Economics School of Louvain - ESL

Economics School of Namur - ESN

Gender and household labour division

An overlapping generations model

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Academic Year 2020-2021

Master in Economics – 120 credits – Focus : Research

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This disertation is dedicated to the women in my life.

merci voor de tijd die we delen.
veel liefs, Thomas

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Acknowledgments

I would like to thank all the women in my life. This is especially true for my mother. *Maman*, you would give up all your time to help others. You are an inspiration to me. Thank you for being there for me.

I would also like to thank my promotor Prof. Dr. David de la Croix.
Professor, thank you for your patience and understanding. I do not take this for granted and I am gratefull for the chance that has been given.

Lastly, I would also like to thank the reader, Prof. Dr. Luca Pensieroso.
Professor, thank you for taking the time to read my dissertation. I also want to thank you for your interesting classes and many discussions, especiallly during the course of "History of Economic thought".

Thomas Lebbe
August, 2021

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1 Introduction

Can the wage gap be explained by expanding a neo-classical growth model with household production? Can domestic service cheques, *titres-services*, help to narrow the gender gap on the labour market? These questions were the starting point of this dissertation.

The gender gap literature has too often focussed on the size of the earning differential and issues of promotion, e.g. the glass ceiling. An alternative, and equally important question is how the gender gap has come to be. It would be interesting to look at the participation-gap: why do women work fewer hours and does this explain part of the wage gap?

If we follow the logic of a representative agent model and we assume that women and men are equal in their ability, then there should be no difference in optimal choice of education, work or retirement. In reality however, there is a difference in terms of working hours, in early retirement, and in pension payments. The easy way out would be to declare men and women are inherently different and hence their outcomes are different. One could easily calibrate a model with a set of parameters for one gender and another set for the other. A simple way would be to assume women have a different taste for leisure than men. This however is not very convincing and looks like an ad-hoc solution.

The true challenge is to find within an existing framework, the limits of explanation of a model. The logic above of equal outcomes only holds when constraints are equal. Here lies the crux of this dissertation: the constraints are not at all the same. By taking into account the gender difference in domestic chores (i.e.: cleaning, cooking, rearing children, helping elders) one sees that men and women have different time constraints. These unequal constraints translate in different choices and thus divergent outcomes.

The key notion is that we all have an endowment of time, but some of us are more constrained in how we use that endowment of time. Household chores are a burden on the available time we have. If women e.g. spend 1/3 of their available time cleaning and cooking, then it leaves them with only 2/3 to spend freely. It is in these contours that situates this paper. I want to investigate the limits and the possibilities of my line of thought. This leads me to pose two research questions: one (i), can household labour division explain the gender gap, in wages and pensions? If so, to what extend? And, secondly (ii), can a subsidy to domestic services reduce these gender gaps?

2 Literature and Contribution

Broadly two reasons explain the increase of women participation in the labour market: a demand- and a supply-side argument. The first combines technological progress with demand for female labour. Improvements in production processes and the rapid increase in demand for goods pushes firms to hire more workers. As a substitute for male labour, demand for female labour increases.

A second line of argumentation explains the increase of women participation through the eyes of the marketization of household production[1]. It is societal progress combined with supply of labour. It is this second line of thought that this paper follows. Household production is a fact of life and is mostly performed by women. Since the rise of female participation during the 1960's society witnesses an increase in market-alternatives to household production. In-living grandparents were sent to retirement homes, home-cooked meals were replaced by pre-cooked meals and fastfood. This ever growing shift has two effects, first (i) female agents gain additional time to spend on other activities and, secondly (ii) more income is needed to be able to buy these new goods, creating a pull-effect.

In the Nordic and the Anglo-Saxon countries, the market has gone the furthest in replacing household production in comparison to other parts of Europe (Biffl, 2007). Think about child and eldercare, but also essential goods like meals. This fast transformation of household to market production has also been translated into the highest female participation rate for women in Scandinavia and the UK as a close second. Likewise, we see the mirror image in the South of Europe were this transformation has not fully set and where female participation rates are relatively low.

A combination of both broad arguments is found in Greenwood, Seshadri and Yorukoglu (2005), these authors link technological progress to the freeing of women from household chores. The introduction of electricity in homes and the labour-saving consumer goods like vacuum cleaners and washing machines rapidly freed women from their stringent time constraints.

So if we continue this line of thought, we must come to an understanding of *the theory of allocation of time* (Becker, 1965). In this work, Becker links time to goods: he links the time to the budget constraint if you wish. The most tangible example is of transportation costs when commuting. Time spend commuting to work is a heavy burden on your endowment of time. A trade-off occurs between a high-paying job with a daily commute or a lower-paying job with no commute. Time is thus clearly linked to the budget constraint in several ways. What is equally relevant for this dissertation is what Becker has to say about the division of labour. He states that "the allocation of time of one household member is greatly influenced by the opportunities open to other members" (Becker, 1965).

[1] definition from Nai and Pissarides (2008): "homeproduction can be marketized, i.e., someone else can be paid to do it and the individual can still derive the same utility from its output.".

Becker(1965) thus hints at the interlocking of time constraints between members of one household unit. it is then only a very small step to my choice of joining the time constraints of men and women (see next section).

It is also enlightening to look at how different countries account for differences in hours worked in a broader framework. Similar reasons or patterns might also (partially) explain the difference in hours worked by gender. The most classic of these studies is done by Prescott (2004). The difference in tax rates seem to be able to explain most of the differences in labour supply in the USA versus Europe. Most notably is that Prescott(2004) shows that, at the aggregate, Europeans would work at comparable levels as that of the USA if they had the same tax rates.

Still at the aggregate level but now including household production, we find the work of Ngai and Pissarides(2008). These authors look and explain long run U pattern of an initial decline, followed by a small rise in hours worked. They explain that hours worked on the balanced growth path follows the dynamics of household production. Interestingly, they also report that off the steady state transitional dynamics push for more leisure and labour supply falling.

Macroeconomic modelling with household production within a gender framework remain few. It is my aim to show that a theoretical model with heterogeneous agents and with home production can explain much of the employment gap still present. By building up this model and including a gender dimensions, I hope to discover a feasible transmission mechanism through which the gender gap can be explained.

I will do this by building an Overlapping Generations model with both an ability and a gender dimension. The remainder of the dissertation has the following outline: In the next part, section 3, I build the model by explaining the constraints and the larger setting. In section 4, I calibrate the model to Belgian data and I make a distinction in parameters determined by calibration and those fixed by the literature. In section 5, I assess the model by doing some external validation: I do a cross-country comparison on recent data. After validation of the model, I introduce a policy shock to the model and I evaluate it's effects on the gender gap, this entails section 6. I finish this dissertation with some concluding remarks, which can be found in section 7. The interested reader can find the technical appendix with first-order conditions at the end of this document.

3 The Model

3.1 Demographics

Agents differ in three ways: gender, age and ability. Agents live for 6 periods, each period corresponding to 15 years. They enter the model at age 15 and face certain death at age 105. The other two dimensions are time-invariant and can only take two values. Within an age group agents are thus divided in four groups:

- Female high ability
- Female low ability
- Male high ability
- Male low ability

Each group is a fraction v_i of the total population, the subscript denotes the subgroup to which the agent belongs. Througout this dissertation the same subscripts are used, L and H for ability types Low and High and F and M for the gender types Female and Male. The total population can be split into males (v_M) and females (v_F) ,and likewise into low (v_L) and high (v_H) ability agents. Ability refers to the potential of accumulating additional human capital through formal higher education. Low ability agents can participate in higher education but are not productive in creating additional human capital. High ability agents on the other hand can productively spend time studying, which yields additional human capital in later periods of life.

Lastly agents face a possibility of dying in each period, this conditional probability to die is between 0 and 1 (excluded). However upon reaching the 6th period of life, their conditional probability to die becomes 1, denoting absolute death. Throughout the model we use ρ_t^j which denotes the conditional probability to survive to the next period j+1 but conditional on surving to age j. In the same lines π_t^j is the unconditional probability to survive to age j at the beginning of life. These survival probablities play a role in discounting futute utility and thus show up in the utility function (eq.10) and in the First-Order Conditions [2].

Demographic change is also modelled through fertility: The youngest cohort at time t relates to the youngest cohort at time t-1. To model this, one can follow the work of de la Croix et al. (2013), Mérette & Georges (2009) and Devriendt & Heylen (2020). The equation below clarifies:

$$N_1^t = f_t N_1^{t-1}$$

f_t is the fertility rate, which is defined as the ratio between the youngest cohort at time t compared to the youngest cohort one period earlier. This fertility rate exogenously enters into the model as shown above.

[2] see appendix A

3.2 Households

To simplify, agents from the two gender types form a unified household using assortative mating. This means that at the household level, high ability females are coupled with high ability males, and that low ability females are coupled with low ability males.

Households are connected through their time constraints. Meaning that agents within the household have liberty to make private decisions given their joint constraint.This implies the use of an explicit or implicit bargaining mechanism between the spouses. This bargaining is simplified into what is called "Bargaining power" and denotes the leverage that one spouse has over the other. I denote this by a share parameter ζ. In perfectly equalitarian households this would result in a perfect split of the bargaining share (and set equal to 0.5).

3.2.1 Time constraints

As mentioned above, agents are connected through their time constraints. These constraints change with age. Two key periods are the 1st period and the 4th period. In the 1st period agents have the ability to accrue additional human capital by studying. But studying implies a time cost on agents, removing time they can spend on working, having leisure or doing household chores. They enter full retirement after some fraction R of the 4th period. From the 5th period onwards they do not work. Model-wise this is done by giving each agent 1 unit of time per period in which they can divide up their time across different activities.

First period in life (ages 15-29):

$$1 = n^g_{1,a} + e^g_{1,a} + h^g_{1,a} + l^g_{1,a} \tag{1}$$

With :
n denoting time spend working, e denoting time spend studying, h denoting time spend on household chores, and l denoting leisure time.
Superscript g denotes gender, with $g = \{M, F\}$ for Male and Female respectively.
Subscript 1 denotes their age (=1st period of life) and subscript a denotes their ability type, with $a = \{L, H\}$ for Low ability and High ability respectively.

For periods 2 up to 4 (ages 30-74):

$$1 = n^g_{j,a} + h^g_{j,a} + l^g_{j,a} \tag{2}$$

With :
all notations are identical to above with exception of subscript j which denotes the age or equivalent period of life.

As noted before, agents in their 4th period of life will spend a part R in which they are active on the labour market. In this part R, time constraints are represented by eq.(2). The remainder (1-R) of the 4th period is spent on retirement. After retirement the time constraint simplifies to eq. (3).

For periods 5 and 6 (ages 75-104):

$$1 = h^g_{j,a} + l^g_{j,a} \tag{3}$$

With :
all notations are identical to previous ones.

In contrast to standard OLG models, this paper introduces an additional constraint throughout the life-cycle: household chores. These chores take up time and do not contribute to utility, as such it seems wise to model them separately from work and leisure. The key motivation is that household chores act as a taxation on leisure. Only true leisure, free of chores, gives rise to utility. What follows is a quick overview of some elements of the time constraint.

Education

Education (e_t) is done to receive additional human capital (x_{t+1}) in the following period of life. This is done by means of spending time in formal education[3]:

$$x_{t+1} = x_t(1 + \phi e^{\sigma}_t) \tag{4}$$

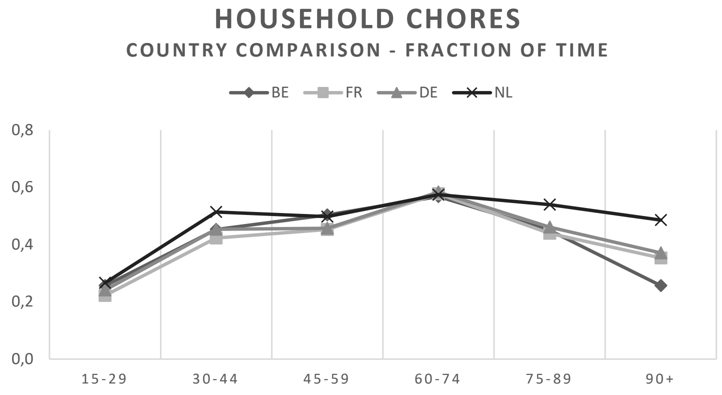
ϕ and σ are the associated efficiency parameters to human capital accumulation. Because of heterogeneity in learning-ability, the high abled agents will receive a higher level of initial human capital x_0. We denote the level of initial human capital by low abled agents as a fraction κ of that of high abled agents. This leaves the initial human capital for low abled agents as such:

$$x^L_0 = \kappa x_0 \tag{5}$$

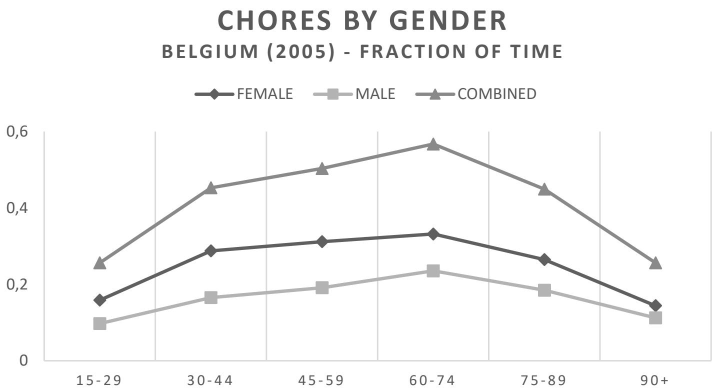
Household chores

Household chores have a life-cycle of their own. Early in life the amount of time spent on housework is relatively small. Domestic work increases around the age of 30 when children are born. Taking care and raising children takes up a lot of time and effort. As children grow up, a decrease in chores occurs. This is partially offset later in life by taking care of grand-children. For the very old we see a decrease in chores. This inverted U – shape of household chores pose a serious reduction in the available time. These chores are put exogenously into the model. This means that implicitly the author assume that these chores do not yield utility.

[3] From here onwards, subscripts for gender and ability types are omitted, unless when it is absolutely necessary to avoid confusion. This is done to increase readability of the readers.



Source: Own calculations and NTTA: Vargha et al. (2016)
Fig. 1: Household chores by couples as fraction of 1 unit of time, averaged over 2005-2010



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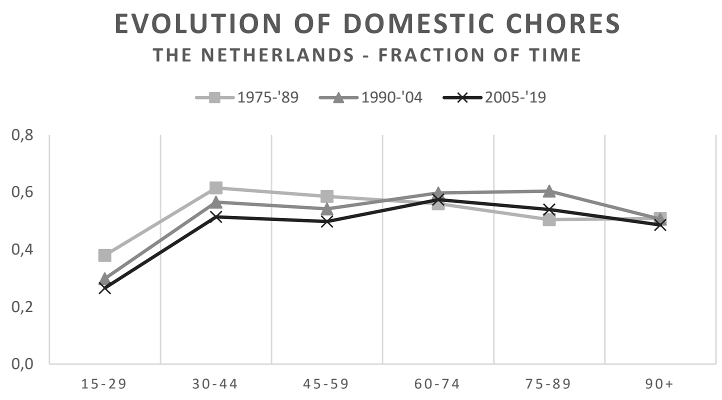
Source: Own Calculations and NTTA: Vargha et al. (2016)
Fig. 2: Household chores by gender as fraction of 1 unit of time.

As the reader notices, household chores take up a considerable amount of time. As data is lacking for many countries and times. This dissertation focusses on Belgium. But comparing it to neighbouring countries we can see that the share of domestic work done by women is very similar across these countries. Also the evolution throughout time shows a stable pattern: it is decreasing slightly



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Source: Own calculations and NTTA: Vargha et al. (2016)
Fig. 3: Evolution of household chores. The case of the Netherlands

for the younger age cohorts and increasing at the tail. The life-cyle pattern however remains fixed.

Luckily for agents, there is a way out: they can hire domestic workers to relief them from their household chores. In the case of childcare, think of nanny's or au-pairs, or generally think of cleaning ladies and cooks and for the elder-care one can think of home nurses.

To model these time constraint, I use the life cycle of agents to differentiate the constraint that each age cohort faces. This is done by including age and time specific chores. The age speficity represents the life-cycle component of household chores. Young to mid-aged adults spend time on child rearing, whilst older adults tend to spend more time on eldercare. This model assumes that childcare and eldercare of both wife and husband are pooled. Meaning that a wife could possible tend for her mother-in-law. Model-wise:

$$\overline{h}_j^t = \underbrace{\zeta\overline{h}_j^t}_{\text{female req. chores}} + \underbrace{(1-\zeta)\overline{h}_j^t}_{\text{male req. chores}} \tag{6}$$

With :
$\overline{h}_j^t$ denoting the total required household chores by a household of age j. This is done to differentiate with the actual housework done by agents, which is simply denoted h_j^t (with bar ommited).

The required domestic chores can either be done by the agent herself, which qualifies as domestic work ($h_{j,a,g}^t$) or can by done by professionals who are hired as domestic servants ($d_{j,a,g}^t$); Below the equation for a female agent is stated:

$$\zeta\overline{h}_j^t = \underbrace{h_{j,a,F}^t}_{\text{domestic work}} + \underbrace{d_{j,a,F}^t}_{\text{domestic services bought}} \tag{7}$$

3.2.2 Budget constraints

Each agent faces a budget constraint of the following generic form:

$$\begin{aligned}(1+\tau_c\ )\ c_t + a_t + d_t w_d(1+\tau_p-\tau_d) = \\ (1+r_t)a_{t-1} + w_a x_t n_t(1-\tau_w) + z_t\end{aligned} \tag{8}$$

Agents can spend their income, on consumption c_t, including taxes (τ_c), savings (a_t), and on domestic services (d_t)which has a cost to it composed out of the wage (w_d) for domestic servants and the associated employer social security contribution(τ_p) and government subsidy (τ_d).

On the Right Hand Side (=RHS) we find the different sources of income: through (i) accumulated wealth (a_{t-1}) and interest payments(r_t), through (ii) labour income: depended on wage rate by ability type a (w_a), accumulated human capital (x_t) and the hours worked (n_t) and after labour taxes (τ_w) and by (iii) lump-sumps (z_t) given by government[4].

For retired agents, labour income is replaced by a pension payment (p_t). In the 4th period of life, the agent has a fraction of time R for which she is still active on the labour market and a fraction (1-R) for which she is retired and receives a pension payment. This all results in transformation of the previous equation into the following:

$$\begin{aligned}(1+\tau_c\ )\ c_t + a_t + d_t w_d(1+\tau_p-\tau_d) = \\ (1+r_t)a_{t-1} + Rw_a x_t n_t(1-\tau_w) + (1-R)*p_t + z_t\end{aligned} \tag{9}$$

3.2.3 Utility

Utility takes log-utility form for consumption, CES for leisure and quadratic for domestic services. Below the reader finds the felicity function for a female[5] agent at age j:

$$\begin{aligned}U(c_t, l_t, d_t) &= ln(c_t) + \gamma_j \frac{l_t^{1-\theta}}{1-\theta} - \frac{\gamma_d}{2}[\zeta \bar{h_t} - d_t]^2 \\ &= ln(c_t) + \gamma_j \frac{l_t^{1-\theta}}{1-\theta} - \frac{\gamma_d}{2}[h_t]^2\end{aligned} \tag{10}$$

Instantaneous utility is derived from three sources: the consumption of goods (c_t), the enjoyment of leisure (l_t) and the lack of domestic chores (h_t), which is achieved by buying domestic services (d_t). γ_j and γ_d denote the relative weights attached to utility gained from leisure and domestic services vis-a-vis consumption. γ_j is age specific and changes over the life-cycle, γ_d stays fixed. Lastly, θ stands for the inverse of the intertemporal elasticity with respect to leisure.

Whilst the modelling for leisure and consumption are quite standard in current macroeconomics, quadratic utility is not often used. They fit more in models of

[4] The lump-sums can be negative and then act as a lump-sum tax

[5] For male agents the utility function only changes in that ζ is replaced by (1-ζ), where ζ denotes the bargaining power of men, see **3.2.1 §household chores**

subsistence economies[6], but in the case of domestic services they make sense.

First (i) demand for domestic services is an atypical good, it follows a bliss-point utility. Agents don't want to accrue an indefinite amount of domestic services. At most agents want to hire domestic services up to the point at which their household chores are met. In lay-man terms: I want to hire a gardener to landscape my backyard, but when the garden is well kept, I have no reason to hire him for additional gardening. A second (ii) reason is more technical in nature, if domestic services would be unbounded, agents would be able to buy additional time beyond their time constraint. They would not only buy to gain more leisure, they would also buy additional time to have more hours in a day.Suddenly agents would have >24 hours in a day. Of course this makes no sense at all.

The agent doesn't optimizes her instantenous utility however,but her lifetime-utility, which yields the following expression:

$$U_L(.) = \sum_{t=0}^{5} \beta^t \pi_t U(c_t, l_t, d_t) \tag{11}$$

Utility is discounted by two ways: (i) by a time discounting rate (β) and (ii) by the unconditional probability to survive (π_t).

π_j can be seen as the product of the conditional survival rates in each period of life from entering the model until age j. Together, β and π_t form the effective time discount rate of the optimizing agent.

3.3 The productive sector

The economy is split in two sectors: a productive sector and a domestic service sector. Firms in the productive sector act perfectly competitive on the output market and maximize profit. Instead of solving a dynamic problem, firms are solving the static problem at each period in time. Technology takes the form of constant returns to scale but decreasing returns to the factors of production.

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{12}$$

The previous equation show that production follows a Cobb-Douglas specification, with α being the capital share in production. A_t the level of technological progress and L_t the total effective labour. Labour-augmenting technological progress (ι_t) is the exogenous driver of long-term growth:

$$A_{t+1} = \iota_t A_t \tag{13}$$

Effective Labour L is itself a CES-function[7].

$$L = (\eta L_L^{\frac{\lambda-1}{\lambda}} + (1-\eta) L_H^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \tag{14}$$

[6] see i.e. critique by Van Daal and Merkies (1989)
[7] time subscripts are dropped for readability

Effective labour consists out a combination of Low-skilled (L_L) and High-skilled (L_H) effective labour. A CES form is used to denote the imperfect substitution between the different types of labour. This is represented by the share parameter η and the elasticity parameter λ.

Effective labour within an ability group is the product of hours worked and the human capital accumulated. A key assumption here is that women and men labour are perfect substitutes. Below the effective labour of high-skilled agents is shown.

$$L_H = \sum_{j=1}^{4} \sum_{M,F} N_{j,H,g} \; n_{j,H,g} \; x_{j,H,g} \tag{15}$$

With $N_{j,g,H}$ denoting the cohort size of age j and gender g and ability H. $n_{j,H,g}$ denotes the hours worked and $x_{j,H,g}$ denote the efficiency units.

Likewise we find the effective labour of low-skilled workers in the productive sector:

$$L_L = \sum_{j=1}^{4} \sum_{M,F} \; N_{j,L,g} \; n_{j,L,g} \; x_{j,L,g} - L_d \tag{16}$$

The reader notes that not all the effective labour of low-skilled workers goes to to the productive sector. A part L_d works in the domestic service sector instead.

3.3.1 Factor prices

Firms will attract capital and effective labour until their marginal costs equalize their marginal benefits, which results in the following FOC's for firms:

$$(1-\alpha)A_t \frac{K_t}{A_t L_t}^{\alpha} \eta \frac{L_t}{L_{a,t}}^{\frac{1}{\lambda}-1} = w_{a,t}(1+\tau_p) \tag{17}$$

Because of the assumption of a competitive labour market, the total demand and supply of effective labour will equalize. The wage cost per effective unit of labour will be equal to the marginal labour productivity. This FOC is shown in the equation above: with the marginal productivity per effective labour of ability type 'a' on the LHS and the wage cost per effective unit of labour of ability type 'a' on the RHS. τ_p is the employer social contribution rate and the firm takes this into account when hiring.

$$[\alpha(\frac{A_t L_t}{K_t}) - \delta](1-\tau_k) = r_t \tag{18}$$

Firms will install capital up to the point where the after-tax (τ_k) marginal product of capital, net of depreciation (δ), equals the interest rate (r_t). If the interest rate is higher than the marginal productivity, the demand for capital will decrease in order to ensure equality. Important to note is that with survival rates, the interest also reflects perfect annuity markets[8]. The supply of capital comes from aggregate savings. A part of this stock is reserved to finance the governments outstanding debt (see later).

3.4 The domestic service sector

Domestic services are modelled as a seperate sector from market production Y_t. Agents transform hours worked linearly into domestic services.

$$D_t^s = A_d L_{d,t} \tag{19}$$

Where D_t^s stands for the aggregate supply of domestic work and $L_{d,t}$ stands for the supply of effective hours worked in the domestic sector. A_d is the technology level in the service sector, which remains fixed over time. For simplicity this has been put to 1.

Some assumptions of the model: (i) only Low-abled agents offer their work to the domestic service sector, (ii) domestic services are a linear function of hours worked, (iii) Low-skilled agents are indifferent in working hours in the productive or in the domestic service sector. With the after tax wage rate determining their labour choice between them.

Free entry to the domestic service sector will ensure that the after tax wages of both types of work equalize:

$$w_{L,t}(1-\tau_w) = w_{d,t}(1-\tau_{w,d}) \tag{20}$$

[8] This to contrast with models with accidental bequests, for which no assumption on annuity markets is required

The demand for domestic services comes from households and is the sum of all individuals optimal choice for domestic services:

$$D_t = \sum_{j=1}^{6} \sum_{g=M,F} \sum_{a=L,H} N_{j,a,g}\ d_{j,a,g} \tag{21}$$

An agent of age j, gender g and ability a decides over her optimal demand for domestic services $d_{j,a,g}$ by optimizing her life-time utility function (eq.11) with respect to domestic services d_t and taking into account her budget (eq.8-9) and time (eq.1-3) constraints.

As domestic servants enter quadratically into the utility function, agents will try to attain a bliss point. This bliss-point is when domestic services cover all required household chores. The associated first order condition for the demand of domestic services can be found in Appendix A.

3.5 Government

The government has a general budget which includes pensions:

$$B_{t+1} - B_t = B_t r_t + G_t + P_t + Z_t - T_{n,t} - T_{p,t} - T_{k,t} - T_{c,t} + T_{d,t} \tag{22}$$

Government debt (B_t) increases when the government expenditures exceed it's income.
The expenditures are: the outstanding debt ($B_t r_t$), Government consumption (G_t), Pension payments (P_t),lump-sum payments (Z_t) and the subsidy to domestic servants ($T_{d,t}$). $T_{d,t}$ takes the form of a tax cut and is hence noted in the same way as other taxes. Income comes from taxation on Labour income ($T_{n,t}$),employers social contributions ($T_{p,t}$) tax on capital income ($T_{K,t}$) and a tax on consumption ($T_{c,t}$).

3.6 General equilibrium

The model assumes a closed economy, the national accounting identity equation reflects this:

$$Y_t + D_t^s = C_t + I_t + G_t + D_t^d \tag{23}$$

Which is a standard Accounting identity, consisting out of aggregate consumption (C_t), investments (I_t) and government consumption (G_t). Added to this is the total consumption of domestic services bought by the households. As demand (D_t^d) and supply (D_t^s) for domestic services cancel out, the accounting equation simplifies to the more common form:

$$Y_t = C_t + I_t + G_t \tag{24}$$

Looking at the equation below, we find that investments are used for accumulating more capital in the next period (1st term RHS) and to compensate for depreciation of the current capital stock (2nd term RHS). Capital depreciates at an exogenous and constant rate δ.

$$I_t = \Delta K_{t+1} + \delta K_t \tag{25}$$

Lastly, the existence of government bonds (B_t) on outstanding debt makes that not all wealth (S_t) gets translated into capital:

$$S_t = K_t + B_t \tag{26}$$

With this last part explained, we have enough information to make a formal definition of the intertemporal equilibrium.

3.6.1 Intertemporal equilibrium

Given the initial endownment of human capital x_0, the initial stock of physical capital K_0, intial population distribution N_0, an equilibrium consists of sequences of prices$\{w_{L,t}, w_{H,t}, w_{d,t}, r_t\}$, aggregate quantities $\{L_{L,t}, L_{H,t}, L_{d,t}, K_t\}$ and decision rules $\{c_t^{j,a,g}, n_t^{j,a,g}, e_t^g, d_t^{j,a,g}, x_{t+1}^{j,a,g}, h_t^{j,a,g}\}$ for each type of agent[9], such that:

1. The household's decision rules $c_t^{j,a,g}, n_t^{j,a,g}, e_t^g, d_t^{j,a,g}, x_{t+1}^{j,a,g}, and\ h_t^{j,a,g}$ maximize utility subject to the constraints (eq.1-10)
2. The firm hires optimally $L_{L,t}$, $L_{H,t}$ and K_t as to maximize profit.
3. The prices w_L, w_H, w_d and r_t are such that markets clear.
   Meaning equations (17),(18), and (20) hold.
4. Human capital evolves in the manner of eq.(4).
5. The government budget is satisfied as in eq.(22).
6. Demand and supply for domestic work equalize,
   as seen by equations (19),(20) and (21).

[9] As a reminder, superscript ^{j} denotes age, superscript ^{a} denotes ability type, and superscript ^{g} denotes gender.

4 Parametrization

In the previous chapter the model was explained. In this chapter values are given to the parameters of the model. There are two sets of parameters: (i) those from literature and (ii) those determined by calibration. At the end of the chapter a table is included with all the parameter values and the exogenous variables.

4.1 Parameters from literature

Many of the following parameters are taken directly from the macroeconomic literature. Others provide a range, such as Rogerson (2007) for the inverse of the intertemporal elasticity of subsitution (θ). This model uses $\theta = 2$, which is taken from Devriendt and Heylen (2020) and is the half-way point set by Rogerson (2007). This corresponds to an elasticity of 0.5, which seems low. As Rogerson and Wallenius (2009) point out however, micro elasticities might not hold in macroeconomic studies.

The physical capital depreciation rate is taken directly from Penn World Table 9.1 (Feenstra, 2015) and is set to the average rate of Belgium. The yearly depreciation rate is 5.07%, which translates in our model to $\delta = 0.458$. Likewise, the capital share in the economy (α) is set to 1/3, which is the average of the labour share in Belgium and is also taken from the Penn World Table.

For human capital accumulation σ is set to 0.3, this is done as to not overestimate the returns to education. Differential returns to education is also not the focus of this dissertation, as such a modest choice was preffered. This value is much lower than Lucas (1990), but falls within the range of Bouzahzah et al.(2002) and Docquier and Paddison (2003). The efficiency parameter in education (ϕ) is calibrated (see section 4.2).

Agents also differ in innate ability, this is denoted by the initial level of human capital stock x_0. The literature is however very sparse, and few guidance exists on determining it. Buyse et al. (2017) use PISA science scores to determine the relative capital stock. This dissertation follows their approach and sets x_0 to 1 and uses PISA scores to set κ. The relative human capital stock of low-educated agents κ is set to 0.744, which corresponds to the relative PISA score of those scoring at the 25th percentile vis-a-vis those scoring at the 75th percentile.

For the elasticity of subsitution between ability types in the CES labour function, this dissertations follows Buyse et al.(2017) and Devriendt and Heylen(2020) and sets $\lambda = 1.5$. This value falls within the range of the empirical labour literature, see Caseli and Coleman (2006).

The population share parameters v_L,v_H are set to 1/3 and 2/3 respectively. v_L set to 1/3 is chosen as an absolute maximum of the population who would consider working as a domestic servant. v_F and v_M are set equal to 0.5 as this is roughly the gender composition of society.

This leaves our model with 8 parameters to be determined, this is done through calibration, which is explained in the next section.

4.2 Parameters through calibration

The remaining parameters are the 4 taste parameters for leisure ($\gamma_1, \gamma_2, \gamma_3, \gamma_4$), a taste parameter for domestic services (γ_d), a share parameter (η) in the CES labour function, the efficiency parameter to education (ϕ) and the time preference factor β.

These parameters are calibrated following the procedure set forth in Ludwig et al.(2011). An initial guess is made to the parameter values and a iterated process of comparing simulated moments to the data allow the parameters to be fitted perfectly for the reference period.

This fitting of the data requires a link between the parameters and the simulated variables. Through the labour-leisure choice, it becomes clear that the taste for leisure γ_i is closely linked to hours worked n_i. As a target we take the average hours worked across men and women. This is in accordance with our premise of identical agents only differing in time-constraints. In a similar fashion, the efficiency parameter to education ϕ is closely linked to hours studied e and the associated target is the average hours studied across men and women.

The taste for domestic services γ_d is linked to the overall percentages of domestic servants in the economy. This is done by comparing hours worked in the domestic service sector in comparison to total hours worked, this is abbreviated by “domes” in table 1.

The share of low-skilled workers η in the CES labour function is linked to the observed pre-tax earnings of workers with different educational attainment, we use the ratio of these earning $\frac{wL}{wH}$ as our target.

Lastly, the time preference factor β is closely linked to aggregate saving behaviour. Aggregate savings in its turn drives capital accumulation. The associated target becomes the capital-output ratio $\frac{K}{Y}$. Combined with demographics (i.e. survival rates) β explains the effective time preference that drives capital accumulation.

The calibration is set to Belgium during the period 1990-2004, in the following section one can see the scatter-plots comparing the real data to the model's prediction. In those plots the reader finds Belgium (1990-2004) is on the 45°-line denoting a perfect fit, which shows that this is the reference period and country. The only exception is γ_d which is calibrated in the period 2005-2019, due to boundary issues of domestic services in 1990-2004.

Tab. 1: Parametrization of the model

Parameter	Description	Value
Taken from literature		
α	Production share of capital	1/3
λ	Elasticity of substitution between workers of different ability	1.5
θ	Inverse of the intertemporal elasticity of substitution	2
δ	Depreciation rate of physical capital	0.458
σ	Elasticity of education to human capital accumulation	0.3
υ_L	Share of lower-skilled agents	1/3
υ_H	Share of higher-skilled agents	2/3
υ_M	Share of men	1/2
υ_F	Share of women	1/2
κ	Relative initial human capital	0.744
Calibrated	Reference: BELGIUM 1990-2004	
γ_1 , γ_2 γ_3, γ_4	Preference for leisure, by age group	0.6961 , 0.0229 0.5056 , 0.3834
γ_d	Preference for domestic services	3.472
η	Share of low-skilled workers (CES labour)	0.4261
ϕ	Efficiency in education	2.504
β	Time preference	0.7661

Targets for calibration		
Hours worked	$n_1 = 0.252$ $n_3 = 0.383$	$n_2 = 0.680$ $n_4 = 0.176$
Domestic services		domes=0.0736
Pre-tax earnings		wL/wH = 0.7103
Studying		e=0.1564
Capital-Output ratio		K/Y = 3.62

4.3 Exogenous variables

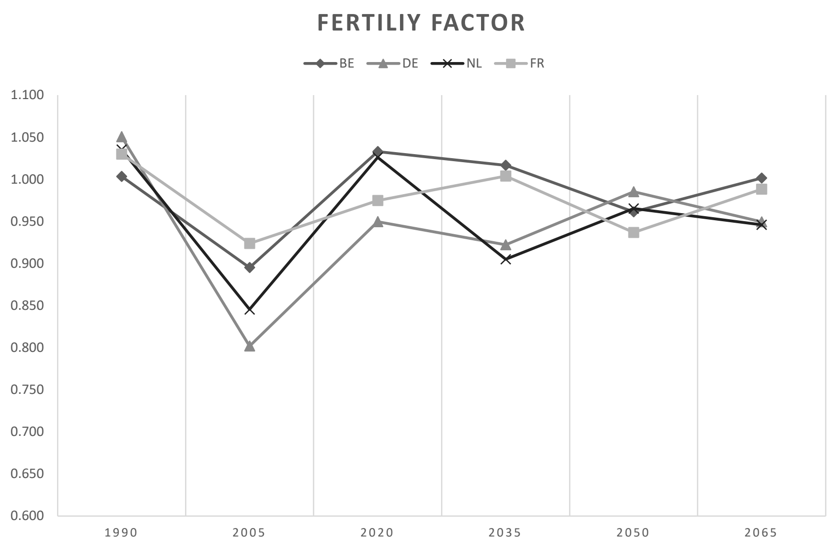
Section 4.3 gives more details on the construction and the sources of the exogenous variables in the model. Graphs show data on Belgium, occasionaly with additional data for France, Germany and the Netherlands. These countries will form an external frame to test the validity of the model (see chapter 5).

This section starts with an overview on demographics and the growth variables, it is followed by the household variables, and ends with the description of fiscal policy variables. At the end of this section a summary is given through a table.

Table 2 gives an overview of the exogenous variables within the model. The table states the values during the calibration period in Belgium (1990-2004).

4.3.1 Demographics

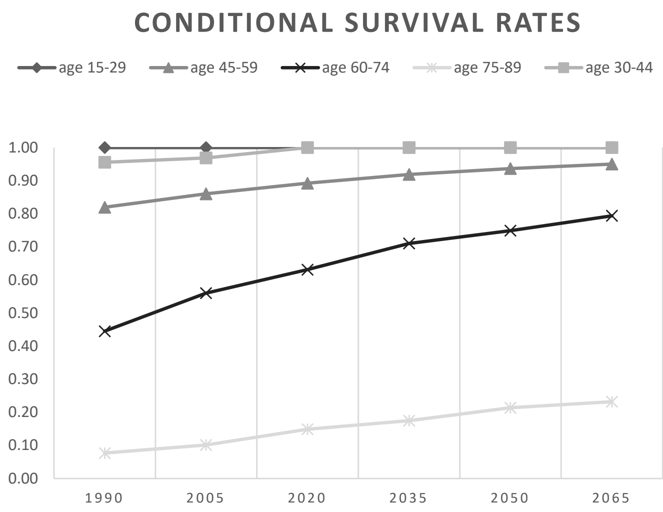
The data on demographic change comes from the World Population Prospect (UN, 2019). This database contains both historic data and projections on population sizes by age group. This data allows to calculate survival and fertility rates. Figure 4 shows the movement of the fertility factor, as society is shrink-



Source: Own calculations and data from United Nations (2019)
Fig. 4: Fertility factor of Western Europe (1990-2065)

ing, the fertiliy factor drops below 1. When population expands, the factor is larger than 1.The reader recalls that fertility is defined as the relative size of the youngest cohorts from one time period to the previous one.

Survival rates are calculated in a similar fashion but tracking the same age cohort across time periods.

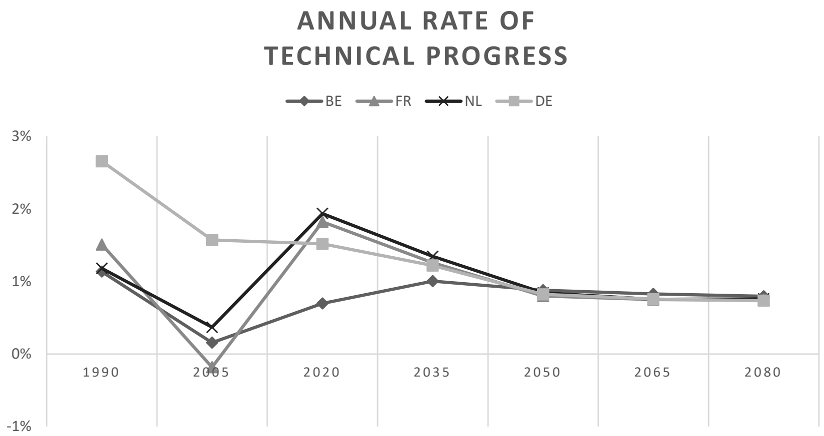


Source: Own calculations and data from United Nations (2019)

Fig. 5: Conditional survival rates of Belgium, by age group (1990-2065)

4.3.2 Technology

The time series on technological progress comes from Penn World Table 9.1 (2015) for which data is used until 2017. From then onwards TFP projections from Cette et al. (2017) are used. These TFP series were adjusted in two ways: (i) a transformation was applied to get the Labour-augmenting technical progress and (ii) the data was smoothed with a HP-filter to obtain a trend rate that excludes cyclical components.



Source: Own calculations and data from Feenstra et al. (2015) and Cette et al.(2017)
Fig. 6: Annual rate of labour augmenting technical progress, Western Europe(1990-2080)

4.3.3 Policy variables

Policy variables are taken from OECD stat, McDaniel(2007) and IMF DataMapper. The labour income tax rate τ_w is taken from "OECD stat - taxation - Taxing Wages - Tax database - Table I.5" and is the All-in tax rate, Combined central and subcentral income tax rate, including employee social security contribution as % of gross wage earnings.

The employers social security contribution rate τ_p is taken from "OECD stat - taxation - Taxing Wages - Tax database - Table I.5".

The corporate income tax rate τ_k is taken from "OECD stat - taxation - Taxing Wages - Tax database - Table II.2".

The consumption tax rate τ_c is taken from McDaniel (20007), the 2014-update.

The subsidy to domestic services is taken from Selleslagh (2019) and is calculated based on the cost of one *titres-services* and the gross compensation for domestic servants. The gap between cost and payement of domestic services is used as the subsidy rate.

The debt to GDP ratio b and the government spending to GDP ratio g is taken from IMF (2021) 's Fiscal Monitor.

Pension variables, such as the net replacement rate rr, the statutory pension age R are taken from the OECD (2005, 2007, 2009, 2011, 2013, 2015, 2017 and 2019) Pension at a Glance publications.

On the next page a summary is given of all the exogenous variables in the model during the calibration period (1990-2004) and fitted to Belgium.

Tab. 2: Exogenous variables of the model - Belgium 1990-2004

Symbol	**Discription**	**Value**
Demographics and growth		
ρ_1 , ρ_2, ρ_3 ρ_4 , ρ_5	Unconditional probability to survive to next period	1 , 0.956 , 0.82 0.445 , 0.077
f_t	Fertility (factor)	1.003
ι	Labour-augmenting technological progress (factor)	1.184
Household		
$\overline{h}_1$, $\overline{h}_2$ $\overline{h}_3$, $\overline{h}_4$ $\overline{h}_5$, $\overline{h}_6$	required household chores of the entire household (units of time)	0.288 , 0.499 0.548 , 0.591 0.503 , 0.267
ζ	Bargaining power of men	0.60
Policy variables		BELGIUM 1990-2004
τ_w	Labour income tax rate (including social security)	$\tau_{wL} = 0.355$ $\tau_{wH} = 0.467$
τ_p	Employers social security contribution	$\tau_{pL} = 0.298$ τ_{pH}=0.322
τ_k	Corporate income tax rate	0.386
τ_c	Consumption tax rate	0.180
τ_d	Domestic service subsidy rate	0
R	Statutorary retirement age for pension (fraction of period)	R_F=0.200 R_M=0.333
rr	Net pension replacement rates	$rr_L = 0.638$ $rr_H = 0.427$
b	government debt as fraction of GDP	1.209
g	government spending as fraction of GDP	0.524

5 Validity of the model

After parametrization of the model, we can gauge the validity of the model by comparing simulated moments to the data. There are two possibilities in approaching this, one could opt for a time-series approach and test the closeness of fit. Alternatively we could do a cross-country comparison. The idea remains the same, we determine whether the model's prediction fit the data.

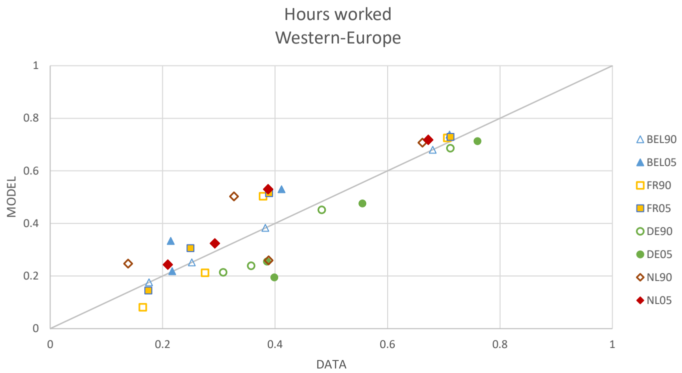
5.1 Cross-country comparison

This dissertation tries to fit the model across a group of 7 countries and over two time periods. This gives us 14 unique markers to assess the closeness of fit. A first group of countries is labelled "Western Europe" and these are the countries in close proximity to Belgium. The aim is to score a good fit for these countries. A second group of countries is denoted as the "Other Europe" group, these countries are more different to Belgium than the Western counterparts.These countries are included to test the limits of the model. The Other Europe group consists of Italy, Spain and the United Kingdom. These three countries try to represent the Southern European and the Anglo-saxon countries respectively. The Western Europe group consists of the Netherlands, France and Germany.

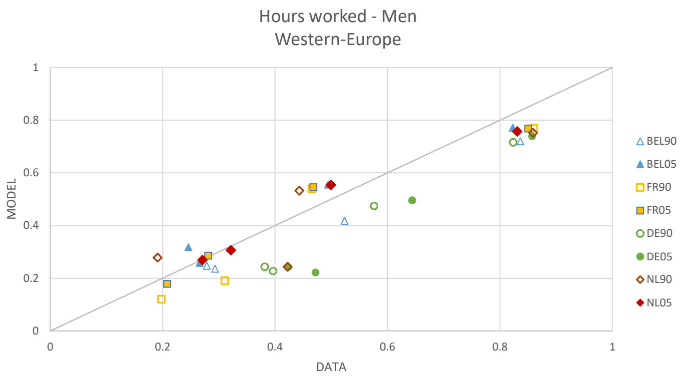
The variables of interest are hours worked, educational attainment, the gender wage gap, the pension gap and the educational wage gap. For hours worked and educational attainment we test on three fronts: total population, males and females.

5.1.1 Hours worked

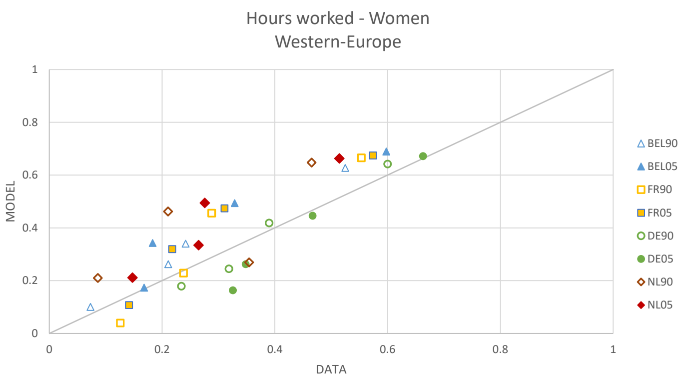
The scatterplot below shows the model's prediction vis-a-vis the data. Each point represents a year and a country. Shapes denote the country, e.g. as can be seen by the triangle representing Belgium. The shading of these shapes in turn provide a clear distinction between the period 1990-2004 (hollow) or the period 2005-2019 (shaded). A 45°-line denotes the perfect fit of the model.



Source: Data from OECD labour statistics.
Fig. 7: Hours worked - Total pop (model/data) - WEU



Source: Data from OECD labour statistics.
Fig. 8: Hours worked - Men (model/data) - WEU

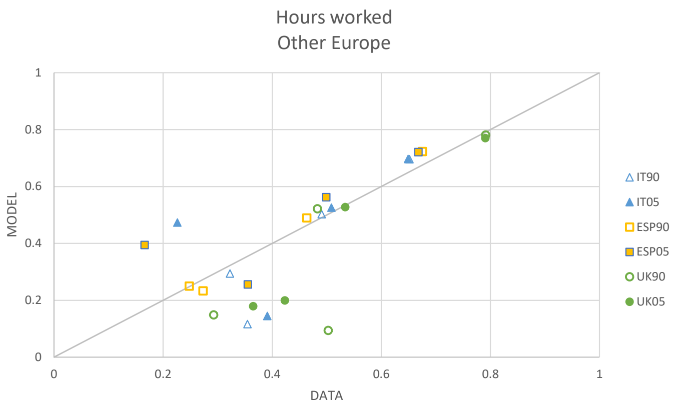


Source: Data from OECD labour statistics.
Fig. 9: Hours worked - Women (model/data) - WEU

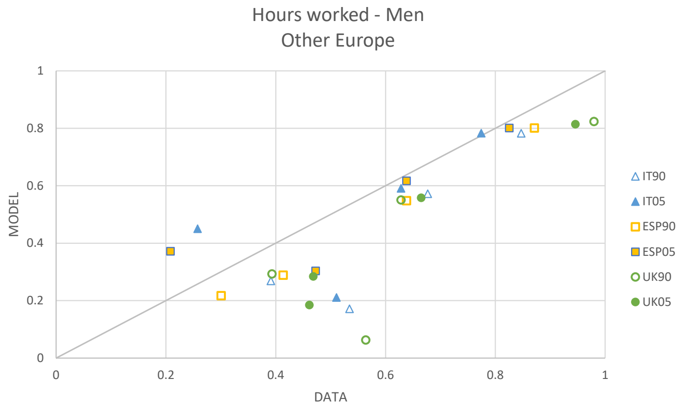
Figures 7 to 9 are all very encouraging. The model seems to predict hours worked very well. The model however overestimates the amount of hours worked by women. This seems to indicate that household labour division along gender lines is not sufficient to entirely explain the level of female labour. It does however give the right direction, predicting higher working hours for men in comparison to women.

Other Europe

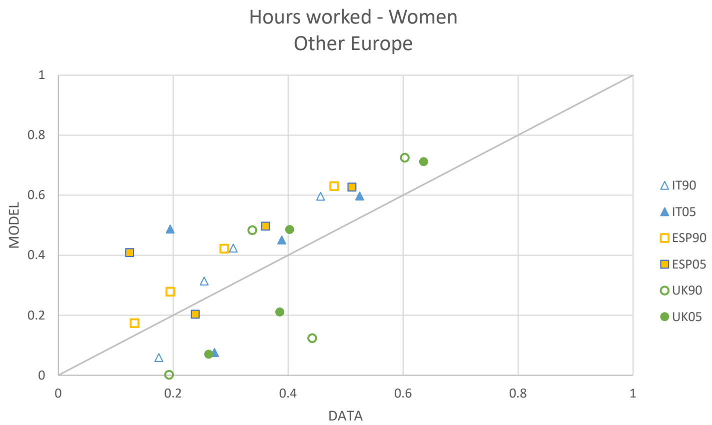
If we compare to above graphs to their Italian, Spanish and British counterparts we find a less convincing story. This is especially true in the bottom left corner of Fig. 10. Here one sees the hours worked of the young (15 to 30 year olds) and the oldest workers(60 to 74 year old), the gap between model's prediction and the data is much larger than the Western European graph above (Fig.7). If we look at prime age workers, the model does seem to fit the data quite well. This pattern repeats itself across the graphs of male and female hours worked. Similar to Western Europe, the model underestimates hours worked by men and overestimates it for women. Strengthening the comment made earlier: gender household labour division is not enough to fully explain the gap in labour participation. Labour participation is vital in the calculation of both the gender wage gap and the gender pension gap. As it stands now, the model will likely underestimate these gaps.



Source: Data from OECD labour statistics.
Fig. 10: Hours worked - Total pop (model/data) - Other EU



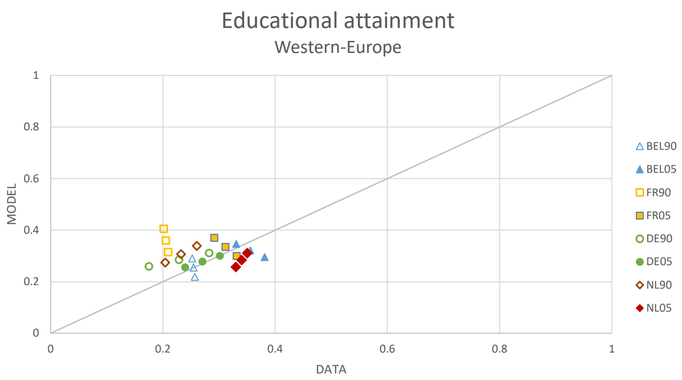
Source: Data from OECD labour statistics.
Fig. 11: Hours worked - Men (model/data) - Other EU



Source: Data from OECD labour statistics.
Fig. 12: Hours worked - Women (model/data) - Other EU

5.1.2 Education

An important note on education: the model is calibrated on hours studied (time use survey of Eurostat). The model however also encompasses education that is transformative, meaning that it results in human capital accumulation. A good comparison would require educational attainment data instead of hours studied. As a result the model's prediction of hours studied are rescaled to match the Belgian data on educational attainment in the tertiary sector for 1990-2004. This can be seen be the hollow triangle landing exactly on the 45°-line. Each series has three data points, the top (vertical scale) gives male education, the middle averaged and the bottom female education. The model closely predicts

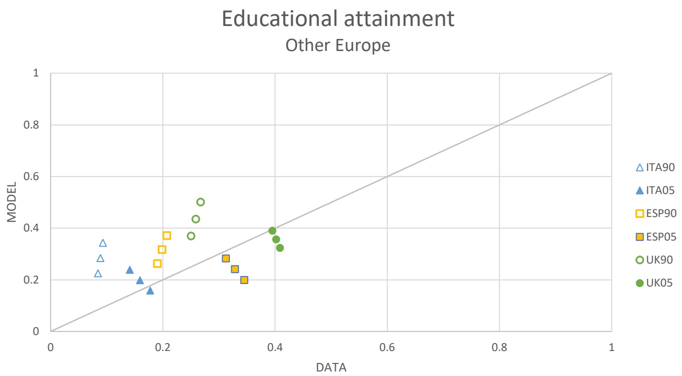


Source: Data from OECD - Education and training.
Fig. 13: Education (model/data) - Western Europe

educational outcomes of most countries. France in the first period is a slight outlier, where the model overestimates education choice of all agents. This however is corrected in the next period, where France fits the data well.

Other Europe

Comparing the model's prediction of education in the Other Europe group, we find the evidence less convincing. The model tends to overestimate the educational attainment in the 1990-2004 period and underestimate it for the 2005-2019 period. Figure 14 gives additional indication that the model is not well fitted for a broader European context



Source: Data from OECD - Education and training.
Fig. 14: Education (model/data) - Other Europe

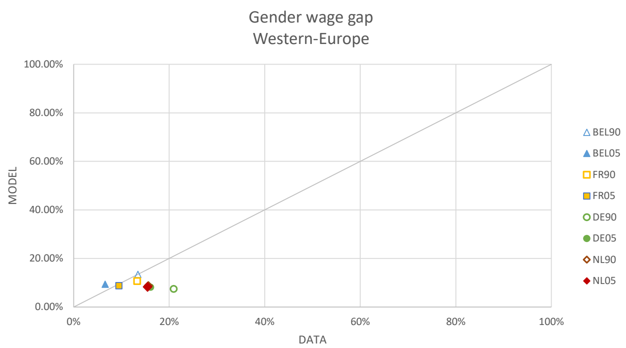
In the next three subsections, the different gaps will be examined. The gaps are important as they are not directly modelled. By looking at the relative earnings, one can calculate the wage and the pension gaps. The aim is to assess whether this simple model accounts for gender and educational disparities.

5.1.3 Gender wage gap

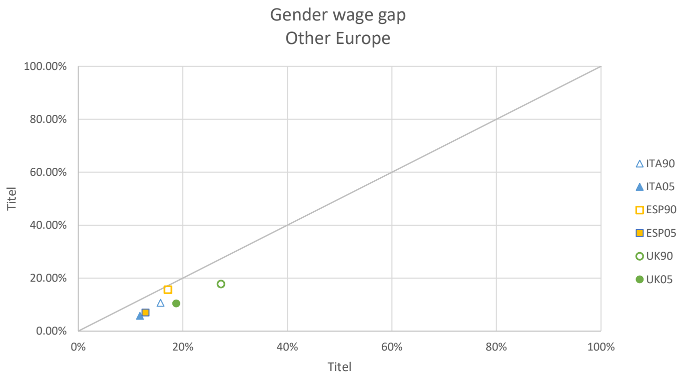
The gender wage gap of the model represents the combined effect of lower labour participation and of lower wages. The data, however, encompases a stylized gender wage gap, only taking into acccount the median income of full-time workers of men and women. This definition is quite different to the model. Hence the model's prediction is rescaled to fit the data for Belgium in 1990-2004 (the calibration reference). This approach is identical to the one used in the previous subsection.

Surprisingly both the Western European and the Other Europe group fit the data very well. The model tends to underestimate the gender wage gap somewhat in Western Europe and persistently underestimates it in Italy, Spain and the UK.

Using the previous results on hours worked, this makes sense. The model overestimates female hours worked and underestimates men's hours. This effect was even stronger among the Other Europe group. If the model had a better fit in gender labour participation it would automatically better fit the gender wage gap.



Source: Data from OECD - Earnings and wages - Gender.
Fig. 15: Gender wage gap (model/data) - Western Europe



Source: Data from OECD - Earnings and wages - Gender.
Fig. 16: Gender wage gap (model/data) - Other Europe

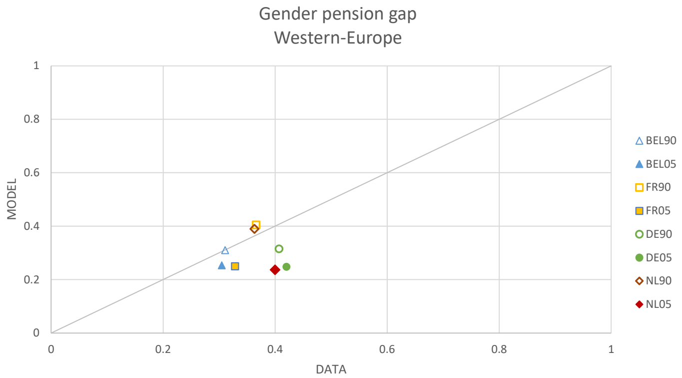
5.1.4 Gender pension gap

A second issue in gender division is the pension gap. The gender pension gap is defined as the fraction of total pensions benefits of women vis-a-vis men. Again a cautionary note, the model only includes the first-pillar pension. The second pension pillar is missing in the model. Savings however continue after retirement and these could be counted as a third pillar pension. Again we use a rescaling to adjust for the definitional differences in pension systems.

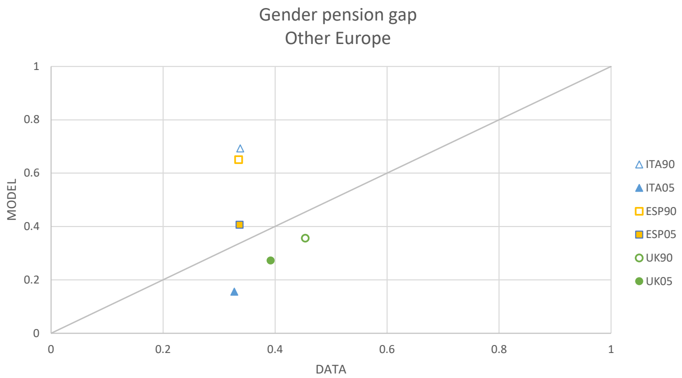
The scatterplot of pensions provide a less convincing story than the wage gap. In the Western-European group the model underestimates the pension gap. In the Other Europe group, the average is about right but the spread between data and prediction is very large.

The underestimation for the Western Europe group can partially be explained. The model uses replacement rates rr_L and rr_H to denote the net pension replacement rates for low and high earners. The model's pension system however ignores eligibility criteria for full pension benifits. In Belgium e.g. a worker is only entitled full pension benefits when she has a full-time career of 45 years. Similar eligibility criteria exist in Europe but is not present in the model.

An ad-hoc solution would be to differentiate the net pension replacement rates by gender. A more general solution would be the inclusion of labour participation rates in the calculation of the replacement rates. This would, however, increase the complexity of the model.



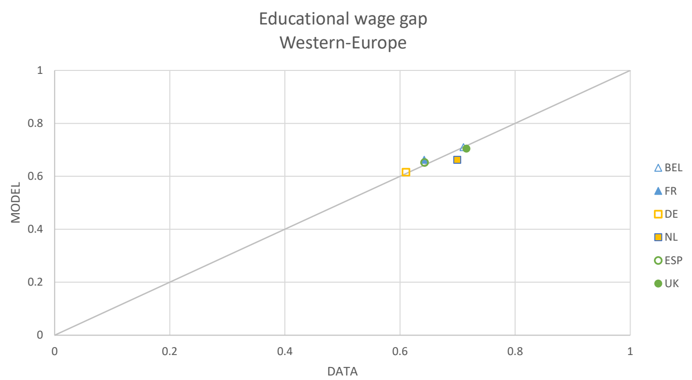
Source: Data from EUROSTAT - Gender Pension gap.
Fig. 17: Gender pension gap (model/data) - Western Europe



Source: Data from EUROSTAT - Gender Pension gap.
Fig. 18: Gender pension gap (model/data) - Other Europe

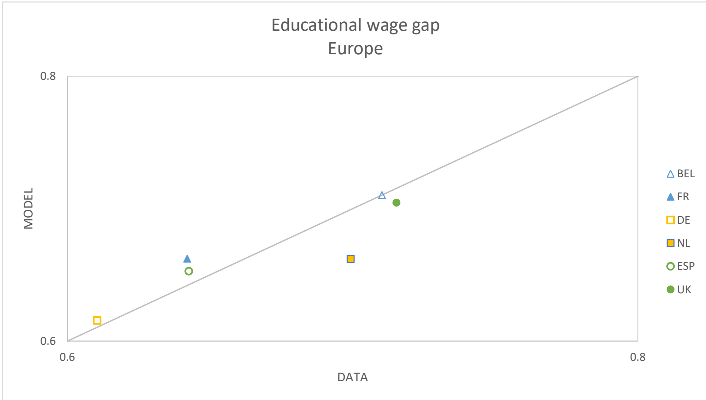
5.1.5 Educational wage gap

The last comparison is the relative returns to schooling. The model's prediction is fitted to data of relative earnings of people with upper-secondary education versus people with tertiary education. This corresponds with the model types of low-skilled and high-skilled labour. Due to data limitation, the only comparison that is possible is for the 2005-2019 period. Below are two graphs, one regular scatterplot and a close-up to assess the differences in greater detail. Despite



Source: Data from OECD - Education and Earnings.
Fig. 19: Educational wage gap (model/data) - Europe

the limitation in observation, the fit of the model to reality is very impressive. A close up of 5-fold magnitude shows that the model explains educational returns very well across both European groups. It remains important to be careful in making claims as this is only a partial comparison.



Source: Data from OECD - Education and Earnings.
Educational wage gap (model/data) - Europe (close-up)

5.2 Summary

The model has mixed results in predicting reality. Overall the model is better fitted to the Western European data. An exception is the educational wage gap, which is fitted excellently across Europe.

A second observation is that the averaged values are better explained than the gender-specific values. In terms of hours worked the model correctly predicts men working more than women, yet the model underestimates the labour participation gap between them.This is a first sign that the model might be too simple to grasp all the nuances in the gender division.

A third observation is that the lack of predictive power in gender labour participation, partially, explains the underestimation of the wage and the pension gap between men and women.

A final observation is that pension system of the model needs to be extended to better grasp the gender dimension. Two possibilities: including pension eligibility criteria or differentiating replacement rates along gender line. The first option is the more general one.

6 Policy evaluation

The first research question can now be answered. *Can household labour division explain the gender gap in wages and pensions?* Yes, to an extend this OLG model shows that household labour division can explain some of the gender differences in hours worked, wages and pensions. The model is imperfect, however, in explaining the size of the gender gaps. The second research question *Can a domestic service subsidy reduce the gender gap?* will be addressed in this chapter.

In this chapter we use the model to simulate a domestic service subsidy. The reasoning being that domestic services can be bough to relax the time constraint of women. More free time means that more time can be spend on working and education, which in turn should decrease the gender wage gap and pension gap.

Giving the good fit for Belgium and the rest of Western Europe, we continue the analysis for this group of countries. The model predictions for Italy, Spain and the United Kingdom is less convincing and so these countries will not be included in further analysis.

6.1 Domestic service subsidy

In order to answer the second research question a permanent policy shock is introduced in the model. Agents will get a subsidy to hire domestic servants. As this type of subsidy already exists in Belgium under the *titres-services* system the shock is modelled after the Belgian case.

This system was introduced in Belgium at the end of 2004. As our model contains periods of 15 years, this works out as a policy shock for Belgium in the period 2005-2019. The subsidy rate is given on top of the employers social contribution rate. In effect allowing low-productivity workers to be hired. What counts is the net subsidy rate $(\tau_{pL} - \tau_d)$. Whilst $\tau_d = 0.4$, the net subsidy factor becomes $(1 + \tau_{pL} - \tau_d) = 0.85$ or correspondingly a net subsidy rate of 0.15. It is this net subsidy rate that will be used as a policy shock for the other countries in our sample.

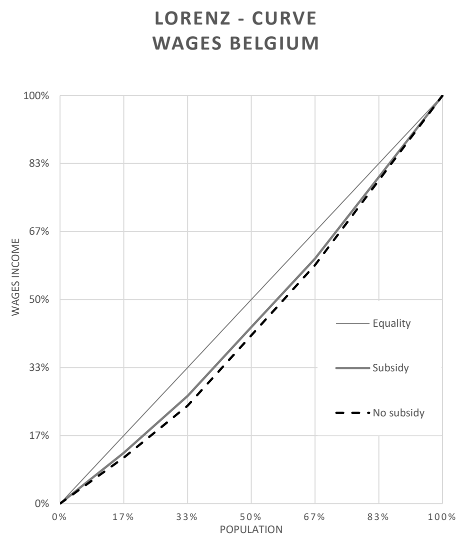
For Belgium, three scenarios are run: (i) a baseline scenario, which includes the *titres-services* system $(\tau_d = 0.40)$, (ii) a counterfactual in which $\tau_d = 0$ and (iii) a benchmark scenario where the bargaining power of men and women are equal $(\zeta = 0.5)$ and $\tau_d = 0$.

For France, The Netherlands and Germany, the same three scenarios are run: (i) a baseline scenario with no domestic service subsidy $\tau_d = 0$, (ii) a counterfactual where $(\tau_{pL} - \tau_d) = -0.15$ and a (iii) benchmark with $\tau_d = 0$ and $\zeta = 0.5$.

In the following section lorenz-curves are plotted for Belgium to show graphically the changes in the gender wage gap and the gender pension gap. At the end of the chapter a table gives an overview of the relative earnings of men versus women among the three scenarios. This table also allows for a cross-country comparison of Western European countries. Finally a table for pensions is presented in the same way.

6.2 Lorenz-curves

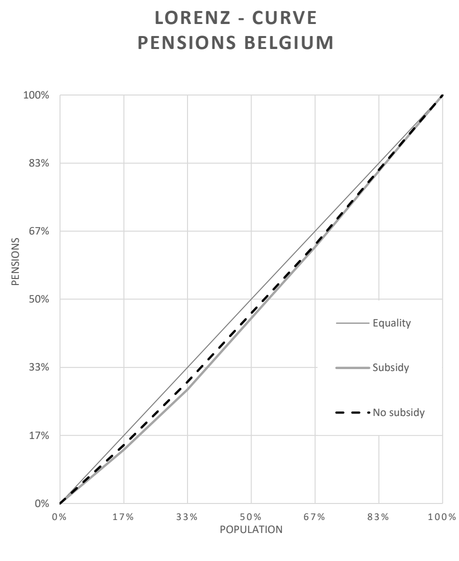
For wages and pensions, two graphs are plotted. The first is a comparison of the the subsidy and the no-subsidy scenario. A second plot compares the subsidy to a benchmark of gender equality. This is obtained by setting the relative bargaining power of men $\zeta = 0.5$.



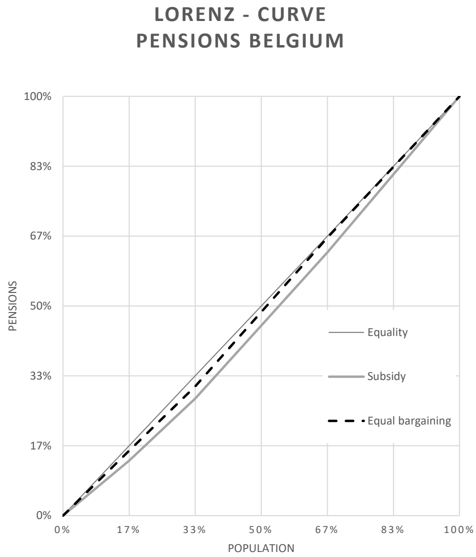
Source: Data from OECD - Earnings and wages - Gender

Fig. 20: Lorenz-curve wages Belgium 2005-2019 (I/II)

Source: Data from OECD - Earnings and wages - Gender
Fig. 21: Lorenz-curve wages Belgium 2005-2019 (II/II)



Source: Data from Eurostat - Gender pension gap
Fig. 22: Lorenz-curve pensions Belgium 2005-2019 (I/II)



Source: Data from Eurostat - Gender pension gap
Fig. 23: Lorenz-curve pensions Belgium 2005-2019 (II/II)

These curves provide some insights into the dynamics of the model. Again we see confirmation that the gender pension gap is not well explained by the model. We also see that the domestic service subsidy has different effects on the pension and on the wage gap. Figure 20 shows that the subsidy pushes the lorenz-curve inwards, which means that the wage inequality has decreased. This is in accordance with expectations of the model. The shift is more visible at the lower side of the income-distribution. Figure 21 indicates that the subsidy performs well: the subsidy line comes in close contact to the benchmark.

Turning to pensions, the subsidy does not seem to help the pension gap. The subsidy increases the pension gap. How can this be explained? The answer can be found in the funding of the government. The subsidy to domestic services is a serious drain to government funds. By design the model keeps the debt-to-gdp ratio constant. If finances change, these are offset by lump-sum taxes. Whilst workers have the possibility to compensate these lump-sum taxes by increasing labour participation, retired agents have no alternative but to see their budget decrease.

6.3 Gender gaps

Below the reader finds an overview of the three simulations. Data is presented in full at the start of the policy shock (2005-2019). In addition the percentage-point differences between the policy shock and the baseline are presented for the period of impact and for the steady state (2060-2075). Each variable is divided in the overall gender gap "(wF/wM)" and a gender gap within the low-earners "(wF/wM) Low" and the high earners "(wF/wM) High". The same distinction is also made for the pension gap.

Tab. 3: Policy evaluation - the relative wages of women and men

Country	Variable	No subsidy	net subsidy rate of 0.15	benchmark	Δ %-point at impact	Δ %-point Steady State
BEL	wF/wM	0.842	0.856	0.975	1.422	2.178
	(wF/wM) Low	0.875	0.886	1.000	1.059	1.960
	(wFwM) High	0.822	0.836	0.953	1.380	2.071
FR	wF/wM	0.844	0.874	0.998	3.016	4.578
	(wF/wM) Low	0.862	0.897	1.000	3.442	5.157
	(wFwM) High	0.832	0.855	0.980	2.297	3.595
NL	wF/wM	0.868	0.903	0.990	3.523	5.024
	(wF/wM) Low	0.881	0.934	1.000	5.334	7.477
	(wFwM) High	0.859	0.875	0.975	1.570	1.898
DE	wF/wM	0.879	0.893	0.991	1.354	1.903
	(wF/wM) Low	0.895	0.904	1.000	0.966	1.426
	(wFwM) High	0.870	0.885	0.977	1.452	2.113

Tab. 4: Policy evaluation - the relative pensions of women and men

Country	Variable	No subsidy	net subsidy rate of 0.15	benchmark	Δ %-point at impact	Δ %-point Steady State
BEL	pF/pM	0.804	0.803	0.826	-0.036	2.165
	(pF/pM) Low	0.828	0.827	0.849	-0.106	1.960
	(pFpM) High	0.778	0.778	0.801	-0.036	2.071
FR	pF/pM	0.823	0.823	0.849	-0.027	4.635
	(pF/pM) Low	0.836	0.834	0.861	-0.171	5.157
	(pFpM) High	0.811	0.812	0.838	0.066	3.595
NL	pF/pM	0.847	0.848	0.865	0.139	5.256
	(pF/pM) Low	0.852	0.854	0.870	0.136	7.477
	(pFpM) High	0.843	0.844	0.861	0.130	1.898
DE	pF/pM	0.866	0.866	0.887	-0.016	1.869
	(pF/pM) Low	0.881	0.879	0.900	-0.125	1.426
	(pFpM) High	0.856	0.856	0.877	0.031	2.113

Tables 3 and 4 provide additional insights into the working of the domestic services subsidy. Whilst the policy shock ($\tau_{pL} - \tau_d$) is the same, the impact of the policy is remarkably different across countries.

The policy has the highest impact on France and the Netherlands. The short run impact of the subsidy on the gender wage gap is uniformly positive across all countries. The effect ranges from a 1 to a 5 percentage-point decrease in the wage gap. In the long-run these numbers are higher, ranging from 1.5 to 7.5 percentages points.

We should be hesitant to take these numbers literally. The model uses a broader definition of the gender wage gap. The most important one being labour participation. The gender-gap decrease in the model is mainly due to higher labour participation.

Focussing on pensions, we get quasi the same results in the long-run as for the wagegap. This is to be expected, as the model calculates pensions as a fraction of the previous labour income. But before the model gets to steady state a different story occurs. At impact the subsidy increases the pension gap in half the cases.

As stated in the previous section, government finances are to blame. The model fixes the debt-to-GDP ratio at the historical data. When a shock occurs, the model balances the budget by imposing a uniform lump-sum tax. As a result, retirees see their disposable income decrease. In the definition of pension we use the first-pension pillar and savings. Whilst not impacting the first pillar, a lump-sum tax does impact the wealth of agents.

Second research question

The first research question was answered positively, but with some concerns. Now we are ready to answer the second research question:

Can a domestic service subsidy reduce the gender gap? Following the results from table 3 and 4, the answer is again "yes". Domestic services allow for an immediate and stable improvement of the gender wage gap. But taking the limitations of the model, one should take the numbers as an indication rather than a projection.

Assessing the effects on the pension gap proves more difficult as the model does not fit the data extremely well. The model also lacks some elements that could greatly improve the predictive power of the pension gap. The most important one being pension eligibility criteria or, alternatively, gender specific replacement rates.

7 Conclusion

An overlapping generation models with household labour division along gender lines has proved to be an interesting tool to assess the gender gaps in society. The simple model in which male and females are deemed identical but differently constrained has been able to reproduce the societal differences in hours work and the wage gap. The size of these participation and wage gaps are however underestimated by the model.

Returning to the two research questions: (i) *Can household labour division explain the gender gap?*, and (ii) *Can a domestic service subsidy reduce the gender gap?* the answers are clear. Yes, household labour division along gender lines can explain the gender gap, but only part of it. Likewise for question two: yes, a subsidy to domestic services can reduce the gender gap. For both questions the model is better able to explain the gender wage gap and labour participation. Unfortunately, the model is limited in predicting the pension gap.

Another limitation to the model is that it can not be externally validated for countries outside Western Europe. Within Western Europe, the model did well in predicting the data.

As stated earlier, the simple model might be too simple. Some extensions to the model, especially in the pension design might increase the closeness of fit and increase the predictive power along gender lines. These extensions are left for future research.

8 References

Becker, G., (1965), A Theory of Allocation of Time, Economic Journal, 75, 493–517.

Biffl, G. (2007) : The Employment of Women in the European Union,WIFO Working Papers, No. 297, Austrian Institute of Economic Research (WIFO), Vienna

Bouzahzah, M., de la Croix, D. and Docquier, F. (2002). Policy reforms and growth in computational OLG economies, Journal of Economic Dynamics and Control, 26, 2093–2113.

Buyse, T., Heylen, F., and Van de Kerckhove, R. (2017). Pension reform in an OLG model with heterogeneous abilities. Journal of Pension Economics and Finance, 16, 144–172.

Caselli, F., Coleman, I., and John, W. (2006). The world technology frontier. American Economic Review, 96(3), 499–522.

Cette, G., Lecat, R. and Ly-Marin, C. (2017): "Long-term growth and productivity projections in advanced countries," Banque de France. Working paper No. 617

de la Croix, D., Pierrard, O., and Sneessens, H. R. (2013). Aging and pensions in general equilibrium: Labor market imperfections matter. Journal of Economic Dynamics and Control, 37(1), 104-124.

Devriendt, W., and Heylen, F. (2020). Macroeconomic and distributional effects of demographic change in an open economy—the case of Belgium. Journal of Demographic Economics, 86(1), 87-124

Docquier, F. and Paddison, O. (2003) Social security benefit rules, growth and inequality. Journal of Macroeconomics, 25, 47–71.

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The next generation of the Penn World Table. American Economic Review, 105(10), 3150–3182

Greenwood, J., Seshadri, A., & Yorukoglu M. (2005). Engines of Liberation. The Review of Economic Studies,Vol. 72, N.1, pp. 109-133

IMF (2021). Fiscal Monitor April 2021. World economic and financial surveys, International Monetary Fund.

Lucas, R. (1990). Supply-side economics: an analytical review. Oxford Economic Papers, 42:292–316.

Ludwig, A., Schelkle, T., and Vogel, E. (2011). Online appendix to "demographic change, human capital and welfare". Review of Economic Dynamics. Technical Appendices 08-168.

McDaniel, C. (2007). Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003. Manuscript, Arizona State University.

Mérette, M., and Georges, P. (2009). Demographic changes and the gains from globalisation: an overlapping generations CGE analysis.

Ngai, L.R., and Pissarides, C. (2008). 'Trends in Hours and Economic Growth'. Review of Economic Dynamics 11(2): 239–56.

OECD (2005), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2007), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2009), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2011), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2013), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2015), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2017), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

OECD (2019), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris.

Prescott, E. (2004). Why do Americans work so much more than Europeans?, Quarterly Review, Federal Reserve Bank of Minneapolis, vol. 28(Jul), pages 2-13.

Rogerson, R. (2007). Taxation and market work: is Scandinavia an outlier? Economic Theory, 32(1):59–85.

Rogerson, R. and Wallenius, J. (2009). Micro and macro elasticities in a life cycle model with taxes. Journal of Economic Theory, 144:2277–2292.

Selleslagh (2019). Hoeveel kost uw poetshulp echt? De Tijd. www.tijd.be/netto/budget/hoeveel-kost-uw-poetshulp-echt/10185884.html

United Nations. (2019). World population prospects 2019: highlights. Department of Economic and Social Affairs, Population Division.

Vargha, L., Šeme, A., Gál, R., I., Hammer, B., Sambt, J. (2016): European National Time Transfer Accounts. Available at: http://www.wittgensteincentre.org/ntadata

Van Daal, J, and Merkies, A. (1989). A Note on the Quadratic Expenditure Model. Econometrica 57(6): 1439.

Appendices

Appendix A First Order Conditions

Appendix A show the different equations underlying optimizing behaviour for households. First comes the Euler equations regarding consumption, followed by the Labour-Leisure choice that agents make. Thirdly, the education choice is shown. Lastly, the choice for domestic services is reported.

A.1 Euler Equation

$$\frac{c^t_{j+1,a,g}(1+\tau_c)}{c^t_{j,a,g}(1+\tau_c)} = \beta\rho^t_j(1+r_{t+j}) \tag{A.1}$$

Note: Subscript j denotes the age of the agent, the superscript t denotes that this agent is born into to the model at time t. The subscript a denotes ability type and subscript g denotes gender.

With $c^t_{j,a,g}$ denoting consumption, τ_c as consumption tax, β time preference discount factor, ρ denoting conditional probability to survive the next period and r denoting the economy-wide interest rate.

A.2 Labour-Leisure choice

Ages 15-59 (for j = 1 to 3)

$$\begin{aligned} \beta^{j-1}\pi_j\frac{\gamma_j}{l^\theta_j} = & \frac{\beta^{j-1}\pi_j}{c^t_{j,a,g}(1+\tau_c)}\big(w_{a,t}x^t_{j,a,g}(1-\tau_w)\big) \\ & + \frac{\beta^3\pi_4}{c^t_{4,a,g}(1+\tau_c)}(1-R^t_g)\frac{\partial p^t_{a,g}}{\partial n^t_{j,a,g}} \\ & + \sum_{i=5}^{6}\frac{\beta^{i-1}\pi_i}{c^t_{i,a,g}(1+\tau_c)}\frac{\partial p^t_{a,g}}{\partial n^t_{j,a,g}} \end{aligned} \tag{A.2}$$

With:

$$\frac{\partial p^t_{a,g}}{\partial n_{j,a,g}} = rr^t_a s_j w_{t,a} x^t_{j,a,g}(1-\tau_w)$$

Where $\frac{\partial p^t_{a,g}}{\partial n_{j,a,g}}$ denotes the return to working to pension benefits. rr^t_a is the net replacement rate on earnings, s_j the pension weight of work during the j^{th} period of life.

Ages 60-74 (for j = 4)

$$\frac{\gamma_4}{l_4^\theta} = \frac{R_g^t}{c_{4,a,g}^t(1+\tau_c)}\big(w_{a,t}x_{4,a,g}^t(1-\tau_w)\big) + \frac{(1-R_g^t)}{c_{4,a,g}^t(1+\tau_c)}\frac{\partial p_{a,g}^t}{\partial n_{4,a,g}^t} + \sum_{i=5}^{6}\frac{\beta^{i-4}\pi_i/\pi_4}{c_{i,a,g}^t(1+\tau_c)}\frac{\partial p_{a,g}^t}{\partial n_{4,a,g}^t} \tag{A.3}$$

With:

$$\frac{\partial p_{a,g}^t}{\partial n_{4,a,g}} = rr_a^t s_4 w_{t,a} x_{4,a,g}^t(1-\tau_w)$$

A.3 Education choice

$$\frac{\gamma_1}{l_1^\theta} - \frac{1}{(1+\tau_c)c_{1,a,g}^t} = \sum_{j=2}^{3}\beta^{j-1}\pi_j\frac{w_{a,t}n_{j,a,g}^t(1-\tau_w)}{(1-\tau_c)c_{j,a,g}^t}\frac{\partial x_{j,a,g}^t}{\partial e_g^t} + \beta^3\pi_4\left[R_g^t w_{a,t}n_{4,a,g}^t(1-\tau_w)\frac{\partial x_{j,a,g}^t}{\partial e_g^t} + (1-R_g^t)\frac{\partial p_{a,g}^t}{\partial e_g^t}\right] + \sum_{i=5}^{6}\beta^{j-1}\pi_j\frac{\partial p_{a,g}^t}{\partial e_g^t} \tag{A.4}$$

With:

$$\frac{\partial x_{j,a,g}^t}{\partial e_g^t} = x_{1,H,g}^t\ \sigma\ \phi(e_g^t)^{\sigma-1}$$

$$\frac{\partial p_{a,g}^t}{\partial e_g} = rr_a^t s_j w_{t,a} n_{j,a,g}^t(1-\tau_w)\frac{\partial x_{j,a,g}^t}{\partial e_g^t}$$

A.4 Domestic services

for males

$$\gamma_d d_{j,a,M} = \gamma_d(1-\zeta)\bar{h}_{j,a} - \left(\frac{w_L x_{j,a,M}^t(1+\tau_p-\tau_d)}{c_{j,a,M}^t(1+\tau_c)}\right) + \gamma_j l_{j,a,M}^{-\theta} \tag{A.5}$$

for females

$$\gamma_d d_{j,a,F} = \gamma_d\zeta\bar{h}_{j,a} - \left(\frac{w_L x_{j,a,F}^t(1+\tau_p-\tau_d)}{c_{j,a,F}^t(1+\tau_c)}\right) + \gamma_j l_{j,a,F}^{-\theta} \tag{A.6}$$