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The Influence of Supply Chain Financing on Operational Decisions

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Abstract

Supply chain financing modes are increasingly investigated in the literature as means to optimize firms cash flow. However few emphasis have been done to understand the risk-taking behavior of agents involved in supply chain financing. We attempt to alleviate this gap by exploring the impact of supply chain financing on the risk taking behavior of members of the supply chain and its consequence. We propose a newsvendor model involving a cash-constrained retailer financed through trade credit by a supplier, which itself relates on an external financing through bankloan. We explore the behavior of actors of the supply chain with respect to different consideration such as variability, risk aversion or bankruptcy risk, under limited liability assumption. We finds overall that supply chain financing leads to an increased leverage of the supply chain and induce actors to increase their exposure to adverse outcomes when making their operational decision. These results provides insight on the needs for decrease information asymmetry between external financier and the members of the supply chain financing arrangement.

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CONTENT

Introduction

Supply chain financing is raising an increased interest in the literature nowadays, in the context of increased complexity and globalization of the supply chain, working capital has become an increasingly important variable of interest as a lever to increase the performance of the firm. Last decades have risen the efficiency of the supply chain from an operational point of view in a significant way. The concepts of just-in-time, lean management and supply chain risk management have allowed the supply chain manager rationalize supply chains in such way to improve dramatically the performance of their firm.

However, supply chain management improvements have focus mainly on the operational flow leaving aside the need of rationalization of both the informational and the financial flow. The development of technologies of information and telecommunication have enable to render the information flow very efficient as well. We note that the financial flow in the supply chain remains relatively less understood. Yet, the literature of supply chain finance is rising since the last decade and is currently developing at a decent pace.

Despite the seminal irrelevance result of **Modigliani Miller (1958)** that assert that financial and operational decisions should not interfere, we observe in the reality that this statement does not hold in a world characterized by many source of market imperfection. This observation leads us to explore event more the issues relating to the interaction between operational and financial decision. The need for developing a well tied up theory of supply chain finance becomes relevant under the conditions of the current imperfect market.

It is especially true in our context where a significant market volatility prevail in the economy, leading toghether with the highly rational and global supply chains to inflate the exposure of the firm to adverse events. Along with this, the tightening of credit resulting from the last crisis and the increased presence of banking regulation have made the issue of capital availability more important nowadays, especially for small and middle enterprises.

That is what leads us to study opportunity of financing through the supply chain as alternative to conventional ways of financing such as bank loans. Therefore, we explore supply chain financing in a quantitative analysis such as to brings further insights on the pros and cons of such decision.

We first present a review of the relevant literature on the related subject and then propose a model to explore the issue of supply chain financing through trade credit instrument.

A first part presents the settings of the problem as well as several insights on the impact of such financing in operational decisions. We introduce several extension in our model such as to grasp even more the mechanism arising from trade credit financing in the supply chain. We then provide in a second part an analysis that focuses more on the risk issues. We explore there variability issues as well an extension that relate on a criteria reflecting the risk aversion of the decision maker.

In this text, we explore the problem of supply chain financing through the use of trade credit. We study the impact of this type of financing configuration on the operational decision of the members of the supply chain. Our research question is "*How does supply chain financing arrangement through trade credit influences operational decisions?"*. We answer to this question through an in depth exploration of each actors of the supply chain under different assumption such as financial constraint or bankruptcy costs. We furthermore examine variability issues and risk aversion for extracting insights mainly risk oriented. We emphasize our analysis on the impact of these variables on the risk-taking behavior of the supply chain's agents.

AS a result of our analysis we found different insights about the impact of trade credit in the behavior of the firm. Among other, we found that trade credit financing help sharing risk between the retailer and the supplier leading the former to take more risky objectives when its initial wealth is low.

In addition, we find that this behavior is consistent with that of the financially constrained supplier, that relates on exernal financing source. Indeed, as its initial wealth is low the supplier tends to induce the retailer to order more, be extending better credit terms (lower interest rate or wholesale price) This mechanism reveals to be reduce when introducing retailer's bankruptcy cost consideration. We found furthermore an independence between financial and operational decision in the case of perfect capital market. However, this statement appears to be wrong. Finally when exploring variability consideration we deduce the existence of a concavity in the relationship between variance of profit and ordering decision.

Before entering into more details about our findings, we begin with a review of the relevant literature.

In this revue, we present the relevant literature related to the concepts surrounding the financing decision in supply chain management as well as the repercussion of this decision on the firm. We explore by turn the fields related with working capital management, capital structure decision, and a series of financing configuration used in supply chain management.

Chapitre 1. Disruption

1.1.1 Definition

First of all, we take a closer look at the concept of disruption that is relevant for to illustrate the purpose of our analysis. The definition of disruption vary in the literature depending on the field. In the financial supply chain management literature disruption is defined by as *"an unplanned event that adversely affects the firm's normal operations"* **(Schmidt & Raman, 2012)**

[i].

We can classify disruption in two broad categories according to the source of disruption: the source is external or internal to the firm **(Schmidt & Raman, 2012).** Among internal disruption we can denote unexpected event occurring directly in the operations in the firms, but also indirectly from within the supply chain through propagation. For example, the bankruptcy of members of the supply chain fall in this category. The second category relate to events outside the control of the supply chain. These events coming from environmental surroundings such as weather, or natural disasters, fall within hazardous factors that do not arise from mismanagement of the firm, and is prone to be punished less severely by the shareholder.

Disruption is a phenomena that occurs frequently within companies. For example **Aydin, Babich, Beil and Yang (2012)**[ii] illustrated that 58% of the companies have suffered from financial losses due to supply disruption within the year preceding the study. This consistent rate led to an increased infatuation on this issue. They present several reason could explain the infatuation to focus on disruption risk. The first reason is the upward trend for globalization, making supply chains more complex and vulnerable to disruptions. A second driver is the increased media attention for such events that lead to sharper reaction of investors, increasing the adverse outcome of disruptions. This Thirdly, they mentioned the influence of academicians and practitioners that increasingly strive to highlights benefits of disruption mitigation in terms of supply chain performance. Finally the increased accountability of managers with respect to the performance forecast and shareholders value protection, initiated by the Sarbanes-Oxley Act of 2002 .

However, even if the interest on these issues is rising, the lack of understanding of risk profile of disruption and its consequence on the financial performance makes the company still unprepared to face unplanned events. **(Babich et al., 2011)**

2.1.1 Market Implication of Disruption

a. General Disruption

Yet many scientists explore the link between supply chain disruption and financial performance. They study among other its impact in the value of the firm. Following the financial theory, and more precisely the discounted cash flow model, the firm's market value evolves according to a trade-off between the perceived risk and reward on investment **(Pinto, Henry, Robinson, & Stowe, 2010)**[iii]. Either operational or financial disruptions in the supply chain can modify this perception. The ability of the firm to generate sufficient revenue to pay its loan and dividends is reconsidered just after the disruption in anticipation of adverse consequence on the revenue resulting therefrom. Indeed, disruption means that demand and supply cannot be matched anymore. Furthermore, the perceived increase in risk make stakeholders require a higher reward for keeping investing in this company putting upward pressure on the cost of capital. These phenomenon tends to decrease the value of the firm. Similarly, debt holders adjust their required rate of return as the risk of lending has increased, due to the disruption. This cost of debt is reflected in the market through assignment of credit rating's downgrade by agencies such as Standards and Poor's, Moody's or Fitch. All these consequences have negative post-disruption effect through worsening of the financing conditions.

In the literature, the effect of disruption on firm's value has been investigated by **Hendricks and Singhal (2003)**[iv]**.** They measured the variability of stock prices prior and post announcement of the supply chain disruption. Empirical evidence from a set of 827 disruptions announcement during the period between 1989 and 2000 shows that disruption affects equity price and volatility. In a period lasting from one year before to two years after announcement, the average equity abnormal return reduces from 33 to 40 percent and stock price volatility increases by 13.50%. They furthermore insist on the findings of another study of **Malatesta**

and Thompson (1984)[v] that highlight the influence of anticipation of disruption on its impact on the stock price. Unexpected disruptions tend to cause sharper shock. In a reworked version **Hendricks & Singhal (2011)**[vi] they add that 62% to 68% of the firms observed that experienced disruptions underperformed their respective their benchmark within the three years period following the disruption. Which show that there is a high likelihood that the impact of disruption really harms the competitiveness of the firm. Indeed, besides the harmful consequences on the shareholder value, from the firm's perspective, other variables are directly impacted by the disruption. This is the case of the total costs (+11%), as well as the declines in sales growth (-7%) that lead in the end the operating income to fall dramatically (-107%) **(Hendricks & Singhal, 2012).**

Disruption have adverse consequence on the firm value but also on other firms. Disruption propagate through the supply chain and worsen performance of firm's partners. In this matter, a study of **Pandit, Wasley and Zach (2011)**[vii] focuses on the externality of disruption announcement on the valuation of companies in the same supply chain. They show that externalities of announcement is reflected in seasonal change in customer revenue, earning persistence or COGS, environmental factor, as well as the mutual dependency of supplier and customer.

b. Financial disruption

The propagation of financial shock along supply chain is studied in various other scope. **Escaith and Gonguet (2011)**[viii] takes a macroeconomic point of view and investigate the transmission of external financial shock throughout supply chain. They explain that financial distress in the banking sector harm primarily weaker firms. But then it propagates throughout the supply chain to stronger firms, reducing in the end their ability to back the banking sector. **Kiyotaki and Moore (1997)**[ix] taking a more microeconomic point of view developed the concept of credit chain that helps to understand the propagation mechanism of financial disruption throughout the supply chain, highlighting the importance of dealing with disruption given its consequence in the entire supply chain.

The link between firm's credit-worthiness and the supply chain risk is also investigated in the litterature. By means of firms' credit ratings data **Cardella (2012)**[x] finds empirical evidences regarding the dependence between supplier and customer and its influence on the credit rating of the firm. To illustrate, a customer with a higher concentration in one supplier experienced a lower credit rating due to exposure to propagation of financial distress. Supporting this evidence **Hertzel, Officer, Li, and Rodgers (2006)**[xi]shows the actual contagions effect of financial distress throughout the supply chain, demonstrating that higher level of dependency between agents lead to worse impact of contagion effect of financial distress. Additionally, they mentioned an horizontal effect that is beneficial for the competitors of the firm in difficulty.

Finally, it is worth mentioning that the different effects of financial distress are dynamic and do not have impact until after the filling date or to some extent prior to this date. The effect vary depending on the economic conditions prevailing at the time of the disruption **(Boissay, 2006)**[xii]

All these examples shows the high dependence between members of the same supply chain. The risk is shared between all parties involved within the same chain. It is relevant to investigate the mechanism that can help to lower this risks propagation and reduce the risks of the entire supply chain. Risk can occur in the physical chain as well as in the financial chain that link companies one to another. The following section focuses mainly on the financial chain and presents the different practices used by companies to manage the financing cost of their operation as well as the risk related to these financing solutions.

Chapitre 2. Working Capital Management

According to the European Central Bank, the scarcity in credit for financing supply chains is rising **(Kerle, 2010)** [xiii]**.** This worsening in credit lending conditions leads to difficulties of financing the working capital of many companies, and especially the SME. Knowing that working capital deficiency results in delay in production, "*suboptimal stocking level, higher prices and financial distress*" **(Tanrisever, Cetinay, Reindorp, & Fransoo, 2012)**[xiv] , working capital management is worth managing in order to add value to the company. Previously, companies focused more on the inventory amid the components of working capital **(Seifert & Seifert, 2009)** [xv]**.** The other parts related to the payment velocity, has nearly not changed, and remain around 30-45 days on average for years. Since recent years, an increased interest in working capital management has pushed the financing of payable as a major issue of the agenda of the companies **(Seifert & Seifert, 2009).**

Recall the definition of working capital from an accounting perspective:

$$Working\ Capital\ =\ Current\ assets -\ Current\ liabilities.$$

There exists several strategies to determine the working capital, however a risk-reward trade-off can helps guiding the decision. On one hand a positive working capital increases the liquidity and mitigate the risk but on the other hand decreases the overall profitability due to higher inventory and financing costs. While negative working capital is a more aggressive strategy, it decreases the funding cost but increases the risk of shortage that can lead to higher goodwill cost for unfilled demand**.(Hofmann and Belin, 2011)**[xvi] The working capital can be managed by taking the operational view of the cash-to-cash (C2C) cycle, also called net operating cycle or cash conversion cycle. It is defined by the CFA Institute **(Clayman, Fridson, Troughton, 2012)**[xvii]

$$N° \; of \; days \; of \; inventory + N° \; of \; days \; of \; receivable - N° \; of \; days \; of \; payables$$

The three variables to optimize are the inventory level, receivables and payables **(Randal & Farris, 2009)**. The number of days of inventory, is defined by **Hoffman and Belin (2011)** as "*[…] the time to convert stock of raw material into work in process and then finished product to be sold".* The two last terms simply refer to the time elapsing between the reception and the repayment of an order. From the C2C we can deduce three means for reducing the working capital: extend days of payables, reduce inventory, or reduce days of receivables. There exists trade-offs for each of these strategies. For example, DPO extension involve shifting working capital upstream in the supply chain, which could hinder relationship with supplier. While a reduction in inventory involve increase in shortage risk and potentially higher transportation cost, especially when using some JIT strategies. Finally, the DSO reduction shifts inventory downstream leading to increase the risk of liquidity constraint for the buyer. **(Hoffman & Belin, 2011).**

Cash-to-cash cycle is an efficient variable to manage the working capital. According to **Howorth and (Westhead)**[xviii] a reduction of 25% of C2C leads on average to an increase of 7.5% of the firm's value. However, this is the result of decentralized decision. By taking a supply chain perspective for optimizing cash-to-cash cycle and working capital, the result has the potential to be even more significant. Indeed several source agree on the idea that supply chain coordination involve financial chain coordination between agents. As consequence, working capital and C2C are indicators of supply chain efficiency **(Hoffman & Belin, 2011)(Farris & Hutchison, 2003)**[xix]**.**

Increasingly there is a need for adopting a supply chain perspective for viewing working capital management through supply chain financing by considering the consequence of financing decision in the entire supply chain. **(Randal & Farris, 2009)**[xx] highlight the adverse impact of local working capital management through delay in payment or early payment discounts, that can lead to sub-optimal decisions. It shows the importance to model the impact of its own C2C optimization decision on the partner working capital.

At this point, it is interesting to dwell a little on the issues of coordination in the supply chain, before going further throught the impact of financial decision on the operations of the firm.

Chapitre 3. Supply chain coordination : decentralization, coordination and efficiency

The literature in supply chain management (SCM) identifies three kinds of supply chain flows: *"[…] the flow of goods and services, the information flow, and the financial flow"* **(Lambert & Pohlen, 2001)[xxi].** That is why it is important to take a supply chain perspective when making financing decision as well. Maximizing a particular transaction in a single function of the system results in a sub-optimized outcome that negatively impacts overall firm performance **(Randal & Farris, 2009).** This statement is true for the each of these supply chain flows. The problem is highlighted throughout the wide literature of supply chain coordination which emphasizes several techniques for improving supply chain efficiency through the use of structured contracts between agents in the supply chain. **Cachon (2003)[xxii]** proposes an extensive review of some of them. The wholesale price contract or price-only contract, widely used in the practice is investigated in depth by **Larivière and Porteus (1999)**. The wholesale price is known to coordinate the supply chain only under the condition that the supplier extracts the entire profit from the sales which means that in practice, coordination is less likely to occurs. A serie of coordinating contracts exists to circumvent this problem and is often based on a risk-sharing mechanism that permits the buyer to share the demand risk he faces, with the supplier and as consequence increase its order. In such way, the risk-sharing mechanism tend to increase the efficiency of the supply chain.

However, coordinate a long supply chain appears to be challenging as **Ding and Chen (2004)[xxiii]** present through the study of a multi-level supply chain under information asymmetry. The supplier at the upstream end of the supply chain is unable to propose a coordinating contract due to the lack of knowledge about the contract established between downstream players. The occurrence of a coordinating contract exists but arises by chance in presence of information asymmetry.

Several drivers are at the origin of uncoordinated supply chain. **Babich et al. (2011)** gives insight on the outcomes resulting from decentralization of the supply chain. For example, there exist misalignment of incentives between suppliers and buyers that result in different objectives to maximize leading to sub-optimality of decentralized decision. Also, in some cases, the information flow is hindered by the separation between the entities, resulting in information asymmetry between buyers and suppliers. The outcome increase the effect of misalignment of incentive by the fact that the supplier cannot anticipate properly the behavior of the buyer and vice-versa. Finally, information asymmetry reduces the effectiveness of the use of the best-response. **Babich et al. (2011)** highlight some other characteristics of decentralization in the supply chain arise when there is an asymmetry between the layers of the supply chain in terms of number of agents. For example, when multiple suppliers rely on the same buyer it can lead to a fierce competition between suppliers resulting in a closer to perfect competition that increase the probability of disruption. The case when there are several buyer for one supplier results in a larger extent of the impact of the supplier's disruption.

Chapitre 4. Capital Structure and Operational Decision: the role of financing configuration in the supply chain management

Working capital can be financed by several means**. Buzacott and Zhang (2004)**[xxiv] identified several strategies for financing the inventory. We classify these sources of financing in three categories. The internal source consists of delayed payment to suppliers as well as acceleration of customers' payment. Secondly, external borrowing without transfer of firm's ownership. Bank loan, factoring, enter in this category. The last category consists of the shareholder capital, raised through issuance of new shares.

Availability of capital being scarced, firms can often not rely only on its internal funding resource to finance the entire working capital. Companies rather rely on external financing such as equity or fixed income. However, one's has to consider the impact of these financing configurations on the operational performance. A large literature relates on the effect of financing configuration on the optimal inventory decision and the coordination of the supply chain.

4.1. Capital constraint

The case of lack of capital for financing the supply chain has been investigated by **Kouvelis and Zhao (2013)**[xxv]which demonstrated that coordinating contracts continue to coordinate under financial constraint as long as there is a competitively priced bank loan. In the same area, **Khouvelis and Zhao (2012)**[xxvi] investigate different financing configurations between supplier early payment discounts, supplier open account financing and bank loan financing. ***(*Kouvelis**

& Zhao,2012). They find under relevant assumptions that optimal order quantity are not modified by the financial constraint. Additionally, they point out that bank credit are more expansive than the optimally priced trade credit. This finding is consistent with the evidence of empirical literature (**Fabbri & Klapper, 2008)**[xxvii]**.** We actually observe generally lower financing costs internally through trade credit facilities than with external financing such as bank loans.

Yan and Sun (2013)[xxviii] design a model with capital constrained supplier that rely on external financing with a finite credit limit. The financing scheme characterized by a credit line level induce the supplier to choose the appropriate wholesale price such as to achieve supply chain coordination. They contrast the findings of **Lee and Rhee (2010)**[xxix] ,testing the coordination mechanism in the case of a cash constrained buyer that rely on external financing with infinite loan and positive cost of borrowing (i.e. positive interest rate charged) from financial institution and shows that all-unit quantity discount, buybacks, two-part tariff and revenue sharing contracts fail to coordinate the supply chain under this assumption. Coordination can indeed not be achieved if unlimited credit line is provided to the retailer. Additionally, **Yan and Sun (2013)** verify that traditional wholesale price still fails to coordinate the supply chain in the case of infinite loan provided by the bank. Under finite loan through the use of a credit line by the financial institution, the traditional wholesale price contract may coordinate the supply chain.

4.2.Capital structure

In any case, these ascertainment refers to the seminal irrelevance result of **Modigliani and Miller (1958)**[xxx] (MM) that emphasizes the irrelevance of the capital structure decisions of the firms. As a result, financing and operational decisions should not interfere and be done jointly. The assumptions of perfect and efficient capital market, no transaction cost, no taxes, no bankruptcy cost and full information are sufficient to accept the results. They do not deny that financial decision could increase the expected value of the firm. Indeed they state: *"For decisions which affect the expected value will also tend to affect the dispersion and other characteristics of the distribution of outcomes. In particular, the use of debt rather than equity funds to finance a given venture may well increase the expected return to the owners, but only at the cost of increased dispersion of the outcomes"* **(Modigliani and Miller, 1958).**

In an extension, **Miller (1977)**[xxxi] introduced taxes effect and still argue that *"even in a world in which interest payments are fully deductible in computing corporate income taxes, the value of the firm, in equilibrium will still be independent of its capital structure."* **(Miller,1977).** He adds: *"Any situation in which the owners of corporations could increase their wealth by substituting debt for equity (or vice versa) would be incompatible with market equilibrium"***(Miller,1977).** Indeed, from the investors' point of view the tax deductibility advantage of corporate interest is counteracted by the income taxes resulting from interest repayment. As consequence, Investors tend to value differently debt and equity lending such as in the end, the law of supply and demand moves the value of each financing mode such as to reach an equilibrium letting the capital decision irrelevant.

Many authors from operations field demonstrate the exactitude of the (MM) theory under relevant hypothesis in the newsvendor settings. However, when relaxing some assumption required for market perfection, it is shown that this statement does not hold anymore. This is the case for **Kouvelis and Zhao (2012)** [xxxii] that shows under bankruptcy costs the firm's financing and operational decisions are interdependent. In this case, the choice of the financing configuration affects the operational decision and an optimal capital structure can be derived. Besides, supplier financing such as trade credit, which is not competitively priced, could be a better alternative for improving supply chain efficiency by increasing ordering quantity. **Xu and Birge (2004)**[xxxiii] similarly study the effect of bankruptcy cost and also introduce tax advantages of the debt. In both case, the introduction of imperfection worsen the company's performance because there exist a difference between the optimal capital structure and the optimal operation decisions.

According to **Ang, Chua, and McConnell (1982)**[xxxiv] three types of costs influences the optimal financing decision:

- *"(1) the direct administrative expenses paid to various third parties involved in the bankruptcy proceedings;*
- *(2) the "shortfall" in realized value when assets are sold in liquidation or the "indirect" costs of reorganization;*
- *(3) the loss of tax credits which the firm would have received had it not gone bankrupt."* **(Ang, Chua & McConnell, 1982)**

In many texts of operation management literatures, bankruptcy costs are often presented as: fixed costs of bankruptcy, which represent the administrative cost; a variable cost on the sales or the collateral securing the loan.

However, besides bankruptcy costs, other sources of imperfections can be mentioned. **Harris and Raviv (1991)**[xxxv] identified the importance of the agency theory in determining the structure of capital. For example, there exist an agency problem between shareholders and debt holder because shareholders are incentivized to underinvest after debt holders have invested due to the asymmetric resulting payoff. Indeed, in this way they can extract the entire upside of the return distribution (if the return is higher than the face value of the debt).

Other theory of capital structure coming from corporate finance explain the optimal financing configuration decisions. The *Pecking Order Theory* (**Myers, 1984)** relates on adverse selection. It states that firms always prefer internal capital, and then the external capital with the lowest information cost (i.e the debt financing, before relying on equity) **(Frank & Goyal, 2003)[xxxvi] .** In fact, the information cost of equity arise due to the difficulty to appropriately price new issue of share without over- or underevaluate the price of new issue. A too low issuing price means that the potential value of fund is exctracted by new investors. But an expensive issuing price leads to a reduction of the firms value that can increase the cost of capital of the firm. The *Pecking Order Theory* fits especially to SME with high growth that experience severe adverse selection problems and information asymmetry the firm and the stock market. They are as consequence likely to indeed prefer the financing mode that minimize the information cost **(Frank & Goyal, 2003).**

The *Static Tradeoff Theory* **(Kraus & Litzenberger, 1973)[xxxvii]** in which the firm choose an optimal debt ratio resulting from a cost benefits analysis of the sources of borrowing. The trade-off between the tax advantage and the bankruptcy risk lead to an optimal financial structure. This optimum moves as the company evolve, and require the company to adjust its structure. Due to the transaction costs, in practice the ratio is rarely optimal.. Instead, *"the firm is viewed as setting a target debt-to-value ratio and gradually moving towards it, in much the same way that a firm adjusts dividends to move towards a target payout ratio."(***Myers, 1984). (Kraus & Litzenberger, 1973)** studied the shape of the relationship between the indebtedness and the value of the firm and found that it can be fundamentally different from the perfect relationship that was widely envisioned in the traditional view. According to the traditional view (the modified net income approach), the relation between the debt and the value of the firm is concave. *"An implication of the concavity of the function is [would be] that if the value is not at its maximum, the firm's market value can [could] be increased by a small change in leverage"*. **(Kraus & Litzenberger, 1973). Xu & Birge (2004)** also verify mathematically in a newsvendor settings, that the decision of indebtedness is influenced by the trade-off that exists between tax advantage and bankruptcy risk. Among their findings, they show that an increased volatility in demand makes the cost of borrowing higher and induces to borrow less. The retailer renounces part of its tax advantage to reduce increased bankruptcy cost.

Empirical investigation have been done to verify the applicability of this theory. **Elkamhi, Ericsson and Parsons (2012)[xxxviii]** find that when comparing the present value of bankruptcy costs and that of tax benefits, the latter overwhelmingly exceeds the former and the theory does not hold because firms should always increase their leverage such as to increase their expected value. However introducing financial distress costs[1] occurring before the bankruptcy, seems offsetting the tax benefits of debt and tend to make the theory better suited. Interestingly, they also find that smaller firms are more influenced by the introduction of fixed bankruptcy costs. Indeed, the present value of these fixed costs tends to offset the tax benefits faster for the small firms.

Chapitre 5. Trade Credit

Trade credit represent a major financing contract among account receivable. Account payables as well as receivables represent an important share of the firm's value. According to **Rajan and Zinghales (1998)[xxxix]** ,in the United States at the end of the last century, around payable accounted for about 17.8% and receivables accounted for 15% of the firm's value. Trade credit remains one of the main component of these accounts. According to **Camerinelli (2014)[xl]** the greater part of B2B transaction (80%) are undertaken mostly on credit terms and 37% of them are facilitated by trade credit. In addition "In 2004, trade credit represented more than one half of short-term liabilities in the United States" **(Boissay, 2006).** All these figures indicate the importance of studying trade credit in depth.

Trade credit is financing instrument has been used for years around the world as a means for financing the working capital **(Yang & Birge, 2013)[xli]** . Trade credit allows the buyer, to delay the repayment of the order that has already been received, incurring an amount of interest agreed with the supplier. The supplier directly deals with the retailer without involving an intermediary and they agree together about the terms of the contract. In practice, trade credits contract often allows the buyer to benefits from early payment discount. In fact, the retailer can repay the supplier without incurring any charge if it is settled within a given period of time defined by the supplier and then, an interest rate is charged to the retailer **(Kouvelis & Zhao, 2012)**.

[1] Financial distress costs relate to all kinds of costs resulting in the degradation of financial health of the company when moving toward the bankruptcy point (lost sales, underinvestment, downgrading).

According to the price discrimination theory of trade credit**,** early payment discount can sometimes be a component of the supplier strategy to perform price discrimination against its customers **(Giannetti, Burkart, & Ellingsen, 2011)**[xlii] .However, some financial arrangements do not allow early payment discount. That is the case for open account financing which is a form of internal financing instrument (**Kouvelis & Zhao, 2012)** in which the upstream agent allows a delayed payment from the retailer without charging interest. In fact, this means that the supplier accepts to bear the entire bankruptcy risk, since he is not compensated by means of an interest rate payment **(Zhou, 2009)**[xliii]**.** In practice, "open account refers to an arrangement whereby sales are with no formal debt contract" **(Zhou, 2009)**

Trade credit appears as a one of the main alternative of for financing the supply chain. The role of trade credit in the supply chain has been investigated by many authors. Among others, we found texts on the impact of trade credit on riskiness of the supply chain. For instance **Ellis, Henry and Shockley (2010)**[xliv] study the impact of supply disruption risk on the buyer perception. This perception and risk aversion could lead the buyer to modify its terms in the trade credit.

Indeed, trade credit is presented by **Yang and Birge (2013)** as a risk-sharing mechanism. The retailer allocates part of the risk to the supplier, and is incentivized to order more. As a result it increase the supply chain efficiency. The supplier shares a part of the demand risk because the trade credit repayment is contingent on the demand realization. In this way, the distressed cost is transferred from the retailer to the supplier. The supplier has furthermore an incentive to propose lower interest to induce the retailer to order more. In addition, **Yang and Birge (2013)** find that the increase in order quantity leads to an increased exposure for the supplier that leads him to reduce the share of trade credit in the contract terms. This behavior could partly explain why we can see a wide dispersion between prices of trade credit in practice. The result of the trade credit arrangement arises from the difference in reservation profit as well as the access to financing between the retailer and supplier, which determine their bargaining power. A financially constrained retailer has a lower reservation profit due to its inability to reach significant profit without relying to an external financing source. An offer from the supplier to extend credit, can significantly improve its profit. Conversely, a high bargaining power from the retailer perspective decreases the marginal effect of trade credit in order quantity.

Fabri and Klapper (2009)[xlv] find a relationship between the use of trade credit and the competitiveness of supplier in terms of horizontal (supplier-supplier) and vertical market power (retailer-supplier). The party with the lower market power tends to offer better conditions, such as lower interest rate or longer terms, to the one with the higher market power which benefits from a higher bargaining power. These findings allow to consider the influence of market power on the supply chain efficiency.

Burkart et al. (2011) study the use of trade credit in relation to the nature of the activity of the firm. They compare services, differentiated and standardized products. For example a supplier providing standardized products have less incentives to extend trade credit and the supplier providing services although extending as much trade credit, is less likely to provide early payment discount than the supplier providing differentiated goods. They also shows support the theory of reservation profit by showing that companies experiencing high growth opportunity or credit constrained, would be more reluctant to propose trade credit due to its higher opportunity cost. As a consequence, the supplier will prefer offering more trade credit to creditworthy buyers

Burkart et al. (2011) present furthermore four reasons encouraging the supplier to extend trade credit even if bank financing is available for the buyer. Firstly, the supplier often has superior information about the customer than the bank. Secondly, the moral hazard is greater for the bank, due to the impact on the trade relationship, the buyer would be more concerned to honor its loan to the supplier to avoid deterioration of the relationship in the long term. Thirdly, in case of liquidation, the supplier, operating in similar business, can potentially extract a higher value from the assets of the insolvent retailer than the bank. Finally, they consider that the supplier would have a lower opportunity cost that the banker.

Raghavan and Kumar (2009)[xlvi] take the viewpoint of the lender and present an interesting behavior resulting from the financing of an upstream agent in the supply chain. They show that under uncertain demand, the bank experience an incentive to extend financing to the downstream firm as well such that it increases its order quantity.

Murfin and Njoroge (2012)[xlvii]study cash constrained suppliers that offer trade credit to their larger customer. They note the asymmetry of the impact of trade credit terms in the cash-to-cash-cycle of both the lender and the borrower in accordance to their relative size. A little reduction of this cycle for large buyer leads to a large augmentation of cash-to-cash of the smaller supplier. This behavior can cause a worsening the financing availability of the cash constrained supplier that can hinder its growth opportunities.

Chod (2014)[xlviii] focus on agency issue and highlights the power of trade credit to that trade credit can help to alleviate agency problems. An agency issue arise after debt holder have provided their capital because shareholders have incentive to maximize the value of equity rather than the total asset value. Another agency problem results from the misalignment of incentive between shareholders and managers that lead the latter to improperly achieve the shareholders desired objectives. In the same field **Birge and Xu (2008)** [xlix] present a newsvendor model that study the relationship between manager's inventory decision and managerial compensation. Their finding showed that product decision are positively correlated with the weight of performance based bonus compensation while the debt usage increase as share of managerial equity ownership increases.

Chen and Wang (2012)[l] present a model that studies the ordering behavior of a retailer with budget constraint. They propose trade credit as a mean for the retailer to finance its purchase. They find that the limited liability in case of bankruptcy, tend to makes the retailer ordering more aggressively. This behavior is even more present as the retailer's initial budget decreases. The retailer with lower budget experience higher asymmetry in the distribution of its outcome, and benefits for larger upside potential. Hence, the retailer is induced to order more through trade credit, shifting more risk to the supplier. Overall, the trade credit has the potential to create a larger value in the supply chain and coordination can be partly achieved, providing that the retailer has limited initial capital.

It is demonstrated in the literature that the supply chain cannot coordinate with external financing with positive cost and unlimited credit line. However, trade credit can serve as a coordination tool. On this issue **Lee & Rhee (2010)** test the influence of trade credit on contracts that are known to achieve supply chain coordination. They present all-unit quantity discount, buybacks, revenue-sharing and two-part tariff contracts and find that solely the revenue-sharing fails to coordinate the supply chain under trade credit. The rational underlying the ability of trade credit to coordinate the supply chain is that trade credit is used as a mean for the supplier to subsidize the retailer cost of inventory financing as well as a risk sharing mechanism. **Lee and Rhee (2011)**[li] present furthermore an in-depth analysis of supply chain coordination under trade credit with markdown allowance. They find that unlike with trade credit, markdown allowance fail to achieve supply chain coordination when the retailer rely on external financing. With trade credit, markdown allowance as risk sharing mechanism compensate the retailer for the increased wholesale price set by the supplier in reaction of trade credit extension and in this way provide sufficient incentive to achieve the supply chain coordination. Finally, it is important to add that trade credit achieves coordination only when the supplier's financing cost is lower than that of retailer, otherwise the retailer should always switch to a better source of external finance.

Jing and Seidmann (2014)[lii] study the financial sourcing decision and its impact on the supply chain coordination. It compared trade credit receipt and bank financing in a one period problem as ways to finance its inventory under uncertain demand. They find that trade credit is better in alleviating double marginalization when production costs are relatively low while bank financing appears to be better in case of high production cost.

Caldentey and Chen (2012)[liii] compare the performance of trade credit versus bank financing for the case of a retailer facing constraint in its working capital. They find that internal financing is more beneficial from the supplier point of view. However the advantage tend to reduce as the retailer's initial budget increases since its order quantity reduces. As consequence, the supplier will offer a lower wholesale price such as to incentivize the retailer to order more. As a result, adverse selection problem can arise since the retailer has an incentive to understate its initial capital available and benefits from lower price, but then order less than what is optimal for the supply chain.

Chapitre 6. Other financial arrangements

6.1. Trade Credit Insurance

"Trade credit insurance (also known as credit insurance, business credit insurance or export credit insurance) is an insurance policy and risk management product that covers the payment risk resulting from the delivery of goods or services." **(Jones, 2010)**[liv]**.** This mechanism allows to secure its trade credit such as to reduce the uncertainty regarding the expected incomes coming from its buyer that could arise from credit risk due to protracted default, insolvency, bankruptcy risk… **(Jones, 2010)**. Trade credit insurance is an efficient risk management mechanism that enables to avoid propagation of financial distress of the buyer to the supplier The credit crisis has contributed to foster the infatuation for these insurance in reaction to the increased exposure to unpaid trade credit resulting from the poor economic conditions**(Morel, 2010).**[lv] This has made the trade credit insurance market growing to 5.3 Billion euro in 2008, at the peak of the credit crisis, covering about 2.6 trillions sales **(Jones, 2010)**. From a macroeconomic point of view, trade credit insurance increases the overall economic stability by enabling suppliers to share risk with credit insurers that are better equipped to absorb them

(Jones, 2010). Jones (2010) identified a list of benefits of trade credit insurance that are summarized in *Appendix Bis 5.1*.

In exchange to the transfer of risk, the trade credit insurer charges a premium that reflect the overall credit worthiness of the covered buyers **(Jones, 2010)** in form of regular payment covering the trade credits extended a defined buyer during a certain period of time. This premium is proportional to the amount of turnover resulting from trade credit as well as the historical expected default loss resulting therefrom **(Jones, 2010).** Generally, the insurer accepts to bear the risk up to a certain credit limit depending of different underlying risk factors. Generally the policy covers over 85-90% of the loss incurred by the payment failure of trade credit **(Jones, 2010).**The purpose is to incentivize the supplier to continue monitoring properly its trade credit, by sharing part of the risk with him. In addition, the trade credit insurance cannot be requested before a determined period of 60 to 180 days following the repayment due date prompting even more the supplier to manage its receivables and reduce payment delays **(Jones, 2010).**

Few papers investigates quantitatively trade credit insurance. In the newsvendor setting, to the best of our knowledge, we only found one recently written article. **Li, Zhen and Cai (2014)**[lvi] provides a first attempt to grasp the trade credit insurance behavior. They study the case of a manufacturer encountering financial constraint and retailer repayment risk that makes the financing decision and at the same time chooses the optimal amount of trade credit insured in response to the probability distribution of repayment. Specifically, the behavior of the bank regarding the trade credit insurance coverage decision is explored under both risk neutrality and risk aversion of the bank Their main findings are that a decrease in interest rate from the bank causes the manufacturer to increase its credit sales and therefore increase the insurance coverage. Secondly, the proportion of credit sales uncovered by trade credit insurance decreases with the interest rate. Then, trade credit insurance as expected reduces supplier risk. But in contrary to what could be expected, it does not always reduce the interest rate charged by the bank. The conclude that overall, trade credit insurance is beneficial for both agents.

6.2.Factoring

"*Factoring is a traditional product that allows a supplier to pre-finance its receivables whereby the factor pays a percentage of the face value of the receivables based upon its assessment of the credit risk and the underlying payment terms."* **(Jones, 2010)**

The supplier experiencing delayed repayment can use factoring as a mean to receive cash directly and in such way manages the match between payables and receivables **(Soufiani 2002).**[lvii] In this case, he sells the trade credit receipt to a third party financier– the factor - who buy it in exchange of a premium depending on the risk incurred by the factor **(SCF working group, 2010)**[lviii]**.** At the receivable due date, the factor receives the payment in lieu of the supplier.

Factoring involves the transfer of ownership and, to some extent, the transfer of risk depending on the type of agreement. We classify it into two main categories: with recourse factoring, and non-recourse factoring. *"With recourse agreement is a legal agreement that provides protection to lenders, as they are assured of having some sort of repayment – either cash or liquid assets – in the event that the borrower is unable to satisfy the debt obligation"* **(Camerinelli, 2014).** With recourse agreement guarantee the factor to receive payment from the seller (supplier) in case of failure of repayment from the borrower (buyer). In this case the repayment risks remains borne by the supplier making the premium charged by the factor cheaper than for without-recourse agreement, that does not involve the supplier's liability. From the factoring company point of view, without-recourse factoring is a more risky proposition. Factoring companies tend to limit the exposure of their portfolio to such contracts. In practice we note that they tend to purchase without-recourse contract only from the highest quality receivable **(Sopranzetti, 1998)**[lix]. For the with-recourse agreements however they also restrict their target by limiting the share of the receivable pool of bankrupt prone sellers to avoid problems of moral hazard **(Sopranzetti, 1998)**

The reasons that motivate supplier to use factoring are investigated by **Summers and Wilson (2000)**[lx]**.** Among the reason, they denoted the transactions costs. If the factor is able to monitor the risk of the buyer better than the supplier, it should induce the supplier to use factoring. For example, when a factor has receivable coming from suppliers that deal with the same pool of customer, they can benefit from economy of scale in acquiring better information from these buyers. Then they mentioned the economy of scale of the supplier when monitoring trade credit. A supplier extending trade credit with many buyers is less likely to benefit from factoring. The need for cash flow is also mentioned as driver of the use of factoring. Finally they mention the impact of the industry practice toward factoring.

6.3. Asset based financing

To avoid prohibitively high bank loan, in many cases, firms use their assets as collateral for their financing needs. This technique is known as asset-based financing. **Buzacott and Zhang (2004)** developed a model incorporating collateral for joint financing and operational decision still in the stream of newsvendor literature taking into account bankruptcy risk. **Kouvelis and Zhao (2012)** studied the joint financial and production decision in a case with asset based financing as well. They proposed a costly bankruptcy and supported the idea that financial decision should be taken with the operational decision in the case of imperfect market. In both cases, they showed that bankruptcy cost affects the interest rate the bank will charge, since the bank concerned by the difficulty of recovering the entire value of the collateral. A reason is that selling goods is not the business of the bank **(Burkart, Giannetti & Ellingsten, 2005)**. As previously mentioned, on this issue, the supplier extending a trade credit for financing the buyer could experience an advantage over the bank to recover most of the amount of the collateral since it is its business to sell the same category of goods than the supplier. That explain for example why suppliers are likely to offer lower interest rate than bank. Seniority issue must be taken into account when pricing both bank- and supplier lending in the case of using collateral. **Longhofer and Santos (2000)**[lxi] proposed a study on that issue state that suppliers are less likely to make a company going bankrupt that the bank. This result from the seniority of the loan that makes the supplier junior to the bank such that in case of liquidation it only receives the residual claim. As consequence, the supplier has also less incentive to monitor the buyer.

6.4. Pre-shipment, Inventory-in-transit, Post-shipment Financing

He et al. (2012) distinguish three category of financing arrangement for the supply chain according to the event trigger for which financing is needed.

- Pre-shipment relate to the financing needed after the reception of the purchase order but before the product has been shipped. This financing is need to cover the working capital and the purchase of raw material. Post-shipment financing takes place to offset the time to receive cash from the buyer after having shipped the good to him. (=>Retrouver la réference)

- In-transit finance relate to financing need that arise between the shipment of the good and the invoice approval. Institution often involve the thrird-party logistic provider in the operation such as to visualize the material flow to better monitor the risk. This kind of financing is even more useful for long distance shipment, such as transportation by boat oversea, which are especially important in the context of offshoring.

- Post-shipment finance includes early-payment discount, account receivable financing, factoring and reverse-factoring. This category of financing allow the supplier to reduce the gap between the invoice approval and the actual payment of the buyer in order to reduce its short terms receivable and improve its working capital position.

6.5.Reverse factoring

An upcoming Supply chain finance solution is reverse factoring in which the supplier with lower creditworthiness is given financing for its working capital at lower cost by transferring the liability to the more creditworthy buyer. In other words, after having delivered the order to the buyer, the supplier discount receivables to the financial institution. It results the transfer of the obligation to the bank that receives the ownership of the receivable. The bank then wait for repayment payment from the customer, which is generally endowed with a higher credit worthiness. The supplier can either wait for the payment scheduled or receive the discounted amount earlier as if it was a credit. The benefits from the supplier point of view is that it is given a lower interest rate by benefiting of the higher rating of its buyer. Indeed, in reverse factoring, the client makes explicit guarantee to the factor that the payment of the obligation will be met **(Klapper, 2006)**[lxii]. The main difference with factoring arrangement is that here the facility is arranged with the buyer, not the supplier. The buyer introduce then the service to its supplier **(Demica, 2009)**[lxiii].

Relatively few literature have investigated supply chain finance solutions quantitatively. To the best of our knowledge, we found three recent texts.

Van der vliet et al. (2014) study reverse factoring, which is an important supply chain finance mechanism. They fund inter alia that extension of payment terms to the buyer’s point of view induces a non-linear financing cost for the supplier. As consequence, the optimal cost increase asymptotically with the extention of the payment terms.

Tanrisever and al. (2012) give insight on the estimation of the value of reverse factoring. They demonstrate that value of reverse factoring depend mostly on the spread between internal (supplier, customer) and external financing cost (bank), the operating characteristics of the supplier and the level of the risk-free rate in the economy.

Research Problem

Now that we have further insights on the literature related to the financing of the supply chain, we will state the direction of our research.

We focus our interest on the behavior of agents of the supply chain with respect to their financing decision. Therefore we are concern about the impact of such decision on their operational decision. We state our research problem as follow:

Research Problem: *How does supply chain financing arrangement through trade credit influences operational decisions?*

In order to explore this problem in a more structured way, we propose different research question that will guide our analysis.

In the supply chain finance system, we consider three main actors, the retailer, the supplier, and the financier (which can be a bank, or another supplier). As consequence we are concerned about the impact of the financial and operational decisions of the supply chain on each of these actors.

That is why we will explore a series of research question in turn

Research Question:

- *What is the impact of trade credit financing on the retailer's operational decision?*
- *What is the impact of trade credit financing on the supplier's operational decision?*
- *What is the impact of leverage on the supplier's operational decision?*
- *How does the bank react to the supply chain financing systems when defining its interest rate?*
- *What is the impact of market perfection on the operation decision? (order quantity, wholesale price, trade credit interest rate)*
- *What is the impact of bankruptcy costs on the retailer, the supplier and the financier?*
- *What is the impact of financial constraint on the retailer's variance of profit?*
- *What is the interaction resulting from the combination of CVaR criteria as objective and financing retailer's financing need?*

We explore these question through our exploratory section and provide proofs for most of them. The reader is invited to refer to the appendix for more details on the results presented.

In our analysis, we consider the trade credit contract. This kind of financing has already been investigated widely in the literature as tool for financing the supply chain. We base our analysis on the findings of **Chen and Wang (2012)** that explore open account financing in the context of retailer's budget constraint. Some similarity are likely to arise regarding the first setting of the model. Next, we extent the analysis by considering trade credit interest rate and its impact on the retailer's ordering decision.

Then we explore the case where the retailer experiences financial constrained as well and relate to an external source of fund to finance its operations as well as those of its retailer. In this way we present a supply chain financing arrangement consisting of a pass through of supplier financing to the supplier by means of delayed payment. We provide insights on the externality of such arrangement on the behavior of the agents in the supply chain. We envisage the case of constant and exogenous financing scheme and the case of competitively priced loan arising from a bank evolving in a perfect capital market without market imperfection such as bankruptcy costs, information asymmetry, transaction costs or taxes. We found that under these conditions, the operational and financial decision appears irrelevant, even in the case of a supplier controlling its wholesale price and interest rate. Next, we consider the introduction of bankruptcy cost in the analysis and see how this market imperfection influence the supplier decision regarding its optimal wholesale price and interest rate. We additionally explore the influence of such imperfection on the relevance of the junction between the operational and financial decision. In a second part, we analysis the issue of variability of the newsvendor profit in the context of financial constraint. We explore the convergence of profit as well as the optimality conditions. Finally, we address the risk aversion through the presentation of the Conditional Value-at-Risk criteria as an objective. In the end, we summarize all our findings.

Chapitre 7. Supply Chain Financing Model

We follow a two-period, two-level newsvendor model, with the supplier (she) acting as the Stackelberg Leader, and the retailer (he) acting as the follower in the game. We consider in our analysis, that supplier and retailer are both financially constrained. Which means that for a level of order (or production) exceeding their initial capital available, they need to relate on another source of financing than their internal capital. We propose a supply chain finance system where both the supplier and the retailer are cash constrained. The supplier finances purchases of the retailer by extending trade credit. For financing her operations, the supplier borrows a loan to the bank (it) (alternatively, we will also consider the case where the supplier finances herself through other financing sources, such as open account financing). The loan is secured by the expected repayment of the retailer. We assume the retailer has only one external source of finance, which is, the supplier's trade credit facility (or open account). The supplier can finance its operations solely through a bank loan (alternatively, a single external financing source such as open account financing or others, depending on the assumptions underlying) or its initial capital. The bank, as the Stackelberg sub-leader determines an endogenous interest rate on the basis of the supplier risk profile arising from her operations. The bank is assumed to operate in a perfect capital market. The singularity of perfect capital market is that the bank proposes a loan such as the marginal cost of capital equates the marginal revenue from this loan. The bank does not earn a margin for granting the loan and end up with zero profit (in expectation) from its operations. We assume all three actors in this model are risk-neutral. Which means that there is no difference in the preferences regarding a deterministic income or an expected revenue. Agents in the model relate only on the first moment of the demand distribution to determine their decision.

The retailer faces an uncertain demand ξ that is common knowledge of the supplier, the retailer and the bank. Hence, we assume perfect information within the supply chain. In other words, there does not exist asymmetry in information among participants, including the bank. The demand follow a probability distribution denoted by $F(x) = P(\xi \leq x)$, with density $f(x) = F'(x)$, and the tail distribution is given by $\bar{F}(x) = 1 - F(x)$. The demand has a finite mean $\mu < \infty$ and is non negative (i.e. $F(0) = 0$).

We focus our analysis on demand function with an increasing failure rate (IFR). As defined by **(Larivière, 2005)**[lxiv]**,** the failure rate is given by $h(\xi) \leq \frac{f(\xi)}{1-F(\xi)} = \frac{f(\xi)}{\bar{F}(\xi)}$. If $h(\xi)$ is weakly increasing for all ξ such that $\Phi\,(\xi) < 1$ we have an increasing failure rate probability distribution function **(Larivière, 2005)**. Among known IFR distributions we have the Uniform distribution, the Normal distribution, the Exponential, the Power distribution for $k \geq 1$, the gamma with parameter $k \geq 1$, the Weidbul distribution with shape parameter $k \geq 1$, the Beta Distribution for parameters $\alpha, \beta \geq 1$ **(Banciu & Mirchandani, 2013**)[lxv] the log-normal distribution with parameter $\lambda > 1$ **(Gupta & Lvin, 2005)**[lxvi]**.** It is worth noticing that IFR distribution are IGFR (Increasing Generalize Failure Rate). The generalized failure rate is given by $\xi h(\xi)$. In the contrary, IGFR do not guarantee that the probability function is IFR.

The sequence of the operations is the following:

1) The supplier anticipates the retailer's best-response order quantity and sets the appropriate wholesale price w and interest rate r^{tc}, in order to maximize her own profit.
2) The retailer reacts to the offer of the supplier and places an order of Q^* units of product to the such as to maximize his own profit, according to his view of the expected demand.
3) The retailer pays upfront the supplier a share of the total amount (or the entire amount) of product. We assume the payment is required prior the production and is used by the supplier to finance her operations.
4) The supplier borrows from the bank the necessary amount of additional fund to its internal capital and the retailer's upfront payment in order to finance its activity. She is charged an interest rate r^b determined by the bank according to its view of the expected supplier's revenue. (Alternatively, the supplier delays payment under open account financing or other fixed financing sources. The rate r^s is assumed fixed and exogeneous)
5) The supplier produces (or alternatively procures) Q^* unit of product. We assume thproduction is instantaneous. We assume infinite production capacity and deterministic supply.
6) The suppliers ships the production to the retailer.
7) The demand is realized and the retailer receives a revenue $pmax[D, Q^*]$.
8) The retailer pays the supplier the remaining fees. If he does not have the sufficient amount of capital to reimburse, the retailer declares bankruptcy, and all his assets are transferred to the supplier.

9) The supplier receives the payment of the retailer, she reimburses the loan to the bank. If she is not able repay its debt, the supplier goes bankrupt.

7.1.Retailer Problem

Let p be the price at which the retailer sells its product to the market. It is exogenously fixed and is taken as given. The retailer is price-taker in the market and the market price is not influenced by the quantity put into the market. Let w the wholesale price at which the retailer procures his product to the supplier. To make the problem interesting, let $p > w(1 + rf)$, in such way, we avoid trivial result. The retailer would never order any quantity if he does not earn profit on it.

The retailer has initially an internal capital K^r and can invest at most this amount without entering into a trade credit contract to delay the remaining payment. Besides, we consider the money have an opportunity cost of $(1 + rf)$ where rf is the risk free rate. This rate is assumed to be the relevant discount rate in this model. An additional assumption here is that risk-free rate is the same for borrowing or lending money. Adopting this view is relevant under the assumption of perfect capital market. In a perfect capital market, banks lend fund at their marginal cost, which is the market borrowing cost. The bid-ask spread is as a consequence inexistent. The only consideration driving the interest rate up, is the risk incurred by the lender to provide funds. We will see later in the banks setting how this interest rate is determined regarding the risk profile of the borrower.

If the retailer does not have sufficient cash to pay the supplier upfront, $wq > K$, he delays payment and enter into a trade credit contract which is a form of loan granted by the supplier to the retailer such as to circumvent the sub-optimal order quantity of the cash-constrained case (as we will see later). Denote the loan under trade credit contract as $L^r = (wQ - K^r)$. The supplier charges the interest rate r^{tc}. Hence, the retailer is expected to pay at the end of the season $L^r(1 + r^{tc})$ to the supplier. For sake of simplicity, we consider only the case where the supplier's interest rate is higher than the risk free rate $(r^{tc} > rf)$. In such case, the retailer is always induced to first use its internal capital to finance its operations and then use trade credit to increase further its order quantity. Note that all interest rate used in the present text are compounded only once in a fixed horizon represented by the selling period. In this way, the loan takes the form of a zero coupon bond. In trade credit, we refer to it as trade credit in net term.

When selling season occurs, the retailer receives $pmin[\xi, Q]$ which is the minimum between demand and the available quantity to be sold. For the remaining of this text, we will consider normalized parameters such that $p = 1$, and $0 < c(1+rf) < w(1+rf) < 1;\ w(1+r^{tc}) < 1$.

We can formulate the retailer profit as following:

$$\pi^r(Q) = min[\xi, Q] + (K^r - wQ)^+(1+rf) - (wQ - K^r)^+(1+r^{tc})$$

1.1.7 First case: $K \geq wQ^*$

First, we analyze the case where the retailer has sufficient fund to finance its optimal ordering decision ($K^r > wQ^*$)

The problem becomes: $\max E[\pi^r(Q)] = \max E\big[min[\xi, Q]\big] + (K^r - wQ)(1+rf)$

$$s.t.\ wQ \leq K^r$$

The optimal order quantity can be fund be deriving the expected profit $E[\,\pi^r(q)]$ according to the order quantity and equating the result to zero. The FOC gives the following result *(see Proof 1.1)*:

$$FOC: \frac{dE[\,\pi^r(Q)]}{dQ} = 0 \;=>\; \bar{F}(Q^*) = w(1+rf)\ ;\ F(Q^*) = 1 - w(1+rf)$$

This result is exactly the same as for the unconstrained newsvendor model. Denote $\tilde{Q} = Q^*$ the result of this problem.

2.1.7 Second case: $K < wQ^*$

Now let explore the behavior of the retailer in the case of insufficient internal capital to order its optimal quantity. First, we present the financially constrained retailer with no external source of financing. It is easy to find that, due to the concavity of the profit function ($\frac{d^2E[\,\pi^r(Q)]}{dQ^2} < 0$), if $Q^* > K^r$, the retailer's optimal order quantity should be at the bound of the feasible set and the constraint is binding. We find that the optimal quantity is: $Q^* = \frac{K}{w}$ .As consequence, for the cash-constrained case, the retailer best response function is given by $Q^* = \min\left[\tilde{Q}, \frac{K}{w}\right]$.

As consequence, with cash constraint, both the expected profit of the retailer and the supplier are lower when $Q^* \geq K^r$, than in the unrestricted case.

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In order to increase the efficiency of the supply chain, and allow the retailer ordering more, the supplier extends financing scheme to the retailer. She grants the loan $L^r = wQ - K^r$ in order to help him purchasing more inventory. He is supposed to reimburse the amount at the end of the season with an interest rate fixed by the supplier: $(wq - K^r)(1 + r^{tc})$.

We can rewrite the retailer's problem as follow:

$$\max E[\pi^r(q)] = E[min[\xi, Q] - (wQ - K)(1 + r^{tc})]^+ - K^r(1 + rf)$$

$$s.t. wQ > K^r$$

Note that the positive sign $(a)^+ = \max[0, a]$ for the profit express the assumption of limited liability of the retailer. In fact, when at the end of the period, the sales does not permit to satisfy the liability, the retailer does not have to compensate the supplier for the deadweight loss the supplier has incurred. The retailer only goes bankrupt, transfer its wealth to the supplier that bears the risk of this loss. The limited liability is an interesting setting when analyzing the behavior of agents in the supply chain. Because it limits the downside consequences of randomness while letting upside potential free. The result is an incentive for the leveraged agent to take more risks than in the case of unlimited liability.

Before going through this problem into more details, it is worth defining the bankruptcy threshold. This threshold represents the level of demand realization under which the retailer does not have sufficient revenue to repay loan and declare bankruptcy. At this threshold, the retailer's assets are transferred to the supplier and he ends up with a terminal value of zero. The supplier receives the asset's value, which is always a smaller amount that the initially proposed loan.

The bankruptcy threshold, which is the minimum quantity needed to be sold in order to avoid going bankrupt is easily fund by setting the profit to zero. $min[\xi, Q] - (wQ - K)^+(1 + r^{tc}) = 0$ since $w < 1 = p$, the threshold is represented by the demand level such that $\xi = (wQ - K)^+(1 + r^{tc})$.

For the remaining of this text, we define the retailer's bankruptcy threshold as:

$$b(Q) = b = \frac{(wQ - K)^+(1 + r^{tc})}{p} = (wQ - K)^+(1 + r^{tc})$$

Now, we can rewrite the retailer problem as following *(see proof 1.2.2)*:

$$\max E[\pi^r(Q)] = -K(1+rf) + \int_{b(q)}^{q} \bar{F}(x)dx$$

Similarly to the previous analysis, we can derive the best response function of the retailer regarding the optimal order quantity *(see Proof 1.2)*. FOC: $\frac{dE[\pi^r(Q^*)]}{dQ^*} = 0$. We find:

$\bar{F}(Q^*) - w(1+r^{tc})\bar{F}\big(b(Q^*)\big) = 0 \Rightarrow \bar{F}(Q^*) = w(1+r^{tc})\bar{F}\big(b(Q^*)\big)$

The best response of the retailer is given by: $Q^* = \bar{F}^{-1}\left(w(1+r^{tc})\bar{F}\big(b(Q^*)\big)\right)$. Note that the FOC possess some similarities with the case without borrowing where we had $\bar{F}(Q^*) - w = 0$. This suggests that provided that the supplier propose a sufficiently low trade credit interest rate r^{tc}, we have $w > w(1+r^{tc})\bar{F}\big(b(Q^*)\big)$ since $0 < \bar{F}\big(b(Q^*)\big) < 1$ and as consequence,$\bar{F}(Q^*)^{trade\ credit} < \bar{F}(Q^*)^{unrestricted}$ which means that the retailer is induced to order more with trade credit than in the unrestricted case. This follows our suggestions about the limited liability. Due to the limited downside risk, the retailer is induced to order more.

With trade credit, the demand risk is shared between the supplier and the retailer due to the possibility of bankruptcy. Under unrestricted internal capital, the retailer does not need trade credit unless it is below the risk-free rate. As consequence, the supplier does not share the risk with the retailer and receives upfront the total amount ordered: wQ^*. The future value of this amount at the end of the season equals: $wQ(1+rf)$ and is deterministic, and known with certainty at the time the retailer places its order.

In our cash constrained case, the retailer relates on trade credit financing and pays upfront only one part of the liability, then at the end of the period, contingent of the demand realization, he repays the loan or declares bankruptcy and transfers all its asset to the supplier. The risk is shared since the supplier cannot predict exactly what amount she will receive from the retailer. The supplier is promised to receive $\min[D, L^r(1+r^{tc})]$ Hence, the expected repayment at the end of the season can be written as follow:

$$E[L^r(1+r^{tc})] = \int_0^{b(Q)} x dF(x) + b(Q)\int_{b(Q)}^{\infty} dF(x)$$

The first term representing the expected revenue from bankruptcy, which comes from the asset pledged by the retailer to secure its loan to the supplier. We assume here no bankruptcy cost, and the supplier being senior creditor since we assumed no other sources of financing. The shareholders of the retailer receives money only after the supplier in case of bankruptcy.

If the retailer had to fund its operation directly from the bank, operating in a perfectly competitive market, the required interest rate from the bank's would have been computed as:

$$\int_0^{b(Q)} xdF(x) + L^r(1+r^b)\int_{b(Q)}^{\infty} dF(x) = L^r(1+rf)$$

Such that the expected the bank is indifferent between granting a loan to the retailer or invest its money at risk free rate. From the supplier's perspective, the rate resulting is different. Since the supplier can benefits from bearing more risk on the loan in order to increase sales, the interest rate becomes here, along the wholesale price, a decision variable to adjust the quantity ordered by the retailer. We can denote (w, r^{tc}), the set of decision variables of the supplier. We will further explore how these variables interact with the retailer's decision and see how they can help the supplier to maximize its own profit.

7.2.<u>Supplier problem</u>

Before entering into more details, let present the settings of the supplier's problem. We assume the supplier has insufficient capital to finance its own operations and rely on a bank loan to complete its budget for operations. She can furthermore use the upfront payment of the retailer to finance its operations as previously presented in the sequence of event (Note that we do not lose insight in the analysis by considering that the supplier would not be able to use the retailer upfront payment to finance its inventory. The result would only be a higher need for financing from the supplier's point of view).

The supplier's profit is given by the following:

$$\pi^s(Q) = L^r(1+r^{tc}) - (cq - K^s - K^r)^+(1+r^b) - (cq - K^s - K^r)^-(1+rf)$$

In the case the supplier has sufficient cash to finance its operations we have:

$$E[\pi^s(Q)] = E[L^r(1+r^{tc})] - (cq - K^s - K^r)(1+r^s)$$

When the retailer orders more than what can be covered by the capital, in the case, the supplier has not enough cash to fund its operations solely with initial capital and retailer's upfront payment (i.e. $K^s + K^r \leq cq$). She enters in a loan with a Bank and its problem can be written as:

$$\max E(\pi^s(Q)) = [L^s\big[E[L^r(1+r^{tc})] - L^{s+}(1+r^b)\big]^+ - \widehat{K}(1+rf)$$

Where $K^r + K^s = \widehat{K}$; and $L^s = cQ - K^r - K^s$, with K^s the supplier's initial capital, L^s the supplier loan, r^s the interest rate charged by the bank to the supplier.

-

Note that in this case, similarly to the retailer, the supplier becomes prone to bankruptcy. We can define the supplier bankruptcy threshold as well:

$$g(Q) = \frac{L^s(1+r^s)}{p} = \frac{(cQ - K^r - K^s)(1+r^s)}{p} = (cQ - K^r - K^s)(1+r^s)$$

This number corresponds to the minimum level of demand required for the supplier to reimburse its loan, for a given level of indebtedness. Note that this level should be bellow which of the retailer's bankruptcy threshold $b(Q)$. Otherwise, the supplier would in any case go bankrupt, even if the retailer would repay the whole amount of trade credit. It should never occurs since we assumed $c < w$, along with the fact that the retailer upfront payment is used to finance the supplier's operation.

A bankruptcy threshold such that $b(Q) = g(Q)$ should therefore represent the highest level of risk permissible from the supplier point of view to be allowed by the bank to enter into a loan. As a consequence $b(Q)$ reprensents an upper bound for the supplier financing. Normally, the threshold should always exceed the retailer's bankruptcy threshold, and therefore, the following inequality must always hold.

$$g(Q) \leq b(Q)$$

This implies that the supplier should never go bankrupt unless the retailer's goes bankrupt. This evidence is consistent with the findings of **(Lee & Rhee, 2011)**[2]

-

Now, let derive the supplier's problem. First, rewriting the expected profit with regards to the bankruptcy threshold indicators, we obtain:

$$E[\pi^s] = \left[\left(\int_0^{b(Q)} x dF(x) + b(Q)\int_{b(Q)}^{\infty} dF(x)\right) - L^s(1+r^s)\right]^+ - \widehat{K}(1+rf)$$

[2] Trade credit contract with limited liability in supply chain with budget constraint

$$E[\pi^s] = \int_{g(Q)}^{b(Q)} [x - g(Q)]dF(x) + [b(Q) - g(Q)] \int_{b(Q)}^{\infty} dF(x)$$

7.3.Retailer's sensitivities

Before deriving the supplier optimal decision, we need to grasp what really happens to the retailer ordering decision as the supplier changes the parameters of the contract(w, r^{tc}). We present sensitivities of the optimal order quantity relative to the wholesale price, and the trade credit interest rate.

1.3.7 Sensitivity to the wholesale price. *(see Proof 2.1)*

First of all, let see the impact of the supplier wholesale price on the retailer's optimal quantity decision. We follow the derivation technique used by **Chen & Wang (2012)** [3]. It consists of making the use of the implicit function theorem: $\frac{dQ^*}{dw} = -\frac{\frac{dG(Q^*)}{dw}}{\frac{dG(Q^*)}{dQ}}$ (Where $G(Q^*) = \bar{F}(q^*) - w(1 + r^{tc})\bar{F}\big(b(q^*)\big) = 0$) This techniques allow us to compute the sensitivity of the retailer's optimal order quantity to the wholesale price set by the supplier. We obtain: $\frac{dQ^*}{dw} = \frac{[1 - wQ(1 + r^{tc})h(b(Q^*))]}{-w[h(Q^*) - w(1 + r^{tc})h(b(Q^*))]} < 0$. It appears to be a negative function in the feasible regions of the wholesale price $c(1 + rf) \leq w(1 + rf) \leq p = 1$ *(see Proof 2.1.2).* As consequence, an increase in the wholesale price lead the retailer to order less, as intuitively expected. The optimization of the revenue from the supplier point of view will result in an arbitrage between size of the order, and the per unit margin.

The particularity of the wholesale price from the retailer's point of view, is that it influences the retailer margin on the whole order, even if it relates to the upfront payment. This is not the case of interest rate as we will see later. Both parameters results in different impact on the order quantity.

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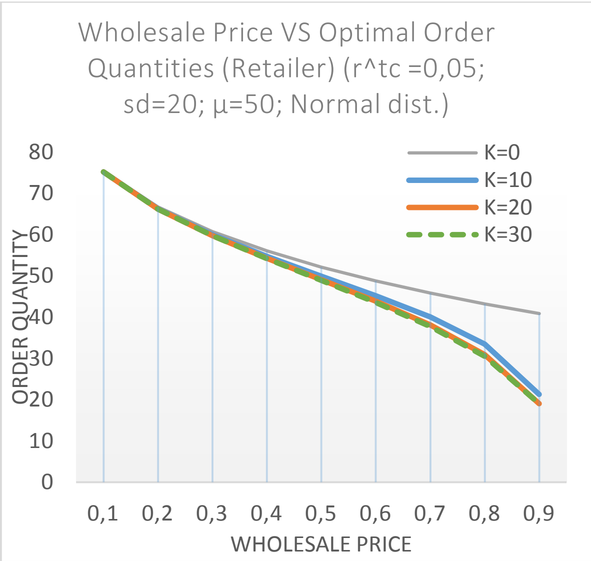


Figure 1.1

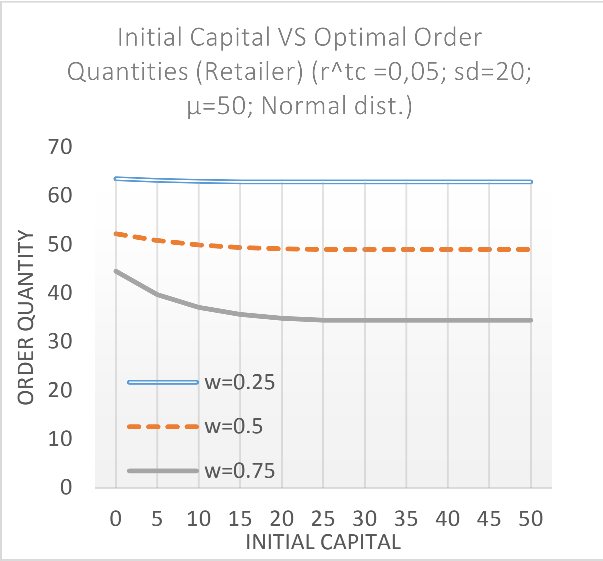


Figure 1.1

The figure above shows the influence of the wholesale price on the order quantity *(Figure 1.1)*. The function is non-linear decreasing and differ depending on the initial wealth. We can see that a higher wholesale price has less impact on the retailer that does not have initial capital. *Figure 1.2* shows the evolution of the order quantity relative to the initial capital for different level of given wholesale price. We can notice that the mitigation mechanism resulting from an increase of the wholesale price reduce as the initial capital is closer to zero. Which means that the wholesale price could be less efficient as a tools for controlling the retailer's order quantity when the retailer experiences significant capital constraint.

2.3.7 <u>Sensitivity to the revenue. (Proof 2.2)</u>

As we said, the supplier decision regarding the revenue takes the form of a trade-off between the quantity sold and the price at which they are sold. An increase in the wholesale price decrease part of the order quantity but decrease the amount at which each quantity is sold. Hence, the relevant measure to take into consideration from the supplier's point of view is the sensitivity of her revenue regarding the wholesale price she sets to the retailer.

From the best-response function of the retailer, the supplier can easily deduce the evolution of its own revenue relative to the wholesale price set to the retailer. Similarly, still following **Chen & Wang (2012)**, we derive the sensitivity of the supplier's revenue to the wholesale price. $\frac{dwQ^*}{dw} = \left(\frac{dQ^*}{dw}\right) w + Q^*$ We find: $\frac{dwQ^*}{dw} = \frac{(1-Q^* h(Q^*))}{-[h(Q^*) - w(1+r^{tc})h(b(Q^*))]}$. This result will be used to compute the supplier optimal wholesale price as presented later.

3.3.7 Sensitivity to the Trade Credit interest rate.(*see Proof 2.3*)

Unlike **Chen & Wang (2012)**, we explore furthermore the evolution of the retailer's order quantity with respect to the interest rate charged for delayed payments. Here, we found the following result: $\frac{dQ^*}{dr^{tc}} = \frac{[1-h(b)(1+r^{tc})(wQ^*-K)]}{-[h(Q)-w(1+r)^2h(b)]} < 0$ *(see Proof 2.3.1)*.The retailer's best response order quantity are negatively correlated with the trade credit interest rate, similarly to the wholesale price.

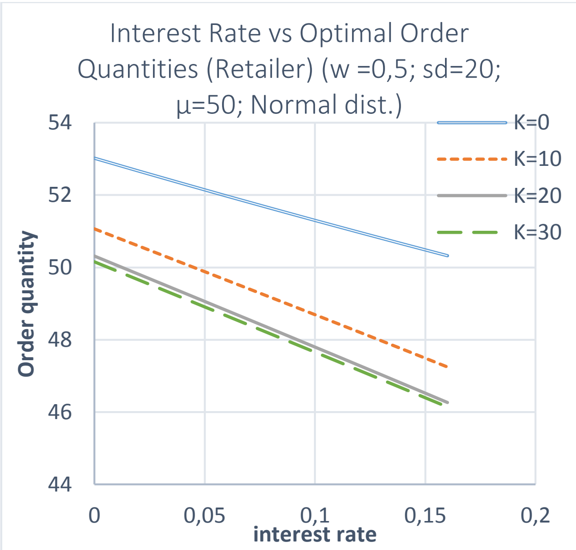


Figure 2.1 *Figure 2.2*

Figure 2.1 illustrates the evolution of the retailer's optimal quantity with respect to the interest rate, for different initial capital. We can note here a linear negative relationship and a slope that does not differ substantially between the different initial wealth. The difference in terms of high is explained by the behavior resulting from the sensitivity to initial capital that we present in the following section. The retailer with the lower initial capital tends to indeed order more, and this is verified for any given level of interest rate. However, the retailer with lower initial capital experiences a sharper reduction of its profit when the interest rate is increased as the effect of interest rate is reflected in the part of the order that is financed through delayed payment *(Figure 2.2)*. Note that with low interest rate, the retailer with low initial capital benefits from the limited downside risk and can foresee a higher opportunity of profit.

4.3.7 Sensitivity to the retailer's internal capital. (Proof 2.4)

Finally, we derive the optimal order quantity with regard to the retailer internal capital. We obtain: $\frac{dQ^*}{dK^r} = \frac{(1+r^{tc})h(b(q))}{-[h(Q)- w(1+r^{tc})h(b(Q))]} < 0$

The retailer's order quantity decreases with its internal capital. This is understandable by looking at the limited liability benefits. When the retailer has very low internal capital, he is induced to order more because he incurs a very large upside possibility and a low downside in its revenues. Providing that the demand distribution remains the same, as the internal capital is lower, the share of the demand distribution that is above the maximum quantity he is able to order with internal fund is bigger. This induces the retailer to order even more than its own resource, especially because he shares the demand risk with the supplier for this part. The probability to have a demand less than what its own capital can afford is smaller, the lesser he has initial capital. As consequence, the importance of the upside potential relative to the downside risk increases and the retailer prefers using external capital. Indeed, he faces a higher probability to gain money than that of losing money due to low demand. Not only in terms of probability but even more in terms of expectation, because for the demand realization that result in bankruptcy, the outcome of the supplier cannot decrease below zero. The lower the internal capital, the lesser the share of the risk the retailer bears. In consequence, the supplier bears an increased risk, which is the result of retailer's bankruptcy event. We will see later, that in order to circumvent this problem, the supplier is induced to increase its interest rate in order to make the retailer ordering less, such as to decreases its risk.

7.4. Supplier optimal wholesale price

1.4.7 Unrestricted problem *(see Proof 3.1)*

Now that we have gained more insights on the reaction of the retailer regarding different variable, we explore further the analysis of the supplier's wholesale price. We will derive the optimal wholesale price such as to maximize the profit of the supplier. We first present the case where the supplier has sufficient initial capital to finance its operations. The result is similar to what has been found by **(Chen & Wang, 2012).**

The supplier objective function can be written as follow:

$$\pi^s = -K^s - K^r + \int_0^{(wQ-K)(1+r^{tc})} \bar{F}(x)dx - cQ(1+rf)$$

We can derive the optimal wholesale price from this profit function through the use of the first order conditions.

$$\frac{d\pi^s(Q,w)}{dw} = \frac{d(wQ^*)}{dw}\bar{F}\big(b(Q^*)\big)(1+r^{tc}) - c\left(\frac{dQ^*}{dw}\right)(1+rf) = 0$$

That gives us

$$\frac{d\pi^s}{dw} = \frac{[1-(1+r^{tc})wQ^*h(b(Q^*))]\left[\frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1-wQ^*(1+r^{tc})h((b(Q^*))} - c(1+rf)\right]}{-w[h(Q^*) - w(1+r^{tc})h((b(Q^*))]}$$

This can be reduced to this unique optimality condition:

$$\frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1-wQ^*(1+r^{tc})h((wQ^*-K^r)(1+r^{tc}))} = c(1+rf)$$

The right-hand side is fixed, then we compute the wholesale price that equates the marginal cost. Note that, $wQ^*(1+r^{tc})h(wQ^*-K^r) < H(Q^*)$ (by IFR assumption) and $\bar{F}(Q^*) \leq 1$. This means that as w increases, the left hand side increases too. Therefore, if the supplier's marginal cost is increased, the supplier is induced to set its wholesale price higher. In such way, at the same time, she transfers part of the additional cost to the retailer such as to compensate the potential loss in margin. Then, the increase in the wholesale price induces the retailer to order less quantity, at a higher price. Due to the risk-sharing mechanism of the trade credit contract, the supplier is concerned about the risk she takes with respect with the demand uncertainty. As consequence, as usually seen in newsvendor problems, the higher the overstocking cost, the higher the risk of resulting from potential left-over inventory at the end of the season, and the lesser the quantity. The supplier's overstocking cost correspond to c, while the understocking cost represent the margin $(w-c)$. These two types of costs will guide the members of the supply chain regarding their operational decision. Here, we see that as the cost increase, the cost of left-over is higher, the cost of loss sales is lower, and the supplier prefers order less.

2.4.7 Cash-constrained supplier(Proof 3.2)

We go further in the analysis by exploring the behavior of the supplier when she is prone to bankruptcy. We explore firstly the problem without considering the response of the financier. This means that it represents the supplier's behavior in the case of exogenous interest rate.

If we reason about this problem, we would find similarities in the behavior of the cash-constrained supplier and the cash-constrained retailer, regarding their risk taking behaviors. The supplier also faces the limited liability. Therefore, she could benefit from a limited downside. The supplier's optimal decision becomes:

$$\frac{\left(1 - H(Q^*)\right)\bar{F}(Q^*)(1 + r^{tc})}{1 - wQ^*(1 + r^{tc})h\left(b(Q^*)\right)} = c(1 + r^s)\bar{F}(g(Q^*))$$

Here, the right hand side is lower ($\bar{F}(g(Q^*)) = \bar{F}(cQ - K^r - K^s) \leq 1$). Which means that for the same settings, the supplier is induced now to decrease its wholesale price such as to encourage the retailer to order more. In fact, when she was not facing bankruptcy risk, she was bearing the entire risk and then was in a dilemma between decreases its wholesale price such as to increase orders but increasing its risk, or increase the wholesale price but decrease order and at the same time the risk. Now, due to the limited downside acquired from the limited liability, the supplier now shares part of its risk with her financier, as a result she is encouraged to make the retailer ordering more. Hence, he will chose to decline its wholesale price.

Here, we present the analysis assuming open account financing for both the supplier, and the retailer. $cQ - K^r - K^s = g(Q)$ represents the bankruptcy threshold of the supplier. This could be extended to the trade credit case by replacing the right-hand side by $c\bar{F}((cQ - K^r - K^s)(1 + r^s))$. In fact, this equation holds as long as we are in the case of a constant financing scheme, independent on the order quantity. For example when there exist information asymmetry between the financier and the supplier, we can envision a practical case where the interest rate is exogenously fixed.

3.4.7 <u>Optimality conditions *(see Proof 3.3 and Proof 3.4)*</u>

Obviously, in order to derive the optimal wholesale price, one's have to demonstrate the unimodality of the relationship between the supplier's profit with respect to its wholesale price. We prove that in both case presented before, the concavity of the function hold in the area of interest. We can prove the unimodality for all $w > w0$, where $w0$ satisfy $[1 - H(Q^*(w0)] = 0$ ($H(x)$ denotes the generalized failure rate. We prove that $\frac{d^2\pi^s}{dw^2} < 0$ is concave and unimodal on $w0(1 + rf) \leq w(1 + rf)$. For the following sections, we will explore alternatives supplier's profit function and prove that they remain unimodal.

7.5. Supplier optimal Trade Credit interest rate

We make the same analysis as for the wholesale price here with the Trade Credit interest rate. Trade credit interest rate is an alternative decision variable for the supplier to control the ordering decision of the retailer. Unlike the wholesale price, it only applies on the loan part of the retailer's ordering decision.

1.5.7 Unrestricted problem *(see Proof 4.1)*

We find the relationship that gives the optimal interest rate to charge according to the retailer's best-response optimal quantity. We consider the wholesale price as constant here. Similarly to the analysis with the wholesale price, the interest rate is set so that to maximize the profit through the following equality:

$$\left[\frac{\left(1-\frac{(wQ^*-K^r)}{w}h(Q^*)\right)\bar{F}(Q^*)}{[1-(wQ^*-K^r)(1+r^{tc})h(b(Q^*)]}\right]=\left[\frac{\left(1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right)\bar{F}(Q)}{[1-H(b(Q^*)]}\right]=c(1+rf)$$

In fact, the interest rate has to be increased as the supplier's production cost increase. So that to reduce the retailer's order quantity. It still follow the intuition behind the trade-off between overstocking and understocking costs. When looking at the classical newsvendor persective, we note indeed that the critical fractile maximizing the profit with respect to the demand distribution is given by $\bar{F}(Q^*)=c$ (in the integrated case). Which means that the as the overstocking cost increases (i.e. c increases), the optimal order quantity decreases. Here, we have the same intuition. Indeed, by extending trade credit, the supplier also faces the demand risk, and only prefers incurring risk associated with high order quantity, in the case that the overstocking cost is low.

2.5.7 Cash-constrained supplier(Proof 4.2)

Similarly to the insights given by the trade credit, similarly to the retailer, the supplier is induced to take more risk when she shares her risk with her financier. In this case we have the following optimality conditions for the supplier:

$$\left[\frac{\left(1-\frac{(wQ^*-K)}{w}h(Q^*)\right)\bar{F}(Q^*)}{[1-b(Q^*)h(b(Q^*))]}=c\bar{F}(cQ^*-K^r-K^s)\right]$$

In fact, the previous results of optimal trade credit interest rate and wholesale price gives insight regarding the supplier's behavior, providing that she finances with constant interest rate or no interest rate (For example, if he finances through open account financing). These results gives insights in such way that it permits to consider the case of a three layer supply chain. Assume, there is a supply chain with a retailer, a manufacturer and a supplier. The manufacturer extends trade credit (or open account) to the retailer, and the supplier extends delayed payment contract to the manufacturer. It is interesting for the supplier to know that by inducing the manufacturer to order more, through trade credit financing, the manufacturer at his turn is induced to set a lower wholesale price (or trade credit interest rate) and induces the retailer to order more. Hence, there exists a propagation of the incentive throughout the supply chain. There exist a multiplicative effect that is worth exploring in a further research.

7.6.<u>Bank's problem</u>

Coming back to our supply chain finance system, we will demonstrate that when the supplier finances through bank loan, in the context of perfect capital market, the former result does not hold anymore. We will prove that the supplier's optimal operational decision ($w^*(Q^*)$) is not influenced by its capital structure. Therefore, the problem reduce to the case of the unrestricted supplier.

In order to grasp this, let present the Bank's problem at first.

The bank, is assumed to operate in a perfect capital market, without information asymmetry, transaction cost, bankruptcy cost, or other market imperfection. In consequence, adapts her interest rate given the size of the loan, such as to guarantee in expectation the risk free rate from its loan.

$$\pi^b(Q) = \int_0^{g(Q)} xdF(x) + L^r(1+r^b)\int_{g(Q)}^{\infty} dF(x) - L^s(1+rf) = 0$$

With $g(Q) = (cQ - K^r - K^s)(1+r^b) = L^s(1+r^b)$

The term $-L^s(1+rf)$ represent the future value of the capital the bank grants to the supplier. In the perfect capital market, the borrowing rate and the lending rate are the same and banks make zero profit. Zero profit is the target profit of the bank, it accepts to grant a loan at a rate such as to guarantee in expectation to make the targeted zero profit. As $g(Q)$ (the supplier's bankruptcy threshold) increases, the interest rate required has to increase in order to compensate the expected loss incurred from bankruptcy $E[(L(1+rf)-x)^+]$. In fact, intuitively, the bank charges the supplier an amount in the interest rate that correspond in expectation to the bankruptcy risk incurred when lending. In this fashion, the supplier is charged for each additional amount of risk undertaken an additional amount of interest rate for the loan. ,

7.7. Independence between operational and financial decision.

In Proof 5.1, we demonstrate that the interest rate adjustment operated by the bank as the loan increases cancel the advantage of taking more risk, from the supplier's perspective.

We can show that the optimality condition of the leveraged case (i.e. $\frac{(1-H(Q^*))\bar{F}(Q^*)}{1-wQ^*h(wQ^*-K^r)} = c\bar{F}(cQ - K^r - K^s)$) reduces to the conditions of the case of perfect capital market (i.e. $\frac{(1-H(Q^*))\bar{F}(Q^*)}{1-wQ^*h(wQ^*-K^r)} = c(1+rf)$)

In fact, the increase in order quantity from the retailer leads the supplier to increase its loan. In the case of open account, trade credit financing or exogenous interest rate, the supplier has the opportunity to increases its expected profit by taking more risk. Indeed, due to the limited liability, the supplier faces limited downside risk. The downside risk is shared with the financier. His financing cost is in this case not appropriately adjusted for the increased risk.

However, when borrowing in a perfect capital market, the supplier is now charged appropriately by the bank for the additional risk. In this way, the risk sharing mechanism of the loan is cancelled and the supplier pays for the total risk taken. As consequence, she is not induced to take more risk than usual. We proved that in fact, the bank financing does not influence the decision of the supplier regarding the retailer. This proof is consistent with the results of **Kouvelis and Zhao (2012)** and **Modigliani and Miller (1958).**

Adopting another perspective, we can further demonstrate that the size of the loan is not relevant as an optimal decision variable. We can see that as the supplier increases its loan, the bank's adjust its interest rate following this function:

$$\frac{dr(L)}{dL} = \frac{(1+rf)}{L^s \int_{g(q)}^{\infty} f(x)dx} - \frac{1+r}{L^s}$$

The supplier, when deriving its profit in relation to the loan size obtain a sensitivity of $\frac{d\pi^s(r(L),L)}{dL} = -(1+rf)$ which is a constant. (*See Proof 5.1.1)*

Taking the second order derivative we obtain $\frac{d^2\pi^s(r(L),L)}{dL^2} = 0$, which cannot prove the existence of an optimum loan size. As consequence, there does not exist an optimum loan size and the supplier is only concerned on the optimal operational decision in terms of order quantity. The loan size does not interfere in her decision making.

Chapitre 8. Effect of Bankruptcy costs.

We will now modify our model and propose an extension that takes into account bankruptcy cost. This relaxation of assumption adds a market imperfection on the analysis. For the remaining of this section, we assume open account financing ($r^{tc} = 0$), to simplify the notations and the analysis.

Let assume simply that the supply chain is cash constrained, and the retailer has proportional bankruptcy cost. The supplier expected profit becomes.

$$E[\pi^s(Q)] = \int_b^{\infty} (wQ - K^r)(1 + r^{tc})\, dF(x) + \int_{g(Q,\alpha)}^{b} \alpha x f(x)\, dx$$

With $g(Q,\alpha) = \frac{cQ-K^r-K^r}{\alpha}$ with $0 \le \alpha \le 1$ being the proportional bankruptcy recovery rate, which is the share of revenue of the sale received by the supplier after the retailer's bankruptcy proceedings. In fact, when the retailer goes bankrupt, the supplier loses part of the proceeds $(1-\alpha)$ proportionally to the revenue resulted by the realized demand. As a consequence, the supplier is more bankruptcy prone as well since its expected revenue from sales decrease for demand realized bellow the retailer bankruptcy threshold. Note that the higher the recovery rate, the lower is the bankruptcy threshold.

The bank problem remains similar, unless the supplier's bankruptcy threshold is increased in accordance with the bankruptcy recovery rate.

$$\pi^b(Q) = \int_0^{g(Q,\alpha)} x dF(x) + L^r(1 + r^b) \int_{g(Q,\alpha)}^{\infty} dF(x) - L^s(1 + rf) = 0$$

We could consider a variant that take into account the fixed bankruptcy cost that could result from the payment of the liquidator and all administrative fees associated to the bankruptcy proceeding.

The amount received by the supplier for any level of demand is$(D\alpha - P)^+$. In order to reimburse her loan, the supplier needs a sufficient demand level given by:

$$g(Q,\alpha,P) = \min\left\{b(Q), \frac{(cQ - K^r - K^s)(1+r^b) + P}{\alpha}\right\}$$

The bank's problem in this case becomes:

$$E[\pi^b(Q)] = \int_{g(Q,\alpha,P)}^{\infty} L^s(1+r^b)f(x)\,dx + \int_{\mathrm{P}/\alpha}^{g(Q,\alpha,P)} (\alpha x - P)f(x)\,dx - L^s(1+rf)$$

Note that the threshold P/α define the demand level at which the supplier cannot extract revenue from retailer's bankruptcy. As consequence, the supplier end up with a terminal wealth of zero. There is nothing to transfer to the bank as well.

When now considering both the retailer and the supplier with bankruptcy cost (fixed and variable), we further have:

$$E[\pi^B(Q)] = \int_{g(Q,\alpha,P^r,P)}^{\infty} (cQ - K^r - K^s)(1+r^b)\,dF(x) + \int_{l}^{g(Q,\alpha,P^r,P)} [\big((\alpha\beta)x - P^r\big) - P^s]f(x)\,dx$$

Where α denote the retailer's bankruptcy recovery rate and $0 \leq \beta \leq 1$ the recovery rate of the supplier. P^r and P^s denote their fixed cost of bankruptcy respectively.

To recover an amount from the loan, we must have that $(\alpha D)\beta - P^r - P^s \geq 0$ which gives $l = \frac{P^r + P^s}{\alpha\beta}$, the demand threshold at which the bank begins to recover something from the loan. Note that this threshold can be simplified to $l = \left(\frac{P}{\alpha}\right)^2$ when $P^r = P^s = P$ and $\beta = \alpha$ (i.e. the retailer and the supplier faces similar bankruptcy costs). We can see that as the number of layer increases between the financier and the customer, the bankruptcy risks increases for the bank. The more the layer, the higher the risk resulting from downstream bankruptcy disruption. It increases exponentially (by a factor $\left(\frac{P}{\alpha}\right)^i$ $for\ i\ layers$). As a result, the interest rate for a loan increases upstream in the supply chain. As an anticipation to the propagation of the bankruptcy disruption, the bank charges an interest rate higher to the upstream members in the supply chain.

8.1. Supplier's Optimal wholesale price.

1.1.8 Unrestricted problem (Proof 6.1)

We continue our analysis to see the impact of bankruptcy cost on the supplier's optimal decision. First we assume that the supplier has sufficient capital to fund the operations and does not rely on external financing. We study here the case with proportional bankruptcy cost.

From the supplier profit function, we find as the First Order Condition with respect to the wholesale price:

$$\frac{\left[\left(1-H(Q^*)\right)\bar{F}(Q^*)\right]-\left[(1-\alpha)wb\left(1-H(Q^*)\right)f(b(Q^*))\right]}{\left(1-wQh(b(Q^*))\right)}=c$$

We can prove that the second term of the left-hand side is always positive and lower that the first term. As consequence, this additional term tends to decrease the nominator of the left-hand side, when comparing to the unrestricted case. For the same given marginal cost c, the supplier is induced to increase the wholesale price, such as to encourage the retailer to order less. These results seems relevant. The supplier faces the demand risk through the bankruptcy risk of the retailer. Since the retailer bankruptcy becomes now more costly, the supplier becomes more reluctant to propose credit sales.

Note that, when the recovery rate α increase, the second term of the left hand side decreases and reach zero when the recovery rate equals one (the initial case).

2.1.8 Cash-constrained supplier(Proof 6.2)

Now, we consider the case where the supplier enter in a bank loan. We find as FOC:

$$\frac{\left[\left(1-H(Q)\right)\bar{F}(Q)\right]-\left[(1-\alpha)wbf(b)\left(1-H(Q)\right)\right]}{\left(1-wQh(b)\right)}=c(\alpha\bar{F}(g'))$$

With $g'=\frac{g}{\alpha}=g(Q,\alpha)$

Comparing with the unlevered case, the supplier benefits from sharing her risk through the loan (open account, trade credit). He is induced to increase its wholesale price and induce the retailer to order more, due to the limited liability. However, she experienced higher optimal wholesale price than in the case of capital-constrained supplier without bankruptcy cost. As the recovery rate increases to one, the wholesale price decreases such until reaching the level of the case without bankruptcy costs. We can note indeed that when $\alpha = 1$, the FOC converge to the levered case without bankruptcy cost.

8.2. Dependency between operational and financing decision (Proof 5.2)

We prove that when adding bankruptcy cost, which is a form of market imperfection, the financing decision becomes relevant when analyzing the operational decision. Indeed, unlike the finding in the context of the perfect capital market, the supplier's problem cannot be reduced to the case without financing.

We obtain as first order condition for this problem

$$\frac{d\pi^s}{dw} = \frac{d(wQ^*)}{dw}\bar{F}(wQ^* - K^r) - \left(\frac{dQ^*}{dw}\right)\left[\frac{c(1+rf)}{\bar{F}(g(Q^*)) - (1-\alpha)L^s g(Q^*)f(g(Q^*))}\right]\bar{F}(g(Q^*))$$

Which is different from the previous result without bankruptcy cost. This suggests that the irrelevance of financing decision does not hold in this case and the outcome of the optimization problem will differ when facing bankruptcy costs.

It is interesting to see that as the recovery rate becomes close to 1, the problem converge to the initial problem without bankruptcy risks.

Chapitre 9. Risk measures

In this section we analyze another important setting in the newsvendor problem, which is the risk. Generally, models proposed assume the agents of the supply chain to be risk-neutral. As consequence, the expected value is the only relevant information for the decision making. However, in the reality, we often care about the risk of the investment when making a decision. That is why it is interesting to analyze the consequences of availability of capital in the supply chain in terms of risk.

First of all, we focus our analysis on a widely used measure of risk: the variance of profit. Then we briefly present the formulation of expected downside formulation in the case of capital constraint. Finally, we explore the Conditional Value at risk Criteria as objective for operational decision.

9.1. Variance

1.1.9 Retailer's Variance:

From **Choi and Chui (2012)**[lxvii], we can define the retailer's variance of profit of the classical newsvendor problem as:

$$Var[\pi^r(Q)] = Q^2\bar{F}(Q) + \int_0^Q x^2 f(x)dx - \left(\int_0^Q \bar{F}(x)dx\right)^2$$

Following their analysis, we know that the variance of the profit is an increasing concave function of the order quantity and is bounded above by v, the variance of the distribution of demand. Indeed, we they prove that $lim_{Q\to\infty} Var[\pi^r(Q)] = v$ [4]assuming a variance with finite moment. Form this analysis, we add some proof that support this statement *(Proof: 7.1.)* by analyzing the first order and second order derivatives relative to the order quantity and find that the variance function, is increasing in the order quantity ($\frac{\partial Var[\pi^r(Q)]}{\partial Q} > 0$) and increase in a concave fashion ($\frac{\partial^2 Var[\pi^r(Q)]}{\partial^2 Q} < 0$.) The variance is increasing up to the convergence to the variance of the demand.

However, when considering a financially constrained newsvendor, the behavior of the variance in relation with the order quantity differ. We explore the change in the variance behavior when considering the limited availability of capital. Considering that the retailer relates on an external source of financing. We consider here a retailer financing through open account. Following the pecking order hypothesis **(Myers, 1984)**, the retailer uses its internal capital at first and then delay the payment for an amount equivalent to $wQ - K^r = B = b(Q)$

The retailer expected profit is written as:

$$E\ [\pi^r(Q)^c] = \int_B^Q [x - (wq - K)]dF(x) + \int_Q^\infty [Q - (wq - K)]dF(x) - K^r$$

The variance of profit in this case is *(Proof 7.2)*:

[4] Note that when considering a different selling price p than 1, the variance of profit becomes $p^2 Var[\pi^r(Q)]$ and the variance of the demand is equal to $p^2 v$

$$VaR\ [\pi^r(Q)] = 2Q\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx - \left(\int_B^Q F(x)dx\right)^2$$

When considering the case where Q tends to the infinity, we find that: $lim_{Q\to\infty} Var[\pi^r(Q)] = 0$. *(Proof 7.3)*. In fact, intuitively, as the retailer's order quantity increases to the infinity, the loan increase in the same fashion. The more the order quantity is above the demand distribution, the less likely the retailer will end up with sufficient revenue to reimburse its loan. He is likely to declare bankruptcy and end up with a terminal value of zero. Due to the limited liability assumption, the retailer is not supposed to finish with a negative terminal wealth. This means that at a sufficiently high level of quantity ordered, the retailer will always end up with a deterministic terminal value of zero. In consequence, the variance of the profit decreases to zero as $Q \to \infty$.

Note that the variance of the capital constrained retailer reduces to the unrestricted case when the initial capital reach the ordering cost. We have: $lim_{K\to wQ}\ Var[\pi^r(Q)^c] = Var[\pi^r(Q)]$. As consequence, as the initial capital increases, we can note that the variance of profit converge slower to zero when $Q \to \infty$.

2.1.9 Expected Downside Deviation of Profit: Retailer

An alternative to variance for the measure of risk is the semi-deviation of profit. It only considers the adverse variation of profit and can be very relevant in the case of asymmetric demand distribution function. The semi-deviation of profit of the classical newsvendor problem is presented by **Choi and Chui (2012)** as following: $\sigma^{downside} = \int_0^{q-\int_0^q F(x)dx}\left[q - \int_0^q F(x)dx - x\right]dF(x)$

We adapt the result for our analysis of the cash constrained retailer and find

$$\sigma^{downside^c} = \int_{b(Q)}^{\int_{b(Q)}^{Q}\bar{F}(x)dx}\left[\int_{b(Q)}^{Q}\bar{F}(x)dx - x\right]dF(x)$$

As for the variance, we do not find the same results with respect to the convergence issue. We find that the expected downside deviation of profit converge to zero as the order quantity tend to the infinity: $\lim_{q\to\infty}\sigma^{downside^c} = 0$. In the unrestricted newsvendor problem, we have as presented by **Choi and Chui (2012)** : $\lim_{q\to\infty}\sigma^{downside} = \int_0^{\int_0^\infty xf(x)dx}(x - \int_0^\infty xf(x)dx)dF(x) = L(e)$. Which is the right linear loss function of the demand distribution.

Once again, as the order quantity tends to the infinity, due to the bankruptcy risk and limited liability, the retailer's outcome becomes deterministic.

3.1.9 Optimality Conditions

In order to better grasp the behavior of the variance relative to the order quantity in the capital constrained case, we perform first order and second order conditions such as to determine the shape of the curve. Analytically, we obtain the following equations:

$$\frac{\partial VaR(\pi^r)}{\partial Q} = 2\int_B^Q F(x)dx - 2(Q-B)wF(B) - 2\left[\left(\int_B^Q F(x)dx\right)\left(F(Q) - wF(B)\right)\right]$$

$$\frac{\partial^2 VaR(\pi^r)}{\partial^2 Q} = 2[F(Q) - wF(B)] - 2\left[\left(wF(B) + Qw^2 f(B)\right) - \left(w^2F(B) + w^2Bf(B)\right)\right]$$
$$- 2\left[\left(F(Q) - wF(B)\right)^2 + \left(\int_B^Q F(x)dx\right)\left(f(Q) - w^2f(B)\right)\right]$$

Analytically, it remains intricate to extract the area in which the second order condition demonstrate the concavity of the function. However, when looking graphically at the financially constrained case, we can note that the variance first increase with the quantity, reaches a maximum, and then decreases to zero. If we analyse the evolution of the second order condition along with the variance when modifying the order quantity, we note that the maximum exist in a region where the relationship is unimodal and concave. There exists a bounded concave area in which the variance function attain its maximum *(Figure 3)*.

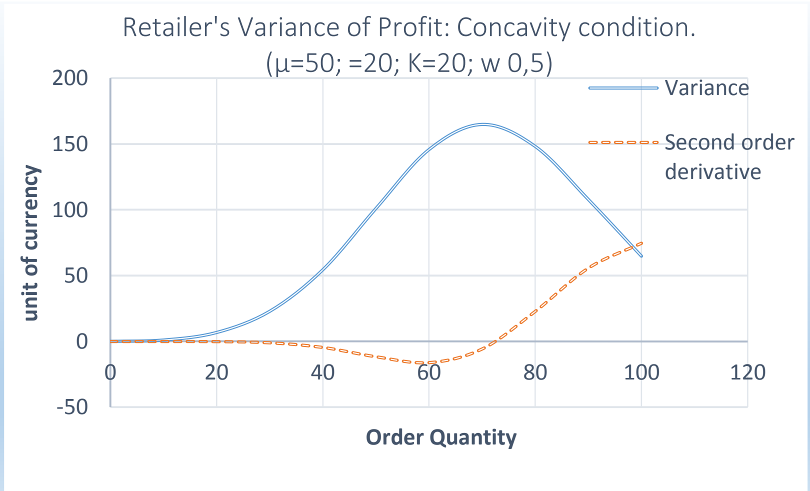


Figure 3

We performed a sensitivity analysis *(Appendix Bis 2)* for different level of initial capital. With a normal distribution of standard deviation = 20, mean = 50, We note that there exist a region in which we can derive the optimal variance in most cases. However, when the internal capital becomes big relative to the wholesale price, the retailer's variance of profit does not reach anymore a maximum expressed by the concavity measures, but level off at the point where it reaches the variance of the demand v[5]. When the wholesale price is sufficiently low and there is enough internal capital, the variance of profit can reach the same level than the that of the variance of demand. In this situation, the second order condition are not respected anymore and the maximum variance of profit is equal to v.

Comparing the variance of profit with the expected value, we discover that the maximum expected profit is not coincident to the maximum of the variance of profit as suggest. To the best of our knowledge, we see that, the variance of profit reaches its maximum at a level of order quantity above the optimal decision of the risk-neutral retailer as illustrated in *Figure 4.* *(see Appendix Bis 3)*

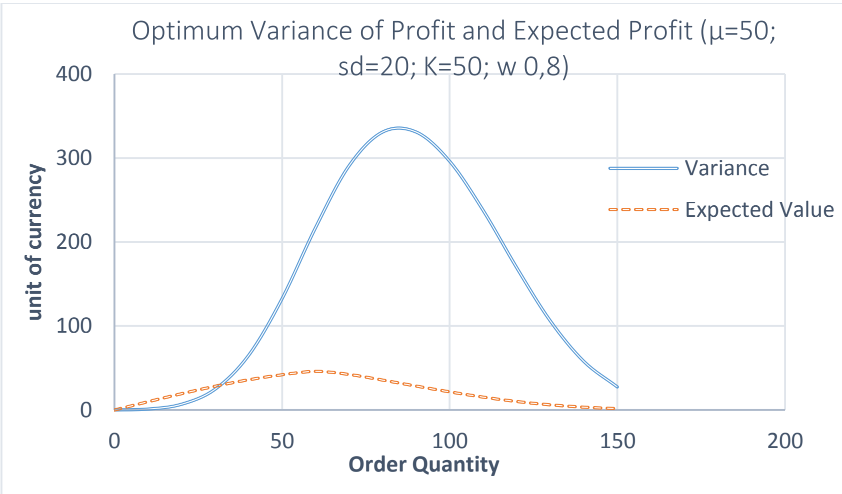


Figure 4

4.1.9 <u>Numerical Analysis with Bankruptcy risk</u>

In this sub-section we present some figures to illustrate deeper the behavior of the variance of profit under bankruptcy risk. We still take the point of view of a cash constrained retailer.

a. *Variance unrestricted case*

First of all, we present the variance of an unrestricted retailer's profit, considering different level of demand volatility. The graph in *Figure 5* present the evolution of the variance of profit

[5] Alternatively p^2v, when the selling price is different to one.

variance of profit in relation to the order quantity chosen by the retailer, we can see that it converges indeed to the variance of the demand as $Q \to \infty$. The lower the standard deviation of the demand, the quicker the variance of profit converge.

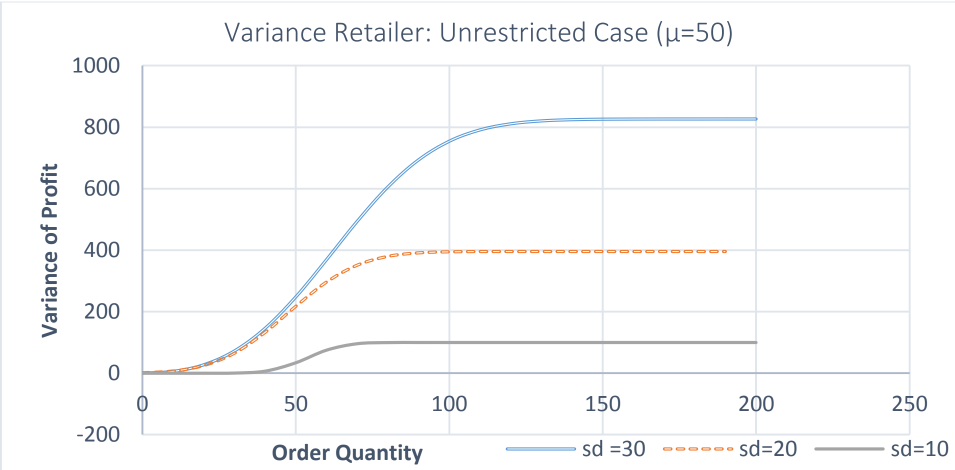


Figure 5

5.1.9 <u>Variance with financial constraint</u>

When we look now at the case with financial constraint, we obtain an interesting result regarding the variance of the retailer.

When looking at the evolution of the variance regarding the order quantity, we can notice that indeed the variance converge to zero as the order quantity increase. However, the variance grow at first, before reaching a maximum and then decrease. We can notice the existence of a region within which the function is unimodal and admit a maximum.

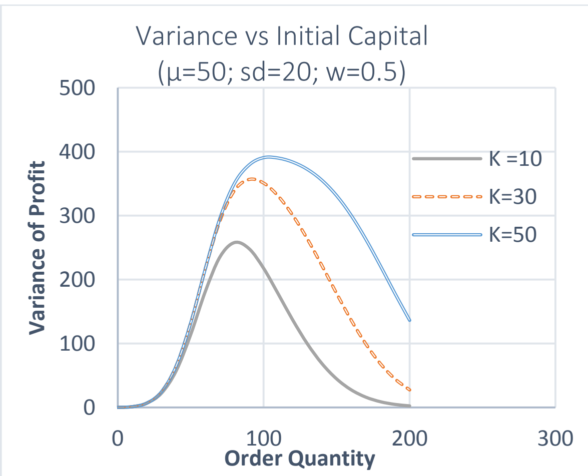


Figure 6.1

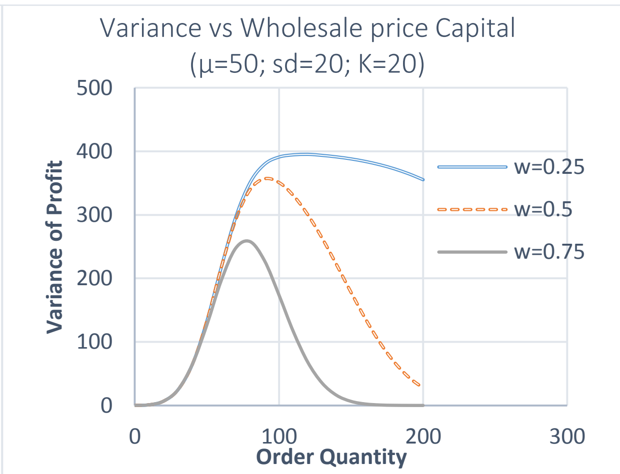


Figure 6.2

In *Figure 6.1* we can see that a lower level of initial capital lead to a reduction of the variance of profit, given the same order quantity. This can be explained by the higher probability of bankruptcy experienced by a retailer with a lower initial capital, given a level of order quantity. The limited liability prevent the outcomes to decreases below zero. As a result, there is a lower uncertainty with respect to the downside risk.

In *Figure 6.2* we analyze the impact of the wholesale price on the variability of profit. Here, the variance of profit converge slower to zero in the case of low wholesale price. Similarly to the intuition behind the influence of the initial capital, we can deduce that a higher wholesale price, involving a lower margin, leads to a higher probability of bankruptcy, for a given level of order quantity. However we can notice a superior variance of profit when the wholesale price is set lower. In one hand, the higher margin have a positive impact on the absolute value of the variance, on the other hand, there is less certainty with respect to the adverse outcome, due to the lower bankruptcy risk. Nonetheless, these absolute measures does not allow to deduce relevant conclusion about the risk taken by the retailer experiencing with respect with these different parameters.

In order to make the results comparable, we analyze the coefficient of variation $\frac{\sigma}{\mu}$ of revenue[6]in order to standardize the variability measures. *Figure 7.1* and *Figure 7.2* below suggest that either a higher understocking cost (a high margin) or a low initial capital are associated with a greater coefficient of variation. Conversely to the prior insights regarding the risk incurred. In fact, indeed the variability of profit is reduced when either the initial capital is low or the margin is low, but it happens at the expense of the expected revenue. When looking at the initial capital, we see that indeed the higher leverage means more risk, in relative terms. In fact both the high wholesale price and the low initial capital leads the expected revenue downward because of bankruptcy risk consideration.

[6] We take the revenue in order to avoid negative values as discouraged for comparability issues. Note that the variance of profit is equivalent to the variance of revenue, the initial capital being constant for equivalent order quantity. When looking at positives value of the coefficient of variation of profit, we found consistent result supporting this analysis. We observe the same ranking in terms of coefficient of variation. *(see Appendix Bis 4)*

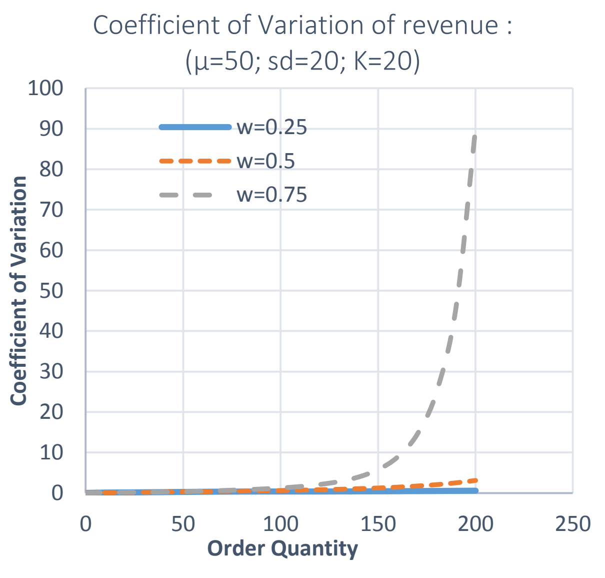


Figure 7.1

Figure 7.2

Concerning the wholesale price, the result show that the decision of the risk-neutral newsvendor with respect to the expected value tend to reduce the risk when choosing his order quantity since he reduces his order quantity as the wholesale price increases. However, when looking at the impact of the internal capital, we can notice that the lower the initial capital, the higher the coefficient of variation. Nonetheless, the risk-neutral retailer's optimal decision tends to increases the order quantity as the internal capital decreases. This means that the decision of the risk-neutral retailer tend to increase the risk of investment.

From these observation, we could deduce the behavior of a risk-averse newsvendor considering the coefficient of variation as a decision variable for adjusting for the risk. We would expect the initial capital to be a variable inducing the retailer to order less, conversely to what we found in the risk neutral case. The wholesale price could be used as a mechanism for at the same time increasing the expected revenue and reducing the coefficient of variation, inducing even more the retailer to increase its order quantity.

9.2.**Conditional Value-at-Risk**

In this section, we provide an analysis of the conditional value at risk, which raise more attention in finance and operation research nowadays. This measure takes into account in some extent the aversion in the decision making. The remaining of this section focuses on the impact of the CVaR confidence level on the operational decision making in the context of supply chain financing.

1.2.9 <u>Retailer</u>

By definition, the CVaR represents the expected value conditional to being bellow a target α level. It represents the expected shortfall. The expected revenue in the worst $\alpha\%$ case. This measure is more informative than the VaR, because it better represents the increased risk arising from the shape of the tails of the distribution. For the remaining, we follow the definition of **Jammernegg and Kischka(2012)**[lxviii]**.**The Conditional Value-at-Risk of profit is given by:

$$CVaR^{\alpha}\big(g(\xi,q)\big) = \frac{1}{\alpha}\int_{0}^{F^{-1}(\alpha)} g(\xi,q)dF(x)$$

Where $g(\xi,q)$ represents the profit function.

The Conditional Value-at-Risk is a measure that allow to take into consideration the measure of risk in the operational decision. This measure appears as an alternative as expected value as a decision criteria for optimizing the supply chain. In fact, the Conditional Value-at-Risk relate to the expected value we could earn bellow a given confidence level $0 < \alpha < 1$. The lower the α, the more the concern is with respect to the downside risk. In fact, the CVaR can be used as an indicator either to modelize the downside limit objective as well as downside limit constraint.

Before studying the CVaR in the context of capital constrain, we recall the first order conditions when using CVaR as objective function in the context of classical unrestricted newsvendor.

The objective: $\max CVaR^{\alpha}\big(g(\xi,q)\big) = \max\frac{1}{\alpha}\Big(\int_0^Q (x - cQ)dF(x) + (1 - c)Q\int_Q^{F^{-1}\alpha} f(x)dx\Big)$

From this problem, we obtain $Q^* = F^{-1}\big(\alpha(1-c)\big) < F^{-1}(1-c)$ [7] (see Proof 8.1)

The parameter α represents the percentile of interest for the decision, the lower we set this value, the higher concern we have relative to the downside risk. As consequence, we can notice that the "CVaR-Newsvendor" is induced to order less such as to guarantee a reasonable conditional expected value. Indeed, when increasing its order quantity, the retailer increases its downside exposure resulting from the possibility of ending up with a significant amount of left-over inventory. When he considered the entire demand distribution, this left-over inventory risk was compensated by the upside of the distribution. As consequence he had an incentive to order more so as to retain the opportunity to benefits from this high demand. There was an equal

[7] The classical Newsvendor critical fractile.

balance between left over inventory and lost sale. However, when considering CVaR criterion, the emphasis is turned to the leftover risk, more than with respect to the opportunity cost of lost sales.

2.2.9 Supplier

We turn our analysis to the supplier problem, we will focus on two scenarios. The first one consider a risk neutral retailer ordering to the loss averse supplier relating in the CVaR criterion. Secondly, we explore the case of a risk neutral supplier dealing with a CVaR newsvendor.

a. *Risk neutral cash constrained retailer with CVaR criterion for the supplier. (see Proof 8.2)*

We will see how the CVaR objective affect the supplier decision. Here we assume that the retailer is still risk neutral in its operational decision. Recall $(wQ - K) = b(Q) = b$[8] The supplier problem is as follow:

$$\max CVaR^{\alpha}(\pi^{s}(Q,w)) = \frac{d}{dw}\left[\frac{1}{\alpha}\left(b\int_{b}^{F^{-1}\alpha} f(x)dx + \int_{0}^{b} xdF(x) - cQ\right)\right] = 0$$

Performing the First Order Conditions, we obtain the following equality at the optimum profit.

$$\frac{\left(1 - H(Q^{*})\right)\left(\bar{F}(Q) - w(1-\alpha)\right)}{1 - wQ^{*}h(wQ^{*} - K^{r})} = c(1 + rf)$$

We can notice that when $\alpha = 1$, which means, risk neutral newsvendor, it reduces to the initial case previously presented. However, as α increases, the nominator effect conteract the denominator effect of an increase in wholesale price. Or similarly, accelerate a decrease in wholesale price. Which suggest that providing that the future value of the production cost remain the same $c(1 + rf)$, the supplier needs to increase her wholesale price even more in order to attain the maximization of the objective.

In fact, the supplier is more concerned about the bad prospect regarding the demand. It is more risk averse and care more about the downside expected value. In fact, by increasing the wholesale price, the supplier induces the retailer to order less, such as to decrease its leverage. Which seems really intuitive.

[8] We consider here open account financing

b. CVaR criterion for the retailer with risk neutral supplier (see Proof 8.3 and 8.4)

Now, we explore the second case where the retailer is risk-averse, and the supplier is risk neutral. The first step is to determine the best-response order quantity function of the retailer with respect to the supplier's wholesale price. The retailer's problem is:

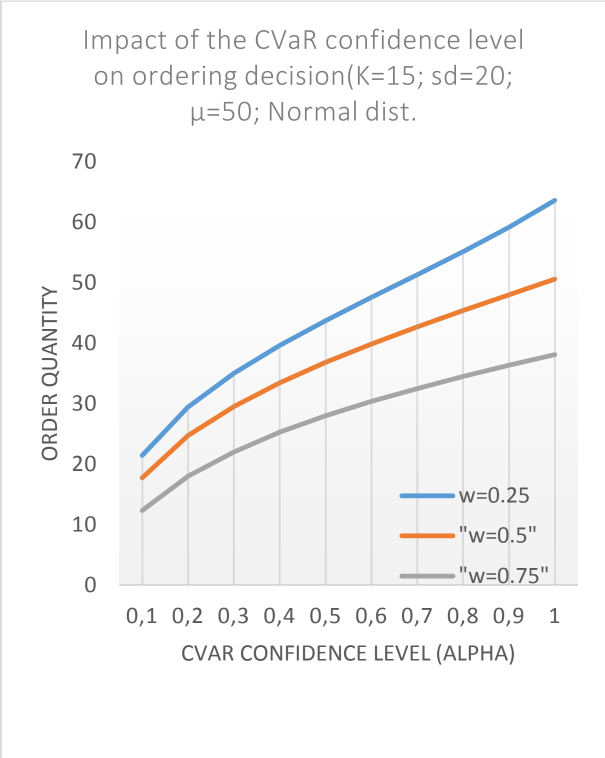
$$\max CVaR^{\alpha}(\pi^{r}(Q,w,\alpha)=\frac{d}{dQ}\left[\frac{1}{\alpha}\left(\int_{L}^{Q}(x-L)dF(x)+(Q-L)\int_{Q}^{F^{-1}\alpha}f(x)dx\right)\right]=0$$

We have,

$$\bar{F}(Q^{*})=w\bar{F}(L)+(1-w)(1-\alpha)$$

Note that $L=b(Q)=b$

When looking at this formula, we can see similarity with the optimality condition of the risk neutral case, in the context of capital constrained retailer. The new part in the equality is $(1-w)(1-\alpha)$. In fact, if we compare with first the classical newsvendor equilibria $\bar{F}(Q^{*})=w$, we first have that, the higher the wholesale price, the lower the optimal order quantity. Then, when adding capital constraint, we note that because $\bar{F}(L)<1$ it reduces the right-hand side and induces the retailer to order more than with its own capital. The intuition behind that have already be presented previously. However, when now changing the criteria, we add $(1-w)(1-\alpha)$ to the best-response function. When having a closer look at this part of the formula we can notice that the lower is the CVaR threshold α, the lesser is the order quantity of the retailer. The retailer cares more about its expected profit at the bottom of the distribution. As consequence, he takes less risks. By increasing the order quantity he would takes more risk and increase its exposure toward the low values Secondly, the higher the wholesale price, the lesser the impact of CVaR criterion. This is due to the fact that, with a high wholesale price the retailer is already induced to order less in the context of expected value criteria. As consequence, the impact of taking a more conservative approach (a lower α) in terms of ordering decision is reduced. The figure bellow illustrate these ascertainments *(Figure 8.1)*.When comparing with the case without financial leverage, we observe that differences arise when the margin decreases *(Figure 8.2)*. This difference results from the incentive of capital constraint retailer to order significantly more when the overstocking cost increases. As the CVaR criteria becomes more risk-averse oriented (low α), the difference is reduced due to the increased emphasis on the low outcomes.



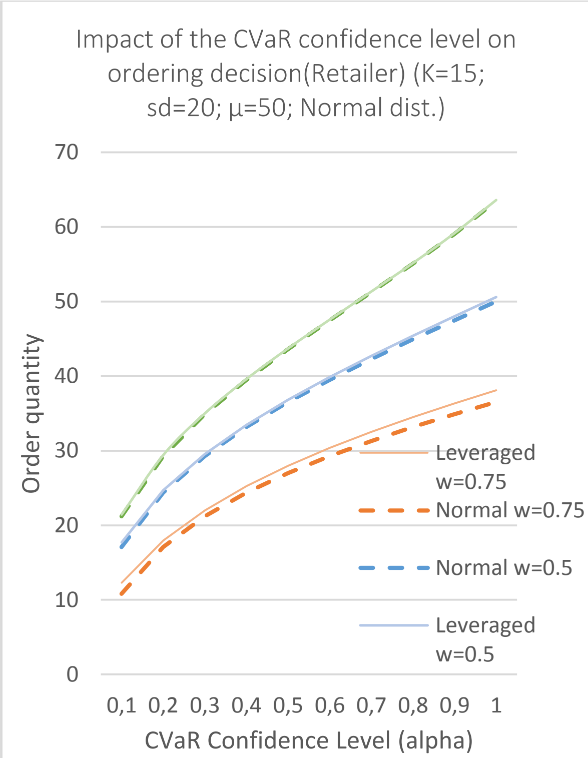


Figure 8.1 *Figure 8.2*

Now, let move to the supplier problem. We can show that the sensitivity of the retailer order quantity with respect to the wholesale price can be given by *(see Proof 8.4):*

$$\frac{dQ^*}{dw} = \frac{[1 - wQh(b(Q^*))] - \dfrac{1-a}{\bar{F}(b(Q^*))}}{-\left[\dfrac{f(Q^*)}{\bar{F}(b(Q^*))} - w^2h(b(Q^*))\right]}$$

From this, we can derive the optimal wholesale price:

$$\frac{\bar{F}(Q^*)(\bar{F}(Q)[1 - H(Q^*)] - (1-a))}{(\bar{F}(b(Q^*))[1 - wQh(b(Q^*))] + (1-a))} - c$$

The $((1-a)$ in the nominator reduce the left hand side, inducing the supplier to charge a higher wholesale price in order to fulfill the equality garanteeing the optimality. Similarly, when looking at the denominator, the $(1-a)$ increases the denominator, such that there is a need for increasing even more the wholesale price in order to attain the equality with the marginal cost c. Both $(1-a)$ reduce the effect of an increase in the whole sale price.

Here is the intuition behind this: the more the retailer is risk averse, the higher we charge to him, because it is more costly to induce him to order more by decreasing the wholesale price.

The marginal benefits of additional quantity decreases. In fact, the supplier must renounce at a higher margin to induce the retailer to order more. As consequence, trade-off between margin and quantity sold turn out to be dominated by the effect of margin.

Note that when the retailer is risk-neutral, which means that $a = 1$, the problem reduce to the original problem of capital constrained newsvendor previously explored.

Finally, We can demonstrate that $\frac{dQ^*}{dw} < 0$ still holds in the context of CVaR criteria(see Proof 8.5).

9.3.<u>Summary of the main findings</u>

- There exist a negative sensitivity of retailer's optimal order quantity to the trade credit interest rate
- There exist a negative sensitivity of retailer's optimal order quantity to the wholesale price
- There exist a negative sensitivity of retailer's optimal order quantity to the initial capital
- The efficiency of trade credit interest rate effect with respect to the retailer's optimal order quantity is more robust to differences in level of retailer's initial capital than wholesale price.
- The lower the retailer's internal capital, the lower the impact of wholesale price as control variable to define the retailer's optimal order quantity
- There exist a unique optimal wholesale price that maximize the supplier's revenue.
- The higher the supplier's marginal cost c, the higher the wholesale price and the lower the retailer's order quantity
- When experiencing financial constraint, the supplier is induced to lower the wholesale price so that to encourage the retailer's increasing its order(under assumption of constant financing scheme)
- The optimality condition of financially constrained supplier remains verified
- Optimality conditions of financially constrained supplier
- There exist a unique optimal interest rate.
- An higher marginal cost lead the supplier to increase the interest rate.
- Cash constrained supplier experience lower interest rate all other things beings equals.

- Under Perfect capital market assumption there exist an independence between the supplier's optimal order wholesale, price, loan, and order quantity.
- Bankruptcy propagation risk lead the upstream members of the supply chain to experience higher loan in case of bankruptcy cost.
- Bankruptcy cost induce the unrestricted supplier to increase its wholesale price, inducing the retailer to order less.
- Under bankruptcy costs, the supplier is still induced to decrease its wholesale price since he share its risk with a financier. However his wholesale price increases as the bankruptcy recovery rate decreases.
- The optimality condition still holds under bankruptcy costs.
- There exist a dependency between operation and financial decision in the context of variable bankruptcy costs
- The variance function of the unrestricted retailer is increasing and concave.
- Convergence of the retailer's variance of profit and expected downside deviation to zero in the case of limited capital.
- The variance of profit function is partially concave for the cash-constrained retailer problem.
- The variance of profit admit a maximum that is always above the maximum expected profit
- A lower initial capital lead to lower variance of profit due to higher probability of bankruptcy for a given order quantity
- Higher wholesale price, higher variance of profit
- Higher coefficient of variation of revenue as the initial capital increases
- A supplier relying on the CVaR criteria when extending trade credit increases his wholesale price such as to makes the retailer ordering less
- A risk-neutral supplier facing a retailer relying on CVaR criteria tends to propose an higher wholesale price to maximize its profit.

Conclusion

We have explored the issues of supply chain financing in the case of trade credit. From this analysis we discover several insights about the relationship between the financing and the operational decision. We study a supply chain in which trade credit is used as a financing instrument to transfer wealth from the supplier to the retailer to circumvent suboptimal outcome resulting from the capital constraint.

We first focused on an analysis of a Stackelberg game where the supplier as leader is the financier of the retailer. Trade credit is presented as a risk sharing mechanism that allow the supplier to back the retailer in case of low demand, due to the limited liability assumption. In this way, the supplier bears the bankruptcy risk and is guarantor for low demand level.

We found that the retailer is more inclined to order important quantity when its initial capital is low due to the limited downside risk it bears under limited liability assumption. The trade credit push the bankruptcy risk to the supplier making the retailer more prone to demonstrate a risk taking behavior. In reaction, the supplier can adjust either the level of the wholesale price or that of the trade credit interest rate such as to mitigate this behavior. However we notice that the effect of wholesale price would be decreasing in the initial capital, making the interest rate potentially more interesting for inducing the retailer to order less, and reduce the supplier exposure.

Then, we extended our analysis to the case where the supplier experienced at its turn a constraint in capital availability to finance its operations but yet still extend trade credit to its retailer. In this case the financier of the supplier acts as a way to mitigate the supplier's exposure to adverse demand realization.

We found that the supplier is induced to encourage the retailer to order more and as consequence increases its risk exposure, due to limited liability in case of bankruptcy. However, when funding to a bank operating to a perfectly competitive market, that does not experience market imperfections such as information asymmetry, transaction costs, bankruptcy costs or taxes, the supplier loses his benefits from increasing its leverage. Indeed, the bank charge the supplier with the appropriate interest rate with respect to its risk profile such as to end up with an amount of profit equivalent to the risk free rate, limiting by this means the risk sharing mechanism of indebtedness. In case of such loan financing the supplier bears its risk entirely. This result in an irrelevance in joining financial and operational decision when setting wholesale price, interest rate, or deciding the optimal order quantities.

Next, we extended our analysis by adding bankruptcy costs and find that it mitigate the risk taking behavior of upstream agents of the financial chain. The supplier becomes more reluctant to induce the retailer to order more since the bankruptcy risk it bears becomes more costly. Similarly, the bank tend to increase its interest rate as the exposure to bankruptcy cost increases. However, it does not appropriately price the loan it grants to the supplier such as to keep joined operational and financial decision irrelevant. As consequence we notice that there exists an influence of financial decision to the operational decision.

In the second part of our analysis, we examined the behavior of the variance of profit in the case of leveraged newsvendor problem in relation to the order quantity. Our findings relates on the existence of a concave area in the variance of profit function in which the maximum value can be extracted. We add that the this maximum variance level arises always at a level of order quantity higher than which that achieved the maximum expected profit.

Finally, in a third part studied the Conditional Value at risk criteria and found that a risk averse supplier relying on CVaR as an objective function tends to propose a higher wholesale price to its retailer to induce him to order less and mitigate its risk exposure. In addition, when considering a risk-neutral supplier dealing with a retailer using CVaR as decision criteria, we note interestingly that the supplier is induced as well to increase its wholesale price to extract a higher amount of profit from the retailer.

Several extension from our result are worth studying. When considering the supplier's set of decision variable, we explored local optimization of each of them without exploring their interaction. It could be interesting to explore the optimality condition of the joined decision of wholesale price and trade credit interest rate setting in the context of financially constrained supply chain relying on financing arrangement.

Another point relate on the performance measures of such supply chain finance arrangement, we did not examine the issue of supply chain efficiency and distribution of the shares of revenue among the players of the supply chain. Especially the impact of supplier's leverage as well as the introduction of bankruptcy costs can brings important insight on the consequence of such arrangement in the supply chain.

From a more practical perspective, our findings contribute to raise important consideration about the propagation mechanism of incentives throughout the financial chain that leads parties of the supply chain that benefits from leverage to increase their exposure to the demand risk by providing better terms to their buyer. We noticed that the financing of operation through the supply chain can lead to an increased risk for the overall supply chain and in the end, on the economy. Indeed, leveraged players by inducing their customer to order more propose greater loan to their customer making them in turn even more likely to increases their exposure and propagate better financing terms in trade credit extended to downstream agent of the chain. When considering the trends of increased complexity and length of supply chain, this mechanism could appears to increase significantly the risks and the exposure to disruptions such as bankruptcy in the supply chain.

These findings gives important insights on the consequence of supply chain financing. When a bank extend payment to a supplier, if its does not have sufficient information with respect to the creditworthiness of the borrower’s customer, we could observe a moral hazard resulting from the supplier’s limited liability. Indeed, by sharing its risk with the bank, the supplier is induced to increases its exposure and take riskier decisions. Therefore, the need for efficient screening mechanism from the financier is becomes relevant as a way for alleviating riskiness of supply chain financing. Indeed, when considering a competitive financial market we found that the influence of financial decision on operation does not hold and the supplier is not anymore induced to increase its exposure due to the appropriate interest charged to him. The absence of information asymmetry can help reducing this problem.

To conclude we call for increasing research on subjects more related to risk issues of supply chain financing and its impact on the supply chain as well as on the overall economy such as to better anticipate the potentially harmful consequence of the unmonitored generalization of this type of financing means.

APPENDIX

Chapitre 10. Notations

$h(x) = \frac{f(x)}{\bar{F}(x)}$ the failure rate

$H(x) = xh(x)$ the generalized failure rate

$\bar{F}(x) = 1 - F(x)$: the tail of the distribution

IFR: Increasing Failure Rate

IGFR: Increasing Generalized Failure Rate.

Chapitre 11. APPENDIX 1 : Retailer

11.1. Proof 1.1 : Retailer's Best Response Optimal Quantities : Unrestricted

$$\pi^r(Q) = min[\xi, Q] + (K - wQ)(1 + rf)$$

$$E[\pi^r(Q)] = \int_0^Q \bar{F}(x) + (K - wQ)(1 + rf)$$

$$\frac{d}{dQ}\left(\int_0^Q \bar{F}(x) + (K - wQ)(1 + rf)\right) = \bar{F}(Q) - w(1 + rf)$$

$$\bar{F}(Q^*) = w(1 + rf)$$

$\frac{d^2}{dQ^2} = -f(Q) < 0$: the function is unimodal and concave.

11.2. Proof 1.2: Retailer's Profit : Cash Constrained

$$E[\pi^r(Q)] = -K(1 + rf) + \int_{b(Q)}^Q \bar{F}(x)$$

$$\frac{d}{dQ}\left(-K(1 + rf) + \int_{b(Q)}^Q \bar{F}(x)\right) = \bar{F}(Q) - w(1 + r^{tc})\bar{F}(b(Q))$$

$$\bar{F}(Q^*) - w(1 + r^{tc})\bar{F}\big(b(Q^*)\big) = 0$$

Where $b(Q) = (wQ - K^r)(1 + r^{tc})$

11.3. Proof 1.2.2: Lagrangian method for optimization

From **Yan and Sun (2013)** we have that the Lagrangian optimization problem can be reduce to an unrestricted optimization problem:

Let $\Lambda = \frac{\partial \mathrm{b(Q)}}{\partial \mathrm{Q}}$

$$\max \pi = \max E[pmin(Q,x) + (K^r + L - wQ) - L(1 + r)]^+$$

$$s.t\ K^r - wQ \leq 0$$

$$\pi = E[pmin(Q,x) - (wQ - K^r)(1 + r)]^+$$

$$\pi = \left[\int_0^q \bar{F}(x)dx - (wQ - K^r)(1 + r)\right]^+$$

$$\pi = \int_{b(Q)}^{Q} \bar{F}(x)dx$$

$$L(Q,\lambda) = \int_{b(Q)}^{Q} \bar{F}(x)dx + \lambda(wQ - K)$$

$$\frac{dL(q,\lambda)}{d\lambda} = wQ^* - K = 0 => Q^* = \frac{K}{w}$$

$$\frac{dL(q,\lambda)}{dQ} = \bar{F}(Q^*) - \Lambda\bar{F}\big(b(Q^*)\big) - \lambda w = 0$$

$$\frac{dL(q,\lambda)}{dQ} = \bar{F}\left(\frac{K}{w}\right) - \Lambda\bar{F}\big(b(Q^*)\big) - \lambda^* w = 0\ if \lambda^* \leq 0. Then\ \lambda^* = 0$$

$$if \lambda = 0 => \frac{dL(Q,\lambda)}{dQ} = \frac{d\pi(Q)}{dQ}$$

11.4. Proof 1.2.3: Retailer's unimodality of the best-response order quantity function

The conditions for unimodality are already verified by **Chen and Wang (2012)** as follow:

$$\pi^r = \int_b^Q \bar{F}(x)dx$$

$$\frac{d\pi^r}{dQ} = \bar{F}(Q^*) - w(1 + r^{tc})\bar{F}\big(b(Q^*)\big) = 0$$

$$\left.\frac{d^2\pi^r}{dQ^2}\right|_{Q=Q^*} = -f(Q) + w^2(1 + r^{tc})^2 f(b(Q))$$

We know that $\frac{f(x)}{\bar{F}(x)} = h(x)$, the failure rate. As a consequence, we have:

$$\frac{d^2\pi^r}{dQ^2} = -h(Q)\bar{F}(Q) + w^2(1 + r^{tc})^2\bar{F}(b(Q))h(b(Q))$$

Since $w\bar{F}(b) = \bar{F}(Q)$, we have :

$$\frac{d^2\pi^r}{dQ^2} = -\bar{F}(Q^*)[h(Q^*) - w(1 + r^{tc})^2 h(b(Q))]$$

Since the demand distribution function is by assumption IFR, we have that $h(Q^*) \geq w(1 + r^{tc})h(b(Q))$. This means that $\left.\frac{d^2\pi^r}{d^2Q}\right|_{Q=Q^*} < 0$ the profit function is concave and admit a unique optimum.

Chapitre 12. APPENDIX 2: Supplier : Sensitivities

12.1. Proof 2.1: Sensitivity wholesale price

$$G(Q^*, w, r^{tc}, K^r) = G(Q^*, w) = \bar{F}(Q^*) - w(1 + r^{tc})\bar{F}\big(b(Q^*)\big) = 0$$

$$\frac{\partial G(Q^*, w)}{\partial w} = -\big[(1 + r^{tc})\bar{F}\big(b(Q^*)\big) - wQ(1 + r^{tc})^2 f\big(b(Q^*)\big)\big] = 0$$

$$\frac{\partial G(Q^*, w)}{\partial Q} = -\big[f(Q^*) - w^2(1 + r^{tc})^2 f\big(b(Q^*)\big)\big] = 0$$

From the implicit function theorem, we have:

$$\frac{dQ^*}{dw} = -\frac{\frac{\partial G(Q^*,w)}{\partial w}}{\frac{\partial G(Q^*,w)}{\partial Q}} = -\frac{-\left[(1+r^{tc})\bar{F}\big(b(Q^*)\big) - wQ(1+r^{tc})^2 f\big(b(Q^*)\big)\right]}{-\left[f(Q^*) - w^2(1+r^{tc})^2 f\big(b(Q^*)\big)\right]}$$

Dividing by $\bar{F}\big(b(Q^*)\big)$, that gives us:

$$\frac{dQ^*(w)}{dw} = \frac{\left[(1+r^{tc}) - wQ(1+r^{tc})^2 h\big(b(Q^*)\big)\right]}{-\left[\frac{f(Q^*)}{\bar{F}\big(b(Q^*)\big)} - w^2(1+r^{tc})^2 h\big(b(Q^*)\big)\right]}$$

Where $h(x) = \frac{f(x)}{1-F(x)}$ the failure rate.

From $G(Q^*,w) = \bar{F}(Q^*) - w(1+r^{tc})\bar{F}\big(b(Q^*)\big) = 0$ we have $\frac{f(Q^*)}{\bar{F}(b(Q^*))} = w(1+r^{tc})h(Q^*)$

Finally,

$$\frac{dQ^*(w)}{dw} = \frac{\left[1 - wQ(1+r^{tc})h\big(b(Q^*)\big)\right]}{-w\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]}$$

12.2. Proof 2.1.2 Negative sensitivity to wholesale price

$$\frac{dQ^*(w)}{dw} = \frac{\left[1 - wQ(1+r^{tc})h\big(b(Q^*)\big)\right]}{-w\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]} < 0$$

We refer to the proof of Proposition 1 **(Chen and Wang, 2012).** We restate this proof for the trade credit case.

We will prove that $\frac{dQ^*}{dw} < 0$, by demonstrating that there does not exist a point in the feasible region where $\frac{dQ^*}{dw} \geq 0$ can be verified.

We assume first that $\frac{dQ^*}{dw} > 0$ and $w0$ satisfy $1 - (1+r^{tc})Q^*(w0)h\big(Q^*(w0)\big) = 0$, assuming r^{tc} set constant. As consequence, wQ^* and $Q^*h(Q^*)$ are also increasing in w.

From the above equation following our assumption, we can see that that when $w < w0$ the equation is negative ($\frac{dQ^*}{dw} < 0$) since we always have $h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big) \geq 0$ by definition of IFR function. Also the result of $[1 - wQ(1 + r^{tc})h(b(Q^*))]$ remain positive. Similarly, when $w > w0$, $\frac{dQ^*}{dw} > 0$. In this way wQ achieve its minimum at $w0$

Recall The feasible region for the present value of the wholesale price is $\left(c, \frac{1}{1+rf}\right)$.

First case: $w0 \geq \frac{1}{1+rf}$. We obtain,

$$Q^*(w)h(Q^*(w)) \leq Q^*\left(\frac{1}{1+rf}\right)h\left(Q^*\left(\frac{1}{1+rf}\right)\right) \leq Q^*(w0)h(Q^*(w0))$$

Which involve that $\frac{dwQ^*}{dw} = \frac{1-Q^*h(Q^*)}{-[h(Q^*)-w(1+r^{tc})h(b(Q^*))]} < 0$ *(see Proof 2.2)*

Contradictory, because when $w < w0$, $\frac{dQ^*}{dw} > 0$. Indeed, $Q^*(w0)h(Q^*(w0)) = 1$ as we have assumed.

Second case: $c < w0 < \frac{1}{1+rf}$,

The same problem occurs here, the revenue wQ^* is decreasing in $(c, w0)$. This not match with the assumption $\frac{dQ^*}{dw} > 0$.

Third case: $c = w0$, As we have of IFR distribution and recall that $w(1 + r^{tc}) < 1$, we have:

$$wQ^*(c)(1 + r^{tc})h(wQ^*(c) - K) < Q^*(c)h(Q^*(c)) = 1$$

This means that for all $K > 0$ this equation cannot hold. Assuming $\frac{dQ^*}{dw} \geq 0$ is contradictory with this equation.

To conclude, in the feasible set of wholesale price contract $\left[c, \frac{1}{1+rf}\right]$, we always have $\frac{dQ^*}{dw} < 0$

12.3. Proof 2.2: Sensitivity Revenue

$$\frac{dwQ^*}{dw} = \left(\frac{dQ^*}{dw}\right)w + Q^*$$

$$\frac{dwQ^*}{dw} = \frac{\left[1 - wQ(1 + r^{tc})h\big(b(Q^*)\big)\right] - Q^*\left[h(Q^*) - w(1 + r^{tc})h\big(b(Q^*)\big)\right]}{-\left[h(Q^*) - w(1 + r^{tc})h\big(b(Q^*)\big)\right]}$$

$$\frac{dwQ^*}{dw} = \frac{1 - Q^*h(Q^*)}{-\left[h(Q^*) - w(1 + r^{tc})h\big(b(Q^*)\big)\right]}$$

12.4. Proof 2.3: Sensitivity Trade Credit interest rate.

$$G(Q^*, r^{tc}, w, K^r) = G(Q^*, r^{tc}) = \bar{F}(Q^*) - w(1 + r^{tc})\bar{F}(b(Q^*)) = 0$$

$$\frac{\partial G(Q^*, r^{tc})}{\partial r} = -\left[w\bar{F}\big(b(Q)\big) + w(1 + r^{tc})(wQ - K)\left(-f\big(b(Q)\big)\right)\right]$$

$$\frac{\partial G(Q^*, r^{tc})}{\partial Q^*} = -f(Q) + w^2(1 + r)^2 f(b)$$

By the implicit function theorem we obtain:

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = -\frac{\frac{\partial G(Q^*, r^{tc})}{\partial r^{tc}}}{\frac{\partial G(Q^*, r^{tc})}{\partial Q}}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = -\frac{-\left[w\bar{F}\big(b(Q)\big) + w(1 + r^{tc})(wQ - K)\big(-f(b)\big)\right]}{-f(Q) + w^2(1 + r)^2 f(b)}$$

Divising by $\bar{F}(b(Q))$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = -\frac{[w - h(b(Q))w(1 + r^{tc})(wQ - K)]}{\left[\frac{f(Q)}{\bar{F}(b(Q))} - w^2(1 + r)^2 h(b(Q))\right]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = -\frac{w[1 - h(b(Q))(1 + r^{tc})(wQ - K)]}{w[h(Q) - w(1 + r)^2 h(b(Q))]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = \frac{[1 - h(b(Q))(1 + r^{tc})(wQ - K)]}{-[h(Q) - w(1 + r)^2 h(b(Q))]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = -\frac{[w - h(b(Q))w(1 + r^{tc})(wQ - K)]}{\left[w(1 + r^{tc})\frac{f(Q)}{\bar{F}(Q)} - w^2(1 + r^{tc})^2 h(b(Q))\right]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = -\frac{w[1 - h(b(Q))(1 + r^{tc})(wQ - K)]}{w\left[(1 + r^{tc})\left(h(Q) - w(1 + r^{tc})^2 h(b(Q))\right)\right]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = \frac{[1 - h(b(Q))(1 + r^{tc})(wQ - K)]}{-(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b(Q))\right)\right]}$$

12.5. Proof 2.3.1: Negative Sensitivity to Trade Credit interest rate.

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = \frac{[1 - h(b(Q))(1 + r^{tc})(wQ - K)]}{-(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b(Q))\right)\right]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = \frac{[1 - b(Q)h(b(Q))]}{-(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b(Q))\right)\right]}$$

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = \frac{[1 - H(b(Q))]}{-(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b(Q))\right)\right]}$$

$(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b(Q))\right)\right]$ is always positive due to IFR distribution and the assumption $w(1 + r^{tc}) < 1$.

For a given wholesale price w, Assume $1 - H(Q^*(r^{0tc})) = 0$, we have always $1 - H(b(Q^*)) > 0$

For a $r^{tc} > r^{0tc}$, we have $\frac{dQ^*}{dr^{tc}} > 0$, for $r^{tc} < r^{0tc}$, $\frac{dQ^*}{dr^{tc}} < 0$,

Recall The feasible region for the trade credit

$$w(1 + rf) \leq w(1 + r^{tc}) < p = 1$$

$$rf \leq r^{tc} \leq \frac{p}{w} - 1 = \frac{1}{w} - 1 = \frac{1 - w}{w}$$

Assume $\frac{dQ^*}{dr^{tc}} > 0$, then $Q^*h(Q^*)$ is increasing in r^{tc}.

Note that $(1 + r^{tc})(wQ - K) < pQ = Q$ and $bh(b) < Qh(Q)$

First case: $r^{0tc} = rf$. We have, following the property of increasing failure rate functions, assuming $w(1 + r^{tc}) < 1$, and $K \geq 0$.

$$(wQ^* - K)((1 + r^{tc}))h(b(Q^*)) < Q^*(w, 1 + rf)h(Q^*(w, 1 + rf)) = 1$$

But since we assumed that Q^* was increasing in r^{tc} we should have $(wQ^* - K)((1 + r^{tc}))h(b(Q^*)) \geq 1$, following the equation $\frac{[1-H(b(Q))]}{-(1+r^{tc})[(h(Q)-w(1+r^{tc})h(b(Q)))]}$.

Second case: $r^{0tc} > \frac{1-w}{w}$

Assume $r^{0tc} > \frac{1-w}{w}$ Due to IFR (=> IGFR) Function, we have:

$$Q^*(r^{tc})f(Q^*(r^{tc}) < Q^*\left(\frac{1-w}{w}\right)f\left(Q^*\left(\frac{1-w}{w}\right)\right) < Q^*(r^{0tc})f(Q^*(r^{0tc})$$

Which induce that for all r^{tc}, we have $1 - Q^*(r^{tc})f(Q^*(r^{tc}) > 0$. There is a contradiction between the assumption and the formulas. Indeed in this case $\frac{[1-H(b(Q))]}{-(1+r^{tc})[(h(Q)-w(1+r^{tc})h(b(Q)))]}$ becomes < 0.

Third case: $rf < r^{0tc} < \frac{1-w}{w}$.

We have that

$$Q^*(r^{tc})f(Q^*(r^{tc}) < Q^*(r^{0tc})f(Q^*(r^{0tc}) < Q^*\left(\frac{1-w}{w}\right)f(Q^*\left(\frac{1-w}{w}\right)$$

For the cases between (rf, r^{0tc}), makes the formula $\frac{[1-H(b(Q))]}{-(1+r^{tc})[(h(Q)-w(1+r^{tc})h(b(Q)))]} < 0$. Which lead to a contradiction.

Finally, the best-response order quantities should be decreasing in the interest rate.

12.6. Proof 2.4: Sensitivity Internal Capital.

$$\frac{\partial G(Q, K^r)}{\partial Q} = w^2(1 + r^{tc})^2 f\big(b(Q)\big) - f(Q)$$

$$\frac{\partial G(Q, K^r)}{\partial K^r} = -\left(-w(1 + r^{tc})^2\left(-f\big(b(Q)\big)\right)\right)$$

$$\frac{dQ^*}{dK^r} = -\frac{\frac{\partial G(Q^*)}{\partial K^r}}{\frac{\partial G(Q^*)}{\partial Q^*}}$$

$$\frac{dQ^*(K^r)}{dK^r} = -\frac{-w(1 + r^{tc})^2 f\big(b(Q)\big)}{-\left[f(Q) - w^2(1 + r^{tc})^2 f\big(b(Q)\big)\right]}$$

Dividing by $\bar{F}(b(Q))$

$$\frac{dQ^*(K^r)}{dK^r} = -\frac{-w(1+r^{tc})^2 h\big(b(Q)\big)}{-\left[\frac{f(Q)}{\bar{F}\big(b(Q)\big)} - w^2(1+r^{tc})^2 h\big(b(Q)\big)\right]}$$

From $G(Q^*, K^r)$

$$\frac{dQ^*(K^r)}{dK^r} = -\frac{-w(1+r^{tc})^2 h\big(b(Q)\big)}{-w(1+r^{tc})\big[h(Q) - w(1+r^{tc})h\big(b(Q)\big)\big]}$$

$$\frac{dQ^*(K^r)}{dK^r} = \frac{(1+r^{tc})h\big(b(q)\big)}{-\big[h(Q) - w(1+r^{tc})h\big(b(Q)\big)\big]}$$

Since $h(x) \geq 0$, and the probability function is assumed to be IFR, $\Big(h(Q) > h\big(b(Q)\big)\Big)$, as well as $w(1+r^{tc}) < 1$, the order retailer's order quantities are decreasing as K^r increases

Chapitre 13. APPENDIX 3: Supplier : Optimal Wholesale prices

13.1. Proof 3.1: Unrestricted Problem

We derive the optimal wholesale price from the supplier's expected profit function

$$E[\pi^s(Q,w)] = \int_0^{(wQ-K)(1+r^{tc})} \bar{F}(x)dx - cQ(1+rf)$$

$$\frac{d\pi^s(Q,w)}{dw} = \frac{d(wQ^*)}{dw}(1+r^{tc})\bar{F}(b(Q^*)) - c\left(\frac{dQ^*}{dw}\right)(1+rf)$$

With $b(Q^*) = (wQ^* - K)(1+r^{tc})$

Recall

$$\frac{d\big(wQ^*(w)\big)}{dw} = Q^* + \frac{dQ^*}{dw}w = \frac{dwQ^*}{dw} = \frac{1 - Q^*h(Q^*)}{-\big[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\big]}$$

$$\frac{dQ^*(w)}{dw} = \frac{\big[1 - wQ(1+r^{tc})h\big(b(Q^*)\big)\big]}{-w\big[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\big]}$$

Which gives

$$\frac{d\pi^s(Q,w)}{dw} = \left(\frac{1 - Q^* h(Q^*)}{-\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]}\right)(1+r^{tc})\bar{F}(b(Q^*)) - c(1 + rf)\left(\frac{[1 - wQ(1+r^{tc})h\big(b(Q^*)\big)]}{-w\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]}\right)$$

$$\frac{d\pi^s(Q,w)}{dw} = \frac{w(1 - Q^* h(Q^*))(1+r^{tc})\bar{F}\big(b(Q^*)\big) - c(1+rf)[1 - wQ(1+r^{tc})h\big(b(Q^*)\big)}{-w\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]}$$

$$\frac{d\pi^s(Q,w)}{dw} = \frac{\left[\frac{w\big(1 - Q^* h(Q^*)\big)(1+r^{tc})\bar{F}\big(b(Q^*)\big)}{1 - wQ(1+r^{tc})h\big(b(Q^*)\big)} - c(1+rf)\right]\left[1 - wQ(1+r^{tc})h\big(b(Q^*)\big)\right]}{-w\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]}$$

$$\frac{d\pi^s}{dw} = \frac{[1 - wQ^* h(b(Q^*))]\left[\frac{\big(1 - H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1 - wQ^*(1+r^{tc})h\big(b(Q^*)\big)} - c(1+rf)\right]}{-w\left[h(Q^*) - w(1+r^{tc})h\big(b(Q^*)\big)\right]}$$

At the optimum,

$$\frac{d\pi^s}{dw} = \frac{\big(1 - H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1 - wQ^*(1+r^{tc})h\big(b(Q^*)\big)} - c(1+rf) = 0$$

We have

$$\frac{\big(1 - H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1 - wQ^*(1+r^{tc})h\big(b(Q^*)\big)} = c(1+rf)$$

The optimal wholesale price can be determined with this formula.

13.2. Proof 3.2: Cash-constrained Supplier

$$\pi^s = -K^r - K^s + \int_{(cQ-K^r-K^s)(1+r^s)}^{(wQ-K^r)(1+r^{tc})} \bar{F}(x)dx$$

$(wQ - K^r)(1+r^{tc}) = b(Q) = b$ and $(cQ - K^r - K^s)(1+r^s) = g(Q); r^s > rf$

$$\frac{d\pi^s}{dw} = \frac{d(wQ^*)}{dw}(1+r^{tc})\bar{F}(b(Q^*)) - c\left(\frac{dQ^*}{dw}\right)(1+r^s)\bar{F}(g(Q^*))$$

At the optimum,

$$\frac{d\pi^s}{dw} = \frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1-wQ^*(1+r^{tc})h\big(b(Q^*)\big)} - c(1+r^s)\bar{F}(g(Q^*)) = 0$$

We have

$$\frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)(1+r^{tc})}{1-wQ^*(1+r^{tc})h\big(b(Q^*)\big)} = c(1+r^s)\bar{F}(g(Q^*))$$

13.3. <u>Proof 3.3: Concavity optimal wholesale price function: Case with unrestricted supplier.</u>

We will restate the approach of **Chen and Wang (2012)** as a proof of unimodality of the supplier's function of profit. We set the interest rate constant. For convenience of notation, we assume the trade credit interest rate $r^{tc} = 0$. This is used as an illustration in order to prove further the behavior of the "leveraged case" and "bankruptcy cost case" function.

We come back to our profit sensitivity formula:

$$\frac{d\pi^s}{dw} = \frac{[1-wQ^*h(b(Q^*))]\left[\frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)}{1-wQ^*h(wQ^*-K^r)} - c(1+rf)\right]}{-w[h(Q^*)-wh(wQ-K^r)]}$$

From $\frac{d\big(wQ^*(w)\big)}{dw} = Q^* + \frac{dQ^*}{dw}w$ and as well as $\frac{dQ^*}{dw}$, we know that $\frac{[1-wQ^*h(b(Q^*))]}{-w[h(Q^*)-wh(wQ-K^r)]} < 0$, which means that it remain to look at $\left[\frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)}{1-wQ^*h(wQ^*-K^r)} - c(1+rf)\right]$ to determine the sign of the equation. Let $\delta(w) = \frac{\big(1-H(Q^*)\big)\bar{F}(Q^*)}{1-wQ^*h(wQ^*-K^r)}$. If $\delta(w) - c(1+rf) > 0$ we have $\frac{d\pi^s}{dw} < 0$ and if $\delta - c(1+rf) < 0$ we have that $\frac{d\pi^s}{dw} > 0$.

Let $w0$ the wholesale price that achieve $1 - H(Q^*) = 0$. It is easy to see that in this case $\delta(w) = 0$ we obtain $-c(1+rf) < 0,$ which means that $\frac{d\pi^s}{dw} > 0$ in this case. If we prove that $\frac{d\delta(w)}{dw} > 0$ always hold when $w > w0$, it is sufficient to prove the unimodality. Since $c(1+r)$ is constant, if $\frac{d\delta(w)}{dw} > 0,$ $\delta(w) - c(1+rf)$ will cross zero only once, at the optimum. In consequence, the sensitivity of the profit function $\frac{d\pi^s}{dw}$ change sign only once. It is a sufficient condition to prove the unimodality of the relationship.

We can prove that $\frac{d\delta(w)}{dw} = \frac{(-H'(Q)[1-wQh(wQ-K)Q')}{[1-wQh(wQ-K^r]^2} + \frac{(1-H(Q))[H'(wQ-K^r)+K^r h'(wQ-K^r)(wQ)')}{[1-wQh(wQ-K^r]^2} > \frac{\left((1-H(Q))\left[-\left(Qh'(Q)-wQh'(wQ-K^r)\right)\right]Q'\right)}{\left(1-wQh(wQ-K^r)\right)^2} > 0$ *(see Proof 3.1.1)*

As consequence, $\delta(w) - c(1+rf)$ crosses zero at the optimal wholesale price $\widehat{w}$. Therefore, the profit function is decreasing when $w < \widehat{w}$, and increasing when $w > \widehat{w}$, and attain its maximum at $\widehat{w}$, when $\delta(w) = c(1+rf)$

13.4. Proof 3.3.1 Details Concavity : Normal problem.

$$\frac{d\delta(w)}{dw} = \frac{d}{dw}\frac{\left(1-H(Q^*)\right)\bar{F}(Q^*)}{1-wQ^*h(wQ^*-K^r)}$$

We already know $\frac{dQ^*}{dw} < 0$, which means that $\frac{d\bar{F}(Q^*)}{dw} = -\frac{dQ^*}{dw}f(Q^*) > 0$. It remains to prove that

$\frac{\left(1-H(Q^*)\right)}{1-wQ^*h(wQ^*-K^r)} > 0$ as well

Let $y = 1 - wQ^*h(wQ^* - K^r)$

$$\frac{d\delta(w)}{dw} = \frac{1}{y^2}\left[-Q'H'(Q)y + \left(1-H(Q)\right)\left(-(wQ)'\left(h(b) + wQ(wQ)'h'(b)\right)\right]\right.$$

$$= \frac{1}{y^2}\left[-Q'H'(Q)y + (wQ)'[h(b) + wQh'(b)](1-H(Q)]\ \right]$$

We can see that $H'(b) = ((wQ-K)h(b))' = (wQ)'h(b) + (wQ-K)h'(b)(wQ)'$

We can therefore rewrite the problem as:

$$\frac{1}{y^2}\left[-Q'H'(Q)y + (wQ)'[H'(b) + K^r h'(b)](1-H(Q)]\ \right]$$

In fact $y < 1$ because $\frac{dQ^*}{dw} < 0$. Which means that when $w > w0$ (as in the case we are concerned for), we have $Q(w)F\left(Q(w)\right) < Q(w0)F\left(Q(w0)\right)$. Recall that $h(wQ-K) \leq\ h(Q)$

by IFR assumption, then $wQ \leq Q$. As consequence, since we have, $1 - H\big(Q(w0)\big) = 0$, alternatively $H\big(Q(w0)\big) = 1$, we must have that $1 - wQ^* h(wQ^* - K^r) < 1$

$$> \frac{1}{y^2}[-Q'H'(Q) + (wQ)'[H'(b) + K^r h'(b)](1 - H(Q)]\,]$$

$$> \frac{1}{y^2}[Q'[-H'(Q) + H'(b) + K^r h'(b)](1 - H(Q)]\,]$$

$$= \big(1 - H(Q)\big)\big[\big(h(b) - h(Q)\big) - \big(Qh'(Q) - wQh'(wQ - K^r)\big)\big]Q'\big]\frac{1}{y^2}$$

$$> \frac{\Big(\big(1 - H(Q)\big)\big[-\big(Qh'(Q) - wQh'(wQ - K^r)\big)\big]Q'\Big)}{\big(1 - wQh(wQ - K^r)\big)^2} > 0$$

See the original proof on Chen and Wang (2012)

13.5. Proof 3.4. Concavity optimal wholesale price : case with financial constraint.

An adaptation of the case without constraint. Let, $b = b(Q)$; $r^s = 0$

$$\frac{d\delta^c(w)}{dw} = \frac{d}{dw}\frac{\big(1 - H(Q^*)\big)\bar{F}(Q^*)}{1 - wQ^* h(wQ^* - K^r)\bar{F}(cQ^* - K^r - K^s)} = \frac{x}{y^c}$$

$$\frac{1}{y^2}\big[Q'H'(Q)y + \big(1 - H(Q)\big)y'\big]$$

$$y^{c\prime} = \big(1 - wQ^* h(wQ^* - K^r)\bar{F}(cQ^* - K^r - K^s)\big)'$$

$$y = 1 - wQ^* h(wQ^* - K^r)$$

$$= (wQ)'[h(b) + wQh'(b)]\bar{F}(cQ^* - K^r - K^s) + \big(1 - wQ^* h(wQ^* - K^r)\big)\big(-cQ'f(g)\big)$$

We have then

$$\frac{1}{y^{c2}}[-Q'H'(Q)y^c + \big(1 - H(Q)\big)\big((wQ)'[h(b) + wQh'(b)]\bar{F}(cQ^* - K^r - K^s) + \big(1 - wQ^* h(wQ^* - K^r)\big)\big(-cQ'f(g)\big)\big]$$

$$= \frac{1}{y^{c2}}[-y[Q'H'(Q)\bar{F}(g) + cQ'f(g)]$$

$$+ \left(1 - H(Q)\right)((wQ)'[h(b) + wQh'(b)]\bar{F}(cQ^* - K^r - K^s)]$$

$$> \frac{1}{y^{c2}}[-[Q'H'(Q)\bar{F}(g) + cQ'f(g)]$$

$$+ \left(1 - H(Q)\right)((wQ)'[h(b) + wQh'(b)]\bar{F}(cQ^* - K^r - K^s)]$$

$$> \frac{1}{y^{c2}}[-[Q'H'(Q)\bar{F}(g) + cQ'f(g)] + \left(1 - H(Q)\right)([h(b) + wQh'(b)]\bar{F}(g)]$$

$$= \frac{1}{y^{c2}}[-[Q'H'(Q)\bar{F}(g) + \frac{cQ'f(g)\bar{F}(g)}{\bar{F}(g)}] + \left(1 - H(Q)\right)([h(b) + wQh'(b)]\bar{F}(g)]$$

$$= \frac{1}{y^{c2}}[-[Q'H'(Q)\bar{F}(g) + cQ'h(g)\bar{F}(g)] + \left(1 - H(Q)\right)([h(b) + wQh'(b)]\bar{F}(g)]$$

$$= \frac{1}{y^{c2}}[-[Q'H'(Q) + cQ'h(g)] + \left(1 - H(Q)\right)\left([h(b) + wQh'(b)]\right]$$

$$> \frac{1}{y^{c2}}[-[Q'H'(Q) + cQ'h(b)] + \left(1 - H(Q)\right)\left([h(b) + wQh'(b)]\right]$$

$$> \frac{1}{y^{c2}}[-[Q'H'(Q) + cQ'h(b)]\left(1 - H(Q)\right)[H'(b) + K^r h'(b)]]$$

$$> \frac{1}{y^{c2}}[\left(1 - H(Q)\right)[H'(b) + K^r h'(b) - H'(Q) - ch(b)]Q']$$

$$= \frac{1}{y^{c2}}[\left(1 - H(Q)\right)\left[\left(h(b) - h(Q)\right) - \left(Qh'(Q) - wQh'(wQ - K^r)\right) - ch(b)\right]Q']$$

Recall $Q' < 0$

$$> \frac{1}{y^{c2}}[\left(1 - H(Q)\right)\left[-\left(Qh'(Q) - wQh'(wQ - K^r)\right) - ch(b)\right]Q']$$

$$> 0$$

As long as $H(Q) < 1$, as assumed since we explore $w > w0$

Chapitre 14. APPENDIX 4 : Supplier : optimal Interest rate.

14.1. Proof 4.1: Unrestricted Problem

Let now see how the retailer optimal order quantities evolve with the interest rate.

Recall *see Proof*

$$\frac{dQ^*(r^{tc})}{dr^{tc}} = \frac{[1 - h(b(Q))(1 + r^{tc})(wQ - K)]}{-(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b(Q))\right)\right]}$$

Now we compute the optimal trade credit interest rate for a given w, assuming the supplier have enough capital $K > cQ$

$$\pi^s = \int_0^{(wQ-K)(1+r)} \bar{F}(x)dx - cQ$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left(\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}}\right)\bar{F}(wQ^* - K) - c\left(\frac{dQ^*}{dr^{tc}}\right)$$

Firstly,

$$\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}} = \frac{d(wQ^* - K)}{dr^{tc}}(1 + r^{tc}) + (wQ^* - K)$$

$$\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}} = \frac{wdQ^*}{dr^{tc}}(1 + r^{tc}) + (wQ^* - K)$$

$$= \frac{w(1 + r^{tc})[1 - h(b)(1 + r^{tc})(wQ - K)] - (wQ^* - K)(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b)\right)\right]}{-(1 + r^{tc})\left[\left(h(Q) - w(1 + r^{tc})h(b)\right)\right]}$$

$$= \frac{[w[1 - h(b)(1 + r^{tc})(wQ - K)] - (wQ^* - K)\left[\left(h(Q) - w(1 + r^{tc})h(b)\right)\right]}{\left[\left(h(Q) - w(1 + r^{tc})h(b)\right)\right]}$$

$$\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}} = \frac{w - (wQ - K)h(Q)}{-\left[\left(h(Q) - w(1 + r^{tc})h(b)\right)\right]}$$

Then, we replace in the profit sensitivity function:

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left(\frac{w-(wQ-K)h(Q)}{-\left[\left(h(Q)-w(1+r^{tc})h(b)\right)\right]}\right)\bar{F}(b) - \left(\frac{c[1-h(b)(1+r^{tc})(wQ-K)]}{-(1+r^{tc})\left[\left(h(Q)-w(1+r^{tc})h(b)\right)\right]}\right)$$

$$= \left(\frac{(1+r^{tc})\left(w-(wQ-K)h(Q)\right)\bar{F}(b) - c[1-h(b)(1+r^{tc})(wQ-K)]}{-(1+r^{tc})\left[\left(h(Q)-w(1+r^{tc})h(b)\right)\right]}\right)$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left(\frac{\left[\frac{(1+r^{tc})\left(w-(wQ-K)h(Q)\right)\bar{F}(b)}{[1-h(b)(1+r^{tc})(wQ-K)]} - c\right][1-h(b)(1+r^{tc})(wQ-K)]}{-(1+r^{tc})\left[\left(h(Q)-w(1+r^{tc})h(b)\right)\right]}\right)$$

$$\left[\frac{(1+r^{tc})\left(w-(wQ-K)h(Q)\right)\bar{F}(b)}{[1-h(b)(1+r^{tc})(wQ-K)]} - c\right]$$

$$\left[\frac{(1+r^{tc})\left(w-(wQ-K)h(Q)\right)\frac{\bar{F}(Q)}{(1+r^{tc})w}}{[1-h(b)(1+r^{tc})(wQ-K)]} - c\right]$$

$$\left[\frac{\left(1-\frac{(wQ-K)}{w}h(Q)\right)\bar{F}(Q)}{[1-h(b)(1+r^{tc})(wQ-K)]} - c\right]$$

Finally, we obtain:

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left[\frac{\left(1-\frac{(wQ-K)}{w}h(Q)\right)\bar{F}(Q)}{[1-bh(b)]} - c\right]$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left[\frac{\left(1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right)\bar{F}(Q)}{[1-H(b(Q^*))]} - c\right]$$

14.2. Proof 4.2: Cash-constrained Supplier

For sake of simplicity, we assume $r^s = 0$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left(\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}}\right)\bar{F}(b) - c\left(\frac{dQ^*}{dr^{tc}}\right)\bar{F}(cQ^* - K^r - K^s)$$

$$\pi^s = \int_{(cQ - K^r - K^s)}^{(wQ - K^r)(1+r)} \bar{F}(x)dx$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left(\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}}\right)\bar{F}(wQ^* - K) - c\left(\frac{dQ^*}{dr^{tc}}\right)\bar{F}(cQ^* - K^r - K^s)$$

$$\frac{d(wQ^* - K)(1 + r^{tc})}{dr^{tc}} = \frac{d(wQ^* - K)}{dr^{tc}}(1 + r^{tc}) + (wQ^* - K)$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left(\frac{w - (wQ^* - K)h(Q^*)}{-\left[\left(h(Q) - w(1 + r^{tc})h(b)\right)\right]}\right)\bar{F}(b)$$
$$- \left(\frac{c[1 - h(b)(1 + r^{tc})(wQ^* - K)]}{-(1 + r^{tc})\left[\left(h(Q^*) - w(1 + r^{tc})h(b)\right)\right]}\right)\bar{F}(cQ^* - K^r - K^s)$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}}$$
$$= \left(\frac{(1 + r^{tc})\left(w - (wQ^* - K)h(Q^*)\right)\bar{F}(b) - c[1 - h(b)(1 + r^{tc})(wQ^* - K)]\bar{F}(cQ^* - K^r - K^s)}{-(1 + r^{tc})\left[\left(h(Q^*) - w(1 + r^{tc})h(b)\right)\right]}\right)$$

$$\frac{d\pi^s(r^{tc})}{dr^{tc}}$$
$$= \left(\frac{\left[\frac{(1 + r^{tc})\left(w - (wQ^* - K)h(Q^*)\right)\bar{F}(b)}{[1 - h(b)(1 + r^{tc})(wQ^* - K)]} - c\bar{F}(cQ - K^r - K^s)\right][1 - h(b)(1 + r^{tc})(wQ^* - K)]}{-(1 + r^{tc})\left[\left(h(Q^*) - w(1 + r^{tc})h(b)\right)\right]}\right)$$

$$= \left[\frac{(1 + r^{tc})\left(w - (wQ^* - K)h(Q^*)\right)\bar{F}(b)}{[1 - h(b)(1 + r^{tc})(wQ^* - K)]} - c\bar{F}(cQ^* - K^r - K^s)\right]$$

$$= \left[\frac{(1+r^{tc})\big(w - (wQ^* - K)h(Q^*)\big)\dfrac{\bar{F}(Q^*)}{(1+r^{tc})w}}{[1 - h(b)(1+r^{tc})(wQ^* - K)]} - c\bar{F}(cQ^* - K^r - K^s) \right]$$

$$= \left[\frac{\left(1 - \dfrac{(wQ^* - K)}{w} h(Q^*)\right)\bar{F}(Q^*)}{[1 - h(b)(1+r^{tc})(wQ^* - K)]} - c\bar{F}(cQ^* - K^r - K^s) \right]$$

Finally,

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left[\frac{\left(1 - \dfrac{(wQ^* - K)}{w} h(Q^*)\right)\bar{F}(Q^*)}{[1 - bh(b)]} - c\bar{F}(cQ^* - K^r - K^s) \right]$$

14.3. **Proof 4.3 Concavity of the relationship between interest rate and supplier's profit.**

$$\frac{d\pi^s(r^{tc})}{dr^{tc}} = \left[\frac{\left(1 - \dfrac{(wQ - K)}{w} h(Q^*)\right)\bar{F}(Q^*)}{[1 - b(Q^*)h(b(Q^*))]} - c \right]$$

Recall that, $\frac{dQ}{dr^{tc}} = -\frac{[1-h(b)(1+r^{tc})(wQ-K)]}{(1+r^{tc})[(h(Q)-w(1+r^{tc})h(b))]} < 0$, which means that the only condition to determine the sign of $\frac{d\pi^s}{dw}$ is to find the sign of $\left[\frac{\left(1-\frac{(wQ^*-K)}{w}h(Q^*)\right)\bar{F}(Q)}{[1-b(Q^*)h(b(Q^*))]} - c\right]$

Here, similarly to the analysis with the wholesale price, let $\delta^{r^t} = \frac{\left(1-\frac{(wQ^*-K)}{w}h(Q^*)\right)\bar{F}(Q^*)}{[1-b(Q^*)h(b(Q^*))]}$. We proved that $\frac{dQ^*}{dr^t} < 0$. *(see Proof 2.3.1)*, which means that $\frac{dQ^*h(Q^*)}{dr^t} < 0, \frac{dwQ^*}{dr^t} < 0$ and $\frac{d\left(Q^*-\frac{K}{w}\right)h(Q^*)}{dr^t} < 0$. Let $w0$ be the wholesale price that satisfy $\frac{(wQ-K)}{w}h(Q) = 0$, when $w < w0$, we have that $\delta^{r^t} < 0$. Alternatively, when $w > w0$, we have that $\delta^{r^t} > 0$. When $w = w0$, then $\delta(r^t) = 0$ and $\delta(r^t) - c < 0$. In the region encompassing $[\delta^0(r^t), c]$ and $[c, \infty]$ we have as wholesale price $w > w0$. Since c is a constant, We need to prove that $\frac{d\ \delta(r^t)}{dr^t}$ is an increasing function when $w > w0$ as sufficient proof to guarantee that $\delta(r^t)$ crosses c only once, involving the unimodality of the function.

-

$$\frac{d\left(\delta(r^t)\right)}{dr^{tc}} = \frac{d}{dr^{tc}}\left(\frac{\left(1-\frac{(wQ^*-K)}{w}h(Q^*)\right)\bar{F}(Q^*)}{[1-b(Q^*)h(b(Q^*))]}\right).$$

We already know that $\frac{d\bar{F}(Q^*)}{dr^{tc}} = -\frac{dQ^*}{dr^{tc}}f(Q^*) > 0$, the problem reduce to $\frac{d}{dr^{tc}}\left(\frac{\left(1-\frac{(wQ-K)}{w}h(Q)\right)}{[1-bh(b)]}\right) = \frac{d}{dr^{tc}}\left(\frac{X}{Y}\right)$ (Let $X = 1 - \frac{(wQ-K)}{w}h(Q)$ and $Y = [1 - bh(b)]$

$$\frac{1}{Y^2}[X'Y - Y'X]$$

$$Y' = -b'H'(b)$$

$$X = (1 - \left(H(Q) - \frac{K}{w}h(Q)\right)$$

$$X' = -Q'[H'(Q) - \frac{K}{w}h'(Q)]$$

We have,

$$\frac{1}{Y^2}[X'Y - Y'X] =$$

$$\left[-Q'[H'(Q) - \frac{K}{w}h'(Q)]\right]Y - \left((-b'H'(b))\left[1 - \left(H(Q) - \frac{K}{w}h(Q)\right)\right]\right)$$

$$> \left[-Q'[H'(Q) - \frac{K}{w}h'(Q)]\right] - \left([-b'H'(b)]\left[1 - \left(H(Q) - \frac{K}{w}h(Q)\right)\right]\right)$$

$$= \left[-Q'[H'(Q) - \frac{K}{w}h'(Q)]\right] + (b'H'(b))\left[1 - \left(H(Q) - \frac{K}{w}h(Q)\right)\right]$$

(Because $\left[1 - \left(H(Q) - \frac{K}{w}h(Q)\right)\right] < 1$ and $b' < Q'$)

$$> -Q'\left[H'(Q)-\frac{K}{w}h'(Q)+H'(b)\right]\left[1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right]>0$$

(since $H'(Q)>h'(Q)$)

Which prove that $\frac{d\left(\delta(r^t)\right)}{dr^{tc}}>0$. Then, the function is unimodal.

14.4. Proof 4.4 Concavity of the relationship between interest rate and profit: cash-constrained case.

$$\frac{1}{Y^2}[X'Y-Y'X]$$

$$Y'=-[b'H'(b)\bar{F}(g)+bH(b)\left(-f(g)\right)]$$

$$X=(1-\left(H(Q)-\frac{K}{w}h(Q)\right)$$

$$X'=-Q'[H'(Q)-\frac{K}{w}h'(Q)]$$

We have,

$$\frac{1}{Y^2}[X'Y-Y'X]=$$

$$\left[-Q'[H'(Q)-\frac{K}{w}h'(Q)]\right]Y$$
$$-\left(\left(-[b'H'(b)\bar{F}(g)+bH(b)\left(-f(g)\right)]\right)\left[1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right]\right)$$

$$>\left[-Q'[H'(Q)-\frac{K}{w}h'(Q)]\right]$$
$$-\left(\left[-[b'H'(b)\bar{F}(g)+bH(b)\left(-f(g)\right)]\right]\left[1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right]\right)$$

$$=\left[-Q'[H'(Q)-\frac{K}{w}h'(Q)]\right]+\left(-[b'H'(b)\bar{F}(g)-bH(b)\left(f(g)\right)]\right)\left[1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right]$$

(Because $\left[1-\left(H(Q)-\frac{K}{w}h(Q)\right)\right]<1$ and $b'<Q'$)

$$> \left[-Q'\left[H'(Q) - \frac{K}{w}h'(Q) + H'(b)\bar{F}(g)\right] + bH(b)\big(f(g)\big)\right]\left[1 - \left(H(Q) - \frac{K}{w}h(Q)\right)\right] > 0$$

$(H'(Q) > h'(Q))$

Chapitre 15. APPENDIX 5 : Independence Financing and Operation in case of Perfect Capital Market.

15.1. Proof 5.1 Independence Financing and Operational decision Supplier

From the bank, we have :

$$G^b = \int_g^\infty L^s(1+r^s)dF(x) + \int_0^g xdF(x) - L^s(1+rf) = 0$$

$$g = (cQ - K)(1+r^s) = L^s(1+r^s)$$

Interest rate derivation relative to the wholesale price.

$$dr^s = \left(\frac{\partial r^s}{\partial Q^*}\right)dQ^* + \left(\frac{\partial r^s}{\partial w}\right)dw$$

$$\frac{dr^s}{dw} = \left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw} + \frac{\partial r^s}{\partial w} = \left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw}$$

$$\frac{\partial r^s}{\partial Q^*} = -\frac{\frac{\partial G^b}{\partial Q^*}}{\frac{\partial G^b}{\partial r^s}} =$$

$$\frac{\partial G^b}{\partial Q^*} = \frac{\partial}{\partial Q^*}L^s(1+r^s)\int_g^\infty dF(x) + xF(x)|_0^g - \int_0^g F(x)dx - L^s(1+rf)$$

$$g'\bar{F}(g) - g\, g'f(g) + g'F(g) + gg'f(g) - g'F(g) - c(1+rf)$$

$$= c(1+r^s)\bar{F}(g) - c(1+rf)$$

$$\frac{\partial G^b}{\partial r^s} = g'\bar{F}(g) - g\, g'f(g) + g'F(g) + gg'f(g) - g'F(g)$$

$$= (cQ-K)\bar{F}(g)$$

$$\frac{\partial r^s}{\partial Q^*} = -\frac{\frac{\partial G^b}{\partial Q^*}}{\frac{\partial G^b}{\partial r^s}} = -\frac{c(1+r^s)\bar{F}(g) - c(1+rf)}{(cQ-K)\bar{F}(g)} = \frac{c(1+rf)}{L^s\bar{F}(g)} - \frac{c(1+r^s)\bar{F}}{L^s}$$

Now

$$\frac{dr^s}{dw} = \left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw}$$

The supplier problem :

$$\frac{d\pi^s}{dw} = \frac{d(wQ^*)}{dw}\bar{F}(wQ^* - K^r) - \left(\frac{d[(cQ^*-K)(1+r^s)]}{dw}\right)\bar{F}((cQ^*-K)(1+r^s))$$

$$\frac{d[(cQ^*-K)(1+r^s)]}{dw} = c\left(\frac{dQ^*}{dw}\right)(1+r^s) + (cQ-K)\frac{dr^s}{dw}$$

We assume the supplier extend open account financing to the retailer here ($r^{tc} = 0$).

$$\frac{dQ^*}{dw} = \frac{[1 - wQh(b(Q^*))]}{-w[h(Q^*) - wh(b(Q^*))]}$$

$$\frac{dwQ^*}{dw} = \frac{1 - Q^*h(Q^*)}{-[h(Q^*) - wh(b(Q^*))]}$$

$$\frac{dr^s}{dw} = \left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw} = \frac{[1 - wQh\big(b(Q^*)\big)]}{-w\big[h(Q^*) - wh\big(b(Q^*)\big)\big]}\left(-\frac{c(1+r^s)\bar{F}(g) - c(1+rf)}{(cQ-K)\bar{F}(g)}\right)$$

$$\frac{d[(cQ^* - K)(1+r^s)]}{dw}$$
$$= c\left(\frac{dQ^*}{dw}\right)(1+r^s) + (cQ-K)\left(\frac{dQ^*}{dw}\right)\left(-\frac{c(1+r^s)\bar{F}(g) - c(1+rf)}{(cQ-K)\bar{F}(g)}\right)$$

$$c\left(\frac{dQ^*}{dw}\right)(1+r^s) + \left(\frac{dQ^*}{dw}\right)\left(-\frac{c(1+r^s)\bar{F}(g) - c(1+rf)}{\bar{F}(g)}\right)$$

$$\left(\frac{dQ^*}{dw}\right)\left[c(1+r^s) + \left(-\frac{c(1+r^s)\bar{F}(g) - c(1+rf)}{\bar{F}(g)}\right)\right]$$

$$\left(\frac{dQ^*}{dw}\right)\left[\frac{c(1+r^s)\bar{F}(g)}{\bar{F}(g)} + \left(-\frac{c(1+r^s)\bar{F}(g) - c(1+rf)}{\bar{F}(g)}\right)\right]$$

$$\left(\frac{dQ^*}{dw}\right)\left[\frac{c(1+rf)}{\bar{F}(g)}\right]$$

Now we put it in the supplier's profit function:

$$\frac{d\pi^s}{dw} = \frac{d(wQ^*)}{dw}\bar{F}(wQ^* - K^r) - \left(\frac{dQ^*}{dw}\right)\left[\frac{c(1+rf)}{\bar{F}(g)}\right]\bar{F}((cQ^* - K)(1+r^s))$$

This reduce to

$$\frac{d(wQ^*)}{dw}\bar{F}(wQ^* - K^r) - c(1+rf)\left(\frac{dQ^*}{dw}\right)$$

Prove the results of Modigliani Miller, and as from Kouvelis and Zhao

15.2. Proof 5.1.1 No influence on operational decision : Supplier optimal loan size

From the bank problem, we have:

$$L^s(1+r)\int_{g(q)}^{\infty} f(x)dx + p\int_0^{g(q)} xf(x)dx - L^s(1+rf) = 0 = G(r,L)$$

$$\frac{\partial G}{\partial r} = L^s \int_{g(q)}^{\infty} f(x)dx$$

$$\frac{\partial G}{\partial L} = (1+r)\int_{g(q)}^{\infty} f(x)dx - (1+rf)$$

$$\frac{dr}{dL} = \frac{(1+rf)}{L^s \int_{g(q)}^{\infty} f(x)dx} - \frac{1+r}{L^s}$$

Recall $g(Q^*) = \frac{L^s(1+r^b)}{p}$

$$\frac{d\pi^s(r(L),L)}{dL} = \frac{dr}{dL}\frac{\partial\pi^s(r(L),L)}{\partial r} + \frac{\partial\pi^s(r(L),L)}{\partial L}$$

We have

$$\pi^s(r(L),L) = -K^s + p\int_{\frac{L^s(1+r(L))}{p}}^{\frac{(wQ-K^r)^+}{p}(1+r^r)} \bar{F}(x)dx$$

$$\frac{\partial\pi^s(r(L),L)}{\partial r} = -L^s\bar{F}\left(\frac{L^s(1+r)}{p}\right)$$

$$\frac{\partial\pi^s(r(L),L)}{\partial L} = -(1+r)\bar{F}\left(\frac{L^s(1+r)}{p}\right)$$

$$\frac{d\pi^s(r(L),L)}{dL} = -L^s\bar{F}\left(\frac{L^s(1+r)}{p}\right)\left[\frac{(1+rf)}{L^s\int_{g(q)}^{\infty} f(x)dx} - \frac{1+r}{L^s}\right] - (1+r)\bar{F}\left(\frac{L^s(1+r)}{p}\right)$$

$$= -(1+rf)$$

No influence of capital structure on operational decision.

15.3. Proof 5.2 Dependence between Financing and Operational decision Supplier when bankruptcy costs.

As simplification, we consider no bankruptcy cost for the retailer.

First, we derive the sensitivity of the interest rate to the wholesale price.

$$dr^s = \left(\frac{\partial r^s}{\partial Q^*}\right) dQ^* + \left(\frac{\partial r^s}{\partial w}\right) dw$$

$$\frac{dr^s}{dw} = \left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw} + \frac{\partial r^s}{\partial w} = \left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw}$$

$$\frac{\partial r^s}{\partial Q^*} = -\frac{\frac{\partial G^b}{\partial Q^*}}{\frac{\partial G^b}{\partial r^s}} =$$

$$G^b = \int_g^\infty L^s(1+r^s)dF(x) + \alpha\int_0^g xdF(x) - L^s(1+rf) = 0$$

$$g = (cQ-K)(1+r^s) = L^s(1+r^s)$$

$$\frac{\partial G^b}{\partial Q^*} = \frac{\partial}{\partial Q^*} L^s(1+r^s)\int_g^\infty dF(x) + \alpha\left[xF(x)|_0^g - \int_0^g F(x)dx\right] - L^s(1+rf)$$

$$g'\bar{F}(g) - g\,g'f(g) + \alpha[g'F(g) + gg'f(g) - g'F(g)] - c(1+rf)$$

$$= c(1+r^s)\bar{F}(g) - (1-\alpha)(cQ-K)(1+r^s)^2 cf(g) - c(1+rf)$$

$$\frac{\partial G^b}{\partial r^s} = g'\bar{F}(g) - g\,g'f(g) + \alpha[g'F(g) + gg'f(g) - g'F(g)]$$

$$= (cQ-K)\bar{F}(g) - (1-\alpha)(cQ-K)^2(1+r^s)f(g)$$

$$\frac{\partial r^s}{\partial Q^*} = -\frac{\frac{\partial G^b}{\partial Q^*}}{\frac{\partial G^b}{\partial r^s}} = -\frac{c(1+r^s)\bar{F}(g) - (1-\alpha)(cQ-K)(1+r^s)^2 cf(g) - c(1+rf)}{(cQ-K)\bar{F}(g) - (1-\alpha)(cQ-K)^2(1+r^s)f(g)}$$

$$= -\frac{c(1+r^s)\bar{F}(g) - (1-\alpha)(cQ-K)(1+r^s)^2 cf(g) - c(1+rf)}{(cQ-K)[\bar{F}(g) - (1-\alpha)(cQ-K)(1+r^s)f(g)]}$$

$$-\frac{c[(1+r^s)\bar{F}(g)-(1-\alpha)L^s(1+r^s)^2 f(g)-(1+rf)]}{L^s[\bar{F}(g)-(1-\alpha)L^s(1+r^s)f(g)]}$$

Now,

$$\frac{dr^s}{dw}=\left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw}$$

The supplier problem becomes :

$$\frac{d\pi^s}{dw}=\frac{d(wQ^*)}{dw}\bar{F}(wQ^*-K^r)-\left(\frac{d[(cQ^*-K)(1+r^s)]}{dw}\right)\bar{F}((cQ^*-K)(1+r^s))$$

$$\frac{d[(cQ^*-K)(1+r^s)]}{dw}=c\left(\frac{dQ^*}{dw}\right)(1+r^s)+(cQ-K)\frac{dr^s}{dw}$$

We assume the supplier extend open account financing to the retailer here ($r^{tc}=0$).

$$\frac{dQ^*}{dw}=\frac{[1-wQh\big(b(Q^*)\big)]}{-w\big[h(Q^*)-wh\big(b(Q^*)\big)\big]}$$

$$\frac{dwQ^*}{dw}=\frac{1-Q^*h(Q^*)}{-\big[h(Q^*)-wh\big(b(Q^*)\big)\big]}$$

$$\frac{dr^s}{dw}=\left(\frac{\partial r^s}{\partial Q^*}\right)\frac{dQ^*}{dw}$$

$$=\frac{[1-wQh\big(b(Q^*)\big)]}{-w\big[h(Q^*)-wh\big(b(Q^*)\big)\big]}\left(-\frac{c[(1+r^s)\bar{F}(g)-(1-\alpha)L^s(1+r^s)^2 f(g)-(1+rf)]}{L^s[\bar{F}(g)-(1-\alpha)L^s(1+r^s)f(g)]}\right)$$

$$\frac{d[(cQ^*-K)(1+r^s)]}{dw}$$

$$=c\left(\frac{dQ^*}{dw}\right)(1+r^s)$$

$$+L^s\left(\frac{dQ^*}{dw}\right)\left(-\frac{c[(1+r^s)\bar{F}(g)-(1-\alpha)L^s(1+r^s)^2 f(g)-(1+rf)]}{L^s[\bar{F}(g)-(1-\alpha)L^s(1+r^s)f(g)]}\right)$$

$$\frac{d[(cQ^* - K)(1+r^s)]}{dw} = c\left(\frac{dQ^*}{dw}\right)(1+r^s) + \left(\frac{dQ^*}{dw}\right)\left(-\frac{L^s c[(1+r^s)\bar{F}(g) - (1-\alpha)L^s(1+r^s)^2 f(g) - (1+rf)]}{L^s[\bar{F}(g) - (1-\alpha)L^s(1+r^s)f(g)]}\right)$$

$$\frac{d[(cQ^* - K)(1+r^s)]}{dw} = c\left(\frac{dQ^*}{dw}\right)(1+r^s) + \left(\frac{dQ^*}{dw}\right)\left(-\frac{c[(1+r^s)\bar{F}(g) - (1-\alpha)L^s(1+r^s)^2 f(g) - (1+rf)]}{[\bar{F}(g) - (1-\alpha)L^s(1+r^s)f(g)]}\right)$$

$$\frac{d[(cQ^* - K)(1+r^s)]}{dw} = +\left(\frac{dQ^*}{dw}\right)\left(\frac{c(1+r^s)[\bar{F}(g) - (1-\alpha)L^s(1+r^s)f(g)]}{[\bar{F}(g) - (1-\alpha)L^s(1+r^s)f(g)]} - \frac{c[(1+r^s)\bar{F}(g) - (1-\alpha)L^s(1+r^s)^2 f(g) - (1+rf)]}{[\bar{F}(g) - (1-\alpha)L^s(1+r^s)f(g)]}\right)$$

$$\frac{d[(cQ^* - K)(1+r^s)]}{dw} = \left(\frac{dQ^*}{dw}\right)\left(-\frac{c(1+rf)}{[\bar{F}(g) - (1-\alpha)L^s(1+r^s)f(g)]}\right)$$

Now we put it in the supplier's profit function:

$$\frac{d\pi^s}{dw} = \frac{d(wQ^*)}{dw}\bar{F}(wQ^* - K^r) - \left(\frac{dQ^*}{dw}\right)\left[\frac{c(1+rf)}{\bar{F}(g) - (1-\alpha)L^s g f(g)}\right]\bar{F}((cQ^* - K)(1+r^s))$$

There is exist a dependency between operation and financing decision.

Note that when $\alpha = 1$ the problem reduce to the case without market imperfection.

Chapitre 16. APPENDIX 6: Bankruptcy: variable cost; Optimal Wholesale price

16.1. Proof 6.1: Optimal Wholesale price: Supplier without constraint.

The supplier's profit function in the context of bankruptcy cost is given by:

$$\pi^s = [K^s + K^r - cQ](1 + rf) + E[L^r]$$

$$E[L^r] = L^r(1 + r^{tc}) \int_{b(Q)}^{\infty} f(x)\,dx + \alpha \int_0^{b(Q)} xf(x)\,dx$$

With $= b(Q) = (wQ - K^r)(1 + r^{tc}) = L^r(1 + r^{tc})$; $0 \leq \alpha \leq 1$, the bankruptcy recovery rate.

Recall the traditional problem when no bankruptcy risk

$$\frac{d\pi^s}{dw} = \frac{d(wQ^*)(1 + r)}{dw} \bar{F}(Q^*) - c\left(\frac{dQ^*}{dw}\right)$$

With $\left(\frac{dQ^*}{dw}\right) = -\frac{\frac{\partial G}{\partial w}}{\frac{\partial G}{\partial Q}}$; with $G = G(Q^*, w, r^{tc}, K^r) = \bar{F}(Q^*) - w(1 + r^{tc})\bar{F}(b(Q^*)) = 0$

For this demonstration we ignore the trade credit interest rate, we assume open account financing and $rf = 0$. Such that,

$$G(Q^*, w, K^r) = \bar{F}(Q^*) - w\bar{F}(b(Q^*)) = 0$$

$$b(Q) = (wQ - K)$$

$$\frac{db(Q^*)}{dw} = \frac{d(wQ^* - K)}{dw} = \frac{dwQ^*}{dw}$$

Here we have

$$\frac{d\pi^s}{dw} = \frac{d}{dw}[K^s + K^r - cQ(w)] + (wQ - K)\int_b^{\infty} f(x)\,dx + \alpha[\,xF(x)|_0^b - \int_0^b F(x)\,dx]$$

$$= \frac{d}{dw}[K^s + K^r - cQ(w)] + b\bar{F}(b) + \alpha[\,bF(b) - \int_0^b F(x)\,dx]$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - b\left(\frac{db}{dw}\right)f(b) + \alpha\left[\left(\frac{db}{dw}\right)F(b) + b\left(\frac{db}{dw}\right)f(b) - \left(\frac{db}{dw}\right)F(b)\right] - c\left(\frac{dQ(w)}{dw}\right)$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - b\left(\frac{db}{dw}\right)f(b) + \alpha\left[b\left(\frac{db}{dw}\right)f(b)\right] - c\left(\frac{dQ(w)}{dw}\right)$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - (1-\alpha)b\left(\frac{db}{dw}\right)f(b) - c\left(\frac{dQ(w)}{dw}\right)$$

$$= \left(\frac{dwQ}{dw}\right)\bar{F}(wQ-K) - (1-\alpha)(wQ-K)\left(\frac{dwQ}{dw}\right)f(wQ-K) - c\left(\frac{dQ}{dw}\right)$$

We know that

$$\frac{d\big(wQ^*(w)\big)}{dw} = Q^* + \frac{dQ^*}{dw}w = \frac{1-Qh(Q)}{-[h(Q)-wh(wQ-B)]}$$

$$\frac{dQ^*}{dw} = -\frac{\frac{dG(Q^*,w,B)}{dw}}{\frac{dG(Q^*,w,B)}{dQ^*}} = \frac{1-wQh(wQ-B)}{-w[h(Q^*)-wh(wQ-B)]}$$

The equation becomes

$$\left(\frac{1-Qh(Q)}{-[h(Q)-wh(b)]}\right)\bar{F}(b) - (1-\alpha)(b)\left(\frac{1-Qh(Q)}{-[h(Q)-wh(b)]}\right)f(b) - c\left(\frac{1-wQh(b)}{-w[h(Q^*)-wh(b)]}\right)$$

$$\frac{\left[w\big(1-Qh(Q)\big)\bar{F}(b)\right] - \left[(1-\alpha)wb\big(1-Qh(Q)\big)f(b)\right] - c\big(1-wQh(b)\big)}{-w[h(Q^*)-wh(b)]}$$

$$\frac{\left[\frac{\left[w\big(1-Qh(Q)\big)\bar{F}(b)\right] - \left[(1-\alpha)wb\big(1-Qh(Q)\big)f(b)\right]}{\big(1-wQh(b)\big)} - c\right]\big(1-wQh(b)\big)}{-w[h(Q^*)-wh(b)]}$$

$$\frac{\left[\frac{\left[w\big(1-H(Q)\big)\bar{F}(Q)\right] - \left[(1-\alpha)wb\big(1-H(Q)\big)f(b)\right]}{\big(1-wQh(b)\big)} - c\right]\big(1-wQh(b)\big)}{-w[h(Q^*)-wh(b)]}$$

$$\frac{\left[\big(1-H(Q)\big)\bar{F}(Q)\right] - \left[(1-\alpha)wb\big(1-H(Q)\big)f(b)\right]}{\big(1-wQh(b)\big)} = c$$

$$\frac{\left[\left(1-H(Q)\right)\bar{F}(Q)\right]}{\left(1-wQh(b)\right)}-\frac{\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]}{\left(1-wQh(b)\right)}=c$$

Assume $w0$ satisfy $\left(1-H(Q)\right)=0$, in the normal case, when $w<w0$ the function $\frac{\left[(1-H(Q))\bar{F}(Q)\right]}{(1-wQh(b))}=\varepsilon(w)<0$, since IFR function and $wQh(b)<Qh(b)<Qh(Q)$ so that if $\left(1-H(Q)\right)=0,\ H(Q)=1$, we have $\left(1-wQh(b)\right)>0$

Prove that:

$$\left[\left(1-H(Q)\right)\bar{F}(Q)\right]\geq\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]$$

$$\bar{F}(Q^{*})\geq(1-\alpha)wbf(b)$$

$$1=\frac{\bar{F}(Q^{*})}{\bar{F}(Q^{*})}\geq\frac{(1-\alpha)wbf(b)}{\bar{F}(Q^{*})}=\frac{(1-\alpha)wbf(b)}{w\bar{F}(b)}=(1-\alpha)bh(b)$$

$$(1-\alpha)bh(b)=(1-\alpha)(wQ-K)h(b)$$

$$wQh(b)<wQh(Q)=wH(Q)$$

Because IFR function are also IGFR.

Since $(1-\alpha)<1$, $wQh(b)<wQh(Q)$, $wQ>K$, we have that $1>(1-\alpha)bh(b)$ and as a consequence, the second term of the left-hand side of the equation is smaller that the first term:

$$\left[\left(1-H(Q)\right)\bar{F}(Q)\right]\geq\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]$$

$$\frac{\left[\left(1-H(Q)\right)\bar{F}(Q)\right]}{\left(1-wQh(b)\right)}-\frac{\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]}{\left(1-wQh(b)\right)}$$

When w increases, at the same time it decreases $\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]$ and increases $\left(1-wQh(b)\right)$

$\frac{\left[(1-H(Q))\bar{F}(Q)\right]}{(1-wQh(b))}$ is increasing in w, and $\frac{\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]}{(1-wQh(b))}$ increasing in w and $-\frac{\left[(1-\alpha)wb\left(1-H(Q)\right)f(b)\right]}{(1-wQh(b))}$ is decreasing in w. The second terms decreases the profit, for the same wholesale price. Which makes the supplier increasing furthermore the wholesale price.

For the same equality $\frac{[(1-H(Q))\bar{F}(Q)]}{(1-wQh(b))} - c = 0$ we now need a higher wholesale price to make $\frac{[(1-H(Q))\bar{F}(Q)]}{(1-wQh(b))} - \frac{[(1-\alpha)wb(1-H(Q))f(b)]}{(1-wQh(b))} = c$

-

The result seems logical, since the bankruptcy risk have increased, the supplier prefer now inducing the retailer to order less, such as to decrease its exposure to bankruptcy. Because now, the bankruptcy has become more costly.

-

If we derive with respect to α, we obtain $\frac{[wb(1-H(Q))f(b)]}{(1-wQh(b))}$ which show the profit is increasing in the recovery rate. The more the recovery rate, the less the effect of bankruptcy in the profit. The less the second term of the left hand side is significant. And for the same c, it is now necessary to give a lower wholesale price to find the optimal expected profit. The supplier does not need really more downside protection, he induce the retailer to order more via lower price.

-

16.2. **Proof 6.2: Optimal Wholesale price ; Supplier with constraint.**

The original problem is

$$g = L^s(1 + r^s) = (cQ^* - K^r - K^s)(1 + r^s)$$

$$b = L^r(1 + r^{tc})$$

$$g' = \frac{g}{\alpha} \neq \frac{dg}{dw}$$

$$\frac{dg'}{dw} = \frac{c}{\alpha}\left(\frac{dQ^*}{dw}\right) ; \frac{db}{dw} = \left(\frac{wQ^*}{dw}\right)$$

We assume set $r^s, r^{tc} = 0$ for this analysis.

When $K^s + K^r < cq$

$$E[\pi^s(Q^*,\alpha,w)] = -K^s - K^r + \left[p\int_0^{wQ-K^r} \bar{F}(x)dx - L^s(1+r^s)\right]^+$$

$$-K^s - K^r + p\int_{L^s}^{wQ-K^r} \bar{F}(x)dx$$

The problem was

$$\max -K^s - K^r + p\int_g^b \bar{F}(x)dx$$

$$L^s + K^r + K^s \geq cq$$

Now with bankruptcy costs we have.

$$E[\pi^s(Q^*,\alpha,w)] = -K^s - K^r + \left[L^r\int_b^\infty f(x)\,dx + \alpha\int_0^b xf(x)\,dx - L^s\right]^+$$

$$\max E[\pi^s(Q^*,\alpha,w)] = -K^s + [(L^r - L^s)\int_b^\infty f(x)\,dx + \alpha\int_{\frac{L^s}{\alpha}=g'}^b \left(x - \frac{L^s}{\alpha}\right) f(x)\,dx$$

$$\frac{dE[\pi(Q^*,\alpha,w)]}{dw} = \frac{d}{dw}\left[(L^r - L^s)\int_b^\infty f(x)\,dx + \alpha\int_{\frac{L^s}{\alpha}=g'}^b (x-g')f(x)\,dx\right] - K^s$$

$$L^r = b$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - b\left(\frac{db'}{dw}\right)f(b) - \left(\alpha\left(\frac{d(g')}{dw}\right)\bar{F}(b) - \alpha g'\left(\frac{dg'}{dw}\right)f(b)\right)$$

$$+ \frac{d}{dw}\left(\alpha\left[xF(x)|_{g'}^{b} - \int_{g'}^b F(x)\,dx\right]\right)$$

$$-\alpha\left(\left(\frac{dg'}{dw}\right)\int_{\frac{L^s}{\alpha}=g'}^b f(x)\,dx + g'\left[\left(\frac{db}{dw}\right)f(b) - \left(\frac{d(g')}{dw}\right)f(g')\right]\right)$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - b\left(\frac{db'}{dw}\right)f(b)$$

$$+ \alpha\left[\left(\left(\frac{db}{dw}\right)F(b) + \left(\frac{db}{dw}\right)bf(b) - \left(\frac{db}{dw}\right)F(b)\right)\right.$$

$$\left. - \left(\left(\frac{d(g')}{dw}\right)F(g') + \left(\frac{d(g)}{dw}\right)\ g'f(g') - \left(\frac{d(g')}{dw}\right)F(g')\right)\right]$$

$$- \left(\alpha\left(\frac{d(g')}{dw}\right)\bar{F}(b) - \alpha g'\left(\frac{dg'}{dw}\right)f(b)\right)$$

$$- \alpha\left(\left(\frac{d(g')}{dw}\right)F(b) - \left(\frac{d(g')}{dw}\right)F(g') + g'\left[\left(\frac{db}{dw}\right)f(b) - \left(\frac{d(g')}{dw}\right)f(g')\ \right]\right)$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - b\left(\frac{db'}{dw}\right)f(b) + \alpha\left[\left(\left(\frac{db}{dw}\right)bf(b)\right) - \right] - \left(\alpha\left(\frac{d(g')}{dw}\right)\bar{F}(b)\right)$$

$$- \alpha\left(\left(\frac{d(g')}{dw}\right)F(b) - \left(\frac{d(g')}{dw}\right)F(g')\right)$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - (1-\alpha)b\left(\frac{db}{dw}\right)f(b) - \alpha\left(-\left(\frac{d(g')}{dw}\right)F(g')\right) - \alpha\left(\frac{d(g')}{dw}\right)$$

$$= \left(\frac{db}{dw}\right)\bar{F}(b) - (1-\alpha)b\left(\frac{db}{dw}\right)f(b) - \alpha\left(\frac{d(g')}{dw}\right)\bar{F}(g')$$

Finaly, the problem becomes:

$$= \left(\frac{dwQ^*}{dw}\right)\bar{F}(b) - (1-\alpha)b\left(\frac{dwQ^*}{dw}\right)f(b) - \alpha c\left(\frac{dQ^*}{dw}\right)\bar{F}(g')$$

$$\left(\frac{1 - Qh(Q)}{-[h(Q) - wh(b)]}\right)\bar{F}(b) - (1-\alpha)b\left(\frac{1 - Qh(Q)}{-[h(Q) - wh(b)]}\right)f(b)$$

$$- \alpha c\left(\frac{1 - wQh(b)}{-w[h(Q^*) - wh(b)]}\right)\bar{F}(g')$$

$$\frac{\left[w\big(1 - Qh(Q)\big)\bar{F}(b)\right] - \left[(1-\alpha)(b)\big(1 - Qh(Q)\big)f(b)w\right] - c(1 - wQh(b))\alpha\,\bar{F}(g'))}{-w[h(Q^*) - wh(b)]}$$

$$\frac{\left[\frac{[w(1-Qh(Q))\bar{F}(b)]-[(1-\alpha)(b)(1-Qh(Q))f(b)w]}{(1-wQh(b))}-c(\alpha\,\bar{F}(g'))\right](1-wQh(b))}{-w[h(Q^{*})-wh(wQ-K)]}$$

$$\frac{\left[\frac{[w(1-H(Q))\bar{F}(b)]-[(1-\alpha)(b)(1-H(Q))f(b)w]}{(1-wQh(b))}-c(\alpha\,\bar{F}(g'))\right](1-wQh(b))}{-w[h(Q^{*})-wh(b)]}$$

$$\frac{[(1-H(Q))\bar{F}(Q)]-[(1-\alpha)wbf(b)(1-H(Q))]}{(1-wQh(b))}=c(\alpha\bar{F}(g'))$$

16.3. Proof 6.3 Concavity Optimal Wholesale Price : Case with bankruptcy costs.

From *Proof 3.3.1,* we know that the first term of the nominator is positive. We just need to prove that the second term $(1-\alpha)wbf(b)(1-H(Q))$ is positive as well but inferior.

$$\frac{d}{dw}\delta^{bc}(w)=\frac{d}{dw}\frac{[(1-H(Q))\bar{F}(Q)]-[(1-\alpha)wbf(b)(1-H(Q))]}{(1-wQh(b))}$$

Let derive the second term: $\frac{d}{dw}\frac{[(1-\alpha)wbf(b)(1-H(Q))]}{(1-wQh(b))}$

$$\frac{1}{y^{2}}\Big[(1-a)[(1-\alpha)wbf(b)(1-H(Q))]'y+y'[(1-\alpha)wbf(b)(1-H(Q))]\Big]$$

With $[(1-\alpha)wbf(b)(1-H(Q))]'=(1-a)[(wbf(b))'(1-H(Q))+wbf(b)(1-H(Q))'$

$$=(1-a)\Big[\big((wb)'f(b)+(wb(wQ)'f'(b))\big)(1-H(Q))+wbf(b)(-Q'H'(Q))\Big]$$

We have

$$\frac{1}{y^{2}}\Big[(1-a)\Big[\big((wb)'f(b)+(wb(wQ)'f'(b))\big)(1-H(Q))+wbf(b)(-Q'H'(Q))\Big]y$$
$$+\Big(-\big(-(wQ)'(h(b)+wQ(wQ)'h'(b))\big)[(1-\alpha)wbf(b)(1-H(Q))]\Big)\Big]$$

$$>\frac{1}{y^{2}}\Big[(1-a)\Big[\big((wb)'f(b)+(wb(wQ)'f'(b))\big)(1-H(Q))+wbf(b)(-Q'H'(Q))\Big]$$
$$+\Big(-\big(-(wQ)'(h(b)+wQ(wQ)'h'(b))\big)[(1-\alpha)wbf(b)(1-H(Q))]\Big)\Big]$$

$$= \frac{1}{y^2}\Big[(1-a)\Big[\Big((wb)'f(b) + \big(wb(wQ)'f'(b)\big)\Big)\big(1-H(Q)\big)\Big]$$

$$+ (1-a)wbf(b)\big(-Q'H'(Q)\big)$$

$$+ \Big(-\Big(-(wQ)'\big(h(b) + wQ(wQ)'h'(b)\big)\Big)\big[(1-\alpha)wbf(b)\big(1-H(Q)\big)\big]\Big)\Big]$$

$$= \frac{1}{y^2}\Big[(1-a)\Big[\Big((wb)'f(b) + \big(wb(wQ)'f'(b)\big)\Big)\big(1-H(Q)\big)\Big]$$

$$+ (1$$

$$- \alpha)wbf(b)\Big(\Big((wQ)'\big(h(b) + wQ(wQ)'h'(b)\big)\Big)\big[\big(1-H(Q)\big)\big]\big(-Q'H'(Q)\big)\Big)\Big]$$

The first part of the second term is also positive

$$> \frac{1}{y^2}\Big[(1-\alpha)wbf(b)\Big(\Big((wQ)'\big(h(b) + wQ(wQ)'h'(b)\big) - H'(Q)\Big)\big[\big(1-H(Q)\big)\big](Q')\Big)\Big]$$

$$= \frac{1}{y^2}\Big[(1-\alpha)wbf(b)\Big(\big(H'(b) + K^r h'(b) - H'(Q)\big)\big[\big(1-H(Q)\big)\big](Q')\Big)\Big]$$

$$> 0$$

Now, if we compare both term we obtain

$$\frac{1}{y^2}\big[\big(1-H(Q)\big)'y + \big(1-H(Q)\big)y' >$$

$$\frac{1}{y^2}\Big[\big[(1-\alpha)wbf(b)\big(1-H(Q)\big)\big]'y + y'\big[(1-\alpha)wbf(b)\big(1-H(Q)\big)\big]\Big]$$

$$= \frac{1}{y^2}\Big[(1-\alpha)\Big(\big[wbf(b)\big(1-H(Q)\big)\big]'y + y'\big[wbf(b)\big(1-H(Q)\big)\big]\Big)\Big]$$

$$(1-a)\Big[\Big((wb)'f(b) + \big(wb(wQ)'f'(b)\big)\Big)\big(1-H(Q)\big) + wbf(b)\big(-Q'H'(Q)\big)\Big]$$

$$\big(1-H(Q)\big)' = -Q'H'(Q)$$

All terms are positive. There still exist a concave relationship between the supplier's optimal wholesale price and its profit.

Chapitre 17. APPENDIX 7: Variance of profit: retailer

17.1. Proof 7.1: Classical Newsvendor variance of profit

The variance $Q^2\bar{F}(Q) + \int_0^Q x^2 f(x)dx - \left(\int_0^Q \bar{F}(x)dx\right)^2$ can be rewritten by some manipulation as: $2Q\int_0^Q F(x)dx - 2\int_0^Q xF(x)dx - \left(\int_0^Q F(x)dx\right)^2$ (proof 7.1.1)

$$\frac{\partial VaR(\pi^r)}{\partial q} = 2Q\int_0^Q F(x)dx - 2\int_0^Q xF(x)dx - \left(\int_0^Q F(x)dx\right)^2$$

For $\frac{\partial}{\partial q}\int_0^Q xF(x)dx$, we know that $\int \frac{d}{dx}\big(u(x)v(x)\big)dx = \int u'(x)v(x)dx + \int u(x)v'(x)dx$

Which gives us $\int_0^Q xF(x)dx = \int_0^Q F(x)dx + \int_0^Q xf(x)dx = \int_0^Q F(x)dx + xF(x)|_0^Q - \int_0^Q F(x)dx = QF(Q)$

$$2\int_0^Q F(x)dx + 2QF(Q) - [2QF(Q)] - 2\left(\int_0^Q F(x)dx\right)F(Q)$$

$$2\big(1 - F(Q)\big)\left(\int_0^Q F(x)dx\right) \geq 0$$

Increasing function

$$\frac{\partial^2 VaR(\pi^r)}{\partial^2 q} = 2\left(\big(-f(Q)\big)\left(\int_0^Q F(x)dx\right) + \big(-F(Q)\big)F(Q)\right)$$

$$-2\left(f(Q)\left(\int_0^Q F(x)dx\right) + \big(F(Q)\big)^2\right) < 0$$

Concave function

Furthermore from **Choi and Chui (2012),** the variance can be explore to the infinity as following.

$$VaR(\pi^r) = Q^2\bar{F}(Q) + \int_0^Q x^2 f(x)dx - \left(\int_0^Q \bar{F}(x)dx\right)^2$$

$$= Q^2\bar{F}(Q) + \int_0^Q x^2 f(x)dx - \left(q - \int_0^Q F(x)dx\right)^2$$

With $q - \int_0^Q F(x)dx = q - qF(q) + \int_0^Q xf(x)dx$

$$VaR(\pi^r) = Q^2\bar{F}(Q) + \int_0^Q x^2 f(x)dx - \left(q - qF(q) + \int_0^Q xf(x)dx\right)^2$$

We know furthermore that $lim_{Q\to\infty} \int_0^Q x^2 f(x)dx = E(x^2)$ and $lim_{Q\to\infty} \int_0^Q xf(x)dx = E(x)$ the the demand expectation. $lim_{Q\to\infty}\bar{F}(Q) = 0$ the demand distribution assumed to be bounded above, $lim_{Q\to\infty}Q\bar{F}(Q) = 0$ even if $lim_{Q\to\infty}Q = \infty$ since $\bar{F}(Q)$ converge faster. $lim_{Q\to\infty}F(Q) = 1$

In conclusion we have

$$lim_{Q\to\infty}Var[\pi^r(Q)] = 0 + E(x^2) - \big(q - q + E(x)\big)^2 = E(x^2) - E(x)^2 = v$$

17.2. **Proof 7.1.1: Retailer Variance Transformation.**

On the basis of **Choi and Chui (2012),** we know that $\pi^r(Q) = \min(x, Q) - cQ = (1-c)Q - (Q-x)^+$

$$Var(\min(\xi, Q) - cQ) = Var(\min(\xi, Q)) = Q^2\bar{F}(Q) + \int_0^Q x^2 f(x)dx - \left(\int_0^Q \bar{F}(x)dx\right)^2$$

$$= Var\big((1-c)Q - (Q-x)^+\big) = Var((Q-x)^+)$$

$$= \int_0^Q (Q-x)^2 dF(x)dx - \left(\int_0^Q (Q-x)dF(x)dx\right)^2$$

First, we have for the first term

$$\int_0^Q [Q^2 - 2xQ + x^2]dF(x)dx$$

$$= Q^2\int_0^Q f(x)dx - 2Q\int_0^Q xf(x)dx + \int_0^Q x^2 f(x)dx$$

$$= Q^2F(Q) - 2Q\left(QF(Q) - \int_0^Q F(x)dx\right) + Q^2F(Q) - 2\int_0^Q xF(x)dx$$

$$= 2Q\int_0^Q F(x)dx - 2\int_0^Q xF(x)dx$$

The second term,

$$\int_0^Q (Q-x)dF(x)dx$$

$$= QF(Q) - \int_0^Q xdF(x)dx$$

$$= QF(Q) - \left(xF(x)|_0^Q - \int_0^Q F(x)dx\right)$$

$$= \int_0^Q F(x)dx$$

Finally,

$$Var[\pi^r(Q)] = 2Q\int_0^Q F(x)dx - 2\int_0^Q xF(x)dx - \left(\int_0^Q F(x)dx\right)^2$$

17.3. Proof 7.2: Cash-constrained retailer Variance

$$B = wQ - K$$

$$E\ [\pi^r(q)] = \int_{wQ-K}^Q [x - (wq - K)]dF(x) + \int_Q^\infty [Q - (wq - K)]dF(x) - K$$

$$= -K + \int_{wQ-K}^Q \bar{F}(x)dx$$

$$E\ [\pi^r(q^2)] = \int_{wQ-K}^Q [x - (wq - K)]^2dF(x) + \int_Q^\infty [Q - (wq - K)]^2dF(x) - K$$

$$\int_B^Q [x - B]^2dF(x) = \int_B^Q [x^2 - 2xB + B^2]f(x)dx$$

With

$$\int_B^Q x^2 f(x)dx = x^2F(x)|_B^Q - 2\int_B^Q xF(x)dx$$

$$-2B\int_B^Q xf(x)dx = -2B\left(xF(x)|_B^Q\ - \int_B^Q F(x)dx\right)$$

$$B^2\int_B^Q f(x)dx = B^2F(x)|_B^Q$$

What gives us

$$x^2F(x)|_B^Q - 2\int_B^Q xF(x)dx - 2B\left(xF(x)|_B^Q\ - \int_B^Q F(x)dx\right) + B^2F(x)|_B^Q$$

$$Q^2F(Q) - B^2F(B) - 2\int_B^Q xF(x)dx - 2BQF(Q) + 2B^2F(B) + 2B\int_B^Q F(x)dx + B^2F(Q) - B^2F(B)$$

$$[Q^2 - 2BQ + B^2]F(Q) + [2B^2 - B^2 - B^2]F(B) + 2B\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx$$

$$[Q^2 - 2BQ + B^2]F(Q) + [2B^2 - B^2 - B^2]F(B) + 2B\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx$$

$$= [Q-B]^2F(Q) + 2B\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx$$

et

$$\int_Q^\infty [Q-B]^2 dF(x) = \int_Q^\infty [Q^2 - 2QB + B^2]f(x)dx = [Q-B]^2\bar{F}(Q)$$

$$E\ [\pi^r(q^2)] = [Q-B]^2 + 2B\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx$$

$$VaR\ [\pi^r(Q)] = [Q-B]^2 + 2B\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx - \left(\int_B^Q \bar{F}(x)dx\right)^2$$

Where $\left(\int_B^Q \bar{F}(x)dx\right) = \left((Q-B) - \int_B^Q F(x)dx\right)^2 = (Q-B)^2 - 2(Q-B)\int_B^Q F(x)dx + \left(\int_B^Q F(x)dx\right)^2$

That gives us

$$VaR\ [\pi^r(Q)] = [Q-B]^2 + 2B\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx$$
$$-\left((Q-B)^2 - 2(Q-B)\int_B^Q F(x)dx + \left(\int_B^Q F(x)dx\right)^2\right)$$

$$VaR\ [\pi^r(Q)] = 2Q\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx - \left(\int_B^Q F(x)dx\right)^2$$

17.4. Proof 7.3 limit $Q \to \infty$

$$\lim_{q\to\infty} VaR\ [\pi^r(q^2)] = 0$$

The variance of profit converge to zero when reaching the infinity since $\lim_{q\to\infty} wQ - k = \lim_{q\to\infty} Q$. Hence, $\lim_{q\to\infty}\left(Q-(wQ-k)\right) = 0$

However, we observe at first an increase in the variability and then it reach a maximum.

17.5. Proof 7.4. Limit $K \to 0$

$$\lim_{K\to wq} VaR\ [\pi^r(q^2)] = [Q-0]^2 + 2(0)\int_0^Q F(x)dx - 2\int_0^Q xF(x)dx - \left(\int_0^Q \bar{F}(x)dx\right)^2$$

$$= Q^2 - 2\int_0^Q xF(x)dx - \left(Q - \int_0^Q F(x)dx\right)^2$$

$$= Q^2 - 2\int_0^Q xF(x)dx - \left(Q^2 - 2Q\int_0^Q F(x)dx + \left(\int_0^Q F(x)dx\right)^2\right)$$

$$= 2Q\int_0^Q F(x)dx - 2\int_0^Q xF(x)dx - \left(\int_0^Q F(x)dx\right)^2$$

Which is equivalent to the variance of profit for the unconstrained newsvendor.

17.6. Proof 7.5 : Concavity of the variance function.

$$B = (wQ - K); \frac{\partial B}{\partial Q} = B' = w$$

$$VaR\,[\pi^r(Q)] = 2Q\int_B^Q F(x)dx - 2\int_B^Q xF(x)dx - \left(\int_B^Q F(x)dx\right)^2$$

$$\frac{\partial VaR(\pi^r)}{\partial Q} = 2\int_B^Q F(x)dx + 2Q[F(Q) - wF(B)] - 2[QF(Q) - wBF(B)] - 2\left[\left(\int_B^Q F(x)dx\right)\left(F(Q) - wF(B)\right)\right]$$

$$\frac{\partial VaR(\pi^r)}{\partial Q} = 2\int_B^Q F(x)dx - 2(Q - B)wF(B) - 2\left[\left(\int_B^Q F(x)dx\right)\left(F(Q) - wF(B)\right)\right]$$

$$\frac{\partial^2 VaR(\pi^r)}{\partial^2 Q} = 2[F(Q) - wF(B)] - 2\left[\left(wF(B) + Qw^2 f(B)\right) - \left(w^2F(B) + w^2Bf(B)\right)\right] - 2\left[\left(F(Q) - wF(B)\right)^2 + \left(\int_B^Q F(x)dx\right)\left(f(Q) - w^2 f(B)\right)\right]$$

We have

$$\frac{\partial^2 VaR(\pi^r)}{\partial Q^2} = 2[F(Q) - wF(B)] - 2[(1 - w)wF(B) + (Q - B)w^2 f(B)] - 2\left[\left(F(Q) - wF(B)\right)^2 + \left(\int_B^Q F(x)dx\right)(f(Q) - w^2 f(B)\right]$$

Prove that $\frac{\partial^2 VaR(\pi^r)}{\partial^2 Q} < 0$

$$2\left[\left(wF(B) + Qw^2 f(B)\right) - \left(w^2F(B) + w^2Bf(B)\right)\right] \geq 0$$

Because $Qw^2 f(B) > Bw^2 f(B)$, $wF(B) > w^2F(B)$ (because $w < 1$)

Then,

$$2\left[\left(F(Q) - wF(B)\right)^2 + \left(\int_B^Q F(x)dx\right)\left(f(Q) - w^2 f(B)\right)\right] \geq 0$$

if$\left(f(Q) - w^2 f(B)\right) \geq 0$

The function is always concave when

$$2[F(Q) - wF(B)] \leq 2\left[\left(wF(B) + Qw^2 f(B)\right) - \left(w^2 F(B) + w^2 B f(B)\right)\right] + 2\left[\left(F(Q) - wF(B)\right)^2 + \left(\int_B^Q F(x)dx\right)\left(f(Q) - w^2 f(B)\right)\right]$$

There exist a region where this inequality hold, the maximum variance can be found in this area.

Chapitre 18. APPENDIX 8 CONDITIONAL VAR

18.1. Proof 8.1 Classic Newsvendor

$$CVaR^\alpha = \frac{1}{\alpha}\left(\int_0^Q (x - cQ)dF(x) + (1-c)Q\int_Q^{F^{-1}\alpha} f(x)dx\right)$$

$$\frac{1}{\alpha}\left(\int_0^Q x dF(x) + cQF(Q) + Q[\alpha - F(Q)] - cQ[\alpha - F(Q)]\right)$$

$$\frac{1}{\alpha}\left(\int_0^Q x dF(x) + Q[\alpha - F(Q)] - cQ\alpha\right)$$

$$\frac{dCVaR^\alpha}{dQ} = \frac{1}{\alpha}\left(Qf(Q) + \alpha - [F(Q) + Qf(Q)] - c\alpha\right) = 0$$

$$\alpha - [F(Q)] - c\alpha = 0$$

$$\frac{dCVaR^\alpha}{dQ} = \alpha - [F(Q)] - c\alpha = 0$$

$$Q^* = F^{-1}(\alpha(1-c))$$

$$\frac{dCVaR^\alpha}{d\alpha} = (1-c)Q$$

18.2. Proof 8.2: Supplier problem (without loan) assuming risk neutral retailer.

$$\pi^s = b\int_b^{F^{-1}\alpha} f(x)dx + \int_0^b xdF(x) - cQ$$

Supplier profit maximization

$$\frac{d\pi^s}{dw} = \frac{d}{dw}\left(b\int_b^{F^{-1}\alpha} f(x)dx + \int_0^b xdF(x) - cQ\right)$$

$$\left(\frac{d(wQ^*)}{dw}\right)F(x)|_b^{F^{-1}(\alpha)} - b\left(\frac{d(wQ^*)}{dw}\right)f(b) + \left(\frac{d(wQ^*)}{dw}\right)bf(b) - c\left(\frac{dQ^*}{dw}\right)$$

$$\left(\frac{d(wQ^*)}{dw}\right)F(x)|_b^{F^{-1}(\alpha)} - c\left(\frac{dQ^*}{dw}\right)$$

$$\left(\frac{1-Q^*h(Q^*)}{-\big[h(Q^*)-wh\big(b(Q^*)\big)\big]}\right)F(x)|_b^{F^{-1}(\alpha)} - c(1+rf)\left(\frac{\big[1-wQh\big(b(Q^*)\big)\big]}{-w\big[h(Q^*)-wh\big(b(Q^*)\big)\big]}\right)$$

$$\frac{w(1-Q^*h(Q^*))F(x)|_b^{F^{-1}(\alpha)} - c(1+rf)\big(1-wQh(b(Q^*))\big)}{-w[h(Q^*)-wh(b(Q^*))]}$$

$$\frac{d\pi^s}{dw} = \frac{[1-wQ^*h(b(Q^*))]\left[\frac{\big(1-H(Q^*)\big)F(x)|_b^{F^{-1}(\alpha)}w}{1-wQ^*h(wQ^*-K^r)} - c(1+rf)\right]}{-w[h(Q^*)-wh(wQ-K^r)]}$$

$$F(x)|_b^{F^{-1}(\alpha)} = \alpha - F(b) = 1 - F(b) - (1-\alpha) = \bar{F}(b) - (1-\alpha)$$

$$\frac{d\pi^s}{dw} = \frac{[1-wQ^*h(b(Q^*))]\left[\frac{\big(1-H(Q^*)\big)\big(\bar{F}(Q)-w(1-\alpha)\big)}{1-wQ^*h(wQ^*-K^r)} - c(1+rf)\right]}{-w[h(Q^*)-wh(wQ-K^r)]}$$

Finally we have

$$\frac{\left(1-H(Q^*)\right)\left(\bar{F}(Q)-w(1-\alpha)\right)}{1-wQ^*h(wQ^*-K^r)}=c(1+rf)$$

18.3. Proof 8.3: Best response order quantity function risk averse retailer.

The retailer's Conditional-Value-at-Risk Objective function:

$$CVaR^{\alpha}=\frac{1}{\alpha}\left(\int_L^Q (x-L)dF(x)+(Q-L)\int_Q^{F^{-1}\alpha} f(x)dx\right)$$

$$CVaR^{\alpha}=\frac{1}{\alpha}\left(\int_L^Q xdF(x)-LF(x)|_L^Q+Q[\alpha-F(Q)]-L[\alpha-F(Q)]\right)$$

$$CVaR^{\alpha}=\frac{1}{\alpha}\left(\int_L^Q xdF(x)-L\left(F(Q)-F(L)\right)+Q[\alpha-F(Q)]-L[\alpha-F(Q)]\right)$$

$$CVaR^{\alpha}=\frac{1}{\alpha}\left(\int_L^Q xdF(x)+LF(L)-QF(Q)+(Q-L)\alpha\right)$$

The problem becomes :

$$\frac{dCVaR^{\alpha}}{dQ}=\frac{1}{\alpha}\frac{d}{dQ}\left(\int_L^Q xdF(x)+LF(L)-QF(Q)+(Q-L)\alpha\right)$$

We know,

$$\frac{d}{dQ}\int_L^Q xdF(x)=\frac{d}{dQ}\left(xF(x)|_L^Q-\int_L^Q F(x)dx\right)=Qf(Q)-wLf(L)$$

Then

$$=\frac{1}{\alpha}\left(Qf(Q)-wLf(L)+wF(L)+wLf(L)-\left(F(Q)+Qf(Q)\right)+(1-w)\alpha\right)$$

$$=\frac{1}{\alpha}(wF(L)-F(Q)+(1-w)\alpha)$$

FOC

$$F(Q)-wF(L)=(1-w)\alpha$$

$$\alpha-F(Q)=w\left(\alpha-F(L)\right)$$

$$\bar{F}(Q) - (1-\alpha) = w\big(\bar{F}(L) - (1-\alpha)\big)$$

$$\bar{F}(Q) - w\bar{F}(L) - (1-w)(1-\alpha) = 0$$

$$\bar{F}(Q) = w\bar{F}(L) + (1-w)(1-\alpha)$$

18.4. Proof 8.4: Risk-neutral supplier problem with risk-averse (CVaR

$$G(Q^*, w) = \bar{F}(Q) - w\bar{F}(L) - (1-w)(1-\alpha) = 0$$

Same as before :

$$\frac{\partial G(Q^*, w)}{\partial Q^*} = -\left[f(Q^*) - w^2 f\big(b(Q^*)\big)\right]$$

$$\frac{\partial G(Q^*, w)}{\partial w} = -\left[\bar{F}\big(b(Q^*)\big) - wQf\big(b(Q^*)\big)\right] + (1-a)$$

$$= -\Big(\left[\bar{F}\big(b(Q^*)\big) - wQf\big(b(Q^*)\big)\right] - (1-a)\Big)$$

$$\frac{dQ^*}{dw} = \frac{\left[\bar{F}\big(b(Q^*)\big) - wQf\big(b(Q^*)\big)\right] - (1-a)}{-\left[f(Q^*) - w^2 f\big(b(Q^*)\big)\right]}$$

$$\frac{dQ^*}{dw} = \frac{\left[1 - wQh\big(b(Q^*)\big)\right] - \frac{1-a}{\bar{F}\big(b(Q^*)\big)}}{-\left[\frac{f(Q^*)}{\bar{F}\big(b(Q^*)\big)} - w^2 h\big(b(Q^*)\big)\right]}$$

Recall: $w\bar{F}\big(b(Q^*)\big) = \bar{F}(Q) - (1-w)(1-\alpha)$

$$\frac{dQ^*}{dw} = \frac{\left[1 - wQh\big(b(Q^*)\big)\right] - \frac{1-a}{\bar{F}\big(b(Q^*)\big)}}{-\left[\frac{f(Q^*)}{\frac{\bar{F}(Q) - (1-w)(1-\alpha)}{w}} - w^2 h\big(b(Q^*)\big)\right]}$$

$$\frac{dQ^*}{dw} = \frac{\left[1 - wQh\big(b(Q^*)\big)\right] - \frac{1-a}{\bar{F}\big(b(Q^*)\big)}}{-w\left[\frac{f(Q^*)}{\bar{F}(Q) - (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

The revenue,

$$\frac{dwQ^*}{dw} = Q^* + w\left(\frac{dQ^*}{dw}\right)$$

$$\frac{dwQ^*}{dw} = \frac{\left(\left[1 - wQh\big(b(Q^*)\big)\right] - \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right) - Q\left[\frac{f(Q^*)}{w\bar{F}\big(b(Q^*)\big)} - wf\big(b(Q^*)\big)\right]}{-\left[\frac{f(Q^*)}{\bar{F}(Q) - (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

$$\frac{\left(\left[1 - wQh\big(b(Q^*)\big)\right] - \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right) - Q\left[\frac{f(Q^*)}{w\bar{F}\big(b(Q^*)\big)} - wh\big(b(Q^*)\big)\right]}{-\left[\frac{f(Q^*)}{\bar{F}(Q) - (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

$$\frac{\left(1 - \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right) - Q\left[\frac{f(Q^*)}{w\bar{F}\big(b(Q^*)\big)}\right]}{-\left[\frac{f(Q^*)}{\bar{F}(Q) - (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

$$\frac{dwQ^*}{dw} = \frac{\left(1 - \left[\frac{Qf(Q^*) + w(1-a)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)}{-\left[\frac{f(Q^*)}{\bar{F}(Q) - (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

$$= \frac{\left(\left[\frac{\bar{F}(Q) - (1-a)(1-w) - Qf(Q^*) - w(1-a)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)}{-\left[\frac{f(Q^*)}{\bar{F}(Q) - (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

$$= \frac{\left(\left[\frac{\bar{F}(Q) - (1-a) - Qf(Q^*)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)}{-\left[\frac{f(Q^*)}{w\bar{F}(b) + (1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}$$

The supplier problem

$$\frac{d\pi^s}{dw} = \left(\frac{dwQ^*}{dw}\right)\bar{F}(b(Q^*)) - c\left(\frac{dQ^*}{dw}\right)$$

$$\left(\frac{\left(\left[\frac{\bar{F}(Q)-(1-a)-Qf(Q^*)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)}{-\left[\frac{f(Q^*)}{\bar{F}(Q)-(1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}\right)\bar{F}(b(Q^*))$$

$$- c\left(\frac{\left[1-wQh\big(b(Q^*)\big)\right] + \frac{1-a}{\bar{F}\big(b(Q^*)\big)}}{-w\left[\frac{f(Q^*)}{\bar{F}(Q)-(1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}\right)$$

$$\left(\frac{\left(\left[\frac{\bar{F}(Q)-(1-a)-Qf(Q^*)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)w\bar{F}(b(Q^*)) - c\left(\left[1-wQh\big(b(Q^*)\big)\right] + \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right)}{-w\left[\frac{f(Q^*)}{\bar{F}(Q)-(1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}\right)$$

$$\left(\frac{\left[\frac{\left(\left(\left[\frac{\bar{F}(Q)-(1-a)-Qf(Q^*)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)\right)w\bar{F}(b(Q^*))}{\left(\left[1-wQh\big(b(Q^*)\big)\right] + \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right)} - c\right]\left(\left[1-wQh\big(b(Q^*)\big)\right] + \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right)}{-w\left[\frac{f(Q^*)}{\bar{F}(Q)-(1-w)(1-\alpha)} - wh\big(b(Q^*)\big)\right]}\right)$$

$$\frac{\left(\left[\frac{\bar{F}(Q)-(1-a)-Qf(Q^*)}{w\bar{F}\big(b(Q^*)\big)}\right]\right)w\bar{F}(b(Q^*))}{\left(\left[1-wQh\big(b(Q^*)\big)\right] + \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right)} - c$$

$$\frac{(\bar{F}(Q)-(1-a)-Qh(Q^*)wF(Q^*))}{\left(\left[1-wQh\big(b(Q^*)\big)\right] + \frac{1-a}{\bar{F}\big(b(Q^*)\big)}\right)} - c$$

$$\frac{(\bar{F}(Q)[1-H(Q^*)]-(1-a))}{\left(\left[1-wQh\big(b(Q^*)\big)\right] + (1-a)\frac{1}{\bar{F}\big(b(Q^*)\big)}\right)} - c$$

$$\frac{(\bar{F}(Q)[1-H(Q^*)]-(1-a))}{\left(\frac{\bar{F}\left(b(Q^*)\right)\left[1-wQh\left(b(Q^*)\right)\right]+(1-a)}{\bar{F}\left(b(Q^*)\right)}\right)}-c$$

$$\frac{\bar{F}\left(b(Q^*)\right)(\bar{F}(Q)[1-H(Q^*)]-(1-a))}{\left(\bar{F}\left(b(Q^*)\right)\left[1-wQh\left(b(Q^*)\right)\right]+(1-a)\right)}-c$$

$$\frac{\bar{F}(b(Q^*))(\bar{F}(Q)[1-H(Q^*)]-(1-a))}{\left(\bar{F}\left(b(Q^*)\right)\left[1-wQh\left(b(Q^*)\right)\right]+(1-a)\right)}-c$$

18.5. <u>Proof 8.5: Sign of the best response order quantities in the context of conditional CVaR.</u>

We verify that $\frac{dQ^*}{dw}<0$

$$\frac{dQ^*}{dw}=-\frac{-\left[1-wQh\left(b(Q^*)\right)\right]+\frac{1-a}{\bar{F}\left(b(Q^*)\right)}}{-w\left[\frac{f(Q^*)}{\bar{F}(Q)-(1-w)(1-\alpha)}-wh\left(b(Q^*)\right)\right]}$$

By the increasing failure rate assumption, we know that $h(Q)>h(b)>wh(b)$. Also, it is easy to show that $h(Q)=\frac{f(Q^*)}{\bar{F}(Q^*)}\leq\frac{f(Q^*)}{\bar{F}(Q^*)-(1-w)(1-\alpha)}$ as long as $\bar{F}(Q)-(1-w)(1-\alpha)\geq 0$ which is garanteed by the first order conditions $\bar{F}(Q)-(1-w)(1-\alpha)=w\bar{F}\left(b(Q^*)\right)$.

Hence, from $h(Q^*)-wh\left(b(Q^*)\right)\geq 0$ we have that $\frac{f(Q^*)}{\bar{F}(Q)-(1-w)(1-\alpha)}-wh\left(b(Q^*)\right)\geq 0$

Then, in order to prove that $\frac{dQ^*}{dw}$ is still >0 in the case of CVaR criteria, we need to prove that $-\left[1-wQh\left(b(Q^*)\right)\right]+\frac{1-a}{\bar{F}\left(b(Q^*)\right)}<0$

$$-\left[1-wQh\left(b(Q^*)\right)\right]+\frac{1-a}{\bar{F}\left(b(Q^*)\right)}<0$$

$$\frac{-\left[\bar{F}\left(b(Q^*)\right)-wQf\left(b(Q^*)\right)\right]+(1-a)}{\bar{F}\left(b(Q^*)\right)}<0$$

$$-(1-F(b)-wQf\left(b(Q^*)\right)+(1-a)<0$$

$F(b)-wQf\left(b(Q^*)\right)-a<0$dd

Which is true since $a > F(b)$

Indeed

$$G(Q^*, w) = \bar{F}(Q) - w\bar{F}(L) - (1 - w)(1 - \alpha) = 0$$

Rewritten

$$G(Q^*, w) = 1 - F(Q) - w\big(1 - F(b)\big) - (1 - w)(1 - a)$$

$$1 - F(Q) - w + wF(b) - 1 + a + w - aw = 0$$

$$-F(Q) + wF(b) + a - aw = 0$$

$$G(Q^*, w) = F(Q) - wF(b) - (1 - w)a = 0$$

Which gives us

$$a = \frac{F(Q) - wF(b)}{1 - w} >= \frac{F(b) - wF(b)}{1 - w} = \frac{(1 - w)F(b)}{1 - w} = F(b)$$

Chapitre 19. APPENDIX BIS

19.1. Appendix Bis 1

VBA Function: Simpson Approximation for integration (Source)

```
Function IntegraleSimpson(f As String, a As Double, b As Double, n As Integer) As Double

 Dim h As Double

 Dim k As Integer

 Dim ak0 As Double, ak1 As Double

 Dim i As Double

 Dim f0 As Double, f1 As Double, f2 As Double



 h = (b - a) / n

 ak0 = a
```

```
f0 = Evaluate("+" & Replace(Replace(f, "xxx", a), ",", "."))

For k = 1 To n

ak1 = a + k * h

f1 = Evaluate("+" & Replace(Replace(f, "xxx", Replace(ak0 + h / 2, ",", ".")), ",", "."))

f2 = Evaluate("+" & Replace(Replace(f, "xxx", Replace(ak1, ",", ".")), ",", "."))

i = i + f0 + 4 * f1 + f2

f0 = f2

ak0 = ak1

Next

IntegraleSimpson = i * h / 6

End Function
```

19.2. Appendix Bis 2 Variance Concavity

Sensitivity analysis of the variance of profit for the capital constrained retailer.

(There exist a bias resulting from the Simpson approximation for integration. We assume that 395.51 means 400, the variance of the demand.)

K=0	sd=0,2	μ=50	Normal Dist															
w	0,90		0,80		0,70		0,60		0,50		0,40		0,30		0,20		0,10	
Q	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance
-			-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5,00	0,00	0,00	0,00	0,01	0,00	0,02	0,00	0,04	0,00	0,06	0,01	0,08	0,01	0,11	0,01	0,14	0,01	0,17
10,00	0,00	0,02	0,00	0,08	0,00	0,16	0,01	0,26	0,01	0,37	0,01	0,50	0,01	0,63	0,01	0,76	0,01	0,88
15,00	-0,00	0,08	-0,00	0,28	-0,00	0,56	-0,00	0,90	-0,00	1,26	0,00	1,63	0,00	1,99	0,00	2,33	0,00	2,65
20,00	-0,01	0,22	-0,02	0,77	-0,03	1,53	-0,05	2,38	-0,06	3,26	-0,06	4,11	-0,07	4,90	-0,07	5,61	-0,08	6,23
25,00	-0,03	0,51	-0,09	1,79	-0,15	3,47	-0,21	5,32	-0,26	7,15	-0,29	8,86	-0,31	10,36	-0,33	11,64	-0,34	12,69
30,00	-0,09	1,04	-0,27	3,59	-0,48	6,92	-0,66	10,49	-0,79	13,93	-0,89	17,00	-0,96	19,61	-1,00	21,72	-1,03	23,37
35,00	-0,21	1,87	-0,67	6,45	-1,18	12,39	-1,63	18,68	-1,98	24,60	-2,22	29,75	-2,37	33,96	-2,46	37,22	-2,52	39,63
40,00	-0,43	3,01	-1,38	10,46	-2,46	20,17	-3,42	30,40	-4,16	39,92	-4,66	48,02	-4,98	54,44	-5,16	59,22	-5,25	62,59
45,00	-0,70	4,38	-2,38	15,43	-4,37	30,08	-6,18	45,59	-7,59	59,98	-8,54	72,05	-9,11	81,38	-9,42	88,09	-9,58	92,60
50,00	-0,96	5,78	-3,47	20,84	-6,66	41,32	-9,73	63,36	-12,14	83,91	-13,78	101,04	-14,75	114,05	-15,25	123,13	-15,49	128,98
55,00	-1,05	6,95	-4,22	25,85	-8,72	52,49	-13,35	81,95	-17,15	109,80	-19,75	133,08	-21,27	150,54	-22,03	162,44	-22,36	169,82
60,00	-0,82	7,66	-4,09	29,54	-9,61	61,90	-15,90	99,03	-21,39	135,00	-25,26	165,36	-27,50	188,04	-28,59	203,19	-29,03	212,28
65,00	-0,19	7,74	-2,66	31,20	-8,42	67,98	-16,12	112,25	-23,46	156,62	-28,86	194,79	-32,02	223,38	-33,51	242,21	-34,07	253,17
70,00	0,75	7,20	0,11	30,57	-4,69	69,80	-13,10	119,85	-22,21	172,29	-29,36	218,67	-33,64	253,75	-35,62	276,65	-36,32	289,65
75,00	1,80	6,19	3,80	27,87	1,33	67,23	-6,69	121,06	-17,22	180,61	-26,27	235,23	-31,87	277,26	-34,44	304,63	-35,29	319,81
80,00	2,69	4,94	7,66	23,72	8,73	61,00	2,41	116,21	-8,92	181,36	-19,90	243,89	-27,06	293,19	-30,35	325,38	-31,39	342,91
85,00	3,24	3,67	10,87	18,93	16,19	52,33	12,90	106,50	1,62	175,36	-11,23	245,12	-20,19	301,88	-24,36	339,24	-25,63	359,27
90,00	3,35	2,54	12,86	14,21	22,46	42,63	23,30	93,59	13,08	164,07	-1,44	240,09	-12,45	304,36	-17,70	347,23	-19,25	369,95

95,00	3,09	1,65	13,40	10,07	26,69	33,11	32,35	79,22	24,29	149,24	8,44	230,30	-4,85	301,99	-11,41	350,68	-13,32	376,29
100,00	2,58	1,01	12,67	6,76	28,55	24,61	39,23	64,84	34,39	132,49	17,75	217,20	2,00	296,08	-6,12	350,91	-8,47	379,60
105,00	1,97	0,58	11,04	4,31	28,22	17,56	43,61	51,48	42,95	115,16	26,23	202,04	7,91	287,73	-2,02	348,97	-4,91	380,97
110,00	1,40	0,32	8,97	2,62	26,17	12,08	45,53	39,78	49,79	98,25	33,87	185,77	13,01	277,74	0,99	345,64	-2,53	381,17
115,00	0,92	0,16	6,85	1,52	22,99	8,02	45,27	29,98	54,92	82,42	40,80	169,09	17,56	266,67	3,21	341,42	-1,05	380,71
120,00	0,57	0,08	4,94	0,85	19,27	5,16	43,25	22,07	58,40	68,06	47,15	152,47	21,84	254,87	4,93	336,60	-0,18	379,89
125,00	0,33	0,04	3,38	0,45	15,49	3,22	39,92	15,90	60,30	55,37	52,98	136,28	26,07	242,57	6,40	331,36	0,31	378,87
130,00	0,18	0,02	2,20	0,23	11,97	1,96	35,72	11,21	60,74	44,40	58,28	120,75	30,38	229,93	7,79	325,76	0,60	377,74
135,00	0,09	0,01	1,37	0,12	8,92	1,15	31,06	7,75	59,81	35,10	62,97	106,06	34,84	217,06	9,22	319,85	0,80	376,54
140,00	0,04	0,00	0,81	0,05	6,42	0,66	26,28	5,26	57,68	27,36	66,97	92,36	39,46	204,06	10,75	313,65	0,97	375,27
145,00	0,02	0,00	0,46	0,03	4,46	0,37	21,66	3,49	54,51	21,03	70,17	79,72	44,21	191,02	12,41	307,18	1,12	373,94
150,00	0,01	0,00	0,25	0,01	3,00	0,20	17,39	2,28	50,53	15,94	72,48	68,21	49,05	178,05	14,22	300,43	1,28	372,55

K=30 sd=0,2 μ=50 Normal Dist

w	0,90		0,80		0,70		0,60		0,50		0,40		0,30		0,20		0,10	
Q	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5,00	0,00	0,20	0,00	0,20	0,00	0,20	0,00	0,20	0,01	0,20	0,01	0,20	0,01	0,20	0,01	0,20	0,01	0,20
10,00	0,00	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01
15,00	-0,01	2,94	-0,01	2,94	-0,01	2,94	-0,01	2,94	-0,00	2,94	-0,00	2,94	-0,00	2,94	-0,00	2,94	0,00	2,94
20,00	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76
25,00	-0,34	13,54	-0,34	13,54	-0,34	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54
30,00	-1,00	24,61	-1,01	24,61	-1,02	24,61	-1,03	24,61	-1,04	24,61	-1,04	24,61	-1,04	24,61	-1,05	24,61	-1,05	24,61
35,00	-2,42	40,69	-2,47	41,34	-2,49	41,34	-2,50	41,34	-2,52	41,34	-2,53	41,34	-2,54	41,34	-2,54	41,34	-2,55	41,34
40,00	-4,83	61,06	-5,09	63,83	-5,19	64,84	-5,22	64,84	-5,24	64,84	-5,27	64,84	-5,28	64,84	-5,29	64,84	-5,30	64,84
45,00	-8,35	86,12	-9,03	91,31	-9,40	94,66	-9,51	95,46	-9,55	95,46	-9,59	95,46	-9,62	95,46	-9,64	95,46	-9,65	95,46
50,00	-12,47	114,05	-14,05	123,13	-14,92	128,98	-15,35	132,49	-15,42	132,49	-15,48	132,49	-15,53	132,49	-15,56	132,49	-15,58	132,49
55,00	-15,92	141,75	-19,17	156,59	-21,01	166,27	-21,93	172,05	-22,24	174,02	-22,33	174,02	-22,40	174,02	-22,45	174,02	-22,48	174,02
60,00	-16,71	165,36	-22,68	188,04	-26,23	203,19	-28,03	212,28	-28,83	217,20	-28,96	217,20	-29,06	217,20	-29,13	217,20	-29,17	217,20
65,00	-12,72	181,13	-22,55	213,59	-28,79	236,03	-32,08	249,72	-33,55	257,13	-33,93	258,84	-34,07	258,84	-34,16	258,84	-34,22	258,84
70,00	-2,50	186,47	-17,03	229,96	-27,11	261,48	-32,71	281,24	-35,25	292,01	-36,08	296,08	-36,26	296,08	-36,39	296,08	-36,47	296,08
75,00	13,91	180,61	-5,27	235,23	-20,27	277,26	-29,20	304,63	-33,38	319,81	-34,95	327,01	-35,18	327,01	-35,35	327,01	-35,45	327,01
80,00	34,71	164,81	12,32	229,18	-8,26	282,45	-21,63	318,88	-28,21	339,63	-30,71	349,52	-31,21	350,91	-31,42	350,91	-31,55	350,91
85,00	56,72	141,88	34,13	213,17	8,18	277,42	-10,73	324,10	-20,62	351,64	-24,47	364,93	-25,37	368,09	-25,63	368,09	-25,79	368,09
90,00	76,20	115,41	57,74	189,70	27,80	263,53	2,57	321,16	-11,68	356,74	-17,42	374,24	-18,91	379,60	-19,23	379,60	-19,42	379,60
95,00	89,98	88,85	80,40	161,81	49,21	242,74	17,46	311,36	-2,24	356,10	-10,57	378,72	-12,90	386,81	-13,27	386,81	-13,50	386,81
100,00	96,18	64,84	99,54	132,49	71,01	217,20	33,41	296,08	7,24	350,91	-4,46	379,60	-7,98	391,02	-8,41	391,02	-8,67	391,02
105,00	94,59	44,92	113,17	104,25	91,88	189,03	50,16	276,60	16,75	342,12	0,77	377,89	-4,16	392,31	-4,85	393,33	-5,15	393,33

110,00	86,47	29,60	120,11	78,91	110,53	160,10	67,53	254,09	26,59	330,44	5,33	374,27	-1,44	392,27	-2,46	394,51	-2,81	394,51
115,00	73,94	18,58	120,12	57,50	125,75	131,99	85,27	229,54	37,11	316,37	9,61	369,18	0,49	391,40	-1,00	395,08	-1,39	395,08
120,00	59,43	11,12	113,86	40,36	136,51	105,92	102,91	203,89	48,65	300,26	14,00	362,82	1,93	389,95	-0,15	395,34	-0,59	395,34
125,00	45,04	6,36	102,67	27,31	142,07	82,72	119,76	177,99	61,38	282,41	18,85	355,28	3,16	388,07	0,32	395,44	-0,17	395,44
130,00	32,26	3,48	88,28	17,82	142,16	62,86	134,96	152,61	75,28	263,09	24,43	346,60	4,39	385,80	0,59	395,49	0,04	395,49
135,00	21,88	1,82	72,53	11,23	137,01	46,48	147,60	128,45	90,15	242,63	30,93	336,77	5,76	383,13	0,74	395,50	0,15	395,50
140,00	14,06	0,91	57,00	6,83	127,35	33,43	156,86	106,07	105,62	221,37	38,48	325,78	7,37	380,06	0,86	395,51	0,20	395,51
145,00	8,57	0,44	42,89	4,01	114,26	23,40	162,13	85,90	121,18	199,71	47,13	313,66	9,28	376,54	0,95	395,51	0,23	395,51
150,00	4,95	0,20	30,92	2,28	99,04	15,94	163,09	68,21	136,25	178,05	56,89	300,43	11,56	372,55	1,04	395,51	0,26	395,51

K=50	sd=0,2	μ=50	Normal Dist															
w	0,9		0,8		0,7		0,6		0,5		0,4		0,3		0,2		0,1	
Q	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance	SOD	Variance
0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	0,00	0,20	0,00	0,20	0,00	0,20	0,00	0,20	0,01	0,20	0,01	0,20	0,01	0,20	0,01	0,20	0,01	0,20
10	0,00	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01	0,01	1,01
15	-0,01	2,94	-0,01	2,94	-0,01	2,94	-0,01	2,94	-0,00	2,94	-0,00	2,94	-0,00	2,94	-0,00	2,94	0,00	2,94
20	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76	-0,08	6,76
25	-0,34	13,54	-0,34	13,54	-0,34	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54	-0,35	13,54
30	-1,00	24,61	-1,01	24,61	-1,02	24,61	-1,03	24,61	-1,04	24,61	-1,04	24,61	-1,04	24,61	-1,05	24,61	-1,05	24,61
35	-2,45	41,34	-2,47	41,34	-2,49	41,34	-2,50	41,34	-2,52	41,34	-2,53	41,34	-2,54	41,34	-2,54	41,34	-2,55	41,34
40	-5,11	64,84	-5,15	64,84	-5,19	64,84	-5,22	64,84	-5,24	64,84	-5,27	64,84	-5,28	64,84	-5,29	64,84	-5,30	64,84
45	-9,33	95,46	-9,39	95,46	-9,46	95,46	-9,51	95,46	-9,55	95,46	-9,59	95,46	-9,62	95,46	-9,64	95,46	-9,65	95,46
50	-15,06	132,49	-15,17	132,49	-15,27	132,49	-15,35	132,49	-15,42	132,49	-15,48	132,49	-15,53	132,49	-15,56	132,49	-15,58	132,49
55	-21,69	174,02	-21,86	174,02	-22,01	174,02	-22,13	174,02	-22,24	174,02	-22,33	174,02	-22,40	174,02	-22,45	174,02	-22,48	174,02
60	-27,26	214,29	-28,28	217,20	-28,49	217,20	-28,67	217,20	-28,83	217,20	-28,96	217,20	-29,06	217,20	-29,13	217,20	-29,17	217,20
65	-29,88	250,48	-32,62	257,51	-33,29	258,84	-33,54	258,84	-33,76	258,84	-33,93	258,84	-34,07	258,84	-34,16	258,84	-34,22	258,84
70	-27,54	279,04	-33,04	290,89	-35,23	296,08	-35,56	296,08	-35,85	296,08	-36,08	296,08	-36,26	296,08	-36,39	296,08	-36,47	296,08
75	-18,60	297,05	-28,71	315,85	-33,23	325,24	-34,28	327,01	-34,65	327,01	-34,95	327,01	-35,18	327,01	-35,35	327,01	-35,45	327,01
80	-2,21	302,83	-19,23	330,95	-27,35	345,59	-30,07	350,91	-30,53	350,91	-30,91	350,91	-31,21	350,91	-31,42	350,91	-31,55	350,91
85	21,63	296,17	-4,70	335,84	-18,32	357,62	-23,72	367,43	-24,54	368,09	-25,01	368,09	-25,37	368,09	-25,63	368,09	-25,79	368,09
90	51,90	278,13	14,59	330,95	-6,81	361,93	-15,82	376,44	-17,91	379,60	-18,48	379,60	-18,91	379,60	-19,23	379,60	-19,42	379,60
95	86,60	250,86	38,29	317,27	6,78	359,40	-7,52	380,15	-11,71	386,81	-12,38	386,81	-12,90	386,81	-13,27	386,81	-13,50	386,81
100	122,66	217,20	65,98	296,08	22,50	350,91	0,88	379,60	-6,59	391,02	-7,37	391,02	-7,98	391,02	-8,41	391,02	-8,67	391,02
105	156,21	180,38	96,85	268,92	40,75	337,20	9,58	375,60	-1,84	391,52	-3,64	393,33	-4,34	393,33	-4,85	393,33	-5,15	393,33
110	183,11	143,52	129,46	237,49	62,00	318,88	19,16	368,61	2,27	390,29	-1,09	394,51	-1,89	394,51	-2,46	394,51	-2,81	394,51
115	199,77	109,29	161,57	203,66	86,48	296,55	30,35	358,87	6,22	387,71	0,56	395,08	-0,35	395,08	-1,00	395,08	-1,39	395,08

120	204,10	79,57	190,36	169,32	113,91	270,84	43,84	346,45	10,51	383,90	1,61	395,34	0,58	395,34	-0,15	395,34	-0,59	395,34
125	195,97	55,37	212,81	136,28	143,32	242,57	60,19	331,36	15,65	378,87	2,28	395,44	1,14	395,44	0,32	395,44	-0,17	395,44
130	177,29	36,82	226,38	106,05	173,10	212,71	79,67	313,63	22,09	372,53	3,56	394,08	1,49	395,49	0,59	395,49	0,04	395,49
135	151,39	23,40	229,53	79,71	201,16	182,36	102,21	293,43	30,21	364,73	5,11	392,33	1,74	395,50	0,74	395,50	0,15	395,50
140	122,22	14,21	222,09	57,84	225,17	152,63	127,38	271,01	40,33	355,35	7,09	390,13	1,96	395,51	0,86	395,51	0,20	395,51
145	93,38	8,26	205,28	40,50	242,99	124,58	154,36	246,81	52,72	344,27	9,63	387,40	2,15	395,51	0,95	395,51	0,23	395,51
150	67,60	4,60	181,43	27,36	252,93	99,09	181,97	221,38	67,54	331,42	12,88	384,08	2,35	395,51	1,04	395,51	0,26	395,51

19.3. Appendix Bis 3 Comparaison Optimum Variance and Expected Value

K=0	sd=0,2	μ=50	Normal Dist															
w	0,90		0,80		0,70		0,60		0,50		0,40		0,30		0,20		0,10	
Q																		
-	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation
5,00	0,02	0,98	0,08	1,96	0,16	2,94	0,26	3,93	0,37	4,91	0,50	5,90	0,63	6,89	0,76	7,88	0,88	8,88
10,00	0,22	1,88	0,77	3,78	1,53	5,70	2,38	7,63	3,26	9,58	4,11	11,54	4,90	13,51	5,61	15,49	6,23	17,47
15,00	1,04	2,58	3,59	5,24	6,92	7,99	10,49	10,80	13,93	13,66	17,00	16,55	19,61	19,48	21,72	22,43	23,37	25,40
20,00	3,01	2,90	10,46	6,05	20,17	9,42	30,40	12,95	39,92	16,63	48,02	20,41	54,44	24,27	59,22	28,17	62,59	32,12
25,00	5,78	2,75	20,84	5,98	41,32	9,64	63,36	13,69	83,91	18,03	101,04	22,61	114,05	27,34	123,13	32,19	128,98	37,11
30,00	7,66	2,18	29,54	5,06	61,90	8,65	99,03	12,90	135,00	17,71	165,36	22,95	188,04	28,51	203,19	34,27	212,28	40,14
35,00	7,20	1,44	30,57	3,67	69,80	6,82	119,85	10,94	172,29	15,96	218,67	21,71	253,75	27,99	276,65	34,62	289,65	41,45
40,00	4,94	0,79	23,72	2,27	61,00	4,75	116,21	8,43	181,36	13,37	243,89	19,42	293,19	26,32	325,38	33,78	342,91	41,54
45,00	2,54	0,35	14,21	1,20	42,63	2,94	93,59	5,97	164,07	10,56	240,09	16,69	304,36	24,07	347,23	32,30	369,95	40,98
50,00	1,01	0,13	6,76	0,55	24,61	1,63	64,84	3,92	132,49	7,94	217,20	13,92	296,08	21,63	350,91	30,55	379,60	40,13
55,00	0,32	0,04	2,62	0,21	12,08	0,81	39,78	2,40	98,25	5,72	185,77	11,33	277,74	19,19	345,64	28,73	381,17	39,19
60,00	0,08	0,01	0,85	0,07	5,16	0,36	22,07	1,37	68,06	3,95	152,47	9,02	254,87	16,86	336,60	26,91	379,89	38,22
65,00	0,02	0,00	0,23	0,02	1,96	0,15	11,21	0,73	44,40	2,62	120,75	7,02	229,93	14,66	325,76	25,12	377,74	37,25

70,00	0,00	0,00	0,05	0,01	0,66	0,05	5,26	0,37	27,36	1,67	92,36	5,34	204,06	12,61	313,65	23,37	375,27	36,29
75,00	0,00	0,00	0,01	0,00	0,20	0,02	2,28	0,17	15,94	1,01	68,21	3,96	178,05	10,73	300,43	21,67	372,55	35,32
80,00	0,00	0,00	0,00	0,00	0,06	0,01	0,92	0,07	8,80	0,59	48,61	2,86	152,63	9,02	286,19	20,01	369,60	34,37
85,00	0,00	0,00	0,00	0,00	0,01	0,00	0,34	0,03	4,60	0,32	33,43	2,01	128,45	7,49	271,01	18,40	366,42	33,41
90,00	0,00	0,00	0,00	0,00	0,00	0,00	0,12	0,01	2,28	0,17	22,19	1,37	106,07	6,14	255,04	16,86	362,99	32,46
95,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,00	1,07	0,08	14,22	0,91	85,90	4,96	238,44	15,37	359,30	31,52
100,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,48	0,04	8,80	0,59	68,21	3,96	221,38	13,96	355,35	30,59
105,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,20	0,02	5,26	0,37	53,09	3,11	204,06	12,61	351,13	29,66
110,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,08	0,01	3,04	0,22	40,50	2,40	186,69	11,34	346,63	28,73
115,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,00	1,70	0,13	30,28	1,83	169,48	10,14	341,84	27,82
120,00	0,00	-	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,92	0,07	22,19	1,37	152,63	9,02	336,78	26,91
125,00	-	-	-	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,48	0,04	15,94	1,01	136,34	7,98	331,42	26,01
130,00	-0,00	-	0,00	-	0,00	0,00	0,00	0,00	0,00	0,00	0,24	0,02	11,23	0,73	120,77	7,02	325,78	25,12
135,00	-	-	-	-	0,00	0,00	0,00	0,00	0,00	0,00	0,12	0,01	7,76	0,52	106,07	6,14	319,86	24,24
140,00	0,00	-	-0,00	-	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,01	5,26	0,37	92,36	5,34	313,66	23,37
145,00	0,00	-	0,00	-	-	0,00	0,00	0,00	0,00	0,00	0,03	0,00	3,49	0,25	79,72	4,61	307,18	22,51
150,00	-	-	-0,00	-	-	-	0,00	0,00	0,00	0,00	0,01	0,00	2,28	0,17	68,21	3,96	300,44	21,67

K=30	sd=0,2	μ=50	Normal Dist																
w	0,90		0,80		0,70		0,60		0,50		0,40		0,30		0,20		0,10		
Q	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
5,00	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	
10,00	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	
15,00	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	
20,00	61,06	30,14	63,83	34,10	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	
25,00	114,05	27,34	123,13	32,19	128,98	37,11	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	
30,00	165,36	22,95	188,04	28,51	203,19	34,27	212,28	40,14	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08	
35,00	186,47	17,53	229,96	23,46	261,48	29,86	281,24	36,55	292,01	43,42	296,08	48,37	296,08	48,37	296,08	48,37	296,08	48,37	
40,00	164,81	12,02	229,18	17,82	282,45	24,54	318,88	31,88	339,63	39,58	349,52	47,47	350,91	49,45	350,91	49,45	350,91	49,45	
45,00	115,41	7,32	189,70	12,44	263,53	19,03	321,16	26,74	356,74	35,15	374,24	43,93	379,60	49,87	379,60	49,87	379,60	49,87	
50,00	64,84	3,92	132,49	7,94	217,20	13,92	296,08	21,63	350,91	30,55	379,60	40,13	391,02	50,00	391,02	50,00	391,02	50,00	
55,00	29,60	1,82	78,91	4,60	160,10	9,56	254,09	16,85	330,44	26,00	374,27	36,28	392,27	47,06	394,51	50,03	394,51	50,03	
60,00	11,12	0,73	40,36	2,40	105,92	6,14	203,89	12,61	300,26	21,67	362,82	32,46	389,95	44,10	395,34	50,04	395,34	50,04	
65,00	3,48	0,25	17,82	1,12	62,86	3,66	152,61	9,02	263,09	17,62	346,60	28,73	385,80	41,15	395,49	50,04	395,49	50,04	
70,00	0,91	0,07	6,83	0,46	33,43	2,01	106,07	6,14	221,37	13,96	325,78	25,12	380,06	38,22	395,51	50,04	395,51	50,04	
75,00	0,20	0,02	2,28	0,17	15,94	1,01	68,21	3,96	178,05	10,73	300,43	21,67	372,55	35,32	395,51	50,04	395,51	50,04	
80,00	0,04	0,00	0,66	0,05	6,83	0,46	40,50	2,40	136,34	7,98	271,01	18,40	362,99	32,46	394,11	48,05	395,51	50,04	
85,00	0,01	0,00	0,17	0,02	2,63	0,19	22,19	1,37	99,09	5,73	238,44	15,37	351,13	29,66	392,34	46,07	395,51	50,04	
90,00	0,00	0,00	0,04	0,00	0,92	0,07	11,23	0,73	68,21	3,96	204,06	12,61	336,78	26,91	390,13	44,10	395,51	50,04	

95,00	0,00	0,00	0,01	0,00	0,29	0,02	5,26	0,37	44,42	2,62	169,48	10,14	319,86	24,24	387,40	42,13	395,51	50,04
100,00	0,00	0,00	0,00	0,00	0,08	0,01	2,28	0,17	27,36	1,67	136,34	7,98	300,44	21,67	384,08	40,17	395,51	50,04
105,00	0,00	0,00	0,00	0,00	0,02	0,00	0,92	0,07	15,94	1,01	106,07	6,14	278,71	19,20	380,06	38,22	395,51	50,04
110,00	0,00	0,00	0,00	0,00	0,00	0,00	0,34	0,03	8,80	0,59	79,72	4,61	255,04	16,86	375,27	36,29	395,51	50,04
115,00	0,00	0,00	0,00	0,00	0,00	0,00	0,12	0,01	4,60	0,32	57,84	3,37	229,95	14,66	369,60	34,37	395,51	50,04
120,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,00	2,28	0,17	40,50	2,40	204,06	12,61	362,99	32,46	395,51	50,04
125,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00	1,07	0,08	27,36	1,67	178,05	10,73	355,35	30,59	395,51	50,04
130,00	-	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,48	0,04	17,84	1,12	152,63	9,02	346,63	28,73	395,51	50,04
135,00	-	-	0,00	0,00	0,00	0,00	0,00	0,00	0,20	0,02	11,23	0,73	128,45	7,49	336,78	26,91	395,51	50,04
140,00	0,00	-	0,00	0,00	0,00	0,00	0,00	0,00	0,08	0,01	6,83	0,46	106,07	6,14	325,78	25,12	395,51	50,04
145,00	0,00	-	-	0,00	0,00	0,00	0,00	0,00	0,03	0,00	4,01	0,29	85,90	4,96	313,66	23,37	395,51	50,04
150,00	-0,00	-	-	-	0,00	0,00	0,00	0,00	0,01	0,00	2,28	0,17	68,21	3,96	300,44	21,67	395,51	50,04

K=50 sd=0,2 μ=50 Normal Dist

w	0,9		0,8		0,7		0,6		0,5		0,4		0,3		0,2		0,1	
Q	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation	Variance	Expectation
0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87	1,01	9,87
10	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45	6,76	19,45
15	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37	24,61	28,37
20	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08	64,84	36,08
25	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06	132,49	42,06
30	214,29	42,12	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08	217,20	46,08
35	279,04	35,59	290,89	42,43	296,08	48,37	296,08	48,37	296,08	48,37	296,08	48,37	296,08	48,37	296,08	48,37	296,08	48,37
40	302,83	28,15	330,95	35,70	345,59	43,51	350,91	49,45	350,91	49,45	350,91	49,45	350,91	49,45	350,91	49,45	350,91	49,45
45	278,13	20,66	330,95	28,56	361,93	37,08	376,44	45,90	379,60	49,87	379,60	49,87	379,60	49,87	379,60	49,87	379,60	49,87
50	217,20	13,92	296,08	21,63	350,91	30,55	379,60	40,13	391,02	50,00	391,02	50,00	391,02	50,00	391,02	50,00	391,02	50,00
55	143,52	8,48	237,49	15,37	318,88	24,23	368,61	34,36	390,29	45,08	394,51	50,03	394,51	50,03	394,51	50,03	394,51	50,03
60	79,57	4,61	169,32	10,14	270,84	18,40	346,45	28,73	383,90	40,17	395,34	50,04	395,34	50,04	395,34	50,04	395,34	50,04
65	36,82	2,20	106,05	6,14	212,71	13,27	313,63	23,37	372,53	35,32	394,08	48,05	395,49	50,04	395,49	50,04	395,49	50,04
70	14,21	0,91	57,84	3,37	152,63	9,02	271,01	18,40	355,35	30,59	390,13	44,10	395,51	50,04	395,51	50,04	395,51	50,04
75	4,60	0,32	27,36	1,67	99,09	5,73	221,38	13,96	331,42	26,01	384,08	40,17	395,51	50,04	395,51	50,04	395,51	50,04
80	1,25	0,10	11,23	0,73	57,84	3,37	169,48	10,14	300,44	21,67	375,27	36,29	395,51	50,04	395,51	50,04	395,51	50,04
85	0,29	0,02	4,01	0,29	30,28	1,83	120,77	7,02	263,12	17,62	362,99	32,46	394,85	49,05	395,51	50,04	395,51	50,04

90	0,06	0,01	1,25	0,10	14,22	0,91	79,72	4,61	221,38	13,96	346,63	28,73	392,34	46,07	395,51	50,04	395,51	50,04
95	0,01	0,00	0,34	0,03	6,00	0,41	48,61	2,86	178,05	10,73	325,78	25,12	388,84	43,11	395,51	50,04	395,51	50,04
100	0,00	0,00	0,08	0,01	2,28	0,17	27,36	1,67	136,34	7,98	300,44	21,67	384,08	40,17	395,51	50,04	395,51	50,04
105	0,00	0,00	0,02	0,00	0,78	0,06	14,22	0,91	99,09	5,73	271,01	18,40	377,77	37,25	395,51	50,04	395,51	50,04
110	0,00	0,00	0,00	0,00	0,24	0,02	6,83	0,46	68,21	3,96	238,44	15,37	369,60	34,37	395,51	50,04	395,51	50,04
115	0,00	0,00	0,00	0,00	0,07	0,01	3,04	0,22	44,42	2,62	204,06	12,61	359,30	31,52	395,51	50,04	395,51	50,04
120	0,00	0,00	0,00	0,00	0,02	0,00	1,25	0,10	27,36	1,67	169,48	10,14	346,63	28,73	395,51	50,04	395,51	50,04
125	0,00	0,00	0,00	0,00	0,00	0,00	0,48	0,04	15,94	1,01	136,34	7,98	331,42	26,01	395,51	50,04	395,51	50,04
130	0,00	0,00	0,00	0,00	0,00	0,00	0,17	0,02	8,80	0,59	106,07	6,14	313,66	23,37	394,11	48,05	395,51	50,04
135	-	0,00	0,00	0,00	0,00	0,00	0,06	0,01	4,60	0,32	79,72	4,61	293,43	20,83	392,34	46,07	395,51	50,04
140	-	0,00	0,00	0,00	0,00	0,00	0,02	0,00	2,28	0,17	57,84	3,37	271,01	18,40	390,13	44,10	395,51	50,04
145	-0,00	-	0,00	0,00	0,00	0,00	0,00	0,00	1,07	0,08	40,50	2,40	246,81	16,11	387,40	42,13	395,51	50,04
150	-0,00	-	0,00	0,00	0,00	0,00	0,00	0,00	0,48	0,04	27,36	1,67	221,38	13,96	384,08	40,17	395,51	50,04

19.4. Appendix Bis 4

Coefficient of Variation of Profit vs Wholesale Price (Retailer)

Coefficient of Variation of Profit vs Initial Capital (Retailer)

19.5. Appendix Bis 5 : Litterature Review

1.5.19 Appendix Bis 5.1

1 Trade Credit Insurance Peter M. Jones

BENEFITS

• It transfers the payment risk to the trade credit insurers, whose credit expertise, diversification of risk and financial strength enable them to assume these risks;

• It provides insured suppliers with access to professional credit risk expertise and related advice;

• It can help prevent insured suppliers from suffering liquidity shortages or insolvency due to delayed or non-payments;

• It reduces earnings volatility of insured suppliers by protecting a significant portion of their assets against risk of loss;

• It facilitates the access by insured suppliers to receivables financing and improved credit terms from lending institutions, some of which will insist on trade credit insurance before providing financing;

• It enables insured suppliers to extend credit to customers rather than requiring payment in advance or on delivery, or requiring security such as a letter of credit, thus allowing the supplier to effectively compete in a global marketplace where many buyers only buy on credit; and

• Allows insured suppliers to move up the value chain and accept direct buyer risk, thus cutting out the wholesaler or auction house.

Jones, P. (2010). *Trade Credit Insurance* (Primer Series on Insurance issue 15). Worldbank, Washington DC

BIBLIOGRAPHY

[i] Schmidt, W., & Raman, A. (2012). *When supply-chain disruptions matter*. Working paper, Harvard Business School, Boston.

[ii] Aydin, G., Babich, V., Beil, D., & Yang, Z. (2012). Decentralized supply risk management. In Kouvelis, P., Dong, L., Boyabalti, O & Li, R. (Eds.) *Handbook of Integrated Risk Management* (pp.389-424). Hoboken: Wiley & Sons.

[iii] Pinto, J., Henry, E., Robinson, T., & Stowe, J.D. (2010). Equity asset valuation (2nd ed.). Hoboken, New Jersey: Wiley & Sons.

[iv] Hendricks, K., & Singhal, V. (2003). An empirical analysis of the effect of supply chain disruptions on Long-run stock price performance and equity risk of the firm. *Production and Operation Management 14*(1), 35-52.

[v] Malatesta, P.H., & Thompson, R. (1984). Partially anticipated events: A model of stock price reaction with application to corporate acquisitions. *Journal of Financial Economics, 14*(2), 237- 250.

[vi] Hendricks, K., & Singhal, V. (2012). The effect of supply chain disruption on corporate performance. In Kouvelis, P., Dong, L., Boyabalti, O & Li, R. (Eds.) *Handbook of Integrated Risk Management* (pp.51-78). Hoboken: Wiley & Sons.

[vii] Pandit, S., Wasley, C., & Zach, T. (2011). Information externalities along the supply chain. *Contemporary Accounting Research, 28*(4), 1304-1343.

[viii] Escaith, H., & Gonguet, F. (2011). International supply chain as a real transmission channels of financial shock. *Journal of Financial Transformation, 3*(31), 83-97.

[ix] Kiyotaki, N., & Moore, J.H. (1997). Credit cycle. *Journal of Political Economy. 105*(2), 211-248.

[x] Cardella, L. (2012). *Credit risk and inter-firm Dependence* (Doctoral dissertation) The University of Arizona. Tucson. Retrieved from http://arizona.openrepository.com/arizona/handle/10150/228116

[xi] Hertzel, M., Officer, M., Li, Zhi., & Rodger C, K. (2006). Inter-firm linkages and the wealth effects of financial distress along the supply chain. *Journal of Financial Economics, 87*(2), 374-387.

[xii] Boissay, F. (2006) Credit chains and the propagation of financial distress. *ECB Working Paper series no. 537*

[xiii] Kerle, P. (2010). Securing growth and supply chain finance. *Credit Control, 31*(5/6), 26-31

[xiv] Tanrisever, F., Cetinay, H., Reindorp, M., & Fransoo, J. (2012). Value of reverse factoring in multi-stage supply chains. Available at SSRN: http://ssrn.com/abstract=2183991

[xv] Seifert, R., & Seifert, D. (2009). Supply chain finance – What's it worth? Retrieved from https://www.imd.org/research/publications/upload/PFM178_LR_Ralf_Daniel_Seifert.pdf

[xvi] Hofmann, E. & Belin, O. (2011). *Supply chain finance solution.* Berlin: Éd. Springer.

[xvii] Clayman, M.R., Fridson, S.F., & Troughton, G.H. (2012). Coporate finance: A practical approach (2nd Ed.). Hoboken, New Jersey: Wiley & Sons.

[xviii] Howorth, CA., & Westhead, P. (2003). The focus of working capital management in the UK small firms. *Management Accounting Research,14,* 94-111.

[xix] Farris, M., & Hutchison, P. (2003). Measuring cash-to-cash performance. *The international Journal of Logistics Management, 14*(2), 83-92.

[xx] Randal, W., & Farris, M. (2009). Supply chain financing : using cash-to-cash variable to strengthen the supply chain. *International Journal of Physical Distribution & Logistics Management,39*(8), 699-689.

[xxi] Lambert, D., & Pohlen, T. (2001). Supply chain metrics. *The international Journal of Logistics Management, 12*(1), 1-19.

[xxii] Cachon, G.P. (2003). Supply chain coordination with contracts. *Handbook for Operations Research and Management Sciences, 11*, 227-339
[xxiii] Ding, D., & Chen, J. (2008). Coordinating a three level supply chain with flexible return policies. *Omega, 36*(5), 865-876.
[xxiv] Buzacott, J., Zhang, R. (2004). Inventory management with asset-based financing. *Management Science 50*(9), 1274-1292.
[xxv] Kouvelis, P., & Zhao, W. (2013). *Supply chain coordination with financial constraints and bankruptcy costs*. Working paper, Washington University, St Louis.

[xxvi] Kouvelis, P. & Zhao, W. (2012). Financing the Newsvendor: Supplier vs. Bank, and the Structure of Optimal Trade Credit Contracts. *Operations Research, 60*(3), 566-580.

[xxvii] Fabbri, D., & Klapper, L. (2008*). Market power and the matching of trade credit terms*. Working Paper, World Bank, Washington DC.

[xxviii] Yan, N., & Sun, B. (2013). Coordinating loan strategies for supply chain financing with limited credit. *Operation Research Spectrum, 35*. 1039-1058.
[xxix] Lee, C., & Rhee, B-D. (2010). Coordination contract in the presence of positive inventory financing costs. *International Journal of Production Economics. 124*(2), 331-339.
[xxx] Modigliani, F. & Miller, M. (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review, 48*(3), 261-297.
[xxxi] Miller, M. (1977). Debt and taxes. *The Journal of Finance, 32*(2). 261-275.
[xxxii] Kouvelis, P., & Zhao, W. (2012). Supply chain finance. In Kouvelis, P., Dong, L., Boyabalti, O & Li, R. (Eds.) *Handbook of Integrated Risk Management* (pp.249-288). Hoboken: Wiley & Sons.
[xxxiii] Xu, X., & Birge, J. (2004). *Joint production and financing decisions: Modelling and analysis*. Working Paper, Evanston Illinois: Northwestern University.
[xxxiv] Ang, J., Chua, J, & McConnell, J. (1982) The administrative costs of corporate bankruptcy: A note. *The Journal of Finance, 37*(1), 219-226.
[xxxv] Harris, M. & Raviv, A. (1991). The theory of capital structure. *The Journal of Finance, 46*(1), 297-355.
[xxxvi] Frank, M., & Goyal, V. (2003). Testing the pecking order theory of capital structure. *Journal of Financial Economics, 67*(2). 217-248.
[xxxvii] Kraus, A., & Litzenberger, R. (1973). A state-preference model of optimal financial leverage. *Journal of Finance, 28*(4), 911-922.
[xxxviii] Elkahmi, R., Ericsson, J., & Parsons, C. (2012). The cost and timing of financial distress. *Journal of Financial Economics, 105*(1), 62-81.
[xxxix] Rajan G., & Zingales, L. (1998). Financial dependence and growth. *American Economic Association, 88*(3), 559-586.

[xl] Camerinelli, E. (2014). A study of the business case for supply chain finance. Retrieved from http://www.accaglobal.com/content/dam/acca/global/PDF-technical/small-business/pol-tp-asitbc.pdf
[xli] Yang, S.A., & Birge, J.R. (2013).*How inventory is (should be) financed: Trade credit in supply chains with demand uncertainty and costs of financial distress*. Working paper, retrieved from http://ssrn.com/paper=1734682.

[xlii] Giannetti, M., Burkart, M., & Ellingsen, T. (2011). What you sell is what you lend? Explaining trade credit contracts. *Review of Financial Studies 24*, 1261-1298.
[xliii] Zhou, J. (2009). *Essays in Interfaces of Operations and Finance in Supply Chains* (Doctoral dissertation). University of Rochester, New York.
[xliv] Ellis, S. C., Henry, R. M., & Shockley, J. (2010). Buyer perceptions of supply disruption risk: A behavioral view and empirical assessment. *Journal of Operations Management, 28*(1), pp. 34-46.

[xlv] Klapper, L., & Fabbri, D. (2009). Trade credit and the supply chain. Unpublished working paper. University of Amsterdam.

[xlvi] Raghavan N.R, & Mishra, V.K. (2011). Short-term financing in a cash-constrained supply chain.

[xlvii] Murfin J, & Njoroge, K. (2012). *Small lending big: The real effects of trade credit on constrained suppliers*. Working paper. Yale University, New Haven.

[xlviii] Chod, J. (2014). *Agency theory of trade credit*. Working paper. Boston College, Boston. Retrieved from http://www.researchgate.net/publication/260943778_Agency_Theory_of_Trade_Credit

[xlix] Birge, J., & Xu, X. (2008). operational decisions, capital structure, and managerial compensation: a news vendor perspective. *The engineering economist, 53*(3), 173-196.

[l] Chen, X., & Wang. (2012). Trade Credit Contract with limited liability in the supply chain with budget constraints. *Annals of Operations Research, 196*(1), 153-165.

[li] Lee, C., & Rhee, B-D. (2011). Trade credit for supply chain coordination. *European Journal of Operational Research, 214*(1), 136-146.

[lii]Bing, J., & Seidmann, A. (2014). Finance sourcing in the supply chain. *Decision Support Systems, 58*, 15-20.

[liii] Caldentey, R., & Chen, X. (2012). The role of financial services in procurement contracts. In Kouvelis, P., Dong, L., Boyabalti, O & Li, R. (Eds.) *Handbook of Integrated Risk Management* (pp.289-326). Hoboken: Wiley & Sons.

[liv] Jones, P. (2010). *Trade Credit Insurance* (Primer Series on Insurance issue 15). Worldbank, Washington DC.

[lv] Morel, F. (2010). Credit insurance in support to international trade: Observation throughout the crisis. In Chauffour, J-P & Malouche M (Eds*), Trade finance during the great collapse* (pp. 337-355). Washington DC: Worldbank.

[lvi] Li, Y., Zhen, X., & Cai, X. (2014). Trade credit insurance, capital constraint, and the behavior of manufacturer and banks. *Annals of Operations Research.* doi: 10.1007/s10479-014-1602-x

[lvii] Soufani, K. (2002). The Decision to Finance Accounts Receivable: The Factoring Option. *Managerial & Decision Economics, 23*(1), 21-32.

[lviii] Supply Chain Finance Working Group. (2010). *Supply chain finance*. Retrieved from https://www.treasurers.org/scf

[lix] Sopranzetti, B. (1998) The economics of factoring accounts receivable. *Journal of Economics and Business*, 50(4), 339-359.

[lx] Summers, B., & Wilson, N. (2000). Trade Credit Management and the Decision to Use Factoring: An empirical Study.

[lxi] Longhofer, S., & Santos, J. (2000). The Importance of Bank Seniority for Relationship Lending. *Journal of Financial Intermediation, 9*(1), 57-89.

[lxii] Klapper, L. (2006). The role of factoring for financing small and medium enterprises. *Journal of Banking & Finance, 30*, 3111-3130.

[lxiii] Demica.(2009).Strenghtening the Links. *Demica Report Series (10).*Retrieved from http://www.demica.com/images/PDFs/supply_chain_finance_strengthening.pdf

[lxiv] Lariviere, M.A. (2006). A note on probability distributions with increasing generalized failure rates. *Operations Research, 54*(3). 602-604.

[lxv] Banciu, M., & Mirchandani, P. (2013) New results concerning probability distributions with increasing generalized failure rates. *Operations Research 63*(2), 925-931.

[lxvi] Gupta, R.C, & Lvin, S. (2005) Reliability functions of generalized log-normal model. *Mathematical and Computer Modelling, 42*(9-10), 939-946.

[lxvii]Choi, T-M., & Chiu, C-H. (2012). Risk Analysis in Stochastic Supply Chains. New York: Springer.

[lxviii] Jammernegg, W., & Kischka, P. (2012) Newsvendor problems with VaR and CVaR Consideration. In Choi, T-M. (Ed.)*Handbook of Newsvendor problem* (pp. 197-216). New York: Springer Science.