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Torsional Response of Multi-storey Buildings

Contribution to the development of the 2nd generation of Eurocode 8

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Abstract

This master thesis aims to provide consistent work to help the development of the second generation of the Eurocode 8 focused on the torsional response of multi-storey buildings. This master thesis is the continuation of the master thesis of Maxence Carlier [1]. The main objective is to extend his analysis focused on single-storey buildings to multi-storey buildings.

Firstly, a literature review is conducted to present the concepts related to torsion and to become familiar with the Eurocode 8. Then the two conditions for the classification of torsionally flexible buildings are analysed. A comparison between the two procedures to compute the torsional radius, the 3D procedure and the simplified method, can be found in this master thesis. As expected, the simplified method is more conservative since it does not consider the horizontal structural elements. A discussion about the reference point of the torsional radius is detailed in this work as well. According to the literature and the results obtained from the numerical analysis, it would be preferred to compute it with respect to the centre of mass. Finally, a linear analysis is performed to analyse the effects of the classification as torsionally flexible on the design of a building. The impacts of the accidental eccentricity are also reviewed. Several numerical models were created and analysed with the software Scia Engineer to achieve these goals.

These results and those obtained by Maxence Carlier for single-storey buildings bring a good overview of the classification of torsionally flexible buildings developed in the second generation of the Eurocode 8. However, all the models presented in this master thesis are regular in elevation. Therefore, to be complete, the analysis should be extended to multi-storey buildings irregular in elevation.

Contents

CHAPTER 1

Introduction

1.1. Motivation

The second generation of the Eurocode 8 is currently under preparation. The first generation has been released in 2004. Since then, calculation methods have evolved and new scientific discoveries have been made. This master thesis aims to make a humble contribution to the development of the second generation of Eurocode 8.

The subject "Torsional Response of Multi-storey Buildings", proposed by Pr. João Pacheco de Almeida and Dr. António Correia, is the continuation of the topic "Seismic response of torsionally flexible buildings" covered by Maxence Carlier in his master thesis last year [1]. The main objective is to extend his analysis focused on single-storey buildings to **multi-storey buildings**.

The topic covers an interesting aspect of the response of a building to an earthquake. During the past years, unfortunately, several buildings collapsed due to their torsional response. The following examples have been taken from the course Earthquake Engineering given by Pr. João Pacheco de Almeida at UCLouvain [2].

Figure 1.1: Collapse after the earthquake in Kobe, 1995, source [3]

In the case shown in Figure 1.1, the building collapsed during the earthquake in Kobe in 1995. This happened due to its natural eccentricity. The building had a continuous fire wall to the right rear as well as more eccentric bracing to the back. This caused the building to twist significantly, and led to the columns in the front failing due to strong deformations.

(a) (b)

Figure 1.2: Collapse after the earthquake in Athens in 1999, source [4]

The example in Figure 1.2 occurred after the earthquake in Athens in 1999. We can see a really stiff structural element in the corner of the building. This produced a torsional response, which twisted the building. Once again, the columns at the opposite corner failed because of too large deformations.

To avoid a collapse due to torsional responses in the future, both generations of the Eurocode 8 provide conditions to classify a building as "torsionally flexible". This means that the building does not provide sufficient torsional resistance. In the examples above (Figure 1.2 and 1.1), the buildings were irregular in plan, therefore, torsion around the vertical axis appeared. This torsion can be significant and in the worst cases lead to the collapse of the building. However, even buildings considered as regular in plan can be classified as torsionally flexible if small eccentricities of the seismic horizontal forces cause large torsional deformations to the storey. For example, it can be the case for buildings with a structural system consisting of flexible frames combined with walls concentrated near the centre of the building in plan. Therefore the torsional stiffness of the building will be low and torsion may appear during an earthquake.

In the first generation of the Eurocode 8, a condition for torsionally flexible buildings is presented in the criteria for regularity in plan. This condition compares the torsional radius to the radius of gyration. However, no procedure is suggested to calculate the torsional radius. The first generation of the Eurocode 8 also limits the natural eccentricity, but again, no procedure is suggested to calculate it.

The second generation of the Eurocode 8 dedicates a whole section to the classification of torsionally flexible buildings. It also presents two procedures to calculate the natural eccentricity and the torsional radius : the simplified method and the 3D procedure. However, the simplified method computes the torsional radius with respect to the centre of stiffness, while with the 3D procedure the torsional radius is calculated with respect to the centre of mass. This inconstancy will be studied further in this work. Therefore a comparison between these two methods and an evaluation of the results will be realised in this master thesis.

A new condition using modal response spectrum analysis is also introduced in the second generation. It allows to directly use the results obtained by the modal response spectrum analysis. However, it requires a 3D model. A comparison between the two conditions will be made to ensure that they give the same conclusion for classification. The classification as torsionally flexible will have an impact on the design of the building. It is therefore important that both conditions lead to the same results.

1.2. Objectives

The objectives of this master thesis are:

- to compare the two procedures introduced in the second generation of the Eurocode 8 to calculate the natural eccentricity and the torsional torsional radius, the simplified method and the 3D procedure;
- to discuss the preferable reference point for the calculation of the torsional radius. Should it be computed with respect to the centre of stiffness as in the simplified method or with respect to the centre of mass as in the 3D procedure?
- to compare the two conditions given in the second generation of the Eurocode 8 for the classification of torsionally flexible buildings;
- to analyse the consequences on the design for a building classified as torsionally rigid;
- to analyse the effect of the accidental torsion.

To achieve the goals stated above, firstly, a literature review will be conducted. It helps us to understand all the concepts used and to become familiar with both generation of the Eurocode 8. Secondly, the results of the torsional radius from the 3D procedure will be analysed and compared with the results obtained with the simplified method. A comparison between the two conditions of torsionally flexible buildings introduced in the second generation will be made. Finally, a response spectrum analysis will be performed to verify the displacements and internal forces of the most critical points of a building. The effects of accidental eccentricity on a building will also be reviewed.

1.3. Organization

This master thesis is divided into four main parts.

Chapter 2

The state of the art is presented in this chapter. It is made up of three sections. **Section 2.1** introduces the important concepts related to the torsion. In addition to a simple definition, the procedures to follow to calculate the torsional radius and the static eccentricity will also be detailed. The **Section 2.2** presents the sections related to the torsion in both generations of the Eurocode 8. The conditions for the classification of torsionally flexible buildings, the consequences this will have on buildings design, as well as an introduction of the accidental eccentricity can be found there. Finally, the **Section 2.3**, introduced the different methods of analysis used in this master thesis.

Chapter 3

The chapter "Case studies and Torsional Classification" is divided in two sections. In the **Section 3.1**, the different models created for this master thesis are shown, as well as the seismic load spectrum used in the analysis. The **Section 3.2** leads to the classification of the different models. In this section, we can find :

- A comparison between the simplified method and the 3D procedure for the calculation of the position of the centre of stiffness and the torsional radius;
- A discussion about the reference point for the calculation of the torsional radius;
- The classification of torsionally flexible buildings following the two conditions introduced in the second generation of the Eurocode 8.

Chapter 4

The chapter "Response spectrum analysis and Discussion" is divided in two sections. The **Section 4.1** shows the results obtained from the modal response spectrum analysis. The consequences on the design for a building classified as torsionally flexible are analysed. For buildings with large natural eccentricity, the limitation of the inter-storey drift is verified. Finally the effects introduced by the accidental torsion are discussed.

In the **Section 4.2**, some observations are discussed in more detail and modest changes are suggested for the development of the second generation of the Eurocode 8.

Chapter 5

This chapter closes the master thesis with a general conclusion of the results obtained and with propositions for further developments.

1.4. Symbols

A_x	gross area of the section
E	Young Modulus of the material
F_b	base shear force
F_{ij}	horizontal seismic force acting at a storey j in a direction i
G_k	permanent load
I	inertia of the section
J	polar moment of inertia in plan with respect to the center of mass of the floor
$K_{x/y,ij}$	lateral stiffness in direction x/y of primary bracing j in the storey below floor i
$K_{x/y,i}$	lateral stiffness in direction x/y of the storey i
$K_{\theta,i}$	torsional stiffness of the storey i
L_e	distance between the two-outermost lateral load resisting members,measured perpendicularly to the direction of the seismic action considered
L_{ij}	length of the floor perpendicular to the direction i of the seismic action at the j-th storey
L_n	is the modal excitation factor for the n-th mode
$M_{a,ij}$	torsional moment from the accidental eccentricity
M_n	is the generalised mass of the n-th mode
Q_k	live load
S	soil factor
$S_e(T)$	elastic response spectrum
$S_d(T)$	design response spectrum
T	vibration period of a linear single degree of freedom system
T_1	fundamental period of vibration of a building
a_g	design ground acceleration on type A ground
a_{gR}	reference peak ground acceleration on type A ground
b	width of the rectangular floor
$c_{p,i}$	correction factor accounting for the torsional effect

$c_{E,i}$	correction factor to correct the higher mode effects in elevation
$d_{et,j}$	values of the control displacement from the linear elastic analysis for the design seismic action and the corresponding displacement at location j in plan
$d_{t,j}$	target displacements associated with the considered limit state and the corresponding displacement at location j in plan
$d_{r,SD}$	inter-storey drift at significant damage limit state
$e_{0x/y,i}$	static eccentricity in x/y direction at the i-th floor
$e_{a,ij}$	accidental eccentricity of the storey mass j in direction i from its nominal location, applied in the same direction at all floors
e_{ij}	maximum between $e_{a,ij}$ and $e_{0,ij}$
f_e	elastic strength
f_y	yield strength
h_i	height of the storey i
$h_{w,i}$	height of the wall i
α_0	prevailing aspect ratio of the walls of the structural system
$\Delta d_{x/y,i}$	inter-storey drift of the centre of mass in x/y direction between the storey $i-1$ and i
$\Delta d_{x/yCS,i}$	inter-storey drift of the centre of stiffness in x/y direction between the storey $i-1$ and i
$\Delta\theta_{x,i}$	difference of rotation about the vertical axis between the storey $i-1$ and i due to the lateral forces applied in the x direction
$\Delta\theta_{y,i}$	difference of rotation about the vertical axis between the storey $i-1$ and i due to the lateral forces applied in the y direction
$\Delta\theta_{z,i}$	difference of rotation about the vertical axis between the storey $i-1$ and i due to the moment about the vertical axis applied at the centre of mass
δ	factor multiplying the action effects of the load resisting elements to consider the accidental torsional effects
γ_I	importance class of a building
η	damping correction factor
μ_d	ductility demand factor
ξ	viscous damping ratio

$\psi_{2,i}$	combination coefficient for the quasi-permanent value of a variable action i
$\psi_{E,i}$	combination coefficient for a variable action i, to be used when determining the effects of the design seismic action
ω	eigen frequency of the mode
ABSSUM	Sum of the Maximum Absolute Values
CM	centre of mass
CS	centre of stiffness
CQC	Complete Quadratic Combination
ISD	inter-storey drift
LFM	Lateral Force Method
RSA	Response Spectrum Analysis
SSRS	Square Root of Sum of Squares

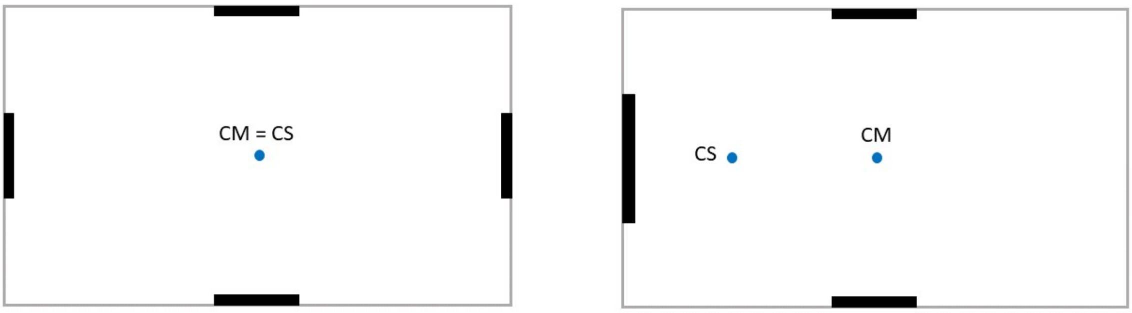
CHAPTER 2

State of the art

2.1. Basic Concepts

2.1.1. Centre of stiffness

The position of the centre of stiffness is important in earthquake engineering. Indeed, for asymmetrical buildings, the position of the centre of stiffness will differ from the position of the centre of mass.



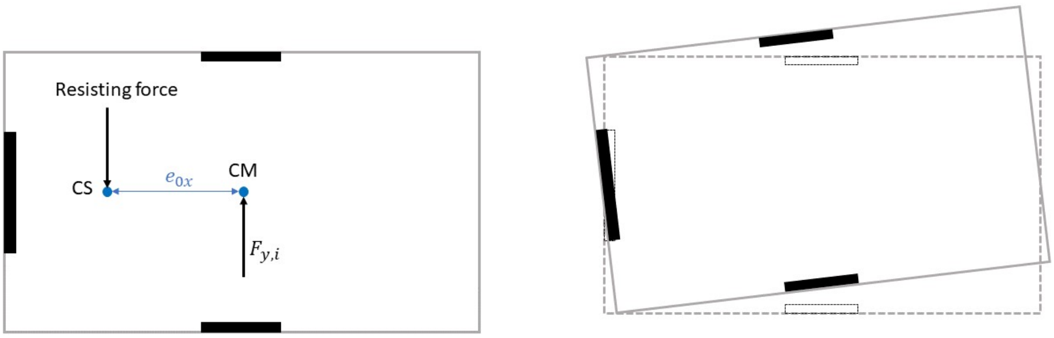
(a) (b)

Figure 2.1: Position of the centre of stiffness for (a) symmetrical storey, (b) asymmetrical storey

When the position of the centre of stiffness is different than the position of the centre of mass, an earthquake will produce a moment around the vertical axis, expressed as follows.

$$M_i = e_{0i} \cdot F_i \tag{2.1}$$

This moment will twist the building (Figure 2.2). Unfortunately, in the worst cases, the torsion can cause the collapse of the building due to excessive deformations (Examples shown in the Section 1).



(a) (b)

Figure 2.2: Torsional response of an asymmetrical building

However, the definition of the centre of stiffness for multi-storey building is not general and only approximations exist. Indeed, the concept of the centre of stiffness (CS) was originally applied for single-storey buildings. Its location deals with several structural properties, which are listed below.

1. It is the point where the application of a lateral load will not cause any rotation of the floor (centre of rigidity);
2. It is the location in plan where the resultant of the story shears passes when there is no rotation of the floor (shear centre);
3. It is the point that remains stationary when the structure is subject to torque loading (centre of twist);
4. In computation, it is the point located as the ratio of the sum of the first moment of the lateral stiffness to the total lateral stiffness (centre of stiffness).

In single-storey buildings, all these centres are located at the same point. However, for multi-storey buildings, these centres are different sets of points and their location depends on the lateral loading distribution with the height. For these reasons, the extension of the concept of the centre of stiffness to multi-storey buildings is not trivial and its location can't be defined exactly. In the literature released during the past years, several definitions were given for the CS.

Poole (1977) [5] suggested to use the shear centre as the centre of stiffness at each floor. Humar (1984) [6] interpreted the centre of stiffness as the point through which the resultant of the lateral loads will not cause any rotation of the floor under consideration (the other floors may rotate). Cheung and Tso (1986) [7] defined the centre of stiffness as the set of points located at the floor level such that the application of the resultant of the lateral load will not cause any rotations of any floor.

For this master thesis, as in the Eurocode 8, the definition in Fardis 2009 [4] will be used : "*The centre of lateral stiffness (CS) is defined as the point in plan through which application to the elastic structure of lateral forces produces only translation of the individual floors, without twisting of any floor.*"

The centre of stiffness is independent of the lateral loading only for single storey buildings. Indeed, Tso (1990) [8] shows that for multi-storey buildings, the CS depends on the distribution of lateral loading with height, that is why only an approximate definition is possible. This problem is all the more present for buildings that are strongly irregular in elevation and/or have a dual (frame-wall) structural system. In these cases, an approximation of the centre of stiffness for the whole building can be obtained as suggested by M.N. Fardis [4] :

1. A set of horizontal force F_i is applied following the equivalent lateral force method. The building is analysed under a set of torques whose values are proportional to these forces: $F_i = T_i$.
2. The centres of twist due to these torques are determined at each floor. The horizontal projection of the centre of twist at the elevation of 80% of the total height of the building may be considered as the centre of stiffness of the whole building.

A lateral load applied at this point will produce translation of the individual storey with minimal rotation. However, the second generation of the Eurocode 8 accepts that the equations (2.2) can be applied independently at each storey when the primary structure is composed of the same type of primary bracing continuous from the basement to the level under consideration.

$$x_{CS,i} = \frac{\Sigma_j x_j K_{x,ij}}{\Sigma_j K_{x,ij}} \qquad y_{CS,i} = \frac{\Sigma_j y_j K_{y,ij}}{\Sigma_j K_{y,ij}} \tag{2.2}$$

Where :

x_j and y_j are the coordinates of the bracing element j at the i-th storey;
$K_{x/y,ij}$ is the lateral stiffness in direction x/y of primary bracing j in the storey below floor i.

The calculation of the lateral stiffness depends on the type of the structure. For primary structure composed of columns (Figure 2.3(a)), $K_{x/y,ij}$ is given by the formula (2.3).

$$K_{x/y,ij} = \frac{EI_{x/y,j}}{h_i^3} \tag{2.3}$$

Where :

$EI_{x/j,j}$ is the flexural stiffness of the column j in the storey below the i-th floor in direction x/y;
h_i is the free height of the column j in the storey below the i-th floor.

While for a primary structure composed of walls and box walls (Figure 2.3(b)), the formula (2.4) will be used.

$$K_{x/y,ij} = \left[\left(\frac{EI_{x/y,j}}{h_i^3} \right)^{-1} + \left(\frac{5 \cdot EA_{x/y,i}}{12 \cdot h_i} \right)^{-1} \right]^{-1} \tag{2.4}$$

Where :

$EI_{x/j,j}$ is the flexural stiffness of the wall j in the storey below the i-th floor in direction x/y;

$A_{x/y,i}$ is the gross area of the section of the wall.

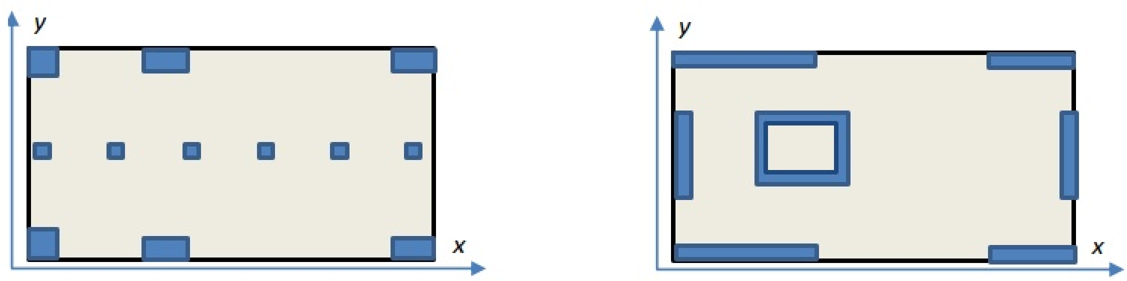
Finally, for primary structure composed of frames with bracing, the lateral stiffness should be determined as given by the formula (2.5).

$$K_{x/y,ij} = \frac{EA_{x/y\alpha,i}}{h_i} \cdot \cos \alpha_i \tag{2.5}$$

Where :

$EA_{x/j\alpha,i}$ is the axial stiffness of the bracing element considered;

α is the angle of the bracing element to the horizontal.



(a) (b)

Figure 2.3: Primary structure composed of (a) columns, (b) walls and box walls, taken from [9]

Using a 3D procedure to find the position of the CS at each storey is also possible. For that, three loads cases should be applied separately to the structure :

- Load case 1 : a profile of lateral forces applied in the x direction;
- Load case 2 : a profile of lateral forces applied in the y direction;
- Load case 3 : a profile of moments about the vertical axis applied at the center of mass of the floor.

Each profile of force should be defined as applying a unit acceleration to the mass of each floor, which means that the values of the lateral forces and the moment applied at i-th storey are equal to the mass of the storey.

The natural eccentricity of each floor is then calculated with the formula (2.6).

$$e_{0x,i} = \frac{\Delta\theta_{y,i}}{\Delta\theta_{z,i}} \qquad e_{0y,i} = \frac{\Delta\theta_{x,i}}{\Delta\theta_{z,i}} \tag{2.6}$$

Where :

$\Delta\theta_{x,i}$ is the difference of rotation around the vertical axis between the storey $i-1$ and i due to the load case 1;
$\Delta\theta_{y,i}$ is the difference of rotation around the vertical axis between the storey $i-1$ and i due to the load case 2;
$\Delta\theta_{z,i}$ is the difference of rotation around the vertical axis between the storey $i-1$ and i due to the load case 3 .

The values in the global coordinated system of the centre of stiffness are expressed with the Equation (2.7).

$$x_{CS,i} = x_{CM,i} + e_{0x,i} \qquad y_{CS,i} = y_{CM,i} + e_{0y,i} \tag{2.7}$$

2.1.2. Torsional radius

For the analysis of the torsional response of a building, we need to define the parameter r, called torsional radius. It is defined as the square root of the ratio of the torsional stiffness K_θ, with respect to the center of stiffness, to the lateral stiffness $K_{x/y}$.

$$r_x = \sqrt{\frac{K_\theta}{K_y}} \qquad r_y = \sqrt{\frac{K_{\theta,}}{K_x}} \tag{2.8}$$

Where, the torsional stiffness is defined as the ratio of the storey torsional moment to the corresponding storey twist. Similarly, the lateral stiffness is the ratio of the storey shear to the corresponding displacement of the storey.

$$K_{\theta,i} = \frac{M_i}{\Delta\theta_i} \qquad K_{x,i} = \frac{F_i}{\Delta d_{x,i}} \tag{2.9}$$

Unfortunately, as for the centre of stiffness, the torsional radius as defined above is unique and independent of the lateral loading only for single-storey buildings. For multi-storey buildings, the CS and r will both depend on the lateral loading with height. For buildings with the same type of primary bracing continuous from the basement to the level under consideration, the second generation Eurocode 8 gives us the formula (2.10) to calculate the torsional radius in x direction of the i-th floor.

$$r_{x,i} = \sqrt{\frac{\sum_j K_{x,ij}(y_j - y_{C,i})^2 + \sum_k K_{y,ik}(x_k - x_{C,i})^2}{\sum_k K_{y,ik}}} \tag{2.10}$$

Where :

$K_{x/y,ij}$	is the lateral stiffness of primary bracing j at the i-th floor in direction x/y, the calculation is detailed in the Section 2.1.1;
$x_{C,i}$ and $y_{C,i}$	are the coordinates of the center of stiffness at the i-th floor;
x_j and y_j	are the coordinates of the bracing element j at the i-th floor.

The second generation of the Eurocode 8 also allows a calculation by a 3D model. For this procedure, three load cases should be applied separately to the structure, as defined in the Section 2.1.1.

The torsional radius is then calculated with the formula (2.11).

$$r_{xCM,i} = \sqrt{\frac{\Delta d_{yCM,i}}{\Delta\theta_{z,i}}} \qquad r_{yCM,i} = \sqrt{\frac{\Delta d_{xCM,i}}{\Delta\theta_{z,i}}} \tag{2.11}$$

Where :

$\Delta d_{x,i}$	is the inter-storey drift of the centre of mass in x direction between the storey $i-1$ and i due to the lateral forces applied in the x direction;
$\Delta d_{y,i}$	is the inter-storey drift of the centre of mass in y direction between the storey $i-1$ and i due to the lateral forces applied in the y direction;
$\Delta\theta_{z,i}$	is the difference of rotation about the vertical axis between the storey $i-1$ and i due to the moment about the vertical axis applied at the centre of mass .

The 3D procedure gives us the torsional radius with respect to the center of mass, unlike the simplified procedure for which the torsional radius is defined with respect to the center of stiffness. This inconsistency was studied in the master thesis of Maxence Carlier [1] and will be further extended for the case of multi-storey buildings in this master thesis.

To calculate the torsional radius with respect to the centre of stiffness after the 3D procedure, we will use the relation (2.12), given in the Annex B of the second generation of the Eurocode 8.

$$\Delta d_{xCS,i} = \Delta d_{xCM,i} - e_{0y,i} \cdot \Delta\theta_{x,i} \tag{2.12}$$

2.1.3. Behaviour factor

Since the classification of a torsionally flexible building will have an impact on the design of buildings, a quick introduction of the behaviour factor (q) is given in this section. We will focus here on the physical meaning of the q factor, the role of the behaviour factor in the representation of the seismic action will be further detailed in the section 2.3.1.

During the past few years, several buildings have resisted to strong ground motion even if they were designed to resist less than the force that would develop if the structure was completely elastic. Therefore, the codes had introduce a reduction factor (behaviour factor q in the Eurocode 8 and response modification factor R in the US codes). These factors are used to evaluate the ability for energy absorption of a structure. In practice, the behaviour factor reduces the elastic response spectrum used to represent a seismic action (Figure 2.4).

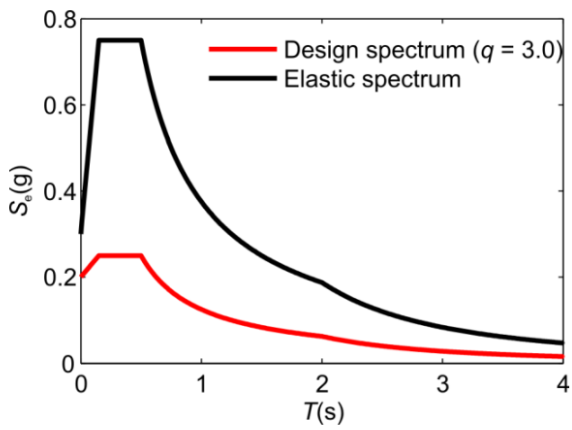


Figure 2.4: Reduction of the elastic response spectrum for $q = 3$

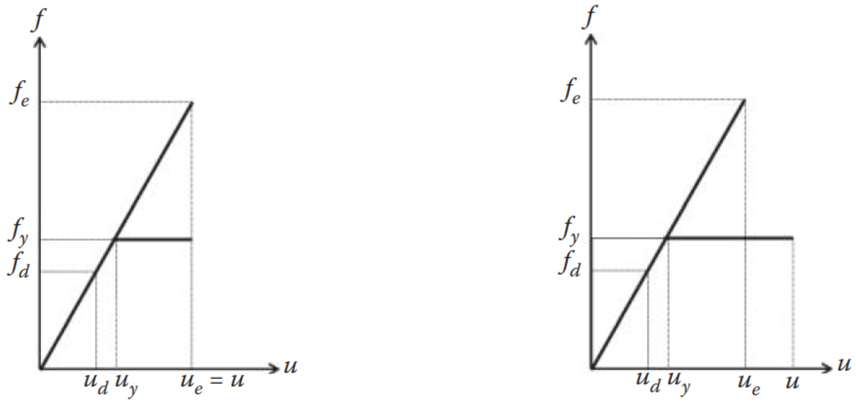
The behaviour factor is defined as the ratio of the elastic strength f_e to a smaller strength called yield strength f_y.

$$q = \frac{f_e}{f_y} \tag{2.13}$$

Since the q factor depends of the ductility class of the building, we have to introduce the ductility demand factor μ_d. It is defined as the ratio of the maximum displacement to the yield displacement (Formula 2.14). A large value of μ_d means that large plastic deformations will occur, while a structure with a small ductility factor will tend to a brittle failure.

$$\mu_d = \frac{u_{max}}{u_y} \tag{2.14}$$

Two main relations were established by Newmark and Hall in 1982 [2] between the behaviour factor and the ductility demand, the equal displacement principle and the equal energy principle.



(a) Equal displacement principle (b) Equal energy principle

Figure 2.5: Idealised force - displacement relationships [10]

The first one (Figure 2.5(a)), is used for buildings having large natural period. In this case, the maximum displacement is the same as the elastic one. Therefore, we can express the following relation between the ductility factor and the behaviour factor.

$$q = \mu_d \tag{2.15}$$

For small periods, the equal energy principle will be applied, which means that the dissipated energy is the same for the elastic system and for the plastic one (Figure 2.5(b)). In this case, the relation between the ductility factor and the behaviour factor becomes :

$$q = \sqrt{2 \cdot \mu_d - 1} \tag{2.16}$$

2.2. Consideration of torsion in Eurocode 8

2.2.1. Torsionally flexible building

The classification of a building as torsionally flexible will have consequences on its design. The main impact is the decrease of the behaviour factor. The implication of this torsional classification for both Eurocode 8 is detailed further in the Section 2.2.2.

In the **first generation** of the Eurocode 8, the torsional classification of buildings is presented in the criteria for regularity in plan. Two main conditions can be found. The first one will limit the natural eccentricity while the second one is a limitation of the torsional radius.

$$e_{0x,i} \leq 0,3 \cdot r_{x,i} \tag{2.17}$$

$$r_{x,i} \geq l_{s,i} \tag{2.18}$$

Where

$e_{0x,i}$ is the static eccentricity, which means the distance between the CM and CS in the direction x at the i-th storey;

$r_{x,i}$ is the torsional radius in direction x at the i-th storey;

$l_{s,i}$ is the radius of gyration of the i-th storey.

The radius of gyration in plan is defined as the square root of the ratio of the polar moment of inertia in plan with respect to the center of mass of the floor (J) to the total overlying mass (m). If the mass of the body is concentrated in this radius around the rotational axis, the moment of inertia of the body will stay unchanged.

$$l_s = \sqrt{\frac{J}{m}} \tag{2.19}$$

The value of the radius of gyration is not fixed or constant, it depends on the distribution of the mass of the body around the rotational axis. In case of a rectangular floor with dimensions l and b and if the mass is well distributed over the area, the radius of gyration can be easily calculated with the formula (2.20).

$$l_s = \sqrt{\frac{l^2 + b^2}{12}} \tag{2.20}$$

Penelis [11] shows that the second condition (Equation 2.18) is equivalent to say that the greater modes should be the translational ones.

$$\omega_\theta < \omega_x, \omega_y \tag{2.21}$$

Indeed $\omega_\theta,\ \omega_x,\ \omega_y$ can be express as :

$$\begin{cases} \omega_x & = \sqrt{\frac{K_x}{m}} \\ \omega_y & = \sqrt{\frac{K_y}{m}} \\ \omega_\theta & = \sqrt{\frac{K_\theta}{J}} \end{cases} \tag{2.22}$$

With m, the mass of the building and J the polar moment of inertia.

The expression (2.21) becomes :

$$\begin{aligned} & \begin{cases} \sqrt{\frac{K_\theta}{J}} & < \sqrt{\frac{K_x}{m}} \\ \sqrt{\frac{K_\theta}{J}} & < \sqrt{\frac{K_y}{m}} \end{cases} \\ \Leftrightarrow & \begin{cases} \sqrt{\frac{K_\theta}{K_x}} & > \sqrt{\frac{J}{m}} \\ \sqrt{\frac{K_\theta}{K_y}} & > \sqrt{\frac{J}{m}} \end{cases} \\ \Leftrightarrow & \begin{cases} r_x & > l_s \\ r_y & > l_s \end{cases} \end{aligned}$$

Unlike the first generation, the **second generation** of the Eurocode 8 dedicates a whole section to the classification of torsionally flexible buildings. Two conditions are given and a verification will be done in this master thesis to ensure that the two conditions give the same results for the classification.

1. A building should be considered as torsionally flexible if in each main horizontal direction, the greatest effective modal mass is not that of the first or second mode, possible local modes and equipment modes being not considered in this classification.

   Note : this condition corresponds to the fact that the first mode (in at least one horizontal direction) is substantially influenced by torsion.

2. Buildings verifying the Formula (2.23) at every storey i, with the possible exception of the highest storey, may be considered as not being torsionally flexible.

$$r_i \geq l_{s,i} \tag{2.23}$$

The first condition in the first generation of the Eurocode 8 (Equation 2.17) was there to provide an acceptable natural eccentricity, and thus to categorise a building as being regular in plan. Since the regularity in plan is only informative in the second generation, this condition can be found in the Annex A of the second generation.

The second condition is the same in both generation of the Eurocode 8. The second generation just specifies that the highest storey should not be considered.

The first condition of the second generation requires a numerical analysis, so that the modal shape and natural frequency of the structure can be computed. The effective modal mass is a measure to classify the importance of a mode shape when a structure is excited by a ground acceleration. Indeed, a high effective mass will lead to a high reaction force at the base while, on the other hand, modes with low effective masses will give low reaction forces at the base. It can be interpreted as the part of the total mass responding to the ground acceleration in mode i. It is calculated as follows :

$$m_{eff,x} = \frac{L_n^2}{M_n} = \frac{(\Phi_n^T M I_x)^2}{\Phi_n^T M \Phi_n} \tag{2.24}$$

Where,

L_n is the modal excitation factor for the n-th mode;
M_n is the generalised mass of the n-th mode.

To illustrate the first condition, the two examples below of modal analysis found in the Scia help centre [12] are used.

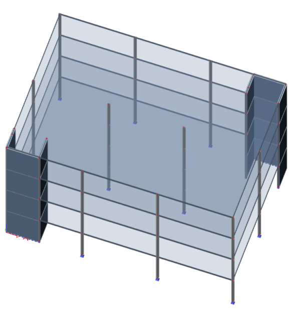


Figure 2.6: Strong system in torsion

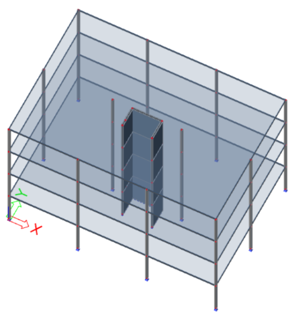


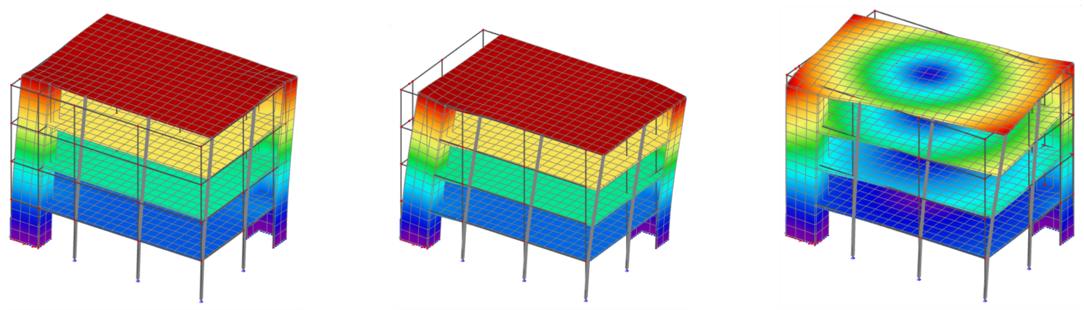
Figure 2.7: Weak system in torsion

Below are the results of the modal shape for the building with a regular system (Figure 2.6).

Mode	ω[rad/s]	T [s]	f [Hz]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	4.94535	1.27	0.79	0.0012	**0.7149**	0	0.1306	0.0022	0
2	7.61599	0.82	1.21	**0.726**	0.0012	0	0	0.0773	0
3	9.65372	0.65	1.54	0	0	0.0002	0	0	**0.7134**

Table 2.1: Modes for the strong system in torsion

For this model, in each main horizontal directions, the greatest effective modal mass is in first position (for y direction) and second (for x direction). According to the first condition, the building is thus considered as torsionally rigid.

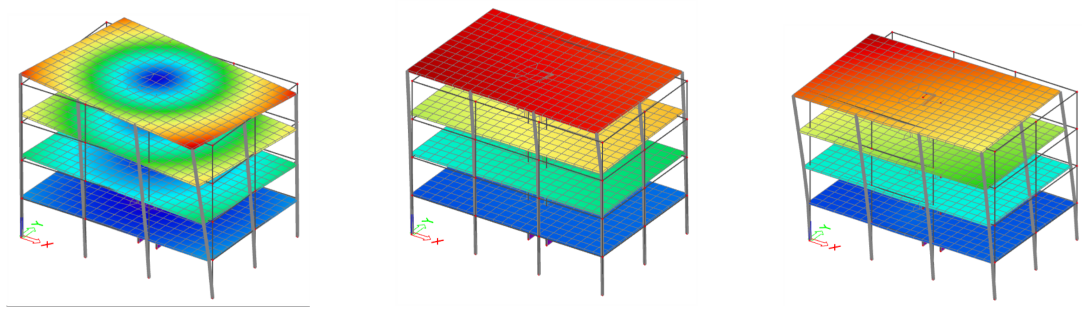


(a) Mode 1 (b) Mode 2 (c) Mode 3

Now, reviewing the modal shape of the second example (Figure 2.7) presented in the table just below, we noticed that the first mode is influenced by the torsional effective modal mass. The greatest effective modal mass in x direction is only for the 5^{th} mode and in y direction for the 6^{th} mode. This means that the building is torsionally flexible. The effective modal masses $m_{eff,z}$, $m_{eff,Rx}$ and $m_{eff,Ry}$ correspond to local mode of the rigid diaphragm and are therefore not considered in the classification.

Mode	ω[rad/s]	T [s]	f [Hz]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	0.745	8.43	0.12	0.0062	0.0034	0	0.0007	0.0006	**0.7729**
2	1.759	3.57	0.28	0.0008	0.0002	0	0.0052	0.0001	0.1504
3	2.291	2.74	0.36	0.0006	0.0014	0	0.0009	0.0002	0.0565
4	2.548	2.47	0.41	0	0.0001	0	0.0005	0	0.0093
5	3.668	1.71	0.58	**0.3884**	0.3448	0.0001	0.0744	0.0522	0.0009
6	3.755	1.67	0.6	0.3294	**0.393**	0.0002	0.084	0.0488	0.0066

Table 2.2: Modes for the weak system in torsion



(d) Mode 1 (e) Mode 5 (f) Mode 6

2.2.2. Consequences on the design

The classification of a building as torsionally flexible will have consequences on the design of the building.

In the **first generation** of the Eurocode 8, the value of the behaviour factor depends on the structural system of the structure. In the following, you can see the different systems of concrete buildings depending on their structural types.

- Frame system;
- Dual system (frame or wall equivalent);
- Ductile wall system (coupled or uncoupled);
- System of large lightly reinforced walls;
- Inverted pendulum system;
- Torsionally flexible system.

Frame, dual or walls system without a minimum torsional rigidity may be classified as torsionally flexible. This means if at any floor, the radius of gyration of the floor mass exceed the torsional radius in at least one direction, the structure is considered as torsionally flexible.

The behaviour factor should be computed with the Formula (2.25).

$$q = q_0 \cdot k_w \geq 1,5 \tag{2.25}$$

Where,

- q_0 is the basic value of the behaviour factor, which depends on the type of the structural system and its regularity in elevation. Its value for system regular in elevation can be find in the Table 2.3;

Structural type	DCM	DCH
Frame system, dual system, coupled wall system	3,0α_u/α_1	4,5α_u/α_1
Uncoupled wall system	3,0	4,0α_u/α_1
Torsionally flexible system	2,0	3,0
Inverted pendulum system	1,5	2,0

Table 2.3: Basic values of the behaviour factor for system regular in elevation in the first generation of the Eurocode 8

- k_w is the factor reflecting the prevailing failure mode in structural systems with walls. It shall be taken as follows :

$$k_w = \begin{cases} 1,00 & \text{for frame and frame - equivalent dual systems} \\ (1+\alpha_0)/3 \in [0,5;1] & \text{for wall, wall-equivalent and torsionally flexible systems} \end{cases} \quad (2.26)$$

  With α_0 which is the prevailing aspect ratio of the walls of the structural system. It is determined with the following expression :

$$\alpha_0 = \frac{\Sigma h_{w,i}}{\Sigma l_{w,i}} \quad (2.27)$$

  Where $h_{w,i}$ is the height of the wall i and $l_{w,i}$ is the length of the section of the wall i.

We can notice that the classification of a building as torsionally flexible will significantly reduced the behaviour factor.

The **second generation** of the Eurocode 8 suggests some modifications of the behaviour factor in case of a torsionally flexible building.

- The behaviour factor should be taken as the minimum of both horizontal directions where the primary bracing is different. While for torsionally rigid buildings, it may be different in the two directions;
- The value of q should be multiplied by 0,8;
- q_R should be taken equal to 1.

Where,

q_R is the behaviour factor component accounting for overstrength due to the redistribution of seismic action effects in redundant structures.

2.2.3. Accidental torsion

Most of the codes introduce an "accidental eccentricity" to ensure a minimum of torsional resistance and stiffness and limit the possible consequences of an unforeseen torsional response. In case of a building with full planwise symmetry of stiffness and nominal masses, the analysis of seismic action will not produce any torsional response. However, these may appear, due to different factors:

- Change of the structure's occupancy, meaning a different case of live loads than the one assume in design is present during the earthquake;
- Incoherent ground-motion characteristic;

– Change of the mass and stiffness contribution due to the infill walls, non-structural element that are not considered during the analysis.

The accidental torsional effects can be obtained by shifting the mass from their nominal position by a distance equal to accidental eccentricity, $e_{a,ij}$. In both generations of the Eurocode 8, this $e_{a,ij}$ is equal to 5% of the floor-dimension of the building (Equation 2.28). It should be taken in both positive and negative sense along the two orthogonal direction of the horizontal seismic action components.

$$e_{a,ij} = \pm 0,05 \cdot L_{ij} \tag{2.28}$$

Where

$e_{a,ij}$ is the accidental eccentricity of the storey mass j in direction i from its nominal location, applied in the same direction at all floors;

L_{ij} is the length of the floor perpendicular to the direction i of the seismic action at the j-th storey.

However, shifting the mass on a 3D model is possible but requires, in general, four different models and is very inconvenient for practical application. For this reason, in its **first generation**, the Eurocode 8 suggests two alternative methods to account the accidental torsional effects:

1. If we use the lateral force method, the accidental torsional effects should be taken into account by multiplying the seismic action by a factor δ, given in the Formula (2.29)

   $$\delta = 1 + 0,6 \cdot \frac{x_j}{L_e} \tag{2.29}$$

   Where,

   x_j is the distance of the member j under consideration from the centre of mass of the building in plan, measured perpendicularly to the direction of the seismic action considered;

   L_e is the distance between the two-outermost lateral load resisting members, measured perpendicularly to the direction of the seismic action considered.

   If the analysis is performed using two planar models, one for each main horizontal direction, the factor $0,6$ should be increase to 1,2.

   $$\delta = 1 + 1,2 \cdot \frac{x_j}{L_e} \tag{2.30}$$

2. For the modal response spectrum analysis, the effect of the accidental eccentricity perpendicular to the direction i should be calculated as resulting from a torsional moment $M_{a,ij}$, which acts around the vertical axis at the storey j.

   $$M_{a,ij} = e_{a,ij} \cdot F_{ij} \tag{2.31}$$

   Where

   F_{ij} is the horizontal force acting on the j-th storey in direction i.

The first method may be more conservative than the second one. However, the two methods will be analysed and compared in this master thesis.

Unlike the first generation, in the **second generation**, there is no distinction between the two methods of analysis. When a force-based approach is used, the Eurocode 8 advises to apply a torsional moment M_{ij}, which acts around the vertical axis of the building given by the Formula(2.32), to consider the effects of a mass eccentricity.

$$M_{ij} = e_{ij} \cdot F_{ij} \tag{2.32}$$

Where
e_{ij} is the maximum between $e_{a,ij}$ and $e_{0,ij}$.

The main difference between the two codes for the analysis of the accidental torsion is that the **second generation** considers this minimal eccentricity only if it exceeds the natural eccentricity of the building, while for the **first generation**, it should be considered for all cases.

The **second generation** introduces a correction factor $c_{p,j}$ and $c_{E,i}$ for non linear-static analysis. The factor $c_{p,j}$ account for the torsional effect, while $c_{E,i}$ is there to correct the higher mode effects in elevation. The effects obtained from analysis should be corrected by multiplying the displacement, as well as the other deformations and stresses, by the product of $c_{p,j}$ and $c_{E,i}$. Since we are focusing on the torsional effect, only the definition of $c_{p,j}$ will be detailed.

If the torsional system is not torsionally flexible and if, in both horizontal direction i at each storey, the two components of the natural eccentricity, $e_{0x,j}$ and $e_{0y,j}$ do not exceed $0,10 \cdot L_{ij}$, the correction factor $c_{p,j}$ should be calculated with the following formula.

$$c_p = 1 + 0,6\frac{x}{L_e} \tag{2.33}$$

Otherwise, c_p should be determined with the Formula (2.34).

$$c_{p,j} = \frac{d_{et,j}}{d_{t,j}}\frac{d_t}{d_{et}} \geq 1 \tag{2.34}$$

Where
j is the index of the structural member;
d_{et} and $d_{et,j}$ are the values of the control displacement from the linear elastic analysis for the design seismic action and the corresponding displacement at location j in plan, respectively;
d_t and $d_{t,j}$ are the target displacements associated with the considered limit state and the corresponding displacement at location j in plan.

2.3. Methods of analysis

In this master thesis, a seismic analysis has been done for different models. For this reason, a quick introduction into the different methods used is given in this section. We focus only on the linear methods : linear static analysis, with the lateral force method, and modal response spectrum analysis.

2.3.1. Representation of the seismic action

The representations of the seismic actions used in this master thesis are based on the **first generation** of the Eurocode 8 [13] since it is the representation implemented in the software Scia Engineer.

Horizontal elastic response spectrum

The basic spectral shape of the horizontal seismic action is composed of four branches : very low period branch, constant acceleration branch, constant velocity branch and finally the constant displacement branch. These branches are separated by three periods, T_B, T_C and T_D (Figure 2.8). The elastic response spectrum $S_e(T)$ is defined by the following expressions :

$$\begin{cases} 0 \leq T \leq T_B: & S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2,5 - 1)\right] \\ T_B \leq T \leq T_C: & S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \\ T_C \leq T \leq T_D: & S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \left[\frac{T_C}{T}\right] \\ T_D \leq T \leq 4\ s: & S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \left[\frac{T_C \cdot T_D}{T^2}\right] \end{cases} \tag{2.35}$$

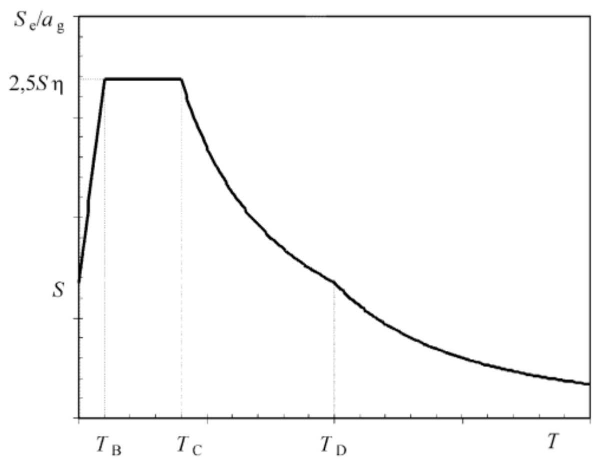


Figure 2.8: Shape of the Horizontal elastic response spectrum

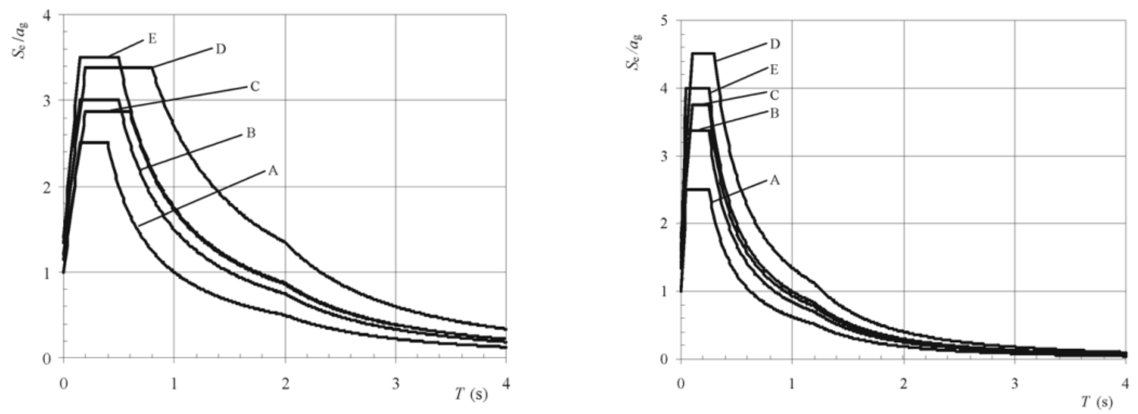
The parameter η is the damping correction factor. Its reference value ($\eta = 1$) corresponds to a viscous damping ratio $\xi = 5\%$. For other values of ξ, it can be calculated with the Formula (2.36).

$$\eta = \sqrt{10/(5+\xi)} \geq 0,55 \tag{2.36}$$

The horizontal spectrum is also defined by the ground acceleration $a_g = a_{gR} \cdot \gamma_I$, which is influenced by the important class of the building γ_I. This factor depends on the consequences of collapse for human life, the importance of the building for public safety and the social and economics consequences of collapse.

The values of the periods T_B, T_C and T_D as well as the soil factor S depend of two main parameters, listed below.

- **Spectrum type** represents two types of earthquakes : Type 1 and Type 2. The Type 1 is preferred for moderate to large magnitude earthquake. While the Type 2 will be used for low magnitude ones (e.g. if the earthquakes that contribute most to the seismic hazard have a surface-wave magnitude not greater than 5.5), at close distance.
- **Ground type** depends of the subsoil type of the ground, the different types are defined in the Annex A.



(a) Type 1 (b) Type 2

Figure 2.9: Shape of the horizontal elastic response spectrum for the different Ground Types (for 5% damping)

Design spectrum for analysis

As introduced in the section 2.1.3, the experience has shown that the ability of buildings to resist seismic actions in the non-linear domain generally allows them to be designed to withstand lower seismic forces than those corresponding to a linear elastic response.

The reduction of the seismic force is accomplished by introducing the behaviour factor. The first generation of the Eurocode 8 [13] define the q factor as "*an approximation of the ratio of the seismic forces that the structure would experience if its response was completely elastic with 5% viscous damping, to the seismic forces that may be used in the design, with a conventional elastic analysis model, still ensuring a satisfactory response of the structure*". The design spectrum is therefore defined by the following expressions :

$$\begin{cases} 0 \leq T \leq T_B : & S_d(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2{,}5}{q} - \frac{2}{3}\right)\right] \\ T_B \leq T \leq T_C : & S_d(T) = a_g \cdot S \cdot \frac{2{,}5}{q} \\ T_C \leq T \leq T_D : & S_d(T) \begin{cases} = & a_g \cdot S \cdot \frac{2{,}5}{q} \left[\frac{T_C}{T}\right] \\ \geq & \beta \cdot a_g \end{cases} \\ T_D \leq T \leq 4\ s : & S_d(T) \begin{cases} = & a_g \cdot S \cdot \frac{2{,}5}{q} \left[\frac{T_C \cdot T_D}{T^2}\right] \\ \geq & \beta \cdot a_g \end{cases} \end{cases} \tag{2.37}$$

Where the factor β gives the lower bound for the horizontal design spectrum. Its recommended value is 0,2.

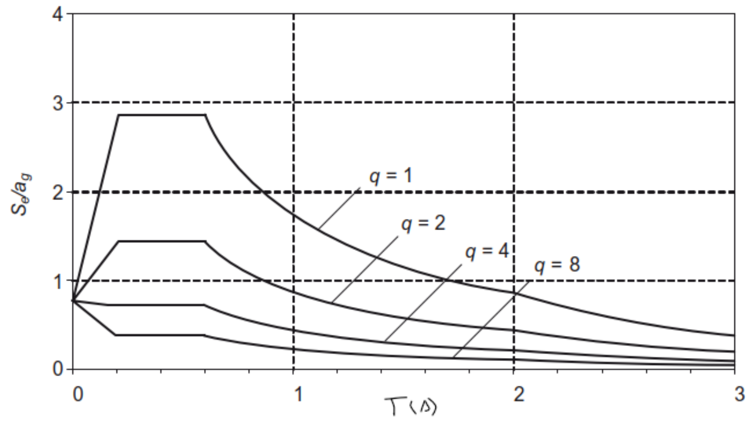


Figure 2.10: Design response spectrum example (Type 1, soil type C) - taken from [14]

2.3.2. Lateral force method

The first approach for the seismic analysis is the lateral force method. It is a simplified approach used for simple structures in seismic standards and codes. It consists of applying a system of static loads at the centre of mass at each storey.

This method takes into account only the fundamental mode of vibration for both main directions. For this reason, it can not be applied to a structure whose response is significantly affected by contributions from modes of vibration higher than the fundamental one. This requirement is assumed true for buildings respected the following conditions.

- The height of the building is less than 30 m.
- Their fundamental period in the two mains conditions is smaller than the following values.

  $$T_1 \leq \begin{cases} 4 \cdot T_C \\ 1,5\ s \end{cases} \tag{2.38}$$

  Where
  T_C is the upper corner of the period of the constant spectral acceleration range.
- They are considered as regular in elevation.

The steps to follow are listed bellow.

1. **Fundamental period of vibration** is determined using approximate methods of structural dynamics.
2. **Base shear force** is calculated using the Formula (2.39).

   $$F_b = S_d(T_1) \cdot m \cdot \lambda \tag{2.39}$$

   Where
   $S_d(T_1)$ is the pseudo acceleration of the design spectrum at period T_1;
   m is the total mass of the building;
   λ is the equal to $0,85$ if $T_1 \leq \min(2 \cdot T_C; 1,2\ s)$ and if the building has more than two storeys, otherwise is equal to 1.
3. **Distribution of the horizontal seismic force** along the height of the structure, applied at each storey i is determined by using the Formula (2.40).

   $$F_i = F_b \cdot \frac{s_i m_i}{\Sigma s_j m_j} \tag{2.40}$$

   Where
   s_i is the displacement of the mass m_i in the fundamental mode shape;
   m_i is the mass of the storey i.

2.3.3. Modal response spectrum analysis

Unlike the 'Lateral Force Method', there is no constraints on the applicability of the modal response spectrum analysis (abbreviate 'RSA'). It also offers a better balance of safety and economic. However, its computation is more complicated to apply and requires a 3D model.

The Eurocode 8 requires the consideration of all the modes of vibration that contribute significantly to the global response. The requirements for this condition are listed below.

- The sum of the effective modal masses for the modes taken into account amount to at least 90% of the total mass of the structure.
- All modes with effective modal masses greater than 5% of the total mass are taken into account.

Three superposition methods exist to combine the modal responses :

- ABSSUM (Sum of the Maximum Absolute Values), can be expressed as follow :

  $$R_{tot} = \sum_{j=1}^{N} |R_{j,max}| \tag{2.41}$$

  This method assumes that all the maxima occur at the same time and in the same direction. This assumption is too conservative, for this reason, the two other methods are preferred.

- SSRS (Square Root of Sum of Squares) can be expressed as follow :

  $$R_{tot} = \sqrt{\sum_{j} R_{j,max}^2} \tag{2.42}$$

  This method is suitable when all the modes are well separated, meaning that each mode has a frequency far enough from its neighbours so the modes are not interfering with each other.

- CQC (Complete Quadratic Combination) can be expressed as follow :

  $$R_{tot} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} R_i \cdot \rho_{i,j} \cdot R_j} \tag{2.43}$$

  With $\rho_{i,j}$, the modal cross correlation coefficients. It is computed with the formula 2.44.

  $$\rho_{i,j} = \frac{8\sqrt{\xi_i \xi_j} \cdot (\xi_i + r\xi_j) \cdot r^{3/2}}{(1-r^2)^2 + 4\xi_i\xi_j r(1+r^2) + 4(\xi_i^2 + \xi_j^2)r^2} \tag{2.44}$$

  Where $r = \frac{\omega_j}{\omega_i}$ and ξ_i and ξ_j are the damping ratio for mode i and j.

  Unlike the SSRS, this method is more general and is suitable in the case of closely-spaced modes, but it is also more complicated and intense to compute.

CHAPTER 3

Case studies and Torsional Classification

3.1. Case studies

The numerical analysis has been done with the commercial software Scia Engineer. This choice was made for its accessibility and range of functionality, which was perfectly suited to a linear dynamic analysis.

The models in this master thesis are based on the following basic model. The walls are drawn in red so that they can be quickly identified.

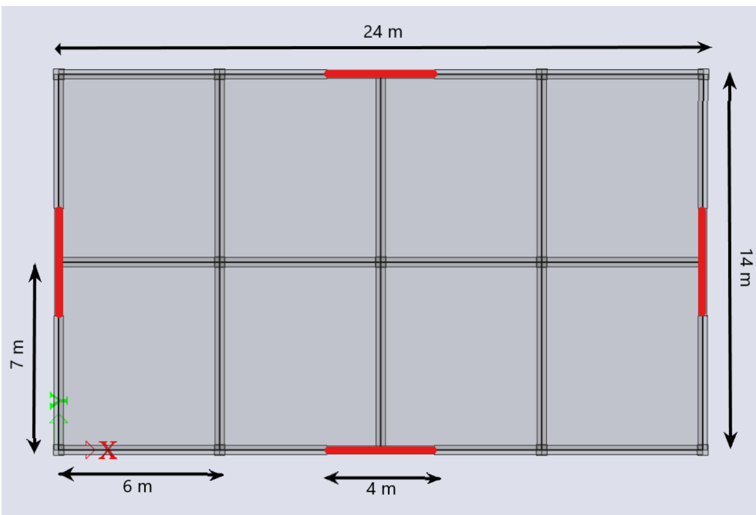
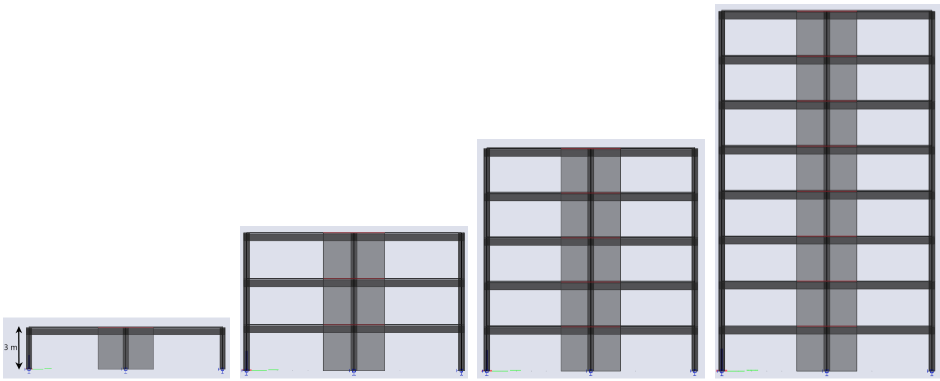


Figure 3.1: Symmetric building with walls at the periphery

All models have the same storey area, 24 m in x direction and 14 m in y direction. All models are also composed of the same horizontal structural elements: reinforced concrete slabs with 150 mm thickness, beams of section 350 mm x 450 mm, with a span of 6 m

in x direction and 7 m in y direction. The columns have a section of 400 mm x 400 mm, and the walls 4 m x $0,3$ m.

All models are **regular in elevation** and will be computed for 1-, 3-, 5- and 8-storey. This allows us to see the effects of the height for the classification of torsionally flexible buildings. The inter storey height is the same at each storey and is equal to 3 m.



(a) (b) (c) (d)

Figure 3.2: Elevation view of the basic model

The material for all structural elements is concrete C30/37 with reinforcement steel S500. Below the important characteristics of these materials are listed.

Concrete C30/37		
f_{ck}	30	MPa
f_{cm}	38	MPa
f_{cd}	20	MPa
$f_{ctk,0,05}$	2	MPa
$f_{ctk0,95}$	3,8	MPa
f_{ctm}	2,9	MPa
E_{cm}	33	GPa
Steel S500		
f_{yk}	500	MPa
f_{yd}	434,8	MPa
E_s	200	GPa

3.1.1. Geometry in plan

The location and shape of the walls change in each model. Firstly, the objective is to keep the models symmetrical and to decrease the torsional stiffness of the building by moving the walls closer to the centre of mass. Secondly, non-regular buildings in plan are analysed to calculate the position of the centre of stiffness and also to compare the torsional radius calculated with respect to the centre of mass and with respect to the centre of stiffness. Finally, the original shape of the rectangular walls was changed to a U-shaped walls since most of the current buildings have U-shaped walls.

Rectangular walls

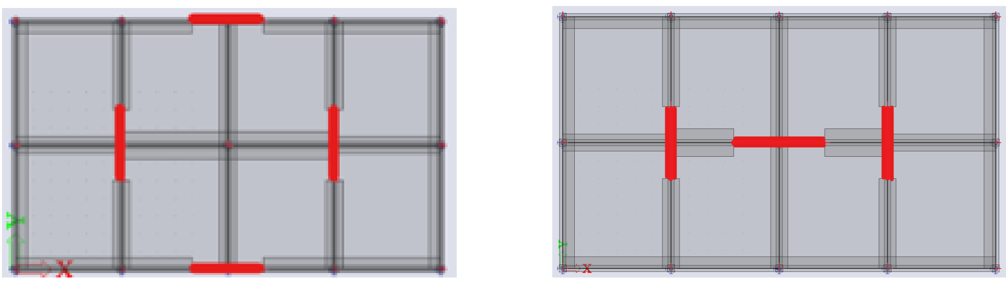
For these models, the walls have a rectangular section. This section can be divided into two major cases, the symmetrical buildings in plan and the asymmetrical ones. In some cases, there is just one wall in one direction instead of two. For the sake of keeping the inertia equal for these models, the length of the wall is increased to $5,04\ m$, but the width is kept equal to $0,3\ m$. The inertia of the floor with two walls in the same direction is given by the formula below.

$$I = 2 \cdot \frac{bh^3}{12} \tag{3.1}$$

$$I = 2 \cdot \frac{0,3 \cdot 4^3}{12} = 3,2\ m^4 \tag{3.2}$$

$$I = \frac{0,3 \cdot 5,04^3}{12} = 3,2\ m^4 \tag{3.3}$$

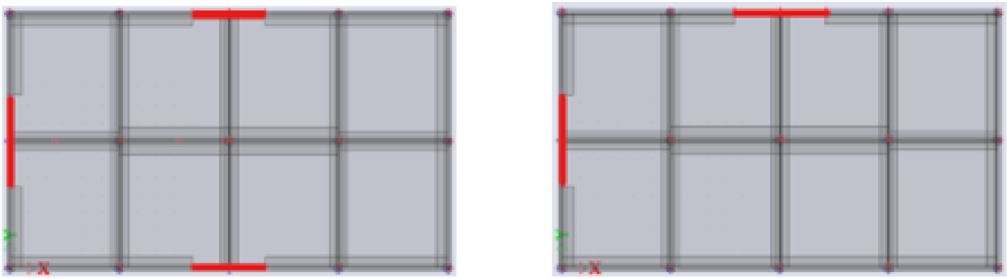
Penelis (2014) [11] defined a torsionally flexible building as a structural system wherein small eccentricities of the seismic horizontal forces cause large torsional deformations. For this reason, some models will remain symmetric but the walls will move closer to the centre of mass to reduce the torsional resistance of the building.



(a) (b)

Figure 3.3: Symmetric buildings with walls moving closer to the CM in (a) one direction (b) two directions

Since we are studying the torsional response of a building after an earthquake, it seems logical to have some asymmetric models as well. These models are interesting to see the effects of the natural eccentricity and also to compare the torsional radius computed with respect to the centre of mass and with respect to the centre of stiffness.

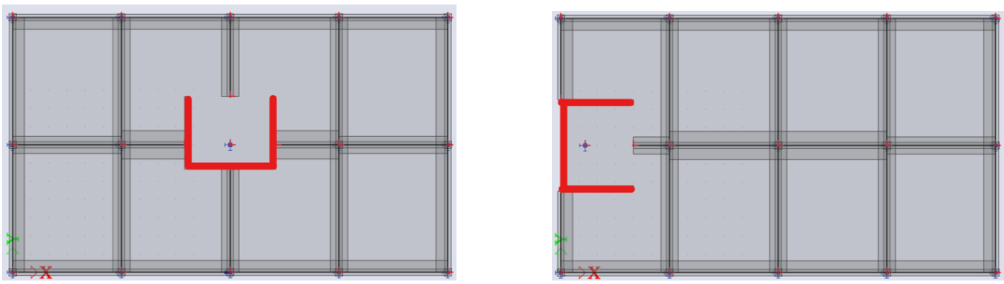


(a) (b)

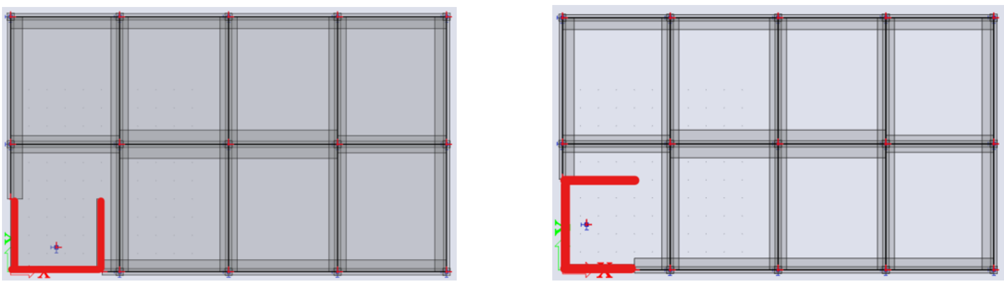
Figure 3.4: Asymmetric buildings in (a) one direction (b) two directions

U-shaped wall

Nowadays, many buildings are built with U-shaped walls. These are very practical because they can be used for lifts, stairs, etc. Therefore, it seemed logical to make some models with them. With these models below, we have a good range of analysis : the model 3.5(a), is the most common in construction, then the models 3.5(b) and 3.5(d) are interesting to analyse the effect of the natural eccentricity in one and two directions, finally, with the both models 3.5(c) and 3.5(d), the effects due to the orientation of the wall will be analysed. The wall is modelled by a columns with a U-shaped section.



(a) (b)



(c) (d)

Figure 3.5: U-shape wall model

3.1.2. Loads

Gravity Loads

The gravity loads can be divided in two categories, the dead loads (G) and the live loads (Q). The table below shows the values for each load.

Structure self-weight	$G_{k,1}$	$\gamma = 25$	kN/m^3
Other permanent loads	$G_{k,2}$	$2,5$	kN/m^2
	$G_{k,2,roof}$	1	kN/m^2
Live loads	Q_k	2	kN/m^2
	$Q_{k,roof}$	1	kN/m^2

These loads should be combined as follows :

$$G_k + \psi_{E,i} \cdot Q_{k,i} \tag{3.4}$$

With $G_k = G_{k,1} + G_{k,2}$ and $\psi_{E,i}$, the combination coefficient for variable action i. This coefficient is calculated with the formula below.

$$\psi_{E,i} = \phi \cdot \psi_{2i} \tag{3.5}$$

The values of ϕ are given in the Table below.

Type of variable action	Storey	ϕ
Categories A-C	Roof	1
	Storeys with correlated occupancies	0,8
	Independently occupied storeys	0,5
Categories D-F and Archives		1

The categories of type of variable action are defined more specifically in the first generation of the Eurocode 8 [13], but for our models, the category A is chosen. This correspond to residential areas. Therefore, for this category, the value of ψ_{2i} is 0,3.

Seismic Load

As explained in Section 2.3.1, the design response spectrum that represents the seismic action depends of several parameters. The choices made in this master thesis are presented below.

- Spectrum type : Type 1 (5% damping), so $a_{gR} = 2,5\ m/s^2$;
- Ground type : Type B defined in the Eurocode 8 [13] as "deposits of very dense sand, gravel, or very stiff clay at least several tens of metres in thickness, characterised by a gradual increase of mechanical properties with depth" (see Annex A). The Table 3.1 shows the values of the periods as well as the soil factor.

Spectrum	Ground	S	$T_B(s)$	$T_C(s)$	$T_D(s)$
Type 1	Type B	1,20	0,15	0,50	2,00

Table 3.1

- Importance classes for building : Class II, for "ordinary buildings, not belonging in the other categories". The value of γ_I for importance class II is equal to 1;
- Behaviour factor will depend on whether the building is classified as torsionally flexible. In case the model is torsionally rigid, the behaviour factor will be equal to 3. Indeed, according to the first generation of the Eurocode 8, q should be calculated as follow :

$$q = q_0 \cdot k_w \tag{3.6}$$

Where,

k_w is equal to $(1+\alpha_0)/3 < 1$ for wall system, with $\alpha_0 = \sum h_{wi}/\sum l_{wi} = 3,75$; therefore $k_w = 1$;

q_0 is equal to 3 for uncoupled wall system design for the ductility class DCM.

For buildings which are torsionally flexible, according to second generation of the Eurocode 8, the value of the behaviour factor will be multiplied by 0,8 ($q = 2,4$).

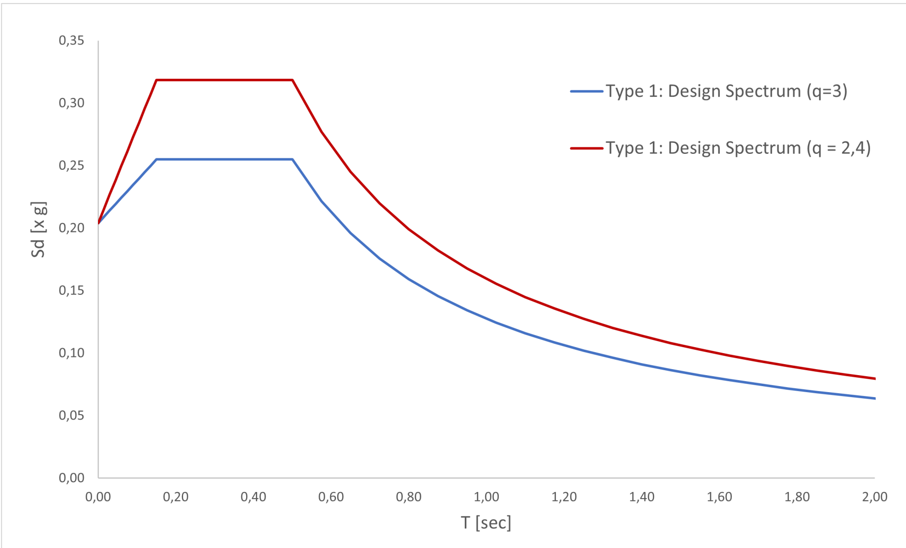


Figure 3.6: Design response spectrum Type 1, ground soil B

3.2. Classification of torsionally flexible building

The simplified procedure described in the second generation of the Eurocode 8 can be used when the primary structure is composed of the same type of primary bracing continuously from the basement to the level under consideration.This condition is met because we are focusing on buildings which are regular in elevation.

The 3D procedure for the calculation of the position of the centre of stiffness and the torsional radius presented in the Sections 2.1.1 and 2.1.2 follows the method described in the second generation of the Eurocode 8. This means that the profile of forces is defined in applying an unit acceleration to the mass of each floor i, and the value of the profile of moment around the vertical axis applied at the centre of mass of the storey, is equal to the values of the lateral forces.

However, the forces and moment as defined above can not be defined simply in the software Scia. For this reason, the Scia help centre [12] suggests to adapt the method by directly using the results of standard equivalent lateral force seismic load cases. According to the "Scia method", two seismic load cases should be defined in direction x and y, the equivalent lateral force method should be activated as well as the accidental eccentricity. The steps to follow are listed just below.

1. **Storey forces** $F_{x,i}$ and $F_{y,i}$ are calculated following the formula 3.7, and can be obtained in the software Scia from the section *Summary storey results* for both seismic load cases.

$$F_i = F_b \cdot \frac{s_i m_i}{\Sigma s_j m_j} \tag{3.7}$$

2. **Storey moments** are corresponding to the accidental eccentricity load cases. Their values can be calculated from the storey forces.

$$M_{z(F_x),i} = e_{a,xi} \cdot F_{xi} \qquad M_{z(F_y),i} = e_{a,yi} \cdot F_{yi} \tag{3.8}$$

Where

$e_{a,xi}$ and $e_{a,yi}$	are the accidental eccentricity at the storey i , by default their value is 5% of the width of the storey to the perpendicular direction of the seismic load case;
$F_{x,i}$ and $F_{y,i}$	are the storey forces applied to the storey i.

Because the values of the moments and the forces are not the same, we have to calculated the inter-storey unit rotation and displacement.

3. **Inter-storey unit rotation and displacement** are calculated with the following formulas for x direction, similar formulas can be used in y direction.

$$\Delta\theta_{F_{x1,i}} = \frac{\Delta\theta_{F_{x,i}}}{F_{R,x,i}} \tag{3.9}$$

$$\Delta\theta_{Mz(F_x)1,i} = \frac{\Delta\theta_{Mz(F_x),i}}{M_{R,z(F_x),i}} \tag{3.10}$$

$$\Delta d_{x1,i} = \frac{\Delta d_{x,i}}{F_{R,x,i}} \tag{3.11}$$

Where

$\Delta\theta_{F_{x,i}}$	is the difference of rotation around the vertical axis between the storey $i-1$ and i due to the seismic load case in x direction;
$\Delta\theta_{Mz(F_x)1,i}$	is the difference of rotation around the vertical axis between the storey $i-1$ and i due to the accidental eccentricity induced by the seismic load case in x direction;
$\Delta d_{x,i}$	is the inter-storey drift of the centre of mass in x direction between the storey $i-1$ and i due to the seismic load case in x direction;
$F_{R,x,i}$	is the resultant force in x direction applied at the floor i;
$M_{R,z(F_x),i}$	is the resultant moment applied at the floor i.

The value of $\Delta\theta_{Mz(F_x)1,i}$ and $\Delta\theta_{Mz(F_y)1,i}$ should be the same in principle, it is therefore not necessary to calculate both. Processing only one accidental eccentricity is sufficient.

4. **Natural eccentricity and torsional radius** are finally obtained with the Equation 3.12.

$$e_{0x,i} = \frac{\Delta\theta_{F_{y1,i}}}{\Delta\theta_{Mz(F_y)1,i}} \qquad r_{x,i} = \sqrt{\frac{\Delta d_{y1,i}}{\Delta\theta_{Mz(F_y)1,i}}} \tag{3.12}$$

For multi-storey buildings, the position of the centre of stiffness and the torsional radius depends on the lateral loading with height. Therefore, the results will differ with the choice of the method. However, even if the values are not exactly the same, the shape of the torsional radius and natural eccentricity with the height is the same for both methods. Therefore, the comments in the following sections stay unchanged regardless of the method.

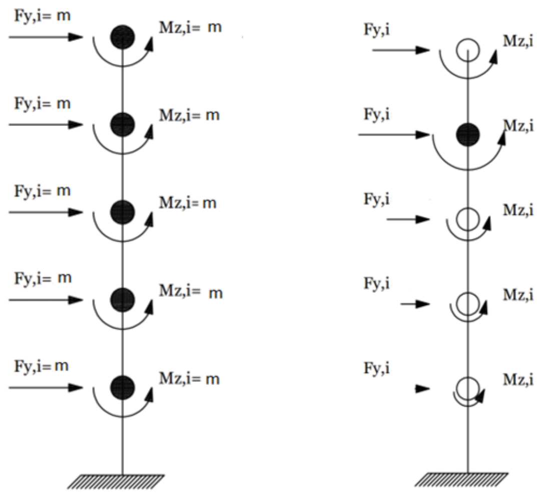


Figure 3.7: Application of the load following the method described in (a) the second generation of the Eurocode (b) Scia

The results shown in the following sections were calculated using the method described in the second generation of the Eurocode 8, however, the results using the method recommended by the Scia help centre can be found in the Annex C.

3.2.1. Natural eccentricity

In this section, the position of the centre of stiffness is calculated for multi-storey buildings using the simplified method and the 3D procedure.

To achieve that, the results are detailed for the model with rectangular walls with asymmetry in both directions x and y (Figure 3.8). The results for the other models with asymmetry in plan can be found in the Annex B.1.

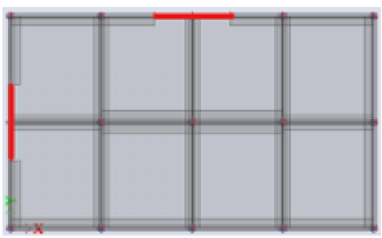


Figure 3.8

Simplified procedure

The formula 3.13 are used to calculate the coordinates of the centre of stiffness.

$$x_{CS,i} = \frac{\Sigma_j x_j K_{x,ij}}{\Sigma_j K_{x,ij}} \qquad y_{CS,i} = \frac{\Sigma_j y_j K_{y,ij}}{\Sigma_j K_{y,ij}} \tag{3.13}$$

Where the lateral stiffness of the columns is calculated with the following formula. Since the columns have a square section, the lateral stiffness is equal in both direction.

$$K_{ij} = \frac{EI_j}{h_i^3} \tag{3.14}$$

With $E = 33\ GPa$, $I = \frac{bh^3}{12} = \frac{0{,}4^4}{12} = 0,0021\ m^4$ and $h_i = 3\ m$,

$$\rightarrow K_{columns} = 2607,41\ kN/m \tag{3.15}$$

And the lateral stiffness of the walls are calculated with the equation (3.16).

$$K_{x/y,ij} = \left[\left(\frac{EI_{x/y,j}}{h_i^3} \right)^{-1} + \left(\frac{5 \cdot EA_{x/y,i}}{12 \cdot h_i} \right)^{-1} \right]^{-1} \tag{3.16}$$

With $A = h \cdot b = 5,04 \cdot 0,3 = 1,512\ m^2$, unlike the columns, the inertia of the walls varying with the direction, therefore a weak inertia and a strong inertia can be computed :

$$I_{strong} = \frac{bh^3}{12} = 3,2\ m^4 \tag{3.17}$$

$$I_{weak} = \frac{hb^3}{12} 0,011\ m^4 \tag{3.18}$$

The lateral stiffness in the weak and strong direction are equal to :

$$K_{strong,walls} = 2\,500\,056\ kN/m \tag{3.19}$$

$$K_{weak,walls} = 13\,831,5\ kN/m \tag{3.20}$$

Therefore, the position of the centre of stiffness with respect to the geometric centre can be calculated as follow.

$$x_C = \frac{(-12) \cdot K_{strong,walls} + 12 \cdot K_{columns}}{1 \cdot K_{strong,walls} + 1 \cdot K_{weak,walls} + 13 \cdot K_{columns}} = -11,76\ m \tag{3.21}$$

$$y_C = \frac{7 \cdot K_{strong,walls} - 7 \cdot K_{columns}}{1 \cdot K_{strong,walls} + 1 \cdot K_{weak,walls} + 13 \cdot K_{columns}} = 6,86\ m \tag{3.22}$$

The position of the centre of mass is $x_{CM} = 11,751\ m$ and $y_{CM} = 7,145\ m$. Thus the natural eccentricity is equal to :

$$e_{0x,i} = -11,52\ m \qquad e_{0y,i} = 6,45\ m \tag{3.23}$$

3D procedure

According to the method described in the second generation of the Eurocode 8, a profile of forces applying a unit acceleration to the mass of the floor has been applied to the structure in both directions x and y. A moment, with a value equal to the mass of the storey, has also been applied. For the model in Figure 3.8, the masses are equal to $m_i = 326,52\ t$ and $m_{roof} = 249,47\ t$.

The natural eccentricity at the i-th floor of a multi-storey building can then be calculated with the formula below.

$$e_{0x,i} = \frac{\Delta\theta_{y,i}}{\Delta\theta_{z,i}} \qquad e_{0y,i} = \frac{\Delta\theta_{x,i}}{\Delta\theta_{z,i}} \tag{3.24}$$

The numerical results for the single-storey building are presented below.

$$\Delta\theta_{x,i} = 0,01388\ mrad \qquad \Delta\theta_{y,i} = -0,02138\ mrad \qquad \Delta\theta_{z,i} = 0,00219\ mrad \tag{3.25}$$

This gives us the following natural eccentricity :

$$e_{0x,i} = -9,763\ mm \qquad e_{0y,i} = 6,338\ mm \tag{3.26}$$

The centre of mass expressed in the global coordinate system is at $(11,751; 7,145)[m]$. Thus the coordinate of the centre of stiffness are $x_C = 1,988\ m$ and $y_C = 13,48\ m$. The following graphs show the results for the multi-storey building (3-, 5- and 8-storey buildings).

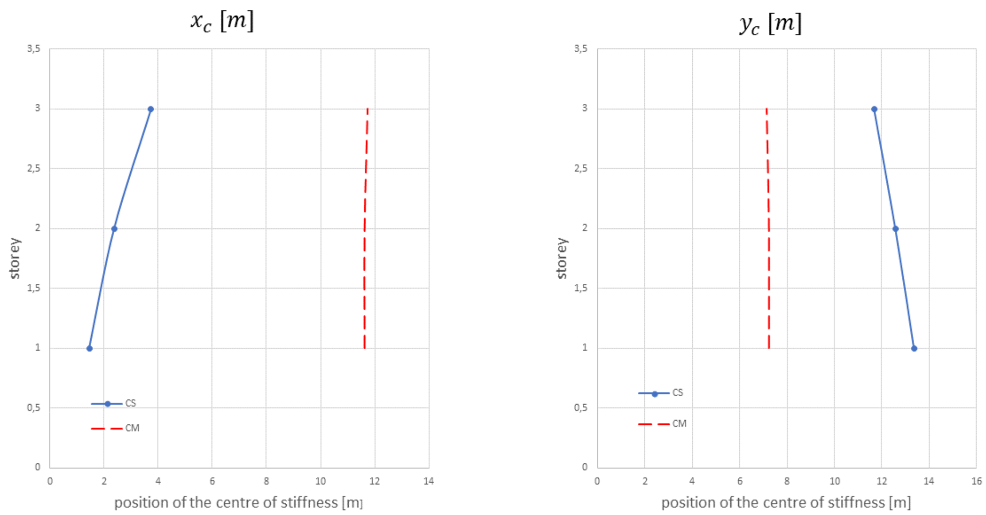


Figure 3.9: position of the centre of stiffness compare to the centre of mass for 3-storey building

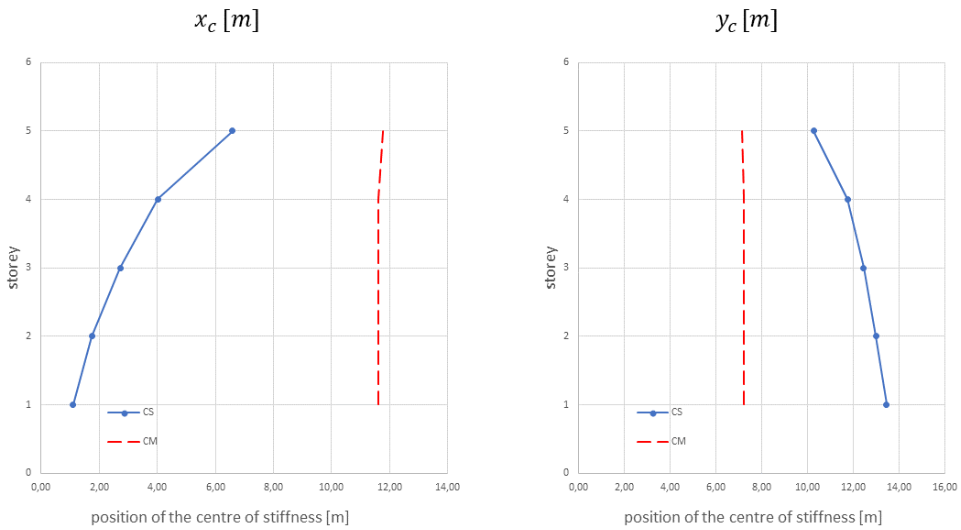


Figure 3.10: position of the centre of stiffness compare to the centre of mass for 5-storey building

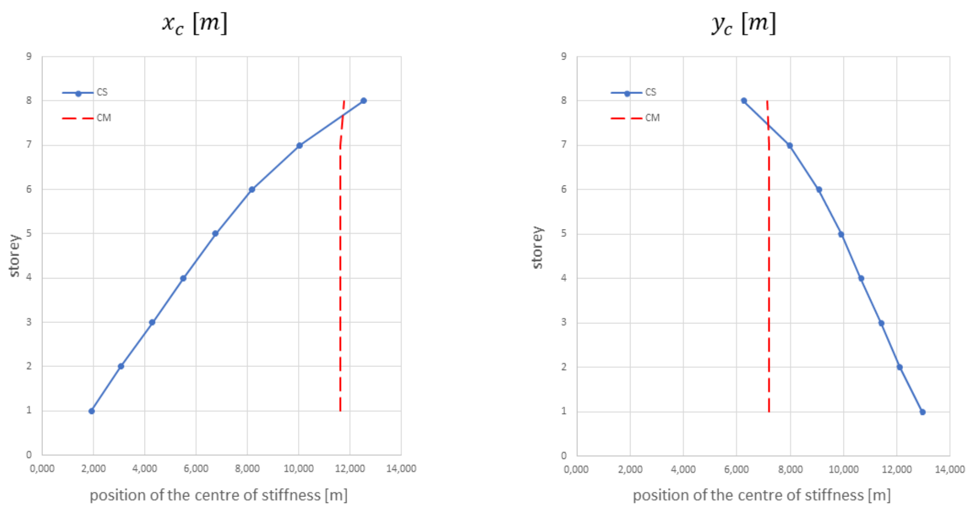
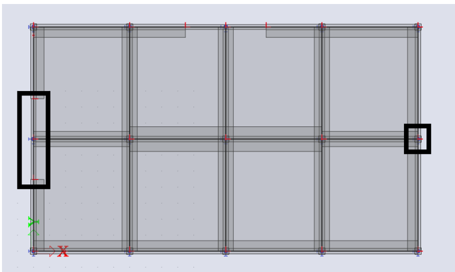


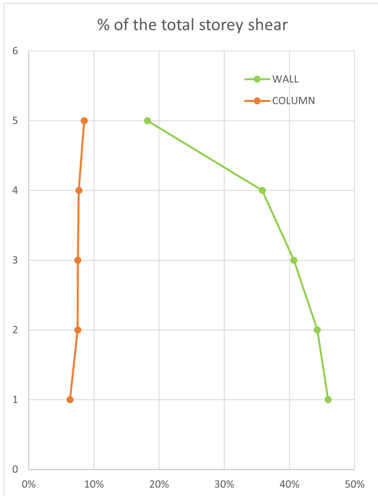
Figure 3.11: position of the centre of stiffness compare to the centre of mass for 8-storey building

Two main observations can be done. In the first one, the position of the centre of stiffness is shifted toward the walls. It was expected since the walls have a bigger lateral and rotation stiffness than the columns and therefore contribute more to the storey stiffness than the columns.

Now comparing the two methods, the simplified method gives bigger natural eccentricities than the 3D procedure. It was expected since the simplified method does not consider the horizontal structural elements. Reviewing the graphs 3.9, 3.10 and 3.11, is that with the **3D procedure**, the position of the centre of stiffness is varying with the height while with the **simplified procedure**, it remains constant. As said just above, the walls contribute more to the storey stiffness than the columns. However, this contribution remains not constant with the height. Indeed, looking at the percentage of the lateral shear (in y direction) taken by the wall and the column at the opposite side (Figure 3.12), we notice that the percentage of the storey shear taken by the wall decreases with the height.



(a) Wall and column analysed



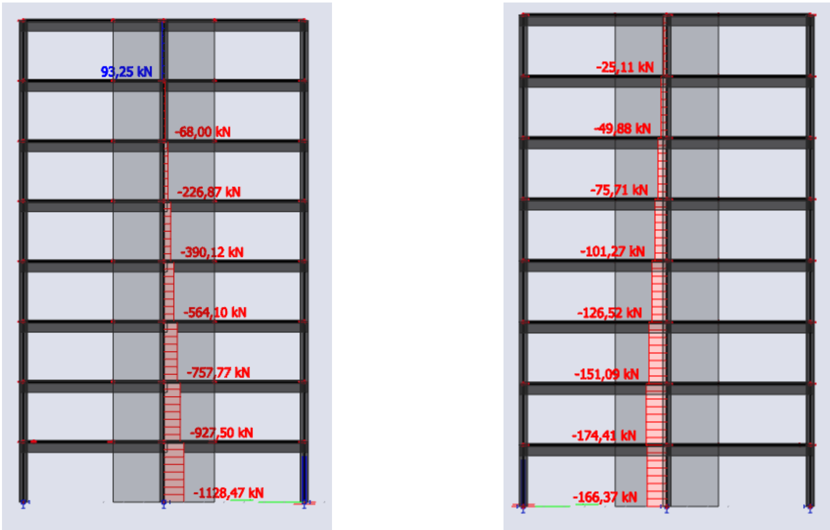
(b)

Figure 3.12: Percentage of the total storey shear taken by the wall and the column - 5 storey building.

$F_{y,storey}$ [kN]	$F_{y,wall}$ [kN]	%	$F_{y,column}$ [kN]	%
1555,57	714,44	46%	116,3	7%
1229,05	543,96	44%	92,24	8%
902,52	367,14	41%	67,96	8%
576,00	206,35	36%	44,25	8%
249,47	45,44	18%	21,26	9%

Table 3.2: Numerical values of the total shear + internal shear force in the wall and column

One last remark, we can notice that on the last storey of the model with 8-storey, the position of the centre of stiffness crosses the centre of mass (Figure 3.11). This would mean that for the last storey, the contribution of the column is bigger than the wall. Looking at Figure 3.13, we can see that the internal force in the wall is positive, this explains why the position of the CS crosses the CM.



(a) (b)

Figure 3.13: Internal shear force in y direction in (a) the wall, (b) the column.

3.2.2. Torsional radius

To classify a building as torsionally rigid, we have to compare the torsional radius to the radius of gyration, l_s. Since all our models have a rectangular floor and the mass is well distributed, l_s is equal to :

$$l_s = \sqrt{\frac{l^2 + b^2}{12}} = \sqrt{\frac{24^2 + 14^2}{12}} = 8,02\ m \tag{3.27}$$

Simplified method

The formula 3.28 can be used to calculate the torsional radius in the x direction (the same formula can be applied in y direction).

$$r_{x,i} = \sqrt{\frac{\sum_j K_{x,ij}(y_i - y_{C,i})^2 + \sum_k K_{y,ik}(x_k - x_{C,i})^2}{\sum_k K_{y,ik}}} \tag{3.28}$$

The results in this section are detailed for the model with rectangular walls close to the centre of mass (Figure 3.14). Since the building is symmetric, the position of the centre of mass is the same as the centre of stiffness. This allows us to compare the results given by the simplified procedure and the 3D procedure without taking into account the difference in the reference point. The values of the torsional radius for all the models are presented in the Annex B.2.

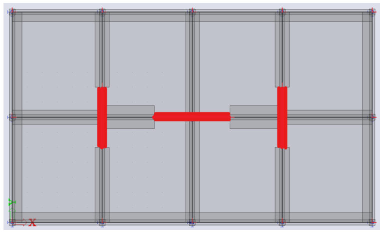


Figure 3.14

The value of the position of the centre of stiffness is calculated with the simplified method. For this model, the position of the centre of stiffness is the same than the centre of mass, which means that $x_C = 12\ m$ and $y_C = 7\ m$. The value of the lateral stiffness for the columns and for the wall with the length $= 5,04\ m$ have been calculated in the previous section.

$$K_{columns} = 2607,41\ kN/m \tag{3.29}$$
$$K_{strong,wall,h=5,04} = K_{x,wall,h=5,04} = 2\,500\,056\ kN/m \tag{3.30}$$
$$K_{weak,wall,h=5,04} = K_{y,wall,h=5,04} = 13\,831,5\ kN/m \tag{3.31}$$

The lateral stiffness for the walls whose length is equal to 4 m can be expressed as follows.

$$K_{x/y,ij} = \left[\left(\frac{EI_{x/y,j}}{h_i^3}\right)^{-1} + \left(\frac{5 \cdot EA_{x/y,i}}{12 \cdot h_i}\right)^{-1}\right]^{-1} \tag{3.32}$$

With $A = h \cdot b = 4 \cdot 0,3 = 1,2\ m^2$, the inertia in weak and strong direction are expressed as :

$$I_{strong} = \frac{bh^3}{12} = 1,6\ m^4 \tag{3.33}$$
$$I_{weak} = \frac{hb^3}{12} 0,009\ m^4 \tag{3.34}$$

The lateral stiffness in the weak and strong direction are equal to :

$$K_{strong,walls,h=4} = K_{y,wall,h=4} = 1\,442\,623\ kN/m \tag{3.35}$$
$$K_{weak,walls,h=4} = K_{x,wall,h=4} = 10\,978\ kN/m \tag{3.36}$$

Therefore, for this model, the torsional radius in x and y direction are equal to :

$$r_{x,i} = \sqrt{\frac{\sum_{12} K_{col,i}(y_i - 7)^2 + \sum_{12} K_{col,k}(x_k - 12)^2 + \sum_2 K_{y,wall,k,h=4}(x_k - 12)^2}{\sum_1 K_{y,wall,h=5,04} + \sum_2 K_{y,wall,h=4} + \sum_{12} K_{col}}} = 6,497\ m \tag{3.37}$$

$$r_{y,i} = \sqrt{\frac{\sum_{12} K_{col,i}(y_i - 7)^2 + \sum_{12} K_{col,k}(x_k - 12)^2 + \sum_2 K_{y,wall,k,h=4}(x_k - 12)^2}{\sum_1 K_{x,wall,h=5,04} + \sum_2 K_{x,wall,h=4} + \sum_{12} K_{col}}} = 6,065\ m \tag{3.38}$$

3D procedure

Applying the three load cases previously defined (for this model, $m_i = 332,09\ t$ and $m_{roof} = 252,26\ t$), the torsional radius can be expressed as follow :

$$r_{xCM,i} = \sqrt{\frac{\Delta d_{yCM,i}}{\Delta\theta_{z,i}}} \qquad r_{yCM,i} = \sqrt{\frac{\Delta d_{xCM,i}}{\Delta\theta_{z,i}}} \tag{3.39}$$

As we can see, the formula for the **3D procedure** computes the torsional radius with respect to the centre of mass, while for the **simplified method**, the reference point is the centre of stiffness. Since, in this section, we focus on a symmetrical building, this will not influence the results but a deeper analysis of an asymmetrical building is done in the next section.

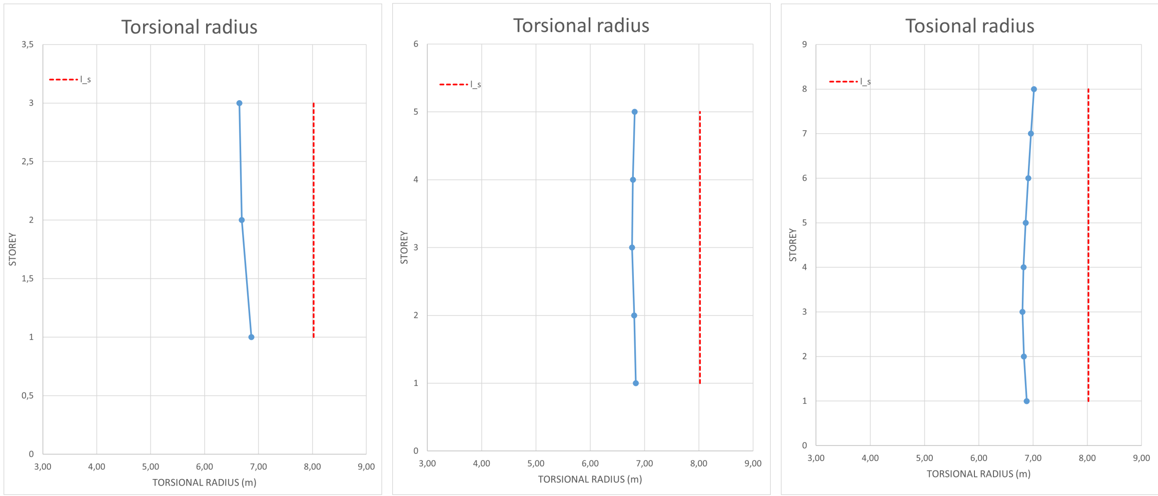
The numerical results for the single-storey building are presented below.

$$\Delta d_{xCM,i} = 0,05298\ mm \qquad \Delta d_{yCM,i} = -0,06306\ mm \qquad \Delta\theta_{z,i} = 0,00115\ mrad \tag{3.40}$$

We can therefore calculated the torsional radius :

$$r_{xCM,i} = 7,41\ m \qquad r_{yCM,i} = 6,79\ m \tag{3.41}$$

The following graphs show the results of the torsional radius for 3-, 5- and 8-storey buildings as well as the radius of gyration $l_s = 8,02\ m$ (in red dotted line in the graphs).



(a) (b) (c)

Figure 3.15: Torsional radius 3D procedure for (a) 3-, (b) 5- and (c) 8-storey building

The value of the torsional radius (Figure 3.15) is lower than the radius of gyration, it would mean that the building is torsionally flexible. However, while we can classify a building as torsionally rigid if $r_i \geq l_s$, the opposite is not true. A quick verification with the modal analysis is necessary. In this case, the first mode is influenced by torsion (see Table 3.3), which means that the building is indeed torsionally flexible.

Mode	T [s]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	0,45	0	0	0	0	0	**0,7546**
2	0,38	0	**0,7301**	0	0,1646	0	0
3	0,36	**0,7568**	0	0	0	0,0677	0

Table 3.3: Effective modal mass for 5-storey building

A closer look at the results obtained with the simplified method and with the 3D procedure shows that the torsional radius calculated from the simplified method is lower than with the 3D procedure. Since a high torsional radius means a higher resistance to the torsion, **the simplified method is more conservative than the 3D procedure**.

The difference between the two values can be explained by the fact that the simplified method does not consider the horizontal elements as the beam or the slab. Indeed, only the stiffness of the vertical elements (walls and columns) is taken into account during the calculation.

3.2.3. Comparison between the torsional radius with respect to the centre of mass and the centre of stiffness

As discussed in the previous section, the simplified procedure in the Eurocode 8 defined the torsional stiffness with respect to the centre of stiffness (Equation 3.42).

$$K_\theta = \sum_j K_{x,ij}(y_i - y_{C,i})^2 + \sum_k K_{y,ik}(x_k - x_{C,i})^2 \tag{3.42}$$

In the 3D procedure, however, the torsional radius is calculated with respect to the centre of mass. This inconstancy between the two definitions has been studied in the master thesis of Maxence Carlier for single storey buildings and will be extended here for multi-storey buildings.

The calculation of the torsional radius computed with respect to the centre of mass could tend to overestimate its value, while on the other hand, the torsional radius computed with respect to the centre of stiffness could be too conservative. The 3D procedure should be used with the condition that the floors may be considered rigid in their planes. Therefore, the rotation due to the moment around the vertical axis applied at the centre of mass of the floor is the same for the centre of mass and the centre of stiffness. Therefore, the torsional radius computed with respect to the centre of stiffness can be obtained with the formula (3.43).

$$r_{x,CS,i} = \sqrt{\frac{\Delta d_{y,CS,i}}{\Delta \theta_{z,i}}} \qquad r_{y,CS,i} = \sqrt{\frac{\Delta d_{x,CS,i}}{\Delta \theta_{z,i}}} \tag{3.43}$$

Where the displacements of the centre of stiffness are obtained with the following formula.

$$\Delta d_{x,CS,i} = \Delta d_{x,CM,i} - \Delta \theta_{x,i} \cdot e_{0y,i} \qquad \Delta d_{y,CS,i} = \Delta d_{y,CM,i} - \Delta \theta_{y,i} \cdot e_{0x,i} \tag{3.44}$$

With,

- $\Delta d_{x/y,CM,i}$ is the displacement of the centre of mass in the x/y direction between the floor $i-1$ and the floor i;
- $\Delta \theta_{x/y,i}$ is the difference of rotation around the vertical axis between the floor $i-1$ and i due to the lateral force in x/y direction;
- $e_{0x/y,i}$ is the natural eccentricity of the floor i in x/y direction.

To make the comparison between the torsional radius computed with respect to the centre of mass and with respect to the centre of stiffness, we analyse the results for the model with rectangular walls with asymmetry in both direction x and y (Figure 3.16). The comparison for the other models can be found in the Annex B.3.

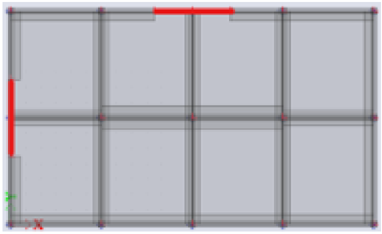
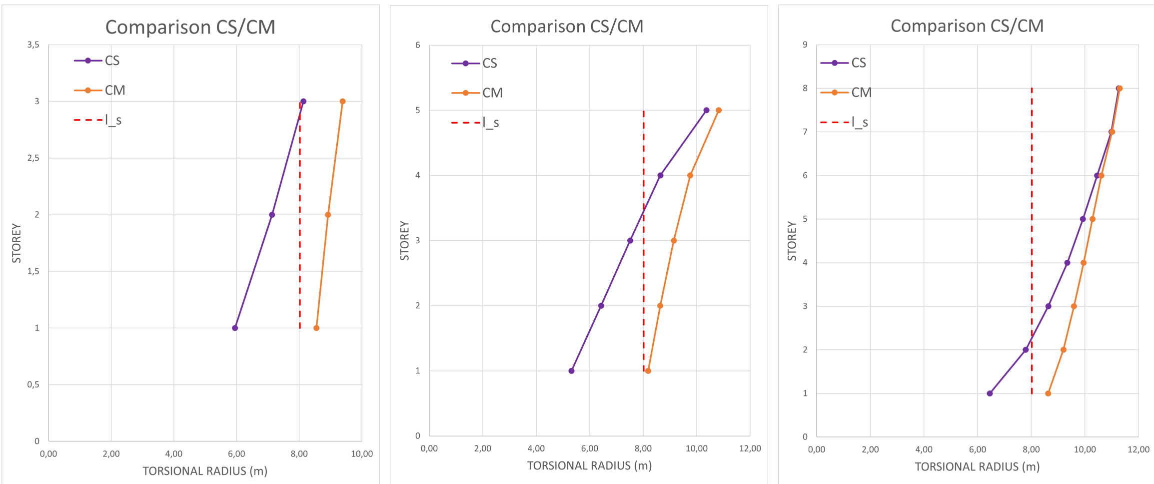


Figure 3.16

Below are the results of the torsional radius, calculated with respect to the centre of mass and with respect to the centre of stiffness, obtained with the 3D procedure.



(a) (b) (c)

Figure 3.17: Comparison of the torsional radius with respect to the centre of mass (CM) and with respect to the centre of stiffness (CS) - 3D procedure for (a) 3-storey building, (b) 5-storey building and (c) 8-storey building

Unlike the model in the previous section (Figure 3.15) where its torsional radius is more or less constant with the height, the torsional radius for this model (Figure 3.17) is increasing with the height. This can be explained by the variation of the centre of stiffness. As shown in the section 3.2.1, the position of the centre of stiffness is not constant with the height due to the relative contribution of the walls at each storey. At the base of the building, the static eccentricity is bigger than at the top, which means that the base of the building has a lower torsional resistance than the top of the building. Since the torsional radius is defined with the Formula (3.45), if K_θ is smaller at the base than at the top, r_i will be smaller at the base than at the top.

$$r_x = \sqrt{\frac{K_\theta}{K_y}} \qquad r_y = \sqrt{\frac{K_\theta}{K_x}} \tag{3.45}$$

The values of the torsional radius computed with respect to the centre of stiffness are smaller than the ones with respect to the centre of mass. This was expected since the displacements of the centre of stiffness are smaller than the displacements of the centre of mass. The difference between the two torsional radius are bigger for large static eccentricity. Looking more specifically at the results for the 8-storey building, we can see that at the two upper storey of the building, $r_{i,CS}$ and $r_{i,CM}$ are close. It is due to the static eccentricity which is small and thus the difference between the displacements is no longer significant.

For this case, the change of the reference point changes the classification of the building. Indeed, the torsional radius with respect to the centre of mass is greater than the radius of gyration and thus the building can be classified as torsionally rigid. However, if r_i is calculated with respect to the centre of stiffness, the torsional radius becomes lower than the radius of gyration. Therefore a verification with the modal analysis is necessary.

Mode	T [s]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	0,87	0,1529	**0,4651**	0,0000	0,0361	0,0059	0,2333
2	0,42	**0,5405**	0,1907	0,0001	0,0,361	0,0476	0,0008
3	0,29	0,0108	0,0411	0,0000	0,1849	0,0326	0,0495
4	0,25	0,0669	0,1458	0,0000	0,0204	0,0047	**0,4923**

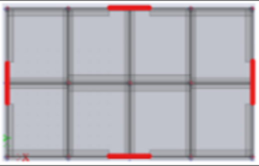
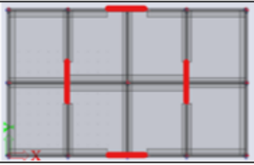
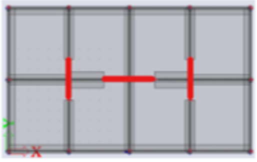
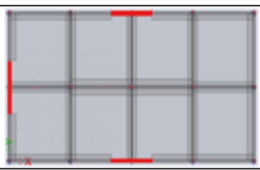
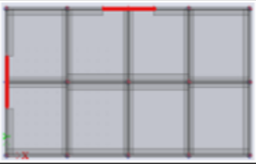
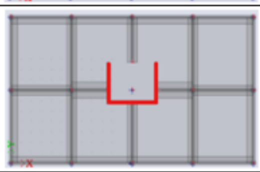
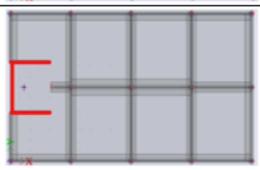
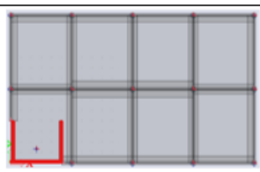
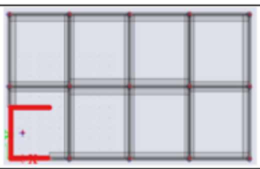
Table 3.4: Effective modal mass for 5-storey building

Since the first and second modes are influenced by the two main horizontal directions, while the greatest effective modal mass of the torsion is at the fourth position (see Table 3.4), the building is classified as torsionally rigid. This classification is the same as the torsional radius calculated with respect to the centre of mass.

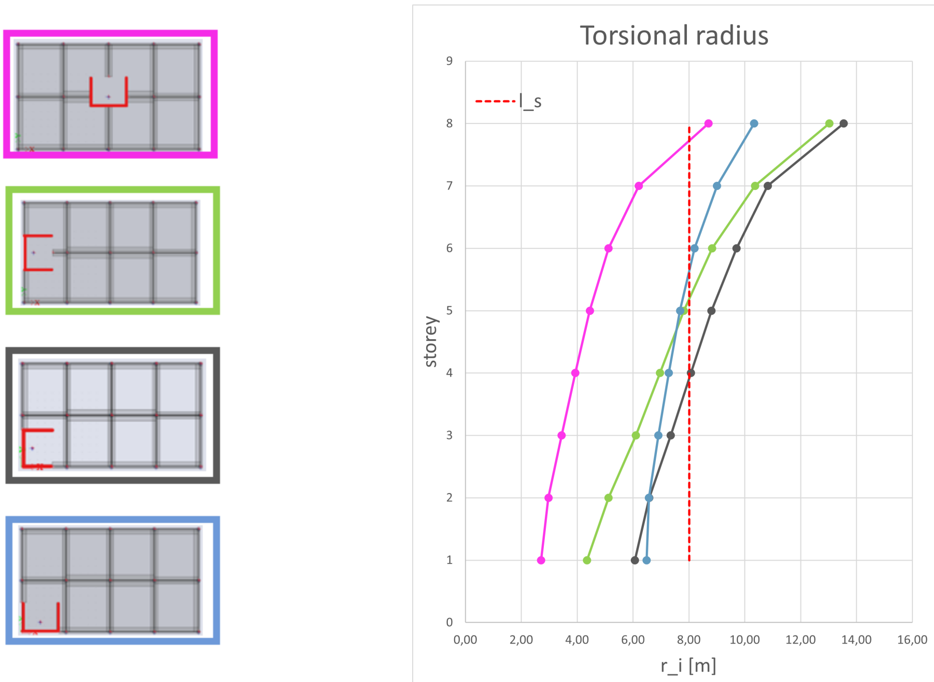
Penelis 2014 [11] says that the calculation of the torsional radius with respect to the centre of stiffness is on the safe side. On the other hand, Fardis 2009 [4] advises calculating the torsional radius with respect to the centre of mass to compare it to the radius of gyration, since l_s is defined with respect to the centre of mass of the floor in plan. Looking at the results in Figure 3.17, I would also suggest calculating the torsional radius with respect to the centre of mass, this remark is similar to the conclusion made by Maxence Carlier for single-storey buildings.

3.2.4. Classification

The following is a summary of the classification of a torsionally flexible building, according to the two conditions of the **second generation** of Eurocode 8. The modal analysis for each model is detailed in the Annex B.4.

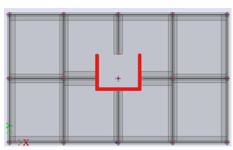
Models	1^{st} Condition	2^{nd} Condition
	$\omega_\theta < \omega_x, \omega_y$	$r_i > l_s$
	$\omega_\theta < \omega_x, \omega_y$	$r_i > l_s$
	$\omega_\theta > \omega_x, \omega_y$	$r_i < l_s$
	$\omega_\theta < \omega_x, \omega_y$	$r_i > l_s$
	$\omega_\theta < \omega_x, \omega_y$	$r_i > l_s$
	$\omega_\theta > \omega_x, \omega_y$	$r_i < l_s$
	$\omega_\theta < \omega_x, \omega_y$	$r_i < l_s$
	$\omega_\theta < \omega_x, \omega_y$	$r_i < l_s$
	$\omega_\theta < \omega_x, \omega_y$	$r_i < l_s$

As said before, we can classify a building as torsionally rigid if $r_i > l_s$, but the opposite conclusion is not possible. Therefore, the three last models with the U-shaped wall in the corner are considered as **torsionally rigid**. In the rest of this section, we will discuss the results obtained for the U-shaped wall models in more detail. The results of the torsional radius versus the radius of gyration and the modal analysis for the four U-wall models with 8-storey are presented below.

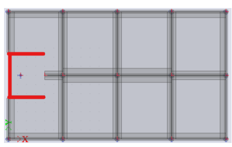


(a) (b)

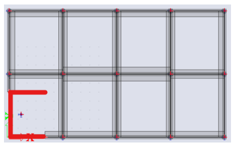
Figure 3.18: Torsional radius for the U-shape walls model



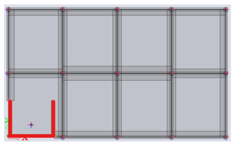
Mode	T [s]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	1,18	0	0	0	0	0	**0,8390**
2	0,71	0	**0,6995**	0	0,2299	0	0,0001
3	0,56	**0,7003**	0	0	0	0,1640	0



Mode	T [s]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	1,35	0	**0,4915**	0	0,0688	0	0,3462
2	0,61	**0,6881**	0	0,0006	0	0,1325	0
3	0,45	0	0,0443	0	0,2907	0	0,0528
4	0,32	0	0,3014	0	0,0783	0	**0,3947**



Mode	T [s]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	1,36	0,0781	**0,4367**	0	0,0597	0,0068	0,3232
2	0,62	**0,6044**	0,0786	0,0001	0,0225	0,1217	0,0036
3	0,45	0,0085	0,0394	0	0,2543	0,0274	0,0490
4	0,32	0,0179	0,2669	0	0,0694	0,0033	**0,4097**



Mode	T [s]	$m_{eff,x}$	$m_{eff,y}$	$m_{eff,z}$	$m_{eff,Rx}$	$m_{eff,Ry}$	$m_{eff,Rz}$
1	1,35	0,1265	**0,3782**	0	0,0519	0,0110	0,3332
2	0,57	**0,3558**	0,2791	0,0001	0,1103	0,0598	0,0519
3	0,45	0,0020	0,0448	0	0,1963	0,0614	0,0552
4	0,36	0,2536	0,0890	0	0,0178	0,0417	**0,3450**

For the first model with the wall in the centre of the building, the first mode is the one with the greatest effective torsional mass ($m_{eff,Rz}$), the model is therefore classified as torsionally flexible. However, it is not the case for the last three others, which should be classified as torsionally rigid, even if their torsional radius is lower than the radius of gyration. It can be noted that for these three models, the first mode is strongly influenced by the torsion but it is not the greatest torsional modal mass.

The values of the torsional radius plotted in the Figure 3.18 are $r_{y,CMi}$ for all the models because $r_{y,CMi}$ is smaller than $r_{x,CMi}$. It means that the value of the torsional radius depends on $\Delta d_{x,CM}$. Using the torsional radius with respect to the centre of stiffness would have been more conservative, as detailed in the section 3.2.3, for this reason, the plotted values of the torsional radius are calculated with respect to the centre of mass.

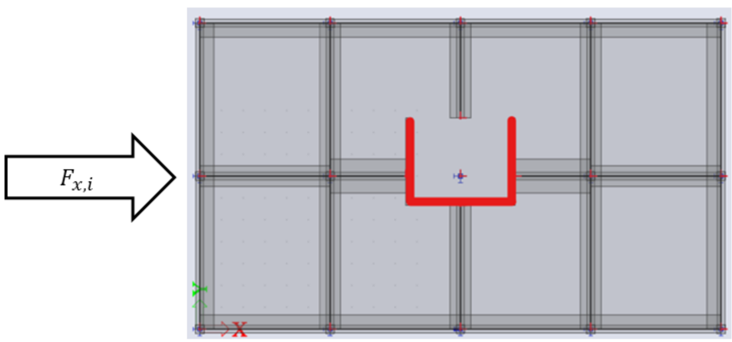
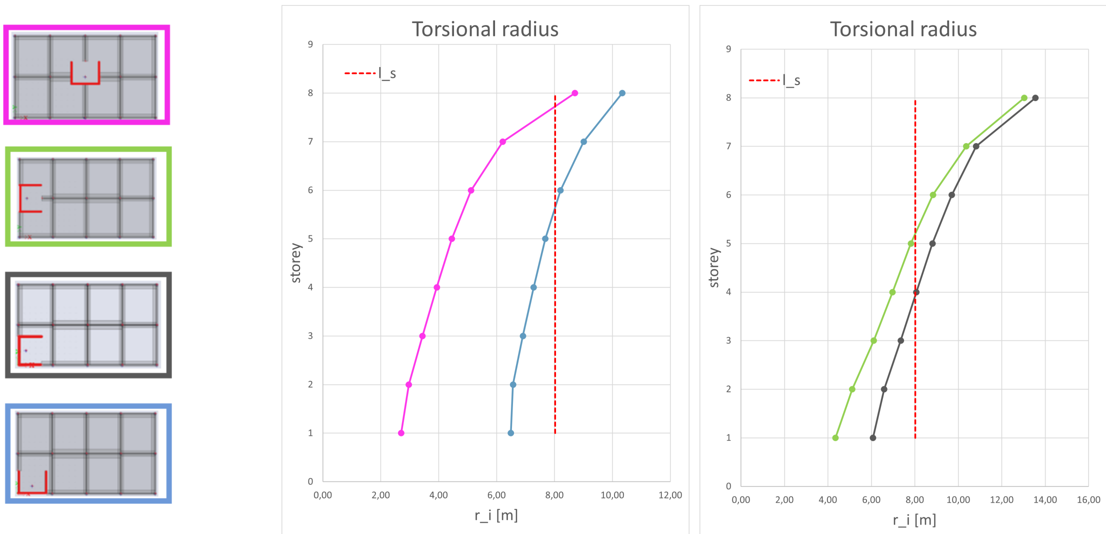


Figure 3.19: Direction of the force for the calculation of $r_{y,CMi}$

Now reviewing the curve of the torsional radius with the height, two trends stand out, depending on the orientation of the wall. For the building with the opening of the U-shaped wall to the top, the torsional radius increases slower than for the other models. It is due to the lateral stiffness of the wall which varies depending on the orientation. The torsional resistance of the storey depends more on the location of the wall than on its orientation.

In figure 3.21, the rotations of the three models with natural eccentricity are quite similar compared to the model with the wall close to the CM.



(a) (b) (c)

Figure 3.20: Torsional radius for the U-shape walls model

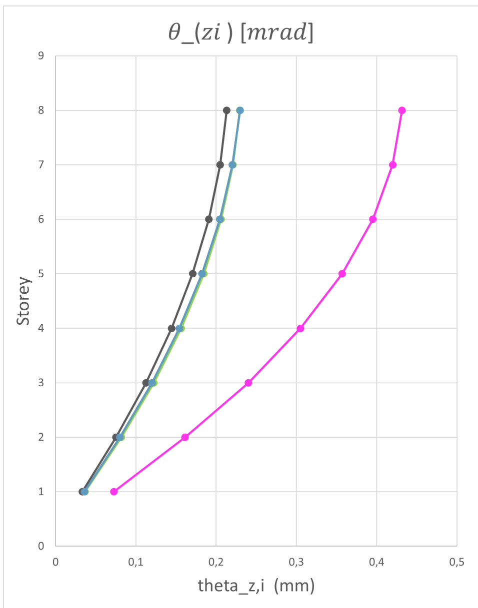


Figure 3.21: θ_{zi} of the centre of mass for U-shaped wall models with 8-storey

CHAPTER 4

Response spectrum analysis and Discussion

4.1. Response spectrum analysis

4.1.1. Accidental torsion

As described in the Section 2.2.3, two methods to take into account the effects of the accidental torsion exist. A comparison between these two methods and a analysis of the results for the building shown in figure 4.1 is done in this section.

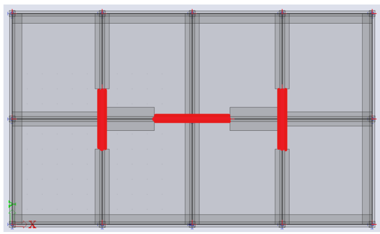


Figure 4.1

The first method consists of multiplying the displacement and stresses by the factor δ, which can be calculated as follows if the analysis is performed using two planar models, one for each horizontal direction.

$$\delta = 1 + 1,2 \cdot \frac{x_j}{L_e} \tag{4.1}$$

Where,

- x_j is the distance of the member j under consideration from the centre of mass of the building in plan, measured perpendicularly to the direction of the seismic action considered;
- L_e is the distance between the two-outermost lateral load resisting members, measured perpendicularly to the direction of the seismic action considered.

The analysis has been made for the column at the upper right corner of the model for 5-storeys (Figure 4.2). Therefore, for a seismic load in y direction, the values of x_j and L_e are determined as follows.

$$x_j = 12\ m$$
$$L_e = 24\ m$$
$$\rightarrow \delta = 1,6$$

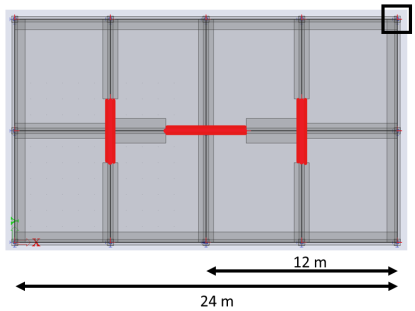
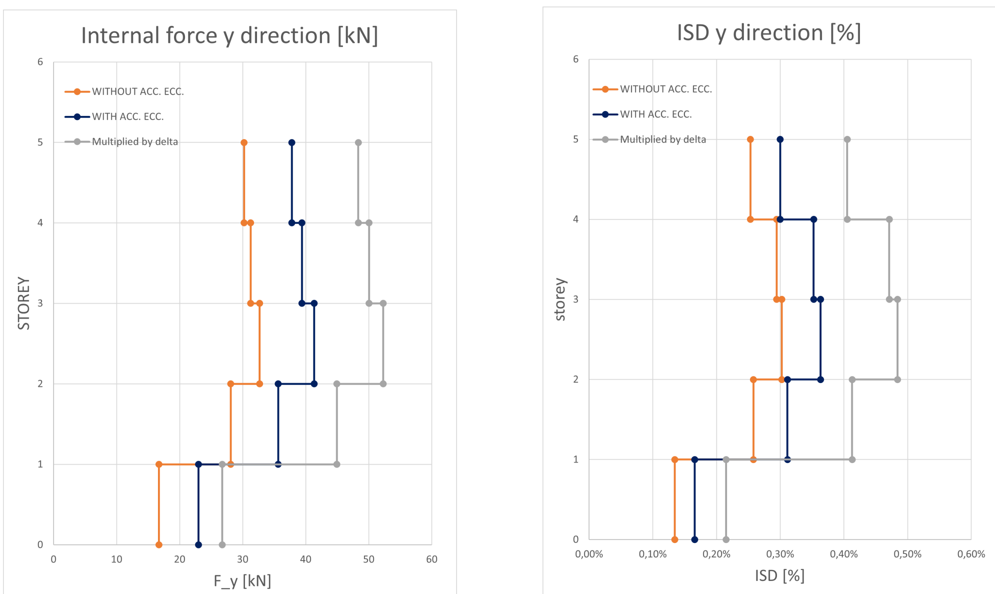


Figure 4.2: Column under consideration

The second method consists of applying a static moment at each floor which acts around the vertical axis of the building given by the following formula.

$$M_{a,ij} = e_{a,ij} \cdot F_{ij} \tag{4.2}$$

This method is implemented in the software Scia. Therefore, when the accidental eccentricity is activated for a seismic load case, a static moment around the vertical axis is applied to the structure. By default, the value of $e_{a,ij}$ is equal to 5% of the length of the floor perpendicular to the direction i of the seismic action at the j-th storey. The internal shear forces in the column in y direction (F_y), as well as the inter-storey drift in y direction, are shown in the graphs below for a 5-storey building.

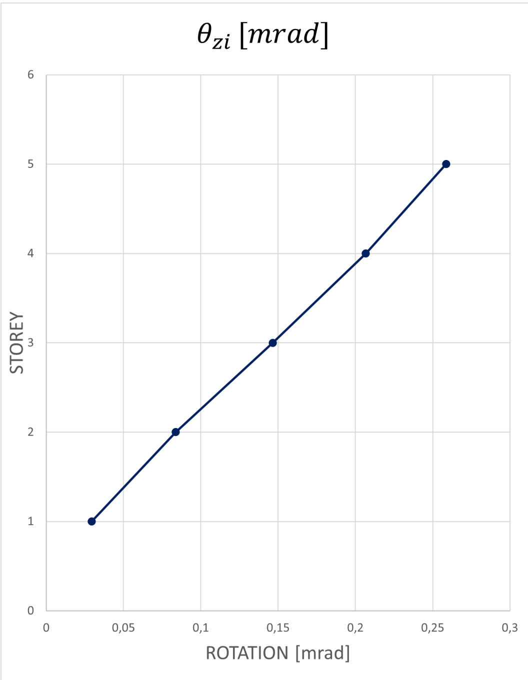


(a) (b)

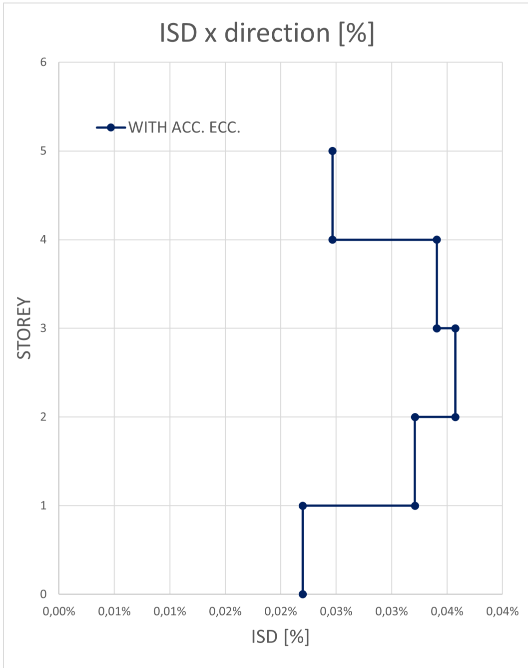
Figure 4.3: Effects of the accidental eccentricity on the column for (a) the internal shear force in y direction, (b) the inter-storey drift

As expected, the method using the δ factor (grey line on the graph) produces higher values of F_y than the other method (blue line on the graph). Multiplying the values of the internal forces and the inter-storey drift by δ increases the values by 60%. On the other hand, applying a static moment increases the values by $\approx 25 - 27\%$ for the internal shear forces and $\approx 20 - 23\%$ for the displacement.

Applying a static moment around the vertical axis produces also a rotation of the centre of mass, which is not present without because the building is symmetric. The moment also introduces a displacement in x direction of the column at the upper corner.



(a)



(b)

Figure 4.4: Effects of the accidental eccentricity for (a) the rotation of the centre of mass, (b) the inter-storey drift in x direction for the critical column

4.1.2. Consequences of the classification of torsionally flexible building

For buildings classified as torsionally flexible, the behaviour factor should be multiplied by 0,8. In this section, we analyse the consequences of changing the value of the behaviour factor for the model Figure 4.5.

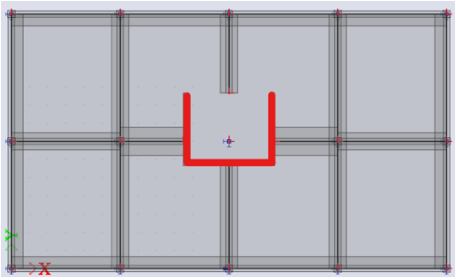
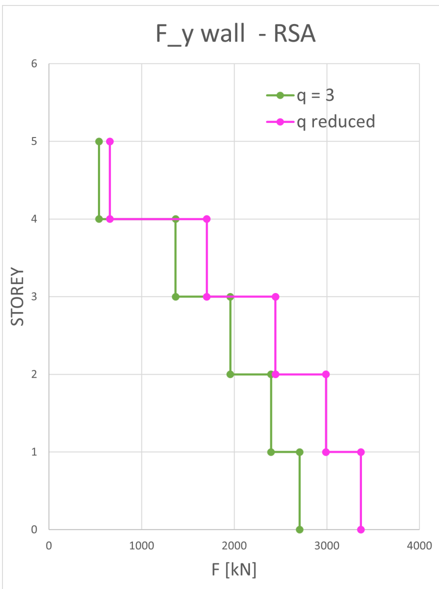
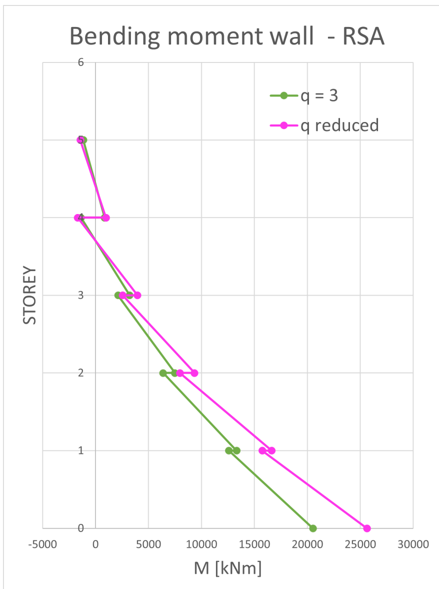


Figure 4.5

The modal response spectrum analysis has been done for a seismic action applied in y direction, with the accidental eccentricity calculated from the static moment. The combination used for the responses is the "Square Root of Sum Squares". The internal forces F_y and the bending moment in the wall are shown in the graphs below.



(a)



(b)

Figure 4.6: Effects of the reduction of the behaviour on the wall for (a) the internal forces, (b) the bending moment

Since the analysis is linear, the difference between the internal force with $q = 3$ and $q = 2,4$ is constant with the height. The value for the behaviour factor reduced is 25% higher than when the behaviour factor is kept equal to 3. This was expected because multiplying the q factor by 0,8 means increasing the seismic action by 25% and thus also the internal forces of the structural elements. The same conclusion can be drawn for the bending moment.

However, the diminution of the behaviour factor does not have any consequences on the inter-storey drift.

4.1.3. Limitation of the inter-storey drift

The second generation of the Eurocode 8 does not limit the static eccentricity, unlike the first generation. However, a large static eccentricity will have an impact on the displacements of the building. As shown in the examples in the introduction, the buildings collapsed because the deformations of the columns located at the opposite side of the stiff element were too large.

In this section, the limitation of the inter-storey drift for the models shown in Figures 4.7 and 4.8 with 5-storey will be verified.

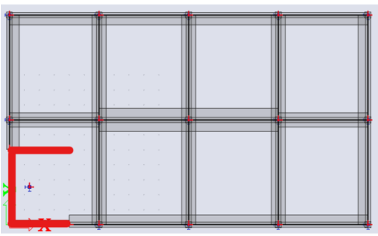


Figure 4.7

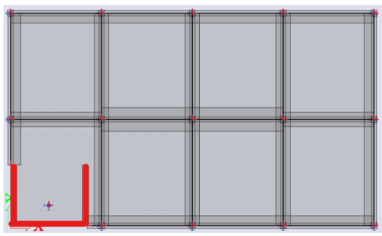


Figure 4.8

Their natural eccentricity is larger than the accidental one. Therefore, according to the second generation of the Eurocode 8, the accidental eccentricity should not be taken into account. The results below have been calculated with the response spectrum analysis, with the combination 'SSRS', for a seismic action in $100\%Y + 30\%X$ direction. Since the models are classified as torsionally rigid, the behaviour factor stay unchanged, $q = 3$.

The second generation of the Eurocode 8 suggests to limit the inter-storey drift to 2% at significant damage limit state. It can be written as follows.

$$d_{r,SD} \leq 0,02 \cdot h_s \tag{4.3}$$

Where,

$d_{r,SD}$ is the design inter-storey drift, defined as the difference of the average lateral displacements d_s at the top and bottom of the storey under consideration at the Significant Damage limit state;

h_s is the height of the storey.

The displacement is computed for the column at the upper right corner, which is the most critical point of the building (Figures 4.9 and 4.10). To verify the limitation of the inter-storey drift, the displacement total of the column is used.

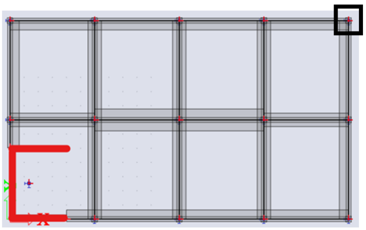


Figure 4.9

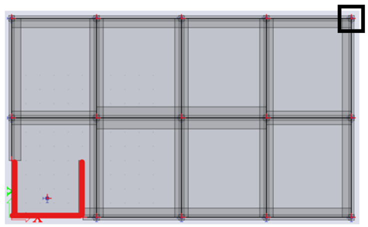
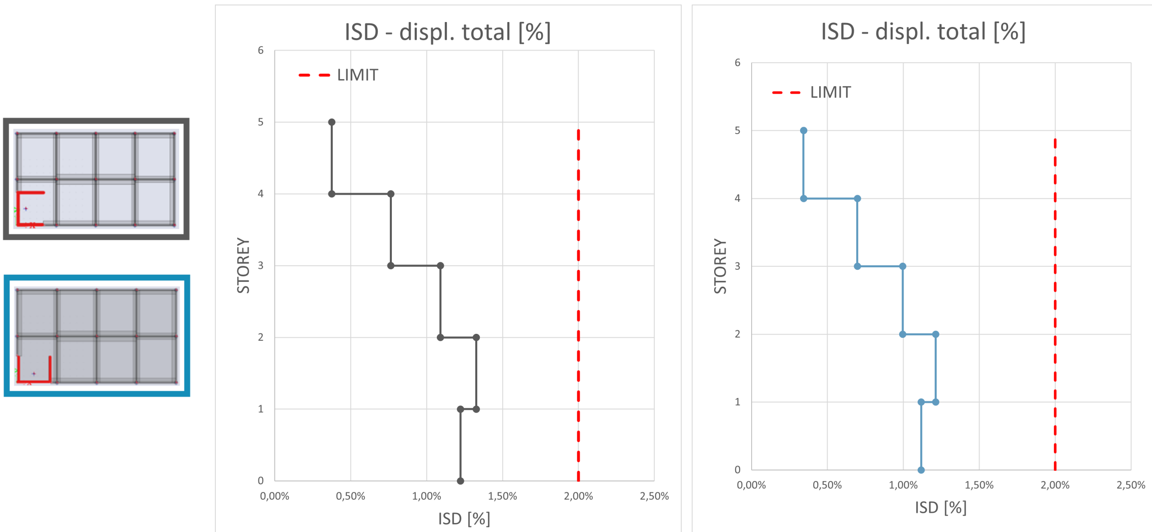


Figure 4.10

The following graphs show the results of the inter-storey drift of the total displacement of the column. The limit of 2% is drawn in red dotted line on the graphs.



(a) (b) (c)

Figure 4.11: Inter-storey drift of the total displacement of the column

Reviewing the results above, the inter-storey drift of the two models is below 2% at each storey. This means that buildings should resist the torsional effect.

4.2. Discussion and conclusions

As the second generation of Eurocode 8 is being developed at the moment, this master thesis aims to make a humble contribution to its development. This section contains the main conclusions as well as a discussion of the results obtained.

4.2.1. Conditions for the classification

In the second generation of the Eurocode, two conditions are introduced.

1. A building should be considered as torsionally flexible if in each main horizontal direction, the greatest effective modal mass is not that of the first or second mode, possible local modes and equipment modes being not considered in this classification.

   Note : this condition corresponds to the fact that the first mode (in at least one horizontal direction) is substantially influenced by torsion.

2. Buildings verifying the Formula (4.4) at every storey i, with the possible exception of the highest storey, may be considered as not being torsionally flexible.

$$r_i \geq l_{s,i} \tag{4.4}$$

Based on the results, for some models, the two conditions do not give the same conclusion about the classification. In these cases, the second condition is more conservative than the first one.

As explained previously, the building can be classified as torsionally rigid if the torsional radius is larger than the radius of gyration. However, the opposite conclusion is not true. If $r_i < l_{s,i}$, a modal analysis should be realised to verify the first condition. Therefore, it may be interesting to add a note for the second condition, stating that $r_i < l_{s,i}$ does not mean that the building is torsionally flexible.

In case the two conditions do not give the same results on classification, the first condition should be preferred, because the second condition tends to be more conservative. Therefore, it may be worth adding a note specifying that a building with $r_i < l_{s,i}$ can not be classified as torsionally flexible.

However, if the goal of the classification is to be conservative, the second condition should be preferred to the first one.

4.2.2. Comparison between the 3D procedure and the simplified method

The second generation of the Eurocode 8 introduces two procedures to calculate the natural eccentricity and the torsional radius. These two methods are detailed in Annex B of the second generation. The formulas for the simplified method and the 3D procedure are presented again below.

Simplified method	3D procedure
$x_{CS,i} = \frac{\Sigma_j x_j K_{x,ij}}{\Sigma_j K_{x,ij}}$	$e_{0x,i} = \frac{\Delta\theta_{y,i}}{\Delta\theta_{z,i}}$
$r_{x,i} = \sqrt{\frac{\sum_j K_{x,ij}(y_j - y_{C,i})^2 + \sum_k K_{y,ik}(x_k - x_{C,i})^2}{\sum_k K_{y,ik}}}$	$r_{xCM,i} = \sqrt{\frac{\Delta d_{yCM,i}}{\Delta\theta_{z,i}}}$

Where the lateral stiffness used in the simplified method is one of the vertical elements only.

In multi-storey buildings, the natural eccentricity calculated with the simplified method and with the 3D procedure is quite different. With the simplified method, the position of the centre of stiffness is constant with the height. While with the 3D procedure, the centre of stiffness approaches the centre of mass with the height. This is explained by the contribution of the storey shear of the wall which decreases with height.

The values obtained with the simplified method are on the safe side compared to the 3D procedure. The simplified method only gives an approximation, since it does not take into account the stiffness of the horizontal structural elements (such as the slab and the beams).

> The simplified method can be a good approximation for an early design to estimate the torsional effect quickly. However, it does not take into account the horizontal structural elements, as well as the change of the contribution of the wall with the height. Therefore, the 3D procedure will give more precise results.

4.2.3. Reference point of the torsional radius

The second condition in the second generation of the Eurocode 8 compares the torsional radius to the radius of gyration. It says that buildings verifying the Formula (4.5) at every storey i, with the possible exception of the highest storey, may be considered as not being torsionally flexible.

$$r_i \geq l_{s,i} \tag{4.5}$$

Where,

$l_{s,i}$ is the radius of gyration of the i-th storey, defined as the square root of the ratio of the polar moment of inertia in plan with respect to the center of mass of the floor (J) to the total overlying mass (m)

The radius of gyration is calculated with respect to the centre of mass, while the reference point for the calculation of the torsional radius in the second generation of the Eurocode 8 is unclear. The simplified procedure computes the torsional radius with respect to the centre of stiffness, while in the 3D procedure, the torsional radius is calculated with respect to the centre of mass.

In order for the second condition to be physically correct, since the radius of gyration is calculated with respect to the centre of mass, the torsional radius should also be calculated with respect to the centre of mass. Therefore, according to Fardis [4] and Penelis [11], the torsional radius calculated with the simplified method could be expressed as follow.

$$r_{xCM,i} = \sqrt{r_{xCS,i}^2 + e_{x0,i}^2} \qquad\qquad r_{yCM,i} = \sqrt{r_{yCS,i}^2 + e_{y0,i}^2} \tag{4.6}$$

Based on the results obtained previously, the torsional radius computed with respect to the centre of stiffness is always smaller or equal to the torsional radius computed with respect to the centre of mass, since the displacements of the centre of stiffness are always smaller or equal to the displacements of the centre of mass.

The difference between the two torsional radius increases with the natural eccentricity. The larger the natural eccentricity, the larger the difference between the torsional radius calculated with respect to the centre of stiffness and with respect to the centre of mass. For asymmetrical multi-storey buildings, since the natural eccentricity decreases with height, the difference will be larger at the base than at the top.

With these three main observations, we can conclude :

If the goal of the Eurocode 8 is to be conservative, the torsional radius should be calculated with respect to the centre of stiffness.

However, if the goal of the Eurocode 8 is to be closer to reality, the torsional radius could be calculated with respect to the centre of mass. Therefore, the simplified procedure could be changed to compute the torsional radius with respect to the centre of mass.

CHAPTER 5

Conclusions and Further Developments

The second generation of the Eurocode 8 is currently under preparation. The aim of this master thesis is to provide consistent work to help its development. The focus lies on the effects of the torsional response after an earthquake. Unlike its first generation, the second generation dedicates a whole section to the classification of torsionally flexible buildings. This master thesis is the continuation of the master thesis of Maxence Carlier, which focused on single-storey buildings. The aim is to extend the subject to multi-storey buildings and compare the conclusions with those of single-storey buildings. Three main conclusions can be drawn.

The first element is the comparison between **the two conditions for the classification of torsionally flexible buildings**. In most cases, the two conditions give the same results. However, for some models with large natural eccentricity, the two conditions do not give the same classification. For these cases, the second condition using the torsional radius is always conservative. However, this inconstancy is not a problem for the classification. Indeed, for buildings with a torsional radius lower than the radius of gyration, modal analysis is necessary to classify a building as torsional flexible. We can not automatically classify a building as torsional flexible if the torsional radius is lower than the radius of gyration.

This observation is the opposite of the one in the master thesis of Maxence Carlier. For his models, the first condition using the modal analysis was always conservative. Therefore, we can not generalise that one condition is more conservative than the other.

The second element analysed is **the two procedures to calculate the torsional radius** suggested in Annex B of the second generation of the Eurocode 8. A comparison between the two shows that the simplified method is always on the safe side compared to the 3D procedure, which gives more precise results. This was expected since the simplified method does not take into account the horizontal structural elements. Moreover, for multi-storey buildings, the simplified procedure does not consider the contribution of the walls for the total storey shear either, which decreases with the height. This has a more important impact on asymmetrical buildings. However, the simplified procedure can be a good approximation in the early design of buildings. As expected, this observation was the same for one-storey buildings.

The third element is a discussion about the **reference point for the calculation of the torsional radius**. The difference can be significant as noticed in section 3.2.3, the torsional radius computed with respect to the centre of stiffness is lower than the radius of gyration, while when it is computed with respect to the centre of mass it was larger. Since the torsional radius calculated with respect to the centre of stiffness is always smaller or equal to the one computed with respect to the centre of mass, using $r_{i,CS}$ can provide additional safety. However, based on the theory of Penelis [11] and Fardis [4], the torsional radius could be computed with respect to the centre of mass if the goal of the Eurocode is to be as close as possible to reality.

These observations about the reference point for the calculation of the torsional radius are the same as those found in the master thesis of Maxence Carlier for single-storey buildings. Therefore, the choice will depend on the philosophy of the CEN.

The second part of the results in this master thesis focuses on the **response spectrum analysis** of the models. The classification of a building as torsionally flexible will have consequences on its design. Indeed, the behaviour factor should be multiplied by 0,8 in this case. This will increase the design spectrum response and therefore the internal forces in the structural elements of the building as a consequence.

The effects of the accidental torsion have also been analysed. In the second generation of the Eurocode 8, the accidental eccentricity should be applied only for buildings whose natural eccentricity is smaller than the accidental one. The first generation of the Eurocode 8 suggests two methods to calculate accidental torsion. The method multiplying the effect of the seismic action by the factor δ is more conservative than the one applying a static moment around the vertical axis at the centre of mass.

Finally, in the second generation of the Eurocode 8, the natural eccentricity is not limited. However, it has an impact on the limitation of the inter-storey drift. All models in this master thesis have an inter-storey drift below the limit of 2% at the significant damage limit state. Therefore, it would be interesting to analyse more models with a larger area or with a bigger natural eccentricity.

Future Developments

Since we focused on multi-storey buildings which are regular in elevation, further analysis can be done for buildings non-regular in elevation. It would be interesting to calculate the position of the centre of stiffness and the torsional radius with the two methods suggested in Annex B of the Eurocode 8. Depending on the conclusion, the methods could be extended to buildings non-regular in elevation. Furthermore, the conclusions shown in this master thesis are drawn from reinforced concrete buildings. Therefore, additional analysis would be interesting to extend the findings to other materials.

The numerical analysis has been done with the lateral force method and/or the response spectrum analysis. These methods of analysis are linear. To obtain results closer to reality, a non-linear analysis could be performed. This would provide more accurate results for the effects of accidental eccentricity, and the consequences for the design of a torsionally flexible building.

APPENDIX A

Ground type in Eurocode 8

Ground type	Description of stratigraphic profile	Parameters		
		$v_{s,30}$ (m/s)	N_{SPT} (blows/30cm)	c_u (kPa)
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface.	> 800	–	–
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of metres in thickness, characterised by a gradual increase of mechanical properties with depth.	360 – 800	> 50	> 250
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of metres.	180 – 360	15 - 50	70 - 250
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil.	< 180	< 15	< 70
E	A soil profile consisting of a surface alluvium layer with v_s values of type C or D and thickness varying between about 5 m and 20 m, underlain by stiffer material with v_s > 800 m/s.			
S_1	Deposits consisting, or containing a layer at least 10 m thick, of soft clays/silts with a high plasticity index (PI > 40) and high water content	< 100 (indicative)	–	10 - 20
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A – E or S_1			

Figure A.1: Ground Type defined in the Eurocode 8 [13]

APPENDIX B
Classification of torsionally flexible buildings

The results shown in this annex have been calculated with the method described in the second generation of the Eurocode 8. The results obtained with the method described in the Scia help centre re presented in the next annex.

B.1. Natural eccentricity

Rectangular walls

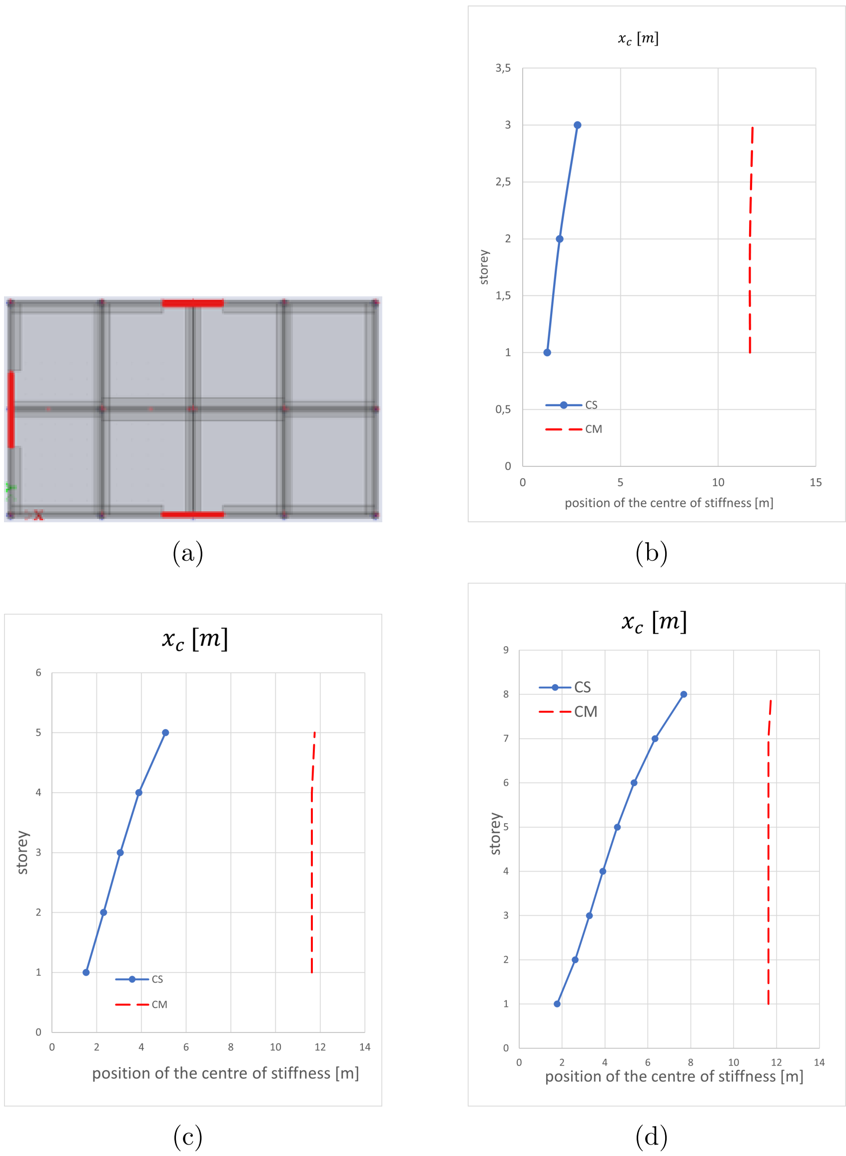
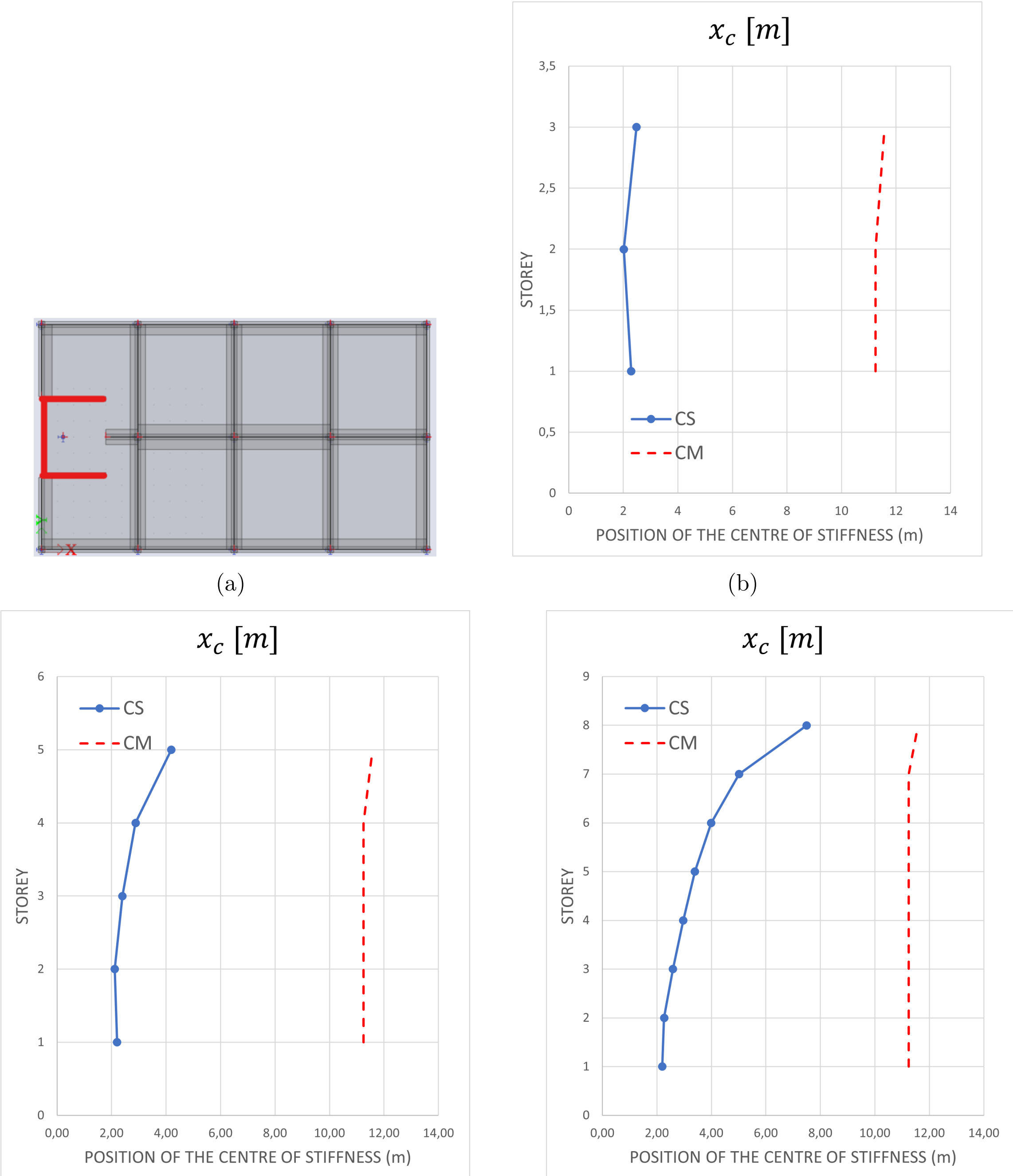


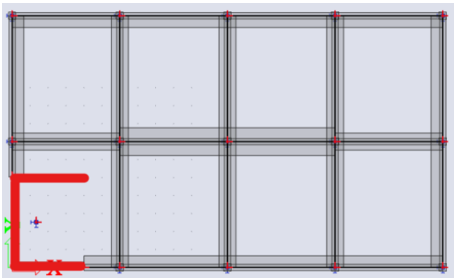
Figure B.1: Position of the centre of stiffness for the model with asymmetry in x direction

U-shape walls

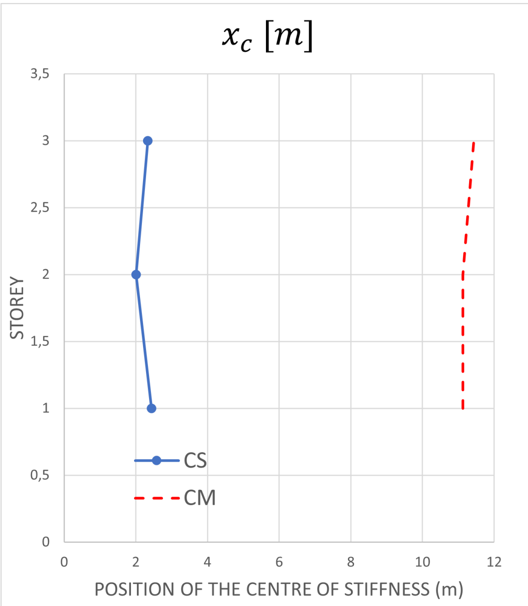


(a) (b) (c) (d)

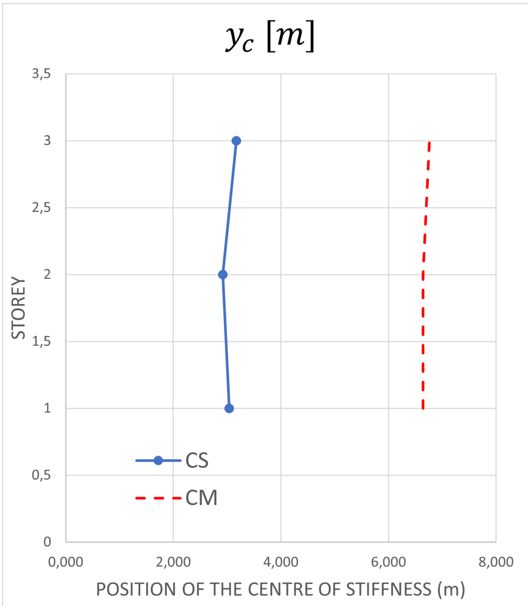
Figure B.2: Position of the centre of stiffness for the model with asymmetry in x direction



(a)



(b)



(c)

Figure B.3: Position of the centre of stiffness for 3 storey buildings

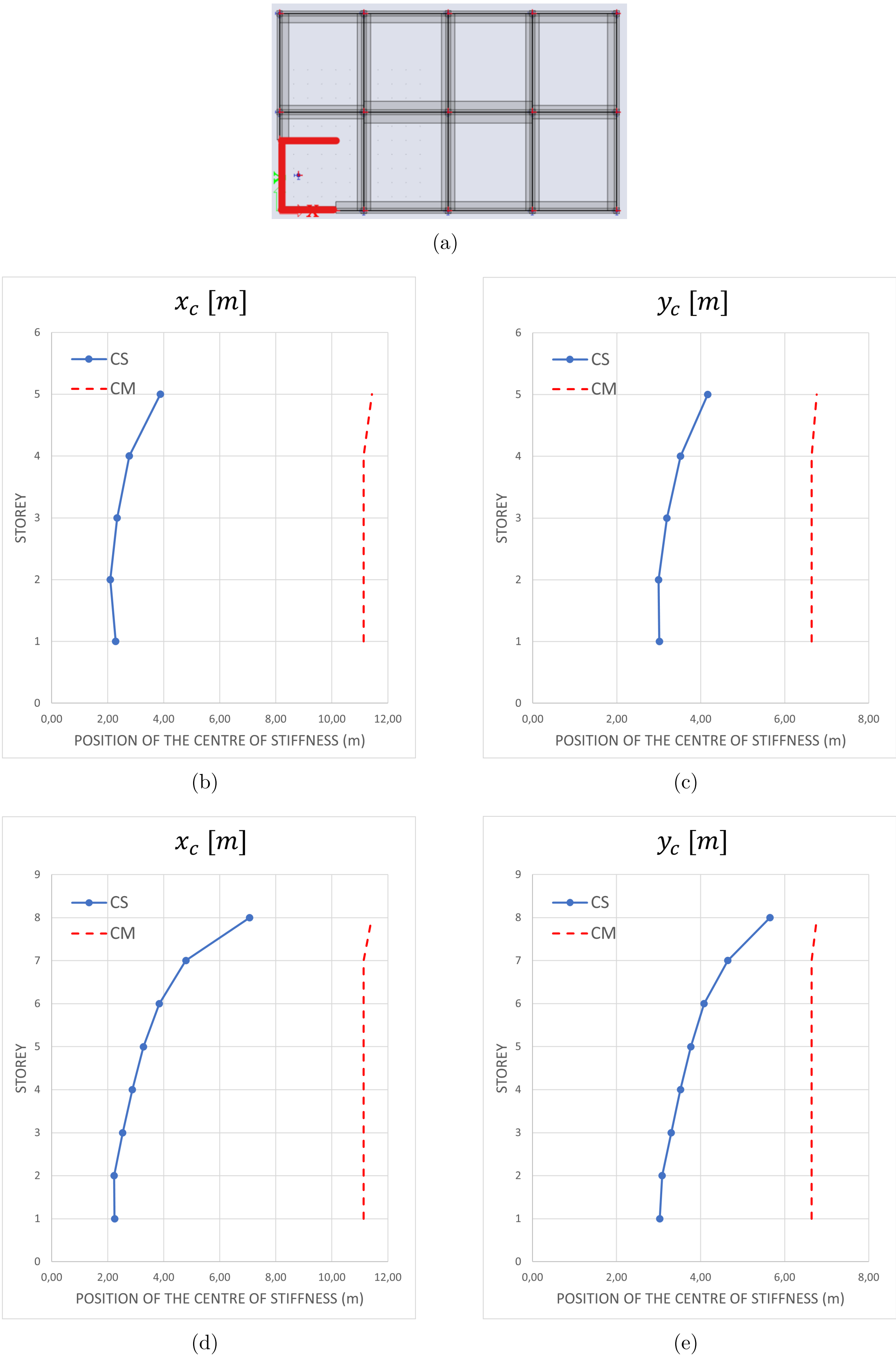
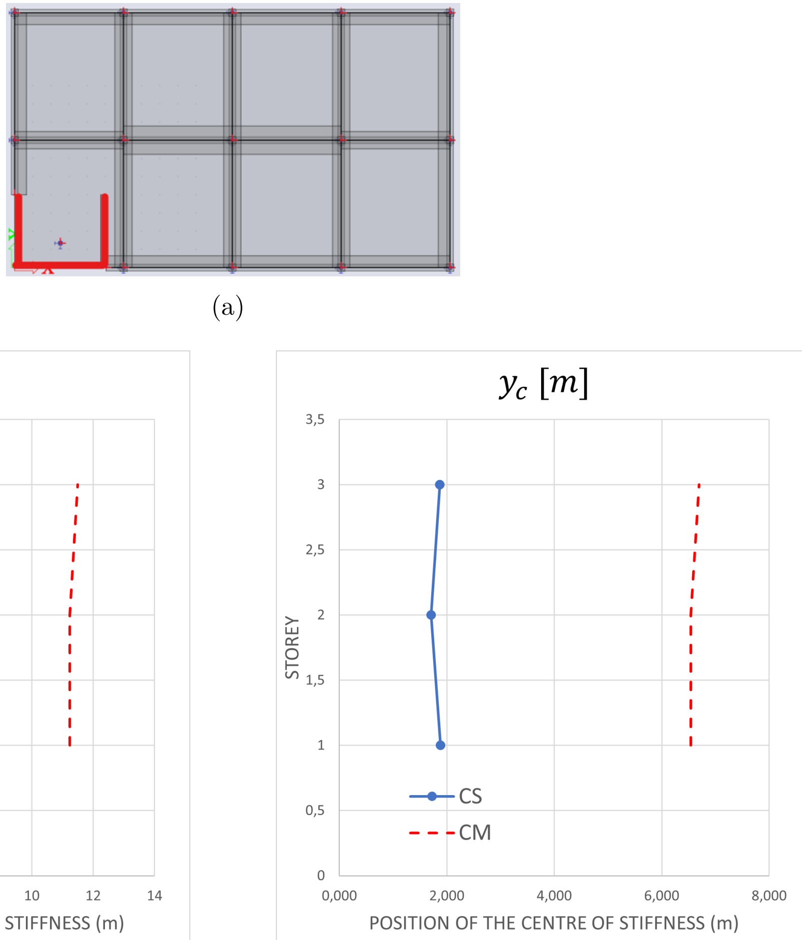


Figure B.4: Position of the centre of stiffness for 5 and 8 storey buildings

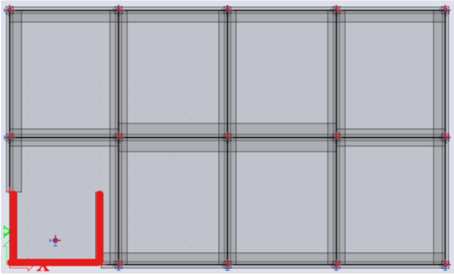


(a)

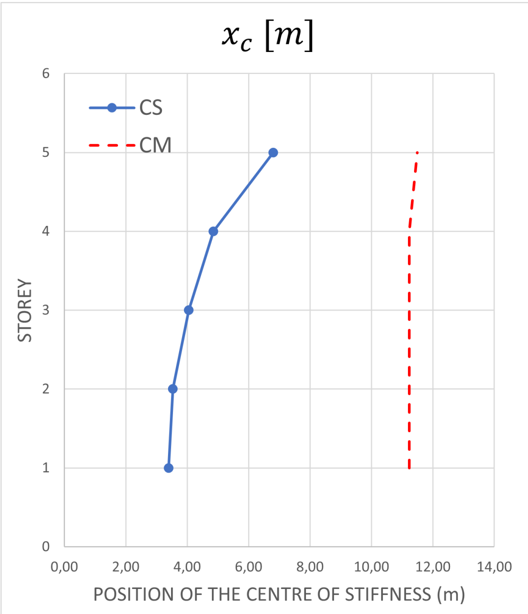
(b)

(c)

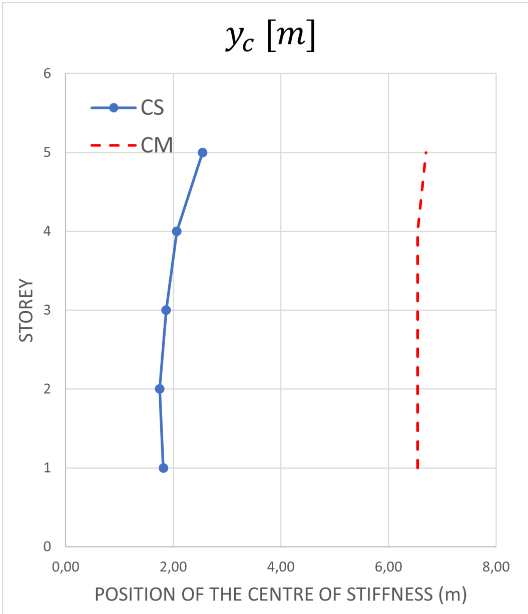
Figure B.5: Position of the centre of stiffness for 3 storey buildings



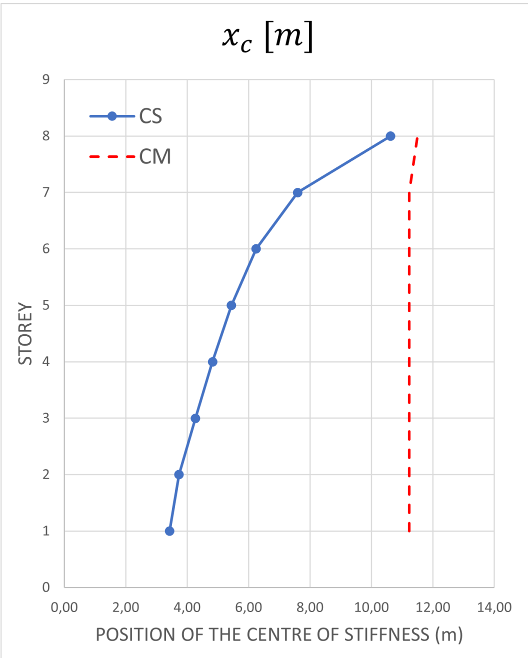
(a)



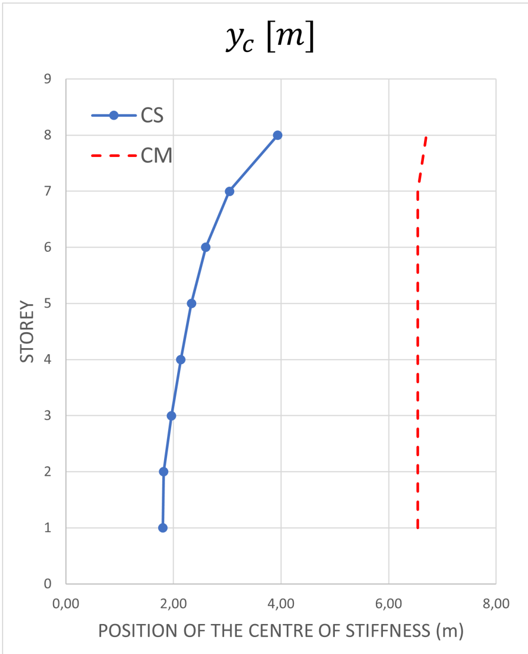
(b)



(c)



(d)

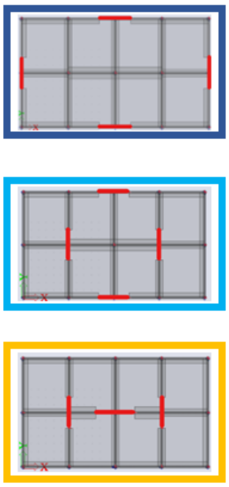


(e)

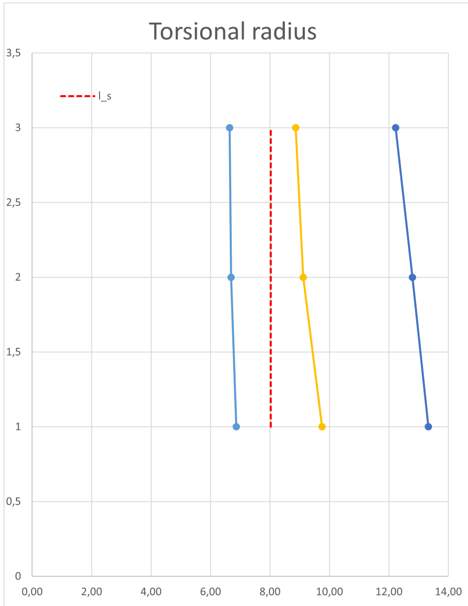
Figure B.6: Position of the centre of stiffness for 5 and 8 storey buildings

B.2. Torsional radius

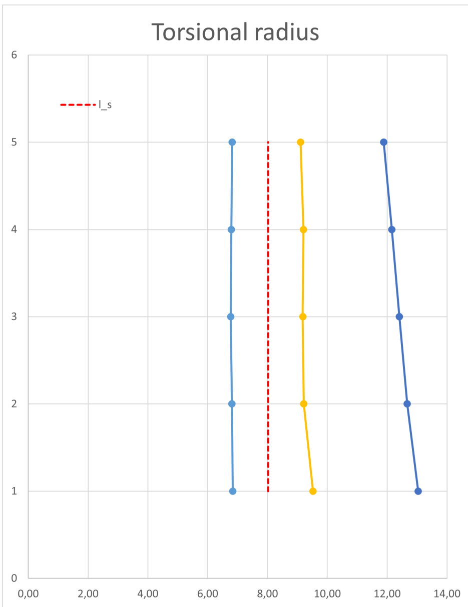
Rectangular walls - Symmetrical buildings



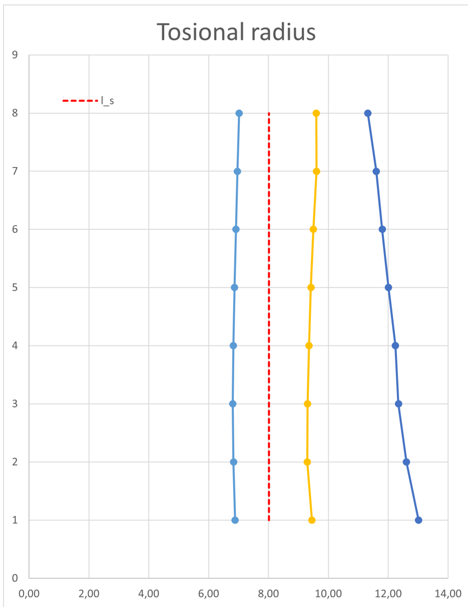
(a)



(b)



(c)



(d)

Figure B.7: Torsional radius for symmetrical buildings at 3-, 5-, 8-storeys

Rectangular walls - Asymmetrical buildings

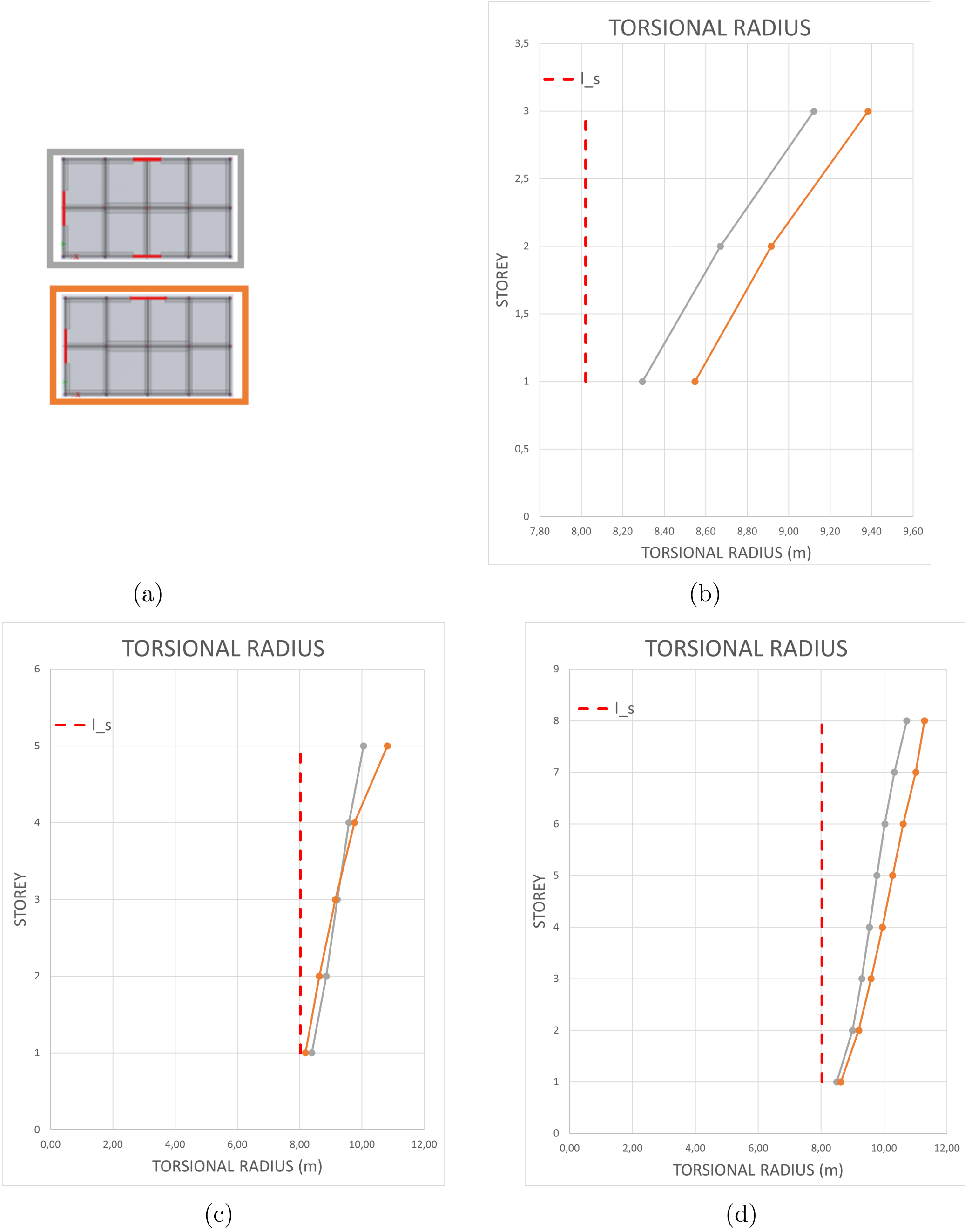
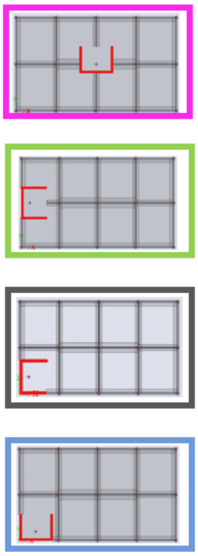
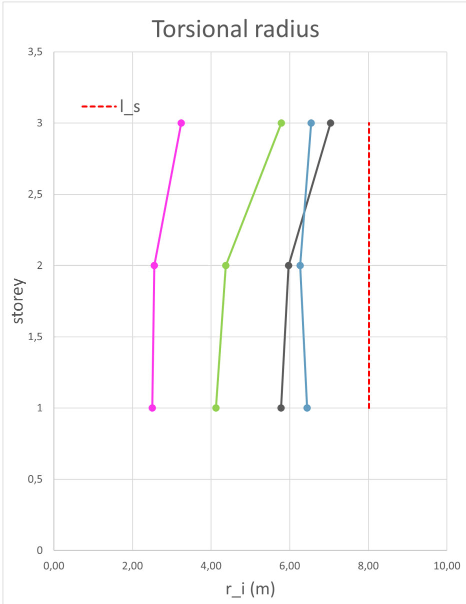


Figure B.8: Torsional radius for asymmetrical buildings at 3-, 5-, 8-storeys

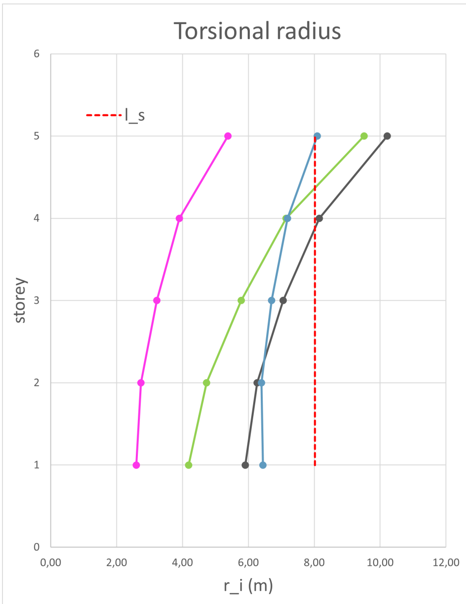
U-shape walls



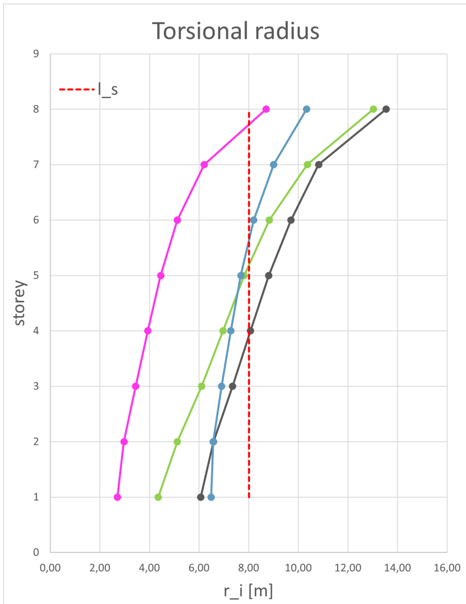
(a)



(b)



(c)

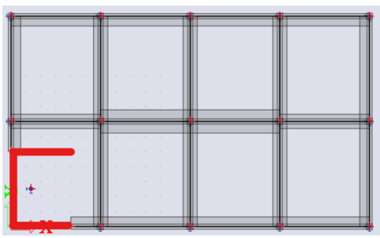


(d)

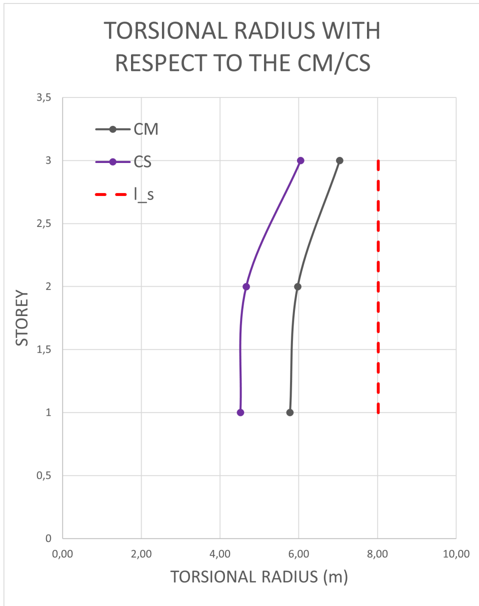
Figure B.9: Torsional radius for buildings with U-shape walls at 3-, 5-, 8-storeys

B.3. Comparison reference point for the calculation of the torsional radius

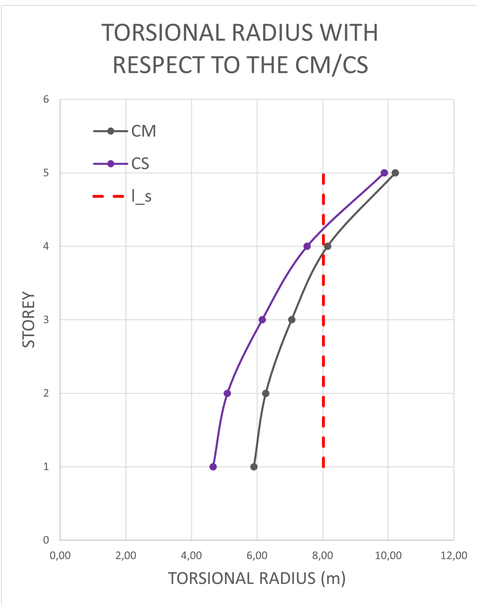
The torsional radius shown on the graph are $r_{y,i}$. Therefore, for the models with no static eccentricity in y direction, the torsional radius stay unchanged. Only the models with eccentricity in y direction will be calculated.



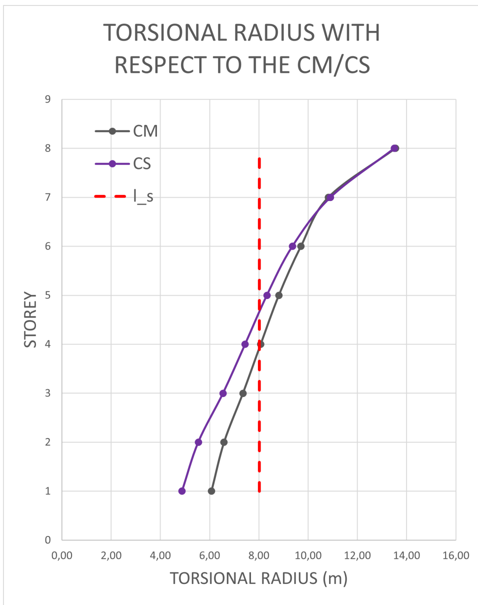
(a)



(b)



(c)



(d)

Figure B.10: Torsional radius computed with respect to the centre of mass and withe respect to the centre of stiffness

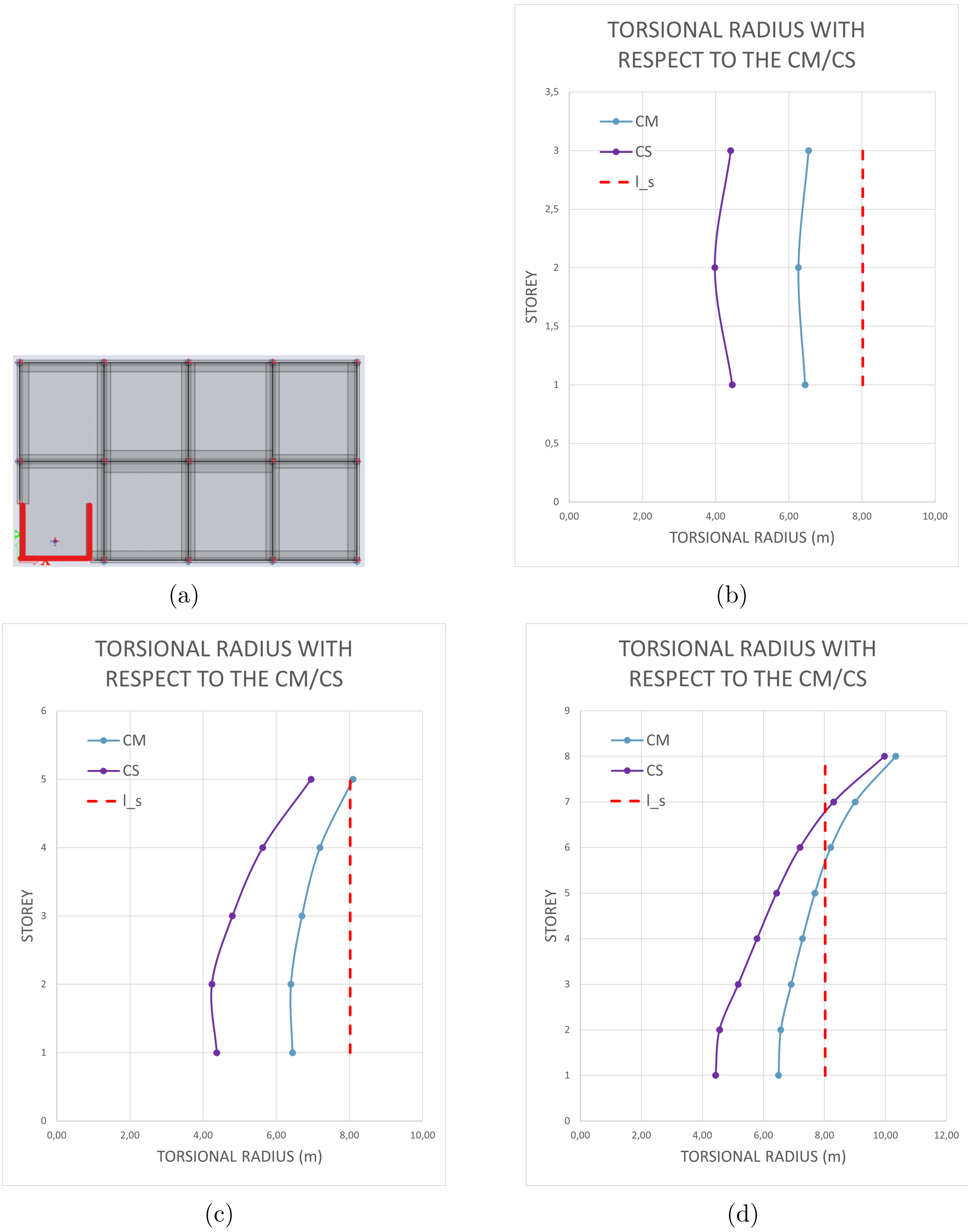
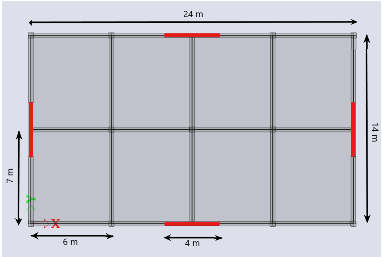


Figure B.11: Torsional radius computed with respect to the centre of mass and withe respect to the centre of stiffness

B.4. Effective modal mass

Symmetrical buildings



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	61.8653	0,10	9,85	0,0000	0,0000	0,0000	0,4124	0,0000	0,0000
2	65.8679	0,10	10,48	0,0003	0,0000	0,0000	0,0000	0,4778	0,0000
3	68.086	0,09	10,84	0,0000	0,0000	0,5690	0,0000	0,0000	0,0000
4	103.394	0,06	16,46	0,0000	0,9487	0,0000	0,0001	0,0000	0,0000
5	116.046	0,05	18,47	0,9776	0,0000	0,0000	0,0000	0,0002	0,0000
6	186.345	0,03	29,66	0,0000	0,0000	0,0000	0,0000	0,0000	0,9336

(b) 1-storey buildings

Relative modal masses 3-storey

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	30.2855	0,21	4,82	0,0000	0,7537	0,0000	0,0610	0,0000	0,0000
2	31.4164	0,20	5,00	0,7561	0,0000	0,0000	0,0000	0,0201	0,0000
3	47.2082	0,13	7,51	0,0000	0,0000	0,0000	0,0000	0,0000	0,7291

(c) 3-storey buildings

Relative modal masses 5-storey

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	15.1592	0,41	2,41	0,0000	0,7232	0,0000	0,1351	0,0000	0,0000
2	15.7974	0,40	2,51	0,7278	0,0000	0,0000	0,0000	0,0659	0,0000
3	23.0618	0,27	3,67	0,0000	0,0000	0,0000	0,0000	0,0000	0,7088

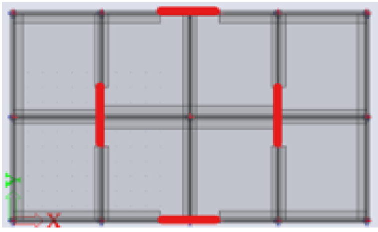
(d) 5-storey buildings

Relative modal masses 8-storey

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	8.14051	0,77	1,30	0,0000	0,7210	0,0000	0,2013	0,0000	0,0000
2	8.55361	0,73	1,36	0,7266	0,0000	0,0000	0,0000	0,1287	0,0000
3	12.2217	0,51	1,95	0,0000	0,0000	0,0000	0,0000	0,0000	0,7104

(e) 8-storey buildings

Figure B.12: Effective modal mass



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	80.5952	0,08	12,83	0,0000	0,0048	0,0000	0,4572	0,0000	0,0000
2	81.6498	0,08	12,99	0,0000	0,0000	0,0000	0,0000	0,5990	0,0000
3	86.9772	0,07	13,84	0,0000	0,0000	0,6032	0,0000	0,0000	0,0000
4	118.265	0,05	18,82	0,9834	0,0000	0,0000	0,0000	0,0001	0,0000
5	121.137	0,05	19,28	0,0000	0,9581	0,0000	0,0011	0,0000	0,0000
6	129.215	0,05	20,57	0,0000	0,0000	0,0000	0,0000	0,0000	0,9679

(b) 1-storey buildings

Relative modal masses 3-storey

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	31.9925	0,20	5,09	0,0000	0,7492	0,0000	0,0846	0,0000	0,0000
2	32.3631	0,19	5,15	0,7605	0,0000	0,0000	0,0000	0,0290	0,0000
3	35.5693	0,18	5,66	0,0000	0,0000	0,0000	0,0000	0,0000	0,7569

(c) 3-storey buildings

Relative modal masses 5-storey

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	16.3018	0,39	2,59	0,0000	0,7296	0,0000	0,1479	0,0000	0,0000
2	16.3786	0,38	2,61	0,7317	0,0000	0,0000	0,0000	0,0763	0,0000
3	18.2635	0,34	2,91	0,0000	0,0000	0,0000	0,0000	0,0000	0,7334

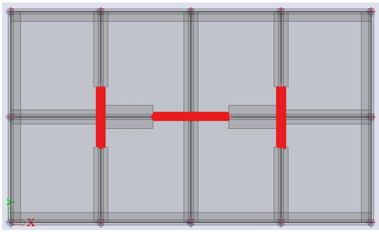
(d) 5-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	8.8962	0,71	1,42	0,0000	0,7305	0,0000	0,2036	0,0000	0,0000
2	8.91208	0,71	1,42	0,7303	0,0000	0,0000	0,0000	0,1390	0,0000
3	10.1348	0,62	1,61	0,0000	0,0000	0,0000	0,0000	0,0000	0,7371

(e) 8-storey buildings

Figure B.13: Effective modal mass



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	83.334	0,08	13,26	0,0000	0,0146	0,0000	0,5025	0,0000	0,0000
2	85.0931	0,07	13,54	0,0000	0,0000	0,0000	0,0000	0,5845	0,0000
3	89.494	0,07	14,24	0,0000	0,0000	0,0000	0,0000	0,0000	0,9738
4	89.5129	0,07	14,25	0,0000	0,0000	0,5858	0,0000	0,0000	0,0000
5	108.275	0,06	17,23	0,9880	0,0000	0,0000	0,0000	0,0001	0,0000
6	122.754	0,05	19,54	0,0000	0,9515	0,0000	0,0053	0,0000	0,0000

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	26.4002	0,24	4,20	0,0000	0,0000	0,0000	0,0000	0,0000	0,7782
2	32.5937	0,19	5,19	0,0000	0,7468	0,0000	0,1041	0,0000	0,0000
3	32.9272	0,19	5,24	0,7890	0,0000	0,0000	0,0000	0,0244	0,0000

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	14.1061	0,45	2,25	0,0000	0,0000	0,0000	0,0000	0,0000	0,7546
2	16.7229	0,38	2,66	0,0000	0,7301	0,0000	0,1646	0,0000	0,0000
3	17.4488	0,36	2,78	0,7568	0,0000	0,0000	0,0000	0,0677	0,0000

(d) 5-storey buildings

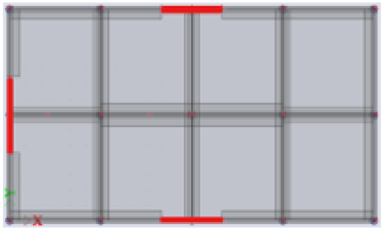
Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	7.96671	0,79	1,27	0,0000	0,0000	0,0000	0,0000	0,0000	0,7531
2	9.09066	0,69	1,45	0,0000	0,7294	0,0000	0,2176	0,0000	0,0000
3	9.78976	0,64	1,56	0,7519	0,0000	0,0000	0,0000	0,1265	0,0000

(e) 8-storey buildings

Figure B.14: Effective modal mass

Asymmetrical buildings



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	54.8892	0,11	8,74	0,0000	0,7693	0,0000	0,0301	0,0000	0,1539
2	60.9308	0,10	9,70	0,0000	0,0580	0,0000	0,3829	0,0000	0,0104
3	63.749	0,10	10,15	0,0001	0,0000	0,0832	0,0000	0,4610	0,0000
4	67.6006	0,09	10,76	0,0001	0,0000	0,4994	0,0000	0,0585	0,0000
5	116.511	0,05	18,54	0,9786	0,0000	0,0000	0,0000	0,0002	0,0000
6	155.288	0,04	24,71	0,0000	0,1487	0,0000	0,0000	0,0000	0,7832

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	18.3297	0,34	2,92	0,0000	0,6233	0,0000	0,0284	0,0000	0,1828
2	31.4553	0,20	5,01	0,7555	0,0000	0,0001	0,0000	0,0227	0,0000
3	50.3357	0,12	8,01	0,0000	0,1790	0,0000	0,0324	0,0000	0,5455

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	10.0657	0,62	1,60	0,0000	0,6041	0,0000	0,0789	0,0000	0,1711
2	15.7912	0,40	2,51	0,7269	0,0000	0,0000	0,0000	0,0700	0,0000
3	24.5022	0,26	3,90	0,0000	0,1479	0,0000	0,0396	0,0000	0,5603

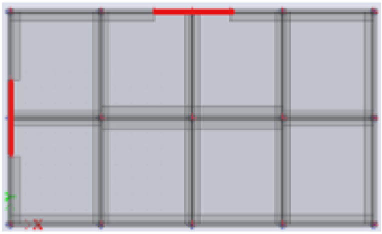
(d) 5-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_F}	W_{yi_R}/W_{ytot_F}	W_{zi_R}/W_{ztot_F}
1	5.80718	1,08	0,92	0,0000	0,6100	0,0000	0,1272	0,0000	0,1602
2	8.53532	0,74	1,36	0,7254	0,0000	0,0000	0,0000	0,1340	0,0000
3	12.7652	0,49	2,03	0,0000	0,1333	0,0000	0,0570	0,0000	0,5702

(e) 8-storey buildings

Figure B.15: Effective modal mass



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_R	Wyi_R/Wytot_R	Wzi_R/Wztot_R
1	32.5171	0,19	5,18	0,1812	0,5385	0,0000	0,0017	0,0002	0,2751
2	60.0817	0,10	9,56	0,0010	0,0013	0,0082	0,4307	0,0008	0,0007
3	63.2626	0,10	10,07	0,0000	0,0004	0,1495	0,0015	0,4064	0,0001
4	68.2444	0,09	10,86	0,0006	0,0002	0,4360	0,0007	0,1083	0,0000
5	95.8311	0,07	15,25	0,7023	0,2735	0,0001	0,0000	0,0002	0,0027
6	173.036	0,04	27,54	0,1007	0,1557	0,0000	0,0001	0,0000	0,6612

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_R	Wyi_R/Wytot_R	Wzi_R/Wztot_R
1	12.0219	0,52	1,91	0,1583	0,4781	0,0000	0,0124	0,0017	0,2443
2	26.5381	0,24	4,22	0,5630	0,1898	0,0001	0,0128	0,0143	0,0002
3	35.1002	0,18	5,59	0,0135	0,0493	0,0000	0,0866	0,0120	0,0337
4	45.4884	0,14	7,24	0,0565	0,1458	0,0000	0,0014	0,0006	0,4954

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_R	Wyi_R/Wytot_R	Wzi_R/Wztot_R
1	7,23868	0,87	1,15	0,1529	0,4651	0,0000	0,0361	0,0059	0,2333
2	15,0667	0,42	2,40	0,5405	0,1907	0,0001	0,0361	0,0476	0,0008
3	21,7483	0,29	3,46	0,0108	0,0411	0,0000	0,1849	0,0326	0,0495
4	25,0421	0,25	3,99	0,0669	0,1458	0,0000	0,0204	0,0047	0,4923

(d) 5-storey buildings

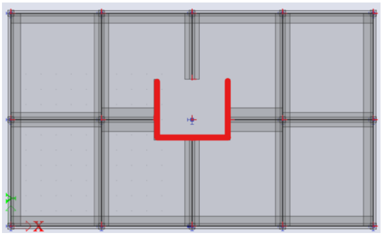
Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_R	Wyi_R/Wytot_R	Wzi_R/Wztot_R
1	4.47573	1,40	0,71	0,1454	0,4799	0,0000	0,0689	0,0128	0,2066
2	6.44947	0,97	1,03	0,5318	0,1667	0,0000	0,0544	0,1033	0,0006
3	9.0901	0,69	1,45	0,0422	0,1049	0,0000	0,0550	0,0139	0,5234

(e) 8-storey buildings

Figure B.16: Effective modal mass

B.4.1. U-shape walls



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_F	Wyi_R/Wytot_F	Wzi_R/Wztot_F
1	36.6515	0,17	5,83	0,0000	0,0000	0,0000	0,0000	0,0000	0,9989
2	74.226	0,08	11,81	0,0000	0,0000	0,0002	0,4881	0,0000	0,0000
3	77.4776	0,08	12,33	0,0000	0,0000	0,0058	0,0000	0,6275	0,0000
4	82.0006	0,08	13,05	0,0000	0,0000	0,5910	0,0000	0,0058	0,0000
5	121.903	0,05	19,40	0,0000	0,9472	0,0000	0,0004	0,0000	0,0000
6	135.68	0,05	21,59	0,9888	0,0000	0,0000	0,0000	0,0001	0,0000

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_F	Wyi_R/Wytot_F	Wzi_R/Wztot_F
1	13.9375	0,45	2,22	0,0000	0,0000	0,0000	0,0000	0,0000	0,8898
2	36.2384	0,17	5,77	0,0000	0,7577	0,0001	0,0731	0,0000	0,0000
3	40.1827	0,16	6,40	0,0000	0,0002	0,0000	0,0000	0,0000	0,0913
4	46.2461	0,14	7,36	0,8095	0,0000	0,0000	0,0000	0,0249	0,0000

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_F	Wyi_R/Wytot_F	Wzi_R/Wztot_F
1	8.45377	0,74	1,35	0,0000	0,0000	0,0000	0,0000	0,0000	0,8576
2	17.3139	0,36	2,76	0,0000	0,7125	0,0000	0,1559	0,0000	0,0000
3	22.6002	0,28	3,60	0,7354	0,0000	0,0000	0,0000	0,0819	0,0000

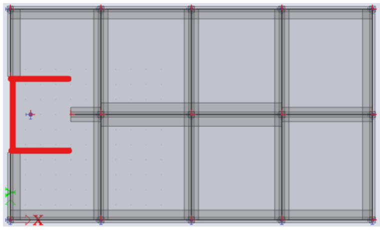
(d) 5-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	Wxi/Wxtot	Wyi/Wytot	Wzi/Wztot	Wxi_R/Wxtot_F	Wyi_R/Wytot_F	Wzi_R/Wztot_F
1	5.30385	1,18	0,84	0,0000	0,0000	0,0000	0,0000	0,0000	0,8390
2	8.86557	0,71	1,41	0,0000	0,6995	0,0000	0,2299	0,0000	0,0001
3	11.3072	0,56	1,80	0,7003	0,0000	0,0000	0,0000	0,1640	0,0000

(e) 8-storey buildings

Figure B.17: Effective modal mass



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	32.2191	0,20	5,13	0,0000	0,5970	0,0000	0,0006	0,0000	0,4013
2	72.4445	0,09	11,53	0,0000	0,0007	0,0000	0,4997	0,0000	0,0004
3	78.7953	0,08	12,54	0,0001	0,0000	0,2528	0,0000	0,3878	0,0000
4	86.9348	0,07	13,84	0,0009	0,0000	0,3852	0,0000	0,1472	0,0000
5	125.574	0,05	19,99	0,9640	0,0000	0,0002	0,0000	0,0002	0,0000
6	188.548	0,03	30,01	0,0000	0,3556	0,0000	0,0000	0,0000	0,5300

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	12,2598	0,51	1,95	0,0000	0,5187	0,0000	0,0144	0,0000	0,3716
2	35,3486	0,18	5,63	0,0000	0,0532	0,0000	0,1113	0,0000	0,0380
3	44,9974	0,14	7,16	0,7862	0,0000	0,0009	0,0000	0,0124	0,0000
4	52,8047	0,12	8,40	0,0000	0,0112	0,0000	0,0057	0,0000	0,0072
5	64,1432	0,10	10,21	0,0000	0,0000	0,0000	0,0041	0,0000	0,0001
6	65,6305	0,10	10,45	0,0000	0,0001	0,0000	0,3086	0,0000	0,0000
7	66,2448	0,09	10,54	0,0000	0,0000	0,3391	0,0000	0,2902	0,0000
8	69,7599	0,09	11,10	0,0000	0,0000	0,0158	0,0000	0,0214	0,0000
9	71,6993	0,09	11,41	0,0000	0,0007	0,0000	0,1541	0,0000	0,0008
10	73,705	0,09	11,73	0,0025	0,0000	0,3031	0,0000	0,1277	0,0000
11	76,4045	0,08	12,16	0,0003	0,0000	0,0001	0,0000	0,0680	0,0000
12	77,0193	0,08	12,26	0,0000	0,0000	0,0238	0,0000	0,0066	0,0000
13	80,3472	0,08	12,79	0,0000	0,3492	0,0000	0,0100	0,0000	0,4877

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	7.42922	0,85	1,18	0,0000	0,4995	0,0000	0,0403	0,0000	0,3580
2	21.281	0,30	3,39	0,7177	0,0000	0,0007	0,0000	0,0577	0,0000
3	22.1512	0,28	3,53	0,0000	0,0524	0,0000	0,2086	0,0000	0,0431
4	36.1786	0,17	5,76	0,0000	0,0224	0,0000	0,0171	0,0000	0,0097
5	40.4853	0,16	6,44	0,0000	0,3116	0,0000	0,0568	0,0000	0,4395

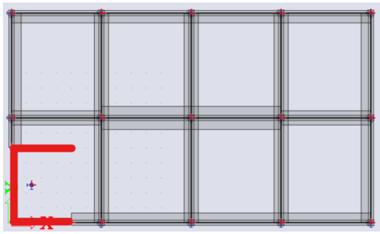
(d) 5-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	4.64788	1,35	0,74	0,0000	0,4915	0,0000	0,0688	0,0000	0,3462
2	10.3527	0,61	1,65	0,6881	0,0000	0,0006	0,0000	0,1325	0,0000
3	13.942	0,45	2,22	0,0000	0,0443	0,0000	0,2907	0,0000	0,0528
4	19.5528	0,32	3,11	0,0000	0,3014	0,0000	0,0783	0,0000	0,3947

(e) 8-storey buildings

Figure B.18: Effective modal mass



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	31.9342	0,20	5,08	0,0939	0,5304	0,0000	0,0005	0,0000	0,3742
2	67.4846	0,09	10,74	0,0003	0,0002	0,0118	0,3633	0,0605	0,0001
3	74.9509	0,08	11,93	0,0000	0,0002	0,0174	0,1016	0,3916	0,0001
4	80.3028	0,08	12,78	0,0000	0,0000	0,5840	0,0115	0,0717	0,0000
5	116.366	0,05	18,52	0,8484	0,0994	0,0000	0,0000	0,0000	0,0074
6	174.21	0,04	27,73	0,0092	0,3280	0,0000	0,0000	0,0000	0,5350

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	12.1724	0,52	1,94	0,0818	0,4617	0,0000	0,0121	0,0009	0,3473
2	35.0631	0,18	5,58	0,0081	0,0476	0,0000	0,0949	0,0069	0,0352
3	44.6175	0,14	7,10	0,7008	0,0907	0,0001	0,0032	0,0134	0,0033
4	52.3055	0,12	8,32	0,0017	0,0098	0,0000	0,0047	0,0004	0,0066
5	61.5549	0,10	9,80	0,0000	0,0000	0,0001	0,0006	0,0000	0,0001
6	62.767	0,10	9,99	0,0001	0,0002	0,0001	0,2374	0,0243	0,0001
7	65.4348	0,10	10,41	0,0000	0,0001	0,0224	0,0146	0,4136	0,0000
8	67.1261	0,09	10,68	0,0000	0,0000	0,0029	0,0209	0,0303	0,0000
9	67.4116	0,09	10,73	0,0000	0,0000	0,3675	0,0941	0,0040	0,0000
10	67.5548	0,09	10,75	0,0000	0,0006	0,2432	0,0506	0,0084	0,0006
11	71.6697	0,09	11,41	0,0000	0,0000	0,0349	0,0006	0,0014	0,0000
12	73.163	0,09	11,64	0,0000	0,0001	0,0028	0,0160	0,0493	0,0001
13	77.5927	0,08	12,35	0,0000	0,0000	0,0290	0,0009	0,0033	0,0000
14	80.3604	0,08	12,79	0,0157	0,3158	0,0002	0,0075	0,0020	0,5023

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	7.37988	0,85	1,17	0,0791	0,4444	0,0000	0,0344	0,0030	0,3345
2	21.1076	0,30	3,36	0,6310	0,0837	0,0001	0,0093	0,0566	0,0048
3	21.9944	0,29	3,50	0,0140	0,0428	0,0000	0,1829	0,0138	0,0393
4	35.8929	0,18	5,71	0,0031	0,0190	0,0000	0,0150	0,0014	0,0096
5	41.1799	0,15	6,55	0,0144	0,2825	0,0000	0,0492	0,0021	0,4535

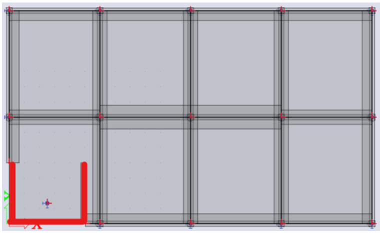
(d) 5-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	4.61834	1,36	0,74	0,0781	0,4367	0,0000	0,0597	0,0068	0,3232
2	10.1411	0,62	1,61	0,6044	0,0786	0,0001	0,0225	0,1217	0,0036
3	13.851	0,45	2,20	0,0085	0,0394	0,0000	0,2543	0,0274	0,0490
4	19.9012	0,32	3,17	0,0179	0,2669	0,0000	0,0694	0,0033	0,4097

(e) 8-storey buildings

Figure B.19: Effective modal mass



(a)

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	32.1106	0,20	5,11	0,1586	0,4459	0,0000	0,0004	0,0000	0,3940
2	67.5942	0,09	10,76	0,0003	0,0001	0,0114	0,3437	0,0652	0,0001
3	74.8402	0,08	11,91	0,0001	0,0001	0,0189	0,1032	0,3996	0,0001
4	80.2892	0,08	12,78	0,0000	0,0000	0,5830	0,0130	0,0718	0,0000
5	113.396	0,06	18,05	0,7751	0,1615	0,0000	0,0000	0,0000	0,0173
6	177.458	0,04	28,24	0,0167	0,3551	0,0000	0,0000	0,0000	0,5139

(b) 1-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	$W_{xi\_R}/W_{xtot\_}$	$W_{yi\_R}/W_{ytot\_}$	$W_{zi\_R}/W_{ztot\_}$
1	12,244	0,51	1,95	0,1373	0,3902	0,0000	0,0099	0,0015	0,3634
2	35,2571	0,18	5,61	0,0160	0,0366	0,0000	0,0781	0,0118	0,0383
3	48,1011	0,13	7,66	0,5270	0,2838	0,0000	0,0111	0,0085	0,0099

(c) 3-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	7.42317	0,85	1,18	0,1310	0,3787	0,0000	0,0288	0,0051	0,3482
2	22.0904	0,28	3,52	0,0430	0,0120	0,0000	0,1733	0,0190	0,0563
3	23.3145	0,27	3,71	0,3603	0,3302	0,0001	0,0273	0,0384	0,0245
4	35.6775	0,18	5,68	0,2572	0,1062	0,0000	0,0118	0,0145	0,3283

(d) 5-storey buildings

Relative modal masses

Mode	mega [rad/s	Period [s]	Freq. [Hz]	W_{xi}/W_{xtot}	W_{yi}/W_{ytot}	W_{zi}/W_{ztot}	W_{xi_R}/W_{xtot_R}	W_{yi_R}/W_{ytot_R}	W_{zi_R}/W_{ztot_R}
1	4.64429	1,35	0,74	0,1265	0,3782	0,0000	0,0519	0,0110	0,3332
2	11.1097	0,57	1,77	0,3558	0,2791	0,0001	0,1103	0,0598	0,0519
3	13.9332	0,45	2,22	0,0020	0,0448	0,0000	0,1963	0,0614	0,0552
4	17.2779	0,36	2,75	0,2536	0,0890	0,0000	0,0178	0,0417	0,3450

(e) 8-storey buildings

Figure B.20: Effective modal mass

APPENDIX C

Torsional radius - "Scia method"

C.1. Torsional radius - graphs

Rectangular walls - Symmetrical buildings

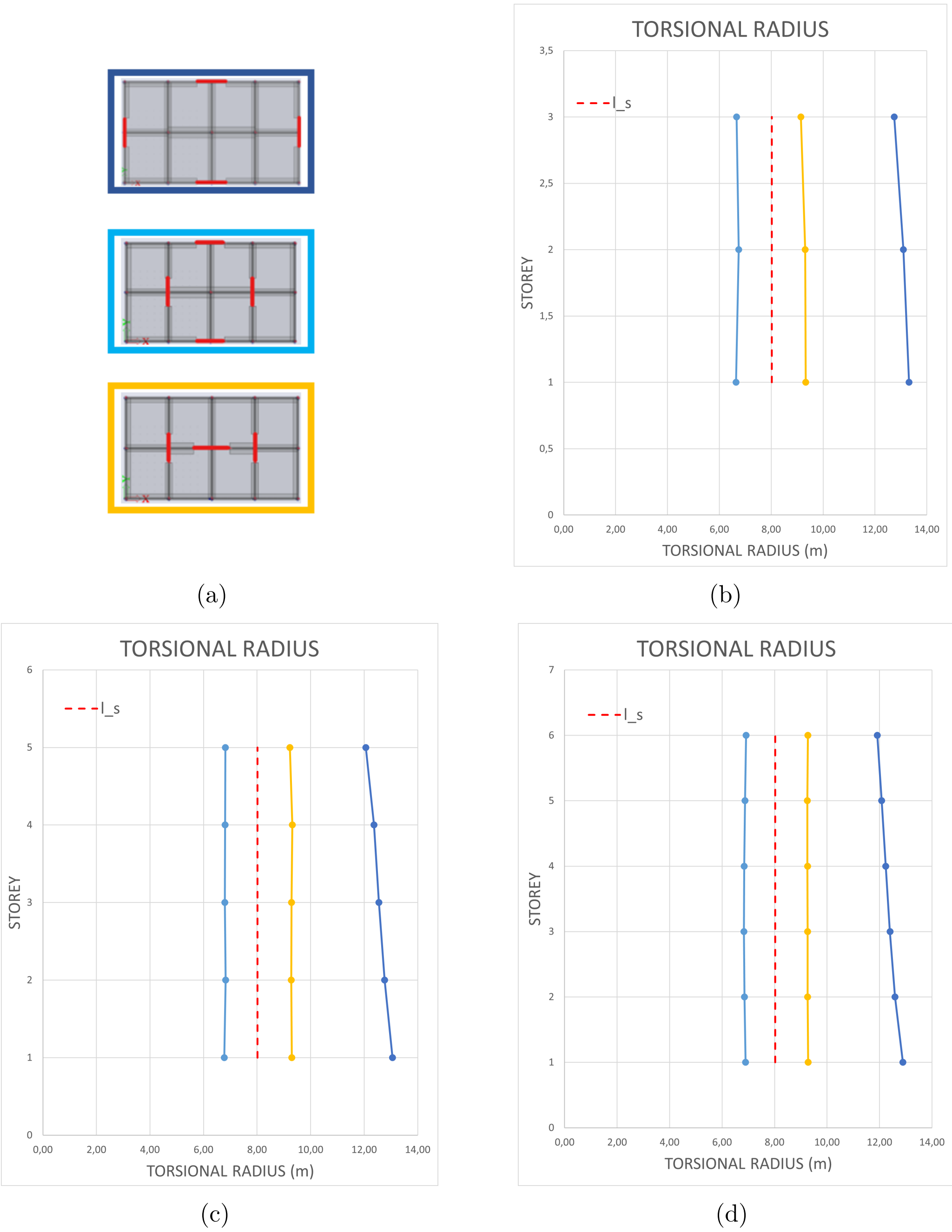
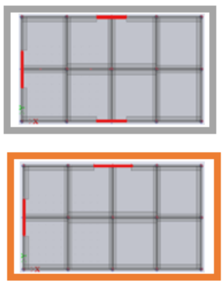
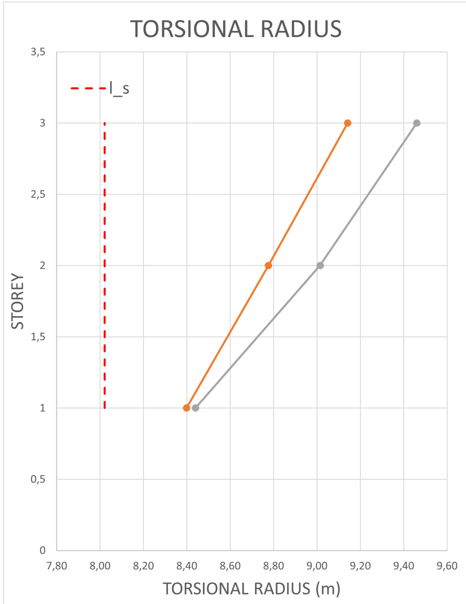


Figure C.1: Torsional radius for symmetrical buildings at 3-, 5-, 8-storeys

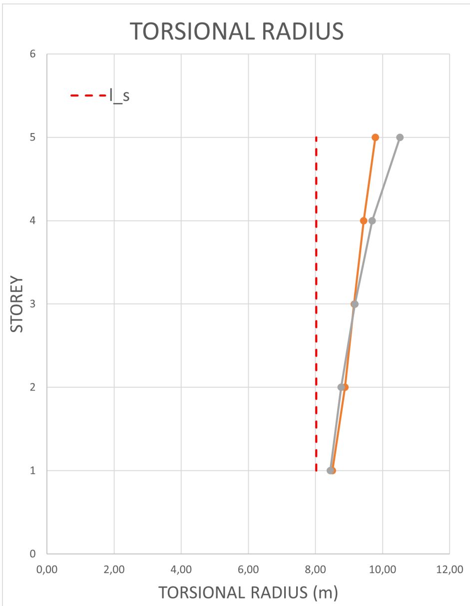
Rectangular walls - Asymmetrical buildings



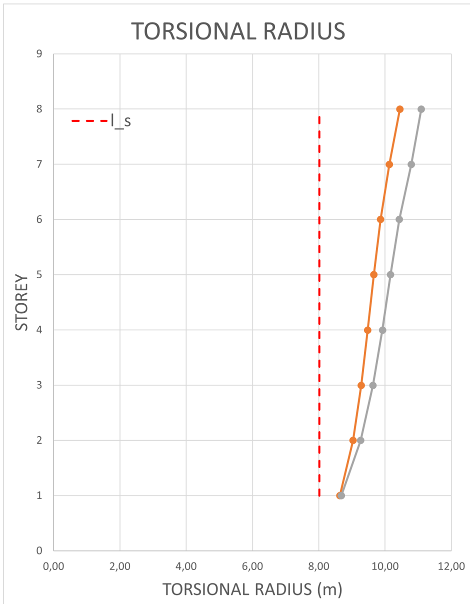
(a)



(b)



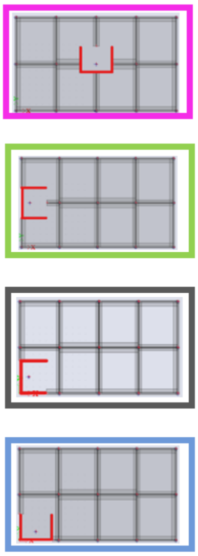
(c)



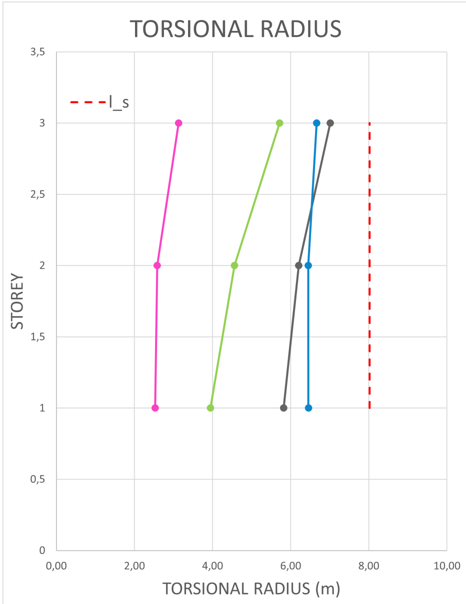
(d)

Figure C.2: Torsional radius for asymmetrical buildings at 3-, 5-, 8-storeys

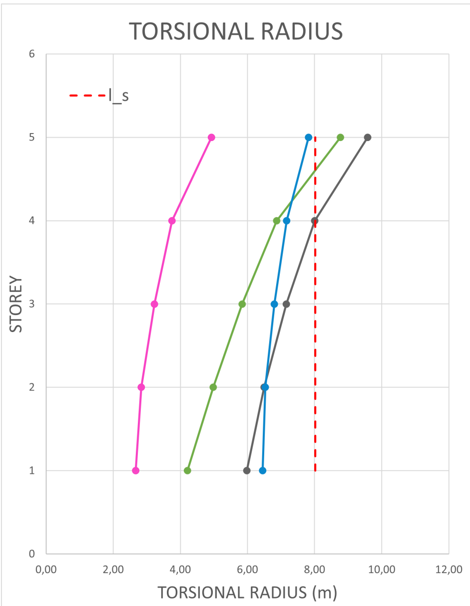
U-shape walls



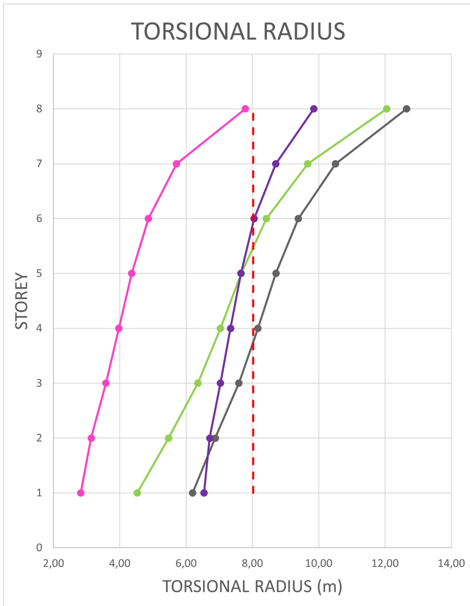
(a)



(b)



(c)



(d)

Figure C.3: Torsional radius for buildings with U-shape walls at 3-, 5-, 8-storeys

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