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Volatility as a Hedging Framework: Portfolio Analysis

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May the financial markets give up their beautiful secrets.

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Part I

Introduction

The goal of every asset manager is to construct robust portfolios combined to great returns through time. However nowadays various regulatory constraints have forced asset managers to track downside risk. Traditional portfolio management introduces negatively correlated assets to the portfolio in order to hedge it. Although asset classes such as bonds and commodities did well in the past for hedging purposes it has not always been the case. Evidences show that asset correlations turned positive during the 2008 crisis, annihilating all portfolio benefits (Szado 2009). In the search of the best asset class offering strong negative correlation with equities and even stronger during market shocks it appears that volatility may offer an answer. In our study we examine the benefits of a long volatility position within a diversified portfolio and consider its application on the market.

Equity volatility trading is a quite recent scope of investment interest. The conventional way to gain equity market exposure is to trade options. By buying some options it's possible to hedge positions or an entire portfolio. However options are directional besides offering the possibility to gain exposure to volatility. It means considerable resources and engineering to construct a delta-hedge portfolio. Another issue of options is the time decay that characterizes them. In other word, their value decreases through the option life obligating managers to constantly roll their positions.

The introduction of VIX index in 2004 did solve the problem of options directionality and time decay to only retain the volatility exposure. VIX index is commonly referred to as the "investor fear gauge" by transcribing the stress of the market. Whaley (2008) defines it as follows: "VIX is *implied* by the current prices of S&P 500 index options and represents expected *future* market volatility over the next 30 calendar days." Likewise options it has a negative correlation with the underlying equities and is explained in the literature through the "leverage effect" and the volatility "feedback effect". We focus on this recent volatility exposure mean to analyze the alternatives to common diversification.

The purpose of the study is constructed around four different axes. Firstly our goal is to propose an up-to-date insight about VIX index performances during recent market turmoils. Due to the asymmetry in volatility VIX index is negatively correlated to S&P 500. We examine this evidence and compare the correlation of VIX and S&P 500 to other asset classes that are normally used to hedge an equity portfolio.

Secondly we constructed an optimized portfolio with a VIX allocation for the period from June 2006 to June 2015 and compared it to an optimal portfolio without VIX allocation. An efficient frontier is generated which determines the optimal weight for the VIX index allocation and serves as a point of reference for further allocation. The "modus operandi" is also used for the recession period. This shed the light on the effect of VIX allocation to a portfolio and his related performances during different periods.

Thirdly based on the results obtained previously an investable portfolio strategy is constructed and compared to a traditional 60/40 strategy. The volatility index strategy has been replicated with encouraging results. Although the strategy isn't return generating like the optimized portfolio with a VIX allocation does, volatility hedge plays his role. It permits to protect the portfolio from downsides in the market. In the light of these results a refined investment strategy is examined to retain the best of both world. A tail hedge strategy that generates returns either in a bull market or a bear market.

Lastly the portfolio is separated into two parts. On the one hand a hedging part and on the other hand a return generating part. Volatility is present as previously in order to hedge the portfolio with an allocation to a leverage volatility position and is also present in the form of the inverse volatility. Last strategy supports our previous observations and generates substantial risk adjusted performances but embeds some relative tail risk.

The construction of portfolio exhibiting less risk and stronger returns also benefits to the lambda investors. It will permit to everyone to invest into robust portfolios without being feared to lose everything during a market shock. The financial market has to gain back the confidence of investors it could be a solution.

Finally our paper is constructed as follows. Part 2. defines volatility as a whole by defining options and the VIX. Part 3. concentrates on the literature review and essential theories of principal volatility mechanisms. Part 4. examines the different portfolios with VIX allocation to test the benefits of volatility when included to a diversified portfolio. Part 5. concludes

Part II

Volatility

Chapter 1

What is an option?

The concept of trading options is very ancient. The philosopher and mathematician Thales of Miletus was probably the first options speculator in the history. 7^{th} century BCE, predicting a bountiful harvest year, he put a deposit on olive presses. When the olive season came, the harvest were more abundant than expected he sold his rights on the presses and made large profit due to an overwhelming presses demand. The concept option was born (Wiley and Sons 1998).

Options popularity increased by the establishment of the Chicago Board Options Exchange (CBOE) in 1973. Exchange-traded markets permitted to organize the market and better define contracts. But not all option contracts are traded on exchange-traded markets, over-the-counter markets became rapidly very popular. Figure 1 points out this situations and depicts the evolution of the international derivatives market.

The first call option contracts started trading in 1973. four years later put option contracts were traded. 1973 was also the year of the discovery of the famous Black-Scholes option pricing model (Black and Scholes 1973). As a result, the understanding around options increased as its interest by the financial world. The interest of options reside in their considerable leverage effect. Possessing a single option means controlling 100 underlying shares. Consequently volatility's options is defined by the implied volatility of the underlying. The complexity of options lies in its constituents. Indeed a winning option hedge or bet should take into account the right movement of the underlying, the magnitude and finally the timing. The appearance of derivatives in the financial world

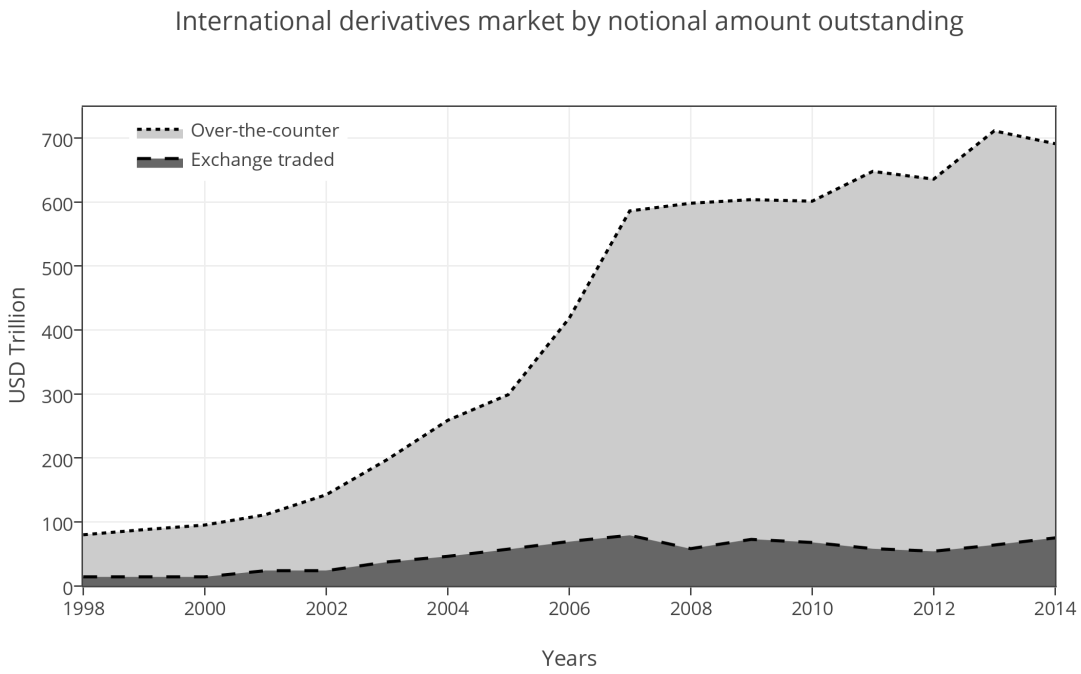


Figure 1: OTC markets pronounced expansion compared to the Exchange traded markets during these last decades. Source:(Bank for International Settlements 2014)

and its versatility opened a wide range of possibilities for large corporation and investors. It allowed them to hedge against sudden market decline or exchange rate fluctuation. It became also very simple for them to speculate or to profit from arbitrage if some exists. A fundamental difference of options over other derivatives is the fact that the option gives the holder the right to buy or sell the underlying but is not required to exercise it. The type of option determines the possibility of exercising. Therefore European options can only be exercised on the expiration date, whereas American options can be exercised during their entire life up to the expiration date (Hull 2009).

The global picture of options having been delimited, let's study into more details the intrinsic features of options. The following section will concentrate on the factor affecting the price of an option, the different types of options, the underlying, the linked costs. Finally an overview of the option trading type will be drawn.

1.1 Types of options

We can distinguish two basic types of options. A call option gives the holder the right to buy an asset for a certain price and within a period of time. On the contrary, a put option gives the holder the right to sell an asset for a certain price and within a period of time (Hull 2009). It's also possible for investors to sell (write[1]) options instead of buying them. An interesting observation is, according to CBOE[2], only 10% of the options are exercised. This asymmetry between buying and writing options is interesting because it's reasonable in this case to think the odds are in investor's favor and to consider option writing as an advantageous naked strategy compared to option buying.

Owing to the versatility that characterize options, buying or selling options could be considered as part of an investment plan. Some hedge funds based their portfolio strategy by being long on stocks and short on a smaller quantity of stocks by the intermediary of options buying. This is the case of the hedge fund Pershing Square Capital Management with his famous activist CEO Bill Ackman[3]. Another strategy is option writing instead of buying option. Warren Buffett used this strategy several times although considering derivatives as *time bombs for the parties that deal in them and the economic system* (Berkshire Hathaway 2003). In April 1993, he wrote out-of-the-money Coca-Cola (KO) put options for a strike price he considered to be a fair price if the put options are exercised. By writing put options Warren Buffett earned $7.5 million for being agree to pay a Coca-Cola (KO) share less than the actual market price (StreetAuthority 2015).

1.1.1 Buying options

Calls

The most basic strategy assimilated with call buying is to benefit from upward price movements. Assuming an investor buying a June call option with a strike price of $40.

[1]Writing naked options is equivalent to writing options without owning the underlying.

[2]Old myth telling 90% of options expire worthless die hard. In fact CBOE clarified the situation as follow: about 10% of options are exercised, 30 to 35% of options expire worthless and approximately 55% to 60% of options are traded out prior their expiration date. It should be noted that 90% of options are unexercised which is different than expiring worthless (Chicago Board Options Exchange 2001)

[3]In 2014, Pershing Square Capital Management was the most profitable hedge fund with a net return of 32,8% (Durden 2015). It detained a portfolio of 18$ billion composed by 9 long positions, one short position on Herbalife and a second short position that has to be disclosed (PSCM 2015)

It gives him the right to purchase 100 stocks at the strike price before option expiration in June [4]. Considering underlying price raises to $45 before option expiration, option's premium will consequently move upwards. Two choices are of interest.

- Closing out the position by selling the in-the-money[5] option back into the market. The difference between premium paid and received determines the profit.
- Exercising the option allows the investor to buy the underlying stocks at the strike price and sell them simultaneously at the market price. The difference between option premium paid and stocks sale determines the trade profitability.

The payoff of a long European call option is represented by:

$$max(S_T - K, 0) \tag{1.1}$$

With S_T representing the spot price of the underlying and K the exercise price. If S_T is greater than K the option will be exercised. On the contrary if S_T is smaller or equal to K the option won't be exercised.

Nevertheless as the underlying stock fall below the strike price of $40, the investor isn't going to exercise the option. It is cheaper to buy the underlying directly at market price rather than at strike price. To partly offset the premium paid it remains possible, to sell the option back into the market before expiration otherwise it will expire worthless. Figure 2 draws the profit function of buying a call option.

Buying call options is advantageous to lock in a share purchase price. Assuming an investor desires to buy 100 stocks of a company at a certain price but doesn't have sufficient capital. Investing in one option gives him this opportunity for less capital as underlines (Black 1975) *If options are priced according to the formula, then the "net money" in a stock position is always more than the net money in an equivalent option position.*

Puts

The investor has the possibility to be protected or to speculate on downward price movements by buying put options. Unlike short selling, when buying put options the risk

[4]Expiration date is pre-determined. US exchange-listed equity stock option contracts expire on third Saturday following third Friday of expiration month.

[5]An option is said in-the-money (ITM) as soon as the price of the underlying exceeds the strike price.

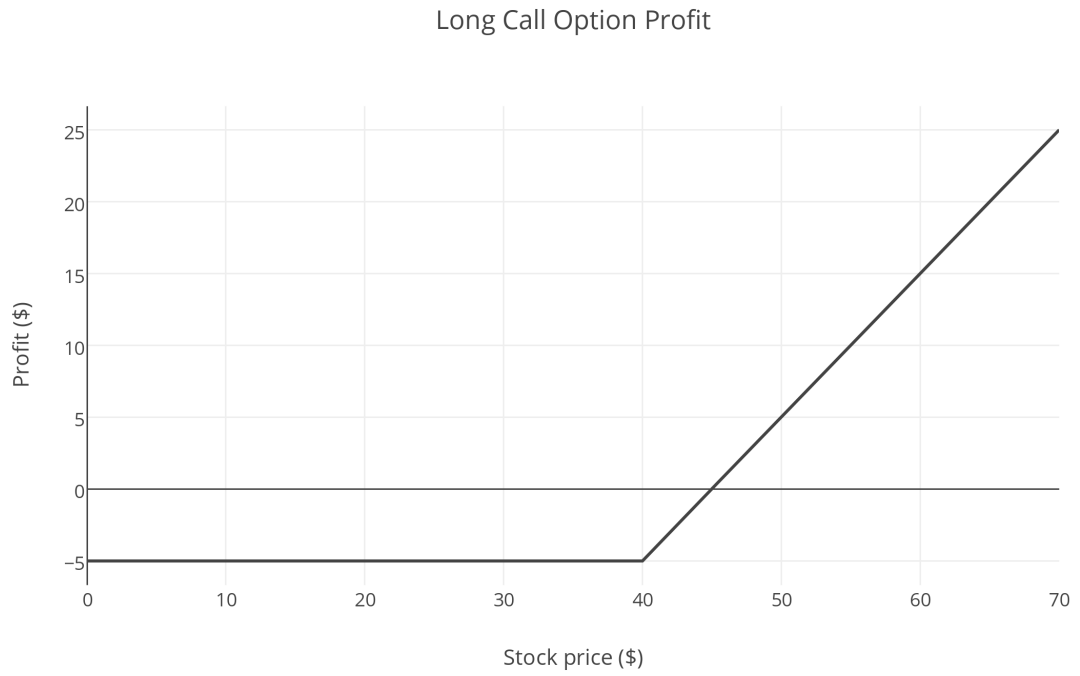


Figure 2: Profit from buying a call option, transaction costs are ignored. Premium = \$5; strike price = \$40.

is limited to the premium paid for the option. Moreover shorting stocks requires a margin account and margin calls could force the investor to cover its position prematurely which is not the case with put options (Cboe 1994). Assuming an investor buying June put option with a strike price of \$40. It gives him the right to sell 100 stocks at the strike price before expiration in June. Considering underlying price fall to \$35 before option expiration, option's premium will consequently move upwards. Two choices are of interest.

- Closing out the position by selling the in-the-money option back into the market. The difference between premium paid and received determines the profit.
- Exercising the option allows the investor to buy the underlying at the market price and sell them simultaneously at the strike price. The difference between option premium paid and stocks sale determines the trade profitability.

The payoff of a long European put option is represented by:

$$max(K - S_T, 0) \tag{1.2}$$

Investors could consider investing into stocks and put options in order to neutralize or limit the downward price movement. This strategy could be assimilated to hedging. Buying naked put options is considered as speculative by betting a fall of underlying prices without any hedge if prices evolve in opposite direction. In that case the loss will simply be the premium paid. Figure 3 presents the profit function of buying a put option.

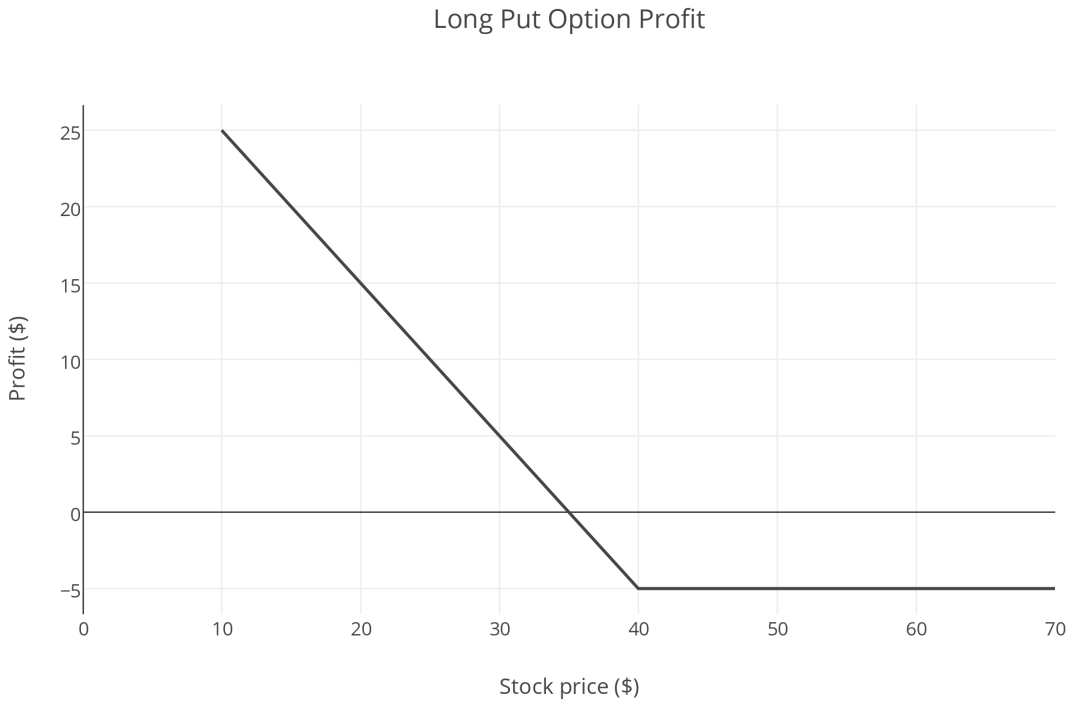


Figure 3: Profit from buying a put option, transaction costs are ignored. Premium = $5; strike price = $40.

1.1.2 Writing options

Calls

On one side there is the buyer of the option, taking a long position and on the other side there is the seller of the option, also know as the writer. He is taking a short position. As being short on a call position, the writer has the obligation to sell at the strike price when the buyer exercises his right. A margin should be deposit by the seller to assure the repurchase of the stock if he is assigned. This obligation is remunerated by a premium paid at the time the call is sold. Because of the underlying stock price infinite rise risk,

the loss is unlimited. A very common strategy when call writing is to cover a long position in the underlying stock. Benefits of using this type of strategy are twice:

- Earning the premium on the call options sold.
- Gaining some protection, limited to the amount of the premium, if the underlying stock declines.

The payoff of a short European call option is represented by:

$$-max(S_T - K, 0) = (K - S_T, 0) \tag{1.3}$$

The left hand side of the equation reflects the exercise of the option if S_T is greater than K and the right hand side reflects non-exercise of the option if S_T is smaller or equal to K. In this last case the only gain to the holder of a short position in a European call option is the premium received at selling.

Figure 4 presents the profit function of writing a call option.

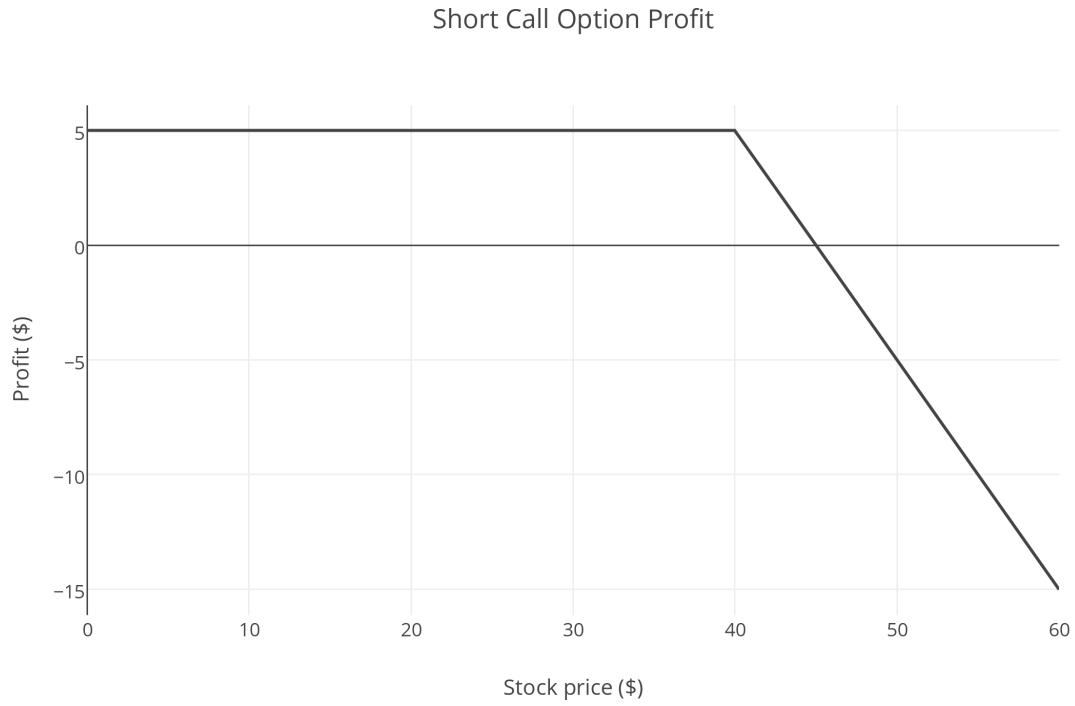


Figure 4: Profit from writing a call option, transaction costs are ignored. Premium = \$5; strike price = \$40.

Puts

As for short position in call options, an investor has the possibility to write put options. It's said he has a short position in a put option. By selling put options the investor is confident about a future increase in the value of the underlying asset. In the long run, if the underlying asset doesn't move much, the short option position will do better than the short stock position. Conversely, if the stock moves much in the short run, the short option position do worse than the short stock position (Black 1975).

Uncovered put writing has potential loss if the option is exercised when the underlying falls below the strike price. Profit is generated by the premium collection. Motivation of selling put options are twice:

- Collecting the premium on the put options sold.
- Acquire stock at a predetermined price below the current market price.

A covered put writing position is created by having an equivalent short stock position. The gain is limited to the premium received and the difference between the stock price of the short position and the strike price of the put. Potential loss is unlimited but partially offset by the premium collected.

The payoff of a short European put option is represented by:

$$-max(K - S_T, 0) = min(S_T - K, 0) \tag{1.4}$$

Figure 5 presents the profit function of writing a put option.

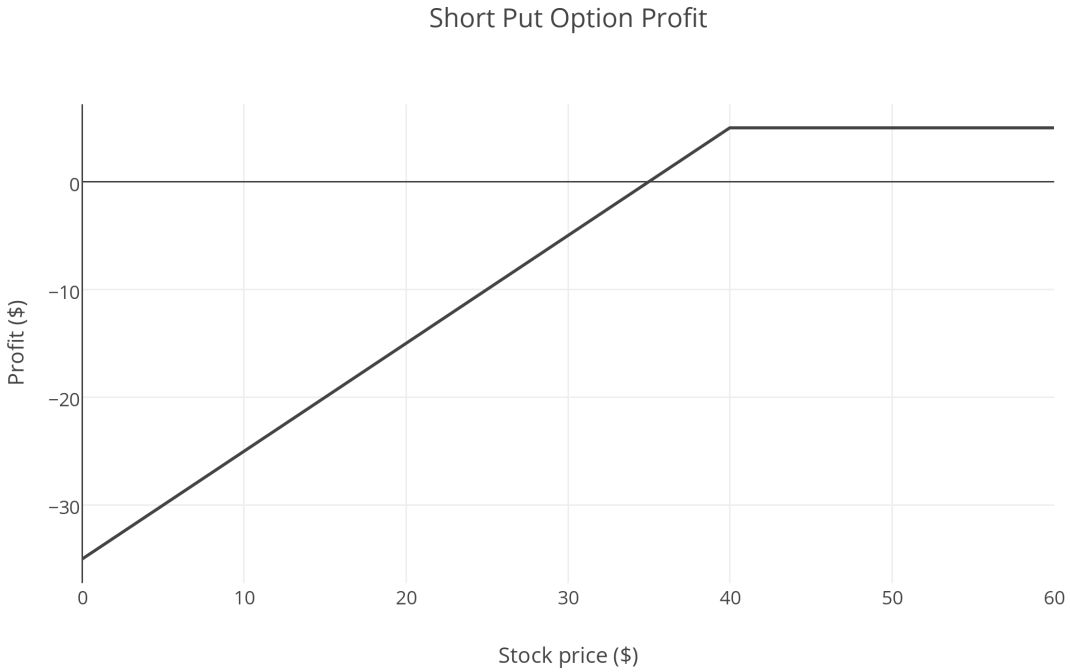


Figure 5: Profit from writing a put option, transaction costs are ignored. Premium = \$5; strike price = \$40.

1.2 Option hedging

When constructing a portfolio managers seek generally to hedge it. One way to hedge it is with options. Options are directional so if one portfolio manager is expecting a downturn in the market he has the choice to hedge his positions individually or via an index. Buy puts is the most direct way of hedging a portfolio but there are plenty of possibilities with strategies having most of the time bird names. The hedging process relies on the asymmetry between volatility and equities (see: chapter 2 2.3). In other words when equities go down volatility increases and conversely.

The Chicago Board Options Exchange created various examples of equities strategies hedged by options like the CBOE S&P 500 BuyWrite Index (BXM) and is defined on CBOE website as follows "The BXM is a passive total return index based on (1) buying an S&P 500 stock index portfolio, and (2) "writing" (or selling) the near-term S&P 500 Index (SPXSM) "covered" call option, generally on the third Friday of each month. The SPX call written will have about one month remaining to expiration, with an exercise price just above the prevailing index level (i.e., slightly out of the money). The SPX call is held until expiration and cash settled, at which time a new one-month, near-the-money call is written" (CBOE 2015).

By selling calls on the index the strategy will profit from downturns in the market. That's precisely what is observable during the 2008 crisis (see: Figure 6) while when the market is bullish the strategy because the options are probably exercised causing a loss even if the strategy is covered by the detention of the underlying stock.

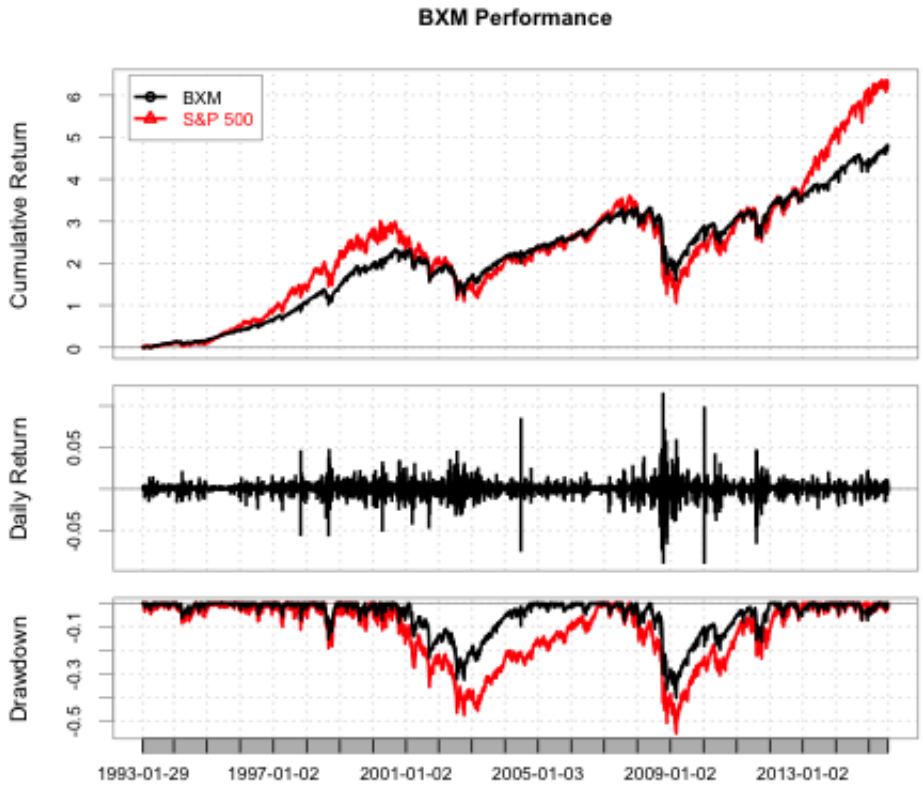

BXM Performance
BXM
S&P 500
Cumulative Return
0
1
2
3
4
5
6
Daily Return
0.05
-0.05
Drawdown
-0.1
-0.3
-0.5
1993-01-29
1997-01-02
2001-01-02
2005-01-03
2009-01-02
2013-01-02

Figure 6

Chapter 2

VIX Index

2.1 History

VIX was launched in 1993 by Professor Robert E. Whaley to provide a picture of market volatility. The intention behind the introduction of VIX index was to provide an index upon which futures and options contracts could be written (Whaley 2008). VIX futures and options contracts were introduced by Chicago Board Options Exchange respectively in May 2004 and February 2006. It worth emphasizing that the volatility index is forward-looking and traduces the future volatility expected by investors. In such Whaley (2008) defines it as follows, "VIX is *implied* by the current prices of S&P 500 index options and represents expected *future* market volatility over the next 30 calendar days".

2.2 Calculation

In 1993 when the VIX index was introduced the index was base on the S&P 100 prices. Moreover the price of the index depended only of eight at-the-money index calls and puts because they were by far the most traded at that time (Whaley 2008). Through years the S&P 500 (SPX) became the most active index option market. Thus in 2003 VIX index calculation changed to take into account this shift of index option trading and the dominance of out-of-the money and at-the-money index puts. This new trend in option market was due to the portfolio insurance needs. In fact Bollen and Whaley (2004) show the importance of out-of-the money and at-the-money SPX index puts as a key driver in

SPX implied volatility measures. For insurance purposes managers included also out-of-the money puts to their portfolio. The new VIX index included this change that contains a lot of information about the market routine. Including individual options prices reduced manipulation about the VIX (Whaley 2008).

VIX index has been qualified as the "Investor Fear Gauge" because it translates the unexpected moves on the S&P 500. Furthermore S&P 500 index option market is dominated by hedgers buying puts on the index in order to face downturns in the market. The demand of insurance increased the price of options hence the increase of the VIX index.

2.3 Relation to the stock market

The asymmetry between VIX index and SPX is explained by the need of insurance. The proportion of puts bought is greater than the proportion of calls bought on the market because investors are generally risk averse. This makes the VIX index more a barometer of fear than a barometer of optimism. The relation explains why VIX index gains negative correlation during downturns in the market. Figure 7 shows the relation between VIX and S&P 500. VIX reacted in 2001 during the dot com bust and 11 September 2001 traducing the panic on the market. The same story happens during 2008 but with a stronger reaction of VIX due to the increased hedging demand. Such a behavior is very beneficial to a portfolio of equities because it reacts in opposite direction.

On a theoretical ground VIX index is a mean-reverting process. The value of VIX tends to reach it's mean and is so be definition a non-generating return process (Whaley 2008; Moran and Dash 2007; Engle 2004).

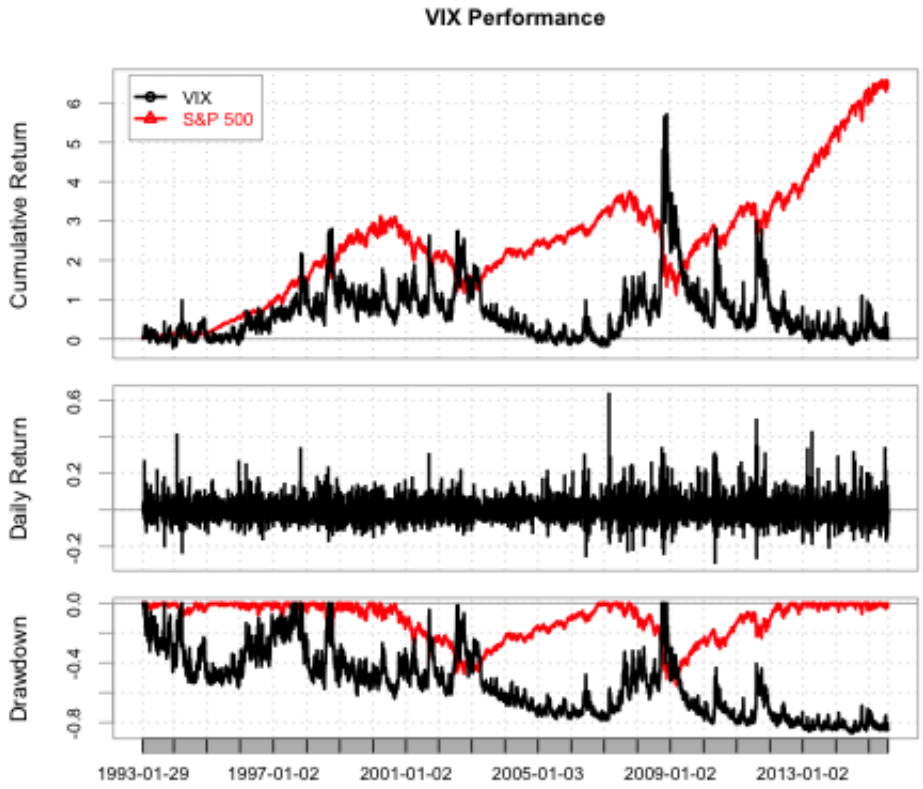


Figure 7: Evolution of VIX and S&P 500 from January 2000 to June 2015.

Part III

Literature Review

Asymmetric volatility

Returns and conditional volatility are negatively correlated. Negative returns are generally associated with an increase in conditional volatility and on the contrary positive returns are generally associated with a decrease in conditional volatility. This mechanism is referred in the current literature as asymmetric volatility (Engle and NG 1993).

The rapid expansion of derivatives on the over-the-counter market with variance swaps and on the exchange traded market with volatility index futures provided investors a direct access to volatility exposure. One reason for investors to invest in volatility is to diversify their portfolio resulting from a negative correlation between volatility and equity returns. Daigler and Rossi (2006) calculate the correlation for the annual period between S&P 500 and VIX from 1992 to 2002 using daily observations. Results are impressive with very high negative correlation comprised between -0,45 to -0,82 for the time period. In the light of these results it is suggested that S&P 500 and VIX move in opposite directions.

But what happened to correlations during a downturn in the market? Asymmetric volatility is more present during stock market downturns. A large drop in the stock market is generally associated with a significant increase in market volatility. Szado (2009) studies the correlation between asset classes during the period ranging from 2004 to 2006 and from 2007 to 2008. During the crisis it shows that every asset classes except managed futures gain positive correlation to S&P 500 when things go wrong. On the contrary the correlation of VIX and S&P 500 is particularly intense during the same period. Correlation is stronger when S&P 500 experiences strong drops.

Moran and Dash (2007) draws the same conclusion after studying the correlation between the S&P 500 and VIX from 1990 through 2006. Daily returns value are equal to -0,66 and monthly returns are equal to -0,61. An increase in the correlation when market drops is also measured. On October 27, 1997 the S&P 500 dropped by 6,9% and VIX increased by 34,3%. Consistent with previous results Alexander and Korovilas (2011) finds a correlation of S&P 500 and VIX volatility index averaging -0,7 before the banking crisis and becomes even more negative during turmoil with a value of -0,85. The negative correlation and even greater negative correlation when the market is affected by

a downturn is from great interest when considered as a portfolio diversifier.

Two different school of thought lead the debate of the negative correlation between equities and equities volatility. On the one hand the "Leverage Effect" is defended by (Black 1976; Schwert 1989) which argue that an increase in equity prices relative to bond prices increases the company leverage thus increasing in the same way the risk to equity holders with the consequence of increasing equity volatility. And conversely a decrease in equity prices relative to bond prices decreases the company leverage thus decreasing in the same way the risk to equity holders which results in an equity volatility increase.

On the other hand and more recently (Bekaert and Wu 2000; Wu 2001; Kim et al. 2004) defend the "Volatility Feedback Effect" view to explain the negative correlation between equities and equities volatility. It incorporates an economic approach and assumes that volatility is part of stock prices. A positive volatility increase will also increase the future required returns on equity and thus stock prices will fall instantaneously. Conversely a negative volatility shock will decrease the future required returns on equity and as a reaction the stock prices will rise simultaneously.

Leverage Effect The negative correlation between equity and volatility highlights unexpected relative reactions. Black (1976) finds an impressive reaction of implied volatility when the price of the related stock goes up. The study estimates that a 1% summed return implies a more than 1% implied volatility change and suggests a stronger volatility effect change for low volatility stocks than for higher volatility stocks. If we think about this last suggestion it makes sense to obtain more volatility change when the initial volatility is low because a change in stock price is less likely to happen for a low volatility stock than for a higher volatility stock. The unexpected price increase of the low volatility stock makes the implied volatility reaction stronger than the high volatility stock.

The approach of (Schwert 1989) is innovative in the sense that it concentrates on a relative approach to explain the leverage effect that affects volatility in stock prices. It takes into account bond prices relative to stock prices and the capital structure of companies. A stock volatility increase is explained by the relative fall of stock prices

compared to bond prices. Or a stock volatility increase is explained by a larger issuance of debt securities in comparison of new equities prior to their capital structure.

Volatility Feedback Effect An alternative explanation of the negative correlation between returns and conditional volatility is the volatility feedback effect. (Bekaert and Wu 2000) compare the leverage effect and the volatility feedback effect. They conclude to a dominance of the volatility feedback to explain the asymmetry on the Japanese stock market. The story is different to the one related by the leverage effect. Return changes lead to shocks in conditional volatility for the leverage effect whereas the volatility feedback effect claims that return changes are caused by conditional volatility changes. Through the GARCH specification the asymmetry generated by shocks seems relatively strong compared to the leverage effect on volatility. High volatility corresponds to decline in the market which generates strong increases in volatility. This mechanism explains the volatility feedback which generates volatility asymmetry.

One year after (Wu 2001) came to a more nuanced conclusion for the analysis of the U.S. stock market. In his paper he develops an asymmetric volatility model with dividend growth and dividend volatility as the two state variable of the economy. Both popular theories that explains asymmetric volatility are tested by the simulated method of moments. The leverage effect and the volatility feedback effect are found to be meaningful components of asymmetric volatility. Moreover the statistical and economical significance of volatility feedback is demonstrated. As noted that both effect doesn't react in the same manner to dividend growth. For the weekly and monthly CRSP value-weighted index, the leverage effect was predominant in the negative correlation determination between return volatility and stock market returns.

Volatility feedback and dividend news contribute both in the return generation process. However the importance of both are relative to the state of the economy. Indeed during volatile market conditions the volatility feedback is very large and leads the return generating process. On the contrary when market conditions are more stable its importance seems secondary while dividend news dominates the return generating process.

The paper of Kim et al. (2004) investigates if a positive correlation exists between

equity premium and stock market when volatility feedback effects of large and continuous market volatility changes are taken into account. The results support the evidence that a positive relationship exists between equity premium and market volatility. In line with previous studies (Wu 2001; Bekaert and Wu 2000) a increase (respectively, decrease) in the stock market volatility tends to decrease (respectively increase) stock prices thus equity premium.

By considering the volatility feedback effect an economic ground is persistent to explain the increase (respectively, decrease) of stock market. Furthermore the negative relation between stock market and implied volatility is a strong indication of the diversification potential of volatility. It contrasts with the portfolio diversification difficulties when a recession period arises and portfolio assets correlation converges to one when it's the most needed (Szado 2009; Alexander and Korovilas 2011). Nevertheless short-term crashes are painful Asness et al. (2011) point out that diversification do not fall on the same time on every market. It's better to focus on long-term returns drivers. Diversification protects against a concentration of assets in one country with low long-term economic performance.

However diversification through volatility positions come at a cost. Evidence show that there is a negative market volatility risk premium. A correspondence stands between the volatility risk premium and the mean delta-hedged portfolio. A long volatility position presents a negative risk premium while conversely a short position presents a positive risk premium. In other words because of asymmetric volatility (Engle and NG 1993) investors are disposed to pay a premium to buy volatility.

In his study of S&P 500 index options Bakshi and Kapadia (2003) investigates delta-hedged portfolios and the resulting risk premium by buying options on the one hand and on the other hand hedging with stocks. Surprisingly the delta-hedged investment strategy does return a negative value. Nonetheless this result is relative to options used in order to delta-hedge the portfolio. When using out-of-the money options the delta-hedged portfolio does less underperform than while delta-hedging the portfolio with at-the-money or in-the-money options. Moreover delta-hedged portfolio performances are worst during higher volatility times. Another underperformance explanation is that the volatility premium affects in a great extent the performances of delta-hedged portfolios.

Through a synthetic combination of option prices to represent variance swap rate Carr and Wu (2009) do quantify the variance risk premium. They explore variance risk premiums on 35 distinctive stocks as well as five stock indexes and asserts of a strong negative variance risk premiums for the Dow indexes and S&P and consistent with (Bakshi and Kapadia 2003). Further analysis lead to the finding that the asymmetric volatility generates strong negative beta but it could only partially explain the negative variance risk premiums. A low beta is consistent with the observations of (Bakshi and Kapadia 2003) and would explain the underperformance of the delta-hedged portfolio. Nevertheless negative risk premiums are justified by a large inefficiency in the market and a willingness from investors to pay substantial premium in order to hedge against increasing volatility. They conclude that the majority of the risk premium is generated by an independent risk factor which is priced strongly in the market.

It also worth noting that volatility has some idiosyncratic features that could explain the negative risk premium of being long volatility. Engle (2004) reveals that when trades tend to cluster volatility is stronger as well as it demonstrates a mean-reverting pattern. Moran and Dash (2007) complete this analysis by underlining the absence of intrinsic value and the expected long-term value of zero when considering volatility.

Portfolio benefits

Asymmetric volatility exhibits some interesting features for portfolio hedging purposes. Daigler and Rossi (2006) determine the benefits of a volatility asset addition to the S&P 500 portfolio with the goal to examine the effect on risk and returns. During the period from 1992 to 2002 they obtain significant results with a risk reduction and a small return reduction compared to the S&P 500 portfolio. Furthermore if the strategy consists of previous year optimal portfolio plus volatility weights replication the optimal portfolio plus volatility will almost have the same risk and returns than the optimal minimum risk portfolio.

The recent introduction of VIX futures and options allows Moran and Dash (2007) to replicate an allocation of spot VIX index to a portfolio and analyze the benefits of this position. They also point out the difference of volatility between spot VIX index and

VIX futures and options. VIX index reacts much stronger to shocks than VIX futures and options moreover it shows more correlation to the S&P 500. However it appears that a small allocation to VIX futures leads to improve the portfolio efficiency and reduce downside risk. Unfortunately the study suffers from a time period limitation because it doesn't include a high volatile period and couldn't therefore estimate the portfolio behavior with a VIX futures allocation during though period.

Guobuzaite and Martellini (2012) conduct their analyze on the European market by examining the benefits of a VSTOXX allocation to EUROSTOXX 50+ portfolio. In the same way they determine the best allocation of VSTOXX futures and VSTOXX options to the EUROSTOXX 50+ portfolio. The results obtained on the European market are consistent with (Moran and Dash 2007) on U.S. market. Strong negative correlation between a long volatility position and the underlying equity portfolio is underlined. They also suggest that the addition of a long volatility position to a portfolio improves their risk-adjusted performance. Moreover this benefit is found strongest when the market goes downside. The comparison with the global minimum variance portfolio shows strong evidence that a portfolio with a long volatility position is a more efficient approach to manage risk. Results also reveals the robustness while adding VIX futures or options to a portfolio and taking into account trading costs.

In their article (Dash and Moran 2005) explore the relationship between hedge funds returns and VIX to determine if an inclusion of VIX into the hedge fund portfolio could lead to a downside risk reduction. A comparison is made between the hedge funds monthly returns lost of more than 3% and the monthly returns of VIX and shows evidence that there is an asymmetry in the correlation between hedge funds and VIX. During hedge funds positive return the correlation with VIX is -5% while for negative return the correlation is -44%. It suggests the VIX to be a good diversifier during tough periods. Equally interestingly the study states that during the highest quartile months for the hedge funds the correlation is positive instead.

Equity index volatility shorting strategies generate high Sharpe ratios and returns. They tend to outperform hedge funds and long equity indexes even in hostile markets. In their study Gregory et al. (2007) consider volatility as an asset class because it offers

significant diversification in hostile markets, returns magnitude that justifies a non-trivial allocation and, returns on the long run are independent of investor or manager's skill.

Volatility is very dynamic and the decision about taking a volatility position should be considered with great care. Cuttler et al. (2007) demonstrate that because of the excessive volatility on particular stocks the best strategy is to short volatility by selling covered out-of-the money calls to gain the premiums with the short position and to gain some stock return on the long position.

More recent studies about diversified portfolios with volatility allocation take the crisis period into account.Gantenbein and Andreas (2013) test diverse volatility weights to a portfolio of assets over the 2006 to 2011 volatile period. They conclude that a 10% allocation to mid-term volatility products would have benefited the equity portfolios in most scenarios. Guobuzaite and Martellini (2012) demonstrate on the European market that by allocating 70% to EUROSTOXX 50+ and 30% in VSTOXX the portfolio has a better risk-adjusted return than the EUROSTOXX 50+ during the 1999 to 2011 time period and is also more robust during 2008 crisis.

This result in corroborated by the study of Szado (2009) that analyses the allocation of volatility in the form of the VIX index, VIX futures and VIX calls through different moneyness. The results converge with previous studies by stating that although long volatility exposure aren't return generating it may be beneficial to a portfolio for hedging purposes and particularly during downturns such as the 2008 crisis. Furthermore the study underlines strong evidence that the better dollar-for-dollar diversification investment resides in VIX calls compared to SPX puts on a volatility basis.

Stronger negative correlation during downturns in the market are pointed out by Siegel (2008) who shows that during a negative market shock investor are more willing to buy out-of-the money puts for hedging purposes. This move increase the implied volatility and consequently the option prices. The increase in volatility is transcribed through VIX index value which amplifies the negative correlation between equity and VIX. Moreover arbitrageurs who sell puts should also sell the underlying stocks to reach delta-neutrality (Siegel 2008).

The study of Lee (2012) concentrates on the downside protection generated by volatility and the effectiveness of variance futures and volatility futures for hedging purpose compared traditional long volatility hedging instruments like put options. Using contract rolling methodologies he shows that VIX volatility futures are more efficient to protect against extreme downside risk than variance futures and traditional out-of-the money put options on the S&p 500.

Through these different studies the portfolio diversification benefits and the hedging framework have clearly been pointed out. Some practitioners consider volatility as an asset (Gregory et al. 2007; Marshall et al. 2012; Cuttler et al. 2007; Gregory and Rangel 2014; Gregory et al. 2013) seeking to enhance investment strategy returns while academics search to analyze the deep relationship between volatility and securities through time (Guobuzaite and Martellini 2012; Szado 2009; Moran and Dash 2007) . Further fertile ground of study is the optimization of portfolios with volatility allocation (Zymler et al. 2011; Alexander et al. 2014; Faias and Santa-Clara 2011; Ghaoui et al. 2003) and their risk examination (Zymler et al. 2012).

Part IV

Empirical Perspective

Chapter 3

Index Level

3.1 Preliminary results

3.1.1 Data

The dataset consists of daily adjusted closing prices[1] collected from Yahoo Finance for each assets. The sample period collection is different for every assets compared to the type of analysis and the inception of the asset class. Nonetheless a common period analysis exists, going from June 2006 to June 2015. Further analysis are based on asset class differentiation representing equities, volatility index, commodities and bonds.

Equities are represented by S&P 500 (GSPC) ticker and covers the period from January 2000 to June 2015. Volatility index is represented by CBOE VIX index (VIX) ticker and covers the period from January 2000 to June 2015. VIX index isn't directly investable. Commodities are represented by the investable iPath S&P GSCI Total Return ETN (GSP) ticker providing exposure to the S&P GSCI Total Return Index. Analysis period goes from inception date June 2006 to June 2015. Bonds are represented by the investable iShares Core US Aggregate Bond (AGG) ticker. This ETF tracks the investment results of the Barclays U.S. Aggregate Bond Index by investing into investment-grade U.S. Treasury bonds, government-related bonds, corporate bonds, mortgage-backed pass-through securities, commercial mortgage-backed securities and asset-backed securities that

[1]We choose to work with adjusted closing price because it takes into account all splits and dividends of a specific asset.

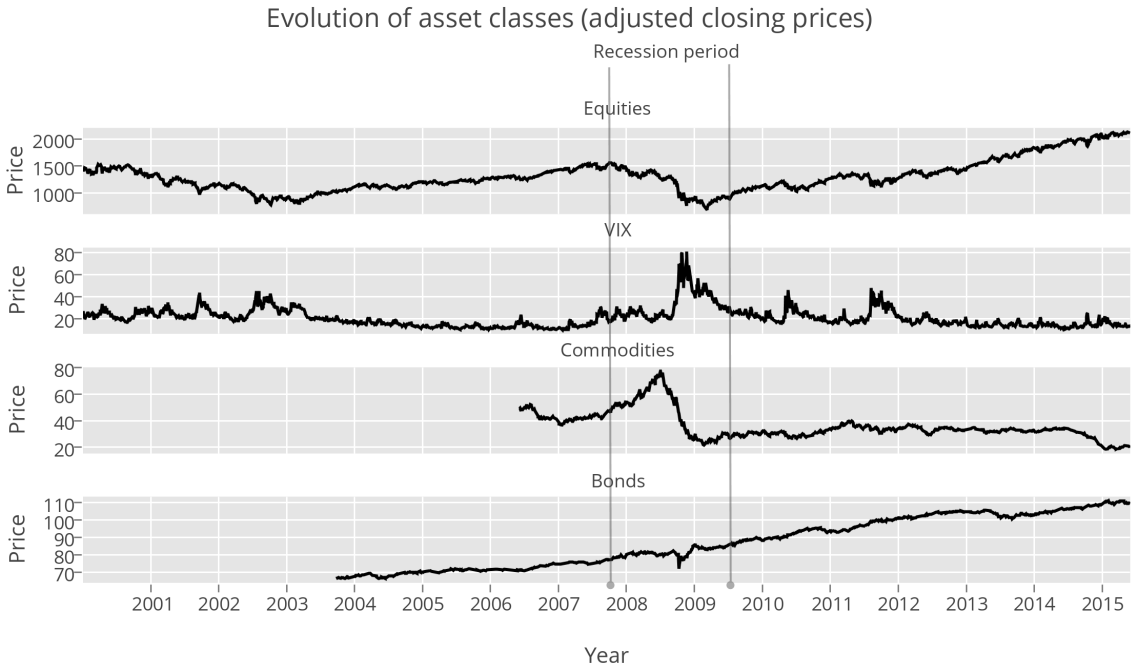


Figure 8: Evolution of 4 asset classes from January 2000 to June 2015.

are publicly offered for sale in the United States. The period covered by iShares Core US Aggregate Bond goes from September 2003 to June 2015.

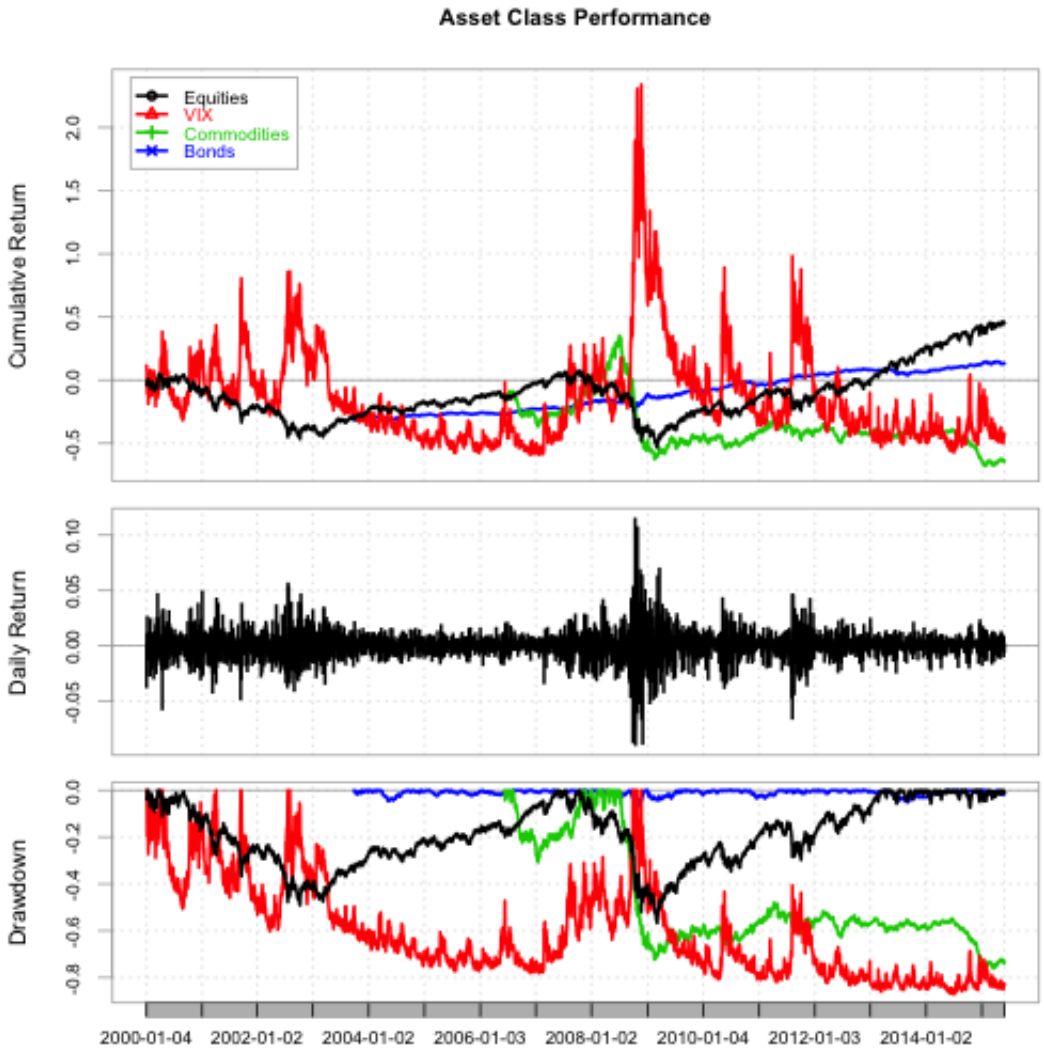
3.1.2 Observations

As a preliminary step an individual analysis is done by plotting the return distribution and a summary of their respective statistics is compiled into a table.

Figure 8 underlines the evolution differences existing through asset classes. At a first sight each asset class exhibit differentiated patterns furthermore some assets do perform in opposite direction to others. It has also to be noted that during the recession period[2] VIX and commodities to a lesser extent, do largely move into opposite direction to equities represented by S&P 500. Figure 25 in appendix draws the individual performances of those four asset classes. Figure 9 gives us a clue about the cumulative return evolution of the four different asset classes from January 2000 to June 2015. Cumulative returns indicate a clear reaction of VIX compared to other asset classes during volatility periods.

[2]Period determined by the National Bureau of Economic Research (NBER)

Figure 9: Asset classes performances from January 2000 to June 2015



VIX reaction during those period and particularly during recession period is very dynamic and much more important than equities, commodities and bonds. If we think about those four assets into a portfolio construction, equities will clearly be the return generator while the other asset classes will tend to smooth the performance of the portfolio by reacting in an opposite direction to equities during increased volatility period.

Turning our analysis on individual asset classes statistics. Table 1 quantifies the major statistics of securities analysis in order to assess and understand their intrinsic performances during two time periods. If we analyze the asset classes between them during

the entire period, it's clear that taking a long position into the implied volatility equity index isn't the best solution to generate returns. As M.Moran and S.Dash precise in their study about VIX Futures and Options (Moran and Dash 2007), a volatility series has an expected long return of zero and is not a return generating process. Consequently the sharpe ratio[3] of the VIX is also equal to zero. Analyzing the distribution of the returns depicted in Figure 26 in appendix, VIX is the only one with a positive value for skewness. An investor with risk adverse preference will be inclined to prefer a positive skewness because of the better chance to obtain positive returns. VIX's excess kurtosis is the second lowest meaning more predictable returns.

The recession period table draws assets' behavior during turmoil. Equities and commodities returns turn negative compared to the entire period. Bonds and VIX returns are the only ones that do not turn negative during turmoil. De facto bonds returns move down from 4,96% to 2,62%. VIX returns are impressive, being the only one asset to react positively. It moves up from -0,43% to 24,04% when volatility increases. Return distributions, characterized by skewness and kurtosis, are more normal during the recession period than during the entire period. Figure 27 in appendix traduces visually this intuition.

Table 3.1 informs us about the correlation of the asset classes during the entire period. Commodities demonstrate a negative correlation with equities during this period. The correlation value is equal to -0,10 which is quite lower than the correlation between bonds and equities which is equal to 0,55[4]. Considering the negative correlation of -0,69 between VIX and equities it reveals VIX as an interesting portfolio diversifier.

When constructing a portfolio it's important to consider the reaction of this last one during large drops. A well constructed portfolio will have the capacity to minimize these drops and to smooth the cumulative return curve. Consistent with H.Markowitz (Markowitz 1952) in order to keep variance low it's important to invest in securities that aren't highly correlated among themselves. Table 3.1 illustrates the correlation between the

[3]Sharpe Ratio considers a normal distribution of returns to assess the return/risk ratio correctly. However we decided to use the Sharpe Ratio to measure the return/risk of the assets even if their distribution aren't normal. We estimated the Skewness and Kurtosis to take into account the non-normality of the returns.

[4]The correlation value is dependent of the ETF investment proportion into High Yield bonds which are more correlated to equities than Lower Yield bonds (Szado 2009)

Table 3.1: Asset class correlation matrix during entire period and recession period

June 2006 to June 2015				
	Equities	VIX	Commodities	Bonds
Equities	1,00			
VIX	-0,65	1,00		
Commodities	-0,10	0,00	1,00	
Bonds	0,55	-0,27	-0,50	1,00
December 2007 to June 2009				
	Equities	VIX	Commodities	Bonds
Equities	1,00			
VIX	-0,80	1,00		
Commodities	0,87	-0,71	1,00	
Bonds	-0,55	-0,05	-0,56	1,00

asset classes during the crisis. Correlation between equity index and its implied volatility are higher during recession period than during the entire period. This result is coherent with the definition of the VIX which estimates the volatility from the prices of S&P 500 index options. On the contrary correlation between commodities and equities shifts from negative during the entire period to positive during recession period. This was a massive concern during the crisis. Much of assets correlation increased during high volatile markets. Bonds and equities correlation turns to negative during recession period which increases diversification benefits.

Finally it has to be noted the increase in negative correlation between VIX & commodities during a recession period. With regards to the asset classes correlation analysis, we can highlight the positive correlation increase of some asset classes when their diversifier characteristics are the most needed. VIX is an exception and gains negative correlation during recession period. A long exposure to VIX could be from great benefit when considering portfolio construction. The results obtained are consistent with (Szado 2009; Alexander and Korovilas 2011).

In summary, the correlation between equities and VIX remains negative regardless of the time period and is even more pronounced during turmoil. Nevertheless taking a long position in VIX only isn't optimal because on the long run the expected return of VIX is equal to zero. Next section aims to appraise the addition of volatility to a portfolio of assets.

3.2 Portfolio construction

This case study considers the benefits of adding volatility by implementing spot VIX index to a diversified portfolio. It takes into account the entire period and the recession period to evaluate if a long volatility position may be a good diversifier particularly during breakdown periods when it's the most desirable. The idea behind this analysis is to enhance portfolios returns with a passive management strategy.

3.2.1 Methodology

Assets used to construct the portfolio are the same as described in the data section 3.1.1. Over a first index level phase, we choose to use a "theoretical" approach by using the spot VIX index it allows us to collect longer data history to analyze portfolios during several time periods. The construction and evaluation of VIX addition to portfolio could be competed by many different manners. We had to make a choice to delimit the possibilities of portfolio evaluation. When managers construct a passive portfolio one of their rule of thumb is not to put all eggs in the same basket. Based on this common mantra we decided to analyze the addition of volatility to an already diversified portfolio. The portfolio is constituted by equities, commodities and bonds with a maximum allocation constraint of 50% for any one asset classes in order to keep a diversified portfolio. To this portfolio, a long position to spot VIX index is added.

Optimal portfolio is determined and compared to a 100% equity portfolio allocation and an optimal portfolio with no long VIX allocation. The path followed to analyze the portfolios doesn't consider any re-balancing and short selling during the life of the portfolio. The portfolio analysis is conducted during both time periods. A long time period ranging from June 2006 to June 2015 will determine the best allocation of volatility to a portfolio during a recession period followed by a non-declining economy. A recession period ranging from June 2007 to December 2009 will inform us about the effect of volatility addition into a portfolio during turmoil. To compare performances of portfolios between each other we used some traditional measure parameters including returns, standard deviation, Sharpe ratio[5], skewness, kurtosis, historical VaR at a 95% threshold and Expected Shortfall at a

[5]Risk free rate is equal to zero for the sharpe ratio computation

95% threshold.

3.2.2 Results

Entire period

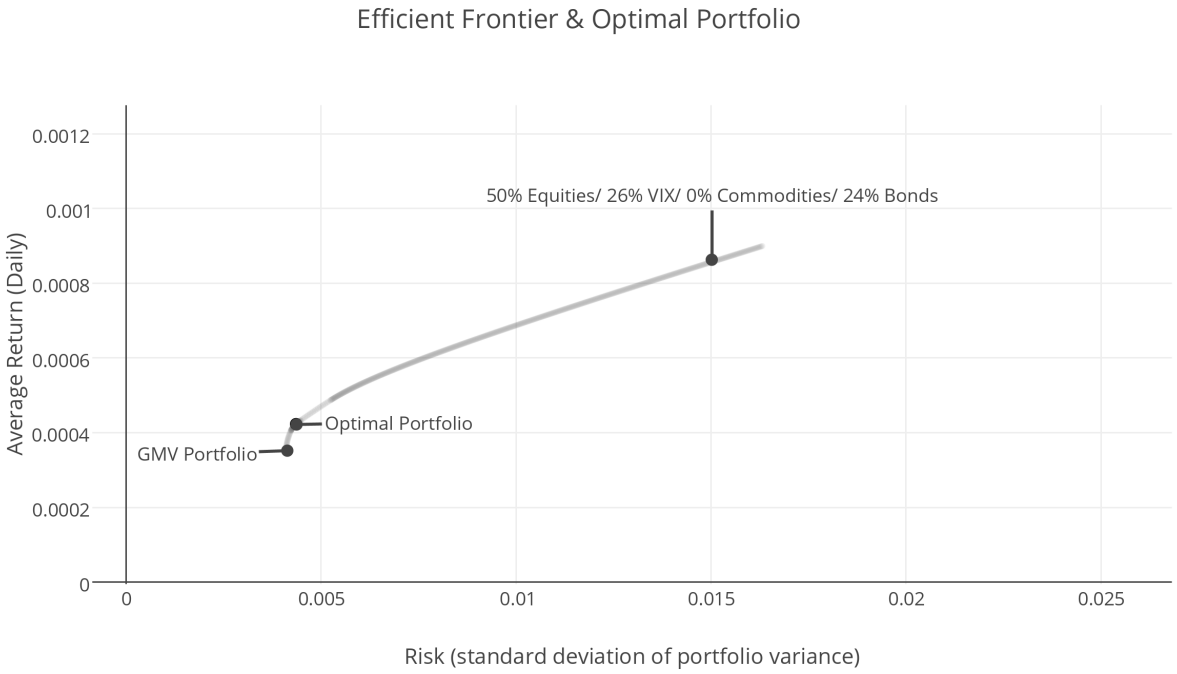
Table 3.2 is a summary statistic of three portfolios with different allocation and inclusion or not of spot VIX index. Equities portfolio is represented by a 100% allocation into S&P 500 (GSPC) index meaning the portfolio is also diversified by with a concentration in the equity asset class. Further details of optimal portfolio without spot VIX allocation are transcribed into Table 2. Similarly Table 4 transcribes allocation details according to the sharpe ratio maximization for the portfolio with a long position into the spot VIX index. The efficient frontier (Figure 10) traduces visually this last allocation matrix. Optimal portfolio maximizes sharpe ratio and allocates 7,60% VIX to get the best risk/return ratio during this time period. On the contrary when the portfolio contains 26% of spot VIX index it increases both Std deviation and returns but it doesn't maximizes the sharpe ratio.

Table 3.2: Summary statistics for different portfolios from June 2006 to June 2015

	Portfolios		
	Equities	Optimal portfolio no VIX	Optimal portfolio VIX
Annualized Return	5,68%	5,45%	4,95%
Annualized Std Dev	21,32%	9,15%	7,29%
Annualized Sharpe	0,27	0,60	0,68
Skewness	-0,08	0,25	0,59
Excess Kurtosis	10,37	17,90	20,74
Historical VaR (daily)	-1,93%	-0,68%	-0,46%
Expected Shortfall (daily)	-3,32%	-1,38%	-1,03%
	Weights		
Equities	100,0%	50,0%	42,4%
VIX	0,0%	0,0%	7,60%
Commodities	0,0%	0,0%	0,0%
Bonds	0,0%	50,0%	50,0%

The optimal portfolio with a 7,60% VIX allocation has smaller return than equity portfolio and optimal portfolio without VIX. On the other hand, Std deviation has the

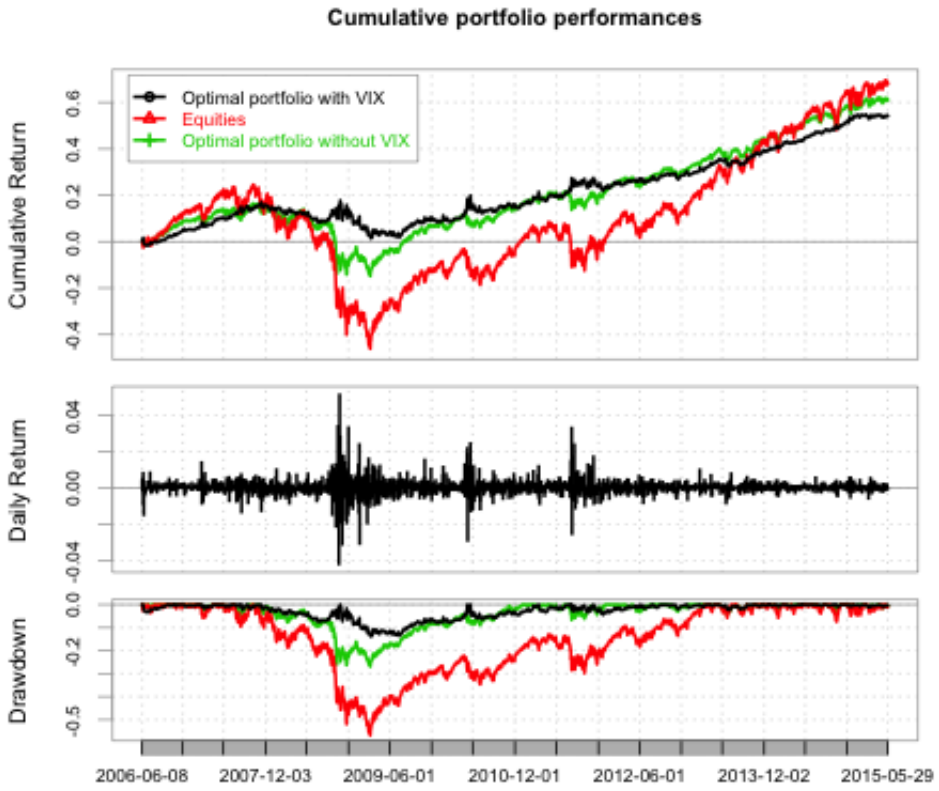
Figure 10: Efficient frontier and optimal portfolio with VIX allocation from June 2006 to June 2015



lowest value between all the portfolios analyzed when VIX is incremented to the portfolio. The combination of high returns and low Std deviation provides the best sharpe ratio among all the portfolios. As noted in (Table 5), the portfolio with the smallest Std deviation (0,41%) is obtained by incorporating 5,80% of spot VIX index which is smaller than the Std deviation of the global minimum variance (GMV) portfolio without VIX (0,62%)[6] (Table 3). In this way the optimal VIX allocation to maximize the sharpe ratio is close to the GMV portfolio. It is from great interest for managers that have a minimum risk policy. Figure 11 exhibits cumulative returns performance of the three portfolios. optimal portfolio with VIX allocation doesn't have the best cumulative returns on the entire period but does the least suffer from the recession period. However if we consider the period going from 2007 to 2012, optimal portfolio with VIX allocation was the portfolio with the best cumulative returns. VIX allocation counterbalances bad returns for equity asset classes. If we look to drawdowns it's clear that the optimal portfolio with VIX allocation is the most robust portfolio when facing turmoil.

[6]Std deviations are calculated on average daily returns

Figure 11: Portfolios cumulative returns from June 2006 to June 2015



Effect of VIX addition on portfolio Sharpe Ratio The return of the optimal portfolio isn't the best. Likewise the Std deviation isn't the lowest between all portfolios with VIX allocation. Nevertheless, by allocating 7,60% of spot VIX index to the portfolio it gives the best sharpe ratio among all the portfolios. Table 4 points out the reduction in sharpe ratio when there is an allocation transfer in the vast majority between bonds and VIX. The increase in spot VIX index allocation and the decrease in bonds allocation amplifies the Std deviation while boosting to a lesser extent the returns of the portfolio. Because of the non-normality of the returns and the difference of returns distribution between portfolios we also considered third and fourth order moments to complete the sharpe ratio measure.

Effect of VIX addition on portfolio Third & Fourth moments Figure 28 in appendix depicts the distributions of the three portfolios. Results indicate that the equity

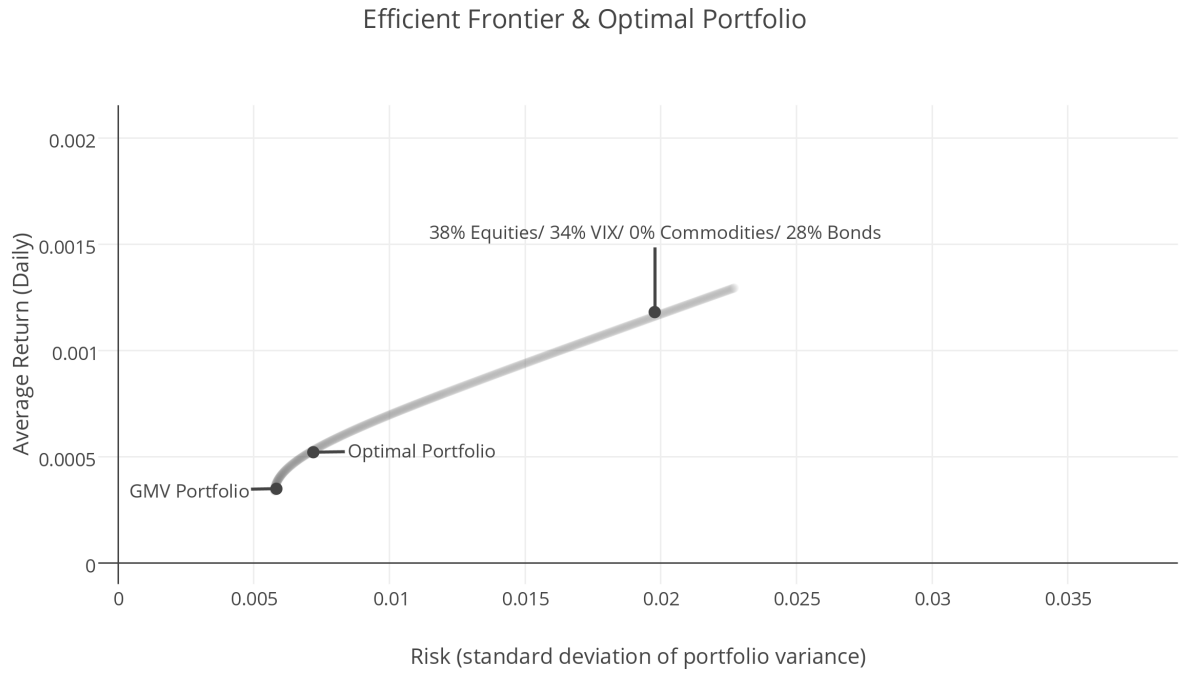
portfolio present a small skewed return distribution. On the contrary OPnV is positively skewed and adding a long volatility exposure to the portfolio increases even more the positive skewness of the portfolio. Positive skewness denotes that the majority of the returns are positive. Positive skewness is in most cases preferred by investors to negative skewness. Besides allocating a part of the portfolio to the spot VIX index also increases the portfolio kurtosis. It drives the return distribution less predictable. Hence an investor would in general prefer a smaller or negative excess kurtosis value. In brief, incorporating spot VIX index to the portfolio increased its skewness and kurtosis which explained by the non-normality of long volatility positions consistent with (Guobuzaite and Martellini 2012; Alexander et al. 2015).

Effect of VIX addition on extreme downside risk Downside risk is of great interest when investing. A turmoil could erodes previous cumulative returns creating year of negative cumulative returns if timing isn't optimal. That's why risk averse investors would in general prefer a smooth cumulative return line and robust portfolios in tough periods. We measured the robustness of the different portfolios by estimating their Historical VaR and Expected Shortfall both at 95% threshold. The addition of spot VIX index is beneficial to the portfolio. It reduces both extreme downside risk. Examining drawdown plot on Figure 11 gives us the same impression. Portfolio containing spot VIX index is more robust to recession periods.

Recession period

Table 3.3 is a summary statistic of three portfolios with different allocation and inclusion or not of spot VIX index. Equities portfolio is represented by a 100% allocation into S&P 500 (GSPC) index meaning the portfolio is also diversified by with a concentration in the equity asset class. Further details of optimal portfolio without spot VIX allocation are transcribed into Table 6. Similarly Table 8 transcribes allocation details according to the sharpe ratio maximization for the portfolio with a long position into the spot VIX index. The efficient frontier (Figure 12) traduces visually this last allocation matrix. Optimal portfolio maximizes sharpe ratio and allocates 13,8% VIX to get the best risk/return ratio during this time period.

Figure 12: Efficient frontier and optimal portfolio with VIX allocation from June 2007 to December 2009



When markets are more turbulent, the impact of spot VIX addition is beneficial in terms of returns. Portfolio returns went from -1,38% to 3,67% for the best portfolios. The positive impact of volatility is impressive during the recession period but could be explained by the increasing negative correlation between VIX and the others assets constituting the portfolio (Table 3.1). Std deviation goes tremendously up when adding a VIX position to the portfolio but it could partially be explained by the overweighting of equities (36,2%) compared to the OPnV (19,9%). However the global minimum variance portfolio with 8,81% VIX allocation exhibits smaller Std deviation (0,58%) (Table 9) than the global minimum portfolio without VIX allocation (0,93% Std deviation)[7](Table 7).

To sum up, introducing spot VIX index into a portfolio enhanced the returns and reduces the Std deviation of the returns. Figure 13 exhibits cumulative returns performance of the three portfolios. The benefits of diversification is clear when comparing the OPnV with the equity portfolio. OPnV does better absorb the turmoil than the equity one.

[7]Std deviation are calculated on average daily returns

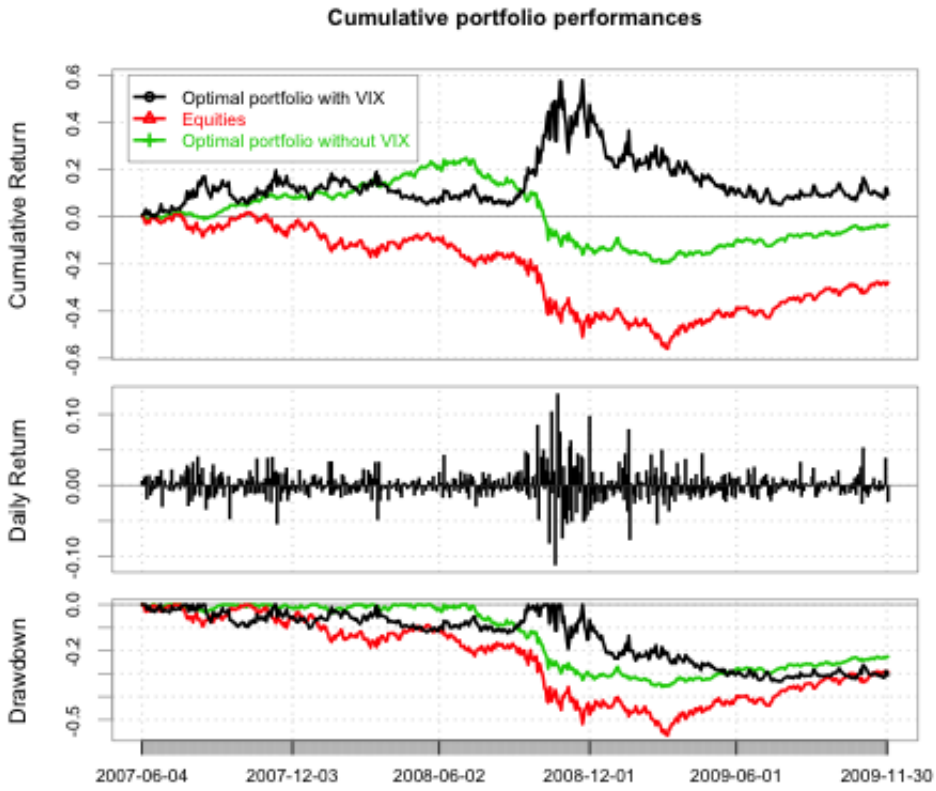
Table 3.3: Summary statistics for different portfolios from June 2007 to December 2009

	Portfolios		
	Equities	Optimal portfolio no VIX	Optimal portfolio VIX
Annualized Return	-12,21%	-1,38%	3,67%
Annualized Std Dev	32,41%	13,61%	32,00%
Annualized Sharpe	-0,38	-0,10	0,11
Skewness	0,09	-0,57	0,63
Excess Kurtosis	5,09	7,11	7,63
Historical VaR (daily)	-3,13%	-1,42%	-2,59%
Expected Shortfall (daily)	-4,93%	-2,08%	-4,41%
	Weights		
Equities	100,0%	19,9%	36,2%
VIX	0,0%	0,0%	13,8%
Commodities	0,0%	30,1%	0,0%
Bonds	0,0%	50,0%	50,0%

optimal portfolio with VIX allocation does the most positively react to the recession due its higher allocation to spot VIX index. It is the only one gaining some positive return during the recession period. The pattern drawn by the portfolio has some similarities with VIX pattern. As noted that the optimal portfolio with VIX allocation reaction to chaos is very dynamic. There is no lag between the market down move and the up move of the optimal portfolio with VIX allocation. However when the market recovers, the optimal portfolio with VIX allocation cumulative returns curve inverts. Drawdown confirms the robustness of the optimal portfolio with VIX allocation during recession period

Effect of VIX addition on portfolio Sharpe Ratio Although the optimal portfolio with VIX allocation does generate large Std deviation, the resulting returns outperform to a large extent the two other portfolios. So it appears that the best sharpe ratio is obtained by allocating 13,8% into spot VIX index. It's the only one portfolio with a positive sharpe ratio when the equity portfolio and OPnV return negative sharpe ratio. Table 8 sheds light on the existing dependence between sharpe ratio and VIX allocation. It tells us that if there is an appetite to enhance returns by taking more risks the solution is to disinvest the bond part to invest the VIX part of the portfolio. Figure 12 helps to visualize the dependencies. Finally it are the indifference curves of every investors that will determine the portfolio risk/return preferences and the optimal portfolio.

Figure 13: Portfolios cumulative returns from June 2007 to December 2009



Effect of VIX addition on portfolio Third & Fourth moments Figure 29 in appendix emphasizes the distribution differences between the portfolios. OPnV is negatively skewed with a skewness of -0,57 when the equity portfolio skewness value is the closest to 0 with 0,09. optimal portfolio with VIX allocation is positively skewed with 0,63 meaning that a majority of the returns is positive. This result is consistent with the return distribution of VIX as an asset class during this period with a skewness of (0,74) (see Table 1 and Figure 27 in appendix). Excess kurtosis increased also when adding spot VIX to the portfolio and constructing the optimal portfolio (7,63) but the increase is minor if compared to the OPnV (7,11). Adding VIX the a portfolio conducts to the same effect on the skewness and excess kurtosis whatever the time period considered. Both increase which is consistent with current literature.

Effect of VIX addition on extreme downside risk If we compare with the entire period, VIX optimal allocation is equal to 7,60% with corresponding historical VaR and expected shortfall being the smallest between all the portfolios. During the recession period, optimal portfolio choose to overweight VIX in order to maximize the sharpe ratio but on the other hand historical VaR and expected shortfall increased also. So overweighting spot VIX index into a portfolio harms historical VaR and expected shortfall. But it remains less risky than a 100% equities concentration for a huge return difference in favor of optimal portfolio with VIX allocation and almost equal Std deviation. Analyzing drawdown plot on Figure 13 supports this impression an add an important analysis. The portfolio containing VIX does almost not suffer from substantial drawdown during the most volatile period. That's a quit desirable effect of VIX addition for portfolio managers fearing drawdowns.

3.2.3 Index volatility puzzle

To sum up, the construction of an optimal portfolio with VIX allocation increases the sharpe ratio for both time periods compared to an optimal portfolio without VIX allocation. It also increases skewness and kurtosis and decreases historical VaR and expected shortfall during the entire period. Furthermore it provides a better managing of downside volatility exposure. VIX inclusion is beneficial to performances and risk reduction of a portfolio. But it doesn't reduce historical VaR and expected shortfall if VIX is overweighted within the portfolio during the recession period. Next section addresses the enhancement of optimal portfolio with VIX allocation by re-balancing the investments. optimal portfolio with VIX allocation with re-balancing is compared to optimal portfolio with VIX allocation without re-balancing and to a portfolio of pure equity investment.

3.3 Trading enhancement

So far investments into the portfolios were made without any re-balancing. It's legitimate the ask the question "how do the portfolios perform when re-balanced ?". The idea behind re-balancing is to keep an optimal dollar value allocated between the positions during the lifetime of the portfolio.

3.3.1 Methodology

First of all in order to keep a pure ground of comparison between portfolio strategies transaction costs are ignored. However we choose to re-balance the portfolio annually to limit trading costs and the bias it could introduce when comparing portfolios performances. To underline the effect of re-balancing the optimal portfolio with VIX allocation and no re-balancing is compared to the performances of the optimal portfolio with VIX allocation with re-balancing during the entire period going from June 2006 to June 2015. The optimal portfolio with VIX allocation with re-balancing is also compared to the re-balanced optimal portfolio without VIX allocation so that the relative performances of both portfolios aren't biased by the transaction costs.

3.3.2 Results

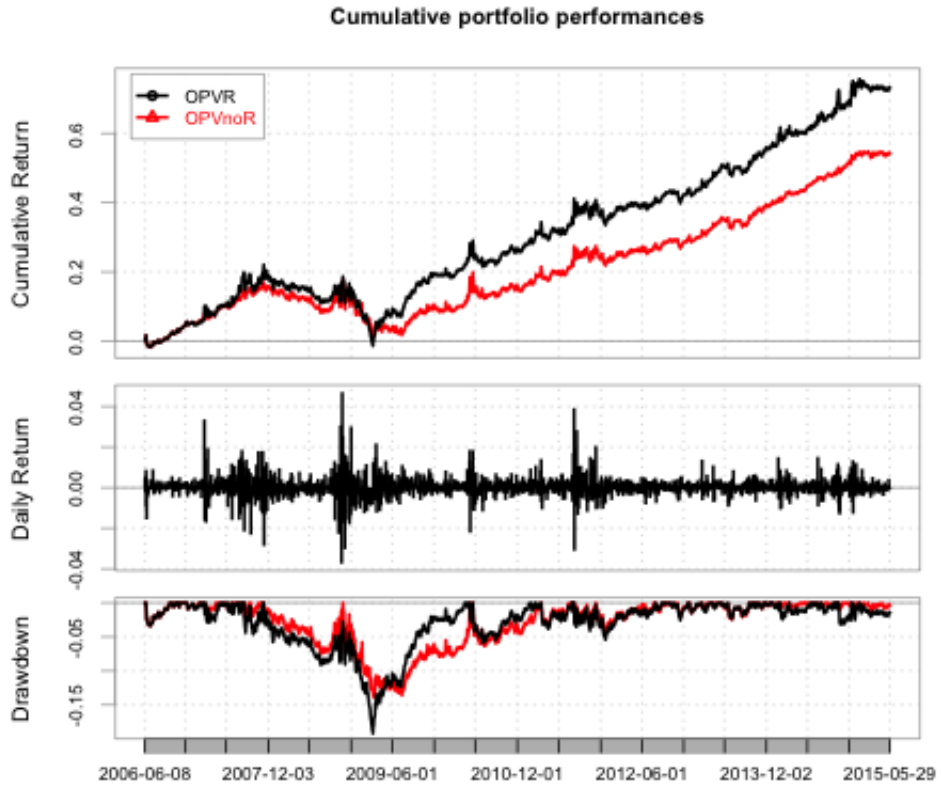
Optimal Portfolio with VIX allocation & Re-balanced

Figure 14 depicts the difference between the two strategies and is quantified by Table 10. More than obviously, re-balancing the portfolio every years tend to increase its performance. Annualized return goes up from 1,36% while in the same time the Std deviation stays rather stable at a 7,82% for the optimal portfolio with VIX allocation and re-balanced.

Effect of re-balancing on portfolio Sharpe Ratio All in all, the increase of return for almost the same Std deviation have for consequences an increase in the sharpe ratio by 0,13 basis points for the optimal portfolio with VIX allocation and re-balanced.

Effect of re-balancing on portfolio Third & Fourth moments On the one hand Re-balancing the portfolio has no effect on the skewness, on the other hand the excess kurtosis decreases when re-balancing. Optimal portfolio with VIX allocation and re-balanced excess kurtosis value is equal to 13,32 on the contrary optimal portfolio with VIX allocation not re-balanced excess kurtosis value is equal to 20,74. Re-balancing tend to make returns more predictable in our case. It substantially corrects the default of VIX addition into a portfolio by reducing the excess kurtosis (see: Figure 30 in appendix).

Figure 14: Re-balancing effect on optimal portfolio with VIX allocation from June 2006 to June 2015

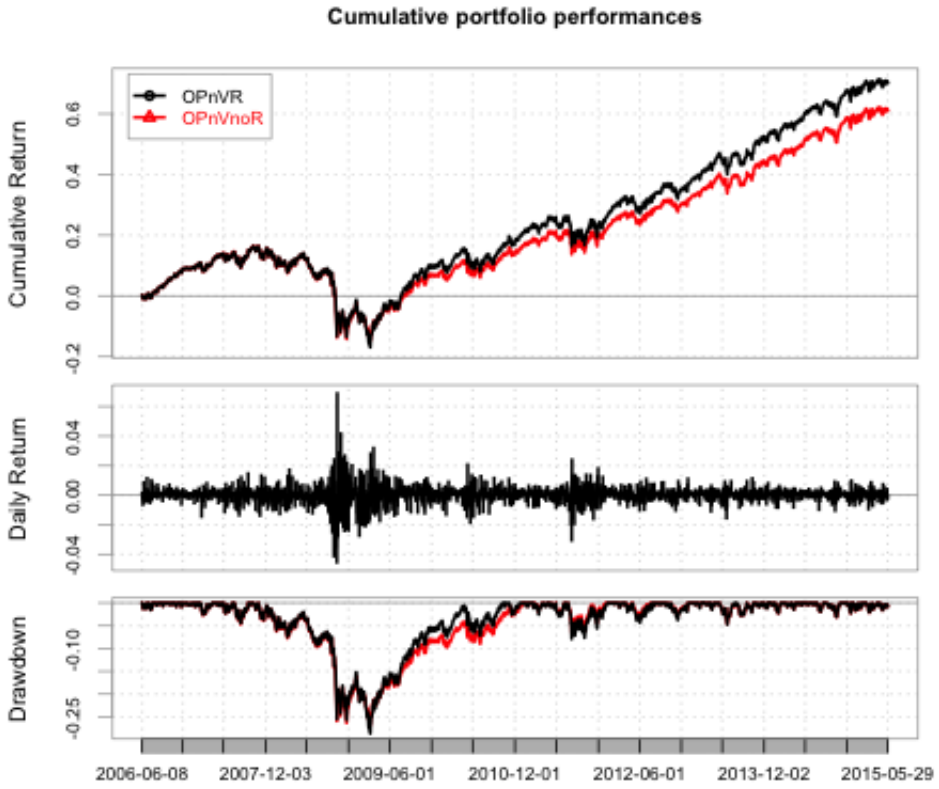


Effect of re-balancing on extreme downside risk Turning our analysis on the risk side of the portfolio brings us to examine historical VaR, expected shortfall and worst drawdown. Each of these three measures characterize the optimal portfolio with VIX allocation and re-balanced as more risky compared to the optimal portfolio with VIX allocation not re-balanced. Worst drawdown peak is observed during the 2008 crisis illustrated on Figure 14 with a value of 0,19 for the optimal portfolio with VIX allocation and re-balanced contrasted with the lower optimal portfolio with VIX allocation not re-balanced value equal to 0,14.

Optimal Portfolio without VIX & Re-balanced

As for optimal portfolio with VIX allocation, re-balancing the portfolio annually does enhance its performances (Figure 15) but results aren't as impressive as for optimal

Figure 15: Re-balancing effect on optimal portfolio without VIX allocation from June 2006 to June 2015

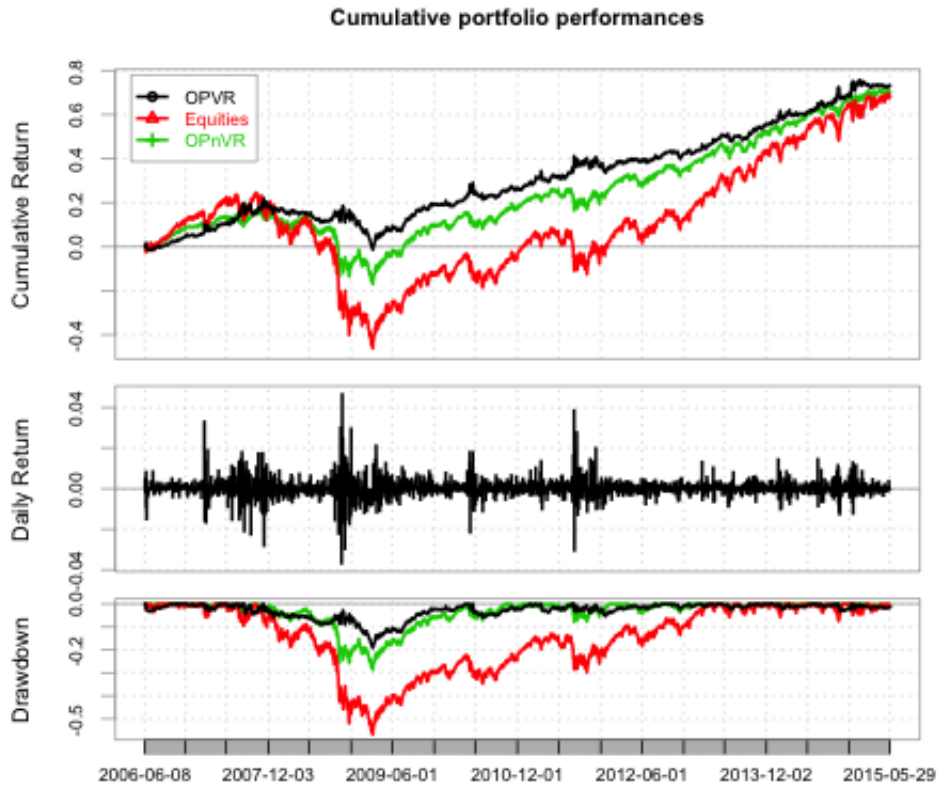


portfolio with VIX allocation re-balancing. Table 11 quantifies the performance increase with a positive annualized return differential of 0,65% in favor of optimal portfolio without VIX allocation but re-balanced. Notwithstanding the Std deviation is slowly increase to reach 9,90% on an annual basis.

Effect of re-balancing on portfolio Sharpe Ratio The return rise is counterbalanced by the Std deviation rise on the other hand leading to an almost unchanged sharpe ratio value.

Effect of re-balancing on portfolio Third & Fourth moments Skewness decreases slightly to reach 0,15. In the same way excess kurtosis decrease is more pronounced going from 17,90 to 13,19 after re-balancing the portfolio. Figure 31 in appendix traces the return distribution of both portfolios.

Figure 16: Portfolio comparison between optimal portfolio with VIX allocation and re-balanced (OPVR) - Optimal portfolio without VIX allocation but re-balanced (OPnVR) - Equities from June 2006 to June 2015



Effect of re-balancing on extreme downside risk Historical VaR, expected shortfall and worst drawdown values rise each indicating a small negative increase in the overall extreme downside risk.

3.3.3 Re-balancing portfolio puzzle

The fact of re-balancing the portfolio annually has from major effect to enhance annualized returns and sharpe ratio. It's all the more true for the portfolio with VIX allocation in contrast to the one without VIX allocation. Furthermore excess kurtosis benefits from re-balancing by reducing its value in the optimal portfolio with VIX allocation and optimal portfolio without VIX allocation cases. However the extreme downside risk suffers on a small scale of re-balancing either the portfolio contains VIX or not.

3.3.4 Final comparison

This final section is a horse race on an index level between three different portfolio strategies namely optimal portfolio with VIX allocation & re-balanced, optimal portfolio without VIX allocation & re-balanced and finally equities. Figure 16 illustrates the superior cumulative performances of optimal portfolio with VIX allocation and re-balanced. It's followed closely by the optimal portfolio without VIX allocation but re-balanced and the equity portfolio. optimal portfolio with VIX allocation and re-balanced reacts at best to turmoil of 2008 and makes even some gains while the two other portfolios cumulative returns drop. Turning our analysis to Table 3.4 gives us a more precise idea of the portfolio performances.

Table 3.4: Summary statistics for different portfolios from June 2006 to June 2015

	Portfolios		
	Optimal portfolio with VIX&Re-balanced	Optimal portfolio without VIX&Re-balanced	Equities
Annualized Return	6,31%	6,10%	5,68%
Annualized Std Dev	7,82%	9,90%	21,32%
Annualized Sharpe	0,81	0,62	0,27
Skewness	0,60	0,15	-0,08
Excess Kurtosis	13,32	13,19	10,37
Historical VaR (daily)	-0,56%	-0,81%	-1,93%
Expected Shortfall (daily)	-1,10%	-1,51%	-3,32%
Worst Drawdown	19,00%	29,00%	57,00%
	Weights		
Equities	42,4%	50,0%	100,0%
VIX	7,6%	0,0%	0,0%
Commodities	0,0%	0,0%	0,0%
Bonds	50,0%	50,0%	0,0%

A side by side comparison of the three most prolific strategies highlights the combination of VIX inclusion to a portfolio and the benefits of annual re-balancing. If we concentrate our analysis on the optimal portfolio with VIX allocation and re-balanced and equity portfolio it's to notice that the Std deviation is divided by three while returns are more pronounced. As a result sharpe ratio is three times higher than a 100% allocation in equities. With a positive skewness of 0,60 returns have more probabilities to be positive, reducing the probability of an extreme downside risk. This is confirmed through historical

VaR, expected shortfall and worst drawdown on Table 3.4 with values each in favor of the optimal portfolio with VIX allocation and re-balanced. Excess kurtosis is the only one value that doesn't fall behind the equity value even after re-balancing the portfolio. Nonetheless recall that all our portfolio results are based on the assumption that re-balancing costs are equal to zero.

Next section goes a step further into the implementation of volatility into a portfolio. The challenge is to find some financial products that capture spot VIX index curve in order to construct an investable portfolio.

Chapter 4

Investable Level

4.1 Preliminary results

4.1.1 Data

Likewise index level data (see 3.1.1) daily closing prices are adjusted to cope with different split and dividends during the time span. Assets inception diverges, going from December 2010 to June 2015. The aim is to replicate the long volatility position at index level with investable assets.

It has been chosen to invest into Exchange Traded Funds with leverage to boost the portfolio returns as suggested by (Long 2014). To this end retained ETF are namely Direxion Daily S&P 500 Bull 3X ETF (SPXL) ticker which seeks to replicate 300% of the S&P 500 index daily return performance (SPY) by investing at least 80% within the same securities with some leveraged positions. Secondly VelocityShares Daily Inverse VIX MT ETN (ZIV) ticker seeking to replicate the inverse of the daily return of the S&P 500 VIX Mid-Term Futures index. Thirdly VelocityShares Daily 2x VIX ST ETN (TVIX) ticker seeking to replicate the returns of 200% the daily performance of the S&P 500 VIX Short-Term Futures index (VXX). Then iPath S&P 500 VIX ST Futures ETN (VXX) offers some exposure to daily rolling long position in the first and second month VIX futures contracts, reflecting the implied volatility on S&P 500. Finally Direxion Daily 20+ Yr Trsy Bull 3X ETF (TMF) ticker tends to replicate 300% the NYSE 20 Year Plus Treasury Bond Index daily return performance (TLT) by investing at least 80% within

U.S. government securities with some leveraged or on the contrary unleveraged positions.

In order to include a recession period and to compare them with index level portfolio, we synthetically generate SPXL, TVIX and TMF until June 2006 by leveraging respectively SPY, VXX and TLT adjusted for discrepancy. VXX and ZIV long historical data come from (Vollmeier 2015b,a).

4.1.2 Observations

First and foremost every investable assets are compared to the assets they are supposed to replaced at index level in order to construct a fully investable portfolio. To this end GSPC is compared to SPXL, VIX is compared to VXX, TVIX and ZIV. At last AGG is compared to TMF.

GSPC vs SPXL

Figure 17 exhibits massive returns but also huge drawdown due to its 300% S&P 500 daily return replication policy. Looking at drawdown needless to say that the investment is also very volatile and risky because of high leverage positions. After the 2008 recession period the ETF did worse than the S&P 500 until end 2013 where cumulative returns became greater. Table 12 shows the correlation between both assets. A value of 0,98 indicates that SPXL reacts almost perfectly to GSPC moves but not necessarily with the same amplitude due to SPXL leveraged positions.

VIX vs VXX vs ZIV vs TVIX

Figure 18 draws the cumulative performances of ETF tracking volatility index. Spot VIX index is quite difficult to track. TVIX and VXX do well to mimic the index but the amplitude is smaller for both ETF's. TVIX is more aggressive than VXX when tracking spot VIX index and suffers from bigger drawdowns because of its highest leverage. ZIV reacts in an opposite direction to spot VIX index and may be considered as return generating and used as a complement to leverage equities when constructing a portfolio. Table 13 shows the correlation matrix between the fourth studied derivatives. It emphasizes

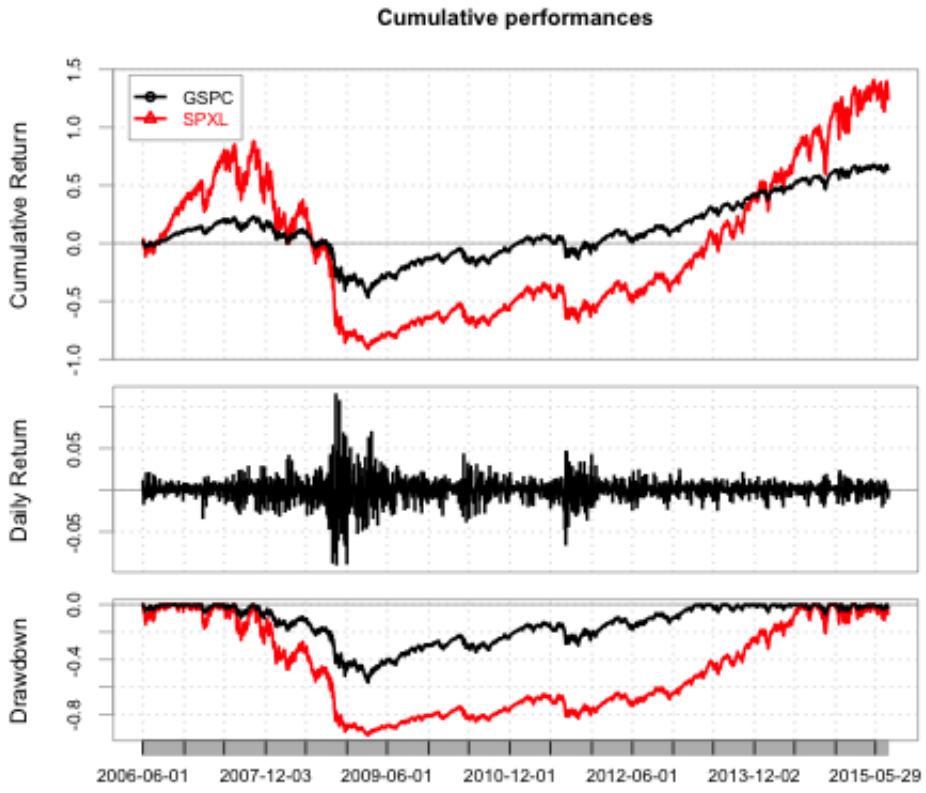


Figure 17: GSPC vs SPXL performances comparison from June 2006 to June 2015

the great positive correlation of (0,87) between VIX and VXX as well as the negative correlation of (-0,74) between VIX and ZIV.

AGG vs TLT vs TMF

Figure 19 demonstrates massive cumulative returns of TMF which is leveraged and tries to track 3 times TLT bond index performances. Correlation between bond index (AGG) used to construct portfolios at index level and TLT/TMF is equal to 0,60[12] see Table 14. More importantly than cumulative returns, drawdown should be considered with great care.

The introduction of bonds into a portfolio are considered as the hedging part of it. Great drawdown undermine the entire portfolio. As seen on Figure 19, timing is essential.

[1]Small discrepancies could persist because of synthetic build up and correction of TMF
[2]This poor result is coherent because TLT and TMF does not exactly track AGG index

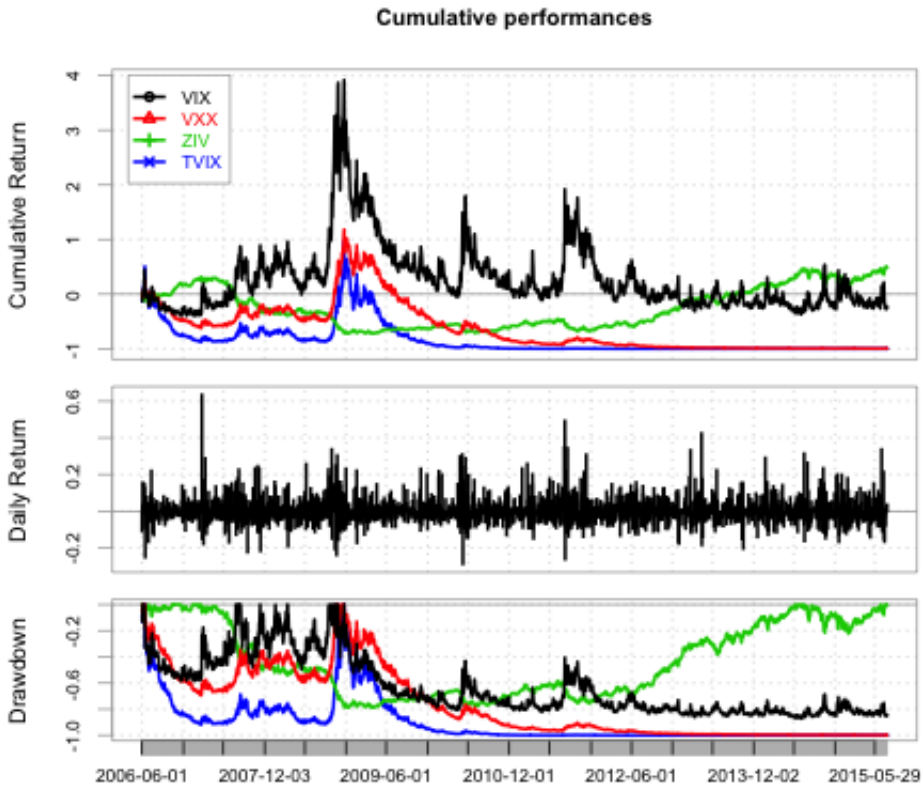


Figure 18: VIX vs VXX vs ZIV vs TVIX performances comparison from June 2006 to June 2015

If TMF position is taken end 2012, somewhat after the ETF suffers from great losses and only recovers in the end of 2014 and generates a cumulative return of zero. The worst drawdown is encounter at end 2013. If the manager exits his position at this precise moment, losses would be huge due to leverage. This is why every managers should not only consider cumulative returns but also drawdowns in order to counterbalance their portfolio with adequate securities and correlation between them.

4.2 Portfolio construction

This section considers the construction and enhancement of the index level portfolio but at an investable level. By constructing different portfolios with leveraged securities we attempt to replicate index level performances and to prove that there exists alternative solutions to bonds and commodities when things go wrong.

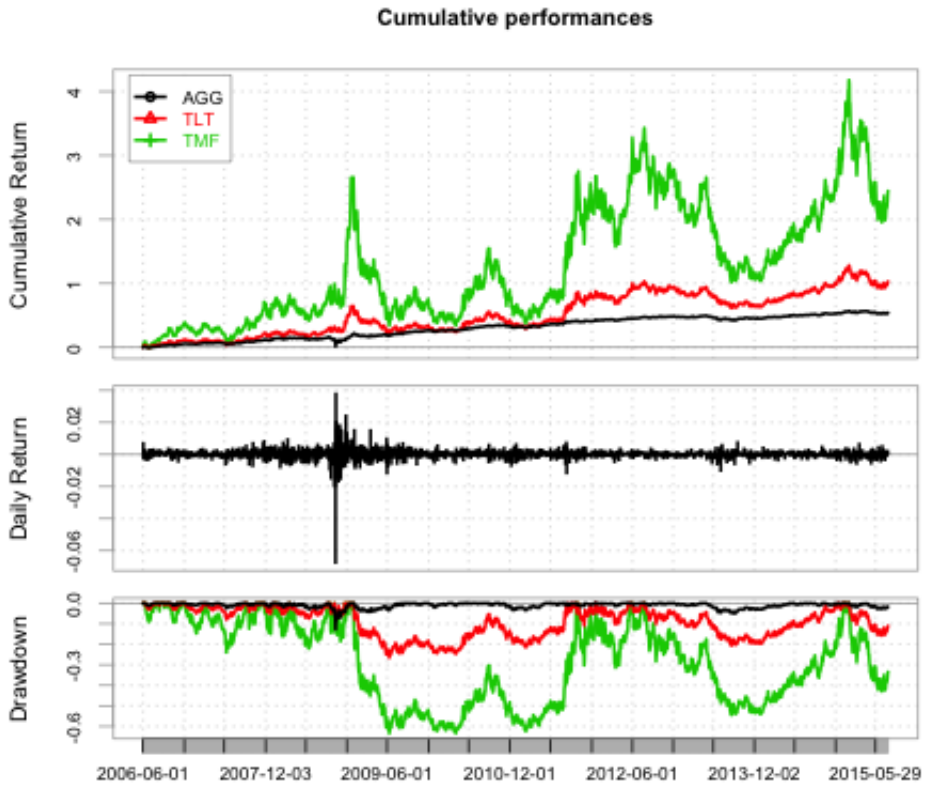


Figure 19: AGG vs TLT vs TMF performances comparison from June 2006 to June 2015

4.2.1 Methodology

Security prices used for this study are detailed in section 4.1.1. Based on previous results at an index level portfolios are re-balanced yearly to enhance their performances. Weights allocated to securities within the portfolio try to respect to weighting previously determined at an index level to compare both portfolios further on a level playing field. The first portfolio is constructed by implementing SPY AGG and VXX[3]. The second one tries to enhance the performances of the first portfolio by investing into leverage securities namely SPXL TMF VXX. The third one is identically the same but VXX is replaced by TVIX. And finally ZIV is added to the third one to compose the fourth portfolio. A global comparison of each strategy is made as well as a comparison with equities[4], with the optimal portfolio at an index level and with a traditional "60% Equity /40% Bonds"

[3]As noted that GSPC has been replaced by SPY to respect the investable policy of each assets.
[4]Equities are represented by S&P 500

investing policy .

As noted that the commodity side of the portfolio vanished because it didn't demonstrate particular diversifier qualities. Its weight was equal to zero into the optimal portfolio at an index level. Similarly to index level section, transaction costs aren't taken into account.

4.2.2 Results

SPY AGG VXX

In order to get a fully investable portfolio, GSPC is replaced by SPY which is an ETF that tracks S&P 500 index (GSPC) and exhibits almost the same performances. As expected and seen when constructing a portfolio with spot VIX index allocation, the portfolio doesn't suffer as much as equities during the 2008 recession period. It draws a smoother cumulative returns curve (Figure 20) than a 100% equity portfolio. Drawdowns are also cut drastically making the portfolio less risky than equities.

The curve of the VXX portfolio exhibit the same pattern as the portfolio with spot VIX index (section 3.2.2 Figure 11) but cumulative returns are less impressive with a correlation of 0,29 between both portfolios (Appendix: Figure 32). Looking at Table 16 reveals a greater skewness and excess kurtosis compared to a 60/40 portfolio which is consistent with our observations at an index level. In the same vein historical VaR, Expected Shortfall and Worst Drawdown are all smaller than equities and than the portfolio at an index level.

60/40 portfolio exhibits great returns principally due to the last 3 years which was particularly rosy for bonds and equities. Volatility allocation does not generate returns on the long run so it couldn't compete on return ground. However sharpe ratio isn't so far from the sharpe ratio of the 60/40 portfolio with 0,40 vs 0,51. But if we look to the distribution of the returns and especially to the risk of the portfolio they are in favor of the VXX portfolio. Skewness is enhanced as well as kurtosis which isn't desirable but is consistent with current studies and our observation at an index level. Std deviation, historical VaR, expected shortfall and worst drawdown are all smaller for the VXX portfolio making the portfolio less subject to extreme downside risk.

Next portfolios refine the strategy by boosting returns but keeping its unique resistance

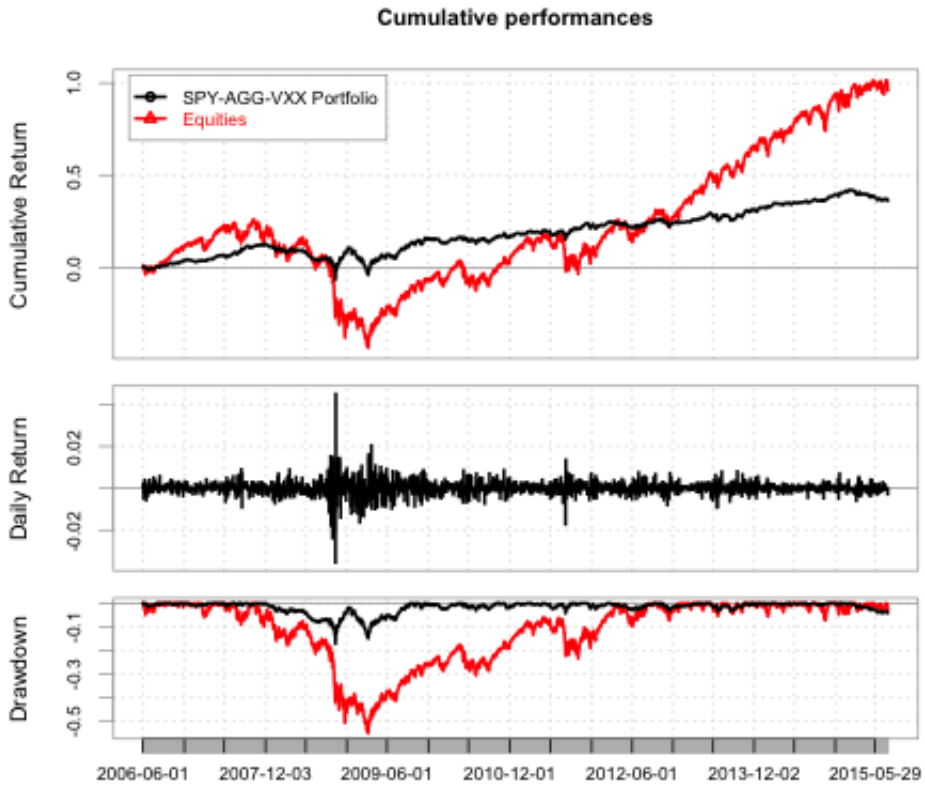


Figure 20: SPY - AGG - VXX Portfolio & Equities from June 2006 to June 2015 during turmoil.

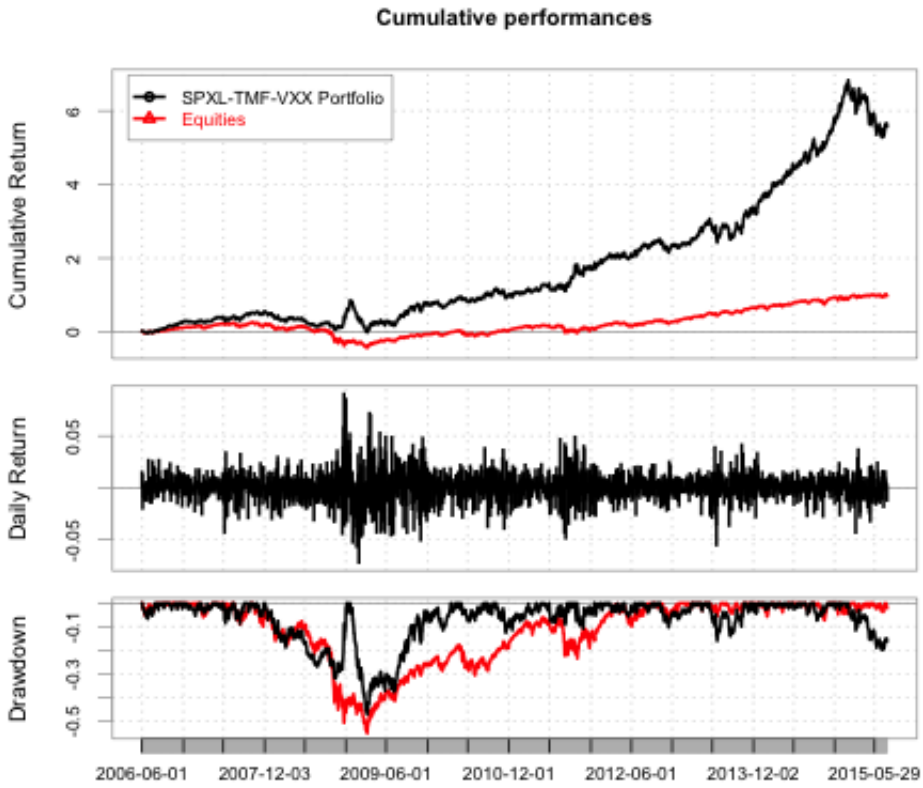


Figure 21: SPXL - TMF - VXX Portfolio & Equities from June 2006 to June 2015

4.2.3 Improving the portfolio strategy

SPXL TMF VXX

This strategy exceed the market since its inception in 2006. During 2008 market volatility the portfolio did well but major drawdowns appear at this period. Even the strategy is still profitable on the long run, timing is crucial. If entering at 2008 spike up and exiting the position at 2008 drawdown, massive losses will result in very short time. This precise rebound is partially generated by VXX allocation within the portfolio. The impressive cumulative returns curve at the end of 2013 is so pronounced because of the combination of the bull equity and bond markets. The inversion of the curve early 2015 is also due to the slowdown of bonds. Hence if equities follow the move[5] VXX portfolio part is going to put things back on an even keel thanks to a negative correlation with equities almost two

[5]A slowdown of long term bonds and equities happened only three times over the last 84 years. During great depression from 1931, during World War II 1941 and during US stagflation in 1969.

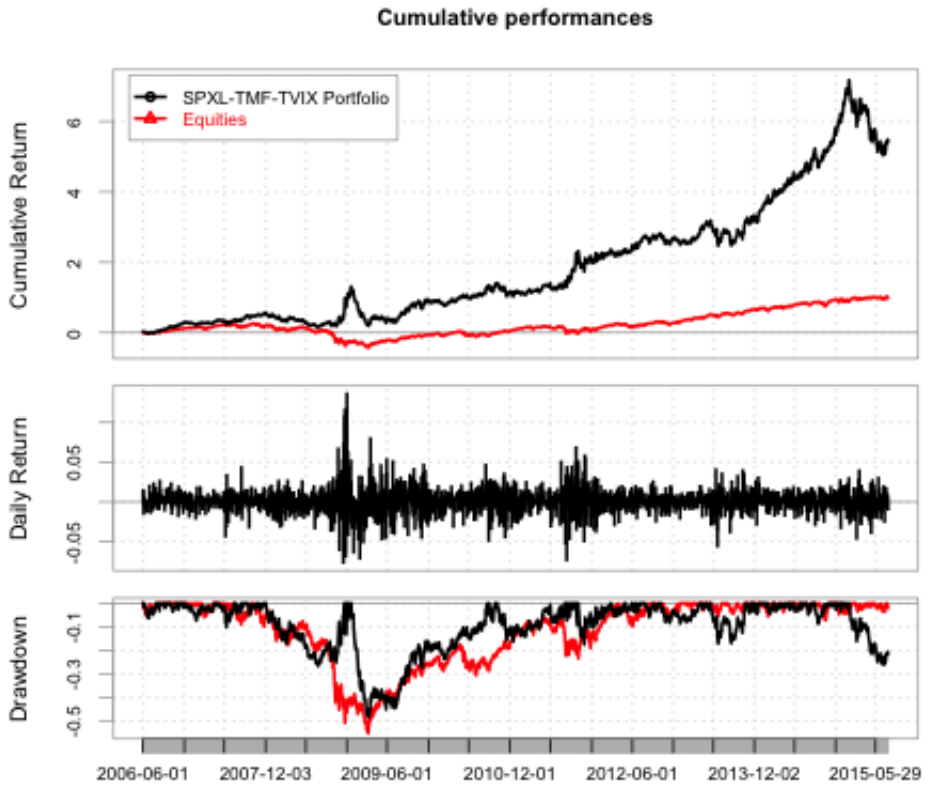


Figure 22: SPXL - TMF - TVIX Portfolio & Equities from June 2006 to June 2015

times greater than bonds and equities (-0,78 vs -0,42). Correlation between the portfolio and the market is equal to 0.20.

Again comparing the porfolio with VXX allocation to 60/40 portfolio it shows a bit less annualized return, Std deviation but close sharpe ratio indeed. Tail risk and worst drawdown are once more better for VXX portfolio (Table 17).

Next portfolio tries to be less reliant to bonds for hedging purpose and reduce further the portfolio correlation with the market.

SPXL TMF TVIX

Again the portfolio containing TVIX has a smaller sharpe ratio than the 60/40 portfolio strategy. Its skewness, kurtosis and tail risk are bigger when TVIX is allocated to the portfolio in substitution of VXX. Figure 22 underlines a more marked positive reaction

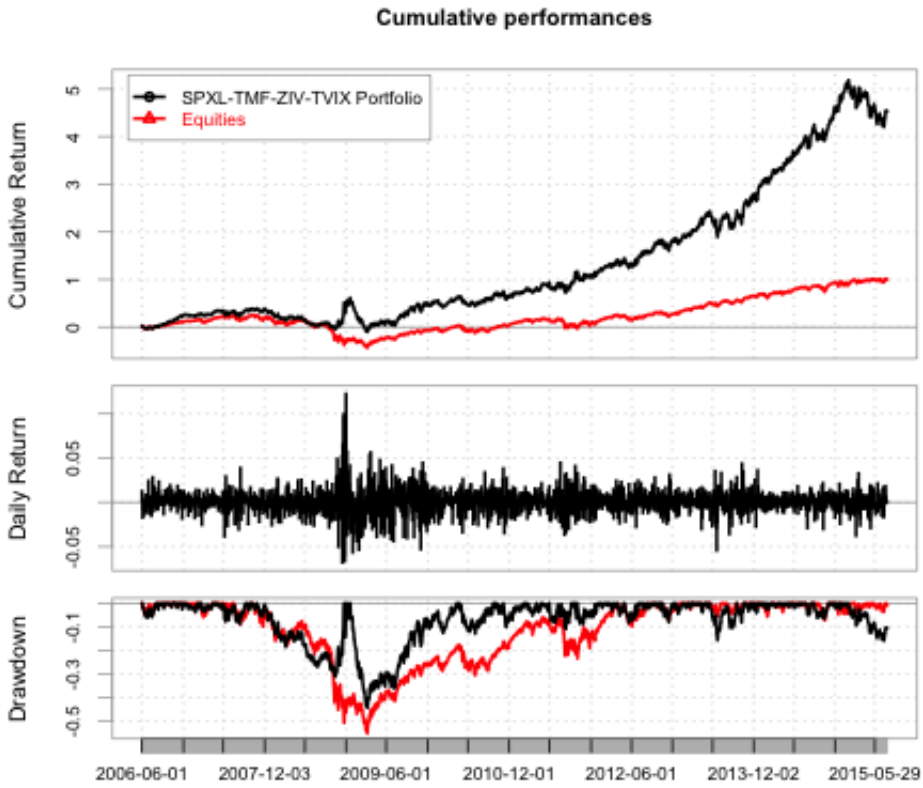


Figure 23: SPXL - TMF - ZIV - TVIX Portfolio & Equities from June 2006 to June 2015

from the portfolio with TVIX allocation during market volatility compared to the one with VXX. The portfolio is better prepared to turmoil and does no longer entirely rely on bonds for diversification. Correlation between the portfolio and the market is reduced from 0,20 to 0.05 with the introduction of TVIX in place of VXX. Even though the investment strategy is almost uncorrelated to the market, it generates much higher cumulative returns than the equity portfolio on the long run. Last but not least, the strategy turns to negative correlation with equities during volatility periods avoiding a collapse of entire portfolio during those tough periods.

Next portfolio tries to smooth the cumulative return curve and be even more robust to turmoil.

SPXL TMF ZIV TVIX

This last portfolio strategy could be separated into two components. On the one hand there is the generating component composed by SPXL and ZIV. On the other hand the hedging component is composed by TMF and TVIX. Correlation table is available in appendix (Table 15). Each security of the generating component is negatively correlated with an asset of the hedging component. SPXL and TVIX have a correlation of -0,77. ZIV and TVIX correlation is equal to -0,86. Furthermore TMF is also negatively correlated to SPXL and ZIV (-0,42 ; -0,37). Weighting of the portfolio is arbitrary but respects an allocation of 60% for the generating part and 40% for the hedging part with respect to previous TVIX and SPXL allocation.

Results on Table 19 indicates the second higher sharpe ratio between all the four volatility portfolios. Skewness and Excess kurtosis are both higher than a 60/40 portfolio strategy. Nonetheless tail risk is the smallest between leveraged portfolios and is also smaller than the traditional 60/40 portfolio. It is almost as small as equity portfolio tail risk but our portfolio is on the other hand highly leveraged. In the same way worst drawdown is the smallest of all leveraged portfolios and even smaller than 60/40 portfolio and equity portfolio. Correlation between the portfolio and the market is gone up to 0,34 due to the addition of ZIV[6] but is still very low. As a comparison the correlation of the 60/40 portfolio and the market is equal to 0,64.

In fact the portfolio strategy is a tail hedge portfolio strategy beating the market during the studied period whatever it is bullish or bearish. In bear markets hedges tend to make the portfolio more robust and smooth the cumulative curve. While in bull markets, leverage of the generating components skyrocket the portfolio.

Global Portfolio comparison

After constructing several different portfolios with their own strengths and weaknesses it's time to plot them against each other to evaluate their behavior along time. A group of portfolio is directly identifiable in Figure 24 these are the leveraged portfolios. They

[6]As a reminder, correlation between SPXL and ZIV is equal to 0,72

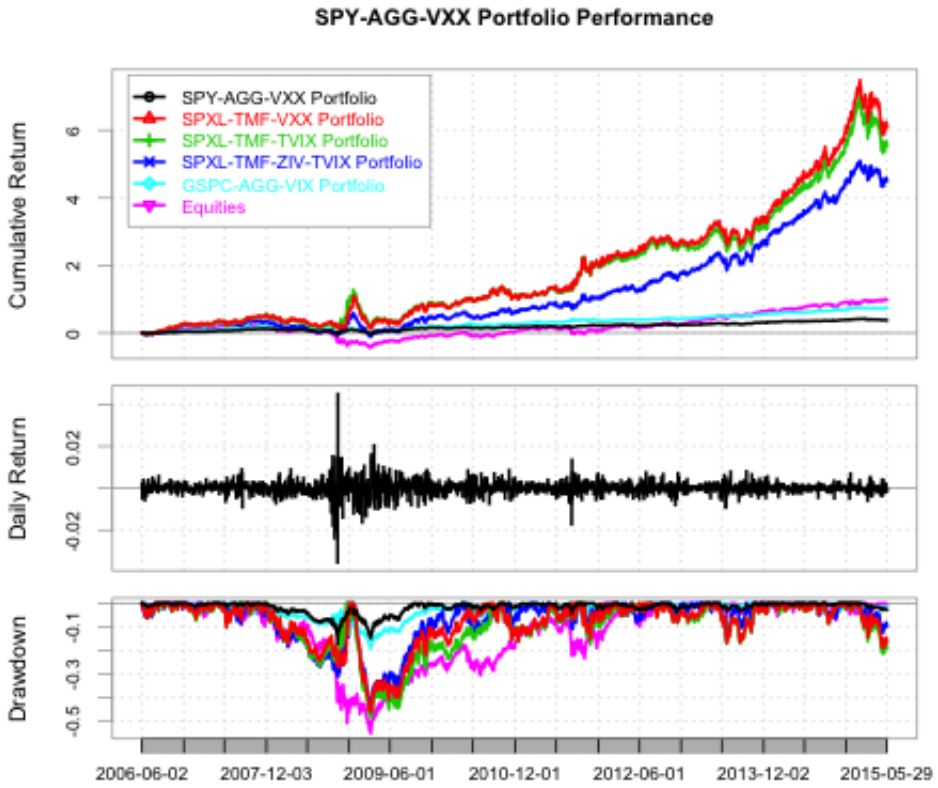


Figure 24: Global portfolio & Equity comparison from June 2006 to June 2015

react exponentially to market events compared to unleveraged portfolios. Hence their cumulative returns are much higher but their volatility and drawdowns are also more important. Within the group of leverage portfolio SPXL-TMF-VXX is very interesting cumulative returns speaking. Although it generates high performances it isn't relatively more risky than SPXL-TMF-TVIX if we concentrate on drawdowns. Safest portfolio is the SPXL-TMF-ZIV-TVIX one by reducing the magnitude of worst drawdowns but it gives up some return in compensation.

The comparison between GSPC-AGG-VIX and SPY-AGG-VXX shed light on the possibility to implement the volatility strategy and its respective performances. The investable portfolio recreates quite well the portfolio at an index level but with less cumulative returns. It does better than equity and index level portfolio during the crisis with a smoother curve and less drawdowns.

4.2.4 Investable volatility puzzle

During our study at an investable level we verified the assertion made at an index level and attempt successfully to replicate those performances at an investable level. Skewness and kurtosis increased when volatility is added to an investable portfolio as well the tail risk decreases. It has also been demonstrated that it is possible to construct a portfolio (SPXL-TMF-ZIV-TVIX) with leverage volatility that exhibits 2,33 bigger CAGR for a more normal distribution of returns than the market, almost the same tail risk and a smaller worst drawdown. This last portfolio is in fact a tail-hedge portfolio strategy that beats the market year after year regardless the state of the market.

Part V

Conclusion Recommendations & Remarks

In past studies it has been proven that there exists a strong negative correlation between stock market and implied volatility becoming even stronger during shock periods while correlation of equities and other asset classes tend to one. The current study supports this analysis and shows that VIX has the highest negative correlation during turmoil and long periods between all asset classes considered as portfolio hedge.

This result is driven by Siegel (2008) analyze who shows that during a negative market shock investor are more willing to buy out-of-the money puts for hedging purposes increasing in the same time the implied volatility and consequently the option prices. The increase in volatility increases VIX index value which amplifies the negative correlation between equity and VIX. Furthermore arbitrageurs who sell puts should also sell the underlying stocks to reach delta-neutrality (Siegel 2008).

Historical negative correlation between stock market and volatility increases in bear markets while decreasing in bull market. We sought to provide benefit evidences of a VIX optimized portfolio compared to an optimal portfolio without VIX allocation and the S&P 500. We found that a greater VIX allocation into the optimized portfolio is preferable during bear markets while a smaller part of the portfolio should be allocated to VIX during bull markets. So if a manager forecasts an increase in the stock market he will eventually choose to reduce his exposure to volatility and conversely.

The study went also beyond the scope of tail risk when a part of the optimized portfolio is allocated to VIX. It has been demonstrated that when VIX is allocated to a portfolio it increases its skewness and kurtosis and on the contrary decreases historical VaR and expected shortfall during the period ranging from June 2006 to June 2015.

Another important result was the effect of re-balancing on portfolio performances. We established that re-balancing has a positive impact on Sharpe ratio but increased to a lesser extent extreme downside risk. While the benefits of a long volatility position combined to a diversified portfolio are substantial we attempted to transpose this investment strategy to the market. Volatility index isn't directly investable thus we sought to replicate spot VIX index through investing in exchange traded funds. The novel approach of the study resides in the fact the different portfolios are optimized instead of arbitrary allocations.

Furthermore their replication on the market is reached through investing in exchange traded funds instead of VIX futures and options.

Investing into VXX instead of VIX to construct an optimized portfolio couldn't reproduce the strong performances of the optimized portfolio with a VIX allocation but it exhibits the same pattern and has similar reactions during market shocks. However the strategy reproduces the tail hedge seen at an index level by being two times less risky than a traditional 60/40 portfolio on an extreme downside basis.

On the long-term volatility has an intrinsic value of zero and doesn't generate any return (Moran and Dash 2007). Thus a portfolio with a volatility allocation is penalized compared to traditional hedged portfolios. In order to cope with those limitations it has been chosen to invest in leveraged generating returns by means of exchange traded fund. Several innovative leveraged strategies have been generated exhibiting all stronger Sharpe ratios than unleveraged optimized portfolios but embedding higher extreme downside risk. However the tail risk of the last strategy invested in SPXL TMF ZIV and TVIX is almost equal to S&P 500 with 2,3 times more return and a smaller worst drawdown value for the period from June 2006 to June 2015.

Our results suggest some benefits to investors seeking extreme hedge during market turmoils. Such diversification means could bring investors back to the financial market. This would be beneficial to the society. We have also seen that if some leverage is added to the portfolio this last one could generate high returns for limited risks. Such portfolios could also find their public and be beneficial to the society because of liquidity flows.

Limitations to study Several assumptions have been made in this study. First we assumed trading transaction costs equal to zero for all time periods. Another limitation was that liquidity problems weren't taken into account although volatility products tends to be very liquid by the time.

Areas for further studies Myriad of further studies could lead to a better understanding of volatility allocation to a traditional portfolio. First, one could examine the addition of other indices to a portfolio to enhance its performances compared to traditional

passively managed portfolio. One could also compare performances of portfolio with volatility allocation on a more frequent re-balancing basis.

Finally because of the dynamic nature of volatility it would be from great interest to analyze the performance of a portfolio with volatility allocation and re-balancing it based on performances instead of a time. During bull market less volatility will be allocated to the portfolio and conversely. Such a strategy could generate more robust portfolios.

Bibliography

Alexander, C., Kapraun, J., and Korovilas, D. (2015). Trading and Investing in Volatility Products. *Financial Markets, Institutions and Instruments*.

Alexander, C. and Korovilas, D. (2011). The Hazards of Volatility Diversification. *Futures*, 44(0):0–23.

Alexander, C., Korovilas, D., and Kapraun, J. (2014). Home-grown Diversification with Volatility Products.

Asness, C. S., Israelov, R., and Liew, J. M. (2011). International diversification works (eventually). *Financial Analysts Journal*, 67(3):24–38.

Bakshi, G. and Kapadia, N. (2003). Delta-Hedged Gains and the Negative Market Volatility Risk Premium. *Review of Financial Studies*, 16(2):527–566.

Bank for International Settlements (2014). Derivatives statistics. http://www.bis.org/statistics/derstats.htm?m=6%7C32, Retrieved on March 25, 2015.

Bekaert, G. and Wu, G. (2000). Asymmetric volatility and risk in equity markets. *Review of Financial Studies*, 13(1):1–42.

Berkshire Hathaway (2003). 2002 Berkshire Hathaway INC. Annual Report. Technical report.

Black, F. (1975). Fact and Fantasy in the Use of Options. *Financial Analysts Journal*, 31(4):36–41.

Black, F. (1976). Studies in stock price volatility changes.

Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3):637–654.

Bollen, N. P. B. and Whaley, R. E. (2004). Does Net Buying Pressure Affect the Shape of Implied Volatility Functions? *Journal of Finance*, 59(2):711–753.

Carr, P. and Wu, L. (2009). Variance risk premiums. *Review of Financial Studies*, 22(646):1311–1341.

Cboe (1994). Understanding Stock Options. *Options*, 4258(312).

CBOE (2015). CBOE S&P 500 BuyWrite Index (BXM) Micro Site.

Chicago Board Options Exchange (2001). Ask the Institute. http://www.cboe.com/LearnCenter/OCShowDocument.aspx?DIR=LCOptionsCorner&FILE=20010611.doc&CreateDate=11.06.2001 , Retrieved on July 22, 2015.

Cuttler, J., Waghorn, J., Della Vigna, M., Courvalin, D., Grant, M., and A. Lacey, M. (2007). Considering All Options. *Global Investment Research*, pages 1–9.

Daigler, R. T. and Rossi, L. (2006). A Portfolio of Stocks and Volatility. *The Journal of Investing*, 15:99–106.

Dash, S. and Moran, M. T. (2005). VIX as a Companion for Hedge Fund Portfolios. *The Journal of Alternative Investments*, 8(3):75–80.

Durden, T. (2015). 2014 Hedge Fund Hall Of Fame: The 100 Best-Performing Hedge Funds | Zero Hedge. http://www.zerohedge.com/news/2015-01-05/2014-hedge-fund-hall-fame-100-best-performing-hedge-funds, Retrieved on March 15, 2015.

Engle, R. (2004). Risk and Volatility: Econometric Models and Financial practicei. *The American Economic Review*, pages 405–419.

Engle, R. and NG, V. (1993). Measuring and Testing the Impact of News on Volatility. *The journal of finance*.

Faias, J. and Santa-Clara, P. (2011). Optimal Option Portfolio Strategies. *SSRN Electronic Journal*, (October).

Gantenbein, P. and Andreas, R. (2013). Volatility as an Asset Class - A Valuable Portfolio Diversifier in Volatile Times ? *GSTF Journal on Business Review*, 2(1):224–230.

Ghaoui, L. E., Oks, M., and Oustry, F. (2003). Worst-Case Value-At-Risk and Robust Portfolio Optimization: A Conic Programming Approach. *Operations Research*, 51(4):543–556.

Gregory, K., Grant, M., and Lui, J. (2007). Volatility as an Asset. Technical report, Goldman Sachs.

Gregory, K., Marshall, J., and Fogertey, K. (2013). The Art of Put Selling : A 10 year study. Technical report, Goldman Sachs.

Gregory, K. and Rangel, J. G. (2014). The Buzz : Who pressed the reset button ? Technical report, Goldman Sachs.

Guobuzaite, R. and Martellini, L. (2012). The Benefits of Volatility Derivatives in Equity Portfolio Management. (May).

Hull, J. (2009). *Options, futures and other derivatives*, volume 7th Editio.

Kim, C., Morley, J., and Nelson, C. (2004). Is there a positive relationship between stock market volatility and the equity premium? *Journal of Money, Credit and banking*, 36(3):339–360.

Lee, B. (2012). Using Volatility Futures as Extreme Downside Hedges.

Long, H. (2014). A Refined All-Long Strategy III. http://seekingalpha.com/article/2627145-a-refined-all-long-strategy-iii , Retrieved on July 20, 2015.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, 7(1):77–91.

Marshall, J., Gregory, K., Rangel, J. G., Fogertey, K., and Jha, A. (2012). Finding alpha : A 16 year study of S & P 500 index overwriting. Technical report, Goldman Sachs.

Moran, M. T. and Dash, S. (2007). VIX Futures and Options : Pricing and Using Volatility Products to Manage Downside Risk and Improve Efficiency in Equity Portfolios. pages 1–10.

PSCM (2015). Annual Update Presentation. Technical report.

Schwert, W. (1989). Why does stock market volatility change over time ? (3290).

Siegel, J. J. (2008). *Stocks for the Long Run: The Definitive Guide to Financial Market Returns and Long-Term Investment Strategies*, volume 2008. McGraw-Hil Education.

StreetAuthority (2015). Buffett's Strategy To Make Millions Without Buying Stock - NASDAQ.com. http://www.nasdaq.com/article/buffetts-strategy-to-make-millions-without-buying-stock-cm466409 , Retrieved on June 16, 2015.

Szado, E. (2009). VIX Futures and options: A Case Study of Portfolio Diversification During the 2008 Financial Crisis. *The Journal of Alternative Investments*, 12(August):68–85.

Vollmeier, H. (2015a). VXX Historical prices. https://dl.dropboxusercontent.com/s/950x55x7jtm9x2q/VXXlong.TXT , Retrieved on July 21, 2015.

Vollmeier, H. (2015b). ZIV Historical prices. https://dl.dropboxusercontent.com/content_link/xbplStJPzgxz98UpaJC8ufHMP2pNKUwN7O0n5xflriJK 0NLNmtwWXzNGRf0jsEmW?dl=1", destfile="longZIV.txt , Retrieved on July 21, 2015.

Whaley, R. E. (2008). Understanding VIX.

Wiley, J. and Sons (1998). A Brief History of Derivatives. *Financial Engineering News*, (Essays in Derivatives).

Wu, G. (2001). The Determinants of Asymmetric Volatility. *Review of Financial Studies*, 14(3):837–859.

Zymler, S., Kuhn, D., and Rustem, B. (2012). Worst-Case Value-at-Risk of Non-Linear Portfolios. pages 1–32.

Zymler, S., Rustem, B., and Kuhn, D. (2011). Robust portfolio optimization with derivative insurance guarantees. *European Journal of Operational Research*, 210:410–424.

Appendices

Table 1: Summary statistics for individual asset classes during entire period and recession period

June 2006 to June 2015				
	Equities	VIX	Commodities	Bonds
Annualized Return	5,68%	-0,43%	-9,21%	4,96%
Annualized Std Dev	21,32%	117,23%	25,01%	5,41%
Annualized Sharpe	0,27	0,00	-0,37	0,91
Skewness	-0,08	1,24	-0,30	-2,82
Excess Kurtosis	10,37	6,06	2,60	83,94
June 2007 to December 2009				
	Equities	VIX	Commodities	Bonds
Annualized Return	-12,21%	24,04%	-10,89%	2,62%
Annualized Std Dev	32,41%	118,57%	34,83%	8,88%
Annualized Sharpe	-0,38	0,20	-0,31	0,84
Skewness	0,09	0,74	-0,26	-2,39
Excess Kurtosis	5,09	2,20	1,04	42,70

Figure 25: Individual asset classes performances

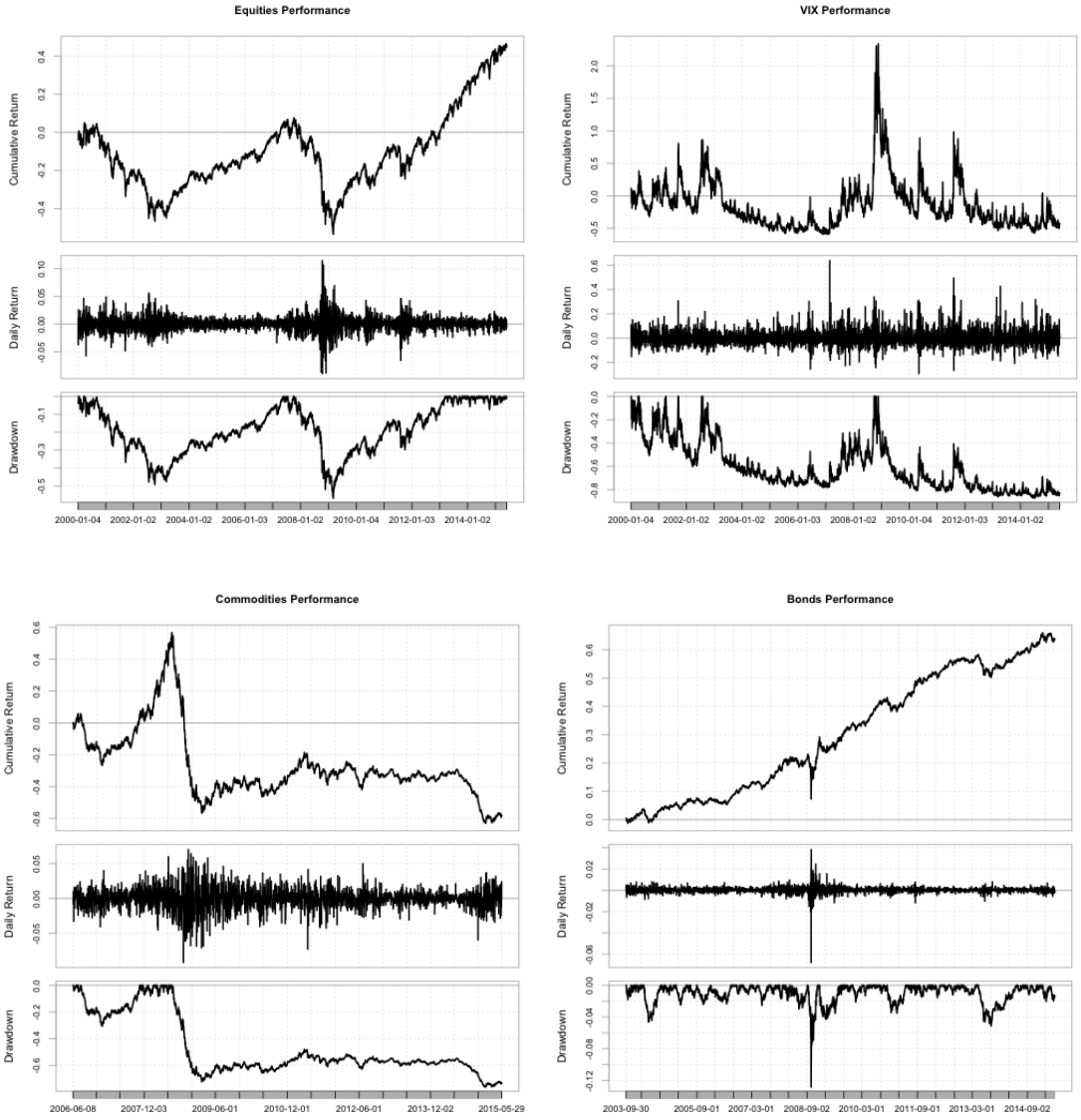


Figure 26: Asset classes return distributions from June 2006 to June 2015

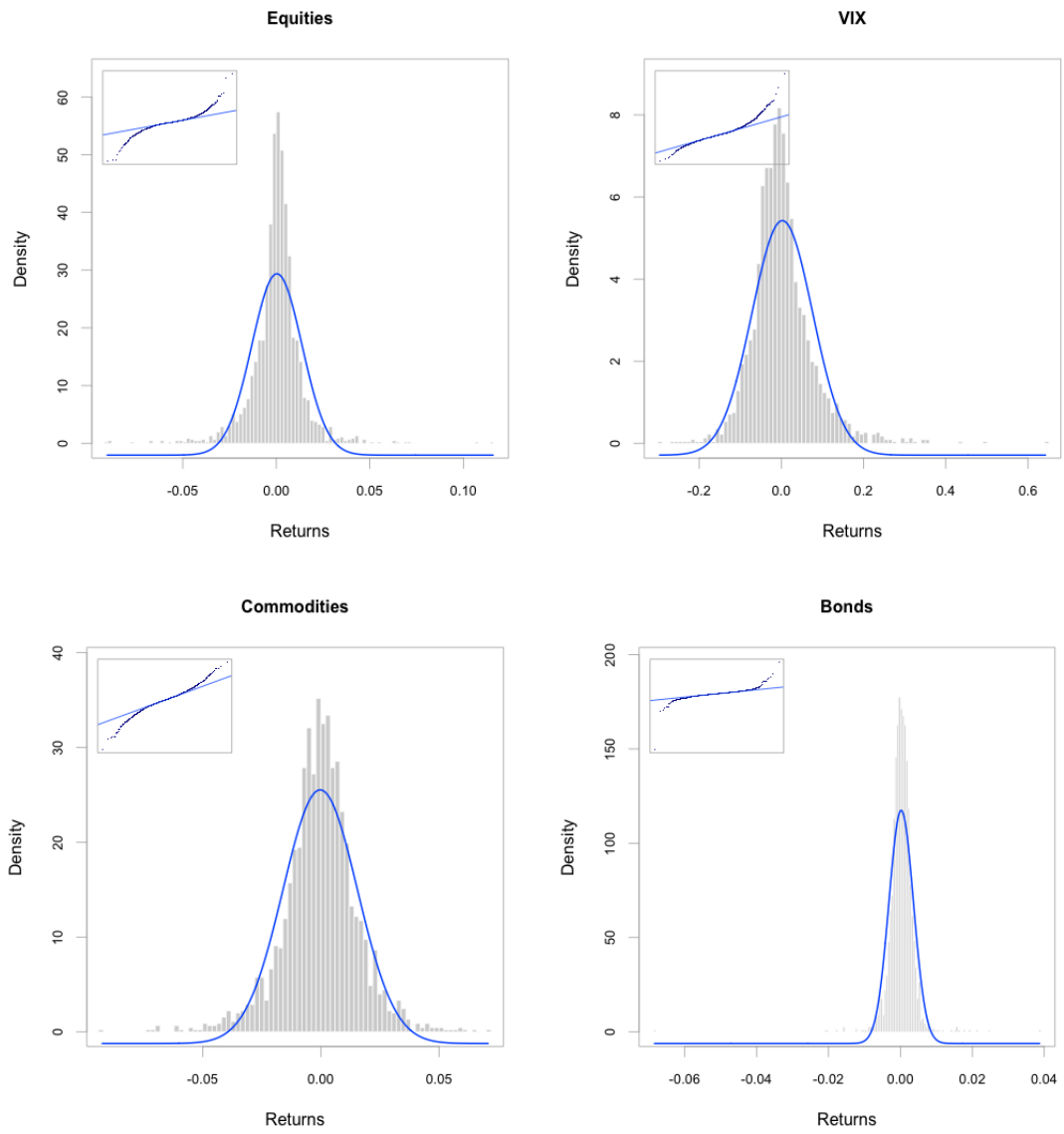


Figure 27: Asset classes return distributions from June 2007 to December 2009

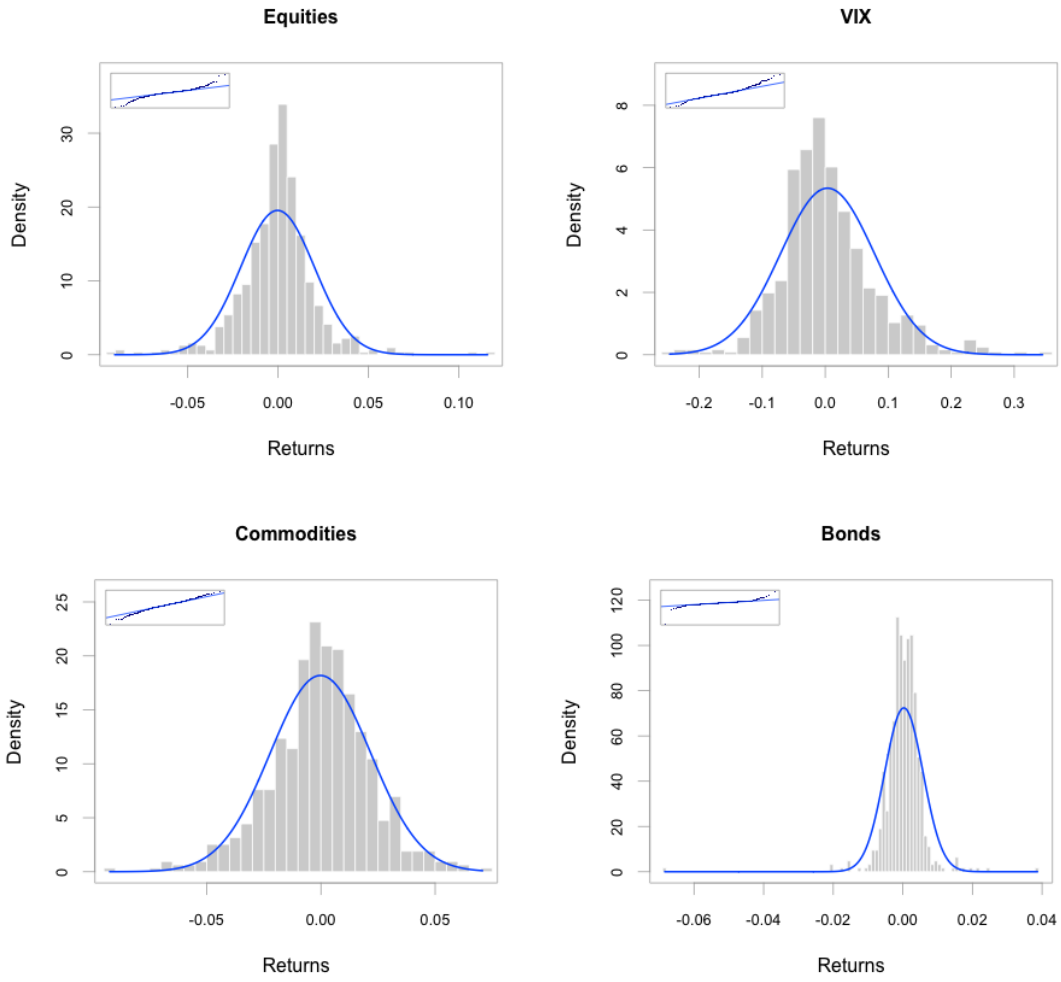


Figure 28: Portfolio return distribution from June 2006 to June 2015

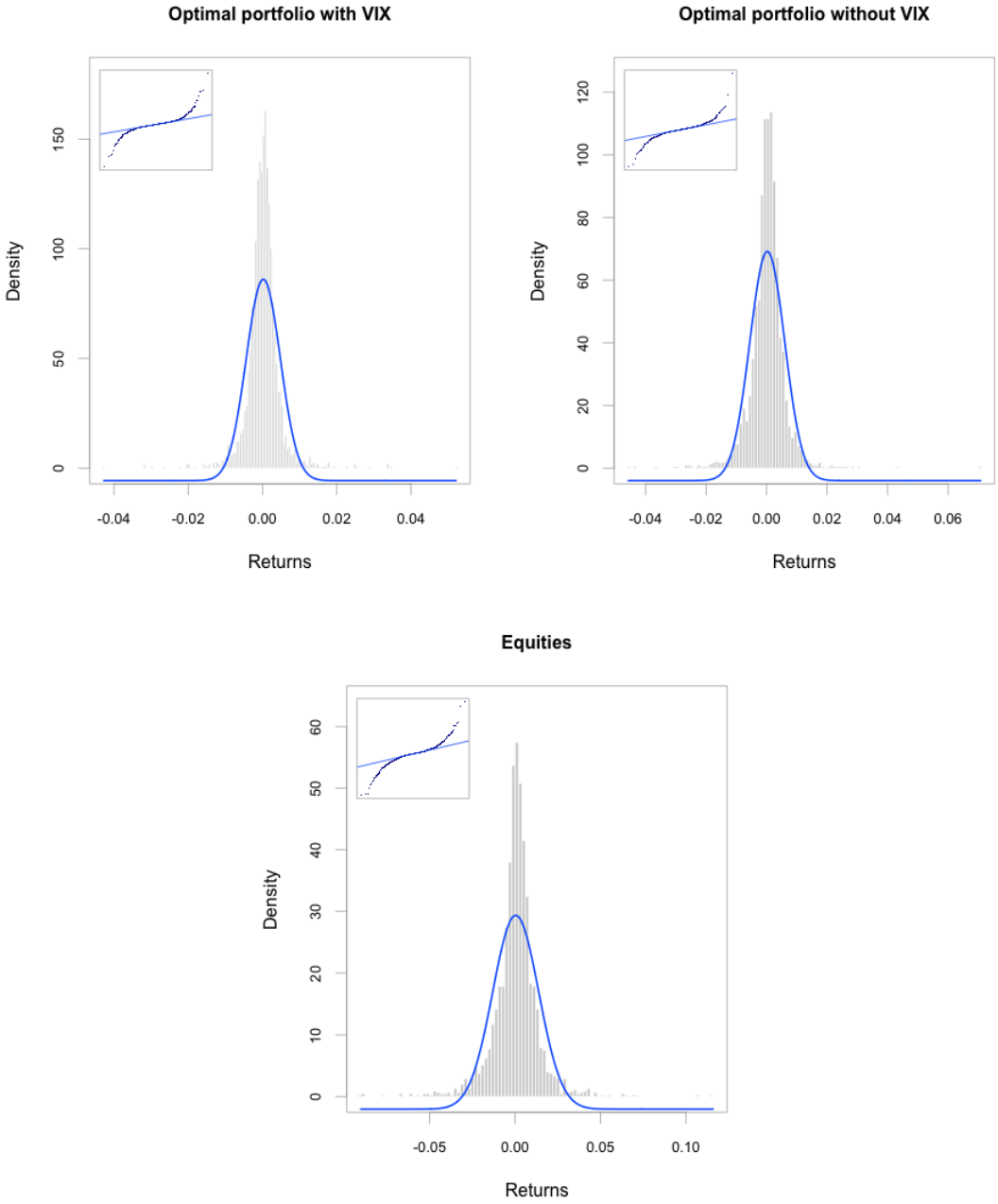


Figure 29: Portfolio return distribution from June 2007 to December 2009

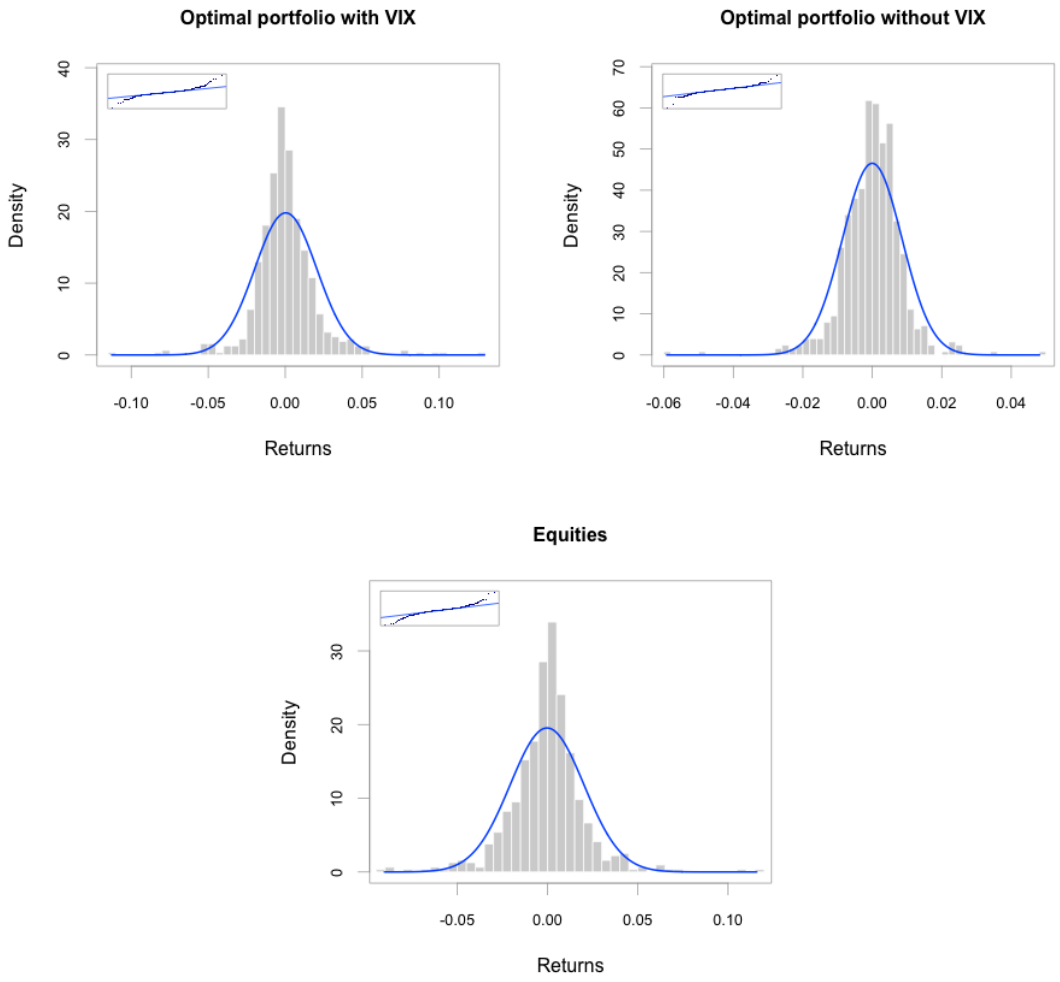


Figure 30: Portfolio return distribution from June 2006 to June 2015

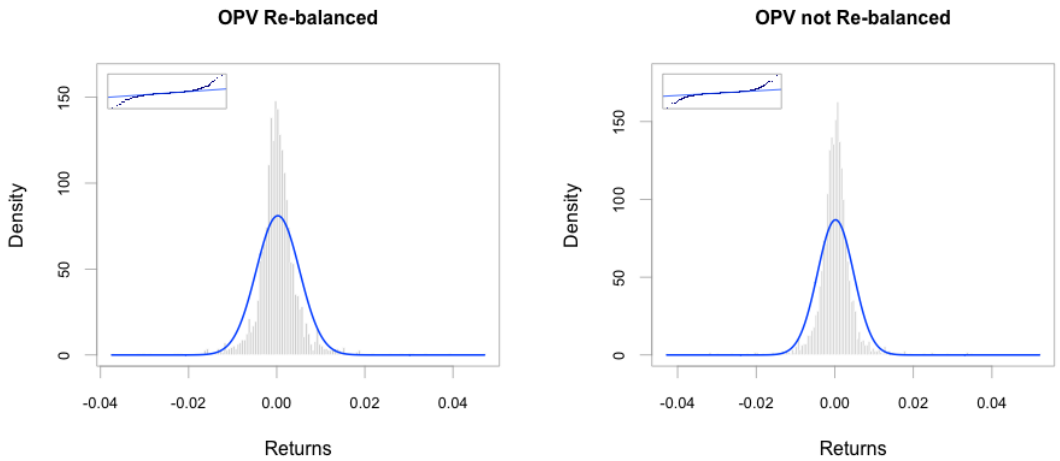


Figure 31: Portfolio return distribution from June 2006 to June 2015

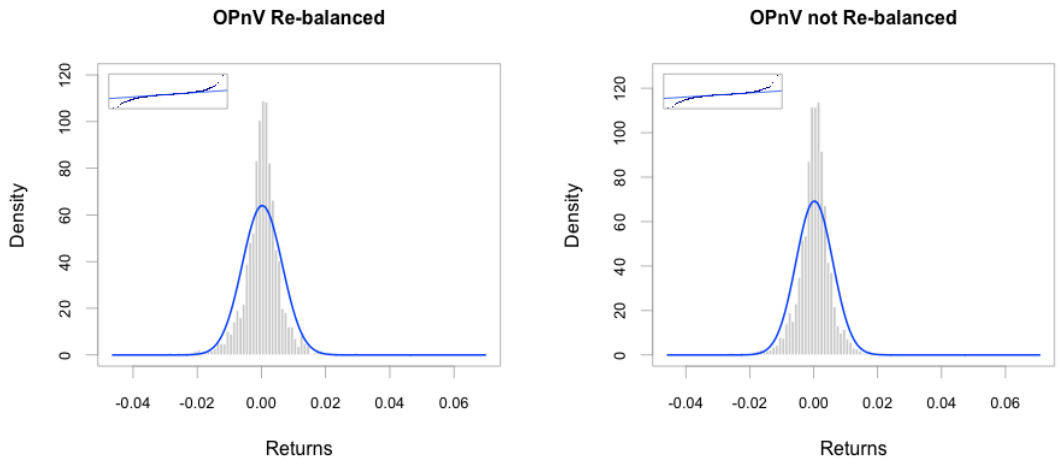


Table 2: Portfolio without VIX allocation efficient frontier from June 2006 to June 2015 (daily) - SR sorted

	Equities	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	50,00%	0,00%	50,00%	0,67%	0,03%	0,039
	...	...	...	...	...	...
Tail	36,23%	13,77%	50,00%	0,62%	0,02%	0,029
	35,99%	14,01%	50,00%	0,62%	0,02%	0,029
	35,75%	14,25%	50,00%	0,62%	0,02%	0,029
	35,51%	14,49%	50,00%	0,62%	0,02%	0,028
	35,27%	14,73%	50,00%	0,62%	0,02%	0,028
	35,03%	14,97%	50,00%	0,62%	0,02%	0,028
	34,79%	15,21%	50,00%	0,62%	0,02%	0,028
	34,55%	15,45%	50,00%	0,62%	0,02%	0,027
	34,30%	15,70%	50,00%	0,62%	0,02%	0,027
	34,06%	15,94%	50,00%	0,62%	0,02%	0,027
	33,82%	16,18%	50,00%	0,62%	0,02%	0,027
	33,58%	16,42%	50,00%	0,62%	0,02%	0,027
	33,34%	16,66%	50,00%	0,62%	0,02%	0,026
	33,10%	16,90%	50,00%	0,62%	0,02%	0,026
	32,86%	17,14%	50,00%	0,62%	0,02%	0,026

Table 3: Portfolio without VIX allocation efficient frontier from June 2006 to June 2015 (daily) - Min Std Deviation sorted

	Equities	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head						
	32,86%	17,14%	50,00%	0,62%	0,02%	0,0259
	33,10%	16,90%	50,00%	0,62%	0,02%	0,0261
	33,34%	16,66%	50,00%	0,62%	0,02%	0,0263
	33,58%	16,42%	50,00%	0,62%	0,02%	0,0265
	33,82%	16,18%	50,00%	0,62%	0,02%	0,0268
	34,06%	15,94%	50,00%	0,62%	0,02%	0,0270
	34,30%	15,70%	50,00%	0,62%	0,02%	0,0272
	34,55%	15,45%	50,00%	0,62%	0,02%	0,0274
	34,79%	15,21%	50,00%	0,62%	0,02%	0,0276
	35,03%	14,97%	50,00%	0,62%	0,02%	0,0279
	35,27%	14,73%	50,00%	0,62%	0,02%	0,0281
	35,51%	14,49%	50,00%	0,62%	0,02%	0,0283
	35,75%	14,25%	50,00%	0,62%	0,02%	0,0285
	35,99%	14,01%	50,00%	0,62%	0,02%	0,0287
	36,23%	13,77%	50,00%	0,62%	0,02%	0,0289
Tail						
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386
	50,00%	0,00%	50,00%	0,67%	0,03%	0,0386

Table 4: Portfolio with VIX allocation efficient frontier from June 2006 to June 2015 (daily) - SR sorted

	Equities	VIX	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	42,43%	7,57%	0,00%	50,00%	0,44%	0,04%	0,097
	42,46%	7,54%	0,00%	50,00%	0,44%	0,04%	0,097
	42,40%	7,60%	0,00%	50,00%	0,44%	0,04%	0,097
	42,49%	7,51%	0,00%	50,00%	0,43%	0,04%	0,097
	42,37%	7,63%	0,00%	50,00%	0,44%	0,04%	0,097
	42,52%	7,48%	0,00%	50,00%	0,43%	0,04%	0,097
	42,34%	7,66%	0,00%	50,00%	0,44%	0,04%	0,097
	42,55%	7,45%	0,00%	50,00%	0,43%	0,04%	0,097
	42,31%	7,69%	0,00%	50,00%	0,44%	0,04%	0,097
	42,58%	7,42%	0,00%	50,00%	0,43%	0,04%	0,097
	42,28%	7,72%	0,00%	50,00%	0,44%	0,04%	0,097
	42,61%	7,39%	0,00%	50,00%	0,43%	0,04%	0,097
	42,25%	7,75%	0,00%	50,00%	0,44%	0,04%	0,097
	42,22%	7,78%	0,00%	50,00%	0,44%	0,04%	0,097
	42,64%	7,36%	0,00%	50,00%	0,43%	0,04%	0,097
	...	...	...	...	...	...	...
Tail	50,00%	27,29%	0,00%	22,71%	1,59%	0,09%	0,056
	50,00%	27,33%	0,00%	22,67%	1,59%	0,09%	0,056
	50,00%	27,38%	0,00%	22,62%	1,60%	0,09%	0,056
	50,00%	27,42%	0,00%	22,58%	1,60%	0,09%	0,056
	50,00%	27,46%	0,00%	22,54%	1,60%	0,09%	0,056
	50,00%	27,50%	0,00%	22,50%	1,60%	0,09%	0,056
	50,00%	27,55%	0,00%	22,45%	1,61%	0,09%	0,055
	50,00%	27,59%	0,00%	22,41%	1,61%	0,09%	0,055
	50,00%	27,63%	0,00%	22,37%	1,61%	0,09%	0,055
	50,00%	27,67%	0,00%	22,33%	1,62%	0,09%	0,055
	50,00%	27,72%	0,00%	22,28%	1,62%	0,09%	0,055
	50,00%	27,76%	0,00%	22,24%	1,62%	0,09%	0,055
	50,00%	27,80%	0,00%	22,20%	1,63%	0,09%	0,055
	50,00%	27,84%	0,00%	22,16%	1,63%	0,09%	0,055
	50,00%	27,89%	0,00%	22,11%	1,63%	0,09%	0,055

Table 5: Portfolio with VIX allocation efficient frontier from June 2006 to June 2015 (daily) - Min Std Deviation sorted

	Equities	VIX	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	37,97%	5,80%	6,24%	50,00%	0,41%	0,03%	0,0849
	38,25%	5,85%	5,90%	50,00%	0,41%	0,04%	0,0856
	38,54%	5,90%	5,56%	50,00%	0,41%	0,04%	0,0864
	38,83%	5,95%	5,22%	50,00%	0,41%	0,04%	0,0871
	39,11%	6,00%	4,88%	50,00%	0,41%	0,04%	0,0878
	39,40%	6,06%	4,54%	50,00%	0,41%	0,04%	0,0885
	39,69%	6,11%	4,21%	50,00%	0,41%	0,04%	0,0891
	39,97%	6,16%	3,87%	50,00%	0,41%	0,04%	0,0898
	40,26%	6,21%	3,53%	50,00%	0,41%	0,04%	0,0904
	40,55%	6,27%	3,19%	50,00%	0,41%	0,04%	0,0910
	40,83%	6,32%	2,85%	50,00%	0,41%	0,04%	0,0916
	41,12%	6,37%	2,51%	50,00%	0,41%	0,04%	0,0922
	41,41%	6,42%	2,17%	50,00%	0,42%	0,04%	0,0927
	41,69%	6,47%	1,83%	50,00%	0,42%	0,04%	0,0933
	41,98%	6,53%	1,49%	50,00%	0,42%	0,04%	0,0938
Tail	...	...	...	...	...	...	...
	50,00%	27,29%	0,00%	22,71%	1,59%	0,09%	0,0557
	50,00%	27,33%	0,00%	22,67%	1,59%	0,09%	0,0557
	50,00%	27,38%	0,00%	22,62%	1,60%	0,09%	0,0556
	50,00%	27,42%	0,00%	22,58%	1,60%	0,09%	0,0556
	50,00%	27,46%	0,00%	22,54%	1,60%	0,09%	0,0555
	50,00%	27,50%	0,00%	22,50%	1,60%	0,09%	0,0555
	50,00%	27,55%	0,00%	22,45%	1,61%	0,09%	0,0555
	50,00%	27,59%	0,00%	22,41%	1,61%	0,09%	0,0554
	50,00%	27,63%	0,00%	22,37%	1,61%	0,09%	0,0554
	50,00%	27,67%	0,00%	22,33%	1,62%	0,09%	0,0553
	50,00%	27,72%	0,00%	22,28%	1,62%	0,09%	0,0553
	50,00%	27,76%	0,00%	22,24%	1,62%	0,09%	0,0552
	50,00%	27,80%	0,00%	22,20%	1,63%	0,09%	0,0552
	50,00%	27,84%	0,00%	22,16%	1,63%	0,09%	0,0552
	50,00%	27,89%	0,00%	22,11%	1,63%	0,09%	0,0551

Table 6: Portfolio without VIX allocation efficient frontier from June 2007 to December 2009 (daily) - SR sorted

	Equities	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	19,95%	30,05%	50,00%	0,95%	0,00%	0,0022
	19,97%	30,03%	50,00%	0,95%	0,00%	0,0022
	19,98%	30,02%	50,00%	0,95%	0,00%	0,0022
	20,00%	30,00%	50,00%	0,95%	0,00%	0,0022
	20,02%	29,98%	50,00%	0,95%	0,00%	0,0022
	20,04%	29,96%	50,00%	0,95%	0,00%	0,0022
	20,06%	29,94%	50,00%	0,95%	0,00%	0,0022
	20,08%	29,92%	50,00%	0,95%	0,00%	0,0022
	20,10%	29,90%	50,00%	0,95%	0,00%	0,0022
	20,11%	29,89%	50,00%	0,95%	0,00%	0,0022
	20,13%	29,87%	50,00%	0,95%	0,00%	0,0022
	20,15%	29,85%	50,00%	0,95%	0,00%	0,0022
	20,17%	29,83%	50,00%	0,95%	0,00%	0,0022
	20,19%	29,81%	50,00%	0,95%	0,00%	0,0022
	20,21%	29,79%	50,00%	0,95%	0,00%	0,0022
Tail	...	...	...	...	...	...
	28,95%	21,05%	50,00%	0,93%	0,00%	0,0014
	28,97%	21,03%	50,00%	0,93%	0,00%	0,0014
	28,99%	21,01%	50,00%	0,93%	0,00%	0,0014
	29,01%	20,99%	50,00%	0,93%	0,00%	0,0014
	29,03%	20,97%	50,00%	0,93%	0,00%	0,0014
	29,05%	20,95%	50,00%	0,93%	0,00%	0,0014
	29,06%	20,94%	50,00%	0,93%	0,00%	0,0014
	29,08%	20,92%	50,00%	0,93%	0,00%	0,0014
	29,10%	20,90%	50,00%	0,93%	0,00%	0,0014
	29,12%	20,88%	50,00%	0,93%	0,00%	0,0014
	29,14%	20,86%	50,00%	0,93%	0,00%	0,0014
	29,16%	20,84%	50,00%	0,93%	0,00%	0,0014
	29,18%	20,82%	50,00%	0,93%	0,00%	0,0014
	29,19%	20,81%	50,00%	0,93%	0,00%	0,0014
	29,21%	20,79%	50,00%	0,93%	0,00%	0,0014

Table 7: Portfolio without VIX allocation efficient frontier from June 2007 to December 2009 (daily) - Min Std Deviation sorted

	Equities	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	29,21%	20,79%	50,00%	0,93%	0,00%	0,0014
	29,19%	20,81%	50,00%	0,93%	0,00%	0,0014
	29,18%	20,82%	50,00%	0,93%	0,00%	0,0014
	29,16%	20,84%	50,00%	0,93%	0,00%	0,0014
	29,14%	20,86%	50,00%	0,93%	0,00%	0,0014
	29,12%	20,88%	50,00%	0,93%	0,00%	0,0014
	29,10%	20,90%	50,00%	0,93%	0,00%	0,0014
	29,08%	20,92%	50,00%	0,93%	0,00%	0,0014
	29,06%	20,94%	50,00%	0,93%	0,00%	0,0014
	29,05%	20,95%	50,00%	0,93%	0,00%	0,0014
	29,03%	20,97%	50,00%	0,93%	0,00%	0,0014
	29,01%	20,99%	50,00%	0,93%	0,00%	0,0014
	28,99%	21,01%	50,00%	0,93%	0,00%	0,0014
	28,97%	21,03%	50,00%	0,93%	0,00%	0,0014
	28,95%	21,05%	50,00%	0,93%	0,00%	0,0014
Tail	...	...	...	...	...	...
	20,21%	29,79%	50,00%	0,95%	0,00%	0,0022
	20,19%	29,81%	50,00%	0,95%	0,00%	0,0022
	20,17%	29,83%	50,00%	0,95%	0,00%	0,0022
	20,15%	29,85%	50,00%	0,95%	0,00%	0,0022
	20,13%	29,87%	50,00%	0,95%	0,00%	0,0022
	20,11%	29,89%	50,00%	0,95%	0,00%	0,0022
	20,10%	29,90%	50,00%	0,95%	0,00%	0,0022
	20,08%	29,92%	50,00%	0,95%	0,00%	0,0022
	20,06%	29,94%	50,00%	0,95%	0,00%	0,0022
	20,04%	29,96%	50,00%	0,95%	0,00%	0,0022
	20,02%	29,98%	50,00%	0,95%	0,00%	0,0022
	20,00%	30,00%	50,00%	0,95%	0,00%	0,0022
	19,98%	30,02%	50,00%	0,95%	0,00%	0,0022
	19,97%	30,03%	50,00%	0,95%	0,00%	0,0022
	19,95%	30,05%	50,00%	0,95%	0,00%	0,0022

Table 8: Portfolio with VIX allocation efficient frontier from June 2007 to December 2009 (daily) - SR sorted

	Equities	VIX	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	36,16%	13,84%	0,00%	50,00%	0,72%	0,05%	0,0739
	36,21%	13,79%	0,00%	50,00%	0,71%	0,05%	0,0739
	36,11%	13,89%	0,00%	50,00%	0,72%	0,05%	0,0739
	36,25%	13,75%	0,00%	50,00%	0,71%	0,05%	0,0739
	36,07%	13,93%	0,00%	50,00%	0,72%	0,05%	0,0739
	36,30%	13,70%	0,00%	50,00%	0,71%	0,05%	0,0739
	36,02%	13,98%	0,00%	50,00%	0,72%	0,05%	0,0739
	36,35%	13,65%	0,00%	50,00%	0,71%	0,05%	0,0739
	35,98%	14,02%	0,00%	50,00%	0,73%	0,05%	0,0739
	36,39%	13,61%	0,00%	50,00%	0,70%	0,05%	0,0739
	35,93%	14,07%	0,00%	50,00%	0,73%	0,05%	0,0739
	35,88%	14,12%	0,00%	50,00%	0,73%	0,05%	0,0738
	36,44%	13,56%	0,00%	50,00%	0,70%	0,05%	0,0738
	35,84%	14,16%	0,00%	50,00%	0,73%	0,05%	0,0738
	36,48%	13,52%	0,00%	50,00%	0,70%	0,05%	0,0738
Tail	...	...	...	...	...	...	...
	39,94%	37,14%	0,00%	22,93%	2,21%	0,13%	0,0574
	39,98%	37,20%	0,00%	22,81%	2,21%	0,13%	0,0573
	40,03%	37,27%	0,00%	22,70%	2,22%	0,13%	0,0573
	40,07%	37,34%	0,00%	22,59%	2,22%	0,13%	0,0573
	40,12%	37,41%	0,00%	22,47%	2,23%	0,13%	0,0573
	40,16%	37,48%	0,00%	22,36%	2,23%	0,13%	0,0572
	40,21%	37,55%	0,00%	22,25%	2,24%	0,13%	0,0572
	40,25%	37,61%	0,00%	22,13%	2,24%	0,13%	0,0572
	40,30%	37,68%	0,00%	22,02%	2,24%	0,13%	0,0572
	40,34%	37,75%	0,00%	21,91%	2,25%	0,13%	0,0572
	40,39%	37,82%	0,00%	21,79%	2,25%	0,13%	0,0571
	40,43%	37,89%	0,00%	21,68%	2,26%	0,13%	0,0571
	40,48%	37,96%	0,00%	21,56%	2,26%	0,13%	0,0571
	40,52%	38,03%	0,00%	21,45%	2,27%	0,13%	0,0571
	40,57%	38,09%	0,00%	21,34%	2,27%	0,13%	0,0570

Table 9: Portfolio with VIX allocation efficient frontier from June 2007 to December 2009 (daily) - Min Std Deviation sorted

	Equities	VIX	Commodities	Bonds	Std.Dev	Exp.Return	Sharpe Ratio
Head	35,98%	8,81%	5,21%	50,00%	0,58%	0,03%	0,0587
	36,00%	8,86%	5,14%	50,00%	0,58%	0,03%	0,0591
	36,02%	8,92%	5,06%	50,00%	0,58%	0,03%	0,0594
	36,05%	8,97%	4,98%	50,00%	0,58%	0,03%	0,0598
	36,07%	9,03%	4,90%	50,00%	0,58%	0,03%	0,0601
	36,09%	9,08%	4,83%	50,00%	0,58%	0,03%	0,0604
	36,12%	9,14%	4,75%	50,00%	0,58%	0,04%	0,0608
	36,14%	9,19%	4,67%	50,00%	0,58%	0,04%	0,0611
	36,16%	9,24%	4,59%	50,00%	0,58%	0,04%	0,0614
	36,19%	9,30%	4,51%	50,00%	0,58%	0,04%	0,0617
	36,21%	9,35%	4,44%	50,00%	0,58%	0,04%	0,0621
	36,23%	9,41%	4,36%	50,00%	0,58%	0,04%	0,0624
	36,26%	9,46%	4,28%	50,00%	0,58%	0,04%	0,0627
	36,28%	9,52%	4,20%	50,00%	0,58%	0,04%	0,0630
	36,30%	9,57%	4,13%	50,00%	0,58%	0,04%	0,0633
Tail	...	...	...	...	...	...	...
	39,94%	37,14%	0,00%	22,93%	2,21%	0,13%	0,0574
	39,98%	37,20%	0,00%	22,81%	2,21%	0,13%	0,0573
	40,03%	37,27%	0,00%	22,70%	2,22%	0,13%	0,0573
	40,07%	37,34%	0,00%	22,59%	2,22%	0,13%	0,0573
	40,12%	37,41%	0,00%	22,47%	2,23%	0,13%	0,0573
	40,16%	37,48%	0,00%	22,36%	2,23%	0,13%	0,0572
	40,21%	37,55%	0,00%	22,25%	2,24%	0,13%	0,0572
	40,25%	37,61%	0,00%	22,13%	2,24%	0,13%	0,0572
	40,30%	37,68%	0,00%	22,02%	2,24%	0,13%	0,0572
	40,34%	37,75%	0,00%	21,91%	2,25%	0,13%	0,0572
	40,39%	37,82%	0,00%	21,79%	2,25%	0,13%	0,0571
	40,43%	37,89%	0,00%	21,68%	2,26%	0,13%	0,0571
	40,48%	37,96%	0,00%	21,56%	2,26%	0,13%	0,0571
	40,52%	38,03%	0,00%	21,45%	2,27%	0,13%	0,0571
	40,57%	38,09%	0,00%	21,34%	2,27%	0,13%	0,0570

Table 10: Summary statistics for different portfolios from June 2006 to June 2015

	Portfolios	
	Optimal portfolio with VIX&Re-balanced	Optimal portfolio with VIX&not Re-balanced
Annualized Return	6,31%	4,95%
Annualized Std Dev	7,82%	7,29%
Annualized Sharpe	0,81	0,68
Skewness	0,60	0,59
Excess Kurtosis	13,32	20,74
Historical VaR (daily)	-0,56%	-0,46%
Expected Shortfall (daily)	-1,10%	-1,03%
Worst Drawdown	19,00%	14,00%
	Weights	
Equities	42,4%	42,4%
VIX	7,6%	7,6%
Commodities	0,0%	0,0%
Bonds	50,0%	50,0%

Table 11: Summary statistics for different portfolios from June 2006 to June 2015

	Portfolios	
	Optimal portfolio without VIX&Re-balanced	Optimal portfolio without VIX&not Re-balanced
Annualized Return	6,10%	5,45%
Annualized Std Dev	9,90%	9,15%
Annualized Sharpe	0,62	0,60
Skewness	0,15	0,25
Excess Kurtosis	13,19	17,90
Historical VaR (daily)	-0,81%	-0,68%
Expected Shortfall (daily)	-1,51%	-1,38%
Worst Drawdown	29,00%	27,00%
	Weights	
Equities	50,0%	50,0%
VIX	0,0%	0,0%
Commodities	0,0%	0,0%
Bonds	50,0%	50,0%

Table 12: GSPC - SPY correlation matrix from June 2006 to June 2015

	GSPC	SPY
GSPC	1	
SPY	0,98	1

Table 13: VIX - VXX - ZIV - TVIX correlation matrix from June 2006 to June 2015

	VIX	VXX	ZIV	TVIX
VIX	1,00			
VXX	0,87	1,00		
ZIV	-0,74	-0,86	1,00	
TVIX	0,87	1,00	-0,86	1,00

Table 14: AGG - TLT - TMF correlation matrix from June 2006 to June 2015

	AGG	TLT	TMF
AGG	1		
TLT	0,60	1	
TMF	0,60	1	1

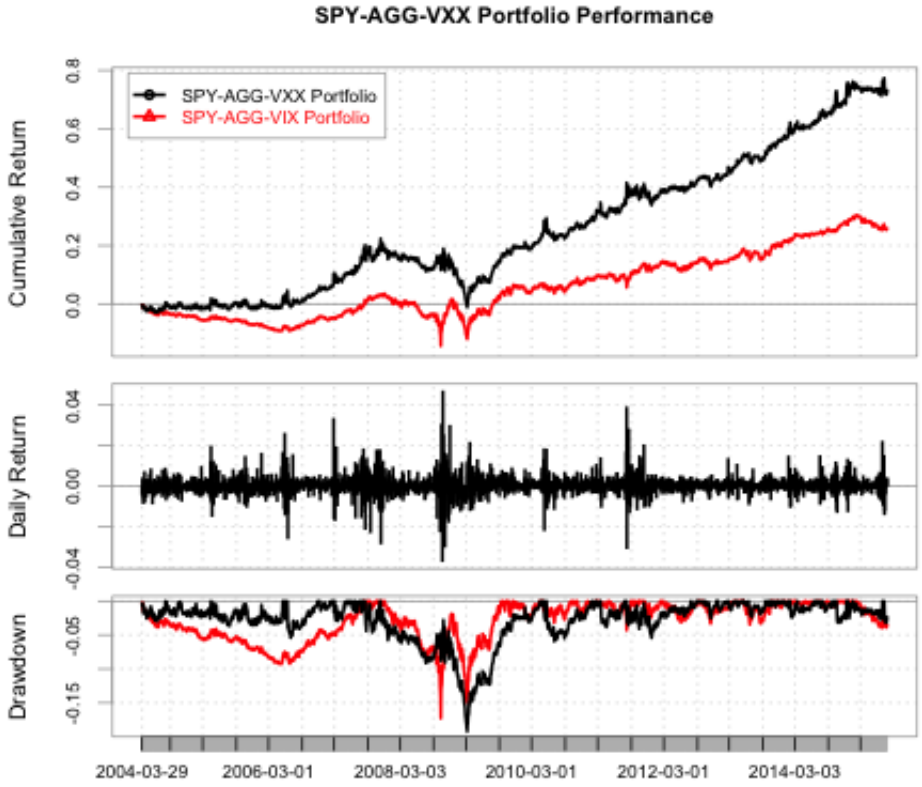


Figure 32: GSPC - AGG - VIX Portfolio & GSPC - AGG - VXX Portfolio from June 2006 to June 2015

Table 15: SPXL - TMF - ZIV - TVIX correlation matrix from June 2006 to June 2015

	SPXL	TMF	ZIV	TVIX
SPXL	1,00			
TMF	-0,42	1,00		
ZIV	0,72	-0,37	1,00	
TVIX	-0,77	0,38	-0,86	1,00

Table 16: Portfolio summary statistics from June 2006 to June 2015

	Portfolios		
	SPY - AGG - VXX	60(SPY)/40(AGG)	Equities(SPY)
Annualized Return	2,08%	5,60%	7,85%
Annualized Std Dev	5,23%	10,99%	19,52%
Annualized Sharpe	0,40	0,51	0,40
Skewness	0,36	0,10	0,23
Excess Kurtosis	22,82	12,01	16,48
Historical VaR (daily)	-0,34%	-0,93%	-1,49%
Expected Shortfall (daily)	-0,75%	-1,68%	-2,98%
Worst Drawdown	17,00%	35,00%	55,00%
	Weights		
Equities	42,4%	60,0%	100,0%
Volatility	7,6%	0,0%	0,0%
Bonds	50,0%	40,0%	0,0%

Table 17: Portfolio summary statistics from June 2006 to June 2015

	Portfolios		
	SPXL - TMF - VXX	60(SPXL)/40(TMF)	Equities(SPY)
Annualized Return	19,99%	25,23%	7,85%
Annualized Std Dev	23,29%	27,96%	19,52%
Annualized Sharpe	0,86	0,90	0,40
Skewness	0,13	-0,02	0,23
Excess Kurtosis	3,43	3,18	16,48
Historical VaR (daily)	-2,17%	-2,70%	-1,49%
Expected Shortfall (daily)	-3,27%	-4,04%	-2,98%
Worst Drawdown	46,00%	56,00%	55,00%
	Weights		
Equities	42,4%	60,0%	100,0%
Volatility	7,6%	0,0%	0,0%
Bonds	50,0%	40 %	0,0%

Table 18: Portfolio summary statistics from June 2006 to June 2015

	Portfolios		
	SPXL - TMF - TVIX	60(SPXL)/40(TMF)	Equities(SPY)
Annualized Return	18,64%	25,23%	7,85%
Annualized Std Dev	24,32%	27,96%	19,52%
Annualized Sharpe	0,77	0,90	0,40
Skewness	0,39	-0,02	0,23
Excess Kurtosis	6,38	3,18	16,48
Historical VaR (daily)	-2,07%	-2,70%	-1,49%
Expected Shortfall (daily)	-3,38%	-4,04%	-2,98%
Worst Drawdown	48,00%	56,00%	55,00%
	Weights		
Equities	42,4%	60,0%	100,0%
Volatility	7,6%	0,0%	0,0%
Bonds	50,0%	40,0%	0,0%

Table 19: Portfolio summary statistics from June 2006 to June 2015

	Portfolios		
	SPXL-TMF-ZIV-TVIX	60(SPXL)/40(TMF)	Equities(SPY)
Annualized Return	18,36%	25,23%	7,85%
Annualized Std Dev	21,49%	27,96%	19,52%
Annualized Sharpe	0,85	0,90	0,40
Skewness	0,39	-0,02	0,23
Excess Kurtosis	6,11	3,18	16,48
Historical VaR (daily)	-1,86%	-2,69%	-1,49%
Expected Shortfall (daily)	-3,01%	-4,04%	-2,98%
Worst Drawdown	44,00%	56,00%	55,00%
	Weights		
Equities	42,4%	60,0%	100,0%
Volatility (TVIX)	7,6%	0,0%	0,0%
Volatility (ZIV)	17,6%	0,0%	0,0%
Bonds	32,4%	40,0%	0,0%