Strategic R&D Networks and Competition Policy Implications

Case of three ex-ante different firms

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Academic year 2017/2018

In order to obtain the Joint Degree
Master 120 en Sciences économiques, Orientation générale, Finalité spécialisée (UCL/UNamur)

and
Dottore magistrale in Economics and Political Science (UNIMI)

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ACKNOWLEDGMENTS

I would like to thank all the people who contributed in some way to the work described in this thesis.

First and foremost, I would like to express my sincere gratitude to my Supervisor Vincent Vannetelbosch, Professor of Game Theory at the University of Louvain, for introducing me to the topic and for the continuous support throughout the learning process of this Master thesis. I am particularly thankful for his readiness to respond to my many questions and for his useful comments and advice.

My grateful thanks are also extended to my Reader Paolo Garella, Professor of Global Firms and Markets at the University of Milan, for his availability to fulfill the obligations of this role and for the interest he demonstrated.

Furthermore, I would like to acknowledge the assistance of Giacomo Albi, Professor of the Mathematics Department at the University of Verona, for his help with the software *Mathematica*, used for the calculations and graphs of the model.

Moreover, I wish to thank my family, especially my parents, for the support and encouragement provided throughout my study, and Enrico, for the assistance during the preparation of this essay.

Finally, I wish to thank my friends and all the people who stood by me in this last year.

TABLE OF CONTENTS

1. INTRODUCTION

The recent developments of increasing market competition and the downward pressures of globalization on the price of the products have make firms thinking about a way to decrease their marginal costs of production or to innovate their products. As a result, many companies found in R&D collaboration the answer to all these challenges.

A growing body of empirical studies has shown the high level of interfirm collaboration in R&D activity, especially in technological sectors, as a way to survive global market competition (Caloghirou et al. (2003), Hagedoorn, J. (2002)). This concept refers to firms' cooperation in projects aimed to improve existing products or services, or to innovate the process of production with the objective of reducing their marginal cost. These collaborations can take different forms, ranging from informal agreements among firms to the equity forms of R&D collaboration, such as Research Joint Ventures. Moreover, these R&D collaborations can operate in different ways, such as sharing information and competence or even sharing assets and resources. Similarly, R&D collaborations can take place between firms competing in the same markets or between firms operating in independent markets; it can take the form of a private partnership between profit-maximizing companies or it can connect private companies with public ones, such as universities; moreover, it can rise between horizontally related firms or between suppliers and manufacturers.

The literature on Research and Development distinguishes two approaches on the topic, the coalition theory and the R&D network concept. In the former case, groups can be of arbitrary size and each firm can belong to one group only. In the latter case, collaboration links are bilateral and embedded in a broader network architecture, which includes other firms with their ties. The main difference with the coalition approach is that links are pairwise and non-exclusive. This "allows a firm to form collaboration relations with other firms without seeking prior permission of current collaborators" (Goyal and Joshi, 2003). Consequently, each firm can unilaterally decide to delete a link and can reach a bilateral agreement with every other firm to form a collaboration. Different structures can arise in this setting, resulting in asymmetric architectures that we cannot find in the other case. Hence, in my study I will only consider the case of R&D network theory.

Contemporary literature has emphasised the benefits of R&D collaborations for social welfare, calculated as the sum of consumer surplus and producers' surplus (Goyal and Moraga-González (2001), Goyal and Joshi (2006), Zikos (2010)). R&D collaborations, through the share of information, increase the chances of innovation that in turn increases productivity and creates wealth in the economy. This generally results in a higher producer surplus by virtue of the decrease in marginal costs of production, as well as in a higher consumer surplus, due to the consequent decrease in the final price or to the wider range of product choice available.

Therefore, the benefits of R&D collaborations are widely accepted. However, the level of collaboration reached by the firms is not always optimal from the social welfare point of view. As shown in the literature (Kesavayuth and Zikos (2012), Goyal and Joshi (2006), Goyal and Joshi (2003)), the incentives of firms to collaborate are not always adequate from the social welfare standpoint. Indeed, when firms collaborate, they face a tradeoff between two opposite effects. On one hand, by collaborating with another firm they manage to decrease their cost of production or to develop a new product, on the other hand their individual R&D efforts reduce the costs of their competitors as well. Consequently, the R&D efforts of firms could be too small compared to the optimum. Nevertheless, there could be also cases in which the level of R&D efforts are higher than the welfare maximizing ones. In this case, profits maximizing firms have incentives to form too much collaborations with other firms than what is socially optimal.

Due to these divergences in the stable network architecture that emerges in equilibrium and the welfare maximizing one, there is the need of a policy intervention to change the incentives of firms to collaborate and increase social welfare. The aim of this work is to find a way to achieve the network configuration with the highest welfare, by means of some policy tools at government's and Competition Authorities' disposal. On one side, the government has the possibility to give subsidies to the firms for each link formed or for each unit of effort put, in order to increase the number of links when they are too few with respect to the efficient condition. On the other side, the government can impose a tax on the links to avoid a too high level of collaboration among firms and thus increase the social welfare. Furthermore, competition authorities can block some agreements when they are excessive for the interests of society. I will discuss later in the paper how these tools can work and the issues to take into account before the intervention.

To reach my objective, I study the different network configurations that could arise in specific settings and I calculate the social welfare in all the possible architectures. Subsequently, I compare the equilibrium network with the welfare maximising one and I examine the possible policy intervention that could change firms' incentives to collaborate, in order to reach the best outcome for society as a whole.

In my analysis I start considering the basic case in which three ex-ante equal firms compete in a homogeneous product market, in which the costs of the links is equal to zero and the knowledge spillovers are perfect between linked firms and null between unconnected firms. To develop this model I rely on Goyal and Moraga-González (2001) study, taking the example of three firms competing horizontally.

A common characteristic that I found in the literature on R&D networks is that the models generally start with ex-ante equal firms, thus firms with the same marginal costs of production, size and market share. They subsequently acquire advantages on their competitors by means of the position they have in the network. However, in the empirical setting (GlaxoSmithKline website, Bower and Whittaker, 1993) firms do not start always with the same characteristics. Some of them may have a lower production cost because of their technology, and this lower cost leads to a biggest market share and size of the firm. As a result, I consider important to develop a model that takes these differences into account. Indeed, my objective is to observe which different network configurations will emerge in the case of ex-ante different firms, depending on their distinct marginal cost of production before the link formation process. Due to the difficulties of developing a general model, I focus on the case of three competing firms and observe how the efficient and the pairwise stable networks vary if the difference in costs among them enlarges or reduces. The results show that there is little correspondence between the equilibrium network and the one that maximises welfare, and therefore I will discuss the possible policy interventions in the cases where they do not coincide.

Afterward, I review the extensions on the basic model of Goyal and Moraga-González (2001) in the literature, considering the case of symmetric networks with n firms (Goyal and Moraga-González, 2001), positive costs of link formation (Goyal and Joshi, 2003), transfers between firms (Goyal and Joshi, 2003), positive spillovers among unlinked firms (Goyal and Joshi, 2006; König et al., 2012; Zikos, 2010; Goyal and Moraga-González, 2001), the presence of a public firm (Zikos, 2010), different types of competition (Goyal and Joshi, 2003), differentiated products (Westbrock, 2008) and vertically related firms (Kesavayuth and Zikos, 2012; Thanh Tran and Zikos, 2017).

For all the extensions in the literature for which is possible to calculate the social welfare, I will briefly examine the policy implications and the way in which governments and competition authorities can act to influence the links and achieve a network configuration with a higher welfare.

2. THE BASIC MODEL

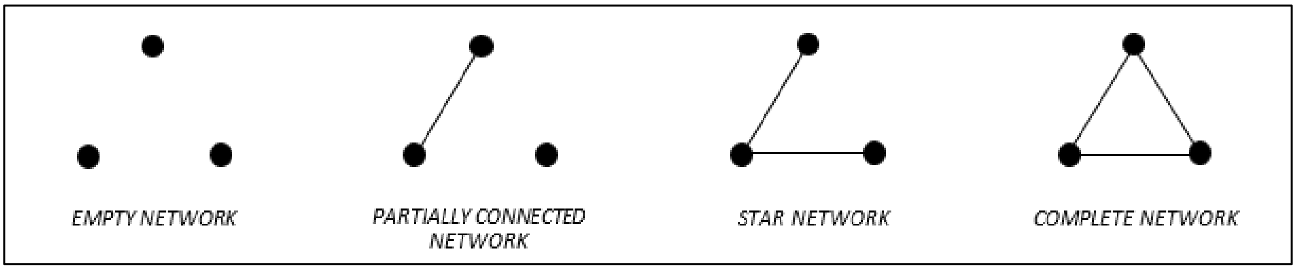
The model I use in this work is an adaptation of Goyal's and Moraga-González (2001) model, in the simple case of three ex-ante equal firms competing in quantities in a homogeneous-product market. It is a model of endogenous network formation, where firms can decide to link to other firms in order to transfer knowledge and lower their marginal costs of production before competing in the market. We assume that the costs of forming links, as well as knowledge spillovers among unconnected firms, are equal to zero. Knowledge spillovers between connected firms are instead perfect, which means that own research is fully shared with linked partners.

The game consists of three stages. In the first stage, firms form pairwise collaboration links. In the second stage, each firm chooses a level of R&D efforts unilaterally. This level of efforts, combined with the efforts of the linked firms, defines the final costs of the firms. In the third stage, firms compete in the market choosing quantity to maximize own profits, taking as given the new costs of production. This model reflects the long-run decision of the link formation, which affects the choice of the level of R&D efforts and the output. In order to solve this model we thus proceed by backward induction, anticipating in stage 1 what the efforts and quantities of the other firms in equilibrium will be and finding the best response to them.

Networks. Let $N = \{1,2,3\}$ be the set of ex-ante equal firms. For any pair of firms $i, j \in N$, the pairwise relationship between the two firms is represented by the binary variable $g_{ij} \in \{0,1\}$. If the two firms are linked, g_{ij} = 1, if they are not linked g_{ij} = 0. The network $g = \{g_{ij}\}_{i,j \in N}$ is the collection of links created by the firms. Let $g - g_{ij}$ denote the network obtained by deleting an existing link between the two firms i and j in the network g, and $g + g_{ij}$ denote the network obtained by adding a collaboration link between the two firms i and j in the network g.

Within the setting of three firms, there are four possible network architectures (see *figure 1*). First there is the empty network, g^e, in which there are no collaboration links and all firms are isolated. On the opposite side there is the complete network, g^c, where all firms are linked to each other's with pairwise collaborations and the level of collaborative activity is maximal. At the intermediate levels of collaboration there is the partially connected network, g^p, in which two firms are linked while the third one is isolated; and the star network, g^s, in which one firm -"hub"- is connected with two others -"spokes", which are not linked between them. As the three firms are ex-ante equally efficient, just four qualitative different network architectures are possible.

Figure 1: possible network architectures with three identical firms



R&D effort levels and spillovers. After the formation of a network g, each firm chooses its own level of R&D efforts unilaterally. I assume that knowledge spillovers between a pair of linked firms are perfect, and therefore, the knowledge that stems from the research of the partner firm is fully absorbed. Hence, each firm benefits from its individual efforts, which reduce both its own and the partners' marginal costs of production, and from the efforts of the connected firms. If instead two firms have not a collaboration link, we assume the knowledge spillovers to be equal to zero. Consequently, only linked firms help reduce the firm's production costs.

Following Goyal and Moraga-González (2001), I assume that firms are initially symmetric with zero fixed costs and identical marginal costs of production $\bar{c}$. Given a network g and a collection of effort levels $\{e_i(g)\}_{\,i\in N}$, the cost of production of firm i is:

$$c_i(\{e_i(g)\}_{\,i\in N}) = \bar{c} - e_i - \sum_{k\in N_i(g)} e_k$$

where e_i are the efforts of firm i's own research and e_k are the efforts of firms linked to firm i, for which the research knowledge is fully absorbed. This total cost reduction represents the *effective* R&D level. In line with the authors, I assume that R&D effort is costly. Given a level $e_i \in [0, \bar{c}]$ of efforts, the cost of efforts follows a quadratic function of $Z(e_i) = \gamma e_i^2$, with $\gamma > 0$ indicating the effectiveness of R&D expenditure. Assuming this, the cost of R&D effort is an increasing function and exhibits decreasing returns, with the parameter γ measuring the curvature of this function. Following the authors, I assume that γ is sufficiently large so that first-order conditions are enough to ensure an interior equilibrium and that second-order conditions hold. In this case it is sufficient to assume that $\gamma > 1$, which avoids corners solutions and the negative values for the efforts and cost functions.

Payoffs. Firms take as given their marginal costs of production $c_i(\{e_i(g)\}_{\,i\in N})$, which depend on their own efforts e_i and on the efforts of connected firms, e_k. The demand in the market is assumed to be linear and given by the function $Q = a - p, \quad a > \bar{c}$. Given these marginal costs and the market demand, firms choose their quantity to maximize profits, conjecturing the quantity produced by the other profit-maximizing competitors. In the homogeneous-product market with three quantity-setting firms, $Q = \sum_{i=1}^{3} q_i$. Thus, profits of firm i in the network g are given by:

$$\pi_i(g) = \left[a - q_i(g) - \sum_{j \neq i} q_j(g) - c_i(g)\right] q_i - \gamma e_i^2(g)$$

Stability. In the literature on R&D networks there are two main definitions of stability. The concept of stability is useful to establish which network configuration will endogenously emerge in equilibrium. The final aim is to check whether the stable network is also the most efficient one or we need to act on firms' incentives to reach a superior outcome. The typical definition of stability applied in the framework of network formation belongs to Jackson and Wolinksy (1996) and is the concept of pairwise stability.

A network is pairwise stable if the following two conditions are satisfied:

(i) If $g_{ij} = 1$, then $\pi_i(g) \geq \pi_i(g - g_{ij})$ and $\pi_j(g) \geq \pi_j(g - g_{ij})$
(ii) If $g_{ij} = 0$ and $\pi_i(g + g_{ij}) > \pi_i(g)$, then $\pi_j(g + g_{ij}) \leq \pi_j(g)$

The first part of this definition says that if i and j are linked, the payoffs of both firms i and j are at least equal to the payoff that they would have in the network in which they are not connected. The second part asserts that if firms i and j are not linked and the payoff of firm i would be higher with the link, the lack of connection is because the payoff of the other firm would be lower with that link. The implication is that, while to create a link both firms must agree, one firm can unilaterally take the decision to sever it.

This is a quite weak concept of stability. It implies that a network is stable if it survives all possible deviations at bilateral level. However, it could be that a coalition of firms would be better off in another network configuration, and thus these firms decide to rearrange their collaborative links altogether. The notion of stability that takes into account this possibility is the concept of strong stability by Jackson and van den Nouweland (2005). As this is a refinement of the concept of pairwise stability, a network to be strongly stable must also be pairwise stable. However, in this study, I will mainly focus on the wider concept of pairwise stability and therefore I assume that firms cannot collude and collaborate with each other to form a new network. Hence, once a configuration is created, only unilateral deviations or bilateral decisions on a single link are possible.

Efficiency. There are multiple definitions of efficiency, which depend on the goal we are interested in. Considering the outcome from the economic point of view, there could be three concepts according to which we can measure the efficiency of a network.

The first concept is the one of producer surplus. This concept refers to the aggregate profits of all the producers in the network considered. Thus, in the case of a homogeneous-product oligopoly with three firms, the producer surplus is given by:

$$PS(g) = \sum_{i=1}^{3} \pi_i \,(g)$$

The second concept is the one of consumer surplus, which measures the difference between what consumers are willing to pay for a given product and its market price. In the setting of homogeneous-product market and linear demand, consumer surplus is given by:

$$CS(g) = \frac{Q(g)^2}{2}$$

The last concept is a combination of the previous ones, and it refers to the social welfare approach. Total welfare is measured as the sum of producer surplus and consumer surplus, and it gives equal weight to the two sides of the economy: producers and consumers. Its formula is:

$$SW(g) = \frac{Q(g)^2}{2} + \sum_{i=1}^{3} \pi_i \,(g)$$

The last two definitions of welfare, i.e. consumer surplus and social welfare, are both taken into account as two possible concepts to evaluate the efficiency of a policy. Following Alboek (2013), the former is usually considered by competition authorities in the European Union, which have the objective to protect consumers from the power of producers. The latter is instead the criterion adopted by economists, who prefer to use this more neutral definition for the welfare to maximize, and then to leave to distributional polices the task to redistribute it.

In this work, I will take the concept of social welfare as the welfare standard to detect the efficient network in each setting. The efficient network is thus the one that maximises social welfare, which considers both consumers and producers. I decided to take this concept because it is a wider measure of the welfare of the economy and does not favour any of the two parts.

As pointed out by Goyal and Moraga-González (2001), the concept of efficiency that we consider is in the spirit of the second best, because the first best would be in the case where firms cooperate to determine quantities and efforts. However, in this model firms choose their level of efforts and quantities uncooperatively, without internalizing the spillovers involved in the collaboration.

Policy recommendation. The objective of this work is to explore the implications of R&D collaborations for competition policy. As I will show later in the paper, the efficient network from the social welfare perspective does not always coincide with the stable network that the profit-maximizing firms would form. This is certainly a problem for the society, which is stuck in an inefficient condition. Thus, I will try to find and suggest some possible solutions to influence firms' behaviour and incentivize them to create a network with a higher total welfare.

As I have already mention before, in the national economies there are two types of authorities that could do something in this respect: the government and the competition authorities. My objective is to advice these two authorities with the best intervention to increase social welfare in different theoretical cases, analysing the effects of the policy tools at their disposal.

3. BASIC MODEL RESULTS

In order to find the best policy tool to reach the highest welfare, we first have to detect whether there is any discrepancy between the stable and the efficient network. I start by computing the social welfare in each of the four possible configurations for a three-firm environment, and then I proceed by calculating the stable and the efficient architecture. Therefore, we can compare these networks and identify the best option for the government or competition authorities to maximize welfare.

In order to compute the social welfare, we must solve the model by backward induction for each of the four network architectures.

Empty network. In the third stage, firms individually choose their quantities to maximize profits taking as given their costs of production, the cost of the research efforts and their collaboration links, if any. In the case of the empty network, the cost reduction of each firm depends only on its own R&D efforts, since firms are all isolated and there are no knowledge spillovers among unlinked firms. Thus, each firm i chooses its individual quantity q_i to maximize its profit.

$$\pi_i(g) = [a - q_1(g) - q_2(g) - q_3(g) - c_i(g)]q_i - \gamma e_i^2(g)$$

From the first order conditions we get the quantity of each firm, which are:

$$q_1 = \frac{1}{2}(a - c + e_1 - q_2 - q_3)$$

$$q_2 = \frac{1}{2}(a - c + e_2 - q_1 - q_3)$$

$$q_3 = \frac{1}{2}(a - c + e_3 - q_1 - q_2)$$

Then we solve a system of these three equations to determine which is the quantity produced by each firm, and we get:

$$q_1 = \frac{1}{4}(a - c + 3e_1 - e_2 - e_3)$$

$$q_2 = \frac{1}{4}(a - c - e_1 + 3e_2 - e_3)$$

$$q_3 = \frac{1}{4}(a - c - e_1 - e_2 + 3e_3)$$

In the second stage, each firm chooses individually its R&D effort levels to maximize its profits, anticipating the quantity it will sell in the market competition stage. From the first order conditions, we get the effort levels of each firm.

$$e_1 = \frac{3(a - c - \mathrm{e}_2 - e_3)}{-9 + 16\gamma}$$

$$e_2 = \frac{3(a - c - \mathrm{e}_1 - e_3)}{-9 + 16\gamma}$$

$$e_3 = \frac{3(a - c - \mathrm{e}_1 - e_2)}{-9 + 16\gamma}$$

In the case of the empty network with ex-ante identical firms, the best response function of each firm given the behaviour of the others is the same, and thus the effort levels are identical for each firm as well. Hence, we can find the equilibrium effort level for each firm simply invoking symmetry rather than solving the system. Therefore, the equilibrium effort levels are:

$$e_1 = e_2 = e_3 = \frac{3(a - c)}{-3 + 16\gamma}$$

As pointed out by Goyal and Moraga-González (2001), we find that efforts of linked firms are strategic complements while efforts of non-linked firms are strategic substitutes. Moreover, the equilibrium level of R&D effort is declining in the level of collaborative activity.

Once we have the effort levels, we can plug them back in the quantity function to see which is the quantity produced by each firm. As firms are ex-ante equal, the quantity that we find is:

$$q_1 = q_2 = q_3 = \frac{4(a - c)\gamma}{-3 + 16\gamma}$$

Given the effort levels and the quantity produced, firms profits are:

$$\pi_1 = \pi_2 = \pi_3 = \frac{(a - c)^2\gamma(-9 + 16\gamma)}{(3 - 16\gamma)^2}$$

Summing up the profits of the three firms, we obtain the producer surplus:

$$PS = \frac{3(a - c)^2\gamma(-9 + 16\gamma)}{(-3 + 16\gamma)^2}$$

Substituting the quantities in the consumer surplus formula we obtain:

$$CS = \frac{72(a - c)^2\gamma^2}{(-3 + 16\gamma)^2}$$

We can now add the consumer surplus to the producer surplus to get the social welfare:

$$SW = \frac{3(a - c)^2\gamma(-9 + 40\gamma)}{(3 - 16\gamma)^2}$$

Complete network. In the complete network, the firms are all linked to each other and they benefit from the research efforts of all the other firms. Therefore, the cost function of each firm takes into account also the reduction in the marginal cost due to the efforts of the other firms.

Following the same procedure that we have used in the empty network case, we can find the equilibrium efforts, quantities and profits in the case of the complete network. Eventually, we can combine these results to get the consumer surplus, producer surplus and social welfare in the complete network. All the calculations are available in the Appendix (*Appendix*).

For our policy intervention advice, we are interested in the social welfare result, which in this case is:

$$SW = \frac{3(a-c)^2\gamma(-1+40\gamma)}{(3-16\gamma)^2}$$

Partially connected network. The same procedure we have used before applies also here, with the only difference that we cannot find the equilibrium effort level by invoking symmetry. In the partially connected network, the cost reduction of the isolated firm depends only on its own R&D efforts, while the other two connected firms share their knowledge and benefit from the partner's research efforts. Therefore, in order to obtain the effort levels of each firm we must solve the system of the three efforts best response functions. Again, I present here the main result, while the intermediate calculations are available in the Appendix.

In the case of the partially connected network, the social welfare is:

$$SW = \frac{(a-c)^2\gamma(-27+228\gamma-434\gamma^2+240\gamma^3)}{2(3-17\gamma+16\gamma^2)^2}$$

Star network. In the star network there is some asymmetry among firms due to the network architecture, like in the partially connected network case. The "hub" firm benefits from its own R&D efforts and from the research efforts of the two "spoke" firms, while these ones only benefit from their own efforts and from the "hub" firm's efforts. Solving the model as in the previous case, we obtain the social welfare result (*calculations in the Appendix*):

$$SW = \frac{(a-c)^2\gamma(-1+20\gamma+124\gamma^2+480\gamma^3)}{(1+2\gamma-32\gamma^2)^2}$$

Stable network. In order to see which network will endogenously emerge in this setting, we can use the concept of pairwise stability by Jackson and Wolinsky (1996) explained before. This definition implies that a network is pairwise stable if it survives all possible deviations at bilateral level. A firm i decides not to delete its existing link with firm j if its payoffs are higher in the current network g than in the network $(g - g_{ij})$ without this link. Two firms i and j will instead decide to link if they both reach higher payoffs in the network $(g + g_{ij})$ than in the current network g. In this particular setting, Goyal and Moraga-González (2001) find that the pairwise stable networks are the complete and the partially connected network, when $\gamma = 1$.

However, in my study I have assumed that $\gamma > 1$ to avoid the case in which in the partially connected network the isolated firm does not produce and is thus driven out of the market. In Goyal and Moraga-González' (2001) this does not happen because they added a parameter $\beta \in [0,1]$ which defines positive knowledge spillovers between firms that have no collaboration links. In this way, the isolated firm in the partially connected network still benefits from the research efforts of the other two firms and therefore manages to stay in the market. In my exemplification, I should also fix a value of γ in order to compare the profits of the firms. I get their same pairwise stability results while avoiding the exit of firms by fixing γ to a value marginally greater than 1.

Indeed, the empty network is never stable because it is dominated in terms of firms' payoffs by the complete network. Neither the star network is stable, because the two spoke firms will have an incentive to form a link between them to increase their profits. The partially connected network is instead stable, with the isolated firm having a significant cost disadvantage. The complete network is also stable, because no firm has an incentive to delete a link that would lower its profits and increase the ones of the hub firm in the emerging star network.

This definition of stability is rather weak, because it does not take into account the possibility of deviations from coalitions of firms. Jackson and van den Nouweland (2005) have introduced the concept of strong stability to allow for the possibility of deviations of more agents at the same time. Since it is a refinement of the pairwise stability concept, a network to be strongly stable must also be pairwise stable. The only candidates are thus the complete and the partially connected networks. The individual payoffs of the linked firms in the partially connected network are higher than the payoffs in the complete network. Hence, two firms in the complete network can decide to delete their links with the third firm at the same time to achieve higher profits and eliminate the competition of the third firm. Thus, the complete network is not strongly stable.

On the contrary, each of the partially connected networks is strongly stable, because the two linked firms have higher payoffs than the ones they could attain forming other links or severing the existing one. Starting from the case of two connected firms, the only possible deviation consists in one of the connected firms deleting the current link and forming a new link with the previously isolated firm. In this case, they will form another partially connected network in which the newly isolated firm is worse off, the newly connected one is better off and the firm which simply changes its link does not change its payoff. Therefore, the latter is indifferent between keeping the existing link and substitute it with the new link. However, even though forming links is not costly, the decision to collaborate is just the first step in a long-run relationship that requires firms to do research and share it with the partners. Once a network is formed and firms have started their research, they have not incentives to change the link with another one that would give them the same profits. Therefore, I assume that once a partially connected network is established, this is also strongly stable. The only issue concerns which one of the three possible partially connected networks is formed, as the three firms are exactly ex-ante equal. The first two that manage to reach an agreement on a collaboration link will create a network that will be than strongly stable against collective deviations.

Efficient network. As I have previously pointed out, I use a social welfare criterion to establish which network is more efficient. In this simple case of three firms, it is enough to compare the social welfare in the four network configurations and see which one is the highest. In order to directly compare the results we should again fix a value of γ marginally greater than 1, which ensures the non-negativity of all variables and no firm exits.

Clearly, social welfare in this framework is maximized in the partially connected network (*calculations in the Appendix*). With different values of γ, the order of welfare results could be different.

Policy recommendation. In this basic case of three ex-ante equal firms and γ marginally greater than 1, the stable and the efficient networks coincide. It means that firms will endogenously create a network that maximises social welfare. Therefore, there is no need of policy intervention by the government.

4. MODEL EXTENSION: CASE OF EX-ANTE DIFFERENT FIRMS

Although the literature on R&D networks has developed several models with different initial assumptions, authors generally agree on starting with ex-ante identical firms. As a result, prior to the decision of setting up collaborations, firms have identical marginal costs of production. Thus, they are assumed to have also the same size and the same market share. By forming a network, they subsequently manage to reduce their production costs and, if they succeed in acquiring a good position in the network, they can become more competitive with respect to their rivals.

However, in the empirical setting firms do not always start with the same characteristics. It frequently happens that some firms have a marginal cost of production lower than their competitors, which allows them to control a bigger market share. For example, in the pharmaceutical industry firms sustain a large amount of R&D collaborations, because the uncertain conditions of the market combined with the rapid decline in prices due to competition makes companies innovate in order to remain viable and to secure competitive advantage (Deloitte Analysis, 2016). The pharmaceutical companies have not all the same size and initial conditions before starting a collaboration agreement with other firms or academic institutions. For instance, the Glaxo Group Research is a big pharmaceutical company and it has collaboration links to share knowledge with other big pharmaceutical firms, with universities, and with small biotechnology companies all over the world (GlaxoSmithKline website). A similar pattern of collaboration is evident in the case of Merck, a German pharmaceutical, chemical and life sciences company, which balances in-house development with long-term collaborations with smaller firms but also with bigger competitors in its programme to address AIDS and HIV (Bower and Whittaker, 1993).

Due to these empirical disparities among firms that share Research and Development knowledge, I consider important to build a theoretical model that takes them into account.

Therefore, in this section I develop a basic model with three competing firms having different production costs before forming the network. The final aim is to observe whether the stable and efficient network architectures differ between them and from the case of ex-ante identical firms. I will also examine the effects on stability and efficiency of an increase or reduction in the cost difference between firms. Furthermore, I will give some policy recommendations to incentivize the creation of the efficient network from the social welfare standpoint in settings in which firms do not endogenously form it. Also in the cases in which the welfare-maximizing network arises, the government could step in with subsidies or taxes to increase or reduce the initial cost difference among firms, if this could make welfare increase even more. In the case of external intervention, we need to make sure that this is legally allowed. It is necessary to consult the law before giving subsidies or imposing taxes, or before taking the action to block some links between firms.

However, since I look at the 3-player case, all the conclusions have to be taken cautiously. Nevertheless, this simple model can help us to understand the mechanisms behind the formation of R&D collaborations and can give us an insight into what would happen in a more complex environment.

4.1 THE MODEL

The main assumptions of this model are the same as in the three-firm example of the previous section. The unique variation is in the marginal costs of production of the three firms prior to the construction of the network. In the previous model, each of the three firms started with a marginal cost $\bar{c}$. Now I fix the marginal cost of the most efficient firm at $\bar{c}$, and I let the cost of the other two firms to be higher than this one and depending on two parameters x and y. I make some assumptions on $\bar{c}$, x and y, specifically $x \geq 0, y \geq 0$ and $\bar{c} + x + y < a$. Moreover, I start directly by setting $\gamma = 1$, which ensures the non-negativity of all variables and simplifies the calculations.

As a result, the final cost functions of the three firms before deciding on the collaboration links are:

Firm 1's marginal cost: $c_1 = c_2 + x = \bar{c} + x + y$

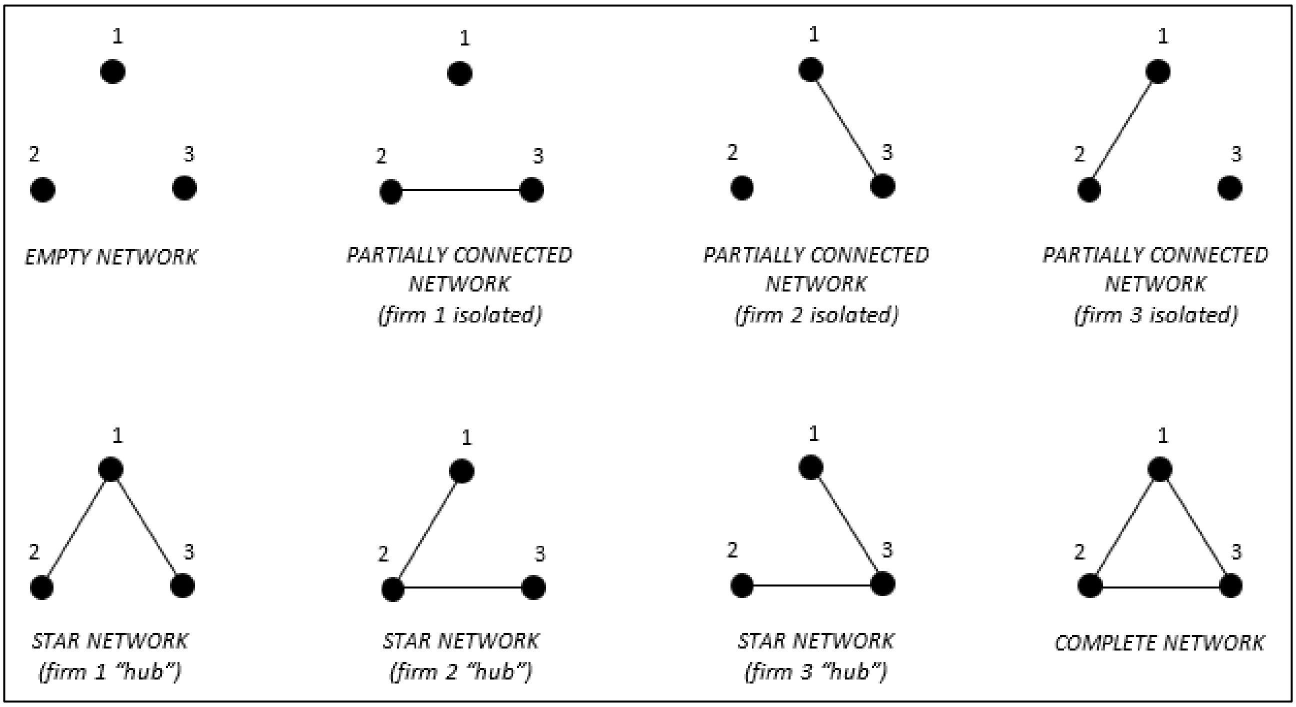
Firm 2's marginal cost: $c_2 = c_3 + y = \bar{c} + y$

Firm 3's marginal cost: $c_3 = \bar{c}$

Therefore, firm 3 is the most efficient firm, followed by firm 2 and firm 1. The difference in efficiency reduces as x and y decrease. In the limit case where $x = 0$ and $y = 0$ we have the same firms' costs that we had in the previous model.

In this setting there are eight possible network architectures, which we can see in the *figure 2*. We have again the empty and complete network as before, but there are now three possibilities for both the star and the partially connected network. In the case of the star network, there are three possible configurations in which respectively the most efficient firm, the middling firm and the least efficient firm are the "hub" firm. For the partially connected network, there are three possible configurations in which the firm with higher, middling and lower costs are the isolated firm.

Figure 2: possible network architectures with three ex-ante different firms



As in the simple model with three ex-ante identical firms, I solve the model by backward induction and I compute the social welfare in each of the eight network configurations (*model results in the Appendix*).

The first thing to be careful with in this model is that firms could be forced to leave the market. Since the marginal costs of production are different for the three firms, the least efficient firms can decide to exit the market if their cost in the current network does not allow them to produce a positive quantity and gain positive profits. As in this model there are not fixed costs of production, firms only consider their marginal costs and the profits when deciding whether to stay in the market or not.

4.1.1 Firms' exit and Social Welfare

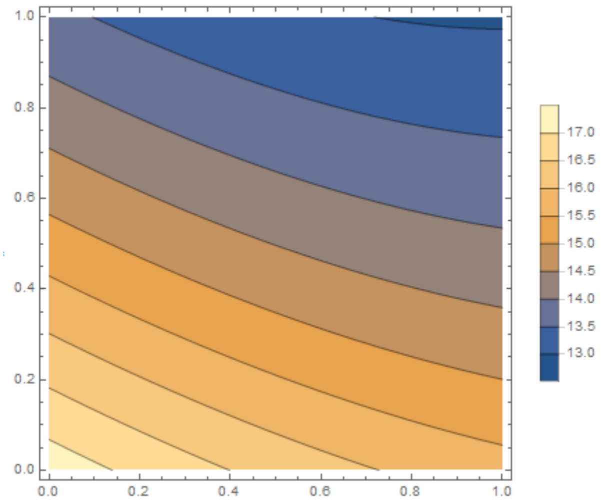
In order to show which firms would decide to stay in the market for each network, I fixed the value or range of the parameters in such a way to avoid an excessive exit from the market and the consequent creation of a monopoly. Therefore, I substituted in the model the values of $a = 10, \bar{c} = 5$ and $x, y \in [0,1]$.

Each firm would decide to compete in the market if the network to which it belongs allows it to produce a positive quantity of the product. Hence, for each network, firms try to maximise their profits deciding the level of R&D efforts and the quantity to produce, taking into account what the other firms do in each case and the reduction in marginal cost they gain from their partners' research. Since they have perfect information on their competitors costs and profits functions, they can find the best response to their actions in each case.

Calculating the quantity of each firm in all the networks with the restricted values of the parameters, I found that all firms find it profitable to stay in the market only in the cases of the complete network and the star network with firm 1 hub. In all the other cases, at least one firm is driven out of the market. Firm 3, which is the most efficient one, always manages to remain in the market except in the partially connected network in which it is isolated and only in the limit case of $x = y = 0$.

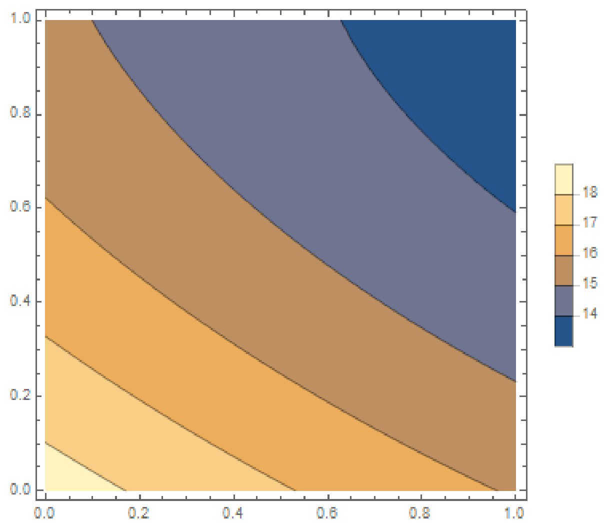
Therefore, you can see below the equations of social welfare in those two networks and the graph showing its value. The horizontal axis shows the values of x, while the vertical axis reports the values of y.

1. COMPLETE NETWORK: $\frac{1}{104}(9000 + 73x^2 + 97xy + 97y^2 + 240(x + 2y) - 480(15 + x + 2y))$



2. STAR NETWORK (firm 1 hub):

$\frac{1}{841}(77875 + 2650x + 461x^2 + 4440y + 726xy + 1665y^2 - 20(3115 + 265x + 444y))$

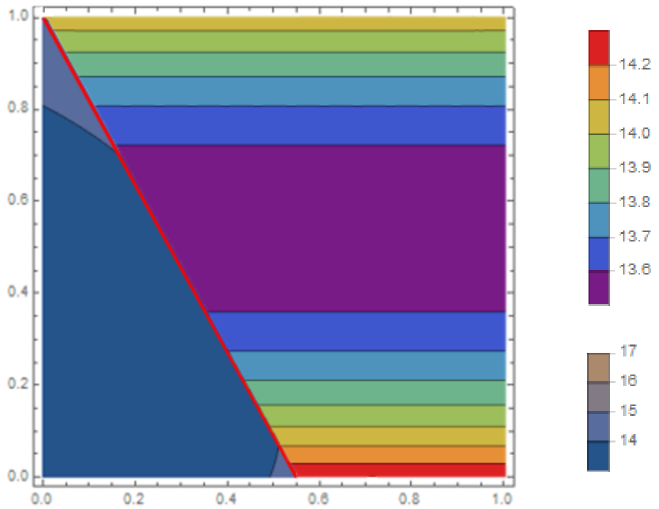


In the empty network, firm 1 chooses to exit if $9x + 5y > 5$ because the revenues it would get would not be enough to cover the costs of production. Firm 2 always chooses to stay in the market, except in the extreme case of $x = 0$ *and* $y = 1$, in which it is equal to firm 1 and they have the maximal difference in costs with respect to firm 3. However, in this case of two equal firms, once the first decides to leave the market, the second one has no incentives to exit anymore because it can now produce a positive quantity. Hence, to calculate the welfare in the empty network we consider the welfare arising with the three unconnected firms if $9x + 5y < 5$, and the welfare of just firms 2 and 3 not connected if $9x + 5y > 5$.

3. EMPTY NETWORK:

$\frac{1}{169}(11625 + 799x^2 + 830xy + 830y^2 + 310(x + 2y) - 620(15 + x + 2y))$, if $9x + 5y < 5$

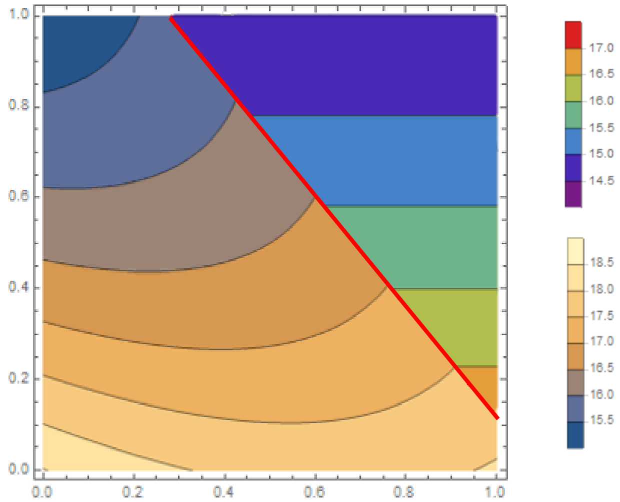
$\frac{1}{14}(1000 + 40y + 37y^2 - 80(10 + y))$, if $9x + 5y > 5$.



In the star network with firm 2 or 3 hub, firm 1 is driven out of the market for high values of x and y. When the hub firm is the middling firm, it manages to reduce its production costs thanks to the collaboration with both the other firms and thus to increase the cost gap with the least efficient firm. As a result, the latter has more possibilities to be driven out of the market. Firm 1 produces a positive quantity if $18x + 15y < 20$ or $18x + 7y < 20$ in the star networks with firm 2 or firm 3 hub respectively. Therefore, in the calculation of welfare in these two star networks, I consider the case of three firms if $18x + 15y < 20$ and $18x + 7y < 20$, and the case of firms 2 and 3 remaining and linked otherwise.

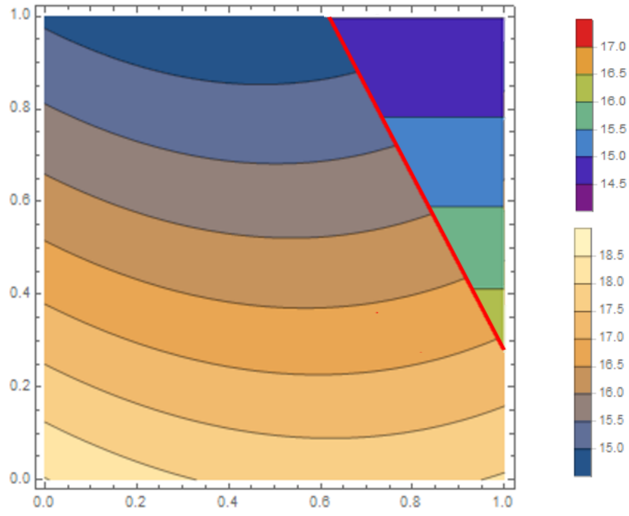
4. STAR NETWORK (firm 2 hub):

$$\frac{1}{841}\left(77875 + 1790x + 1400x^2 + 4440y + 2604xy + 1665y^2 - 20(3115 + 179x + 444y)\right),$$
$$if\ 18x + 15y < 20$$
$$\frac{1}{882}(76500 + 3060y + 545y^2 - 6120(10 + y)), \quad \text{if } 18x + 15y > 20.$$



5. STAR NETWORK (firm 3 hub):

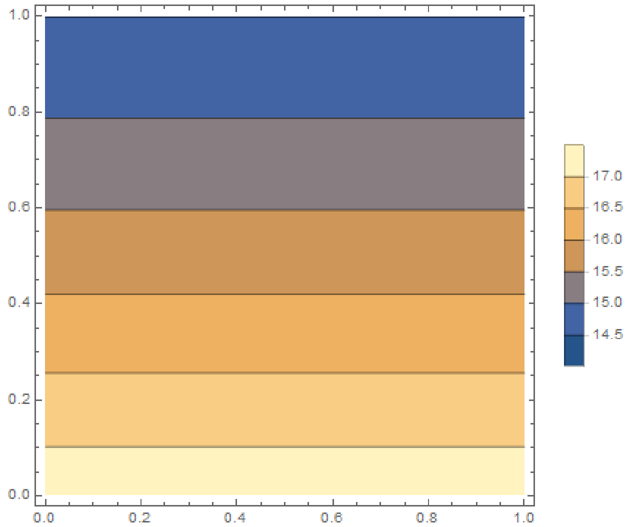
$$\frac{1}{841}\left(77875 + 1400x^2 + 554xy + 554y^2 + 1790(x + 2y) - 20\left(3115 + 179(x + 2y)\right)\right), \quad \text{if } 18x + 7y < 20$$
$$\frac{1}{882}(76500 + 3060y + 545y^2 - 6120(10 + y)), \quad \text{if } 18x + 7y > 20$$



In the partially connected network with firm 1 isolated, the latter never survives market competition. This firm already starts with a cost disadvantage, which increases when the other two firms form a collaboration and manage to decrease their production costs even more. Hence, this network cannot exist and we are left again with the case of firms 2 and 3 remaining and linked.

6. COMPLETE NETWORK WITH ONLY FIRMS 2 AND 3 REMAINING:

$$\frac{1}{882}(76500 + 3060y + 545y^2 - 6120(10 + y))$$

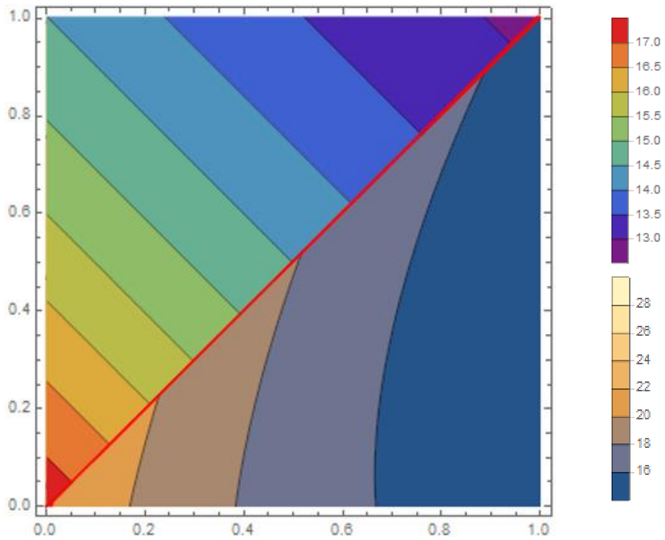


In the partially connected network with firm 2 isolated, it is the middling cost firm which has now the risk to be driven out of the market. It happens whenever $x < y$, thus when it has a smaller cost difference with the least efficient firm, which however benefits from the collaboration with the most efficient one. For $x < y$, I consider the welfare arising in the case of just firms 1 and 3 remaining and connected.

7. PARTIALLY CONNECTED NETWORK (firm 2 isolated) :

$$\frac{1}{32}\left(3500 + 380x + 147x^2 - 100y - 170xy + 99y^2 - 40(70 + 19x - 5y)\right), \quad \text{if } x > y$$

$$\frac{1}{882}(76500 + 3060(x + y) + 545(x + y)^2 - 6120(10 + x + y)), \quad \text{if } x < y$$

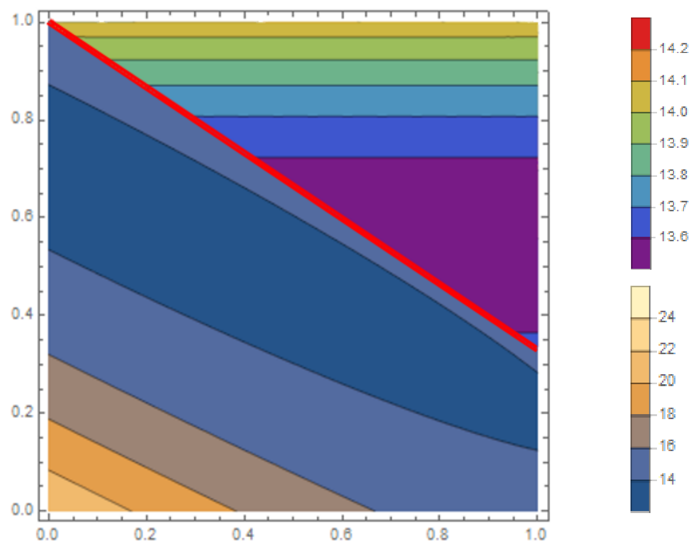


In the partially connected network with firm 3 isolated, firm 1 is forced to leave the market if $7x + 10y > 10$. Once the least efficient firm exits, the other two remain and are not linked. This is also the unique network architecture in which firm 3 can exit the market. However, it can only happen in the extreme case of $x = y = 0$, namely when it loses any cost advantage with respect to the other two firms.

8. PARTIALLY CONNECTED NETWORK (firm 3 isolated) :

$\frac{1}{32}(700 + 147x^2 + 540xy + 540y^2 + 380(x + 2y) - 760(x + 2y))$, if $7x + 10y < 10$.

$\frac{1}{14}(1000 + 40y + 37y^2 - 80(10 + y))$, if $7x + 10y > 10$



4.2 STABILITY

As already mention above, in this study I will focus on the wide concept of pairwise stability, assuming that firms cannot collaborate with each other to form another network once a configuration is created. Therefore, only unilateral deviations or bilateral decisions on a single link are possible to change an existing network.

In order to find the pairwise stable network as defined by Jackson and Wolinsky (1996), I should evaluate the incentives of firms to delete an existent link or create an additional one for each values of x and y. Since developing this model with two changing parameters would be too complicated, I will fix the value of one of the two and see what happens as the other varies between 0 and 1. Therefore, I will first focus on the case of $y = 0$ and then on the one of $x = 0$ to find the stable network. In the first setting, there are only two different types of firms: two equally most efficient firms and a less efficient one. In the second case, there are instead two least efficient firms and a more efficient one.

In both cases, to find the pairwise stable network I need to focus on the profits that firms could gain in each configuration. For each network, I consider each firm individually and I compare the profits in the current setting with the profits in a configuration in which it adds or deletes a link (*see Appendix for the graph*). Starting from a specific network, if no firm has economic incentives to sever one link and no pair of firms have incentives to form an additional link, it means that this network is pairwise stable. Since pairwise stability depends on the initial conditions, there can be more than one pairwise stable network.

4.2.1 Case of $y = 0$ (firms 2 and 3 equal)

Starting from the empty network, firm 1 remains in the market if $x \leq 0.56$. For these values of x, every pair of two firms has an incentive to connect. Hence, the empty network is not stable. If $x > 0.56$, only firms 2 and 3 remain. Profits for both firms are higher if they are linked, thus the empty network does not survive even in this case.

In the complete network, firms 2 and 3 have no incentives to break their link to form the star network 1 "hub", because their profits would decrease. Firm 2 (or 3, as they are equal) has an incentive to break the link with firm 1, forming the star network 3 (or 2) hub, when its profits are higher here than in the complete network, i.e. when $x > 0.68$. Therefore, *the complete network is stable if $x \leq 0.68$.*

In the partially connected network with firm 1 isolated, the latter always exits from the market because its marginal costs of production are higher than its revenues. Thus, in this network, just firms 2 and 3 remain and they are linked, and they have no incentives to break this link. If firms 1 and 2 (3) decide to link, the star network 2 (3) hub will be formed. Firm 1 always wants to connect, because otherwise it is driven out of the market. Firm 2 (3) also prefers to connect with firm 1, forming the star network with firm 2 (3) hub, if $x > 0.13$. Therefore, the partially connected network with firm 1 isolated never exists and in its place there is the *complete network with only firms 2 and 3 remaining. This network is stable for $x \leq 0.13$.*

Starting with the partially connected network with firm 2 (3) isolated, firm 1 has never the incentive to break the link with firm 3 (2), as its profits would be lower in the empty network and for $x > 0.56$ it would also be driven out of the market. Firm 3 (2), on the other side, has incentives to break the link if $x > 0.87$, because it would have higher profits in the empty network with only firms 2 and 3 (as firm 1 would exit) than in the partially connected network with firm 2 (3) isolated. Firm 2 (3) always wants to form the link with firm 1 to form the star network firm 1 hub, and firm 1 wants to form this link only if $x > 0.19$. Firm 2 (3) also has always an incentive to link with firm 3 (2) to form the star network 3 hub. Firm 3 has this incentive if $x > 0.21$. In the case in which $x = 0$, the three firms are equal. In this case, firm 2 (3), which is isolated, has a too great cost disadvantage and is driven out of the market. Therefore only firms 1 and 3 (2) remain and are connected. If they decide to break their collaboration, firm 2 would enter the market again and the empty network with three firms will be formed. As the profits would be lower for both, they decide to keep their link. From the complete network with only firms 1 and 3 (2), if firm 1 decides to form a link with firm 2 (3), the latter would be in the market and the new architecture would be the star network with firm 1 hub. However, firm 1 would have lower profits and thus this link will not be formed. The same would happen for firm 3 (2) creating a link with firm 2 (3). As a result*, the partially connected network with firm 2 (3) isolated is stable for* $0 < x \leq 0.19$*, and the complete network with just firms 1 and 3 (2) is stable for* $x = 0$.

In the star network firm 1 hub, profits of firms 2 and 3 are lower than in the complete network, so they have an incentive to form a link. The link between firms 1 and 2 will break from small values of x (≈ 0.2), forming the partially connected network with firm 2 isolated, because firm 1 would increase its profits. The same for the link between firms 1 and 3. Thus, the star network 1 hub is not stable for any value of x.

In the star network firm 2 (3) hub, firm 1 has an incentive to connect with firm 3 (2) and form the complete network for every value of x, while firm 3 (2) has an incentive only if $x < 0.68$. If the link between firms 1 and 2 (3) breaks, the partially connected network with firm 1 isolated will be formed. In this case firm 1 always exits, thus it has never the incentive to break this link. For firm 2 (3), we consider the case in which there are only firms 2 and 3 connected. In this case firm 2 (3) has an incentive to break the link with firm 1, forming the complete network with firms 2 and 3 linked if $x < 0.13$. Firm 2 (3) has an incentives to break the link with firm 3 only if $x < 0.21$, even if firm 3 (2) has never this incentive. Therefore, *the star network 2 (3) hub is thus stable if* $x \geq 0.68$.

The following table sums up the results on stability in the case of $y = 0$. In the cut-off values of the columns, firms that can decide on forming or deleting a link have the same profits in the two possible networks and are thus indifferent between the two. Therefore, I assume that once one of the architecture is formed, this is stable. Since the decision to form a link is a long-run decision, firms do not have incentives to change the current network for another one if the profits would remain the same.

Table 1: STABILITY ($y = 0$)

$\boldsymbol{x = 0}$	$\boldsymbol{0 < x \leq 0.13}$	$\boldsymbol{0.13 \leq x \leq 0.19}$	$\boldsymbol{0.19 \leq x \leq 0.68}$	$\boldsymbol{0.68 \leq x \leq 1}$
Complete network	*Complete network*	*Complete network*	*Complete network*	*Star network with 2 or 3 hub*
complete network with firms 1 and 3 (2) remaining	*Partially connected network with 2 or 3 isolated*	*Partially connected network with 2 or 3 isolated*		
Complete network with only firm 2 and 3 remaining	*Complete network with only firm 2 and 3 remaining*			

As we can see in the table summarizing the results (*table 1*), for small values of x there are multiple equilibria. The one that will be actually chosen depends on the initial conditions and firms agreements, and we cannot forecast it using the concept of pairwise stability. This concept of stability only assures that once the network is created, no firm would have a unilateral incentive to delete a link and no pair of firm would have higher profits creating a new link. If we would like to know whether a network is also stable against coalitional deviations, we should employ the concept of strong stability by Jackson and van den Nouweland (2005). I leave this topic for further research.

4.2.2 Case of $x = 0$ (firms 1 and 2 equal)

In the case of the empty network, firm 1 (or firm 2, as they have the same initial production cost) produces a positive quantity for every value of $y \neq 1$. If firms 2 (or 1) and 3 form a link, firm 1 (2) is driven out of the market and only firms 2 (1) and 3 remain. Firm 2 (1) has always incentives to connect with firm 3 to increase its profits, while firm 3 has incentives only if $y < 0.74$. Firms 1 and 2 have always reasons to link to form the partially connected network with firm 3 isolated. In the extreme case of $y = 1$, firms 1 and 2 are extremely disadvantaged with respect to firm 3 and, even if they try to connect, one of the two has to exit. If firm 1 (2) leaves the market, the architecture that will arise is the empty network with only firms 2 (1) and 3, because, despite firm 2 (1) has incentive to connect with firm 3, the latter has no incentive to do it. Therefore, *the empty network is stable only if $y = 1$, and this is the case of only firms 2 (1) and 3 remaining.*

In the complete network, every firm manages to make positive profits. Firms 1 and 2 have never the incentives to break their link. Firm 3 has an incentive to break its link with firm 2 (1) to form the star network 1 (2) hub, only if $y > 0.76$, while firm 2 (1) has no incentives to break this link. Hence, *the complete network is stable if $y \leq 0.76$.*

The partially connected network with firm 1 (2) isolated never exists because firm 1 (2) is always driven out of the market. As a result, only firms 2 (1) and 3 remain and are connected. If they decide to sever this link, firm 1 (2) enters the market again and the empty network with three firms would arise. Firm 2 (1) has no incentives to break this link, while firm 3 wants to delete it if $y > 0.74$. If firm 1 (2) and 2 (1) decide to link, the star network with firm 2 (1) hub will be formed. Firm 1 (2) has always incentives to link with firm 2 (1), because otherwise it is forced to leave the market. If > 0.12 , also firm 2 (1) has higher profits if it connects with firm 1 (2), forming the star network with firm 2 (1) hub. If firm 1 (2) and 3 decide to link, the star network with firm 3 hub will be created. Firm 1 (2) has always incentives to link with firm 3 because otherwise it exits, while firm 3 has never this incentive. Hence, *the complete network with only firms 2 (1) and 3 connected is stable only if $y \leq 0.12$.*

The partially connected network with firm 3 isolated exists for all values of $y \in (0,1)$. If $y = 1$, one of the two least efficient firms would exit and only firms 1 (or 2) and 3 remain and are not linked. In this case, they both have incentives to start a collaboration and create the complete network with only firms 1 (2) and 3. On the other side, firm 1 (2) wants to connect with firm 3, forming the star network 1 (2) hub when $y > 0.10$.

However, firm 3 agrees only if $y < 0.66$. If $y = 0$, there are no initial difference in the three firms' costs. In this particular case, firm 3 is driven out of the market and the complete network with only firms 1 and 2 remains. When this network exists, firms 1 and 2 have never incentives to break their link to form the complete network with the three firms. Moreover, firm 1 (2) has no incentives to form an additional link with firm 3 to create the star network 1 (2) hub when $y = 0$. Therefore, this configuration is stable in this particular case. As a result, *the partially connected network with firm 3 isolated is stable for* $y \in (0,0.10]$ *and for* $y \in [0.66,1)$. *The complete network with only firms 1 and 2 remaining is stable for* $y = 0$.

In the star network with firm 1 (2) hub, firm 2 (1) always wants to connect with firm 3 to form the complete network, but firm 3 only accepts if $y < 0.76$. If firms 1 (2) and 2 (1) break their link, the partially connected network with firm 2 (1) isolated arises. In this case, firm 2 (1) always exits and only firms 1 (2) and 3 remain and are connected. Therefore, firm 2 (1) never wants to break this link. Firm 1 (2) wants only if $y < 0.12$. If firms 1 (2) and 3 break their link, the partially connected network with firm 3 isolated is formed. Firm 1 (2) wants to break the link if $y < 0.10$. Firm 3 has incentives to break the link for $y > 0.66$. If $y = 1$, firm 1 (or 2) exit from the market and the empty network with firms 2 (or 1) and 3 remains. Hence, *the star network with firm 1 (2) hub is never stable.*

If firms start from the configuration of the star network with firm 3 hub, firms 1 and 2 always want to connect to form the complete network, because they would have higher profits. Moreover, firm 3 has also always the incentives to delete the link with firm 2 (1), to drive firm 1 (2) out of the market and form the complete network with only firm 1 (2) and 3. In both these cases, we can see that *the star network 3 hub is never stable.*

The following table (*table 2*) sums up the results of stability in the case of $x = 0$. Also in the setting of $x = 0$, there is more than one pairwise stable network. Therefore, we cannot know a priori which one firms will form, relying solely on the concept of pairwise stability.

Table 2: STABILITY ($x = 0$)

$\boldsymbol{y = 0}$	$\boldsymbol{0 < y \leq 0.10}$	$\boldsymbol{0.10 \leq x \leq 0.12}$	$\boldsymbol{0.12 \leq x \leq 0.66}$	$\boldsymbol{0.66 \leq x \leq 0.76}$	$\boldsymbol{0.76 \leq x \leq 1}$	$\boldsymbol{y = 1}$
Complete network	*Complete network*	*Complete network*	*Complete network*	*Complete network*	*Partially connected with firm 3 isolated*	*Empty network with only firms 2 (1) and 3*
Complete network with only firms 2 (1) and 3	*Complete network with only firms 2 (1) and 3*	*Complete network with only firms 2 (1) and 3*		*Partially connected with firm 3 isolated*		
Complete network with only firms 1 and 2	*Partially connected with firm 3 isolated*					

4.3 EFFICIENCY

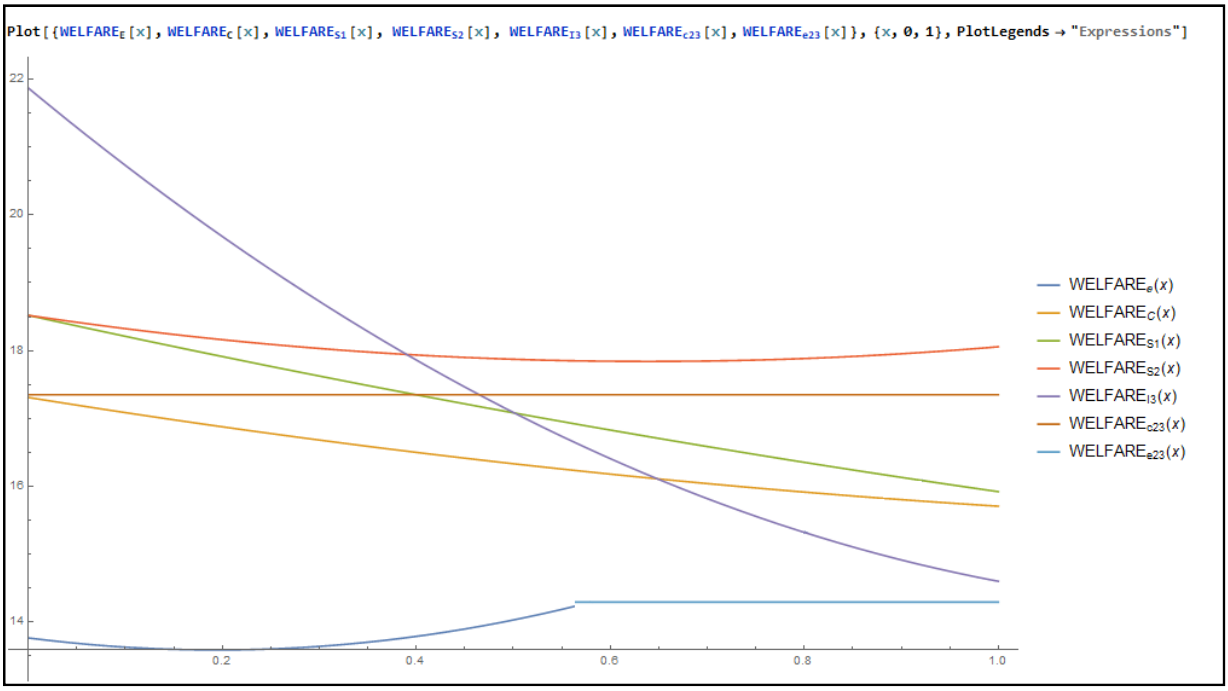
The efficient network, given all the assumptions of the model, is the one that produces the highest social welfare. Since the value of social welfare is conditional on the values of x and y, the efficient network architecture could vary depending on the difference in firms' initial marginal costs. As the final aim is to give policy recommendations by comparing the stable and the efficient networks, I will focus again on the previous two cases, fixing the values of one variable at 0 and letting the other vary in the interval $[0,1]$.

In order to calculate the efficient network, I have simply plotted the social welfare for the different networks in a Cartesian plane, having the value of x $(or\ y)$ on the horizontal axis and the social welfare on the vertical axis.

4.3.1 Case of $y = 0$ (firms 2 and 3 equal)

Below you can see the graph for $y = 0$, with all the networks plotted. The horizontal and vertical axis measure the values of x and of the social welfare respectively. Remember that in this case firms 2 and 3, the most efficient firms, are identical. What varies in this setting is the efficiency in marginal costs with respect to firm 1, which increases as x grows. In this graph the social welfare is plotted in the interval $x \in [0,1]$. As already explained before and as it is clear from the diagram below, some of those architectures do not exist for some values of y because one firm is forced to leave the market.

Figure 3: welfare for $y = 0$



e = empty network, c = complete network, s1 = star network with firm 1 hub, s2 = star network with firm 2 (or 3) hub, i3 = partially connected network with firm 2 (or 3) isolated, c23 = complete network with firms 2 and 3 remaining, e23 = empty network with firms 2 and 3 remaining

From the graph we can see that the partially connected network with firm 3 (or 2) isolated is efficient for small values of x, which I calculated to be the values of $0 < x \leq 0.39$. When $x = 0$, firm 3 (or 2) exits and the complete network with only firms 1 and 2 (3) remains. The value of welfare in $x = 0$ with just these two firms is the same as the welfare of the complete network with firms 2 and 3, since in this point all the three firms are equal. In $x = 0$, the efficient network is thus the star network with any of the firms hub. For higher values of x, namely when $x \geq 0.39$, the star network with firm 2 or 3 hub becomes efficient. All the other networks are never efficient.

Analysing the diagram more in depth, we can compare the networks according to the welfare. The worst architecture is the empty network, which is composed by all the firms for $x < 0.56$ and only by firms 2 and 3 for $x \geq 0.56$. This network has a social welfare of around 14, which becomes slightly greater when firm 1 exits.

The partially connected network with firm 3 (or 2) isolated starts at a welfare of almost 22 for small values of x but higher than 0, because here it does not exist. As x increases, firm 1 becomes more and more disadvantaged with respect to the other two firms. The two linked firms, 1 and 2 (3) put lower efforts with respect to the isolated firm because they benefit from the research of the partner, and they also decrease their efforts as x increases. On the contrary, the isolated firm increases its efforts when the other two firms decrease them. The profits of the two linked firms decrease as x increases, due to the lower efforts to reduce the marginal costs of production; while the profits of firm 3 increase following the increase in x, due to the higher efforts. As a result, the social welfare declines considerably as x grows.

The star network with firm 2 hub and the complete network with only firms 2 and 3 remain quite stable at a value of social welfare around 18 and 17.5 respectively. The star network with firm 1 hub starts at the same point as the star network firm 2 hub, but it decreases faster as x increases. The complete network is always better than the empty one, but it is always dominated by the star networks and by the complete networks with only firms 2 and 3.

Considering the efficient networks only, the highest social welfare is reached as x approaches 0 in the case of the partially connected network with firm 3 (or 2) isolated, in which it nearly touches a value of 22. Therefore, the more the firms are similar, the higher the social welfare is. However, the isolated firm should have a slight advantage with respect to the least efficient firm, otherwise it is driven out of the market. After the value of $x = 0.39$, when the partially connected network with firm 3 (or 2) isolated becomes efficient, the value of welfare remains almost constant at around 18. This is because the big increase in the profits of firm 3 (or 2) is almost completely offset in the social welfare equation by the decrease in the consumer surplus and in the profits of the other two firms.

The following table shows the efficient network from the welfare standpoint for $y = 0$ and $x \in [0,1]$.

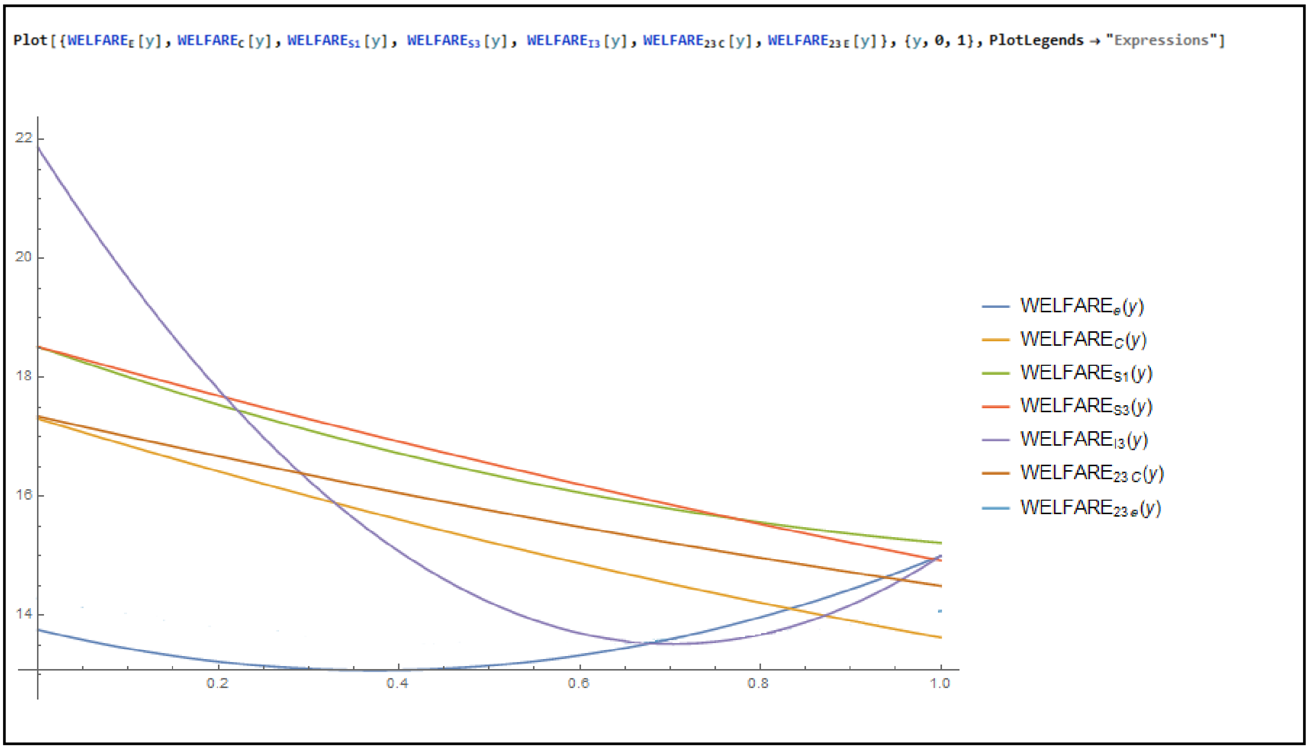
Table 3: EFFICIENCY ($y = 0$)

$x = 0$	$0 < x \leq 0.39$	$0.39 \leq x \leq 1$
Star network with any hub firm (they are all equal)	*partially connected network with firm 3 (or 2) isolated*	*star network with firm 2 (or 3) hub*

4.3.2 Case of $x = 0$ (firms 1 and 2 equal)

Below you can see the graph for $x = 0$, with all the networks plotted. The horizontal and vertical axis measure the values of y and of the social welfare respectively. Remember that in this case firms 1 and 2, the least efficient firms, are identical. What varies in this setting is the difference in marginal costs with respect to firm 3, which decreases as y declines, making the three firms more similar.

Figure 4: welfare for $x = 0$



e = empty network, c = complete network, s1 = star network with firm 1 (or 2) hub, s3 = star network with firm 3 hub, i3 = partially connected network with firm 3 isolated, 23c = complete network with firms 2 (or 1) and 3 remaining, 23e = empty network with firms 2 (or 1) and 3 remaining

From the graph we can see that for small values of y, namely for $y \in (0,0.21]$, the star network with firm 3 isolated is efficient. From this point onwards, the star network with firm 3 hub becomes efficient, until $y = 0.77$. Here, the star network with firm 1 (or 2) hub becomes efficient until the value of $y = 1$. In $y = 0$, the partially connected network with firm 3 isolated does not exist anymore because the isolated firm is driven out of the market. Hence, in this point we should consider only firm 1 and 2 linked. In this point all the three firms are identical, therefore the social welfare is the same that we can see in the graph in the case of firms 2 and 3 connected. This value is around 17.3 and is thus lower than the welfare of the star network with any firm hub, which is then the efficient network in $y = 0$.

Focusing on the performance of the individual networks, we can see that the star networks follow almost the same decreasing path, starting from a welfare of 18.5 when $y = 0$ and reaching a welfare of around 15 when $y = 1$. The complete network with the three firms and the complete network with only firms 1 (or 2) and 3 have also a decreasing pattern, but they both perform worse than the star networks. In particular, the complete network with all the firms is dominated by the star architectures and by the complete network with the two firms illustrated above. The empty network starts at a value of social welfare of almost 14, being the lowest value among all the networks. This value decreases even more as y acquires values of around $[0.2,0.5]$, then it raises a bit as y approaches 1, because firm 3 acquires competitive advantage with respect to its rivals and increases its profits more. However, when $y = 1$ one of the least efficient firms is forced to leave the market, leaving the other two firms there and not connected. Here, the value of welfare is lower and it is about 14.

The strangest pattern is the one of the partially connected network with firm 3 isolated. In this case, the social welfare starts at the most efficient value of almost 22 for y slightly higher than 0 when it begins to exist. However, the welfare drops substantially as y increases, because the profits of the least efficient firms and the consumer surplus considerably decrease. Then, social welfare reaches a value of about 15.5 when $y \sim 0.7$, turning this network into the least efficient one. After this point, the welfare increases again as y approaches one. In $y = 1$ this network does not exist anymore because one of the least efficient firms exits the market.

Considering the efficient networks only, the highest social welfare is reached as x approaches 0 in the case of the partially connected network with firm 3 isolated, in which it nearly touches a values of 22 as in the case of $y = 0$ and $x \in [0,1]$. Again, the more the firms are similar, the higher the social welfare is. However, the isolated firm should have a slight advantage with respect to the least efficient firm, otherwise it is driven out of the market. For $y > 0$, the value of social welfare decreases monotonically as y increases.

The following table shows the efficient network from the welfare standpoint for $x = 0$ and $y \in [0,1]$.

Table 4: EFFICIENCY ($x = 0$)

$\boldsymbol{y = 0}$	$\boldsymbol{0 < y \leq 0.21}$	$\boldsymbol{0.21 \leq y \leq 0.77}$	$\boldsymbol{0.77 \leq y \leq 1}$
Star network with any hub firm (they are all equal)	*Partially connected network with firm 3 isolated*	*Star network firm 3 hub*	*Star network with firm 1 (or 2) hub*

4.4 POLICY RECOMMENDATION

In this chapter I will discuss the stability and efficiency of the potential networks, comparing them and focusing on the policy implications. As already mention before, the aim of my study is to suggest to the relevant authorities – the government and the Competition Authorities - some policy interventions to increase the social welfare. I consider the objective of those authorities the one of maximising the welfare of both the consumers and the producers, namely the social welfare.

It is widely recognized in the literature that markets typically generate a suboptimal amount of R&D. Therefore, some ways to solve this problem have been studied both theoretically and with their practical implications. A study by Takalo (2013) examines the benefits of R&D subsidies as a policy tool in the hands of the government in Finland to decrease the marginal cost of research, thus increase R&D investments and thereby firm profits and consumer surplus. Another paper by Koehler (2018) analyses the benefits of R&D subsidies for Germany and reports the welfare gains for the society. Although these papers do not use the same basic assumptions that I considered in my model, they show the effective empirical use of government subsidies for R&D and remarks the benefits of this policy tool for the economy.

Government's R&D subsidies can be used for different purposes. Zikos (2010) analyses the case in which government pays a percentage of firms' R&D effort costs, i.e. it provides a subsidy per unit of R&D effort in place. This intervention can be effective when the high costs of efforts make firms invest less in research or form too few links, and therefore social welfare is not at the optimal level. In models in which forming a link is costly (Goyal and Joshi, 2003), as we will see in the last section of this paper, the government could intervene by paying part of the cost of link formation to increase firms' incentives to collaborate. Another possible action could be subsidize every link formed, setting the contribution to a level that makes firms endogenously create the efficient network from the social welfare standpoint.

However, there could be cases in which firms form too many collaboration links with respect to the social optimum. In this case, the government has the power to intervene by taxing the formation of a link. As the cost of forming a link is now positive, firms should have incentives to decrease the number of their links. Again, there are various ways to impose a tax. It can be set on specific types of links, after a certain number of links or on the number of collaborations. It the latter case, the tax can be constant or decreasing in the number of connections.

The other regulator that could act to increase total welfare is the Competition Authority. Each country has its own national competition authority, which in the case of European Union cooperate with each other in the context of the European Competition Network to ensure the proper functioning of the European internal market (*ec.europa.eu website*). The role of this agency is to regulate and enforce competition law, mainly with the purpose of protecting consumers. The way in which they operate and the policy tools at their disposal can vary slightly according to the country, however I do not focus on the specific cases but on a more general setting. A tool that Competition Authorities frequently have is the possibility to prohibit some agreements if they can have negative consequences for the society. For example, the competition law aims at consumers' protection, prohibiting mergers that lead to the formation of monopolies and blocking other agreements that could affect consumers negatively. In the European Union, the aim of competition policy is to safeguard a competitive market and the competition authorities can act by banning certain conducts, requiring remedial action or imposing a fine (*ec.europa.eu website*). In the specific case of R&D networks, this regulation authority can act by prohibiting collaboration agreements that can lead to an excessive exit from the market or hinder competition among firms.

In the following paragraphs I will thus compare the results for the efficient and pairwise stable networks obtained before, to check whether they already coincide or whether some authority's intervention is required. When the stable and the efficient networks are different, the government should step in to attain a better outcome for the society as a whole. For those values of x and y for which firms already form the welfare maximising architecture, the political action can target the marginal costs of production of the firms. In this case, the intervention aims at making them more similar to move to a part of the graph where the social welfare is higher.

Therefore, the possible policy interventions that I consider are of two types. The first intervention consists on giving subsidies or taxing some links, or even on prohibiting some of them, to change firms' incentives and make them create a network with a higher social welfare. The second option is instead to subsidise the marginal cost of production of some firms, to make them more similar among them and thus moving to the scenario that attains a higher welfare. These two policy interventions could also be combined to reach a better outcome.

However, before interfering in this matter, the government and the competition authorities have to consider some issues. First, subsidising the production costs of one or more firms or alternatively paying firms for forming or deleting a link, comes with a cost for the government. If this expense is higher than the total increase in social welfare, implementing this policy is pointless. In this case, if the government believes that one side of the economy is disadvantaged, it could just rearrange the surplus through a change in taxation. Second, the government or the competition authorities need to be legitimated to intervene in this issue. It means that they should follow some rules in their actions, which require them to have a reason to ban or incentivise some agreements. Even though the final aim of reaching a higher social welfare is fair and legitimate, pursuing this objective can give advantages to some firms at the expense of others. While taxing the number of links or giving equal subsidies to all the firms can be accepted, a policy that taxes a specific link between two firms or subsidises the production costs of only one of them is not always legal. In order to implement this strategy, the government should justify its actions towards the firms. A small advantage to the least efficient firms can be considered fair because it creates a more balanced environment, while a subsidy to the most productive firm may not be allowed, especially if it drives the disadvantaged firms out of the market or if it decreases their profits considerably. Therefore, it is important to contemplate all the possible complaints when deciding on a policy. Moreover, the government should consider the consequences of the subsidies for all the actors. If the social welfare after the intervention is higher but one firm is worst off with respect to the previous outcome, this policy can be perceived as unfair and unacceptable. The new outcome should be at least Pareto efficient, and possibly, it should maintain the order of profits among firms. If the most efficient firm becomes now the disadvantaged one, the intervention is not legitimate. A redistribution of the surplus is thus necessary in the policy action.

As I have mention before, there are also other options to change firms' incentives to create another network, like taxing or subsidizing the costs of efforts, or allowing the firms to make transfers among them. I will briefly discuss these options in *Chapter 5,* which explores the extensions of the basic model in the literature on strategic R&D networks.

4.4.1 Case of $y = 0$ (firms 2 and 3 equal)

As before, I first focus on the case in which $y = 0$, namely when firm 1 is disadvantaged with respect to the other two. The following table summarises and compares the results for the efficient and the stable networks, showing the correspondences between the two. Additionally, it also shows how the value of social welfare (SW) changes in any of the possible cases.

Table 5: STABILITY AND EFFICIENCY ($y = 0$)

	$\boldsymbol{x = 0}$	$\boldsymbol{0 < x \leq 0.13}$	$\boldsymbol{0.13 \leq x \leq 0.19}$	$\boldsymbol{0.19 \leq x \leq 0.39}$	$\boldsymbol{0.39 \leq x \leq 0.68}$	$\boldsymbol{0.68 \leq x \leq 1}$
STABILITY	*Complete network* ($SW \approx 17.3$) *Complete network with only two firms remaining (they are all equal)* ($SW \approx 17.3$)	*Complete network* ($SW \approx 17.3 \rightarrow 17$) *Partially connected network with firm 3 (or 2) isolated* ($SW \approx 21.8 \rightarrow 20.5$) *Complete network with only firm 2 and 3 remaining* ($SW \approx 17.3$)	*Complete network* ($SW \approx 17 \rightarrow 16.8$) *Partially connected network with firm 3 (or 2) isolated* ($SW \approx 20.5 \rightarrow 19.5$)	*Complete network* ($SW \approx 16.8 \rightarrow 16.5$)	*Complete network* ($SW \approx 16.5 \rightarrow 16$)	*Star network with firm 2 (or 3) hub* ($SW \approx 18$)
EFFICIENCY	*Star network with any hub firm (they are all equal)* ($SW \approx 18.5$)	*Partially connected network with firm 3 (or 2) isolated* ($SW \approx 21.8 \rightarrow 20.5$)	*Partially connected network with firm 3 (or 2) isolated* ($SW \approx 20.5 \rightarrow 19.5$)	*Partially connected network with firm 3 (or 2) isolated* ($SW \approx 19.5 \rightarrow 18$)	*Star network with firm 2 (or 3) hub* ($SW \approx 18$)	*Star network with firm 2 (or 3) hub* ($SW \approx 18$)
COMPARISON	*No correspondence*	*One corresponds*	*One corresponds*	*No correspon-dence*	*No correspon-dence*	*Perfect correspon-dence*

Legend: *$SW \approx A \rightarrow B$: Social welfare changing from a value of around A to a value of around B in the interval considered*

As we can see in the table summarising the results for stability and efficiency, there is little correspondence between the two assuming $y = 0$. The unique correspondence is in the event of $0 < x \leq 0.19$ or when $0.68 \leq x \leq 1$. In the first case the partially connected network with firm 3 (or 2) isolated is both pairwise stable and efficient. However, this architecture is not the uniquely stable network. Indeed, there are other networks that are pairwise stable but attain a lower welfare, namely the complete network in the entire interval and the network with only firms 2 and 3 linked for $0 < x \leq 13$. Therefore, we cannot know in advance which network will arise and hence there is not guarantee that firms endogenously create the welfare maximising one. For $0.68 \leq x \leq 1$, the star network with firm 2 (or 3) hub is both efficient and the uniquely pairwise stable network. With the assumptions that firms cannot collude with other firms to create a new network and that they decide their efforts and quantities non-cooperatively, they create the network with the highest possible welfare by themselves in this interval. Hence, in these circumstances there is no need of policy intervention to increase the welfare.

We can now compare the stable and efficient network with reference to welfare. If $x = 0$, the two pairwise stable networks, namely the complete structures with two or three firms, reach a welfare of around 17.3. This is more than one point lower than the welfare we could attain if firms decided to form the star network with any hub firm. The complete architecture continues to be stable until $x = 0.68$, with the social welfare decreasing from a value of 17.3 to a value of around 16. The efficient network until $x = 0.39$ is again the partially connected network with firm 3 (or 2) isolated, which starts from a social welfare of 21.8 when x approaches 0, and decreases to 18 when $x = 0.39$. Therefore, the welfare difference between the stable and the efficient configuration goes from 4.5 to 1.5 point in that interval. In this case, firms form too few links with respect to the social optimum. Between $x = 0.39$ and $x = 0.68$ the complete and pairwise stable network reaches a welfare of around 16.5 to 16 points, which is almost two points lower than the value attained by the star network with firm 2 (or 3) hub. Even in this case firms endogenously form too few link as compared with the efficient architecture.

These results show that the highest welfare is reached when there is little difference in the initial marginal costs between the three firms and they form the partially connected network with firm 3 (or 2) isolated. Fortunately, this network is also pairwise stable, even though it is not the only one. If the firms start with an initial cost difference which is small, i.e. higher than 0 but lower than 0.19, the best policy measure is the one that makes firms create that network. In order to do that, the government or competition authorities can use different strategies, making sure that they are allowed to do that by law.

In the interval $x \in (0,0.13]$, the other pairwise stable networks are the complete network with three firms or with only firms 2 and 3 remaining. In case the complete network arises, there are too many links with respect to the welfare-maximizing network. In case firms 2 and 3 decide to collaborate, firm 1 becomes extremely disadvantaged and it is forced to leave the market. One way to avoid both those networks to arise is to prohibit the collaboration link between the two most efficient firms. In this way, the network that will emerge is the only other pairwise stable architecture remaining, i.e. the partially connected network with firm 3 (or 2) isolated, which is also the welfare-maximizing one. This solution could work well if the government is allowed to block some specific agreements. A more legitimate solution would be to allow only a certain number of links for each firm, so that the intervention is more general and does not affect any particular firm. Unfortunately, a more general ban would not work in this case: if we allow firms to form maximum a link, firms 2 and 3 would decide to collaborate between them and to exclude the least efficient firm from the market. If instead would be possible to block the agreement between the two most efficient firms, they would prefer the partially connected network with the other efficient firm isolated. Indeed, if they are the isolated firm, their profits drop to almost 0. Among the stable networks, firm 1 is better off in the partially connected network with one of the good firms isolated, where it manages to reach higher profits with respect to the other two structures. Moreover, also the consumer surplus is higher in the partially connected network with firm 3 (or 2) isolated.

As we have just seen, the welfare-maximizing network is not Pareto efficient among the pairwise stable networks, because the efficient firm that is isolated is worse off in this architecture. The competition authorities could still justify the ban on the connection between the two lowest-cost firms with the aim to attain a higher consumer surplus. As I mention before, this is actually the objective of the Competition Authorities in the European Union, and therefore this prohibition could be set following this purpose. However, I concentrate on the wider objective of attaining a higher social welfare as the government target. Moreover, I assume that the consumer surplus justification is not viable also because one of the most efficient firms would lose a lot from this policy and the government has no rights to reverse the profit order on purpose and disadvantage one of the firms that has probably already invested much in innovation to become efficient. Therefore, a possible option to make this policy viable could be compensating the losing firm with some transfers. If the creation of the efficient network increases welfare enough, we can redistribute the surplus to the worse off part not to negatively affect them. In this case, the only firm that loses from the partially connected network is the isolated firm. Indeed, its profits become almost 0 because it does not benefit from the research efforts of the other two firms, which instead get benefits from the collaboration.

Hence, I tried to check whether this strategy is feasible. I just tried with two basic examples, setting the values of x to 0.01 and 0.1, i.e. two values at the beginning and at the end of the interval respectively. Doing all the calculations for the profits, consumer surplus and social welfare, I confirmed that the only part which is worse off in the partially connected network is the efficient firm which remains isolated, for any value of x. Therefore, I tried to check how much each part gains or loses from the potential policy of banning the agreement between firms 2 and 3. In the case of $x = 0.01$, there are two different cases depending on the network we start from. If we start from the complete network, the profits of the isolated firm drop by 2.22 points, while the net increase in social welfare is 4.48 points. Hence, there is room for some redistribution by government. Three parts are better off in this case: both firm 1 and firm 2 (or 3) increase their profits by 2.43 points each, while the consumer surplus increases by 1.84. Here, the government could impose a tax on the profits of these two firms and on the consumers in order to collect at least 2.22 among the three parts and then redistribute this amount to the isolated firm. In this way, social welfare increases and the new equilibrium is Pareto efficient as the three firms and the consumers are better off with respect to the complete network structure. However, the issue is the effective possibility to tax each firm differently without changing their incentives on production levels and the fact that the two most efficient firms have now different revenues. The network that will actually arise depends on the bargaining among firms and the decision of firm 1, which now becomes the crucial firm despite being the least efficient one. Indeed, with this measure, the latter becomes more powerful and earns a lot, which is probably not fair given the initial situation.

If the starting point is the complete network with only firms 2 and 3, the profits of the isolated firm in the partially connected network would be 4.08 points lower. Again, there is room for redistribution to attain a fair outcome. As before, the government could collect taxes from the other two firms and from consumers and redistribute the collected amount to the disadvantaged firm, in such a way to make all of them better off with respect to the initial architecture. The same problems analysed above arise also in this setting.

In the case of $x = 0.1$, there are again the same two cases. Starting from the complete network, the profits of the newly isolated firm are 2.26 points lower and the increase in social welfare is 3.65 points. Compared to the previous scenario in which the three firms were almost ex-ante equal due to the value of x close to 0, in this case of more differentiated firms the least efficient firm gains less profits and the other two firms earn more profits in the complete network. The consumer surplus and the social welfare are slightly reduced with respect to the case of $x = 0.01$. If they form the partially connected network with firm 3 (or 2) isolated, the increase in profits is 2.05 and 2.13 for firm 1 and 2 (or 3) respectively. The consumer surplus increases by 1.75 points. As in the previous case, there is room for redistribution and the government could collect some taxes from these agents and give the surplus to the third firm to reach a Pareto efficient outcome. Starting instead from the complete network with only firms 2 and 3, the isolated firm loses 4.06 while the increase in the net total welfare is 3.39. Firms 1 and 2 (or 3) gain 4.05 and 0.33 respectively, while consumer surplus increases by 3.07. Firm 1 gains a lot because it would be out of the market otherwise, and also the consumer surplus increases consistently. Collecting enough taxes by these two actors to pay the cost of the change for firm 3 (or 2), every firms is better off and the new solution is therefore Pareto efficient. Again, the problem concerns the power given to the previously disadvantaged firm and the need to do the redistribution in such a way that the least efficient firm does not become more advantaged than the other two. The redistribution process must also guarantee that firms' incentives on the efforts and quantities do not change, which is hard to attain given that the objective of the firms is maximise their profits. However, if future research finds a way to overcome this limit, every time there is a net increase in the social welfare there is room for a redistribution policy, which consists on collecting taxes from the actors that would gain from the new network and redistribute them to the worst off parts to compensate them for the loss. If this redistribution is done in the correct way, every firm and consumers are better off with the new setting and therefore they have no reasons for complaining for the policy intervention. The issue is the possibility to actually collect taxes in such a targeted way and the fact that this tax does not change firms' incentives on links and efforts.

As a result, if it is proven to be viable and effective, the relevant authorities could block some specific agreements – in this case the collaboration between the two most efficient firms - and boast the welfare of the society as a whole. As the partially connected network with firm 2 (or 3) isolated is the only other stable network that firms can form in this interval after this ban, they will form the efficient network. The same reasoning is valid for the interval $0.13 \leq x \leq 0.19$.

If instead there is an initial difference in marginal costs between firms that makes the least efficient firm particularly disadvantaged, the policy intervention can be of two types.

The first option is the one that we have just explored above, which consists in making firms create the welfare maximising network of that value of x. In this case, the intervention could be blocking or encouraging some agreements, tax or subsidize some links, or change firms' incentives in other ways. Again, we must be careful with the legitimization of the action and be sure that law allows it. As before, if there is a net increase in social welfare there is theoretically room for a redistribution to make every firm benefiting from the new network, provided that the redistribution system does not change firms' incentives on links and efforts. As we can see in *table 5*, the stable architecture in the interval $0.19 \leq x \leq 0.68$ is the complete one, while the efficient networks are the partially connected with firm 3 (or 2) isolated for $x \in [0.19, 0.39]$ and the star with firm 3 (2) hub for $x \in [0.39, 0.68]$. In both cases, firms form too many links with respect to the efficient architecture. The problem here is that, contrary to the previous case, the efficient network is not pairwise stable. Therefore, I should check which collaborations firms would like to form and which ones should be blocked to make them reach the efficient network. After that, I should check whether the government is allowed to block these agreements.

In the interval $0.19 \leq x \leq 0.39$, the network that would arise is the complete network. The efficient one is instead the partially connected network with firm 3 (2) isolated. Here, the only intervention that would work is prohibiting by law the links between firm 1 and 2, and between 2 and 3. In this way, firms would form the partially connected network with firm 2 isolated because firms 1 and 3 prefer it to the empty architecture. Of course, such a policy is too strong and it is not allowed as it impedes one specific firm (firm 2) to form any collaboration, while allowing an equal firm (firm 3) to do it. As there is no reason to target one firm instead of the other, probably there would be problems in the implementation of this policy even though the welfare would be higher and could be redistributed to make everyone better off. For the values of x between 0.19 and 0.21, it is enough to set a policy that provides for a maximum of one link for each firm. In this way, firms would form the partially connected network with one of the efficient firm isolated, as no one would prefer the empty network and the efficient firm prefers a link with firm 1 than with the other efficient firm. As a result, depending on the outcome of the bargaining among firms, the resulting architecture would be the partially connected network with firm 3 or 2 isolated. It is true that this law disadvantages one of the most efficient firms, but it does not target one of the three firms specifically and leaves them more freedom. Being a more general prescription, it is easier to implement and to justify with the aim of achieving a higher welfare. Moreover, the redistribution policy that would follow this action could offer another reason to implement it.

After $x = 0.21$, this general ban would not work, as the resulting network would be the complete network with the two most efficient firms and firm 1 driven out of the market. The objective of the government is also to help the less efficient enterprises in competition, and hence this policy is not justifiable. What can be justified with this aim is instead the interdiction of the collaboration within the most cost-efficient firms, to help the development and innovation of the high-cost companies and thus create a more balanced environment. However, the outcome would not be the partially connected network with firm 3 (or 2) isolated, but the star network with firm 1 hub. The welfare of this network is higher than in the complete network, therefore there is still an improvement. The issue is that now the profits of the two efficient firms are lower than the ones of the least efficient firm. This is not fair, since the two other firms have probably invested a lot in innovation and research before, to be bigger and more efficient. Hence, even if this policy would attain a higher welfare and helps the less efficient firm, it is too biased towards it.

Starting from the star network with firm 2 (or 3) hub, which is the efficient network for $0.39 \leq x \leq 0.68$, firm 1 and 3 (or 2) have the incentives to link and form the complete network. If the government prohibits such collaboration, the star network with firm 2 (3) hub is now stable. Indeed, no firm would have incentive to delete any other link. Therefore, with the above ban this star architecture becomes stable. However, the government has no legitimization to prohibit the link between one efficient firm and the least efficient firm, as there are two of these links. If the government would prohibit such an agreement and justify it, it should block both. Maybe, there is the possibility to consider a legislative proposal that includes the redistribution of the surplus in such a way to justify the ban on a specific link, which is probably a too strong intervention.

As we have seen, in this case firms form too many collaboration with respect to the ones that lead to the highest possible welfare non-cooperatively. Therefore, another option is to change their incentives by taxing some of the links to discourage their formation. Again, it is important to set the tax to the right level. A deepest analysis is necessary to check which is the right amount of the tax that could discourage some agreements (same result as banning them) or that can make government gain some money to redistribute to the losing parts. Even in this case there is the problem of taxing some specific agreements, and making sure that no one loses from the policy intervention.

We have just seen the option of making firms forming the welfare-maximising network given the initial cost difference among firms. An alternative option to increase welfare is to subsidize the cost of firm 1 to make it more similar to the other two firms and then get back to the previous situation. In this case, there are two things to take into account. The first one is the legitimization of this intervention. This can come from the final aim of reaching a higher welfare, or from the government's task of helping the less efficient companies to survive market competition and of avoiding the formation of monopolies or strong duopolies. A problem would arise only in case the new outcome is not Pareto efficient, and therefore one of the parts is worst off. In the analysis, we should check if any actor is losing something and if the government has the right to intervene anyway. Otherwise, there could still be the possibility of compensating the part that loses from this intervention by giving it a share of the surplus generated through the increase in welfare, in order to make it achieve at least the same profits/surplus it had in the situation before the intervention. As we have seen before, when there is an increase in social welfare there is always room for a theoretical redistribution of the surplus that could make any agent better off. In this case, it is important that the option of the redistribution does not change firms' incentives in the choice of the links or of the efforts. An even more desirable outcome would be respecting the profit ordering of firms, namely to avoid that the least efficient firm earns now more than its efficient competitors do.

The second issue is hence the overall benefit of this intervention. If it is undoubtedly true that there is a net increase in the social welfare in the new network, subsidizing the marginal cost of some firms implies an additional cost for the government. As a result, the government's support is worth it only if it increases the net total welfare, namely if it raises the welfare more than the costs disbursed by the government.

Now, I will thus check the right amount of the subsidy to help the smallest firms and reach a higher net welfare. The highest welfare in this network is attained when there is a narrow cost inequality between the high-cost and low-cost firms, and they decide to form the partially connected network with firm 3 (or 2) isolated. This network is both efficient and one of the pairwise stable architecture, therefore it is easy for the authority to intervene and make firms form it. The subsidy that the government should give to the least efficient firm to make it compete with the other ones depends on the initial cost difference among firm and the quantity it would produce in the new stable network. Indeed, the subsidy must be given to any of the units of goods the firm should produce in the new setting. In order to check whether this intervention is worth it, we should check if the increase in the social welfare is higher than the subsidy disbursed by government.

In the case of $x = 0$ all the three firms are equal, and thus there is no specific firm in need of assistance. In this setting, the government cannot intervene to change the marginal costs of just some of the firms, because it would be unfair. The only thing it could do is helping all the firms equally, but I will not explore this case because it is beyond the purposes of this work. However, even if the pairwise stable and the efficient networks do not correspond, the difference in the welfare attained in the two cases is of just slightly more than one point. Since it is not possible to help one firm at the expenses of the others - being all the enterprises identical - the government should simply leave the situation as it is.

In the interval $0 \leq x \leq 0.19$, the cost difference between firms is small enough to attain a sufficiently high social welfare in the stable network by following the steps I have already mentioned above. For higher values of x the government can give a subsidy to the least efficient firm.

If we start from the highest cost difference possible in this model, i.e. from a value of $x = 1$, firms form the star network with firm 2 or 3 hub, and the value of social welfare is 18.05. The objective of the government could be to decrease this cost difference to a value of 0.1, which would increase the welfare up to 20.73 points in case firms create the partially connected network with one of the efficient firms isolated. This outcome can be reached by banning the agreement mentioned before, once we have the new cost difference among firms. The increase in welfare due to the change in the cost difference and consequently in the architecture created is of 2.68 points. In order to decrease the cost of firm 1 to 0.1, the government should pay a subsidy of 0.9 for each unit produced by this firm. The government foresees that firm 1, benefiting from a subsidy of 0.9 on each unit and being part of the partially connected network with firm 3 (or 2) isolated, would produce a quantity of 2.352 in equilibrium (the quantity is a decimal number, but it becomes an integer if we consider it to be expressed in thousands). Hence, the total subsidy that the government should give to the least efficient firm in this case is $2.325 * 0.9$, which is equal to 2.09. As a result, the money spent by the government to aid firm 1 are less than the social welfare growth. Consequently, the net social welfare, i.e. the value of welfare after subtracting the subsidy, becomes higher than before and implementing this policy could make sense in this case. In order to implement it the government should follow again the same steps of banning the collaboration between the most efficient firms as already discussed before.

Nonetheless, there are some issues to take into account. The first point in question concerns the determination of the right amount of taxes towards the parts that gain from this policy and the subsequent redistribution of the amount collected to the losing part. Here, it is important to check how this taxation system should be designed to be effective and legitimate. It could be that there are no ways to achieve this legally. The second point relates to the profits of the firms in the new setting. Even if, through taxation and redistribution, the authority manages to make everyone better off with respect to the previous situation in which there was a higher cost difference among firms, this would not be considered fair if the initial order of profits is not respected. If the initially disadvantaged firm manages to earn more than the other firms after the government subsidy, these two firms could complain for the aid and ask to be helped as well. Indeed, if the purpose of helping the company more in need can be justified, this aid cannot be exaggerate and subvert the initial structure.

If instead we start from a medium initial cost difference, as for instance $x = 0.5$, firms would form the complete network reaching a value of social welfare of 16.33. If we want to achieve the situation in which firm 1 has a marginal cost disadvantage of 0.1 again, the government should give a subsidy of 0.4 on each unit produced by firm 1 in the new equilibrium. As before, this quantity is 2.325, which leads to a total subsidy of 0.93. The social welfare in the equilibrium network with the new costs is 20.73, which corresponds to an increase of 4.4 points with respect to the initial welfare. Even subtracting the costs of the subsidy, the raise in welfare is still positive and corresponds to a net increase of 3.47 points. Here there is a higher social gain and a great margin for a fair redistribution. The same reasoning is valid for many other cases.

Below you can find a table summarising the recommendations for $y = 0$. As I have mention before, most of the policy recommendations involves some problems of legitimacy and viability. Further research is needed to check their actual implementation.

Table 6: recommendations for $y = 0$

	Outcome	*Policy proposal*	*Welfare attained*	*Problems*
$\boldsymbol{x = 0}$	No correspondence between the stable and efficient network.	All the firms are equal: no possibility to intervene in general terms	≈ 17.3	Difficult to justify any targeted intervention
$\boldsymbol{0 < x \leq 0.13}$	The efficient network is not the uniquely stable architecture	Prohibit the link between the two efficient firms	$\approx 21.8 \rightarrow 20.5$ ($+4.5 \rightarrow$ 3.2 points w.r.t the previous equilibrium)	Difficult to justify the ban. Possibility of a redistribution if there exists a suitable tax system that does not change firms' incentives.
$\boldsymbol{0.13 \leq x \leq 0.19}$	The efficient network is not the uniquely stable architecture	Prohibit the link between the two efficient firms	$\approx 20.5 \rightarrow 19.5$ ($+3.5 \rightarrow$ 2.7 points w.r.t the previous equilibrium)	Difficult to justify the ban. Possibility of a redistribution if there exists a suitable tax system that does not change firms' incentives.
$\boldsymbol{0.19 \leq x \leq 0.39}$	No correspondence between the stable and efficient network. Too many links.	If $0.19 \leq x \leq 0.21$, allow maximum a link for each firm --- Subsidy to the least efficient firm	≈ 19.5 (+ 2.7 points) --- Depends on the starting point, but the net social welfare increases	--- Back to the case of $0 < x \leq 0.19$, same problems
$\boldsymbol{0.39 \leq x \leq 0.68}$	No correspondence between the stable and efficient network. Too many links.	Prohibit the link between firm 1 and 2 (3) --- Subsidy to the least efficient firm	≈ 18 ($+1.5 \rightarrow$ 2 points) --- Depends on the starting point, but the net social welfare increases	No justification to target that specific link. --- Back to the case of $0 < x \leq 0.19$, same problems
$\boldsymbol{0.68 \leq x \leq 1}$	Perfect correspondence between the stable and efficient networks	No intervention needed --- Subsidy to the least efficient firm	≈ 18 --- Depends on the starting point, but the net social welfare increases	--- Back to the case of $0 < x \leq 0.19$, same problems

Legend: *$SW \approx A \rightarrow B$: Social welfare changing from a value of around A to a value of around B in the interval considered*

4.4.2 Case of $x = 0$ (firms 1 and 2 equal)

Now I will focus on the case in which $x = 0$, namely when firm 3 has a initial cost advantage with respect to the other two firms. The following table summarises and compares the results for the efficient and the stable networks, showing the correspondences between the two. Moreover, it also shows the value of social welfare (SW) in any of the possible cases.

Table 7: STABILITY AND EFFICIENCY ($x = 0$)

	$y = 0$	$0 < y \leq 0.10$	$0.10 \leq y \leq 0.12$	$0.12 \leq y \leq 0.21$	$0.21 \leq y \leq 0.66$	$0.66 \leq y \leq 0.76$	$0.76 \leq y \leq 0.77$	$0.77 \leq y \leq 1$	$y = 1$
STABILITY	*Complete network* $(SW \approx 17.3)$ *Complete network with only two firms remaining (they are all equal)* $(SW \approx 17.3)$	*Complete network* $(SW \approx 17.3 \rightarrow 16.7)$ *Complete network with only firms 2 (1) and 3* $(SW \approx 17.3 \rightarrow 17)$ *Partially connected with firm 3 isolated* $(SW \approx 21.8 \rightarrow 19.7)$	*Complete network* $(SW \approx 16.7)$ *Complete network with only firms 2 (1) and 3* $(SW \approx 17)$	*Complete network* $(SW \approx 16.7 \rightarrow 16.4)$	*Complete network* $(SW \approx 16.4 \rightarrow 14.7)$	*Complete network* $(SW \approx 14.7 \rightarrow 14.4)$ *Partially connected with firm 3 isolated* $(SW \approx 13.6)$	*Partially connected with firm 3 isolated* $(SW \approx 13.6)$	*Partially connected with firm 3 isolated* $(SW \approx 13.6 \rightarrow 15)$	*Empty network with only firms 2 (1) and 3* $(SW \approx 14.1)$
EFFICIENCY	*Star network with any hub firm (they are all equal)* $(SW \approx 18.5)$	*Partially connected network with firm 3 isolated* $(SW \approx 21.8 \rightarrow 19.7)$	*Partially connected network with firm 3 isolated* $(SW \approx 19.7 \rightarrow 19.1)$	*Partially connected network with firm 3 isolated* $(SW \approx 19.1 \rightarrow 17.6)$	*Star network firm 3 hub* $(SW \approx 17.6 \rightarrow 16)$	*Star network firm 3 hub* $(SW \approx 16 \rightarrow 15.7)$	*Star network firm 3 hub* $(SW \approx 15.7)$	*Star network with firm 1 (or 2) hub* $(SW \approx 15.7 \rightarrow 15.3)$	*Star network with firm 1 (or 2) hub* $(SW \approx 15.3)$
COMPARISON	*No correspon-dence*	*One corresponds*	*No correspon-dence*	*No correspon-dence*	*No correspon-dence*	*No correspon-dence*	*No correspon-dence*	*No correspon-dence*	*No correspon-dence*

Legend: *$SW \approx A \rightarrow B$: Social welfare changing from a value of around A to a value of around B in the interval considered*

As we can see in the table summarizing the results, when $x = 0$ the stable network never corresponds with the efficient network, except in the case of $0 < y \leq 0.10$. However, even in this case there are other pairwise stable networks that could arise in equilibrium. When $x = y = 0$ all the three firms are equal and this case has already been analyses in the previous table. When $y \in (0,0.10]$, the partially connected network with firm 3 isolated is both stable and efficient, reaching a value of social welfare between 21.8 and 19.7. However, the complete network and the network formed only by firms 2 (or 1) and 3 linked are also pairwise stable. The social welfare they attain is between 4.5 and 1.3 points lower than the efficient one, with higher differences as y goes to 0. As a result, it could happen that firms create a network that is not the best from the social welfare standpoint. In the interval $0.10 \leq y \leq 0.12$, the same two complete networks discussed before are stable, but the efficient one remains the partially connected network with firm 3 isolated. In this range of y there is no correspondence between the stable and the efficient network, which causes a welfare loss of around 2 points.

For $0.12 \leq y \leq 0.66$, only the complete architecture is pairwise stable. However, the highest welfare in this interval is reached in the partially connected network with firm 3 isolated and in the star network with firm 3 hub, before and after the point of $y = 0.21$ respectively. Between $y = 0.66$ and $y = 0.77$, the star network with firm 3 hub is efficient. The pairwise stable network is again the partially connected network with firm 3 isolated and, until $y = 0.76$, also the complete network. The social welfare of the efficient network is decreasing as y increases, because the least efficient firms are more disadvantaged. Therefore, the difference in welfare between the stable and the efficient network is reduced in this interval.

For higher values of y, the star network with firm 1 (or 2) hub becomes efficient, while the partially connected network with firm 3 isolated remains stable. As y approaches 1, the difference in welfare between these two architectures reduces a lot, reaching a value of just 0.3. In this case, even though the stable and the efficient networks do not correspond, there is probably small room for intervention as the costs of a policy are likely to be higher that the benefits of the increase in social welfare. In the extreme case in which $y = 1$, the difference in marginal cost between firms becomes too big and one of the least efficient firm is forced to exit the market. Therefore, the pairwise stable network in this case is the empty network with only firms 1 (or 2) and 3 remaining.

The possible policy interventions are similar to the ones that I have analysed above for the case of $y = 0$, therefore I will be shorter in the explanation and I will mainly focus on the differences with respect to the previous case. The outcomes are reported in the *table 8* at the end of the chapter, summarising the results.

In the interval where the efficient network is one of the pairwise stable networks, i.e. for $0 < y \leq 0.10$, the government could block the agreements for the advantaged firm and then tax the other two firms and consumers and redistribute the surplus to firm 3. However, if there is no doubt that the redistribution could make also firm 3 better off, the fact of prohibiting every possible agreement for this firm is probably a too strong intervention. This outcome could instead be obtained by incentivising the two other equal and disadvantaged firms to link to each other and not to the efficient firm, through a policy of clarifications of the objectives of the government and explaining them the benefits they would have in that setting.

For higher values of y there is no correspondence between the stable and the efficient network. In the setting of $y = 0$, firms were forming too many links with respect to the welfare-maximising network and the intervention went in the direction of banning some links or taxing them prohibitively. Even in the case of $x = 0$, the firms create too many connection for $y \leq 0.76$. However, since the efficient network is not stable, it is more difficult to attain it with some banning. General recommendations are not sufficient to make firms create the welfare-maximising network, but they can still be useful to increase social welfare. For example, in the interval $x \in [0.10, 0.21]$, the policy of allowing maximum a link for each firm does not make them create the partially connected network with firm 3 isolated, i.e. the efficient network; however, the network that arises, namely the complete network with only firms 2 (1) and 3, attains a higher welfare than the equilibrium architecture. The unique problem is that it comes at the cost of excluding one firm from the market, which is a side effect of many government policies for a higher welfare. For higher values of x a targeted banning would be the unique solution, however this is not legal and viable.

Moreover, for $0.76 \leq y < 1$, firms in equilibrium are too linked compared to the efficient outcome. Hence, the intervention in this case cannot be banning some links, but it should focus on incentivising more collaborations. This can be attained through subsidies on efforts or on every link (or additional link) formed. In this particular case, this specific policy does not lead to the desired outcome, unless the subsidy is targeted to some specific links, which is not feasible. However, this solution could be good in some setting and therefore is something that the government should consider in some extensions of these model, as I will discuss in the final chapter.

Another possible decision of the government can head towards subsidizing the marginal costs of the least efficient firms. The policy of subsidisation can be only for the highest cost firms, legitimized by the fact that it constitutes a help for the disadvantaged part. This intervention is allowed because the aim of the government is to help who is more in need. As in this case there are two ex-ante disadvantaged firms, the potential subsidy must be the same for both, as they are equal and there is no reason for discriminating one of them. Hence, the cost of the government is doubled and it is more difficult to offset it with the increase in social welfare. Therefore, I will now explore the outcome of the subsidisation policy to check whether it is viable or not.

My objective in this case is to reach a difference in marginal cost of just 0.10 points, regardless of the starting point, by giving the appropriate subsidy for every unit produced to the least efficient firms. If the new y is 0.10, the stable networks are the complete network, the complete network with only firms 2 (1) and 3, and the partially connected network with firm 3 isolated. Among them, only the latter is efficient. As we have seen before, making firms create this network requires a strong intervention, which I believe is not legal. However, I will now consider that in the case of a cost difference of 0.10 points firms would create this last architecture. I look at this network because it is the efficient one and it makes sense to check whether the subsidies to reach it are worth it. If they are not worthwhile for this network, they are consequently not a good idea in all the other cases. Moreover, even if I did not find any viable solution to make firms reach that network, maybe there are other incentives that I did not explore and that could be effective, probably combined with some redistribution system.

As in the previous analysis, I start to examine the case of moving from $y = 1$ to a difference of just $y = 0.10$. In this case, the government has to pay a subsidy of 0.90 for every unit that each of the two least efficient firm produces in the stable network of the net setting. As I have just explained, I take as the stable network in the case of $y = 0.10$ the partially connected network with firm 3 isolated. In this network, each of the least efficient firm produces 2.25 units of goods. The total subsidy that the government has to pay out is therefore $2 * 2.25 * 0.90 = 4.0$. In order to see if this is viable, we have to check whether the increase in social welfare is greater than the subsidy. In case of $y = 1$, firms in equilibrium would create the empty network with only firms 2 (1) and 3, and the welfare attained is 14.07. If we manage to reduce the cost difference to 0.10 and make firms create the partially connected network with firm 3 isolated, the social welfare reached is instead 19.67. The gross gain in social welfare with this policy is thus 5.6 point, which is greater than the cost for the public finance. Hence, even with a so great cost difference, this policy is feasible and could increase the net welfare.

However, it could be that this happens because, in the particular case of $y = 1$, one of the least efficient firm is driven out of the market. If the initial cost difference is slightly smaller than one, as for instance 0.95, firms would form the partially connected network with the efficient firm isolated. In this case, social welfare would be 14.9 and the government subsidy $2 * 2.25 * 0.89 = 4.0$. The difference in the two welfare is instead 4.77, producing a net increase of 0.77. Hence, this policy seems to be effective and to leave margin for an improvement even in the case of a doubled cost for the government. Due to the fact that the subsidy must be given to two firms, the net increase in welfare is often smaller than in the case of $y = 0$.

This method can be used also for a smallest cost difference, in order to decrease it more and attain a higher welfare. For most of the cases in this setting it is allowed, however, in the real setting with greater cost differences, it is likely that giving a great subsidy to help more than one firm has prohibitive costs for the government. Nonetheless, even a small help could be crucial for changing firms incentives and increase social welfare.

Below you can find a table summarising the recommendations for $x = 0$.

Table 8: recommendations for $x = 0$

	Outcome	***Policy proposal***	***Welfare attained***	***Problems***
$\boldsymbol{y = 0}$	*See previous table in the $x = 0$ case*			
$\boldsymbol{0 < y \leq 0.10}$	The efficient network is not the uniquely stable architecture	Prohibiting the agreements for the efficient firm. --- Incentivize the least efficient firms to link only to one another	$\approx 21.8 \rightarrow 18.8$ ($+4.8 \rightarrow 2$ points) --- $\approx 21.8 \rightarrow 18.8$ ($+4.8 \rightarrow 2$ points)	Too strong intervention. Justification and redistribution needed. --- Difficult to convince them, redistribution needed.
$\boldsymbol{0.10 < y \leq 0.21}$	No correspondence between the stable and efficient network	Allowing maximum one link for each firm	$\approx 17 \rightarrow 16.7$ ($+0.2 \rightarrow$ 0.4 points)	One firm is driven out of the market, side effect
$\boldsymbol{0.21 < y \leq 0.9}$	No correspondence between the stable and efficient network	Subsidy to the least efficient firms	Depends on the case	Same problems of reaching the efficient network that we had for a small cost difference
$\boldsymbol{y}$ ***approaches*** $\boldsymbol{1}$	No correspondence between the stable and efficient network.	Small room for intervention: similar welfare in the stable and efficient network. --- Subsidy to the least efficient firms to get to $y = 0.1$	≈ 15 (-0.3 oints w.r.t. efficient network) --- ≈ 15.7 ($+0.8$ points)	Political cost of intervention probably higher than the benefits. --- Same problems of reaching the efficient network that we had for a small cost difference
$\boldsymbol{y = 1}$	No correspondence between the stable and efficient network.	Small subsidy to the least efficient firms, just to avoid market exit. --- Subsidy to the least efficient firms to get to $y = 0.1$	≈ 15.3 ($+1.2$ points) --- ≈ 15.7 ($+1.6$ points)	--- Same problems of reaching the efficient network that we had for a small cost difference

Legend: *$SW \approx A \rightarrow B$: Social welfare changing from a value of around A to a value of around B in the interval considered*

5. EXTENSIONS IN THE LITERATURE AND RESULTS

In the literature there are several models on R&D networks, which however are not always directly comparable because of the different assumptions they start with. As my primary objective is to suggest policy interventions to increase the value of social welfare, I will concentrate my attention on those models that focus on it.

This basic model of Goyal and Moraga-González (2001) has been widely used in the literature on R&D networks as a starting point. Many authors developed possible extensions of this model, considering for example different competition settings (Goyal and Joshi, 2003), knowledge spilllovers in the network (Goyal and Joshi, 2006; König, Battiston, Napoletano, and Schweitzer, 2012), different costs of link formation (Goyal and Joshi, 2003), and adding other variables to the original design. In this chapter I will thus focus on some of these extensions, which use the same main assumptions of the authors and for which is possible to calculate and directly compare the social welfare. For each of them, I will suggest some possible policy tools that governments and Competition Authorities could use to improve the efficiency of the outcome and I briefly state some issue to take into account in the intervention. This is not an in-depth analysis, but it aims only at exploring the aspects to consider in different settings.

In each of the following paragraphs, I will analyse one of these extensions: the case of symmetric networks with n firms, the presence of positive costs of link formation, the possibility of transfers between firms, a positive spillover rate among unconnected firms, the presence of a public firm in the market, the outcome of different types of competition, the market with differentiated products and the case of vertically related firms.

5.1 SYMMETRIC NETWORK OF N FIRMS

In the basic model I started with, I analysed the simplest case of three firms competing in a market of homogeneous products. Goyal and Moraga-González (2001), at the beginning of their paper, studied the broader case of n firms. Due to the difficulties of computing a general model, they concentrate on the case of symmetric network structures, namely the networks in which every firm has the same number of collaborative ties.

The model they use is the same Cournot-competition model that I explained in the three-firm example, but they extended the set of firms to $N = \{1,2,\dots,n\}$, with $n \geq 3$. $N_i(g)$ is the set of firms linked to firm i, and $\eta_i(g)$is the cardinality of this set. The assumptions on marginal costs of production, costs of efforts, payoffs and spillovers do not change. Since a network is symmetric when every firm has the same number of collaboration links, it follows that $\eta_i(g) = \eta_j(g) = k$, where k is the degree of the network g.

Stable network. In the analysis of the network that will endogenously arise in this model where firms unilaterally decide the level of R&D efforts, Goyal and Moraga-González (2001) observe that the R&D efforts of connected firms are strategic complements, while the efforts of firms without collaboration links are strategic substitutes. Moreover, they find that the equilibrium level of research efforts is decreasing when the degree k of the network increases. As regards the number of the link, we can notice that an increase in k reduces firms own production costs on one side, but it also decrease the costs of the competitors in the market. The conclusion on pairwise stability is that the complete network is the uniquely stable architecture.

Efficient network. In this paper the authors also analyse which is the efficient network from the social welfare point of view. Looking at how the welfare changes when k varies, Goyal and Moraga-González (2001) show that it exists an intermediate level of k, that they label $\tilde{k}$, that maximizes social welfare in the case of symmetric network structures. This value of $\tilde{k}$ depends on n and is $0 < \tilde{k} < n - 1$. Assuming n and k are continuous variables, the authors try to maximize $W(g^k)$ with respect to k. They find that the links are more desirable when n increases. Moreover, the higher the cost of R&D efforts, the higher the number of collaborations must be to maximize social welfare.

Policy recommendation. In this setting there is no correspondence between the stable and the efficient network. The government has thus the possibility to intervene to increase the social welfare and benefit society as a whole. Since the stable network is the complete architecture, it follows that firms endogenously form too many links with respect to the social optimum. The task of the government is therefore to decrease the number of collaborations among firms.

One way to do this is to tax the collaborations. If this tax is imposed on every link, it is likely that firms would reduce their collaborations due to the increase in their costs. In order to attain a higher welfare it is necessary to calculate the right amount of this tax, to avoid an excessive reduction in the research efforts. However, it is possible that the increase in the costs of the link makes collaboration not worth it or decrease firms' profits excessively. It can be also that this increase in the cost makes firms set a higher price, which is negative for the consumers. To avoid these problems, the government could instead decide to tax the links after a certain number. In this way, firms would necessarily form the collaborations for which they do not pay the tax - as they reach higher profits having more links - and probably decide not to form any additional link for which they have to pay. The decision on whether to form additional links or not depends also on the amount of the tax, which must be chosen following the objective that the government wants to reach. As the government is aware of the profit functions of the firms, it can decide the best tax accordingly. In case the tax is too low that firms would still be better off by forming more agreements than the efficient number from the social welfare standpoint, the government would still benefit from the tax and could redistribute it to the consumers.

If the authorities want to make sure that some collaboration agreements are not created, they could prohibit them by law. A good strategy could be prohibiting firms to form more than $\tilde{k}$ collaboration links, which is the number that maximises social welfare. As firms are better off forming the maximum number possible of agreements, they would necessarily form $\tilde{k}$ collaborations and the welfare would thus be maximised. Since the networks formed are symmetric, no firm is disadvantaged with respect of the others with this policy intervention.

Another way to increase the welfare could be by subsidizing the cost of efforts. If the research costs are lower, firms would do more research, share more knowledge and therefore reduce the costs of production more. If the costs are lower, the final price is also lower and this would benefit consumers. As paying for firms' efforts is costly for the government, this strategy should be used only in the cases in which the costs of efforts are so high that firms have no enough incentives to innovate. The cost of the final product is not the only thing that could benefit consumers and increase consumer surplus: it can also increase if there are new innovative product and a larger amount of choice for the final clients.

Alghamdi (2016) studies in its paper the equilibrium outcome of a stable network depending on the number of competitors. Using similar assumptions to the ones of Goyal and Moraga-González' (2001), he finds that total welfare decreases as the number of firms increase. Therefore, small stable networks are more desirable than bigger ones. As a result, in sectors where the number of firms is huge and the market demand follows the author's function, government should find a way to create more small networks instead of just a great one.

5.2 COST OF FORMING LINKS

In our basic model, I assumed that there are no costs of forming a collaboration link. However, firms sometimes have to pay some costs to form a link, such as a fixed investment to start the collaboration. The literature has widely investigated the case of positive costs of link formation.

A good analysis has been made by Goyal and Joshi (2003). Their study start with the basic three-stage game of Goyal and Moraga-González (2001), with no knowledge spillovers across unlinked firms and the same linear demand function. The difference is that the collaborations agreements, which help to lower the costs of production, are now costly. Their model also considers the wider setting of n firms, which complicates a bit the calculations. In this broader case, the possible architectures are the complete and the empty network, the dominant group architecture and the inter-linked star. The dominant group architecture is a network composed by one complete non-singleton component and other isolated firms, where a complete component is a group of firm inside the network with all the possible links within each other. The inter-linked star is instead a network with a central group of firms forming a complete component among them, and a group of peripheral firms with every firm connected to all the central firms but not among them.

To simplify the model, the authors consider ex-ante symmetric firms with identical constant returns to scale cost functions and fixed costs of link formation. The marginal costs of each firm is assumed to be linearly decreasing with the number of links, following the equation

$$c_i(g) = \gamma_0 - \gamma\eta_i(g), i \in N,$$

where η_i is the number of firms firm i has a collaboration with, $\gamma_0 > 0$ is the firm's marginal costs when it has no links, and γ is the cost reduction for each link formed. They make some assumptions to ensure that all firms are producing a positive quantity in equilibrium, thus no firm is driven out of the market.

Goyal and Joshi (2003) focus on the case in which the fixed costs of forming a link are significant. This hypothesis is plausible in those cases where the collaboration involves the commitment of some funds for the firms that decide to cooperate in the research activity. They suppose that each link involves a cost of $f > 0$ on each firm, and each firm pays for its own cost of link formation.

Stable network. In their analysis, Goyal and Joshi (2003) reveal that, in the case of significant costs of link formation, the stable network depends on the fixed costs of establishing a link. When the costs are small, the complete network is uniquely stable. For medium values of f, the stable networks are the dominant group architectures with considerably large dominant groups. When f assumes higher values, the network that endogenously arises has medium size dominant group. When the costs are so high that collaborations are not worth it, the empty network becomes stable. Hence, it is clear that the fixed costs of R&D has a non-monotonic effect on the size of the dominant group.

Efficient network. Due to the difficulty of presenting a general characterization of the efficient network, the authors study welfare in a four-firm example. The result is that the number of links in the efficient network decreases when the fixed cost of link formation increases. Thus, for small values of f social welfare is higher with more links, while for high values of f social welfare is higher with less links. Moreover, when a symmetric and asymmetric network have the same number of links, social welfare is higher in the case of the asymmetric network. The paper does not provide more specific results.

Policy recommendation. In this setting, the government's action can target the fixed cost of link formation. As we have seen above, in the case of a small cost of link formation, social welfare increases with the number of links. In this case, firms endogenously form the complete network, thus the one with the highest number of links possible. As the cost of link formation increases, the welfare-maximising number of links decreases, following the path of the efficient architecture. However, the number of links in the stable networks probably falls faster than in the efficient network, leading to the formation of the empty architecture for high values of f. Therefore, in this case the government could subsidise the cost of the collaborations to incentivise firms' cooperation in the research and thus increase the benefits for the society. However, it is important to consider the costs of this policy before implementing it, to make sure that the net welfare is higher than the one generated before. This means that the costs that the government has to pay to subsidise the links must be smaller than the increase in social welfare. In any case, a clarification on the efficient and stable network for any n is needed in order to have a full picture of the situation and to intervene accordingly.

The other point that arises in the authors' analysis is that, for an equal number of links, the asymmetric network provides a higher welfare. The difficulty is that the government cannot promote networks that create inequalities among firms, benefiting some firms at the expenses of the others. Hence, any type of intervention that moves in this direction is not allowed because it is not Pareto efficient, notwithstanding the social benefits it could give for the society as a whole. In *chapter 5.3* a way out of this situation is considered.

5.3 TRANSFERS

In the previous analysis, we have seen that sometimes the welfare-maximising network does not arise because one or more firms are worst off and they do not agree with that architecture. However, if some firms gain a lot from that network, they could decide to induce their competitors to form more links by paying them the costs of the new network. Goyal and Joshi (2003) analyse the possibility of transfers across firms in the case of fixed costs of link formation. Since links provide increasing returns, firms with many links may have incentives to make transfers to firms with less collaborations to convince them to collaborate. The transfer consists of paying the fixed costs of forming links also for the partner firms. Hence, the definition of stability is now adapted to take the possibility of transfers into account.

Stable network. In their analysis, the authors found that in the class of symmetric networks, i.e. in the networks where all the firms have the same number of links, the unique architectures stable against transfers are the empty and the complete structure. In the case of asymmetric networks, the inter-linked star arises in the pairwise equilibrium. This network is formed by some "central firms" which are linked to all the other firms, and the remaining "peripheral" firms connected just to the central ones. However, it can also be that the external firms have some connections among them. In the special case of just one central firm, we have the star network. Without the possibility of transfers, each peripheral firm would delete its link with the central one. Therefore, the latter has incentives to subsidise the cost of link formation for the peripheral firms. However, this transfer is worth it for the central firm only if the fixed costs of collaboration are not too high and it manages to make higher net profits even with the transfer. Moreover, the authors found that if a network stable against transfers, it is also stable without.

Efficient network. The efficiency of the network is the same as without transfers, since they do not change the payoffs of the firms in each network but are only a way to make them form a network with higher profits. As in the case of costly links in the basic example of four firms, the number of links in the efficient network decreases when the fixed cost of link formation increases. When the fixed cost of collaboration is relatively small, the social welfare is higher with more firms in the network.

Policy recommendation. Allowing firms to make transfer can be consider itself a government policy to make firm create a network with higher social welfare. If the costs of link formation are sufficiently high, they would create an inter-linked star instead of the dominant group architecture. The former has more link, because all the central firms are connected also to the peripheral firms. Hence, the welfare attained is higher than in the case of no transfers. Even if the authors do not analyse in depth this result, the profits of the firms in the case of transfers must be necessarily higher than in case they are not allowed. If what changes between the two cases is that the peripheral firms decide to collaborate with the central ones thanks to the transfer received, it means that both the central firms and the peripheral firms manage to attain higher profits. Otherwise they would not have accepted this agreement and the transfer would not have taken place. Hence, the producer surplus is higher in this case. This is probably due to a decrease in the marginal cost of production of all firms due to the sharing of knowledge, which leads also to a higher consumer surplus. Simply allowing transfers among firms results in a Pareto improvement and a higher social welfare, without the need for the government to intervene with costly subsidies which could also not be legally possible.

5.4 KNOWLEDGE SPILLOVERS

In the basic model of three firms with ex-ante different marginal costs, I considered the case of perfect knowledge spillovers between linked firms and no spillovers between unconnected firms. Nevertheless, the research on R&D networks also explores the case of positive knowledge spillovers between unlinked firms. The literature examines different possibilities to model knowledge spillovers in the network.

For instance, Goyal and Joshi (2006) study a model with a set $N = \{1,2,\dots,n\}$ of firms, in which the cost reduction of the firms does not depend only on linked firms' efforts. They consider two different types of games, which they call respectively *playing the field* and *local spillover* games. In the first case, the marginal profits of each firm are determined by the number of own links and the number of links of the rest of the firms. In the second case, the marginal returns of each firm depend on own links and links of the connected partner of the firm.

König et al. (2012) conduct another good analysis of the role of knowledge spillovers. In their article spillovers are indirect and network-dependent, thus they depend on the network to which a firm belongs. They define a connected component as a set of firms with a path of links that connect any two of them. The reduction in marginal costs of production is the same for every firm in the same component and it depends on the total number of links in the component. As firms only pay for direct connections, they would prefer to have less direct links but benefit from the links of their partners, which contribute to the reduction in their costs but are not costly. This way of seeing knowledge spillovers is in line with the idea of knowledge recombination.

A different way to conceive knowledge spillover is analysed by Zikos (2010), who assumes that the rate of knowledge transmission is determined by the distance connecting two firms, defined as the smaller number of links in the path between them. In this case, the closer two firms are, the more they benefit from the research of the partner firm, with the maximum gain when the two firms are directly connected.

Goyal and Moraga-González (2001) themselves, in the last section of their paper, analyse the case of knowledge spillovers among unlinked firms. As their model is directly comparable with the basic version I started with, I will now focus on this to draw the policy implications for the indirect spillover case. They consider a three-firm market for homogeneous goods and introduce a parameter $\beta \in [0,1]$, which indicates positive knowledge spillovers among firms without collaboration links. The final cost function of firm i is therefore:

$$c_i(\{e_i(g)\}_{i \in N}) = \bar{c} - e_i - \sum_{k \in N_i(g)} e_k - \beta \sum_{l \notin N_i(g)} e_l$$

where e_k are the efforts of linked firms, which are fully absorbed; e_l are the research efforts of unlinked firms and they are partially absorbed depending on the parameter β. As before, the authors calculate the equilibrium effort levels and profits in each network configuration, which only depend on the parameter β. They define β_1 as the solution to the equation $\pi_l(g^p) = \pi_h(g^s)$, where $\pi_l(g^p)$ is the payoff of the linked firm in the partially connected network, while $\pi_h(g^s)$ is the payoff of the hub firm in the star network.

Stable network. Using the definition of pairwise stability, they found that the complete network is stable for every value of $\beta \in [0,1]$, while the partially connected network is stable for small values of β, namely for $\beta \in [0, \beta_1]$. The star and the empty network are instead never stable. From the graph we can see that the level of β_1 is lower than 0.1.

They first observe that public spillovers influence the stability of the network. When spillovers among unlinked firms are small, the partially connected network is stable. In this architecture, one firm has a significant cost disadvantage with respect to the other two. An important implication is that, if this divergence in marginal costs becomes substantial, the isolated firm becomes too inefficient to survive market competition and is forced to exit the market. This is valid only for very low values of β, as the partially connected network is destabilized quickly as the spillover parameter grows. This happens because for sufficiently high public spillovers, the isolated firm benefits from the other two firms research efforts without paying their costs, thus the linked firms have incentives to form an additional link with the isolated firm and form the complete network. The latter remains instead pairwise stable for all values of β, but it also becomes less strong when spillovers increase, as the losses from severing a link become smaller. As a result, for some values of β there is more than one pairwise stable network and it is not possible to anticipate which one will arise. Another important consideration is that the difference in firms' profits in the different networks decreases as β increases, becoming equal to zero in the extreme case of perfect spillovers among unconnected firms, i.e. when $\beta = 1$.

Efficient network. In order to find the efficient network from the social welfare standpoint, the authors compute the welfare in the four networks and compare them to find the configuration that maximises social welfare. After plotting the welfare levels in the four architectures, they define β_2 as the solution to the equation $W(g^p) = W(g^e)$, where $W(g^p)$ and $W(g^e)$ are respectively the social welfare in the partially connected and in the empty network. From the graph we can see that the level of β_2 is around 0.5.

Their results show that two networks can be efficient depending on the level of knowledge spillovers among unlinked firms. If $\beta \in [0, \beta_2]$ the partially connected network is efficient, while when $\beta \in [\beta_2, 1]$ the empty network becomes efficient. The complete network is also always dominated by the partially connected and the star networks. The partially connected network is efficient for low β because the two linked firms benefit from the difference in cost with respect to the isolated firm and exert high effort levels to increase their advantage. On the contrary, when public knowledge spillovers are greater, these can work as substitutes for the collaborative link and, as they lower R&D efforts only slightly, the empty network becomes efficient.

Policy recommendation. To give the appropriate policy recommendation, we need to compare the stable and the efficient network for every value of β and see whether they coincide or not.

For $\beta \in [0, \beta_1]$, the partially connected network is both stable and welfare maximising, but it is not the uniquely stable one. If the other stable network - the complete architecture - arises, the government should intervene to increase the welfare for society. An effective intervention in this case is again the prohibition to form an excessive number of links. This involves no costs for the government and can be legitimated by the objective to increase social welfare. As all the firms are ex-ante equal, no firm would be disadvantage a priori by this intervention itself. Moreover, thanks to knowledge spillovers between unconnected firms, also the firm that would remain isolated would profits from a part of the knowledge created by the other two firms.

For $\beta \in [\beta_1, \beta_2]$, the complete network is stable, while the welfare maximising one is still the partially connected network. Hence, for intermediate values of β, there is an excessive level of collaboration with respect to the efficient one. For $\beta \in [\beta_2, 1]$, the stable network is again the complete network, while the socially efficient one is the empty network. Therefore, for high values of public knowledge spillovers there is also an excessive level of collaboration activity. As a result, private incentives to collaborate are unquestionably excessive with respect to the efficient ones, and thus, there is room for policy intervention to address these failures. In this case, the policy intervention could block some agreements or tax them to make less attractive for firms to collaborate. It is however important to give incentives to firms to innovate and create new products or new production process, without being unfavourable for consumers.

5.5 PUBLIC FIRM

Another extension explored in the literature is the introduction of public firms in the market to observe how this influences the network architecture. Zikos (2010) carried out a good analysis of this particular setting, investigating the basic case of three firms in the market: one public and two private. According to this article, the main difference between the private and the public firm is in the maximization problem. While the former tries to maximize its own profits, the latter aims at maximising social welfare, weighting equally consumer surplus and aggregate producer profits. The main assumptions that the author uses are the same as our basic model, with the same three-stage game, the same quadratic costs of efforts and the same payoff function.

However, in this model, the spillover rate depends on the distance among collaborating firms and in particular it is determined by the shortest path, measured in the number of links, connecting the two firms. The author denotes $t(ij)$ the number of links in the shortest path between the two firms i and j, and sets $t(ij) = \infty$ to indicate the absence of a path between them. In this way, the closer the two firms are, the higher the knowledge spillover they get from the other firm. Thus, firm i's effective R&D efforts are given by:

$$E_i(\{e_i(g)\}_{i \in N}) = e_i + \beta \left[\frac{e_j}{t(ij)} + \frac{e_k}{t(ik)} \right], \qquad i \neq j \neq k$$

where j and k are firm i's research partners. In the cost of efforts, the author includes a quadratic term $q_i^2(g)$ to avoid the case in which the private firms are both forced out of the market. This assumption is just a way to allow for cost differences between the firms in equilibrium, giving the public firm a higher marginal cost.

Since firms are not ex-ante equal, the possible network configurations in the three-firm case become six. As before, we have the empty and the complete network, but two possibilities for the star and partially connected network. The star network can have either a private or a public firm as "hub" firm, which the author calls respectively "private-hub" and "public-hub" star network. Likewise, the partially connected network can have either a private or a public isolated firm, which the author denominates "private partial network" and "public partial network".

The main results of Zikos (2010) are that while the public firm's R&D efforts increase with the number of own links, the private firm's efforts decrease with the number of links except in the case in which they move from the empty to the public partial network and knowledge spillovers are large enough. These results are more marked when the spillover rate grows, as the possibility of sharing knowledge increases.

Stable network. In order to establish which network configuration will endogenously emerge, the author adapts the definition of pairwise stability from Jackson and Wolinsky (1996) to accommodate it to the presence of a public firm. What he founds is that, in the mixed oligopoly context, the unique pairwise stable network is the complete network. To determine if this network is stable also against cooperative deviations from a coalition of agents, the author further checks for the strong stability requirements of Jackson and van den Nouweland (2005) definition. After analysing all the possible deviations from the complete network, Zikos (2010) concludes that this network is also strongly stable.

Efficient network. In this work, the notion of efficiency that the author uses is the one of Pareto efficiency. According to this definition, a network is efficient when it is not possible to increase the payoff of a firm without decreasing the payoff of another firm. As we have seen before, the payoffs of the private firm are its own profits, while the public firm tries to maximise the social welfare. In my definition of efficiency, the unique thing that matters is social welfare; therefore, I only consider Zikos' (2010) calculations on public firm's payoffs to establish which network is efficient. In the graph combining welfare levels with the level of spillovers β, it is clear that the complete network is always the efficient one.

Policy recommendation. In this particular example with two private and one public firm, the stable network always coincides with the efficient network from a social welfare perspective and it corresponds to the complete network. As a result, "the presence of a public firm can reconcile individual incentives to form R&D networks with the objective of efficiency" (Zikos, 2010). Afterwards, the author analyses the broader case of four firms, three private and one public, forming symmetric collaboration networks. A network is symmetric when each firm has the same number of links. Even in this case, the complete network is the unique stable architecture and it is also efficient from the social welfare standpoint. Therefore, the public firm can incentivize the formation of the efficient network even in the case of four firms forming symmetric networks.

Hence, there is no need of policy intervention in this case, as the public firm already provides the incentives to create the best network from the social welfare perspective. Hence, this result is important because it presents another possible policy intervention in case of a divergence between the stable and the efficient network, which consists of introducing a public firm in the market. Even though in this case the firms will already create the best network from the social welfare point of view, Zikos (2010) also analyses what would happen in the case of subsidies. He assumes that, in the second stage of the game, the regulation authority provides a subsidy per unit of R&D efforts with the objective to maximise social welfare net of subsidies. The result is that welfare would increase even more due to the subsidy, as this addresses the underinvestment problems due to imperfect competition. Moreover, the subsidy solves the inefficiencies due to the imperfect public spillovers, giving firms incentives to invest more in R&D. What should be considered in this intervention is the net social welfare after the subsidy, to be sure that the increase in welfare is higher than the cost of the policy.

5.6 TYPE OF COMPETITION

In all the previous examples, I have considered the case of firms competing in quantity in a homogeneous-product market. However, the literature also studies models from the perspective of firms competing in prices, expecting that the nature of market competition could affect the structure of the networks arising in the equilibrium. Goyal and Joshi (2003) investigated the differences in the stable and efficient networks in the two cases of Cournot and Bertrand competition. I will now consider the latter case. In a price competition setting, payoffs are not distributed among firms as in the Cournot case, but only the lowest cost firm manages to gain positive profits. This is a context of extreme competition, and firms must consider even more the profits of the other firms in the decision of collaboration. Again, the equations for marginal costs and demand are respectively $c_i = \gamma_0 - \gamma\eta_i(g)$ and $p = \alpha - \sum_{i \in N} q_i$, $with\ \alpha > 0$. The marginal costs are linearly declining in the number of links.

Stable network. With the previous assumptions and n firms in the market, the authors found that the empty network is the only stable architecture. This happens precisely because just the minimum cost firm manages to make positive profits. Therefore, no firm wants to collaborate with another firm, knowing that only the partner would benefit from the link. However, if the minimum cost firm is not unique, they have incentives to collaborate among them to lower their costs of production and achieve higher profits. The arising network in this case is a structure with a dominant group architecture composed by some or all minimum cost firms.

Efficient network. In this setting of extreme competition, the authors find that social welfare is maximised in the case of an inter-linked star with two central firms and the other firms connected just with these two. The value of social welfare depends on the marginal cost of the minimum-cost firm, and it is maximized when the product is sold at the minimum attainable cost of a firm in any network architecture. Two firms with a lowest cost are necessary and sufficient to have a market price equal to the minimum cost level.

Policy recommendation. In the case of Bertrand competition, there is a conflict between the stable and the efficient architecture. If there is only a minimum cost firm, no firm has incentive to form a link with another firm, helping it to achieve higher profits. In this setting, the unique solution to make firms collaborate is to help them with a subsidy to make them start with the same production cost, so that they prefer to collaborate to lower their marginal costs even more. This subsidisation is not needed if there exists already a group of firms more efficient than their competitors. The issue concerns the possibility to implement this policy, as the government cannot discriminate among firms and help randomly only certain firms. Even if this policy would be viable or firms already start with the required distribution of marginal cost, the inter-linked star would not arise. Indeed, the least efficient firms have no incentives to collaborate with the efficient ones, helping them decreasing their costs even more. Although this collaboration would reduce their own costs too, they do not gain from this partnership in the case of Bertrand competition. The fact that only the lowest-cost firms manage to gain positive profits creates an aggressive competition environment, which impedes the formation of links among firms. As a result, in a price competition setting, it is harder for the government to intervene and to achieve a higher level of welfare.

5.7 DIFFERENTIATED PRODUCTS

In all the previous examples, the focus was on models with homogeneous products. This means that the goods produced by firms are perfectly substitutes and therefore the consumers decide to buy the ones with the smallest price. In the case of quantity competition, the price is thence the same for all the products, and what changes is the mark-up and the quantity produced by each firm. However, in the real economies not all the products are the same. There can be two types of product differentiations. We call horizontal differentiation the one based on consumers' tastes: if two products are sold at the same price, consumers differ in their choice according to their preferences. We call instead vertical differentiation the one based on quality: if two products are sold at the same price and all the consumers buy the same product, this has a higher quality. Of course, this is just a general delineation that does not consider other variables which can affect the choice of a product.

In this chapter, I will therefore relax the assumption of all goods being substitutable. In particular, I will report the results of Westbrock (2008), who considers profit-maximising firms producing different products. Differently from the previous models, this one consists of two stages and there is a social planner who decides on the optimal firms' productions. In the first stage, firms form a collaboration network and the planner makes link-specific R&D investments to decrease their marginal costs; in the second stage, firms compete in the market. This model is solved by backward induction as the previous ones.

In this model, the representative consumer maximises its quasi-linear utility function, taking into account its budget constraint, the price of the goods, the degree of substitutability between products and the market size. The degree of substitutability between goods is represented by the parameter $\beta \in (0,1]$ in the equation, with $\beta = 1$ indicating a market with homogeneous and perfectly substitutable goods, and $\beta = 0$ designating the case of firms operating in independent markets where products are not substitutable. By maximising the utility function with respect to quantities q, the author obtains the inverse demand function $p_i = \alpha - q_i - \beta \sum_{j \neq i} q_j\ ,\ with\ i \in N$. The assumptions in this model are perfect spillovers between connected firms and no spillovers between unlinked firms, constant returns to scale up to a threshold level, no exit from the market and small costs of link formation. When two firms decide to collaborate, the put the maximum efforts in the research due to the convexity property of investments, and the R&D investment costs are equally shared between the two. Therefore, this model becomes equivalent to a simpler game of link formation.

Stable network. The profit function of the firms are convex in the number of links and the connections are strategic substitute. The author found that, if the goods produced by firms are sufficiently close substitutes, and there is an intermediate level of fixed costs of collaboration in a medium-size market, the equilibrium network that arises has a dominant group architecture and exhibits concentration. Compared to the quantity competition, concentration is more typical in the current setting, except when β and n are large.

Efficient network. In this model with differentiated products, the efficient network is typically highly concentrated, due to the fact that the social welfare and consumer surplus are convex and submodular with respect to collaborative ties.

This study stresses the social benefits of concentrating the activities of Research and Development on a small groups of firms. Anyway, the efficient network is not unique and depends on the market conditions such as the costs of link formation, the number of firms, the degree of substitutability between products, and the market shares of the firms prior to the collaborations. Westbrock (2008) found that the efficient architectures could be the empty or the complete network, the dominant group architecture or the core-periphery network. The one that will arise depends on the initial conditions of the market. A feature of the irregular architecture that could emerge is that they are highly concentrated. This is due to convexity property, which makes it socially beneficial to collaborate with firms having already a lot of links; and to submodularity, a characteristic of a market with non-substitutable products, that weakens the advantages of collaborating with poorly linked firms. Therefore, from the society point of view, excluding some firms from the collaborations in case of high fixed costs and the dominance of a few firms is not necessarily detrimental. Maximal concentration is therefore efficient, both for society and for consumers as well.

Policy recommendation. Comparing the stable and the efficient network, what we find is that both typically exhibit a high level of concentration of links. However, a direct comparison is not straightforward because there are too many variables to consider and that can change the results, such as the number of firms, the degree of substitutability and the cost of link formation. In order to directly compare the two networks, Westbrock (2008) analyses the case of seven firms with a moderate degree of substitutability ($\beta = 0.8$) and changes the level of linking costs. What he found is that the equilibrium networks shows too little concentration with respect to the welfare maximizing one. To make them coincide, the government should find a policy that increases the concentration of firms' links. However, even though this intervention could increase the social welfare, it is not a Pareto improvement since it comes to the detriment of the poorly connected firms. Hence, any policy pursuing this result is probably not legitimate, unless the government finds a way to compensate the small firms through taxation and redistribution. A deeper analysis is needed.

5.8 VERTICALLY RELATED FIRMS

In my adaptation of Goyal's and Moraga-González' (2001) model, firms compete horizontally in a market. However, final good manufacturing firms do not generally produce their inputs themselves, but they often take the outputs of other firms, called suppliers, as their own inputs. Hence, R&D networks can arise also in vertically related industries, that is, where there are suppliers and manufacturers. Within this framework, the literature on R&D networks analyses different possible models.

Kesavayuth and Zikos (2012) consider a model where manufacturers and suppliers can form horizontal R&D networks. In this model, they consider three upstream firms and three downstream firms in an industry, where each supplier has an exclusive relation with one manufacturer. Each unit of input is sold by the suppliers to the manufacturer at price w_i, and it produces one unit of output. The decision of forming links is made in the two tiers at the same time, so that each horizontal network does not have an impact on the stability of the other horizontal network. In this model there is a new third stage, which consists in the suppliers choosing simultaneously their wholesale quantity before the manufacturers choose their output quantity.

Another interesting model of vertically related firms has been studied by Thanh Tran and Zikos (2017). They analyse an industry with two suppliers and two manufacturers, where they allow for any possible combination of links among them. Therefore, R&D networks can be both horizontal and vertical, and vertically related firms do not necessary have exclusive relations with one another. In this model the authors consider an industry consisting in two ex-ante equal suppliers and two ex-ante equal manufacturers. Here, they examine both the case of exclusive and non-exclusive relations between the upstream and the downstream firm. I will now focus my attention on the latter case.

The step of the model are almost the same than in our basic model, and this will be solved backwards. In the first stage firms can form pairwise links with any of the firms in the other tier, that is, each supplier can link with any manufacturer and even with both. Therefore, ten qualitative different network architectures can arise. In the second stage, each firm decides unilaterally the level of R&D efforts to maximise own profits. After that, suppliers choose their quantities followed by manufacturers choosing their level of output. The manufacturers follow the same linear market demand $p(Q) = a - Q$ as in our first model and they operate under constant return to scale production function that converts each unit of input in one unit of output. The marginal costs of the upstream and downstream firm are respectively $\bar{u} > 0$ and $\bar{d} > 0$. The cost of R&D efforts follows the same quadratic function as in Goyal and Moraga-González (2001), which is γs_i^2 and γm_i^2 for suppliers and manufacturers respectively. The authors then set $\gamma = 2$ to ensure that all equilibrium variables are non-negative and the profit functions are concave. The spillover rate among linked firms is not perfect and it can be different for suppliers and manufacturers, while there is no transfer of R&D knowledge between unlinked firms.

Stable network. Using the definition of pairwise stability by Jackson and Wolinsky (1996), the network that will endogenously arise is the complete network for any value of knowledge spillovers.

Efficient network. The efficient network we pursuit is the one that maximises social welfare, which in this case of a two-tier industry is the sum of consumer surplus, and the profits of both suppliers and manufacturers. As Thanh Tran and Zikos (2017) show, every new link decreases the marginal cost of production of both firms, thus leading to a lower input price, which contributes to a reduction in the final price. Therefore, the welfare-maximising network is the complete network, regardless of the value of knowledge spillovers.

Policy recommendation. In this particular setting, where firms are not required to engage in exclusive relations, the incentives to form collaboration links are the same that lead to the highest-welfare network. Therefore, there is no need to intervene to reach the welfare maximising configuration.

As an extension of this model, Thanh Tran and Zikos (2017) analyse also the case of n suppliers and n manufacturers. Due to the difficulties of developing a general model, they focus on symmetric networks and three configurations of spillover rates. Calling θ the spillover rate from suppliers to manufacturers and δ the rate from manufacturers to suppliers, the three cases considered are $\theta = 1\ and\ \delta = 0$, $\theta = 0\ and\ \delta = 1$, $\theta = \delta = 1$. The result in this wider setting is again that the complete network is both pairwise stable and efficient.

6. CONCLUSION AND DISCUSSION

In this work, I have analysed the case of firms with different levels of efficiency cooperating in R&D with the objective of sharing knowledge to decrease their production costs. In this model, firms first decide their collaboration links bilaterally - with the possibility of severing a link unilaterally - anticipating the level of efforts and quantities that their competitors will set non-cooperatively. The final aim of each firm is to maximise its own profits.

Despite being a very simplified model of just three firms competing in a homogenous-product market without fixed costs of production and under simplified assumptions for the market conditions and demand, this is to my knowledge the first attempt to consider the implications of having firms with different levels of efficiency prior to the collaboration. Furthermore, in addition to the stability and efficiency analysis generally present in the papers on strategic R&D networks, my work also focuses on the possible policies that the relevant authorities could implement in order to increase the social welfare of the economy. A short analysis on the policy implications, only with some general considerations, has been realized also for the extensions of the basic model in the literature. Therefore, these are the main contributions of my work.

According to the findings, it results that the stable architecture that firms create uncooperatively does not generally correspond to the efficient architecture from the social welfare standpoint. Indeed, firms only look at their own profits in the decision of links and efforts, and this usually results in configurations that are not optimal. Moreover, the structure that maximises welfare usually provides very low profits for the firm that has a disadvantaged position in the network, and thus it contributes to the emergence of disparities among firms, despite the increase in total welfare. Hence, in the efficient network, some firms benefit much more than others do.

Another result shows that the more firms are similar, the higher the social welfare is. Indeed, when there are great disparities among firms it is likely that the most efficient one can force the disadvantaged firm into an unfair agreement that can lead to higher profits just for itself. Instead, being more similar, they have the same market power and, for an equal network, social welfare is higher.

Some findings relate to the possible interventions that the relevant authorities could set in order to attain the efficient network and increase the social welfare. As we have seen, the possible intervention depend on the specific problem to solve. If firm are forming too many collaborations with respect to the social optimum, the intervention could focus on blocking some agreements or make them costly to reduce the number of links. If instead firms are too little connected, the government could incentivise collaboration with a subsidy on the links or on each unit of efforts. The subsidy could also be used to make firms more similar, as it usually increases the value of social welfare. The literature explores other useful interventions, such as including a public firm in the model or allowing transfers among firms, which could increase social welfare without a strong government involvement. As we have seen before, the main problems are the legitimacy to implement these interventions, since some of them are too strong or targeted towards some specific firms, and therefore they are not perceived to be fair.

Certainly, my model presents some limitations. First, it starts from very simplified assumptions such as analysing a market with just three firms, considering a linear demand function, taking the fixed cost of link formation equal to zero and no knowledge spillovers among unconnected firms. In the last chapter of my work, I have analysed the literature on R&D networks examining some of the comparable models that move beyond some of those assumptions, considering for example a market with n firms, positive fixed costs of production, positive knowledge spillovers between unconnected firms and other extensions that make the model closer to the real setting. The next step would be to combine these extensions with the assumption that firms have different marginal costs of production before creating the collaboration network. This would be a good improvement of my model that I leave for future research.

A consequence of the use of very simplified assumptions is that this model does not represent a good approximation of the real environment, and therefore its development is more a theoretical exercise rather than a tool to apply in the economy. Concerning this, we must take reckon that every model is just a simplification of reality and it is impossible to take into account all the nuances of the real setting. However, my simplified model can be helpful for the government because it suggests that it is important to create a balanced environment to increase the welfare benefits, it proposes some possible policy interventions and indicates some issues to consider in the implementation. As we have seen before, the government and Competition Authorities have different problems to deal with. Specifically, they face some trade-offs in the choice of the best strategy. In pursuing the objective of increasing social welfare, they have to consider that giving subsidies to the least efficient firms can make them create a most efficient network, but it comes with a cost for the public finances that can be higher than the benefits in some settings. Moreover, some strategies that the government could implement, such as blocking some agreements or taxing them prohibitively, can be good for the social welfare but it is not always legally applicable. Particularly, the policies that do not reach a Pareto superior outcome, reverse the order of payoffs of firms and consumers or are excessively targeted towards some specific firms are not legitimate nor legal, even if they could reach the specified objective. A possible loophole to implement some of these strategies is compensating the worse-off parts through a system of taxation and redistribution. Again, this plan of action has the same problems of legitimacy and it is also important to check whether, once in place, it does not change firms' incentives on the choice of efforts or links. In any case, my work highlights the general mismatch between the stable and the efficient networks, and therefore the importance of Competition Authorities' and government's role to increase social welfare.

Another limit of my model is that, for reasons of simplicity, I investigated the outcome for the stable networks just for the case of $y = 0$ and $x = 0$, and I only considered the pairwise stability approach. However, it would be good to have a wider analysis on what happens when both x and y vary simultaneously to obtain a more general result. As regards the choice of the stability criterion, I used the pairwise stability concept (Jackson and Wolinksy, 1996), given that this is the most used in the literature and it is easier to analyse. However, empirically firms could decide to collude with other firms to create a network and change more than one link simultaneously. In this case, the concept of strong stability by Jackson and van den Nouweland (2005) would be a better approach.

Another issue of my analysis concerns the creation of the network. I considered three firms in the market that decide their collaborations and efforts to maximize their profits. However, I did not examine what would happen if a new firm wants to enter the market when the network is already set. Therefore, another possible extension of this model could look at how current members of a network can exclude the entry of rivals to their agreement in order to maintain market power, and at the strategies that the government could implement to avoid an excessive barrier to competition.

A final remark concerns the approximation of the extensions to the economic reality. In the last chapter, I analysed the case of a symmetric network of n firms: this model extends the number of firms in the network, improving the similarities with the real setting. However, assuming that firms can form only symmetric architectures, the study loses a bit of accuracy. The second improvement considers positive costs of link formation, which is likely in the hypothesis that firms need to commit some funds before the collaboration. When also transfers among firms are allowed or differentiated products are considered, the model increases its strengths. Moreover, the assumption of zero knowledge spillovers among unlinked firms is also too restrictive, since there is the possibility of copying the rivals' products or processes of production. The models that consider the potential benefits of one firm's research on the other firms are therefore more realistic. However, the best way to model knowledge spillovers depends on the industry and the specific setting. The introduction of a public firm in the model is also a good idea, since the collaboration with universities or other companies not seeking profits is not exceptional. Considering the case of Bertrand competition could also be important for those settings in which capacity can be adjusted easily, such as for low-skilled services. Finally, considering the case of vertically related markets is very important to improve the plausibility of the model, since the final seller usually collaborates with its input suppliers to implement a better sales strategy. However, despite adding some good assumptions to the original model, all these extensions consider just the case of ex-ante equal firms, which is not likely to occur. This is an important deficit of all these models, and the next improvements should necessary take into account these disparities among firms to be worthwhile. Moreover, a theoretically good model should combine all these extensions and consider more plausible assumptions for the marginal costs and for market conditions. Unfortunately, it is probably unrealistic that such a model could be developed.

Further research could focus on overcoming the above limitations and improve the model with other extensions that I have not analysed.

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