol{\sigma}' = \mathbf{D}'_c \boldsymbol{\varepsilon} = \begin{pmatrix} \bar{E}_{c,2} & 0 & 0 \\ 0 & \bar{E}_{c,1} & 0 \\ 0 & 0 & \bar{G}_c \end{pmatrix} \boldsymbol{\varepsilon} \quad (6.15)$$

For the assembly, $\mathbf{D}'_c$ has to be expressed from its reference axis into the global axis. This is done using the following transformation matrices, for a rotation angle of ψ ($c = \cos \psi$, $s = \sin \psi$):

$$\mathbf{T}_\varepsilon = \begin{pmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{pmatrix} \quad \mathbf{T}_\sigma = \begin{pmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{pmatrix} \quad (6.16)$$

such that for $\psi = \pi/2 - \theta + \beta$ (see section 6.1),

$$\boldsymbol{\sigma}' = \mathbf{T}_\sigma \boldsymbol{\sigma} \quad \boldsymbol{\varepsilon}' = \mathbf{T}_\varepsilon \boldsymbol{\varepsilon} \quad (6.17)$$

The stress in the element resulting from the strain field can then be found :

$$\boldsymbol{\sigma} = \mathbf{T}_\varepsilon^T \mathbf{D}'_c \mathbf{T}_\varepsilon \boldsymbol{\varepsilon} = \mathbf{D}_c \boldsymbol{\varepsilon} \quad (6.18)$$

6.4.1.3 Deformations

The principal strains are required to compute the previous equations. They are determined by solving the following eigenvalue problem ($i = 1, 2$) [44, 45] :

$$\begin{pmatrix} \varepsilon_x - \varepsilon_i & \gamma_{xy}/2 \\ \gamma_{xy}/2 & \varepsilon_y - \varepsilon_i \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{for } i = 1, 2 \quad (6.19)$$

leading to :

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2} \quad (6.20)$$

and inclination :

$$\theta = \frac{1}{2} \arctan \left(\frac{\gamma_{xy}}{\varepsilon_y - \varepsilon_x} \right) \quad \text{for } \varepsilon_y - \varepsilon_x > 0 \quad (6.21)$$

$$\theta = \frac{\pi}{2} + \frac{1}{2} \arctan \left(\frac{\gamma_{xy}}{\varepsilon_y - \varepsilon_x} \right) \quad \text{for } \varepsilon_y - \varepsilon_x < 0 \quad (6.22)$$

This is computed in the program using an iterative subroutine using the Jacobi method.

6.4.2 Steel

For the steel reinforcement, a bilinear relationship is used to model the hardening effect. For $i = x, y$:

$$f_{si} = E_{si} \varepsilon_{si} \quad 0 \leq \varepsilon_{si} \leq \varepsilon_{yi} \quad (6.23)$$

$$f_{si} = f_{yi} + \frac{f_{ui} - f_{yi}}{\varepsilon_{ui} - \varepsilon_{yi}} \quad \varepsilon_{yi} \leq \varepsilon_{si} \leq \varepsilon_{ui} \quad (6.24)$$

with

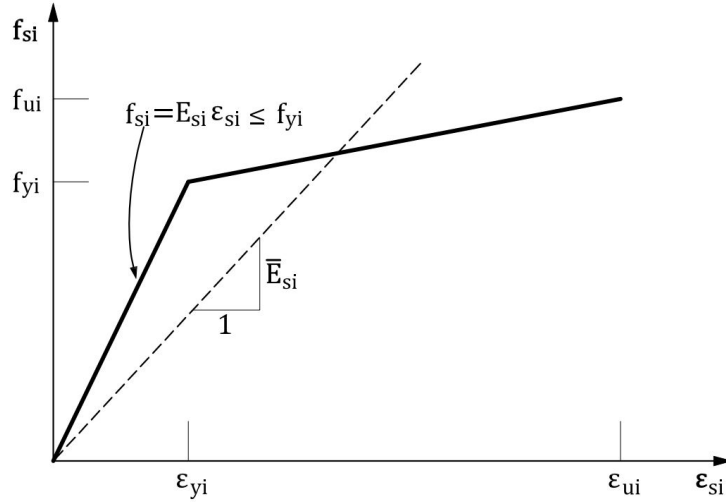


Figure 6.7: Behaviour of the ordinary steel reinforcement [4, 5]

f_{yi} the yielding strength and its corresponding strain ε_{yi} ;

f_{ui} the ultimate strength and its corresponding strain ε_{ui} ;

ε_{si} the strain within the i -th layer of reinforcement.

If $f_{yi} = f_{ui}$, it becomes a perfectly elastoplastic relationship, as is the case for the original formulation of the MCFT.

The secant modulus for the steel is then :

$$\bar{E}_{si} = \frac{f_{si}(\varepsilon_{si})}{\varepsilon_{si}} \quad (6.25)$$

leading to the determination of the stress inside the layer :

$$\sigma'_{s,i} = D'_{s,i} \varepsilon' = \begin{pmatrix} \rho_{si} \bar{E}_{si} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \varepsilon' \quad (6.26)$$

with ρ_{si} the reinforcement ratio for layer i .

By plugging $\psi = \beta + \alpha_i$ (see section 6.1) into the T_{ε} matrix of equation (6.16), the stresses are found in a similar fashion than for the concrete :

$$\sigma = T_{\varepsilon_i}^T D'_{s_i} T_{\varepsilon_i} \varepsilon = D_{s_i} \varepsilon \quad (6.27)$$

6.4.3 Composite element

The global secant stiffness matrix for the concrete element containing smeared reinforcement is found with :

$$D = D_c + \sum_{i=1}^n D_{s_i} \quad (6.28)$$

The global stress-strain relationship for a composite element is then :

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} = \left(\mathbf{T}_\varepsilon^T \mathbf{D}'_c \mathbf{T}_\varepsilon + \sum_{i=1}^n \mathbf{T}_{s_i}' \mathbf{D}_{s_i}' \mathbf{T}_{s_i} \right) \boldsymbol{\varepsilon} \quad (6.29)$$

with $\mathbf{D}$ full, symmetrical and dependent of the orientation of the element with respect to (X, Y) .

6.4.4 Prestressing

The behaviour of the steel used for the prestressing is similar to that of the ordinary reinforcement, see figure 6.9.

An effective way of adding prestressing without being bound to the edges of the finite elements is to treat it as being embedded into them, as represented on figure 6.8. The displacement of points A and B in the direction of the cable fragment, respectively s_A and s_B , are determined with :

$$s_A = q_{xA} \cos \alpha_e - q_{yA} \sin \alpha_e = \mathbf{T} q_A = \mathbf{T} \mathbf{N}|_A \mathbf{Q} \quad (6.30)$$

$$s_B = q_{xB} \cos \alpha_e - q_{yB} \sin \alpha_e = \mathbf{T} q_B = \mathbf{T} \mathbf{N}|_B \mathbf{Q} \quad (6.31)$$

allowing to define the deformation of this embedded element :

$$\varepsilon_b = \frac{s_A - s_B}{L} = \frac{1}{L} \mathbf{T} (\mathbf{N}|_A - \mathbf{N}|_B) \mathbf{Q} \quad (6.32)$$

with

$\mathbf{T}$ the rotation vector for α_e ;

$\mathbf{N}|_i$ the shape functions evaluated at point $i = A, B$;

q_i the vector containing the displacements of point i ;

L the length of the embedded element.

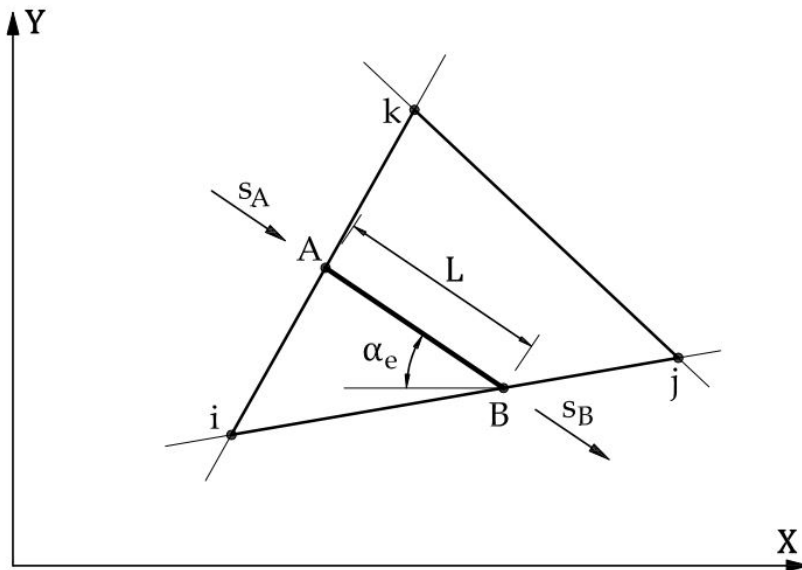


Figure 6.8: Representation of an embedded element [4, 5]

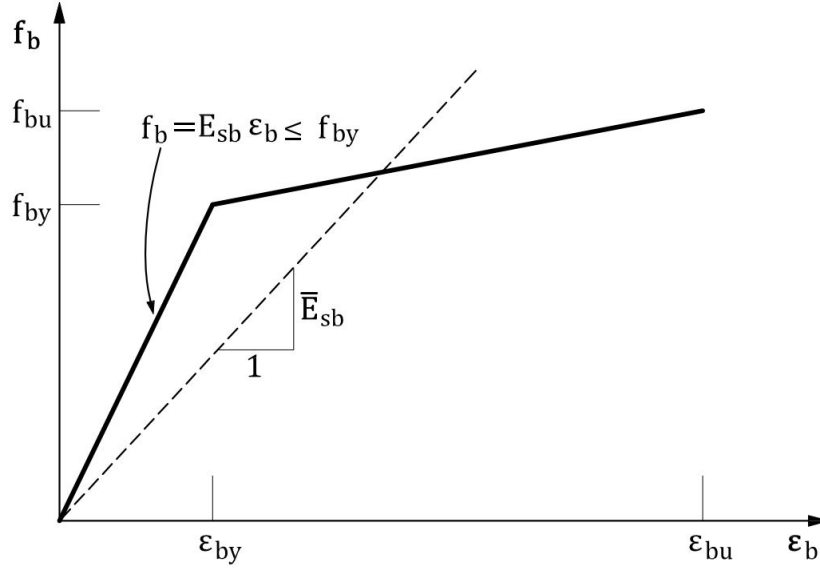


Figure 6.9: Behaviour of the prestressing steel [4, 5]

The stress inside the embedded element is :

$$f_b = E_{sb} (\varepsilon_b - \varepsilon_{bi}) \quad (6.33)$$

with ε_{bi} the inelastic deformation used to model the effects of prestressing.

Using virtual works¹, the rigidity matrix of the element can be determined as :

$$\mathbf{K}_b = (\mathbf{N}|_A - \mathbf{N}|_B)^T \mathbf{T}^T \mathbf{T} (\mathbf{N}|_A - \mathbf{N}|_B) \frac{E_{sb} A}{L} \quad (6.34)$$

with

$$\mathbf{T}^T \mathbf{T} = \begin{pmatrix} \cos^2 \alpha_e & -\cos \alpha_e \sin \alpha_e \\ -\cos \alpha_e \sin \alpha_e & \sin^2 \alpha_e \end{pmatrix} \quad (6.35)$$

and the equivalent forces created by the prestressing are then :

$$\mathbf{F}_p = E_{sb} A (\mathbf{N}|_A - \mathbf{N}|_B)^T \mathbf{T}^T \varepsilon_{bi} \quad (6.36)$$

The secant modulus for the embedded element is :

$$\bar{E}_{sb} = \frac{f_b(\varepsilon_b)}{\varepsilon_b} \quad (6.37)$$

6.4.5 Localised reinforcement

An isolated rebar of cross section area A is defined by its two nodes i and j , each possessing two degrees of freedom (figure 6.10). The stress inside the bar is :

$$f_t = E_{st} \varepsilon_t \quad (6.38)$$

with

¹The theory surrounding virtual works and how they are applied for the implementation of the NLFEA can be found in [4, 5] and will not be reminded in this paper.

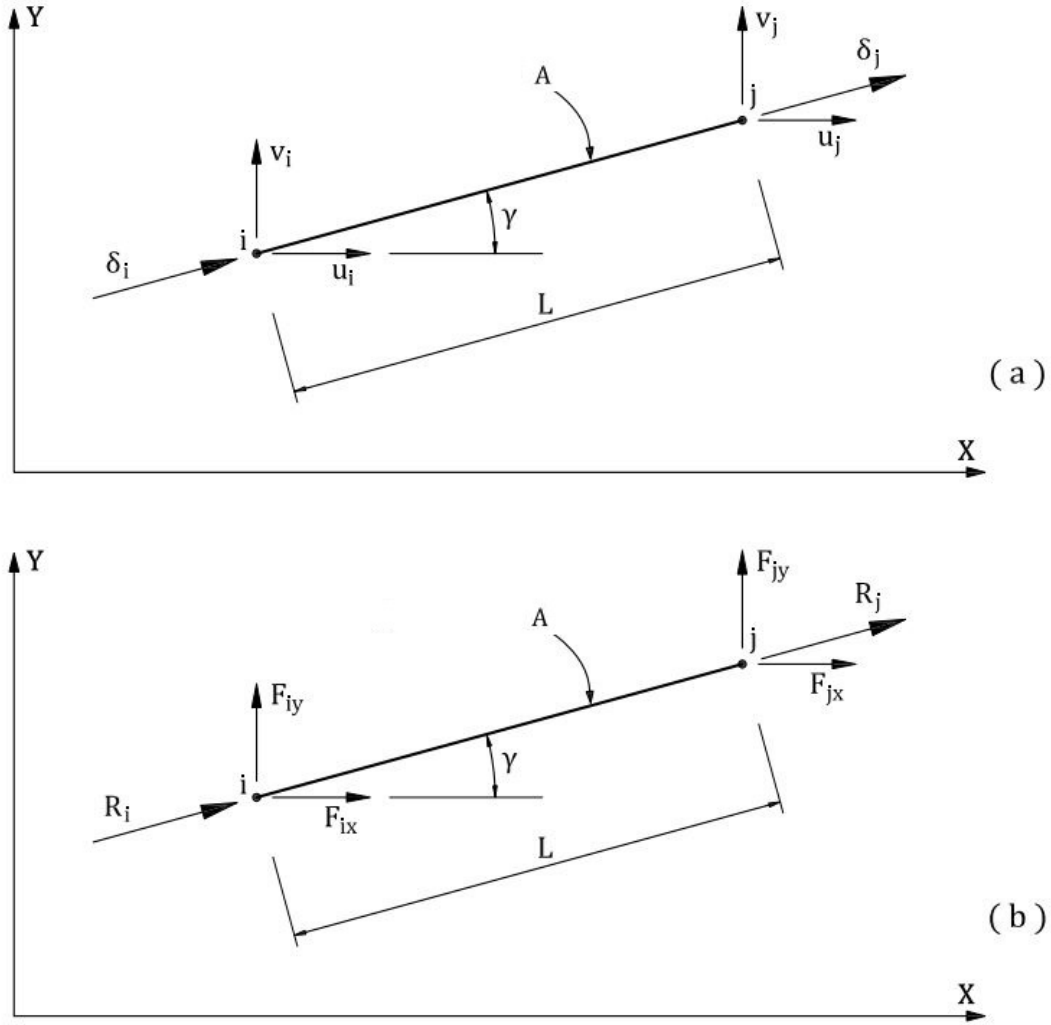


Figure 6.10: Representation of the characteristics of the isolated bar element : a) local and global displacements ; b) local and global forces [4, 5]

ε_t the deformation of the bar ;

$$\varepsilon_t = \frac{du}{dx} = \frac{\delta L}{L} \quad (6.39)$$

E_{st} the Young modulus of the steel used for the bar ;

L the length of the bar.

The required equilibrium in translation allows to define the stiffness matrix $\mathbf{K}'_t$, R being the force that the rebar sustains for a strain of ε_t :

$$R = f_t A = \frac{E_{st} A}{L} \delta L = k \delta L \quad (6.40)$$

$$\begin{pmatrix} R_i \\ R_j \end{pmatrix} = \begin{pmatrix} k & -k \\ -k & k \end{pmatrix} \begin{pmatrix} \delta_i \\ \delta_j \end{pmatrix} = \mathbf{K}'_t \begin{pmatrix} \delta_i \\ \delta_j \end{pmatrix} \quad (6.41)$$

Again, a rotation matrix is required to express the rigidity matrix in the general axis ($c = \cos \gamma$, $s = \sin \gamma$) :

$$\mathbf{T}_t = \begin{pmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{pmatrix} \quad (6.42)$$

$$\mathbf{K}_t = \mathbf{T}_t^T \mathbf{K}'_t \mathbf{T}_t = \frac{E_{st}A}{L} \begin{pmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{pmatrix} \quad (6.43)$$

The resulting forces for each degree of freedom, see figure 6.10(b), can then be found with :

$$\mathbf{F} = \mathbf{K}_t \mathbf{Q} \quad (6.44)$$

Finally, the secant stiffness for the isolated bar is defined as :

$$\bar{E}_{st} = \frac{f_t(\varepsilon_t)}{\varepsilon_t} \quad (6.45)$$

6.5 Solution of the non-linear problem

6.5.1 Formulation

Using the principle of virtual works, the problem to solve can be written as :

$$\sum_e \delta \mathbf{Q}^T \mathbf{K}_e \mathbf{Q} = \sum_e \delta \mathbf{Q}^T \mathbf{F}_e + \sum_e \delta \mathbf{Q}^T \mathbf{f}_e \quad (6.46)$$

with

$\mathbf{K}_e$ the rigidity matrix of element e ;

$\mathbf{F}_e$ the vector of nodal volume loads on e ;

$\mathbf{f}_e$ the vector of nodal surface loads on e .

To determine the nodal displacements of the whole structure, the following condensed matrix formulation of the previous equation is solved by the NLFEA :

$$\mathbf{Q} = \mathbf{K}^{-1} \mathbf{F} \quad (6.47)$$

with

$\mathbf{K}$ the global rigidity matrix ;

$\mathbf{F}$ the nodal forces (containing $\mathbf{f}_e$, $\mathbf{F}_e$ and $\mathbf{F}_p$).

The obtained displacement field is continuous over the entire structure and the borders between its elements, and respects the boundary conditions and equilibrium. However, because of the use of virtual works and an interpolation matrix $\mathbf{N}$, this equation is necessary but not sufficient to find an exact solution to the problem.

6.5.2 Solving method : membrane element

The solution is obtained through an iterative process constantly updating the secant stiffnesses, using the geometrical and material data given as input (nodal coordinates, concrete compressive strength, etc.) as well as the loading applied on the structure. The secant technique has advantages but also shortcomings, such as the inability to represent the formation of individual and localised crack lines. However, it has proved itself robust and relatively accurate in addition to having short processing times.

The first iteration uses the tangent Young modulus of the material, providing a first vector corresponding to a linear elastic state :

$$Q_0 = K_0^{-1}F \quad (6.48)$$

The global rigidity matrix is inverted using the skyline method [46], as it is in general a sparse matrix with non-zero values packed in a band around the main diagonal, and is always symmetrical. The skyline solver allows the inverting of $\mathbf{K}$ by storing only a limited number of its values, reducing the size of the array and thus allowing for better accuracy, denser meshes and a refinement of the output graphics. The numbering of the nodes plays an important role in this method (figure 6.11), as good numbering lowers the size m_K of the non-zero band.

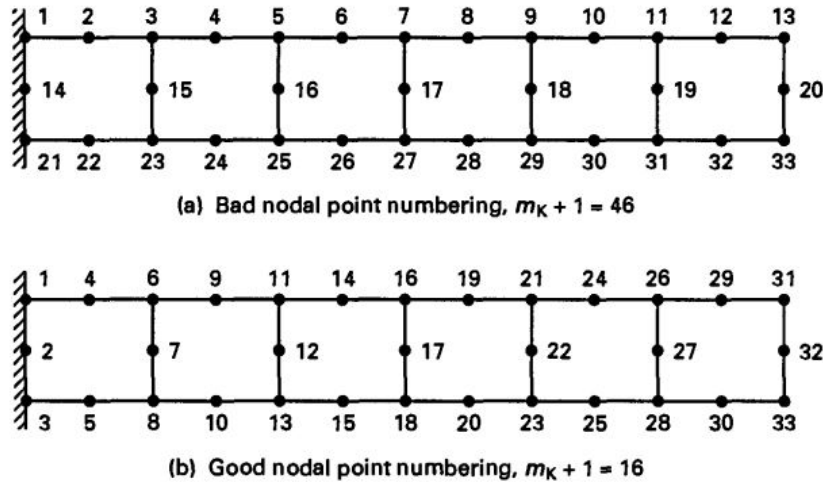


Figure 6.11: Node numbering pattern : a) incorrect ; b) correct [47]

Using the Jacobi method, ε_1 and ε_2 are derived, as well as the crack inclination θ . The new secant moduli are calculated, allowing to find the next Q_i . The process is repeated until the chosen convergence criteria are fulfilled (discussed in the next section). For the first iteration of the i -th step, the secant stiffness matrices used are the ones found in iteration $i - 1$, which accelerates the convergence rate.

The loading is applied incrementally to prevent a possible deviation that can occur when applying in a single step, illustrated on figures 6.12 and 6.13. The first load interval being $\beta_1 F_1$, the consecutive increments are :

$$F_n = \sum_{i=1}^n \beta_i F_1 \quad (6.49)$$

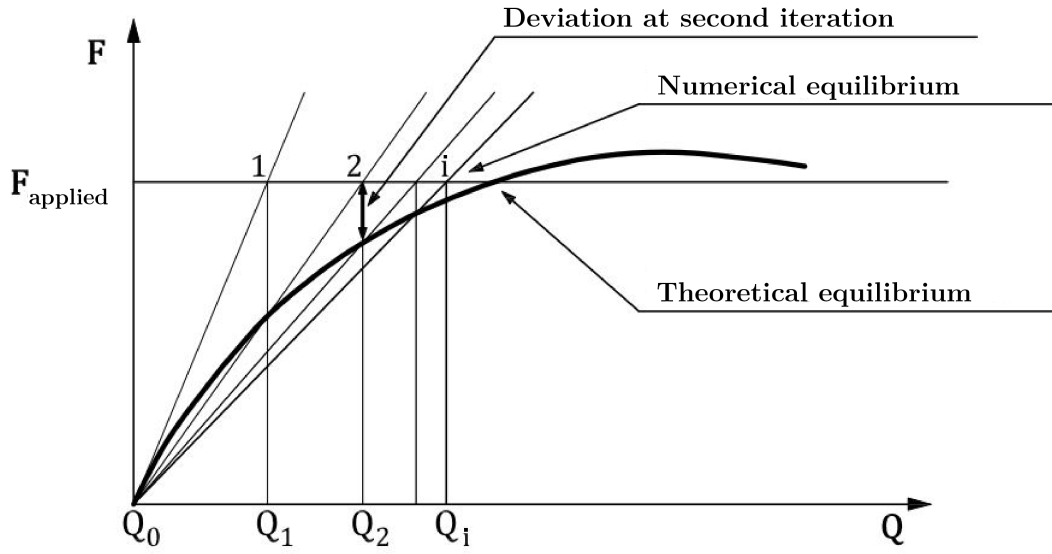


Figure 6.12: Description of the iteration process with the loading applied at once [4, 5]

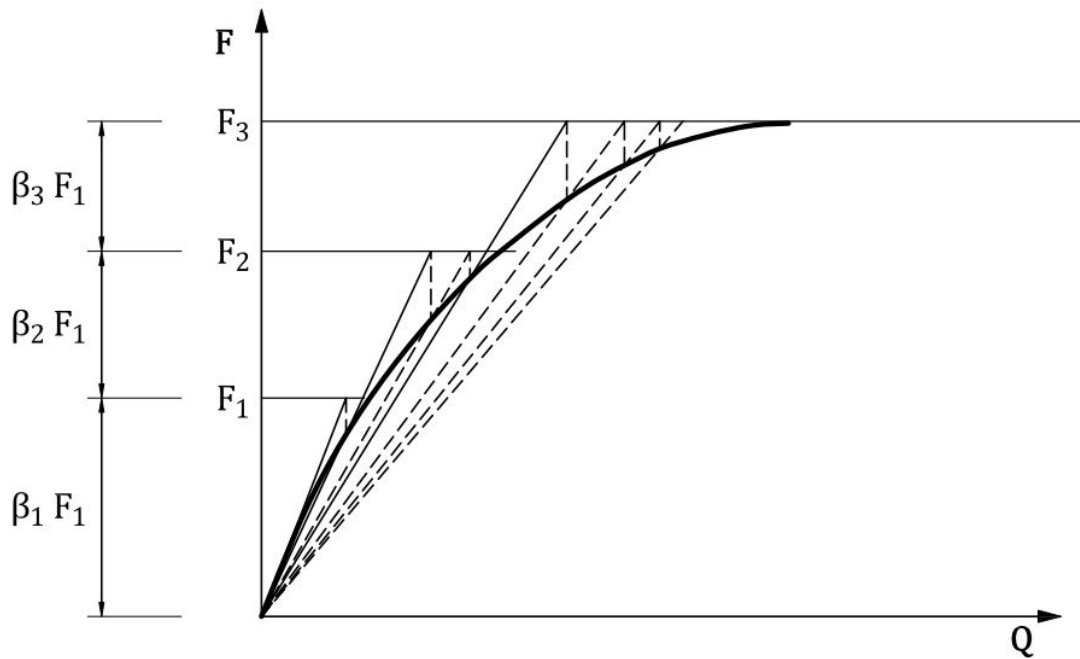


Figure 6.13: Representation of the iteration process for a loading applied by successive increments [48]

6.5.3 Convergence control

6.5.3.1 Numerical control

The process continues until a convergence control criterion δ has been satisfied [49]. Three different criteria are presented here :

1. Comparison between the norm² of the displacement increase and the norm of the current

²The Euclidian norm is generally used in these expressions.

displacement ;

$$\frac{\|Q_i - Q_{i-1}\|}{\|Q_i\|} \leq \delta_1 \quad (6.50)$$

2. Comparison between the norm of the unbalanced residual forces and the norm of the external load ;

$$\frac{\|F_i - R_{i-1}\|}{\|F_i\|} \leq \delta_2 \quad (6.51)$$

3. Comparison between the norm of the work done and the norm of the initial work.

$$\frac{\|(F_i - R_{i-1})^T(Q_i - Q_{i-1})\|}{\|(F_i - R_i)^T Q_i\|} \leq \delta_3 \quad (6.52)$$

The degree of accuracy is chosen by the operator, keeping in mind that too much accuracy can lead to a large number of iterations for a small gain in precision, while too little gives unacceptable results. It is usually recommended to take a value of δ of around 1% (with a common range of 0.5% to 2%). In the case that the solution oscillates around its equilibrium point, higher tolerance can be applied.

6.5.3.2 Criteria to stop the iterative process

The iterative process is stopped when one of the following situations occur [50] :

1. the solution converges ;
2. the structure collapses due to one of the materials reaching an ultimate state ;
3. there is no convergence after an expected number of iterations (usually around 20), for example due to a load condition exceeding the bearing capacity of the structure. If a load close to that of collapse is applied in one single increment, it can take up to 30 iterations to achieve convergence.

6.5.3.3 Further controls

Using certain types of elements, for example with no transverse reinforcement such as the beam presented on figure 6.14, can lead to inaccurate results arising from the constitutive assumptions of the model. To improve this accuracy, several controls have been put into place. A summary is presented here, see [4, 5] for more details.

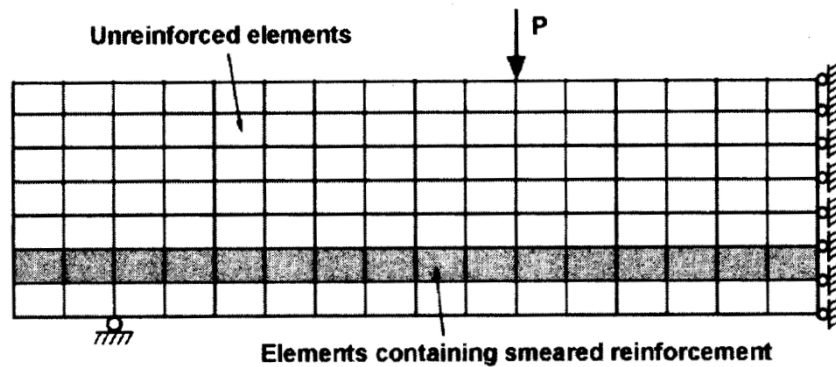


Figure 6.14: Mesh representation of a beam with no transverse reinforcement [51]

Crack width limit : for beams with a low stirrup reinforcement ratio, a few large cracks tend to form between the support and load application point, with the rest having a negligible width. In this type of element, it is recommended to neglect the participation of compressive stresses in the resistance. For cracks exceeding a width w of 2 mm, an additional crack width control is implemented alongside the softening effect, to further reduce the initial compressive strength f'_{c2} of equation (6.6).

$$f_{c2} = f'_{c2} \left(5 - \frac{w}{3} \right) \quad \text{for } 2 \leq w \leq 5 \text{ mm} \quad (6.53)$$

$$f_{c2} = 0 \quad \text{for } w > 5 \text{ mm} \quad (6.54)$$

Crack check control : in the NLFEA, the tension stiffening effect and aggregate interlock can be taken into account in the post-cracking phase. However, a crack check control is required to ensure that the tensile forces can indeed be transmitted through the cracks. For beams with low amounts of stirrup reinforcement (less than 0.05%), the horizontal smeared reinforcement is modelled as a thin strip concentrated in the lower part of the element as shown on figure 6.14. For this type of element, the crack check control does not take into account any possibility of stress transmission across cracks, thus underestimating the bearing capacity. To overcome this, a minimum amount of tensile strength is assumed :

$$f_{c1} \geq 0.1 f_{cr} \quad (6.55)$$

Regularisation techniques : implementing the softening branch of a material has several difficulties, as this creates an unstable behaviour. Since the finite-element based analysis provides results dependent of the discretisation of the mesh, this can cause a localisation of the damage in a band, of width equal to the characteristic dimension L_r of the elements, when the stress-strain diagram assumes the downward slope representing the softening branch. Thus, if the mesh becomes finer, the band decreases in width and would create a cancellation of energy dissipation, which is physically unrealistic. To avoid this problem, regularisation methods are used [52] to control the area where the damage is concentrated.

One method that does not completely eliminate the problem but is very competitive in terms of computational requirements is the one based on fracture energy [25, 53]. The parameters governing the softening branch are modified with the size of the mesh to ensure that the fracture energy associated with the element remains constant, by requiring that it does, in the following way :

$$\begin{aligned} f_{c1}^a &= E_c \varepsilon_1 & \text{for } \varepsilon_1 < \varepsilon_{cr} \\ f_{c1}^a &= f_{cr} \left(1 - \frac{\varepsilon_1 - \varepsilon_{cr}}{\varepsilon_{ts} - \varepsilon_{cr}} \right) & \text{for } \varepsilon_{cr} < \varepsilon_1 \leq \varepsilon_{ts} \\ f_{c1}^a &= 0 & \text{for } \varepsilon_1 > \varepsilon_{ts} \end{aligned} \quad (6.56)$$

with

$$\begin{aligned} \varepsilon_{ts} &= 1.1 \varepsilon_1 & \text{for } \varepsilon_{ts} \leq 1.1 \varepsilon_{cr} \\ \varepsilon_{ts} &= \frac{2 G_f}{L_r f_{cr}} & \text{for } 1.1 \varepsilon_{cr} < \varepsilon_{ts} \leq 10 \varepsilon_{cr} \\ \varepsilon_{ts} &= 10 \varepsilon_{cr} & \text{for } \varepsilon_{ts} > 10 \varepsilon_{cr} \end{aligned} \quad (6.57)$$

with

f_{c1}^a the tensile strength of concrete adapted for the softening mechanism ;

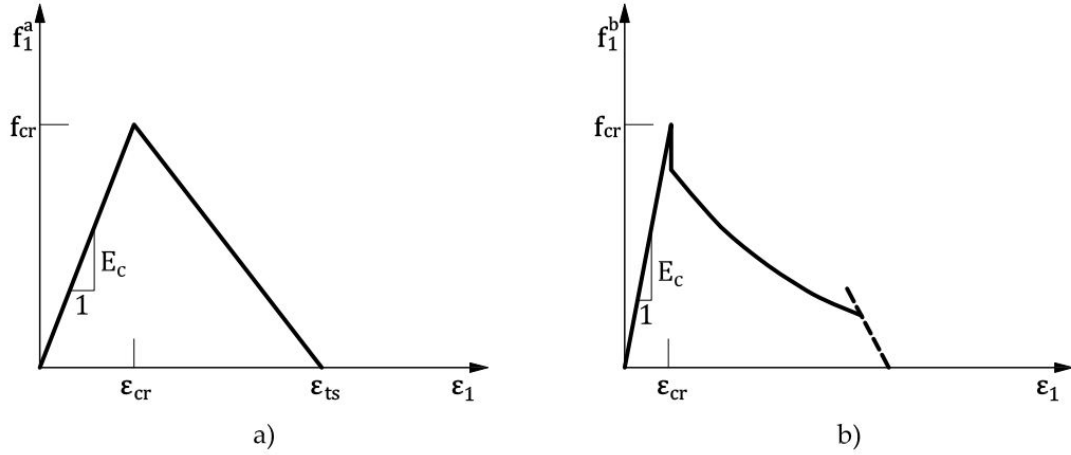


Figure 6.15: Tensile behaviour of concrete for a) tension stiffening ; b) tension softening [25]

G_f the fracture energy required to form a crack of unit area (determined experimentally).

G_f represents the resistance that the material oppose to its cracking, equal to the area under the softening phase of the graph. For ordinary concrete, the typical value for G_f is taken as 75 N/m [25].

If f_{c1}^b is the tensile strength derived from the tension stiffening effect of equation (6.9), the f_{c1} to take into account to determine the secant modulus $\bar{E}_{c1}$ by equation (6.13) becomes (figure 6.15) :

$$f_{c1} = \max(f_{c1}^a, f_{c1}^b) \quad (6.58)$$

6.5.4 Solving method : degradation

As they are, the relationships for the penetration of aggressive substances and the resulting corrosion cannot be directly introduced into the analysis, as they do not take into account the possibility of variation of the diffusion coefficients or corrosion rate at any time t . A development in series is performed by MSG&I to allow the reevaluation of those parameters year after year, based on the changes in constitutive quantities.

6.5.4.1 Carbonation

Both equations (2.2) and (2.3) are developed in series, depending on whether the element is intact or cracked. The following first order Taylor series provides the approximations used in the NLFEA to determine the carbonation depth $x(t)$. A first order has been proven sufficiently accurate, as depicted on figure 6.16.

Intact element : equation (2.2) is developed as

$$x_1(t + \Delta t) = x_1(t) + \frac{dx_1(t)}{dt} \Delta t + \mathcal{O}(t^2) = x_1(t) + \frac{1}{2} K \frac{1}{\sqrt{t}} \Delta t + \mathcal{O}(t^2) \quad (6.59)$$

Cracked element : equation (2.3) is developed as

$$x_2(t + \Delta t) = x_2(t) + \frac{dx_2(t)}{dt} \Delta t + \mathcal{O}(t^2) = x_2(t) + \frac{50}{4} \sqrt{w} t^{-3/4} \Delta t + \mathcal{O}(t^2) \quad (6.60)$$

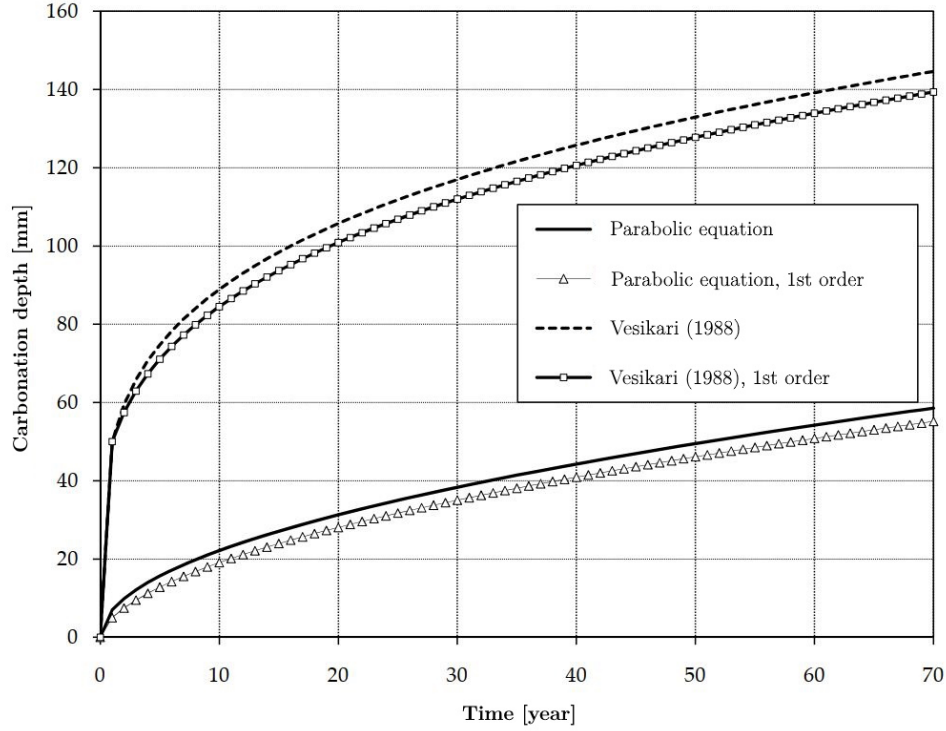


Figure 6.16: Comparison between the depth of carbonation obtained with equation (2.2), theoretical and approximated in Taylor's series ($K = 7 \text{ mm/year}^{1/2}$) ; and comparison between the depth of carbonation obtained with equation (2.3), theoretical and approximated in the Taylor series ($w = 1 \text{ mm}$) [4, 5]

6.5.4.2 Chloride penetration

For the penetration of chloride ions, a first order Taylor series is also used for equations (2.5) and (2.6).

$$\begin{aligned}
 C(x, t + \Delta t) &= C(x, t) + \frac{dC(x, t)}{dt} \Delta t + \mathcal{O}(t^2) \\
 &= C(x, t) + C_s \frac{2}{\sqrt{\pi}} e^{-\left(\frac{x}{2\sqrt{D_i} t}\right)^2} \frac{x}{4t\sqrt{D_i} t} \Delta t + \mathcal{O}(t^2)
 \end{aligned} \tag{6.61}$$

with

$$\begin{aligned}
 D_i &= D_{app} && \text{for } w \leq 0.3 \text{ mm} \\
 D_i &= D_{av} && \text{for } w > 0.3 \text{ mm}
 \end{aligned} \tag{6.62}$$

6.5.4.3 Corrosion of ordinary steel

The reduction of the initial cross sectional area $A_s(t_0)$ expressed by equation (2.9) is also developed in the same way.

$$\begin{aligned}
 A_s(t + \Delta t) &= A_s(t_0) + \frac{dA_s(t)}{dt} \Delta t + \mathcal{O}(t^2) \\
 &= \frac{N_s \pi D_0^2}{4} + \frac{N_s \pi}{4} (2n^2 i_{corr}^2 (t - t_0) - 2D_0 n i_{corr}) + \mathcal{O}(t^2)
 \end{aligned} \tag{6.63}$$

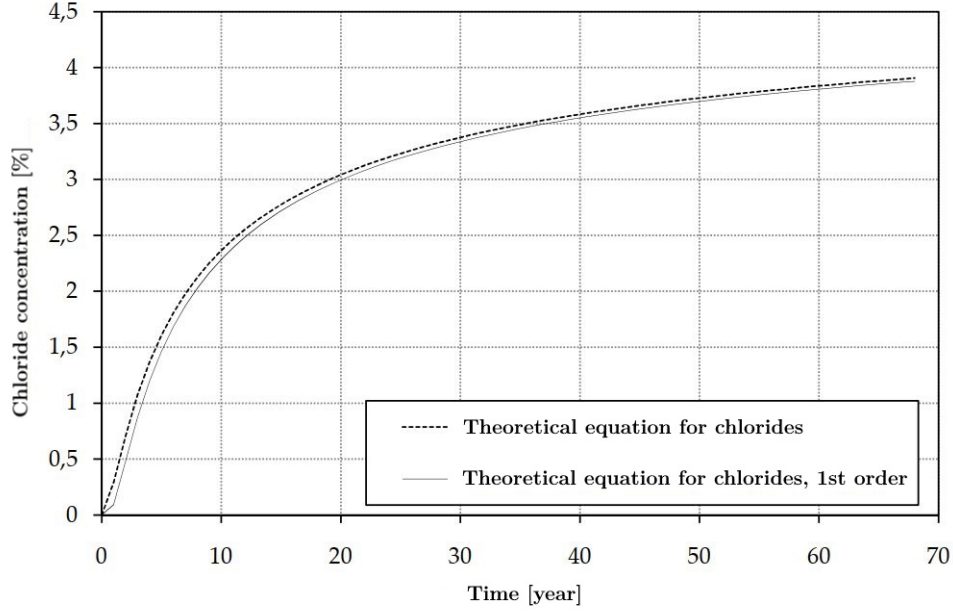


Figure 6.17: Comparison between the concentration of chlorides obtained through equation (2.5) and approximated by the Taylor series ; $x = 50$ mm, $C_s = 5\%$, $D_{app} = 5 \times 10^{-13}$ m²/s, $D_{cr} = 10^{-9}$ m²/s, $s = 150$ mm, $w = 1$ mm [4, 5]

For smeared reinforcement, the geometrical reinforcement ratio at time t can be determined with :

$$\rho(t) = \frac{N_s \pi (D_0 - n i_{corr}(t - t_0))^2}{4bh'} \quad (6.64)$$

with

b the width of the element ;

h' the height of influence of the reinforcement (an example for a typical beam is found figure 6.18).

Its corresponding Taylor series is then :

$$\begin{aligned} \rho(t + \Delta t) &= \rho_0 + \frac{d\rho(t)}{dt} \Delta t + \mathcal{O}(t^2) \\ &= \frac{N_s \pi D_0^2}{4bh'} + \frac{N_s \pi}{4bh'} (2n^2 i_{corr}^2 (t - t_0) - 2D_0 n i_{corr}) + \mathcal{O}(t^2) \end{aligned} \quad (6.65)$$

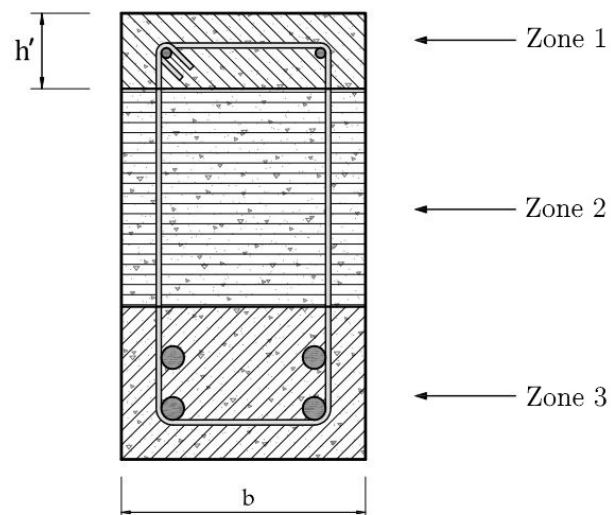


Figure 6.18: Representation of the height of influence h' of the smeared reinforcement zones for a typical beam cross section

7 | Results

A major disadvantage of the use of dapped-end beams is linked to the water tightness of the joint created [54]. A poorly executed or degraded joint allows the seepage of water, often containing chloride ions, and stagnation on the lower nib can occur. Additionally, maintenance is rendered difficult due to limited access to the half-joint [55], as only the outer vertical faces are accessible for inspection, hence the importance to determine how the resisting capacity of the nib is affected by this stagnation of chloride-rich water, as well as the carbonation due to the exposition to the carbon dioxide present in the atmosphere.

The goal of this study is thus to determine the decrease in load-carrying capacity of the nib of a DEB induced by the aggressive environment that surrounds it. It is conducted in two parts, first with the ST model of chapter 5 coupled with the degradation equations of chapter 2 using a Python program, then by running the non-linear finite-element analysis written in FORTRAN77 by MSG&I. They are respectively referred to as the first degradation model (FDM) and second degradation model (SDM), for simplicity.

The impact of this degradation is evaluated through the reduction of reinforcement diameter and thus ultimate load throughout the years. The cracking patterns obtained for each DEB model right before failure are presented, as well as the deformed shape of the nib and its usual stress flow pattern.

Two chloride surface concentrations C_s are taken : 0.2% to represent a moderately to highly aggressive environment, and 1% to represent a heavy stagnation of chloride-rich water on the element.

7.1 Selected beams

Four geometries are selected to study the durability requirements presented in section 2.3. Each nib model, called a B-element, will be referred to as BX-Y, with X the concrete strength in [MPa] and Y the cover thickness in [mm]. The selected B-elements are :

B45-50 : this beam of C45/50 and 50 mm of cover thickness satisfies the durability conditions of XC3 for S5 ;

B35-55 : this beam of C35/45 and 55 mm of cover thickness satisfies the durability conditions of XC3 for S6 ;

B35-30 : this beam of C35/45 and 30 mm of cover thickness does not satisfy the durability conditions of XC3 for S6 ;

B35-05 : this beam of C35/45 and 5 mm of cover thickness does not satisfy the durability conditions of XC3 for S6.

These B-elements, presented with their strut-and-tie reinforcement on figures 7.1 to 7.4, were selected to study the repercussions of not following the minimum cover thickness requirement provided by the Eurocode (B35-55 vs B35-30 and B35-05), the reduction in cover thickness that is allowed when using a higher concrete grade (B45-50 vs B35-55) and the consequences of only protecting the reinforcement with a particularly thin cover (B35-05).

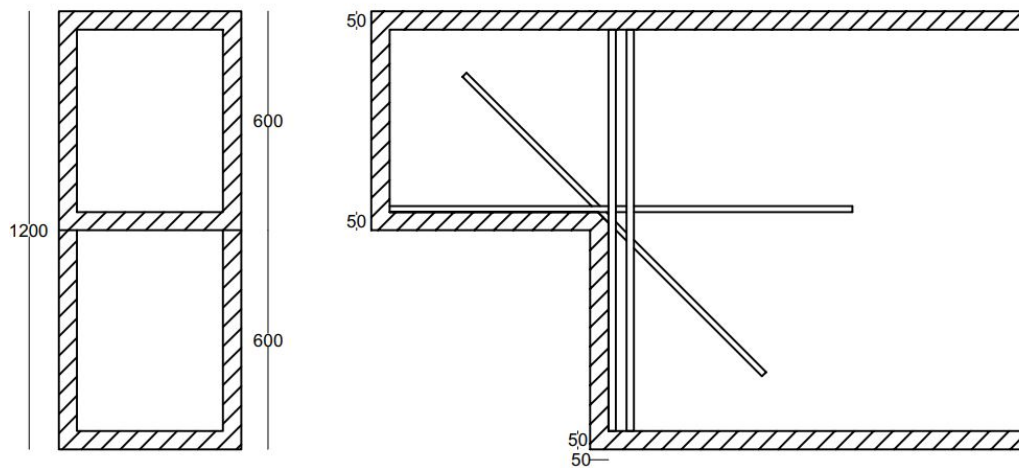


Figure 7.1: Dapped-end beam model B45-50

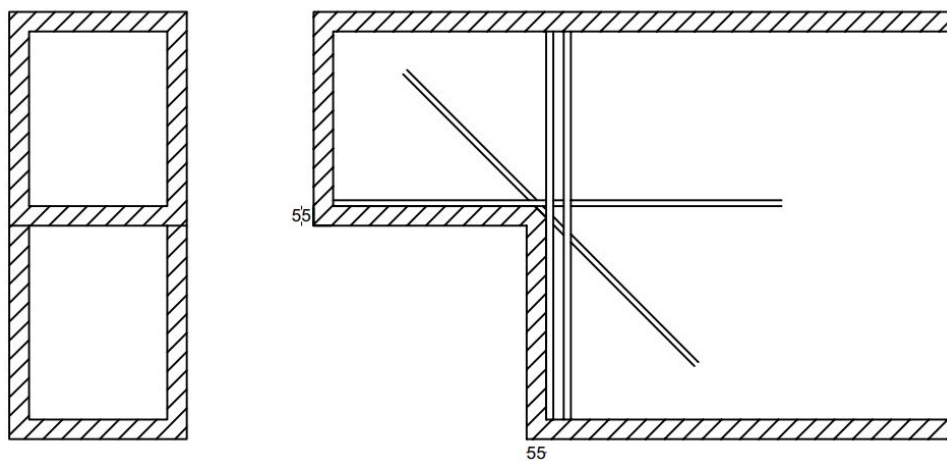


Figure 7.2: Dapped-end beam model B35-55

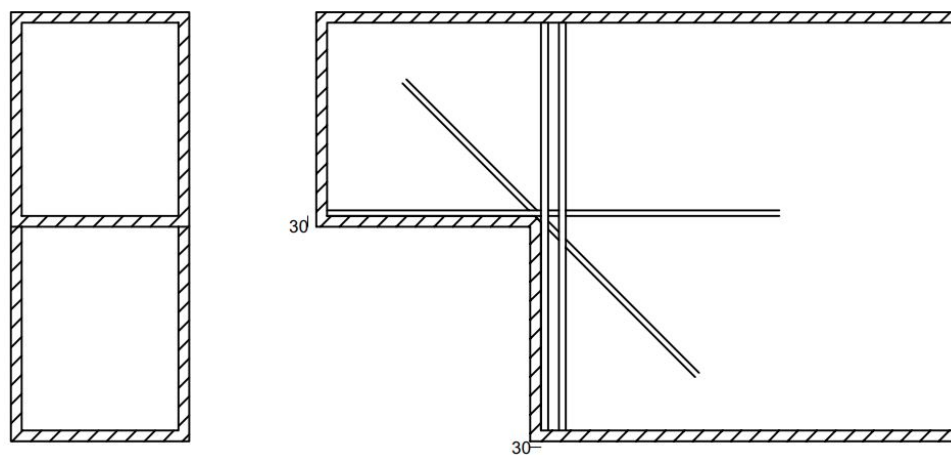


Figure 7.3: Dapped-end beam model B35-30

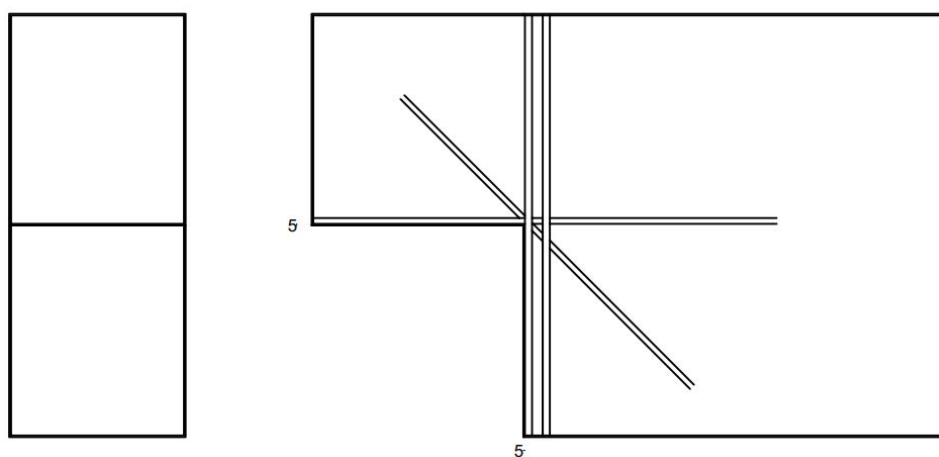


Figure 7.4: Dapped-end beam model B35-05

7.2 First degradation model

The steps to evaluate the degradation using the strut-and-tie method are as follows :

1. the general geometry of the concrete B-element is chosen ;
2. the ultimate load that the nib has to sustain is selected ;
3. using the ST model described in section 5.3, the reinforcement required to withstand that load is found using the load-to-section method ;
4. the nib geometry with its reinforcement is plugged into the section-to-load method to determine the actual¹ ultimate strength of the element ;
5. the reduction in diameter of the reinforcement is obtained using the Python degradation program ;
6. the reduced reinforcement area at time t is put into the section-to-load method to determine the new ultimate strength at that time t .

7.2.1 Geometry and reinforcement of the nib

Geometry of the concrete element : a large scale beam is chosen, represented on figure 7.5.

The cover thickness c is variable and influences a_D , a_V and d . a_D is chosen as equal to the distance between the load application point and the interface between the nib and the full depth of the beam, while a_V is the distance between the application point and the centre of gravity of the stirrups.

Ultimate load : an initial ultimate load of $V_u = 1000$ [kN] is selected, with the assumption that half of its value is sustained by the diagonal reinforcement ($\alpha_D = 0.5$).

Reinforcement : using the ST method, with a reduction coefficient for the strength of the concrete struts k_c of 0.50 for every B-element², the required reinforcement to sustain this load is found as ($D = \text{diameter}$) :

- horizontal : $4 D16 = 804 \text{ mm}^2$
- vertical : 2 closed stirrups D20, spaced by 30 mm = 1257 mm^2
- diagonal : $5 D16 = 1005 \text{ mm}^2$

for a yield strength f_y of 500 MPa and an ultimate strength f_u of 650 MPa (no safety coefficient applied).

New ultimate load : the actual ultimate load of the four B-elements is given in table 7.1.

¹Using standardised European rebar diameters, the final steel area is somewhat larger than the required area obtained in step 3, leading to a slightly higher capacity than what was chosen in step 2.

²Average value given by the Eurocode for the two concrete strength used (C35/45 and C45/50), see table 5.1. Using the real interpolated values for each grade has a negligible influence on the final ultimate load.

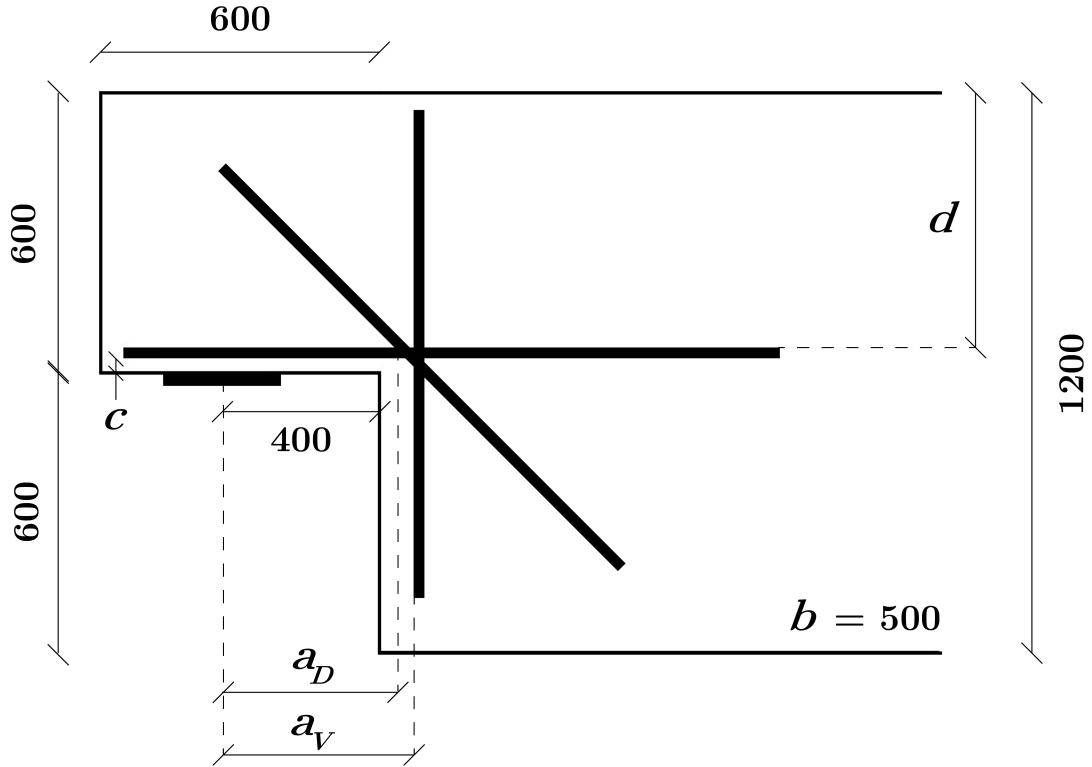


Figure 7.5: Simplified geometry of the extremity of the dapped-end beam for the strut-and-tie model ; measurements in mm

Table 7.1: B-elements and their initial strut-and-tie ultimate capacity

B-element	Cover [mm]	Grade	V_u [kN]
B45-50	50	C45/50	1045
B35-55	55	C35/45	1000
B35-30	30	C35/45	1072
B35-05	5	C35/45	1150

7.2.2 Degradation of the ultimate load

Now that the reinforcement has been determined, the degradation analysis can be conducted. The following fixed parameters are chosen :

K , carbonation coefficient = 8 mm/year^{1/2} (which represents a moderately compacted concrete) ;

C_{crit} , critical chloride concentration at the reinforcement = 0.5% (RILEM criterion) ;

w , crack width = 0.3 mm ;

s , crack spacing = 100 mm ;

D_{app} , chloride diffusion coefficient in intact concrete = 5×10^{-7} mm²/s ;

D_{cr} , chloride diffusion coefficient in cracked concrete = 10^{-3} mm²/s ;

i_{corr} , corrosion rate = 0.055 (only carbonation) and 0.12 (with chlorides) mm/year.

Using these parameters, the diameter reduction in time produces the residual ultimate load-carrying capacities given in tables 7.2 and 7.3 and illustrated on figures 7.6 and 7.7³, for respectively $C_s = 0.2$ and 1%. The percentages next to the ultimate load values represent the diminution compared to the initial load at $t = 0$.

Table 7.2: Reduction of the ultimate load in time for the FDM ; $C_s = 0.2\%$

Year	B45-50		B35-55		B35-30		B35-05	
0	1045		1000		1072		1150	
20	987	6%	976	2%	953	11%	1016	12%
40	864	17%	860	14%	832	22%	887	23%
60	747	29%	749	25%	717	33%	763	34%
80	636	39%	641	36%	608	43%	647	44%

At 80 years, the maximum difference in reduction between the B-elements is of 8%.

Table 7.3: Reduction of the ultimate load in time for the FDM ; $C_s = 1\%$

Year	B45-50		B35-55		B35-30		B35-05	
0	1045		1000		1072		1150	
20	987	6%	977	2%	864	19%	859	20%
40	750	28%	792	21%	612	43%	598	44%
60	514	51%	559	44%	396	63%	375	65%
80	316	70%	357	64%	220	79%	198	82%

At 80 years, the maximum difference in reduction between the B-elements is of 18%.

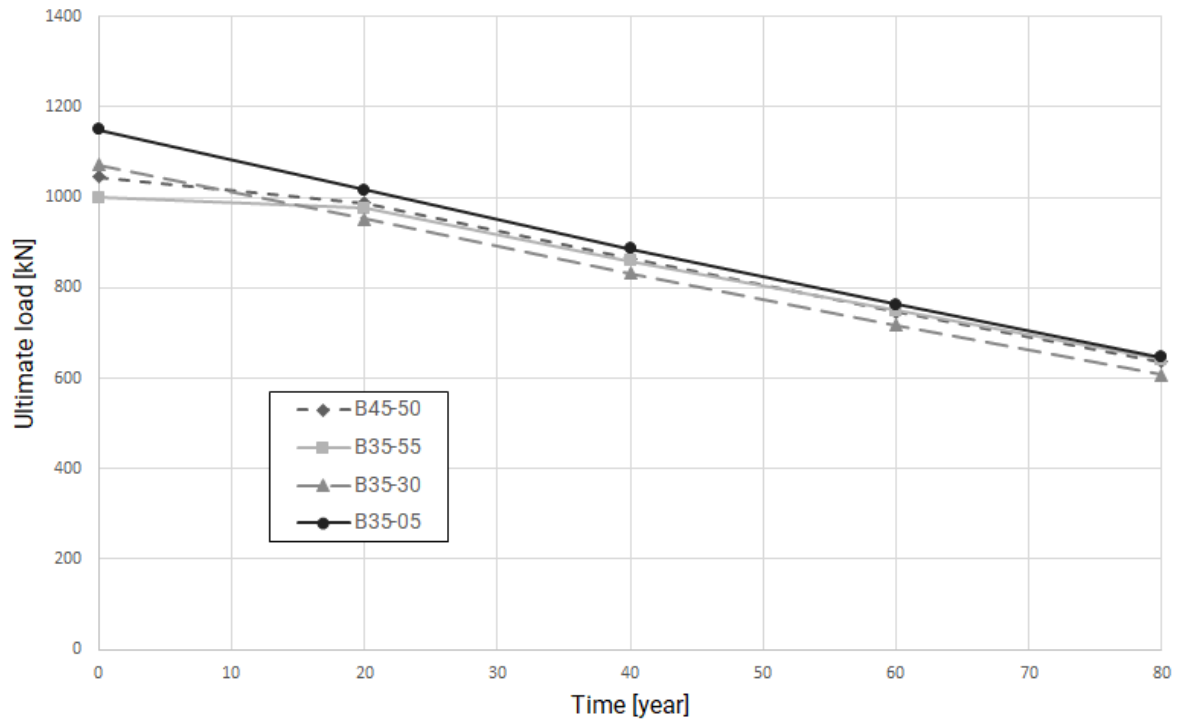


Figure 7.6: Reduction of the ultimate load in time for the FDM ; $C_s = 0.2\%$

³Note : the lines between the values do not represent the actual continuous decrease in time, but were placed there to better visualise the evolution for each element.

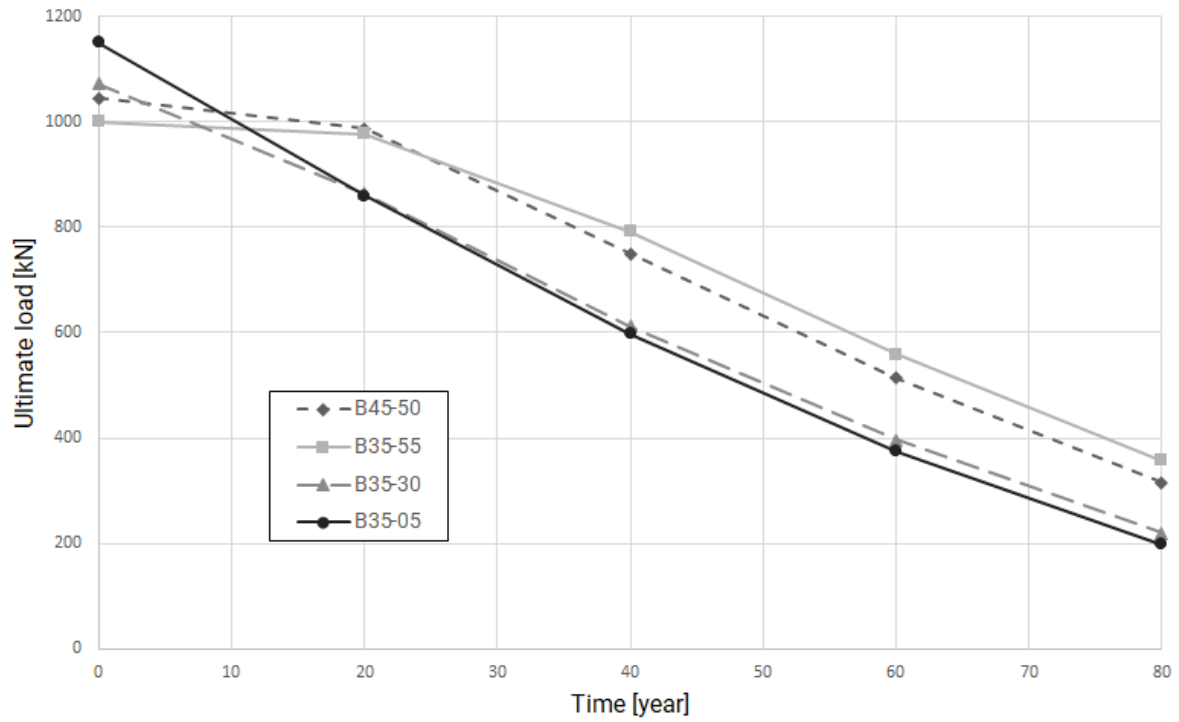


Figure 7.7: Reduction of the ultimate load in time for the FDM ; $C_s = 1\%$

7.3 Second degradation model

The same geometries and parameters of the FDM are used for the SDM, with the exception of the fixed crack pattern, as the non-linear finite-element analysis calculates the evolution of the cracking due to the applied loading and gradual corrosion of the reinforcement.

Several more variables are however required, such as the smeared reinforcement ratios inside the membrane elements and additional localised reinforcement compared to the one found with the ST, see figure 7.9. A mesh consisting of nodes and triangular elements has to be created, depicted on figure 7.8. This mesh is used for every B-element described in section 7.1. It is not completely regular due to the placement of the diagonal reinforcement and stirrups.

To represent the smeared reinforcement, the zones of figure 7.10 are characterised by a selected horizontal reinforcement ratio of $\rho_H = 0.5\%$ and the following vertical ratios ρ_V :

- nib (zone 1) : $0.36\% = 4$ stirrups of 12 mm ;
- full-depth (zone 2) : $0.54\% = 1$ stirrup of 16 mm every 150 mm.

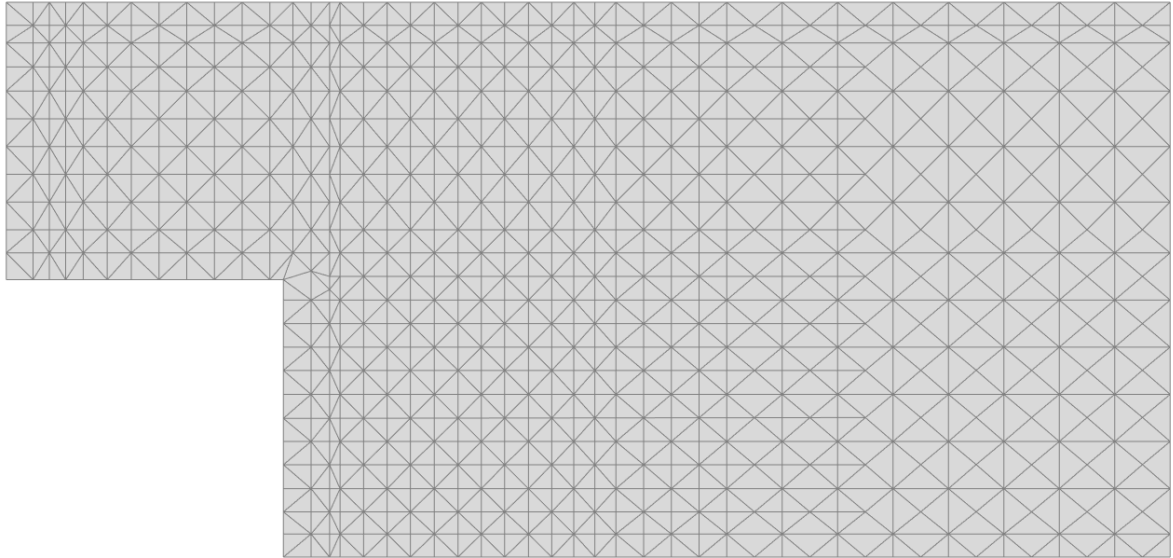


Figure 7.8: Mesh introduced in the non-linear finite-element analysis

7.3.1 Degradation of the ultimate load

To determine the initial ultimate load of each B-element, all the geometrical and material parameters are inputted into the program, with the degradation set to zero. The load is then gradually increased at $t = 1$ year until the program does not converge anymore.

It is considered that a permanent load of 60% of the initial capacity of each B-element is applied on them during their lifetime, which is the percentage used by MSG&I for their study of the degradation of prestressed beams [4, 5]. The initial ultimate capacities obtained through the MCFT-based program are given in table 7.4.

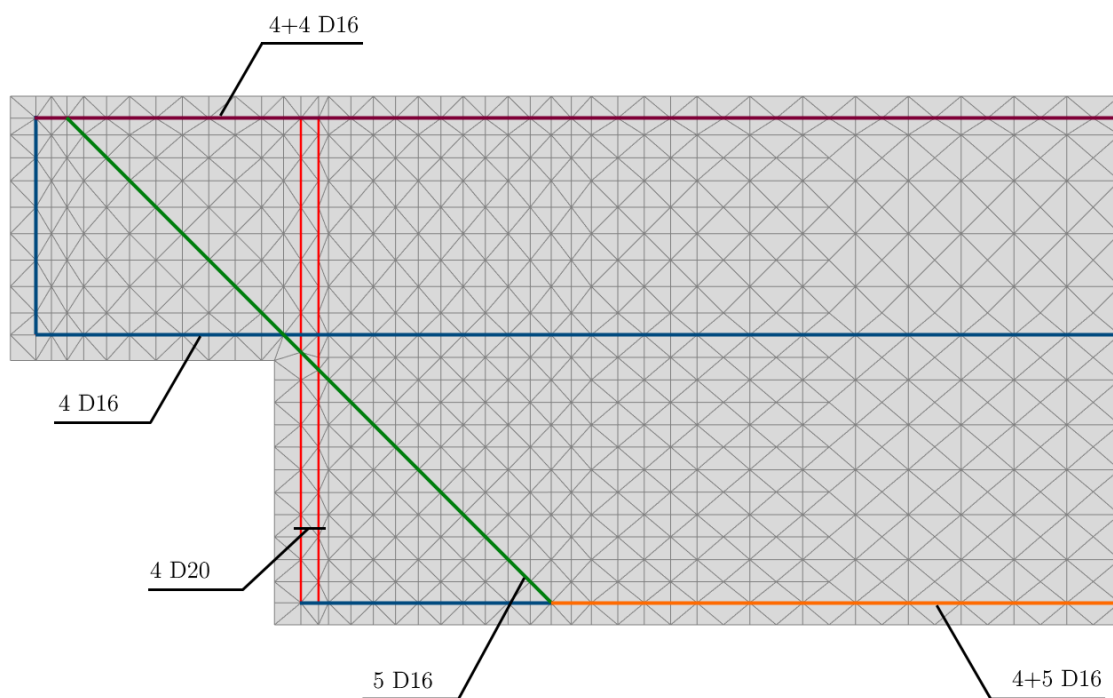


Figure 7.9: Main reinforcement inside the extremity of the dapped-end beam

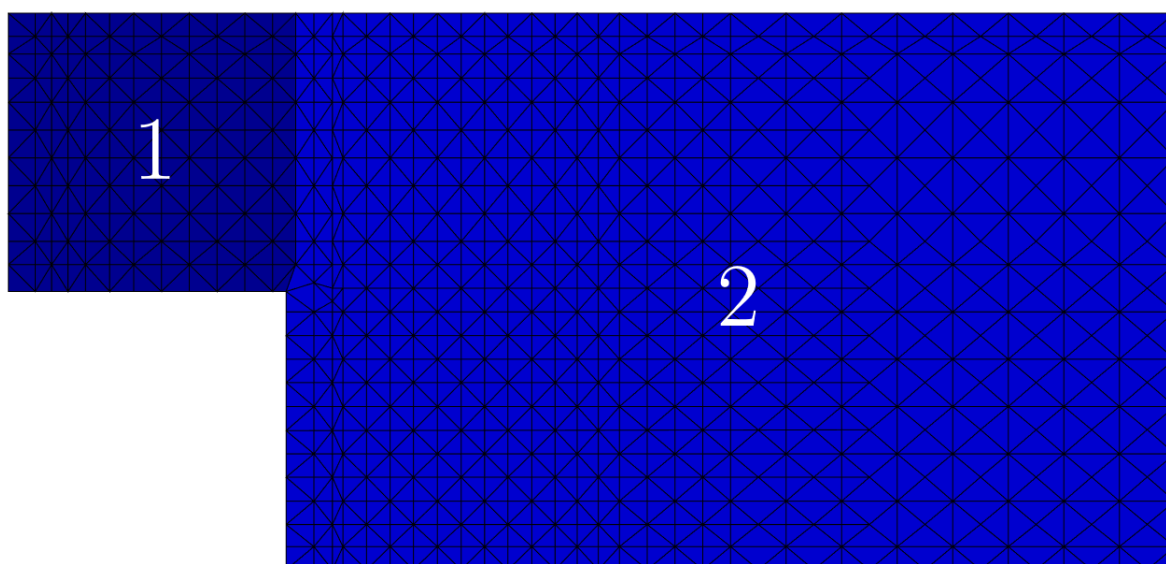


Figure 7.10: Smeared reinforcement zones inside the extremity of the dapped-end beam

Table 7.4: B-elements and their initial MCFT ultimate capacity

Beam	Cover [mm]	Strength	V_u [kN]
B45-50	50	C45/50	984
B35-55	55	C35/45	954
B35-30	30	C35/45	954
B35-05	5	C35/45	912

The SDM is conducted under the permanent load until convergence is not achieved anymore, signifying collapse. The diminishing localised rebar areas and reinforcement ratios are extracted. The residual load is then found in the same manner as the initial capacity after inputting the reduced reinforcement. The values are given in tables 7.5 and 7.6, and depicted on figures 7.11 and 7.12.

Table 7.5: Reduction of the MCFT ultimate load in time due to corrosion ; $C_s = 0.2\%$

Year	B45-50		B35-55		B35-30		B35-05	
0	984		954		954		912	
20	972	1%	894	9%	870	12%	816	17%
40	912	7%	858	13%	768	22%	732	26%
60	810	18%	786	20%	684	30%	654	34%
80	720	27%	708	28%	612	38%	588	40%

At 80 years, the maximum difference in reduction between the B-elements is of 13%.

Table 7.6: Reduction of the MCFT ultimate load in time due to corrosion ; $C_s = 1\%$

Year	B45-50		B35-55		B35-30		B35-05	
0	984		954		954		912	
20	756	23%	726	26%	708	28%	702	29%
40	600	39%	612	38%	570	42%	570	42%
60	522	47%	486	51%	480	51%	480	51%
80	450	54%	408	59%	414	58%	414	58%

At 80 years, the maximum difference in reduction between the B-elements is of 5%.

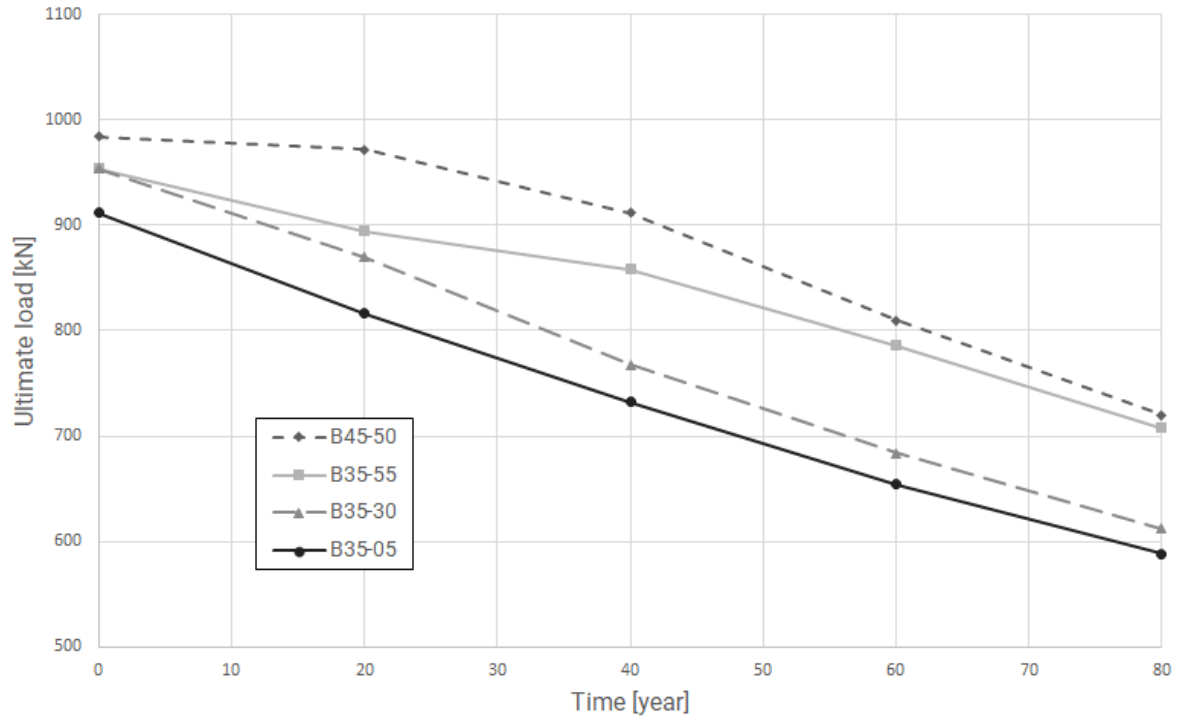


Figure 7.11: Reduction of the MCFT ultimate load in time due to corrosion ; $C_s = 0.2\%$

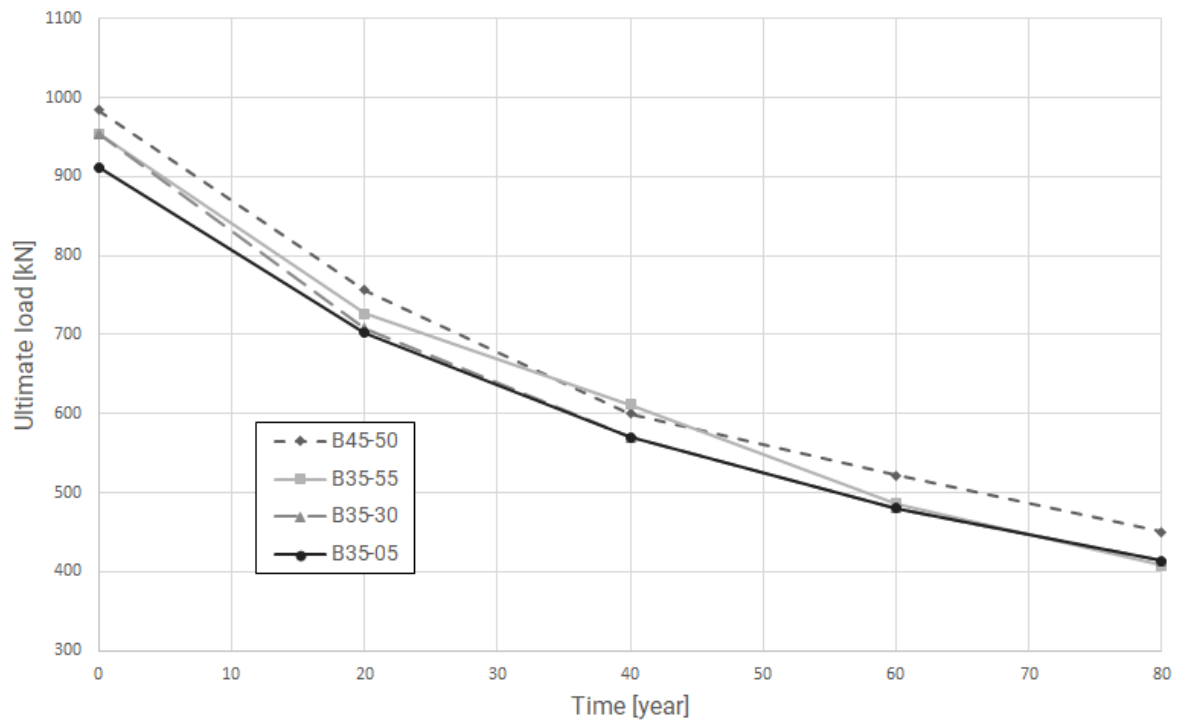


Figure 7.12: Reduction of the MCFT ultimate load in time due to corrosion ; $C_s = 1\%$

7.3.2 Reduction of reinforcement

As an indication, the reduction of reinforcement for the B45-50 and B35-05 subjected to $C_s = 0.2\%$ elements is given in this section. As for the previous tables, the second column with the percentage represents the reduction compared to year 0.

Table 7.7: Reduction of the localised reinforcement for B45-50 ; $C_s = 0.2\%$

Year	2 D20		4 D16		5 D16		4+4 D16		4+5 D16	
0	628 mm ²		804 mm ²		1005 mm ²		1608 mm ²		1809 mm ²	
20	628	0%	804	0%	950	5%	1608	0%	1727	5%
40	628	0%	804	0%	833	17%	1608	0%	1568	13%
60	538	14%	702	13%	724	28%	1463	9%	1418	22%
80	452	28%	602	25%	623	38%	1317	18%	1274	30%

Table 7.8: Reduction of the localised reinforcement for B35-05 ; $C_s = 0.2\%$

Year	2 D20		4 D16		5 D16		4+4 D16		4+5 D16	
0	628 mm ²		804 mm ²		1005 mm ²		1608 mm ²		1809 mm ²	
20	534	15%	697	13%	885	12%	1455	9%	1647	9%
40	447	29%	598	26%	773	23%	1310	19%	1492	18%
60	368	41%	506	37%	668	34%	1173	27%	1345	26%
80	297	53%	422	48%	571	43%	1043	35%	1206	33%

Table 7.9: Reduction of the smeared reinforcement ratio for B45-50 ; $C_s = 0.2\%$

	Zone 1				Zone 2			
Year	Hor.		Vert.		Hor.		Vert.	
0	0.50%		0.36%		0.50%		0.54%	
20	0.50%	0%	0.36%	0%	0.50%	0%	0.54%	0%
40	0.50%	0%	0.36%	0%	0.50%	0%	0.54%	0%
60	0.34%	32%	0.24%	32%	0.34%	32%	0.41%	25%
80	0.20%	59%	0.15%	59%	0.20%	59%	0.29%	47%

Table 7.10: Reduction of the smeared reinforcement ratio for B35-05 ; $C_s = 0.2\%$

	Zone 1				Zone 2			
Year	Hor.		Vert.		Hor.		Vert.	
0	0.50%		0.36%		0.50%		0.54%	
20	0.33%	33%	0.24%	33%	0.33%	33%	0.40%	26%
40	0.20%	60%	0.14%	60%	0.20%	60%	0.28%	48%
60	0.10%	80%	0.07%	80%	0.10%	80%	0.18%	66%
80	0.05%	90%	0.05%	86%	0.05%	90%	0.11%	80%

At 80 years, the hanger and horizontal reinforcements are the most affected by the reduction in strength and cover thickness, with a difference of reduction between the B-elements of respectively 25 and 23%. There is on average 30% more reduction in the smeared reinforcement for B35-05 than for B45-50.

7.3.3 Cracking pattern

The deformed shape of the B-elements and their stress trajectories under the permanent load before failure are shown on figures 7.13 and 7.14, respectively. The cracking patterns⁴ for each B-element under both chloride surface concentrations are presented on figures 7.15 to 7.22. For better visualisation, animated superposed pictures are provided in sections 10.1 and 10.2.

A comparison of crack openings close to failure is done for four finite elements shown on figure 7.23. The resulting graphs are presented on figures 7.24 and 7.25 and their superposed comparison can be visualised in section 10.3.

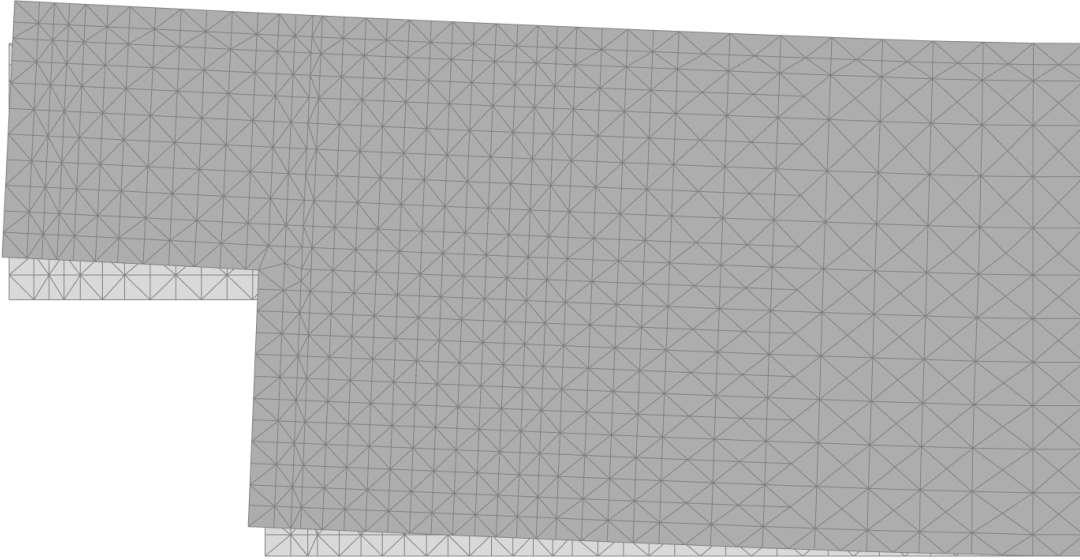


Figure 7.13: Deformed shape of the nib at failure

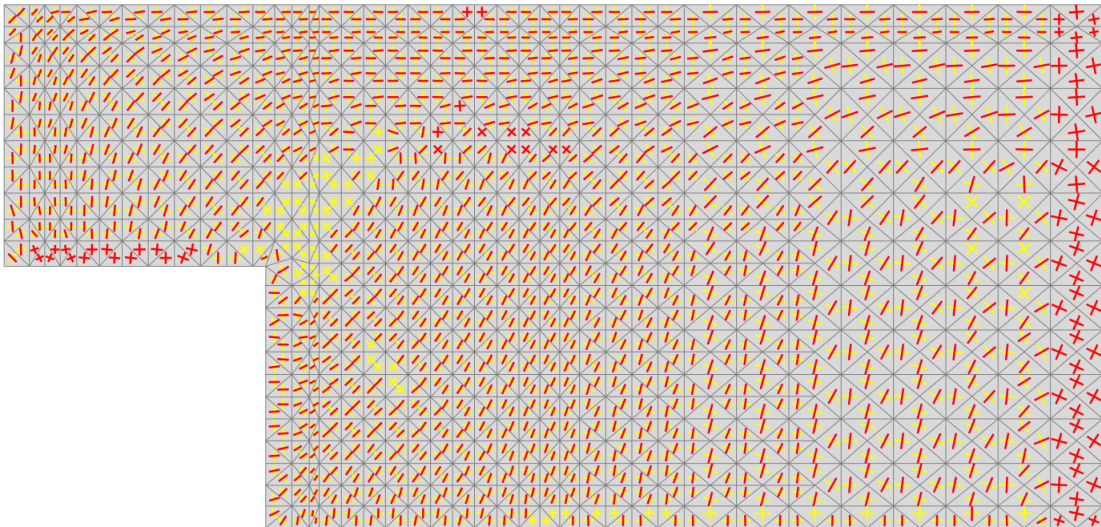


Figure 7.14: Usual stress flow pattern inside the nib ; compression in red, red crosses for unaffected or elastic elements

⁴The attention should be focused on the left-hand two-thirds of the cracking patterns, as the large cracks forming at the far right are caused by the support conditions, placed along the height of the element to represent the attachment to the rest of the beam, which was not introduced in the analysis.

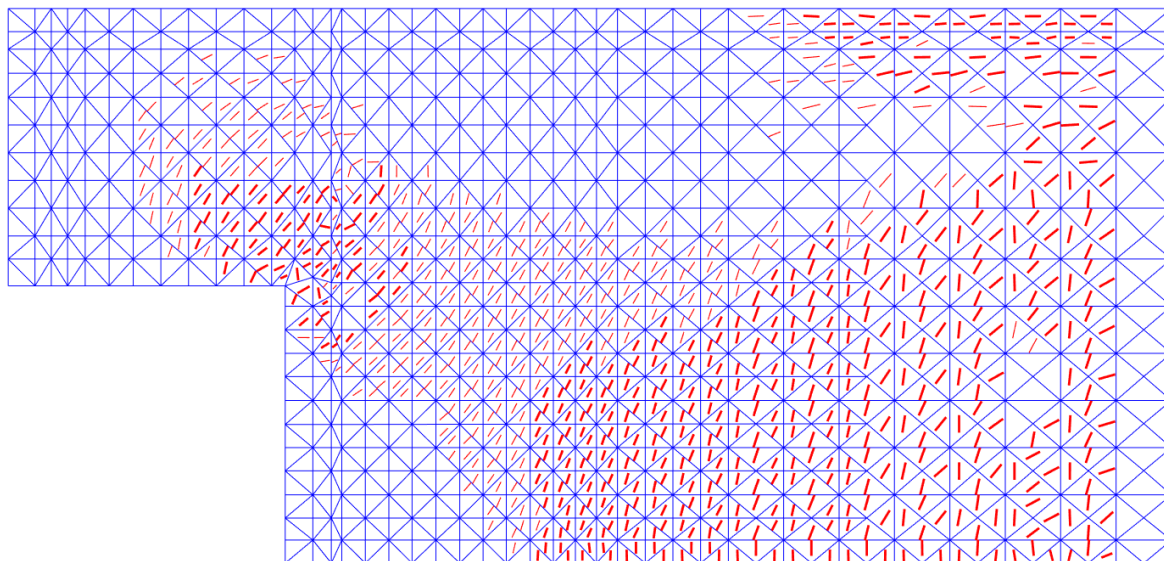


Figure 7.15: B45-50 cracking pattern before failure ; $C_s = 0.2\%$

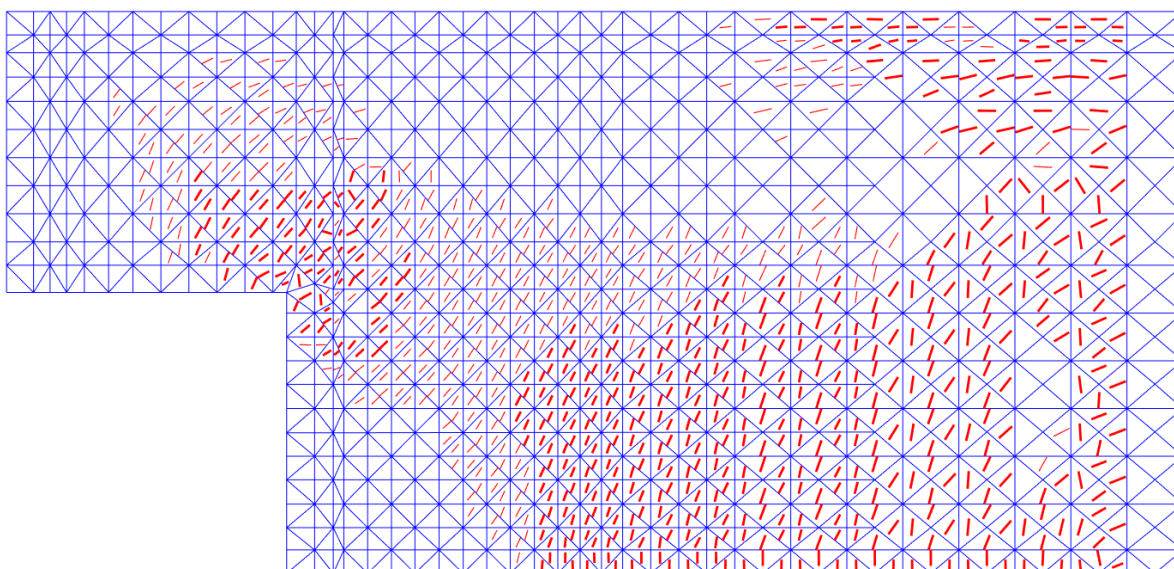


Figure 7.16: B45-50 cracking pattern before failure ; $C_s = 1\%$

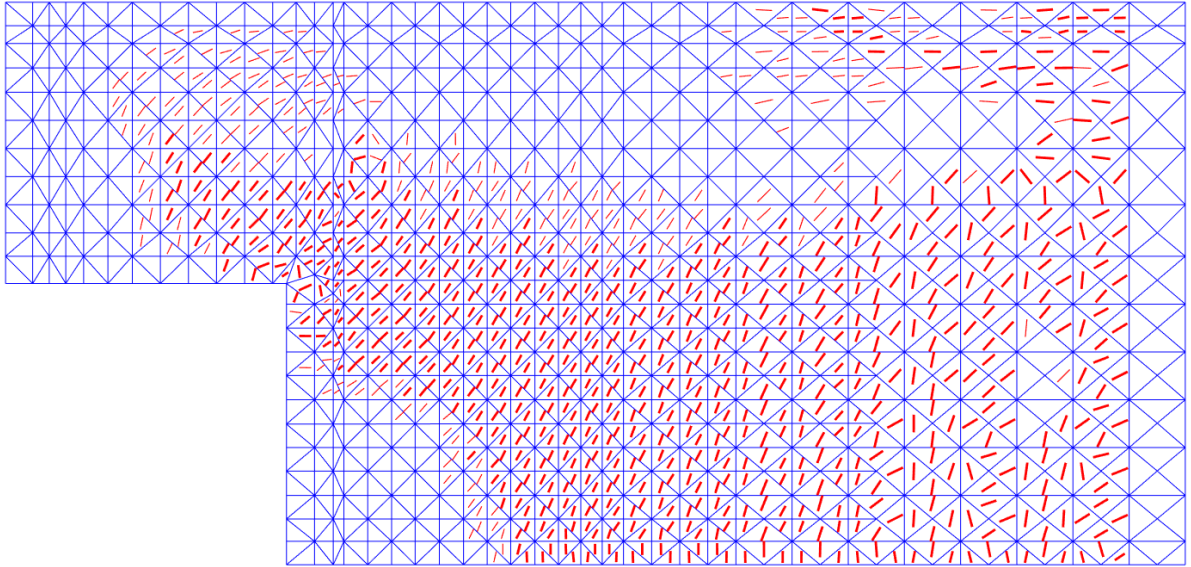


Figure 7.17: B35-55 cracking pattern before failure ; $C_s = 0.2\%$

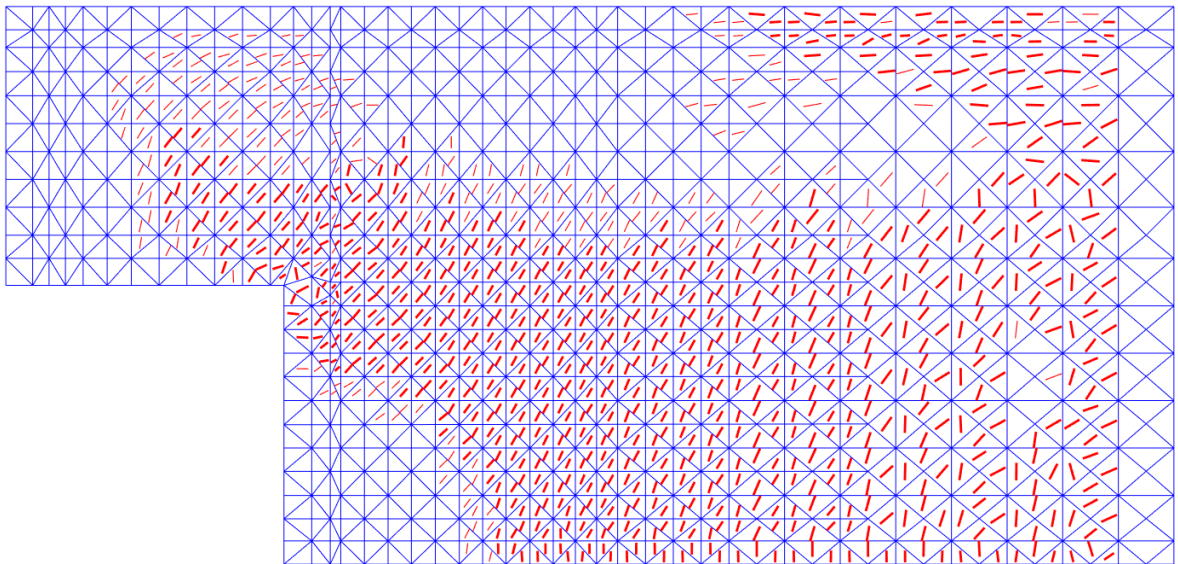


Figure 7.18: B35-55 cracking pattern before failure ; $C_s = 1\%$

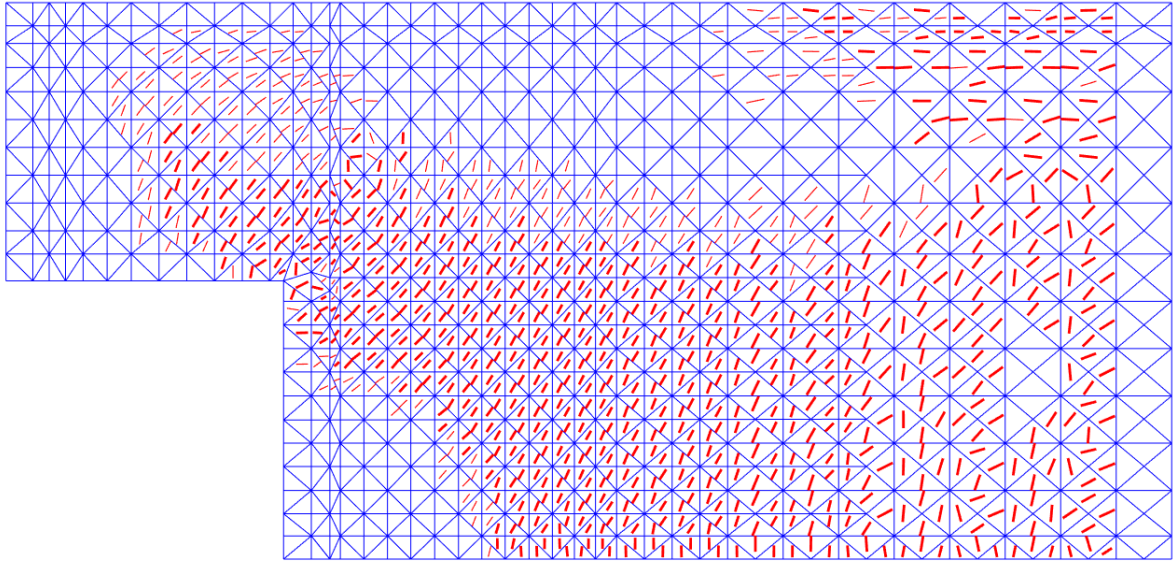


Figure 7.19: B35-30 cracking pattern before failure ; $C_s = 0.2\%$

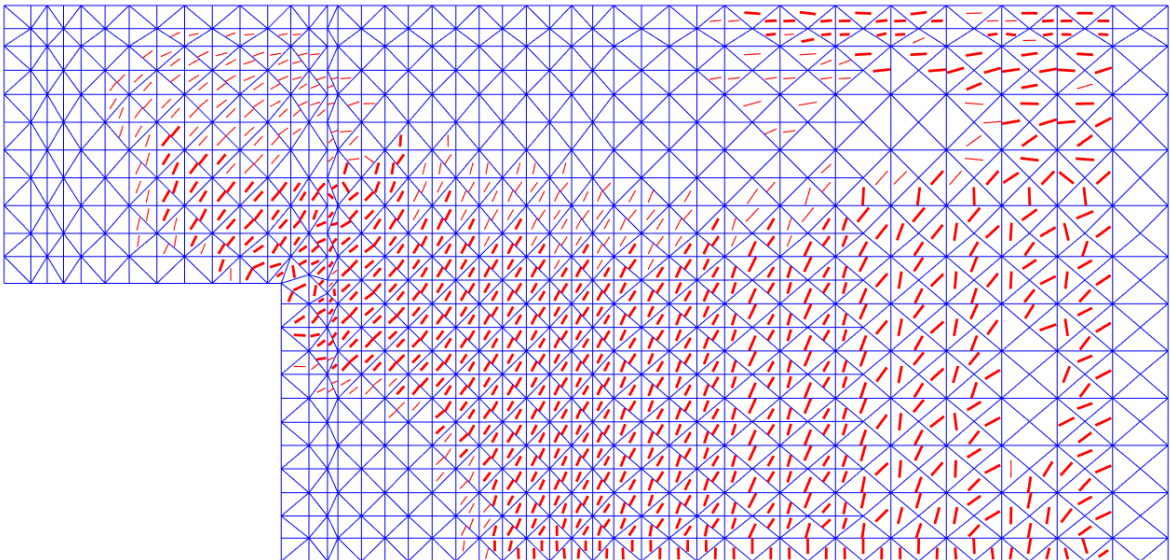


Figure 7.20: B35-30 cracking pattern before failure ; $C_s = 1\%$

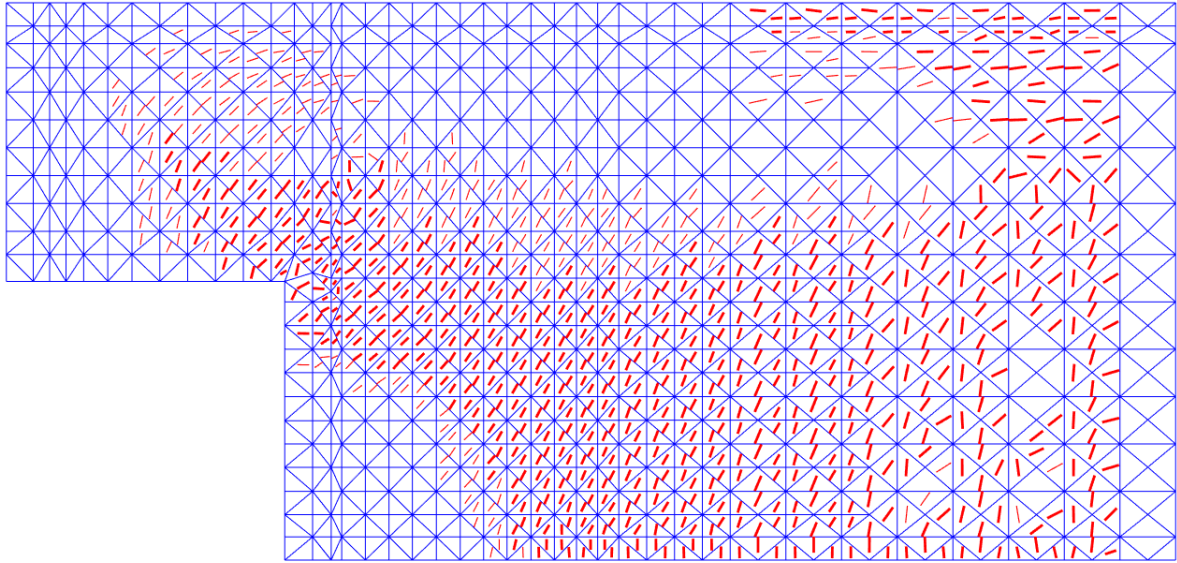


Figure 7.21: B35-05 cracking pattern before failure ; $C_s = 0.2\%$

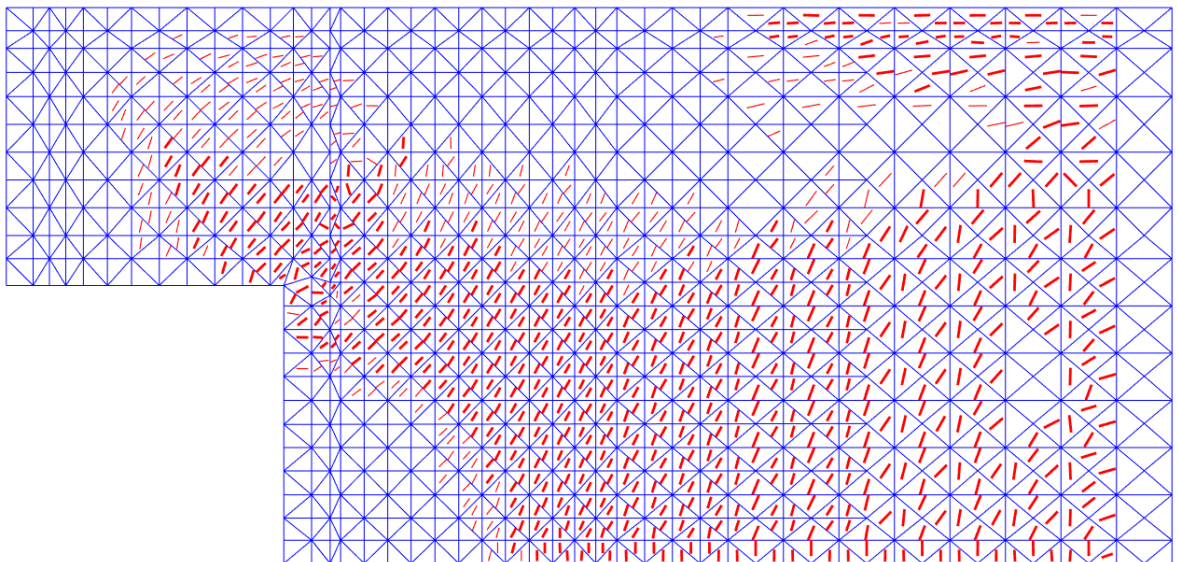


Figure 7.22: B35-05 cracking pattern before failure ; $C_s = 1\%$

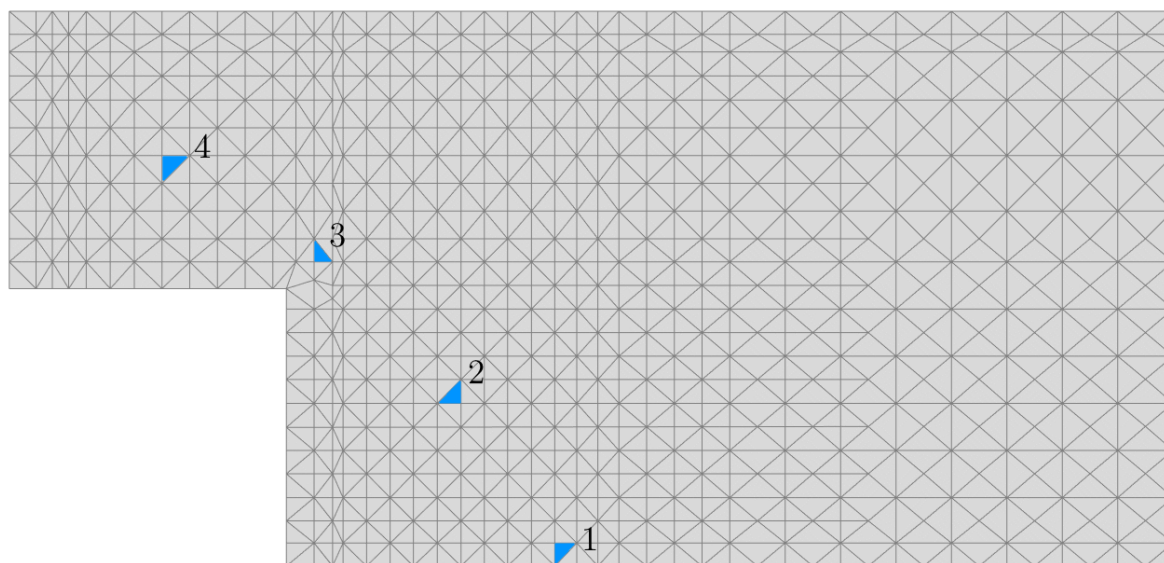


Figure 7.23: Elements selected to compare the crack width

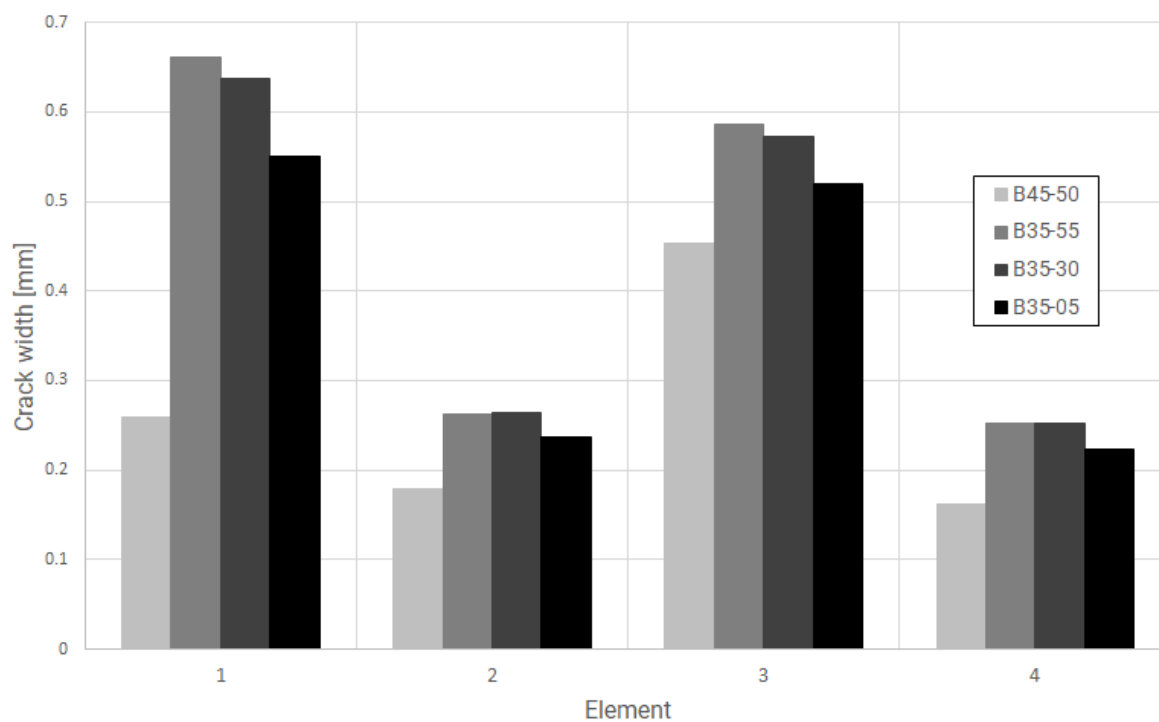


Figure 7.24: Comparison of the crack opening before failure for several elements of the mesh ; $C_s = 0.2\%$

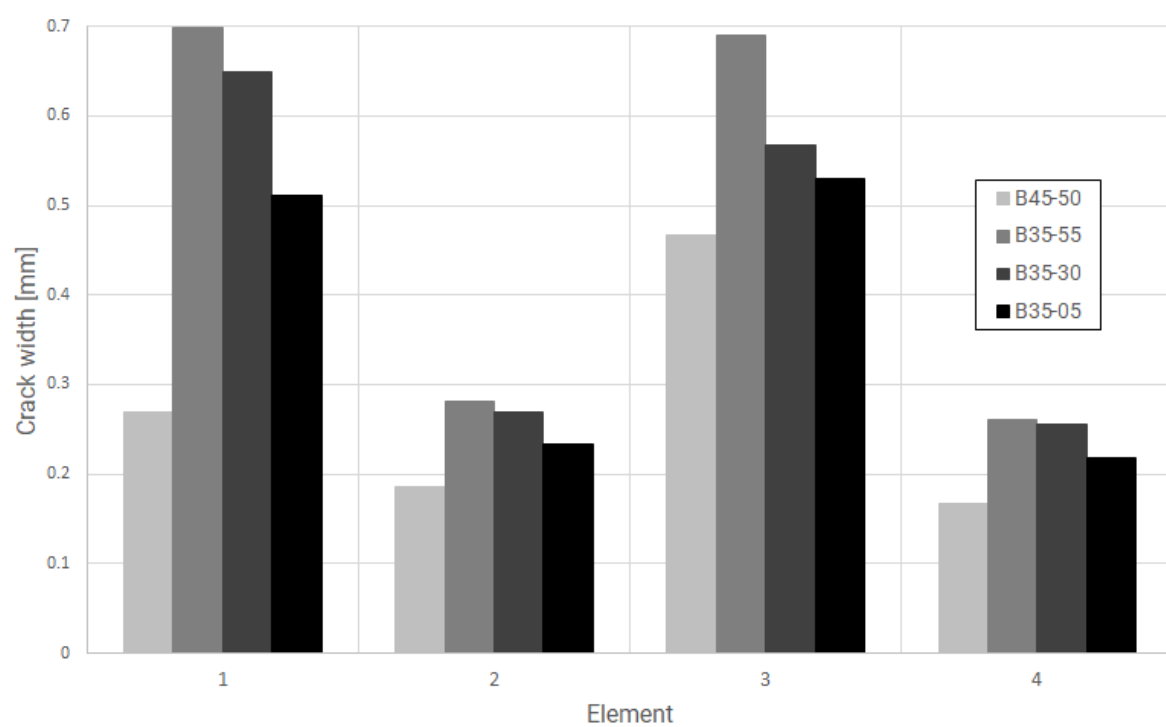


Figure 7.25: Comparison of the crack opening before failure for several elements of the mesh ; $C_s = 1\%$

7.4 Comparison between the two models

Tables 7.2 and 7.3 compare the results of the degradation analysis for both methods, reminding also the initial load and the final load at $t = 80$ years. It should be kept in mind that the FDM uses a fixed crack opening of 0.3 mm.

The difference between initial loads obtained through the ST method and MCFT-based analysis range from 4.6% (B35-55) to 20.7% (B35-05).

For visibility, the difference in reduction over time is plotted on figures 7.26 to 7.29 (note : on the legend, STM refers to the FDM and MCFT to the SDM).

Table 7.11: Comparison of the ultimate load reduction for the FDM and SDM ; $C_s = 0.2\%$

Year	B45-50		B35-55		B35-30		B35-05	
	FDM	SDM	FDM	SDM	FDM	SDM	FDM	SDM
<i>Initial [kN]</i>	1045	984	1000	954	1072	954	1150	912
20	6%	1%	2%	9%	11%	12%	12%	17%
40	17%	7%	14%	13%	22%	22%	23%	26%
60	29%	18%	25%	20%	33%	30%	34%	34%
80	39%	27%	36%	28%	43%	38%	44%	40%
<i>Final [kN]</i>	636	720	641	708	608	612	647	588

The largest difference between predicted loads at 80 years happens for B45-50, with the SDM giving a 11.7% higher final load than the FDM. An inverse tendency is observed for the B35-05, with a FDM final load 9.1% higher than for the SDM.

Table 7.12: Comparison of the ultimate load reduction for the FDM and SDM ; $C_s = 1\%$

Year	B45-50		B35-55		B35-30		B35-05	
	FDM	SDM	FDM	SDM	FDM	SDM	FDM	SDM
<i>Initial [kN]</i>	1045	984	1000	954	1072	954	1150	912
20	6%	23%	2%	26%	19%	28%	20%	29%
40	28%	39%	21%	38%	43%	42%	44%	42%
60	51%	47%	44%	51%	63%	51%	65%	51%
80	70%	54%	64%	59%	79%	58%	82%	58%
<i>Final [kN]</i>	316	450	357	408	220	414	198	414

This time, the largest difference between predicted loads at 80 years happens for B35-05, with the SDM giving a 52.2% higher final load than the FDM. The SDM predicts higher residual loads for every B-element.

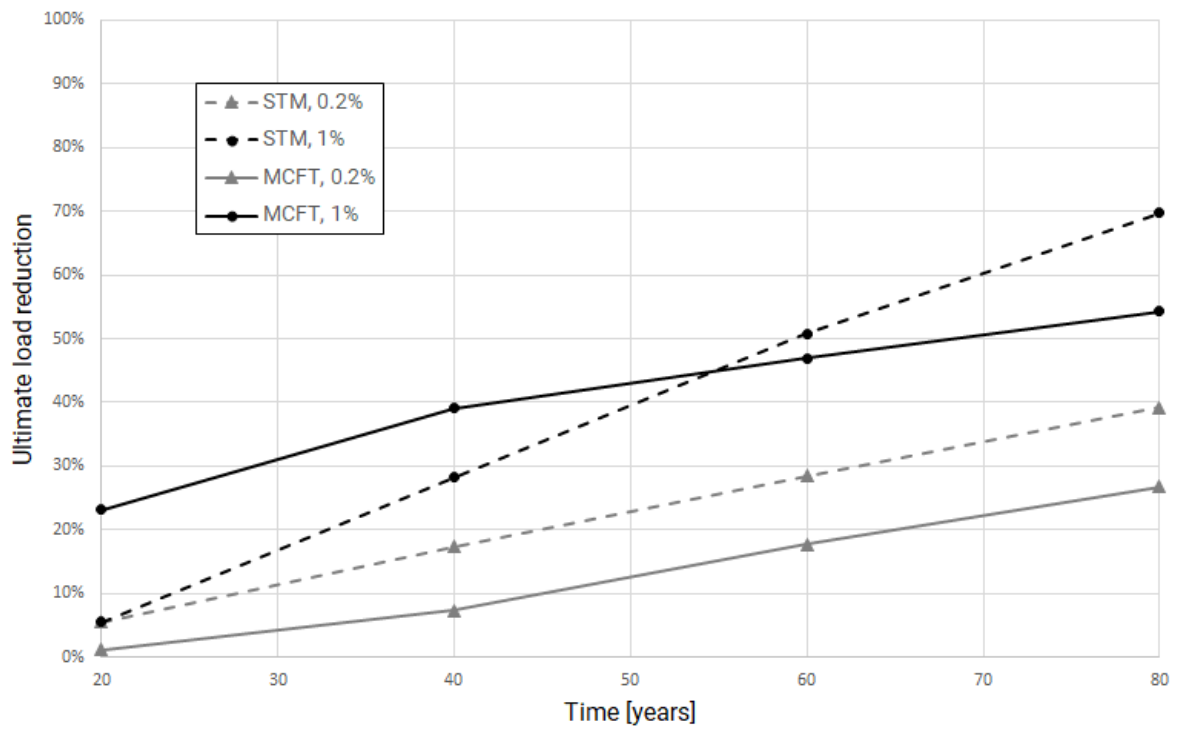


Figure 7.26: B45-50 : comparison of the reduction in ultimate load for both methods ; $C_s = 0.2$ and 1%

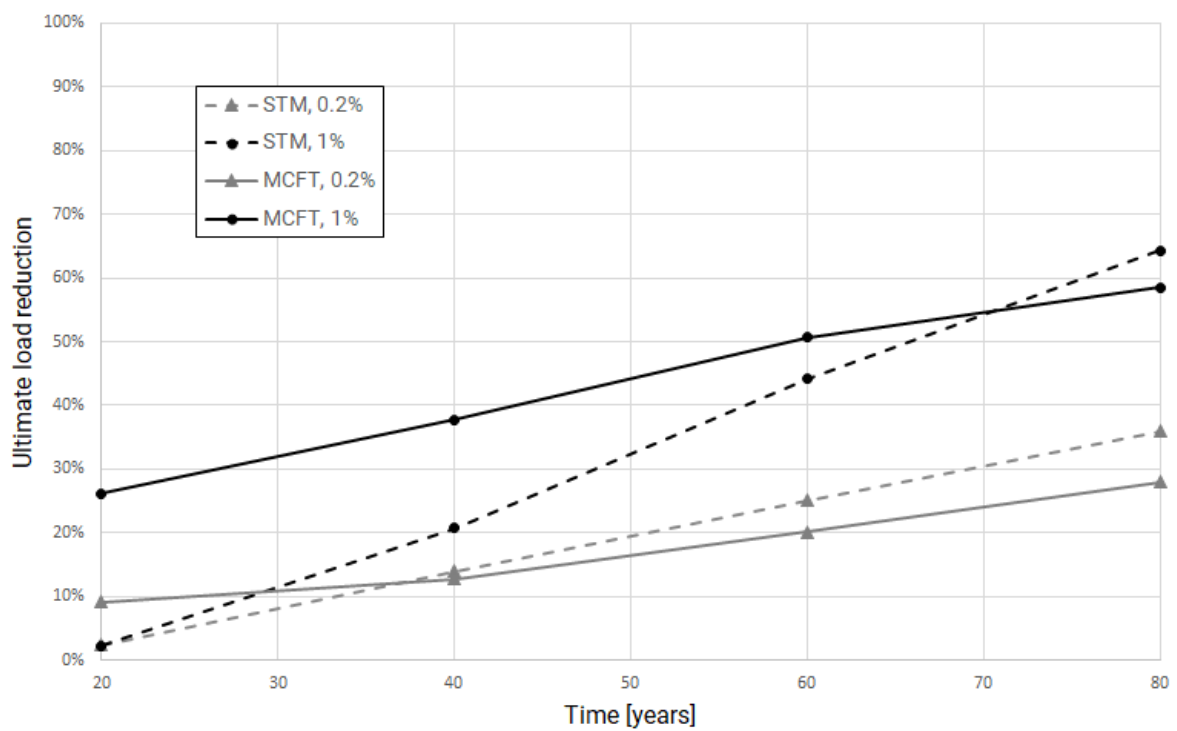


Figure 7.27: B35-55 : comparison of the reduction in ultimate load for both methods ; $C_s = 0.2$ and 1%

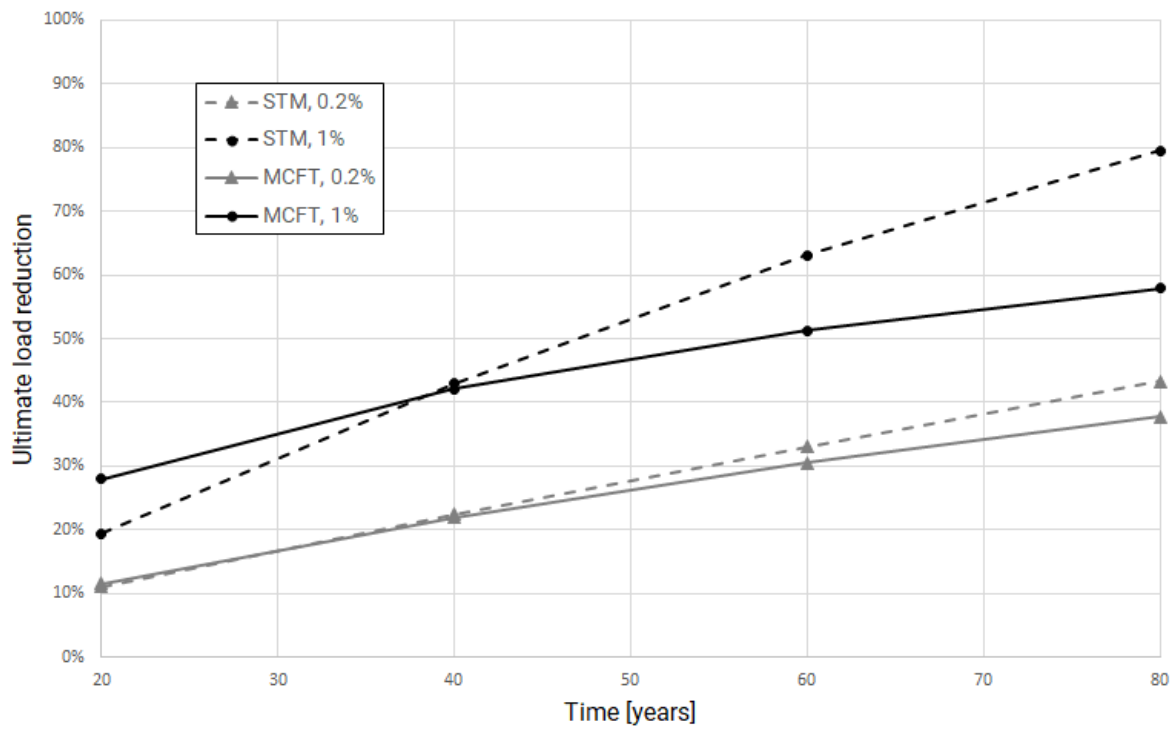


Figure 7.28: B35-30 : comparison of the reduction in ultimate load for both methods ; $C_s = 0.2$ and 1%

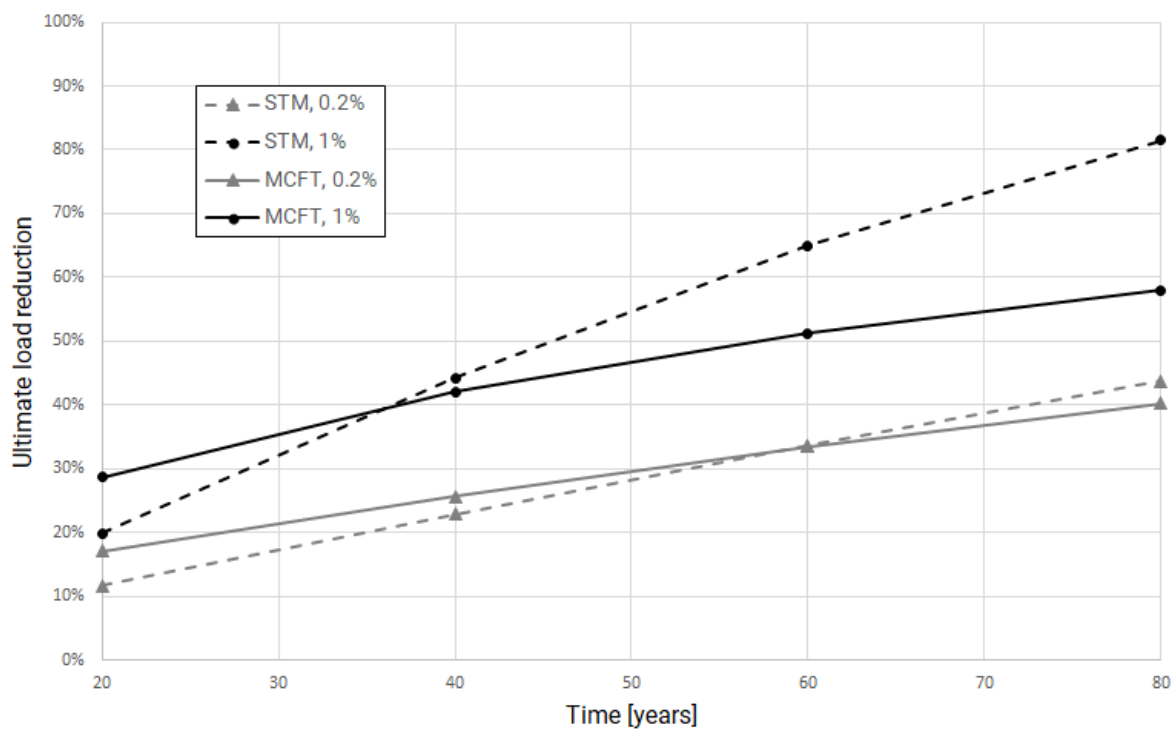


Figure 7.29: B35-05 : comparison of the reduction in ultimate load for both methods ; $C_s = 0.2$ and 1%

8 | Analysis and comparison of results

8.1 Limitations

This analysis is subjected to many limitations which only allow for general conclusions and hypotheses.

- The diffusion coefficients D_{app} and D_{cr} for the penetration of chlorides inside the concrete matrix heavily depend its properties, especially the water/cement ratio. Each structure has a unique diffusion coefficient, which makes it difficult to pick a representative value for the wide range of structures that have been and will be built.

Dhir et al. [56] determined that the strength, cement content and w/c ratio cannot indicate on their own the durability of the concrete against chloride penetration. According to them, D should take into account the material, production and curing of the concrete, and thus be determined from the on-site mix itself. They advise using a diffusion test, as they found penetration tests to be slow and not especially reliable.

In addition, the model used to estimate the penetration of chloride ions into the concrete matrix only accounts for diffusion. Besides, using Fick's Second Law to describe the rate of penetration involves taking a constant diffusion coefficient, which was shown to not accurately describe the real process, as D is also strongly dependent on the period of exposure [57].

- The chloride surface content was taken as uniform over the surface and in time. This is not realistic, especially for $C_s = 1\%$, as water cannot stagnate on vertical or bottom surfaces. Moreover, the use of de-icing salts is confined to winter months, leading to a concentration peak confined to that period.
- The carbonation coefficient is also influenced by many parameters (section 2.2.3.2). In the manner of chloride ions, the amount of carbon dioxide present in the environment is not constant. A fixed coefficient was used during the analysis, choosing a value suggesting a concrete of medium compaction to represent the congestion that the high amount of reinforcement creates at the extremity of the beam.
- Perfect adhesion between the steel reinforcement and the concrete matrix was assumed during the whole period of analysis. However, the deterioration of the bond strength seems to be the major cause leading to the reduction of the load-carrying capacity of the structure [58]. If half of the bar lose its bond with the concrete matrix, it was established that it does not participate anymore in the tensile resistance of the element [12].
- The rate of corrosion is also considered constant in time and space, generating a uniform steel loss along the reinforcement, thus not taking into consideration the possibility of the presence of localised pits of corrosion reducing the diameter of the rebars more rapidly.

- Only the extremity of the beam was introduced into the MCFT-based computer program, which created a cantilever effect that led to flexure cracking where the supports simulating the rest of the beam were placed (bottom right on the cracking patterns). A way to avoid this would be to model the complete beam, with the support placed at the nib and the load applied on the span of the beam, as is usually done during experimentation [7, 31].
- The analysis is purely simulated and is not based or verified on elements used in real-life experimentation. However, the accuracy of the non-linear finite-element analysis was verified by MSG&I through comparison with experiments conducted by the Laboratoire Matériaux et Durabilité des Constructions in Toulouse (France) [4, 5].

8.2 Results

For a lifetime loading of 60% of the ultimate capacity and assuming that the structure is not maintained during its working life,

1. The strut-and-tie method used provides higher initial load-carrying capacities than obtained with the Modified Compression Field Theory (tables 7.1 and 7.4). However, the degradation advances faster in the case of the FDM and the final load at 80 years is most often lower than for the SDM (tables 7.11 and 7.12). The analysis with the FDM thus becomes more conservative as time goes on and the chloride surface concentration increases. The only exception is for B35-05 with $C_s = 0.2\%$.
2. For the FDM at $C_s = 0.2\%$ (figure 7.6), all B-elements have a similar ultimate load through time. However, for $C_s = 1\%$ (figure 7.7), B45-50 and B35-55, which possess the largest cover depths, have a greater residual strength than the thinner covers.
3. In the case of the SDM, for $C_s = 0.2\%$ (figure 7.11), there is a marked difference between the B-elements throughout the years, with again the thicker covers retaining the highest final load-carrying capacities. For $C_s = 1\%$ (figure 7.12), the reduction follows a similar slope for every element. Their final load is almost identical apart from B45-50, for which it is slightly higher. This suggests that the cover thickness does not play much of a protective role for such a high chloride surface content.
4. B35-30 and B35-05 have very similar degradation rates (figures 7.28 and 7.29), hinting at the fact that a cover depth below a certain value does not offer significant protection anymore. It is reminded that their thicknesses are both well under the minimal requirements of the Eurocode of section 2.3.
5. The bottom edge of the full-depth section of the beams is heavily cracked for both surface concentrations, with only B45-50 respecting the width requirement for durability (0.3 mm, see figure 2.8). At the re-entrant corner, this higher concrete grade does not prevent the width from exceeding that threshold. From observing figures 7.24 and 7.25, increasing the concrete cover seems to have an adverse effect on the crack width. The elements which are situated deeper into the element (2 and 4 on figure 7.23) have comparable values for both surface concentrations, hinting at the fact that they are only due to the loading and are not influenced by the degradation.
6. The inclined concrete strut inside the nib is clearly visible on the stress flow pattern of figure 7.14. This compression trajectory becomes parallel to the top of the beam when leaving the nib, in accordance with the previously described spalling failure on top of the hanger reinforcement.

7. The cracking pattern of B45-50 (figures 7.15 and 7.16) is less pronounced than the other B-elements in the diagonal strip between the re-entrant corner and bottom of the full-depth beam. This strip has a thinner band of wide cracks (represented by thicker lines on the figures) in the case of B35-05 (figures 7.21 and 7.22), compared to the other B35 elements. The horizontal cracks close to the top of the nib do not reach the finite-element band representing the cover for the B45-50. For the B35 nibs, their presence at this location suggest spalling of the cover at failure.
8. If failure is deemed to happen as soon as the load reduction reaches 40%, all elements reach their ultimate state before 50 years for $C_s = 1\%$. For $C_s = 0.2\%$, B45-50 and B35-55, possessing the highest covers, attain roughly 40% of reduction at 80 years, while the others fail sooner. The results were not extended to 100 years due to each element collapsing before that time was reached.
9. Comparing the localised reinforcement loss of B45-50 and B35-05 (tables 7.7 and 7.8), while the whole reinforcement immediately starts degrading for B35-05, only the one placed in high cracking zones does for the B45-50.
10. In the case of the smeared reinforcement (tables 7.9 and 7.10), B45-05 is unaffected for the first 40 years, while B35-05 sees its ratio almost reduced by a third after 20 years.

8.3 Comparison with previous studies

Rasheeduzzafar et al., 1992 [59] investigated the influence of the bar diameter and the concrete cover and quality on the rate of corrosion. They concluded that the quality (w/c ratio and cement content) of the concrete and the cover depth play an important role for durability ; concrete mixes with a high cement content and low w/c ratio provide better resistance against corrosion, and an increment of cover thickness becomes more protective as the quality increases.

As for the bar diameter, bigger sizes of reinforcement were found to greatly increase the rate of cracking due to the greater expansive forces that their corrosion products create.

They determined that using the ratio of concrete cover on rebar diameter gave a better indication for the cracking time than either of them separated, with c/d over 2.5 giving a significant improvement. In this paper, the c/d are, using the largest diameter of 20 mm : 2.5 (B45-50), 2.75 (B35-55), 1.5 (B35-30) and 0.25 (B35-05).

Bertolini, 2008 [60] indicates that increasing the cover depth lengthens the initiation time for corrosion, but that thicknesses over 60-70 mm can favour cracking. This puts them at risk of spalling more quickly, uncovering the reinforcement early and thus negating the effect of using a thicker cover. The importance of controlling the variability of the cover depth during construction is reminded in the paper.

Yu et al., 2014 [61] concluded from their experiments on the relationship between the resistivity of the concrete and the rate of corrosion that, although the initiation time can be prolonged by raising the cover depth, this can also reduce the resistivity by providing more passages for the aggressive substances. However, they found that, under a critical value for the relative humidity, the corrosion rate does decrease as the cover gets thicker.

Borosnyói and Snóbli, 2010 [62] demonstrated that the surface crack width is dependent on the concrete cover depth, following a power function, but that this dependence disappears

at the level of the reinforcement. For the type of concrete and reinforcement they experimented on, the surface crack width doubled when going from a cover of 40 to 60 mm, also going over the width durability requirement of a maximum width of 0.3 mm.

8.4 Majors takeaways

Keeping in mind the limitations presented at the beginning of the chapter, the results of the analysis and the comparisons with previous research can lead to several conclusions.

The protective nature of a proper cover thickness against the corrosion of the steel reinforcement is demonstrated once again. The comparison between the crack opening widths of the elements also evokes the negative influence of using a thicker cover on the cracking.

Obviously, increasing the content of aggressive substances, may it be carbon dioxide or chloride ions, raises the rate of their penetration and shortens the corrosion initiation time.

The FDM and SDM provided quite similar reductions of the load-carrying capacity in time for a moderately aggressive environment and concrete strength of C35/45. In this case, the FDM would seem adequate as a first estimation of the degradation of the nib, for a cracking pattern of 0.3 mm wide cracks spaced by 100 mm.

For $C_s = 1\%$, both methods have different slopes and up to 24% of difference between results at 80 years. Therefore, the FDM does not seem adequate for such a high chloride concentration, unless conservative results are desired.

The re-entrant corner is critical for the durability and load-carrying capacity of the nib of the dapped-end beam. In the case of the lower nib, water can penetrate inside the major diagonal crack rapidly through gravity. If the water is charged with chloride ions, the degradation begins very quickly. Since the nib is the part of the beam subjected to the highest shear stress, due to being at the support, it is crucial for the reinforcement to be able to sustain the loading applied on the structure for the entirety of the chosen working life.

However, during this numerical experimentation, none of the elements lasted for the design period of 100 years under a moderately aggressive environment and considering a critical chloride content of $C_{crit} = 0.5\%$. They thus do not fulfil the durability requirement that the sum of the periods of initiation and propagation must be longer than the design working life.

9 | Conclusions

Many studies have been conducted in the past decades to determine the impact of corrosion on public infrastructures such as bridges, due to the deterioration that has been witnessed since their construction, sometimes very early in their working life. This degradation has caused collapses, not only leading to economic losses but also provoking human casualties.

It is difficult to evaluate the real extend of deterioration in a specific structure, as the rate of corrosion depends on a plethora of parameters, from the cement mix to the local weather, and is not uniform over the whole surface. The layout of certain elements also complicates the inspection process, by creating tight spaces or blocking the access to certain surfaces.

This is the case for dapped-end (half-joint) beams, whose height at the extremities has been shortened to create a support, called a nib, reducing the floor height and allowing the placement of an expansion joint. However, water can seep into this joint and stagnate on the lower nib if the drainage system and waterproofing are not adequate. During winter, when de-icing salts are spread on roads, this water becomes charged with chloride ions, which are the leading cause of corrosion in reinforced concrete structures.

This support is placed at the location of highest shear stress and thus tends to crack rather early at its re-entrant corner. This crack, which can grow quite wide, becomes a direct path for the ingress of aggressive substances ; predominantly chloride ions and carbon dioxide. As corrosion progresses, other cracks form and accelerate the process further. The most common type of failure due to this cracking is the detachment of the concrete cover on top of the interface between the nib and the full depth of the beam. Prestressing could be introduced inside the beam to close the cracks, but cables are more susceptible to pitting corrosion, which is more aggressive than the uniformly distributed corrosion that was assumed in the analysis.

The nibs are critical elements, due to their reduced cross-section area and position at a high stress zone. This change in geometry creates a disturbed region on which the classical beam theory cannot be applied, due to the non-linear distribution of strains inside the nib. One available method to calculate this type of region is the strut-and-tie method, that was used to determine the required reinforcement inside the nib to sustain the selected ultimate load. Many strut-and-tie models can be made ; the one that was chosen was introduced by Mata-Falcón et al. [7]. It is of utmost importance to select the general reinforcement layout inside the nib accordingly, as omitting certain elements can lead to high declines in the ultimate load [31], as the region relies heavily on the embedded steel and the way it transmits the stress to the full depth of the beam.

The strut-and-tie method can also be used to find the ultimate load that given reinforcement cross sections can withstand. Degradation was applied on the chosen diameters and the corroded area was introduced back into the strut-and-tie model to determine the residual load-carrying capacity of the element for a fixed cracking state. This process was named the First Degradation Model.

P.G. Malerba, L. Sgambi, G. Gotti and D. Ielmini developed a model to analyse a reinforced

concrete element subjected to corrosion, coupling mechanical behaviour and degradation. The Modified Compression Field Theory introduced by Vecchio and Collins [6], with some added improvements, was used to model the reinforced concrete material. Corrosion through carbonation and penetration of chloride ions was evaluated using the same equations as for the FDM, but now with an evolving cracking pattern. This analysis was termed the Second Degradation Model.

They had used this model to determine how localised corrosion influences the mode of failure of prestressed beams. This paper applied it on the nibs of dapped-end beams to study the impact of a uniform corrosion attack on the load-bearing capacity of the element.

The critical chloride threshold at the reinforcement is chosen based on the RILEM criterion of 0.5%. The influence of the cover depth and of using a higher concrete grade was investigated. The conclusions were in accordance with previous studies about the fact that an increase of cover depth lengthened the corrosion initiation time. However, the negative effect of higher cover thicknesses on cracking was also demonstrated. Keeping in mind the limitations of the model, it would thus seem more efficient to use a higher concrete strength than to raise the cover past a certain threshold.

The FDM and SDM gave similar reductions of the load-carrying capacity as degradation progressed in time for a moderately aggressive environment surrounding a beam made of concrete of medium compaction and strength of C35/45, and subjected to a lifetime permanent load of 60% of its ultimate capacity. The SDM gave lower residual ultimate loads at the end of the analysis for the beams that did not fulfil the cover thickness durability requirements of the Eurocode (section 2.3).

For a higher surface chloride concentration, or a higher concrete grade, the results of both analysis diverged, with the FDM giving more conservative results at the end of the period.

Extra considerations could be taken into account for a more in-depth study of the degradation of the load-carrying capacity of reinforced concrete elements. This analysis for example did not take into account the loss of bond between the steel and concrete induced by corrosion, which greatly weakens the element [63].

An interesting question to bring up is how corrosion affects the resistance of infrastructure against natural disasters such as earthquakes. The loss of bond strength proved to have little influence on the seismic behaviour if the structural elements are properly confined [64, 65]. However, a great deal of today's infrastructure was built decades ago and thus does not follow current quality standards. The opposite question is also relevant : how do earthquakes impact the rate of corrosion of a reinforced concrete element ? The experiments of Cui et al. [66] on columns have for example found their corrosion rate magnified after applying the first earthquake event.

Traffic loads have also changed throughout the years, due to the increasingly high presence of trucks on the roads, which means that the design capacity may be exceeded for older infrastructure. For example, if 90% of the load capacity of the dapped-end beams studied was applied on them, failure would occur before 40 years for the elements possessing the highest cover thicknesses and sooner than 20 years for the ones with insufficient depths.

A major aspect of corrosion damage is the detachment of the concrete cover, found to be mostly influenced by the concrete grade and the cover thickness [67]. The loss of section being closely related to the stiffness of the element and thus the ductility, its evaluation is needed to truly determine the deformation of the corroded element [63]. Also, this variation in cross-section can lead to stress redistribution if the element does not degrade uniformly. To properly evaluate the damage, this aspect should also be introduced during degradation analyses.

Due to the innate tendency of the concrete to crack, using corrosion inhibitors in the concrete mix, a proper cover thickness or reducing the permeability will not prevent the onset of corrosion. Those precautions can increase the initiation time, but the reinforcement itself has to be protected to ensure long-lasting durability. This can be done physically by using organic or metallic coatings, or by using corrosion-resistant alloys. However, the coating may also degrade due to environmental actions, especially on the coast. The best line of action against degradation is thus to use protection both on the concrete and the reinforcement.

For existing structures, impressed-current cathodic protection is a permanent solution to corrosion, but requires maintenance. Another method to rehabilitate a bridge is to execute an electrochemical chloride extraction, which does not involve a permanent wiring system but does not prevent the salts from penetrating again [60, 68].

Polder et al. [69] investigated the costs of full surface corrosion surveys versus those of damage detection through visual inspection. They concluded that the former is less costly in the long term. Visual detection of corrosion is only possible once the damage is already in an advanced state and thus requires heavy repairs. Indeed, during the first part of the corrosion process, the naked eye cannot detect the ingress of aggressive substances. Bertolini [60] suggests defining inspection procedures in the design phase, adopting a monitoring system or programming maintenance in advance for critical parts of the structure.

In conclusion, as stated by many researchers before, corrosion has to be taken into account from the design phase, by selecting the proper concrete mix, cover depth, reinforcement, etc. and by integrating a maintenance planning for the new structures.

Due to the complex geometry of the joint created by the two nibs and the maintenance difficulties, it is of utmost importance to reduce the cracking potential of the element and prevent avoidable contact with aggressive substances through proper drainage and waterproofing of the deck. Since this joint is a sensitive region that is limited in size, a suggestion would be to invest in corrosion-resistant reinforcement for the detailing of the nib, especially close to the re-entrant corner.

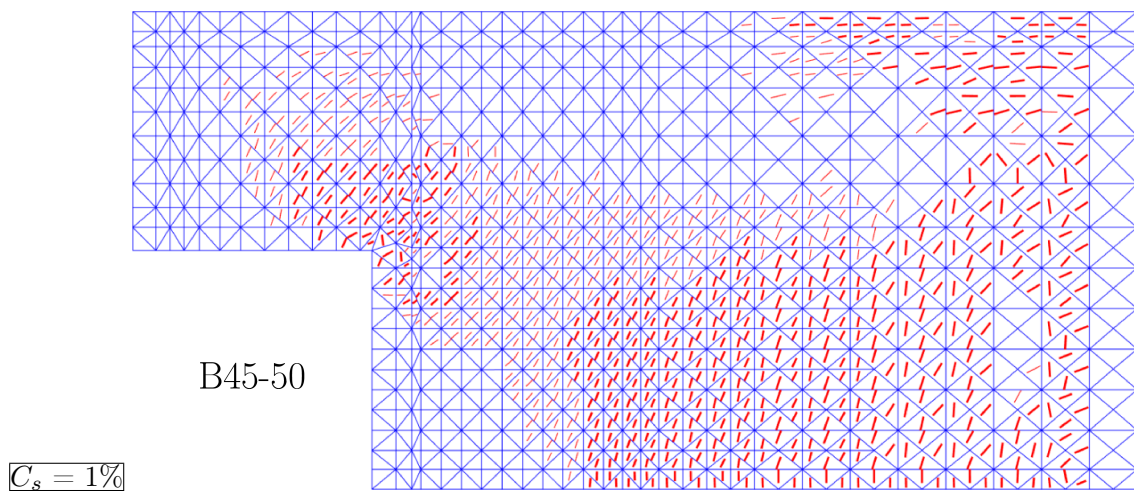
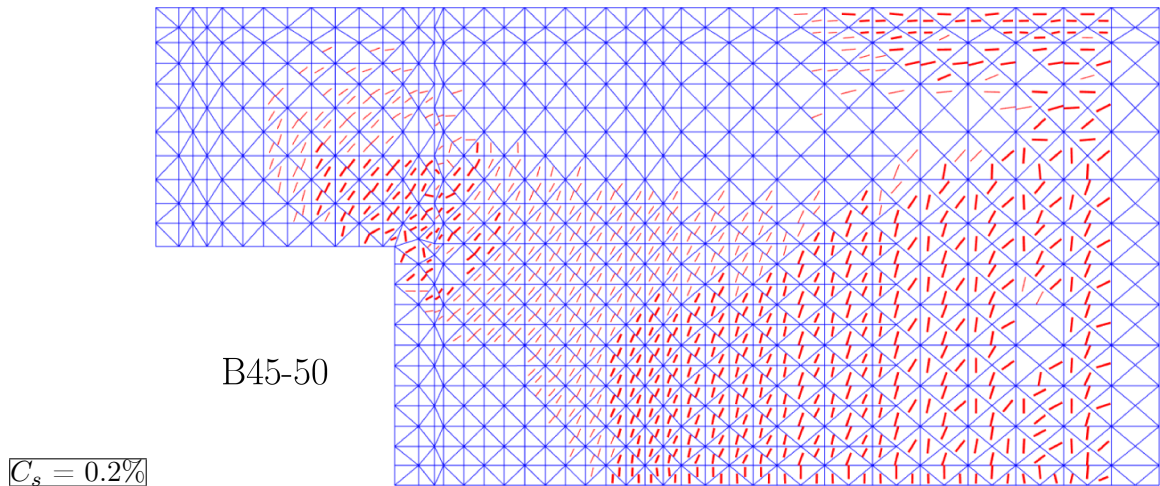
"It has been shown that prevention of degradation is a complex task that involves competences of structural engineers, materials experts, corrosion specialists, constructors, etc. Durability can only arise from the cooperation among these professional figures. This implies that nobody should think that durability is someone else's job, but everybody should concentrate on the final aim of providing all the reasonable features necessary to guarantee the design service life at the lowest cost."

Luca Bertolini [60]

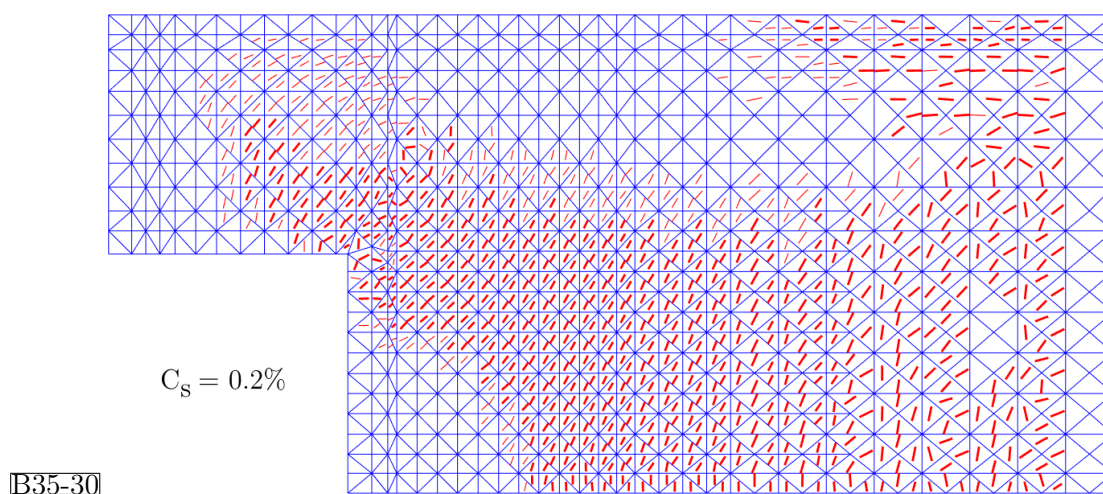
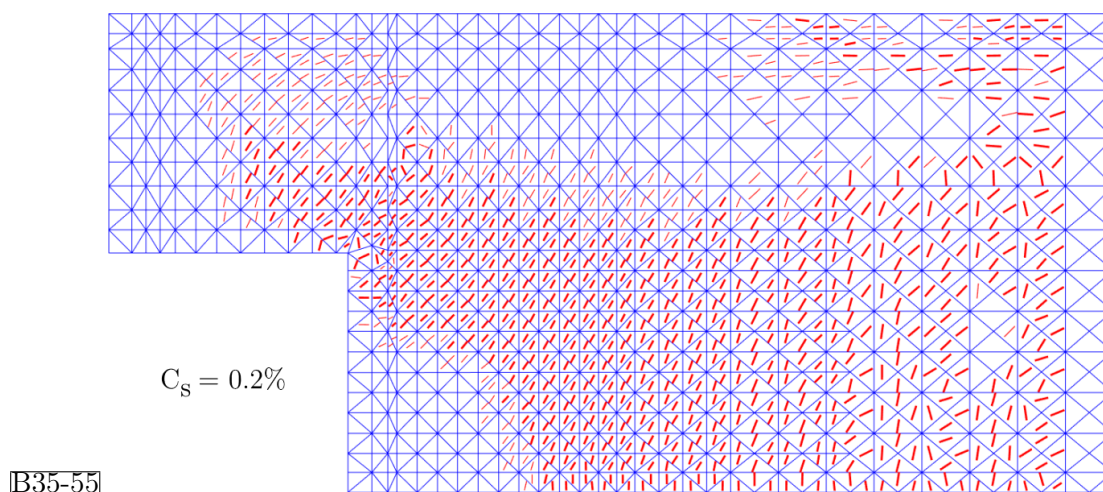
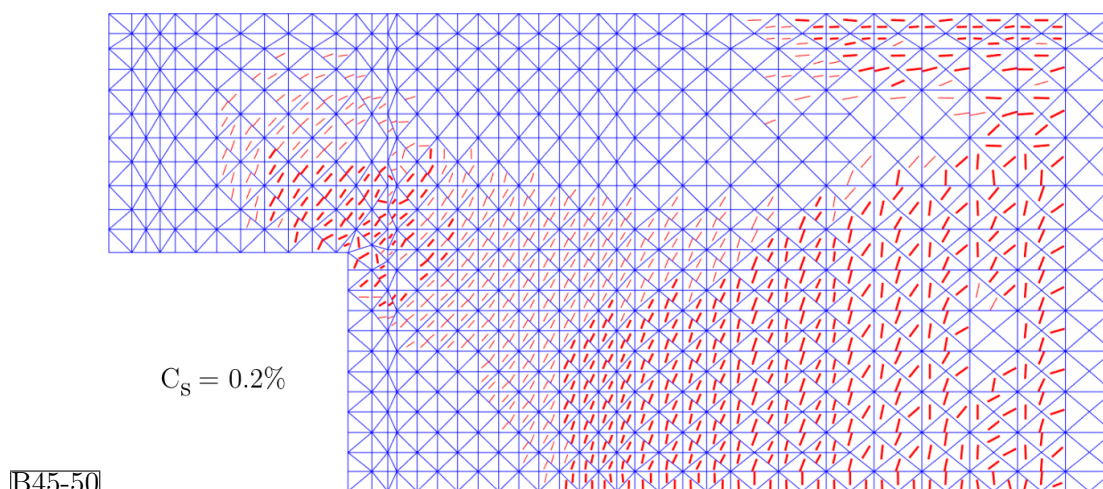
10 | Interactive figures

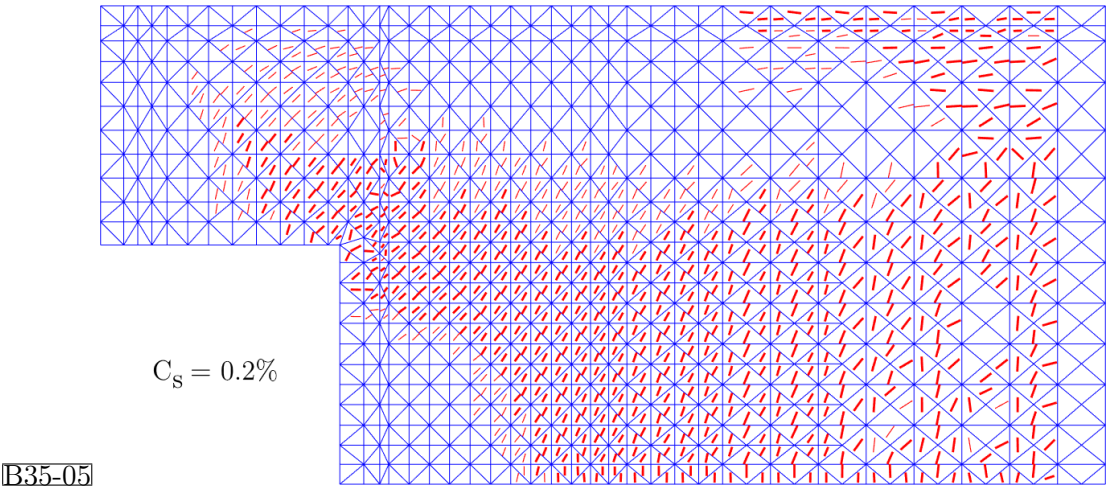
To switch between frames, simply click on the picture. This may not work on every PDF reader.

10.1 Comparison of cracking patterns

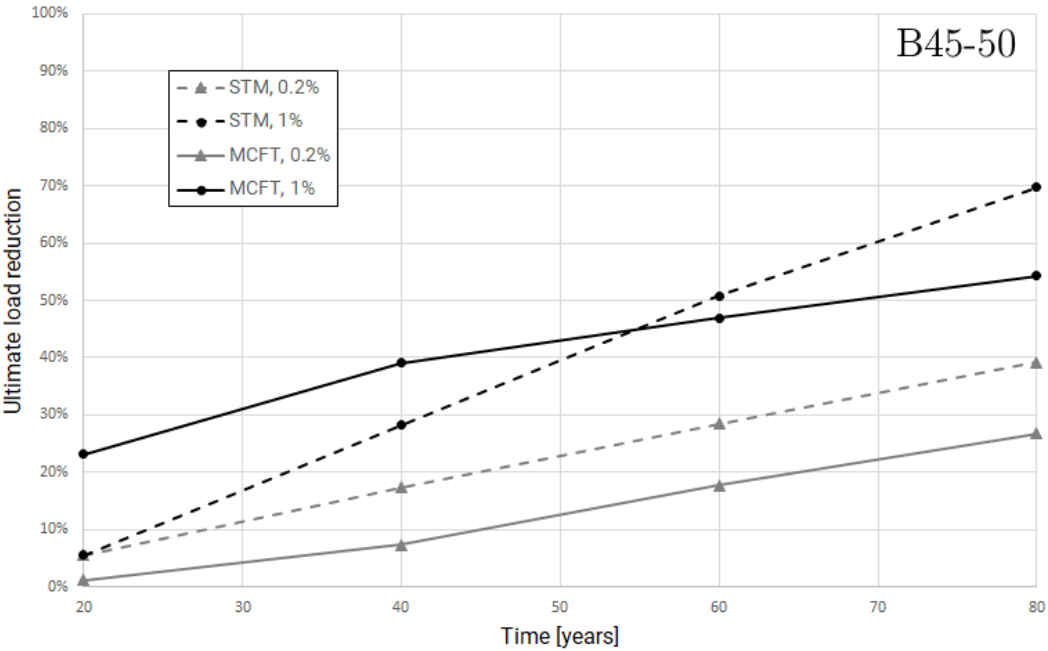


10.2 Cracking pattern for each individual element





10.3 Reduction rates



□

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