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COVID-19 U.S. Inflation:

A Fiscal Theory of the Price Level Perspective

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COVID-19 U.S. Inflation: A Fiscal Theory of the Price Level Perspective

By Sandy Vo

While the pandemic may have ended quite some time ago, the debate as to whether the staggering rise in inflation the U.S. experienced during that period was attributable to the slow reaction of the Federal Reserve or the generosity and frequency of the stimulus packages continues. In an attempt to settle this debate, I simulate a canonical New Keynesian model with short-term debt in the vein of the fiscal theory of the price level and try to match the IRFs to the data. Depending on your political leaning, the results may or may not surprise you: both authorities are responsible for the rise in inflation during the COVID period in the U.S; consumer preference shocks appear to have the most impact on inflation, followed by general government spending shocks. Cost-push shocks adequately matches the overall inflation trend but fails in tracking output movement.

I. Introduction

Giovanni et al. (2022) measured the impact of the COVID-19 pandemic on the Eurozone and other countries' inflation (e.g. the U.S.), and tried to identify the most relevant channels through which inflation was propagated. Amongst their many findings, they found that while supply chain shocks were more significant drivers of inflation in the Eurozone, aggregate demand shocks were a comparatively larger factor in explaining inflation in the U.S. during 2019Q4 - 2021Q4.

However, cost-push and aggregate demand shocks are obviously not the only drivers of inflation during this period of time in the U.S. Many lay blame not only on the slow reaction of the Feds but also on the the liberal spending carried out by the government throughout the pandemic (Hubbard, 2022; Tankersley, 2023; Cutsinger, 2022). Without claiming to be particularly affiliated with either of the two major American political parties, I would like to test if this claim can be substantiated using the fiscal theory of the price level (FTPL) as John Cochrane purportedly suggested that it does in a FT op-ed (Hubbard, 2022).

The FTPL is a theory which asserts that the price level is determined by the interaction between fiscal policy and monetary policy; not merely by monetary policy alone. While the initial conception of the theory could be traced back to Neil Wallace in his 1981 paper on the application of the Modigliani-Miller theorem for open-market operations (Wallace, 1981), it was Leeper (1991) who further refined its theoretical foundations and introduced the notion of "active" and "passive" fiscal/monetary policies in a dynamic general equilibrium framework. Due to its parsimonious yet insightful framework, his work has since become the foundation upon which many macroeconomists have built their analyses and research.

Following Leeper (1991), a policy (either monetary or fiscal) is "active" when it is unconstrained by the burden of having to balance the government's budget. When the opposite is true, the policy is "passive." The only regimes with unique equilibria are the ones wherein monetary and fiscal policy complement each other—i.e. when one is "active," the other is "passive." The passive monetary/passive fiscal policy (PM/PF) regime yields multiple equilibria, which results in the indeterminacy of the price-level. The active monetary/active fiscal (AM/AF) regime yields no equilibrium. To develop the intuition behind the theory, we will be using a basic New Keynesian model in Section III to demonstrate the core dynamics. Then, we will simulate the model using MATLAB Dynare and try to match the impulse responses to FRED COVID-19 data.

The research is organized as follows: Section I is the introduction; Section II recaps and highlights certain U.S. COVID monetary and fiscal policies to give context to our current analysis; Section III briefly goes over the relevant literature to the topic; Section IV outlines the basic model and explains the intuition behind the theory; Section V presents the simulation procedures, results, and discusses them; Section VI concludes.

II. U.S. Fiscal and Monetary Policy during COVID-19

Over the COVID period, the U.S. government expended considerable effort and resources to ensure employment and output stability through various stimulus and relief packages all the while not adjusting its taxes to reflect its ballooning budget deficit. Concurrently, around the beginning of the pandemic in the U.S., the Feds lowered its fed funds target rate (FFTR) and carried out open-market operations until interest rates hit 0.25% then continued its open-market operations to keep interest rates hovering over the zero lower bound. Forward guidance that hoped to keep inflation expectations around the 2% level was introduced on March 15th (Clarida, Duygan-Bump and Scott, 2021). Clarida, Duygan-Bump and Scott (2021) provides a wonderfully detailed timeline of The Federal Reserve's COVID-19 response.

The first laboratory confirmed COVID-19 case was in January 20, 2020 (CDC, n.d.). In terms of fiscal policies, the first COVID relief package was signed on March 6, 2020 by President Trump and allocated $8.3B to vaccine research and national and international spread prevention efforts. Officially, it is called "The Coronavirus Preparedness and Response Supplemental Appropriations Act." However, for clarity purposes, we will refer to this as Package 1 (U.S. Congress, 2020*c*).

Not long after on March 18th, Package 2, "the Families First Coronavirus Response Act (FFCRA)," was signed into law. It was effective from April 1 to December 31, 2020, and provided transfers to families that relied on free school lunches, mandated paid sick leave for COVID-stricken workers, funded states' unemployment insurance coverage, free coronavirus testing, and the increased federal Medicaid funding (U.S. Congress, 2020*d*). Although the estimation of the cost of this program is difficult due to its flexible, need-based nature, the Congressional Budget Office and the staff of the Joint Committee on Taxation (JCT) concluded that the "increase federal deficits by $192 billion over the 2020-2030 period, mostly in fiscal years 2020 and 2021" (The Congressional Budget Office, The staff of the Joint Committee on Taxation, 2020).

Package 3, the CARES Act, was signed into Law on March 27, 2020. The package cost a whopping $2.3 trillion and is the largest single relief package in U.S. history (U.S. Congress, 2020*e*). In general, the package was aimed at SMEs, schools and universities, and individuals. The most notable expenditure in the package is the direct cash payment of $1,200 per person, plus $500 per child in addition to the extra $600 of unemployment per week until July 31, 2020. In addition, the government made out roughly $849B in loans to companies, either through their Paycheck Protection Program (PPP) or otherwise (U.S. Congress, 2020*b*).

Package 4, "The Consolidated Appropriations Act of 2021," was signed on December 27th, 2020, and comprised of $900B of direct transfers of $600 per person to people making less than $75,000 a year, including dependents ages 16 and younger. Further details of this act is omitted because they are largely irrelevant for our purposes (U.S. Congress, 2020*a*).

Package 5, "the American Rescue Plan Act of 2021," was signed on March 11, 2021 by President Biden and cost $1.9 trillion. It also comprised of direct cash transfers of up to $1,400 for individuals earning less than $75,000 a year in addition to $1,400 per dependent. For those with income over $75,000. the amount decreases gradually with a cut-off at $100,000 (U.S. Congress, 2021).

Overall, in addition to the general expenditures, there were multiple efforts to alleviate the debt burden for businesses and individuals. Relief measures included, for example, foreclosure and eviction moratoriums for single-family homeowners whose mortgages were FHA-insured (Federal Housing Administration) or backed by Fannie Mae or Freddie Mac (Federal Housing Finance Agency, 2020). Criteria for debt forgiveness, debt repayment deferment periods, debt maturity dates, payroll and individual tax due dates, etc. were also further relaxed and extended in a variety of ways (U.S. Congress, 2020*f*; U.S. Internal Revenue Services, 2020; Alpert, 2023).

The timing of the stimulus packages are presented in Figure 1 along with quarterly inflation and the effective FFR. A separate Figure 2 for GDP is included for the purposes of selecting the model that best describes the overall dynamics of these variables in the U.S. economy during the pandemic. The timing of the stimulus and relief packages are the dashed magenta vertical lines. Due to the close timing of Package 1, 2, and 3, they are all represented by the farmost left vertical line in the graph. This is convenient for my purposes because I am interested in seeing what happens when shocks to the private sector are compensated by the government through transfers and the three vertical lines in Figure 1 represent the timing of the direct COVID stimulus cash payments the U.S. government made to households. This will help me match the impulse response functions for the shocks in the simulated model to the data later in the paper.

III. Literature Review

Extremely unsurprisingly, the usage of the FTPL to explain U.S. inflation data has been done before. Bianchi and Ilut (2017) estimated a model that accommodated monetary/fiscal policy mix changes in addition to agents' beliefs with respect to what regime will persist in the future and applied it to U.S. data during the '60s, '70s, and '80s to explain why high inflation was present during the '60s and the '70s, and the disinflationary attempts during the late '70s into the very early '80s were unsuccessful.

They concluded that, with respect to the issue of persistently high inflation, one of the reasons was due to the fact that during the '60s and most of the '70s, the dominant regime was that of PM/AF. The rationale goes: even if the government debt level was low (as they were in the '70s), for example, a series of expenditure shocks could (and did) result in significant inflationary pressure because fiscal policy was active and therefore, unresponsive to the government's budgetary needs. Should the government elect not to raise taxes and commit to repaying their debts (as the U.S. government is), then the only thing they could do in order to continue borrowing is to inflate away debt.

Furthermore, the reason given for the failed disinflationary attempts (i.e. there was overall comovement between the real interest rates and inflation in the same upward direction) was that the both monetary and fiscal policies were active during that period in time. It was only when monetary policy was also backed by the fiscal authorities (AM/PF regime) that inflation started going down in the early '80s.

Given the fact that my periods of interest only go from 2019Q4 - 2021Q4, I will not be examining what happened before and after that time to see if there was any regime switching. However, this would be a feasible line of research for those interested.

That being said, there is no shortage of academic papers published by people more capable than yours truly with the goal of analyzing the ups and downs and all-arounds of macroeconomic variables during the COVID-19 period. For example, Harding, Lindé and Trabandt (2023) set out to explain the significant rise in US inflation rates during the Great Recession and post-COVID period, which they identified to be 2022Q1, by proposing a macroeconomic model with a nonlinear Phillips Curve and a quasi-kinked demand schedule that allowed for a larger transmission of shocks when inflation is high.

Regarding works on optimal policy, Chafwehé et al. (2021) utilizes the FTPL structure to work out optimal monetary policy rules under potentially constrained government budgets. For an deep dive into the FTPL and its economic applicability, Cochrane (2023) seems to offer thorough insight. For an accessible,

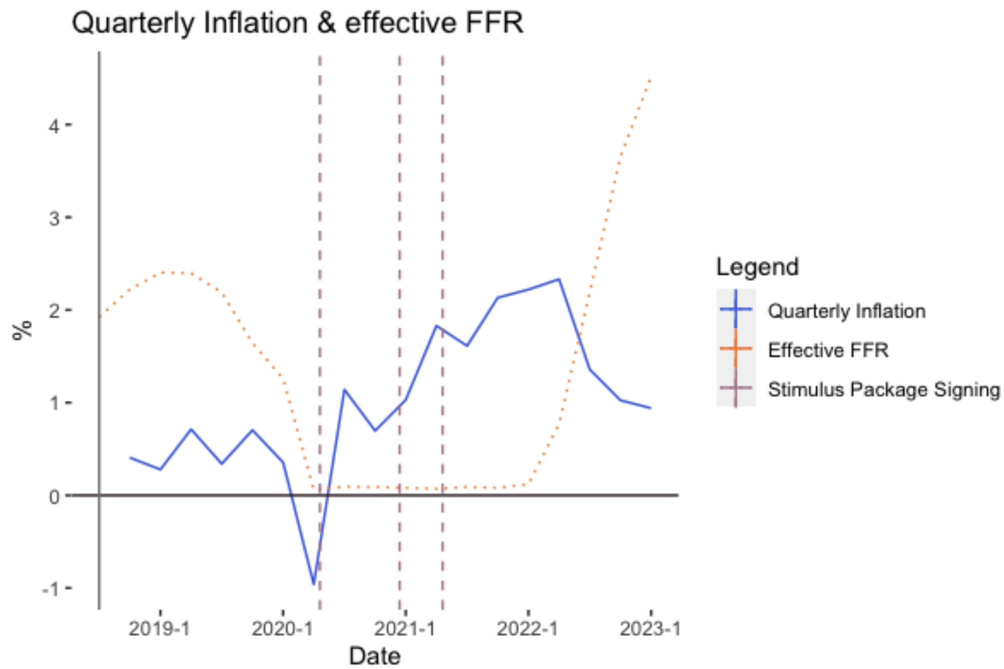


Figure 1. : Quarter-on-quarter inflation is calculated using the FRED CPI data; effective FFR data is also taken from FRED and quarter-ized

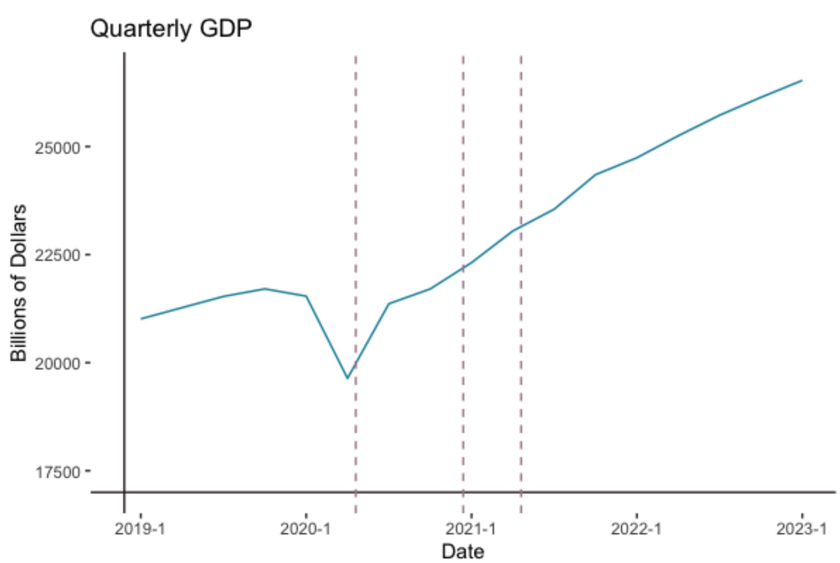


Figure 2. : Quarterly GDP is taken from FRED

non-technical introduction to the theory, Cochrane (2021) is extremely useful.

IV. The Model

To fix ideas I now use a simple Fisherian model to define active/passive monetary/fiscal policies. Though this model constraints the preferences of the representative household to be quasilinear in consumption, it is particularly tractable, the so called Euler equation of the model features an exogenous real rate, and this enables me to derive simple analytical formular.

The model is otherwise a standard New Keynesian model with $j \in [0,1]$ monopolistically competitive firms and prices that are subjected to adjustment costs following Rotemberg (1982).

A. *Firms*

The final good is a CES aggregate of the intermediate good $Y_t(j)$:

$$Y_t = \left(\int_0^1 Y_t(j)^{\left(\frac{1+\eta}{\eta}\right)} dj \right)^{\left(\frac{\eta}{1+\eta}\right)} \tag{1}$$

The firms' technology is linear in labor: $Y_t(j) = h_t(j)$. Each firm maximizes profits according to the following optimization problem:

$$\max_{P_t(j)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{W_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - A\, C_{t+s}(j) \right) \tag{2}$$

subject to:

$$Y_{t+s}(j) = \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{\eta} Y_{t+s} \tag{3}$$

$$A\, C_{t+s}(j) = \frac{\theta}{2} \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1 \right)^2 Y_{t+s} \tag{4}$$

where (2) is the demand schedule and (3) is the Rotemberg price adjustment cost with θ being the degree of the adjustment cost, $P_{t+s}(j)$ is the price of good j in period t+s, P_{t+s} is the aggregate price level, $Y_{t+s}(j)$ is the production of good j at time t+s, $W_{t+s}(j)$ is the wage rate for the sector (or firm) producing good j at time t+s, which is equal to the marginal cost of production.

The FOCs gives the following non-linear Phillips Curve:

$$\theta\,(\,\pi_t - 1)\,\pi_t = 1 + \eta\left(\,1 - \frac{W_t}{P_t}\right) + \beta\,\theta\,\mathbb{E}_t\frac{Y_{t+1}}{Y_t}\,(\,\pi_{t+1} - 1)\,\pi_{t+1} \tag{5}$$

B. Households

Households are representative and optimizes:

$$\max_{C_t, x_t} \qquad \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(C_t - \frac{(1-x_t)^{1+\gamma}}{1+\gamma}\right) \tag{6}$$

subject to

$$P_tC_t + P_t^1B_{t,1} \leq B_{t-1} + (1-\tau_t)W_t(1-x_t) + D_t$$

where C_t is consumption in period t, x_t is hours of leisure, D_t is firms' profits redistributed to households, P_t is the aggregate price level, B_t is a short-term one period bond that pays one unit of income at maturity, and $0 \leq \tau_t \leq 1$ is the distortionary tax rate on labour income.

For simplicity, in the analytical results of this section, I will only be considering short-term bonds. The model can be extended to long-term bonds as in Chafwehé et al. (2021), which adds additional nuance to the FTPL that we will discuss briefly at the end of this thesis.

FOCs w.r.t. C_t and x_t, respectively:

$$1 - x_t = (1-\tau_t)\frac{W_t}{P_t} \tag{7}$$

$$P_t^1 = \beta\mathbb{E}_t\frac{P_t}{P_{t+1}} = \beta\mathbb{E}_t\frac{1}{\pi_{t+1}} \tag{8}$$

(7) is the standard labour supply condition equating the marginal disutility of labour (assuming also that $\gamma = 1$, a common calibration of this parameter) with the payoff of exerting additional work effort, $(1-\tau_t)\frac{W_t}{P_t}$. (8) is the standard Euler equation. Note that since household preferences are linear in C, consumption growth does not enter in the Euler equation. Therefore, the price of the short term bond in t is equal to β times the expected value of the inverse of the inflation rate. Higher expected inflation reduces the real return of investing in the bond, and so the price needs to increase to compensate the household.

C. The Government

The government strives to balance the following budget constraint:

$$P_t^1 B_{t,1} = B_{t-1,1} + G_t - W_t \tau_t (1 - x_t) \tag{9}$$

To get the constraint in real terms, we divide by P_t to get:

$$P_t^1 b_{t,1} = b_{t-1,1} \frac{1}{\pi_t} + \underbrace{(\ g_t - w_t \tau_t (1 - x_t))}_{=-s_t} P_t \tag{10}$$

Inserting (8) into (10) and iterating it forward:

$$\begin{aligned} \beta \mathbb{E}_t \frac{1}{\pi_{t+1}} b_{t,1} &= b_{t-1,1} \frac{1}{\pi_t} - s_t \\ &= \underbrace{\lim_{l \to \infty} \pi_l \beta^{l+1} \mathbb{E}_t b_{t+l,1}}_{=0} + \sum_{i=0} \beta^i \mathbb{E}_t s_{t+i} \end{aligned} \tag{11}$$

The above expression is the intertemporal budget constraint of the government which needs to hold as an equality at all dates and states of the world. It essentially tells us that the government can finance its (real) debt either by guaranteeing future surpluses or through inflating away public debt.

D. Monetary and Fiscal Rules

The focus of this thesis is to study the interactions of monetary/fiscal policies and the effects on the propagation of shocks to inflation. As in much of the literature, I will assume that monetary and fiscal policies follow ad hoc rules. More specifically, monetary policy follows a simple inflation targeting rule:

$$1 + i_t = \pi_t^{\phi_\pi} \tag{12}$$

Moreover, taxes respond to the face value of the debt that has been issued in the previous period:

$$\tau_t = b_{t-1,1}^{\phi_\tau} \tag{13}$$

E. Policy Mixes

It is not easy to derive analytical results based on the non-linear model. Therefore we resort to a log-linear approximation of the above conditions. Log-linearizing (5) using the labor supply condition, (8) with $i_t = -logP_t$ then (10) (12), and (13), we get, respectively:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \tag{14}$$

with $\kappa_1 = -\frac{\eta}{\theta}$ and $\kappa_2 = -\frac{\eta}{\theta}\frac{\bar{\tau}}{1-\bar{\tau}}$

$$\hat{i}_t = \mathbb{E}_t \hat{\pi}_{t+1} \tag{15}$$

$$\beta \bar{b}(\hat{b}_{t,1} - \mathbb{E}_t \hat{\pi}_{t+1}) = \bar{b}(\hat{b}_{t+1} - \hat{\pi}_t) - \bar{s}\hat{s}_t \tag{16}$$

$$\hat{\tau}_t = \phi_\tau \hat{b}_{t-1,1} \tag{17}$$

$$\hat{i}_t = \hat{\pi}_t \phi_\pi \tag{18}$$

To get a log-linearized expression for government surplus that's not dependent on w_t and x_t, we need to use the labor supply condition in (7) and the fact that production is linear in technology:

$$\bar{s}\hat{s}_t = -\bar{g}\hat{g}_t + \bar{r}\left((\gamma+1)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\bar{\tau}} \right) \tag{19}$$

Combining the Fisher equation and the interest rule, we get:

$$\phi_\pi \hat{\pi}_t = \mathbb{E}_t \hat{\pi}_{t+1} \tag{20}$$

Next, I will consider the solutions to the models for different ϕ_π, ϕ_τ in the interest rate rule and the tax rule.

Supposing that the Taylor principle holds, $\phi_\pi > 1$

Let me focus first on the case where the monetary authority focuses on the goal of stabilizing inflation. I assume in this paragraph that the so called Taylor principle is satisfied, i.e. $\phi_\pi > 1$.

Solving (20) forward we can obtain.

$$\hat{\pi}_t = \frac{1}{\phi_\pi}\mathbb{E}_t \hat{\pi}_{t+1} = \frac{1}{\phi_\pi^2}\mathbb{E}_t \hat{\pi}_{t+2} = ... = \lim_{h\to\infty} \frac{1}{\phi_\pi^h}\hat{\pi}_{t+k} = 0 \tag{21}$$

This result basically states that inflation in the Fisherian model is equal to 0 regardless of the shocks that hit the economy. This is not surprising. In the absence of any feedback from output through the real interest rate, shocks will not impact inflation which is pinned down through the Euler equation and the interest rate rule.

Utilizing the above result and the Phillips Curve, we get:

$$\hat{Y}_t = \frac{-\kappa_2}{\kappa_1}\hat{\tau}_t \tag{22}$$

and thus output fluctuations derive from changes in distortionary taxes in this model.

Inserting this into the log-linearized government budget constraint, we can write:

$$\hat{b}_{t,1} - \frac{\bar{g}\hat{g}_t}{\beta\bar{b}} = \frac{1}{\beta\bar{b}} \underbrace{\left\{\bar{b} - \bar{r}\left[(\gamma+1)\right]\left(\frac{-\kappa_2}{\kappa_1}\right) + \frac{1}{1-\hat{\tau}}\phi_\tau\right\}}_{\rho} \hat{b}_{t-1} \tag{23}$$

In order to get a unique, stable rational expectations equilibrium, we need a stable root in the above equation—i.e. $\rho < 1$ (otherwise government debt will display explosive dynamics). Manipulating the expression above we find that $\rho < 1$ requires that

$$\phi_\tau > \frac{(1-\beta)\bar{b}}{\bar{r}\left[(\gamma+1)\right]\left(\frac{-\kappa_2}{\kappa_1}\right) + \frac{1}{1-\hat{\tau}}} \tag{24}$$

Therefore the response of taxes to lagged debt must be sufficiently strong to ensure that debt is a stationary process.

The active monetary/passive fiscal regime comprises of this equilibrium. As stated in Leeper (1991), when the authority in question is free to act as it wishes without being constrained by governmental budgetary concerns (the monetary authority in this case), it can set the coefficient for its decision rule depending on past, current, or expected future variables. Therefore, the interest rate rule (and thereby, inflation) is forward-looking under the AM/PF regime. The monetary policy is free to react strongly to inflation to aim for price stability. This is the regime that is reflected in the standard New Keynesian model.

Should $\rho > 1$, however, the system of the competitive equilibrium equations is explosive. This is what Leeper termed the active monetary/active fiscal (AM/AF) regime. In this regime, both authorities are too focused on determining prices that they completely ignore budgetary needs. Intuitively, when there is a shock to government spending and the government continually borrows to finance its activities without adjusting its taxes, resulting in the need to inflate away debt, while the monetary authority is raising interest rates curbing inflation. Households will then not be incentivized to hold government debt as this would violate the transversality condition.

Another way of looking at the problem with the AM/AF regime is that: let's suppose that the government continually borrows without concern as to whether or not they will be able to pay back their debt. Then, the consumer's expectations of the reliability of government bonds would fall, causing them to not only not continue to invest in government bonds but also to sell the ones that they do own, driving up inflation. When this is the case, the monetary authority can have little impact on the economy in terms of disinflation attempts. This way of framing the FTPL incorporates an aspect of the theory that we are unable to demonstrate quantitatively through the simple model. However, Bianchi and Ilut (2017) offers a great read on the topic.

Supposing that the Taylor principle *does not* hold, $\phi_\pi < 1$

Let me now turn to the case of passive fiscal policy, $\phi_\pi < 1$. In this case, we can write actual inflation at time t+1 as expected inflation plus a white noise (mean zero) shock and iterate backwards:

$$\begin{aligned}\hat{\pi}_{t+1} &= \mathbb{E}_t \hat{\pi}_{t+1} + \eta_{t+1} \\ &= \phi_t \hat{\pi}_t + \eta_{t+1} \\ &= \phi_\pi(\phi_\pi \hat{\pi}_{t-1} + \eta_t) + \eta_{t+1} \\ &= \ldots \\ &= \phi_\pi \hat{\pi}_0 + \sum_{l=0}^{t} \phi_\pi^l \eta_{t+1-l}\end{aligned} \tag{25}$$

Here, the monetary authority is passive—monetary policy is not reacting sufficiently to inflation in order to stabilize it. Therefore, inflation can be driven by sunspot η shocks. Thus, the interest rate (thereby, inflation) is backward-looking. In this case, if the fiscal authority is also passive, then there would exist multiple equilibria due to the fact that either of or both authorities can satisfy the budget constraint.

However, if fiscal policy is active, inflation can be pinned down by the government budget constraint. Let us suppose (for analytical tractability) that taxes are constant: $\phi_\tau = 0$.

Log-linearizing (11) we get:

$$\begin{aligned}\bar{b}(\hat{b}_{t-1,1} - \hat{\pi}_t) &= \mathbb{E}_t \sum_{i=0} \beta^i \bar{s}\hat{s}_{t+i} \\ &= \mathbb{E}_t \sum_{i=0} \beta^i \left\{ -\bar{g}\hat{g}_{t+i} + \bar{r}\left[(\gamma+1)\hat{Y}_{t+i} + \frac{\hat{\tau}_{t+i}}{1-\bar{\tau}} \right] \right\}\end{aligned} \tag{26}$$

Using the Phillips Curve and the fact that taxes are constant:

$$\hat{Y}_t = \frac{(1-\beta\phi_\pi)\hat{\pi}_t}{\kappa_1} \tag{27}$$

Then, using

$$\mathbb{E}_t\hat{\pi}_{t+i} = \mathbb{E}_t\phi_\pi\hat{\pi}_{t+i-1} = E_t\phi_\pi^2 E_t\phi_\pi^2\hat{\pi}_{t+i-2} = ... = \mathbb{E}_t\phi_\pi^i\hat{\pi}_t \tag{28}$$

we get

$$\begin{aligned}\sum_{i\geq 0}\beta^i\mathbb{E}_t\bar{r}\left[(\gamma+1)\hat{Y}_t\right] &= \sum_{i=0}\beta^i\mathbb{E}_t\bar{r}\left[(\gamma+1)\frac{(1-\beta\phi_\pi)}{\kappa_1}\hat{\pi}_{t+i}\right] \\ &= \sum_{i\geq 0}\beta^i\bar{r}\frac{(\gamma+1)}{\kappa_1}(1-\beta\phi_\pi)\phi_\pi^i\hat{\pi}_t \\ &= \frac{\bar{r}(\gamma+1)}{\kappa_1}\hat{\pi}_t\end{aligned} \tag{29}$$

(26) becomes:

$$\bar{b}(\hat{b}_{t-1,1}-\hat{\pi}_t) = \mathbb{E}_t\sum_{i\geq 0}\beta^i(-\bar{g}\hat{g}_{t+i}) + \frac{\bar{r}(\gamma+1)\hat{\pi}_t}{\kappa_1} \tag{30}$$

Therefore, inflation can be written as:

$$\left(\bar{b}+\frac{\bar{r}(\gamma+1)}{\kappa_1}\right)\hat{\pi}_t = \bar{b}\hat{b}_{t-1,1} - \mathbb{E}_t\sum_{i\geq 0}\beta^i(-\bar{g}\hat{g}_{t+i}) \tag{31}$$

The above equation states that inflation is a function of lagged debt (higher debt implying higher inflation) and of the discounted future surpluses of the government (the spending shocks under the assumption of constant taxes). Higher spending will induce an increase in the inflation rate which ensures in this model that debt is not explosive. Therefore under passive monetary policy, fiscal deficits are financed through inflation in the unique rational expectations equilibrium.

V. U.S. COVID-19 Inflation, FFR, and Output Dynamics

The previous analytical results offer useful insights into the two types of equilibria that characterize monetary and fiscal interactions. However, in order to bring more realism to the exercise I want to consider, I need to now turn to the canonical New Keynesian model in which household preferences are not linear in consumption and, in turn, the real interest rate is a function of consumption and therefore output growth.

The model (log-linear) equations now become:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_1 \hat{Y}_t + \kappa_2 \hat{G}_t + \kappa_3 \hat{\tau}_t + u_t$$

where $\kappa_1 = -\frac{1+\eta}{\theta}\overline{Y}(\gamma + \sigma\frac{\overline{Y}}{\overline{C}})$, $\kappa_2 = -\frac{1+\eta}{\overline{Y}\theta}\sigma\frac{\overline{G}}{\overline{C}}$ and $\kappa_3 = -\frac{1+\eta}{\overline{Y}\theta}\frac{\overline{\tau}}{1-\overline{\tau}}$.

Parameter σ denotes the inverse of the intertemporal elasticity of substitution of consumption whereas $\overline{Y}$ and $\overline{C}$ are the steady state values of consumption and output. The final term on the RHS of the above equation is a cost push shock (a shifter of the Phillips curve relation).

Moreover, we have:

$$\hat{i}_t = E_t \hat{\pi}_{t+1} + \sigma E_t \left(\frac{\overline{Y}}{\overline{C}} (\hat{Y}_{t+1} - \hat{Y}_t) - \frac{\overline{G}}{\overline{C}} (\hat{G}_{t+1} - \hat{G}_t) - \hat{\xi}_t \right)$$

as the Euler equation in this model where $\hat{\xi}_t$ is the consumer preference shock.

Quite evidently, under the new system of equations, we have non trivial effects of output fluctuations (that can be driven by the cost push or the government spending shocks or consumer preference shocks) on the Euler equation and therefore, on inflation.

Though in the previous Fisherian version of the model, an interest rate rule targeting inflation and satisfying the Taylor principle would guarantee that inflation is equal to 0 at all times, regardless of the shocks that hit the economy, this is no longer the case. Government spending shocks in this model act as demand shocks (entering the Euler equation), the consumer preference and the cost push shocks that have been added will drive the inflation-output tradeoff even when monetary policy sets the nominal rate via an inflation targeting rule.

The government budget constraint in the canonical New Keynesian model becomes:

$$\beta \overline{b}(\hat{b}_t - \sigma E_t \frac{\overline{Y}}{\overline{C}} (\hat{Y}_{t+1} - \hat{Y}_t) + \sigma E_t \frac{\overline{G}}{\overline{C}} (\hat{G}_{t+1} - \hat{G}_t) - E_t \hat{\pi}_{t+1} - \hat{\xi}_t) = \overline{b}(\hat{b}_{t-1} - \hat{\pi}_t) + \overline{G}\hat{G}_t - \hat{T}_t - \overline{\tau}\overline{w}\overline{Y}(\hat{\tau}_t + \hat{Y}_t + \hat{w}_t)$$

Note that now the bond price is a function of expected output growth (i.e. the LHS of the above equation). Moreover, the term $\hat{T}_t$ (expressed in level deviations from the steady state which is normalized to 0) is a transfer from the government to the private sector. The goal here is to consider scenarios where a positive cost push shock or a negative consumer preference shock is compensated with such a transfer.

A. MATLAB Dynare Simulations

I now simulate the behavior of the economy under the alternative monetary/fiscal policy regimes. For the experiments that I will show in this section I set $\sigma = 1$, a standard calibration for this parameter in the relevant literature.

Moreover, I assume that monetary policy follows an interest rate rule of the form:

$$\hat{i}_t = \phi_R \hat{i}_{t-1} + (1 - \phi_R)\phi_\pi \hat{\pi}_t$$

This inertial interest rate rule turns into a simple interest rates rule when the interest rate smoothing parameter ϕ_R is equal to 0. However, it is worthwhile to consider cases the ϕ_R coefficient is positive and potentially quite close to 1, since, as discussed previously, a characteristic of the current downturn is the sluggish reaction of monetary policy to macroeconomic conditions (inflation).

Note also that the previous results regarding active and passive policies are applicable here. In particular, monetary policy is active either when $\phi_\pi > 1$ (for any coefficient $\phi_R \in [0, 1)$). Analogously, policy is passive if and only if $\phi_\pi < 1$.

In the experiments below I will be varying parameters ϕ_π, ϕ_τ, and ϕ_R to discern the effects of shocks on inflation and interest rates under passive/active policies. In total, there are six specifications that are worth taking a look at for each of the three shocks—cost-push, government spending, and consumer preference:

- The AM/PF regime with $\phi_R = 0, \phi_\pi = 1.2, \phi_\tau = 0.2$
- The PM/AF regime with $\phi_R = 0, \phi_\pi = 0.9, \phi_\tau = 0$
- The PM/AF regime with $\phi_R = 0, \phi_\pi = 0.1, \phi_\tau = 0$
- The AM/PF regime with $\phi_R = 0.9$, $\phi_\pi = 1.2, \phi_\tau = 0.2$
- The PM/AF regime with $\phi_R = 0.9$, $\phi_\pi = 0.9, \phi_\tau = 0$
- The PM/AF regime with $\phi_R = 0.9$, $\phi_\pi = 0.1, \phi_\tau = 0$

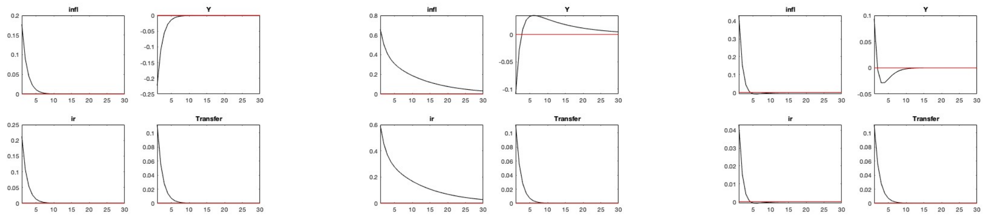
I will simulate the canonical New Keynesian model in MATLAB using Dynare for each specification and shock, present the IRFs, and analyze how well each model specification is able to the COVID data in the U.S.

B. Results

Overall, the results outlined below are robust to changes in coefficients that govern government spending and consumer preference shocks. Each figure below comprises of IRFs for three different model specifications at either $\phi_R = 0$ or $\phi_R = 0.9$: (a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$; (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$; and (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

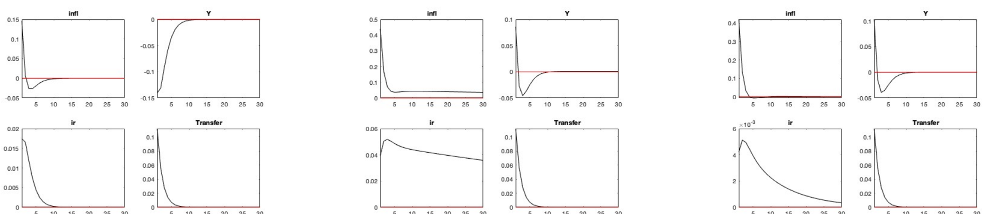
Cost-push Shock

Figures 3 and 4 are the IRFs according to each regime and specification in reaction to a cost-push shock.



(a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$ (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$ (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

Figure 3. : IRFs for a cost-push shock with $\phi_R = 0$



(a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$ (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$ (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

Figure 4. : IRFs for a cost-push shock with $\phi_R = 0.9$

Government Spending Shock

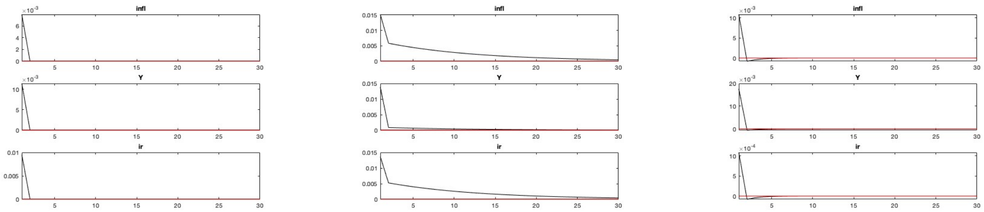
Figures 5 and 6 are the IRFs according to each regime and specification in reaction to a government spending shock.

Consumer Preference Shock

Figures 7 and 8 are the IRFs according to each regime and specification in reaction to a consumer spending shock.

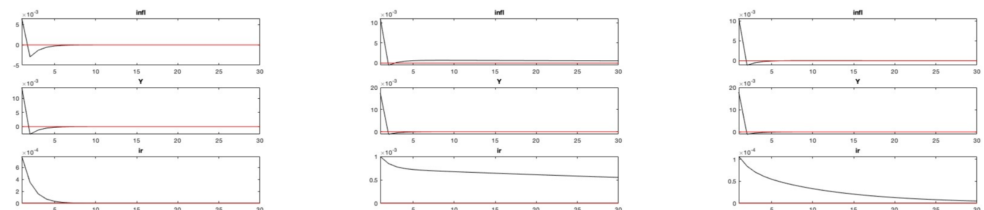
C. Discussion

To begin the discussion of the results, I would like to note that the IRFs for the interest rates in any specification for any given shock shows interest rates



(a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$ (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$ (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

Figure 5. : IRFs for a government spending with $\phi_R = 0$



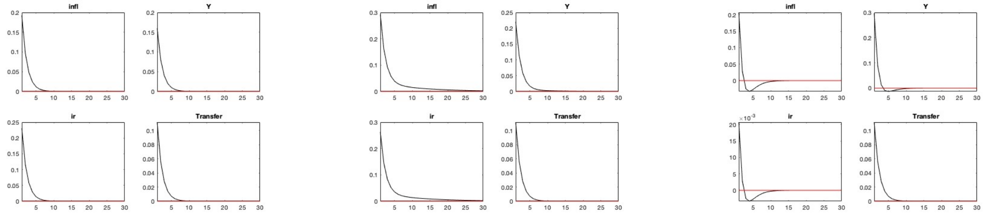
(a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$ (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$ (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

Figure 6. : IRFs for a government spending shock with $\phi_R = 0.9$

increasing. However, since the effect FFR hovers above the ZLB around 0.25% throughout the entire 2019Q4-2021Q4 period, it would not benefit my analysis if I were to use them as a model fit criteria. Therefore, they will be ignored in the analysis.

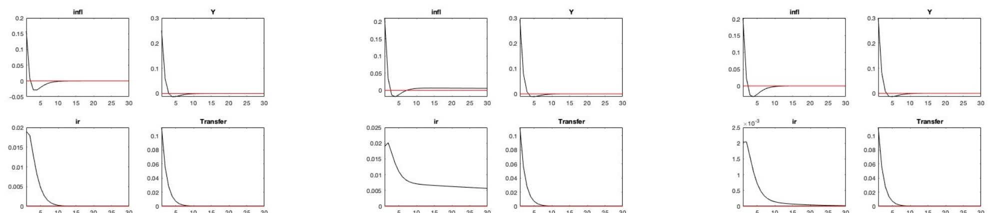
I try to match the IRFs to the signing of each of the stimulus and relief package. The specifications that best fit the COVID data is that of the PM/AF regime with $\phi_R = 0.9$ and $\phi_\pi = 0.9, \phi_\tau = 0$ under either a consumer preference shock or a government spending shock. To be able to come to this conclusion, I employed a straightforward process of elimination.

Immediately, there is a mismatch between the AM/PF specifications not only with respect to the data due to the lack of persistent inflation generation but also with respect to what the Federal Reserve has themselves claimed, which is that they have chosen to assume an accommodative position for the COVID period in order to reach its full employment and price stability goals (Board of Governors of the Federal Reserve, 2020*a*,*b*). This, of course, does sound as if the Fed is taking an "active" (as defined by the FTPL) role in achieving its goals. However, the model does not support this due to the fact that inflation in both the AM/PF and the PM/AF regimes with $\phi_\pi = 0.1$ is not persistent when there is any type of shock.



(a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$ (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$ (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

Figure 7. : IRFs for a consumer preference shock with $\phi_R = 0$



(a) AM/PF; $\phi_\pi = 1.2, \phi_\tau = 0.2$ (b) PM/AF; $\phi_\pi = 0.9, \phi_\tau = 0$ (c) PM/AF; $\phi_\pi = 0.1, \phi_\tau = 0$

Figure 8. : IRFs for a consumer preference shock with $\phi_R = 0.9$

Therefore, according to this model, the U.S. economy during the COVID period was operating under a PM/AF regime with the monetary authority reacting to inflation relatively strongly (but not necessarily timely) while being still "passive."

Next, Figure 3b is the only model specification in the PM/AF $\phi_\pi = 0.9$ group that features a drop in output upon impact, which is not observed in the data. In addition, Figure 4b's IRF for the output shows it increasing then overcorrecting by an arguably significant amount before returning to the steady state. This is not exhibited in the GDP data. Thus, I can exclude these specifications from further consideration. I am left to consider Figures 5b, 6b, 7b, and 8b.

The IRFs for inflation in the PM/AF regime with $\phi_\pi = 0.9$ all exhibit persistence. However, for Figure 6b and 8b, IRFs with $\phi_R = 0.9$ for the government spending and consumer preference shock, respectively, depicts inflation rising then overcorrecting before rising to a new higher steady state. IRFs for inflation in the other specifications feature an increase followed by a gradual drop in inflation that may or may not stabilize around the steady state.

There are two perspectives I can take on in this situation. If I were to want to match the overall (as opposed to quarter-on-quarter) rising trend of inflation in the U.S. during the COVID period, then Figures 5b and 7b can generally explain inflation as observed in the data. Given that my model is not calibrated using real data, it would be difficult to say exactly which one of the two fare better under this view.

However, if I were to want to match the quarter-on-quarter inflation movement as in Figure 1, I would conclude that Figure 6b and 8b are a better fit. This is because if I were to look at the timing of the stimulus and relief packages in Figure 1, I would be able to see that from the signing date of each package except for package 5, inflation for the current and subsequent quarter is higher than the quarter preceding the package signing but then it falls in the second subsequent quarter. The specifications in 5b and 7b yields inflation that is quite persistent but of which gradually returns to the steady state and does not exhibit the overcorrection then rise to a higher inflation level behavior.

That being said, taking into consideration the miniscule overcorrection behavior in the IRFs for output in 6b and 8b, I am more inclined to favor 6b and 8b due to the fact that the GDP data in Figure 2 also exhibits small fluctuations in the quarter after the signing of each stimulus package, even if the overall GDP trend is growing.

Therefore, I conclude that the model specification that best fit the data is the PM/AF regime with $\phi_R = 0.9$, $\phi_\pi = 0.9, \phi_\tau = 0$. In addition, the shocks that are most relevant to explaining inflation in the U.S. during the COVID period are the government spending and consumer preferences shocks with consumer spending shocks exhibiting a more significant impact on the macroeconomic variables. While the cost-push shock is able to explain the data in terms of inflation, it does not track the movement of the GDP data well at all. not This result is in line with with Giovanni et al. (2022) wherein they observed that aggregate demand shocks are more relevant compared to cost-push shocks in the U.S. than to the Eurozone. Finally, both 6b and 8b are models that features $\phi_R = 0.9$. Ergo, the sluggish reaction of the Feds to inflation definitely plays a key role in explaining COVID U.S. inflation dynamics and persistence.

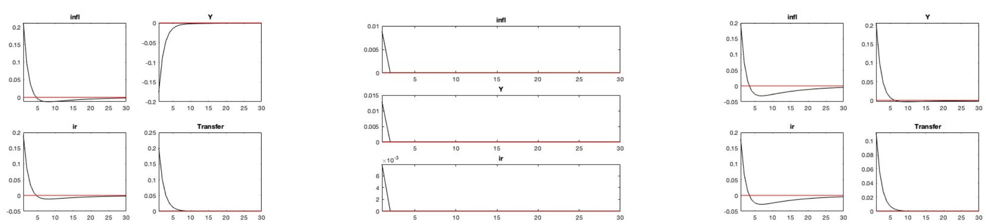
Also, a detail of note here is that while the IRFs for inflation were able to match the inflation response to the first four stimulus packages quite well, they cannot explain the fall in the quarter-on-quarter inflation that occurred after the fifth stimulus package. My hypothesis is that there was a delayed reaction by the private sector to the fifth and final stimulus package and the fall in inflation that is observed is attributable to the fourth package instead.

D. Model Extension with Long-term Debt

The question I pose in this subsection is quite straightforward: what would happen if bonds were long-term instead of short-term? The model derivation is included in Appendix B.

Let $\delta \in (0, 1)$ be the coefficient that governs the maturity of the debt. Naturally, I will be disregarding the discussion about the IRFs when δ is low and focusing on what happens when δ is high. Also, for brevity, only the IRFs for the PM/AF regime with $\phi_R = 0, \phi_\pi = 0.9$ for the three different shocks are included here. In general, they represent the dynamic of what happens if debt becomes long-term quite well and are sufficient for me to expound on the intuition behind the relationship between debt maturity and inflation according to the FTPL. Most of the rest of the IRFs with long-term debt have been relegated to Appendix A.

Unsurprisingly, there is little change to the IRFs under the AM/PF regime due to fiscal policy being passive and reacting to debt in the previous period by adjusting taxes; therefore, diminishing the impact of long-term bonds on the government's budget constraint significantly. The situation gets a little bit more nuanced in the PM/AF regime specifications. Below are the selected IRFs.



(a) Cost-push Shock (b) Gov. Sp. Shock (c) Cons. Pref. Shock

Figure 9. : IRFs in the PM/AF regime with $\phi_R = 0.9, \phi_\pi = 0.9, \delta = 0.9$

The short-term bond counterpart for these IRFs are 3b, 5b, and 7b, which were the IRFs that generated the most persistent inflation. In contrast, when the bond maturity is very high, there is no sign of persistent inflation. In fact, for 9b, there is persistent deflation from around period 5 onward. Similarly, the switch to long-term debt results in all inflation IRFs for all model specifications to feature temporary inflation only; except for the PM/AF $\phi_\pi = 0.9, \phi_\pi = .9, \tau = 0$ with a cost push shock model specification. Why is this?

According to the FTPL, the government intertemporal budget constraint requires that the present value of the outstanding government debt is equal to the present value of the future primary surpluses, which is equal to the difference between government revenues and non-interest expenditures. The presence of long-term debt puts emphasis on households' expectations and belief of the government's credibility when it comes to the government's ability to repay their debt. Households will only choose to own long-term government bonds if they believe that the government will be able to pay it back in the future. Therefore, when there is a shock to government spending, households will perceive the shock as being temporary and expect that the government will adjust its spending behavior accordingly in order to to ensure long-term debt sustainability. This would result in inflation not being as persistent as in the model where there are only short-term bonds.

From the government's perspective, when debt maturity is high enough, it can leverage subsequent deflation to fund its current increased expenditures and satisfy the budget constraint. Therefore, it would have incentive to keep inflation low in future periods whether it be by cooperating with monetary authorities to keep interest rates high, reducing its future spending so as to reduce aggregate demand and therefore, the price level, or influencing market expectations.

Because of the low inflation persistence generated by this extended model, it does not fare well against the original model in terms of being able to explain the data.

VI. Conclusion

The motivation behind this thesis was that I wanted to see if the the high and persistent inflation in the U.S. during the COVID-19 period is primarily due to the slow reaction of the Feds, or to the overall excessive government spending, or both; more specifically, I wanted to identify what shocks that impacted government spending were the most relevant to the U.S. economy during the COVID period. To explore these questions, I utilized the fiscal theory of the price level framework and simulated a canonical New Keynesian model with short-term debt and three types of shocks: consumer preference, cost-push, and government spending.

The analysis carried out seems to suggest that: inflation is the outcome of both the Feds' slow response to inflation and the overall excessive government spending, and that consumer preference shocks are more relevant than general government spending shocks when the cost of the consumer shocks are rebated by the government to the households. Cost-push shocks can explain the rise and persistence in inflation but do not adequately match output movements. In addition, during the COVID period, the U.S. government appear to be operating in a passive monetary/active fiscal regime for if they were under an active monetary/passive fiscal regime, inflation would not have been as persistently increasing as it was in the data.

While my employment of the FTPL is able to explain the inflation in the COVID data relatively well, it is by no means a perfect analysis and there exist a plethora of avenues for further research into this topic using the theory and perhaps a more complex model with calibrations using real data.

In reaching its goal of output growth and employment stability Mutikani (2020); Larrimore, Mortenson and Splinter (2021), inflation was sacrificed by both the Feds and the fiscal authorities in the U.S. The COVID-19 pandemic was undoubtedly a time ripe with economic instability and unpredictability. Deciding on a course of action was the burden that the economic authorities had to bear and not everything could be saved and kept stable. That is to say, in helping to settle the debate as to whether inflation was the fault of the Feds or of the government spending, my analysis arrived at the conclusion that they are both guilty of causing inflation but perhaps not at fault.

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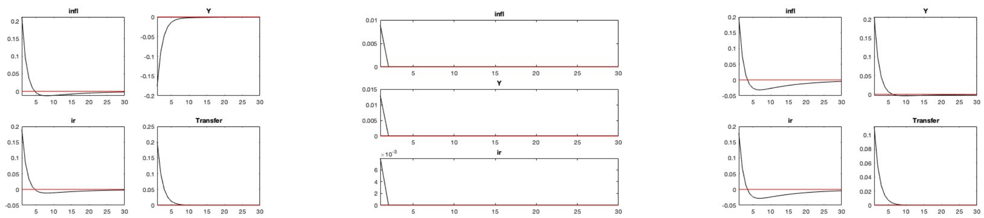
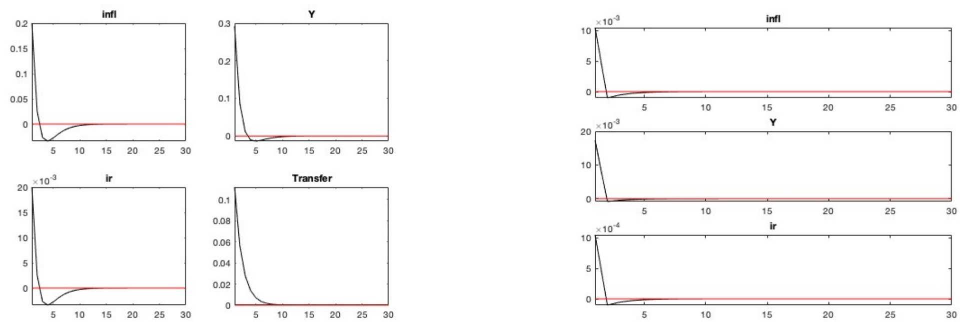
Appendix A: Selected IRFs for the Canonical NK Model with Long-Term Bonds; PM/AF Regimes Only



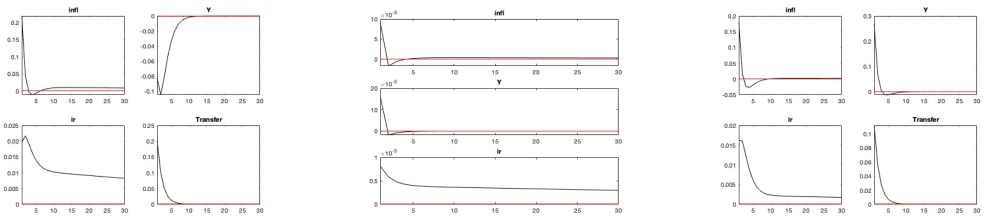
(a) Cost-push shock (b) Gov. Sp. Shock (c) Cons. Pref. Shock

Figure 1. : IRF $\phi_R = 0, \phi_\pi = 0.9, \phi_\tau = 0$



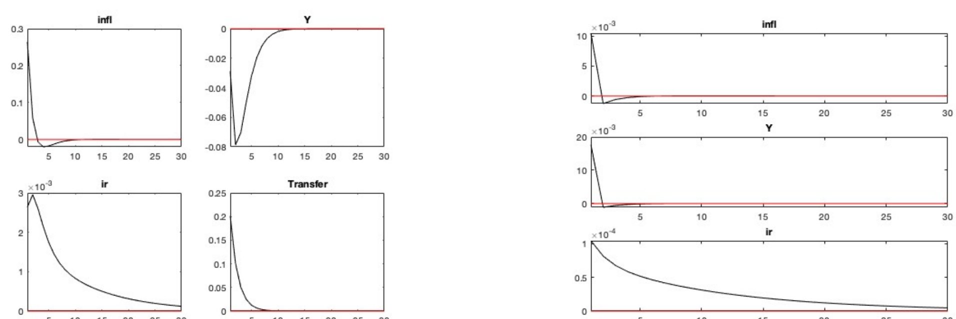
(a) Cost-push shock (b) Gov. Sp. Shock

Figure 2. : IRF $\phi_R = 0, \phi_\pi = 0.1, \phi_\tau = 0$



(a) Cost-push shock (b) Gov. Sp. Shock (c) Cons. Pref. Shock

Figure 3. : IRF $\phi_R = 0.9, \phi_\pi = 0.9, \phi_\tau = 0$



(a) Cost-push shock (b) Gov. Sp. Shock

Figure 4. : IRF $\phi_R = 0.9, \phi_\pi = 0.1, \phi_\tau = 0$

Appendix B: Derivation of the Extended Model with Long-term Bonds

Suppose now that the only bonds households can hold in the economy are long-term government bonds which are perpetuities with coupon payments decaying at the rate $0 < \delta < 10$ and price $P_{t,L}$. The consumer's budget constraint is now:

$$P_t C_t + P_{t,\delta} B_{t,\delta} \leq (1-\tau_t) W_t (1-x_t) + P_t D_t + (1+\delta P_{t+1,\delta}) B_{t-1,\delta} \tag{1}$$

FOCs w.r.t. C_t and x_t, respectively:

$$1 - x_t = (1-\tau_t)\frac{W_t}{P_t} \tag{2}$$

$$P_{t,\delta} = \beta \mathbb{E}_t \frac{P_t}{P_{t+1}}(1+\delta P_{t,\delta}) = \beta \mathbb{E}_t \frac{1}{\pi_{t+1}}(1+\delta P_{t+1,\delta}) \tag{3}$$

The firm's problem remains the same.

The government's budget constraint in real terms is now:

$$P_{t,\delta} b_{t,\delta} = (1+\delta p_{t,\delta})\frac{b_{t-1,\delta}}{\pi_t} - \underbrace{(\ g_t - w_t \tau_t (1-x_t))}_{=-s_t} P_t \tag{4}$$

Iterating the Euler equation (34) forward, we get the price of the long-term bond:

$$P_{t,\delta} = \sum_{j \geq 1} \mathbb{E}_t \beta^j \delta^{j-1} \frac{1}{\prod_{l=1}^{j} \pi_{t+1}} \tag{5}$$

Substituting (36) into (35) and log-linearizing:

$$\begin{aligned} &\frac{\beta \bar{b}}{1-\beta\delta}\hat{b}_{t,\delta} - \sum_{j \geq 1} \mathbb{E}_t \beta^j \delta^{j-1} \frac{1}{\prod_{l=1}^{j} \pi_{t+1}} \\ &= \frac{\bar{b}}{1-\beta\delta}(\hat{b}_{t+1,\delta} - \hat{\pi}_t) - \bar{s}\hat{s}_t - \delta \bar{b} \sum_{j \geq 1} \mathbb{E}_t \beta^j \delta^{j-1} \frac{1}{\prod_{l=1}^{j} \pi_{t+1}} \end{aligned} \tag{6}$$

Supposing that the Taylor principle holds, $\phi_\pi > 1$

Following the same derivation logic of this part in the original model, we would end up with $\hat{\pi}_t = t$ for every period and the following threshold level for the tax rule coefficient:

$$\phi_\pi > \frac{(1-\beta)\frac{\beta\bar{b}}{1-\beta\delta}}{\bar{r}\left[(\gamma+1)\right]\left(\frac{-\kappa_2}{\kappa_1}\right)+\frac{1}{1-\hat{\tau}}} \tag{7}$$

Therefore, as δ increases, the numerator increases, forcing taxes to be more responsive to previous debt in order to for the government budget to balance.

Supposing that the Taylor principle *does not* hold, $\phi_\pi < 1$

Again, following the general logic of the same part in the original model, we get the forward-iterated and log-linearized government budget constraint:

$$\sum_{j\geq 1}\mathbb{E}_t\beta^j\bar{s}\hat{s}_{t+j} = \frac{\bar{b}}{1-\beta\delta}\hat{b}_{t-1,\delta} - \bar{b}\sum_{j\geq 1}\mathbb{E}_t\beta^j\delta^j\sum_{l=1}^{j}\pi_{t+1} \tag{8}$$

Substituting in (19) and (29):

$$\bar{b}\sum_{j\geq 1}\mathbb{E}_t\beta^j\delta^j\sum_{l=1}^{j}\pi_{t+l} + \frac{\bar{r}(\gamma+1)}{\kappa_1}\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\hat{b}_{t-1,1} + \mathbb{E}_t\sum_{i\geq 0}\beta^i(\bar{g}\hat{g}_{t+i}) \tag{9}$$

Using (28):

$$\left(\frac{\bar{b}}{1-\beta\delta}\frac{1-\phi_\pi^j}{\phi_\pi(1-\phi_\pi)} + \frac{\bar{r}(\gamma+1)}{\kappa_1}\right)\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\hat{b}_{t-1,1} + \mathbb{E}_t\sum_{i\geq 0}\beta^i(\bar{g}\hat{g}_{t+i}) \tag{10}$$