A model of sustainability initiatives generation and allocation in a profit-maximizing firm environment

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1 Abstract

This project aims at developing a simple model that is able to account for the fact that modern firms choose to invest part of their resources into sustainable actions. With this last term, I refer to a series of initiatives that companies undertake to the advantage of others. Thus, I am concerned with the voluntary generation of positive externalities whose effect has no direct consequence when it comes to helping the firm in being more efficient in its core business (publicly available technology) and at the same time represent an additional cost. Indeed, this choice seems in contradiction with the classical assumption of profit maximization. A key feature of the present analysis, hence, is to allow for the existence of active sustainable effort (i.e. a nonzero allocation of resources to activities that are simply that) in an equilibrium that arise from the firm's solution to a profit maximization problem.

The structural framing is done keeping in mind classical micro-economic approaches, integrated by results from stakeholder's theory and corporate social responsibility (CSR) literature. Four major stakeholders will be taken into consideration: consumers, suppliers, employees and shareholders. Each of them is assumed to mediate its relationship with the firm in a market-like structure (output, input, labor and capital respectively), with each of the markets yielding a partial equilibrium that will specify the value of the sustainable actions toward the stakeholder at hand, together with the usual good price and quantity.

The fact that consumers, employees and other stakeholders are positively benefited by these actions is explained with the help of results from behavioural and institutionalist economics, with particular attention to the inducement-contribution paradigm. In particular, all the relevant stakeholder are assumed to care for the sustainable actions taken by the firms in their specific favour. Within classes of stakeholders, heterogeneous agents are employed in order to display different sensitivities. This means that different strategies (high sustainable effort vs low extra costs due to sustainability expenses) applied in order to compete for different parts of the stakeholder distributions are in principle all possible.

At first each market will be studied in isolation in order to reconstruct the properties of its partial equilibrium. Then, all markets will be investigated jointly in a unique problem where the firm chooses all the quantities at the same time, maximizing profits. The resulting system of first order conditions yields maxima consistent with positive sustainable effort.

Furthermore, in order to investigate optimal strategies in a competitive environment, a duopoly case will be taken into consideration with two identical firms modeled according to the results developed with a single firm. Developing best responses, different specialized behaviours emerge: cost efficient strategies and zero sustainable effort alternate premium price strategies with superior sustainable effort. However, resulting Nash equilibria imply symmetric behaviour and positive sustainable effort.

Finally, the framework will be put to the test with real data, coming from publicly available information when it comes to quantities and prices, and employing the Golden repository of sustainable initiatives by the Leonardo research centre, part of Imperial College - Imperial Business School, for the sustainable effort data. The results of the simple falsification test employed here points to the consistency of the supposed non-zero effect of sustainable actions.

2 Theoretical Background

The description and modeling of firm behaviour is something that can be tackled from many point of views. This somewhat enriches and at the same time complicates a thorough enquiry about the reasons why sustainable actions are increasingly part of firms' pool of strategic choices. Nonetheless, I tried to investigate such a specific enquiry without leaving any of the major approaches stemming from different theories unaddressed.

Henceforth, the research question pursued in this work might seem too broad and potentially inconclusive on one end, and on the contrary too narrow, hence uninteresting. These criticisms, I believe, are quite far from the truth and I would like to dedicate some space to address them in their version that I consider most reasonable, hoping to cast some light over where a research gap might have been recently opened in this area. Some of the points I make here might be better understood after having given a quick look at how the model actually works, but for logical ordering I prefer having this section before the modeling part: I sincerely apologise to the reader if some back and forth is required.

The criticism of the first type stems from a hereditary flaws of many of the so-called "theories of the firm" (at least, when these theories have been applied to the wrong problems). Indeed, one might think that the model presented in the following paragraph is purposelessly complicated to treat any firm-level decision making, and yet it fails completely in being *realistic*, in the sense of mimicking the reality of decision processes. In this regard, one of my professors used to say "theories must be simple, but not *simpler*", with which he intended that there is a minimum degree of complexity that a theory needs to abide when treating any particular matter. Such a "complexity lower bound", which is of course topic dependent, cannot be crossed but needs to be attained, in order to avoid useless hindrances. I do believe that this work satisfies this paradigm. The contrary belief can be due to the fact that the topic upon which this discussion insists is at the crossroad of many fields of study that compete (and complete) with one another with different approaches, and some hybrid components concurred in the formation of the present work (an account of the early-to-mid stages of this academic debate can be found in Machlup (1967)). To better outline this fact here below three of the major relevant theoretical flows that inspired this paper's model are briefly outlined.

The first of these flows is what comes from orthodox micro-economic theory, for the part it considers firm behaviour. The approach here used is the one historically called *marginalism*. It revolves around the idea of every firm as an individual decision maker that is subject to certain constraints (namely, technology) while it transforms some inputs into some outputs, with the only rationale of maximizing the residual value of the outputs after using some of it to cover the costs of the inputs used (both value and costs assigned by some external pricing), a variable commonly known as *profits*. This setup allows for the greatest mathematisation among the ones particularly suited for the problem at hand and as such it plays a key role in the model presented. Although many more recent extensions exist, the baseline approach here considered in this regard is the one of Mas-Colell, Whinston and Green (1995), both for its relevance in the world of microeconomics and the flexibility it allows when formalising components. In this paper, many tools will be mutuated from marginalist analysis and, in the end, there is an individual decision maker that maximises profits. However, it is debatable if this decision maker, here simply called "the firm", is the entirety of the *organisation.* Indeed, classical micro-economic analysis do not consider employees, shareholders, suppliers or even consumer as separate entities, if at all. This is because the firm in this case is a mere mean to convert some inputs into some outputs under market condition rather than being an object of study in its *actual* behaviour and environment. The real object of study are (changes in) the goods production and behaviour. This restrictive view of the firm is clearly not rich enough to encompass the possibility of existence of sustainable actions, and as such a dialogue with this other entities is needed. The core component of this work is that these entities, stakeholders (in the sense of Freeman, 1984) from here on, are all considered somewhat "external" to the firm, and their relationship with it is based on a market-like structure. In terms of wording, one could easily state that what I call firm throughout the paper is actually "the management", as it will be clear in the next paragraph. What stands from a marginalist perspective, however, is that, although with more elaborate constraints, the workhorse of the model remains a decision maker with profit-maximization in mind.

The second key theoretical approach that left its marks on the following dissertation comes from a more institutionalist perspective (for a general overview see Miner, 2005). In this respect, the firm is viewed as a many-goals organisation and its individual components are members of a coalition, and only the coalition as a whole can be regarded as "the firm". Thus, each coalition member needs to be rewarded, i.e. satisfied in its own goals, for staying in the coalition (with the threat of leaving) and at the same time it offers its contributions to the joint coalition effort. This is essentially the core of the so-called Inducement-Contribution theory, whose early developments are due to Simon (1947) and later better framed by March and Simon (1967) as milestone for organisational survival. These ideas are translated into the model here developed by assuming that individual coalition members goals are *constraints* from the firm's (as a whole) point of view, except for the one member (usually regarded as the *management*) whose goal is profit-maximization. Such a goals/contraints duality stems again from the view of Simon (1976). About the alignment between actual profit-maximization and management goals much could be said, but it is reasonable to assume that it is a good approximation, due to their strive for perpetuation (to be confirmed by the shareholders) and usually to the way their personal wages are set. On the contrary, the main point of departure is the way internal bargain among coalition members is dealt with. Indeed, there are many behavioural paradigms that try to describe how such a process might work, but if one wants to be as realistic as possible, a proper mathematisation can't be attained: in fact, due to many contingent reasons the allocation of inducements and the procurement of contributions might be subject to any sort of inefficiencies or misalignments. This is what is known in the literature as *organisational slack* and it is essentially due to Cyert and March (1963). Although interesting per se in many respects (see for instance the work of Nohria and Gulati, 1996), from the point of view of the present work it is detrimental to the complete formulation of a formally complete model. Hence, since organisational slack is bearer of inefficiencies and in the context of heavy competition it might threaten survival, here it is considered something that firms actively minimise and as such assumed (quite strongly) ex-post negligible. This allows to reinterpret these coalition bargains *as if they were* market structures, where a unit of contribution is sold and bought for relevant "hard" counterparts (e.g. wage, money) and “sustainable” ones (measured in sustainable effort), in a de-facto formalised way. This might sound as a very unrealistic approach to what actually is the complex decision-making processes in a firm is, but a couple of things can be said to support this conceptualisation. First of all, the structural occasions of bargaining and their functioning are not the object of study of this work, rather the effects of behavioural effort are: the structure is a working minimal approach in this regard as the standard marginalist view was for its problem. Then, alternative (and more complicated) approaches might produce mechanisms more one-to-one comparable with the actual ones, but they would necessarily be not at all parsimonious, not formalised and potentially untestable: a tradeoff that goes against the spirit of this work. To better see where these alternative, more *managerial* perspectives would lead also in terms of problems, a good example is the widely known *garbage can* model, which stems again from the work of Cyert and March (1963), and its honest critique by Vibert (2004).

The last theoretical approach I would like to mention due to its contribution in shaping the structure of the model here below comes from the so-called resource based view (one of the milestones is Barney, 1991). In particular, developments in this respect challenge the traditional concept of *residual clamant*. Indeed, according to historical interpretations and the formal way contracts are usually been signed, profits are what is left after all other “contributors” are rewarded with their fixed (and agreed upon in contracts) quota. As such, profits should be linked with shareholders, since they are those among whom supposedly everything that remains is divided. However, this is not the case in the present model: they are treated essentially as any other stakeholder. Indeed, their remuneration is quite far from the theoretical/old description, especially in the case of publicly traded companies. How much is being payed out in terms of dividends is more and more a strategic managerial decision, and holding gains have little to do with this residuality paradigm. On the contrary, any stakeholder that provide access to strategic resources (hence, resource based view), might force its way through residual claims (in the case of finance this can be found in Zingales, 2000). This of course does not mean that resource based approaches invariably explicitly renounce to this “shareholder supremacy”, but such seems the frontier in this area (see Barney, 2018. The expression on quote is his). This discussion affects the model in the sense that all stakeholders are considered equally capable of producing vital resources (for the case of shareholders particularly, capital) and can be remunerated accordingly as a strategic decision by the firm.

Now that all the main theoretical influences have been fully disclosed, I hope that the way in which this research is pursued and this model built can safely be exonerated from the accusation of being purposelessly complicated and broad: it is a minimal mathematical setup, complex enough to (at least partially) accommodate for the many theoretical instances that animate various fields in economics that I considered relevant for the problem at hand.

Moving to the second kind of criticism, it is possible to consider the sustainable effort of firms at best marginal compared to the activities that modern organisations do every day. This might be true in absolute terms, but such a perspective fails to see an exploding trend and a shifting culture that is more and more incline to reward (punish) particularly virtuous (vitius) companies, so much that active sustainable effort seems more and more a survival necessity. The day-by-day evolution and the global database in development at Imperial College (which is used here) are some of the motives that, in my opinion, suggest the presence of a growing phenomenon and the contemporaneous surge of the means to investigate it.

3 The Model

In this work the firm's surrounding environment (or, equivalently, existing stakeholder engagements) has been defined as a set of four relationships that are not interdependent if not through the constrained decisions of the firm itself (i.e. its trade-off between allocating resources among them). The situation is illustrated in figure 1.

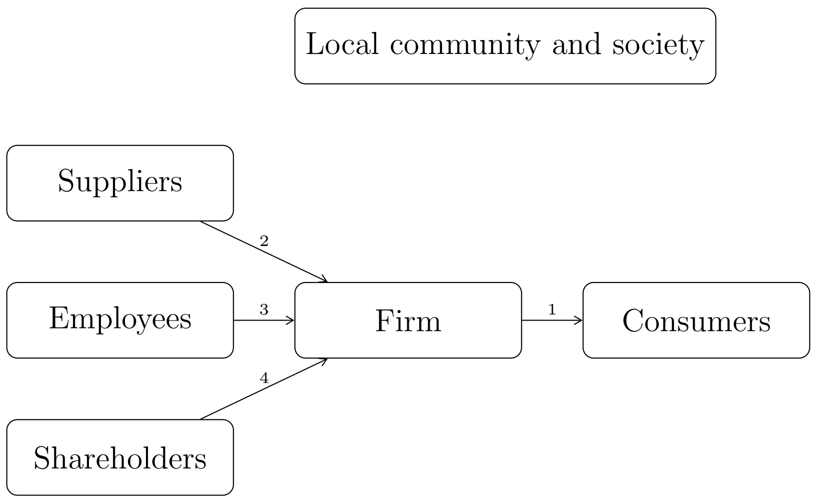


Figure 1: Firm's surrounding environment

Each of the relationships numbered from 1 to 4 is then modeled as a market where the company operates in, bearing in mind its overall profit maximising approach. Moreover, the firm is assumed risk neutral, so that its profit maximization procedure pertains, whenever relevant, expected profits. The description of each of these markets maintain many constant characteristics, but it is worth to consider them separately, at least at the beginning.

3.1 Final good market: consumers

The first stakeholder is defined as a unit mass of agents indexed by $i \in [0, 1]$, each making a decision c_i that represent their stance with respect to the company,

$$c_i \quad i \in [0, 1] \tag{1}$$

and we allow c_i to be a continuum between 0 and 1 that represents the likelihood of the agent's decision to be or not to be a consumer of the firm, where being a consumer means relying entirely on the firm's goods in order to cover one's individual demand for final goods (and all consumers' individual demands are assumed to be the same infinitesimal fraction of total demand M) . In the extreme cases, $c_i = 1$ means that consumer i will purchase from the company with certainty, while $c_i = 0$ means that, with certainty, consumer i will resort entirely to some default alternative option.

$$c_i \in [0, 1] \qquad \forall i \tag{2}$$

So that the firm's market share for the final good can be written as:

$$d_1 = \int_0^1 c_i \mathrm{d}i \tag{3}$$

Which allows total demand for the firm's good to be Md_1. Such market share (and, implicitly, demand) is considered as if it were certain. Indeed, although individual consumers have a probabilistic behaviour, there are so many of them (actually, infinitely many) that ex-post demand (when all consumers decided in the end to buy or not from the firm) is equivalent to ex-ante demand.

Now it is possible to move forward introducing the main novelty if this work, the amount of publicly known sustainability effort that the company can decide to sustain having in mind the needs of the involved stakeholder (here, consumers) $e_1 \in \mathbb{R}_+$. With it, one can specify the decision rule of each consumer $c_i : \quad \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow [0, 1]$:

$$c_i = f_1(p_1)\varphi_1^i(e_1) \tag{4}$$

Where the first component is a fixed element, common to all consumers, that is price dependent (where price is, as we will see, a strategic choice of the firm while maximising profits) and the second component is i-dependent and revolving around sustainable effort (allowing for more and less caring consumers). Moreover, both functions are assumed to take values between 0 and 1, with the first one decreasing in its parameter and the second one increasing:

$$f_1(p_1) : \mathbb{R}_+ \longrightarrow [0, 1] \qquad \frac{\mathrm{d}f_1(p_1)}{\mathrm{d}p_1} \leq 0 \tag{5}$$

$$\forall i, \quad \varphi_1^i(e_1) : \mathbb{R}_+ \longrightarrow [0, 1] \qquad \frac{\mathrm{d}\varphi_1^i(e_1)}{\mathrm{d}e_1} \geq 0 \tag{6}$$

It is maybe worth pondering for a second on the definition just provided. Indeed, it restricts quite a bit the possible interrelation between the price effect and the sustainability effect. As a matter of fact, a certain degree of independence is assumed in this formulation (notice that $\frac{\partial c_i(p_1,e_1)}{\partial p_1} = \frac{\partial f_1(p_1)}{\partial p_1}\varphi_1^i(e_1)$ and $\frac{\partial c_i(p_1,e_1)}{\partial e_1} = f_1(p_1)\frac{\partial \varphi_1^i(e_1)}{\partial e_1}$). This implicit extra imposition is however quite reasonable: one can think about the sustainable component as a "modifier" that affect how the price-based default convenience of the good is perceived. In this sense, it is de facto a behavioural component that is rich enough to encompass different levels of relevancy that different consumers will assign to the sustainability issue (more demanding consumers will require higher levels of e_1 have to display the same modifier as less demanding ones). A functional form that could fit this description and provide a working example is the following:

$$\varphi_1^i(e_1) = 1 - e^{-\frac{e_1}{ki}} \tag{7}$$

Defined, only for positive values of e_1 and with the convention that when $i = 0$, the amount $-\frac{e_1}{ki}$ approaches minus infinity and as such $e^{-\infty} = 0$: the consumer with index $i = 0$ does not care at all about sustainability and its $\phi_1^0 = 1$ always, meaning that she cares only about pricing issues ($c_0 = f_1(p)$). From there on, consumers with larger indexes start caring more and more about sustainability, and total maximum variability is regulated with the positive parameter k (meant to adjust to the relative scale on which e_1 is provided in real settings). Values for $k = 10$ and $k = 30$ are shown in figure 2.

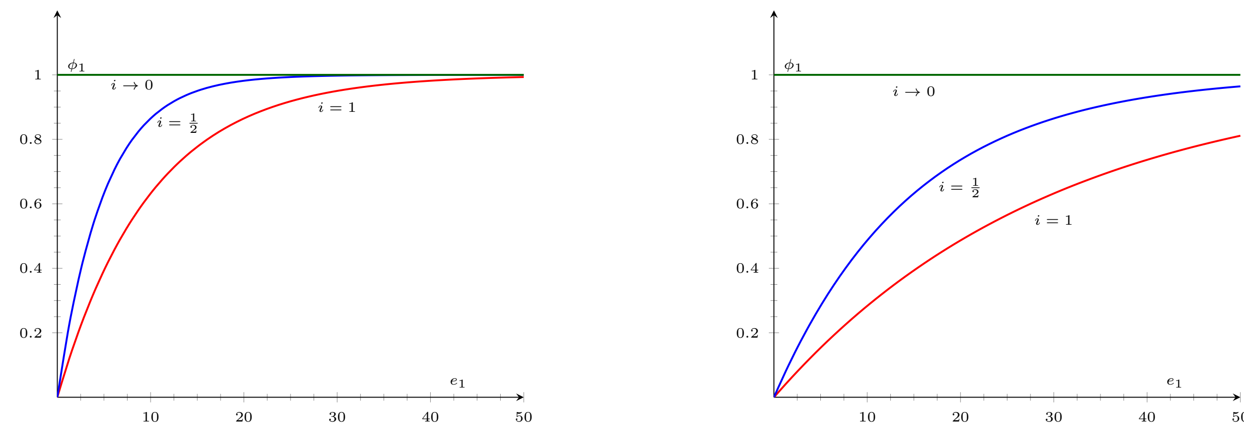


Figure 2: $\varphi_1^i(e_1) = 1 - e^{-\frac{e_1}{ki}}$ with $k = 10$ (right) and $k = 30$ (left). For the same level of sustainable effort. More demanding consumers (increasing in i) will give harsher penalties. Changing k implies only the re-scaling of the e_1 axis.

At this point it is clear that market share as expressed in equation 3 is p_1 and e_1 dependent $d_1(p_1, e_1)$.

The market can be then closed, in the isolated case, with a supply decision rule by the firm which will maximise profits:

$$\max_{p_1, e_1} \Pi = p_1 M d_1(p_1, e_1) - c_1 e_1 \tag{8}$$

Where c_1 is the cost of the behavioural effort, which is for simplicity normalizable to 1 (if we adjust the way $\varphi_1^i(\cdot)$ is constructed, in the example above changing k). One could notice that there are no references to costs for producing exactly the quantity demanded, results of the other bargains or other elements in this profit function. This is because in this simple part market isolation is allowed and as such only the decisions that pertains the final good market exclusively (p_1 and e_1, were considered. Henceforth, a good way to picture the isolation is that there was no production required due to an endless stock.

At this point, the same machinery can be put in place for the other stakeholders (as it is done in the following sub-sections), before looking for a joint profit function that sets equilibrium quantities altogether.

3.2 Input good market: Suppliers

Moving to the second stakeholder here considered, suppliers, some further discussion needs to be made. For a starter, we need to anticipate what later on will be the *production function*. In the previous subsection, indeed, it was not needed as the focus was on the pricing and sustainability choices, but the choice of inputs is necessarily linked to what is going to be made with them. In this setting, a production function is the modeling tool that represent the technology that the firm employs to transform inputs to outputs. In particular, it yields the maximum quantity of outputs attainable for a given set of inputs:

$$F : \boldsymbol{Q} \times L \times K \longrightarrow G \tag{9}$$

$$\boldsymbol{Q} \subseteq \mathbb{R}_{+}^{N} \quad L, K, G \subseteq \mathbb{R}_{+} \tag{10}$$

Where L, K and G (the spaces of possible labor, capital and output values respectively) are for now not very relevant and as such left for the following sections to handle. For now only $\boldsymbol{Q} = Q_1 \times \ldots \times Q_N$, the space of the N (physical) inputs, is considered. It is indeed reasonable to consider the number of suppliers of the company as finite, N hereafter, each of them providing a unique input $q_i, \forall i \in \{1, \ldots, N\}$ (in vector notation $\boldsymbol{q} = [q_1 \ldots q_N]$). This is however not critical and one could employ a unit mass of suppliers and get exactly the same notation as consumers (in the subsection above). A couple of further simplifying assumptions needs then to be made in their description. The first one is that every suppliers observe and value the total sustainable effort e_2 made towards their category, as consumers did. This is somewhat of a shortcoming since, contrary to this last group, it is much more conceivable that some initiatives may be carried out with only a subgroup of suppliers or that they assign different degrees of importance to different subsets of the actions that compose the total sustainable effort. There might be thus room for a more detailed theoretical description of these relationships in further research. Here this path is not pursued, firstly because it is far too specific for the purpose of this work, and secondly because keeping such an environment general enough means almost certainly introducing a great amount of theoretical objects that would prove to be irretrievable from real data.

It is allowed, however, for suppliers to care more or less about sustainable actions. Indexing them as their unique provided input:

$$i \in \{1, \ldots, N\}$$

A similar, but opposite in behaviour, modifier / penalty function $\varphi_2^i(e_2)$ is introduced:

$$\forall i \quad \varphi_2^i : \mathbb{R}^+ \longrightarrow [0, 1]$$

$$\frac{\mathrm{d}\varphi_2^i(e_2)}{\mathrm{d}e_2} \leq 0 \tag{11}$$

Which decreases in e_2. This is due to the interpretation that this function assumes: it actually serves as a synergy effect that improves the transacting relationship between the company and the single supplier, aiming at minimising costs.

As such, defining $p_2 = [p_{2,1}, \ldots, p_{2,N}]^T$ the given prices of each input (the firm is *price taker* in all inputs, an eventual reward to the suppliers in terms of paying off more than the market is encompassed in e_2), the actual cost sustained in the specific transaction with supplier i, C_i, is:

$$C_i = q_i \varphi_2^i(e_2) p_{2,i} \tag{12}$$

Where the main difference from above is that, because of the price taking scenario, the p_1 dependent term, $f(p_1)$, has been replaced with fixed ones, q_i.

And thus it is possible to outline the overall actual cost $C = \sum_{i=1}^{N} C_i$ for the company:

$$C = \sum_{i=1}^{N} q_i p_{2,i} \varphi_2^i(e_2) + c_2 e_2 \tag{13}$$

With c_2, cost of sustainable effort targeting suppliers, as only new term. It is, like it was for c_1 the previous subparagraph, normalizable to 1 provided the proper adjustments on $\varphi_2^i(\cdot)$.

With all this machinery in place, in isolation the following cost minimisation problem (for given level of labor l, capital k and output $g \geq F(\boldsymbol{q}, l, k)$) emerges:

$$\min_{\boldsymbol{q}, e_2} C = \sum_{i=1}^{N} q_i p_{2,i} \varphi_2^i(e_2) + c_2 e_2 \qquad s.t. \quad F(\boldsymbol{q}, l, k) \geq g \tag{14}$$

It is interesting to notice, moreover, that during the discussion and in (14), contrary to what was implied in the description of the final goods mechanism, $\varphi_2^i(\cdot)$ seem a "discount" that acts on the price. Some comments on this are in order. On one hand, also the one of consumers can be interpreted as a premium price rather than a demand effect, with the right mix of shuffling and redefinitions. On the other, although in a less intuitive way, also this framework can be made equivalent to the one above with the help of some matrix notation:

$$\min_{\boldsymbol{q}, e_2} \sum_{i=1}^{N} q_i p_{2,i} \varphi_2^i(e_2) + c_2 e_2 \qquad s.t. \quad F(\boldsymbol{q}, l, k) \geq g \tag{15}$$

$$\min_{\boldsymbol{q}, e_2} \sum_{i=1}^{N} q_i \varphi_2^i(e_2) p_{2,i} + c_2 e_2 \qquad s.t. \quad F(\boldsymbol{q}, l, k) \geq g \tag{16}$$

$$\min_{\boldsymbol{q}, e_2} \sum_{i=1}^{N} d_{2,i}(q_i, e_2) p_{2,i} + c_2 e_2 \qquad s.t. \quad F(\boldsymbol{q}, l, k) \geq g \tag{17}$$

$$\min_{\boldsymbol{q}, e_2} p_2' d_2(\boldsymbol{q}, e_2) + c_2 e_2 \qquad s.t. \quad F(\boldsymbol{q}, l, k) \geq g \tag{18}$$

Where I use the apostrophe *in lieu* of ^{T} in formulae to indicate transposed vectors (there will be cases in which it serves also to indicate first derivative, but the context will make the meaning clear). It is important to notice that $d_2(e_2)$ is quite different from the original (3). First of all, in is not p_2 dependent, but most importantly it is a multidimensional object ($d_2(\boldsymbol{q}, e_2) = [d_{2,1}(q_1, e_2) \ldots d_{2,N}(q_N, e_2)]^T$). Indeed, the smooth aggregation that was possible above with the integral is no longer attainable, due to the fact that each input has individual pricing (it is easy to see that if all the inputs had the same price, the problem would merely become a discretized version of the one of the final good.

3.3 Labor market: Employees

The treatment of employees (and, later on, the one of shareholders) is ultimately based on the one in place for suppliers, with the notable difference that the firm is assumed to have access to an incredibly large number of employees (this is usually the case, since relevant sustainable companies tend to be very large) and as such they are described, like consumers, with a unit mass. Again the assumption that the wage ratio for personnel, p_3, is given by the overall market condition is maintained (exactly as with suppliers, extra wage policies would be part of the sustainable effort, e_3 in this case). Interpreting again the (variable) effect of sustainable effort as a cost reduction φ_3^i are defined following (11):

$$\forall i \in [0,1] \quad \varphi_3^i : \mathbb{R}^+ \longrightarrow [0,1]$$
$$\frac{\mathrm{d}\varphi_3^i(e_3)}{\mathrm{d}e_3} \leq 0 \tag{19}$$

An alternative, maybe more natural, interpretation that is based on increased productivity rather than cost reduction could be made explicit by invoking a different definition, based instead on the one of consumers, that modifies (in the sense of "acts on") the argument of the production function rather than the cost portion. Being the two equivalent from a mathematical point of view and the former less dispersive, the model has been set up accordingly.

This means that a working definition of the cost per worker is:

$$C_i = l\phi_3^i(e_3)p_3 \tag{20}$$

where l is the (*density of*) labor per worker. Thus the overall cost becomes:

$$C = \int_0^1 l\phi_3^i(e_3)p_3 \mathrm{d}i \tag{21}$$

It is maybe worth pondering for a second on definitions (20) and (21), since they are not that obvious on their own. Indeed, they are essentially a shortcut in order to avoid linking an overall "quantity of labor" with the number of workers (each with the same productivity). Actually, the simple and more natural case just described can be equivalently represented by linking the total quantity of labor to the choice of each worker (density of) productivity, which of course has to be linearly remunerated according to p_3. Figure 3 exemplifies such an equivalency.

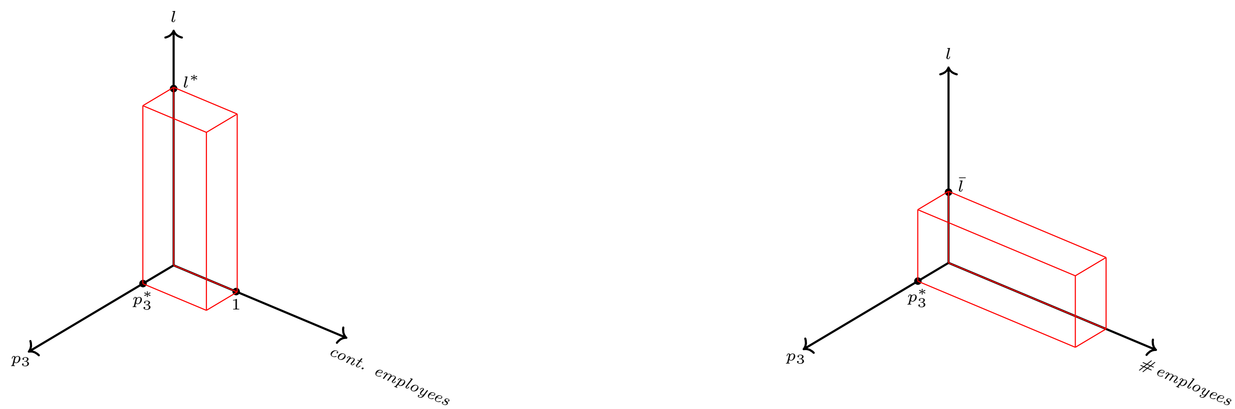


Figure 3: Continuum of empolyees between 0 and 1 with choice of productivity (left) and fixed productivity with choice of number of empolyees (right). In is clear that, with the same fixed wage p_3, total costs (the area of the parallelepiped) behaves in the same way when changing quantity of labor ($1 \times l^*$ for the left one and $\bar{l} \times \#$ employees for the right one). For any productivity choice in the continuum (i.e. making the left parallelepiped taller or shorter) there exist a corresponding choice for number of employees (i.e. making the right parallelepiped wider or narrower) that yield same quantity of labor, and it will imply same total costs.

Henceforth, the overall isolated minimisation problem for a given level of $\boldsymbol{q}$, k and y reads:

$$\min_{l,e_3} C = \int_0^1 l\phi_3^i(e_3)p_3 \mathrm{d}i - c_3e_3 \quad s.t. \quad F(\boldsymbol{q}, l, k) \geq y \tag{22}$$

quite symmetrically to what happened in the previous subsection. In order to remove the integral, it is again possible to introduce $d_3(\cdot)$ as a counterpart to the previous $d_2(\cdot)$ and $d_1(\cdot)$:

$$\begin{aligned} &\min_{l,e_3} \quad C = \int_0^1 l\phi_3^i(e_3)p_3 \mathrm{d}i - c_3e_3 \quad s.t. \quad F(\boldsymbol{q}, l, k) \geq y \\ &\min_{l,e_3} \quad l\int_0^1 \phi_3^i(e_3)\mathrm{d}i \quad p_3 - c_3e_3 \quad s.t. \quad F(\boldsymbol{q}, l, k) \geq y \end{aligned}$$

and with $d_3(l, e_3) = l\int_0^1 \phi_3^i(e_3)\mathrm{d}i$:

$$\min_{l,e_3} \quad d_3(l, e_3)p_3 - c_3e_3 \quad s.t. \quad F(\boldsymbol{q}, l, k) \geq y \tag{23}$$

With the usual comment that even c_3 in both (22) and (23) is for simplicity normalizable to 1 acting properly on $\varphi_3^i(\cdot)$.

3.4 Capital market: Shareholders

It can be easily noticed by now that, although with different interpretations, I tried to be as consistent as possible with mathematical objects and modeling tools. As such, shareholders description will resemble closely the one of employees, but different interpretations are in order. In this case, investors in the capital market are modeled as a unit mass since the company is assumed to be publicly traded in a stock market of the modern world, and the remuneration, p_4, that the firm needs to pay for a unit of capital, k, is determined again by market condition and taken as given by the firm.

In this framework, it is reasonable to think about where debt-owners and corporate debt lie. Their position depends essentially on the assumption one makes about which type of sustainability initiatives they care about: those targeting suppliers or those targeting shareholder. Indeed, in the first case, they can be simply be regarded as some k' supplier among the total N, debt costs as the relative $p_{2,k'}$ and have the production function accordingly using its contribution $q_{k'}$ be utilised in a quasi-capital fashion (it may even be that the case that in it appears only the joint $k + k'$). In the second case, instead, one needs for each level of capital an additional maximising mechanism that chooses the best mix between equity and debt and the subsequent price according to the linear combination of the starting two, weighted for their specific relevance in the mix. Here for simplicity the first route is pursued, leaving the second for further inquiry and research. Thus, only actual shareholders are here considered.

Adopting the same reasoning as the previous subparagraph, the overall cost function in this case assumes the form:

$$C = p_4 \int_0^1 k\phi_4^i(e_4)\mathrm{d}i - c_4 e_4 \tag{24}$$

Where e_4 is the total behavioural effort whose target are shareholders, and as usual its cost c_4 is normalizable to 1.

Now with (24) in mind and following (22), the isolated minimisation, given $\boldsymbol{q}$, l and y, reads:

$$\min_{k,e_4} C = p_4 \int_0^1 k\phi_4^i(e_4)\mathrm{d}i - c_4 e_4 \quad s.t. \quad F(\boldsymbol{q}, l, k) \geq y \tag{25}$$

And introducing the notation $d_4(k, e_4) = \int_0^1 k\phi_4^i(e_4)\mathrm{d}i$, then (25) becomes:

$$\min_{k,e_4} C = p_4 d_4(k, e_4) - c_4 e_4 \quad s.t. \quad F(\boldsymbol{q}, l, k) \geq y \tag{26}$$

3.5 The firm as a whole: general equilibrium across stakeholder markets

Once the partial equilibrium quantities have been outlined, a further step is taken here in order to achieve their integration and define the complete problem that the firm faces at once.

The production function has already been introduced in order to present the constraint of the cost minimisation problem of the different input type, but it is maybe worth here spend some more time on its characteristic and properties. As said, it yields some quantity of output $g \in G \subseteq \mathbb{R}_+$, that represent the maximum attainable *within* G (i.e. *produced*) from given quantities of (types of inputs) $\boldsymbol{q} \in \boldsymbol{Q} = Q_1 \times \dots Q_N \subseteq \mathbb{R}_+ \times \dots \times \mathbb{R}_+ = \mathbb{R}^N$, $l \in L \subseteq \mathbb{R}_+$ and $k \in K \subseteq \mathbb{R}_+$. In order to better outline the properties it is (reasonably) assumed to have, it is worth to built an alternative description of the technology of the firm resorting to a *production set*.

$$Y = \{(g, -\boldsymbol{q}, -l, -k, -e_1, -e_2, -e_3, -e_4) : g \leq F(\boldsymbol{q}, l, k)\} \tag{27}$$

Which is clearly, by the way it is defined, a subset of $\mathbb{R}^{N+7}$. It is thus composed by vectors y in which outputs (from the point of view of the company) take the plus sign, while inputs take negative ones. This has been done to keep the notation consistent with the production set described in Mas-Colell, Whinston and Green (1995).

For the following discussion, the following assumptions on Y are considered:

- nonempty, so that there exists a viable technology for the firm two transform inputs into outputs. $Y \neq \emptyset$.
- closed, so that if $y_n \in Y \quad \forall n$ and $y_n \to y$, then $y \in Y$.
- free disposal, so that it is always possible to use additional units of inputs in order to reach the same level of output (basically "wasting" the extra units employed: $Y - R_+^{N+7} \subset Y$.

Another relevant requirement to add to the specific setting is that the firm is allowed to overproduce (i.e. having stocked items left), but it needs to produce at least enough output to cover the market share that it claims. This is key since, contrary to other settings, here the final good market does not infinitely absorb any quantity of output. Rather, remembering from subsection 3.1, the demanded quantity depends on the price (p_1 chosen by the firm) and the sustainable initiatives towards consumers (e_1) which jointly determine the market share, and overall demand multiplying it with overall quantity demanded M. It is possible to assume, thus, that calling (as before) g the overall quantity produced, we have, mutuating notation from (3):

$$g \geq Md_1(p_1, e_1) \tag{28}$$

With these two premises, having in mind (8), (18), (23) and (26) the integrated maximization process becomes:

$$\begin{aligned} \max_{p_1, \boldsymbol{q}, l, k, e_1, e_2, e_3, e_4} \quad \Pi = & Md_1(p_1, e_1)p_1 - d_2(\boldsymbol{q}, e_2)p_2 - d_3(l, e_3)p_3 - d_4(k, e_4)p_4 \\ & -c_1e_1 - c_2e_2 - c_3e_3 - c_4e_4 \\ & s.t. \quad Md_1(p_1, e_1) \leq g \leq F(\boldsymbol{q}, l, k) \end{aligned} \tag{29}$$

And the notation can be eased since now g is completely redundant (as long as the quantity produced is both feasible and coherent with the created demand, since the value of eventual remaining stock is not considered), and the following matrix notation can be added:

$$\boldsymbol{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad \boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} p_{2,1} \\ \dots \\ p_{2,N} \\ p_3 \\ p_4 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \quad d(\boldsymbol{q}, l, k, \boldsymbol{e}_{-1}) = \begin{bmatrix} d_2(\boldsymbol{q}, e_2) \\ d_3(l, e_3) \\ d_4(k, e_4) \end{bmatrix}$$

With the further definitions of $\boldsymbol{e}_{-1}$, that stands stands for all behavioural efforts *except* the one targeting consumers, and $\boldsymbol{p}_{-1}$, which is the vector of all prices excluding the final good selling price.

Henceforth, expression (29) can be rephrased as:

$$\max_{p_1, \boldsymbol{q}, l, k, \boldsymbol{e}} M d_1(p_1, e_1) p_1 - \boldsymbol{p}'_{-1} \begin{bmatrix} d(\boldsymbol{q}, l, k, \boldsymbol{e}_{-1}) \\ \boldsymbol{e} \end{bmatrix} \quad s.t. \quad M d_1(p_1, e_1) \leq F(\boldsymbol{q}, l, k) \tag{30}$$

Furthermore, it is possible to define a *profit function* $\pi(\cdot)$, that yields the value of the maximization problem (30) for all the levels of $\boldsymbol{p}_{-1}$:

$$\pi(\boldsymbol{p}_{-1}) : \mathbb{R}_+^{N+2} \longrightarrow \mathbb{R} \tag{31}$$

And, conversely, the argmax of the same problem as $y(\cdot)$:

$$y(\boldsymbol{p}_{-1}) : \mathbb{R}_+^{N+2} \longrightarrow \mathcal{P}(Y) \subseteq \mathcal{P}(\mathbb{R}^{N+7}) \tag{32}$$

Where $y(\cdot)$ is a *correspondence* and as such insists on the power set of Y. This happens both because apriori there might be different regions of Y that attain the same level of profits, but most importantly because g is always allowed to be in the range $d_1(p_1, e_1) \leq g \leq F(\boldsymbol{q}, l, k)$ and as such, other components being equal, there might be more than one compatible level of actually produced output g.

Since the presence of extra values in such correspondence due to this last reason seems redundant, it is possible to leave only interesting solutions by assuming that the firm will always chose to produce exactly the quantity demanded without wasting any inputs (which seems reasonable so it does not harm the framework that much):

$$Md_1(p_1, e_1) = F(\boldsymbol{q}, l, k) = g \tag{33}$$

And as such it is possible to forget any complementary slackness condition when framing the problem with Lagrangian multipliers:

$$\mathcal{L} = Md_1(p_1, e_1)p_1 - \boldsymbol{p}'_{-1} \begin{bmatrix} d(\boldsymbol{q}, l, k, \boldsymbol{e}_{-1}) \\ \boldsymbol{e} \end{bmatrix} - \lambda(Md_1(p_1, e_1) - F(\boldsymbol{q}, l, k)) \tag{34}$$

So that, provided differentiability of all the φ-functions, the f_1 and the F functions, any optimum $y^* \in y(\boldsymbol{p})$ satisfies the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial p_1} = 0 \Longrightarrow f(p_1) = (\lambda - p_1)f'(p_1) \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial q_i} = 0 \Longrightarrow p_{2,i}\varphi_2^i(e_2) = \lambda \frac{\partial F(\boldsymbol{q}, l, k)}{\partial q_i} \quad \forall i \in \{1, \dots, N\} \tag{36}$$

$$\frac{\partial \mathcal{L}}{\partial l} = 0 \Longrightarrow p_3 \int_0^1 \varphi_3^i(e_3)\mathrm{d}i = \lambda \frac{\partial F(\boldsymbol{q}, l, k)}{\partial l} \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial k} = 0 \Longrightarrow p_4 \int_0^1 \varphi_4^i(e_4)\mathrm{d}i = \lambda \frac{\partial F(\boldsymbol{q}, l, k)}{\partial k} \tag{38}$$

$$\frac{\partial \mathcal{L}}{\partial e_1} = 0 \Longrightarrow (p_1 - \lambda)\left(f_1(p_1) \int_0^1 \varphi_1^{i'}(e_1)\mathrm{d}i\right) = 1 \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = 0 \Longrightarrow \sum_{i=1}^{N} q_i p_{2,i} \varphi_2^{i'}(e_2) = -1 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial e_3} = 0 \Longrightarrow lp_3 \int_0^1 \varphi_3^{i'}(e_3)\mathrm{d}i = -1 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial e_4} = 0 \Longrightarrow kp_4 \int_0^1 \varphi_4^{i'}(e_4)\mathrm{d}i = -1 \tag{42}$$

Noting that derivatives and integrals have been swapped ($\frac{\partial}{\partial e}\int_0^1 \varphi^i(e)\mathrm{d}i$ to $\int_0^1 \varphi^{i'}(e)\mathrm{d}i$) thanks to the Leibnitz rule, which in its most general version works as follows:

$$\frac{\partial}{\partial x}\int_{a(x)}^{b(x)} f(x, y)\mathrm{d}y = b'(x)f(x, b(x)) - a'(x)f(x, a(x)) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, y)\mathrm{d}y \tag{43}$$

Equations (35)-(42), given the assumptions, completely characterize the solutions in $y(\boldsymbol{p})$.

3.6 Towards a testable theory

A final addition to this work's theoretical part that pertains the single firm regards the possibility of falsifying the framework presented. With this in mind, a good starting point is noticing that (from (34)) $\pi(\cdot)$ can be expressed in matrix notation $\forall y^* \in y(\boldsymbol{p}_{-1})$ as:

$$\pi(\boldsymbol{p}_{-1}) = \boldsymbol{p}' Y(y^*)\psi(y^*) \tag{44}$$

Since $\lambda(Md_1(p_1, e_1) - F(\boldsymbol{q}, l, k)) = 0$ in equilibrium. The newly defined objects in (44) are outlined as follows:

$$Y(y^*) = \begin{bmatrix} Mf_1^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_1^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -q_N^* & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -l^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -e_1^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_2^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_3^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4^* \end{bmatrix} \qquad \psi(y^*) = \begin{bmatrix} \int_0^1 \varphi_1^i(e_1^*)\mathrm{d}i \\ \varphi_2^i(e_2^*) \\ \dots \\ \varphi_2^N(e_2^*) \\ \int_0^1 \varphi_3^i(e_3^*)\mathrm{d}i \\ \int_0^1 \varphi_4^i(e_4^*)\mathrm{d}i \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{45}$$

Thus, they are respectively $(N+7) \times (N+7)$ matrix and $(N+7) \times 1$ vector. The way that these elements are grouped is not casual. Indeed, this decomposition is built such that all the elements are easily observable *except for those that compose* ψ.

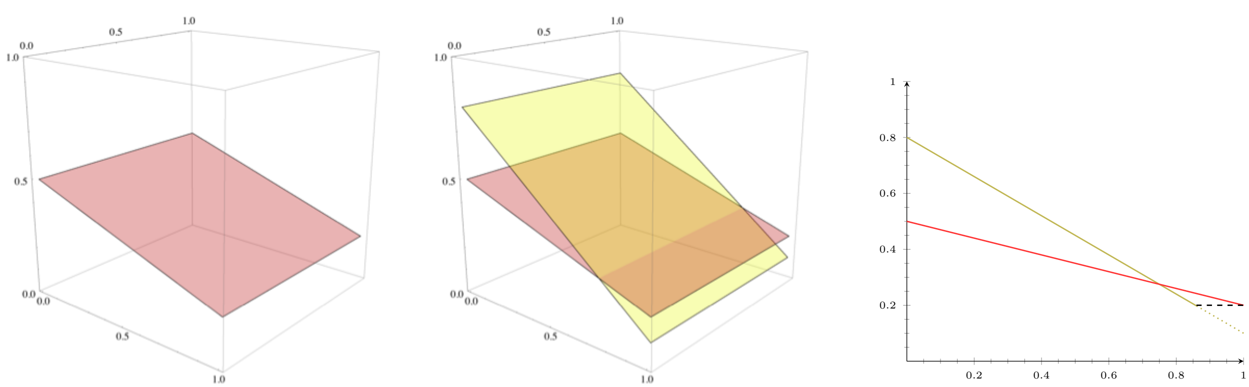
Future directions of research might involve hunting for ψ with relevant field experiments in order to get some estimate $\bar{\psi}$, but here, retrieving data everywhere they are already available. the following linear system with unknown vector ψ arises:

$$\bar{\pi} = \boldsymbol{p}' \bar{Y} \psi(y^*) \tag{46}$$

Which is of the form $a = \boldsymbol{v}'\boldsymbol{u}$ where a is a scalar and v and u are equally shaped vectors. In such problems, there are infinite solutions and in this particular case ψ is left not identified. This however does not mean that nothing can be said about ψ. Indeed, although infinite, all the possible value for ψ (i.e. solutions of the problem (46)) lie in an hyper-plane due to the loss of a degree of freedom. Not only that, but the solution must be within an hyper-cube of area 1, since by the way the individual elements are built (i.e. all within 0 and 1, and the last four exactly 1). Figure 4 exemplifies the situation.

It is possible to reduce the possible values of ψ even further. Indeed, one can rely on multiple observations: assuming the various $\varphi(\cdot)$ constant across years and within sector, it is possible to record various $y \in y(\boldsymbol{p})$ and each of them will yield a different hyper-plane of possible ψ. Unfortunately, no ψ need to be on these two hyper-planes *at the same time*, since it is important to remember that different argmax will possibly present different $\boldsymbol{e}$, altering the "true " value of ψ (since, as it was specified at the beginning, it depends on $\boldsymbol{e}$). Thus, the intersection of these plane carry no particular meaning, but it is still possible to shave off some regions from them, thanks to the fact that the sign of the first derivatives of the various φ is known.

To illustrate the concept clearly, it might be worth to think about a simplified case with two of these different observations and only three dimensions (instead of $N + 7$), none of which is of the kind that has fixed value 1. Considering ψ_1 as the dependent variable in this reduced case, a generic plane for *psi* under observation 1 would be as in figure 4 (a). A second observation might induce a different plane (figure 4 (b). Now suppose that from the observed data it is a known fact that e_1 is higher in the second observation than it was in the first one. Using the assumption on the derivative of ϕ_1 (having it with positive sign in this simplified setting), one can conclude in which direction ψ_1 must have moved from observation 1 to observation 2. In particular, since e_1 is higher in observation 2, all the regions of its plane that assign a lower value to ψ_1 to the minimum value that ψ_1 assumes in observation 1's plane can be removed, since they are not consistent with the framework (ψ_1 cannot be *strictly lower*, figure 4 (c)). According to the same reasoning, since e_1 is lower in observation 1, all values of the corresponding plane that assigns to ψ_1 a higher value than the maximum value that ψ_1 assumes under observation 2 can as well be removed (there are none in the example of figure 4).



(a) Observation 1

(b) Observation 2 (yellow) plotted together with the first observation

(c) Lateral view of the ψ space. The portion of Observation 2 below the minimum of Observation 1 is removed

Figure 4: Process of using multiple empirical evidences in order to restrict the possible values of ψ.

The process just described can easily be applied to the actual ψ and be iterated as many times as there are data available that maintain with certain confidence the same stakeholder framework, technology and stakeholder's caring about sustainability. The results will imply narrower possible regions of ψ *for some level of* $\boldsymbol{e}$ and this can help assess the relevancy of sustainable effort.

Indeed, if this were not the case (i.e. sustainability does not matter) we should observe $\psi = \mathbf{1}$ a constant vectors of 1s and the framework described in this work would shrink quite rapidly to a classical marginalist approach exercise. Moreover, it would indirectly mean that any firm that pursue positive behavioural effort e in any way is not profit maximising (at least not in the way here presented).

Thus, a first and quite basic test with real data to address the solidity of this approach is to check whether or not regions that arise from this iterative process are consistently distinct from the top-left corner of the hyper-cube. This approach is pursued in the last section, where sampled data are examined through the lens of the described procedure.

4 Feasibility of multiple strategies: a duopolistic approach

The test just described is based on the fact that multiple approaches and strategies are non dominated when it comes to profit maximization. Indeed, this has to be the case in order to witness different $y^* \in y(\boldsymbol{p})$, given the same framework (ideally, some with high behavioural effort and premium prices and some more cost efficient). To check the soundness of this assumption a step further is required. If one remembers subsection 3.1, the likelihood of the consumers to buy or not from the company was defined having in mind *some default alternative option*. This is now called into question: a second, identical firm is introduced to share the final good market. To further simplify the concept, the two are assumed to fully divide the market among themselves in terms of market share (i.e. no other competitor) and that their choices do not affect the total demanded quantity (i.e. no possibility of no purchase), but only their relative share. Moreover, all the other stakeholder relationships apart from the final good market are considered separate, so that there is no strategic interaction and competition for employees, shareholders and suppliers: in these respects the two firms act independently and according to the previous setup.

There are several ways to treat such a scenario, and one of the most common is certainly based on representative consumer models (with the most insightful probably being Dixit & Stiglitz, 1977). In order to stay close to the framework presented above, however, the solution here presented is closer to what could be considered a "spatial model" (like the one illustrated in Mas-Colell, Whinston and Green, 1995, or, more specifically in Salop, 1979). Unique features that distinguish the present work in this sense from the cited one is twofold. First of all, the concept of consumer *likelihood to buy* drastically modifies the interpretation. Secondly, competition happens both on a price level, choosing p_1 (à la Bertrand), but at the same time on the side quantities of sustainable actions.

4.1 Reaching explicit functional forms for the final good market

This discussion has already imposed some restrictions on the structure of the market and it might be worthwhile to make them explicit, so that the problem can be completely characterized in its relevant variables. In the following discussion, the superscripts j and $-j$ will be used to refer to the one and the other firms in the duopoly. Moreover, results will be derived only for j, with $-j$ being completely symmetric (inverting superscripts suffices, since in principle the two firms are identical).

A first modification pertains the nature of total demand M. Indeed, in the one firm case the fact that the firm was not affecting it was due to the fact that it was dealing with a much bigger and broader environment. In the duopoly case this does not happen. On the contrary, it makes sense to assume the existence of a substitute good that shrinks total demand in the final good market the less competitive it becomes. Thus, the effect of the firms' strategic choices is twofold: there is an overall effect on total demand, and a relative effect on market shares. Now two main ways are available to treat this case: having consumers modeled with an explicit utility function (see for instance Singh & Vives, 1984) or reaching equivalently rational result introducing reasonable assumptions on what their actual behaviour in order to make it coherent with the homo oeconomicus expected one. Here the second one has been chosen, essentially because it is a shortcut on the analysis implied by the former and allows for a more direct treatment of consumers' reactions when facing with both a strategic choice in a quantity and one in a price. In this sense M is assumed to be directly decreasing in both prices made by the firms (coherent with Fanti, Gori and Sodini, 2015):

$$M = M(p_1^j, p_1^{-j}) = m - \frac{(p_1^j + p_1^{-j})^2}{4} \tag{47}$$

Moving to consider market shares and likelihoods, suppose for now that the two firms undertake the same behavioural effort ($e_1^j = e_1^{-j}$). In such a case, it is reasonable to assume that the price of the final good influences market share completely, through $f_1(\cdot)$. The following must therefore hold:

1. $f_1^j(p_1^j, p_1^{-j}) : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow [0, 1]$
2. $f_1^j(x, y) = f_1^{-j}(x, y)$
3. $\frac{\partial f_1^j(p_1^j, p_1^{-j})}{\partial p_1^j} < 0$
4. $\frac{\partial f_1^j(p_1^j, p_1^{-j})}{\partial p_1^{-j}} > 0$
5. $\lim_{p_1^j \to 0} f_1^j(p_1^j, p_1^{-j}) \to 1 \quad \forall p_1^{-j} \neq 0$
6. $\lim_{p_1^{-j} \to 0} f_1^j(p_1^j, p_1^{-j}) \to 0 \quad \forall p_1^j \neq 0$
7. $\lim_{p_1^j \to +\infty} f_1^j(p_1^j, p_1^{-j}) \to 0 \quad \forall p_1^{-j} \neq +\infty$
8. $\lim_{p_1^{-j} \to +\infty} f_1^j(p_1^j, p_1^{-j}) \to 1 \quad \forall p_1^j \neq +\infty$
9. $f_1^j(p_1^j, p_1^{-j}) + f_1^{-j}(p_1^{-j}, p_1^j) = 1$

The first requirement ask for the same codomain as in the single firm setting, de facto building upon the previous $f_1(\cdot)$. The second is a symmetry requirement: being equivalent in anything, there is no reason for consumers to discriminate among the two firms when their price are swapped (i.e if firm's j price were to become firm $-j$ price and also the reverse swap happened, then the new firm's j market share would be the before-swap firm's $-j$ market share, and the contrary as well). According to the third and fourth requirements, market share increase the more the company's price is competitive (lower), and conversely suffers from increased competitiveness of the rival. The fifth and sixth requirements pertain limit behaviour in a price almost zero scenarios: the fifth ask for the market share to approximate one when the firm continues to lower its price (until basically giving the good for free) and the competitor keeps a strictly positive one, while the sixth ask conversely that if it is the competitor to lower and lower its price while the firm keeps a positive one, then own market share should approach zero. The seventh and eighth requirements dictate requirements for the opposite limit in the domain of p_1: when one of the two firms pushes its price up to infinity while the other keeps a finite one, the markets share of the first converges to 0. Finally, the ninth requirement imposes that for any level of prices, the two market shares saturate the market. Total demand is assumed insensible to firm level choices and as such market share should be multiplied by total demand $M(p_1^j, p_1^{-j})$.

Now, there are many functions that satisfy the requirements above, but in order to produce interesting results the simplest and more minimal one has been chosen:

$$f_1^j(p_1^j, p_1^{-j}) = \frac{(p_1^{-j})^2}{(p_1^{-j})^2 + (p_1^j)^2} \tag{48}$$

And obviously the converse $f_1^{-j}(p_1^{-j}, p_1^j) = \frac{(p_1^j)^2}{(p_1^{-j})^2 + (p_1^j)^2}$. It is clear that both prices cannot be jointly 0. For simplicity, both of them are required to be arbitrarily small but still strictly positive, justifying the usage of the limit notation above.

In principle, all $f_1^j(p_1^j, p_1^{-j}) = \frac{(p_1^{-j})^k}{(p_1^{-j})^k + (p_1^j)^k}, k \in \mathcal{N}_+$ satisfy the nine requirements and the specific choice of the power of 2 is purely pragmatic, since it was the one that yielded the simplest computations. Different k do not however alter the behaviour of the framework significantly: the satisfaction of the requirements already imply a well-behaved function for the applications here considered.

Moving to consider the effect of the behavioural effort, however, the translation in this new environment from the default choice option is trickier. Indeed, before it was simply a penalty on the likelihood that depended on the relative caring of the particular individual and the actual effort produced by the firm. Now, instead, the gain (loss) in terms of market share by a firm needs to be *perfectly counterbalanced* by an opposite loss (gain) of the other. Moreover, ideally the effect of behavioural effort should be more decisive when prices are closer together. All these complications imply henceforth extra care when building $\varphi_1^{i,j}(\cdot)$ and $\varphi_1^{i,-j}(\cdot)$ than what was requested for f_1^j and f_1^{-j}.

Reasoning step by step, $\varphi_1^{i,j}(\cdot)$ needs a core that increases in sustainable effort, decrease in the other firm's sustainable effort (indirectly, since this last one increases its share with it) and as such is anti-symmetric. Moreover, it is required, as modifier of $f_1^j(\cdot)$ which is between 0 and 1, to be bounded (for simplicity between -1 and 1):

1. $core^j(e_1^j, e_1^{-j}) : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow [-1, +1]$
2. $core^j(e_1^j, e_1^{-j}) + core^{-j}(e_1^{-j}, e_1^j) = 0$

3. $\frac{\partial core^j(e_1^j, e_1^{-j})}{\partial e_1^j} > 0$

4. $\frac{\partial core^j(e_1^j, e_1^{-j})}{\partial e_1^{-j}} < 0$

5. $\lim_{e_1^j \to 0} core^j(e_1^j, e_1^{-j}) \to -1 \quad \forall e_1^{-j} \neq 0$

6. $\lim_{e_1^{-j} \to 0} core^j(e_1^j, e_1^{-j}) \to 1 \quad \forall e_1^j \neq 0$

7. $\lim_{e_1^j \to +\infty} core^j(e_1^j, e_1^{-j}) \to 1 \quad \forall e_1^{-j} \neq +\infty$

8. $\lim_{e_1^{-j} \to +\infty} core^j(e_1^j, e_1^{-j}) \to -1 \quad \forall e_1^j \neq +\infty$

Where the limit behaviour is again straightforwards: the firms gets the maximum "behavioural bonus" either when its effort is infinitely large or when, although finite, it is the only one existing (i.e. the competitor is not engaging in sustainable activities). Conversely, it attains the maximum "behavioural malus" (all the bonus is awarded to the rival and as such the firms share is negatively affected) when the competitor is the only one with positive sustainable effort or it is infinitely large. The easiest functional form that satisfy all this is:

$$\frac{e_1^j - e_1^{-j}}{e_1^j + e_1^{-j}} \tag{49}$$

And the converse $\frac{e_1^{-j} - e_1^j}{e_1^{-j} + e_1^j}$. This expression if however not ready to be directly multiplied by $f_1^j(\cdot)$ to yield consumer i likelihood as it is, at it would potentially yield negative values. This modifier needs thus to be re-scaled and re-centered. The rescaling is a perfect occasion to satisfy also the wanted property of the behavioural component becoming more crucial when prices are close together. It is sufficient in this sense to use the competitor's market share as scale. The economic interpretation of this choice relies upon the notion that such share is the total possible "prize" that investing in sustainable effort can provide. The reason why this also implies that the effect is more relevant the closer the price will be clear in a moment. Finally, it is needed also the i dependence and the translation that allow the function to be centered in 0. The overall $\varphi_1^{i,j}(\cdot)$ now reads:

$$\varphi_1^{i,j}(e_1^j, e_1^{-j}) = 1 + \frac{(p_1^j)^2}{(p_1^{-j})^2 + (p_1^j)^2} \frac{e_1^j - e_1^{-j}}{e_1^j + e_1^{-j}} i \tag{50}$$

It is worth noticing that, contrary to the single firm above, this function is not necessarily bounded to one. This happens because while above $\varphi_1^{i,j}(\cdot)$ was merely a penalty, here the penalty of someone needs to be the reward of someone else. However, thanks to the rescaling and recentering the overall likelihood behaves exactly as the one defined in the one firm case. It reads:

$$c_i^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}) = f_1^j(\cdot)\varphi_1^{i,j}(\cdot) = \frac{(p_1^{-j})^2}{(p_1^{-j})^2 + (p_1^j)^2} \left(1 + \frac{(p_1^j)^2}{(p_1^{-j})^2 + (p_1^j)^2} \frac{e_1^j - e_1^{-j}}{e_1^j + e_1^{-j}} i\right) \tag{51}$$

Which does assumes values only between 0 and 1 (no negative likelihoods and "more than certain" ones) and it also satisfy the market completeness requirement $c_i^j(\cdot) + c_i^{-j}(\cdot) = 1$, where $c_i^{-j}(\cdot)$ is defined symmetrically in the usual way:

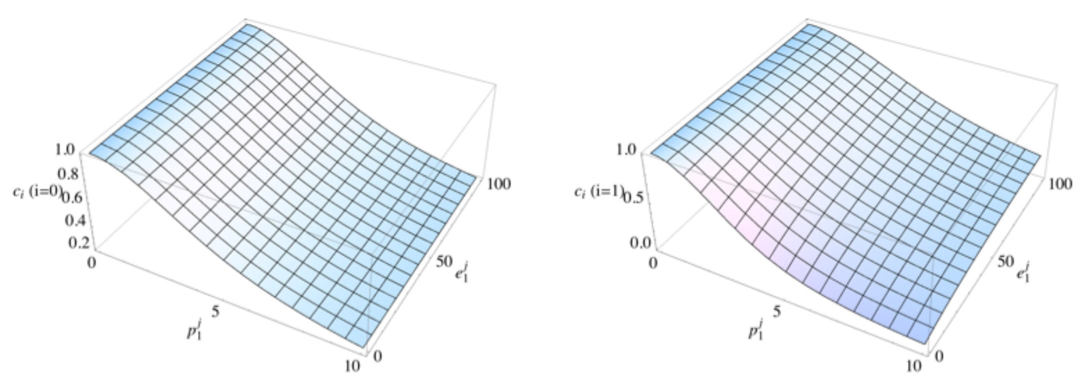
$$\begin{aligned} &\forall i, \quad \forall p_1^j, p_1^{-j}, e_1^{-j}, e_1^j \\ &c_i^j + c_i^{-j} = 1 \\ &c_i^j, c_i^{-j} \in [0,1] \end{aligned}$$

It is now possible to see why incidentally the scale chosen also satisfy the "relevance with closer prices" hypothesis. Indeed, expression (51) can be set equivalently by doing the multiplication as:

$$c_i^j(\cdot) = f_1^j(\cdot) + f_1^j(\cdot) f_1^{-j}(\cdot) \frac{e_1^j - e_1^{-j}}{e_1^j + e_1^{-j}} i \tag{52}$$

Where the product that premultiplies the core, being composed by two functions in $[0,1]$ and one being the complement to one of the other, reaches its maximum when its two components are both $\frac{1}{2}$ and strictly decreases from there.

With this a minimal working framework for the likelihood behaviour in the duopoly case is thus provided. This formulation will be used in further subsection to investigate more in depth the possible strategic actions that the two firms will rationally undertake. What is actually happening on the likelihood among consumers with different types can be better seen graphically with some excitation numbers for the competitor's behaviour and the market structure. Figure 5 shows the behaviour of the insensible consumer $i = 0$ and the most caring one $i = 1$:



(a) Likelihood of consumer $i = 0$ (b) Likelihood of consumer $i = 0$

Figure 5: Likelihood evolution varying p_1^j and e_1^j, for consumers $i = 0$ and $i = 1$

As it can be seen, consumer $i = 0$ is completely insensible to changes in the e_1^j axis. Conversely, consumer $i = 1$ shapes her behaviour in a much more transitional way when it comes to changes in both e_1^j and p_1^j.

A last point that needs to be made explicit in this subsection, however, is the explicit formulation of d_1s (the actual market shares, with demand being that value times M, the fixed value of total demand), which follow pretty straight-forwardly from (51):

$$\begin{aligned} d_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}) &= \int_0^1 c_i^j(\cdot)\mathrm{d}i = \int_0^1 f_1^j(\cdot)\varphi_1^{i,j}(\cdot)\mathrm{d}i \\ &= \int_0^1 \frac{(p_1^{-j})^2}{(p_1^{-j})^2 + (p_1^j)^2}\left(1 + \frac{(p_1^j)^2}{(p_1^{-j})^2 + (p_1^j)^2}\frac{e_1^j - e_1^{-j}}{e_1^j + e_1^{-j}}i\right) i \end{aligned} \tag{53}$$

And, by the way they are defined (again, $d_1^{-j}(\cdot)$ is the same but with opposite superscripts), they maintain all of the properties of c_i^j and c_i^{-j}, de facto behaving equivalently (in the firms own choice parameter) to the one of the single firm scenario:

$$\begin{aligned} &\forall p_1^j, p_1^{-j}, e_1^{-j}, e_1^j \\ &d_1^j + d_1^{-j} = 1 \\ &d_1^j, d_1^{-j} \in [0, 1] \end{aligned}$$

4.2 Minimisation, maximization and best responses

Since the two firms compete directly only in the final good market, it is possible to consider separately a minimisation problem that yields the minimum level of all other costs present in (29) for each level of required output g:

$$\begin{aligned} \min_{\boldsymbol{q}, l, k, e_2, e_3, e_4} \quad & d_2(\boldsymbol{q}, e_2)p_2 + d_3(l, e_3)p_3 + d_4(k, e_4)p_4 + c_2e_2 + c_3e_3 + c_4e_4 \\ & s.t. \quad F(\boldsymbol{q}, l, k) = g \end{aligned} \tag{54}$$

and calling this value, depending on g and the external prices $\boldsymbol{p}_{-(p_1,c_1)}$ (the price vector defined in the previous paragraph, *without* the element that collected the unitary cost of the behavioural effort in the final good market), $c(g, \boldsymbol{p}_{-(p_1,c_1)})$, it is possible to reinterpret the profit maximization problem as a two stage process, the first one being the identification of $c(g, \boldsymbol{p}_{-(p_1,c_1)})$. The full discussion on this equivalence can be found on Mas-Colell, Whinston and Green (1995), but as far as this work is concerned the following rewriting of (29) using (54) holds:

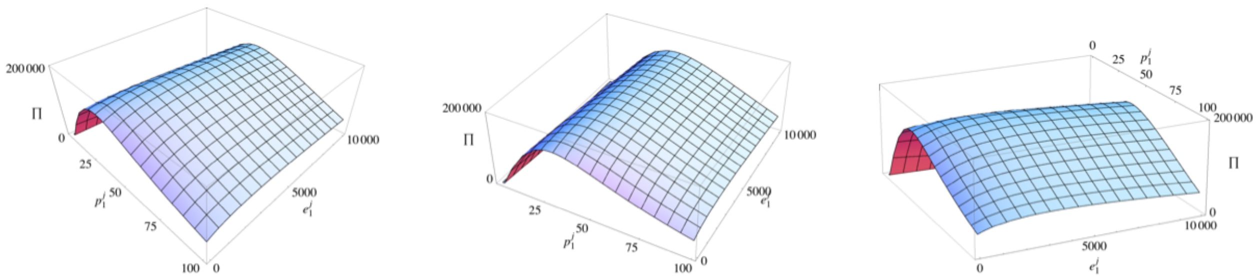
$$\max_{p_1, e_1} p_1 M(\cdot) d_1(\cdot) - c_1 e_1 - c(M(\cdot)d_1(\cdot), \boldsymbol{p}_{-(p_1,c_1)}) \tag{55}$$

Which means that from the point of view of j (as said, $-j$ is symmetric) the profit maximization problem can be rewritten as:

$$\max_{p_1^j, e_1^j} \Pi = p_1^j M(p_1^j, p_1^{-j}) d_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}) - c_1 e_1 - c(M(p_1^j, p_1^{-j}) d_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}), \boldsymbol{p}_{-(p_1,c_1)}) \tag{56}$$

This is extremely useful, since the inner working of $c(\cdot)$ is still developed in isolation and the other firm's choices do not affect it (apart from the effect through output). Henceforth, from the strategic interaction point of view problem (56) (and its $-j$ counterpart) are all that is required.

Again, a better understanding of the forces at play can be achieved taking into consideration some graphical insight. Figure 6 plots the profit function for a fixed behaviour of $-j$, where the axis represent the two choices p_1^j and e_1^j.



(a) Profit function, balanced view

(b) Profit function, side of p_1^j

(c) Profit function, side of e_1^j

Figure 6: Profit function varying p_1^j and e_1^j, from different angles

As it can be seen, the profit function is initially steeply increasing in price, with a slower decreasing part after the peak. Steepness of decline and point of peak are also influenced by the relative amount of e_1^j, but for higher and higher values also in that direction a decreasing trend dominates the graph.

Given the assumptions (and as it can be clearly seen in the figure), the first order conditions are necessary and sufficient to identify the point of maximum. As such, the quest for optimal values can proceed putting the first derivative of the profit function with respect to p_1^j equals to zero:

$$\frac{\partial \Pi}{\partial p_1^j} = 0 \Longrightarrow \frac{(p_1^j d_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}))}{\partial p_1^j} - \frac{\partial c(\cdot)}{\partial p_1^j} = 0 \tag{57}$$

and the same goes for e_1^j:

$$\frac{\partial \Pi}{\partial e_1^j} = 0 \Longrightarrow \frac{(p_1^j M d_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}))}{\partial p_1^j} - c_1^j - \frac{\partial c(\cdot)}{\partial e_1^j} = 0 \tag{58}$$

In order to treat these it is necessary to assume something for $\frac{\partial c(\cdot)}{\partial p_1^j}$ and $\frac{\partial c(\cdot)}{\partial e_1^j}$. Indeed, in order to find these one would need to completely specify the functional form of the minimisation problem as well. However, since it is not the focal point of the strategic interaction and due to the vast amount of assumptions one would need to make on the production function and the other φs, it is possible to avoid such hindrances with a single assumption on $c(\cdot)$ itself.

Indeed, one can easily notice that the composed functions derivative rule applies here:

$$\frac{\partial c(\cdot)}{\partial p_1^j} = \frac{\partial c(Md_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}), \boldsymbol{p}_{-(p_1,c_1)})}{\partial Md_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j})} \frac{\partial Md_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j})}{\partial p_1^j} \tag{59}$$

$$\frac{\partial c(\cdot)}{\partial e_1^j} = \frac{\partial c(Md_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j}), \boldsymbol{p}_{-(p_1,c_1)})}{\partial Md_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j})} \frac{\partial Md_1^j(p_1^j, p_1^{-j}, e_1^j, e_1^{-j})}{\partial e_1^j} \tag{60}$$

Where the second component of both is already known with the assumptions made on $d_1(\cdot)$ and $M(\cdot)$. What remains to be determined is the growth of the cost function over its own constraint. In the two equations above, this corresponds to the first component of the splitting. For simplicity of treatment, it is assumed constant k. Indeed, it is not needed a strictly convex function in this case in order to assure the existence of an optimum coming from the first order conditions, due to the fact that collapsing market share as price increases already implies the fact that the profit function is well-behaved (and given the cost function convex, even if not strictly).

Reducing the system generated by equations (58) and (58) in order to isolate p_1^j and e_1^j would mean find the best responses of the choices depending on the competitor ones. This however is quite tricky since, substituting all the values in the derivatives, one has:

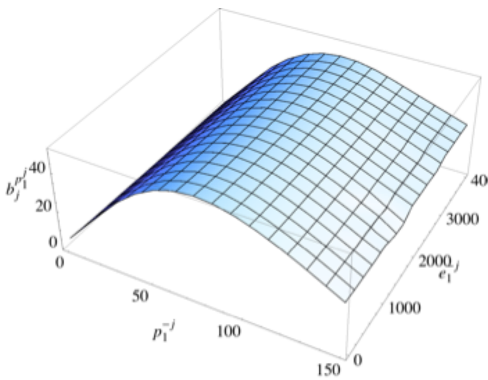
$$\begin{aligned}\frac{\partial \Pi}{\partial p_1^j} = &\frac{(p_1^{-j})^2}{8((p_1^{-j})^2 + (p_1^j)^2)^3(e_1^{-j} + e_1^j)}[e_1^{-j}(-2(p_1^{-j})^6 + 4k(p_1^{-j})^5 + 8m(p_1^{-j})^4 \\ &- 2(p_1^{-j})^2(4(p_1^{-j})^3 + k(p_1^{-j})^2 - 12km)(p_1^j) - 3(p_1^{-j})^2((p_1^{-j})^2 + 2k(p_1^{-j}) + 4m)(p_1^j)^2 \\ &- 2(k(p_1^{-j})^2 - 4km)(p_1^j)^3 - 2((p_1^{-j})^2 + k(p_1^{-j}) + 2m)(p_1^j)^4 - (p_1^j)^6) + \\ &+ (-2(p_1^{-j})^6 + 4k(p_1^{-j})^5 + 8m(p_1^{-j})^4 + (-8(p_1^{-j})^3 + 2k(p_1^{-j})^2 + 8km)(p_1^{-j})^2 x + \\ &(-9(p_1^{-j})^2 + 6k(p_1^{-j}) + 12m)(p_1^{-j})^2(p_1^j)^2 + (-16(p_1^{-j})^3 + 2k(p_1^{-j})^2 + 24km)(p_1^j)^3 - \\ &- (10(p_1^{-j})^2 + 6k(p_1^{-j}) + 12m)(p_1^j)^4 - 3(p_1^j)^6)y]\end{aligned} \tag{61}$$

$$\frac{\partial Pi}{\partial e_1^j} = -c - \frac{e_1^{-j}(p_1^{-j})^2(k - (p_1^j))(p_1^j)^2(((p_1^{-j}) + (p_1^j))^2 - 4m)}{4((p_1^{-j})^2 + (p_1^j)^2)^2(e_1^{-j} + e_1^j)^2} \tag{62}$$

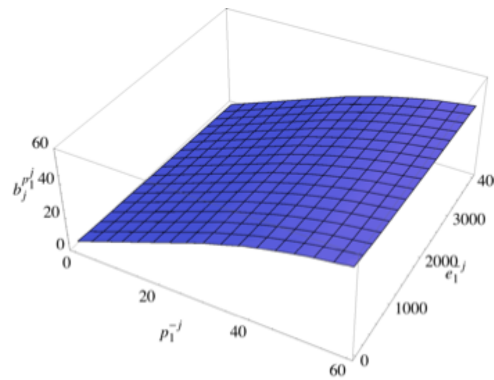
Unfortunately, the system represented by (61) and (62) jointly equal to zero has no closed form solutions for p_1^j and e_1^j due to the many interplays between the variables and the high degree of the resulting polynomials. It is however possible to approach them with numerical methods and as such retrieving the best responses nonetheless.

Rather than showing long and complicated numerical roots, a graphical approach is pursued in order to understand the economic behaviour that undergoes the analysis. Starting from prices, the best response for p_1^j for different competitors choices p_1^{-j} and p_1^{-j} is shown in figure 7

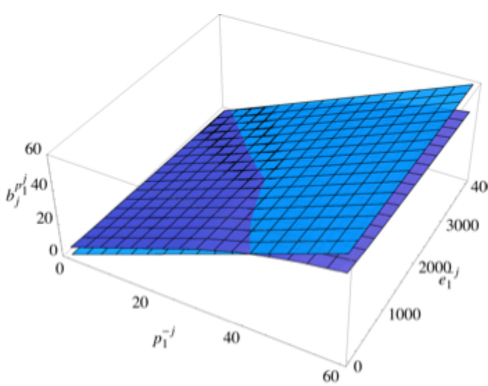
From the first two pictures it is possible to notice the slowing pace at which the firm's best price choice increase as the price of the rival increases, while at the same time increasing



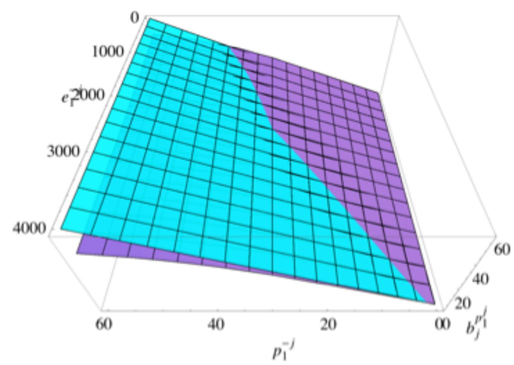
(a) Optimal choice for p_1^j, varying the rival's $-j$ choices



(b) Optimal choice for p_1^j, varying the rival's $-j$ choices, zoom in the increasing part



(c) Optimal choice for p_1^j, varying the rival's $-j$ choices and its intersection with the $p_1^j = p_1^{-j}$ plane



(d) Optimal choice for p_1^j, varying the rival's $-j$ choices and its intersection with the $p_1^j = p_1^{-j}$ plane (reverse view)

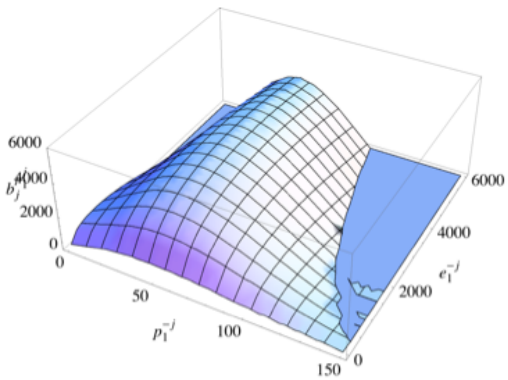
Figure 7: Optimal choice for p_1^j, varying the rival's $-j$ choices, from different angles

behavioural effort e_1^{-j} unequivocally points to lower and lower optimal p_1^j. This happens as a defence from the increasing grips that the rival has on the market share thanks to the behavioural component.

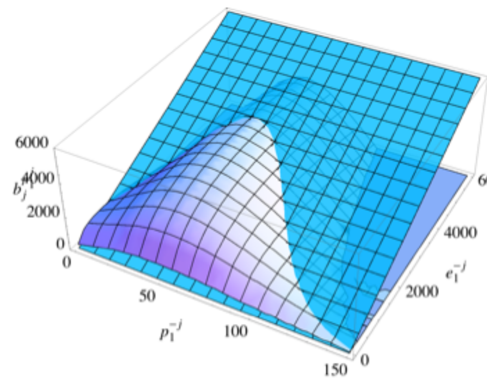
The last two pictures of figure 7 provide further insights on price behaviour with the addition of the plane $p_1^j = p_1^{-1}$. In this way, two very different regions appear, the ones where the graph is *above* the plane, which imply a premium price approach, and the one *below* the plane, which instead focus on being more cost efficient than the competitor.

In this sense, two main comments are in order. First of all, the spaces where each one of the strategies is the winning one imply a degree of *specialisation* from the competitor: indeed, the premium price strategy is the optimal one when the rival prices are lower (i.e. the rival is pursuing the cost efficient strategy), while the converse is true for the cost efficient strategy, in the sense that it is most effective with higher prices from the rival (i.e. when they are pursuing a premium price strategy themselves). Secondly, the effect of the sustainable effort is extremely relevant: the premium price strategy is the most efficient the lower the sustainable effort of the rival is (i.e. the biggest relative sustainable effort the firm has), while for higher and higher competitor's effort one reaches a point where the premium price strategy is no longer viable for any level of p_1^{-j}.

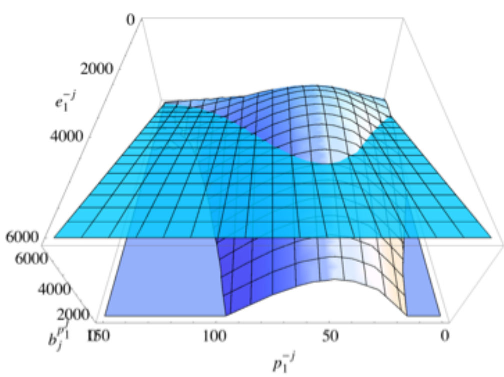
Moving now to consider the best response when it comes to e_1^j, again the graphical analysis proves to be the most insightful. Figure 8 represents it varying p_1^{-j} and e_1^{-j}.



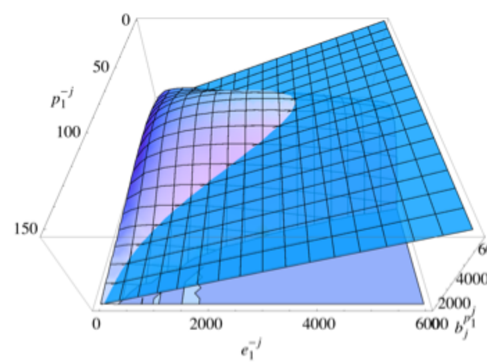
(a) Optimal choice for e_1^j, varying the rival's $-j$ choices, balanced view



(b) Optimal choice for e_1^j, varying the rival's $-j$ choices and its intersection with the $e_1^j = e_1^{-j}$ plane, balanced view



(c) Optimal choice for e_1^j, varying the rival's $-j$ choices and its intersection with the $e_1^j = e_1^{-j}$ plane, p_1^{-j} side



(d) Optimal choice for e_1^j, varying the rival's $-j$ choices and its intersection with the $e_1^j = e_1^{-j}$ plane, p_1^{-j} side

Figure 8: Optimal choice for e_1^j, varying the rival's $-j$ choices, from different angles when p_1^j is set to 50

From the first picture it is clear that, as wanted, the maximum effect of the sustainable effort is achieved with moderate values of p_1^{-j} (while extreme price competition reduces its relevancy) and there it is more critical to have higher effort. As such, that is the region with the highest optimal e_1^j. Conversely, when the sustainable effort form the rival grows more and more, at first the best response strategy is to keep an higher value, but as soon as the values of e_1^{-j} reach a certain level, only a minimal defensive amount stays in place and there is no longer competition to be on top. This is again is a sign of *specialisation*, with one firm keeping a low amount of sustainable effort mainly for defensive purposes while the other takes the lead. The best sustainable effort quickly becomes zero if the firm is also able to differentiate in terms of prices, and the higher the sustainable effort of the rival, the lower price difference is needed to have $e_1^j = 0$ as part of the best strategy. This is even clearer when the plane $e_1^j = e_1^{-j}$ is included as comparison, in the remaining three figures. Indeed, the area of the graph above the plane is the one where actually the firm takes the lead in terms of effort, while beyond that lower or zero effort becomes the dominant strategy.

Overall, the picture that the two best responses draw (with these, but minimal modification happens with different parameters choices) is quite eloquent. Indeed, depending on the environment it faces (here represented by a single competitor, but the notion can be easily generalised), two completely different approaches are observed and justify the approach that has been followed in designing the small test presented in section 3.6. The possibility of specialisation is indeed incredibly powerful in this context, because it helps explaining the great variety in behaviour that it is observed nowadays, provided that an equilibrium has not yet been reach (in this case, with the competitor reaction).

Building on this observation, a final question that is of interest in this section is what happens *in the long run*. Indeed, sustainable goals and actions at the firm level made their appearance relatively recently in the economic world. One could argue that as time goes by all the companies that are currently not engaging in sustainable effort (and, in this sense, are lagging behind in their optimisation approach), will start considering it as a possibility and a possible tool for profit maximization. As such, the best response of a firm with a *given* environment (here represented with a single competitor) can be pushed further: with the entire market efficient, reciprocal best responses can be taken into account to observe the *equilibrium* one reaches among rational players. In this simplified duopolistic approach, this means hunting for the Nash Equilibrium of the two firms maximization game.

4.3 Nash Equilibria

At this point the duopoly game is investigated further taking into consideration its Nash equilibria. In this case, this means looking for the strategic choices quartet:

$$\{p_1^j, e_1^j, p_1^{-j}, e_1^{-j}\} \quad s.t. \qquad \{p_1^j, e_1^j\} \in b_j(p_1^{-j}, e_1^{-j}) \wedge \{p_1^{-j}, e_1^{-j}\} \in b_{-j}(p_1^j, e_1^j) \tag{63}$$

In words, the requirement is that the strategic choices of any firm must be best responses for what the other will play (which will be in their turn best responses, so that overall both are *mutual* best responses).

A key step forward in this regard is represented by the fact that the numerical analysis of the best responses for the p_1^j and e_1^j parts (presented in the previous section) showed that the root is singular and as such they are function and not *correspondences*. It is thus possible to write:

$$b_j(p_1^{-j}, e_1^{-j}) = \{b_j^p(p_1^{-j}, e_1^{-j}), b_j^e(p_1^{-j}, e_1^{-j})\} = \{p_1^{j*}, e_1^{j*}\} \tag{64}$$

$$b_{-j}(p_1^j, e_1^j) = \{b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j)\} = \{p_1^{-j*}, e_1^{-j*}\} \tag{65}$$

Moreover, the two firms are identical: b_j and b_{-j} take the same functional form, they are effectively distinguishable only by the arguments they take (or better, by which firm those arguments are linked to). This implies that Nash equilibria will be symmetric and the two firms will undertake the same strategy, just like a function has intersection with its inverse only in the first diagonal.

Considering the four-dimensional space $\{p_1^j, e_1^j, p_1^{-j}, e_1^{-j}\}$ in which Nash equilibria lies, (64) and (65) are two different conditions that identify two different 4D curves:

$$\{x, y, z, w\} \longrightarrow \{b_j^p(p_1^{-j}, e_1^{-j}), b_j^e(p_1^{-j}, e_1^{-j}), p_1^{-j}, e_1^{-j}\}$$
$$\{x, y, z, w\} \longrightarrow \{p_1^j, e_1^j, b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j)\}$$

And mutual best responses imply finding points that satisfy both constraints. In order words, Nash equilibria lies at the intersection of these surfaces. It is however quite cumbersome to represent properly four-dimensional objects in a 2D paper and as such some clever second-bests are employed. Figure 9 plots the two surfaces described above, with the w axis as color, with warmer corresponding to higher values.

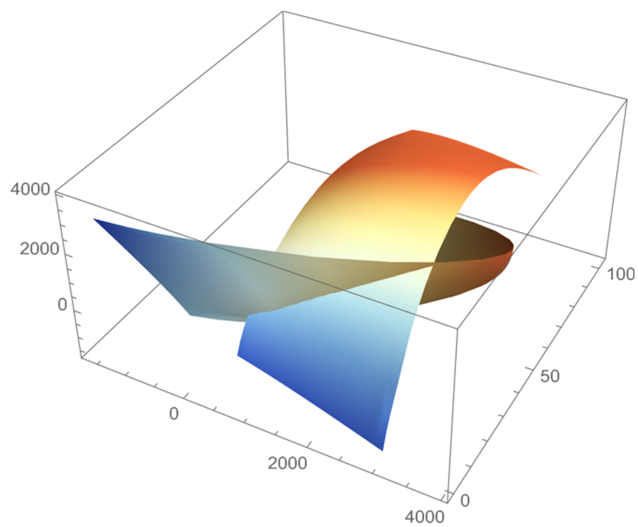


Figure 9: 4D surfaces identified by the best responses, their intersection (including color) represents the Nash equilibrium

The intersection is not as easy to be evaluated as it might seem, however, since a perfect matching in the fourth dimension (color) is required to actually be a Nash equilibrium. Indeed, although closer to what is normally done in order to retrieve Nash equilibria, this approach might be insightful but not the most efficient in terms of producing actual results when best responses are only numerical.

One of the easiest way to look for a Nash equilibrium in this case, provided that only computational means are available, is to exploit the mutual best responses definition as follow: any p_1^j and e_1^j that constitute a Nash equilibrium must be the best response of some p_1^{-j} and e_1^{-j}, which were in their turn played as a best response to those same $p_1^j$and e_1^j. This use of the first mutual nature of the definition allows for the employment of equations only on p_1^j and e_1^j:

$$\{p_1^j, e_1^j\} = \{b_j^p(p_1^{-j}, e_1^{-j}), b_j^e(p_1^{-j}, e_1^{-j})\}$$
$$\{p_1^j, e_1^j\} = \left\{ b_j^p\left(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j) \right), b_j^e\left(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j) \right) \right\}$$

which imply that separately:

$$p_1^j = b_j^p\left(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j) \right) \tag{66}$$
$$e_1^j = b_j^e\left(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j) \right) \tag{67}$$

So that any p_1^j and e_1^j that satisfy both (66) and (67) together with their best responses of $-j$ are the Nash equilibria.

The solutions of the outlined system can be found purely numerically, but here a solution visualisation approach is employed in order to better expose the inner working of the equilibria in this work.

Starting from (66), one can plot both the right and left-hand sides of the equation in a 3D graph, where the first two dimensions are p_1^j and e_1^j and the third is the value each of the sides assume:

$$\{x, y, z\} \longrightarrow \left\{ p_1^j, e_1^j, b_j^p\big(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j)\big) \right\}$$
$$\{x, y, z\} \longrightarrow \{p_1^j, e_1^j, p_1^j\}$$

So that the the values of p_1^j and e_1^j for which the two graph have the same z (i.e., their *intersection points*) will also be the ones that satisfy (66). Such a graphic approach is shown in figure 10.

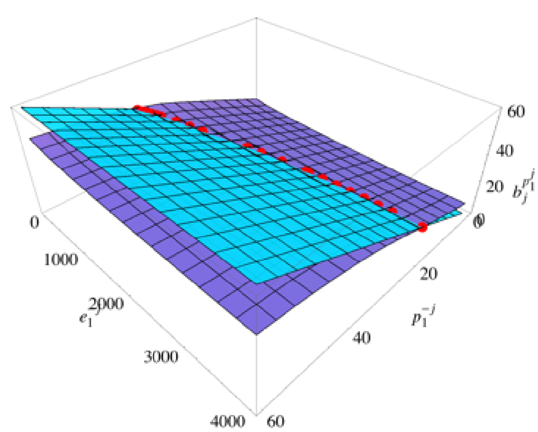


Figure 10: Plane $z = p_1^j$ and curve $z = b_j^p\big(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j)\big)$, the points signaled by red dots were manually checked.

An identical reasoning is applied to equation (67) for e_1^j and $b_j^e(\cdot)$, with the result shown in figure 11.

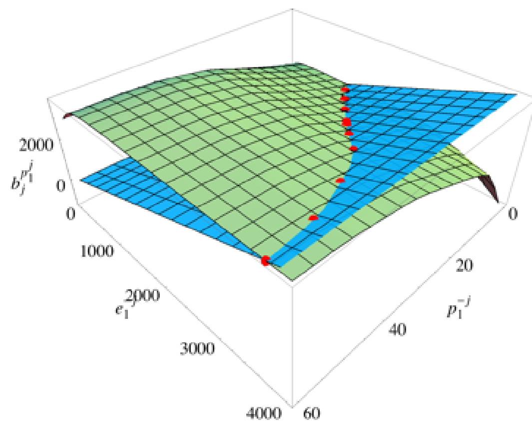


Figure 11: Plane $z = e_1^j$ and curve $z = b_j^p\big(b_{-j}^p(p_1^j, e_1^j), b_{-j}^e(p_1^j, e_1^j)\big)$, the points signaled by red dots were manually checked.

Now, among all the intersection points of the first and second graph, Nash equilibria are only those which jointly satisfy both conditions. Thus, the two intersection lines are in turn plotted together to find their own intersection. The resulting p_1^j and e_1^j will satisfy both (66) and (67), ultimately being part of a Nash quartet, together with $-j$ best responses over them. This operation is shown in figure 12.

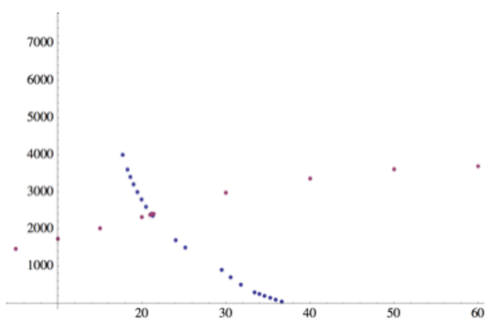


Figure 12: Intersection of the lines of points that solve (66) and (67). Only manually checked values displayed. The intersection is the Nash equilibrium of the problem).

As it can be seen, a unique Nash equilibrium arise (specifically with values:

$$\{21.4893748378538996472534 1,\ 2442.8986721574847165594034823484165 2,\ 21.4893748378538996472534 1,\ 2442.8986721574847165594034823484165 2\}$$

with the parameter choices employed in this visualisation, i.e $m = 10^4$, $c_1 = 5$ and $k = 1$). Consistently with the previous observations, such a Nash equilibrium exists and it is symmetric. Not only that, but for reasonable levels of c_1 it implies positive amounts of sustainable effort.

This is a crucial conclusion for the hypothetical evolution of such effort in tomorrow's world. Indeed, according to the framework that has been set up in this world, the only stable level of strategic choices impose to firms to contribute to the global (private) sustainable effort, which is thus consistent with profit maximization in a strategic environment.

5 Falsification test with real data

In this final part the results of the simple falsification test presented in subsection 3.6 are shown. Data from sustainable effort has been retrieved from the Golden for Impact database by the Leonardo Research Centre (Imperial Business School - Imperial College London). At the time of the writing of this work, the database is still at its early stages and the ability of automatically produce a large quantity of reliable ordered data through artificial intelligence and automatic information scanning is currently undergoing fine tuning. A small working sample, however, is ready for analysis and has been made available for this test. Data are structured in terms of distinct initiatives to which various level of categorization are assigned. In particular, each of them is assigned a "stakeholder recipient" flag that allows for the present distinction between e_1, e_2, e_3 and e_4.

This is however not sufficient to have a complete estimate of the vector $\boldsymbol{p'Y}$, and some other publicly available databases are needed in order to reach the full testing environment. It may be best to report again the structure required in order to comment on how each estimate is reached:

$$Y(\bar{y}) = \begin{bmatrix} \bar{M}f_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\bar{q} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\bar{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{k} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{e_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\bar{e_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\bar{e_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{e_4} \end{bmatrix} \qquad \bar{\boldsymbol{p}} = \begin{bmatrix} \bar{p_2} \\ \bar{p_3} \\ \bar{p_4} \\ \bar{c_1} \\ \bar{c_2} \\ \bar{c_3} \\ \bar{c_4} \end{bmatrix} \qquad (\bar{\boldsymbol{p}}'\bar{Y})' = \begin{bmatrix} \bar{M}f_1\bar{p_1} \\ \bar{q}\bar{p_2} \\ \bar{l}\bar{p_3} \\ \bar{k}\bar{p_4} \\ \bar{e_1} \\ \bar{e_2} \\ \bar{e_3} \\ \bar{e_4} \end{bmatrix} \tag{68}$$

From this writing of $\bar{\boldsymbol{p}}'\bar{Y}$ (which was transposed just for spacing reasons), two implicit simplifications immediately appear. Indeed, first of all the N suppliers now are substituted for a single one "average" supplier that provide the input bundle. This is not too much detrimental since showing that there is an appreciable sustainability effort effect on this fictitious global suppliers implies showing that the average was affected, and as such at least one of them registered some behavioural answer. Secondly, due to the discussion made throughout section 3, all the costs of the behavioural effort have been normalized to 1.

The following choices have been made to retrieve the values, with yearly datapoints for all the variables (including $\boldsymbol{e}$ from the Golden Database):

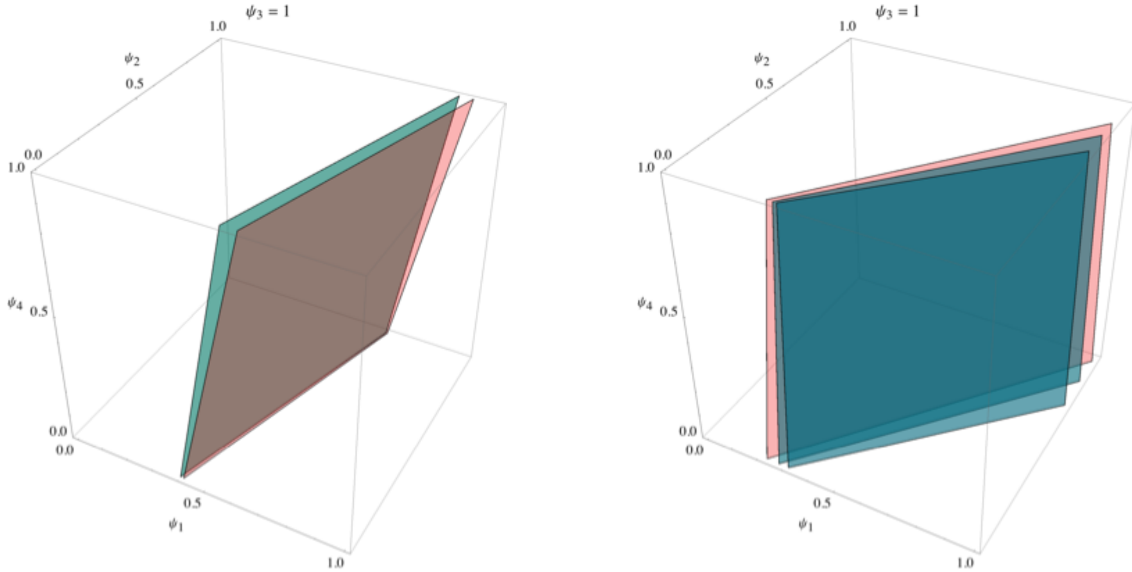
- Profits π have been proxied by earning before taxes (since the sample is multinational this helps avoid distortions)
- $\bar{l}$ has been proxied by number of employees times sector average yearly working hours (equivalent to the present setting, in light of comments from section 3.3), while $\bar{p_3}$ by average sector nominal hourly labor cost.
- $\bar{k}$ has been proxied by the company's total market capitalisation, while $\bar{p_4}$ by the sector average dividend yield.
- $q\bar{p}_2$ has been *jointly* proxied by total production cost, excluding labor costs
- As far as revenues is concerned, the premium price interpretation proves easier to implement, rather than the extra market share one. As such, $\bar{f}_1$ is proxied by the company's market share (as anticipated, sustainable companies tend to be very big worldwide and as such all sampled companies have appreciable market shares in the global market of their sector). Conversely, $\bar{M p_1}$ are jointly proxied by total sector revenues worldwide.

The gathering of such data was possible, aside from the Golden Database already mentioned, with the use of data from Orbis, Eurostat, Statistica, International Labour Organisation (ILOSTAT), OFX and WorldBank.

Now, as anticipated, the output of the analysis consists of hypercubes and tests on hypercubes. This is however not an easy structure to represent visually. The easier solution in this case is to keep fixed one of the (relevant) dimensions of ψ, namely ψ_1, ψ_2, ψ_3 and ψ_4 that correspond to the behavioural effect on revenues, input, labor and capital respectively (see definition (45)). The choice of the value at which fix the excluded dimension is quite easy: fixing it to 1, if the remaining plane also is close enough to the point $(1, 1, 1)$ then a negligible effect of the sustainable effort cannot be excluded. Since there is no particular reason to chose one fixed element or the other, visualisation drawn from all the possibilities are pursued throughout.

The overall number of investigated companies is 17, many of them across multiple years, arriving to a total of 47 data-points (mainly across 2010-2012).

Among them, the results of four of them was found consistent with the supposed irrelevancy of the behavioural components. Its the case of BT Group in the years 2010, 2011 and 2014, and IBM in the years 2010 and 2011, as reported in figure 13.



(a) Planes for IBM in the years 2010 (red), 2011 (blue) and 2014 (orange), keeping $\psi_3 = 1$

(b) Planes for Telefonica in the years 2010 (red), 2011 (green) and 2014 (blue), keeping $\psi_3 = 1$

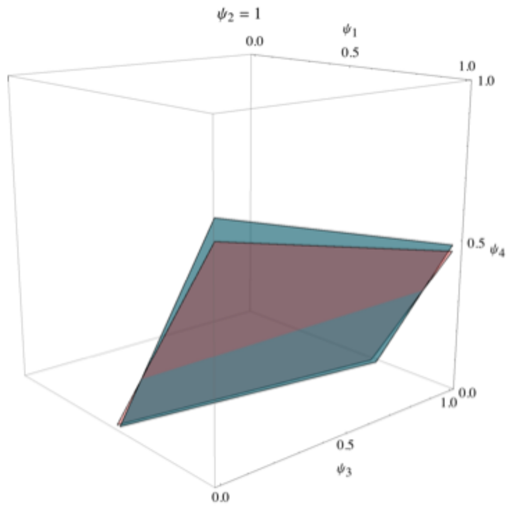
Figure 13: Planes representing the possible vector ψs for British Telecom and Telefonica

Indeed, in both cases the planes pass quite near the "corner of irrelevancy", with a converging inclination as well. In this case, it is not possible to exclude the possibility of ψ being a vector of ones. Other companies that underwent the same destiny were Telefonica and DeutscheTelekom.

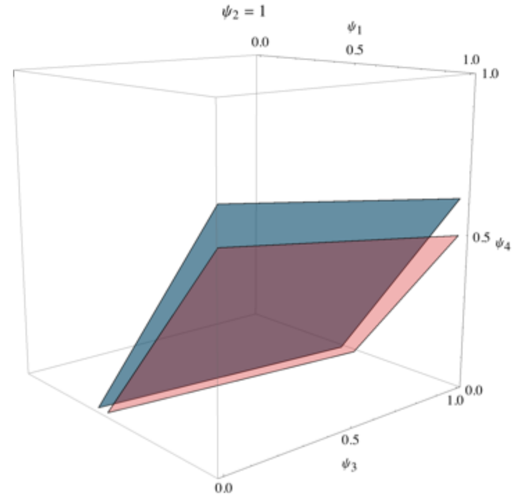
Conversely, all the other companies in the sample behaved in a way that is possible to exclude $\psi = \mathbf{1}$ in their cases. Here, the examples of Ericsson, AT&T, Magyar T. and Emirates Telecomm are reported (figure 14).

Some further comments are needed in order to consider the results of the four firms that do not exclude the irrelevancy hypothesis. Indeed, they can be divided into two groups according to the strategies that they pursue. In the recorded years, BT Group was seriously underperforming in terms of sustainable effort compared to the rest of the sample. This might mean that the irrelevancy recorded is due to a low effort overall, where the low recorded effect is not due to the strength of the ϕ functions, but rather due to the low value of the argument. The other three companies, instead, were seriously investing in sustainable effort in the recorded years, which means that their results are more puzzling and longer time series would help clarify the situation.

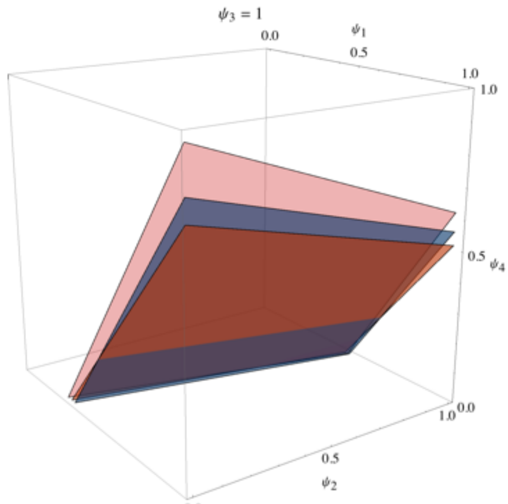
However, due to the results coming from the vast majority of the firms in the sample, it seems that an effect that needs to be addressed is actually present. Moreover, the fact that it *might* fade away in a small portion of the cases is not sufficient to disprove the need of a theoretical modeling regarding this matter. The framework presented in this work can be considered a preliminary effort in this respect, but once data are made more abundant and widespread, more sophisticated mechanisms can be put in place to more closely investigate the effects of sustainable effort.



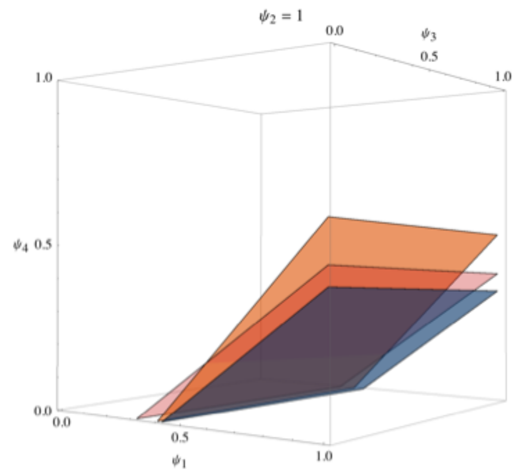
(a) Planes for Ericsson in the years 2010 (red) and 2011 (yellow), keeping $\psi_2 = 1$



(b) Planes for AT&T in the years 2010 (red) and 2011 (yellow), keeping $\psi_2 = 1$



(c) Planes for Magyar T. in the years 2010 (red), 2011 (blue) and 2017 (pink), keeping $\psi_3 = 1$



(d) Planes for Emirates Telecomm in the years 2010 (red), 2011 (orange) and 2016 (yellow), keeping $\psi_2 = 1$

Figure 14: Planes representing the possible vectors ψs for Ericsson, AT&T, Magyar T. and Emirates Telecomm

6 Conclusions

This work addressed the description of firm's behaviour by enriching a standard profit maximization problem with the addition of behavioural components to the various stakeholders it faces when buying and selling quantities: consumers, suppliers, employees and shareholders.

As a mean to dialogue with the behavioural components, the firm was allowed to invest in an arbitrary amount of sustainable actions, which are specialised with respect to their *stakeholder recipient*, i.e the specific entity to which the action is aimed at. The results of the interplay between each stakeholder and the firm was shown to also determine the amount of sustainable actions directed towards that stakeholder, recorded in an aggregate fashion with the term sustainable effort. Depending on the parametric specification, not only positive effort was possible in equilibrium in each relationship, but also when considering the overall profit maximization problem of the firm with all the relationships determined at once this result did not disappear.

Such isolated case was then evolved into a strategic game between two identical firm in a duopoly for the final good market. The best responses that appeared in the game implied the existence of a cost efficient strategy (with low effort and lower price than the competitor) and a premium price one (with high effort and higher price than the competitor). Nash equilibria were also computed in order to investigate a possible strategically stable firm behaviour. The results showed that implied equilibria are symmetric and do not exclude the positive (and equal among firms) sustainable effort.

Finally, a simple falsification test was implemented in order to look for consistency between the described framework and real data. Its results showed that the effects of positive sustainable effort cannot confidently be considered negligible and through the lens of the employed framework it is a valid strategic choice for profit-maximizing firms.

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