Appendix B

Characteristic lengths and weak localization

This chapter is inspired from Ref. [45].

The Fermi wavelength λ_F is the length associated to the wave nature of the electron at Fermi level. It determines the momentum k of the electron :

$$k = \frac{h}{\lambda_F} \tag{B.1}$$

where h is the Planck constant.

The electron mean free path λ_E is the average distance which is travelled by an electron between two collisions. These collisions may be elastic (the kinetic energy is conserved) or inelastic (some kinetic energy is lost). Inelastic collisions happen when an electron meets another electron or when a phonon is implied. Elastic collisions happen between electrons and static defects.

The coherence length l_ϕ is the average distance which is travelled by an electron between two changes of the phase associated to its wave function. These changes are due to inelastic collisions. Intuitively, these three characteristic lengths are such that

$$\lambda_F < \lambda_E < l_\phi. \tag{B.2}$$

When the dimensions of the medium through which the electrons travel are greater than these characteristic lengths, the transport is said to be diffusive. It is limited by diffusion and fixed by the applied electric field. The resistance depends on the length of the sample parallel to the field.

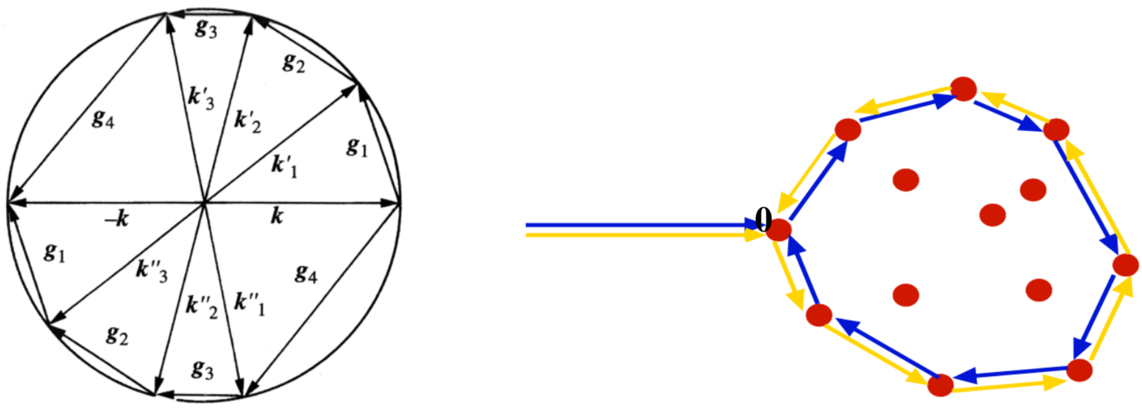
When the dimensions of the medium are smaller than the coherence length, the transport itself is coherent : electrons keep a memory of their phase and interference phenomena begin to appear. This is the mesoscopic regime.

When the dimensions of the medium are smaller than the electron mean free path, the transport is ballistic. It is not limited by diffusion anymore and is fixed by the limits of the medium. The resistance (or the conductance) does not depend on the length of the sample.

If there is a small disorder at the scale of the crystal in a material, electrons can be weakly localized at low temperature. This leads to an increase of the resistance when the temperature decreases, which cannot be explained with a semi-classical theory (which predicts a plateau of residual resistance due to interactions between electrons and static defects of the material).

As this phenomenon is directly linked to quantum interferences, it is observable when $\lambda_E \ll l_\phi$. This means that electrons keep their phase over many collisions. It is possible to understand why the resistance is higher when a quantum approach is followed instead of a semi-classical one. An electron of wave vector $\mathbf{k}$ can be back-scattered to the state $-\mathbf{k}$ through different scattering sequences. Two of them are represented in Fig. B.1. Once the electron is back-scattered, its velocity is opposite to the one it had before the scattering, which leads to a resistance of the material to the propagation of the electron. If the probability amplitude of back-scattering is A, then the total amplitude of probability is nA where n is the total number of possible back-scattering processes. The probability to back-scatter is then n^2A^2. In a classic representation of transport, the total probability would be nA^2, which is indeed lower than the probability computed within a quantum approach. This explains the higher value of resistivity that appears at low temperature in a mesoscopic system.

When temperature increases, the time between two inelastic collisions decreases (there are more phonons), the number of back-scattering processes decreases and resistivity decreases. When a magnetic field is applied, it decreases the effects of constructive interferences which reduces the resistivity of the material. Nevertheless, when the temperature is really low, universal conductance fluctuations appear (oscillation of the conductance with the magnetic field), but this phenomenon will not be described here.



(a) (b)

Figure B.1: Back-scattering representation (a) in k-space and (b) in real space [45].