Economics School of Louvain - ESL

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The regressivity of carbon taxation when green consumption is a luxury and agents are heterogeneous

A two-sector economy featuring heterogeneous agents with non-homothetic preferences

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Academic Year 2020-2021

Master in Economics – 120 credits – Focus : Research

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MASTER'S THESIS

The regressivity of carbon taxation when green consumption is a luxury and agents are heterogeneous

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August 16, 2021

Abstract

Carbon taxation is often reported as regressive in the literature, yet the macroeconomic models for its analysis tend to neglect its distributional impact. We aim to fill this gap with a model featuring heterogeneous agents and two sectors. The polluting sector is subject to carbon taxation since it generates CO_2 emissions, while the green sector is environmentally neutral due to expenditure on abatement costs. Consumers' priority is to secure some basic consumption, keeping environmental issues as a secondary concern. Therefore, less affluent households are in a worse position to switch to more expensive green goods. As a consequence, raising carbon taxation will impact these individuals more severely. In this framework, the role played by the government in determining the optimal tax and the use of tax revenues becomes particularly important, as it has to deal with a trade-off between the attainment of environmental goals and the reduction of inequality.

Keywords: heterogeneous agents, carbon taxation, green goods, inequality.

* **Acknowledgments:** I want to express my gratitude to my supervisor, Rigas Oikonomou, for his confidence in this project and his help throughout the year, which has brought me closer to my learning goals. I also want to thank my classmates, especially Inês Teixeira da Cunha and Joan Jose, for their sincere interest and moral encouragement in this tough year. Luis Herrera Bravo deserves a mention as well, since I deeply appreciate the relevant and inspiring suggestions that he always gives me. Finally, I must acknowledge the support of my friends and family. It is them that motivate me at all times.

1 Introduction

Green products are increasingly present on retail shelves and their consumption is growing, especially in consumer packaged goods, but they still represent a low share of the market (UBA, 2017; Kronthal-Sacco and Whelan, 2019). There seems to be a gap between household attitudes and behaviour towards green consumption, with the proportion of individuals showing concern about climate change being significantly larger than the actual fraction of green consumers (WBCSD, 2008). Gleim, Smith, Andrews, and Cronin Jr (2013) place the price as the main barrier for the demand of these products. Indeed, the price of low-carbon goods includes a considerable premium (Kavilanz, 2008). Other influential aspects are a lack of trust in companies' environmental claims, selfishness and insufficient understanding or awareness (WBCSD, 2008).

Furthermore, several studies argue that the decision to consume eco-friendly products is positively related to income, such as Scott and Willits (1994), Clark, Kotchen, and Moore (2003) and Welsch and Kühling (2009). Conversely, Elliott (2013) and Halder, Hansen, Kangas, and Laukkanen (2020) report a positive but not significant effect of income. In any case, these studies focus on the decision to consume green products or not, neglecting how income affects the percentage that green purchases represent in a household's consumption bundle. Our analysis relies on the assumption that this percentage increases with income or, at least, that richer individuals are better positioned to substitute consumption of dirty goods by green goods.

As environmentalists raise awareness and climate change effects become more notorious, governments are increasingly including environmental policies in their agenda. Measures vary sharply across countries. Lately, we have seen countries adopt ingenious and unconventional regulation. For example, Norway has forbidden commercial relations with firms causing deforestation, while Sweden encourages reuse of old items by offering tax exemptions for repairs. Similarly, bans on single-use plastics and car-free areas are growing in popularity. Still, the more traditional approach of emissions cap and trade prevails (Baldwin, 2008).

We study the effects of one particular environmental policy: carbon taxation. Although it is not as extended, it has been considered by many governments. Indeed, it is in force in a number of countries, like the Scandinavians since the nineties or México and France since the last decade. In others, like Germany and Canada, its implementation has been ultimately dismissed after years of negotiations. Harrison (2010) argues that governments might favour cap and trade due to its lower political costs, but he highlights the convenience of carbon taxation. Moreover, this policy instrument is a popular choice between economists and has been repeatedly employed in macroeconomic models since the end of the last century.

In this framework, the *Integrated Assessment Models* (IAM) emerged seeking to jointly evaluate the environmental and economic implications of governmental policies and agents' actions. The Nobel laureate economist William D. Nordhaus opened the door to this type of analysis with his DICE and RICE models (Nordhaus, 1991; Nordhaus and Yang, 1996), laying the foundation for posterior models like those of Hassler and Krusell (2012) or Golosov, Hassler, Krusell, and Tsyvinski (2014). These models can become considerably intricate, as they try to incorporate environmental mechanisms, but they still raise severe criticism and some authors cast doubts on the validity of their results (Pindyck, 2013).

Other researchers simplify the environmental side of the analysis and focus on the purely economic implications of environmental policies. A common approach relies on extending the RBC model with the introduction of pollution or, more particularly, CO_2 emissions. These emissions are assumed necessary for production, but they also generate an externality that ends up reducing final output. The introduction of pollution requires additional assumptions about how it interacts with the economic model. Fischer and Springborn (2011) consider an additional intermediate input, which could be interpreted as energy, and model emissions as a secondary effect directly proportional to the level of the new input. Nordhaus (2008) and Heutel (2012) avoid additional inputs in the production function by considering CO_2 emissions as a byproduct of production. They also offer the possibility for firms to invest on abatement, using a certain abatement technology. Similarly, Angelopoulos, Economides, and Philippopoulos (2010) also assume that emissions are directly correlated with total output, although in this case it is the government who can invest on abatement.

This paper continues in the same vein, reducing the environmental considerations to the treatment of pollution as a byproduct of production with a negative externality associated. Deviating from the aforementioned models, we prepare the ground for the introduction of an inequality dimension in the analysis. For that purpose, we implement Aiyagari's (1994) scheme, in which agents are subject to borrowing constraints, as in Deaton (1991), and to idiosyncratic productivity shocks. Following Fullerton and Heutel (2007), agents can consume two different goods, one environmentally clean and another one that pollutes. We assume that consumers have a preference for the clean good, but the composition of their consumption bundle will depend on their income, as they also care about their total consumption. This heterogeneity in consumption decisions gives room to an uneven impact of the carbon tax across the wealth distribution.

Carbon taxation has been repeteadly reported as regressive in the literature. Some examples are Wier, Birr-Pedersen, Jacobsen, and Klok (2005), for Denmark; Hassett, Mathur, and Metcalf (2009), for the United States, and Callan, Lyons, Scott, Tol, and Verde (2009), for Ireland. The intuition behind is that most products intensive in pollution are necessities. Therefore, these basic consumption goods, such as transport fuel, food, heating and electricity, are subject to the tax (Andersson and Atkinson, 2020).

The effect of the carbon tax is two-fold. While it provides a direct incentive for firms to reduce their CO_2 emissions, the levy is also partially passed on to the final customer, affecting the prices of dirty goods and thus encouraging substitution toward green products, by making them more competitive. Accepting the premise that polluting goods are basic necessities, as we do in the model, the impact of the policy is unequal. On the one hand, richer households respond as desired, adapting the composition of their consumption bundle to the change in relative prices. On the other hand, individuals with fewer resources need to meet their basic consumption needs and cannot afford to switch to eco-friendly alternatives, which are more expensive. That is, the raise in its price due to the policy makes low-income individuals allocate more resources on the consumption of the dirty good, instead of replacing it with the clean alternative. In summary, the policy might even end up forcing low-income individuals to give up units of the green good.

As environmental policies seem to be building momentum, the implications for inequality might be unavoidable. For this reason, the use of tax revenues is worthy to be analysed, as it could allow to compensate the effects of the tax on inequality (Metcalf, 1999, 2009), to reduce other taxes (Poterba, 1991) or to further boost cuts on emissions (Marron and Morris, 2016).

2 The model

2.1 Households

Households obtain utility from consumption, which they fund with the remunerations that they receive from providing production factors to the private sector. However, they face idiosyncratic shocks to their productivity, l_t, and they are borrowing constrained, so that their assets, a_t, cannot be below a certain value $-b$. This specification is based on Aiyagari's (1994) extension of Deaton (1991) for a unit mass of agents.

We introduce some particularities. There are two goods to consume: a clean good, c_t^c, and a "dirty" good, c_t^d. Individuals see the former as a luxury, while the latter represents a necessity. Therefore, they will only consume the clean good once their basic consumption needs are satisfied. It is then that they will start including units of the clean good in their bundle set, because they are environmentally conscious and they believe green companies' claims about being environmentally neutral. Hence, they prefer the clean alternative over the dirty good and are willing to pay its price premium.

We aim at capturing the underlying mechanism previously described using a simple specification. For this purpose, we use the same preferences as in Claudio (2018):

$$U(c^c, c^d) = \frac{1}{1-\zeta}\left[(c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(c^c)^{1-\phi}\right]^{\frac{1-\zeta}{1-\lambda}}. \tag{1}$$

In Appendix II it is shown that this utility function represents non-homothetic preferences. It imposes different expenditure elasticities for each good. While the dirty good shows a value lower or equal to one, it is greater or equal to one for the clean good. This implies that, for consumers, the dirty good is basic and the clean good is luxurious. In consequence, the share of green products in total household's consumption will increase or, at least, remain the same as the household raise its expenditure. With these preferences, the consumption bundle of agents at the lower part of the wealth distribution will be mainly composed of units of the dirty good, while the eco-friendly good will gain share in the consumption bundle as income increases. This is consistent with the mechanism described previously, which implies that consumers start caring for environmental protection once they have their basic consumption needs satisfied.

Using these preferences, the consumer's problem is:

$$\begin{aligned} V(a,l) = \max_{\{c^c, c^d, a'\}} \quad & \frac{1}{1-\zeta}\left[(c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(c^c)^{1-\phi}\right]^{\frac{1-\zeta}{1-\lambda}} + \beta E V(a', l'), \\ \text{s.t.} \quad & p^c c^c + c^d + a' = (1+r)a + wl, \\ & a' \geq -b. \end{aligned}$$

Incorporating the budget constraint, the problem reduces to:

$$V(a,l) = \max_{\{c^c>0, a' \geq -b\}} \frac{1}{1-\zeta} \left\{ [(1+r)a + wl - p^c c^c - a']^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(c^c)^{1-\phi} \right\}^{\frac{1-\zeta}{1-\lambda}} + \beta EV(a',l'). \qquad (2)$$

2.2 Firms

The production side of this model is based on Fullerton and Heutel (2007), with some features imported from Heutel (2012). Hence, we implement a two-sector economy, with one sector emitting pollution and the other being environmentally neutral. Green companies are able to signal that their production process is not harmful to the environment. We assume that consumers believe those signals and that they are to some extent eco-conscious and have a certain preference for goods produced cleanly.

2.2.1 Polluting firms

The polluting sector is composed of traditional firms producing the dirty good, Y_t^d, and operating under perfect competition. The representative polluting firm has a standard Cobb-Douglas technology with two inputs: capital, K_t^d, and labour, L_t^d.

$$Y_t^d = (1 - d(x_t))A_t(K_t^d)^{\alpha}(L_t^d)^{1-\alpha}. \qquad (3)$$

The net output is thus diminished due to the loss of potential output caused by pollution. This effect is captured by the damage function $d(x_t)$, which ranges between 0 and 1 and whose input, x_t, is the stock of pollution at time t. This production technology differs from Fullerton and Heutel (2007), who introduce emissions in the production function. Instead, we follow Heutel (2012) and assume that emissions are a byproduct of production, described by a function h:

$$e_t^d = h(Y_t^d), \qquad (4)$$

These firms do not possess any technology for abatement of pollution. Therefore, all their emissions are subject to the carbon tax imposed by the government. They are also liable for the remuneration of capital and workers. Hence, polluting firms that seek to choose the levels of capital and labour that maximise their profits face the following profit-maximisation problem:

$$\begin{aligned} \max_{\{K_t^d, L_t^d\}} \quad & \Pi_t^d = Y_t^d - r_t K_{t-1}^d - w_t L_t^d - \tau e_t^d, \\ \text{s.t} \quad & Y_t^d = (1 - d(x_t))A_t(K_t^d)^{\alpha}(L_t^d)^{1-\alpha}, \\ & e_t^d = h(Y_t^d). \end{aligned}$$

Following Heutel (2012), the environmental impact caused by the firm is an externality, not internalised in its optimisation problem. Therefore, the firm makes its decisions taking the damage function, $d(x_t)$, as exogenous, neglecting its own influence on it.

The solution to this problem, developed in Appendix I, leaves:

$$r_t = \left[1 - \tau h'(Y_t^d)\right] f_{K,t}^d - \delta, \tag{5}$$

$$w_t = \left[1 - \tau h'(Y_t^d)\right] f_{L,t}^d, \tag{6}$$

2.2.2 Green firms

Unlike Claudio (2018), we introduce a rigidity in the green sector (luxury sector in their analysis) that will enable $P_t^c > 1$. Sana (2020) studies a model of price competition between green and non-green firms, pointing out the existence of such price premium of eco-friendly goods. To capture this premium, we implement a simpler scheme of monopolistic competition in the clean sector.

There is a unit mass of clean good producers. Each intermediate producer j produces a differentiated clean good using the same Cobb-Douglas technology:

$$y_t^c(j) = (1 - d(x_t))A_t K_t^c(j)^\alpha L_t^c(j)^{1-\alpha}. \tag{7}$$

Observe that green firms suffer from the negative externality caused by dirty production. Identically to the firms of the other sector, the production of the clean good entails CO_2 emissions directly correlated with output. The difference is that firms in this sector are allowed to invest in abatement, $\mu_t(j)$, to erase their environmental footprint:

$$e_t^c(j) = (1 - \mu_t(j))h(y_t^c(j)). \tag{8}$$

This scheme is borrowed from Heutel (2012) and implies that green firms face some abatement costs, $z_t^c(j)$, given by:

$$z_t^c(j) = g(\mu_t(j)) \cdot Y_t^c(j). \tag{9}$$

For simplicity, we assume that green firms are environmentally neutral; they invest for complete abatement of their emissions: $\mu_t(j) = 1$ and $e_t^c(j) = 0$ for all j. Consequently, they are not subject to carbon taxation.

There is a final clean good producer behaving competitively. It takes the differentiated clean goods as inputs and aggregates them into the composite green good sold for household consumption:

$$Y_t^c = \left[\int_0^1 y_t^c(j)^{\frac{\omega-1}{\omega}} dj\right]^{\frac{\omega}{\omega-1}},$$

where ω represents the elasticity of substitution between clean goods. The demand for each differentiated good is given by:

$$y_t^c(j) = \left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega} Y_t^c, \tag{10}$$

with the final price for the clean good defined as:

$$P_t^c = \left[\int_0^1 p_t^c(j)^{1-\omega} dj\right]^{\frac{1}{1-\omega}}.$$

In this framework, intermediate green companies face two different problems. They can affect their market share by adjusting their prices in a profit maximisation problem. Once they fix their demand, they want to reach the target production with minimum costs. Both problems are briefly outlined below and solved in Appendix I.

On the one hand, the problem of minimisation of costs for any intermediate producer j is:

$$\begin{aligned} \min_{\{K_t^c(j), L_t^c(j)\}} \quad & r_t K_t^c(j) + w_t L_t^c(j) + z_t^c(j), \\ \text{s.t} \quad & (1 - d(x_t)) A_t K_t^c(j)^\alpha L_t^c(j)^{1-\alpha} \leq \left(\frac{p_t^c(j)}{P_t^c} \right)^{-\omega} Y_t^c, \\ & z_t^c(j) = g(1)(1 - d(x_t)) A_t K_t^c(j)^\alpha L_t^c(j)^{1-\alpha}. \end{aligned}$$

Since the green firm abates all its emissions, the damage associated with pollution, $d(x_t)$, is exogenous in this problem.

On the other hand, the problem of maximisation of profits is the following:

$$\begin{aligned} \max_{\{p_t^c(j)\}} \quad & \Pi_t^c(j) = (p_t^c(j) - MC_t^c(j))\, y_t^c(j), \\ \text{s.t} \quad & y_t^c(j) = \left(\frac{p_t^c(j)}{P_t^c} \right)^{-\omega} Y_t^c, \end{aligned}$$

where $MC_t^c(j)$ is the marginal cost derived from the problem of minimisation.

As shown in Appendix I, the optimal allocation of production inputs in the clean sector will satisfy:

$$r_t = (\xi_t - g(1)) f_{K,t}^c - \delta, \tag{11}$$

$$w_t = (\xi_t - g(1)) f_{L,t}^c. \tag{12}$$

$$P_t^c = \frac{\omega}{\omega - 1} MC_t^c. \tag{13}$$

2.3 Government

The government collects the tax on CO_2 emissions and uses these resources to finance consumption of the dirty and clean goods in the same proportions as households. We expect to continue this work and extend the role of authorities, considering different potential uses for the revenues collected with the tax.

2.4 Environmental Mechanisms

Following Heutel (2012), the pollution stock decays naturally following a linear function:

$$x_t = \eta x_{t-1} + e_t + e_t^{row}, \tag{14}$$

where e_t denotes the emissions of the domestic economy in period t, which in our case will correspond to the emissions of the polluting sector: $e_t = e_t^d$. The other variable, e_t^{row}, represents the emissions from the rest of the world and is exogenous.

2.5 Competitive Equilibrium

The competitive equilibrium consists of a value function V, as well as a set of decision rules $A(a,l)$, $C(a,l)$ and $D(a,l)$, which characterise an agent's optimal savings and consumption of clean and dirty goods, respectively. In addition, it comprises sequences of quantities $\{K, L, Y^c, Y^d\}$, prices $\{w, r, P^c\}$, a tax rate (τ) and a stationary distribution of individuals (ψ) across a and l such that:

1. Given prices and taxes, households behave optimally, adopting the optimal policies that solve the Bellman equations defined in Equation 2.

2. Firms maximise profits, their optimal use of inputs being described by Equations 5 and 6, in the case of traditional firms, and by Equations 11 and 12, in the case of clean firms. Their productions, Y^d and Y^c, are given by Equations 3 and 7, respectively.

3. The government balances its budget, returning tax income to the economy.

4. All markets clear:

$$K^c + K^d = \int_{a\in[-b,\infty],\ l\in[l_{min},l_{max}]} A(a,l)d\psi(a,l), \tag{15}$$

$$L^c + L^d = \int_{a\in[-b,\infty],\ l\in[l_{min},l_{max}]} l d\psi(a,l), \tag{16}$$

$$Y^c = \int_{a\in[-b,\infty],\ l\in[l_{min},l_{max}]} C(a,l)d\psi(a,l), \tag{17}$$

$$Y^d = \int_{a\in[-b,\infty],\ l\in[l_{min},l_{max}]} D(a,l)d\psi(a,l). \tag{18}$$

5. The measure ψ is consistent. In particular, ψ solves the following:

$$\psi(\mathcal{A},\mathcal{L}) = \int_{a\in[-b,\infty],\ l\in[l_{min},l_{max}]} d\psi(a,l).$$

3 Calibration

We follow Heutel (2012) to choose the form of the environmental functions of the model. The relation between emissions and output is assumed to be described by an iso-elastic function:

$$h(Y_t^j) = (Y_t^j)^{1-\gamma}, \quad j \in \{c,d\}. \tag{19}$$

This assumption could be problematic in our two-sector economy because it is not additively separable. Still, since one sector is carbon-neutral, we keep it. The abatement cost function comes from Nordhaus (2008):

$$g(\mu_t) = \theta_1 \mu_t^{\theta_2}. \tag{20}$$

Given our assumption that $\mu_t = 1\ \forall t$, the abatement cost function is constant and equal to θ_1. Therefore, the total abatement spending of green companies is given by:

$$z_t^c = \theta_1 Y_t^c. \tag{21}$$

The damage function is quadratic:

$$d(x_t) = d_2 x_t^2 + d_1 x_t + d_0. \tag{22}$$

Our baseline case is the scenario with no carbon taxation, $\tau = 0$. We run additional simulations with taxes of 5% and 10% to study the implications of the introduction of the tax. Table 1 shows the values chosen for the rest of parameters. It should be noted that there is still much room for improvement in the choice of parameters. In future developments, the election should be refined to enhance the correspondence between simulated data and reality. For example, the price of the clean good and the ratio of green consumption with respect to traditional consumption should be more accurately calibrated. Besides, it would be convenient to test other characterisations of the idiosyncratic shock. Aiyagari (1994) considers values of 0.2 and 0.4 for the variance of the shock, as well as 0, 0.3, 0.6 and 0.9 for its persistence. Given that the focus of this analysis is on the long run and we do not account for technological change, we normalise $A_t = 1$.

Table 1

Calibration of the parameters

Parameter	Value	Reference	Description
b	0		Borrowing constraint
β	0.95		Subjective discount factor
ζ	2	3 (Claudio, 2018)	Equivalent to γ in (Claudio, 2018)
λ	20	11.5 (Claudio, 2018)	Curvature parameter 1
ϕ	1.5	1.1 (Claudio, 2018)	Curvature parameter 2
η	50	75 (Claudio, 2018)	Utility weight on clean consumption
α	0.36		Capital share of income
δ	0.03		Capital depreciation
ω	1.2	Calibrated for $P_t^c > 1$	Elasticity of substitution between clean goods
θ_1	0.05607	(Heutel, 2012)	Parameter of abatement costs
d_0	$1.3950 \cdot 10^{-3}$	(Heutel, 2012)	Parameter of the damage function
d_1	$-6.6722 \cdot 10^{-6}$	(Heutel, 2012)	Parameter of the damage function
d_2	$1.4647 \cdot 10^{-8}$	(Heutel, 2012)	Parameter of the damage function
γ	0.304	(Heutel, 2012)	Parameter of emissions
n_l	4		Number of nodes of the idiosyncratic shock
ρ_l	0.95		Persistence of the idiosyncratic shock
σ_l	0.1561		Variance of the idiosyncratic shock

4 Computation

Our simulations were carried out in MATLAB®. To reduce the computation time, some simplifications were needed. We have worked with small decision grids, to the detriment of the smoothness of our results. Besides, the model described above required guessing not only the equilibrium value of r but also of w, since both depended on the level of emissions. To avoid the double process of convergence and the time it would entail, we used a simpler version of the tax in our code. Instead of being linear on emissions, the tax is linear on the sum of production inputs on the polluting sector, $K_t^d + L_t^d$. Consequently, Equation 24 simplifies so that the wage is pinned down by r and does not need a guess of its own:

$$w_t = [1 - d(x_t)]^{\frac{1}{1-\alpha}} (1 - \alpha) \left(\frac{r_t + \tau + \delta}{\alpha} \right)^{\frac{\alpha}{\alpha - 1}} - \tau. \tag{23}$$

The algorithm employed to solve the model is standard and it is described below.

1. We discretize the income process l, originally an AR(1), into 4 nodes. For this purpose, we follow Adda and Cooper (2003). From the distribution of l, we can obtain the aggregate level of labour, L.

2. Then we generate two grids for the decision variables: assets, a, and clean consumption, c^c. The lower limit is the borrowing constraint in the case of assets and 10^{-6} in the case of clean consumption. The upper limit is high enough not to be binding.

3. We pick an initial r and derive the corresponding w via Equation 23 and P^c via Equations 13 and 26.

4. Iteratively, we solve the Bellman equation (Equation 2) until the value function converges. After this sequence of optimisations, the decision rules $A(a_t, l_t)$ and $C(a_t, l_t)$ are generated by taking the element in each grid that leads to the optimal solution.

5. From $A(a_t, l_t)$ and $C(a_t, l_t)$ we derive $D(a_t, l_t)$, making use of the budget constraint.

6. With the decision rules, it is possible to find the long run distribution of assets with the following algorithm:

   (a) Start from a guess $\psi_0(\cdot)$.

   (b) Update $\psi_1(a_i, l_k) = \sum_{l:l'=l_k \; a:a'=a_i} \pi(l_k, l)\psi_0(a, l)$, where $\pi(l_k, l)$ is the probability of transition from state l to l_k.

   (c) If $|\psi_1(a_i, l_k) - \psi_0(a_i, l_k)| \leq \epsilon_1$, for a given value of ϵ_1, then stop. Otherwise, continue iterating. We chose $\epsilon_1 = 0.0001$.

7. Once we have an invariant measure $\psi(a_i, l_k)$, we can calculate aggregate capital, K.

8. With the decision rules and the long run distribution, it is possible to compute the long run aggregate demand for the clean good, Y^c, and the dirty good, Y^d.

9. With the aggregate input quantities, K and L, as well as good demands, Y^c and Y^d, and taking into account Equation 27, it is possible to derive the allocation of resources in each sector: K^c, K^d, L^c and L^d.

10. Finally, we can obtain the input prices in each sector: r^d, r^c, w^d and w^c.

11. If $r - r^d \leq \epsilon_2$, for a given value of ϵ_2, then stop. Otherwise, we need to update the guess for r and repeat the process from step 3. In our case, we picked a value of $\epsilon_2 = 0.0001$ for all the simulations that we ran.

5 Simulations

5.1 Baseline: no carbon taxation

In our baseline case, we assume that the government does not intervene ($\tau = 0$). Figure 1 shows households' average optimal decision on clean and dirty consumption given their initial stock of assets. Noteworthy, it reflects the mechanisms that we described in Section 2.1. Households with low assets at the beginning of the period choose consumption bundles intensive in the basic good, whereas the consumption of clean good increases substantially with wealth. Figure 2 shows that the expenditure on traditional consumption is also stable for different realisations of the idiosyncratic shock. In contrast, there is a substantial gap on green consumption between individuals on the upper and lower bounds of the income distribution.

Figure 3 characterises the asset decision rule. We find that lucky individuals who receive the highest realisation of the income shock use it partly to enlarge their stock of assets. This is especially true for individuals close to the borrowing constraint. However, the propensity to save those additional earnings decreases in wealthier individuals, who prefer to just maintain their already high stock of assets and spend the extra income on consumption. Conversely, unlucky agents will smooth the impact of low realisations of the idiosyncratic shock on their consumption by reducing their stock of assets.

Consistent with the previous graphs, Figure 4 shows that dirty consumption is fairly evenly spread across households in this model. Nonetheless, green consumption and asset accumulation are substantially liable to inequalities. The Gini coefficient that our baseline simulation returns is 0.3445, which is however low when we compare it with the value we obtain when we simulate a simple one-sector Aiyagari model with the same calibration as our baseline. Table 2 offers a comparison of the output of the different simulations we ran. Our two-sector model features a lower share of agents borrowing constrained. On the other hand, the richest 1% hold a larger share of total assets and the ratio of capital to output is considerably greater.

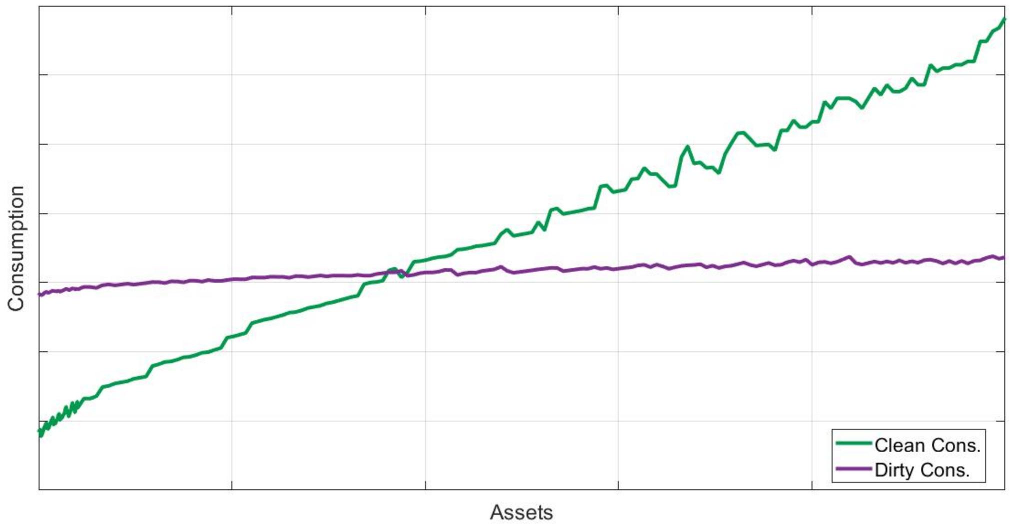
Table 2

Simulation outputs

Variable	Baseline ($\tau = 0\%$)	Model 2 (Reduced)	Model 3 ($\tau = 5\%$)	Model 4 ($\tau = 10\%$)
r	0.0454	0.0475	0.0327	0.0156
w	1.5285	1.5185	1.1696	0.9558
P^c	1.4840	-	1.1400	0.8505
Gini index	0.3445	0.4756	0.3939	0.3885
Fraction ≤ 0 wealth	0.0106	0.0379	0.0141	0.0232
Ratio K/Y	7.0109	4.6434	4.7229	3.7826
Ratio Y^c/Y^d	1.1715	-	0.8189	0.6170
Asset Share Top 1%	0.0747	0.0401	0.0319	0.0280

Figure 1

Consumption decisions in the baseline case ($\tau = 0$)



Note. The curves represent the average consumption given a, without accounting for differences in l.

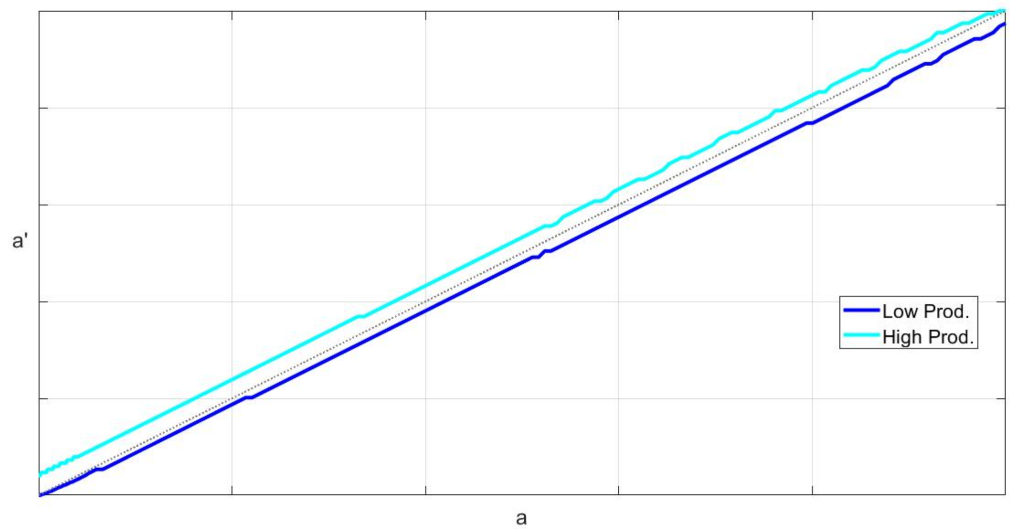
Figure 2

Consumption decisions and idiosyncratic shock in the baseline case ($\tau = 0$)

Note. Lighter and darker tones represent, respectively, the upper and lower bounds of the income shock.

Figure 3

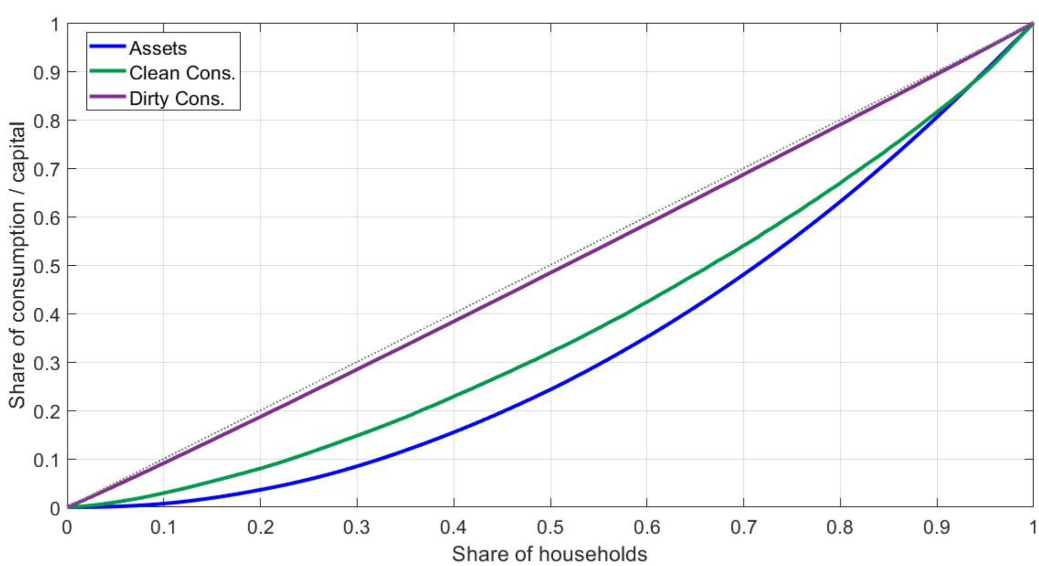
Asset decision in the baseline case ($\tau = 0$)



Note. Lighter and darker tones represent, respectively, the upper and lower bounds of the income shock.

Figure 4

Lorenz curves in the baseline case ($\tau = 0$)



5.2 Taxing emissions

We run two additional simulations with only one modification: we introduce a carbon tax of 5% and 10%, in each case. The results are also included in Table 2. Taxing pollution in this model leads to an increase in wealth inequality, as the Gini index reveals. Indeed, we observe a raise in the fraction of agents for which the borrowing constraint is binding, although the share of assets held by the richest 1% declines. Perhaps counter intuitively, the ratio of clean consumption to dirty consumption falls, implying that the burden of the tax is being shifted to the green sector. The input prices receive downward pressure. Similarly, the relative price of green consumption decreases as the marginal costs of the dirty good grow. The ratio of capital to output decreases, even below the value of the reduced one-sector simulation.

Figure 5 shows the impact of introducing a 10% carbon tax on the consumption of the two goods, keeping our baseline case as reference for the comparison. We observe that the consumption of the dirty good declines across all households. The fall is more evident in less affluent households, unable to counterbalance the impact with a reduction of their asset stock. In the case of clean consumption, the fall is much more pronounced between less wealthy individuals, while richer households increase their consumption of the green good. The reason is that poorer agents now need more resources to secure their intake of the traditional good, a necessity. Therefore, they must redirect funds that were previously allocated to purchase the green good. On the contrary, wealthier individuals do not see their basic consumption at risk. Hence, they can raise their green demand, which has become relatively cheaper.

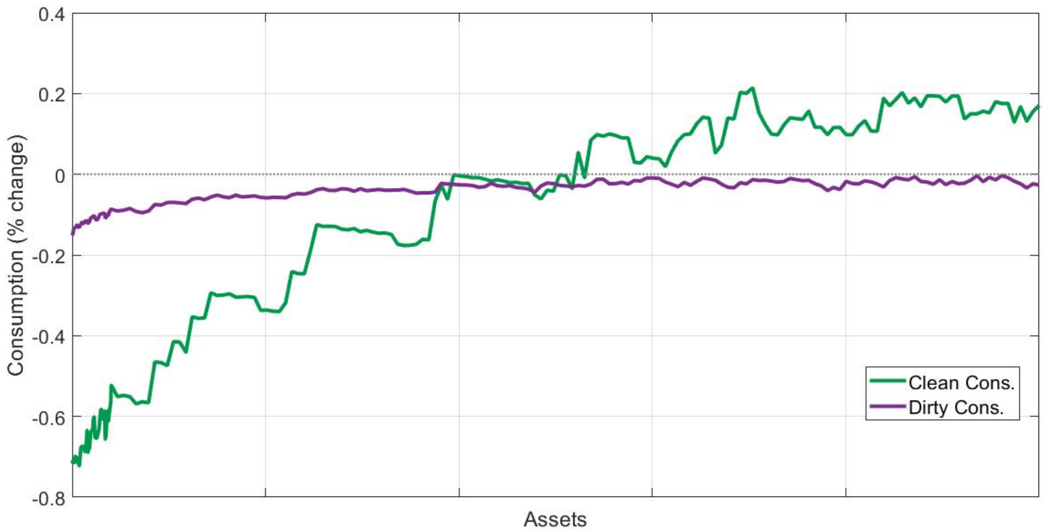
The impact of the policy on asset decisions is depicted in Figure 6. There is not a significant change for individuals suffering a low realisation of the idiosyncratic shock. However, we observe that highly productive agents will spend less of their extra earnings on accumulating assets. This is especially true for poor individuals, given that their basic consumption needs require now a more substantial share of their budget.

In Figure 7 we observe the optimal ranges of consumption decision for each level of initial assets. Noteworthy, dirty consumption will remain almost unchanged for highly productive individuals, regardless their assets. This suggests that basic consumption has the highest priority for households, and therefore they will give up more units of the clean good or lower their stock of assets before letting their dirty consumption fall. That is why only the poorest and low productive individuals show a significant fall in dirty consumption. In such case, it was unavoidable, taking into account that they have no assets to sell and that, as the graph shows, they have already foregone most of their green consumption.

Figures 8, 9 and 10 show the Lorenz curves of clean consumption, dirty consumption and asset accumulation, respectively. In each graph it is possible to compare the curve without tax and with a 10% carbon tax. Inequality increases slightly in traditional consumption, since now there is a larger share of agents not meeting their basic consumption needs. Similarly, wealth is more unequally distributed. Even so, the change in both cases is almost negligible compared to the huge shift that we observe in the Lorenz curve of green demand. As the previous graphs anticipated, the introduction of the tax reinforces the condition of the clean good as a luxury, only affordable for wealthier households with their basic consumption needs satisfied.

Figure 5

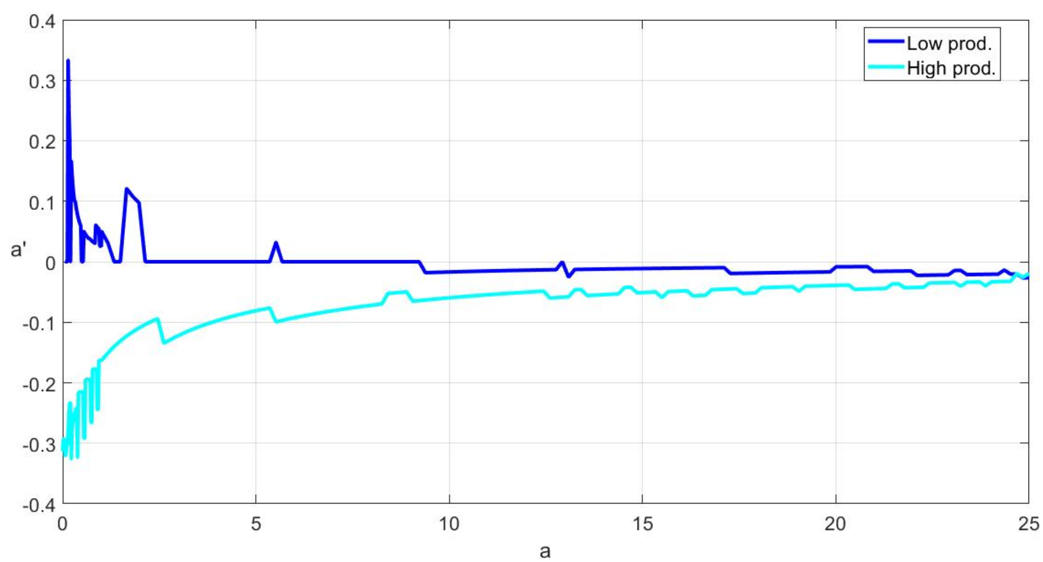
Impact of a carbon tax ($\tau = 10\%$) on consumption



Note. Average percentage change of each type of consumption in Model 2 ($\tau = 10\%$) with respect to the baseline case ($\tau = 0\%$).

Figure 6

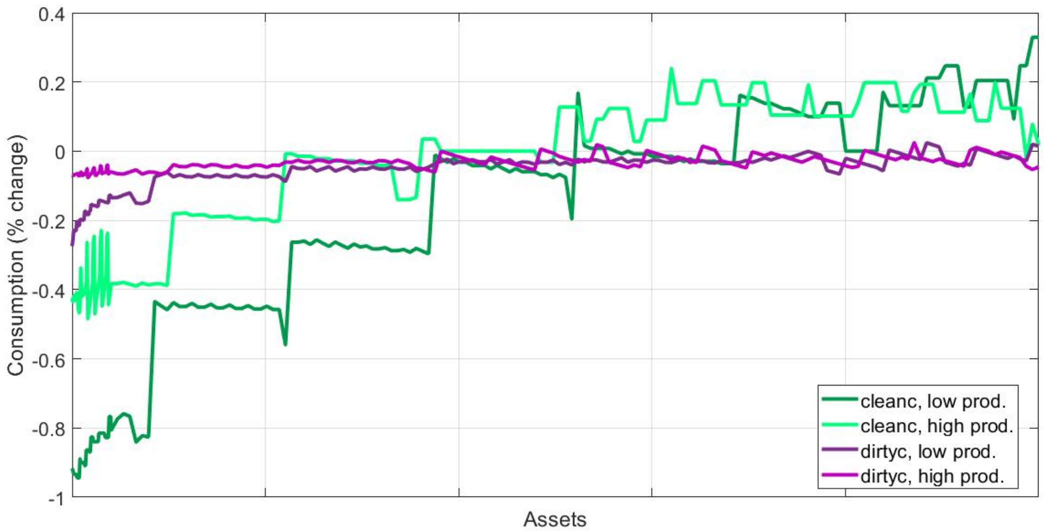
Impact of a carbon tax ($\tau = 10\%$) on asset accumulation



Note. Average change of asset accumulation in Model 2 ($\tau = 10\%$) with respect to the baseline case ($\tau = 0\%$).

Figure 7

Impact of a carbon tax ($\tau = 10\%$) and income shock on consumption decisions



Note. Change of each type of consumption in Model 2 ($\tau = 10\%$) with respect to the baseline case ($\tau = 0\%$), accounting for the realisation of the shock.

Figure 8

Lorenz curves of clean consumption with and without carbon taxation

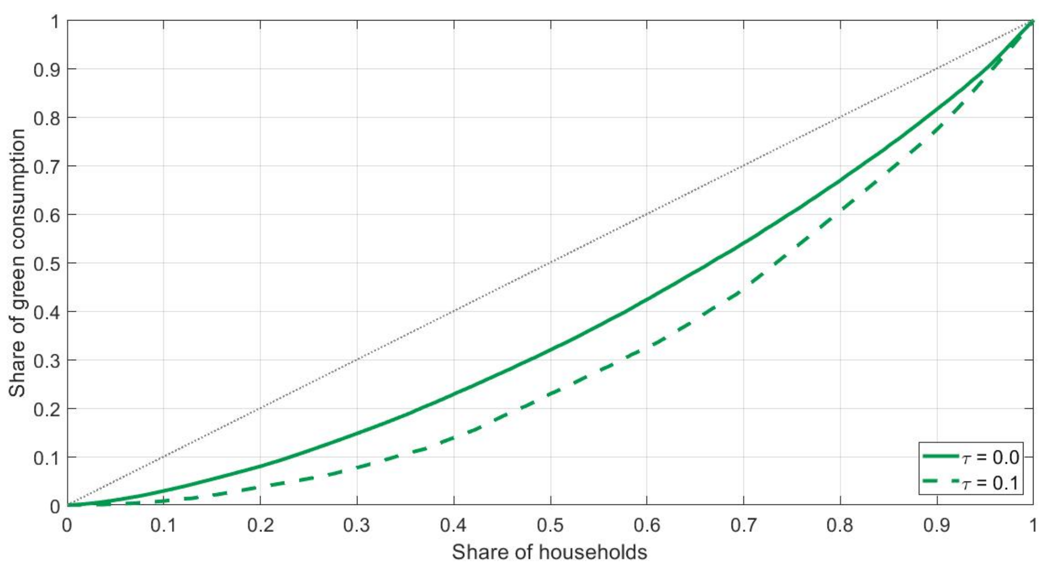


Figure 9

Lorenz curves of dirty consumption with and without carbon taxation

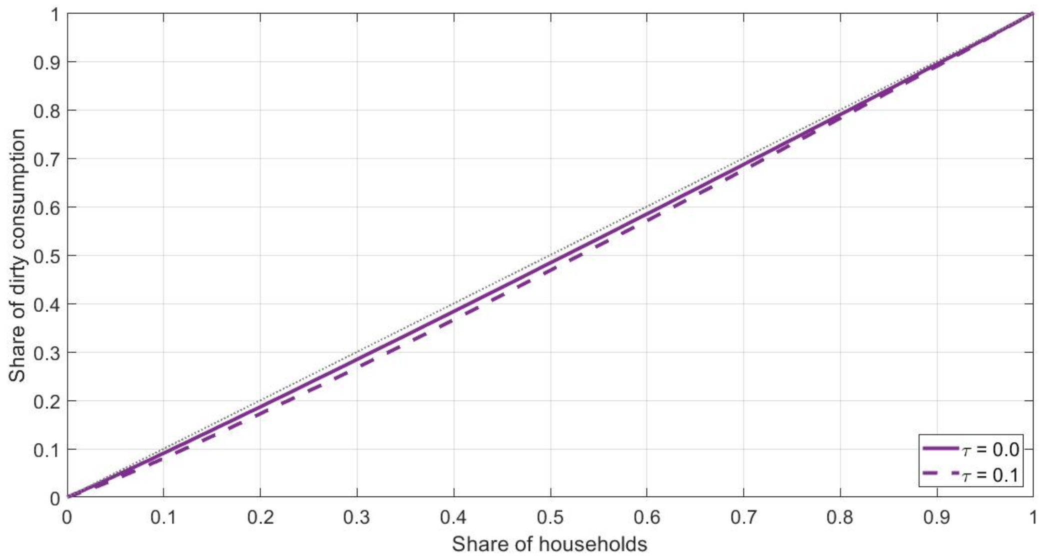
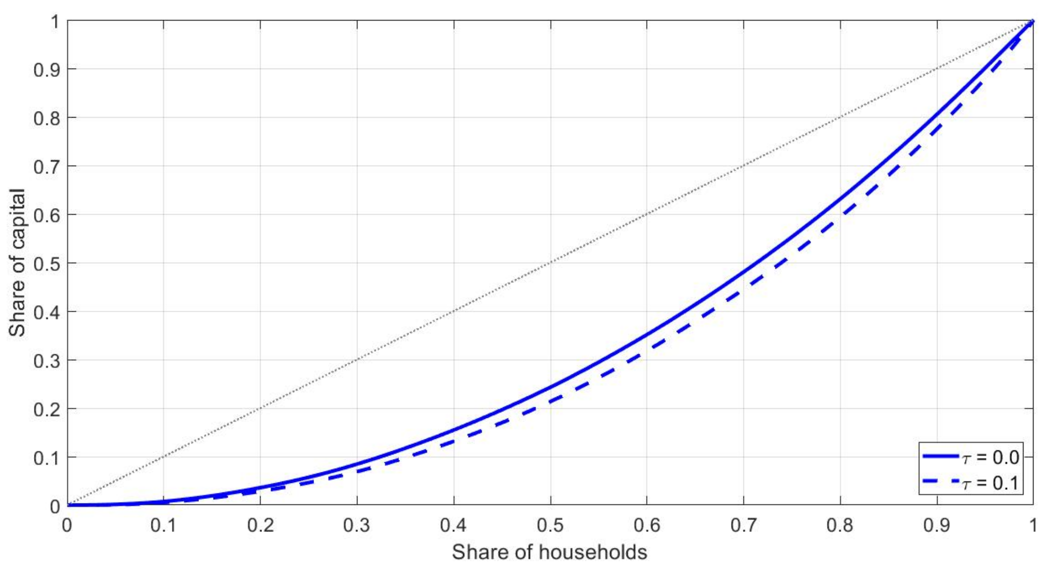


Figure 10

Lorenz curves of asset accumulation with and without carbon taxation



6 Conclusion

We assume that clean goods are luxury goods, whose share in the consumption bundle increases with income. The intuition behind is that, due to the necessity nature of the dirty good, individuals with lower income will be more reluctant to substitute it with green consumption, because that could mean forgoing their basic consumption needs. As income increases, so does consumption and the marginal utility of additional consumption declines. It is then when consumers can be more interested in introducing green goods in their consumption bundle, because their basic needs are satisfied and they have a preference for the green good.

In this framework, the establishment of a carbon tax, which only affects the dirty good, can have important consequences for the distribution of wealth. The dirty good is more inelastically consumed by individuals at lower layers of the wealth distribution. Thus, the carbon tax will only have the desired effects on households of higher income, which are willing to switch to environmentally friendly products. Conversely, less affluent individuals will minimise the reduction of their total consumption given the new prices. This will cause them to cut back on their green purchases because now their basic consumption, which is mostly composed by the dirty good, is more expensive. Against this background, the use that the government makes of the tax revenues gains remarkable importance to attain a more equalitarian sharing of the economic burden of tackling climate change with this policy instrument.

The contribution of this work lies in providing a model for the study of environmental policy instruments, more precisely of carbon taxation, and their implications in an economy in which not all goods are environmentally harmful and not all agents have the same resources at their disposal. Implementing two sectors, one polluting and another clean, opens up the possibility for the government to try to reduce greenhouse gas emissions without shrinking the entire economy. Nevertheless, heterogeneity of agents in their income and the composition of their consumption bundles prevents the measure from being socially neutral.

We carry out several different simulations of the model with and without government's intervention. The results show that the model does capture the mechanisms that we are describing. Indeed, the introduction of the tax causes relatively larger falls in consumption between less affluent households. Since they need to secure their basic consumption, they shift the taxation burden to the green sector, lowering their demand of clean good. Wealthier individuals, in contrast, are able to smooth the impact by reducing their stock of capital and by substituting dirty consumption with the clean good. As a result, the policy instrument only features the desired effects on the richer segments of the economy. Overall, the tax leads to lower ratios of green consumption to total consumption. Moreover, it reinforces the luxury condition of the green good, only within the reach of the most affluent agents, and it penalises the poorest households with disproportionate consumption cuts.

In conclusion, the model proposed brings a new dimension to the analysis of environmental policies, suggesting that the carbon tax alone might impose a high social toll without even achieving the environmental objectives for which it was conceived. The presence of idiosyncratic income shocks and the condition of the traditional good as a necessity are the most important features of the model, and the cause that carbon taxation has such unintended effects. In the model, the measure is clearly regressive, with the less affluent agents bearing the brunt while rich households dodge it by switching to green consumption. This feature is in line with the standard literature and shifts the onus of environmental action to the management of tax revenues.

7 Further development

A natural continuation of this work involves a more relevant role for the authorities. Different uses for tax revenue should be considered. For simplicity, we considered that government's consumption is neutral. However, there are other possibilities. For example, in Heutel (2012), it is returned to the consumer, while in Angelopoulos et al. (2010), it is invested in abatement of emissions. Nevertheless, taking into account the uneven distributional impact of the tax, it seems convenient that the revenue is used progressively. In our model, the tax is successful between wealthier households, who effectively replace dirty consumption with clean consumption. Hence, a potential use for tax revenue is the emission of green consumption vouchers inversely proportional to wealth. This policy might be the stimulus that the poorer strata of society need to switch to green consumption as well.

In order to enhance the usability of the model, considerable effort to refine the model's calibration is required. New simulations should be run, testing different parameter values and trying to adjust the output to an actual economy. Other specifications for the households' utility might turn out to be more adequate and are worthy to be tested.

The model could also be extended in several ways. The assumption that all individuals care about environmental protection and that they do so to the same extent seems too strict. It can be relaxed by introducing heterogeneity in the parameters of the utility function, what could lead to more realistic demands for green goods. It could be complemented with a life-cycle scheme, to capture not only differences in environmental concern but also in economic solvency across generations. Finally, the shift from traditional to eco-conscious production systems is arguably fraught with social, business and environmental complexities that this simple model cannot fully capture.

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Appendices

Appendix I Firms' optimal behaviour

I.1 Polluting firms

The problem of the traditional firm can be rewritten as:

$$\max_{\{K_t^d, L_t^d\}} \quad \Pi_t^d = (1-d(x_t))A_t(K_t^d)^{\alpha}(L_t^d)^{1-\alpha} - r_t K_{t-1}^d - w_t L_t^d - \tau\left[(1-d(x_t))A_t(K_t^d)^{\alpha}(L_t^d)^{1-\alpha}\right]^{1-\gamma}.$$

The corresponding first-order conditions are:

$$\frac{\partial \Pi_t^d}{\partial K_t^d} = 0 \implies f_{K,t}^d - r_t - \tau(1-\gamma)\left(Y_t^d\right)^{-\gamma} f_{K,t}^d = 0 \implies r_t = \left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right] f_{K,t}^d,$$
$$\frac{\partial \Pi_t^d}{\partial L_t^d} = 0 \implies f_{L,t}^d - w_t - \tau(1-\gamma)\left(Y_t^d\right)^{-\gamma} f_{L,t}^d = 0 \implies w_t = \left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right] f_{L,t}^d,$$

with

$$f_{K,t}^d = \alpha(1-d(x_t))A_t(K_t^d)^{\alpha-1}(L_t^d)^{1-\alpha},$$
$$f_{L,t}^d = (1-\alpha)(1-d(x_t))A_t(K_t^d)^{\alpha}(L_t^d)^{-\alpha}.$$

The FOCs can be rewritten as:

$$\frac{K_t^d}{L_t^d} = \left[\frac{r_t}{\left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right]\alpha(1-d(x_t))A_t}\right]^{\frac{1}{\alpha-1}},$$
$$\frac{K_t^d}{L_t^d} = \left[\frac{w_t}{\left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right](1-\alpha)(1-d(x_t))A_t}\right]^{\frac{1}{\alpha}}.$$

Combining both:

$$\left[\frac{r_t}{\left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right]\alpha(1-d(x_t))A_t}\right]^{\frac{1}{\alpha-1}} = \left[\frac{w_t}{\left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right](1-\alpha)(1-d(x_t))A_t}\right]^{\frac{1}{\alpha}}.$$

Operating,

$$\frac{w_t}{\left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right](1-\alpha)(1-d(x_t))A_t} = \left[\frac{r_t}{\left[1-\tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right]\alpha(1-d(x_t))A_t}\right]^{\frac{\alpha}{\alpha-1}}.$$

Solving for the wage,

$$w_t = \left\{ \left[1 - \tau(1-\gamma) \left(Y_t^d\right)^{-\gamma} \right] (1 - d(x_t)) A_t \right\}^{\frac{1}{1-\alpha}} (1-\alpha) \left(\frac{r_t}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}. \tag{24}$$

I.2 Green firms

Minimisation of costs

The problem of minimisation of costs of the intermediate green firm can be rewritten as:

$$\begin{aligned} \max_{\{K_t^c(j), L_t^c(j)\}} \quad & -r_t K_t^c(j) - w_t L_t^c(j) - g(1)(1-d(x_t)) A_t K_t^c(j)^{\alpha} L_t^c(j)^{1-\alpha}, \\ \text{s.t} \quad & (1-d(x_t)) A_t K_t^c(j)^{\alpha} L_t^c(j)^{1-\alpha} = \left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega} Y_t^c. \end{aligned}$$

The associated Lagrangian is:

$$\begin{aligned} \mathcal{L}_t(j) = & - r_t K_t^c(j) - w_t L_t^c(j) - g(1)(1-d(x_t)) A_t K_t^c(j)^{\alpha} L_t^c(j)^{1-\alpha} + \\ & + \xi_t(j) \left[(1-d(x_t)) A_t K_t^c(j)^{\alpha} L_t^c(j)^{1-\alpha} - \left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega} Y_t^c \right]. \end{aligned}$$

The corresponding first-order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}_t(j)}{\partial K_t^c(j)} = 0 &\implies r_t = (\xi_t(j) - g(1)) f_{K,t}^c(j), \\ \frac{\partial \mathcal{L}_t(j)}{\partial L_t^c(j)} = 0 &\implies w_t = (\xi_t(j) - g(1)) f_{L,t}^c(j), \\ \frac{\partial \mathcal{L}_t(j)}{\partial \xi_t(j)} = 0 &\implies (1-d(x_t)) A_t K_t^c(j)^{\alpha} L_t^c(j)^{1-\alpha} = \left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega} Y_t^c \end{aligned}$$

with

$$\begin{aligned} f_{K,t}^c(j) &= \alpha(1-d(x_t)) A_t K_t^c(j)^{\alpha-1} L_t^c(j)^{1-\alpha}, \\ f_{L,t}^c(j) &= (1-\alpha)(1-d(x_t)) A_t K_t^c(j)^{\alpha} L_t^c(j)^{-\alpha}. \end{aligned}$$

Note that the first two FOCs correspond to Equations 11 and 12. Dividing them with respect to the production inputs, we find that:

$$\frac{K_t^c(j)}{L_t^c(j)} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}. \tag{25}$$

All elements on the right-hand side are not firm-specific. Therefore, the ratio capital to labour will be the same across firms and we can get rid of the index j.

The FOCs can be rewritten as:

$$\frac{K_t^c}{L_t^c} = \left[\frac{r_t}{(\xi_t(p) - g(1))\alpha(1 - d(x_t))A_t}\right]^{\frac{1}{\alpha-1}},$$
$$\frac{K_t^c}{L_t^c} = \left[\frac{w_t}{(\xi_t(p) - g(1))(1-\alpha)(1 - d(x_t))A_t}\right]^{\frac{1}{\alpha}}.$$

Since $\xi_t(j)$ is the only term dependent on j, we can assert that it will not vary across firms either and therefore the index j is negligible. We can reduce the two FOCs to one expression:

$$\left[\frac{r_t}{(\xi_t - g(1))\alpha(1 - d(x_t))A_t}\right]^{\frac{1}{\alpha-1}} = \left[\frac{w_t}{(\xi_t - g(1))(1-\alpha)(1 - d(x_t))A_t}\right]^{\frac{1}{\alpha}}.$$

Rearranging,

$$(\xi_t - g(1))^{\frac{1}{1-\alpha}}\left[\frac{r_t}{\alpha(1 - d(x_t))A_t}\right]^{\frac{1}{\alpha-1}} = (\xi_t - g(1))^{\frac{-1}{\alpha}}\left[\frac{w_t}{(1-\alpha)(1 - d(x_t))A_t}\right]^{\frac{1}{\alpha}}.$$

Operating,

$$(\xi_t - g(1))^{-\alpha}\left[\frac{r_t}{\alpha(1 - d(x_t))A_t}\right]^{\alpha} = (\xi_t - g(1))^{1-\alpha}\left[\frac{w_t}{(1-\alpha)(1 - d(x_t))A_t}\right]^{\alpha-1}.$$

Solving for ξ_t, and taking into account that ξ_t represents the marginal cost of the intermediate firms (as proven in Appendix II), we obtain that the marginal cost is the same for all intermediate firms, and it is given by:

$$MC_t^c(j) = MC_t^c = \left[\frac{r_t}{\alpha(1 - d(x_t))A_t}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)(1 - d(x_t))A_t}\right]^{1-\alpha} + g(1). \tag{26}$$

Maximisation of profits

This problem can be simplified to:

$$\max_{\{p_t^c(j)\}} \Pi_t^c(j) = (p_t^c(j) - MC_t^c(j))\left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega} Y_t^c.$$

And the corresponding FOC is:

$$\frac{\partial \Pi_t^c(j)}{\partial p_t^c(j)} = 0 \implies (1-\omega)\left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega} Y_t^c + \xi_t\omega\left(\frac{p_t^c(j)}{P_t^c}\right)^{-\omega-1}\frac{1}{P_t^c}Y_t^c = 0.$$

Taking into account the demand function (Equation 10),

$$(1-\omega)y_t^c(j) + \xi_t\omega\frac{1}{p_t^c(j)}y_t^c(j) = 0.$$

Solving for the price:

$$p_t^c(j) = \frac{\omega}{\omega - 1}\xi_t.$$

Thus, all firms will optimally set the same price at each period, which is a constant mark-up over the marginal cost they face.

I.3 Equilibrium

From Equations 5, 6, 11 and 12, and taking into account that production factors receive the same retribution regardless the sector in which they are exploited, the following must hold:

$$\left[1 - \tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right] f_{K,t}^d = (\xi_t - g(1)) f_{K,t}^c.$$

$$\left[1 - \tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}\right] f_{L,t}^d = (\xi_t - g(1)) f_{L,t}^c.$$

Consequently,

$$\frac{f_{K,t}^d}{f_{K,t}^c} = \frac{f_{L,t}^d}{f_{L,t}^c}.$$

Note that:

$$\frac{f_{K,t}^d}{f_{K,t}^c} = \frac{\alpha^{Y_t^d/K_t^d}}{\alpha^{Y_t^c/K_t^c}} \quad \text{and} \quad \frac{f_{L,t}^d}{f_{L,t}^c} = \frac{(1-\alpha)^{Y_t^d/L_t^d}}{(1-\alpha)^{Y_t^c/L_t^c}}.$$

Hence,

$$\frac{K_t^d}{K_t^c} = \frac{L_t^d}{L_t^c}.$$

And, given the production technologies,

$$\frac{Y_t^d}{Y_t^c} = \left(\frac{K_t^d}{K_t^c}\right)^{\alpha} \left(\frac{L_t^d}{L_t^c}\right)^{1-\alpha},$$

which implies that:

$$\frac{Y_t^d}{Y_t^c} = \frac{K_t^d}{K_t^c} = \frac{L_t^d}{L_t^c}. \tag{27}$$

As a result,

$$\frac{f_{K,t}^d}{f_{K,t}^c} = \frac{f_{L,t}^d}{f_{L,t}^c} = 1,$$

leaving:

$$\xi_t - g(1) = 1 - \tau(1-\gamma)\left(Y_t^d\right)^{-\gamma}.$$

Appendix II Proofs

II.1 Non-homothetic preferences

A monotone preference relation is homothetic if all indifferent sets related by proportional expansion along rays; that is, if $x \sim y$, then $\alpha x \sim \alpha y$ for any $\alpha \geq 0$ (Mas-Colell, Whinston, Green, et al., 1995). Consequently, preferences are homothetic if they can be represented by a utility function which is homogeneous of degree 1. Therefore, the utility function needs to satisfy $U(\alpha c^c, \alpha c^d) = \alpha U(c^c, c^d)$. This is not the case of our utility function, as we prove below:

$$U(\alpha c^c, \alpha c^d) = \frac{1}{1-\zeta}\left[(\alpha c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(\alpha c^c)^{1-\phi}\right]^{\frac{1-\zeta}{1-\lambda}} =$$

$$= \frac{1}{1-\zeta}\left[\alpha^{1-\lambda}(c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}\alpha^{1-\phi}(c^c)^{1-\phi}\right]^{\frac{1-\zeta}{1-\lambda}}.$$

To continue, we need to assume that $\lambda = \phi$. In such case,

$$U(\alpha c^c, \alpha c^d) = \frac{1}{1-\zeta}\left\{\alpha^{1-\lambda}\left[(c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(c^c)^{1-\phi}\right]\right\}^{\frac{1-\zeta}{1-\lambda}} =$$

$$= \frac{1}{1-\zeta}\alpha^{1-\zeta}\left[(c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(c^c)^{1-\phi}\right]^{\frac{1-\zeta}{1-\lambda}}.$$

Finally, if we assume that $\zeta = 0$, we obtain:

$$U(\alpha c^c, \alpha c^d) = \alpha\frac{1}{1-\zeta}\left[(c^d)^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi}(c^c)^{1-\phi}\right]^{\frac{1-\zeta}{1-\lambda}} = \alpha U(c^c, c^d).$$

In conclusion, our utility function will represent homothetic preferences if and only if $\lambda = \phi$ and $\zeta = 0$. However, since we are imposing $\lambda > \phi$ and $\zeta > 0$, we can conclude that our model features non-homothetic preferences.

II.2 Marginal cost and Lagrange multiplier

We offer the proof that the Lagrange multiplier of the intermediate firm's costs minimisation problem represents the marginal cost that the firm faces. Differentiating the objective function with respect to output, we get:

$$\frac{\partial C_t(j)}{\partial y_t^c(j)} = \frac{\partial C_t(j)}{\partial K_t(j)}\frac{\partial K_t(j)}{\partial y_t^c(j)} + \frac{\partial C_t(j)}{\partial L_t(j)}\frac{\partial L_t(j)}{\partial y_t^c(j)} + g(1).$$

Since we have shown that the marginal cost will be the same across firms, we can continue without considering the index j.

$$\frac{\partial C_t}{\partial y_t^c} = \left(r_t + g(1)\frac{\partial y_t^c}{\partial K_t^c}\right)\frac{\partial K_t}{\partial y_t^c} + \left(w_t + g(1)\frac{\partial y_t^c}{\partial L_t^c}\right)\frac{\partial L_t}{\partial y_t^c} + g(1).$$

Observe that the first order conditions (Equations 11 and 12) establish that $r_t = (\xi_t - g(1))^{\partial y_t^c}/_{\partial K_t^c}$ and $w_t = (\xi_t - g(1))^{\partial y_t^c}/_{\partial N_t^c}$. Therefore, substituting into the previous expression and simplifying,

$$\frac{\partial C_t^c}{\partial y_t^c} = \xi_t \left(\frac{\partial y_t^c}{\partial K_t^c} \frac{\partial K_t^c}{\partial y_t^c} + \frac{\partial y_t^c}{\partial L_t^c} \frac{\partial L_t^c}{\partial y_t^c} \right).$$

Now, let us totally differentiate the production function with respect to output:

$$\frac{\partial y_t^c}{\partial K_t^c} \frac{\partial K_t^c}{\partial y_t^c} + \frac{\partial y_t^c}{\partial L_t^c} \frac{\partial L_t^c}{\partial y_t^c} = 1.$$

Taking this result into account, the previous expression reduces to:

$$\frac{\partial C_t}{\partial y_t^c} = \xi_t.$$

In conclusion, ξ_t represents the marginal cost.

Appendix III MATLAB code

III.1 Main

```
% CarbonAiya_simpletax_v5.m
% In this version, the tax is linear on dirty capital and dirty labour.
% v5: - change in computation of cleanc (indexed to assets and shock)
%     - correction in computation of aggdirtyc
% +v3: greater variance of income shock + lower upper bound on asset grid.
% +v1: approximating rdirty.

clc
clear
diary

%% Parameters

% Model parameters
    borrow=.0;  % borrowing constraint ad hoc.
    tau = 0;  % carbon tax

% Household
    % Preferences
        beta=.95; % take a value which we use in the RBC literature.. The
horizon here is annual.
        zita=2; % gamma in Claudio (2018)
            %The coefficient γ that determines the curvature of the period
utility index defined
            %over the maximized intra-temporal utility of consumption
expenditures is set at a
            %value of 3, corresponding to the intermediate risk-aversion case
in Aiyagari (1994).
        lambda=20; %curvature parameter 1 (11.5)
        phi=1.5; %curvature parameter 2 (1.1)
        eta=50; %utility weight on cleanc (75)
            %These 3 values come from Claudio (2018), need to calibrate to
            %green consumption data.
            %The curvature parameters determine how the expenditure share
            %of basic goods varies over the consumption distribution.
        subscc=0; %to shift down clean consumption (start consuming clean
with higher asset)
    % Shock
        nl=4; % number of nodes for uncertainty.
        rhol=.95; % persistence of the idiosyncratic shock
        %sigmal=.2*(1-rhol^2)^(1/2); % coefficient of variation of the shock
        sigmal=0.5*(1-rhol^2)^(1/2);
    % Household decision grids:
        n1=100;
        n2=300;
        asset=[linspace(-borrow,1,n1),linspace(1.01,25,n2)];
        na=length(asset);
        cleanc=[linspace(0.000001,1,n2),linspace(1.1,3,n1)];

% Firm
```

```
    alpha=.36;
    %delta=.0125*4;
    delta=0.03;
    elas=1.2; % Elasticity of substitution between clean goods. It determines
the price mark-up.

% Steady State
    rRBC= 1/beta -1; % The equilibrium interest rate in the RBC model
    % (from alpha K^(alpha-1) LL^(1-alpha)  +(1-delta) =1/beta)
    wRBC= (1-alpha)*(((rRBC+delta)/alpha)^(1/(alpha-1)))^alpha;

% Parte medioambiental
    % these values come from Heutel (2012). Take care! Quarterly in his
    % case
    theta1 = 0.05607; % abatement cost
    d0 = 1.3950*10^(-3); % damage function parameter
    d1 = -6.6722*10^(-6); % damage function parameter
    d2 = 1.4647*10^(-8); % damage function parameter
    gamma = 0.304; % parameter of emissions
    %polpers = 0.9979; % persistence of pollution. eta in Heutel (2012).

% Otros
    jl = 1; % por si decido añadir otro bucle para comparar distintos valores
de un parámetro

%% Idiosyncratic shock
% Discretising an AR(1):
    [Z,PI] = addacooper(nl,0,rhol,sigmal); % this returns the 4 states and
the
        % matrix of transition probabilities
        % inputs: nl states, 0 average, rhol persistence, sigmal st. dev.
    ll=exp(Z); % because it is a logarithmic AR(1)
    prob=PI; % just change the name
% Distribution:
    distributionl=prob^1000*[1;0;0;0];
    LL=distributionl(:)'*ll(:); % weighted sum of each labour supply and
probability (25%)

%% Initial guess
% punto de partida para encontrar el equilibrio general.
% Remember that beta*(1+r) < 1. That's probably the reason why rhighAiya
% must also be lower than rRBC (since beta*(1+rRBC) = 1).
rhighAiya=rRBC-0.000001;
rlowAiya=0.000001;

V=zeros(na,nl); % initial guess for x the value function
optdirtyc=zeros(na,nl); % initialising the matrix of dirtyc given a' and
shock
% parte medioambiental:
carbon0 = 800;
damage0 = d2*carbon0^2 + d1*carbon0 + d0;
% erow_ss = 0; % Heutel (2012) calibrates it to 3 times the SS of e.

%% Loop for the general equilibrium
distar=1;
```

```
limdistar=0.0003;
iter=0;
while distar>=limdistar
    iter = iter+1;
    % guessed value
    r=(rhighAiya+rlowAiya)/2; % current interest rate (guess).
    % w is pinned down by r:
    w=(1-damage0)^(1/(1-alpha)) * (1-alpha) * ((r + tau +
delta)/alpha)^(alpha/(alpha-1)) - tau;
    % marginal cost - clean good
    cleanmc = ((r + delta)/((1-damage0)*alpha)^alpha)*((w/((1-damage0)*(1-
alpha)))^(1-alpha)) + theta1;
    % price of the clean good (from clean firm's focs)
    pcc = elas/(elas-1) * cleanmc;

    %% Nested loop for the consumer's problem: solving the individual value
function.
    dist=1;
    limdist=0.0001;
    while dist>=limdist %.0001
        for i=1:na % we need to make the computations for each possible
initial a
            for j=1:nl % we need to make the computations for each possible
state of l
                %cc=-asset +(1+r)*asset(i)+w*ll(j); % consumption (This can
be negative the way it is defined here- shouldnt be_
                dirtyc=-(asset'*ones(1,na)) - pcc*(ones(na,1)*cleanc) +
(1+r)*asset(i) + w*ll(j);
                indcon=find(dirtyc<0); % to keep track of non desired values
                dirtyc=max(dirtyc,.000000000000000000000000000001); % making
sure consumption is non negative so that the problem  is defined....
                % when dirtyc is 0.000000001 we re certain that the agent
will not choose this value.
                nextV=(V*prob(j,:)'); % Continuation Utility.
                %nextV(indcon)=-10000; % ensuring that the non-desired values
are not chosen
                % utility function (Bellman):
                util = 1/(1-zita) * (dirtyc.^(1-lambda) + ...
                   eta*(1-lambda) / (1-phi) *
((ones(na,1)*cleanc)+subscc).^(1-phi)).^((1-zita)/(1-lambda)) + ...
                   beta*(nextV(:)*ones(1,na)); % nextV only depends on a',
that's why it can be equal for each level of cleanc
                util(indcon) = -10000;
                [utilcc,ind]=max(util); % maximize over the asset grid
                [va,indcc]=max(utilcc) % maximize over the cleanc grid
                inda = ind(indcc)
                V1(i,j)=util(inda,indcc); % update the value function
                aprime(i,j)=asset(inda); % decision rule on assets
                indexaprime(i,j)=inda;
                optcleanc(i,j)=cleanc(indcc); % decision rule on clean
consumption
                %indexcleanc(i,j)=indcc;
                % matrix of dirty consumption given optimal a' (rows) and
                % optimal cleanc (columns)
                optdirtyc(i,j)=dirtyc(inda,indcc);
```

```
            end
        end
        dist=max(max(abs(V1-V)))
        V=V1;
    end

    %% Distribución de Estado Estacionario
    % Inicializar las variables
        problongruna=ones(na,nl)/(na*nl);
%         problongruncc=ones(na,nl)/(na*nl);
        Qa1=zeros(na,na);
        Qa2=zeros(na,na);
        Qa3=zeros(na,na);
        Qa4=zeros(na,na);
%         Qcc1=zeros(na,na);
%         Qcc2=zeros(na,na);
%         Qcc3=zeros(na,na);
%         Qcc4=zeros(na,na);
    % Dar valores a las nuevas variables: Qa1 indica dónde se encuentra la
    % decisión óptima de a' para cada nivel de a, cuando el individuo se
    % encuentra en el 1er estado de l.
    for i=1:na
        Qa1(i,indexaprime(i,1))=1;
        Qa2(i,indexaprime(i,2))=1;
        Qa3(i,indexaprime(i,3))=1;
        Qa4(i,indexaprime(i,4))=1;
%         Qcc1(i,indexcleanc(i,1))=1;
%         Qcc2(i,indexcleanc(i,2))=1;
%         Qcc3(i,indexcleanc(i,3))=1;
%         Qcc4(i,indexcleanc(i,4))=1;
    end
    % Bucle anidado para encontrar la distribución de Estado Estacionario.
    % activos:
    distprob=1;
    while distprob>=.000000000000001
        % probabilidad de cada nivel de activos condicionado al estado de l
        problongrun1= Qa1'*problongruna(:,1);
        problongrun2= Qa2'*problongruna(:,2);
        problongrun3= Qa3'*problongruna(:,3);
        problongrun4= Qa4'*problongruna(:,4);
        % actualización de la matriz de probabilidades de largo plazo
        problongrunnew=[problongrun1,problongrun2,problongrun3,problongrun4];
        problongrunnew=problongrunnew*prob;
        % calculando la distancia con respecto a la matriz inicial
        distprob=max(max(abs(problongrunnew-problongruna)));
        problongruna=problongrunnew;
    end
    % consumo limpio:
%     distprob=1;
%     while distprob>=.000000000000001
%         % probabilidad de cada nivel de activos condicionado al estado de l
%         problongrun1= Qcc1'*problongruncc(:,1);
%         problongrun2= Qcc2'*problongruncc(:,2);
%         problongrun3= Qcc3'*problongruncc(:,3);
%         problongrun4= Qcc4'*problongruncc(:,4);
```

```
%         % actualización de la matriz de probabilidades de largo plazo
%
problongrunnew=[problongrun1,problongrun2,problongrun3,problongrun4];
%         problongrunnew=problongrunnew*prob;
%         % calculando la distancia con respecto a la matriz inicial
%         distprob=max(max(abs(problongrunnew-problongruncc)));
%         problongruncc=problongrunnew;
%     end

    %% Niveles agregados
    amean=sum((asset(:)'*problongruna));
    aggcleanc=sum(optcleanc.*problongruna,'all'); % revisar
    aggdirtyc=sum(optdirtyc.*problongruna,'all'); % revisar

    %% Production inputs in each sector
    ratio = aggdirtyc/aggcleanc;
    Kc = amean/(1+ratio);
    Kd = amean - Kc;
    Lc = LL/(1+ratio);
    Ld = LL - Lc;

    %% Condición de equilibrio general
    % valor para r, por el lado de la empresa
    rdirty=(1-damage0) * alpha * Kd^(alpha-1) * Ld^(1-alpha) - delta - tau;
    rgreen=(cleanmc - theta1)*(1-damage0)*alpha*Kc^(alpha-1)*Lc^(1-alpha)-
delta;
    % valor para w, por el lado de la empresa
    wdirty=(1-damage0) * (1-alpha) * Kd^alpha * Ld^(-alpha) - tau;
    wgreen=(cleanmc - theta1)*(1-damage0) * (1-alpha) * Kc^alpha * Lc^(-
alpha);
    % distancia con respecto a la conjetura inicial
        %distar=max(abs(rdirty-r),abs(rgreen-r));
        distar=abs(rdirty-r);
    % actualizar la conjetura
    if (rdirty>r) % in this case capital accumulation is too low, we need to
increase the interest rate
        rlowAiya=r;
    elseif (rdirty<r)
        rhighAiya=r;
    end
%     % obtain matrix of variable values in each iteration
%     mcleanc(iter) = aggcleanc;
%     mdirtyc(iter) = aggdirtyc;
%     masset(iter) = amean;
%     mkc(iter) = Kc;
%     mkd(iter) = Kd;
%     mL(iter) = LL;
%     mLc(iter) = Lc;
%     mLd(iter) = Ld;
%     mpcc(iter) = pcc;
%     mr(iter) = r;
%     mrdirty(iter) = rdirty;
%     mrgreen(iter) = rgreen;
%     mw(iter) = w;
%     mwdirty(iter) = wdirty;
```

```
%     mwgreen(iter) = wgreen;
%     mutil(iter) = va;
end

%% Distribución de activos en el largo plazo
LRdist=sum(problongruna'); % sums the probability of each level of asset,
regardless the state l
indexposa=find(LRdist>0); % revela qué niveles de a pueden darse en el largo
plazo
LRdist1=LRdist(indexposa); %  igual que LRdist pero sin probabilidades nulas
asset1=asset(indexposa); % igual que asset pero sin opciones con probabilidad
0

%% Long-run distribution of clean consumption
% CCdist=sum(problongruncc');
% indexposcc=find(CCdist>0);
% CCdist1=CCdist(indexposcc);
% cleanc1=cleanc(indexposcc);

%% Otras variables
    % Cash in hand
    cih = (w*ll*ones(1,na))' + (1+r)*asset';
    cih = [cih(:,1);cih(:,2);cih(:,3);cih(:,4)];
    [cih, auxind] = sort(cih); %cih is a vector with ordered values of cash
in hand
    % Optimal clean consumption indexed to cih
    optcleanc2 =
[optcleanc(:,1);optcleanc(:,2);optcleanc(:,3);optcleanc(:,4)];
    optcleanc2 = optcleanc2(auxind); % to order it consistently with cih
    % Optimal dirty consumption indexed to cih
    optdirtyc2 =
[optdirtyc(:,1);optdirtyc(:,2);optdirtyc(:,3);optdirtyc(:,4)];
    optdirtyc2 = optdirtyc2(auxind); % to order it consistently with cih
    % Optimal assets indexed to cih
    optasset2 = [aprime(:,1);aprime(:,2);aprime(:,3);aprime(:,4)];
    optasset2 = optasset2(auxind); % to order it consistently with cih
    % Probabilities of distribution of assets
    probacuma(1) = 0;
    for i=1:length(LRdist1)
        probacuma(i+1) = probacuma(i) + LRdist1(i);
    end
    % Distribution of assets
    Sa(1)=0;
    for i=2:length(asset1)+1;
        Sa(i)=Sa(i-1)+asset1(i-1)*LRdist1(i-1);
    end
    % Distribution of cleanc
    cc = sum((optcleanc.*problongruna),2);
    cc1 = cc(indexposa);
    Scc(1)=0;
    for i=2:length(asset1)+1;
        Scc(i)=Scc(i-1)+cc1(i-1);
    end
    % Distribution of dirtyc
    dc = sum((optdirtyc.*problongruna),2);
```

```
    dc1 = dc(indexposa);
    Sdc(1)=0;
    for i=2:length(asset1)+1;
        Sdc(i)=Sdc(i-1)+dc1(i-1);
    end

%% Algunos índices
% Índice de Gini
    GINI=0;
    for i=2:length(asset1)+1;
        GINI=GINI+LRdist1(i-1)*(Sa(i-1)+Sa(i))/Sa(end);
    end
    display('GINI COEFFICIENT')
    GINI=1-GINI
    GINIjl(jl)=GINI;
% Individuos con a <= 0
    display(' Fraction with Zero or Negative net Worth')
    fractionzero(jl)=sum(LRdist1(asset1<=0))
% Ratio producción / capital
    display('Capital Output Ratio')
    display('IM   RBC')
    [amean/(amean^(alpha)*LL^(1-alpha)), ((1/beta-1+delta)/alpha)^(-1)]
    Capitaloutput(jl)=amean/(amean^(alpha)*LL^(1-alpha));
% Retorno al capital (r)
    display('Returns to Capital')
    display('IM   RBC')
    [rdirty, rRBC]
    returnnew(jl)=rdirty;
% Fracción de riqueza poseída por el 1% más rico
    richestonepercent= min(find(cumsum(LRdist1)>=.99));

wealthrich=(LRdist1(richestonepercent:end)*asset1(richestonepercent:end)')/am
ean;
    display('Fraction of Wealth to the Richest One percent')
    wealthrichjl(jl)=  wealthrich

% SAVE RESULTS
save simpletax_v5_tau0.mat
```

III.2 Graphs

```
% CarbonAiya_graphs.m
% Generating the graphs

%% Preliminares
close all
clc
clear

%% Data with tau=0
load simpletax_v5_tau0.mat

caseA.aprime = aprime; % optimal a' indexed to initial asset and to the shock
caseA.optcc = optcleanc; % optimal cleanc indexed to initial asset and to the
shock
caseA.optdc = optdirtyc; % optimal dirtyc indexed to initial asset and to the
shock
caseA.cih = cih; % cash in hand
caseA.cihcc = optcleanc2; % optimal clean consumption indexed to cih
caseA.cihdc = optdirtyc2; % optimal dirty consumption indexed to cih
caseA.accprob = probacuma; % cumulative probability (asset distribution)
caseA.Sa = Sa; % cumulative optimal asset
caseA.Scc = Scc; % cumulative optimal cleanc
caseA.Sdc = Sdc; % cumulative optimal dirtyc
caseA.K = amean; % total capital
caseA.L = LL; % total labour
caseA.CC = aggcleanc; % total clean consumption
caseA.DC = aggdirtyc; % total dirty consumption

clearvars -except caseA

%% Data with tau=0.10
load simpletax_v5_tau10.mat

caseB.aprime = aprime; % optimal a' indexed to initial asset and to the shock
caseB.optcc = optcleanc; % optimal cleanc indexed to initial asset and to the
shock
caseB.optdc = optdirtyc; % optimal dirtyc indexed to initial asset and to the
shock
caseB.cih = cih; % cash in hand
caseB.cihcc = optcleanc2; % optimal clean consumption indexed to cih
caseB.cihdc = optdirtyc2; % optimal dirty consumption indexed to cih
caseB.accprob = probacuma; % cumulative probability (asset distribution)
caseB.Sa = Sa; % cumulative optimal asset
caseB.Scc = Scc; % cumulative optimal cleanc
caseB.Sdc = Sdc; % cumulative optimal dirtyc
caseB.K = amean; % total capital
caseB.L = LL; % total labour
caseB.CC = aggcleanc; % total clean consumption
caseB.DC = aggdirtyc; % total dirty consumption

clearvars -except caseA caseB asset

%% General settings
```

```
lwidth = 3;
lwidth2 = 1.5;
fsize = 16;
green1 = [0 0.6 0.3];
green2 = [0 1 0.5];
dirty1 = [0.4940, 0.1840, 0.5560];
dirty2 = [0.75, 0, 0.75];
gris = [0.5 0.5 0.5];

%% Calculando tasas de variación
vara = caseB.aprime./caseA.aprime - 1;
varcc = caseB.optcc./caseA.optcc - 1;
vardc = caseB.optdc./caseA.optdc - 1;

%% GRÁFICOS CASO BASE
%% Consumo según riqueza
legnames = {'Clean Cons.', 'Dirty Cons.'};
figure(1)
    plot(asset,mean(caseA.optcc'),'linewidth',lwidth, 'Color', green1)
    hold on
    plot(asset,mean(caseA.optdc'),'linewidth',lwidth, 'Color', dirty1)
    grid on
    xlabel('Assets')
    xticklabels([])
    ylabel('Consumption')
    yticklabels([])
    set(gca,'FontSize',fsize)
    hold off
    legend(legnames, 'Location', 'Best', 'FontSize', fsize)

%% Decisión de activos
legnames = {'Low Prod.', 'High Prod.'};
figure(2)
    plot(asset,caseA.aprime(:,1),'linewidth',lwidth, 'Color', 'b')
    hold on
    plot(asset,caseA.aprime(:,4),'linewidth',lwidth, 'Color', 'c')
    plot([0:1:25], [0:1:25], ':', 'linewidth', lwidth2, 'Color', gris)
    grid on
    xlabel('a')
    xticklabels([])
    ylabel('a''', 'Rotation', 0, 'HorizontalAlignment', 'right')
    yticklabels([])
    set(gca,'FontSize',fsize)
    hold off
    legend(legnames, 'Location', 'Best', 'FontSize', fsize)

%% Consumo según riqueza y productividad
figure(3)
    plot(asset,caseA.optcc(:,1),'linewidth',lwidth, 'Color', green1,
'DisplayName', 'cleanc, low prod.')
    hold on
    plot(asset,caseA.optcc(:,4),'linewidth',lwidth, 'Color', green2,
'DisplayName', 'cleanc, high prod.')
    plot(asset,caseA.optdc(:,1),'linewidth',lwidth, 'Color', dirty1,
'DisplayName', 'dirtyc, low prod.')
```

```
    plot(asset,caseA.optdc(:,4),'linewidth',lwidth, 'Color', dirty2,
'DisplayName', 'dirtyc, high prod.')
    grid on
    xlabel('Assets')
    xticklabels([])
    ylabel('Consumption')
    yticklabels([])
    ylabel('Consumption')
    set(gca,'FontSize',fsize)
    hold off
    legend('Location', 'Best', 'FontSize', fsize)

%% Distribuciones (Curvas de Lorenz)
legnames = {'Assets', 'Clean Cons.', 'Dirty Cons.'};
figure(4)
    plot(caseA.accprob,caseA.Sa/caseA.K,'linewidth',lwidth, 'Color', 'b')
    hold on
    plot(caseA.accprob,caseA.Scc/caseA.CC,'linewidth',lwidth, 'Color',
green1)
    plot(caseA.accprob,caseA.Sdc/caseA.DC,'linewidth',lwidth, 'Color',
dirty1)
    plot([0:0.05:1], [0:0.05:1], ':', 'linewidth', lwidth2, 'Color', gris)
    grid on
    xlim([0 1])
    ylim([0 1])
    xlabel('Share of households')
    ylabel('Share of consumption / capital')
    set(gca,'FontSize',fsize)
    hold off
    legend(legnames, 'Location', 'Best', 'FontSize', fsize)

%% GRÁFICOS COMPARATIVOS
%% Variaciones en el consumo (alternativa)
legnames = {'Clean Cons.', 'Dirty Cons.'};
figure(5)
    plot(asset,mean(varcc,2),'linewidth',lwidth, 'Color', green1)
    hold on
    plot(asset,mean(vardc,2), 'linewidth',lwidth, 'Color', dirty1)
    plot(asset,zeros(length(asset),1), ':', 'linewidth', lwidth2, 'Color',
gris)
    grid on
    xlabel('Assets')
    xticklabels([])
    ylabel('Consumption (% change)')
    set(gca,'FontSize',fsize)
    hold off
    legend(legnames, 'Location', 'Best', 'FontSize', fsize)

%% Variaciones en activos
figure(6)
    plot(asset,vara(:,1),'linewidth', lwidth, 'Color', 'b', 'DisplayName',
'Low prod.')
    hold on
    plot(asset,vara(:,4), 'linewidth', lwidth, 'Color', 'c', 'DisplayName',
'High prod.')
```

```
    grid on
    xlabel('a')
    ylabel('a''', 'Rotation', 0, 'HorizontalAlignment', 'center')
    set(gca, 'FontSize', fsize)
    hold off
    legend('Location', 'Best', 'FontSize', fsize)

%% Distribución del bien verde
legnames = {'\tau = 0.0', '\tau = 0.1'};
figure(7)
    plot(caseA.accprob,caseA.Scc/caseA.CC,'linewidth',lwidth, 'Color',
green1)
    hold on
    plot(caseB.accprob,caseB.Scc/caseB.CC, '--', 'linewidth',lwidth, 'Color',
green1)
    plot([0:0.05:1], [0:0.05:1], ':', 'linewidth', lwidth2, 'Color', gris)
    grid on
    xlim([0 1])
    ylim([0 1])
    xlabel('Share of households')
    ylabel('Share of green consumption')
    set(gca,'FontSize',fsize)
    hold off
    legend(legnames, 'Location', 'SouthEast', 'FontSize', fsize)

%% Distribución del bien tradicional
legnames = {'\tau = 0.0', '\tau = 0.1'};
figure(8)
    plot(caseA.accprob,caseA.Sdc/caseA.DC,'linewidth',lwidth, 'Color',
dirty1)
    hold on
    plot(caseB.accprob,caseB.Sdc/caseB.DC, '--', 'linewidth',lwidth, 'Color',
dirty1)
    plot([0:0.05:1], [0:0.05:1], ':', 'linewidth', lwidth2, 'Color', gris)
    xlim([0 1])
    ylim([0 1])
    grid on
    xlabel('Share of households')
    ylabel('Share of dirty consumption')
    set(gca,'FontSize',fsize)
    hold off
    legend(legnames, 'Location', 'SouthEast', 'FontSize', fsize)

%% Distribución de activos
legnames = {'\tau = 0.0', '\tau = 0.1'};
figure(9)
    plot(caseA.accprob,caseA.Sa/caseA.K, 'linewidth', lwidth, 'Color', 'b')
    hold on
    plot(caseB.accprob,caseB.Sa/caseB.K, '--', 'linewidth', lwidth, 'Color',
'b')
    plot([0:0.05:1], [0:0.05:1], ':', 'linewidth', lwidth2, 'Color', gris)
    grid on
    xlim([0 1])
    ylim([0 1])
    xlabel('Share of households')
```

```
ylabel('Share of capital')
set(gca,'FontSize',fsize)
hold off
legend(legnames, 'Location', 'SouthEast', 'FontSize', fsize)
```

III.3 Adda-Cooper

```
function [Z,PI] = addacooper(n,mu,rho,sigma);
% Approximate n-state AR(1) process following Tauchen (1986) and Tauchen &
Hussey (1991).
% See Adda & Cooper (2003) pp 57-.
%
% Z(t+1) = mu*(1-rho) + rho*Z(t) + eps(t+1)
%
% where  std(eps) = sigma
%
% Martin Flod�n, 2005

sigmaUNC = sigma/sqrt(1-rho^2);
E  = zeros(n+1,1);
Z  = zeros(n,1);
PI = zeros(n,n);
MFPI = zeros(n,n);

E(1)   = -1E6;
E(end) = 1E6;
for i = 2:n
    E(i) = sigmaUNC*norm_inv((i-1)/n) + mu;
end
for i = 1:n
    Z(i) = n*sigmaUNC*(norm_pdf((E(i)-mu)/sigmaUNC) - norm_pdf((E(i+1)-
mu)/sigmaUNC)) + mu;
end
for i = 1:n
    for j = 1:n
        E1 = E(j);
        E2 = E(j+1);
        th_fcn  = @(u) n/sqrt(2*pi*sigmaUNC^2) * (exp(-(u'-mu).^2 /
(2*sigmaUNC^2)) .* ...
                      (norm_cdf((E2-mu*(1-rho)-rho*u')/sigma) - norm_cdf((E1-
mu*(1-rho)-rho*u')/sigma)));
        PI(i,j) = quadl(th_fcn,E(i),E(i+1),1e-10);
        MFPI(i,j) = norm_cdf((E(j+1)-mu*(1-rho)-rho*Z(i))/sigma) -
norm_cdf((E(j)-mu*(1-rho)-rho*Z(i))/sigma);
    end
end
for i = 1:n
    PI(i,:) = PI(i,:) / sum(PI(i,:));
    MFPI(i,:) = MFPI(i,:) / sum(MFPI(i,:));
end
function c = norm_cdf(x)
    c = 0.5 * erfc(-x/sqrt(2));
function p = norm_pdf(x)
    p = 1/sqrt(2*pi) * exp(-(x)^2/2);
function y = norm_inv(x)
    f = @(x0,a) (0.5 * erfc(-x0/sqrt(2))-a);
    y = fzero(f,0,[],x);
```