Louvain School of Management

What is the impact of the introduction of cryptocurrencies on the performance of an investment portfolio?

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Academic year 2022-2023
Dissertation for the master of Management
Master subject and focus on cryptocurrencies
Daytime schedule/Staggered schedule in September

Acknowledgment

I would like to express my sincere gratitude to my supervisor, Pr. Catherine D'Hondt, for taking time to help me with rigorous feedbacks and precious advice during the redaction of this master thesis.

Furthermore, I would like to express my sincere thanks to the academic faculty and staff who have contributed to my academic journey.

My gratitude also extends to my family and friends whose encouragement and understanding provided me with the motivation and strength to persevere through the challenges of this endeavour.

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Abbreviations

ETFs: Exchange-traded-funds

BTC: Bitcoin

ETH: Ethereum

ADA: Cardano

SOL: Solana

RPL: Ripple

EUEA: iShares EURO STOXX 50 UCITS ETF

EUNH: iShares Core € Govt Bond UCITS ETF

EXXT: iShares NASDAQ-100 UCITS ETF

IEF: iShares 7-10 Year Treasury Bond ETF

IEMG: iShares Core MSCI Emerging Markets ETF

IGSB: iShares 1-5 Year Investment Grade Corporate Bond ETF

SHY: iShares 1-3 Year Treasury Bond ETF

TCBT: VanEck iBoxx EUR Corporates UCITS ETF

TFLO: iShares Treasury Floating Rate Bond ETF

TLT: iShares 20+ Year Treasury Bond ETF

URTH: iShares MSCI World ETF

WFSPX: iShares S&P 500 Index Fund

VaR: Value-at-Risk

ES: Expected Shortfall

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Chapter 1 Introduction

In recent years, cryptocurrencies have emerged as a disruptive and transformative force in the global financial landscape, marking a turning point, as these digital assets rapidly gained prominence by offering an alternative to traditional monetary systems, capturing the curiosity and interest of investors, academics, the general public alike and students like me. With their emergence, cryptocurrencies brought many challenges for investors, with risks such as price volatility, regulatory uncertainties, and security concerns.

Though initially unfamiliar with the subject, I found myself intrigued by the potential of cryptocurrencies and their impact on investment portfolios. This fascination led me to embark on a journey of exploration and understanding through my master thesis. Specifically, I aimed to discern the implications of incorporating cryptocurrencies into investment portfolios and determine whether they hold value for investors who may not be well-versed in this innovative realm.

The study of the impact of the introduction of cryptocurrencies on the performance of classical investment portfolios bears practical relevance for both individual investors and financial institutions. In an era of increasing interest in diversification and the pursuit of alternative investments, comprehending how portfolios perform with the inclusion of cryptocurrencies can facilitate the development of robust investment strategies. Moreover, by analysing the behaviour of cryptocurrencies within the broader investment landscape, it allows to gain insights into their potential as hedges against traditional asset classes.

Numerous articles in the past five years have examined the behaviour of cryptocurrencies, focusing on their volatility through univariate and multivariate volatility models, as well as their potential performance with the use various return ratios. My research seeks to take a more comprehensive approach by delving deeper into the behaviour of cryptocurrencies with traditional assets, as well as the potential risks and return faced regarding the introduction of those assets in investment portfolios.

Chapter 2: Literature review

2.1 Emergence of cryptocurrencies

2.1.1 History

Before going deeper into this topic, it is important to first analyse the history of cryptocurrencies. According to "Cryptocurrency Adoption: Status, Opportunities and Unsolved Challenges" (Al-Amri et al., 2019), cryptocurrencies are virtual currencies designed as an alternative to standard fiat currencies, allowing consumers to make digital payments for goods and services without the need of intermediaries.

The idea behind the creation of cryptocurrencies is not a new concept. In 1983, American cryptographer David Chaum introduced the concept of anonymous cryptographic electronic money. This revolutionary form of currency promises to enable untraceable transactions between different parties without relying on the oversight of centralized institutions such as banks in the current traditional monetary system. In 1995, Chaum continued to fulfil his vision by creating the first-ever cryptocurrency, called Digicash.

To understand cryptocurrencies, it is necessary to understand their fundamental purpose: building decentralized networks. Unlike traditional electronic money, which operates under the centralised control of banks, cryptocurrencies operate on a network model akin to the decentralised nature of the internet itself. This decentralized structure empowers users to maintain control over their financial interactions, ensuring a degree of privacy and autonomy that aligns with the principles of democracy and the preservation of individual information.

It is a form of decentralised system: there is no single entity, the server, which gives access to all those who want to connect to the Internet, and which stores all the data, but a network, in which each computer, server, is connected to other computers and servers, enabling the Internet to function as a network.

In an interview with Scott Melker on YouTube, David Chaum elucidated his motivations for pioneering a decentralized digital currency: "*In the late 70s, I was at Berkley as a graduate student and I started to realize that the informational world was going to unfold, and that control over information about people would ultimately be a key aspect for democracy and the future of the civilization. I will not be easy for people to keep control over their own information and sometimes it can have serious abuses. […] For democracy, there are really 3 key aspects:*

communication privacy (confidentiality of message content, traffic analysis data about the persons you talk to), payments (who pays you, who you pay, …), and the third area is the privacy related to information about your interactions with organizations and institutions".

In his pursuit of a decentralized monetary system, Chaum aimed to empower individuals to safeguard their information and privacy, a prescient concern in the contemporary era characterized by the rise of prominent tech conglomerates and the growing significance of personal data.

With this understanding of the origins and motives behind cryptocurrencies, we can now embark on a comprehensive exploration of their multifaceted realm.

2.1.2 Emergence of Bitcoin and the blockchain

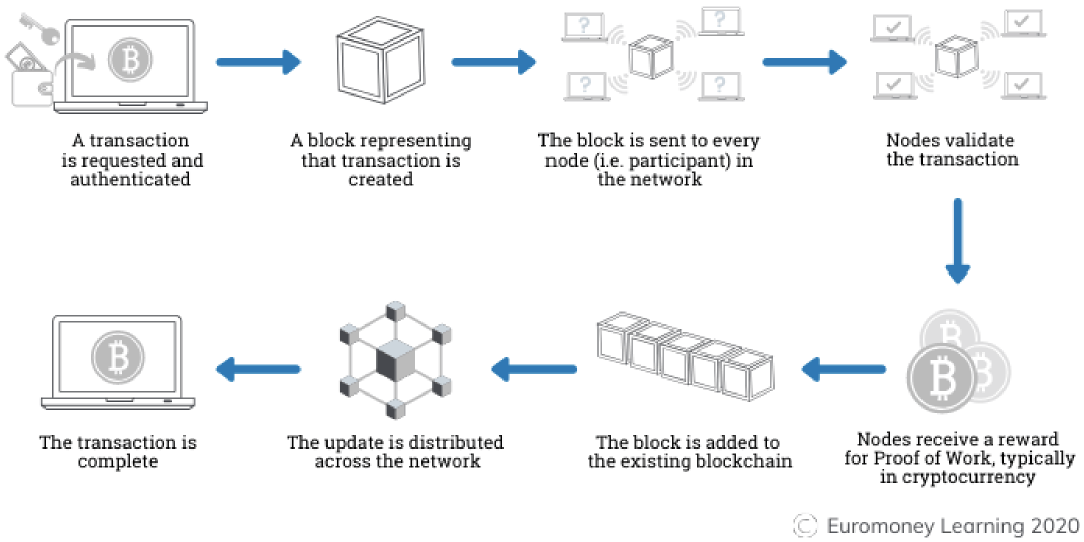
The popularization of cryptocurrencies can be traced back to the emergence of Bitcoin in 2008, when an anonymous individual using the pseudonym Satoshi Nakamoto published a groundbreaking article titled "A peer-to-peer electronic cash system". The article shed light on the potential risks associated with the traditional monetary system, particularly the lack of trust faced by growing internet commerce that relied solely on financial institutions as trusted third parties, often entailing significant transaction fees.

Today, Bitcoin (BTC) reigns as the most renowned cryptocurrency, boasting the highest market capitalization and commanding nearly 39.5% of the global capitalization of all cryptocurrencies as of January 2023. Introduced in 2009, Bitcoin witnessed approximately 260 thousand daily transactions by 2022. One of its distinctive features lies in the innovative blockchain technology it employs, notably utilizing the « Proof of Work » system.

Bitcoin operates on the foundation of "blockchain" technology, a system enabling peer-to-peer transactions without the need for a central authority to authenticate and authorize transactions (eliminating the necessity for traditional banks to control payments). To ensure the legitimacy of each transaction, a two-step process is employed. First, cryptographic keys (a unique string of data similar to a password) authenticate the parties involved in the transaction.

Subsequently, the "Proof of Work" phase comes into play when the transaction is mutually agreed upon by the users. During this phase, individuals owning computers in the network must solve a complex mathematical problem to add a block to the chain, thereby securing the completion of the transaction. To provide further clarity, the figure 1 below illustrates the sequential steps involved in a Bitcoin transaction.

Figure 1: Validation of a transaction through the blockchain technology.



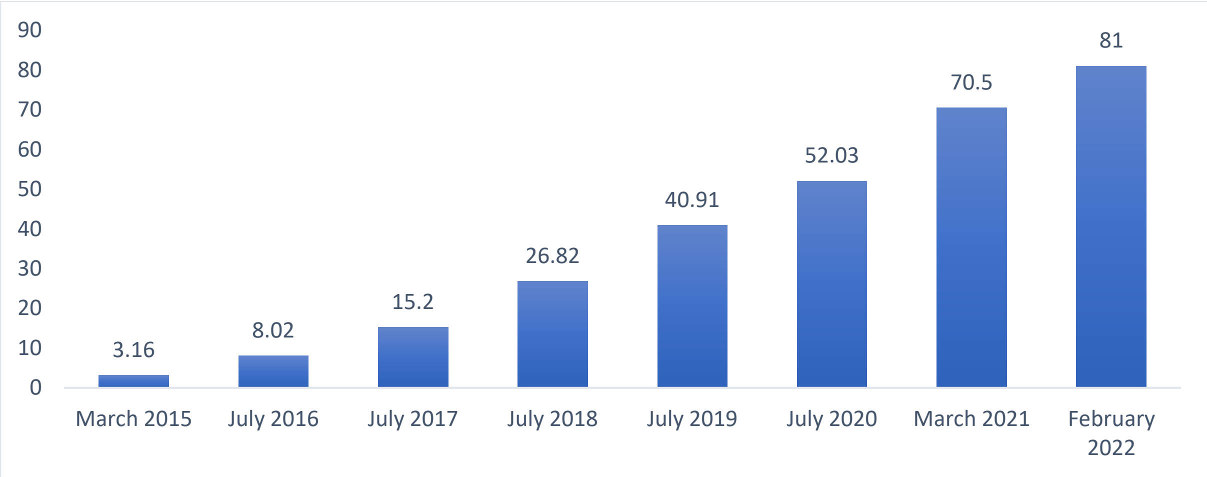
Source: Euromoney Learning 2020

Section 2.2 Cryptocurrency adoption

Over the last decade, cryptocurrencies have experienced a remarkable surge in popularity, leading to widespread adoption worldwide. As highlighted by B.Silbert, the founder of Bitcoin Investment Trust, a prominent digital asset management firm and one of the first investment vehicles specifically designed to enable traditional investors to gain exposure to Bitcoin, this adoption has progressed through several distinct phases.

The initial stage was marked by an experimentation phase, where early pioneers and enthusiasts explored the possibilities of cryptocurrencies. This was followed by the early adopter's phase, during which a more significant number of individuals and businesses began to recognize and embrace the potential of digital currencies. As cryptocurrencies gained traction, the venture capital phase ensued, attracting substantial investments, and further propelling their growth. Subsequently, the Wall Street phase saw traditional financial institutions and institutional investors acknowledge the importance of cryptocurrencies as an asset class, further legitimizing their position in the financial landscape. Finally, cryptocurrencies entered the global consumer adoption phase, where a growing number of individuals from all walks of life started using digital currencies for various transactions and investment purposes.

Figure 2: Number of Blockchain users worldwide from 2015 to 2022.



Source: Statista

Figure 2 reveals a significant growth in the number of blockchain users over a seven-year period, escalating from slightly over 3 million to 81 million. While the number of blockchain users might not directly correlate to the number of investors, it can serve as an indicator of the growing interest and engagement with cryptocurrencies. For a more accurate estimation of cryptocurrency investors, the number of wallet users can be considered as a proxy, with each unique wallet address representing an individual user.

Cryptocurrency exchange platforms also provide valuable insights into the number of users. For instance, Coinbase, a prominent cryptocurrency exchange that also offers wallet services, reported having over 13 million active users in 2019. By 2022, this number skyrocketed to an impressive 103 million verified users, indicating a substantial expansion of its user base within a three years period. Similarly, Binance, another major cryptocurrency exchange, witnessed a substantial increase in its user count. From 2017 to 2021, the number of users on the platform had around boasted from 1.5 million to 28.6 million, illustrating a significant surge in user participation.

To comprehend the various aspects of cryptocurrency adoption, the application of innovation acceptance theories can provide valuable insights. Bharadwaj and Deka (2021) used the "Diffusion of Innovation theory" and the "Unified Theory of Acceptance" to understand the acceptation of cryptocurrencies.

The "Diffusion of Innovation theory" was developed by Rogers (1995) and offers a framework for understanding how new ideas, innovations, and technologies spread among individuals and societies. Likewise, the "Unified Theory of Acceptance" and "Use of Technology model", devised by Venkatesh et al. (2012), is specifically designed to comprehend consumers' acceptance and usage of novel technologies, including cryptocurrencies. According to the Diffusion of Innovation theory, users can be motivated to adopt Bitcoin due to their technological curiosity, while the second theory indicates that performance, effort expectations and social influence constructs are significant for African countries.

Section 2.3 Characteristics of cryptocurrency investments

With the increasing popularity of cryptocurrencies, some characteristics related to them became general knowledge, as their high volatility and return behavioural characteristics.

2.3.1 Basics

Higher Volatility

Tsyvinski & Liu, in their article "Risks and returns of cryptocurrency" (2018), present the risk-return trade-off of cryptocurrencies. With a data period from 2011 to 2018, cryptocurrencies, represented in that article by Bitcoin, Ethereum and Ripple, showed high volatility in general compared to traditional assets. As example with Bitcoin, the standard deviation means of that period was set at 5.55 percent in a daily frequency, 16.64 in a weekly frequency and 69.46 percent for a monthly frequency, which is of higher magnitude compared to traditional assets. While the return mean was set for daily, weekly and monthly at respectively 0.52, 3.79 and 21.6 percent, the Bitcoin's monthly Sharpe ratio was similar quoted compared to traditional assets such as stocks for the same time period, but substantially higher on a daily and weekly frequency. The results founded were similar with Ethereum and Ripple.

Positively skewed return distribution

Regarding the performance distribution, Tsyvinski & Liu also showed a positively skewed return distribution for the cryptocurrencies, which is fundamentally different from the general negative skewness of the return distribution of stocks. This was in line with the results obtained in other similar research and public articles, such as the publication of Momtaz "The Pricing and Performance of Cryptocurrency".

Szczygielski et al. (2020), described the return distribution of cryptocurrencies as departed from the classical normal distribution with excess kurtosis and thick tails. While their study involved 15 different cryptocurrencies, some of them had specific distribution, such as Cardano. They determined that the most suitable distribution for the remaining data is the Cauchy distribution, known for its thick tails and a higher peak in contrast to the normal distribution.

2.3.2 Risks

Because of their inherent volatile characteristics, cryptocurrencies represent a certain amount of uncertainty and risk.

Regulatory/legal risk

The cryptocurrency market emerged in the start of the 2010 decade and with the hype wave submerging all over the world, the regulatory framework related to that specific and new market is currently far from been widely shared. Here is the regulatory situation as of January 2023:

In countries such as the US, Canada, United Kingdom, China or South Korea, cryptocurrencies are not considered as legal tender, while in Australia, Japan, Switzerland, European Union, cryptocurrencies dispose from a legal basis. In South America and Africa, laws diverge country from country. This demonstrated the diverging opinions of legally stated entities over the world. Nevertheless, in most country in which crypto are not considered as legal tender, cryptocurrencies exchanges are legally accepted under certain conditions.

Regarding the US, the country is making progress in developing a federal-level cryptocurrency legislation. The Financial Crimes Enforcement Network (FinCEN) does not consider cryptocurrencies to be legal tender but considers cryptocurrency exchanges to be money transmitters on the basis that cryptocurrency tokens are “other value that substitutes for currency.”

Market risk

Momentum:

Different studies underline the momentum effect occurring on cryptocurrency market. The article “common risk factors in cryptocurrency” (Liu et al., 2022) analysed the performance of the zero-investment long-short strategies, which consist of short-selling positions, based on the one-, two-, three-, four-, eight-, sixteen-, fifty-, and one-hundred-week momentum factors. All strategies are rebalanced weekly, and they found that the one-, two-, three-, and four-week momentum factors generate statistically significant long-short strategy returns.

In “Momentum trading in cryptocurrencies: Short-term returns and diversification benefits” (Tzouvanas and al., 2020) used the J/K strategy (Jegadeesh and Titman, 1993) to construct the momentum portfolios, where *J* represents the formation period (in number of days), and K is the holding period. They ranked cryptocurrencies based on their returns in the past *J*-day and create two portfolios: the top 6 cryptocurrencies are in the “winner portfolio”, whereas the bottom 6 are in the “loser portfolio”. The momentum strategy buys the winner portfolio and sells the loser portfolio. They found out that positive returns can be derived from momentum strategies in the short run (when *J* is equal to seven and fifteen days), which is aligned on the results of the study of Grobys and Sapkota (2019), describing the cryptocurrency market as inefficient in the short-term but efficient over the longer term.

Integration of the crypto market:

In Liu and Serletis (2019) studied the internal cryptocurrency market. Their results provide strong evidence of integration and interdependencies within the cryptocurrency market, especially in the countries where cryptocurrencies are more accepted and used. They also suggest that cryptocurrency can provide some of the advantages that are proper to both stocks and bonds in the financial markets and therefore be a useful tool for portfolio management, risk analysis, and market sentiment analysis. Most aspects of cryptocurrency are similar to financial assets as they react to similar variables in the GARCH models and possess similar hedging capabilities when react to good and bad news.

Bubbles:

Several studies concluded that the cryptocurrency market is prone to speculative bubbles. Fry (2018) developed a rational bubble model which combine heavy-tails measures with other measures of risk and return and incorporates default risk with a potential crash of cryptocurrency when there is no centralized authority. He found evidence of bubbles in Bitcoin and Ethereum, while for other cryptocurrencies such as Ripple there was no such evidence.

Similarly, Agosto and Cafferata (2020) conducted an analysis to examine the presence of bubbles in the top five cryptocurrencies by market capitalization. The authors analysed the co-movements of these cryptocurrencies during different phases of their price behaviour to identify potential co-explosivity effects. The results of the study revealed that not only Bitcoin, but also other crypto assets experienced several periods of explosiveness, corroborating the findings of Bouri et al. (2019). Furthermore, the empirical analysis confirmed the existence of significant interdependence within the cryptocurrency market, aligning with the conclusions drawn in recent studies such as Corbet et al. (2018) and Yi et al. (2018), which focused on volatility spillover effects.

Interestingly, the study identified noteworthy relationships between the explosive behaviours of cryptocurrencies. Ethereum emerged as the cryptocurrency exerting the most influence on others, with its positive-signed effects rapidly increasing during bubble periods. Although Bitcoin played a significant role in explaining the dynamics of other cryptocurrencies' prices, the study did not find strong evidence of its effect during explosive phases.

Diversification:

Different research has been made regarding the diversification effect of cryptocurrencies and the findings from these studies indicated that cryptocurrencies can offer diversification benefits. For instance, studies by Gandal et al. (2014) and Bouri et al. (2017) suggest that the inclusion of cryptocurrencies in traditional portfolios can lead to an increase in expected return per unit of risk. Additionally, Briere et al. (2015) utilized Mean-Variance spanning tests and found that investments in Bitcoin can provide significant diversification benefits.

In Anyfantaki and al. (2019), they employed a stochastic spanning methodology to test whether cryptocurrencies offer diversification benefits to risk-averse investors. Speculative bubble tests are applied to four cryptocurrencies and only Ethereum is found to show a non-locally explosive price behaviour for the full sample. They also conduct an analysis both in- and out-of-sample by constructing and comparing optimal portfolios derived from two respective asset universes: one that includes only the traditional asset classes (equities, bonds and cash) and one that is augmented with Ethereum. They found out that the augmented portfolio showed a relative outperformance, with the Ethereum market is segmented from equity and bond markets.

Goodell and Goutte (2020) examined the diversifying effect of cryptocurrencies during the pandemic COVID-19 crisis. With data from 01 May 2019 to 11 May 2020, they included a set of seven cryptocurrencies with sixteen indices from different geographic marketplaces. The concluded that most cryptocurrencies are negatively correlated with the VIX index, arguing against the efficacy of cryptocurrencies as diversifiers during economic downturns. Bitcoin futures was the exceptions, with co-movements lagged of two weeks to one month and always positively correlated (right arrows) with the VIX, suggesting a diversifying effect during extreme economic downturns. Regarding the other indices, cryptocurrencies showed positive correlation and co-movements with traditional markets indices, affirming that most cryptocurrencies are not safe havens and does not offer interesting diversifying effect.

Section 2.4 Risk modelling

To capture the performance of the investments portoflios and to analyse the impact of the introduction of cryptocurrencies, various models have been implemented to quantify those performances.

Initially, I focused on risk quantification using both basic and more advanced models implemented in the financial industry. Subsequently, I delved into a deeper examination of return performance by employing widely used return ratios.

2.4.1 Volatility and standard deviation

Volatility is the most crucial aspect to measure and understand risk in the financial industry. Markowitz (1952) developed what will be called the modern portfolio theory, emphasing the relation between expected return and risk, risk being defined by the volatility. Black and Scholes (1973) also concentrate on volatility to valuing and pricing option contracts and derivatives.

Many investors regularly use basic statistical metrics such as variance and standard deviation to have first incentives regarding the volatility of a particular security or a portfolio.

The Variance quantifies the average squared deviation of individual data points from the arithmetic mean. Mathematically, for a data set of size n, the variance is calculated as the sum of squared differences between each data point and the mean, divided by (n-1):

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Where $\bar{x}$ represents the mean, x_i the observerd data.

The standard deviation is simply the root square of the variance, define as:

$$\sigma = \sqrt[2]{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Nevertheless, these variables are not considered as coherent and proper risk measures. Artzner and al. (1999) affirm that a property risk measure should satisfy the following four properties:

1) Translation-invariance:

For any constant c, $R(L+c) = R(L) + c$

- If losses increase by c, risk measure increases by c

2) Positive homogeneity:

For any constant c > 0, $R(cL) = cR(L)$

- If losses are multiplied by c, risk measure is multiplied by c

3) Monotonicity :

Given two losses L_1, L_2 such that $L_1 \leq L_2$ with probability one, we have $R(L_1) \leq R(L_2)$

- Positions that lead to higher losses in all scenarios are more risky

4) Subadditivity:

Given two losses L_1, L_2, $R(L_1 + L_2) \leq R(L_1) + R(L_2)$

- Risk can be reduced by diversification ;
- Subadditivity allows decentralization of risk management: if total loss L is made of loss of two business units L_1 and L_2, ensuring that $R(L) \leq M$ can be done by imposing the constraints $R(L_1) \leq M_1$ and $R(L_2) \leq M_2$ with $M_1 + M_2 = M$ to each business unit.

Both variance and standard deviation do not satisfy the property of translation invariance, neither monotonicity; variance do not satisfy the sub-additivity property, neither positive homogeneity. Therefore, other risk measure has to be taken into account in our analysis.

2.4.2 Univariate volatility modelling

In financial time series, volatility is a time-varying phenomenon due to different factors impacting the market (illiquidity, news announcements, uncertainty, …). Due to the restrictive reprensentation of the volatily set by both traditional variance and standard deviation, using more advanced volatility model such as the univariate volatility model allow a better representation of the volatility of each variable.

2.4.2.1 ARCH

The ARCH (AutoRegressive Conditional Heteroskedasticity) model developped by Engle (1982) is set as a standard tool to anlyze the volatility. It is defined as follow:

$$r_t = \mu_t + \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2$$

$$\varepsilon_t = \sigma_t e_t$$

$$e_t \sim N(0, 1)$$

Where μ_t is the conditional mean; ε_t is the variance of the shock, depending on the past shocks $\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-P}$

The major drawback of the model is the necessity of lags to capture the dependence structure encountered, which require to estimate many parameters before executing the model. To overpass that issue, a general version of the ARCH model has been set up.

2.4.2.2 GARCH

Bollerslev (1986) proposed an alternative version of the model called GARCH (Generalized AutoRegressive Conditional Heteroskedasticity), more flexible, defined as follow:

$$r_t = \mu_t + \epsilon_t$$

$$\sigma_t^2 = \omega + \sum_{p=1}^{P} \alpha_p \varepsilon_{t-p}^2 + \sum_{q=1}^{Q} \beta_q \sigma_{t-q}^2$$

$$\varepsilon_t = \sigma_t e_t$$

$$e_t \sim N(0, 1)$$

Where μ_t is the conditional mean; ε_t is the variance of the shock, depending on the past shocks ε_{t-P}; Q is the lags of the conditional variance σ_{t-q}^2.

2.4.3 Dependence

Once we have a better comprehension of the volatility of each cryptocurrency, the following step was to understand the relationship between these assets and traditional ones. One way to do so is by capturing their relative dependence. Various measures can be used for this purpose.

2.4.3.1 Regular Correlation

Correlation serves as a static measurement tool, capturing the presence of a linear relationship between two variables. Its primary objective is to establish connections in a straightforward manner, devoid of any cause-and-effect analysis. The correlation coefficient quantifies the strength of this relationship, providing insights into the statistical significance of a sample. The coefficient ranges between -1 and 1, where -1 indicates a perfect negative correlation, 1 indicates a perfect positive correlation, and 0 indicates no correlation between the variables.

Mathematically, the correlation coefficient (Pearson correlation) between two variables, X and Y, with a dataset of n data points, can be calculated using the following formula:

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\ \sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

Where X_i and Y_i are the individual data points of X and Y, respectively; $\bar{X}$ and $\bar{Y}$ are respectively the means of the X and Y datasets.

However, one important drawback of the regular correlation is the unique value reprenting the relation between the two variables, which does not provide a broader and time-varying view on that relationship. To cover that isse, I will use the Rolling Correlation, which add a time-varying element on the correlation coefficients.

2.4.4 Value-at-Risk (VaR)

One first coherent and highly used risk measure is the Value-at-Risk, pioneered by JPMorgan in 1993. The Value-at-Risk at level α (VaR_α) is the minimum loss of the α worst losses one could expect to lose over a specific period.

Let be L a loss distribution function with confidence level $\alpha \in]0,1[$; the VaR of L is:

$$VaR_\alpha(L) = min\{x \mid F_L(x) \geq \alpha\}$$

With $F_L(x)$ the cumulative distribution function of L.

Because loss distributions can have different shape, different version of the VaR can be performed.

2.4.4.1 Gaussian VaR

A first way of estimating VaR is to assume that the loss L follow a Gaussian distribution with mean μ_L and standard deviation σ_L. For normally distributed random variables, VaR is proportional to the standard deviation.

Let be X a random variable that models returns distributed normally with mean μ and standard deviation $\sigma, i.e. X \sim N(\mu, \sigma^2)$. This variable can be written as a function of the standard normal variable $z \sim N(0,1)$ as follows:

$$X = \mu + Z\sigma$$

denote by z_α the standard Gaussian quantile at threshold α α, which is to say, $F_z(z_\alpha) = \alpha$, where F is the cumulative distribution function (cdf) in the standard normal case. The quantile q_α for X is then identified by $F_X(q_\alpha) = \alpha$ for the corresponding distribution function FX of the normal variable X and can then be written as follow:

$$q_\alpha = \mu + z_\alpha \sigma$$

Then, the expression the VaR is:

$$VaR = -q_\alpha$$

However, many market variables, including cryptocurrencies, do not necessarily follow a Gaussian distribution. To overcome that problem, a second alternative of the VaR can be implemented, called the Cornish-Fisher VaR, which incorporates skewness and kurtosis characteristics of distributions.

2.4.4.2 Corn-Fisher VaR

The Corn-Fisher VaR is an expansion of the normal VaR, and is defined as follows:

Let be z_α and $z_{CF,\alpha}$ the Gaussian and the resulting Cornish-Fisher quantiles, respectively we obtain the following expression for the normalized Cornish-Fisher quantile:

$$q_{CF,\alpha} = \mu_L + \sigma_L \left(z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)\,\varsigma_L + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)\,\kappa_L - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)\,\varsigma_L^2 \right)$$

Where ς_L is the skewness and κ_L the excess kurtosis.

Then, the expression the VaR is:

$$VaR = -q_{CF,\alpha}$$

A major drawback of the Value at Risk measure is that VaR ignores the magnitude of the loss in case the VaR threshold is crossed. To overcome that issue, we will use the Expected Shortfall.

2.4.5 Expected Shortfall (ES)

The Expected Shortfall (ES) is another highly used measure in financial risk management. It measures the expected loss (the average loss) when the loss is known to be worse than the VaR at a certain confidence level, here denoted 1-α.

It is a very complementary measure with VaR because it addresses the drawbacks of the VaR by accounting losses beyond the VaR and respect the subadditive property of coherent risk measure. The mathematical definition is:

$$ES_\alpha(L) = E(L|L \geq VaR_\alpha(L))$$

Also equal to average VaR above probability α:

$$ES_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_q(L)dq$$

2.4.5.1 Gaussian Expected Shortfall

If the loss distribution is assumed Gaussian, $L \sim N(\mu_L, \sigma_L)$, then the expected shortfall can be shown to be:

$$ES_\alpha(L) = -\left(\mu_L + \frac{\sigma_L}{1-\alpha}\phi(z_\alpha)\right)$$

Where ϕ is the density of the $N(0, 1)$,

$$\phi(z_\alpha) = \frac{1}{\sqrt{2\pi}}e^{-z_\alpha^2/2}$$

But as discussed in the Gaussian VaR, financial market variables such as cryptocurrencies does not follow the Gaussian distribution. Therefore, we will use again the Cornish-Fisher expansion, which might provide better estimates.

2.4.5.2 Corn-Fisher Expected Shortfall

When the loss distribution is not assumed to be gaussian, one alternative is the Corn-Fisher ES, which more flexible in accommodating non-normal distributions. Mathematically, the Cornish-Fisher Expected Shortfall (ES) can be approximated as follows:

$$ES_{\alpha}(L) = -(\mu_L + w_{\alpha}\sigma)$$

With:

$$w_{\alpha} = \frac{1}{\alpha} \times \phi(z_{\alpha}) \left[1 + z_{\alpha}\left(\frac{\varsigma_L}{6}\right) + (1 - 2z_{\alpha}^2)\left(\frac{\varsigma_L^2}{36}\right) + (-1 + z_{\alpha}^2)(\frac{\kappa_L}{24})\right]$$

Section 2.5 Modelling return

After modelling the risk, the next step was to capture the performance in term of absolute return. Therefore, return ratios have been implemented.

2.5.1 Sharpe Ratio

The Sharpe ratio represents the excess return per risk unit. It is computed by taking the difference between the return of the portfolio and the performance of the benchmark indicator, with which we compare the performance of the portfolio, over a certain period of time, divided by the standard deviation of the portfolio.

Ex-Post Sharpe ratio

Let R_{Pt} represent the return of the portfolio in period t, R_{Bt} the return on the benchmark portfolio or security in period t , and D_t the differential return in period t:

$$D_t = R_{Pt} - R_{Bt}$$

Let D-bar be the average value of D_t over the historic period from *t*=1 through T:

$$\bar{D} = \frac{1}{T}\sum_{t=1}^{T} D_t$$

and $sigma_D$ be the standard deviation over the period:

$$\sigma_D = \sqrt{\frac{\sum_{t=1}^{T}(D_t - \bar{D})^2}{T-1}}$$

The ex-post, or historic Sharpe Ratio (S_h) is:

$$S_h = \frac{\bar{D}}{\sigma_D}$$

The Sharpe ratio of a single portfolio, analysed alone, do not offer a comprehensive understanding of the risk-adjusted return of that portfolio. But it is the comparison of multiple Sharpe ratios of different portfolios that allows for a more insightful interpretation of the risk-adjusted performance. Furthermore, the Sharpe ratio is based on the total risk of an investment. Therefore, its use is most appropriate when an investor intends to place all (or nearly all) of his wealth in one security or portfolio.

2.5.2 Treynor ratio

The Treynor ratio is another measure of the performance. While the Sharpe ratio focus on the total risk of the portfolio, the Treynor ratio instead measures the excess return over the systematic risk. When an investor is considering the addition of an investment to a well-diversified portfolio, the Treynor ratio is more appropriate, because it is based only on systematic risk, removing the idiosyncratic risk of the portfolio.

Let R_{Pt} represent the return of the portfolio in period t, R_{Bt} the return on the benchmark portfolio or security in period t , and D_t the differential return in period t:

$$D_t = R_{Pt} - R_{Bt}$$

Let D-bar be the average value of D_t over the historic period from *t*=1 through T:

$$\overline{D} = \frac{1}{T}\sum_{t=1}^{T} D_t$$

And $Beta_\beta$ be the ratio of the covariance (Cov) between the portfolio and the market benchmark over the variance of the market benchmark:

$$\beta_{Pt} = \frac{Cov(R_{Pt}, R_{Bt})}{\sigma_{Bt}^2}$$

Where $Cov(R_{Pt}, R_{Bt})$ is defined by the multiplication between the coefficient of correlation between the portfolio and the market benchmark, the standard deviation of the portfolio and the standard deviation of the market benchmark:

$$Cov(R_{Pt}, R_{Bt}) = \rho_{Pt,Bt}\sigma_{Pt}\sigma_{Bt}$$

The Treynor ratio is:

$$T_h = \frac{\overline{D}}{\beta_{Pt}}$$

2.5.3 Jensen's Alpha

The last ratio implemented is the Alpha of Jensen, a performance ratio that measure the difference between actual return and predicted return (by the CAPM).

The Jensen's Alpha is defined as:

$$\alpha_P = R_{Pt} - [R_{Ft} + \beta_{Pt}(R_{Bt} - R_{Ft})]$$

Chapter 3: Methodology

Section 3.1. Data set & Portfolios

After defining the models used to capture the performance of the investment portfolios, I operated the selection of the cryptocurrencies and traditional assets that will be implemented in these investment portfolios. Then, I defined the Strategic Asset Allocation method used to determine the allocation within each investment portfolio.

3.1.1 Cryptocurrencies

In order to make the selection of the cryptocurrencies that will be implemented in this master thesis, I selected 5 of the biggest market capitalization cryptocurrencies as of January 15, 2023, and that recurring in published research related to cryptocurrencies. Here is the list of those cryptocurrencies with a short description of their main characteristics:

Description:

- Bitcoin: the most famous crypto money by far, with the highest market capitalization, representing almost 39.5% of the global capitalization of all crypto currencies. The first bitcoins were available in 2009. In 2022, there were around 260 thousand daily transactions of BTC. Its main characteristic is the specific blockchain technology it uses it is one of the first crypto using the “Proof of Work” system.
- Ethereum: ETH is the second highest market capitalization of the crypto currencies. Additionally, it is the most traded one, with around 1.1 million transaction per day. One of the key aspects of the transaction volumes are the relatively low transactions costs compared to other crypto currencies. Contrary to Bitcoin, which only a form of currency, Ethereum is way broader that been a simple currency: is a decentralized platform that enables smart contracts and decentralized applications (called “dApps”) to be built and run without any downtime or interference from a third party. Smart contracts relate to other services: basically, a computer program automatically executes the terms of a contract when certain conditions are met, and it is self-executing with the terms of the agreement written. They can be used in a variety of industries and applications, such as supply chain management, real estate, and financial services.
- Ripple: XRP is a digital currency and a blockchain-based payment protocol that aims to enable fast, affordable, and reliable cross-border payments. Ripple's main use case is

for facilitating international money transfers for financial institutions and other organizations, such as banks and payment providers. Its protocol allows for near-instant, low-cost transactions between different currencies, including both fiat and cryptocurrencies. Ripple has partnerships with many well-known financial institutions, such as Santander and American Express.

- Cardano: ADA is, as Ethereum, a decentralized platform that runs smart contracts and decentralized applications. The specificity of Cardano is to have a unique multi-layer architecture that separates the computation layer, which runs the smart contracts, from the settlement layer, which records transactions on the blockchain.
- Solana: SOL is a high-performance blockchain platform that facilitates the execution of decentralized applications and smart contracts. Solana's architecture is built to handle high-performance decentralized applications and decentralized finance (DeFi) solutions. Decentralized Finance (DeFi) is a term used to describe financial applications and services that are built on blockchain technology, and operate on a decentralized, open-source platform. The applications and services provided include a wide range of financial products and services, such as lending and borrowing platforms, exchanges, stablecoins, insurance, …

Overall, we can see there are different specific characteristics and goals related to each crypto currency: Bitcoin act as a proper independent monetary system; Ethereum, Solana and Cardano offer broader services than the basic use of currency such as smart contracts or decentralized financial services; Ripple has been made up for international trade.

ETFs

The cryptocurrency market, as seen in the introduction, still an unregulated market in many countries. In order to reflect at best the investment opportunities available to the public regarding the cryptocurrency market and to satisfy to the exigence a master thesis has to face to, I will use Exchange-Traded-Funds (ETFs) securities on crypto, which is one of the single means to invest in cryptocurrencies while benefiting from the protection offered by the regulation of those investment vehicles. ETF are modern and recently introduced investment securities becoming popular to the public. Therefore, one of the biggest drawbacks of using cryptocurrency ETF is the quite short and limited period to perform analysis, with on average 2 years of data history. Therefore, the interpretation of the results obtained from ETF data has to be confirmed using raw exchange rates.

The ETF of the 5 cryptocurrencies selected are: 21Shares Bitcoin; 21Shares Ethereum; 21Shares Solana; 21Shares Ripple, and WisdomTree Cardano. To understand the impact of cryptocurrencies on the performance of "classical" investment portfolios, "classical" portfolios had to be set using traditional assets which are famously shared on the market.

3.1.2 Traditional assets

The primary objective of this master thesis is to explore the relationship between cryptocurrencies and traditional assets. To achieve a comprehensive analysis, it is crucial to include a diverse range of popular traditional assets that are widely adopted. Focusing on specific individual stocks or bonds related to a single company would limit the understanding of these relations. Instead, I have opted to use ETFs based on popular indices, which offer a broader representation of asset classes.

For the fixed income assets, given the vast array of securities available worldwide, a careful selection of the most actively traded ones was made, ensuring a geographically diverse investment landscape. ETFs serve as excellent proxies to invest in various types of fixed income securities, enabling exposure to a large variety of individual securities. The chosen 7 ETFs span short-term, mid-term, and long-term durations, and they include both corporate and government bonds from the US and the EU.

The first three ETFs selected are iShares 1-3 Years Treasury Bond, iShares 7-10 Year Treasury Bond, and iShares 20+ Year Treasury Bond. These ETFs provide exposure to short, medium, and long-term US government bonds. To include exposure to US corporate bonds, I also incorporated iShares 1-5 Year Investment Grade Corporate Bond and iShares Treasury Floating Rate Bond, which include floating rate bonds. For exposure to the European market, two ETFs were chosen: iShares Core Euro Government Bond UCITS, for government bonds, and VanEck iBoxx EURO Corporates UCITS, for corporate European bonds.

Moving on to the equity asset class, I once again selected 5 ETFs to represent US, EU, and emerging market securities. These ETFs are as follows: iShares Euro Stoxx 50 UCITS, which invests in the Euro Stoxx 50, comprising the 50 highest capitalizations in Europe; iShares Core MSCI Emerging Markets, offering exposure to emerging markets; iShares S&P500 Index Fund, which tracks the S&P 500; iShares Nasdaq-100 UCITS EUR, providing access to the Nasdaq-100 converted into euro currency; and iShares MSCI World, which follows the MSCI World index.

The data for cryptocurrencies, equity, and fixed income ETFs were sourced from Euronext.com and Investing.com websites. To compute the risk and performance models, a historical period from June 9th, 2021, to June 9th, 2023, was selected. However, given their recent appearance in the market, some cryptocurrency ETFs disposed from relatively short data period. To ensure a meaningful analysis, investment portfolios are constructed using a consistent period of data available for all ETFs. Therefore, the investment portfolio data is based on a period from May 1st, 2022, to June 9th, 2023, with daily data. This selection allows for a comprehensive assessment of the relationships between cryptocurrencies and traditional assets, while accounting for the varying data availability of cryptocurrency ETFs.

In this analysis, it is important to emphasize that no specific cash instruments were used in the construction of the investment portfolios. The focus of this master thesis was intentionally directed towards comprehending the relationships between cryptocurrencies and traditional assets, specifically represented by Equity and Fixed Income securities. Introducing cash instruments would necessitate the consideration of various currencies frequently used in investment, potentially complicating the analysis and diverting attention from the primary objective.

Moreover, it is important to note that some of the short-term fixed income ETFs included in the portfolios do offer exposure to cash-like securities. These ETFs typically invest in a diversified range of high-quality, short-term debt instruments that exhibit characteristics similar to cash investments. By incorporating such ETFs, the analysis acknowledges the presence of cash-like securities in the portfolio without explicitly introducing specific cash instruments.

Section 3.2 Investment portfolios

The following step was to establish the classical investment portfolios that would be used as a basis, before introducing cryptocurrencies and analyse the performance variation. The Strategic Asset Allocation method, defined by Brennan and al. (1997) to describe the established asset allocation of a portfolio seeking specific exposure to the market, has been used to determine the asset allocation within the portfolios.

To define the asset allocation of the investment portfolios, I used « An asset allocation puzzle » (Canner et al., 1994) to serve as a foundational reference. The authors presented a table illustrating the strategic asset allocation used by four different agents in the financial industry for investment portfolios with varying level of risk: Fidelity Investments, Merrill Lynch, Jane Bryant Quinn, and The New York Times. The table provide insights on how these agents allocate weights of three different asset classes (cash, bonds, and stocks), based on the different risk profiles of the portfolios.

Table 1: Asset allocation table from financial advisors

	Percent of Portfolio		
	Cash	Bonds	Stocks
Fidelity			
Conservative	50	30	20
Moderate	20	40	40
Aggressive	5	30	65
Merill Lynch			
Conservative	20	35	45
Moderate	5	40	55
Aggressive	5	20	75
Jane Bryant Quinn			
Conservative	50	30	20
Moderate	10	40	50
Aggressive	0	0	100
The New York Times			
Conservative	20	40	40
Moderate	10	30	60
Aggressive	0	20	80

Source: Canner and al. (1994). An Asset Allocation Puzzle

Based on the asset allocation strategies of Fidelity and Merrill Lynch, I have defined the composition of three portfolios with varying level of risk as follows: an aggressive portfolio with 70% allocated to stocks (equity) and 30% to bonds, a moderate portfolio with an equal split of 50% in stocks and 50% in bonds, and a conservative portfolio with 30% in stocks and 70% in bonds. To ensure a balanced representation within each asset class, I have equally distributed the weights among the securities present in each asset class.

In order to examine the impact of introducing cryptocurrencies on portfolio performance, I will introduce the crypto asset class in the asset allocation of the portfolios. For this, I have defined three weight allocations for cryptocurrencies: 5%, 10%, and 15%. Each of the three classical portfolios will undergo testing with the inclusion of each individual cryptocurrency separately, and with a diversified basket representing an equal split of the selected cryptocurrencies. Here is a table of the different portfolio's asset allocation with the introduction of cryptocurrencies:

Table 2: Asset allocation of the portfolios

	Percent of Portfolio		
Portfolios	Crypto	Bonds	Stocks
Conservative	0.00	70.00	30.00
Moderate	0.00	50.00	50.00
Aggressive	0.00	30.00	70.00
Conservative 5%	5.00	67.50	27.50
Moderate 5%	5.00	47.50	47.50
Aggressive 5%	5.00	27.50	67.50
Conservative 10%	10.00	65.00	25.00
Moderate 10%	10.00	45.00	45.00
Aggressive 10%	10.00	25.00	65.00
Conservative 15%	15.00	62.50	22.50
Moderate 15%	15.00	45.00	45.00
Aggressive 15%	15.00	22.50	62.50

Source: internal computations

Due to the recent appearance of cryptocurrencies, data periods variated among cryptocurrency ETFs. Consequently, when analysing the impact of introducing a single cryptocurrency, I used the widest period available. However, when comparing different portfolios that include cryptocurrencies, a common period from April 1, 2022, to June 1, 2023, was employed. This allowed for a standardized and consistent comparison across the portfolios containing cryptocurrencies.

Section 3.3 Risk modelling

The Value-at-Risk (VaR) and Expected Shortfall (ES) metrics have been computed on all the portfolios in order to compare the results between the classical portfolios and those including cryptocurrencies. This analysis aims to assess the effect of introducing cryptocurrency at different level on the overall risk performance of each portfolio. The VaR and ES have been tested at different confidence level, 99% and 99.9%, which are levels used in the Basel regulations established for financial institutions.

Section 3.4 Benchmark of return ratios

The benchmarks used in the computation of the three return ratios are the three classical portfolios. Because the objective of this thesis is to understand the influence of introducing cryptocurrencies on the performance of a classical investment portfolio, evolves around comparing the performance variation between the three portfolios that do not include cryptocurrencies and those that are augmented with cryptocurrencies. Therefore, the conservative portfolio will serve as the benchmark for all conservative portfolios augmented with cryptocurrencies, while the moderate and aggressive portfolios will be benchmarks for the augmented alternatives in their respective investment profiles.

Chapter 4: Empirical analysis

Section 4.1 Univariate GARCH

Initially, I conducted a Univariate GARCH (1,1) model to compare the volatility of cryptocurrencies with traditional assets. The results revealed distinct volatility profiles among the different ETFs analysed.

In the case of cryptocurrencies, Ethereum exhibited relatively high and persistent volatility, as indicated by its coefficient of 0.7801. On the other hand, Solana, Cardano, and Ripple displayed relatively lower volatility levels, with constant volatility coefficients of 0.2649, 0.0128, and 0.0693, respectively. It is worth noting that the Univariate GARCH model did not yield statistically significant results for the Bitcoin ETF.

When considering Equity ETFs, both the iShares NASDAQ 100 (EXXT) and iShares MSCI World (URTH) exhibited higher volatility compared to the Cryptocurrency ETFs throughout the observed period. Specifically, they demonstrated constant volatility coefficients of 1.5346 and 1.1243, respectively. Additionally, it was observed that both Cardano and Ripple displayed lower volatility than the iShares EURO STOXX 50 and iShares Core Emerging Markets, with constant volatility coefficients of 0.163 and 0.1057, respectively.

Analysing the Fixed Income ETFs, the iShares Core Euro Government Bond (EUNH) exhibited a relatively low volatility coefficient of 0.0754. In contrast, the iShares 20+ Year Treasury Bond displayed a higher volatility coefficient of 1.0038. The iShares 7-10 Years Treasury Bond had a coefficient of 0.1124, while the iShares 1-5 Year Investment Grade Corporate Bond had a coefficient of 0.0022. The iShares 1-3 Year Treasury Bond and iShares Treasury Floating Rate Bond exhibited even lower volatility, with coefficients of 0.00018 and 0.00014, respectively.

While the results challenge the conventional wisdom and demonstrate that the volatility of crypto ETFs does not uniformly surpass that of traditional assets over the observed period, conclusion have to be cautious due to the economic environment on which was performed the analysis.

Section 4.2 Correlation and dependence

To investigate the relationships and dependencies between cryptocurrencies and the chosen traditional assets, I performed a correlation analysis. The results of this analysis have unveiled several significant findings that shed light on the intricate connections between these financial instruments.

4.2.1 Regular Correlation

Initially, the analysis uncovers that most cryptocurrencies demonstrate substantial positive correlations with a variety of ETFs representing traditional assets. This finding suggests a degree of co-movement between cryptocurrencies and these specific assets. Specifically, Bitcoin, Ethereum, Solana, and Ripple demonstrate strong positive correlations with ETFs such as EUNH, EXXT, IEF, IEMG, IGSB, SHY, TCBT.DE, TLT, URTH, and WFSPX. The correlation values range from 0.83 to 0.91 for Bitcoin, signifying a robust relationship between Bitcoin and the traditional assets under consideration. Similar patterns of strong positive correlations are observed for Ethereum, Solana, and Ripple, implying a potential synchronization in price movements with these ETFs.

Figure 3: Coefficient correlation between cryptocurrencies and traditional assets.

Cryptocurrencies \ Traditional assets	EUEA	EUNH	EXXT	IEF	IEMG	IGSB	SHY	TCBT	TFLO	TLT	URTH	WFSPX
BTC	0.31	0.90	0.83	0.89	0.90	0.91	0.90	0.90	-0.77	0.91	0.84	0.85
ETH	0.42	0.77	0.85	0.75	0.72	0.73	0.74	0.78	-0.68	0.79	0.85	0.88
ADA	-0.31	0.80	0.55	0.65	0.61	0.56	0.66	0.77	-0.43	0.78	0.40	0.53
SOL	0.33	0.92	0.81	0.90	0.90	0.91	0.91	0.92	-0.80	0.92	0.84	0.86
RPL	0.35	0.83	0.76	0.81	0.85	0.84	0.83	0.84	-0.70	0.83	0.82	0.82

Source: internal computations

It is important to highlight that Cardano displays lower correlation strengths compared to other cryptocurrencies. In contrast to the positive correlations observed with most ETFs, an intriguing observation arises concerning the relationship between cryptocurrencies and TFLO (iShares Treasury Floating Rate Bond). Interestingly, all the analysed cryptocurrencies display negative correlations with TFLO.

Furthermore, another significant finding revolves around the negative correlation between Cardano and EUEA (iShares EURO STOXX 50), in contrast to the positive correlations exhibited by the other cryptocurrencies. This divergence indicates that Cardano may have a distinct relationship with European equity markets compared to Bitcoin, Ethereum, Solana, and Ripple. Such divergence may emanate from disparities in market dynamics, investor sentiment, or other unique factors specifically impacting Cardano.

Overall, the correlation analysis offers valuable insights into the relationships between cryptocurrencies and traditional assets. The positive correlations observed highlight the presence of co-movement between cryptocurrencies and the selected ETFs, providing potential opportunities for diversification and risk management strategies.

However, one limitation of this analysis lies in the singular and non-time-varying nature of the correlation values between each cryptocurrency and traditional assets. Additionally, it is essential to recognize that correlation coefficients solely measure linear relationships between variables and do not establish causation.

As a result, in order to gain a comprehensive view of the evolving correlation between cryptocurrencies and traditional assets ETFs and to identify underlying patterns, I undertook an analysis of the rolling correlation. This approach enables us to examine how the correlations change over time, providing dynamic insights into the evolving interactions between these financial instruments.

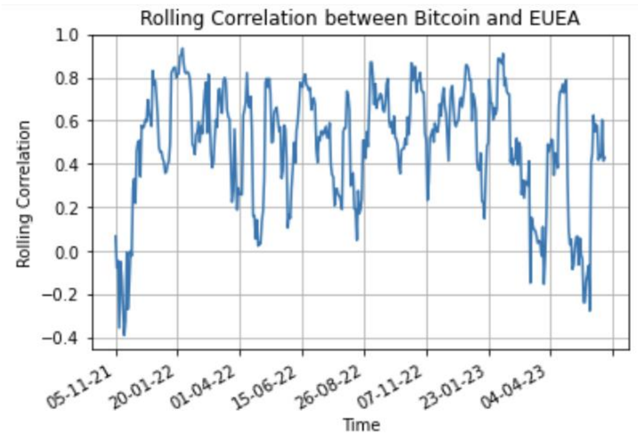
4.2.2 Rolling Correlation

I planned to use the rolling correlation analysis with different window sizes provides a comprehensive understanding of the correlation patterns between cryptocurrencies and traditional assets over varying time horizons. In this study, I aimed to explore these relationships by utilizing window sizes of 10, 30, and 60 days.

The selection of window sizes is a crucial aspect of the analysis as it determines the time frame over which correlations are calculated. By including different window sizes, I can capture correlations at different temporal scales, allowing for a more nuanced interpretation of the data. The choice of a 10-day window aims to examine short-term correlation dynamics, while a 30-day allows me to capture medium-term effects and 60-day window long-term ones. The selection of these window sizes is somewhat arbitrary, but it provides a balance between capturing short-term dynamics, medium-term trends, and longer-term relationships.

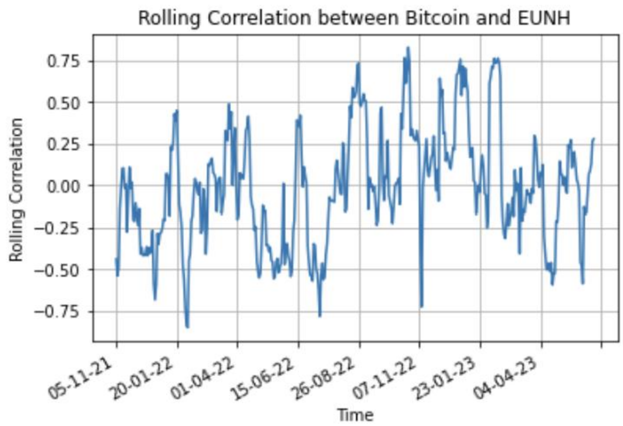
On the 10-day window size, rolling correlations of all cryptocurrencies with traditional assets are hugely volatile, with correlation range bumping between very high positive correlation value and negative correlation. The figure X and XX serve as illustration in the case of Bitcoin. While the regular correlation between BTC and EUNH is set at 0.9, the rolling correlation on a 10-day window is not align with the precedent value, with coefficient evolving in negative territory. This high level of volatility in the rolling correlations suggests that the relationships between cryptocurrencies and traditional assets within short period are not constant and subject to rapid changes. There was no clear trend to draw from this window size, which could be attributed to the inherent volatile nature of cryptocurrencies and/or the non-correlated movements of traditional assets and cryptocurrencies in reaction of news announcement in the short term.

Figure 4: Rolling Correlation 10-day between BTC and EUEA.



Source: internal computations

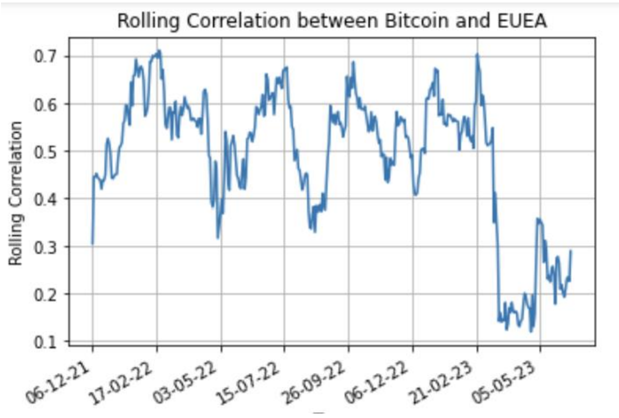
Figure 5: Rolling Correlation 10-day between BTC and EUNH.



Source: internal computations

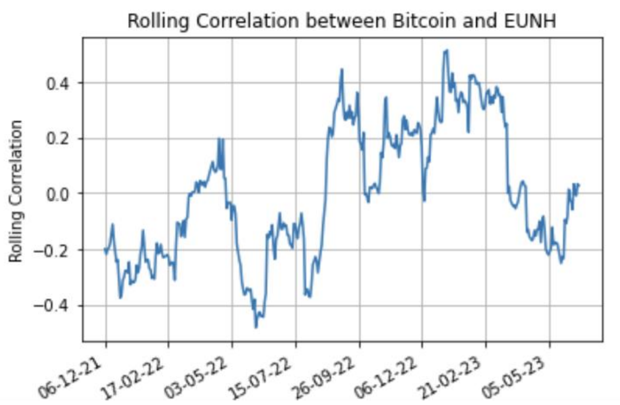
In the 30-day window of the rolling correlation analysis, we observed a reduction in the spread of correlation values with slightly more stabilized relationship between cryptocurrencies and some traditional assets. Specifically, for Bitcoin, positive correlations are evident with assets such as EUEA, EXXT, IEMG, URTH, and WFSPX. Nevertheless, there still lots of volatility in the coefficients and a divergence between regular correlation value and what is observed in the rolling correlation charts. As example, while BTC presents a lower correlation value regarding its relationship with EUEA (0.31) compared to EUNH (0.91), the rolling 30-day correlation tend to show the opposite results with a stronger relationship with EUEA.

Figure 6: Rolling Correlation 30-day between BTC and EUEA.



Source: internal computations

Figure 7: Rolling Correlation 30-day between BTC and EUNH.



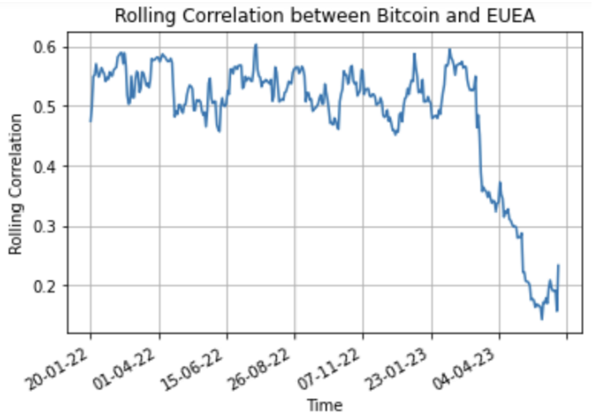
Source: internal computations

Similarly, Ethereum exhibits a similar pattern of positive correlations with EUEA and EXXT, further confirming the convergence of correlations over the medium-term. However, for Cardano, Solana, and Ripple, no clear trends can be identified, suggesting that these cryptocurrencies do not consistently react in a similar manner to traditional assets within this timeframe.

Moving to the 60-day window, the spreads in the rolling correlations become even narrower for some relationship, signifying a more stable and consistent relationship between cryptocurrencies and some traditional assets. As with the 30-day window, Bitcoin and Ethereum maintain their positive correlation trends with certain assets, providing further support for their medium-term associations with the mentioned assets. Nevertheless, the pattern displayed in the 30-day rolling correlation regarding BTC with EUEA and EUNH accentuated, as showed by figures X and XX.

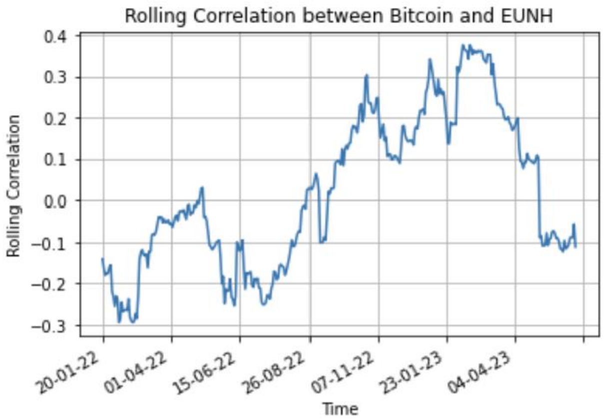
For Cardano and Ripple, trends begin to emerge with certain traditional assets, indicating some alignment over the longer term. However, for Solana, the relationships remain less defined, making it difficult to identify clear trends within this timeframe.

Figure 8: Rolling Correlation 60-day between BTC and EUEA.



Source: internal computations

Figure 9: Rolling Correlation 60-day between BTC and EUNH.



Source: internal computations

When comparing the results of the rolling correlations with the basic correlations computed earlier, there are notable differences regarding the relationship between cryptocurrencies and TFLO. In the initial analysis, TFLO exhibited negative correlations with all cryptocurrencies. However, in the rolling correlation analysis, this trend was not observed, suggesting that the relationship between TFLO and cryptocurrencies may be subject to change over different time horizons.

Overall, because the results obtained from the rolling correlation do not perfectly match those from the regular correlations, cautious is needed when interpreting the relationship between cryptocurrencies and traditional assets. No specific traditional asset classes displayed clear opposite relationship with cryptocurrencies that can be used as potential hedging in the risk of an investment portfolio. While cryptocurrencies presented strong negative correlations with TFLO,

Section 4.3 Value-at-Risk and Expected Shortfall

After analysing the correlation between the selected cryptocurrencies and traditional assets to understand their relationships, the next step is to assess the potential risk associated with introducing cryptocurrencies within an investment portfolio. After conducting the computation of the VaR and ES for various portfolios, distinct trends emerged from the analysis.

Figure 10: VaR and ES metrics of classical portfolios

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive	0.472291	0.473435	-0.514605	-0.511123
Portfolio Moderate	0.361435	0.362309	-0.393816	-0.391152
Portfolio Conservative	0.253803	0.254417	-0.276542	-0.274671

Source: internal computations

For the conservative portfolios, introducing cryptocurrencies worsen the risk metrics of the portfolios, whatever the level of at which cryptocurrencies has been added into the asset allocation. Furthermore, VaR and ES measures worsen as the weights allocated to cryptocurrencies in the asset allocation increase. Compared one to each other, Ethereum is the cryptocurrencies that have the lower negative impact of the risk metrics of the portfolios. The introduction of the cryptocurrency basket does not provide better results that cryptocurrencies alone, with the exception of Solana.

Regarding the moderate portfolios, the results show the same patterns as for the conservative portfolio, with VaR and ES values deteriorating with the increase of the weights of the cryptocurrencies. Once again, portfolios augmented with Ethereum displayed the best results among the other cryptocurrencies, as well as the basket of crypto.

Concerning aggressive portfolios, we can find the first improved metrics when introducing cryptocurrencies in the asset allocation. More precisely, as shown in figure 11 below, portfolios augmented with Ethereum at 5% level of the asset allocation displayed better risk metrics. At 10% and 15%, however, the positive effect of the introduction of Ethereum is removed. These findings suggest a potential diversification benefit of including Ethereum at small weights in the asset allocation of a portfolio.

Figure 11: VaR and ES metrics of classical portfolios augmented with ETH

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% ETH	0.465486	0.467973	-0.507189	-0.499664
Portfolio Moderate 5% ETH	0.380857	0.382892	-0.414979	-0.408822
Portfolio Conservative 5% ETH	0.2869	0.288433	-0.312604	-0.307966
Portfolio Aggressive 10% ETH	0.474509	0.477044	-0.517021	-0.50935
Portfolio Moderate 10% ETH	0.387283	0.389353	-0.421981	-0.41572
Portfolio Conservative 10% ETH	0.290216	0.291767	-0.316217	-0.311525
Portfolio Aggressive 15% ETH	0.485225	0.487817	-0.528697	-0.520852
Portfolio Moderate 15% ETH	0.395852	0.397967	-0.431317	-0.424917
Portfolio Conservative 15% ETH	0.296699	0.298284	-0.32328	-0.318484

Source: internal computations

The use of different methods when computing VaR and ES (Gaussian and Corn-Fisher) does not affect the conclusion of the findings. The only notable deviation pertains to the ES of the high-risk portfolio, enhanced with a 10% allocation to Ethereum, as observed in the Corn-Fisher extension. Furthermore, the choice of confidence level employed had no influence on the findings.

Section 4.4 Performance ratio

To measure the return performance of the portfolios, I implemented the three ratios described in the methodology (Sharpe ratio, Treynor ratio and Jensen's Alpha) and compared the performance between the portfolios (see appendix 40 to 45 for more precision on the results).

4.4.1 Sharpe ratio

When computing the Sharpe ratios, the analysis revealed a consistent pattern across the various cryptocurrencies. When introducing cryptocurrencies at 5% level of the asset allocation, the portfolios exhibit negative value, emphasizing a degradation of the performance of the portfolios.

More precisely, introducing BTC at 5% of the total asset allocation, the Sharpe ratios exhibit negative values of -0.319, -1.315, and -3.310 for the aggressive, moderate, and conservative portfolios, respectively. Furthermore, as the allocation of BTC increases to 10% and 15%, the Sharpe ratios continue to deteriorate, indicating a decrease in risk-adjusted returns.

Similarly, the portfolios with 5% allocation to ETH exhibit negative Sharpe ratios of -0.368, -1.375, and -3.387 for the aggressive, moderate, and conservative portfolios, respectively. The introduction of higher levels of ETH (10% and 15%) leads to a further decline in the portfolios' Sharpe ratios, reflecting diminished risk-adjusted performance.

In the case of ADA, RPL and the mixed basket, results are in line with ETH and BTC, showing worsening Sharpe ratio as the weight allocated to the cryptocurrency asset class increase. Throughout the analysis, portfolio augmented with RPL displayed the best ratios compared to other cryptos, while SOL showed the most negative results, whatever the portfolios risk profile.

These findings demonstrate that as the portfolio adopts a lower risk profile, its performance suffer from a greater deterioration upon the inclusion of cryptocurrencies within its allocation. Adjusted performances also decline as the allocation weights increase.

4.4.2 Treynor ratio

The analysis of the Treynor ratios follows the patterns found previously with the Sharpe ratios.

For portfolios with 5% allocation to BTC, the respective Treynor ratios for the aggressive, moderate, and conservative portfolios are -0.0031, -0.0033, and -0.0036. As the allocation to

BTC increases to 10% and 15%, we notice a deterioration in the Treynor ratios, indicating a decline in risk-adjusted returns for all three portfolio types.

Similarly, when considering portfolios with 5% allocation to ETH, the respective Treynor ratios for the aggressive, moderate, and conservative portfolios are -0.0041, -0.0043, and -0.0046. As the allocation to ETH increases to 10% and 15%, the Treynor ratios continue to deteriorate, suggesting diminishing risk-adjusted returns.

In the case of ADA, RPL and the mixed basket, results are in line with ETH and BTC, showing worsening ratios as the allocation weight allocated to cryptocurrencies increase. As for the Sharpe ratio analysis, RPL and SOL respectively showed the best and worst performance among the augmented portfolios.

4.4.3 Jensen’s Alpha

The Jensen’s alphas of the portfolios exhibit outcomes consistent with the trends underlined.

For portfolios with a 5% allocation to BTC, the computed Jensen's Alpha values for the aggressive, moderate, and conservative portfolios are -0.0026, -0.0040, and -0.0067, respectively. These negative Alpha values indicate that these portfolios underperformed compared to their expected returns based on their risk exposures. When the allocation to BTC is increased to 10% and 15%, the portfolios' Jensen's Alpha further deteriorates. This suggests that incorporating a higher percentage of BTC in the portfolios led to a decline in their risk-adjusted performance.

The same trend is observed when considering portfolios with different allocations to other cryptocurrencies such as ETH, ADA, SOL, and RPL. Regardless of the cryptocurrency chosen, increasing the allocation percentage leads to a deterioration in Jensen's Alpha, indicating a decline in risk-adjusted performance.

Moreover, when evaluating a mixed basket of cryptocurrencies with a 5% allocation, the portfolios exhibit negative Jensen's Alpha values of -0.0092, -0.0112, and -0.0146 for the aggressive, moderate, and conservative portfolios, respectively. At 10% and 15%, results deteriorate.

Chapter 5: Conclusion

The last part of this master thesis will be composed of a conclusion over the results and findings obtained from the different analysis, as well as the limits faced during the analysis and suggestions for future in-depth research on the subject.

Findings

The empirical results obtained throughout different analysis reveal that the inclusion of cryptocurrencies in investment portfolios may lead to a deterioration of its performance.

Indeed, cryptocurrencies do not offer specific hedging opportunities against equity or fixed income asset classes. Their overall strong positive correlation with traditional assets diminishes their potential as effective hedging instruments. The geographical location of the traditional assets does not influence the results. Furthermore, these correlations are very sensible over time, with high fluctuations and unstable relationship.

From a risk perspective, most cryptocurrencies accentuate the magnitude of extreme potential losses when assessed in portfolios. Only Ethereum, when marginally included in the asset allocation of high risk-profile portfolios, provided a mitigating effect on VaR and ES metrics. Additionally, the risk metrics deteriorate as the weights of the cryptocurrency asset class in the total allocation of the portfolio the grow up.

Finally, the comprehensive analysis conducted on the risk adjusted performance of the portfolios augmented with cryptocurrencies consistently demonstrated that the introduction of cryptocurrencies in portfolios result in lower adjusted performance, regardless of the risk profile of the portfolio.

The research findings deviate from the existing literature and indicate a restricted diversification potential presented by cryptocurrencies, cautioning against their consideration as investment prospects for inclusion within investment portfolios. Nonetheless, I would refrain from endorsing such conclusion and recommend exercising caution given the diverse array of findings gathered given the limitations faced during the redaction of this research.

Limitations

First, the recent appearance of Cryptocurrencies ETFs does not allow to use a wide period of past performance to have a more comprehensive view on the overall analysis. A longer period of analysis would have provided more accurate estimations, particularly regarding correlations and risk measures.

Subsequently, the economic environment prevailing for most of the data period might have had a huge influence on the results, which is for me the principal cause of the diverging findings of this research from the literature. When performing the Univariate GARCH models at the start of the empirical analysis, some traditional assets, such as the ETFs on the NASDAQ 100, MSCI World and 20+ Year Treasury Bond, exhibited higher volatility than the cryptocurrencies over the analysed period. Those results are distinct from the literature and displayed abnormal level of volatility on certain assets.

The Ukrainian war started in April 2022 created shocks in financial markets. Furthermore, the inflation presents since January 2022, reaching at some point 40 years record high, followed by massive rate hike, and the banking crisis throughout March and April 2023, marked by the bankruptcy of Silicon Valley Bank and the near bankruptcy of Crédit Suisse, added stress in financial markets, resulting abnormal high volatility in the different assets classes. Operate similar research during a different economic environment could have displayed different results.

From a personal point of view, the second limitation was decisive in the results obtained throughout this research, which prevented me from addressing the central question on which this study is based.

Suggestions

Regarding the suggestions towards potential future research on the subject, I would recommend at first similar research in the next 2-3 years with a longer data period available for ETFs on cryptocurrencies in order to compare the new findings with this research.

Another suggestion would be to include other alternative asset classes in the research, such as real estate, raw materials, and gold. This could provide a broader view on the relationship between cryptocurrencies and different investments.

Finally, a last suggestion would be to use a different prism in the interpretation of cryptocurrencies, not as a specific alternative asset class, but as proper currencies, giving access to different environments of investment, such as smart contracts for example. Exploring the environment of the Decentralized Finance (DeFi), which is currently becoming increasingly popular, would offer other perspectives of investments.

Appendix

Appendix 1: BTC Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    BTC   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -931.426
Distribution:                  Normal   AIC:                           1870.85
Method:            Maximum Likelihood   BIC:                           1886.87
                                        No. Observations:                  405
Date:                Wed, Jun 14 2023   Df Residuals:                      404
Time:                        17:45:39   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu             6.3728      0.782      8.153  3.554e-16 [  4.841,  7.905]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega          0.0844      0.580      0.146      0.884 [ -1.052,  1.221]
alpha[1]       0.9670      5.402      0.179      0.858 [ -9.621, 11.555]
beta[1]    7.9963e-14      5.204  1.536e-14      1.000 [-10.201, 10.201]
========================================================================
```

Appendix 2 : ETH Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    ETH   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -1453.04
Distribution:                  Normal   AIC:                           2914.08
Method:            Maximum Likelihood   BIC:                           2930.92
                                        No. Observations:                  498
Date:                Wed, Jun 14 2023   Df Residuals:                      497
Time:                        17:52:38   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            18.4672      0.408     45.244      0.000 [ 17.667, 19.267]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.7801      0.445      1.753  7.953e-02 [-9.190e-02,  1.652]
alpha[1]       0.9745      0.271      3.593  3.270e-04     [  0.443,  1.506]
beta[1]    1.7277e-16      0.192  9.015e-16      1.000     [ -0.376,  0.376]
============================================================================
```

Appendix 3 : ETH Univariate GARCH (1,1)

```
================================================================================
Dep. Variable:                    ETH   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -1453.04
Distribution:                  Normal   AIC:                           2916.08
Method:            Maximum Likelihood   BIC:                           2937.14
                                        No. Observations:                  498
Date:                Wed, Jun 14 2023   Df Residuals:                      497
Time:                        17:52:59   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            18.4672      0.407     45.334      0.000 [ 17.669, 19.266]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega          0.7800      0.761      1.024      0.306 [ -0.712,  2.272]
alpha[1]       0.9745      0.270      3.613  3.027e-04 [  0.446,  1.503]
alpha[2]   9.2700e-14      0.560  1.656e-13      1.000 [ -1.097,  1.097]
beta[1]        0.0000      0.611      0.000      1.000 [ -1.197,  1.197]
========================================================================
```

Appendix 4 : ADA Univariate GARCH (1,1)

```
================================================================================
Dep. Variable:                    ADA   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -349.039
Distribution:                  Normal   AIC:                           706.078
Method:            Maximum Likelihood   BIC:                           720.840
                                        No. Observations:                  296
Date:                Wed, Jun 14 2023   Df Residuals:                      295
Time:                        17:53:51   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu             4.2896  4.114e-02    104.263      0.000 [  4.209,  4.370]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.0128  3.782e-03      3.394  6.885e-04  [5.424e-03,2.025e-02]
alpha[1]       0.9700  5.161e-02     18.794  8.474e-79     [  0.869,  1.071]
beta[1]        0.0300  2.672e-02      1.124      0.261 [-2.232e-02,8.241e-02]
=============================================================================
```

Appendix 5 : SOL Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    SOL   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -1581.18
Distribution:                  Normal   AIC:                           3170.37
Method:            Maximum Likelihood   BIC:                           3186.49
                                        No. Observations:                  416
Date:                Wed, Jun 14 2023   Df Residuals:                      415
Time:                        17:55:36   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            13.5632      0.252     53.750      0.000 [ 13.069, 14.058]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.2649      0.134      1.980  4.772e-02 [2.658e-03,  0.527]
alpha[1]       0.7987      0.182      4.386  1.154e-05   [  0.442,  1.156]
beta[1]        0.1838      0.183      1.004      0.315   [ -0.175,  0.543]
==========================================================================
```

Appendix 6 : RPL Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    RPL   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -946.566
Distribution:                  Normal   AIC:                           1901.13
Method:            Maximum Likelihood   BIC:                           1917.23
                                        No. Observations:                  413
Date:                Wed, Jun 14 2023   Df Residuals:                      412
Time:                        17:56:39   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            10.7723  6.265e-02    171.934      0.000 [ 10.649, 10.895]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.0693  2.719e-02      2.550  1.079e-02 [1.603e-02,  0.123]
alpha[1]       0.7575  8.890e-02      8.520  1.591e-17   [  0.583,  0.932]
beta[1]        0.2425  9.210e-02      2.634  8.450e-03 [6.203e-02,  0.423]
==========================================================================
```

Appendix 8: EUEA Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                EUEA.AS   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -2594.25
Distribution:                  Normal   AIC:                           5196.51
Method:            Maximum Likelihood   BIC:                           5216.50
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:04:35   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            36.0748      0.178    202.891      0.000 [ 35.726, 36.423]
                             Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.1630  3.650e-02      4.466  7.975e-06 [9.147e-02,  0.235]
alpha[1]       1.0000  7.167e-02     13.954  2.988e-44   [  0.860,  1.140]
beta[1]        0.0000  6.788e-02      0.000      1.000   [ -0.133,  0.133]
==========================================================================
```

Appendix 9: EXXT Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                EXXT.DE   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -4380.87
Distribution:                  Normal   AIC:                           8769.74
Method:            Maximum Likelihood   BIC:                           8789.73
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:06:40   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu           110.3718      0.464    237.929      0.000 [1.095e+02,1.113e+02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega          1.5346      0.510      3.011  2.608e-03    [  0.536,  2.534]
alpha[1]       0.8566  9.728e-02      8.805  1.306e-18    [  0.666,  1.047]
beta[1]        0.1390  9.880e-02      1.407      0.159 [-5.460e-02,  0.333]
===========================================================================
```

Appendix 10 : IEMG Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                   IEMG   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -2948.17
Distribution:                  Normal   AIC:                           5904.33
Method:            Maximum Likelihood   BIC:                           5924.32
                                        No. Observations:                 1094
Date:              Wed, Jun 14 2023     Df Residuals:                     1093
Time:                        18:07:47   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            51.3661      0.111    461.080      0.000 [ 51.148, 51.584]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.1057  4.140e-02      2.554  1.065e-02 [2.459e-02,  0.187]
alpha[1]       0.6126      0.107      5.744  9.255e-09   [  0.404,  0.822]
beta[1]        0.3874      0.116      3.349  8.096e-04   [  0.161,  0.614]
==========================================================================
```

Appendix 11: URTH Univariate GARH (1,1)

```
==============================================================================
Dep. Variable:                   URTH   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -4099.58
Distribution:                  Normal   AIC:                           8207.16
Method:            Maximum Likelihood   BIC:                           8227.16
                                        No. Observations:                 1094
Date:              Wed, Jun 14 2023     Df Residuals:                     1093
Time:                        18:09:07   Df Model:                            1
                               Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu           114.3988      0.727    157.325      0.000 [1.130e+02,1.158e+02]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          1.1243      0.569      1.976  4.816e-02 [9.093e-03,  2.240]
alpha[1]       0.8029      0.191      4.213  2.522e-05   [  0.429,  1.176]
beta[1]        0.1874      0.195      0.959      0.337   [ -0.196,  0.570]
==========================================================================
```

Appendix 12: WFSPX Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                  WFSPX   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -5693.54
Distribution:                  Normal   AIC:                           11395.1
Method:            Maximum Likelihood   BIC:                           11415.1
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:10:06   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu           466.1662      0.839    555.342      0.000 [4.645e+02,4.678e+02]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega         20.1717     15.446      1.306      0.192 [-10.102, 50.445]
alpha[1]       1.0000      0.447      2.237  2.532e-02 [  0.124,  1.876]
beta[1]        0.0000      0.431      0.000      1.000 [ -0.844,  0.844]
------------------------------------------------------------------------
```

Appendix 13: EUNH Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                EUNH.DE   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -2865.43
Distribution:                  Normal   AIC:                           5738.87
Method:            Maximum Likelihood   BIC:                           5758.86
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:11:14   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu           132.3414  8.723e-02   1517.172      0.000 [1.322e+02,1.325e+02]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------
omega          0.0754  1.871e-02      4.028  5.621e-05 [3.871e-02,  0.112]
alpha[1]       0.8568  6.141e-02     13.953  3.037e-44   [  0.736,  0.977]
beta[1]        0.1432  6.185e-02      2.315  2.063e-02 [2.193e-02,  0.264]
========================================================================
```

Appendix 14: IEF Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    IEF   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -3224.38
Distribution:                  Normal   AIC:                           6456.76
Method:            Maximum Likelihood   BIC:                           6476.75
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:12:02   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu           114.1908      0.221    516.665      0.000 [1.138e+02,1.146e+02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega          0.1124  4.331e-02      2.595  9.471e-03  [2.748e-02,  0.197]
alpha[1]       0.8899  7.422e-02     11.990  3.995e-33    [  0.744,  1.035]
beta[1]        0.1101  7.799e-02      1.411      0.158 [-4.279e-02,  0.263]
===========================================================================
```

Appendix 15: IGSB Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                   IGSB   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -1522.17
Distribution:                  Normal   AIC:                           3052.34
Method:            Maximum Likelihood   BIC:                           3072.33
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:13:49   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            53.6606  1.772e-02   3029.003      0.000 [ 53.626, 53.695]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      2.2416e-03  7.274e-04      3.082  2.057e-03 [8.160e-04,3.667e-03]
alpha[1]       0.8218  7.473e-02     10.997  3.963e-28     [  0.675,  0.968]
beta[1]        0.1782  7.746e-02      2.301  2.141e-02   [2.639e-02,  0.330]
============================================================================
```

Appendix 16: SHY Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    SHY   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -1254.30
Distribution:                  Normal   AIC:                           2516.60
Method:            Maximum Likelihood   BIC:                           2536.59
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:14:11   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            86.2746  9.718e-03   8878.124      0.000 [ 86.256, 86.294]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      1.8402e-04  8.981e-05      2.049  4.047e-02 [7.993e-06,3.601e-04]
alpha[1]       0.6051  8.979e-02      6.739  1.597e-11     [  0.429,  0.781]
beta[1]        0.3949  9.156e-02      4.313  1.611e-05     [  0.215,  0.574]
============================================================================
```

Appendix 17: TFLO Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                   TFLO   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                1925.14
Distribution:                  Normal   AIC:                          -3842.27
Method:            Maximum Likelihood   BIC:                          -3822.28
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:15:06   Df Model:                            1
                               Mean Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
mu            50.2900  8.333e-04  6.035e+04      0.000 [ 50.288, 50.292]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      1.4094e-04  6.341e-06     22.225 1.975e-109 [1.285e-04,1.534e-04]
alpha[1]       0.2000  9.918e-03     20.165  1.969e-90     [  0.181,  0.219]
beta[1]        0.7800  9.928e-03     78.569      0.000     [  0.761,  0.799]
============================================================================
```

Appendix 18: TLT Univariate GARCH (1,1)

```
==============================================================================
Dep. Variable:                    TLT   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -4202.23
Distribution:                  Normal   AIC:                           8412.45
Method:            Maximum Likelihood   BIC:                           8432.44
                                        No. Observations:                 1094
Date:                Wed, Jun 14 2023   Df Residuals:                     1093
Time:                        18:15:47   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu           139.1409      0.497    280.068      0.000 [1.382e+02,1.401e+02]
                              Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega          1.0038      0.607      1.654  9.815e-02 [ -0.186,  2.193]
alpha[1]       0.8771      0.179      4.892  9.961e-07 [  0.526,  1.229]
beta[1]        0.1229      0.186      0.659      0.510 [ -0.242,  0.488]
------------------------------------------------------------------------
```

Appendix 19: Value-at-Risk and Expected Shortfall of classical portfolios.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive	0.472291	0.473435	-0.514605	-0.511123
Portfolio Moderate	0.361435	0.362309	-0.393816	-0.391152
Portfolio Conservative	0.253803	0.254417	-0.276542	-0.274671

Appendix 20: Value-at-Risk and Expected Shortfall of portfolios augmented with BTC.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% BTC	0.498931	0.502208	-0.543631	-0.533744
Portfolio Moderate 5% BTC	0.408688	0.411372	-0.445302	-0.437204
Portfolio Conservative 5% BTC	0.308162	0.310186	-0.33577	-0.329664
Portfolio Aggressive 10% BTC	0.50598	0.509303	-0.551312	-0.541285
Portfolio Moderate 10% BTC	0.411893	0.414598	-0.448795	-0.440634
Portfolio Conservative 10% BTC	0.306571	0.308585	-0.334037	-0.327963
Portfolio Aggressive 15% BTC	0.513932	0.517307	-0.559976	-0.549792
Portfolio Moderate 15% BTC	0.415858	0.41859	-0.453116	-0.444875
Portfolio Conservative 15% BTC	0.305615	0.307622	-0.332996	-0.32694

Appendix 21: Value-at-Risk and Expected Shortfall of portfolios augmented with ETH.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% ETH	0.465486	0.467973	-0.507189	-0.499664
Portfolio Moderate 5% ETH	0.380857	0.382892	-0.414979	-0.408822
Portfolio Conservative 5% ETH	0.2869	0.288433	-0.312604	-0.307966
Portfolio Aggressive 10% ETH	0.474509	0.477044	-0.517021	-0.50935
Portfolio Moderate 10% ETH	0.387283	0.389353	-0.421981	-0.41572
Portfolio Conservative 10% ETH	0.290216	0.291767	-0.316217	-0.311525
Portfolio Aggressive 15% ETH	0.485225	0.487817	-0.528697	-0.520852
Portfolio Moderate 15% ETH	0.395852	0.397967	-0.431317	-0.424917
Portfolio Conservative 15% ETH	0.296699	0.298284	-0.32328	-0.318484

Appendix 22: Value-at-Risk and Expected Shortfall of portfolios augmented with ADA.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% ADA	0.520531	0.525223	-0.567166	-0.553086
Portfolio Moderate 5% ADA	0.426056	0.429897	-0.464227	-0.452703
Portfolio Conservative 5% ADA	0.31965	0.322531	-0.348288	-0.339641
Portfolio Aggressive 10% ADA	0.520531	0.525223	-0.567166	-0.553086
Portfolio Moderate 10% ADA	0.426056	0.429897	-0.464227	-0.452703
Portfolio Conservative 10% ADA	0.31965	0.322531	-0.348288	-0.339641
Portfolio Aggressive 15% ADA	0.526698	0.531445	-0.573885	-0.559639
Portfolio Moderate 15% ADA	0.427591	0.431445	-0.465899	-0.454334
Portfolio Conservative 15% ADA	0.315347	0.318189	-0.343599	-0.335069

Appendix 23: Value-at-Risk and Expected Shortfall of portfolios augmented with SOL.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% SOL	0.498692	0.501804	-0.54337	-0.533973
Portfolio Moderate 5% SOL	0.41324	0.415819	-0.450263	-0.442475
Portfolio Conservative 5% SOL	0.318638	0.320627	-0.347186	-0.341181
Portfolio Aggressive 10% SOL	0.522562	0.525824	-0.569379	-0.559532
Portfolio Moderate 10% SOL	0.440237	0.442985	-0.479679	-0.471383
Portfolio Conservative 10% SOL	0.352312	0.354511	-0.383876	-0.377237
Portfolio Aggressive 15% SOL	0.554395	0.557855	-0.604064	-0.593616
Portfolio Moderate 15% SOL	0.47847	0.481456	-0.521337	-0.51232
Portfolio Conservative 15% SOL	0.40259	0.405102	-0.438658	-0.431072

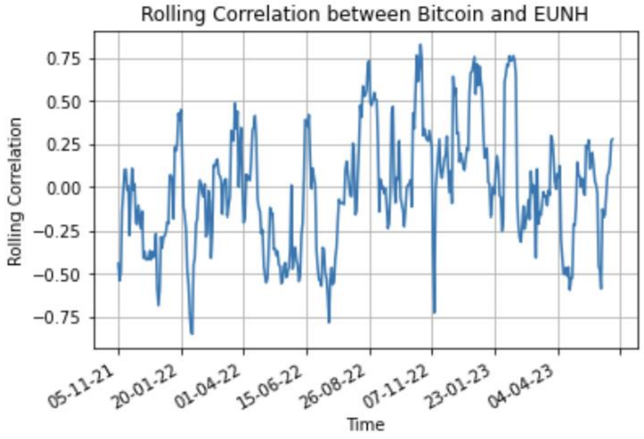
Appendix 24: Value-at-Risk and Expected Shortfall of portfolios augmented with RPL.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% RPL	0.496396	0.499577	-0.540868	-0.531268
Portfolio Moderate 5% RPL	0.402421	0.402421	-0.438474	-0.430691
Portfolio Conservative 5% RPL	0.296955	0.296955	-0.323559	-0.317816
Portfolio Aggressive 15% RPL	0.502453	0.505673	-0.547469	-0.537751
Portfolio Moderate 15% RPL	0.40442	0.407012	-0.440653	-0.432831
Portfolio Conservative 15% RPL	0.293993	0.295877	-0.320332	-0.314646

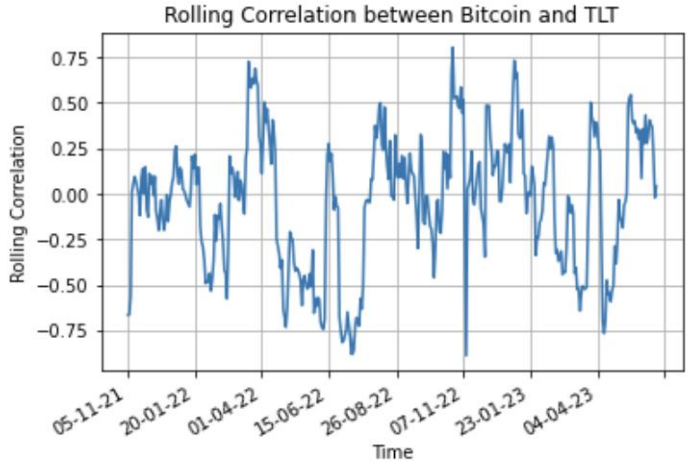
Appendix 25 : Value-at-Risk and Expected Shortfall of portfolios augmented with a basket of cryptocurrencies.

	Confidence level 99.9%			
	Gaussian VaR	Corn-Fisher VaR	Gaussian ES	Corn-Fisher ES
Portfolio Aggressive 5% mixed	0.511992	0.516607	-0.557861	-0.544013
Portfolio Moderate 5% mixed	0.423408	0.427225	-0.461342	-0.449889
Portfolio Conservative 5% mixed	0.324169	0.327091	-0.353212	-0.344444
Portfolio Aggressive 10% mixed	0.518757	0.523433	-0.565234	-0.551202
Portfolio Moderate 10% mixed	0.426378	0.430222	-0.464578	-0.453045
Portfolio Conservative 10% mixed	0.322175	0.325079	-0.35104	-0.342325
Portfolio Aggressive 15% mixed	0.526462	0.531207	-0.573628	-0.559388
Portfolio Moderate 15% mixed	0.430187	0.434065	-0.468728	-0.457092
Portfolio Conservative 15% mixed	0.320918	0.323811	-0.349669	-0.340989

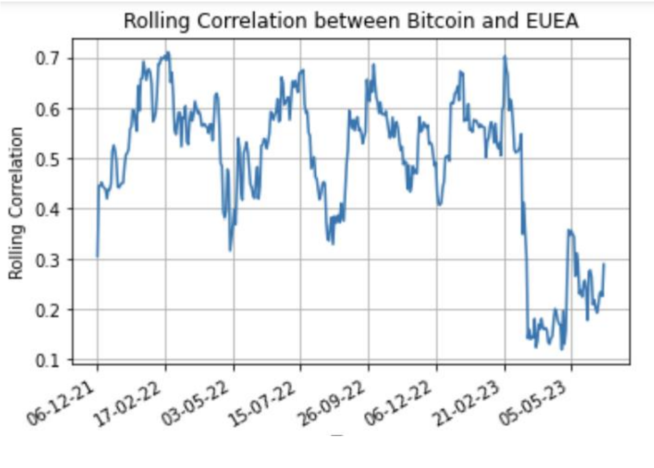
Appendix 26: 10-day rolling correlation between BTC and EUNH



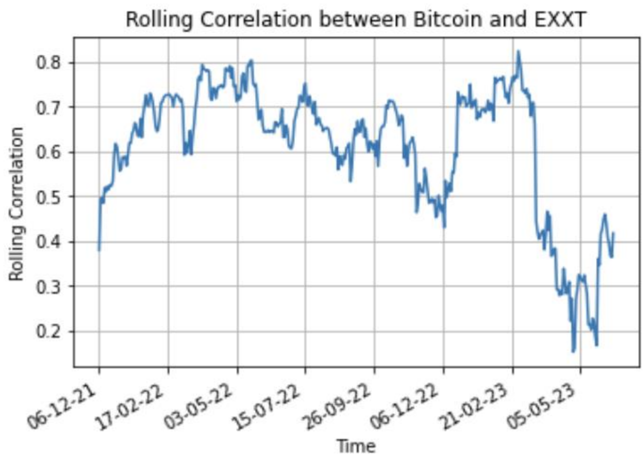
Appendix 27: 10-day rolling correlation between BTC and TLT



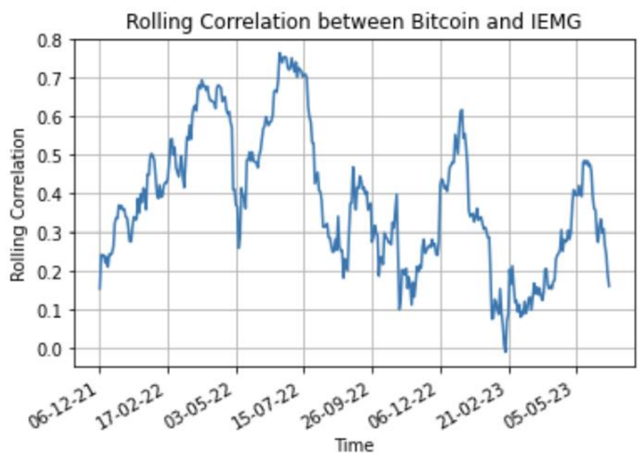
Appendix 28: 30-day rolling correlation between BTC and EUEA



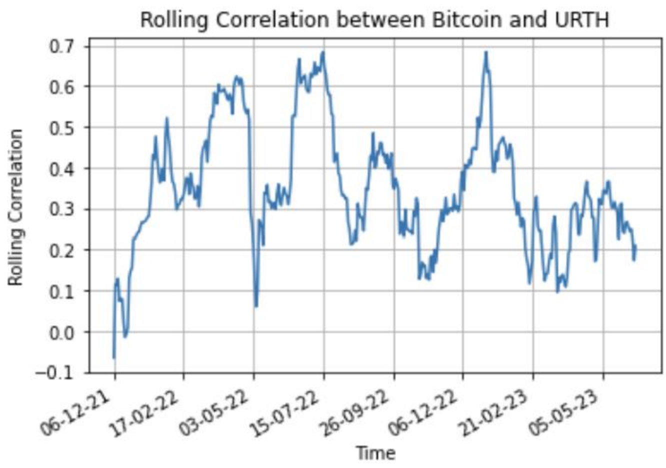
Appendix 29: 30-day rolling correlation between BTC and EXXT



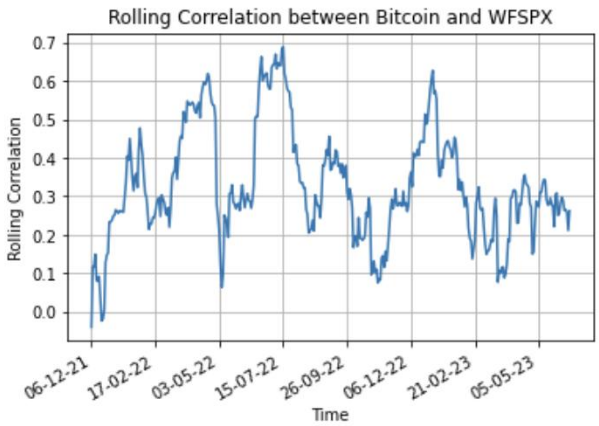
Appendix 30: 30-day rolling correlation between BTC and IEMG.



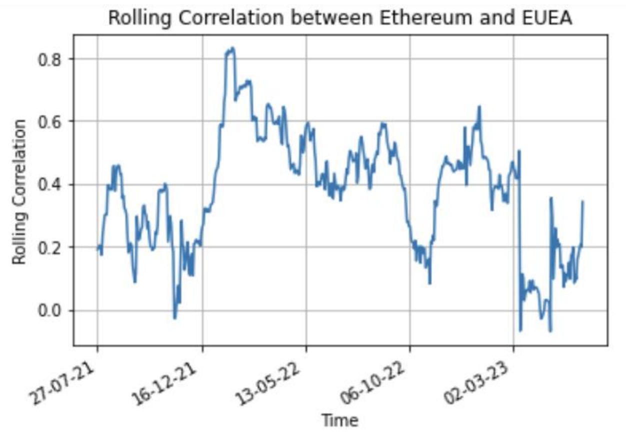
Appendix 31: 30-day rolling correlation between BTC and URTH.



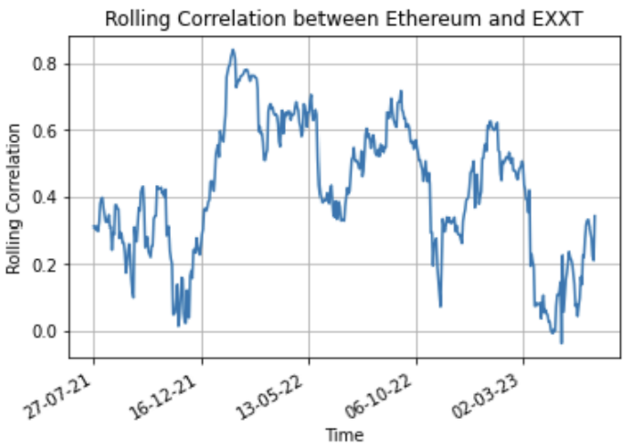
Appendix 32: 30-day rolling correlation between BTC and WFSPX.



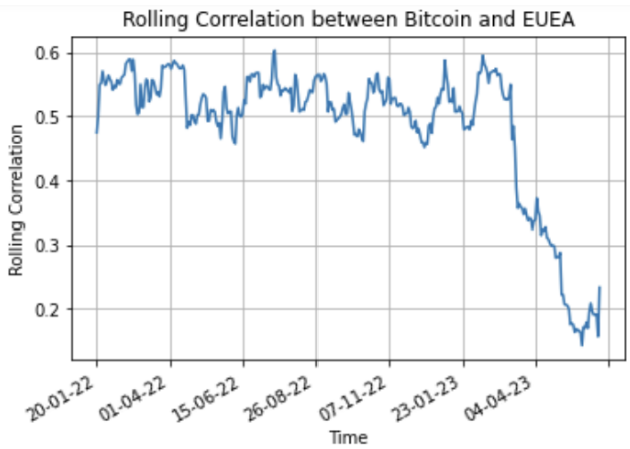
Appendix 33: 30-day rolling correlation between ETH and EUEA.



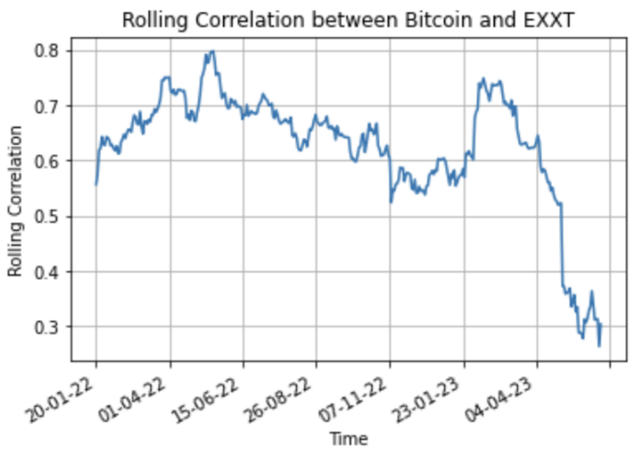
Appendix 34: 30-day rolling correlation between ETH and EXXT.



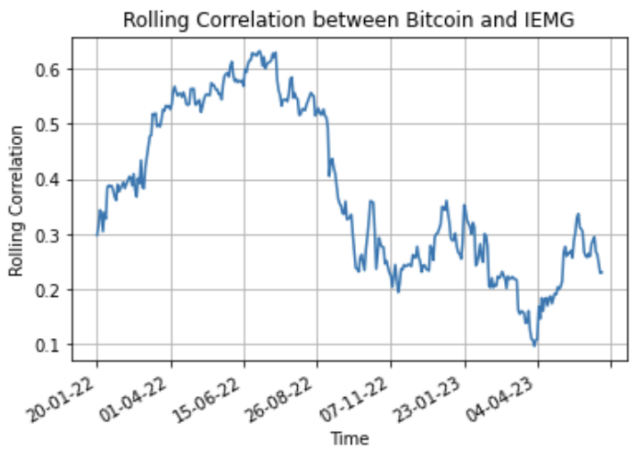
Appendix 35: 60-day rolling correlation between BTC and EUEA.



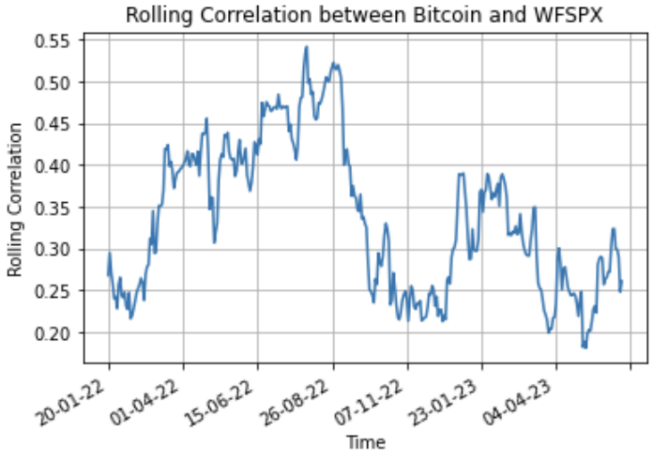
Appendix 36: 60-day rolling correlation between BTC and EXXT.



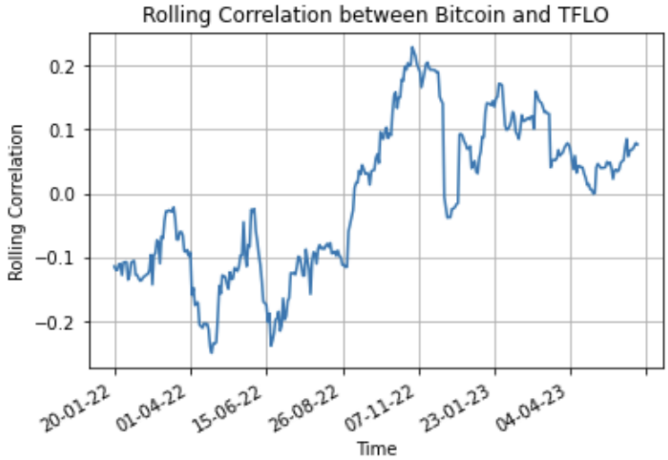
Appendix 37: 60-day rolling correlation between BTC and IEMG.



Appendix 38: 60-day rolling correlation between BTC and WFSPX.



Appendix 39: 60-day rolling correlation between BTC and TFLO.



Appendix 40: Performance ratios of portfolios augmented with BTC.

	Sharpe ratio	Treynor ratio	Jensen's Alpha
Portfolio Aggressive 5% BTC	-0.319	-0.003	-0.003
Portfolio Moderate 5% BTC	-1.315	-0.003	-0.004
Portfolio Conservative 5% BTC	-3.310	-0.004	-0.007
Portfolio Aggressive 10% BTC	-0.657	-0.006	-0.005
Portfolio Moderate 10% BTC	-1.827	-0.007	-0.008
Portfolio Conservative 10% BTC	-4.234	-0.008	-0.014
Portfolio Aggressive 15% BTC	-1.015	-0.010	-0.008
Portfolio Moderate 15% BTC	-2.384	-0.010	-0.013
Portfolio Conservative 15% BTC	-5.279	-0.012	-0.022

Appendix 41: Performance ratios of portfolios augmented with ETH.

	Sharpe ratio	Treynor ratio	Jensen's Alpha
Portfolio Aggressive 5% ETH	-0.368	-0.004	-0.003
Portfolio Moderate 5% ETH	-1.375	-0.004	-0.004
Portfolio Conservative 5% ETH	-3.387	-0.005	-0.006
Portfolio Aggressive 10% ETH	-0.751	-0.008	-0.006
Portfolio Moderate 10% ETH	-0.193	-0.009	-0.008
Portfolio Conservative 10% ETH	-4.345	-0.010	-0.014
Portfolio Aggressive 15% ETH	-1.149	-0.077	-0.009
Portfolio Moderate 15% ETH	-2.527	-0.087	-0.013
Portfolio Conservative 15% ETH	-5.370	-0.100	-0.021

Appendix 42: Performance ratios of portfolios augmented with ADA.

	Sharpe ratio	Treynor ratio	Jensen's Alpha
Portfolio Aggressive 5% ADA	-0.234	-0.002	-0.002
Portfolio Moderate 5% ADA	-1.203	-0.002	-0.003
Portfolio Conservative 5% ADA	-3.161	-0.003	-0.006
Portfolio Aggressive 10% ADA	-0.483	-0.004	-0.003
Portfolio Moderate 10% ADA	-1.597	-0.005	-0.006
Portfolio Conservative 10% ADA	-3.910	-0.005	-0.012
Portfolio Aggressive 15% ADA	-0.749	-0.007	-0.005
Portfolio Moderate 15% ADA	-2.027	-0.008	-0.010
Portfolio Conservative 15% ADA	-4.768	-0.009	-0.019

Appendix 43: Performance ratios of portfolios augmented with SOL.

	Sharpe ratio	Treynor ratio	Jensen's Alpha
Portfolio Aggressive 5% SOL	-1.860	-0.019	-0.020
Portfolio Moderate 5% SOL	-3.390	-0.018	-0.024
Portfolio Conservative 5% SOL	-6.351	-0.018	-0.029
Portfolio Aggressive 10% SOL	-3.709	-0.037	-0.039
Portfolio Moderate 10% SOL	-5.925	-0.037	-0.046
Portfolio Conservative 10% SOL	-10.192	-0.037	-0.057
Portfolio Aggressive 15% SOL	-5.519	-0.056	-0.059
Portfolio Moderate 15% SOL	-8.391	0.055	-0.069
Portfolio Conservative 15% SOL	-13.829	-0.056	-0.085

Appendix 44: Performance ratios of portfolios augmented with RPL.

	Sharpe ratio	Treynor ratio	Jensen's Alpha
Portfolio Aggressive 5% RPL	-0.201	-0.002	-0.003
Portfolio Moderate 5% RPL	-1.160	-0.002	-0.004
Portfolio Conservative 5% RPL	-3.101	-0.003	-0.007
Portfolio Aggressive 10% RPL	-0.414	-0.004	-0.004
Portfolio Moderate 10% RPL	-1.500	-0.005	-0.007
Portfolio Conservative 10% RPL	-3.766	-0.005	-0.013
Portfolio Aggressive 15% RPL	-0.640	-0.006	-0.006
Portfolio Moderate 15% RPL	-1.870	-0.007	-0.010
Portfolio Conservative 15% RPL	-4.517	-0.009	-0.020

Appendix 45: Performance ratios of portfolios augmented with a basket of cryptocurrencies.

	Sharpe ratio	Treynor ratio	Jensen's Alpha
Portfolio Aggressive 5% mixed	-0.605	-0.006	-0.009
Portfolio Moderate 5% mixed	-1.708	-0.006	-0.011
Portfolio Conservative 5% mixed	-3.896	-0.006	-0.015
Portfolio Aggressive 10% mixed	-1.238	-0.012	-0.015
Portfolio Moderate 10% mixed	-2.623	-0.012	-0.019
Portfolio Conservative 10% mixed	-5.425	-0.012	-0.026
Portfolio Aggressive 15% mixed	-1.899	-0.018	-0.020
Portfolio Moderate 15% mixed	-3.596	-0.018	-0.026
Portfolio Conservative 15% mixed	-7.103	-0.019	-0.037

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