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Measuring Base Erosion and Profit Shifting

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Abstract

In the permanent race for profitability, Multinational Enterprises (MNEs) seek to reduce their production cost, their wage cost and ultimately their overall tax bill. They develop strategies to reduce their global tax liability by circulating their profit through various financial vehicles and using profitable detours to exploit differences between national corporate income tax regimes. Nowadays, measures are taken by governments to strike down aggressive tax planning. This thesis, at the boundary of applied mathematics and taxation, develops a mathematical model of the least taxed path problem when a MNE wants to repatriate a profit from a source to a destination country. The flow can circulate in the form of three different vehicles, namely interests, dividends and royalties, with change of vehicle permitted in any country on the path. The model is then extended to comply with selected anti-avoidance rules and we emphasise the need for transparency. It shows that thin-capitalization rules combined with transparency are able to reduce Base Erosion and Profit Shifting (BEPS). Based on these models, two measures of BEPS are provided, one focusing on the source country and the other looking from a global point of view.

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Contents

Introduction

In the permanent race for profitability, Multinational Enterprises (MNEs) seek to reduce their production cost, their wage cost and ultimately their overall tax bill. They develop strategies to reduce their global tax liability by generating Base Erosion and Profit Shifting (BEPS) and they decide the distribution of the tax base among countries in order to take the best of each national legislation. MNEs manage to take advantage, among others, of the lack of coordination and transparency between countries.

Following 2008 financial crisis, most governments increased the pressure on individual taxpayers prompting citizens, NGO's, green political fractions and journalists to conduct inquiries and request for more transparency on the MNEs' tax behaviour.

With the disclosure of the tax avoidance strategies of digital giants (Apple, Google and others) allowing them to escape billions of dollars of taxes, evidence shows deficiencies in the current international tax system. States have lost their sovereignty with respect to taxes and fail to impose MNEs on the profit generated on their territory. The fast emergence of the digital landscape has raised new challenges for fiscal legislation originally designed for a much less globalized economy.

When enterprises started to extend their activity to foreign jurisdictions, the interaction between independent tax regimes led to double taxation. Modifications to the original national corporate income tax regimes have been created to alleviate the adverse effects of double taxation for the global trade by means of bilateral tax treaties. Although double taxation was indeed alleviated, double non taxation appeared. MNEs started to take advantage of tax treaties by using what is called treaty shopping: making detours via specific countries offering a wide range of tax treaties even if no activity is actually conducted in this country. Circulating profit between countries has been facilitated at a point that enables MNEs to basically choose the country where they want to be taxed. This highlights the need of a coordinated fiscal legislation to prevent double non taxation.

In 2013, called by G20 finance ministers, the OECD set up a comprehensive package of 15 actions to tackle BEPS structure by addressing root causes rather than local symptoms: the BEPS Action Plan. This thesis falls within the framework of action 11:

Establish methodologies to collect and analyse data on BEPS and the actions to address it

By building on the work done by Gérard and Gillard [28], current work provides a mathematical model, based on graph and network theory, of the European tax system and extends to some non EU jurisdictions. The idea is to create a model able to discover the least taxed path between any pair of countries to identify weaknesses in tax regimes. The first contribution of this work is to allow a profit to circulate in three different financial vehicles (interests, dividends and royalties) with change of vehicle permitted in any country. Secondly, the model is extended to comply with selected anti-avoidance rules, mostly thin-capitalization rule. Eventually, it provides an interactive interface to visualize the results in an attractive way.

The first chapter of this work is an introduction to international taxation and provides the basis needed to understand the model and its limitations. The next two chapters are dedicated to the discovery of the least taxed path between countries, first, in chapter 2, without taking into account anti-avoidance rules and, second, in chapter 3, by requiring compliance with selected anti-avoidance rules. The fourth chapter look at the tax avoidance problem from another point of view. The goal is not to reach a specific destination country but only to escape taxation, no matter where the profit completes its route. Eventually, the fifth chapter presents the interactive interface.

Chapter 1

Context

Before any modelling work, it is always necessary to have a sufficient understanding of the system we will work with.

We firstly start this chapter by showing a real example of a tax avoidance strategy: Google's tax planning. Secondly we summarize the basis of international taxation including double taxation reliefs and Patent Box regimes. This section answers the following question: how is an international transaction taxed?

Afterward, we introduce base erosion and profit shifting, highlight the major tools used by tax planners to avoid taxation and explain the different measures to tackle this problem. More precisely, we introduce anti-avoidance rules and the Base Erosion and Profit Shifting (BEPS) Action Plan.

Thereafter, we present a literature review of what has been done in the study of tax planning and more precisely at the intersection of tax planning discovery and graph and network theory.

Eventually, we introduce the data we will work with.

1.1 Learning from examples: the Google case

Google, as many Multinational Enterprises (MNEs), has been using complex tax planning to reduce its overall tax bill. According to [23], Google has saved around $3.1 billion between 2007 and 2010, allowing to boost its 2009 overall earnings by 26%.

Google tax avoidance strategy is complex and we depict here one of the main part called the Double Irish with Dutch Sandwich. The first step for Google was to create an Irish Company called Google Ireland Holdings and to transfer its Intellectual Property (IP) rights (for usage outside US) to this new Irish company. Google managed to license its IP at a very low cost such that Google Ireland Holding doesn't send much money back in the US. This Irish company is incorporated in Ireland but managed and controlled from Bermuda. Thanks to the residency definition in Irish legislation, the company is not resident of Ireland for tax purpose and therefore not taxed there.

Another Irish subsidiary called Google Ireland Ltd. was created and is resident of Ireland for tax purpose. The role of this subsidiary is to collect Google's overseas revenue. Thanks to European Directives, the profit generated by Google in Europe can reach Google Ireland Ltd. without being taxed.

The overseas revenue collected by Google Ireland Ltd. would be taxed in Ireland should it remain there. The last subsidiary is called Google Netherlands Holdings BV, incorporated in the Netherlands. The trick is to send the revenue of Google Ireland Ltd. to Google Netherlands Holding BV. This Dutch subsidiary serves as a routing company between Google Ireland Ltd., collecting Google's overseas revenue, and Google Ireland Holding located in Bermuda. The Netherlands are used because of their advantageous treatment of royalties' payments. The tax planning is represented in figure 1.1, found on [26].

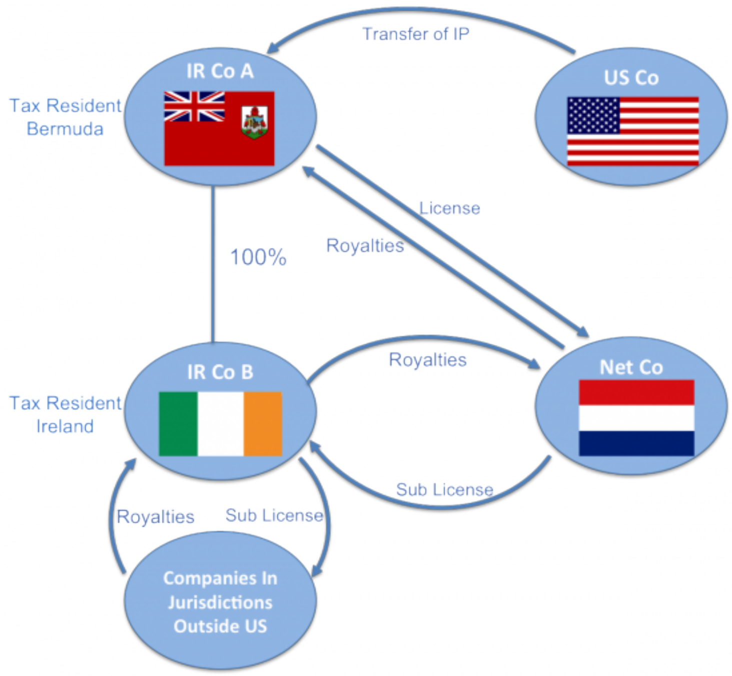

Figure 1.1: Double Irish with Dutch Sandwich

Google is not the only company involved in aggressive tax practices, one can also cite Apple, Starbucks, Ikea, Microsoft, Pfizer,... Apple also used the double Irish with Dutch sandwich structure with a special ruling granted by Ireland. However, the European Commission has concluded, after investigation, that Ireland granted undue tax benefit to Apple [12]. Ireland now has to recover the illegal aid. Moreover, things are moving in Ireland as the pressure coming from other EU Member States has been growing. They denounce unfair tax competition according to [10]. Ireland has changed its residency definition as from the 1st January of 2015, such that companies incorporated in Ireland are deemed resident for tax purposes. Companies already using this tool before this date can maintain it until 2020.

1.2 International taxation

Suppose you are an American citizen willing to travel towards Canada. You have several options: taking the train, taking the plane or driving. Each option will have a specific cost and you may need to pay visa fees to enter Canada depending on why you are entering the country. You have several possible costs to take into account for this trip.
If you are in Europe willing to travel between EU Member States, thanks to Schengen Agreement, you will get rid of all the visa complications. As we will see throughout this section, for an international transaction, you can also choose several vehicles, each having its own cost and you may need to pay some additional fee when crossing a border.

In this study, we allow a transaction to be made in three different financial vehicles: dividends, interests and royalties. While interests and royalties payments are generally considered as a cost and consequently deductible from the tax base in the source country, dividends are not.

We start this section by a brief overview of the global taxation on a dividend payment, before introducing double taxation relief methods and a specific regime concerning royalties' income called Patent Box regimes.

1.2.1 How is an international transaction taxed?

Let us consider a general repatriation of dividends from the country where the subsidiary is operating to the country of residence of the parent company[1], as shown on figure 1.2. We assume that the subsidiary has a permanent establishment in country B and is resident of this country for tax purpose.

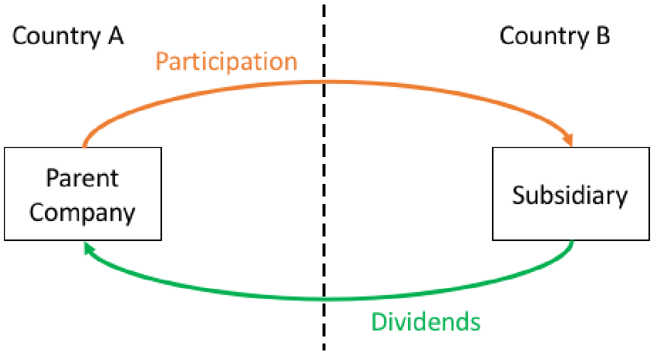

Figure 1.2: International Transfer of Dividends

The income generated by the subsidiary in B can be taxed in country B, where the value is created, this is called the source principle or in country A, where the entity receiving this income is resident for tax purpose, this is called the residence principle.

We assume here that country A applies Residence Principle while country B applies Source Principle. Consequently, when generating a profit in B, the subsidiary is, first, subject to the corporate income tax in B at a rate τ_B. Secondly, when a dividend payment is remitted to another jurisdiction, here country A, country B may levy a withholding tax on the payment at a rate t_{BA}. This rate may depend on a possible tax treaty signed by A and B. Finally, dividends received by the parent company are part of its income and are therefore subject to corporate income tax in country A at a rate τ_A. Therefore, the total tax paid for a profit generated in B and sent to the shareholder company in A in the form of dividends is given by:

$$\text{Total tax} = \tau_B + (1 - \tau_B)t_{BA} + (1 - \tau_B)(1 - t_{BA})\tau_A$$

It is to note that the same profit has been taxed as part of the income in two different jurisdictions. This is called double taxation.

1.2.2 Double taxation relief

Several mechanisms have been settled to eliminate or at least alleviate double taxation. These mechanisms can be unilateral if decided by a country for all its residents, bilateral if they are

[1] The notion of company is intentionally vague here as it is defined differently from country to country. By company, we mean the entity on which the considered taxes apply.

the result of an agreement between two countries or multilateral if they are the result of an agreement between a set of countries. There are two main mechanisms on which we focus in the following: the crediting method and the exemption method.

The crediting method

Under the crediting method, the destination State A, will see the taxes paid in the source State B as if they were paid to itself. Therefore the total tax due to State A will be dropped by the tax paid upstream on the path. Although, tax credit is limited in some cases, we assume here that the whole tax paid upstream is deductible from the tax burden in State A as long as the tax due to A stays non negative. For example, assume the subsidiary has an income of 100€. It pays the corporate income tax in B at a rate of 20% and after that a withholding tax of 10%. The remaining profit entering A is 100 - 20 - 8 = 72 €. Eventually, A applies the crediting method and claims the difference between what the subsidiary would have paid in A if it was operating in A minus what has been paid upstream. By operating in A, the total tax would have been of 35%, the corporate income tax rate of A or 35€. The subsidiary already paid 28€of taxes and the parent company therefore has to pay 35 - 28 = 7€ to State A. More generally, as developed in [28], the effective tax rate in State A is given by:

$$t_A^{crediting} = max\{0, \frac{\tau_A - [(1-\tau_B)t_{BA} + \tau_B]}{(1-t_{BA})(1-\tau_B)}\}$$

The crediting method is for example applied by the United States.

The exemption method

The idea behind the exemption method is that the only State entitled to tax the profit in the situation of figure 1.2 is the source country, namely State B. The destination country, A in our example, simply exempts fully or partially the received dividends from the corporate income tax. The effect of the exemption method is that the income is taxed where it is generated. If State A grants an exemption of received dividends such that only a fraction μ_{AB} is taxed, the effective tax rate in State A is given by:

$$t_A^{exemtpion} = \mu_{AB}\tau_A$$

Thanks to the EU Parent Subsidiary directive [16], EU Member States grant an exemption of 95 or 100% of received dividends from other EU Member States. For example, Belgium taxes 5% of the received dividends from EU source while Spain doesn't tax such dividends.

1.2.3 Patent Box regimes

As explained in [5], "Patent Box regimes grant corporate revenues from Intellectual Property (IP) a preferential tax treatment". MNEs locating their R&D in a given territory generate employment on that territory. States have therefore design incentives for MNEs to locate such activities on their territory by reducing the effective tax rate levied on received royalties. As R&D eventually creates valuable IP, the company may license this IP to others and receive in return royalties. In practice a Patent Box regime exempts a given fraction of the incoming royalties from corporate income tax. As an example, table 1.1 groups the percentage of exemption granted by a sample of EU Member States (2015 data).
As explained in section 1.3.2, such Patent Box regimes, without substance requirement, have been abolished.

	Corporate income tax rate	exemption of royalties	Effective tax rate
Belgium	33.99%	80 %	6.8%
Cyprus	12.5%	80%	2.5%
Netherlands	25%	80%	5%
Luxembourg	29.22%	80%	5.8%
Spain	28%	60%	11.2%

Table 1.1: Patent Box regimes

1.2.4 Summary

The tax levied on a transaction between country A and country B depends on both countries but also on the financial vehicle of the transaction as well as on the financial vehicle that previously entered A.
The tax liability in an intermediate country can be decomposed in two parts: the corporate income tax and the withholding tax.

Firstly, the corporate income tax paid in country A is determined by both the entering and the exiting vehicle. If we define τ_S as the corporate income tax rate, μ_S as the percentage of received dividends taxed[2] and θ_S as the percentage of received royalties taxed in the source country[3], we can summarize in table 1.2 how a profit flowing through a country is taxed. On the one hand, interests and royalties payment are considered as a cost and are consequently deductible from the taxable base[4]. On the other hand, dividends payment is not. Hence, as long as we circulate from country to country either in the form of interests or royalties, the tax liability remains zero. The main question is therefore to find where the profit should be turned into dividends.

Entering vehicle \ Exiting vehicle	Interests	Dividends	Royalties
Interests	0	τ_S	0
Dividends	0	$\mu_S\ \tau_S$	0
Royalties	0	$\theta_S\ \tau_S$	0

Table 1.2: Corporate Income Tax rate as a function of the entering and the exiting financial vehicle

Secondly, the withholding tax is fixed by both the source and the destination country as well as by the vehicle of the transaction.

1.3 Base Erosion and Profit Shifting

"Base erosion and profit shifting (BEPS) refers to tax avoidance strategies that exploit gaps and mismatches in tax rules to artificially shift profit to low or no-tax locations." [35]

[2]according to the exemption method explained in 1.2.2
[3]according to possible Patent Box regimes as explained in 1.2.3
[4]Assuming we comply with potential anti-avoidance rules

1.3.1 Tools

As developed in [13], there are three major tools used by MNEs to channel their profit from high tax to low tax jurisdiction: debt shifting, intangible assets location and transfer pricing.

Firstly, debt shifting can be achieved by means of intra-group loans. Suppose we have two companies, one located in a high-tax jurisdiction and another located in a low-tax jurisdiction such that both companies are part of the same international group. The company located in the high tax jurisdiction takes out a loan from the company located in the low tax jurisdiction. When making profit, the debtor company sends interests to the creditor company. Since interests' payments are deductible from the tax base, the debtor company reduces its tax base in the high tax jurisdiction. The global income of the creditor company located in the low tax jurisdiction is increased and the profit has well been shifted.

Secondly, as introduced previously, some countries implement Patent Box regimes making them more attractive for the location of MNE's intellectual property. A MNE can therefore decide to locate its intellectual property in such a country, routing the profit of the operating subsidiaries in the form of royalties to this country and reducing its global tax liability.

Thirdly, a company having several subsidiaries may need that one of the subsidiaries buys some products from another. The price of such a transaction between related subsidiaries is referred to as transfer price. If this sale was achieved between unrelated companies, the price would be fixed by the market. However, when both companies are under the same common ownership, the group may decide to artificially increase the price of the transaction such that the buying subsidiary located in a high tax jurisdiction pays a large amount to the selling subsidiary reducing its tax base in the high tax jurisdiction. The regulator implemented rules to ensure that such intra-group transactions are done at a market price. Nevertheless, some transactions are so specific that no comparable transaction can be found on the market making the situation even more complex.

The study of transfer pricing is beyond the outreach of this work and we will focus on the first two channels for profit shifting: debt shifting and intangible assets location.

Both debt shifting and intangible assets location are strategies to benefit from tax advantages but do not reflect an economic reality. Moreover, they take advantage of differences between the different tax policies in different jurisdictions.

1.3.2 Anti-Avoidance rules

As explained in [41], anti-avoidance rules are a provision of last resort that national tax authorities can invoke to dismantle unacceptable tax avoidance practices. The aim is to strike down strategies where the taxpayer abused a specific law giving him tax benefits unrelated to any economic reality. We first introduce the general arm's-length principle before focusing on two types of anti-avoidance rules: thin-capitalization rules and substance requirement to benefit from Patent Box regimes. Nevertheless, this is not an exhaustive list.

Arm's-length Principle

The arm's-length principle states that the intra-group price of a transaction should be comparable to the price between unrelated companies.
For example, when a company signs a royalty agreement contract with a related company, the price should be comparable to the price at which this contract would have been signed with a third party. This principle seems quite intuitive however its application is not always easy.

In practice, to determine the acceptable price of such a royalty agreement contract, the only solution is to look for similar contracts signed in the past. Due to the high specificity of many Intellectual Properties licensed, there is a high probability that no similar contract has ever been signed, this is especially true in the digital economy. The challenge is thus to estimate a proper price without having reference contracts.
In this work, we assume that a company can send between 3 and 6% of its profit as royalties, not more. For example, according to [6], Ikea limits its Ikea stores royalties payment to 3%.

Thin-Capitalization rules

Thin-capitalization rules aim to limit the debt-to-equity ratio of a company. A project can be either financed by debt or by equity. There are pros and cons for both types of financing and this debate is beyond the scope of this work. However, due to the deductibility of interests, there is a tax incentive to finance a project by debt. As long as debt is contracted with a third party, we can expect the debt-to-equity ratio to remain within acceptable range. Indeed, the lender will not accept to take a huge counterparty risk and will ask for solvency guarantee. In the case of intra-group loan however, there is not the same counterparty risk and the group can decide to finance a subsidiary by means of intra-group loan only. In such a tax planning the subsidiary will send a huge part of its profit for interests' payment. The profit is then shifted. To tackle this strategy, there are two main types of thin-capitalization rules.

Firstly, one can cite rules limiting the debt-to-equity ratio of a company to a fixed ratio. For example, in Belgium, the debt-to-equity ratio of a company is limited to 5:1. It means that for 1€ of equity, the company is eligible to deduct interests corresponding to a total loan of 5€. Secondly, another approach is to limit the deductibility of interests to a given percentage of the Earning Before Interests, Taxes, Depreciation and Amortization (EBITDA). For example, in Germany, a company can deduct interests for at most 30% of its EBITDA.

Even if today both types of thin-capitalization rules are present in Europe, following the European Directive [14], all EU Member States should enforce a thin-capitalization rule limiting the deductibility of interests to 30% of the EBITDA by the start of 2019.

Nexus approach for Patent Box regimes

The development of Patent Box regimes has raised concerns since in practice, as developed in [5], they have had a significant effect on patents' location but not on real research activity. In the context of BEPS, the OECD and G20 member countries agreed on a new approach to Patent Box regimes: a modified Nexus approach [36].

This modified nexus approach aims to require substantial activity before granting the benefits from a Patent Box regime. There must be a direct nexus between the income receiving tax benefits and the activity contributing to that income [7]. Although old Patent Box regimes have been closed to new entrants by 30 June 2016, they are still available until 30 June 2021 for taxpayers benefitting from these regimes before the closing date, according to [36].

1.3.3 BEPS Action Plan

The BEPS action plan [35], developed by the OECD called by the G20 finance ministers in 2013, is a comprehensive and coordinated battle plan of 15 actions to tackle Base Erosion and Profit Shifting. Aggressive tax avoidance strategies harm individual taxpayer, governments and business itself around the world. This explains why more than 100 jurisdictions participated in the BEPS action plan development to reduce BEPS while meeting the challenges raised by the digital economy. The 15 actions aim to achieve three main objectives:

- "*Establish international coherence of corporate income taxation*" by, in particular, limiting base erosion via interests or other vehicles deduction and taking into account transparency and substance to grant favourable tax regimes.

- "*Restoring the full effects and benefits of international standards*" by, in particular, preventing from treaty abuse and taking into account potential profitable detours.

- "*Ensure transparency while promoting increased certainty and predictability*" by, in particular, developing methodologies to collect data and measure BEPS and requiring disclosure of aggressive tax planning.

1.4 Literature review

Base erosion and profit shifting has mainly be studied tool by tool. Firstly, concerning debt shifting, [11] showed, based on German multinationals data, that it was used in practice to shift profit from high-tax jurisdictions to low-tax jurisdictions. However, the use of debt shifting highly depends on the location of the parent company and the internal rules of this country to fight tax avoidance. Indeed, [9] showed a significant impact of thin-capitalization rules on the capital structures of companies.

Secondly, the location of intangible assets within multinational firms has been extensively studied, [22], [32] as well as [5] showed that MNEs tend to distort the location of their patents in favour of low tax jurisdictions.
The study of the European Commission [13] exposed some aggressive tax planning structures, some using several tools. The study stands itself apart from the others as it allows vehicle changing, meaning that a profit can first circulate as royalties before being turned into dividends and finishing its route for example, and profitable detours.

The need for solutions to tackle base erosion and profit shifting is addressed in several papers. One promising solution is to extend taxation in the source country, where the value is created. Arguments in favour of extending taxation in the source country have been highlighted by [27] and [25]. The later suggests restricting interests and royalty deductibility still without imposing double taxation which is the real challenge. Extending taxation at source is in the goals of the BEPS action plan described above. Also, [17] empirically showed that losses due to base erosion and profit shifting are of even greater importance for developing countries emphasizing the importance that so many jurisdictions are collaborating in the BEPS action plan.

To the best of our knowledge, the first attempt to model the tax network using graph and network theory and taking into account the interactions between the independent national tax legislations has been made by [28]. They addressed the problem of finding the least taxed path between two given countries by using a modified Dijkstra algorithm. They considered two financial vehicles, interests and dividends, with change of vehicle permitted in any country on the path. In his master thesis, [19], build up on this work and add a third vehicle, the royalties. However, the vehicles where considered one by one, without change permitted. Finally, [40] also used a network approach to model the circulation of dividends, profitable detours and treaty shopping on a large set of jurisdictions.

Contribution

The contribution of this thesis is to extend the model developed in [28]. The model is first extended to consider three financial vehicles, namely interest, dividend and royalty with change of vehicle permitted at any time (chapter 2). Secondly, it is extended to comply with specific anti-avoidance rules: thin-capitalization rules limiting the deductibility of interests to a fixed percentage of the EBITDA and limit on the deductibility of royalties (chapter 3). Eventually, the last feature added to the model is the possibility to discover strategies where the profit is stored in some advantageous countries (chapter 4).

The model developed in this thesis allows to rapidly discover all the least taxed paths for all pairs of countries and consequently to detect possible vulnerabilities in the tax system in an automatic way.

1.5 Data

The data used in this work have been collected in the KPMG Global Corporate Tax Handbook 2015 [33]. We first focus on data about EU Member States before extending the database to non EU jurisdictions. It is important to note that we consider, in this thesis, the theoretical tax rates, we do not take into account special rulings or benefits that might be granted.

Our whole database consists of 30 countries, including the EU28 countries as well as the United States and Bermuda. Firstly, EU28 countries are selected to model what could happen in terms of tax planning when repatriating a profit from an EU Member State to a second one without even leaving the European Union. Secondly, a tax haven was added to the database to take into account phenomenon due to highly favourable tax regimes. Among tax havens, Bermuda was selected as it is part of Google strategy. Eventually, since tax planning of many American companies has been disclosed, we included the United States in the database. While the full database is presented in appendix A, we present an overview in this section.

It is to note that in many jurisdictions, the CIT rate is defined by ranges. We always consider here the top range, with the highest CIT rate. This can be justified since developing a tax planning implied fixed costs and therefore only companies with sufficiently large revenue will do so.

The corporate income tax rate of EU Member States ranges from 10 to 35%. Table 1.3 shows Member states offering the lowest and the highest corporate income tax rate.

Rank	Country	Corporate Income Tax Rate (%)
1	Bulgaria	10
2	Ireland	12.5
3	Cyprus	12.5
...	...	...
26	France	33.33
27	Belgium	34
28	Malta	35

Table 1.3: Corporate Income Tax rates - Highest and lowest in EU

In order to compute, for each country the values of table 1.2, we need the percentage of incoming dividends and royalties that are taxed according to the EU Parent-Subsidiary directive and possible Patent Box regime. The data for four EU Member States are presented in table 1.4, the full table is available in appendix A. Interests are always fully taxable while dividends are either fully exempt or exempt for 95%. This exemption is only granted if the parent company has a minimum holding in the capital of the subsidiary. We assume here that this condition is always met.

Country	Interests (%)	Dividends (%)	Royalties (%)
Belgium	100	5	20
Netherlands	100	0	20
Luxembourg	100	0	20
Ireland	100	0	100

Table 1.4: Taxation of each vehicle - Selected countries

The treatment of royalties varies more. Table 1.5 shows the most advantageous tax rates offered to royalties across Europe, Cyprus leading.

Country	Tax Rate on royalties (%)
Cyprus	2.5
Netherlands	5
Luxembourg	5.8
Belgium	6.8
United Kingdom	10

Table 1.5: Patent Box regimes - Top 5

Thanks to both the EU Parent-Subsidiary Directive [16] and the EU Interest and Royalties Directive [15], there is no withholding tax for transactions within the EU. However, for transactions outside the EU, the source country may levy a withholding tax. Each country has a general withholding tax rate for payment remitted to foreign jurisdictions that can be reduced tanks to possible tax treaty signed with the destination country. Therefore, we collected the withholding tax rates levied for each pair of countries, for each of the three considered vehicles.

An overview of the withholding tax rates on dividend payments for some pairs of countries is presented in table 1.6. The withholding tax rates on interest and royalty payments are presented, for some pairs of countries, respectively in table 1.7 and table 1.8. Each row corresponds to a source country and each column to a destination country. The withholding tax is collected by the source country.

Again, the withholding tax rates may differ depending on how much capital of the subsidiary is hold by the parent company. We again assume that the parent company holds as much capital as required and consider the lowest possible withholding tax under the tax treaties.

Source / Destination	Belgium	Luxembourg	United States	Bermuda
Belgium	-	0	0	25
Luxembourg	0	-	0	15
United States	5	5	-	30
Bermuda	0	0	0	-

Table 1.6: Withholding tax rate on dividends (%) - Selected countries

Source / Destination	Belgium	Luxembourg	United States	Bermuda
Belgium	-	0	0	25
Luxembourg	0	-	0	0
United States	0	0	-	30
Bermuda	0	0	0	-

Table 1.7: Withholding tax rate on interests (%) - Selected countries

Source / Destination	Belgium	Luxembourg	United States	Bermuda
Belgium	-	0	0	25
Luxembourg	0	-	0	0
United States	0	0	-	30
Bermuda	0	0	0	-

Table 1.8: Withholding tax rate on royalties (%) - Selected countries

Chapter 2

Least Taxed Path

The goal is to find the least taxed path between any two countries. We make a profit in country A and need to repatriate this profit in country B. What is the cheapest way in terms of taxes to do so?

The first model is our basis model which does not take into account anti-avoidance rules. It provides an upper bound on BEPS or kind of a worst case scenario from governments' point of view.

As discussed in chapter 1, we consider three financial vehicles: interests, dividends and royalties. Each transaction between any two countries can be in the form of any of these three vehicles. On top of it, change of vehicle is permitted inside any country. Moreover, we assume each subsidiary created in a country to be a permanent establishment and a resident of this country for tax purpose.

In this chapter we compute two measures of BEPS, the total tax liability avoided by the MNE as a percentage, denoted by $BEPS_{total}$ and the amount of tax shifted away from the source country, again as a percentage, denoted by $BEPS_{source}$. These measures are defined as:

$$BEPS_{total} = \frac{\tau - \theta}{\tau} \tag{2.1}$$

$$BEPS_{source} = \frac{\tau_S - \theta_S}{\tau_S} \tag{2.2}$$

where τ and τ_S are respectively the global effective tax rate and the effective tax rate in the source country without strategic behaviour. θ and θ_S are respectively the global effective tax rate and the effective tax rate in the source country using the strategic behaviour.

2.1 Model

We first introduce some preliminary notions, based on [8], to understand the modelling tool used in this work, namely what is a graph, before actually modelling international transactions.

2.1.1 Preliminary notions

We give here the definition of a graph and define a way to represent a graph in a computer memory. The reader familiar with graphs can safely skip this part.

Definition 1 (Graph)**.** A **graph** $G(V, E)$ is composed of a set of vertices V (or nodes) and a set of edges E connecting the vertices.
A graph can be **directed** or **undirected** depending on whether an edge can be followed in both ways or not. A graph can be **weighted** if each edge has an associated cost.

An example of a directed and weighted graph is presented on figure 2.1. A graph can be represented by its **adjacency matrix**.

Definition 2 (Adjacency Matrix of a weighted graph)**.** The adjacency of a weighted graph with N nodes is a $N \times N$ matrix A where the entry $A_{i,j}$ is the cost of going directly from node i to node j in the graph, with $A_{i,i} = 0$ for all i. If there is no edge connecting i and j, $A_{i,j}$ is set to $+\infty$.

For example, figure 2.1 shows a weighted and directed graph having as adjacency matrix A:

$$A = \begin{pmatrix} 0 & 1 & 5 & \infty \\ \infty & 0 & 1 & \infty \\ \infty & \infty & 0 & 2 \\ 3 & 2 & \infty & 0 \end{pmatrix}$$

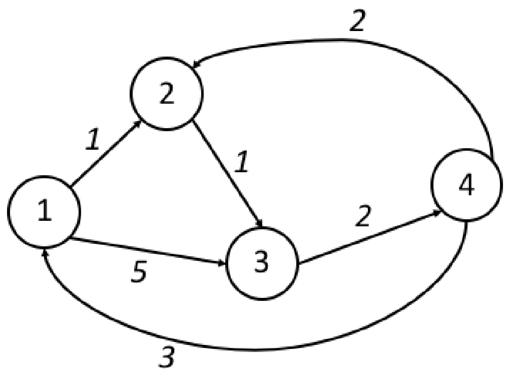

Figure 2.1: Example of weighted and directed graph

2.1.2 Modelling international transactions

In order to model the tax network by a graph, we propose to represent a country by six nodes. We have three entry nodes and three exit nodes, one per vehicle as shown on figure 2.2. Using this approach, we can easily compute both the corporate income tax (taking into account possible exemption, if the income enters in the form of dividends or royalties) and the withholding tax.

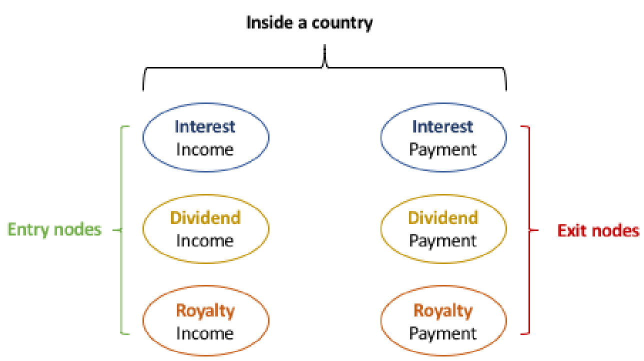

Figure 2.2: Nodes representing a country

Indeed, the three entry nodes are connected to the exit nodes by edges called intra-jurisdictional edges in the later, as shown on figure 2.3. The cost associated to these edges is the tax paid in the intermediate country, taking into account the entering and the exiting vehicle, as in table 1.2. This is the cost of the vehicles changing. One can notice that entry nodes corresponding to interests or dividends are not linked to the exit node corresponding to royalties. The reason is that for making a royalties payment, you should either be an intermediate company routing the royalties or be a company making a profit thanks to a real economic activity in the considered country. Our main assumption is that the only country where the MNE has a real economic activity is the very first country on the path, not any intermediate country. Therefore, turning incoming interests or dividends into royalties is not permitted.

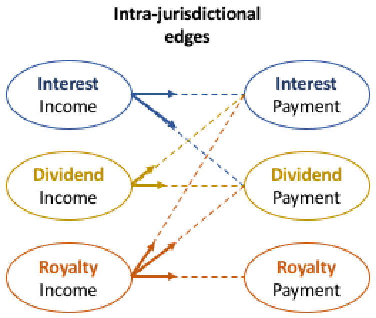

Figure 2.3: Intra-jurisdictional edges

Concerning the withholding tax, it is the cost associated to inter-jurisdictional edges shown on figure 2.4. We know the source and destination country as well as the vehicle used for the transaction and therefore the associated withholding tax. This is the cost of the payment.

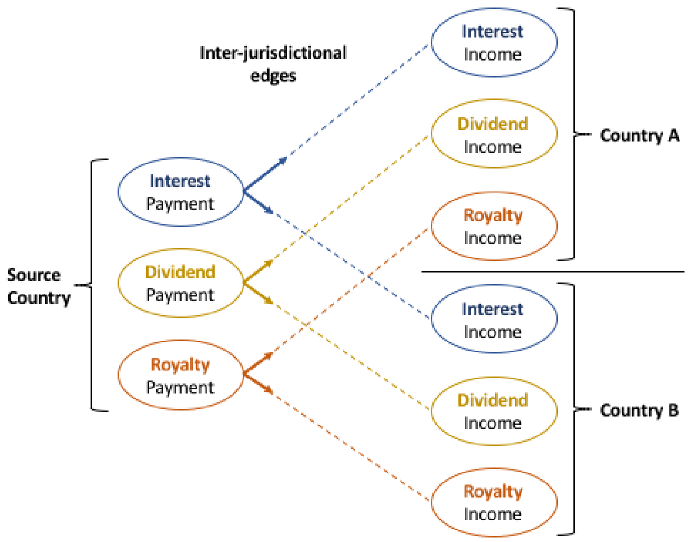

Figure 2.4: Inter-jurisdictional edges

The source country, where the original profit is generated, is different from the intermediate country in the sense that no payment is arriving from other countries. Instead of using the six nodes model described before, we use only four nodes to represent the source country as shown on figure 2.5. Concerning the destination country, no payment will be done and we represent it also using four nodes as shown on figure 2.6.

Such a representation suggests that, for each pair of countries, we would need to create a new graph. This is in practice not the case, as we can use a small trick to use only one graph for all source-destination pairs.
On the one hand, the profit generated in the source country is fully taxable in this country unless it leaves the country in a deductible vehicle like interests or royalties. Consequently, it has the same fiscal properties, in this context, as an interest's income (see table 1.2). On the other hand, when arriving in the destination country, no more possible deductible payment will be made and there is no possible interests or royalties payment deduction. This is similar, in this context, to a dividend payment.
Therefore, we can build a graph where all the countries are represented like the previously depicted intermediate countries. When looking for the least taxed path between country A and B, we will look at the least taxed path between interest income node of A and dividend payment node of B in the graph.

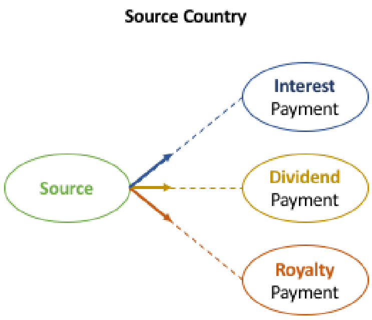

Figure 2.5: Source country representation

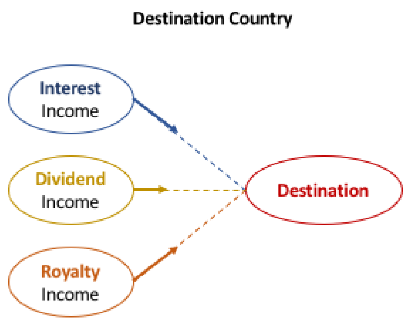

Figure 2.6: Destination country representation

To conclude the description of this graph model, it is interesting to highlight some of its properties assuming it is built for N countries.
First, this is a weighted and directed graph G = (V,E). The number of nodes $|V|$ and the number of edges $|E|$ are given by the following equations.

$$|V| = 6N$$
$$|E| = \underbrace{7(N-2)}_{\substack{\text{Intra-jurisdictional}\\\text{edges}\\\text{intermediate country}}} + \underbrace{3N(N-1)}_{\substack{\text{inter-jurisdictional}\\\text{edges}}} + \underbrace{6}_{\substack{\text{intra-jurisdictional}\\\text{edges}\\\text{source and destination}}}$$

Second, we can compute the density of this directed graph, standardly defined by equation 2.3. The density is a measure of the connections in a graph. When the number of edges increases, the density increases. This measure ranges from 0 to 1 and is useful from an algorithmic point of view, to select the most appropriate algorithm for our needs. The density of our graph with 30 countries is 0.08, meaning that our graph is very sparse.

$$D = \frac{|E|}{|V|(|V|-1)} \tag{2.3}$$

(2.4)

2.2 Algorithm

We have a weighted and directed graph $G = (V, E)$ and the goal is to find the least taxed path between any pair of countries. Thanks to some modifications, it is possible to reduce this problem to a Shortest Path Problem which is a well-known problem in graph theory.

Firstly, the cost of the edges needs to be adapted. The cost of an edge is defined from a graph theory point of view. It is not necessarily the actual cost of the transaction but should allow to compute it. We need to define the cost for the edges such that the total cost of a path is the sum of the cost of its edges. In our case, we know the tax rate applied to each transaction. However, setting the cost equal to the tax rate doesn't work. Indeed, let us consider the two transactions path shown on figure 2.7. On this figure, the tax rate τ_1 and τ_2 are the tax rate applied on a transaction respectively from A to B and from B to C. If we follow the path, the tax paid for transaction 2 is not τ_2 but rather $(1 - \tau_1)\tau_2$. The amount of taxes paid upstream is not taxable since it never arrives in B.

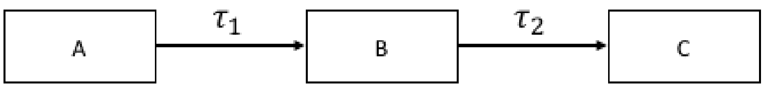

Figure 2.7: 2 edges path

Therefore, we use the approach used in [28] and define the cost of an edge as follows:

$$c = |\ln(1 - \tau)|$$

where τ is the tax rate of the transaction. Such a definition leads to a total cost of the path shown in figure 2.7 of:

$$\begin{aligned} \text{Total Cost} &= c_1 + c_2 \\ &= |\ln(1 - \tau_1)| + |\ln(1 - \tau_2)| \end{aligned}$$

To compute the desired result for the total tax paid on this path, we use the following formula:

$$\begin{aligned} \text{Total Tax} &= 1 - e^{-\text{Total Cost}} \\ &= 1 - e^{-|\ln(1-\tau_1)|}e^{-|\ln(1-\tau_2)|} \\ &= 1 - (1 - \tau_1)(1 - \tau_2) \\ &= \tau_1 + \tau_2 - \tau_1\tau_2 \\ &= \tau_1 + \tau_2(1 - \tau_1) \end{aligned}$$

Secondly, it seems reasonable to assume that a MNE will try to find the shortest path both in terms of total tax liability and in terms of number of countries visited. Indeed, setting up a subsidiary in a country creates fixed costs. Consequently, we assume a path to be strictly better than another if its total tax liability is strictly lower or if its tax liability is equal and the total number of countries visited along the path is strictly smaller.

We now have edges' cost defined such that the cost of a path is the sum of its edges and a proper way to compare different paths. To find the least taxed path between two countries, we can use a dynamic programming approach and more precisely Dijkstra algorithm [21]. In our case, we are not only interested in the least taxed path between two given countries. We would like to find the least taxed path between any two countries. This could be done by repeating the Dijkstra algorithm for every source country or by using the Floyd algorithm designed to find the shortest path between any pair of nodes (see [39]). To distinguish between these two approaches, we compute their respective worst-case time complexity. The reader not familiar with complexity can refer to appendix C based on [20].

Firstly, Dijkstra algorithm is presented in pseudo-code 1. The first step is to initialize the distance and then add all the nodes in queue Q. The time complexity of this operation depends on the data structure used to store Q. If this is a simple list, adding an element can be done in $\mathcal{O}(1)$, leading to a global complexity of $\mathcal{O}(|V|)$ to add all nodes. Then, as long as Q remains non empty, we have to find the node in Q with the smallest distance to the source node. Again, if Q is a simple list, we will have to pass through all the nodes in the list to find the minimum, leading to a worst-case time complexity of $\mathcal{O}(|V|)$. Since we need to do it for all nodes in Q, we obtain $\mathcal{O}(|V|^2)$. Eventually, we have to check for all neighbours of the selected node if the detour is profitable. Basically, we will check each edge of the graph exactly once and obtain a time complexity of $\mathcal{O}(|E|)$. Therefore, the global worst-case time complexity of Dijkstra algorithm when Q is implemented as a simple list is $\mathcal{O}(|E| + |V|^2)$; when repeating it for every possible source node we obtain $\mathcal{O}(|E||V| + |V|^3)$.
However, we can do better with another implementation of the queue. Indeed, with the simple list, the insert operation is done in $\mathcal{O}(1)$ while the extraction of the minimum is done in $\mathcal{O}(|V|)$. By implementing the queue by a binary heap (for details see [30]), we can achieve $\mathcal{O}(\log |V|)$ for both operation and obtain a worst-case time complexity of $\mathcal{O}(|E| + |V| \log |V|)$ for Dijkstra. Since we need to repeat it for every possible source node, computing the least taxed path between any two countries is done in $\mathcal{O}(|E||V| + |V|^2 \log |V|)$ in worst-case.

Secondly, Floyd algorithm is presented in pseudo-code 2. The study of its time complexity is rather simple as it is a triple loop through all nodes leading to a worst-case time complexity of $\mathcal{O}(|V|^3)$.

As a conclusion, as long as the number of edges $|E|$ remains smaller than $|V|^2$, Dijkstra implemented with a binary heap seems more efficient than Floyd. As discussed in section 2.1, our graph is quite sparse and therefore Dijkstra implemented with a binary heap is chosen. A final advantage of Dijkstra over Floyd in this case is that we are not interested in the shortest path from any to any node. Indeed, a country is defined by 6 nodes, and as explained in the model description (2.1.2), to find the shortest path from country A to country B, we look for the shortest path between the interest income node of A and the dividend payment node of B. For 30 countries, we are only interested in $30 \times 29 = 870$ cases instead of $(6 \times 30)^2 = 32400$.

Algorithm 1: Dijkstra Algorithm

```
for each vertex v in g do
    dist[v] = ∞
    predecessor[v] = None
end
dist[src] = 0
Q = set of all nodes of g
while Q is not empty do
    u = arg min dist[u]
    remove u from Q
    for each neighbour v of u do
        current = dist[u] + distance(u,v)
        if current < dist[v] then
            dist[v] = current
            predecessor[v] = u
        end
    end
end
return dist, predecessor
```

Algorithm 2: Floyd Algorithm

```
dist = A
predecessor = None
for k = 1 to N do
    for i = 1 to N do
        for j = 1 to N do
            if dist_ik + dist_kj < dist_ij then
                # profitable detour
                dist_ij = dist_ik + dist_kj
                predecessor_ij = k
            end
        end
    end
end
return dist, predecessor
```

2.3 Results

In this section we first describe a reference route being the direct path, when the MNE is not engaged in any strategic behaviour. Secondly, we present the result obtained by applying the repeated Dijkstra algorithm to the graph defined in section 2.1.

2.3.1 Direct Path, no strategic behaviour

The direct path, without strategic behaviour, from a subsidiary operating in country A to its parent company established in country B is simply a dividend payment from the subsidiary to the parent company. The path on the graph is presented on figure 2.8. The total tax liability is distributed as in table 2.1.

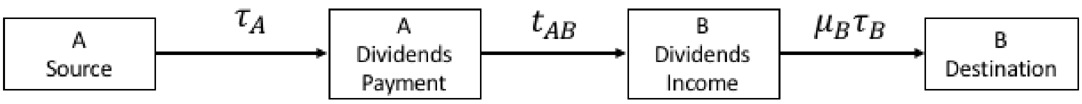

Figure 2.8: Taxation rate of each transaction in the direct path

Country A	Country B
$\tau_A + (1-\tau_A)t_{AB}$	$(1-\tau_A)(1-t_{AB})\mu_B\tau_B$

Table 2.1: Tax liability distribution among countries

2.3.2 Strategic behaviour

The results are presented in three categories. The first one concerns least taxed paths between EU Member States, the second one concerns routes between EU Member State and the United

States, and the third one concerns routes between EU Member State and Bermuda, our tax haven.

Routes within EU

For routes between EU Member States, we assume that the profit stays at all time within EU. We could find cheaper paths by reaching first a tax haven before entering again the EU, however, such routes would be obviously aggressive and taxed when re-entering the EU. Therefore, we discard all routes leaving the EU and remove from the graph described in subsection 2.1 all nodes representing non EU countries as well as associated edges. Since we stay in EU, the withholding tax is zero for any transfer thanks to Parent-subsidiary EU Directive [16] and Interests and Royalties EU Directives [15].
Applying the repeated Dijkstra algorithm to the reduced graph highlights one specific country: Cyprus. With its Patent Box regime exempting 80% of the received royalties and its corporate income tax rate of 12.5%, Cyprus offers to incoming royalties an effective tax rate of 2.5%, the lowest in EU.
The least taxed path between any two EU countries is represented on figure 2.9. The profit escapes from the source country in the form of royalties. These royalties are considered as a cost and are deductible from the taxable base at source. The taxable base reaches zero in the source country and no tax is paid where the value is created. The royalties enter Cyprus where they are turned into dividends paying 2.5% to Cyprus. The dividends finally leave Cyprus for the destination country where they are fully or partially exempted from taxation (typically 0 or 5% are taxed) thanks to the EU Parent Subsidiary Directive [16].

Figure 2.9: Least taxed path between EU countries

In practice, to implement this strategy, the company based in the destination country should create a subsidiary in Cyprus and transfer to this subsidiary its Intellectual Property (IP). The Cypriot company licenses the rights to use this IP to the subsidiary in the source country. According to this royalty agreement contract, the subsidiary in the source country has to pay royalties to the Cypriot company who need in its turn to send dividends to its parent company in the destination country.
We can compute the two measures $BEPS_{total}$ and $BEPS_{source}$ defined in 2.1 and 2.2 for this strategy:

$$BEPS_{total} = \frac{\tau - (2.5\% + (1-0.025) x_D \tau_D)}{\tau} \quad BEPS_{source} = \frac{\tau_S - 0\%}{\tau_S} = 100\%$$

where x_D is the percentage of dividends taxed in the destination country and τ_D is the corporate income tax rate of the destination country.
Taking the average over all pairs of EU countries, we obtain the following results:

$$B\bar{E}PS_{total} = 82.8\% \quad B\bar{E}PS_{source} = 100\%$$

This strategy works for any pair of EU countries. When Cyprus is either the source or the destination country, the path is even simpler since no intermediate country is needed.

Another way to highlight the attractiveness of Cyprus is to compute a centrality measure in the EU network. We consider a specific centrality measure called the 'betweenness' defined hereafter.

Definition 3 (Betweenness)**.** In a graph $G = (V, E)$, let $n^i_{s,t}$ be the number of shortest paths from s to t that pass through node i and let $n_{s,t}$ be the total number of shortest paths from s to t in the graph.
The Betweenness of node i is defined by:

$$b_i = \sum_{s \in V} \sum_{t \in V} \frac{n^i_{s,t}}{n_{s,t}}$$

As we can see on table 2.2 showing the average betweenness of the six nodes of each country, Cyprus is the most central country. It is consistent with the least taxed routes found above. All EU Member States have the same average betweenness.

Rank	Country	Average Betweenness
1	Cyprus	243
2	Netherlands	9
3	Belgium	9
3	France	9
...	...	...

Table 2.2: Average Betweenness

Routes between EU and US

Generally, dividends received by US Corporations from foreign affiliates are not deductible. However, a tax credit corresponding to the amount of tax paid abroad may be granted. Since the top range corporate income tax rate in the US is 35%, the global tax liability will be the maximum between 35% and the tax paid before on the path. The optimal tax strategy for the MNE is not unique. Once arriving in the US, if the total tax paid upstream is smaller than 35%, then a tax is paid in the US and the global tax liability is 35%. If the tax paid upstream is grater or equal than 35%, then no tax is paid in the US, thanks to the crediting method. Therefore, the MNE can decide to follow any path as long as the total tax paid before entering the US is smaller or equal than 35%. The tax paid at source could be anywhere in the compact set $[0, 1]$ according to the MNE choice, leading to a $BEPS_{source}$ ranging from 1 to 0. The global effective tax rate of these strategies is always 35%, and the $BEPS_{total}$ measure is equal to 0 since no EU country has a corporate income tax rate strictly above 35%.

We can see in this strategy the superiority of MNEs over States. Indeed, MNEs fully decide how they distribute their global tax liability over the countries. The profit is taxed where the value is created if the MNE decides so. However, we could assume that for reputational risk concerns as well fixed costs relative to the creation of intermediate companies, the MNE decides in such case to follow the direct path, without any strategic behaviour. The crediting method applied by the US seems to ensure that the global tax liability is a least 35%.

Routes between EU and Bermuda

When the goal is to fly from the EU to Bermuda, since the corporate income tax rate is 0 in Bermuda, there is no need to turn the profit into dividends before reaching the tax haven. The main question is to avoid any withholding tax.

By applying Dijkstra to our graph, we find that for instance the Netherlands are a profitable intermediate country. The Netherlands does not levy any withholding tax on interests or royalties payment irrespective of the destination country. Therefore, an optimal strategy to channel a profit from an EU Member State towards Bermuda is presented on figure 2.10.
The global tax liability is reduced to 0 and therefore, the BEPS measures $BEPS_{total}$ and $BEPS_{source}$ are both equal to 1, irrespective of the EU source country.

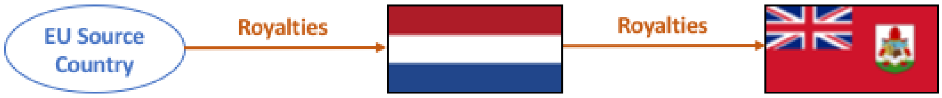

Figure 2.10: Least taxed path from EU to Bermuda

It is interesting to notice that the optimal path from EU to Bermuda is not unique. We could also channel the profit using the same route via the Netherlands but in the form of interests. Moreover, we could use other intermediate countries as Luxembourg, Malta, Hungary or Sweden for routing royalties or interests at no cost and Cyprus or Austria for routing interests at no cost.

2.4 Conclusion

In this chapter, we described a first tool to automatically detect advantageous tax planning strategies. We considered the EU28 countries, the United States and Bermuda as well as 3 financial vehicles: interests, dividends and royalties. We showed that when all transactions are permitted, the tax base at source is easily reduced to zero and the global effective tax rate of transactions is drastically reduced. The United States, with their crediting method, stand apart. Indeed, when trying to reach the United States, the global effective tax rate of any optimized strategy is equal to the one of the direct path, namely 35%. However, the MNE decides where to pay taxes in this case and the base erosion and profit shifting at source can still be of 100%. The global effective tax rates for routing a profit between selected pairs of countries are presented in table 2.3, the full results table can be found in appendix B. On average, we observe a 87 % decrease on the global effective tax rate within the EU in our data base.

Source / Destination	Belgium	Netherlands	Luxembourg	United States	Bermuda
Belgium	-	33.99% / 2.5%	33.99% / 2.5%	35% / 35%	50.49% / 0%
Netherlands	26.27% / 4.2%	-	25% / 2.5%	35% / 35%	36.25% / 0%
Luxembourg	30.42% / 4.2%	29.22% / 2.5%	-	35% / 35%	39.84% / 0%

Table 2.3: Direct Path / Optimized Path global effective tax rates

An important assumption, in this chapter, is that each subsidiary is a permanent establishment (PE) in the country where it operates. It means that in any country, we consider the regulation relative to PE. If we relax this assumption, another country stands out from the crowd: Malta. Indeed, Malta seems to exempt the incoming royalties from corporate income tax, for non-resident. As described in the article [34], Malta seems indeed to be used for royalties routing.

The situation depicted in this chapter seems too good to be true from a MNE point of view. In the real world, as described in the first chapter of this thesis, States enforce anti-avoidance requirements preventing MNEs from using the strategies of this chapter. In the next chapter, we introduce a new model to discover strategies complying with anti-avoidance rules.

Chapter 3

Impact of anti-avoidance rules

In our basic model we considered that any transaction is admissible, however the regulator prevents from using this strategies by imposing tax planning to reflect an economic reality. For example, we considered that the whole profit generated at source was allowed to leave as royalties. It is in practice not the case. Royalties' agreement contracts are subject to the arm's length principle meaning that the intra-group price should be comparable to the inter-group price as developed in chapter 1. Moreover, interests' deductibility is subject to thin-capitalization rules as described in chapter 1 as well.

The aim of this chapter is to include in the basic model thin-capitalization rules limiting, in each country, the deductibility of interests to a given fraction of the EBITDA. Although this is not the only type of thin-capitalization rule, according to the EU directive 2016/1164/EU ([14]), EU Member States will have to limit the deductibility of interests to at most 30% of the EBITDA by the start of 2019. This rule is already enforced in Germany. We also add a constraint in the source country limiting royalties payment to 6% of the profit.

This chapter describes first how to adapt the basic model to enforce these rules and second the results obtained. Secondly, we consider another modelling tool: Markov Chains and Decision Processes. We continue this chapter by analysing the impact of a variation of the limits of 30% or 6% respectively for interests and royalties. Eventually, we focus on what could happen if tax exempt income is part of the EBITDA and if no transparency is required.

3.1 Model

The profit is not anymore allowed to completely flow through interests or royalties payment edges. These edges have now a limited capacity. The profit does not flow towards the destination country along one path it is rather split among several paths. We can see the profit routing as water flow in a pipe network. Each pipe section has a specific cost due to maintenance, all the water entering a node must leave this node, possibly using several exit pipes. The goal is to find the cheapest routing of the water flow through the network or in our case the least taxed routing of the profit.

In the basic model, at each node, we had to decide which edge to follow. Now, we have to decide the amount we send through each of the edges in order to minimize the total tax liability and fulfil the constraints. The problem is now more complex and the model need to be adapted.

Let's introduce a new node called 'Taxes' in our graph. Each time we decide to send money from source to destination, we have to pay the corresponding tax. This part of the profit will flow to this 'Taxes' node and will be lost while the net profit will flow towards the destination. Figure 3.1 shows the flow corresponding to the direct path (without strategic behaviour) in this new graph.

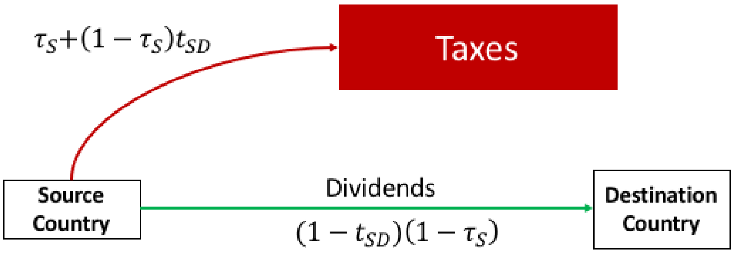

Figure 3.1: Direct Path

More precisely, the three entry nodes of each country are connected to the 'Taxes' node and the eventual corporate income tax will flow through these edges. The three exit nodes are also connected to the 'Taxes' node to represent the withholding tax that may apply. The new representation of a country in the graph is shown on figure 3.2.

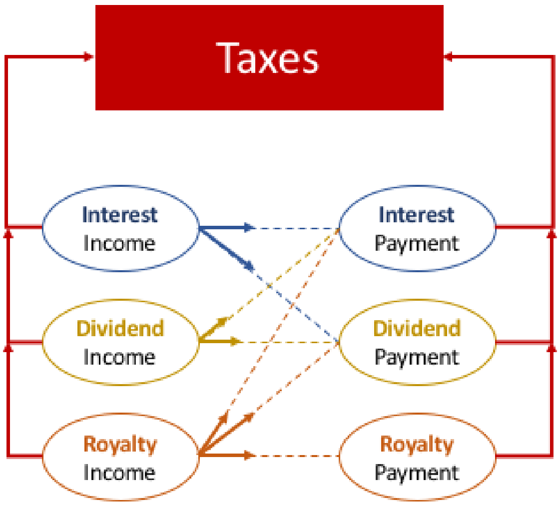

Figure 3.2: Representation of a country

In order to minimize the global tax liability, we have to minimize the part of the profit arriving in the 'Taxes' node. As in the basic model, we inject a profit in the interest income node of the source country and want to channel it towards the target node which is dividends payment node of the destination country. There are only two nodes in which the profit can stay: the 'Taxes' node and the target node. In any intermediate node, everything that enters must leave. Nevertheless the profit cannot circulate freely as in the basic model. The routing must comply with the thin-capitalization rules in any country on the path. Finally, on top of it, only 6% of the profit in the source country can leave as royalties.

Let's now describe this optimization problem more formally. Based on the graph described in chapter 2 augmented by the 'Taxes node, we introduce the following variables:

- $x_{c,v_{in},v_{out}}$: flow along the intra-jurisdictional edge inside country c, transforming a profit arriving in vehicle v_{in} to a payment in vehicle v_2.

- $y_{src,dst,v}$: flow along the inter-jurisdictional edge going from country src to country dst in financial vehicle v.

- $CIT_{c,v_{in}}$: the corporate income tax paid in country c for a profit arriving in vehicle v_{in}.

- $WHT_{c,v}$: the withholding tax paid in country c for a payment in vehicle v.

We define two sets C and V that represent respectively the set of countries and the set of vehicles. The vehicles are referred to in a numeric way, from 1 to 3 for interests, dividends and royalties. For the sack of simplicity, we denote by T the sum of the flows arriving in the 'Taxes' node:

$$T = \sum_{c \in C} \sum_{v \in V} \left[CIT_{c,v} + WHT_{c,v} \right]$$

Using these variables, and assuming a profit P in the source country, the constraints can be formulated as follows.

1. The amount circulating through each edge should be non-negative.

$$x, y, CIT, WHT \geq 0$$

2. The amount entering any intermediate node should be equal to the amount leaving that node.

(Entry nodes) $$\sum_{s \in C} y_{s,c,i} - \sum_{j=1}^{3} x_{c,i,j} = 0 \quad \forall (c,i) \in C \times \{1,2,3\} \text{ such that } (c,i) \neq (s,1)$$

(Exit nodes) $$\sum_{i=1}^{3} x_{c,i,j} - \sum_{d \in C} y_{c,d,j} = 0 \quad \forall (c,j) \in C \times \{1,2,3\} \text{ such that } (c,j) \neq (t,2)$$

3. The profit must exit the source country completely.

$$\sum_{c \in C} y_{c,s,1} - \sum_{j=1}^{3} x_{s,1,j} = P$$

4. The profit, minus the taxes paid, must enter the target node completely.

$$\sum_{i=1}^{3} x_{t,i,2} - \sum_{d \in C} y_{t,d,2} = P - T$$

5. The taxes corresponding to any transaction must be paid.

(Incoming interests leaving as dividends) $$x_{c,1,2}\ \tau_c = CIT_{c,1}(1 - \tau_c) \quad \forall c \in C$$

(Incoming dividends leaving as dividends) $$x_{c,2,2}\ \tau_c\ \mu_c = CIT_{c,2}(1 - \tau_c\ \mu_c) \quad \forall c \in C$$

(Incoming royalties leaving as dividends) $$x_{c,3,2}\ \tau_c\ \theta_c = CIT_{c,3}(1 - \tau_c\ \theta_c) \quad \forall c \in C$$

(Withholding tax) $$y_{c_1,c_2,v}\ t^v_{c_1,c_2} \leq WHT_{c,v}\ (1 - t^v_{c_1,c_2}) \quad \forall c \in C$$

6. The amount leaving a country in the form of interests should be less or equal than a fraction κ of the total profit in this country.

$$\kappa \sum_{i=1}^{3} \sum_{j=1}^{3} x_{c,i,j} \geq \sum_{d \in C} y_{c,d,1} \quad \forall c \in C$$

7. The amount leaving the source country in the form of royalties should be less or equal than a fraction λ of the total profit in this country.

$$\lambda \sum_{i=1}^{3} \sum_{j=1}^{3} x_{s,i,j} \geq \sum_{d \in C} y_{s,d,3}$$

8. An intermediate country cannot transform interests or dividends into royalties.

$$x_{c,1,3} = 0 \text{ and } x_{c,2,3} = 0 \ \ \forall c \in C \backslash \{s\}$$

Constraint 6 implements the thin-capitalization rule where the deductibility of interests is limited to a percentage κ of the profit, it should be enforced in any country. This percentage can be different from country to country. Constraint 7 limits the deductibility of royalties in the source country to a percentage λ of the profit, it should only be enforced in the source country.

The problem seems similar to the well-known Minimum Cost Flow Problem, however, the major difference is that the capacity of the edges leaving a country is not constant but rather depends on the total flow entering this country. If we increase the total profit in a country, we also increase the possibility to deduct interests. Therefore, it could be appealing to artificially inflate the profit in a country by following cycle on the graph. However, again, the regulator prevents from using tax planning that does not reflect a real economic activity and these strategies are not suitable. To prevent from artificial inflation of the profit, we add the following constraint:

9. The total profit in any country should be less or equal than the original profit.

$$\sum_{n_1=1}^{3} \sum_{n_2=1}^{3} x_{c,n_1,n_2} \leq P \ \ \forall c \in C$$

If we denote the admissible set by $\mathcal{A}$ which is the polyhedron defined by the linear constraints enumerated above, the optimization problem is the following:

$$T^* = \min T \quad \text{such that} \quad (x, y, CIT, WHT) \in \mathcal{A}$$

The most important feature of this optimization problem is its linearity. Indeed, the objective function as well as all the constraints are linear. Finding the optimal channelling of a profit from a source to a destination country while respecting the anti-avoidance requirements reduces to solving a linear optimization problem. This can be efficiently done using the Simplex method introduced by Dantzig in 1947 [18]. In practice it has been solved using the Gurobi solver within

Python.

Another characteristic of this problem is that there are multiple available free detours. By free detours we mean that, for example, routing royalties through several EU Member States doesn't lead to any taxation. However, as mentioned in chapter 2, even if such detours are tax free, they suppose that a company has been settled in each intermediate country, and, this has a cost. Therefore, we want to minimize the number of visited countries while keeping the global tax liability at its minimal level found by solving the optimization problem described above, T^*. This task is achieved by solving a second optimization problem. The idea is that the number of visited countries is intrinsically linked to the number of edges used and therefore to the number of non-zero flows. Minimizing the number of non-zero flows means minimizing the L^0 norm of the vector of flows. However, the L^0 norm, classified as a norm by abuse of notation, is non-linear and even non convex. Due to the complexity of solving non convex optimization problems, we decided to approximate the L^0 norm by the L^1 norm which is by definition the sum of all the flows and is therefore linear. The second optimization problem is then given by:

$$\begin{aligned}
&\text{minimize} && \sum_{c \in C} \sum_{i=1}^{3} \sum_{j=1}^{3} x_{c,i,j} + \sum_{c_1 \in C} \sum_{c_2 \in C} \sum_{i=1}^{3} y_{c_1,c_2,i} \\
&\text{such that} && T = T^* \\
& && (x, y, t) \in \mathcal{A}
\end{aligned}$$

3.2 Results

As in the previous chapter, the results, obtained by solving the optimization problem, are presented in three categories, channelling between EU Member States, between an EU Member State and the US and between an EU Member State and Bermuda. To obtain these results we set $\kappa = 30\%$ and $\lambda = 6\%$.

3.2.1 Routes within EU

The optimal way to channel a profit between two different EU Member States is to split the profit into the 3 vehicles and to maximize the total profit leaving in the form of deductible either interests or royalties. Therefore, as shown on figure 3.3, 30% of the profit leaves the source country in the form of interests towards Bulgaria because this country offers the lowest corporate income tax rate of the EU. On top of it, 6% of the profit escapes the source country in the form of royalties towards Cyprus to benefit from its Patent Box regime. In fact, these 6% follow the optimal path previously found with our basic model in section 2.3.2. The remaining 64% of the profit have no choice but to leave as dividends and there is no interest to take a detour via another country in this case. The tax liability distribution of this strategy is summarized in table 3.1 and a numeric example for channelling a profit from Belgium to Ireland is given in table 3.2.

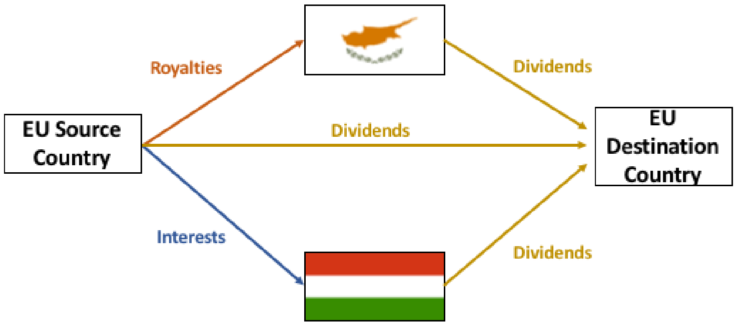

Figure 3.3: Optimal route within EU

	Source Country	Cyprus	Bulgaria	Destination Country
Tax Liability	64% τ_{source}	6% θ_{Cyprus} τ_{Cyprus}	30% $\tau_{Bulgaria}$	$\mu_{destination}$ $\tau_{destination}$

Table 3.1: Tax liability distribution for routes within EU

	Belgium	Cyprus	Bulgaria	Ireland
Tax Liability	21.75%	0.15%	3%	0

Table 3.2: Channelling from Belgium to Ireland

The problem with this strategy is that although Bulgaria is a EU Member State, it is not part of the euro zone. Therefore, by having some profit flowing through Bulgaria, the company may face a currency risk. There are several instruments for hedging currency risk. For example, subscribing to insurance or buying currency forwards but an analysis of these hedging mechanisms is beyond the scope of this work. However, these instruments come at a cost and we can assume the company will favour another channelling of its profit remaining in the euro zone. The corporate income tax rate of Bulgaria is 10%, the lowest in EU, but other countries offer a corporate income tax rate of 12.5%, namely Cyprus and Ireland. Therefore, the company could decide to go through Bulgaria and accept the currency risk, go through Bulgaria and use an instrument for hedging currency risk or to cannel the profit through either Cyprus or Ireland eliminating any currency risk. The optimal path is in this case presented on figure 3.4.

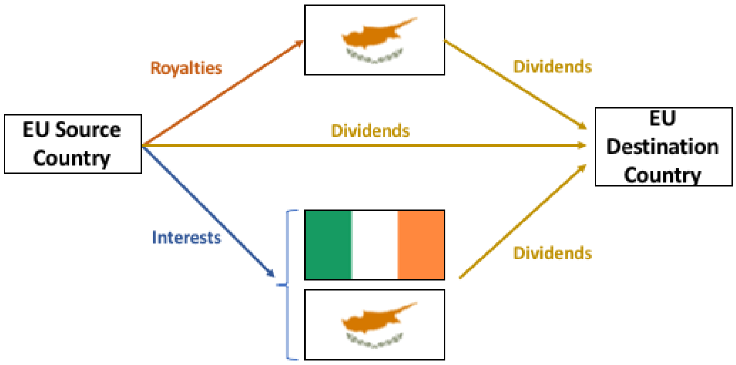

Figure 3.4: Optimal route within EU - without currency risk

The two measures of BEPS are given, for an EU source country S, by:

$$BEPS_{total} = \frac{\tau_S - 64\%\tau_S - 30\%10\% - 6\%2.5\%}{\tau_S} = \frac{36\%\tau_S - 30\%10\% - 6\%2.5\%}{\tau_S}$$

$$BEPS_{source} = 36\%$$

For example, if the source country is Belgium, we have $\tau_S = 33.99\%$ and therefore $BEPS_{total} = 26.73\%$.

3.2.2 Routes between EU and US

Remember that the United States fully tax the received dividends but grant a tax credit for the taxes paid upstream. Since the corporate income tax rate of the US is of at most 35%, as long as we do not exceed the global tax rate of 35% we can follow any path we want. The global effective tax rate of any solution of the optimization problem for such routes is 35%.

Although any path could be a priori chosen, following tax planning that does not reflect an economic reality at least generates a reputational risk for the company. Therefore, as long as the effective tax rate of direct path defined in section 2.8 remains smaller than 35%, it should be preferred. The majority of EU countries signed a tax treaty with the United States reducing or eliminating the withholding tax levied on dividends remitted to the United States. The direct path is optimal starting from all EU Member States except Italy and Spain. For Italy and Spain, table 3.3 summarizes the information about the direct path. Depending on the choice of the MNE to use or not the direct path, the $BEPS_{source}$ measure ranges from 0 to 36% while the $BEPS_{total}$ is of 0%.

Country	Corporate Income Tax Rate	Withholding tax on dividends towards the US	Effective tax rate
Italy	32%	5%	35.4%
Spain	28%	10 %	35.2%

Table 3.3: Source countries where the direct path is not optimal

For these two source countries, in order to reduce the effective tax rate, a strategy is to route 30% of the profit in the form of interests via an EU country with low corporate income tax rate while the remaining 70% follows the direct path. It is even not necessary here to use the Patent Box regime of Cyprus for 6% of the profit, the effective tax rate is sufficiently reduced using the interests. The solution is highly non unique here since interests can be channelled towards the US via several countries at the same global cost. A solution could be to route the interests via Cyprus.

3.2.3 Routes between EU and Bermuda

Since Bermuda has a corporate income tax rate of zero, there is no need to turn the profit into dividends before entering Bermuda. The challenge is here to escape EU at minimum cost no matter which vehicle is used. Most of EU countries levy a withholding tax on all the financial vehicles when the payment is remitted to Bermuda. However some EU Member States do not. EU Member States that do not levy withholding tax on some or all vehicles are listed in table 3.4.

Country	Interests	Dividends	Royalties
Netherlands	×		×
Luxembourg	×		×
Cyprus	×	×	
Hungary	×	×	×
Austria	×		
United Kingdom		×	
Malta	×	×	×
Sweden	×		×
Estonia	×		

Table 3.4: EU Member States **without** withholding tax towards Bermuda

As we can see in table 3.4, there are several categories of countries. First, Malta and Hungary do not levy any withholding tax on any of the three vehicles irrespective of the destination country. Therefore, for these 2 countries, the routing is simple. The profit is split into the three vehicles, 30% as interests, 6% as royalties and 64% as dividends. Both interests and royalties are deductible and reach Bermuda without being taxed. The dividends are taxed at the corporate income tax rate of either Malta or Hungary before reaching Bermuda. The routing is presented in figure 3.5 and the global effective tax rate of this strategy is equal to 64% of the corporate income tax rate of the source country, giving or respectively for Malta and Hungary.

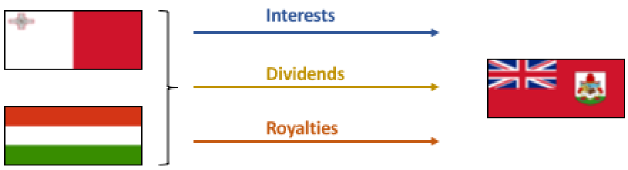

Figure 3.5: From Malta or Hungary to Bermuda

Secondly, there are countries that do not levy withholding tax on interests but on either dividends or royalties or both. For these countries, the routing of dividends and/or royalties must be adapted in order to avoid withholding tax. The idea is to make a detour of these two vehicles via an EU Member State that do not levy a withholding tax. An example is shown on figure 3.6 for a profit leaving Austria. The global effective tax rate of this strategy is again 64% of the corporate income tax rate of the source country, giving in the case of Austria.

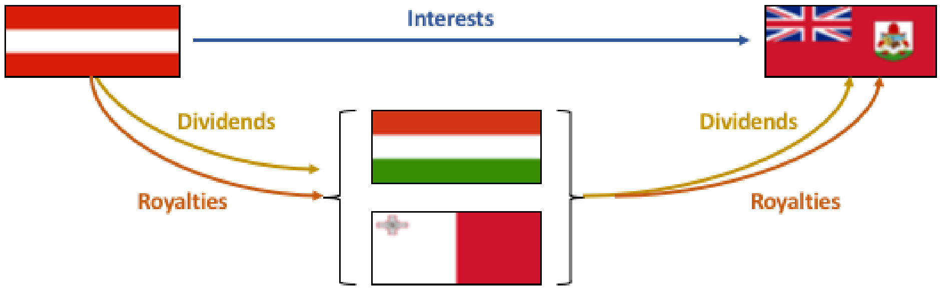

Figure 3.6: From Austria to Bermuda

Eventually, countries that do levy a withholding tax on interests require more sophisticated strategy to escape towards Bermuda. Indeed, the 30% of the profit allowed to leave as interests needs to be routed via another EU Member State to avoid the withholding tax. Nevertheless, when entering this new country, only 30% will then be allowed to leave as deductible interests, the remaining amount will be subject to the corporate income tax of this country. A way to avoid this problem is to follow the strategy presented on figure 3.7.

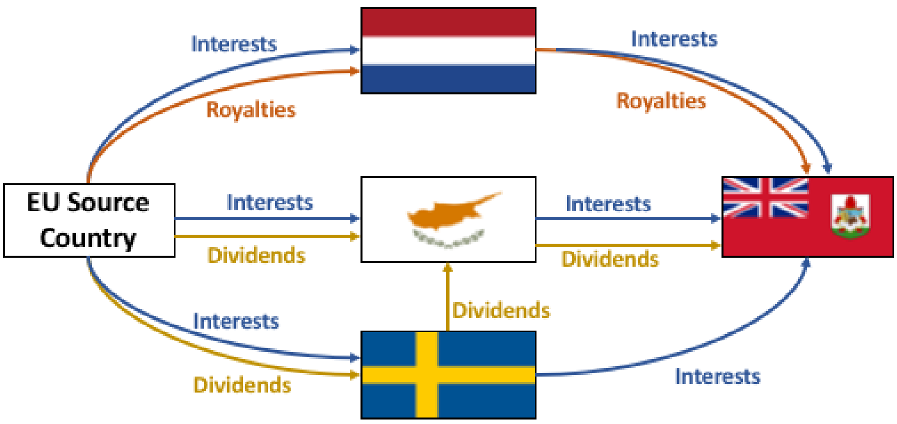

Figure 3.7: General optimal routing strategy

The royalties are channelled via the Netherlands where no withholding tax is levied and enters Bermuda untaxed. The interests are split in three parts and sent to the Netherlands, Cyprus and Sweden while the dividends are sent towards Cyprus and Sweden. A first part of the interests can be channelled via the Netherlands. The idea is to send, to the Netherlands, interests that will be allowed later to leave the Netherlands as deductible interests. The tax base in the Netherlands will be equal to the incoming interests plus the royalties. To be deductible, interests cannot exceed 30% of this tax base. If we denote by $I_{\text{Netherlands}}$ the maximum amount that can flow through the Netherlands in the form of deductible interests, we obtain:

$$\begin{aligned} 0.3(I_{\text{Netherlands}} + 0.06) &= I_{\text{Netherlands}} \\ I_{\text{Netherlands}} &= \frac{3}{7}0.06 = 2.57\% \end{aligned}$$

Therefore, 2.57% of the original profit can flow untaxed from the source country towards Bermuda via the Netherlands.

The remaining interests need to be channelled via another path. The idea is to send the right proportion of dividends and interests to Sweden such that the interest can keep flowing as deductible interests from there and reach Bermuda untaxed. To achieve it, we need to send dividends towards Sweden. However, Sweden levies a withholding tax on dividends. Therefore these dividends need to do an additional detour before going to Bermuda. For this purpose they are sent to Cyprus where no withholding tax is levied. The remaining dividends and interests are channelled via Cyprus. Of course the values of the different flows are given by the optimization problem. However, we can obtain an analytical solution depending only on the corporate income tax rate of the source country. If we denote by I_{Cyprus} and I_{Sweden} the amount of interests flowing through respectively Cyprus and Sweden and D_{Cyprus} and D_{Sweden} for the dividends, the following system of equation should be satisfied:

- The total sum of interest payments should be equal to 30% of the original profit:

$$I_{\text{Cyprus}} + I_{\text{Sweden}} + I_{\text{Netherlands}} = 0.3$$

- The interests flowing via Sweden should remain deductible in Sweden and therefore less or equal than 30% of the total profit in Sweden:

$$I_{\text{Sweden}} \leq 0.3(I_{\text{Sweden}} + D_{\text{Sweden}})$$

- The interests flowing via Cyprus should remain deductible in Cyprus and therefore less or equal than 30% of the total profit in Cyprus:

$$I_{\text{Cyprus}} \leq 0.3(I_{\text{Cyprus}} + D_{\text{Cyprus}} + D_{\text{Sweden}})$$

- All the profit minus the taxes should leave the source country:

$$D_{\text{Cyprus}} + D_{\text{Sweden}} = 0.64(1 - \tau_S)$$

Based on the fact that we can always find a solution of a linear optimization problem being a vertex of the admissible set, we can transform the inequalities into equalities and solve this linear system of equations to obtain:

$$I_{\text{Cyprus}} = \frac{48}{175}(1 - \tau_S) \quad (3.1)$$

$$I_{\text{Sweden}} = \frac{48}{175}\tau_S \quad (3.2)$$

$$D_{\text{Cyprus}} = \frac{16}{25}(1 - 2\tau_S) \quad (3.3)$$

$$D_{\text{Sweden}} = \frac{16}{25}\tau_S \quad (3.4)$$

By following this strategy, 36% of the profit reaches Bermuda untaxed while the remaining 64% are taxed at the corporate income tax rate of the source country but do not suffer from additional taxes.

The Netherlands are used because no withholding tax is levied neither on interests nor on royalties. By looking at table 3.4, The Netherlands can be replaced by either Luxembourg or Sweden without affecting the global tax liability. Cyprus is used because it does not levy any withholding tax neither on interests nor dividends and because it fully exempts received dividends from EU Member States. It can be exchanged with either Malta or Hungary at no cost. Finally, Sweden is used because it fully exempt received dividends from EU Member States and does not levy any withholding tax on interests' payment irrespective of the destination country. It can be replaced by Austria, Cyprus, Estonia, Hungary Luxembourg, Malta or The Netherlands. Nevertheless, two distinct countries must be used for replacing Cyprus and Sweden.

With this strategy, both measures of BEPS, $BEPS_{total}$ and $BEPS_{source}$ are equal to 36%.

3.2.4 Comments

Through this chapter we assumed that incoming dividends are part of the EBITDA. However, since incoming dividends form a tax exempt income, according to European Directive [16] published in 2016, they are not part of the EBITDA.

Consequently, strategies depicted in section 3.2 of this chapter, remains unchanged for routes between EU Member States and for routes between an EU Member States and the United States. Nevertheless, for routes between a EU Member States and Bermuda, the strategy to leave a country levying a withholding tax on interest payments (figure 3.7) is not valid anymore. According to the abovementioned Directive, dividends cannot be used to increase the EBITDA. The dividends will therefore directly flow towards Bermuda while interests will not be able to reach Bermuda untaxed. Interests will be considered as non-deductible at some point, increasing the global effective tax rate.

3.3 Modelling with Markov Chains and Decision Processes

The reader not familiar with Markov Chains (MC) or Markov Decision Processes (MDP) should first read appendix D.
The various strategies found in the previous section can be seen as Markov Chains. The fraction of the profit following each edge can be seen as a probability. Suppose we are in a country A, having a CIT rate of 20%, and we receive interests. We can send dividends to country B but in this case we will have the corporate income tax of A to pay. In this case, 20% of the profit will end up in the Taxes node while the remaining 80% will reach destination country B. If we look at the situation from another point of view, we could also say that by making that decision in country A, we have a 20% probability to end up in the Taxes node and a 80% probability to end up in the desired country.

Let us take an example to show how to transform the strategies into a Markov Chain. The states of the MC are simply the nodes of the graph. To fully define the MC, we need to define the probability of transition from each state to any other state. On figure 3.8, we can see a 2 transactions path with, on each edge, the amount flowing through that edge, as we could obtain by the linear optimization problem described in the previous section.

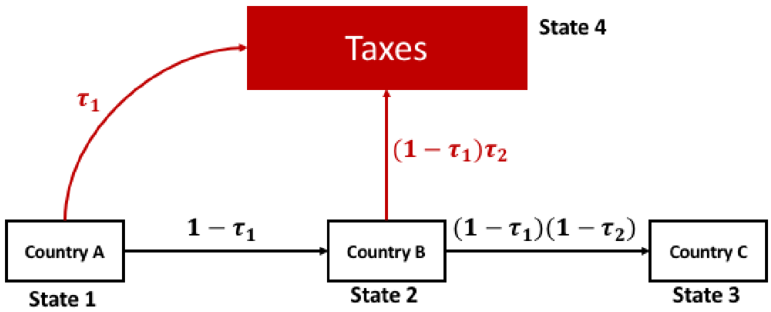

Figure 3.8: From flows to probabilities

Starting from country A, we pay a tax equal to τ_1 and reach country B. The probability of transition from state 1 to state 4 is τ_1, while the probability of transition from state 1 to state 2 is $1-\tau_1$. The probability of transition to the other states is zero. Once we reach state 2, the probability of transition from 2 to 4, is not $\tau_2(1-\tau_1)$ but rather τ_2. Indeed, we have:

$$\mathbb{P}[\text{reach state 4}|\text{we are in state 1}] = \tau_2(1-\tau_1) \tag{3.5}$$

$$\mathbb{P}[\text{reach state 4}|\text{we are in state 2}] = \tau_2 \tag{3.6}$$

The probability of transition from state 2 has to be defined as the conditional probability of transition knowing that we are in state 2, like in equation 3.6. The probability of transition is equal to the tax rate levied on the transaction and not to the amount of taxes paid. By doing so, for each node, we obtain, for this example the following transition matrix:

$$T = \begin{pmatrix} 0 & 1-\tau_1 & 0 & \tau_1 \\ 0 & 0 & 1-\tau_2 & \tau_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We can notice that the MC defined in this way is an absorbing MC, meaning that, at some point, we reach a state from which we cannot leave. It is the case for all the strategies described in this chapter. Indeed, the profit ends up somewhere, either in the taxes node or in the destination country. An interesting fact about absorbing MC, is that we can compute absorbing probabilities. This is, for each absorbing node, the probability that we end up in that node. In our case, the absorbing probabilities, given that we started in state 1, are:

$$B = \begin{pmatrix} (1-\tau_1)(1-\tau_2) & \text{(State 3)} \\ \tau_1 + (1-\tau_1)\tau_2 & \text{(State 4)} \end{pmatrix}$$

The global tax liability of a strategy is equal to the absorbing probability of the Taxes node in the Markov Chain described above.

Knowing that the optimal strategies are Markov Chains, we could imagine that they are the optimal policy of a more general Markov Decision Process. In a MDP, we have a decision to take at each state. In our case, the decision could be the financial vehicle of the transaction but also the destination country. However, once we select a decision, the transition probabilities (or the tax rate of the transaction) are not known. To know them, we need to look upstream on the path to see in which vehicle we entered in the current country. The current state does not give the full information required to predict further movements, the history of the path is also relevant. This statement is in contradiction with the Markov property or memoryless property. Consequently, the problem of finding the optimal tax strategy cannot be modelled in the Markov Decision Processes framework in an easy way. In order to remove the dependence on the history of the path, we could imagine a model with a continuum of states and actions. However, this model is discarded as it significantly increases the complexity of the model compared to a linear optimization problem.

3.4 Sensitivity Analysis

In this section, we analyse the impact of a variation of the anti-avoidance rules parameters: κ and λ. They are respectively the limitation on the deductibility of interests in all countries and the limit on the royalties' payment from the source country.

To this aim, we use the complementary slackness theorem (see [29] for more details):

Theorem 1 (Complementary Slackness)**.** Assume the primal problem (P) has a solution x^* and its dual problem (D) a solution y^*, we have the following:

1. If $x_i^* > 0$ then the ith constraint in (D) is tight.

2. If the ith constraint in (D) is not tight, then $x_i^* = 0$.

3. If $y_j^* > 0$, then the jth constraint in (P) is tight.

4. If the jth constraint in (P) is not tight, then $y_j^* = 0$

When considering a linear program, at the optimal solution either a constraint is tight or the corresponding dual variable is zero, or both. For this section we consider the routing of a profit between two EU Member States although results are the same for other routes. On the one hand, figure 3.9 represents the effective tax rate of the optimal strategy for routing a profit from Belgium to Ireland as a function of the percentage of profit allowed to leave as deductible interests (parameter κ). On the other hand, figure 3.10 represents the effective tax rate of the optimal strategy for the same routing but as function of the percentage of profit allowed to leave as deductible royalties (parameter λ).

First, let us look at the impact of an increase of κ when λ is fixed at 6%. As long as κ remains smaller than 94%, the basis (i.e. the non-zero flows at optimum) stays the same. The profit follows 3 different routes, one for each vehicle as described in section 3.2. On figure 3.9, we see that the effective tax rate decreases linearly with a slope of -0.23. The slope is actually given by the dual variable corresponding to the limit of deductibility of interests in the source country (constraint 6).

Once κ reaches 94%, combined with λ fixed at 6%, the whole profit can leave in the form of a deductible vehicle, meaning that the tax base in the source country is reduced to zero. Therefore, nothing is channelled in the form of dividends and the basis changes. Moreover, we can see that the effective tax rate remains constant. It is due to the fact that the routing of royalties suffers a tax rate of 2.5% while this is 10% for interests. The constraint limiting the deductibility of interests isn't tight anymore, the corresponding dual variable is then zero as a consequence of the complementarity slackness theorem. It explains why the effective tax rate remains constant and equal to 9.55% .

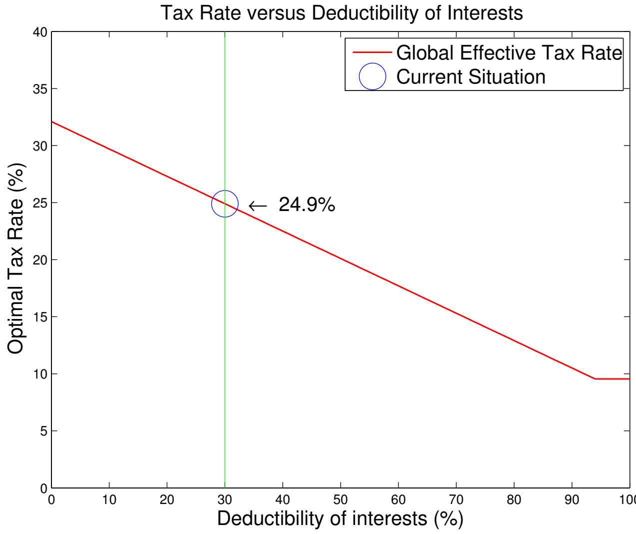

Figure 3.9: Sensitivity to the deductibility of interests

Secondly, let us look at what happen when λ increases for κ fixed at 30%. For λ between 0 and 69%, the fraction of the profit allowed to leave in a deductible form is less than 1, meaning that the dividend vehicle is used. Again, the slope for this range of λ is given by the value of the dual variable corresponding to the deductibility of royalties' constraint.

Once λ reaches 70%, the whole profit is allowed to leave in a deductible vehicle, the basis is then changed leading to another slope. Since the route for royalties is cheaper than the route for interests, further increase of λ reduces the effective tax rate and the constraint is still tight. The slope is however reduced (in absolute value) since the route of interests is not as bad as the route of dividends. Finally, when λ reaches 100%, we are back in the unconstraint case of chapter 2, the whole profit is channelled in the form of royalties via Cyprus leading to an effective tax rate of 2.5%.

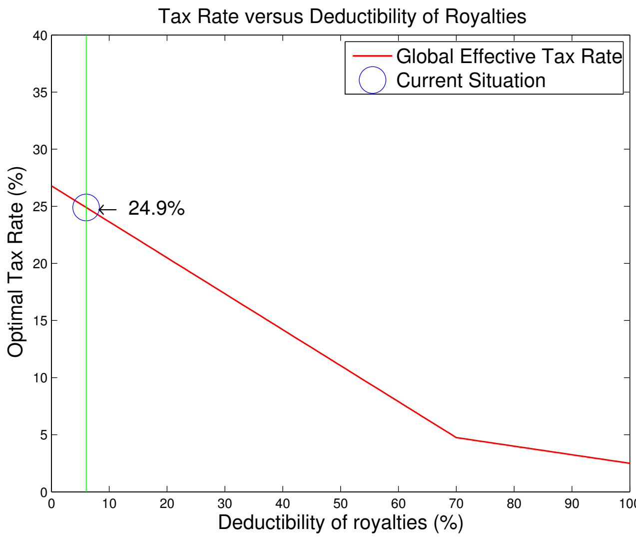

Figure 3.10: Sensitivity to the deductibility of royalties

3.5 EBITDA calculation and transparency

So far in this chapter, we discarded strategies that artificially increase the profit in the source country in order to increase the deductibility of interests. These strategies are judged too obviously aggressive because of the exchange of information between states and the increasing transparency requirements coming from the BEPS action plan.

In this section, we assume that, although incoming dividends are tax exempt income, they are part of the EBITDA. Therefore, we do not comply with the European Directive [14]. On top of it, we assume zero transparency. We mean that states can see what is entering and what is leaving their country but cannot have a look at what happened upstream.

The goal is to demonstrate why it is important to exclude tax exempt income from the EBITDA but also the need for transparency. We describe two strategies that are using cycles to artificially inflate the profit in the source country in order to reduce the tax base to zero in this country.

Strategy 1

We are looking for the least taxed path between a source and a destination country when the source country limits the deductibility of interests to 30% of the profit.
The idea is to channel the profit in the form of interests to the country offering the lowest corporate income tax rate, turn it into dividends and then reach the destination country. Due to the limitation on the deductibility of interests, only 30% of the profit can follow this path at first sight. However, the dividends leaving the intermediate country may enter again the source country, allowing another part of the profit to leave as interests. Each time we turn interests into dividends, we pay the corporate income tax of the intermediate country. Suppose we have 1 $ profit in the source country, following N times this cycle enable us to channel:

$$\begin{aligned} x_{interests} &= \kappa + \kappa^2(1-CIT) + \kappa^3(1-CIT)^2 + ... + \kappa^N(1-CIT)^{N-1} \\ &= \sum_{i=1}^{N} \kappa^i(1-CIT)^{i-1} = \kappa \sum_{i=0}^{N-1} \kappa^i(1-CIT)^i = \kappa \frac{1-\kappa^N(1-CIT)^N}{1-\kappa(1-CIT)} \end{aligned}$$

where κ is the limitation of the deductibility of interests and CIT is the lowest corporate income tax rate.
When N tends to infinity, we obtain $x_{interests} = \frac{\kappa}{1-\kappa(1-CIT)}$. In our case, since $\kappa = 30\%$ and CIT = 10%, applying the formula gives $x_{interests} = 0.41$ and a part of the profit is stuck in the source country. Let's see in which case we won't be able to channel all the profit using this path:

$$\begin{aligned} \frac{\kappa}{1-\kappa(1-CIT)} &< 1 \\ \implies \kappa(2-CIT) &< 1 \end{aligned}$$

For CIT fixed at its actual value of 10%, as long as κ remains smaller than 52.6%, the profit cannot be evacuated using this strategy. For κ fixed at 30%, a part of the profit remains in the source country irrespective of CIT.

Therefore, this first strategy is not suitable to channel the whole profit and we need to develop a more complex strategy.

Strategy 2

We now introduce another strategy in two phases as we can see on figures 3.11 and 3.12. The goal is to artificially inflate the profit at source until the whole original profit is allowed to leave as interests. The total tax base needed in the source country, T, is given by:

$$T = \frac{1}{\kappa}$$

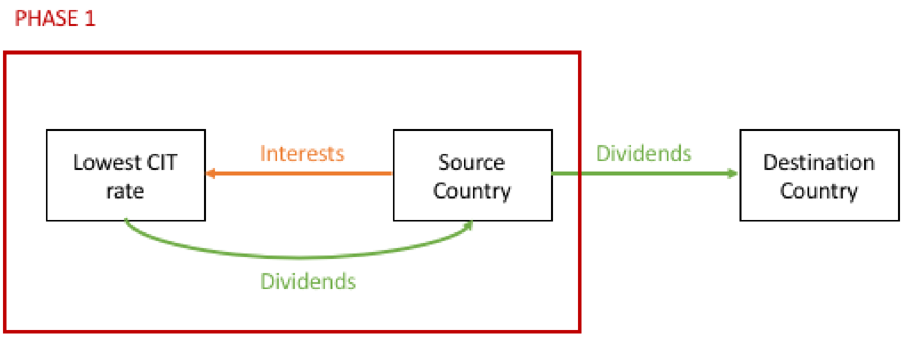

Figure 3.11: Strategy 2: Phase 1

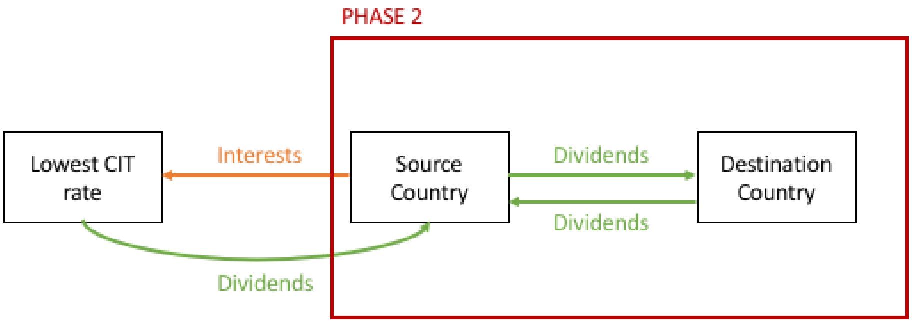

Figure 3.12: Strategy 2: Phase 2

During the first phase, we send a percentage I of our profit in the form of interests towards the intermediate country where we pay a tax equal to CIT. Therefore, κ(1-CIT) are coming back in the source country. This is a first inflation of the profit.

During the second phase, the received dividends from the intermediate country are sent to the destination country. The destination country could keep these dividends, but we suppose it sends them back to the source country, inflating once again the profit at source. These dividends can leave again from source to destination repeating this cycle. The goal is to repeat this second phase until all the original profit is allowed to exit the source country in the form of interests.

Each time we go through the source country, $d_1\%$ (typically 0 or 5%) of the dividends are taxed at S, the corporate income tax rate in the source country. We assume that dividends are fully exempt in the destination country. If not, we can use another intermediate country for the exchange of dividends. For N exchanges, the profit increases to:

$$\begin{aligned}
\text{Total Profit} &= \underbrace{1}_{\text{Intitial profit}} + \overbrace{\kappa(1-CIT)}^{\text{Phase 1}} \\
&+ \underbrace{\kappa(1-CIT)(1-d_1S) + \kappa(1-CIT)(1-d_1S)^2 + ... + \kappa(1-CIT)(1-d_1S)^N}_{\text{Phase 2}} \\
&= 1 + \kappa(1-CIT) + \sum_{i=1}^{N} \kappa(1-CIT)(1-d_1S)^i \\
&= 1 + \sum_{i=0}^{N} \kappa(1-CIT)(1-d_1S)^i \\
&= 1 + \kappa(1-CIT)\frac{1-(1-d_1S)^{N+1}}{d_1S}
\end{aligned}$$

We can now compute the number N of exchanges needed to obtain a total profit equal to T. We proceed to $\lfloor N \rfloor$ exchanges. The last exchange will be slightly different. The destination country will send back to the source country only the amount needed to obtain the total profit T and keep what remains.

Total tax paid with this strategy:

$$\begin{aligned}
\text{Total Tax} &= CIT + \sum_{i=1}^{\lfloor N \rfloor} \kappa(1-CIT)(1-d_1S)^{i-1} d_1S + \mu d_1S \\
&= CIT + \kappa(1-CIT)d_1S \sum_{i=0}^{\lfloor N \rfloor -1} (1-d_1S)^i + \mu d_1S \\
&= CIT + \kappa(1-CIT)d_1S \frac{1-(1-d_1S)^{\lfloor N \rfloor}}{d_1S} + \mu d_1S
\end{aligned}$$

where μ is the amount needed for the very last exchange:

$$\begin{aligned}
\mu &= T - 1 - \kappa(1-CIT) \sum_{i=0}^{\lfloor N \rfloor -1} (1-d_1S)^i \\
&= T - 1 - \kappa(1-CIT) \frac{1-(1-d_1S)^{\lfloor N \rfloor}}{d_1S}
\end{aligned}$$

This strategy uses 3 ingredients: the possibility to deduct interests' payment, a low corporate income tax rate for channelling the interests and the exemption of 95% of the incoming dividends in the source country. To prevent a MNE from using this strategy, from the source country point of view, the solutions are to decrease its corporate income tax rate, to reduce the percentage of exemption for dividends or to reduce the deductibility of interests. To obtain the following graphs, we use Belgium as the source country.

Decrease corporate income tax rate at source As we can see on figure 3.13, strategy 2 stays better than the fair route as long as S stays above 11%. Basically, all countries should reduce their corporate income tax rate to the lowest rate offered in Europe. This is definitely not a suitable solution. Having such strategies available would definitely encourage the race to the bottom on corporate tax.

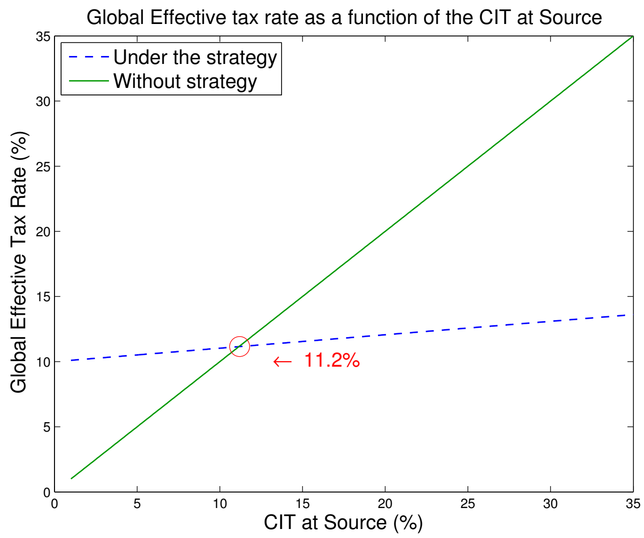

Figure 3.13: Global Tax as a function of Corporate Income Tax rate at source

Reduce the percentage of dividends exempt Currently, 0 or 5% of received dividends are taxed. We can see on figure 3.14 that the source country should raise this percentage up to 24% to prevent companies from using this strategy. Since the low tax on dividends has been established to avoid double taxation, increasing it again will lead to substantial double taxation which can have adverse effects on global trade.

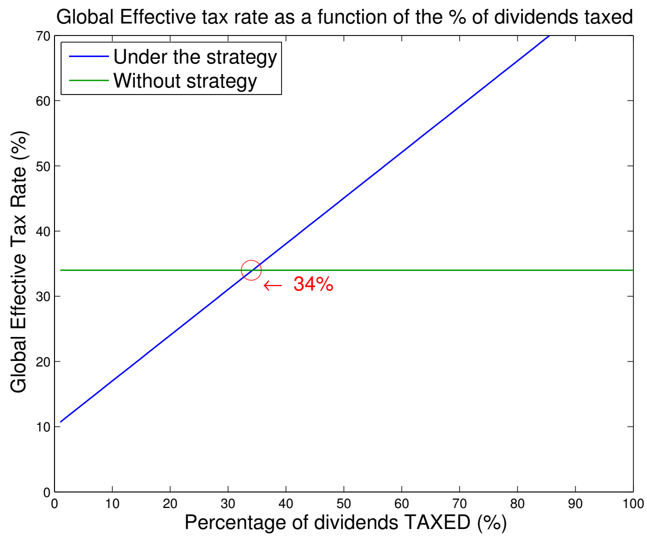

Figure 3.14: Global Tax as a function of the percentage of dividends taxed at source

Reduce the deductibility of interests We can see on figure 3.15 that the deductibility of interests should be limited to 9% of the profit in a country having a CIT rate of 20%. Knowing that a limitation to 30% is already one of the strictest rule in Europe, reducing it to 9% seems not suitable.

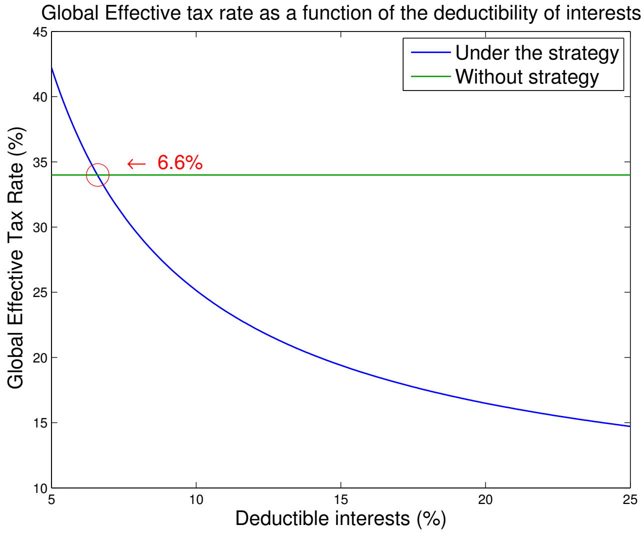

Figure 3.15: Global Tax as a function of deductibility of interests

The three actions would lead the source country (Belgium in the example) to reduce its corporate income tax rate to 11%, increase the percentage of dividends taxed to 34% or decrease the deductibility of interests to 6% of the EBITDA. As explained above, this is not suitable for the source country. We showed that the thin-capitalization rule limiting the deductibility of interests to a fixed percentage of the EBITDA can be abused if tax exempt income (like dividends) is taken into account in the EBITDA. Even if incoming dividends are included in the EBITDA, strategies depicted here could be strike down if transparency is required. Indeed, such a planning would be too obviously aggressive.

3.6 Conclusion

In all optimal strategies presented in this chapter, 64% of the profit is taxed at the corporate income tax rate in the source country, where the value is created. It shows that a coordinated scenario, in which all the EU Member States limit the deductibility of interests to 30% of the EBIDTA, ensure that the BEPS cannot be more than 36%. The global effective tax rates for routing a profit between selected pairs of countries are presented in table 3.5, the full results table can be found in appendix B. The global effective tax rate is on average reduced by 25.5%.

Source / Destination	Belgium	Netherlands	Luxembourg	United States	Bermuda
Belgium	-	33.99% / 24.9%	33.99% / 24.9%	35% / 35%	50.49% / 21.8%
Netherlands	26.27% / 20.5%	-	25% / 19.2%	35% / 35%	36.25% / 16%
Luxembourg	30.42% / 23.2%	29.22% / 21.9%	-	35% / 35%	39.84% / 18.7%

Table 3.5: Direct Path / Optimized Path global effective tax rates

To reuse the notation of chapter 2, we have, for a profit generated in any of the EU Member States:

$$BEPS_{source} = 36\%$$

Compared to the case of chapter 2 where we had a $BEPS_{source}$ of 100%, this is a significant improvement.

We also showed that, although thin-capitalization rule limiting the deductibility of interests to a maximum percentage of the EBITDA are effective to reduce BEPS, they can be abused if tax exempt income is included in the EBITDA and if there is no transparency requirement.

Chapter 4

The best storage place for profit

In chapters 2 and 3, we focused on how to route a profit between two given countries. How should a MNE route its profit from its subsidiary to the parent company? The goal was double: reaching a given country and minimizing the tax liability. But what if the MNE doesn't care about the destination country but pursue the only goal of minimizing its global tax liability? These strategies achieve the least possible tax. The profit is allowed to remain in any foreign country, even in several foreign countries. This is similar to the Apple or the Google case where the profit is channelled towards Bermuda and remains there.

4.1 Model

The model of chapter 3 needs to be adapted as we do not require the profit to enter a specific node at the end. To allow the profit to remain in any country, we introduce new nodes in the graph. Each country will have a reserve node corresponding to a profit remaining in that country, as shown on figure 4.1. The new reserve node is connected to the dividend payment node of the corresponding country. Indeed, if a given amount stays in a country, the corporate income tax has to be paid (according to the incoming vehicle) and the fiscal properties are then in our situation similar to a dividend payment.

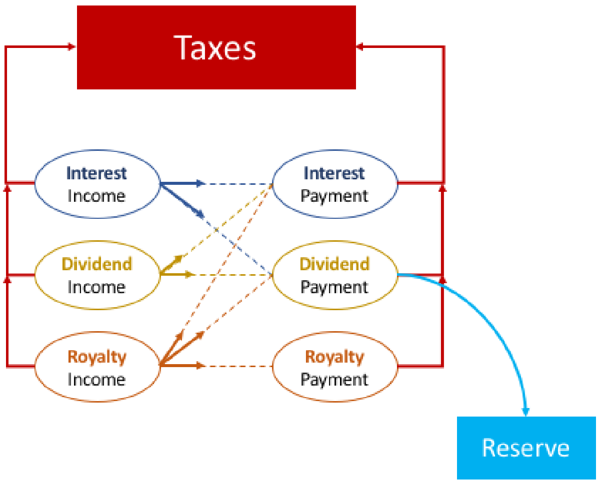

Figure 4.1: New representation of a country

Even if the objective function to be minimized, namely the taxes paid remains the same, the optimization problem described in chapter 3 needs to be adapted. Firstly, since new nodes and edges have been added to the graph, new variables need to be created. One variable per country is added corresponding to the flow going through the reserve edge. Secondly, the flow conservation constraints relative to dividends payment nodes has to take into account the profit flowing to the reserve node. A flow entering a reserve node will remain there. The only constraint relative to the reserve variables is their non negativity. Thirdly, as the profit doesn't need to enter a specific destination country, we remove constraint 4.

As in chapter 3, this optimization problem is linear and can be solved using the Simplex method. For this purpose, the Gurobi solver is used once again within Python.

4.2 Results

4.2.1 Without anti-avoidance rule

If no thin-capitalization rule is enforced in any State, we simply set κ and λ parameters equal to 1. Intuitively, when we want a profit to remain in a given country, this profit is subject to the corporate income tax of this country depending on the incoming vehicle. It means that at some point on the path, the profit will need to be turned into dividends and pay the corresponding tax. Consequently, we look for the jurisdiction offering the lowest income tax rate which is, in our dataset, Bermuda (0%). The result obtained by solving the optimization problem described above is consistent with this intuition, as we can see on figure 4.2.

US corporations could use such a strategy to route the profit generated by its EU subsidiaries towards Bermuda. Sending directly this profit to the US parent company would cost the US corporate income tax (35%). Another approach is to store this profit in Bermuda until the US government decide to open a reduced tax time period. According to [31], President Donald Trump plans to offer a reduced rate of 10% on repatriated dividends from offshore jurisdictions. With such a reduced repatriation rate, US corporations could be able to reduce their effective tax rate on their EU profit to 10%. The situation is mitigated by thin-capitalization rules.

Figure 4.2: Best Strategy to escape Belgium - Without thin-capitalization rule

4.2.2 With anti-avoidance rule

When anti-avoidance rules are enforced, we cannot achieve a 0% tax rate. Nevertheless, the idea is that what is allowed to escape the source country in a deductible form should do so. As we already saw in section 3.2 when describing optimal routing of a profit from EU to Bermuda, the 36% of the profit allowed to leave in a deductible vehicle (interests and royalties) are able to reach Bermuda untaxed. If we decide to stop this part of the profit in an intermediate EU country on the path, it will be subject to the corporate income tax rate of this country depending on the incoming vehicle. Therefore, the optimal solution is to let this profit flow towards Bermuda.

Due to thin-capitalization rules, a part of the profit (64%) cannot escape from the corporate income tax rate in the source country. As we also saw in section 3.2, this part of the profit minus the taxes paid in the source country can reach Bermuda without any additional tax. However, even if leaving the EU is simple in that case, if for any reason this profit has to be sent back in the EU once in Bermuda, it will be much more complicated. The best option is therefore to keep this part of the profit in the source country (or in an EU Member State that fully exempt incoming dividends). A schema representing this optimal strategy is presented in figure 4.3. In this schema, percentages are expressed in terms of the original profit generated in Belgium.

Back to the case of a US corporation, according to the announced repatriation tax rate by President Donald Trump, the effective tax rate on their EU profit would be equal to 64% of the corporate income tax rate of the source country plus 10% on the remaining 36%. For a profit generated in Belgium, the effective tax rate would be 25% rather than the 35% using the direct path.

Again, the impact of anti-avoidance rules is substantial.

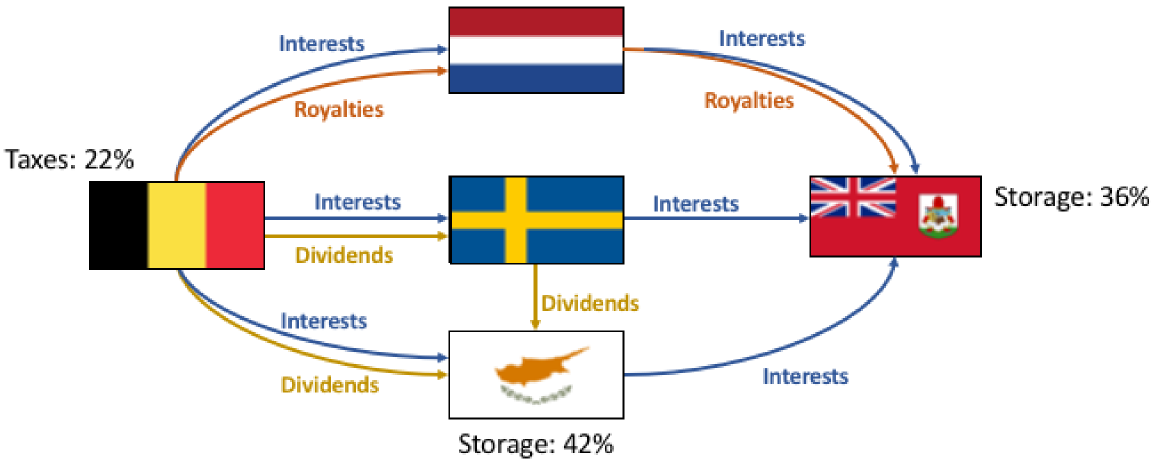

Figure 4.3: Best Strategy to escape Belgium - With thin-capitalization rule

Based on these results, we can compute the effective tax rate applied on a profit realised in each EU Member State: $64\%\tau_S$ where S is the considered source country. Consequently, for each EU Member State, when a profit is generated on its territory, the BEPS at source is 36%.

We can directly see that any EU Member State allowing a higher percentage of interests to be deductible will increase its BEPS at source.

Chapter 5

Interactive Interface

The aim of this chapter is to briefly present the architecture of the Python code used to obtain the results of chapters 2 to 4 and to show a preview of the `HTML/JavaScript` interface. The code is available at `https://bitbucket.org/csautelet/masterthesis/overview`.

5.1 Python Code

The code developed in this work is mainly composed of 3 different classes:

- **TaxNetwork**: class that, based on data files, build the adjacency matrix of the graph by computing the cost of each possible transaction. This class does not depict any strategic behaviour and is used to compute the taxes paid in direct path.

- **StrategicUncapacitated**: model the strategic behaviour when no anti-avoidance rules are enforced. This class is used to compute the shortest path between all pairs of countries using Dijkstra algorithm and compute the associated BEPS measure: $BEPS_{total}$ and $BEPS_{source}$.

- **StrategicCapacitated**: model strategies complying with anti-avoidance rules. The least taxed flow is computed by solving linear optimization problems. It uses the Gurobi Solver [3].

Both `StategicUncapacitated` and `StrategicCapacitated` inherit from `TaxNetwork`. All the graph visualization work has been done using the `Networkx` [4] which is a Python language software package providing data structures for graphs.

5.2 Local Python Server

In order to enable communication between the Python code and the interactive interface, we build a local Python server. To this aim, we use the `Flask` module [2] in Python which is a web framework or micro-framework.

We start by creating instances of the classes described above so that data structures are build and the response to request coming from the interface can be faster. At launch, on the one hand, we compute the shortest path between all pairs of countries when no anti-avoidance rules are enforced. When the interface requests such a path, the answer is therefore immediate. On the other hand, we do not compute all the shortest paths complying with anti-avoidance rules.

The reason is that the launch would take too much time and the memory required to store the results of all these optimization problems would be huge. To convince ourselves, the number of variables for each optimization problem is, for 30 countries, around 3000 and $30 \times 29 = 870$ such paths should be stored. Therefore, when the user will ask for a strategy complying with anti-avoidance rules, there will be a small delay to get the answer, the server will have to solve the corresponding optimization problem.

When the application is launched, the local Python server waits for a request from the interface.

5.3 Interface

The interface is coded in `HTML`. It uses `JavaScript` for the animation and `CSS` for the design aspects. The graphs are drawn using the `D3.js JavaScript` library which has been designed to manipulate documents based on data [1].

The interface is first composed of a control panel, shown on figure 5.1, allowing the user to select the source and destination country as well as the conditions under which the shortest path must be build. The user can choose to use or Patent Box regimes and whether he wants to reach the destination country or just escape from taxation. On top of it, the user can choose if the strategy should comply with anti-avoidance rules. By anti-avoidance rules, we consider here the coordinated scenario in which all EU Member States limit the deductibility of interests to 30% of the EBITDA and where only 6% of the profit can escape the source country in the form of royalties.

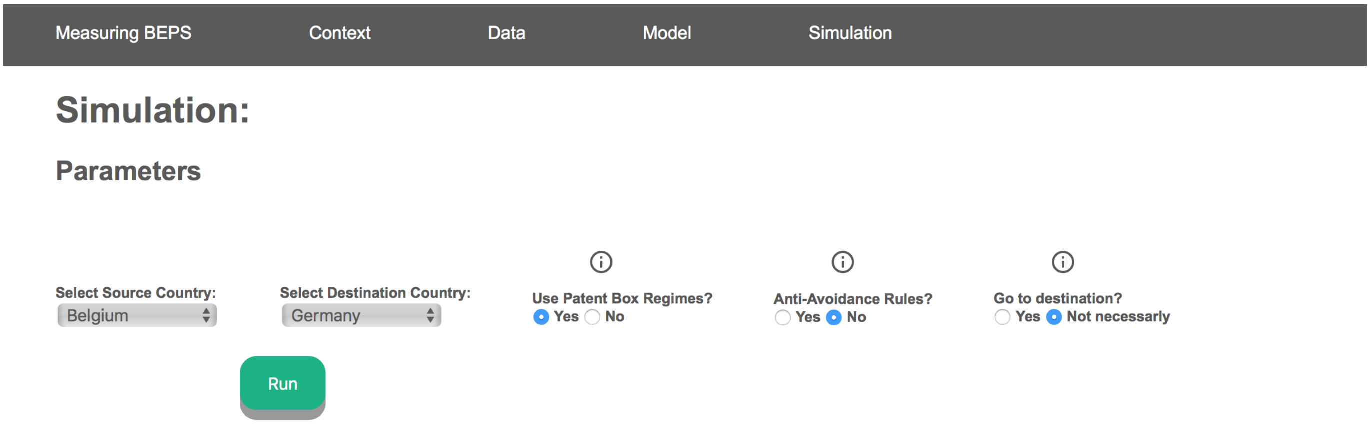

Figure 5.1: Control Panel

Secondly, once the user has selected the parameters, the JavaScript code of the interface performs an asynchronous HTTP (Ajax) request to communicate with the local python server. It basically sends to Python the parameters selected by the users and wait for Python to send back the corresponding result.

The communication between the Python server and the Interface is done by means of string (sequence of characters) formatted in a `JavaScript Object Notation` (JSON) which is a standard file format in computer science. Eventually, the results are displayed.

Let us take an example. Imagine we are interested in the least taxed path between Belgium and Germany. Firstly, we do not care about anti-avoidance rules and we want to obtain a lower bound on the taxes we must pay. To do so, we select in the control panel Belgium as the source country and Germany as the destination country, as we can see on figure 5.2. We also select the desired parameters concerning the anti-avoidance rules as shown on figure 5.2.

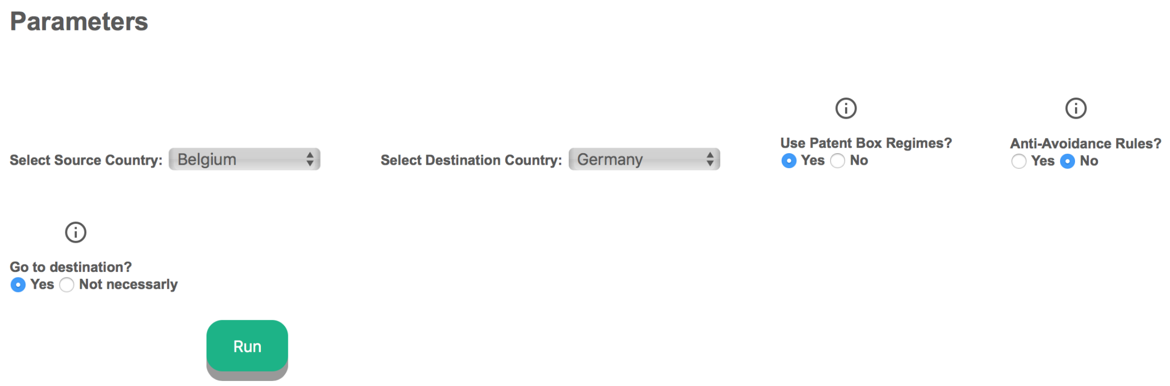

Figure 5.2: Control Panel - Without anti-avoidance rules

Once the parameters are selected, we press `Run` and the results are displayed. On figure 5.3, we can see the optimal strategy to follow. On figure 5.4, we can see the taxes paid in each country of the path, on the left when following the direct path and on the right when following the strategic path.

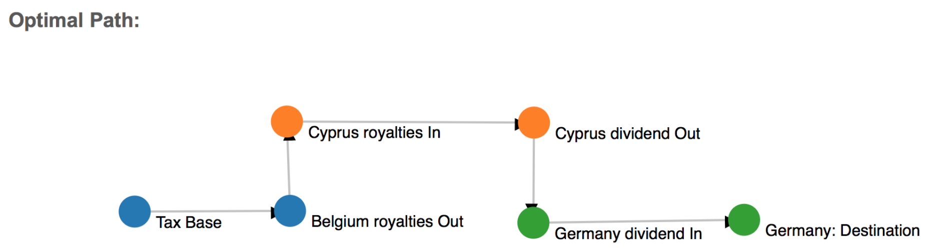

Figure 5.3: Optimal Strategy - Without anti-avoidance rules

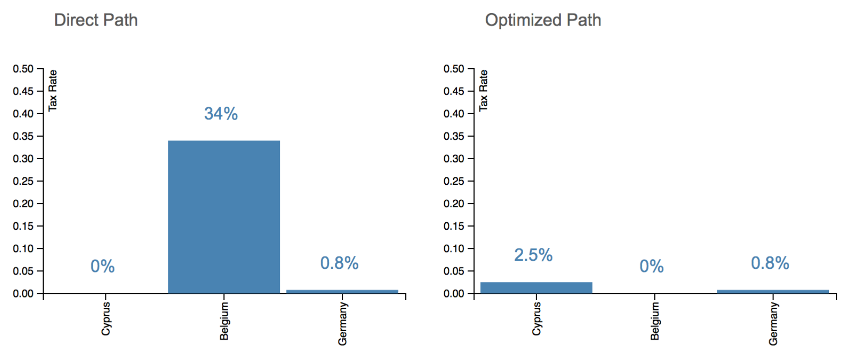

Figure 5.4: Distribution of the tax liability - Without anti-avoidance rules

Secondly, suppose we want to find a strategy complying with anti-avoidance rules (as described in chapter 3). We must select the parameters accordingly, as shown on figure 5.5. When pressing again on `Run`, previous results are removed and the optimal strategy complying with anti-avoidance rules is displayed, as we can see on figure 5.6. The distribution of the tax liability is displayed on figure 5.7. On the left, it corresponds to the direct path and, on the right, to the strategic path.

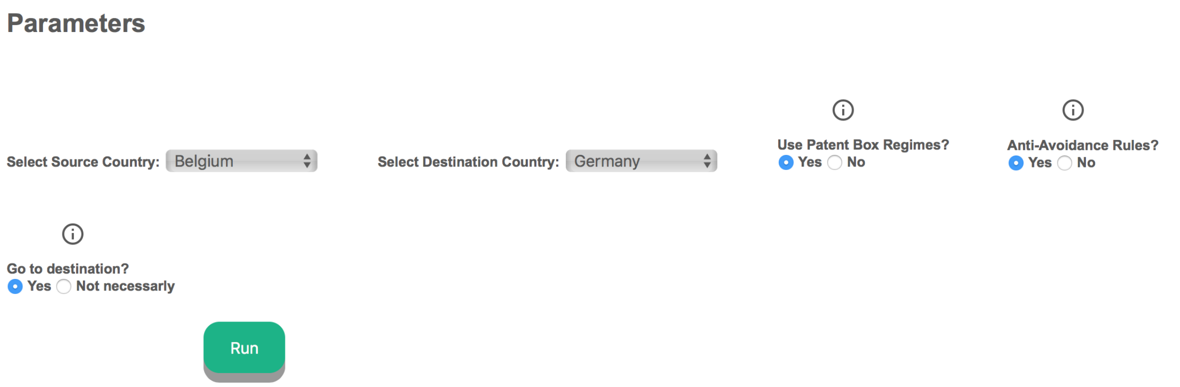

Figure 5.5: Control Panel - With anti-avoidance rules

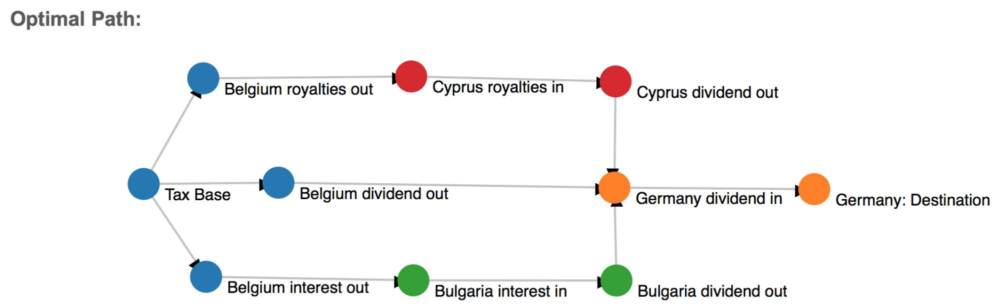

Figure 5.6: Optimal Strategy - With anti-avoidance rules

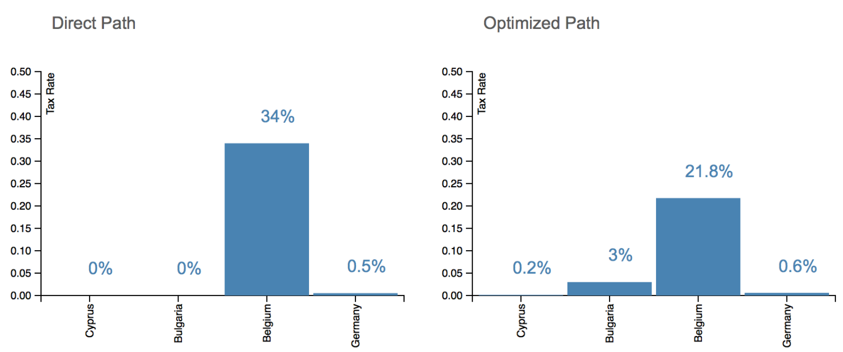

Figure 5.7: Distribution of the tax liability - With anti-avoidance rules

Eventually, suppose our only goal is to reduce our tax liability no matter the destination country reached. In this case, again, the corresponding parameters must be selected on the control panel as we can see on figure 5.8. Pressing `Run` displays the new optimal strategy as we can see on figure 5.9. This request is different from the other in the sense that we do not know in advance where the profit will complete its journey. Consequently, another graph is displayed, showing where the profit is stored, as we can see on figure 5.10.

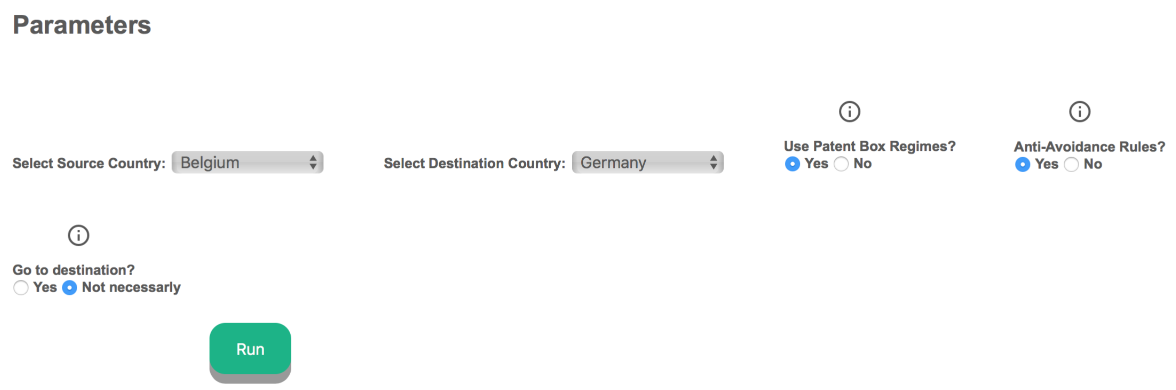

Figure 5.8: Control Panel - Best Storage Country

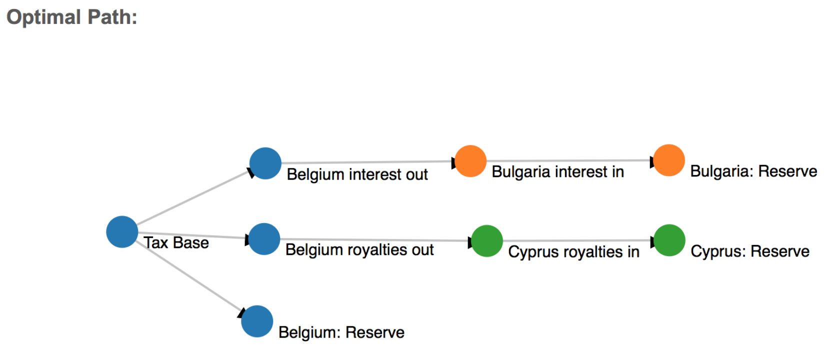

Figure 5.9: Optimal Strategy - Where to store the profit?

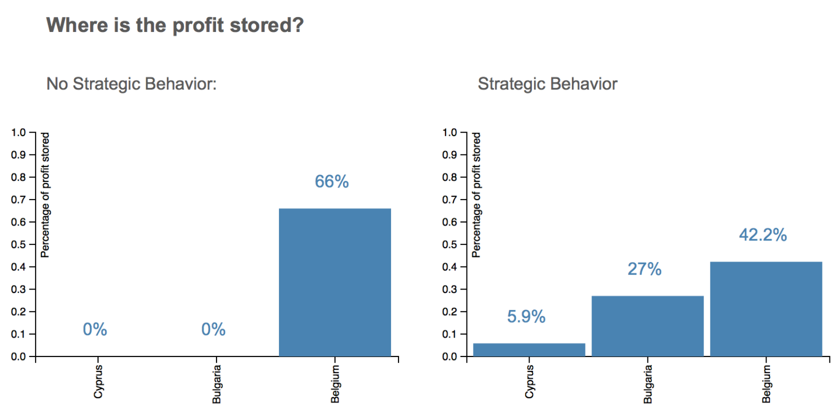

Figure 5.10: Profit Location

Conclusion

Let us recall the original goal of this master thesis: create a mathematical model of BEPS allowing to measure it.

In chapter 2, we designed a graph model of the tax network and used Dijkstra algorithm to discover the least taxed path between any pair of countries, allowing to automatically detect optimal tax strategies. We showed that, without anti-avoidance rules, the profit can fully escape the source country leading to a 100% BEPS at source. For routes within the EU, the discovered strategy does not even leave the EU: it highlights royalties routing via Cyprus.

In chapter 3, we extended the model to comply with selected anti-avoidance rules. We considered a coordinated thin-capitalization rule where all the countries limit the deductibility of interests to 30% of the EBITDA. On top of it, royalties' payment from the source country is limited to 6%.

Firstly, we showed that these selected anti-avoidance rules ensure that 64% of the profit is taxed in the country where it is generated, immediately setting an upper bound of 36% for the BEPS at source. We also showed that this kind of thin-capitalization rule can be abused when there is no transparency requirement and when tax exempt income is part of the EBITDA.
Secondly, we highlighted two countries for routes within EU: Cyprus and Bulgaria. The Patent Box regime offered by Cyprus allows 6% of the profit flowing in the form of royalties to only suffer a tax rate of 2.5%. Bulgaria, with the lowest CIT rate in the EU, is attractive for turning into dividends the 30% of the profit flowing in the form of interests. However, because Bulgaria is not part of the euro zone, MNEs can choose to replace Bulgaria by Cyprus or Ireland offering a slightly higher corporate income tax rate. The selected country collects taxes on a profit that has not been generated in its territory. We highlighted two ways for a country to become an advantageous intermediate country: offering an interesting Patent Box regime or having a low CIT rate. It can be one source of tax competition between states. Although it is common to think about tax haven when talking about tax avoidance, we presented strategies that reduce the overall tax liability, again, without even leaving the EU.

In chapter 4, we provided a modified model to discover strategies to avoid taxation without trying to reach a predefined country. We showed that even when anti-avoidance rules are enforced, 36% of the profit can escape the source country and, more generally, the EU without being taxed anywhere. This part of the profit can then reach any tax haven and remain untaxed there.

Although the results presented in this thesis are obtained based on 2015 data, the whole code can easily be run on updated data and could be used to see the impact of new possible future tax rates.

Potential future work may try to model the other major tool used in tax avoidance strategies: transfer price. The tax competition implied by the found tax avoidance strategies could also be analysed.

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Glossary

BEPS	Base Erosion and Profit Shifting.
CIT	Corporate Income Tax.
EBITDA	Earnings Before Interests, Taxes, Depreciation and Amortization.
EU	European Union.
IP	Intellectual Property.
MNE	Multinational Enterprise.
NGO	Non-Governmental Organization.
PE	Permanent Establishment.
R&D	Research and Development.
WHT	Withholding Tax.

Appendix A

Data

A.1 Countries' profile

Table A.1: Countries' profile

Country	CIT	Interests	Dividends	Royalties	EU Member State
Belgium	0.3399	1	0.05	0.2	1
Netherlands	0.25	1	0	0.2	1
Ireland	0.125	1	0	1	1
Luxembourg	0.2922	1	0	0.2	1
France	0.3333	1	0.05	0.4356	1
United Kingdom	0.2	1	0	0.5	1
Germany	0.1583	1	0.05	1	1
Spain	0.28	1	0	0.4	1
Portugal	0.21	1	0	0.5	1
Malta	0.35	1	0	1	1
Austria	0.25	1	0	1	1
Bulgaria	0.1	1	0	1	1
Croatia	0.2	1	0	1	1
Cyprus	0.125	1	0	0.2	1
Czech Republic	0.19	1	0	1	1
Denmark	0.235	1	0	1	1
Estonia	0.2	1	1	1	1
Finland	0.2	1	0	1	1
Greece	0.26	1	0	1	1
Hungary	0.19	1	0	1	1
Italy	0.3221	1	0.05	0.7	1
Latvia	0.15	1	0	1	1
Lithuania	0.15	1	0	1	1
Poland	0.19	1	0	1	1
Romania	0.16	1	0	1	1
Slovakia	0.22	1	0	1	1
Slovenia	0.17	1	0.05	1	1
Sweden	0.22	1	0	1	1
Bermuda	0	0	0	0	0
United States	0.35	1	1	1	0

A.2 Withholding tax on interests

Table A.2: Withholding tax on interests from EU Member States to Bermuda and to the United States (%)

Source/Destination	Bermuda	United States
Belgium	25	0
Netherlands	0	0
Ireland	20	0
Luxembourg	0	0
France	75	0
United Kingdom	20	0
Germany	26.38	0
Spain	20	0
Portugal	35	10
Malta	0	0
Austria	0	0
Bulgaria	10	5
Croatia	20	15
Cyprus	0	0
Czech Republic	35	0
Denmark	25	0
Estonia	0	10
Finland	20	0
Greece	15	0
Hungary	0	0
Italy	26	10
Latvia	10	10
Lithuania	10	10
Poland	20	0
Romania	16	10
Slovakia	35	0
Slovenia	15	5
Sweden	0	0

Table A.3: Withholding tax on interests from the United States to EU Member States (%)

Destination/Source	United States
Belgium	0
Netherlands	0
Ireland	0
Luxembourg	0
France	0
United Kingdom	0
Germany	0
Spain	10
Portugal	10
Malta	10
Austria	0
Bulgaria	5
Croatia	30
Cyprus	10
Czech Republic	0
Denmark	0
Estonia	10
Finland	0
Greece	0
Hungary	0
Italy	10
Latvia	10
Lithuania	10
Poland	0
Romania	10
Slovakia	0
Slovenia	5
Sweden	0

A.3 Withholding tax on dividends

Table A.4: Withholding tax on dividends from EU Member States to Bermuda and to the United States (%)

Source/Destination	Bermuda	United States
Belgium	25	0
Netherlands	15	0
Ireland	20	5
Luxembourg	15	0
France	75	0
United Kingdom	0	0
Germany	26.38	0
Spain	20	10
Portugal	35	5
Malta	0	0
Austria	25	5
Bulgaria	5	5
Croatia	12	12
Cyprus	0	0
Czech Republic	35	5
Denmark	27	0
Estonia	20	5
Finland	20	0
Greece	10	10
Hungary	0	0
Italy	26	5
Latvia	15	5
Lithuania	15	5
Poland	19	5
Romania	16	10
Slovakia	35	5
Slovenia	15	5
Sweden	30	0

Table A.5: Withholding tax on dividends from the United States to EU Member States (%)

Destination / Source	United States
Belgium	0
Netherlands	0
Ireland	5
Luxembourg	5
France	0
United Kingdom	0
Germany	0
Spain	10
Portugal	5
Malta	5
Austria	5
Bulgaria	5
Croatia	30
Cyprus	5
Czech Republic	5
Denmark	0
Estonia	5
Finland	0
Greece	30
Hungary	5
Italy	5
Latvia	5
Lithuania	5
Poland	5
Romania	10
Slovakia	5
Slovenia	5
Sweden	0

A.4 Withholding tax on royalties

Table A.6: Withholding tax on royalties from EU Member States to Bermuda and to the United States (%)

Source/Destination	Bermuda	United States
Belgium	25	5
Netherlands	0	0
Ireland	20	0
Luxembourg	0	0
France	75	0
United Kingdom	20	0
Germany	15.83	0
Spain	24	8
Portugal	35	10
Malta	0	0
Austria	20	0
Bulgaria	10	5
Croatia	20	15
Cyprus	10	0
Czech Republic	35	10
Denmark	25	0
Estonia	10	5
Finland	20	0
Greece	20	0
Hungary	0	0
Italy	22.5	5
Latvia	15	5
Lithuania	10	5
Poland	20	10
Romania	16	10
Slovakia	35	0
Slovenia	15	5
Sweden	0	0

Table A.7: Withholding tax on royalties from the United States to EU Member States (%)

Destination/Source	United States
Belgium	0
Netherlands	0
Ireland	0
Luxembourg	0
France	0
United Kingdom	0
Germany	0
Spain	8
Portugal	10
Malta	10
Austria	0
Bulgaria	5
Croatia	30
Cyprus	0
Czech Republic	10
Denmark	0
Estonia	5
Finland	0
Greece	0
Hungary	0
Italy	5
Latvia	5
Lithuania	5
Poland	10
Romania	10
Slovakia	10
Slovenia	5
Sweden	0

Appendix B

Results

B.1 Direct Path

Source / Destination	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Belgium (1)	-	34	34	34	35.1	34	34.5	34	34	34	34	34	34	34	34
Netherlands (2)	26.3	-	25	25	26.2	25	25.6	25	25	25	25	25	25	25	25
Ireland (3)	14	12.5	-	12.5	14	12.5	13.2	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5
Luxembourg (4)	30.4	29.2	29.2	-	30.4	29.2	29.8	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2
France (5)	34.5	33.3	33.3	33.3	-	33.3	33.9	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3
United Kingdom (6)	21.4	20	20	20	21.3	-	20.6	20	20	20	20	20	20	20	20
Germany (7)	17.3	15.8	15.8	15.8	17.2	15.8	-	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8
Spain (8)	29.2	28	28	28	29.2	28	28.6	-	28	28	28	28	28	28	28
Portugal (9)	22.3	21	21	21	22.3	21	21.6	21	-	21	21	21	21	21	21
Malta (10)	36.1	35	35	35	36.1	35	35.5	35	35	-	35	35	35	35	35
Austria (11)	26.3	25	25	25	26.2	25	25.6	25	25	25	-	25	25	25	25
Bulgaria (12)	11.5	10	10	10	11.5	10	10.7	10	10	10	10	-	10	10	10
Croatia (13)	21.4	20	20	20	21.3	20	20.6	20	20	20	20	20	-	20	20
Cyprus (14)	14	12.5	12.5	12.5	14	12.5	13.2	12.5	12.5	12.5	12.5	12.5	12.5	-	12.5
Czech Republic (15)	20.4	19	19	19	20.3	19	19.6	19	19	19	19	19	19	19	-
Denmark (16)	24.8	23.5	23.5	23.5	24.8	23.5	24.1	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5
Estonia (17)	21.4	20	20	20	21.3	20	20.6	20	20	20	20	20	20	20	20
Finland (18)	21.4	20	20	20	21.3	20	20.6	20	20	20	20	20	20	20	20
Greece (19)	27.3	26	26	26	27.2	26	26.6	26	26	26	26	26	26	26	26
Hungary (20)	20.4	19	19	19	20.3	19	19.6	19	19	19	19	19	19	19	19
Italy (21)	33.4	32.2	32.2	32.2	33.3	32.2	32.7	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2
Latvia (22)	16.4	15	15	15	16.4	15	15.7	15	15	15	15	15	15	15	15
Lithuania (23)	16.4	15	15	15	16.4	15	15.7	15	15	15	15	15	15	15	15
Poland (24)	20.4	19	19	19	20.3	19	19.6	19	19	19	19	19	19	19	19
Romania (25)	17.4	16	16	16	17.4	16	16.7	16	16	16	16	16	16	16	16
Slovakia (26)	23.3	22	22	22	23.3	22	22.6	22	22	22	22	22	22	22	22
Slovenia (27)	18.4	17	17	17	18.4	17	17.7	17	17	17	17	17	17	17	17
Sweden (28)	23.3	22	22	22	23.3	22	22.6	22	22	22	22	22	22	22	22
Bermuda (29)	34	25	12.5	29.2	33.3	20	16.5	28	21	35	25	10	10	12.5	15
United States (30)	36.1	35	38.3	38.3	36.1	35	35.5	41.5	38.3	38.3	38.3	38.3	54.5	38.3	38.3

Table B.1: Direct Path, Global Effective Tax Rate (%)

Source / Destination	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Belgium (1)	34	34	34	34	34	35.1	34	34	34	34	34	34.6	34	50.5	35
Netherlands (2)	25	25	25	25	25	26.2	25	25	25	25	25	25.6	25	36.2	35
Ireland (3)	12.5	12.5	12.5	12.5	12.5	13.9	12.5	12.5	12.5	12.5	12.5	13.2	12.5	30	35
Luxembourg (4)	29.2	29.2	29.2	29.2	29.2	30.4	29.2	29.2	29.2	29.2	29.2	29.8	29.2	39.8	35
France (5)	33.3	33.3	33.3	33.3	33.3	34.4	33.3	33.3	33.3	33.3	33.3	33.9	33.3	83.3	35
United Kingdom (6)	20	20	20	20	20	21.3	20	20	20	20	20	20.7	20	20	35
Germany (7)	15.8	15.8	15.8	15.8	15.8	17.2	15.8	15.8	15.8	15.8	15.8	16.5	15.8	38	35
Spain (8)	28	28	28	28	28	29.2	28	28	28	28	28	28.6	28	42.4	35
Portugal (9)	21	21	21	21	21	22.3	21	21	21	21	21	21.7	21	48.6	35
Malta (10)	35	35	35	35	35	36	35	35	35	35	35	35.6	35	35	35
Austria (11)	25	25	25	25	25	26.2	25	25	25	25	25	25.6	25	43.8	35
Bulgaria (12)	10	10	10	10	10	11.4	10	10	10	10	10	10.8	10	14.5	35
Croatia (13)	20	20	20	20	20	21.3	20	20	20	20	20	20.7	20	29.6	35
Cyprus (14)	12.5	12.5	12.5	12.5	12.5	13.9	12.5	12.5	12.5	12.5	12.5	13.2	12.5	12.5	35
Czech Republic (15)	19	19	19	19	19	20.3	19	19	19	19	19	19.7	19	47.3	35
Denmark (16)	-	23.5	23.5	23.5	23.5	24.7	23.5	23.5	23.5	23.5	23.5	24.2	23.5	44.2	35
Estonia (17)	20	-	20	20	20	21.3	20	20	20	20	20	20.7	20	36	35
Finland (18)	20	20	-	20	20	21.3	20	20	20	20	20	20.7	20	36	35
Greece (19)	26	26	26	-	26	27.2	26	26	26	26	26	26.6	26	33.4	35
Hungary (20)	19	19	19	19	-	20.3	19	19	19	19	19	19.7	19	19	35
Italy (21)	32.2	32.2	32.2	32.2	32.2	-	32.2	32.2	32.2	32.2	32.2	32.8	32.2	49.8	35
Latvia (22)	15	15	15	15	15	16.4	-	15	15	15	15	15.7	15	27.8	35
Lithuania (23)	15	15	15	15	15	16.4	15	-	15	15	15	15.7	15	27.8	35
Poland (24)	19	19	19	19	19	20.3	19	19	-	19	19	19.7	19	34.4	35
Romania (25)	16	16	16	16	16	17.4	16	16	16	-	16	16.7	16	29.4	35
Slovakia (26)	22	22	22	22	22	23.3	22	22	22	22	-	22.7	22	49.3	35
Slovenia (27)	17	17	17	17	17	18.3	17	17	17	17	17	-	17	29.5	35
Sweden (28)	22	22	22	22	22	23.3	22	22	22	22	22	22.7	-	45.4	35
Bermuda (29)	23.5	36	20	26	19	32.2	15	15	19	16	0	17	22	-	35
United States (30)	35	50.6	35	54.5	38.3	39.2	38.3	38.3	38.3	41.5	38.3	38.8	35	54.5	-

Table B.2: Direct Path, Global Effective Tax Rate (%)

B.2 Without complying with anti-avoidance rules

B.2.1 Using Patent Box Regimes

Table B.3: Without Anti-Avoidance rules and using Patent Box regimes

Source / Destination	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Belgium (1)	-	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Netherlands (2)	4.2	-	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Ireland (3)	4.2	2,5	-	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Luxembourg (4)	4.2	2,5	2,5	-	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
France (5)	4.2	2,5	2,5	2,5	-	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
United Kingdom (6)	4.2	2,5	2,5	2,5	4,1	-	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Germany (7)	4.2	2,5	2,5	2,5	4,1	2,5	-	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Spain (8)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	-	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Portugal (9)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	-	2,5	2,5	2,5	2,5	2,5	2,5
Malta (10)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	-	2,5	2,5	2,5	2,5	2,5
Austria (11)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	-	2,5	2,5	2,5	2,5
Bulgaria (12)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	-	2,5	2,5	2,5
Croatia (13)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	-	2,5	2,5
Cyprus (14)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	-	2,5
Czech Republic (15)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	-
Denmark (16)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Estonia (17)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Finland (18)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Greece (19)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Hungary (20)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Italy (21)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Latvia (22)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Lithuania (23)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Poland (24)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Romania (25)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Slovakia (26)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Slovenia (27)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Sweden (28)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
Bermuda (29)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
United States (30)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5

Table B.4: Without Anti-Avoidance rules and using Patent Box regimes

Source / Destination	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Belgium (1)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Netherlands (2)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Ireland (3)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Luxembourg (4)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
France (5)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
United Kingdom (6)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Germany (7)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Spain (8)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Portugal (9)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Malta (10)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Austria (11)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Bulgaria (12)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Croatia (13)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Cyprus (14)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Czech Republic (15)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Denmark (16)	-	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Estonia (17)	2,5	-	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Finland (18)	2,5	2,5	-	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Greece (19)	2,5	2,5	2,5	-	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Hungary (20)	2,5	2,5	2,5	2,5	-	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Italy (21)	2,5	2,5	2,5	2,5	2,5	-	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Latvia (22)	2,5	2,5	2,5	2,5	2,5	4,1	-	2,5	2,5	2,5	2,5	3,3	2,5	0	35
Lithuania (23)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	-	2,5	2,5	2,5	3,3	2,5	0	35
Poland (24)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	-	2,5	2,5	3,3	2,5	0	35
Romania (25)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	-	2,5	3,3	2,5	0	35
Slovakia (26)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	-	3,3	2,5	0	35
Slovenia (27)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	-	2,5	0	35
Sweden (28)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	-	0	35
Bermuda (29)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	-	35
United States (30))	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	0	-

B.2.2 Without Patent Box Regimes

Table B.5: Without Anti-Avoidance rules and Without Patent Box regimes

Source / Destination	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Belgium (1)	-	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Netherlands (2)	11,5	-	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Ireland (3)	11,5	10	-	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Luxembourg (4)	11,5	10	10	-	11,5	10	10,7	10	10	10	10	10	10	10	10
France (5)	11,5	10	10	10	-	10	10,7	10	10	10	10	10	10	10	10
United Kingdom (6)	11,5	10	10	10	11,5	-	10,7	10	10	10	10	10	10	10	10
Germany (7)	11,5	10	10	10	11,5	10	-	10	10	10	10	10	10	10	10
Spain (8)	11,5	10	10	10	11,5	10	10,7	-	10	10	10	10	10	10	10
Portugal (9)	11,5	10	10	10	11,5	10	10,7	10	-	10	10	10	10	10	10
Malta (10)	11,5	10	10	10	11,5	10	10,7	10	10	-	10	10	10	10	10
Austria (11)	11,5	10	10	10	11,5	10	10,7	10	10	10	-	10	10	10	10
Bulgaria (12)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	-	10	10	10
Croatia (13)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	-	10	10
Cyprus (14)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	-	10
Czech Republic (15)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	-
Denmark (16)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Estonia (17)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Finland (18)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Greece (19)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Hungary (20)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Italy (21)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Latvia (22)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Lithuania (23)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Poland (24)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Romania (25)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Slovakia (26)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Slovenia (27)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Sweden (28)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
Bermuda (29)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
United States (30)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10

Table B.6: Without Anti-Avoidance rules and Without Patent Box regimes

Source / Destination	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Belgium (1)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Netherlands (2)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Ireland (3)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Luxembourg (4)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
France (5)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
United Kingdom (6)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Germany (7)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Spain (8)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Portugal (9)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Malta (10)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Austria (11)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Bulgaria (12)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Croatia (13)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Cyprus (14)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Czech Republic (15)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Denmark (16)	-	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Estonia (17)	10	-	10	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Finland (18)	10	10	-	10	10	11,4	10	10	10	10	10	10,8	10	0	35
Greece (19)	10	10	10	-	10	11,4	10	10	10	10	10	10,8	10	0	35
Hungary (20)	10	10	10	10	-	11,4	10	10	10	10	10	10,8	10	0	35
Italy (21)	10	10	10	10	10	-	10	10	10	10	10	10,8	10	0	35
Latvia (22)	10	10	10	10	10	11,4	-	10	10	10	10	10,8	10	0	35
Lithuania (23)	10	10	10	10	10	11,4	10	-	10	10	10	10,8	10	0	35
Poland (24)	10	10	10	10	10	11,4	10	10	-	10	10	10,8	10	0	35
Romania (25)	10	10	10	10	10	11,4	10	10	10	-	10	10,8	10	0	35
Slovakia (26)	10	10	10	10	10	11,4	10	10	10	10	-	10,8	10	0	35
Slovenia (27)	10	10	10	10	10	11,4	10	10	10	10	10	-	10	0	35
Sweden (28)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	-	0	35
Bermuda (29)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	-	35
United States (30)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	0	-

B.3 Complying with Anti-Avoidance rules

B.3.1 Using Patent Box Regimes

Source / Destination	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Belgium (1)	-	24,9	24,9	24,9	26,2	24,9	25,5	24,9	24,9	24,9	24,9	24,9	24,9	24,9	24,9
Netherlands (2)	20,5	-	19,2	19,2	20,5	19,2	19,8	19,2	19,2	19,2	19,1	19,2	19,2	19,2	19,2
Ireland (3)	12,7	11,2	-	11,1	12,6	11,1	11,9	11,2	11,2	11,2	11,1	11,2	11,2	11,1	11,2
Luxembourg (4)	23,2	21,9	21,9	-	23,2	21,9	22,5	21,9	21,9	21,9	21,9	21,9	21,9	21,9	21,9
France (5)	25,8	24,5	24,5	24,5	-	24,5	25,1	24,5	24,5	24,5	24,5	24,5	24,5	24,5	24,5
United Kingdom (6)	17,4	15,9	16	15,9	17,4	-	16,6	15,9	15,9	15,9	15,9	15,9	15,9	15,9	15,9
Germany (7)	14,8	13,3	13,3	13,3	14,7	13,3	-	13,3	13,3	13,3	13,3	13,3	13,3	13,3	13,3
Spain (8)	22,4	21,1	21,1	21,1	22,4	21,1	21,7	-	21,1	21,1	21,1	21,1	21,1	21,1	21,1
Portugal (9)	18	16,6	16,6	16,6	18	16,6	17,3	16,6	-	16,6	16,6	16,6	16,6	16,6	16,6
Malta (10)	26,8	25,6	25,5	25,6	26,8	25,5	26,1	25,6	25,5	-	25,5	25,5	25,5	25,5	25,5
Austria (11)	20,5	19,2	19,2	19,2	20,5	19,2	19,8	19,2	19,1	19,2	-	19,1	19,2	19,2	19,2
Bulgaria (12)	11,1	9,6	9,5	9,5	11,1	9,5	10,3	9,6	9,6	9,6	9,6	-	9,6	9,6	9,6
Croatia (13)	17,4	16	15,9	15,9	17,4	15,9	16,6	15,9	15,9	15,9	15,9	15,9	-	15,9	16
Cyprus (14)	12,8	11,3	11,3	11,3	12,8	11,3	12	11,3	11,3	11,3	11,3	11,3	11,3	-	11,3
Czech Republic (15)	16,7	15,3	15,3	15,3	16,7	15,3	16	15,3	15,3	15,3	15,3	15,3	15,3	15,3	-
Denmark (16)	19,6	18,2	18,2	18,2	19,6	18,2	18,8	18,2	18,2	18,2	18,2	18,2	18,2	18,2	18,2
Estonia (17)	17,4	15,9	16	15,9	17,4	16	16,6	15,9	16	15,9	15,9	15,9	16	16	15,9
Finland (18)	17,4	15,9	16	15,9	17,4	15,9	16,6	15,9	15,9	15,9	15,9	15,9	15,9	16	15,9
Greece (19)	21,2	19,8	19,8	19,8	21,1	19,8	20,4	19,8	19,8	19,8	19,8	19,8	19,8	19,8	19,8
Hungary (20)	16,7	15,3	15,3	15,3	16,7	15,3	16	15,3	15,3	15,3	15,3	15,3	15,3	15,3	15,3
Italy (21)	25,1	23,8	23,8	23,8	25	23,8	24,4	23,8	23,8	23,8	23,8	23,8	23,8	23,8	23,8
Latvia (22)	14,2	12,8	12,7	12,7	14,2	12,7	13,4	12,7	12,8	12,7	12,7	12,7	12,7	12,7	12,7
Lithuania (23)	14,2	12,8	12,7	12,7	14,2	12,7	13,4	12,7	12,8	12,7	12,8	12,7	12,7	12,7	12,7
Poland (24)	16,7	15,3	15,3	15,3	16,7	15,3	16	15,3	15,3	15,3	15,3	15,3	15,3	15,3	15,3
Romania (25)	14,9	13,4	13,4	13,4	14,8	13,4	14,1	13,4	13,4	13,4	13,4	13,4	13,4	13,4	13,4
Slovakia (26)	18,6	17,2	17,2	17,2	18,6	17,2	17,9	17,2	17,2	17,2	17,2	17,2	17,2	17,2	17,2
Slovenia (27)	15,5	14	14	14	15,5	14	14,7	14	14	14	14	14	14	14	14
Sweden (28)	18,6	17,2	17,2	17,2	18,6	17,2	17,9	17,2	17,2	17,2	17,2	17,2	17,2	17,2	17,2
Bermuda (29)	4.2	2,5	2,5	2,5	4,1	2,5	3,3	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5
United States (30)	23,7	22,4	22,4	22,4	23,7	22,4	23	22,4	22,4	22,4	22,4	22,4	22,4	22,4	22,4

Table B.7: Complying with Anti-Avoidance rules and using Patent Box regimes

Source / Destination	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Belgium (1)	24,9	24,9	24,9	24,9	24,9	26,1	24,9	24,9	24,9	24,9	24,9	25,5	24,9	21,8	35
Netherlands (2)	19,2	19,2	19,2	19,2	19,2	20,5	19,2	19,2	19,2	19,2	19,2	19,8	19,2	16	35
Ireland (3)	11,1	11,1	11,2	11,1	11,2	12,6	11,1	11,1	11,2	11,2	11,1	11,9	11,2	8	35
Luxembourg (4)	21,9	21,9	21,9	21,9	21,9	23,1	21,9	21,9	21,9	21,9	21,9	22,5	21,9	18,7	35
France (5)	24,5	24,5	24,5	24,5	24,5	25,7	24,5	24,5	24,5	24,5	24,5	25,1	24,5	21,3	35
United Kingdom (6)	15,9	15,9	15,9	15,9	16	17,3	15,9	16	15,9	15,9	15,9	16,7	15,9	12,8	35
Germany (7)	13,3	13,3	13,3	13,3	13,3	14,7	13,3	13,3	13,3	13,3	13,3	14	13,3	10,1	35
Spain (8)	21,1	21,1	21,1	21,1	21,1	22,3	21,1	21,1	21,1	21,1	21,1	21,7	21,1	17,9	35
Portugal (9)	16,6	16,6	16,6	16,6	16,6	17,9	16,6	16,6	16,6	16,6	16,6	17,3	16,6	13,4	35
Malta (10)	25,5	25,5	25,5	25,6	25,6	26,7	25,6	25,6	25,5	25,5	25,6	26,2	25,5	22,4	35
Austria (11)	19,2	19,2	19,2	19,1	19,2	20,5	19,2	19,2	19,2	19,2	19,2	19,8	19,2	16	35
Bulgaria (12)	9,6	9,6	9,6	9,6	9,6	11	9,6	9,6	9,6	9,6	9,6	10,3	9,6	6,4	35
Croatia (13)	15,9	15,9	15,9	15,9	16	17,3	16	16	16	16	16	16,7	15,9	12,8	35
Cyprus (14)	11,3	11,3	11,3	11,3	11,3	12,7	11,3	11,3	11,3	11,3	11,3	12,1	11,3	8	35
Czech Republic (15)	15,3	15,3	15,3	15,3	15,3	16,7	15,3	15,3	15,3	15,3	15,3	16	15,3	12,2	35
Denmark (16)	-	18,2	18,2	18,2	18,2	19,5	18,2	18,2	18,2	18,2	18,2	18,9	18,2	15	35
Estonia (17)	16	-	16	15,9	15,9	17,3	16	15,9	15,9	15,9	15,9	16,7	15,9	12,8	35
Finland (18)	15,9	15,9	-	15,9	15,9	17,3	15,9	16	15,9	16	15,9	16,7	15,9	12,8	35
Greece (19)	19,8	19,8	19,8	-	19,8	21,1	19,8	19,8	19,8	19,8	19,8	20,5	19,8	16,6	35
Hungary (20)	15,3	15,3	15,3	15,3	-	16,7	15,3	15,3	15,3	15,3	15,3	16	15,3	12,2	35
Italy (21)	23,8	23,8	23,8	23,8	23,8	-	23,8	23,8	23,8	23,8	23,8	24,4	23,8	20,6	35
Latvia (22)	12,7	12,8	12,7	12,8	12,8	14,2	-	12,7	12,7	12,7	12,7	13,5	12,7	9,6	35
Lithuania (23)	12,8	12,8	12,7	12,7	12,8	14,2	12,8	-	12,8	12,8	12,7	13,5	12,7	9,6	35
Poland (24)	15,3	15,3	15,3	15,3	15,3	16,7	15,3	15,3	-	15,3	15,3	16	15,3	12,2	35
Romania (25)	13,4	13,4	13,4	13,4	13,4	14,8	13,4	13,4	13,4	-	13,4	14,1	13,4	10,2	35
Slovakia (26)	17,2	17,2	17,2	17,2	17,2	18,6	17,2	17,2	17,2	17,2	-	17,9	17,2	14,1	35
Slovenia (27)	14	14	14	14	14	15,4	14	14	14	14	14	-	14	10,9	35
Sweden (28)	17,2	17,2	17,2	17,2	17,2	18,6	17,2	17,2	17,2	17,2	17,2	17,9	-	14,1	35
Bermuda (29)	2,5	2,5	2,5	2,5	2,5	4,1	2,5	2,5	2,5	2,5	2,5	3,3	2,5	-	35
United States (30)	22,4	37,9	22,4	22,4	22,4	23,6	22,4	22,4	22,4	22,4	22,4	23,1	22,4	22,4	-

Table B.8: Complying with Anti-Avoidance rules and using Patent Box regimes

B.3.2 Without Patent Box Regimes

Source/Destination	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Belgium (1)	-	25.4	25.4	25.4	26.6	25.4	25.9	25.4	25.4	25.4	25.4	25.4	25.4	25.4	25.4
Netherlands (2)	21	-	19.6	19.6	20.9	19.6	20.2	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6
Ireland (3)	13.1	11.6	-	11.6	13.1	11.6	12.3	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6
Luxembourg (4)	23.6	22.3	22.3	-	23.6	22.3	22.9	22.3	22.3	22.3	22.3	22.3	22.3	22.3	22.3
France (5)	26.2	24.9	24.9	24.9	-	24.9	25.5	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9
United Kingdom (6)	17.8	16.4	16.4	16.4	17.8	-	17.1	16.4	16.4	16.4	16.4	16.4	16.4	16.4	16.4
Germany (7)	15.2	13.7	13.7	13.7	15.2	13.7	-	13.7	13.7	13.7	13.7	13.7	13.7	13.7	13.7
Spain (8)	22.9	21.5	21.5	21.5	22.8	21.5	22.1	-	21.5	21.5	21.5	21.5	21.5	21.5	21.5
Portugal (9)	18.49	17	17	17	18.4	17	17.7	17	-	17	17	17	17	17	17
Malta (10)	27.3	26	26	26	27.2	26	26.6	26	26	-	26	26	26	26	26
Austria (11)	21	19.6	19.6	19.6	20.9	19.6	20.2	19.6	19.6	19.6	-	19.6	19.6	19.6	19.6
Bulgaria (12)	11.5	10	10	10	11.5	10	10.7	10	10	10	10	-	10	10	10
Croatia (13)	17.8	16.4	16.4	16.4	17.8	16.4	17.1	16.4	16.4	16.4	16.4	16.4	-	16.4	16.4
Cyprus (14)	13.1	11.6	11.6	11.6	13.1	11.6	12.3	11.6	11.6	11.6	11.6	11.6	11.6	-	11.6
Czech Republic (15)	17.2	15.8	15.8	15.8	17.2	15.8	16.4	15.8	15.8	15.8	15.8	15.8	15.8	15.8	-
Denmark (16)	20	18.6	18.6	18.6	20	18.6	19.3	18.6	18.6	18.6	18.6	18.6	18.6	18.6	18.6
Estonia (17)	17.8	16.4	16.4	16.4	17.8	16.4	17.1	16.4	16.4	16.4	16.4	16.4	16.4	16.4	16.4
Finland (18)	17.8	16.4	16.4	16.4	17.8	16.4	17.1	16.4	16.4	16.4	16.4	16.4	16.4	16.4	16.4
Greece (19)	21.6	20.2	20.2	20.2	21.6	20.2	20.9	20.2	20.2	20.2	20.2	20.2	20.2	20.2	20.2
Hungary (20)	17.2	15.8	15.8	15.8	17.2	15.8	16.4	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8
Italy (21)	25.5	24.2	24.2	24.2	25.5	24.2	24.8	24.2	24.2	24.2	24.2	24.2	24.2	24.2	24.2
Latvia (22)	14.7	13.2	13.2	13.2	14.6	13.2	13.9	13.2	13.2	13.2	13.2	13.2	13.2	13.2	13.2
Lithuania (23)	14.7	13.2	13.2	13.2	14.6	13.2	13.9	13.2	13.2	13.2	13.2	13.2	13.2	13.2	13.2
Poland (24)	17.2	15.8	15.8	15.8	17.2	15.8	16.4	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8
Romania (25)	15.3	13.8	13.8	13.8	15.3	13.8	14.5	13.8	13.8	13.8	13.8	13.8	13.8	13.8	13.8
Slovakia (26)	19.1	17.7	17.7	17.7	19.1	17.7	18.3	17.7	17.7	17.7	17.7	17.7	17.7	17.7	17.7
Slovenia (27)	15.9	14.5	14.5	14.5	15.9	14.5	15.2	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5
Sweden (28)	19.1	17.7	17.7	17.7	19.1	17.7	18.3	17.7	17.7	17.7	17.7	17.7	17.7	17.7	17.7
Bermuda (29)	11,5	10	10	10	11,5	10	10,7	10	10	10	10	10	10	10	10
United States (30)	23.7	22.4	22.4	22.4	23.7	22.4	23	22.4	22.4	22.4	22.4	22.4	22.4	22.4	22.4

Table B.9: Complying with Anti-Avoidance rules and without Patent Box regimes

Source/Destination	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Belgium (1)	25.4	25.4	25.4	25.4	25.4	26.6	25.4	25.4	25.4	25.4	25.4	26	25.4	21.8	35
Netherlands (2)	19.6	19.6	19.6	19.6	19.6	20.9	19.6	19.6	19.6	19.6	19.6	20.3	19.6	16	35
Ireland (3)	11.6	11.6	11.6	11.6	11.6	13	11.6	11.6	11.6	11.6	11.6	12.4	11.6	8	35
Luxembourg (4)	22.3	22.3	22.3	22.3	22.3	23.6	22.3	22.3	22.3	22.3	22.3	23	22.3	18.7	35
France (5)	24.9	24.9	24.9	24.9	24.9	26.1	24.9	24.9	24.9	24.9	24.9	25.6	24.9	21.3	35
United Kingdom (6)	16.4	16.4	16.4	16.4	16.4	17.7	16.4	16.4	16.4	16.4	16.4	17.1	16.4	12.8	35
Germany (7)	13.7	13.7	13.7	13.7	13.7	15.1	13.7	13.7	13.7	13.7	13.7	14.5	13.7	10.1	35
Spain (8)	21.5	21.5	21.5	21.5	21.5	22.8	21.5	21.5	21.5	21.5	21.5	22.2	21.5	17.9	35
Portugal (9)	17	17	17	17	17	18.4	17	17	17	17	17	17.7	17	13.4	35
Malta (10)	26	26	26	26	26	27.2	26	26	26	26	26	26.6	26	22.4	35
Austria (11)	19.6	19.6	19.6	19.6	19.6	20.9	19.6	19.6	19.6	19.6	19.6	20.3	19.6	16	35
Bulgaria (12)	10	10	10	10	10	11.4	10	10	10	10	10	10.8	10	6.4	35
Croatia (13)	16.4	16.4	16.4	16.4	16.4	17.7	16.4	16.4	16.4	16.4	16.4	17.1	16.4	12.8	35
Cyprus (14)	11.6	11.6	11.6	11.6	11.6	13	11.6	11.6	11.6	11.6	11.6	12.4	11.6	8	35
Czech Republic (15)	15.8	15.8	15.8	15.8	15.8	17.1	15.8	15.8	15.8	15.8	15.8	16.5	15.8	12.2	35
Denmark (16)	-	18.6	18.6	18.6	18.6	20	18.6	18.6	18.6	18.6	18.6	19.3	18.6	15	35
Estonia (17)	16.4	-	16.4	16.4	16.4	17.7	16.4	16.4	16.4	16.4	16.4	17.1	16.4	12.8	35
Finland (18)	16.4	16.4	-	16.4	16.4	17.7	16.4	16.4	16.4	16.4	16.4	17.1	16.4	12.8	35
Greece (19)	20.2	20.2	20.2	-	20.2	21.5	20.2	20.2	20.2	20.2	20.2	20.9	20.2	16.6	35
Hungary (20)	15.8	15.8	15.8	15.8	-	17.1	15.8	15.8	15.8	15.8	15.8	16.5	15.8	12.2	35
Italy (21)	24.2	24.2	24.2	24.2	24.2	-	24.2	24.2	24.2	24.2	24.2	24.9	24.2	20.6	35
Latvia (22)	13.2	13.2	13.2	13.2	13.2	14.6	-	13.2	13.2	13.2	13.2	13.9	13.2	9.6	35
Lithuania (23)	13.2	13.2	13.2	13.2	13.2	14.6	13.2	-	13.2	13.2	13.2	13.9	13.2	9.6	35
Poland (24)	15.8	15.8	15.8	15.8	15.8	17.1	15.8	15.8	-	15.8	15.8	16.5	15.8	12.2	35
Romania (25)	13.8	13.8	13.8	13.8	13.8	15.2	13.8	13.8	13.8	-	13.8	14.6	13.8	10.2	35
Slovakia (26)	17.7	17.7	17.7	17.7	17.7	19	17.7	17.7	17.7	17.7	-	18.4	17.7	14.1	35
Slovenia (27)	14.5	14.5	14.5	14.5	14.5	15.9	14.5	14.5	14.5	14.5	14.5	-	14.5	10.9	35
Sweden (28)	17.7	17.7	17.7	17.7	17.7	19	17.7	17.7	17.7	17.7	17.7	18.4	-	14.1	35
Bermuda (29)	10	10	10	10	10	11,4	10	10	10	10	10	10,8	10	-	35
United States (30)	22.4	37.9	22.4	22.4	22.4	23.6	22.4	22.4	22.4	22.4	22.4	23.1	22.4	22.4	-

Table B.10: Complying with Anti-Avoidance rules and without Patent Box regimes

Appendix C

Time Complexity

When comparing algorithms performing the same task, a criterion is the execution time. The time required by an algorithm depends on the size of the input. Intuitively, if you have to run 10km, it will take you longer than running 1km. The goal of this section is to introduce a measure to discriminate between several algorithms for the same size of input.

As we are assessing algorithms and not machines, we do not want our measure to depends on CPU performances. Moreover, distinct input of the same size can be more or less complex to handle for the algorithm. Again, running 1km on a steep road is more difficult than running the same distance on a flat road. Therefore, we will consider the worst possible input of a given size for assessing the performances of each algorithm. Consequently, the measure considered here, called the big O notation, is an upper bound for any input of a given size.

Given an input of size n sufficiently large, if an algorithm runs in $\mathcal{O}(g(n))$, it means that the number of steps performed by the algorithm is not greater than $cg(n)$, where c is a constant. More formally, we have:

$$T(n) \in \mathcal{O}(g(n)) \text{ if } \exists c, n_0 \text{ such that } \forall n \geq n_0, |T(n)| \leq c|g(n)|$$

Given this notation, the smallest function $g(n)$, the more efficient algorithm we have.

Properties

Upper Bound

The big $\mathcal{O}$ notation is an upper bound. It means that if an algorithm belongs to the $\mathcal{O}(n)$ class, it also belongs to the $\mathcal{O}(n^2)$ class.

Additivity

If we have an algorithm in two parts T_1 and T_2 such that $T_1(n) \in \mathcal{O}(g_1(n))$ and $T_2(n) \in \mathcal{O}(g_2(n))$, then the algorithm runs in $T_1(n) + T_2(n) \in \mathcal{O}(g_1(n) + g_2(n))$.

Neglect lower order terms

To introduce this property, we just show an example:

$$n^5 + n^3 \in \mathcal{O}(n^5)$$

Appendix D

Markov Chains and Decision Processes

This section is based on the course [38]. The aim is to introduce Markov Chains (MC) and Decision Processes (MDP).

A Markov Chain is a set of states and the possible transitions between those states are described by a probability transition matrix. To be classified as a Markov Chain, the process must fulfil the Markov property according to which "the future is independent of the past given the present". It means that the current state captures all relevant information from the history of the process. Once we know the current state, the history doesn't bring any additional information.

Firstly, this property can be defined in terms of conditional probabilities as follows:

Definition 4 (Markov Property)**.** A State S_t is Markov if and only if

$$\mathbb{P}[S_{t+1}|S_t] = \mathbb{P}[S_{t+1}|S_1, S_2, ..., S_t]$$

Secondly, a Markov Chain is defined by:

Definition 5 (Markov Chain)**.** A Markov Chain is a tuple (S,T) where:

- S is a finite set of states

- T is a state transition probability matrix: $T_{ss'} = \mathbb{P}[S_{t+1} = s'|S_t = s]$

A state S of a MC is said to be absorbing ([37] and [24]) if once we reach that state, leaving it is impossible. In other words, $T_{SS} = 1$. A MC is absorbing if it has at least one absorbing state and if, from any state, it is possible to reach an absorbing state, not necessarily in one step. In an absorbing MC, non-absorbing states are called transient. If we order the states such that the first states are the transient states and the following states are absorbing, we can decompose the transition matrix as follows:

$$T = \begin{pmatrix} Q & R \\ 0 & I \end{pmatrix}$$

Using this decomposition, starting from transient state i, the probability of reaching transient state j in exactly k steps is given by $Q^k_{i,j}$. We can then compute matrix N where entry (i,j) is the expected number of times the chain is in state j before reaching an absorbing state knowing that we started from state i. This matrix N is given by:

$$N = \sum_{k=0}^{\infty} Q^k = (I - Q)^{-1}$$

Eventually, we can compute the probability of absorption by each absorbing state and for each starting point. The probability of being absorbed by state j knowing that we started in state i is the entry (i,j) of matrix B defined by:

$$B = NR$$

A Markov Decision Process is a process where, at each state, you select an action. The selected action determines the transition probabilities. On top of it, a reward is associated to each state. Consequently, the goal is to find the optimal policy (i.e. best action to select at each state) to maximize the total reward.

Definition 6 (Markov Decision Process)**.** A Markov Decision Process is a tuple (S,A,T,R,γ) where:

- S is a finite set of states

- A is a finite set of actions

- T is a transition probability matrix:

$$T^a_{ss'} = \mathbb{P}[S_{t+1} = s' | S_t = s \ and \ A_t = a]$$

- R is a reward function:

$$R^a_s = \mathbb{E}[R_{t+1} | S_t = s \ and \ A_t = a]$$

- γ is a discount factor between 0 and 1

A discount factor is used to model the fact that earning a reward today is more interesting that earning the same reward tomorrow.

Given a Markov Decision Process, the optimal policy can be found using either the Policy Iteration algorithm or the Value Iteration algorithm.

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