APPENDIX K

DYNAMICS: SPRINGS IN PARALLEL

The pneumatic and electrical motors are fixed to the rotor through a mechanical bellows coupling. This latter induces constraints on the axial movement as it presents a mechanical stiffness K_s. Let us also consider that the unwanted Eddy currents (see section 4.5.1) induce an axial force F_c. Therefore, the axial movement of the complete system is governed by:

$$M\ddot{z} + C\dot{z} = F + F_c - K_s(z - z_{0,s}) + F_e, \tag{K.1}$$

where $z_{0,s}$ is the equilibrium point of the mechanical coupling and F_e is the axial load. In quasi-static conditions, both forces F and F_c can be replaced by their spring equivalent forces. Their stiffnesses are respectively K_f and K_c while their equilibrium point is identical, i.e. $z = 0$. Hence, in quasi-static, the system composed of these three springs in parallel has its own equilibrium point $z_{0,t}$ that can be determined by cancelling the derivatives in the relation (K.1) and replacing the forces F and F_c by their spring equivalent forces, yielding:

$$0 = -K_f(\omega)z_{0,t} - K_c(\omega)z_{0,t} - K_s(z_{0,t} - z_{0,s}) + F_e, \tag{K.2}$$

where the three stiffnesses are positive. Hence, the total equilibrium point $z_{0,t}$ can be calculated as:

$$z_{0,t}(\omega) = z_{0,s} \cdot \frac{K_s}{K_s + K_f(\omega) + K_c(\omega)} + \frac{F_e}{K_s + K_f(\omega) + K_c(\omega)}. \tag{K.3}$$

The total stiffness of the bearing is thus the sum of these of the proper functioning and of the Eddy currents, i.e. $K_{tot} = K_f + K_c$. Let us assume the axial load F_e as being equal to zero. When the spin speed is zero, the stiffness K_{tot} is equal to zero, yielding $z_{0,t} = z_{0,s}$. By constrast, when the spin speed increases, the total stiffness K_{tot} increases up to reach a limit value, as stated in section 3.7.3, meaning that the equilibrium position $z_{0,t}$ gets closer to zero, namely the equilibrium position of the bearing.

K.1 Open circuit

Considering that the windings are left in open circuit, the stiffness K_f is equal to zero. Besides, the displacement sensor allows to measure the equilibrium position $z_{0,t}$. Hence, by inverting the relation (K.3) and replacing $K_f(\omega)$ by zero, the axial stiffness $K_c(\omega)$ induced by the Eddy currents can be calculated as:

$$K_c(\omega) = K_s\left(\frac{z_{0,s}}{z_{0,t}(\omega)} - 1\right) + \frac{F_e}{z_{0,t}(\omega)}. \tag{K.4}$$

This stiffness can thus be determined experimentally.

K.2 Short-circuited

Considering now that the windings are short-circuited, both stiffnesses K_f and K_c are present. Therefore, assuming that the stiffness $K_c(\omega)$ has already been determined experimentally, the stiffness $K_f(\omega)$ can be calculated as:

$$K_f(\omega) = K_s\left(\frac{z_{0,s}}{z_{0,t}(\omega)} - 1\right) + \frac{F_e}{z_{0,t}(\omega)} - K_c(\omega). \tag{K.5}$$