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Forecasting Belgian and Norwegian CPI Inflation with Commodity Indexes

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Summary

This paper examines whether the inclusion of several commodity indexes in multivariate models could improve the forecast of CPI inflation for Belgium and Norway. The results from the multivariate forecast models are compared to three different univariate models: an AR(1), an Atkeson & Ohanian model, and an other autoregressive model with specific lags chosen.

The second objective of this thesis, is to assess if the economic profile of a country would provide better forecasting results. In this case, due to the importance of commodities for the Norwegian economy (almost 70% of their total exports), would the forecast results be better than those of Belgium ?

The variable forecasted is the consumer price index inflation while the commodity indexes originate from two sources: the Commodity Research Bureau (CRB) and the International Monetary Fund (IMF). In total, four commodity indexes are chosen: two generic indexes and two sub-indexes. The data set ranges from January 1992 till May 2015. The forecasts are performed at four different time horizons.

The root mean square error (RMSE) is the main indicator used to compare the results. In some case (for Belgium), the multivariate models with the commodity indexes provide better results than the univariate benchmark models. However, those results are not significant enough to affirm that the inclusion of commodity indexes provides more accurate forecasting results than univariate models. Finally, the fact that commodities represent an important share of the total export of Norway does not seem to provide better forecasting results.

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1 Introduction

1.1 The importance of forecasting inflation

Maintaining price stability is the primary objective of most central banks. For the European Central Bank this means keeping the “inflation rates of below but close to, two percent over the medium term”. (European Central Bank, n.d.)[1] The recent financial crisis and global recession, which followed, have led to important upward and downward risks likely to endanger this stability.

On the one hand, the upward factors can include the inflation pressures that could arise if key interest rates remained at such low level for too long, if the emergency measures were abandoned too slowly or if public finances were not consolidated on time. On the other hand, the downward factors can include deflationary pressures produced by prolonged output gaps (Cunningham, Desroches, & Santor, 2010).

Mastering these various risks for price stability is crucial for the central banks. In this regards anticipation in inflation may reveal valuable information regarding the normalization of their monetary policy. Furthermore in order to achieve their price stability, it is important for central banks to contain inflation expectations through their interventions. Consequently, forecasting inflation is necessary for central banks, so that their monetary policy can depend not only on the current level of inflation but also on its anticipated value. As it is confirmed by Ben Bernanke several times during his speeches: “ ...the control of inflation is central to good monetary policy. Price stability, which is one leg of the Federal Reserve’s dual mandate from the Congress, is a good thing in itself.” (Bernanke, 2007), therefore: “ ...this activity [inflation forecasting at the Federal Reserve] provides critical inputs into the making of monetary policy” (Bernanke, 2007).

We are far from the double-digit inflation rate that ruled during the 1970s. In Belgium,

[1] European Central Bank. *Monetary Policy*. https://www.ecb.europa.eu/mopo/html/index.en.html (Consulted on 04/05/2015).

the inflation rate was above the 10% threshold during April 1974 and February 1976. While Norway experienced some spikes in inflation rate during the the 1970s and early 1980s. Figure 1 illustrates those erratic changes in CPI inflation for Belgium and Norway. The CPI inflation rates of the ECB as well as the one of the U.S. are also depicted in the graphic for comparison purpose.

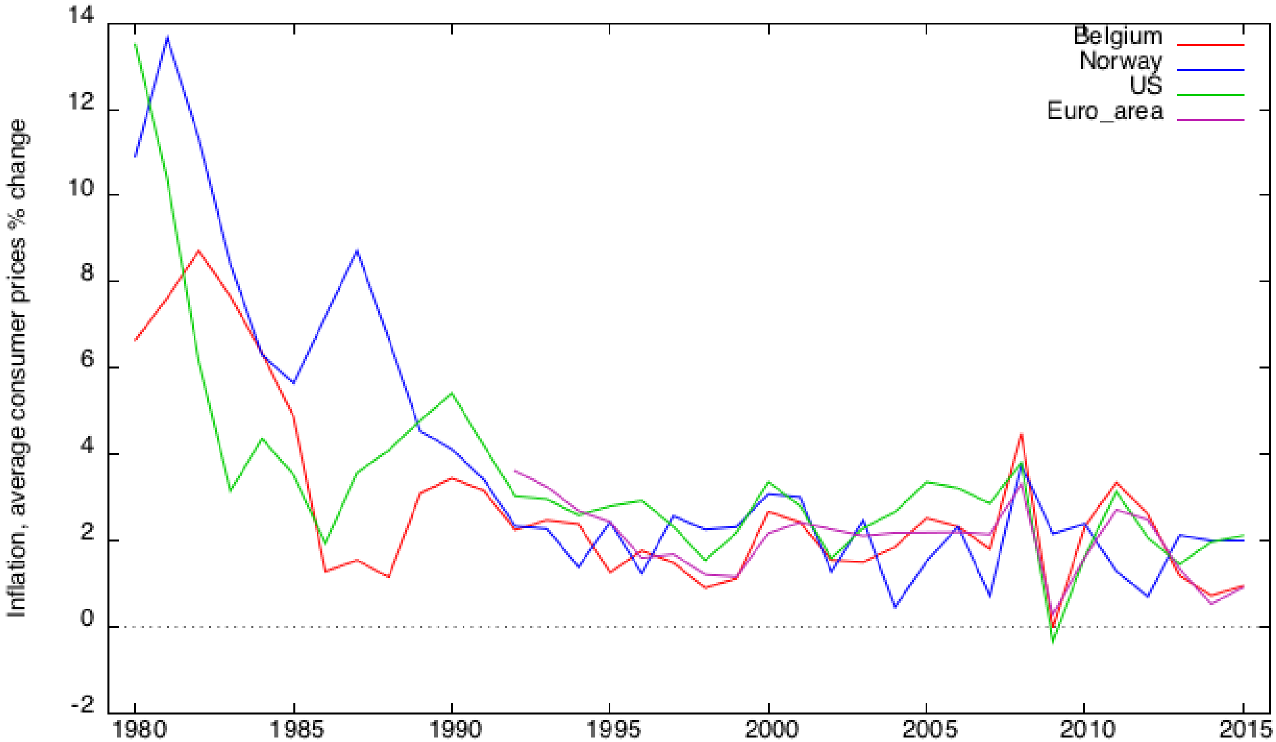


Figure 1: Inflation (average CPI % change for Belgium, Norway, US and the Eurozone

The inflation is much less volatile than it was in the 1970s and the early 1980s. Due to this decrease, inflation has become easier to forecast (Stock & Watson, 2006). However, the accuracy of multivariate time-series forecasting models has decreased compared to the univariate forecast models (Stock & Watson, 2006). For example, the backward Phillips curve, which represents the relationship between the rate of inflation and the unemployment rate, is widely used as an inflation forecasting tool. In this case, Phillips curve forecast refers to forecast models that include some activity variables (Stock & Watson, 2008).

However it appears that the Phillips curve has lost its ability to forecast with precision inflation as it is stated by Atkeson and Ohanian (2001): "Given the weak theoretical and empirical underpinnings of the various incarnations of the Phillips curve, we conclude that the search for yet another Phillips curve-based forecasting model should be abandoned." (Atkeson & Ohanian, 2001, p. 10).

1.2 The role of commodity indexes

More recently, attention has been focused on the role of the commodity prices as a tool for forecasting inflation. Commodities are in general raw materials, goods, and manufactured products, such as electronic components, agricultural intermediate products, current products of the chemical industry. Those types of products serves mainly used as an input in the production of a country. In this paper, the commodities include in the index are mainly raw materials, agricultural goods, metals, and energy products such as natural gas or crude petroleum.

Ben Bernanke, Federal Reserve Chairman, has defended this relationship between commodity prices and inflation in 2008 during the Federal Reserve Bank of Boston's 53rd Annual Economic Conference. According to him (Bernanke, 2008):

> Rapidly rising prices for globally traded commodities have been the major source of the relatively high rates of inflation we have experienced in recent years, underscoring the importance for policy of both forecasting commodity price changes and understanding the factors that drives those changes.

Furthermore, asset prices are more reliable as a forecasting variable than monetary aggregates such as the Phillips curve. Indeed, new financial instruments are constantly introduced, thus a close monitoring of the monetary aggregates is vital when forecasting inflation. On the opposite, the measurement error is thin when using asset prices to forecast inflation (Stock & Watson, 2003).

1.3 Relationship between commodity prices and consumer prices

Blomberg and Harris developed three theories regarding the linkage between commodity prices and inflation (1995). Firstly, they compared the relationship between commodity prices and inflation with the tortoise-and-hare fable. "Like the hare in Aesop's famous fable, commodity prices tend to take a quick, early lead in inflation cycles, but ultimately lose the race, falling in real terms." (Blomberg & Harris, 1995, p. 22). Due to an increase in the demand for final goods, an increase for commodity inputs should arise which will lead to inflation. Consequently, commodity prices have the ability to act as an early signal.

Furthermore, commodity prices are set in auction markets while, on the opposite, industrial prices are determined by sellers and are modified gradually (Acharya, Gentle, Mishra, & Paudel, 2008). Thus, commodity prices tend to react immediately on news about inflation movement (Boughton & Branson, 1988), (Furlong & Ingenito, 1996). It is also stated by Edelstein (2007) that a macroeconomic shock will directly affect commodity prices while it will affect consumer prices but after a certain time, a certain lag.

Finally, Bordo (1980) declared that agricultural products tend to react more quickly to a change in the monetary policy than industrial products. Those conclusions were the results of a study led on 14 industries.

Secondly, the link between commodity prices and inflation can come from the fact that commodities represent a valuable input in the production of the country. For the United States it represents one-tenth of their production (Blomberg & Harris, 1995). As we can observe from Table 1, commodities also represent an important share of the total export for Norway. Almost all of the first ten most exported products are commodities for Norway. For Belgium, commodities represent a smaller share of total exports compared to Norway. However, the goal of this paper, is to assess whether commodities are useful in order to forecast CPI inflation. Therefore, if commodity indexes are useful predictors, the forecast accuracy for the Norwegian inflation should be better due to the high importance of commodities in the production of the country. The data are provided by the Observatory of Economic Complexity (Observatory of

Economic Complexity, n.d.-a) and (Observatory of Economic Complexity, n.d.-b) for Belgium and Norway.

Table 1: Products exported by Belgium and Norway

	Belgium[2]		Norway[3]	
#	Product	% of total export	Product	% of total export
1	Refined Petroleum	9.3%	Crude Petroleum	36%
2	Cars	6.4%	Petroleum Gas	26%
3	Package Medicaments	5.0%	Refined Petroleum	5.3%
4	Diamonds	2.9%	Non-fillet Fresh Fish	2.4%
5	Human or Animal Blood	2.3%	Raw Aluminium	1.9%
6	Vehicle Parts	1.6%	Fish Fillets	1.3%
7	Ethylene Polymers	1.6%	Non-fillet Frozen Fish	1.1%
8	Coated Flat-Rolled Iron	1.3%	Raw Nickel	0.99%
9	Nitrogen Heterocyclic Compounds	1.2%	Ferroalloys	0.90%
10	Propylene Polymers	1.1%	Passenger and Cargo Ships	0.72%

The third linkage results from the previous two. Indeed, because commodity prices can be used as an early signal for inflation pressure, investors may heavily rely on them for inflation hedge. Generally, agents protect themselves from anticipated inflation by buying commodity future contracts (Boughton & Branson, 1988). Genuinely, commodities represent the perfect asset due to their intrinsic value. Usually, precious metal such as gold has been used as inflation hedge. However, commodity assets do not represent the only solution for inflation hedge. For example, government bonds are an appropriate hedge against long-term inflation (Watts, 2014).

Even though several links exist between commodity prices and consumer prices, an empirical link is difficult to establish (Edelstein, 2007). Firstly, it is possible that the higher commodity prices will not automatically lead to higher consumer prices. For example, the Fed governor Ben Bernanke argued in his speech to the American Economic Association in 2004 (Bernanke, 2004):

> ...the direct effects of commodity price inflation are empirically minuscule, both because raw materials costs are a small portion of total costs and because part of any

[3]Observatory of Economic Complexity. *Learn More About Trade in Belgium*. https://atlas.media.mit.edu/en/profile/country/blx/ (Consulted on 07/05/2015).

[3]Observatory of Economic Complexity. *Learn More About Trade in Norway*. https://atlas.media.mit.edu/en/profile/country/nor/ (Consulted on 07/05/2015).

> increase in the cost of materials tends to be absorbed in the margins of final goods producers and distributors. Accelerations in commodity prices comparable to or larger than the most recent one occurred following the 1981-82 and 1990-91 recessions, as well as in 1986-87 and 1999, with no noticeable impact on inflation at the consumer level.

However this point of view is challenged by the Fed governor Donald Kohn' speech during the National Economists Club luncheon meeting in 2004 (Kohn, 2004):

> Commodities and imports are only a small part of what we consume, and changes in their prices as well as in the price of energy usually do not affect measured core consumer inflation very much. But the recent situation has been notable for the size and number of price shocks going in the same direction, so that even with limited pass-thought of individual price movements, the total effect probably has been significant. Judging from the results of statistical models incorporating the factors we have been examining, increases in commodity, energy and import prices together might have boosted core consumer inflation on the order of roughly 1/4 to 1/2 percentage point over the past four quarters.

Secondly, it may be possible that an increase of the demand for a commodity might tamper the relationship between the commodity prices and the consumer prices. Let us take the example of an increase in aggregate demand for agricultural manufactured goods. This economic event will have two consequences. On the one hand, it will increase the final prices. On the other hand, it will decrease the prices of agricultural commodities due to the increase in demand. Consequently, the relationship between the commodity prices and the consumer prices will be broken. Indeed, their relationship would be negative (Furlong & Ingenito, 1996).

1.4 Structure of the thesis

The goal of this thesis is to assess whether the inclusion of commodity indexes inside our forecast models provides better results than the common univariate models. Two countries are studied in this paper: Belgium and Norway. Those two countries are relatively small commodity-exporting countries. However, the primary exported products of Norway are commodities while for Belgium, the primary exported products contains manufactured goods. Therefore, it will be interesting to assess whether commodity indexes are more useful to predict Norwegian inflation than Belgian's inflation due to the higher importance of commodities as an input for the Norwegian economy.

In order to assess the usefulness of commodity indexes for forecasting inflation, pseudo out-of-sample forecast will be executed with four different models. An autoregressive distributed lag model (ARDL) will be performed as our multivariate time series model which incorporate our commodity indexes. To compare our results, two benchmark univariate time series models, which are an AR(1) process and an Atkeson and Ohanian random walk process (AO) will be executed. In addition, an other autoregressive model with six lags of the dependent variables is estimated. The AR(1) is well known as a benchmark model (Chen, Turnovsky, & Zivot, 2014), (Stock & Watson, 2004). Even though, the AO model is quite simple, it showed promising results by improving the forecast accuracy upon the AR(1) benchmark for the period 1984-1999 for the United States (Stock & Watson, 2008).

Our sample ranges from January 1992 and May 2015. We shortened our sample of one year in order to perform our pseudo out-of-sample forecast. Forecasts are performed over four different time periods: 1-month, 3-month, 6-month, and 12-month. Before modeling, the time series need to be stationary. Our commodity indexes series contained one unit root, therefore the first difference was applied on the data to make it stationary.

The results of the different models are compared with the root mean squared error (RMSE) relative to the AR(1) benchmark. First of all, the AO model outperformed the AR(1) for all the time horizon except at the 1-month horizon forecast for Belgium, which is in line with the results of Stock and Watson (2008). Secondly, for Belgium the inclusion of commodity indexes in the forecasting model improves the forecast accuracy. However, those improvements are not significant. Besides, for Norway, it is the AR(6) with specific lags models that provides the best forecast results. Furthermore, it does not seem that the forecast accuracy with the commodity indexes is better for Norway than Belgium.

The thesis is structured in this way. Section 2 is dedicated to the literature survey. The link between inflation of consumer price index and commodity indexes were presented in the Introduction. The data used are depicted in section 3. In this section, both the commodity indexes and the CPI inflation data are presented. Furthermore, the stationarity of the time series is evaluated in this section as well. The methodology used in this paper is explained in section 4. Our forecasting models are presented in section 5. The forecasting results are analyzed in section 6. Finally, the limitation of our study and the conclusion are presented in section 7.

2 Literature survey

Koskinen provided in 2014 a table in order to categorize the different researches already done. He classified those researches according to their forecasting performance. Three categories were developed: successes, mixed results, and failures (Koskinen, 2014). Successes occur when the commodity prices or indexes used are judged as a useful inflation forecast indicator. When the improvement of the forecast is only marginal, the study is ranked as mixed results. Finally, the study is a failure when the commodity used does not bring any improvement to the forecast (Koskinen, 2014). New researches/studies have been added to Table 2 as well as new columns. Compared to the table of Koskinen (2014), a commodity index and an inflation measure columns have been added. The first one depicts the index used by the author in their forecast model. The second one illustrates the inflation measure taken into account for the forecast.

Table 2: Summary table of past researches

Author	**Year**	**Sample period**	**Frequency**	**Countries**	**Model**	**Commodity index**	**Inflation measure**
Acharya, Gentle, Mishra & Paudel	2008	1957-2005	Annual	US	VAR	CRB	CPI
Balcilar, Katzke & Gupta	2015	1968-2013	Monthly	South Africa	Bayesian-Dynamic Conditional Correlation	Gold and platinium	CPI
Blomberg & Harris	1995	1970-1994	Monthly	US	VAR	CRB, JOC, crude PPI, NAPM, PHIL, Gold, Food and Oil	CPI/PPI
Boughton & Branson	1988	1962-1987	Monthly	G7	Polynomial distributed lag	IMF (commodities division)	CPI
Browne & Cronin	2007	1959-2007	Quaterly	US	VECM	CRB	CPI
Cechetti, Chu & Steindel	2000	1975-1984	Quarterly	US	ADL	JOC, NAPM and oil price	CPI
Chen, Turnovsky & Zivot	2012	1983-2010	Monthly	5 countries	ADL	CRB sub-indexes	CPI/PPI
Edelstein	2007	1993-2004	Monthly	US	Several models[4]	46 individual commodity prices	CPI
Eugeni & Kruger	1994	1970-1994	Monthly	US	ADL	CRB, JOC, SMPS	PPI
Furlong & Ingenito	1996	1955-1995	Monthly	US	VAR	CRB and CRBRAW	CPI
Garner	1995	1973-1994	Monthly	US	ADL	Gold, CRB, JOC, CIBCR, PW	CPI
Gospodinov & Ng	2011	1983-2008	Monthly	G7	Convenience yields	CRB	CPI
Koskinen	2014	2008-2014	Monthly	Finland	ADL	IMF and DJUBS	CPI/PPI
Roth	1986	1949-1980	Monthly	US	VAR	JOC	CPI
Sims	1992	1957-1991	Monthly	5 countries	VAR	6 variables	CPI
Stock & Watson	2003	1959-1999	Quarterly	7 OECD countries	ADL	Commodity price index & oil price	CPI
Webb	1988	1954-1988	Monthly	US	VAR	JOC and CRB	CPI

2.1 Choice of the country

As we can notice from Table 2, many researches have been conducted about the relationship between commodity prices and consumer prices in the past. Most of those research papers focus on the US situation and only a few tackles the problematic for other countries. Amongst those we can mention Balcilar, Katzke, and Gupta (2015) who focused on the relationship between precious metal prices, such as gold and platinum, and South African inflation.

[4]Bagging, Bayesian model averaging, shrinkage estimation and factor models

Boughton and Branson (1988) tried to forecast inflation for the G7[5] countries by using commodity prices.

Chen et al. (2014) focused on five small commodity-exporting countries.[6] Different criteria have been taking into account in the choice of the countries. The monetary policy of these countries must be transparent. They have to be a commodity producing country and their exports must rely heavily on them. Finally, forecasting inflation is one of their primary concerns seeing that they have inflation-targeting policies (Chen et al., 2014).

Koskinen (2014) focused on forecasting Finnish inflation by using commodity indexes.

Sims (1992) estimated a forecast of the inflation for five countries[7] using not only commodity indexes but also other independent variables. Stock and Watson (2003) analyses 43 different economic variables commodity included from seven developed countries.[8]

2.2 Choice of the index

The choice of commodity indexes as the forecasting tool differs likewise amongst the different studies. The most used one is the CRB index. Indeed, it is used by Acharya et al. (2008), Blomberg and Harris (1995), Browne and Cronin (2010), Chen et al. (2014), Eugeni and Krueger (1994), Furlong and Ingenito (1996), Garner (1995), Gospodinov and Ng (2011), and Webb (1988).

The Commodity Research Bureau computes this index. This index was released for the first time in 1940 at the request of the U.S. Department of the Treasury. This index measures the price movement of 23 sensitive basic commodities. Those basic commodities are chosen due to their sensibility to economic changes. Consequently, they can serve as an early signal for change in business activity (Commodity Research Bureau, n.d.)[9]. The Commodity Research Bureau provides also six sub-indexes: metals, textiles, raw industrials, foodstuffs, fats and oils,

[5]The United States, Japan, the Federal Republic of Germany, France, the United Kingdom, Italy and Canada.

[6]Australia, Canada, Chile, New Zealand and South Africa.

[7]France, Germany, Japan, the United Kingdom and the United States.

[8]Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States

[9]CRB. *CRB BLS Spot Indices*.
`http://www.crbtrader.com/crbindex/spot\_background.asp` (Consulted on 10/05/2015).

and livestock. Chen et al. (2014) used these six sub-indexes for their forecasting model.

Another widely used index is the one from the Journal of Commerce (JOC). The Journal of Commerce-Economic Cycle Research Institute index of global industrial prices (JOC-ECRI Index) is “a leading indicator of inflation based on a broad assortments of raw materials used in industrial production. The IPI growth rate compares latest week’s index with the IPI’s average over the past year.” (Journal of Commerce, 2013)[10].

Blomberg and Harris (1995), Cecchetti, Chu, and Steindel (2000), Eugeni and Krueger (1994), Garner (1995), Roth (1986) and Webb (1988) are the authors who used the JOC Index for their research. According to Garner (1995), the JOC Index seems to be a better leading indicator for inflation than the CRB Index because of the lower share of agricultural prices. Consequently, the JOC Index would be less shake up by a supply shock and hence, less volatile compared to the CRB Index (Garner, 1995). Cecchetti et al. (2000) also argued that the JOC Index increases the accuracy of the inflation forecast. However, they also discovered a negative relationship between the JOC Index and inflation. Those results seem a bit odd because one would expect that a rise in the JOC Index would lead to a rise in inflation as it is stated in Section 1.3.

The NAPM index and IMF Primary Commodity Prices are two other indexes often used by the authors in their research papers. The National Association of Purchasing Mangers (NAPM) Index which is now known as the ISM (Institute of Supply Management) Index, is actually a survey realized amongst 18 diverse sectors of the U.S. manufacturing economy. It helps to indicate if the U.S. manufacturing sector is going to expand or to shrink. The International Monetary Fund Primary Commodity Prices Index tracks the price of international traded commodities including non-fuel commodities such as food, beverages, agricultural, raw materials, and metals, and energy commodities. Non-fuel commodities account for 36,9% of the Index, while energy commodities represent 63,1% (International Monetary Fund, n.d.). According to Cecchetti et al. (2000), the NAPM Index is not a reliable forecast indicator as it: “repeatedly

[10]JoC. *JOC-ECRI Industrial Price Index*. `http://www.joc.com/economy-watch/joc-ecri-industrial-price-index\_20130314.html` (Consulted on 10/05/2015).

reduced the accuracy of the forecast generated." (Cecchetti et al., 2000, p. 4).

Finally, several authors used individual commodity prices such as gold and platinium (Balcilar et al., 2015) or gold, food, and oil (Blomberg & Harris, 1995). In conclusion, it seems that none of those commodity indexes possess a superior forecasting power compared to the others. The high correlation between the different commodity prices may be the reason. Even tough it appears that the CRB Index has a little advantage compared to the others. Indeed Browne and Cronin (2010) and Gospodinov and Ng (2011) used this index and it was judged as a useful forecast indicator. However, Blomberg and Harris (1995) found that the CRB Index was a weak forecasting indicator since mid-1980, even though it was a leading one for the period 1970-1980. In conclusion, the choice of commodity index does seem to bear little importance.

2.3 Choice of the model

The frontier between successes and middle results or failures clearly lies in the model used. On the one hand, many of the authors used more traditional Autoregressive Distributed Lag (ADL) and Vector Autoregression (VAR) models. On the other hand, the authors who used more complex models are the ones who performed the best and achieved the best results. It is the case with Edelstein (2007) who used several methods such as bootstrap aggregating, also called bagging, Bayesian model averaging, factor models, and Bayesian shrinkage estimation. He found that: "Forecasting models with commodity prices are superior not only to inflation-only models. I find that there is information in commodity prices not captured by the leading principal components of a broader set of macroeconomic and financial variables." (Edelstein, 2007, p. 25). Browne and Cronin (2010) also find a positive relationship between commodity prices, consumer prices and money. According to them: "Long-run proportional relationships between money and, in turn, consumer prices and commodity prices are not rejected by the Johansen procedure." (Browne & Cronin, 2010, p. 22). In order to achieve those results, they used a Vector Error Correction Model (VECM).

Gospodinov and Ng (2011) found that the convenience yields of some commodity prices were able to improve the inflation prediction while some others were not. Consequently, they averaged the convenience yields of (i) cocoa, orange juice, and copper (positive effect), and (ii) soybeans, oats, and silver (negative effect). As a result, it was demonstrated that those two new variables were good predictors of inflation (Gospodinov & Ng, 2011).

Amongst the mixed results, we can count Stock and Watson (2003) who found that asset prices provide some improvements in inflation forecast compared to the AR benchmark. However, those improvements are rarely significant. Cecchetti et al. (2000) found that the indicators chosen do not improve the inflation forecast compared to the forecasts made with the past value of inflation with autoregressive models. Nevertheless, they observed that some indicators were well correlated with inflation such as the JOC Index. However, the negative relationship observed and explained above does not allow us to rely solely on those indicators. Acharya et al. (2008) found that the CRB Index relationship with inflation is still significant despite the results of earlier studies. According to Blomberg and Harris (1995), the commodity indexes used (CRB, JOC, NAPM, PHIL, gold, food, and oil) were able to forecast short-run changes in inflation for the full period 1970-1994. However, this relationship between commodity prices and consumer prices deteriorated in the mid-1980s. Blomberg and Harris (1995) support three explanations regarding the diminished signalling power of commodities. The first explanatory factor is the decrease of the commodity proportion in the U.S. production. This is due to a decrease in the demand (Blomberg & Harris, 1995). Secondly, commodities are not seen anymore as a reliable inflation hedge (Blomberg & Harris, 1995). Finally, the third explanation could be an example of the Goodhart's law. This theory states that: " ...when a measure become the target, it can no longer be used as the measure." (Business Dictionnary, n.d.).

Regarding the papers categorized as failures, Koskinen (2014) place five of those in this category, Boughton and Branson (1988), Eugeni and Krueger (1994), Garner (1995), Furlong and Ingenito (1996), and Mahdavi and Zhou (1997). Boughton and Branson (1988) rejected the fact that the commodity prices and consumer prices were cointegrated, thus, the hypothesis of a long-run relationship is rejected. Moreover, the inclusion of commodity prices in the model improved the in-sample regression. However those results are not reliable enough to improve out-of-sample forecast (Boughton & Branson, 1988). According to Eugeni and Krueger (1994): "Economic indicators have value only to the extent that they possess unique and independent information." (Eugeni & Krueger, 1994, p. 3). The conclusion of their research demonstrated that the use of the three commodity indexes (CRB, JOC, and SMPS) in the bivariate model do not perform any better than the univariate model that relies solely on the past values of consumer prices (Eugeni & Krueger, 1994). For Garner (1995), composite indicators have more forecasting power than single ones. However, none of the indicators chosen (CRB, JOC, gold, CIBCR, and PaineWebber) has been able to predict inflation magnitude (Garner, 1995). Furlong and Ingenito (1996) are in line with Blomberg and Harris (1995) as they demonstrated that since mid-1980s commodity prices perform poorly as an inflation forecasting indicator. Koskinen (2014) can be added to this category. Indeed, he was unable to obtain significant improvements by using multivariate models with inclusion of commodity indexes compared to the results of univariate models for forecasting Finnish inflation.

3 Data

3.1 Software

In this thesis, the results were obtained with gretl. Gretl which stands for Gnu Regression, Econometrics, and Time-series Library is an open-source multi-platform software dedicated to the econometric analysis and developed by Allin Cottrell (Cottrell & Lucchetti, 2015). The software is driven through a graphical user interface or through a command line. The command line may be used in two different ways: either by introducing commands interactively or by subjecting scripts (text files) containing the instructions.
This software has been evaluated several times in different magazines such as the Journal of Applied Econometrics or the Journal of Statistical Software. Yalta and Yalta (2007) argued in their review that: "gretl is a high-quality, feature-rich, and accurate econometrics package... furthermore, the program proves to be as good or even better in terms of numerical precision compared to other, widely used alternatives." (Yalta & Yalta, 2007, p. 853). Furthermore, Rosenblad (2008) made a complete review of the software. His conclusion was: "gretl is an easy-to-use statistical software that offers most of the features necessary for performing econometrics and time series analyses... It can be recommended to both students and professionals." (Rosenblad, 2008, p. 12). Finally, gretl was also positively reviewed by Baiocchi and Distaso (2003), and Mixon and Smith (2006) in the Journal of Applied Econometrics.

3.2 Commodity indexes

The data for the commodity indexes were acquired from two different sources: the Commodity Research Bureau and the International Monetary Fund. The index for the Commodity Research Bureau is the CRB (BLS) Spot Index. As already mentioned in section 2.2, this index was created in 1934 and released to the public in 1940. Table 38 in the appendix F; provides a description of the 23 basic commodities used by the index. The CRB (BLS) spot index includes only raw materials that have not been transformed. Consequently, they are still at the very beginning of the production stage. The spot price is the price set for immediate delivery and is paid in cash market. According to the CRB, sometimes the bid or asked price is been used in the absence of spot price.

Three indexes have been chosen from the IMF: the IMF commodity price index that encompasses all the commodities used by the IMF, the IMF non-fuel price index, and the IMF fuel price index. The IMF non-fuel price index includes edibles, food and beverages, and industrial inputs, agricultural and raw materials, and metals. The IMF fuel price index includes crude oil (petroleum), natural gas, and coal. Table 39 in the appendix F provides the different weights of those sub-indexes. It is important to notice the importance of the IMF fuel price index compared to the IMF non-fuel price index. Indeed, the first one accounts for 63,1% of the total primary commodity index.

Both the data from the CRB and the IMF are monthly and denominated in US dollars. Since the IMF data are available since January 1992, our sample period will be January 1992 till May 2015.

The data from the IMF and the CRB did not share the same base year. Indeed, for the IMF data it is 2005 = 100 base while for the CRB (BLS) Spot it is a 1967 = 100 base. The data from the CRB were rebased to 2005 =100 base according to this formula:

$$\frac{1967 \quad based \quad index \quad in \quad 1967}{1967 \quad based \quad index \quad in \quad 2005} = base \quad year \quad conversion \quad factor \tag{1}$$

Then each value of the old series is converted to the new series by multiplying its value with the base year conversion factor.

3.3 Inflation measure: consumer price index

The consumer price index (CPI) has been chosen to be the measure of inflation. This index is designed to measure the evolution in the cost of living. As an economic indicator, it measures changes in prices of a basket of goods and services purchased by households and representatives of their consumer habits. The index does not actually measure the level of prices but it measures the fluctuation between periods. Each year the basket of products can be updated to reflect any changes in households consumption profile. Consequently, the index will take into account new products on the market in order to remain as representative as possible.

For the Belgian inflation, data were collected from Statistics Belgium and for Norway it was collected for Statistics Norway. Both indexes are updated on a monthly basis and available online.

The collection of data for the CPI of Norway is based on electronic data from different firms, questionnaires, turnover information, commodity trade statistics, and budget shares. More than 650 goods are used for the computation of the CPI, while more than 14.000 goods are used for the sub-indexes (Statistics Norway, 2015).

Since the 30th of January 2014, the CPI for Belgium is calculated according to a new method and has 2013 = 100 reference period. The basket is made with 611 different goods and the weight bestowed to each of the goods depends on the household expenditures (Statistics Belgium, n.d.).

The data that will be forecasted is the CPI inflation. Consequently, the inflation formula has been applied to our data. The formula used was:

$$i = \frac{CPI_n - CPI_{n-1}}{CPI_{n-1}} \times 100 \tag{2}$$

We took the growth rate of the CPI in order to have stationary data as it is explained in section 3.4. Another way could have been to apply a logarithmic transformation to the data and then the first difference in order to have stationary variables.

3.4 Test for stationarity

Many financial institutions or researchers are working with time series data such as inflation rates, asset prices, exchange rates, and consumer price indexes. One of the major issues in the study of time series is to assess whether they follow a stationary process. It means that the structure of the underlying process evolves with time or not. If the structure remains the same, the process is said to be stationary. In short, a stationary series is “one with a constant mean, constant variant and constant autocovariance for each given lag.” (Brooks, 2014, p. 353). Verbeek (2004) distinguishes two type of stationarity, a weak and a strong one.

> A stochastic process is said to be strictly stationary if its properties are unaffected by a change of time origin; in other words, the joint probability distribution at any set of

times is not affected by an arbitrary shift along the time axis. (Verbeek, 2004, p. 258)

While the weak stationarity only takes into account the means, variances, and covariances of the time series and not the entire distribution as the strong stationarity (Verbeek, 2004). In time series analyses it is required to have stationary data. Before performing test, a time series plot needs to be done in order to have a general overlook of the data and to assess if the graph appears to be trending downward and if it wanders around some constant (Adkins, 2010).

The presence of a unit root in the time series is the cause of non-stationarity. A $I(0)$ series is stationary while a $I(1)$ series contains one unit root. Consequently, a non-stationary series y_t must be differenced d times in order to become stationary, and thus to become a $I(d)$ series (Brooks, 2014). This can be written as if $y_t = I(d)$ then $\Delta^d y_t = I(0)$ (Brooks, 2014, p. 360). In this way, a $I(2)$ series contains two unit roots and needs to be differenced twice in order to become a $I(0)$ series. Usually, most of the time series contains one unit root (Brooks, 2014). In order to assess the presence of a unit root, several tests exists. The most common is the Augmented Dickey-Fuller test. Before performing the Augmented Dickey-Fuller test, several decisions need to be taken based on the plots. By looking at the graphics we need to assess if the time series includes a constant and/or a time trend and/or seasonal dummies. The null hypothesis H_0 of the ADF (Augmented Dickey-Fuller) test is that the time series has one unit root and therefore is not stationary. If the null hypothesis is rejected, then the time series has no unit root and is stationary. If the null hypothesis is not rejected, or accepted, then the time series has a unit root and is not stationary. To reject the null hypothesis, the p-value needs to be less or equal to a specified significance level, in this case 0.05 (5%).

Kwiatkowski, Phillips, Schmidt, and Shin (1992) developed an alternative test in order to confirm the results of the ADF test. In opposition with the Augmented Dickey-Fuller test, stationarity is the null hypothesis and the existence of a unit-root is the alternative. Consequently, if the test statistics of the KPSS test is greater than the $p - value$, we would reject the null hypothesis of stationarity.

The next decision is to assess the number of lags that we want to include in the ADF regressions. One idea is to use the frequency of the time series data to decide for the appropriate number of lags. Another one suggested by Ng and Perron (1995) implies this following approach in order to assess the correct lag length with minimal power loss. First we need to set an upper bound p_{max} for p. Then we need to make our ADF test regression with $p = p_{max}$. Finally:

> ..if the absolute value of the t-statistics for testing the significance of the last lagged difference is greater than 1.6 then set $p = p_{max}$ and perform the unit root test. Otherwise, reduce the lag length by one and repeat the process. (Zivot & Wang, 2006, p. 121).

Schwert (1989) determined a common rule of thumb for determining the p_{max}:

$$p_{max} = [12 \times (\frac{T}{100})^{1/4}] \tag{3}$$

3.4.1 CPI Inflation of Belgium and Norway

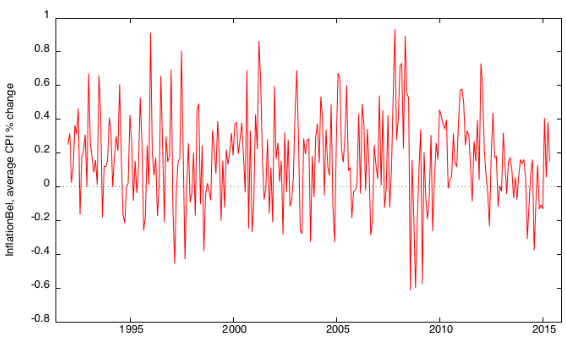


Figure 2: % change of CPI over time for Belgium

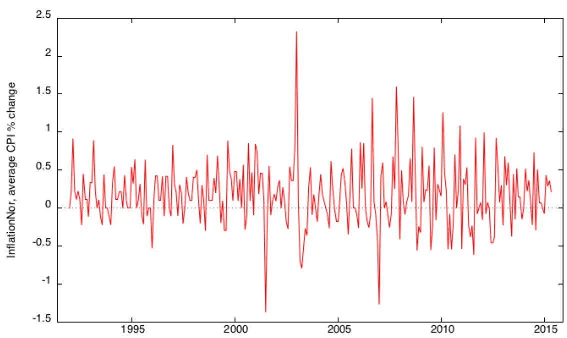


Figure 3: % change of CPI over time for Norway

Clearly in Figure 2 and 3 the data of CPI inflation for Belgium and Norway appears to have a constant mean and a constant variance. Consequently, we might to think that $d = 0$ and that the data are stationary.

The ADF test confirms those interpretations. According to Schwert (1989) equation we chose 15 lags for our tests and compute the ADF test with a constant. Table 3 displays the results of the ADF test of CPI inflation for Belgium and Norway. The $p - value$ in both case is smaller than the significance level of 5%. Therefore, both time series have no unit root, are $I(0)$ process, and are stationary. The KPSS test confirms both the ADF test and the graphical interpretations. Indeed, Table 20 in the appendix A, exhibits the results. The KPSS statistics is 0,07 for Belgium and Norway respectively. Both results are lower than the 99% quantile. Therefore, the null that both time series are $I(0)$ is accepted at the 1% level.

Table 3: ADF results for CPI inflation for Belgium and Norway

Augmented Dickey-Fuller test for InflationBel	**Augmented Dickey-Fuller test for InflationNor**
including 12 lags of (1-L)InflationBel	including 12 lags of (1-L)InflationNor
(max was 15, criterion AIC)	(max was 15, criterion AIC)
sample size 268	sample size 268
unit-root null hypothesis: $a = 1$	unit-root null hypothesis: $a = 1$
test with constant	test with constant
model: $(1 - L)y = b0 + (a - 1) * y(-1) + ... + e$	model: $(1 - L)y = b0 + (a - 1) * y(-1) + ... + e$
1st-order autocorrelation coeff. for $e : -0.011$	1st-order autocorrelation coeff. for $e : 0.002$
lagged differences: $F(12, 254) = 2.280[0.0091]$	lagged differences: $F(12, 254) = 6.638[0.0000]$
estimated value of $(a - 1) : -0.747433$	estimated value of $(a - 1) : -1.56513$
test statistic: $tau\_c(1) = -4.21454$	test statistic: $tau\_c(1) = -5.30755$
asymptotic $p - value0.0006184$	asymptotic $p - value4.596e - 06$

3.4.2 Commodity indexes: CRB, IMF, IMFNF, IFMM

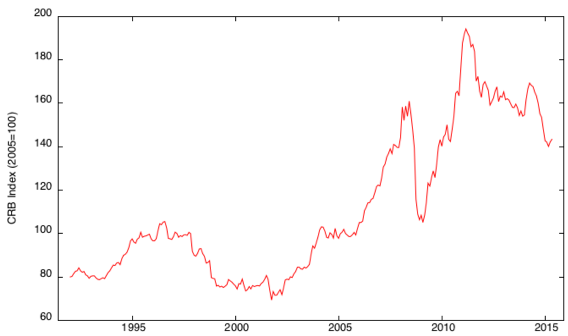


Figure 4: CRB index over time

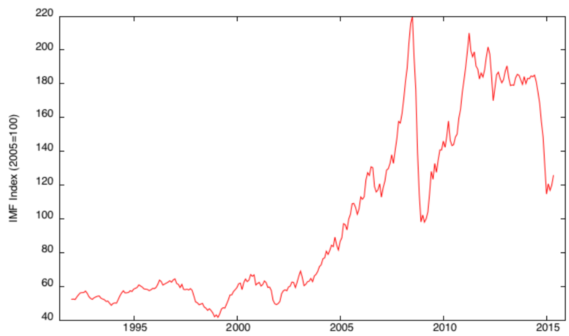


Figure 5: IMF index over time

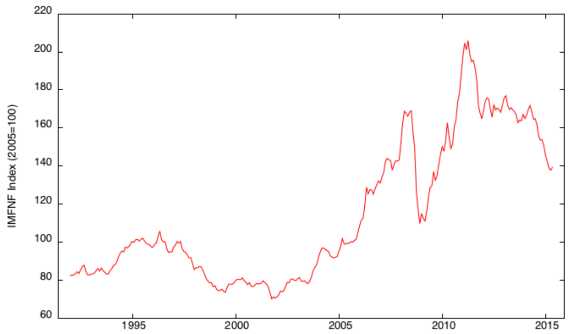


Figure 6: IMFNF index over time

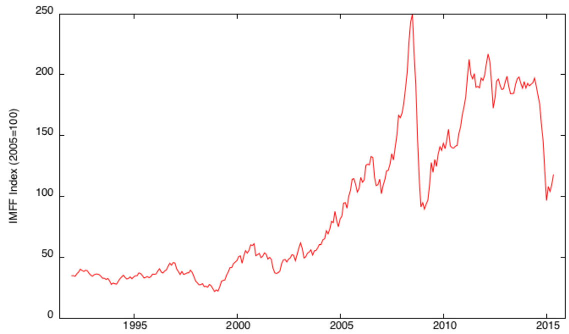


Figure 7: IMFF index over time

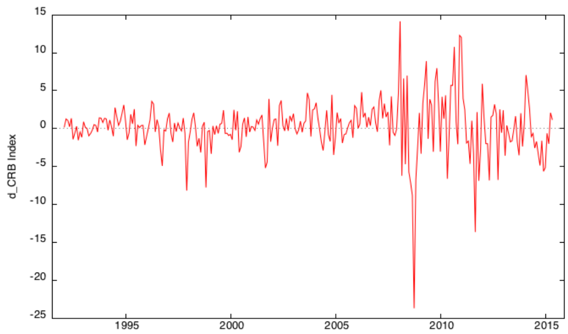


Figure 8: d_CRB index over time

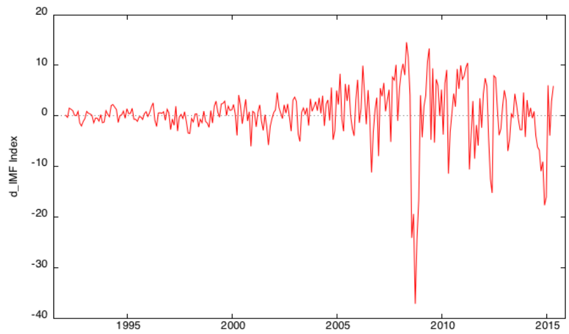


Figure 9: d_IMF index over time

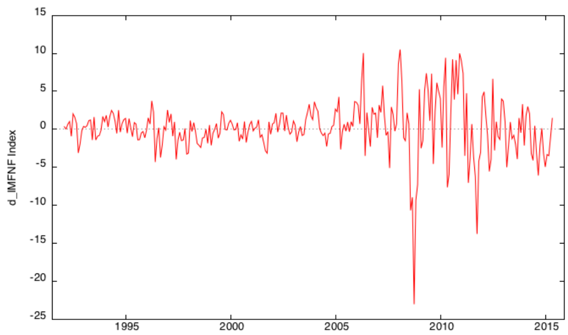


Figure 10: d_IMFNF index over time

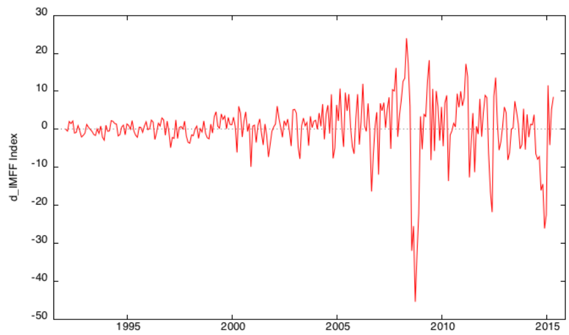


Figure 11: d_IMFF index over time

We can assess that the graphic indicates random walk behavior. Despite some irregularities, we can assume that the series have an upward trend. Besides the graphics representing the first difference series illustrate a white noise process; which indicates a constant. Consequently we can include a constant and a trend in our ADF test regression.

The null hypothesis of the ADF test for the CRB and the IMFNF series is not rejected.

Therefore, the test is applied on the first difference of the series. Table 4 exhibits the results of the ADF test. The KPSS results (see Table 21 in appendix A) confirm the previous ones.

For the IMF series, the null hypothesis of the ADF test is rejected. However, the null hypothesis of the KPSS test is also rejected. Because of those different results and due to the fact that the $p - value$ of the ADF test is close to the significance level of 5%, the first difference is also applied to the IMF series. For the IMFF series, the conclusions are the same. The null hypothesis of the ADF is rejected so is the null hypothesis of the KPSS test. Therefore, the first difference is also applied to those series. In conclusion, the four commodity indexes series present one unit root and are integrated of order one, denoted $I(1)$ because they become stationary after first differencing. Table 5 provides the results of the ADF test applied to the first differenced commodity indexes and the results of the KPSS test are avaible in the appendix in Table 22.

Table 4: ADF results for the 4 commodity indexes

Augmented Dickey-Fuller test for CRB	**Augmented Dickey-Fuller test for IMF**
including 12 lags of (1-L)CRB (max was 15, criterion AIC) sample size 268 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1 - L)y = b0 + b1 * t + (a - 1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.014 lagged differences: $F(12, 253) = 4.886[0.0000]$ estimated value of $(a - 1)$: -0.0221588 test statistic: $tau_c t(1) = -1.9125$ asymptotic $p - value 0.6479$	including 2 lags of (1-L)IMF (max was 15, criterion AIC) sample size 278 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1 - L)y = b0 + b1 * t + (a - 1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.007 lagged differences: $F(2, 273) = 46.269[0.0000]$ estimated value of $(a - 1)$: -0.0409913 test statistic: $tau_c t(1) = -3.41566$ asymptotic $p - value 0.04925$
Augmented Dickey-Fuller test for IMFNF	**Augmented Dickey-Fuller test for IMFF**
including 13 lags of (1-L)IMFNF (max was 15, criterion AIC) sample size 267 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1 - L)y = b0 + b1 * t + (a - 1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.008 lagged differences: $F(13, 251) = 7.388[0.0000]$ estimated value of $(a - 1)$: -0.0175741 test statistic: $tau_c t(1) = -1.79154$ asymptotic $p - value 0.7092$	including 2 lags of (1-L)IMFF (max was 15, criterion AIC) sample size 278 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1 - L)y = b0 + b1 * t + (a - 1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.003 lagged differences: $F(2, 273) = 43.048[0.0000]$ estimated value of $(a - 1)$: -0.0532606 test statistic: $tau_c t(1) = -3.75441$ asymptotic $p - value 0.01893$

Table 5: ADF results for the first difference of the 4 commodity indexes

Augmented Dickey-Fuller test for d_CRB	**Augmented Dickey-Fuller test for d_IMF**
including 11 lags of (1-L)d_CRB (max was 15, criterion AIC) sample size 268 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1-L)y = b0 + b1 * t + (a-1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.013 lagged differences: $F(11, 254) = 3.311[0.0003]$ estimated value of $(a-1) : -0.798357$ test statistic: $tau_c t(1) = -5.17506$ asymptotic p-value 8.114e-05	including 12 lags of (1-L)d_IMF (max was 15, criterion AIC) sample size 267 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1-L)y = b0 + b1 * t + (a-1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.001 lagged differences: $F(12, 252) = 2.446[0.0050]$ estimated value of $(a-1) : -0.901264$ test statistic: $tau_c t(1) = -5.84474$ asymptotic p-value 2.736e-06
Augmented Dickey-Fuller test for d_IMFNF	**Augmented Dickey-Fuller test for d_IMFF**
including 12 lags of (1-L)d_IMFNF (max was 15, criterion AIC) sample size 267 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1-L)y = b0 + b1 * t + (a-1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for e : 0.007 lagged differences: $F(12, 252) = 2.430[0.0053]$ estimated value of $(a-1) : -0.688241$ test statistic: $tau_c t(1) = -4.81094$ asymptotic p-value 0.0004129	including 5 lags of (1-L)d_IMFF (max was 15, criterion AIC) sample size 274 unit-root null hypothesis: $a = 1$ with constant and trend model: $(1-L)y = b0 + b1 * t + (a-1) * y(-1) + ... + e$ 1st-order autocorrelation coeff. for $e : -0.003$ lagged differences: $F(5, 266) = 2.279[0.0472]$ estimated value of $(a-1) : -0.735591$ test statistic: $tau_c t(1) = -7.76431$ asymptotic p-value 1.145e-11

4 Methodology

It exists a multitude of inflation forecasting models but usually those models can be regrouped into four different categories according to Stock and Watson (2008). The first category encompasses the forecasts based on the previous inflation values. This category includes univariate time series models such as the ARIMA (Stock & Watson, 2008). Usually, this category serves as forecasting benchmark in most of the research papers. The second category is made of forecast models based on activity measures such as the unemployment rate or the activity growth (Stock & Watson, 2008). This category is referenced as the Phillips curve forecasts. The third category, is the forecasts based on the forecasts of others. Therefore, in this category, the forecasting models rely extensively on other computed forecasts (Stock & Watson, 2008). For example, Mishkin (1990) uses the term structure of interest rates in order to forecast inflation. The final category is the forecast based on other predictors than activity or expectations variables (Stock & Watson, 2008). However, according to Stock and Watson (2008) those type of forecasts perform poorly compared to the three other categories and especially compared to the AR benchmark.

In this paper we are going to focus on both univariate and multivariate time series models. Univariate time series models rely solely on information contained on the past values of the variables (Brooks, 2014). On the opposite, multivariate models, also called structural models, linked different variable together and try to explained the change in a dependent variable with both the past values of the dependent variables and past values of an independent variable. Structural model may be inappropriate for forecasting. Indeed, Brooks (2014) states that univariate time series models are more useful for out-of-sample forecasting. In this paper we will focus on both models. Therefore at the end, we will be able to assess if the inclusion of commodity indexes in our multivariate models improves the forecast results for Belgian and Norwegian inflation.

4.1 Univariate time series model

The univariate time series models used in this paper belong to the family of autoregressive integrated moving average (ARIMA) models.

4.1.1 Autoregressive process

An autoregressive process relies solely on the previous values of the variable forecasted plus an error term (Brooks, 2014). The equation of an $AR(p)$ model is:

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + u_t \tag{4}$$

where u_t represents a white noise disturbance term. Consequently, the present value of y_t can be found thanks to the past value of the dependent variable plus a random shock.

4.1.2 Moving average process

"A moving average model is simply a linear combination of white noise processes, so that y_t depends on the current and previous values of a white noise disturbance term." (Brooks, 2014, p. 256). An $MA(q)$ process equation can be written as:

$$y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + ... + \theta_q u_{t-q} \tag{5}$$

where u_t represents the white noise terms.
In oppostion with an $AR(p)$ model which includes lagged terms of the previous values of the variables, a $MA(q)$ model includes lagged values of white noise terms.

4.1.3 ARIMA and ARMA process

An $ARMA(p,q)$ model combines both an $AR(p)$ and a $MA(q)$ process. Consequently, this model states that the current value of a dependent time series y_t depends on both its own previous values and the previous values of a white noise term. The model is therefore written

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + ... + \theta_q u_{t-q} \tag{6}$$

In the above equation, we clearly depicts the $AR(p)$ and $MA(q)$ models that compose the $ARMA(p,q)$. The ARIMA model is a type of ARMA model which is useful in the case of non-stationarity, thus time series which have been first differenced in order to become stationary. Consequently, an ARMA model is a linear stationary model as well as the $AR(p)$ and $MA(q)$ models. While an ARIMA model is a linear non-stationary model. The i stands for integrated. Consequently, an ARIMA is typically written as $ARIMA(p,d,q)$. The p is for the number of autoregressive terms, the d is for the order of differencing, and the q is for the number of moving average terms.

4.1.4 Atkeson and Ohanian

In 2001, Atkeson and Ohanian introduced a new simple univariate benchmark based on a random walk process. The model is quite simple, as it is simply an arithmetic average of the past 12 months. The AO model can be written as:

$$\pi_{t+j} = \frac{1}{12}\sum_{i=1}^{12}\pi_{t-i} + v_{t+j} \tag{7}$$

This model beats the classic AR(1) benchmark for the period 1984-1999 and consequently the Phillips curve based models (Atkeson & Ohanian, 2001) and (Stock & Watson, 2008). Faust and Wright (2013) made a roundup of the different forecasting models used to forecast inflation in which they listed the Atkeson and Ohanian random walk model.

4.1.5 The Box-Jenkins model building process

Box and Jenkins proposed in 1976 a forecasting technique for univariate time series which is based on the notion of ARIMA process. This technique is divided into three steps : identification, estimation, and verification (Box, Jenkins, & Reinsel, 2008).

The first step is to assess the $ARIMA(p,d,q)$ model which could lead to the series. Consequently, it is important to transform the series to make it stationary. Then we need to identify the $ARMA(p,q)$ of the series by using the correlogram and partial correlogram. Usually the graphics of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) allow us to determine the order of the ARMA model (Box et al., 2008).

The second step is to estimate the ARIMA model by using non-linear method and the objective is to minimize the sums of square errors (Box et al., 2008).

The third step is to check whether the estimated model reproduces the model that generated the data. For that, the residuals obtained are used to check if they behave like white noise errors (Box et al., 2008).

The final step is to use the estimated model in order to generate a forecast of the time series (Box et al., 2008).

4.1.6 The autocorrelation function (ACF) and the partial autocorrelation function (PACF)

The autocorrelation function (ACF) allows us to determine the correlation coefficient between the time series and its own lags. Therefore, we can assess the lenght and strenght of the process (Verbeek, 2004). The partial autocorrelation function (PACF) measures the correlation between an observation from the past and the current observation after removing the effect of intermediate observations (Brooks, 2014).

The correlogram of the ACF and the PACF should allow us to determine which model is adequate for our time series. According to Brooks (2014), an AR process has a geometrically declining ACF and the number of non-zero points on the PACF graph indicates the AR order. For the MA process, it has a geometrically declining PACF and the number of non-zero points of the ACF graph indicates the MA order. Finally, the ARMA is represented by a geometrically declining ACF and PACF (Brooks, 2014).

However, the ACF and PACF are very complicated to interpret when using real economic data. In this case we must rely on information criteria. It exists several information criteria. However the Akaike (1974) information criterion (AIC), the Schwarz (1978) Bayesian information criterion (SBIC), and the Hannan-Quinn criterion (HQIC) (Hannan & Quinn, 1979) are the three information criteria most commonly used. The equation of those three information criteria is divided into two terms: one which is function of the residual sum of square (RSS) and one penalty term due to the addition of extra parameters (Brooks, 2014). Consequently, the addition of an extra parameter will have two opposite effects. On the one hand, it will decrease the residual sum of squares (RSS). On the other hand, it will increase the penalty term. In conclusion, the addition of an extra parameter is useful if the fall of the RSS balances with the increased value of the penalty term (Brooks, 2014). The equations (Brooks, 2014) of the AIC, SBIC, and HQIC are respectively:

$$AIC = ln(\hat{\sigma}^2) + \frac{2k}{T} \tag{8}$$

$$SBIC = ln(\hat{\sigma}^2) + \frac{k}{T} lnT \tag{9}$$

$$HQIC = ln(\hat{\sigma}^2) + \frac{2k}{T} ln(ln(T)) \tag{10}$$

The model that minimizes the information criteria is the most suitable. However, according to Brooks (2014), no information criterion prove to be superior to another.

4.2 Multivariate time series model

The multivariate time series model used in this paper is the autoregressive distributed lag model (ARDL). The ARDL is an extension of the autoregressive model (AR). In this model, both lags of the dependent variable and lags of independent variables are included (Adkins, 2010). This model was developed by Almon (1965). In the equation, p is the maximum distributed lag and q is the maximum autoregressive lag (Adkins, 2010). The model is therefore:

$$y_t = \delta + \delta_0 x_t + \delta_1 x_{t-1} + ... + \delta_q x_{t-q} + \phi_1 y_{t-1} + ... + \phi_p y_{t-p} + v_t \tag{11}$$

where v_t is a random disturbance term and δ is the constant term.

Eugeni and Krueger (1994) used the ARDL model in their research. Their modus operandi was divided into three steps. First, they forecast inflation based on historical data. They used an autoregressive model with 12 lags and measure the forecast accuracy with the root mean square error (RMSE). Then they added the commodity indexes into their models which became a bivariate model, and both the inflation and the index were lagged 12 months (Eugeni & Krueger, 1994). Finally, they were able to compare the average forecast error between the model with and without commodity indexes.

4.3 Evaluation of the forecast accuracy

Different indicators are useful in order to assess the accuracy of the forecast model. The mean squared error (MSE) is the arithmetic average of the squared errors between forecasts and observations. According to Brooks (2014), the MSE provides a quadratic loss function which is useful when the large forecast error is much more important than the smaller one. The equation (Brooks, 2014) can be written as:

$$MSE = \frac{1}{T-(T_1-1)}\sum_{t=T_1}^{T}(y_{t+s}-f_{t,s})^2 \tag{12}$$

where $f_{t,s}$ equal the s-step ahead forecast made at time t.

The value of the MSE needs to be minimized. The mean absolute error (MAE) is the arithmetic mean of the absolute values of the difference. Consequently, it measures the absolute forecast error and its equation is therefore:

$$MAE = \frac{1}{T-(T_1-1)}\sum_{t=T_1}^{T}|y_{t+s}-f_{t,s}| \tag{13}$$

A final indicator is the root mean squared error (RMSE) which represents the square root of the MSE.

These three indicators are often used to compare different forecasting models in relation to a time series. However, the RMSE is the one mostly used in the research papers. Most of the time, the different forecasting models are compared to the AR benchmark with a relative RMSE.

4.4 In-sample and out-of-sample forecast

When performing a forecast of a time series, two methods can be used: the in-sample forecast or the out-of-sample forecast. The in-sample analysis used the same data as the one used to estimate the parameters of the model (Brooks, 2014). When the forecast analysis is done, the fitted values are compared with the actual realizations and it is, then, possible to compute the accuracy of the forecast thanks to the three indicators: MSE, MAE, and RMSE. The out-of-sample forecast aims to determine how an adjusted model predicts future values of the dependent variable based on the regressors (Gujarati, 2003). Consequently, this method is time consuming because in order to assess the accuracy of the forecast we have to wait until the next period in order to record the forecast and to compute the forecast error. In this paper, because of the availability of the data, static forecasts will be performed. Static forecast relies on the actual values of the dependent value to perform the forecast. While dynamic forecast distinguishes itself from static forecast by using the already forecasted values to forecast the next ones. Dynamic forecast is an iterated process mainly used for out-of-sample forecast.

4.5 Next steps

In this paper four forecasting models are going to be performed. Amongst those four models, three will be univariate time series model and one will be a multivariate time series model that will include the different commodity indexes. The univariate model consists of the AR(1), which will be used as our benchmark model, an AO model, and an AR model with specific lags chosen according the ACF, PACF, and the AIC, BIC criterion. The lag orders of the models are determined by the information criteria in view of the difficulty of interpretation of the correlograms. The multivariate model will be an ARDL with specific lags of the dependent and independent variables. Four models are tested, each including one of the four commodity indexes.

Both the AR and ARDL models are fitted through ordinary least squares (OLS). Even though it is common to use OLS model for time series data, normally OLS models are more appropriate for cross-sectional data (Pickett, Reilly, & McIntyre, 2005). However, the OLS models must satisfy several conditions in order to obtain a sufficient forecast model (Pickett et al., 2005). If the OLS models do not fulfill the conditions, there is a risk that the models will " ...contain violations of the underlying assumption of independence." (Pickett et al., 2005, p. 14). Those violations may lead to an inaccurate forecast (Pickett et al., 2005). First, the coefficients must be statistically significant. Secondly, the residuals must have a constant mean around zero. Thirdly, the residuals must have a constant variance. Fourthly, no autocorrelation can be found amongst the residuals. Fifthly, the residuals must be normally and independently distributed (Pickett et al., 2005). Therefore, all the coefficients of the models are statistically signifiant and test for autocorrelation, heteroscedasticity, and normality are performed on the residuals of the OLS models. The graphics of the residuals against time will be display in the appendix in order to confirm the second and the third conditions.

In this paper, a pseudo out-of-sample forecast will be performed. Forecast procedures will be performed on an 1-month, 3-month, 6-month, and one year horizon. In order to perform a pseudo out-of-sample forecast, our data sample will be reduced by one year. Therefore, our new sample is from January 1992 till May 2014 and our forecast period will be from June 2014 till May 2015. Thanks to this pseudo out-of-sample forecast we will be able to compute the forecast errors. Finally, we will compare the average forecast error from each model that encompasses a commodity index. If the forecast error of one of the multivariate model is significantly smaller than the AR(1) benchmark, then we would be able to assert that the multivariate model with the incorporation of a commodity index improves the forecast accuracy.

5 Models

5.1 Univariate time series model

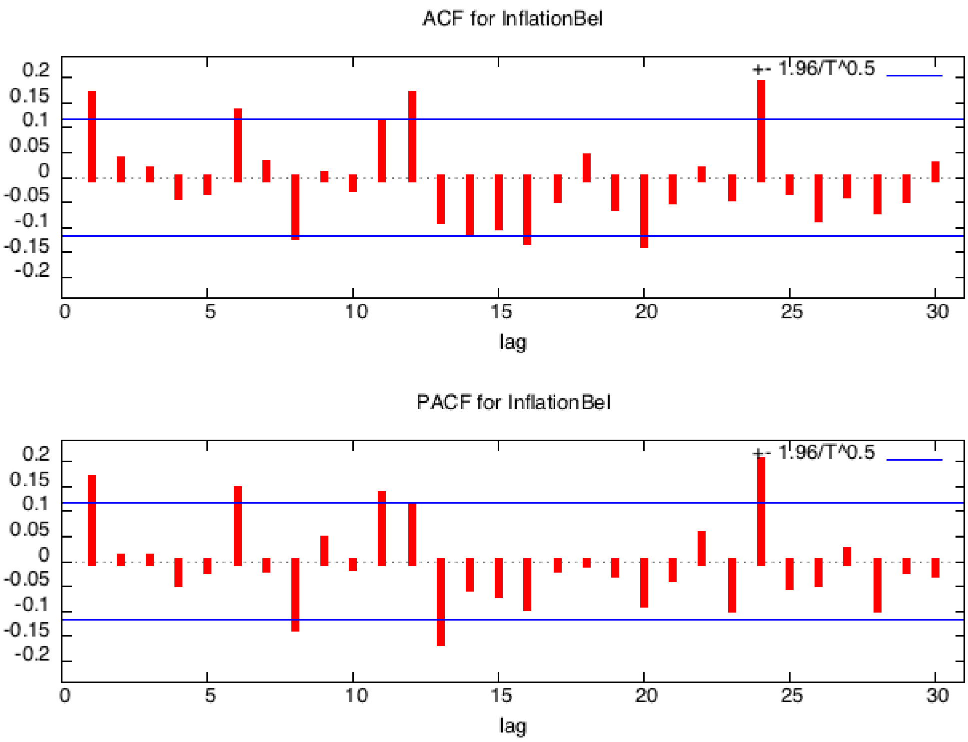


Figure 12: Correlogram of the ACF and PACF for Belgium

Figure 12 and 13 represents the correlogram of the autocorrelation function and the partial autocorrelation function for Belgium and Norway. Both graphics appear to be quite erratic and difficult to interpret especially to assess the lag order of our model. However, those graphics allow us to determine which lags are explanatory enough to merit inclusion in the AR model.

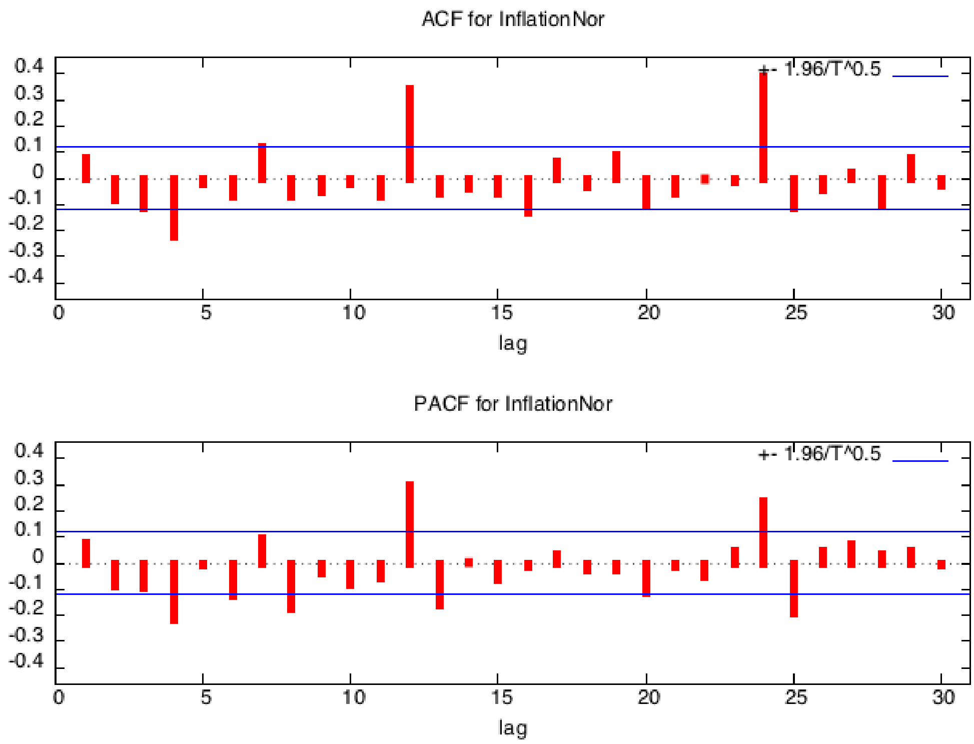


Figure 13: Correlogram of the ACF and PACF for Norway

5.1.1 Autoregressive models

As we can observe from the graphics, it is difficult to assess whether the correlograms display an AR or MA signature. However, because the PACF displays a sharp cut-off and the first lag autocorrelation is positive, an AR model seems more useful (Brooks, 2014). This is the case especially for Belgium. Normally, the ACF of an $AR(p)$ process should decay and the number of non-zero points on the PACF should indicate the lag order p. Consequently, if the PACF is significant till lag k then we should try fitting an $AR(k)$ process (Brooks, 2014). Furthermore the information criteria are useful in order to confirm our lag order choice. In this paper, we rely firstly on the AIC criteria even though, it does not seem that an agreement has been reached on which is better. Indeed, the AIC is recommended by Burnham and Anderson (2004) to be used in general as it exists evidence of its good performance. However, Medel and Salgado (2013) tested the estimation and forecasting power of both information criteria and they argue that the

BIC is a better information criteria to model stationary time series with an autoregressive model. Therefore, the BIC is used as a confirmation.

Firstly, we set the number of lags according to the PACF graphic. Therefore, the appropriate lag number is the one beyond which the partial autocorrelation is all zero. Unfortunately, for both Belgium and Norway there is no maximum lag beyond which the partial autocorrelation is all zero. Consequently, we tested different models according to the lags with spike. For Belgium we tested four different autoregressive models with different lags chosen. In order to assess which model was the best fitted, we rely on the information criteria. Table 6 provides the model that minimizes both the AIC and BIC information criterion. For this model the lags chosen were 1, 6, 8, 11, 12, and 13.

According to Table 7 all the coefficients are significant except the eleventh lag because the $t - ratio$ is lower than 2 and because the $p - value$ is lower than the significance level of 5%. However, if we take out the lag from the model, the AIC and the BIC criterion increases. Furthermore, the significant coefficients were the one with a high partial autocorrelation function. Therefore the AR model equation for Belgium can be written as follow:

$$y_{Bel,t} = \mu + \phi_1 y_{Bel,t-1} + \phi_2 y_{Bel,t-6} + \phi_3 y_{Bel,t-8} + \phi_4 y_{Bel,t-11} + \phi_5 y_{Bel,t-12} + \phi_6 y_{Bel,t-13} + u_t \quad (14)$$

For Norway an $AR(6)$ is also fitted. In this case the lags chosen were 2, 4, 6, 7, 12, and 24 according to the correlogram. Therefore the equation for the autoregressive model for Norway can be written:

$$y_{Nor,t} = \mu + \phi_1 y_{Nor,t-2} + \phi_2 y_{Nor,t-4} + \phi_3 y_{Nor,t-6} + \phi_4 y_{Nor,t-7} + \phi_5 y_{Nor,t-12} + \phi_6 y_{t-24} + u_t \quad (15)$$

Table 8 provides the $AR(6)$ model for Norway. As it was the case for Belgium, all the coefficient of the regression are statistically significant, which is in line with the condition of Pickett et al. (2005).

Because the observation of patterns is difficult on the ACF and PACF, several models have been tested and compared according to the Akaike and Schwarz criterion. It is only after comparison that the AR(6) with specific lags was the best fitted model. Table 6 provides the different models and their respective AIC and BIC.

Table 6: Search for the best model according to the AIC and BIC criterion - univariate models

Belgium	**AIC**	**SC (BIC)**	**Norway**	**AIC**	**SC (BIC)**
AR(1)	64,90	72,08	AR(1)	300,98	308,17
AR(2)	67,41	78,17	AR(2)	300,06	310,82
AR(6*)[12]	**50,79**	75,61	AR(6*)[12]	**222,95**	247,46
AR(6)	70,06	95,06	AR(6)	285,29	310,30
AR(12)	68,96	115,09	AR(12)	257,16	303,30
ARMA(1,1)	68,25	82,63	ARMA(1,1)	299,61	313,99
ARMA(1,2)	70,25	88,23	ARMA(1,2)	292,70	310,68
ARMA(2,1)	66,00	83,98	ARMA(2,1)	291,51	309,48
ARMA(2,2)	62,40	83,96	ARMA(2,2)	293,08	314,64

Clearly, the autoregressive models with the specific lags are the best fitted according to the Akaike and Schwarz criterion.

Eventhough, the AR(1) is not the best fitted model according to the AIC and BIC criteria, it is widely used as an univariate benchmark model. Therefore, the model will be used in this paper. The AR(1) model is a pretty straight forward model. The two AR(1) benchmark models are depicted in Table 23 and 24 in the appendix B. The equations of the AR(1) models for Belgium and Norway are respectively:

$$y_{Bel,t} = \mu + \phi_1 y_{Bel,t-1} + u_t \tag{16}$$

$$y_{Nor,t} = \mu + \phi_1 y_{Nor,t-1} + u_t \tag{17}$$

[12] AR(6*) corresponds to the model with 6 specific lags chosen.

Table 7: AR(6) process for Belgium

Model 1: OLS, using observations 1993:02–2014:05 (T = 256)
Dependent variable: InflationBel

	Coefficient	Std. Error	t-ratio	p-value
const	0.119232	0.0282677	4.2180	0.0000
InflationBel_1	0.193151	0.0605906	3.1878	0.0016
InflationBel_6	0.130855	0.0601705	2.1747	0.0306
InflationBel_8	−0.138112	0.0595667	−2.3186	0.0212
InflationBel_11	0.103181	0.0602334	1.7130	0.0880
InflationBel_12	0.128261	0.0616777	2.0795	0.0386
InflationBel_13	−0.157719	0.0607620	−2.5957	0.0100

Mean dependent var	0.161996	S.D. dependent var	0.277233
Sum squared resid	17.30560	S.E. of regression	0.263629
R^2	0.117007	Adjusted R^2	0.095730
$F(6, 249)$	5.499214	P-value(F)	0.000023
Log-likelihood	−18.39742	Akaike criterion	50.79485
Schwarz criterion	75.61109	Hannan–Quinn	60.77585
$\hat{\rho}$	−0.017774	Durbin's h	−1.159381

Breusch-Pagan test for heteroskedasticity (robust variant) –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 10.1172
with p-value = $P(\chi^2(6) > 10.1172)$ = 0.119804

LM test for autocorrelation up to order 12 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.34678
with p-value = $P(F(12, 237) > 1.34678)$ = 0.193046

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 1.61253
with p-value = 0.446524

Table 8: AR(6) process for Norway

Model 2: OLS, using observations 1994:01–2014:05 (T = 245)
Dependent variable: InflationNor

	Coefficient	Std. Error	t-ratio	p-value
const	0.133619	0.0346749	3.8535	0.0001
InflationNor_2	−0.124113	0.0573364	−2.1646	0.0314
InflationNor_4	−0.195530	0.0566788	−3.4498	0.0007
InflationNor_6	−0.108975	0.0576065	−1.8917	0.0597
InflationNor_7	0.108233	0.0559782	1.9335	0.0544
InflationNor_12	0.198369	0.0588912	3.3684	0.0009
InflationNor_24	0.316284	0.0598001	5.2890	0.0000

Mean dependent var	0.167616	S.D. dependent var	0.434115
Sum squared resid	33.65763	S.E. of regression	0.376057
R^2	0.268045	Adjusted R^2	0.249592
$F(6, 238)$	14.52606	P-value(F)	4.08e–14
Log-likelihood	−104.4752	Akaike criterion	222.9504
Schwarz criterion	247.4592	Hannan–Quinn	232.8200
$\hat{\rho}$	0.200737	Durbin–Watson	1.594679

Breusch-Pagan test for heteroskedasticity (robust variant) –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 6.56751
with p-value = $P(\chi^2(6) > 6.56751)$ = 0.3627

LM test for autocorrelation up to order 24 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.14039
with p-value = $P(F(24, 214) > 1.14039)$ = 0.301975

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 56.0537
with p-value = 6.73127e-013

The model residuals were tested for autocorrelation, heteroskedasticity, and normality. Those tests allow us to determine the quality of our models. The results of those models are depicted at the bottom of Table 7 and 8. The autocorrelation was tested with Ljung-Box test. For Belgium 12 lags were added to the test while for Norway 24 lags were added, and no trace of autocorrelation was found for both countries. Indeed, the Ljung-Box $p-values$ for Belgium and Norway were respectively $0,19$ and $0,3$. For the heteroskedasticity the White test was performed and it did not find any heteroskedasticity. The test for normality of the residuals was concluding for Belgium, but it was not the case for Norway. However, the assumptions of independence and homoskedasticity are more important than normality. This is pointed out by Box (1976) in his paper *Science and Statistics* as he argued that:

> ... the statistician knows, for example, that in nature there never was a normal distribution, there never was a straight line, yet with normal and linear assumptions, known to be false, he can often derive results which match, to a useful approximation, those found in the real world. (Box, 1976, p. 792)

The plots of the residuals provided in appendix C.1 demonstrate that the mean of the residuals is constant around zero and that they have a constant variance.

5.2 Multivariate time series model

5.2.1 Autoregressive distributed lag model

For the ARDL model, the lags of the four commodity indexes are added. The commodity indexes are the independent variables in the model. The number of commodity lags differs essentially between the CRB commodity index and the commodity indexes from the IMF. After a thorough search for the best model, the lags of the different commodity indexes were chosen for each of the countries. The commodity lags for the CRB are for Belgium 1, 2, 8, and 12 while for Norway they are 12, 15, and 34. For the IMF commodity indexes the lags chosen for Belgium are 1, 7, 11, and 12 and for Norway, those lags are 2, 8, 17, and 24. For the non-fuel commodity index of the IMF (IMFNF) the lags chosen are, for Belgium, 1, 5, 14, 17, and 22. On the opposite, those lags for Norway are 2, and 17. Finally, the lags for the IMFF for Belgium are the same as the IMF while for Norway, those are 1, 8, 23, and 24. Table 9 lists the AIC and BIC of the different ARDL models. For comparison purpose, ARDL models with 12 lags of both the dependent and independent variables have been added. Eugeni and Krueger (1994) tested three multivariate models with three different commodity indexes and for those three models, both the inflation and the index were lagged 12 months.

Table 9: Search for the best model according to the AIC and BIC criterion - multivariate models

Belgium	**AIC**	**SC (BIC)**	**Norway**	**AIC**	**SC (BIC)**
ARDL CRB (1,2,8,12)	32,76	71,76	ARDL CRB (12,15,34)	204,63	239,18
ARDL IMF (1,7,11,12)	24,63	63,62	ARDL IMF (2,8,17,24)	213,85	252,32
ARDL IMFNF (1,5,14,17,22)	34,52	76,58	ARDL IMFNF (2,17)	217,73	249,24
ARDL IMFF (1,7,11,12)	22,32	61,32	ARDL IMFF (1,8,23,24)	209,26	247,73
ARDL (12,12) CRB	50,65	139,28	ARDL (12,12) CRB	269,63	358,26
ARDL (12,12) IMF	39,28	127,91	ARDL (12,12) IMF	262,53	351,16
ARDL (12,12) IMFNF	65,20	153,83	ARDL (12,12) IMFNF	267,66	356,29
ARDL (12,12) IMFF	35,03	123,66	ARDL (12,12) IMFF	260,14	348,77

As for example, the equation of the ARDL model with inclusion of the CRB index for Belgium can be written as follow:

$$\begin{aligned} y_t = \delta + \delta_1 x_{CRB,t-1} + \delta_2 x_{CRB,t-2} + \delta_3 x_{CRB,t-8} + \delta_4 x_{CRB,t-12} \\ + \phi_1 y_{Bel,t-1} + \phi_2 y_{Bel,t-6} + \phi_3 y_{Bel,t-8} + \phi_4 y_{Bel,t-11} + \phi_5 y_{Bel,t-12} + \phi_6 y_{Bel,t-13} + v_t \end{aligned} \quad (18)$$

The majority of the coefficient of the different multivariate models are statistically significant as the $t - ratio$ is higher than 2 and the $p - value$ lower than the significance level of 5%. Heteroscedasticity, autocorrelation, and normality have been tested and the results are displayed on the bottom of Tables 10, 11, 12, and 13 for Belgium and on the bottom of Tables 14, 15, 16, and 17 for Norway. The tests were conclusive, seeing that no trace of heteroscedasticity or autocorralation has been found. However, as for the autoregressive models, the residuals of the ARDL models for Belgium are normally distributed while it is not the case for Norway. This may arise when working with real data (Box, 1976). The plots of the residuals are provided in appendix C.2. As for the autoregressive models, the residuals of our autoregressive distributed lag models have a constant mean around zero and a constant variance.

Table 10: ARDL CRB Belgium

Model 3: OLS, using observations 1993:02–2014:05 (T = 256)
Dependent variable: InflationBel

	Coefficient	Std. Error	t-ratio	p-value
const	0.109382	0.0279392	3.9150	0.0001
d_CRB_1	0.0192303	0.00476910	4.0323	0.0001
d_CRB_2	−0.00387603	0.00478421	−0.8102	0.4186
d_CRB_8	0.00973891	0.00473765	2.0556	0.0409
d_CRB_12	0.00834640	0.00471283	1.7710	0.0778
InflationBel_1	0.146532	0.0611503	2.3963	0.0173
InflationBel_6	0.153834	0.0598540	2.5702	0.0108
InflationBel_8	−0.129902	0.0585769	−2.2176	0.0275
InflationBel_11	0.0521686	0.0598189	0.8721	0.3840
InflationBel_12	0.162744	0.0606408	2.6837	0.0078
InflationBel_13	−0.133031	0.0608930	−2.1847	0.0299

Mean dependent var	0.161996	S.D. dependent var	0.277233
Sum squared resid	15.63246	S.E. of regression	0.252598
R^2	0.202376	Adjusted R^2	0.169820
$F(10, 245)$	6.216238	P-value(F)	1.82e–08
Log-likelihood	−5.382272	Akaike criterion	32.76454
Schwarz criterion	71.76150	Hannan–Quinn	48.44897
$\hat{\rho}$	0.000305	Durbin's h	0.023636

Breusch-Pagan test for heteroskedasticity (robust variant) –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 2.46125
with p-value = $P(\chi^2(10) > 2.46125)$ = 0.991428

LM test for autocorrelation up to order 12 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.38337
with p-value = $P(F(12, 233) > 1.38337)$ = 0.174607

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 1.24537
with p-value = 0.536503

Table 11: ARDL IMF Belgium

Model 4: OLS, using observations 1993:02–2014:05 (T = 256)
Dependent variable: InflationBel

	Coefficient	Std. Error	t-ratio	p-value
const	0.102683	0.0282616	3.6333	0.0003
d_IMF_1	0.0160633	0.00346489	4.6360	0.0000
d_IMF_7	0.0101885	0.00319367	3.1902	0.0016
d_IMF_11	0.00926410	0.00344814	2.6867	0.0077
d_IMF_12	−0.00789598	0.00356899	−2.2124	0.0279
InflationBel_1	0.0229996	0.0674935	0.3408	0.7336
InflationBel_6	0.116708	0.0605318	1.9280	0.0550
InflationBel_8	−0.0846179	0.0587116	−1.4412	0.1508
InflationBel_11	0.0719643	0.0635857	1.1318	0.2588
InflationBel_12	0.222767	0.0654128	3.4056	0.0008
InflationBel_13	−0.0814720	0.0595574	−1.3680	0.1726

Mean dependent var	0.161996	S.D. dependent var	0.277233
Sum squared resid	15.14325	S.E. of regression	0.248615
R^2	0.227337	Adjusted R^2	0.195800
$F(10, 245)$	7.208530	P-value(F)	5.78e–10
Log-likelihood	−1.312602	Akaike criterion	24.62520
Schwarz criterion	63.62216	Hannan–Quinn	40.30963
$\hat{\rho}$	−0.036214	Durbin–Watson	2.066794

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 73.1698
with p-value = $P(\chi^2(65) > 73.1698)$ = 0.227692

LM test for autocorrelation up to order 12 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.55002
with p-value = $P(F(12, 233) > 1.55002)$ = 0.107597

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 3.4701
with p-value = 0.176391

Table 12: ARDL IMFNF Belgium

Model 5: OLS, using observations 1993:12–2014:05 (T = 246)
Dependent variable: InflationBel

	Coefficient	Std. Error	t-ratio	p-value
const	0.108294	0.0281948	3.8409	0.0002
d_IMFNF_1	0.0133986	0.00493622	2.7143	0.0071
d_IMFNF_5	0.00973604	0.00466446	2.0873	0.0379
d_IMFNF_14	0.0150338	0.00488517	3.0774	0.0023
d_IMFNF_17	−0.00826028	0.00455950	−1.8117	0.0713
d_IMFNF_22	0.0149424	0.00471611	3.1684	0.0017
InflationBel_1	0.112728	0.0630904	1.7868	0.0753
InflationBel_6	0.168635	0.0608352	2.7720	0.0060
InflationBel_8	−0.127987	0.0602375	−2.1247	0.0347
InflationBel_11	0.104710	0.0599487	1.7467	0.0820
InflationBel_12	0.147428	0.0614477	2.3992	0.0172
InflationBel_13	−0.179480	0.0604740	−2.9679	0.0033

Mean dependent var	0.161124	S.D. dependent var	0.279371
Sum squared resid	15.03238	S.E. of regression	0.253458
R^2	0.213862	Adjusted R^2	0.176907
$F(11, 234)$	5.787073	P-value(F)	2.85e–08
Log-likelihood	−5.258502	Akaike criterion	34.51700
Schwarz criterion	76.58098	Hannan–Quinn	51.45421
$\hat{\rho}$	−0.047718	Durbin's h	−5.186660

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 73.0133
with p-value = $P(\chi^2(77) > 73.0133)$ = 0.607569

LM test for autocorrelation up to order 12 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.12595
with p-value = $P(F(12, 222) > 1.12595)$ = 0.340054

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 4.32504
with p-value = 0.115035

Table 13: ARDL IMFF Belgium

Model 6: OLS, using observations 1993:02–2014:05 (T = 256)
Dependent variable: InflationBel

	Coefficient	Std. Error	t-ratio	p-value
const	0.104875	0.0282177	3.7166	0.0003
d_IMFF_1	0.0117741	0.00259943	4.5295	0.0000
d_IMFF_7	0.00766256	0.00238144	3.2176	0.0015
d_IMFF_11	0.00754310	0.00258232	2.9211	0.0038
d_IMFF_12	−0.00741151	0.00267549	−2.7702	0.0060
InflationBel_1	0.0276318	0.0672321	0.4110	0.6814
InflationBel_6	0.108600	0.0598263	1.8153	0.0707
InflationBel_8	−0.0908075	0.0582564	−1.5588	0.1203
InflationBel_11	0.0762703	0.0635998	1.1992	0.2316
InflationBel_12	0.226820	0.0646227	3.5099	0.0005
InflationBel_13	−0.0809216	0.0591704	−1.3676	0.1727

Mean dependent var	0.161996	S.D. dependent var	0.277233
Sum squared resid	15.00762	S.E. of regression	0.247499
R^2	0.234258	Adjusted R^2	0.203003
$F(10, 245)$	7.495098	P-value(F)	2.15e–10
Log-likelihood	−0.160989	Akaike criterion	22.32198
Schwarz criterion	61.31893	Hannan–Quinn	38.00641
$\hat{\rho}$	−0.040364	Durbin–Watson	2.075339

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 74.9491
with p-value = $P(\chi^2(65) > 74.9491)$ = 0.186874

LM test for autocorrelation up to order 12 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.52131
with p-value = $P(F(12, 233) > 1.52131)$ = 0.117254

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 2.83067
with p-value = 0.242844

Table 14: ARDL CRB Norway

Model 7: OLS, using observations 1994:12–2014:05 (T = 234)
Dependent variable: InflationNor

	Coefficient	Std. Error	t-ratio	p-value
const	0.130063	0.0348562	3.7314	0.0002
d_CRB_12	0.0115668	0.00656593	1.7616	0.0795
d_CRB_15	−0.0242197	0.00661496	−3.6614	0.0003
d_CRB_34	0.0200524	0.00707201	2.8355	0.0050
InflationNor_2	−0.145176	0.0566119	−2.5644	0.0110
InflationNor_4	−0.218105	0.0562461	−3.8777	0.0001
InflationNor_6	−0.111216	0.0567940	−1.9582	0.0514
InflationNor_7	0.142337	0.0554611	2.5664	0.0109
InflationNor_12	0.214402	0.0579914	3.6971	0.0003
InflationNor_24	0.310949	0.0588229	5.2862	0.0000

Mean dependent var	0.167561	S.D. dependent var	0.441990
Sum squared resid	30.15842	S.E. of regression	0.366928
R^2	0.337436	Adjusted R^2	0.310815
$F(9, 224)$	12.67563	P-value(F)	3.09e–16
Log-likelihood	−92.31535	Akaike criterion	204.6307
Schwarz criterion	239.1839	Hannan–Quinn	218.5625
$\hat{\rho}$	0.202701	Durbin–Watson	1.594080

White's test for heteroskedasticity –
  Null hypothesis: heteroskedasticity not present
  Test statistic: LM = 37.5487
  with p-value = $P(\chi^2(54) > 37.5487) = 0.956759$

LM test for autocorrelation up to order 24 –
  Null hypothesis: no autocorrelation
  Test statistic: LMF = 1.27852
  with p-value = $P(F(24, 200) > 1.27852) = 0.182049$

Test for normality of residual –
  Null hypothesis: error is normally distributed
  Test statistic: $\chi^2(2) = 60.6817$
  with p-value = 6.65487e-14

Table 15: ARDL IMF Norway

Model 8: OLS, using observations 1994:02–2014:05 (T = 244)
Dependent variable: InflationNor

	Coefficient	Std. Error	t-ratio	p-value
const	0.139684	0.0342418	4.0793	0.0001
d_IMF_2	0.00946351	0.00432951	2.1858	0.0298
d_IMF_8	0.00725789	0.00442518	1.6401	0.1023
d_IMF_17	−0.00899429	0.00440604	−2.0414	0.0423
d_IMF_24	−0.00832743	0.00451848	−1.8430	0.0666
InflationNor_2	−0.142526	0.0570377	−2.4988	0.0132
InflationNor_4	−0.173185	0.0563886	−3.0713	0.0024
InflationNor_6	−0.124180	0.0570066	−2.1783	0.0304
InflationNor_7	0.0982586	0.0553385	1.7756	0.0771
InflationNor_12	0.192315	0.0580250	3.3143	0.0011
InflationNor_24	0.316825	0.0600677	5.2745	0.0000

Mean dependent var	0.169205	S.D. dependent var	0.434293
Sum squared resid	31.63972	S.E. of regression	0.368501
R^2	0.309663	Adjusted R^2	0.280035
$F(10, 233)$	10.45163	P-value(F)	1.48e–14
Log-likelihood	−97.00492	Akaike criterion	216.0098
Schwarz criterion	254.4787	Hannan–Quinn	231.5030
$\hat{\rho}$	0.186337	Durbin–Watson	1.626929

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 48.6849
with p-value = $P(\chi^2(65) > 48.6849)$ = 0.93475

LM test for autocorrelation up to order 24 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 0.948727
with p-value = $P(F(24, 209) > 0.948727)$ = 0.535843

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 67.0522
with p-value = 2.75302e-15

Table 16: ARDL IMFNF Norway

Model 9: OLS, using observations 1994:01–2014:05 (T = 245)
Dependent variable: InflationNor

	Coefficient	Std. Error	t-ratio	p-value
const	0.144844	0.0345066	4.1976	0.0000
d_IMFNF_2	0.0135875	0.00657068	2.0679	0.0397
d_IMFNF_17	−0.0157614	0.00661884	−2.3813	0.0180
InflationNor_2	−0.149920	0.0572185	−2.6201	0.0094
InflationNor_4	−0.204458	0.0560269	−3.6493	0.0003
InflationNor_6	−0.128453	0.0571890	−2.2461	0.0256
InflationNor_7	0.120764	0.0554219	2.1790	0.0303
InflationNor_12	0.192039	0.0580822	3.3063	0.0011
InflationNor_24	0.301413	0.0593410	5.0793	0.0000

Mean dependent var	0.167616	S.D. dependent var	0.434115
Sum squared resid	32.41497	S.E. of regression	0.370610
R^2	0.295069	Adjusted R^2	0.271173
$F(8, 236)$	12.34805	P-value(F)	9.48e–15
Log-likelihood	−99.86681	Akaike criterion	217.7336
Schwarz criterion	249.2449	Hannan–Quinn	230.4232
$\hat{\rho}$	0.182341	Durbin–Watson	1.631734

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 32.3663
with p-value = $P(\chi^2(44) > 32.3663)$ = 0.902746

LM test for autocorrelation up to order 24 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 0.844691
with p-value = $P(F(24, 212) > 0.844691)$ = 0.676816

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 60.5962
with p-value = 6.94536e-14

Table 17: ARDL IMFF Norway

Model 10: OLS, using observations 1994:02–2014:05 (T = 244)
Dependent variable: InflationNor

	Coefficient	Std. Error	t-ratio	p-value
const	0.110368	0.0342618	3.2213	0.0015
d_IMFF_1	0.00730923	0.00323186	2.2616	0.0246
d_IMFF_8	0.00648597	0.00320885	2.0213	0.0444
d_IMFF_23	0.0106240	0.00370668	2.8662	0.0045
d_IMFF_24	−0.0118583	0.00380985	−3.1125	0.0021
InflationNor_2	−0.0999766	0.0561480	−1.7806	0.0763
InflationNor_4	−0.157028	0.0561389	−2.7971	0.0056
InflationNor_6	−0.0928747	0.0562755	−1.6504	0.1002
InflationNor_7	0.110724	0.0548967	2.0169	0.0448
InflationNor_12	0.180143	0.0579206	3.1102	0.0021
InflationNor_24	0.357277	0.0594701	6.0077	0.0000

Mean dependent var	0.169205	S.D. dependent var	0.434293
Sum squared resid	30.77653	S.E. of regression	0.363439
R^2	0.328497	Adjusted R^2	0.299677
$F(10, 233)$	11.39827	P-value(F)	7.41e–16
Log-likelihood	−93.63026	Akaike criterion	209.2605
Schwarz criterion	247.7294	Hannan–Quinn	224.7536
$\hat{\rho}$	0.195511	Durbin–Watson	1.608838

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 52.4184
with p-value = $P(\chi^2(65) > 52.4184)$ = 0.869594

LM test for autocorrelation up to order 24 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.19563
with p-value = $P(F(24, 209) > 1.19563)$ = 0.248614

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 64.1284
with p-value = 1.18768e-14

6 Results

The inflation previsions of the different models are depicted in appendix D. Tables 18 and 19 provide the RMSE and the relative RMSE. A low RMSE value indicates a good performance of the forecasting model. The relative RMSE aims at comparing the errors of the multivariate models from the AR(1) benchmark. If the value is higher than 1, it means that the model was unable to beat the benchmark. However, if the value is lower than 1, the model provided better forecast results compared to the benchmark. Table 37 in appendix E resumes all the forecasting errors, which are MSE, MAE, and RMSE, for all the time horizons.

Table 18: RMSE of the forecasts

RMSE	**Belgium**				**Norway**			
	1-month	3-month	6-month	12-month	1-month	3-month	6-month	12-month
AR(1)	0,040	0,312	0,259	0,238	0,379	0,505	0,395	0,308
AO	0,051	0,247	0,195	0,235	0,368	0,479	0,370	0,294
AR(6)	0,059	0,294	0,248	0,246	0,020	0,430	0,333	0,255
ARDL CRB	0,017	0,251	0,217	0,230	0,095	0,595	0,438	0,335
ARDL IMF	0,086	0,202	0,183	0,268	0,110	0,502	0,382	0,294
ARDL IMFNF	0,107	0,314	0,257	0,263	0,000	0,462	0,349	0,274
ARDL IMFF	0,117	0,201	0,174	0,266	0,338	0,460	0,362	0,282

Table 19: Relative RMSE of the forecasts

Rel. RMSE	**Belgium**				**Norway**			
	1-month	3-month	6-month	12-month	1-month	3-month	6-month	12-month
AR(1)	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
AO	1,263	0,793	0,755	0,986	0,969	0,949	0,937	0,954
AR(6)	1,457	0,942	0,959	1,035	0,054	**0,851**	**0,842**	**0,827**
ARDL CRB	**0,421**	0,804	0,838	**0,965**	0,251	1,178	1,108	1,088
ARDL IMF	2,141	0,649	0,707	1,128	0,291	0,995	0,967	0,954
ARDL IMFNF	2,668	1,006	0,992	1,106	**0,001**	0,914	0,884	0,889
ARDL IMFF	2,900	**0,645**	**0,674**	1,120	0,892	0,910	0,916	0,914

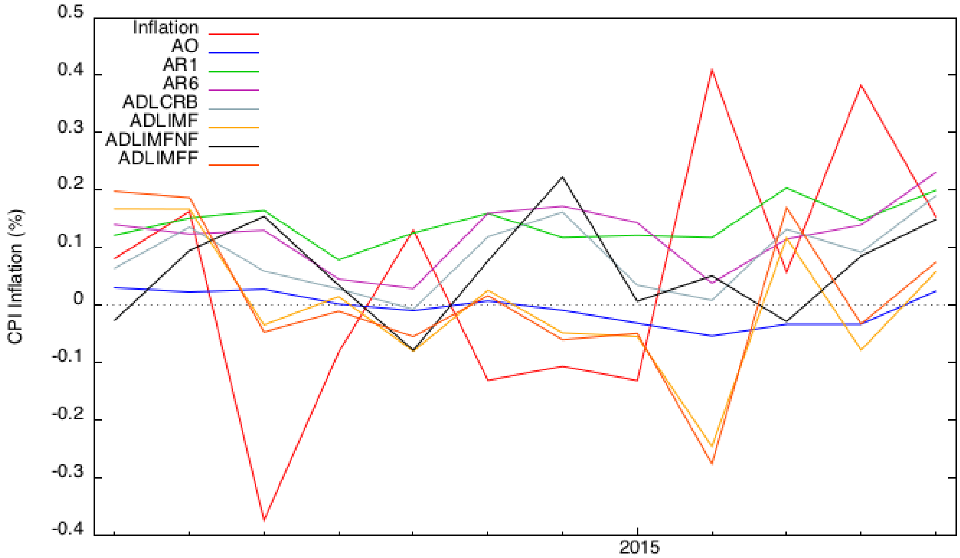
The values in bold in table 19 represent the forecast models with the lowest RMSE. Firstly, it seems that the AO benchmark improved slighlty over the AR benchmark. Indeed, except for the 1-month forecast of Belgium, the AO model provides better results than the AR both for Belgium and Norway at all the time horizons. This is consonant with Stock and Watson (2008) who demonstrated that the AO model provided better forecast results than the AR benchmark for the period of 1984-1999.

Secondly, if we take the case of Belgium, it seems that the multivariate models perform better than the three univariate models at all the forecast horizons. Indeed, for the 1 and 12-months horizon, the autoregressive distributed lag model with the CRB commodity index performs the best. While for the 3 and 6-months horizon, it is the autoregressive distributed lag model with IMF fuel price index which beats the other models and the univariate benchmark models. However, if we compared those results with the Atkeson & Ohanian model, the improvements are rather small except for the 1-month horizon. Indeed, at the 3-month horizon, the ARDL IMFF is 0,202 while it is 0,247 for the AO model. At the 6-month horizon, the ARDL IMFF is 0,174 while it is 0,195 for the AO. And at the 12-month horizon, we improve the accuracy from 0,235 (AO) to 0,230 (ARDL CRB). It is as stated only for the 1-month horizon that the RMSE for the ARDL CRB is two times lower than the RMSE of the AR(1), while all the other models are unable to improve the forecast accuracy at 1-month horizon. Therefore, the improvements does not seem significant enough to be able to conclude that the inclusion of commodity indexes in multivariate models allows to obtain better forecast results than the univariate benchmark models and especially the Atkeson & Ohanian model.

In the case of Norway, the results are even clearer. Indeed, besides the ARDL IMFNF which performs exceptionally well at the 1-month horizon, it is the AR(6) model with specific lags which performs the best in the three other time horizons. Furthermore, because of the important role played by commodities in the domestic economy of Norway, one would have thought than the IMF fuel price index would provide better and more accurate forecast results. Indeed, the IMFF commodity index is only composed of energy commodities such as spot crude petroleum, natural gas, and coal (see Table 39). And the three most exported products for Norway are crude petroleum, petroleum gas, and refined petroleum, the three of them account for more than 67% of the total Norwegian exportation (see Table 1). Therefore, the second point of the theory of Blomberg and Harris (1995) could have been applied in this case. As a reminder, the link between commodities and consumer price index comes from the fact that commodities represent a valuable input in the production of the country.

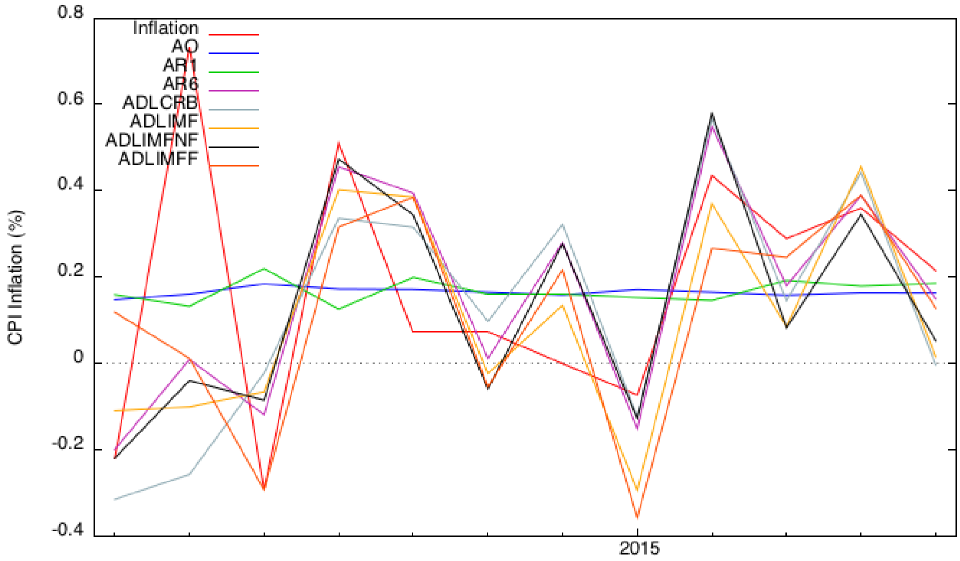
However, it was not the case, all the multivariate models with the IMF, IMFNF, and IMFF index were able to perform slightly better than the AR(1) benchmark but were not able to beat the AO model results. And it was, the ARDL IMFNF model which provides the best results amongst the multivariate with commodity index for the International Monetary Fund. Moreover, the ARDL CRB model was not even able to perform better than the AR(1) benchmark.

Figure 14: Forecast results of all the models for Belgium



Figures 14 and 15 depict the forecast results for all the models tested. The red line in both graphics represents the actual value of the CPI inflation. From an empirical point of view and after observation of both graphics, it seems that the forecast models are able to capture more easily changes in the Norwegian CPI inflation curve than changes in the Belgian one. The individual graphics are available in appendix D.3. Only the forecast model results and the actual CPI inflation values are represented in those graphics. The 95% confidence intervals are also incorporated in the graphics as error bars. From those graphics it is important to mention that the actual value of the CPI inflation is always included in the 95% confidence interval.

Figure 15: Forecast results of all the models for Norway



Therefore, the conclusions applied to the Belgium's case are the same for Norway. Multivariate autoregressive distributed lag models with the inclusion of commodity indexes are not able to produce better and significant forecasting results than univariate models.

However, the improvements are not strong enough to be considerate as significant. The difference between actual inflation and the forecasted value of inflation for all the different models is rather small. Thus, aside some exceptions, forecast errors for the multivariate models follow the same trend and is almost identical as the forecast errors of the univariate models.

To conclude, as Eugeni and Krueger (1994) explained: "Economic indicators have value only to the extent that they possess unique and independent information. In addition, they can be useful forecasting tools if they reliably and consistently satisfy the purpose for which they were designed." (Eugeni & Krueger, 1994, p. 3). In this case, the economic indicators chosen, commodity indexes, included in our forecast model were not able to significantly improve the forecast accuracy provided by the univariate models.

7 Conclusion

As mentioned, Central Banks rely heavily on forecasted inflation value to establish their monetary policy. Therefore, it is crucial to develop models that are the most accurate as possible. Several ideas came up in the past in order to reinforce those forecasting models. Frequently, those ideas relied on the inclusion of an economic indicator bound with CPI inflation and able to procure more predictive power than past values of inflation used in univariate models.

One of those economic indicators are commodities. Many past researches or papers focused on the inclusion of commodity indexes in forecasting models. It has been found that commodity indexes possessed predictive power in the past. Indeed, forecast models with commodity indexes were able to positively predict inflation movements in the 1970s and early 1980s (Blomberg & Harris, 1995). However, it has been proven that those predictive powers have been lost.

In this paper, we try to reinvestigate the link between commodities and consumer price index by focusing on forecasting CPI inflation for two small european economy countries. Our aim is to test whether the inclusion of commodity indexes in multivariate autoregressive distributed lag models produced better and more accurate forecast than univariate models.

Four models have been produced, three univariate models and one multivariate model. Amongst, the univariate models, an AR(1), an Atkeson & Ohanian model, and an AR with 6 specific lags have been fitted by OLS. Autoregressive distributed lag models with the commodity indexes have also been fitted by OLS. In order to compare the accuracy of our different models, we relied on the root mean squared error.

Our conclusion was that multivariate models produced better forecast than univariate models, but those improvement were not statistically significant. While for Norway, the multivariate models were not able to beat the results of the univariate benchmark models. Regarding the univariate models, Stock and Watson (2008) found that the AO model performed better than the

AR(1) model over the 1984-1999 period in the U.S.. Our results follow the same path for Belgium and Norway, as the AO model beats the AR(1) at every time horizons except at 1-month for Belgium.

Therefore, the results are in line with most of the studies performed in the past. A link clearly exists between commodities and consumer price (Blomberg & Harris, 1995), however this link is not strong enough to produce significant improvement in forecast accuracy compared to univariate models. Consequently, as seen as the use of multivariate models does not seem to improve the inflation forecast accuracy, it would maybe be better to focus solely on more elaborate univariate models such as the unobserved components-stochastic volatility (UC-SV) model developed by Stock and Watson (2006) which provided good results.

7.1 Limitations of the study

The models used in this thesis are rather simple and straightforward. However, past studies have demonstrated that the used of more complex models helps to achieve better results. Edelstein (2007), Browne and Cronin (2010), and Gospodinov and Ng (2011) are good examples.

Furthermore, only pseudo-out-of-sample forecasts have been performed in this paper. Even though, pseudo out-of-sample forecast is the standard in order to test the forecasting models, out-of-sample forecasts need to be performed in order to assess the robustness of the models. However, this method is time consuming and the lack of time prevented us from achieving it.

Finally, only generic index from the Commodity Research Bureau and the International Monetary Fund has been used. The use of sub-indexes or specific commodities could have produced better forecast. For exmple, Chen et al. (2014) used the seven sub-indexes provided by the CRB, Balcilar et al. (2015) focused on the predictive power of precious metal in South Africa, and, Blomberg and Harris (1995) used some generic commodity index as well as gold, food, and oil.

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A KPSS tests results

Table 20: KPSS results for CPI inflation for Belgium and Norway

KPSS test for InflationBel	**KPSS test for InflationNor**
T = 281 Lag truncation parameter = 15 Test statistic = 0.073647 Critical values: 0.348 (10%) 0.462 (5%) 0.741 (1%) P-value > .10	T = 281 Lag truncation parameter = 15 Test statistic = 0.0704348 Critical values: 0.348 (10%) 0.462 (5%) 0.741 (1%) P-value > .10

Table 21: KPSS results for the 4 commodity indexes

KPSS test for CRB (including trend)	**KPSS test for IMF (including trend)**
T = 281 Lag truncation parameter = 15 Test statistic = 0.302957 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value < .01	T = 281 Lag truncation parameter = 15 Test statistic = 0.263475 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value < .01
KPSS test for IMFNF (including trend)	**KPSS test for IMFF (including trend)**
T = 281 Lag truncation parameter = 15 Test statistic = 0.308057 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value < .01	T = 281 Lag truncation parameter = 15 Test statistic = 0.227928 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value < .01

Table 22: KPSS results for the first difference of the 4 commodity indexes

KPSS test for d_CRB (including trend)	**KPSS test for d_IMF (including trend)**
T = 280 Lag truncation parameter = 15 Test statistic = 0.0775038 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value > .10	T = 280 Lag truncation parameter = 15 Test statistic = 0.0960095 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value > .10
KPSS test for d_IMFNF (including trend)	**KPSS test for d_IMFF (including trend)**
T = 280 Lag truncation parameter = 15 Test statistic = 0.0983624 Critical values: 0.120 (10%) 0.148 (5%) 0.217 (1%) P-value > .10	T = 280 Lag truncation parameter = 15 Test statistic = 0.0877195 Critical values: 0.120 (10%) 0.148 (5%) 0.217(1%) P-value > .10

B Benchmark models

Table 23: AR(1) process for Belgium

Model 11: OLS, using observations 1992:02–2014:05 (T = 268)
Dependent variable: InflationBel

	Coefficient	Std. Error	t-ratio	p-value
const	0.138563	0.0194478	7.1248	0.0000
InflationBel_1	0.159958	0.0606262	2.6384	0.0088

Mean dependent var	0.165203	S.D. dependent var	0.275123
Sum squared resid	19.69453	S.E. of regression	0.272102
R^2	0.025503	Adjusted R^2	0.021840
$F(1, 266)$	6.961362	P-value(F)	0.008820
Log-likelihood	−30.44894	Akaike criterion	64.89788
Schwarz criterion	72.07985	Hannan–Quinn	67.78250
$\hat{\rho}$	−0.000242	Durbin's h	−0.032437

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 2.66043
with p-value = $P(\chi^2(2) > 2.66043)$ = 0.264421

LM test for autocorrelation up to order 11 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.5396
with p-value = $P(F(11, 255) > 1.5396)$ = 0.117591

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 1.91326
with p-value = 0.384185

Table 24: AR(1) process for Norway

Model 12: OLS, using observations 1992:02–2014:05 (T = 268)
Dependent variable: InflationNor

	Coefficient	Std. Error	t-ratio	p-value
const	0.152998	0.0277839	5.5067	0.0000
InflationNor_1	0.0913873	0.0610450	1.4970	0.1356

Mean dependent var	0.168359	S.D. dependent var	0.423670
Sum squared resid	47.52508	S.E. of regression	0.422689
R^2	0.008355	Adjusted R^2	0.004627
$F(1,266)$	2.241154	P-value(F)	0.135566
Log-likelihood	-148.4918	Akaike criterion	300.9836
Schwarz criterion	308.1656	Hannan–Quinn	303.8682
$\hat{\rho}$	0.009566	Durbin's h	4.342444

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 1.27417
with p-value = $P(\chi^2(2) > 1.27417)$ = 0.528831

LM test for autocorrelation up to order 3 –
Null hypothesis: no autocorrelation
Test statistic: LMF = 1.39321
with p-value = $P(F(3,263) > 1.39321)$ = 0.245243

Test for normality of residual –
Null hypothesis: error is normally distributed
Test statistic: $\chi^2(2)$ = 42.9542
with p-value = 4.70549e-10

C Residuals plot from the OLS models

C.1 Autoregressive models

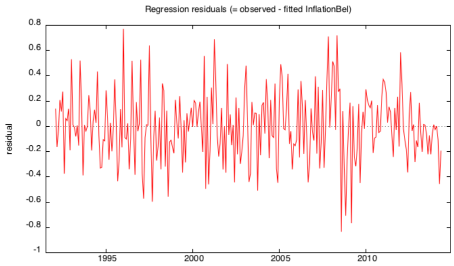

Figure 16: Residual plot from the AR(1) model for Belgium

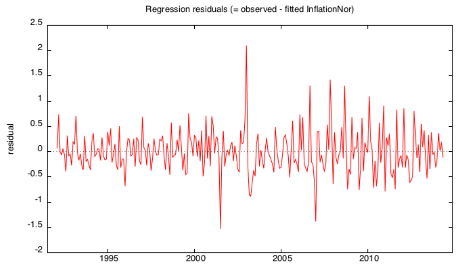

Figure 17: Residual plot from the AR(1) model for Norway

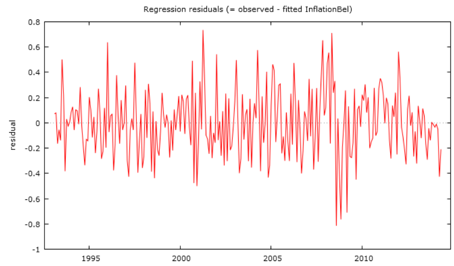

Figure 18: Residual plot from the AR(6) model for Belgium

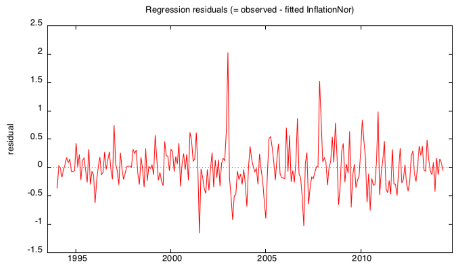

Figure 19: Residual plot from the AR(6) model for Norway

C.2 Autoregressive distributed lag models

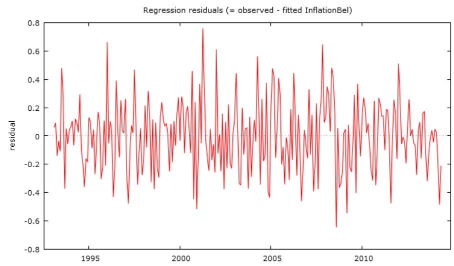

Figure 20: Residual plot from the ARDL CRB model for Belgium

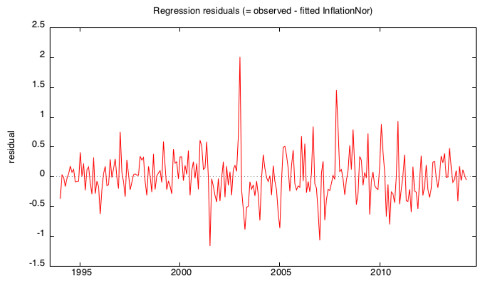

Figure 21: Residual plot from the ARDL CRB model for Norway

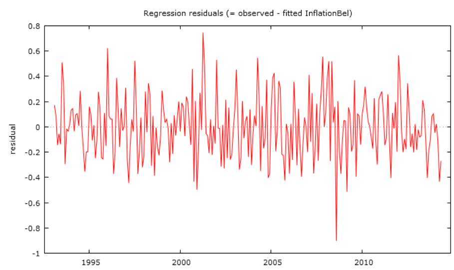

Figure 22: Residual plot from the ARDL IMF model for Belgium

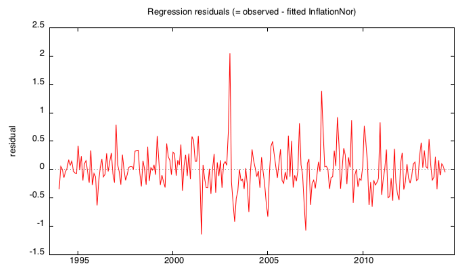

Figure 23: Residual plot from the IMF ARDL model for Norway

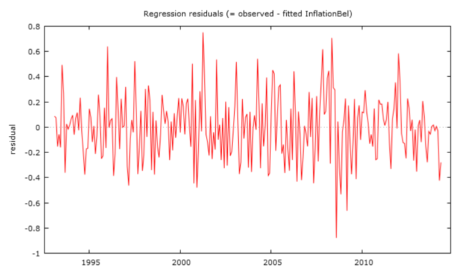

Figure 24: Residual plot from the ARDL IMFNF model for Belgium

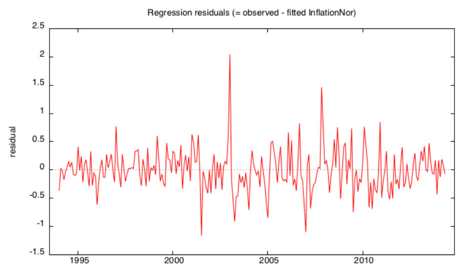

Figure 25: Residual plot from the ARDL IMFNF model for Norway

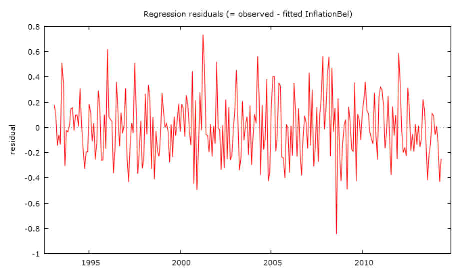

Figure 26: Residual plot from the ARDL IMFF model for Belgium

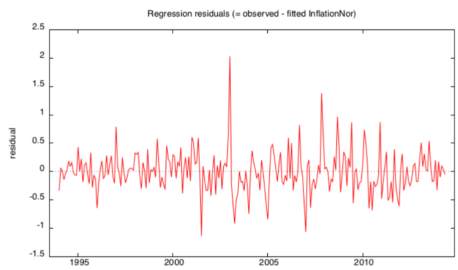

Figure 27: Residual plot from the ARDL IMFF model for Norway

D Forecast results

D.1 Belgium

D.1.1 AR(1) model

Table 25: CPI inflation forecast for Belgium with AR(1)

For 95% confidence intervals, $t(266, 0.025) = 1.969$

Obs	InflationBel	prediction	std. error	95% interval	
2014:06	0.081453	0.121643	0.273109	−0.416087	0.659373
2014:07	0.162774	0.151592	0.272658	−0.385251	0.688434
2014:08	−0.373771	0.164600	0.272609	−0.372147	0.701346
2014:09	−0.081559	0.078775	0.274570	−0.461833	0.619382
2014:10	0.130602	0.125516	0.273024	−0.412047	0.663079
2014:11	−0.130431	0.159453	0.272618	−0.377310	0.696217
2014:12	−0.106114	0.117699	0.273203	−0.420217	0.655615
2015:01	−0.130740	0.121589	0.273110	−0.416144	0.659321
2015:02	0.409098	0.117650	0.273204	−0.420269	0.655568
2015:03	0.057040	0.204001	0.273006	−0.333526	0.741528
2015:04	0.382767	0.147687	0.272690	−0.389219	0.684592
2015:05	0.154146	0.199789	0.272924	−0.337577	0.737156

Forecast evaluation statistics

Mean Error	−0.096227
Mean Squared Error	0.056583
Root Mean Squared Error	0.23787
Mean Absolute Error	0.18669
Mean Percentage Error	64.468
Mean Absolute Percentage Error	120.57
Theil's U	0.79457
Bias proportion, U^M	0.16365
Regression proportion, U^R	0.01621
Disturbance proportion, U^D	0.82014

D.1.2 AO model

D.1.3 AR(6) model

Table 26: CPI inflation forecast for Belgium with AR(6)

For 95% confidence intervals, $t(249, 0.025) = 1.970$

Obs	InflationBel	prediction	std. error	95% interval	
2014:06	0.081453	0.139999	0.265052	−0.382030	0.662028
2014:07	0.162774	0.123860	0.264657	−0.397392	0.645112
2014:08	−0.373771	0.130218	0.264577	−0.390877	0.651312
2014:09	−0.081559	0.045398	0.266794	−0.480063	0.570859
2014:10	0.130602	0.029005	0.266382	−0.495644	0.553653
2014:11	−0.130431	0.159542	0.265138	−0.362657	0.681740
2014:12	−0.106114	0.172170	0.266319	−0.352354	0.696695
2015:01	−0.130740	0.143450	0.265130	−0.378733	0.665634
2015:02	0.409098	0.038708	0.267043	−0.487243	0.564660
2015:03	0.057040	0.114935	0.266490	−0.409926	0.639797
2015:04	0.382767	0.139551	0.268185	−0.388649	0.667751
2015:05	0.154146	0.230813	0.267631	−0.296295	0.757922

Forecast evaluation statistics

Mean Error	−0.076032
Mean Squared Error	0.060639
Root Mean Squared Error	0.24625
Mean Absolute Error	0.20172
Mean Percentage Error	84.788
Mean Absolute Percentage Error	121.97
Theil's U	0.85305
Bias proportion, U^M	0.095333
Regression proportion, U^R	0.17428
Disturbance proportion, U^D	0.73039

D.1.4 ARDL models

Table 27: Belgian CPI inflation forecast with CRB

For 95% confidence intervals, $t(245, 0.025) = 1.970$

Obs	InflationBel	prediction	std. error	95% interval	
2014:06	0.081453	0.064518	0.254758	−0.437277	0.566313
2014:07	0.162774	0.135804	0.253896	−0.364293	0.635901
2014:08	−0.373771	0.059336	0.254745	−0.442434	0.561106
2014:09	−0.081559	0.028383	0.255909	−0.475681	0.532446
2014:10	0.130602	−0.006761	0.258531	−0.515988	0.502467
2014:11	−0.130431	0.119057	0.257211	−0.387570	0.625683
2014:12	−0.106114	0.161683	0.257189	−0.344901	0.668267
2015:01	−0.130740	0.034969	0.255654	−0.468592	0.538529
2015:02	0.409098	0.008591	0.261989	−0.507448	0.524629
2015:03	0.057040	0.132226	0.260216	−0.380319	0.644771
2015:04	0.382767	0.092109	0.258687	−0.417426	0.601644
2015:05	0.154146	0.189993	0.258774	−0.319713	0.699699

Forecast evaluation statistics

Mean Error	−0.03872
Mean Squared Error	0.052733
Root Mean Squared Error	0.22964
Mean Absolute Error	0.18413
Mean Percentage Error	81.865
Mean Absolute Percentage Error	107.71
Theil's U	0.86796
Bias proportion, U^M	0.028431
Regression proportion, U^R	0.092882
Disturbance proportion, U^D	0.87869

Table 28: Belgian CPI inflation forecast with IMF

For 95% confidence intervals, $t(243, 0.025) = 1.970$

Obs	InflationBel	prediction	std. error	95% interval	
2014:06	0.081453	0.128851	0.255304	−0.374040	0.631741
2014:07	0.162774	0.158143	0.255943	−0.346008	0.662294
2014:08	−0.373771	0.002064	0.256894	−0.503959	0.508086
2014:09	−0.081559	0.031478	0.257872	−0.476472	0.539429
2014:10	0.130602	−0.084210	0.258284	−0.592971	0.424551
2014:11	−0.130431	−0.043439	0.258933	−0.553479	0.466600
2014:12	−0.106114	0.019606	0.259369	−0.491292	0.530504
2015:01	−0.130740	−0.118390	0.261785	−0.634047	0.397267
2015:02	0.409098	−0.251835	0.265460	−0.774731	0.271061
2015:03	0.057040	0.033084	0.266511	−0.491881	0.558050
2015:04	0.382767	−0.036752	0.264106	−0.556980	0.483477
2015:05	0.154146	0.014064	0.263362	−0.504700	0.532828

Forecast evaluation statistics

Mean Error	0.05855
Mean Squared Error	0.071583
Root Mean Squared Error	0.26755
Mean Absolute Error	0.18544
Mean Percentage Error	78.911
Mean Absolute Percentage Error	88.61
Theil's U	1.1237
Bias proportion, U^M	0.04789
Regression proportion, U^R	0.34162
Disturbance proportion, U^D	0.61049

Table 29: Belgian CPI inflation forecast with IMFNF

For 95% confidence intervals, $t(244, 0.025) = 1.970$

Obs	InflationBel	prediction	std. error	95% interval	
2014:06	0.081453	0.083753	0.263727	−0.435720	0.603226
2014:07	0.162774	0.076065	0.262718	−0.441419	0.593549
2014:08	−0.373771	0.062918	0.263383	−0.455877	0.581712
2014:09	−0.081559	0.063316	0.264692	−0.458057	0.584690
2014:10	0.130602	−0.070672	0.266428	−0.595463	0.454120
2014:11	−0.130431	0.083110	0.263939	−0.436781	0.603000
2014:12	−0.106114	0.129880	0.265004	−0.392107	0.651867
2015:01	−0.130740	0.091981	0.263468	−0.426980	0.610943
2015:02	0.409098	−0.045343	0.266682	−0.570635	0.479949
2015:03	0.057040	−0.041554	0.267252	−0.567970	0.484862
2015:04	0.382767	−0.000786	0.269586	−0.531799	0.530227
2015:05	0.154146	0.120607	0.268744	−0.408748	0.649962

Forecast evaluation statistics

Mean Error	0.00016581
Mean Squared Error	0.064591
Root Mean Squared Error	0.25415
Mean Absolute Error	0.20952
Mean Percentage Error	121.78
Mean Absolute Percentage Error	122.25
Theil's U	1.0539
Bias proportion, U^M	4.2562e-007
Regression proportion, U^R	0.45862
Disturbance proportion, U^D	0.54138

Table 30: Belgian CPI inflation forecast with IMFF

For 95% confidence intervals, $t(116, 0.025) = 1.981$

Obs	InflationBel	prediction	std. error	95% interval	
2014:06	0.081453	0.154258	0.268143	−0.376833	0.685349
2014:07	0.162774	0.199194	0.268512	−0.332627	0.731016
2014:08	−0.373771	0.028875	0.270152	−0.506196	0.563946
2014:09	−0.081559	0.062587	0.271213	−0.474585	0.599758
2014:10	0.130602	0.134178	0.274599	−0.409700	0.678055
2014:11	−0.130431	0.061527	0.279650	−0.492356	0.615410
2014:12	−0.106114	0.037621	0.275868	−0.508770	0.584012
2015:01	−0.130740	−0.157414	0.281399	−0.714761	0.399932
2015:02	0.409098	−0.168111	0.284671	−0.731938	0.395716
2015:03	0.057040	0.198237	0.282119	−0.360534	0.757009
2015:04	0.382767	0.053867	0.280089	−0.500885	0.608619
2015:05	0.154146	0.072282	0.279058	−0.480427	0.624991

Forecast evaluation statistics

Mean Error	−0.010153
Mean Squared Error	0.059645
Root Mean Squared Error	0.24422
Mean Absolute Error	0.17926
Mean Percentage Error	38.732
Mean Absolute Percentage Error	102.47
Theil's U	0.92081
Bias proportion, U^M	0.0017283
Regression proportion, U^R	0.21995
Disturbance proportion, U^D	0.77832

D.2 Norway

D.2.1 AR(1) model

Table 31: CPI inflation forecast for Norway with AR(1)

For 95% confidence intervals, $t(266, 0.025) = 1.969$

Obs	InflationNor	prediction	std. error	95% interval	
2014:06	−0.219459	0.159688	0.423516	−0.674182	0.993559
2014:07	0.733138	0.132942	0.424137	−0.702150	0.968035
2014:08	−0.291121	0.219998	0.424879	−0.616556	1.056551
2014:09	0.510949	0.126393	0.424403	−0.709224	0.962011
2014:10	0.072622	0.199692	0.423993	−0.635118	1.034502
2014:11	0.072569	0.159635	0.423517	−0.674237	0.993506
2014:12	0.000000	0.159630	0.423517	−0.674242	0.993501
2015:01	−0.072516	0.152998	0.423601	−0.681039	0.987035
2015:02	0.435414	0.146371	0.423731	−0.687923	0.980665
2015:03	0.289017	0.192789	0.423791	−0.641622	1.027201
2015:04	0.360231	0.179411	0.423541	−0.654509	1.013330
2015:05	0.215363	0.185919	0.423639	−0.648194	1.020031

Forecast evaluation statistics

Mean Error	0.0075616
Mean Squared Error	0.094962
Root Mean Squared Error	0.30816
Mean Absolute Error	0.25582
Bias proportion, U^M	0.00060212
Regression proportion, U^R	0.3697
Disturbance proportion, U^D	0.6297

D.2.2 AO model

D.2.3 AR(6) model

Table 32: CPI inflation forecast for Norway with AR(6)

For 95% confidence intervals, $t(238, 0.025) = 1.970$

Obs	InflationNor	prediction	std. error	95% interval	
2014:06	−0.219459	−0.199103	0.379820	−0.947341	0.549135
2014:07	0.733138	0.009541	0.379958	−0.738969	0.758051
2014:08	−0.291121	−0.118141	0.379253	−0.865264	0.628981
2014:09	0.510949	0.455743	0.381271	−0.295353	1.206839
2014:10	0.072622	0.394695	0.378991	−0.351911	1.141301
2014:11	0.072569	0.011983	0.379134	−0.734904	0.758870
2014:12	0.000000	0.279888	0.379524	−0.467768	1.027544
2015:01	−0.072516	−0.150606	0.379912	−0.899025	0.597813
2015:02	0.435414	0.548760	0.380540	−0.200897	1.298418
2015:03	0.289017	0.180185	0.378552	−0.565556	0.925926
2015:04	0.360231	0.389517	0.378416	−0.355955	1.134990
2015:05	0.215363	0.149969	0.377260	−0.593227	0.893164

Forecast evaluation statistics

Mean Error	0.012814
Mean Squared Error	0.064887
Root Mean Squared Error	0.25473
Mean Absolute Error	0.16914
Bias proportion, U^M	0.0025307
Regression proportion, U^R	0.095186
Disturbance proportion, U^D	0.90228

D.2.4 ARDL models

Table 33: Nowegian CPI inflation forecast with CRB

For 95% confidence intervals, $t(234, 0.025) = 1.970$

Obs	InflationNor	prediction	std. error	95% interval	
2014:06	−0.219459	−0.240581	0.381375	−0.991948	0.510785
2014:07	0.733138	0.021420	0.380687	−0.728591	0.771432
2014:08	−0.291121	−0.146703	0.380314	−0.895980	0.602573
2014:09	0.510949	0.420157	0.382210	−0.332855	1.173170
2014:10	0.072622	0.390866	0.383049	−0.363799	1.145531
2014:11	0.072569	0.026028	0.382126	−0.726819	0.778875
2014:12	0.000000	0.246471	0.382013	−0.506153	0.999096
2015:01	−0.072516	−0.192427	0.381889	−0.944807	0.559953
2015:02	0.435414	0.535026	0.387821	−0.229042	1.299094
2015:03	0.289017	0.163502	0.382580	−0.590239	0.917242
2015:04	0.360231	0.371426	0.379942	−0.377118	1.119970
2015:05	0.215363	0.110785	0.378887	−0.635681	0.857251

Forecast evaluation statistics

Mean Error	0.033353
Mean Squared Error	0.062616
Root Mean Squared Error	0.25023
Mean Absolute Error	0.17001
Bias proportion, U^M	0.017766
Regression proportion, U^R	0.088241
Disturbance proportion, U^D	0.89399

Table 34: Nowegian CPI inflation forecast with IMF

For 95% confidence intervals, $t(232, 0.025) = 1.970$

Obs	InflationNor	prediction	std. error	95% interval	
2014:06	−0.219459	−0.253410	0.376904	−0.996001	0.489182
2014:07	0.733138	0.044764	0.378200	−0.700381	0.789910
2014:08	−0.291121	−0.122515	0.379178	−0.869587	0.624557
2014:09	0.510949	0.324403	0.380422	−0.425120	1.073926
2014:10	0.072622	0.309442	0.378333	−0.435965	1.054849
2014:11	0.072569	−0.101680	0.377568	−0.845580	0.642220
2014:12	0.000000	0.136230	0.380047	−0.612555	0.885014
2015:01	−0.072516	−0.302075	0.384739	−1.060104	0.455954
2015:02	0.435414	0.333587	0.388065	−0.430996	1.098169
2015:03	0.289017	0.079208	0.388215	−0.685670	0.844086
2015:04	0.360231	0.365764	0.379577	−0.382094	1.113621
2015:05	0.215363	0.085696	0.377881	−0.658821	0.830214

Forecast evaluation statistics

Mean Error	0.10057
Mean Squared Error	0.063931
Root Mean Squared Error	0.25285
Mean Absolute Error	0.19176
Bias proportion, U^M	0.15819
Regression proportion, U^R	0.024809
Disturbance proportion, U^D	0.817

Table 35: Norwegian CPI inflation forecast with IMFNF

For 95% confidence intervals, $t(233, 0.025) = 1.970$

Obs	InflationNor	prediction	std. error	95% interval	
2014:06	−0.219459	−0.169321	0.384543	−0.926946	0.588305
2014:07	0.733138	−0.028687	0.382758	−0.782796	0.725421
2014:08	−0.291121	−0.150024	0.382182	−0.902999	0.602951
2014:09	0.510949	0.421970	0.383600	−0.333799	1.177738
2014:10	0.072622	0.349525	0.383714	−0.406468	1.105518
2014:11	0.072569	−0.042319	0.382656	−0.796227	0.711588
2014:12	0.000000	0.248715	0.381775	−0.503458	1.000888
2015:01	−0.072516	−0.204275	0.383329	−0.959509	0.550959
2015:02	0.435414	0.470677	0.385625	−0.289080	1.230434
2015:03	0.289017	0.095812	0.383332	−0.659428	0.851052
2015:04	0.360231	0.291630	0.382674	−0.462314	1.045573
2015:05	0.215363	0.062217	0.381094	−0.688613	0.813048

Forecast evaluation statistics

Mean Error	0.063357
Mean Squared Error	0.070545
Root Mean Squared Error	0.2656
Mean Absolute Error	0.18871
Bias proportion, U^M	0.056902
Regression proportion, U^R	0.074617
Disturbance proportion, U^D	0.86848

Table 36: Norwegian CPI inflation forecast with IMFF

For 95% confidence intervals, $t(233, 0.025) = 1.970$

Obs	InflationNor	prediction	std. error	95% interval	
2014:06	−0.219459	−0.251233	0.376715	−0.993435	0.490969
2014:07	0.733138	0.045579	0.377864	−0.698888	0.790046
2014:08	−0.291121	−0.132384	0.379216	−0.879514	0.614745
2014:09	0.510949	0.340936	0.379902	−0.407545	1.089417
2014:10	0.072622	0.309639	0.376486	−0.432113	1.051392
2014:11	0.072569	−0.125867	0.378070	−0.870740	0.619006
2014:12	0.000000	0.106750	0.381091	−0.644075	0.857575
2015:01	−0.072516	−0.319653	0.387065	−1.082247	0.442942
2015:02	0.435414	0.324674	0.389941	−0.443587	1.092934
2015:03	0.289017	0.102206	0.391336	−0.668803	0.873215
2015:04	0.360231	0.373800	0.380479	−0.375819	1.123419
2015:05	0.215363	0.109060	0.378231	−0.636130	0.854251

Forecast evaluation statistics

Mean Error	0.10189
Mean Squared Error	0.062877
Root Mean Squared Error	0.25075
Mean Absolute Error	0.1879
Bias proportion, U^M	0.16511
Regression proportion, U^R	0.027911
Disturbance proportion, U^D	0.80697

D.3 Graphical results

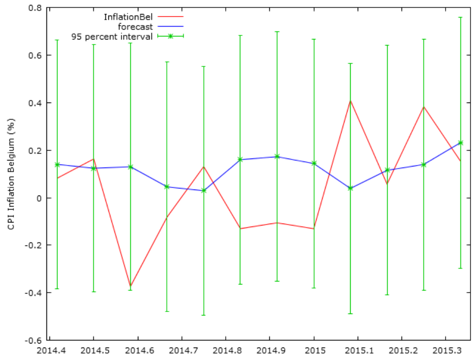

Figure 28: Belgian CPI inflation forecast with AR(6) model

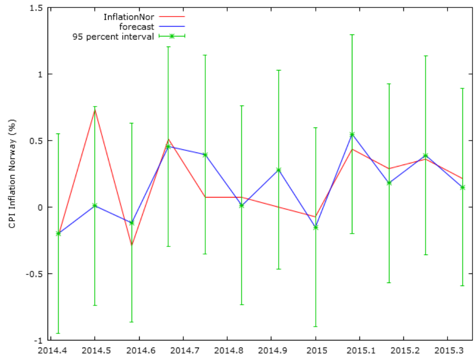

Figure 29: Norwegian CPI inflation forecast with AR(6) model

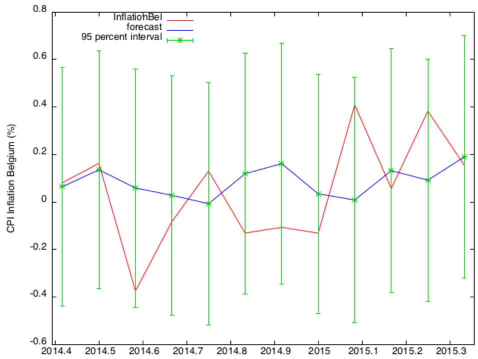

Figure 30: Belgian CPI inflation forecast with CRB commodity index

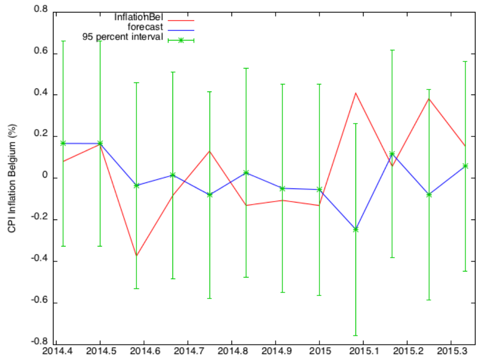

Figure 31: Belgian CPI inflation forecast with IMF commodity index

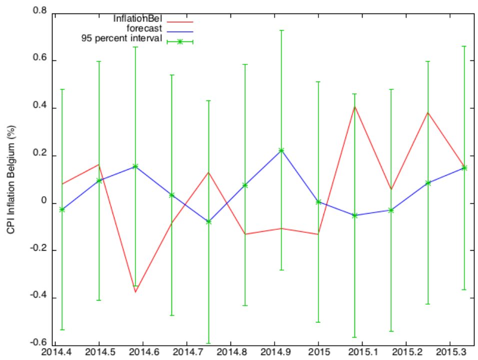

Figure 32: Belgian CPI inflation forecast with IMFNF commodity index

Figure 33: Belgian CPI inflation forecast with IMFF commodity index

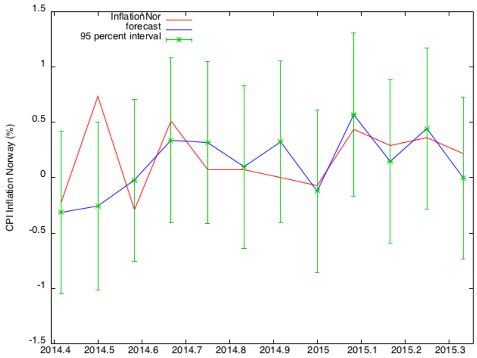

Figure 34: Norwegian CPI inflation forecast with CRB commodity index

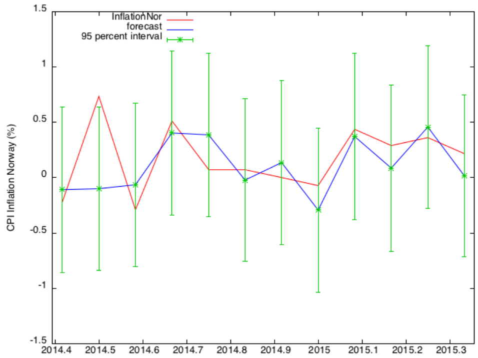

Figure 35: Norwegian CPI inflation forecast with IMF commodity index

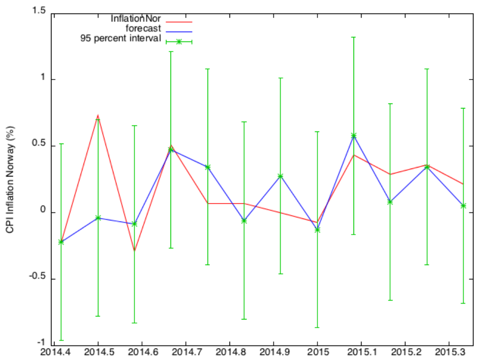

Figure 36: Norwegian CPI inflation forecast with IMFNF commodity index

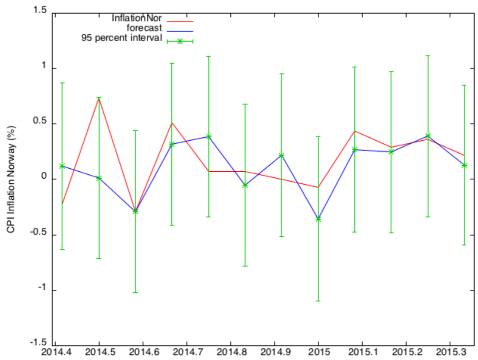

Figure 37: Norwegian CPI inflation forecast with IMFF commodity index

E Forecast errors

Table 37: Summary of the forecasting errors

Forecast errors	**Belgium**											
	RMSE				MAE				MSE			
	1-month	3-month	6-month	12-month	1-month	3-month	6-month	12-month	1-month	3-month	6-month	12-month
AR(1)	0,040	0,312	0,259	0,238	0,040	0,197	0,174	0,187	0,002	0,097	0,067	0,057
AO	0,051	0,247	0,195	0,235	0,051	0,197	0,159	0,187	0,003	0,061	0,038	0,055
AR(6)	0,059	0,294	0,248	0,246	0,059	0,200	0,187	0,202	0,003	0,086	0,062	0,061
ARDL CRB	0,017	0,251	0,217	0,230	0,018	0,153	0,150	0,178	0,000	0,063	0,047	0,053
ARDL IMF	0,047	0,202	0,183	0,268	0,086	0,143	0,149	0,192	0,007	0,041	0,033	0,072
ARDL IMFNF	0,107	0,314	0,257	0,263	0,107	0,234	0,206	0,213	0,012	0,098	0,066	0,069
ARDL IMFF	0,117	0,201	0,174	0,266	0,117	0,156	0,145	0,191	0,014	0,040	0,030	0,071
	Norway											
	RMSE				MAE				MSE			
	1-month	3-month	6-month	12-month	1-month	3-month	6-month	12-month	1-month	3-month	6-month	12-month
AR(1)	0,379	0,505	0,395	0,308	0,379	0,497	0,348	0,256	0,144	0,255	0,156	0,095
AO	0,368	0,479	0,370	0,294	0,368	0,472	0,324	0,250	0,135	0,230	0,137	0,086
AR(6)	0,020	0,430	0,333	0,255	0,020	0,306	0,226	0,169	0,000	0,185	0,111	0,065
ARDL CRB	0,095	0,595	0,438	0,335	0,095	0,451	0,299	0,229	0,009	0,354	0,192	0,112
ARDL IMF	0,110	0,502	0,382	0,294	0,110	0,390	0,281	0,217	0,012	0,252	0,146	0,086
ARDL IMFNF	0,000	0,462	0,349	0,274	0,000	0,326	0,236	0,190	0,000	0,213	0,122	0,075
ARDL IMFF	0,338	0,460	0,362	0,282	0,338	0,353	0,282	0,210	0,114	0,211	0,131	0,079

F Components of the commodity indexes

Table 38: Components of the CRB BLS spot index

Commodity	Market	Source
Burlap (01)	New York	USDA
Butter (02)	Chicago	USDA
Cocoa beans (CC)	New York	USDA
Copper scrap (HG)	New York	USGS
Corn (C-)	Central Illinois	USDA
Cotton (06)	7 markets	USDA
Hides (30)	Chicago	USDA
Hogs (LH)	Iowa S. Minn.	USDA
Lard (09)	Chicago	USDA
Lead scrap (10)	New York	USGS
Print cloth (11)	New York	USDA
Rosin (12)	New York	USDA
Rubber (13)	New York	USDA
Soybean Oil (BO)	Central Illinois	USDA
Steel Scrap (57)	Chicago	USGS
Steers (LC)	Tex. Okla.	USDA
Sugar (17)	New York	USDA
Tallow (18)	Chicago	USDA
Tin (54)	New York	USGS
Wheat (MW)	Minneapolis	USDA
Wheat (KW)	Kansas City	USDA
Wool tops (22)	Boston	USDA
Zinc (23)	New York	USGS

Table 39: Components of the IMF commodity index with the weight of the main categories

All primary commodities				
Non-Fuel Commodities	Edibles	Food	Cereals	Wheat
36,9%	**18,5%**	**16,7%**		Maize
				Rice
				Barley
			Vegetable oils/protein meals	Soybeans
				Soybean meal
				Soybean oil
				Palm oil
				Sunflower/safflower oil
				Olive oil
				Fishmeal
				Groundnuts
				Rapeseed oil
			Meat	Beef
				Lamb
				Swine meat
				Poultry
			Seafood	Fish
				Shrimp
			Sugar	Free Markets
				United States
				EU
			Bananas	
			Oranges	
		Beverages	Coffee	Other milds
		1,8%		Robusta
			Cocoa Beans	
			Tea	
	Industrial inputs	Agricultural row materials	Timber	Hardwood
	18,4%	**7,7%**		Softwood
			Cotton	
			Wool	Fine
				Coarse
			Rubber	
			Hides	
		Metals	Copper	
		10,7%	Aluminium	
			Iron ore	
			Tin	
			Nickel	
			Zinc	
			Lead	
			Uranium	
Energy	Spot crude petroleum	U.K. Brent		
63,1%	**53,6%**	Dubai Fateh		
		West Texas Intermediate		
	Natural Gas	Russian Border Price		
		Henry Hub		
		Indonesian		
	Coal	Australian Coal		