

École polytechnique de Louvain

A study of the landing phase of the Juventas CubeSat on the moon of the binary asteroid system Didymos

**Juventas CubeSat in support of the HERA
mission**

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“Dreams can come true, but there is a secret. They’re realized through the magic of persistence, determination, commitment, passion, practice, focus and hard work. They happen a step at a time, manifested over years, not weeks.”

Elbert Hubbard

Abstract

Asteroids present a threatening danger for humans. In the past, many have crashed on Earth, causing damage to structures and injuring people, including being a possible cause of extinction of the dinosaurs. Their exploration is also motivated by a great scientific interest; considered as the building blocks of our Solar System, exploring them should allow scientists to better understand its evolution and formation. They also present a great astrodynamic challenge due to their complex low-gravity dynamics.

The Master Thesis discusses design solutions for the ballistic landing and bouncing of a nanosatellite on the moon of a binary asteroid system, identifies the crucial design parameters of a direct landing strategy and analyses the sensitivity to the parameters of a binary asteroid system. The proposed framework is generic and can be applied to the landing of a spacecraft on any binary asteroid system.

The work is focused on the Juventas CubeSat, a nanosatellite taking part in the Asteroid Impact & Deflection Assessment (AIDA) collaboration, jointly led by the National Aeronautics and Space Administration (NASA) and the European Space Agency (ESA). The mission, set to start in 2022, will assess the possibility of deflecting asteroids using a kinetic impactor on the smaller of the two bodies and increase our knowledge of the Solar System. The chosen target for the mission is the Didymos binary asteroid system.

The binary system is modeled using the Circular-Restricted Three Body Problem, further completed by adding gravity perturbations as well as solar radiation pressure. The interaction of Juventas with the surface of the moon is also modeled. The landing phase analysis aims at proposing reliable direct landing trajectories and bouncing on the moon of the binary system, characterized by a high success rate and a low settling time. The solutions found are presented and their characteristics are discussed through a Monte Carlo analysis, allowing to take into account uncertainties related to the mission. A sensitivity analysis of the trajectories to the parameters of Didymos is also performed, discussing the robustness of the solutions found.

The key deliverable of this work are: provide a propagator allowing to simulate the motion of a nanosatellite in the Didymos system and additional numerical tools allowing to study and optimize landing trajectories, identify the set of parameters leading to a successful landing and low settling time and finally, propose a set of robust landing trajectories for Juventas.

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Acronyms

The following list groups all the acronyms used throughout the work, in alphabetical order.

6U	6 Units	LFR	Low Frequency Radar
AGEX	Asteroid Geophysical Explorer	LICIA	Light Italian CubeSat for Imaging of Asteroids
AIDA	Asteroid Impact and Deflection Assessment	LT	Landing Trajectory
AIM	Asteroid Impact Mission	MC	Monte-Carlo
APEX	Asteroid Prospection Explorer	MONTE	Mission Analysis, Operations and Navigation Toolkit Environment
AU	Astronomical Unit	NAIF	Navigation and Ancillary Information Facility
CMA-ES	Covariance Matrix Adaptation - Evolution Strategy	NASA	National Aeronautics and Space Administration
CoM	Center of Mass	NEXT	NASA Evolutionary Xenon Thruster
CoR	Coefficient of Restitution	PDF	Probability Density Function
CR3BP	Circular Restricted Three Body Problem	POP	Proximity Observation Phase
DART	Double Asteroid Redirection Test	RAAN	Right Ascension of the Ascending Node
EoM	Equations of the Motion	ROB	Royal Observatory of Belgium
ES	Evolution Strategy	SNR	Signal to Noise Ratio
ESA	European Space Agency	SoI	Sphere of Influence
GA	Gravity Anomaly	SRP	Solar Radiation Pressure
GNC	Guidance, Navigation and Control system	SSTO	Self-Stabilized Terminator Orbit
GP	Gravity Perturbation	UTC	Coordinated Universal Time
GOP	Global Observation Phase		
IMU	Inertial Measurement Unit		
ISL	Inter-Satellite Link		
Isp	Specific Impulse		
J2000	Julian 2000		
JPL	Jet Propulsion Laboratory		

Nomenclature

The following list groups all the nomenclature used throughout the work in Greek and Roman alphabetical order.

<i>Greek alphabet</i>		f	Restitution coefficient of the tangential velocity
α, β, γ	Semi-axes of Didymoon	F_i	Gravitational force exerted by body <i>i</i> on Juventas
λ	Longitude of Juventas	G	Universal gravitational constant
μ	Mass parameter of Didymos	J_n	Harmonics of coefficient n
μ_i^*	Standard gravitational parameter of planet i	K	Juventas kinetic energy
ξ_i	Relative distance between Juventas and body <i>i</i>	L_i	Lagrange point i
ϕ	Latitude of Juventas	m	Juventas mass
ω	Angular velocity of Didymos system	m_{body}	Mass of the studied body
<i>Roman alphabet</i>		m_i	Mass of body i
a	Absolute acceleration of Juventas	P(x)	Legendre polynomial of order <i>n</i>
A_{sun}	Juventas effective area wrt. Sun	PSR@1AU	Nominal pressure exerted by the SRP at 1 AU
a_{SRP}	Acceleration due to the SRP	r	Juventas position vector
C	Jacobi constant	r_i	Distance between the barycenter and body <i>i</i>
c_R	Reflectively coefficient surface	R_i	Radius of the circumscribing sphere of body <i>i</i>
e	Restitution coefficient of the normal velocity	r₁₂	Distance between Didymain and Didymoon
E	Specific energy of Juventas	r_{SoI}	Radius Sphere of Influence
e_{cc}	Eccentricity of Didymoon orbit	R_{Sun-body}	Distance between the sun and the studied body
e_{ij}	Didymoon eccentricity in plane i-j	t	time
		U	Total potential energy of Juventas
		U_i	Potential energy of Juventas w.r.t. body <i>i</i>
		v	Absolute velocity of Juventas

Chapter 1

Introduction

On April 29th 2020, the asteroid nicknamed 1998 OR2 came 6.2 million kilometers close to Earth. The size of the celestial body was scary: 4 km long and 2 km wide, enough to wipe out the human civilization if it crashed on the surface of the Earth [60]. Hopefully, there were no consequences, but in the past, many asteroids have crashed on Earth. For example, 66 million years ago, the Chicxulub impactor, a 11 to 81 km large asteroid is said to be the cause of the mass extinction of the dinosaurs [25]. More recently, in 2013 the 19m wide Duende asteroid entered the atmosphere above the region of Chelyabinsk in Russia. The object exploded, approximately 20 km above the ground, releasing 26 to 33 times as much energy as the Hiroshima atomic bomb. It caused damage to hundreds of buildings and non-fatally injured nearly 1500 people [58]. Such examples prove that the impact of asteroids is a long-term threat to humans and that we should be ready to face one of them in the future.



Figure 1.1: Representation of a threatening asteroid, from [32].

Asteroids are also considered as the current frontier of space exploration. Among all the them, the binary systems of asteroids are of great interest since 1993,

when the first one of its kind, 243 Ida was discovered by the Galileo spacecraft [10]. Characterized by complex low-gravity environments, binary systems have interesting dynamical properties, making them an ideal target for space exploration to study gravitational dynamics. Also considered as the building blocks of the early Solar System, exploring them should also enable us to better understand the formation and evolution of the Solar System [36].

Asteroids are therefore considered as interesting targets for space missions. It is within this idea that NASA and ESA decided to join forces: the AIDA collaboration was born. The mission, set for 2022, has two main objectives: assess the possibility of deflecting asteroids using a kinetic impactor and increase our knowledge in the Solar System. It will be the humankind's first investigation of a planetary defense technique. For the mission, a binary asteroid system was identified: 65803 Didymos. The system is composed of a small body, orbiting around a bigger main one, respectively named Didymoon and Didymain. A total of 5 spacecrafts will contribute to the mission. One of them, the Juventas 6 units CubeSat¹, on which the Master Thesis focuses, is developed by a European consortium under ESA contract. After orbiting around the system of asteroids, it will attempt a direct landing on Didymoon.

Previous work review

Several papers and private companies have already studied the landing of nanosatellites on asteroids. A brief overview is proposed:

Fabio Ferrari et al. have studied the ballistic landing of the MASCOT-2 nanosatellite on the moon of Didymos in their paper "*Ballistic landing design on binary asteroids: The AIM case study*" [6]. In their work, an effective landing strategy for MASCOT-2, modeled as a sphere, is proposed, taking advantage of the complex dynamics of the binary asteroid system. Nevertheless, the bouncing phase was not analyzed.

A few years ago, the Asteroid Geophysical Explorer (AGEX), a 3U CubeSat was among the proposals for the Asteroid Impact Mission (AIM) from ESA, a mission that was headed to characterize Didymos. The AGEX development team carried out a landing and bouncing study [39], considering the lander as a cube and took into account different rock models for the surface of the moon as proposed in the ESA reference model for the Itokawa asteroid [27]. The weaknesses of this work are that the CubeSat is a 3U and that the mission was canceled.

¹Class of nanosatellite made up of 10 cm x 10 cm x 10 cm cube units. Each unit weights less than 1.36 kg. [42]. They are commonly named after the number of cube units they are made of (e.g. 6U CubeSat for a 6 units nanosatellite.)

More recently, the Juventas 6U CubeSat was proposed in support of Hera's ESA mission to Didymos [38]. GOM Space, GMV, the Royal Observatory of Belgium and many other teams are currently working under contract with ESA on the Juventas mission, developing all the mission phases and the CubeSat [29].

Master Thesis outline

In this context, the Royal Observatory of Belgium is focused on studying the approach of Juventas to Didymoon and its behavior on the surface of the asteroid, so as to perform the surface measurements. For this purpose, the ROB needs several things. First, a numerical tool allowing to model the behavior of a nanosatellite in the vicinity of Didymos. Then, a set of reliable landing trajectories and finally a first analysis of the bouncing of Juventas on Didymoon. These numerical tools and studies are proposed in the present work.

In this Master Thesis, the dynamical environment of Didymos is first modeled, using the Circular-Restricted Three Body problem. Then, robust direct landing trajectories, characterized by a high success rate and low settling time on Didymoon are proposed. Using the best trajectories, the bouncing on Didymoon is studied through a Monte Carlo analysis, allowing to take into account the uncertainties related to the mission. The critical parameters of the landing phase are also discussed. A sensitivity analysis of the trajectories to the parameters of Didymos is also performed, allowing to discuss the robustness of the solutions found. The Master Thesis consists of 9 chapters.

Chapter 1 introduces the work, reviews the previous studies and sets the outline of the Master Thesis.

Chapter 2 sets the context of the Juventas mission. First, the AIDA collaboration between NASA and ESA is presented, followed by a description of the chosen asteroid system: Didymos. Then, the Juventas CubeSat, the science objectives, main constraints of the mission and preliminary trajectories are provided.

Chapter 3 is entirely dedicated to the dynamical environment of Didymos. The equations of motion of Juventas are derived, followed by a presentation of the dynamical characteristics of Didymos. Then, the orbital perturbations to which Juventas is subject are implemented and discussed.

Chapter 4 gives an overview of the numerical implementation of the propagator using MATLAB® and of the SPICE TOOLKIT, a tool used to assist scientists in space mission design. Finally, all the dynamics considered in the study are summarized.

Chapter 5 is entirely dedicated to the energetic verification and validation of the implemented propagator, by comparing obtained landing results with data from companies developing Juventas and a space mission design software developed by NASA.

Chapter 6 then presents additional numerical tools developed for the upcoming studies. First, the behavior of Juventas on Didymoon is implemented and different models for the topography of Didymoon are discussed. Then, a Monte Carlo algorithm is developed and all the uncertainties related to the landing phase of Juventas are provided. Finally, an optimization method, the Covariance Matrix Adaptation - Evolution Strategy is implemented in the context of the Juventas mission.

Chapter 7 provides a landing phase analysis. The landing strategy to Didymoon is chosen and a backward propagation is performed so as to suggest many possible landing approaches, based on several strict selection criteria. Then, a Monte Carlo analysis is done, taking into account the uncertainties of the thrusters of Juventas and the Didymoon surface. It allows to assess the robustness of the solutions and to study the propagation of the uncertainties. The bouncing of Juventas on Didymoon is also studied. Based on those analysis, the impact of the design parameters of the landing on the results is discussed and two trajectories are proposed for Juventas.

Chapter 8 provides an additional study of the sensitivity of the landing trajectories to the main Didymos parameters.

Finally, Chapter 9 concludes this Master Thesis and suggests future work that could be realized on the Juventas mission.

Chapter 2

Mission context

In this chapter, the context of the Master Thesis is presented. The AIDA international collaboration is firstly introduced: a planetary defense mission jointly led by scientists from the NASA and the ESA. Then, the chosen binary asteroid system for the mission is presented, Didymos. Finally, more details are given concerning the European mission, more commonly called the Hera mission and its 6 units CubeSat, Juventas. The nanosatellite structure, as well as its science objectives, the main constraints and the preliminary trajectories are discussed.

2.1 AIDA collaboration

The AIDA mission is an international collaboration that aims to study the effects of impacting at high velocity the moon of a binary system. The overall mission is divided into two missions: the Double Asteroid Redirection Test (DART) mission for the Americans and the Hera mission for the Europeans. The DART mission has two spacecrafts: The DART impactor and the Light Italian CubeSat for Imaging of Asteroids (LICIA). The Hera mission has three spacecrafts: the mothercraft Hera and two nanosatellites, Juventas and the Asteroid Prospection Explorer (APEX). The AIDA collaboration will then see a total of 5 spacecrafts flying to Didymos and together, they should prove if the method is effective to deflect asteroids that could possibly impact Earth. This will be the first time humans modify the dynamics of a celestial body of the Solar System in a measurable way.[24] [18]

NASA will launch its DART impactor in February of 2022, by the means of a SpaceX Falcon 9 rocket. The impact on Didymoon is planned for early October 2022. The secondary spacecraft LICIA, a 6U CubeSat developed by the Italian Space Agency will travel on-board of DART. It will be released before the impact and will study the crater formation and the debris. The DART spacecraft will deflect the moon of Didymos by crashing at a relative speed of approximately 6.6

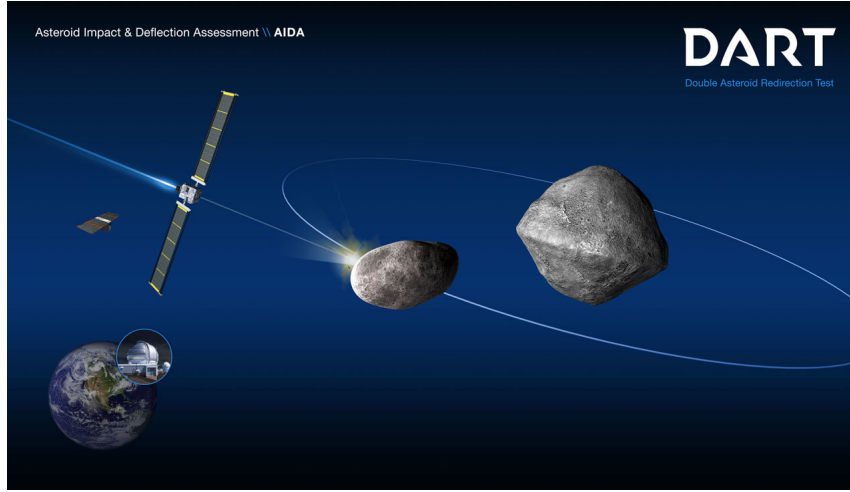


Figure 2.1: DART impacting Didymos, from [24].

km/s. The spacecraft is propelled by the NASA Evolutionary Xenon Thruster¹ (NEXT). The collision will change the orbital speed of Didymoon by a fraction of 1%, changing the orbital period by several minutes, enough to be observed from Earth. The immediate consequences of the impact will be hidden by a cloud of dust. As the field of vision becomes clearer and clearer, different measurements will be performed from the Earth, at a distance of approximately 11 million km from Didymos. Due to the distance, it has been calculated that the measurements will have a 10% residual uncertainty. Radar investigation and ground-based observation will evaluate the change in momentum in Didymoon. Nevertheless, the impact crater and other important information need to be obtained, such as the exact mass of Didymoon, its porosity and its internal structure. For this, close observation is needed. This is where the European spacecrafts come in to play. The HERA mission will play a key role in providing clear information about the impact [13; 41].

The European spacecraft HERA will start its mission in 2024, launched from an Ariane 6 rocket. It will arrive to Didymos in 2027, 5 years after DART's impact on the moon of Didymos. Once in the vicinity of the asteroids, HERA will deploy the two 6U CubeSats; APEX and Juventas and will conduct a post-impact survey. In addition to assessing the possibility of deflecting asteroids, the HERA mission has two main science goals [32; 50]:

¹The NEXT engine uses electricity to accelerate xenon particles up to speeds of 40 km/s. It is projected to be three times as powerful as current engines of the same type. More information can be found at [31]

- Technology demonstration: HERA will pave the way to future interplanetary missions by using innovating real-time navigation systems combining complex embedded algorithms and a series of sensors, allowing HERA to screen its environment and produce a detailed picture of it in a complex low-gravity environment [56].
- Asteroid Science: HERA will also be responsible for many science objectives such as: defining the mass of Didymoon, its porosity, surface topography, internal structure and crater investigation. The collected information will refine our understanding of the Solar Systems and might lead to new paradigms in terms of Solar System formation [19].

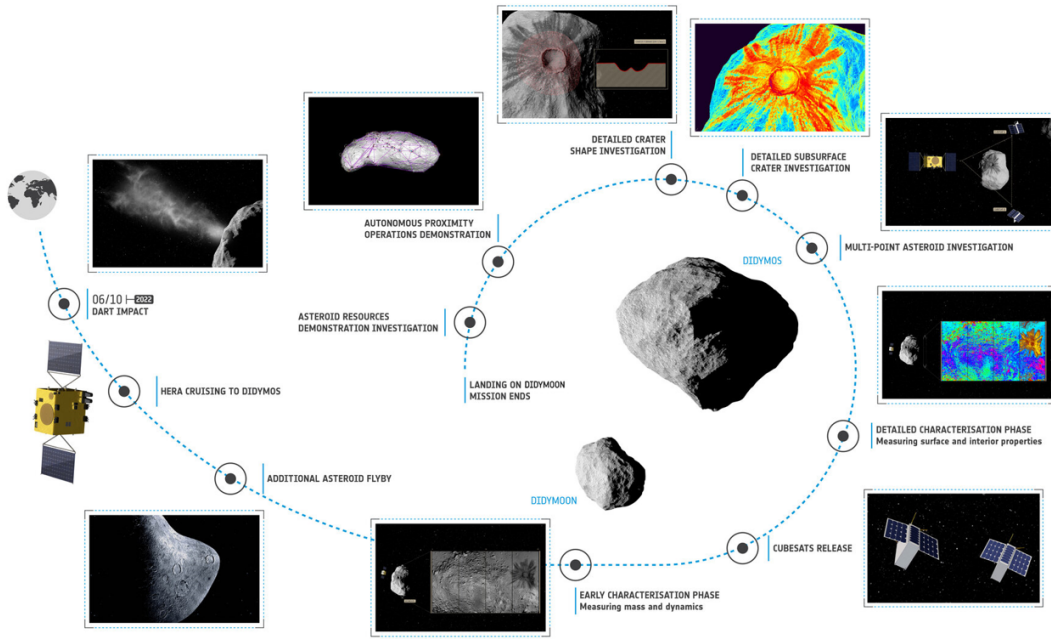


Figure 2.2: HERA mission objectives, from [32].

Figure 2.2 gives a detailed picture of the sciences objectives. APEX is being developed by Sweden, Finland, Germany and Czech Republic while Juventas is developed by the private companies GOM Space and GMV, as well as additional partners such as the Royal Observatory of Belgium, which is working on the Juventas bouncing on the surface of Didymoon. Both CubeSats will attempt to land on the surface of the moon asteroid and gather additional data on Didymoon and relay information through HERA [34; 33].

To summarize, the AIDA collaboration will see a total of five spacecrafts flying to Didymos. Two mothercrafts: HERA and DART. Three 6U CubeSats, Juventas, APEX and LICIA. They will contribute to the understanding of the formation of

binary asteroids and will assess the possibility of defending Earth by deflecting asteroids using a kinetic impact. Table 2.1 summarizes the information of the five spacecrafts:

Space Agency	Launch Date	Spacecraft	Size	Dry mass [kg]	Main Goal
NASA	February 2022	DART	$12.5 \text{ m} \times 2.4 \text{ m} \times 2.4 \text{ m}^*$	500	Impact Didymoon
		LICIA	6U Cubesat	_**	Study crater formation and ejecta
ESA	2024	HERA	_**	350	Characterize Didymos
		Juventas	6U Cubesat	12	Internal Structure of Didymoon characterization
		APEX	6U Cubesat	_**	Characterize Didymoon

Table 2.1: Spacecrafts taking part in the AIDA collaboration.

* With extended solar arrays. ** Confidential information.

2.2 Didymos system

Didymos main asteroid, Didymain, was initially discovered in 1996 by the University of Arizona' Observatory, while the smaller asteroid, Didymoon orbiting around the primary, was only discovered recently, in 2003. The binary asteroid system is called "Didymos", the Greek word for twin due to its binary nature. Didymos orbits around the Sun with a period of 770 days and a distance ranging from 1 to 2.3 AU [37].

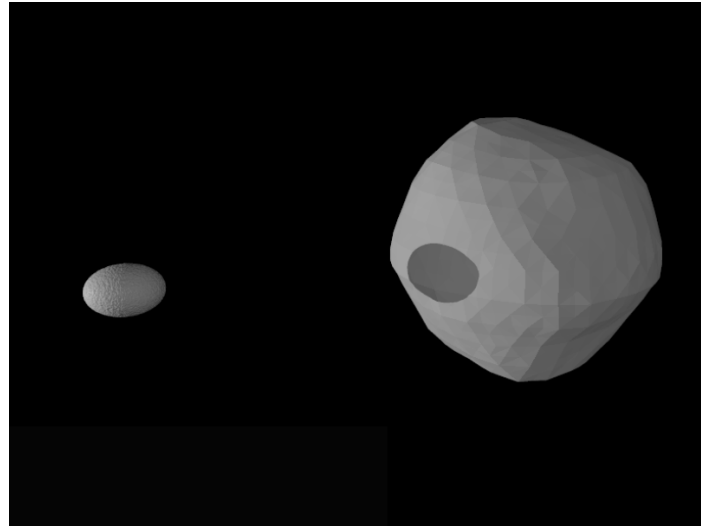


Figure 2.3: Didymain and Didymoon at scale, from [27].

The shape model of Didymain was obtained thanks to radar observations and light-

curve data. Due to very weak SNR², radar cannot provide a model of Didymoon. The shape of Didymoon is therefore assumed to be a triaxial ellipsoid. The long axis is assumed to be oriented on average in the direction that connects the center of mass of Didymain and Didymoon [27]. The model is not a final one, but from the above, one concludes that it is a good approximation for performing preliminary studies. The current model is shown in Figure 2.3.

The main parameters of the system are summarized in Table 2.2. Due to the lack of information, it is considered that both the primary and secondary asteroids have the same constant bulk density. This assumption might change in future updated shape models.

	Value	Units
<i>Physical parameters</i>		
Mass parameter	9.214 e-3	-
Total mass	5.278 e11	kg
Primary ellipsoid semi-axes	399,392,380	m
Secondary ellipsoid semi-axes	103, 79, 66	m
Primary spin rate	7.723 e-4	rad/s
Secondary orbital period	11.92	h
Secondary orbital eccentricity	0.03 (upper limit)	-
Primary density	2104	kg/m ³
<i>Binary orbit</i>		
Semi-major axis	1180	m
Pole ecliptic longitude	310	deg
Pole ecliptic latitude	-84	deg
<i>Heliocentric orbit</i>		
Semi-major axis	1.645	AU
Eccentricity	0.384	-
Inclination	3.408	deg
Argument of perigee	319.20	deg
RAAN	73.256	deg
Mean anomaly	195.97	deg
Epoch	2007/01/31	UTC

Table 2.2: Parameters of the Didymos system, from [55; 5; 47].

²Signal to Noise ratio, measure of the level of signal to the level of noise.

2.3 Juventas

Once released from HERA, Juventas will start its mission. Equipped with a low frequency radar, it will characterize the binary system of asteroids. Juventas is designed to use HERA as a proxy: information will be relayed from Juventas to the Earth through the mothercraft. Once the radar and radio science objectives will be met, the nanosatellite will attempt to land on Didymoon to investigate the dynamical properties of the asteroid.

This section aims at defining the role of Juventas in the HERA mission: the sciences objectives are presented, the CubeSat payload and structure, the mission design and the main constraints are discussed.

2.3.1 Science objectives

Juventas has a total of four objectives, with the first three considered as primary and a secondary one [38]:

- Determine the gravity field of Didymoon
- Determine the interior structure of Didymoon
- Determine the surface properties of Didymoon and crater characterization
- Determine the dynamical properties of Didymoon

Setting these objectives is essential, as they can be broken-down in investigations and measurements, needed to define the payload of Juventas.

2.3.2 Juventas CubeSat

Juventas is a 6U CubeSat with a wet mass of 12 kg. Its small size is needed to fit in the Hera mothercraft and will be deployed using a specific deployer interface needed for low velocities releases, in addition to the standard rail interface. The payload of Juventas includes a Low Frequency Radar (LFR), a 3-axis gravimeter, an Inter-Satellite Link radio (ISL), a visible camera, accelerometers and gyros (IMU) as well as electronics needed for the Guidance, Navigation and Control (GNC) system. The attitude determination will use the star tracker as well as the sensors, the gyros and the reaction wheels. The propulsion subsystem is a butane cold-gas system [38]. The full internal structure is presented in Figure 2.4a.

Figure 2.4b shows Juventas in a deployed configuration. The two large solar arrays are needed to cover the operations. The antennas will extend up to 1.5 m in order

to support the low frequency radar operations. The main Juventas parameters are summarized in Table 2.3.

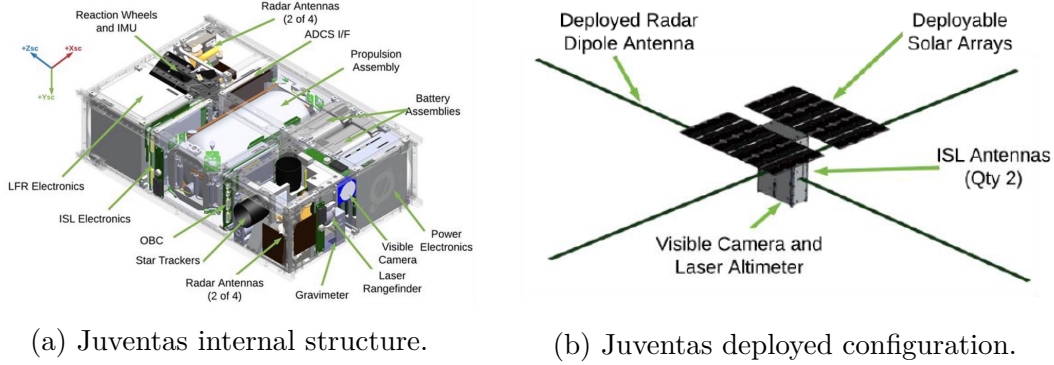


Figure 2.4: Juventas CubeSat, from [38; 29].

	Value	Units
<i>Technical parameters</i>		
Wet mass	12	kg
Reflective Surface	0.464	m ²
Reflective coefficient	1.5	-
Isp	50	sec

Table 2.3: Parameters of Juventas CubeSat, from [29].

2.3.3 Mission design

After being released from Hera, the Juventas mission begins. It will be released from a distance of $\sim 10km$ from Didymain with a very low velocity. The mission is divided in three phases: the Commissioning, the Observations and the Impact/Landing. In order to commission the spacecraft and check all the subsystems, one week is estimated before starting the proper payload observations. The complete timeline of the mission is summarized in Table 2.4.

Once Juventas is pronounced operational, it will inject itself into a Self-Stabilized Terminator Orbit (SSTO). Three days are estimated to be needed to transfer Juventas to the SSTO. This orbit has the characteristic of being approximately perpendicular to the Didymoon orbit around Didymain and the ecliptic plane³ of the system of asteroids. Such an orbit also minimizes the DeltaV required to maintain the orbit and simplifies the ground flight dynamics planning of the

³The ecliptic plane is defined as the plane of Didymos's orbit around the Sun.

Mission Phase	Initial Epoch	Final Epoch	Duration [days]
Release and commissioning Phase	1st June 2027	8th June 2027	7
Transfer to SSTO	8th June 2027	11th June 2027	3
Global Observation Phase	11th June 2027	11th July 2027	30
Proximity Observation Phase	11th July 2027	10th August 2027	30
Landing Phase	10th August 2027	10th August 2027	<1
Total			71

Table 2.4: Mission timeline.

navigation team [38]. Two SSTO are currently studied; the first one on a 3 km orbit, in order to perform the Global Observation Phase (GOP) and the second one at a 2 km distance from Didymain, allowing to perform the Proximity Observation Phase (POP). Figure 2.5 shows the trajectories of Juventas during the two observation phases. The plane of the SSTO can be seen to be perpendicular to the Sun direction.

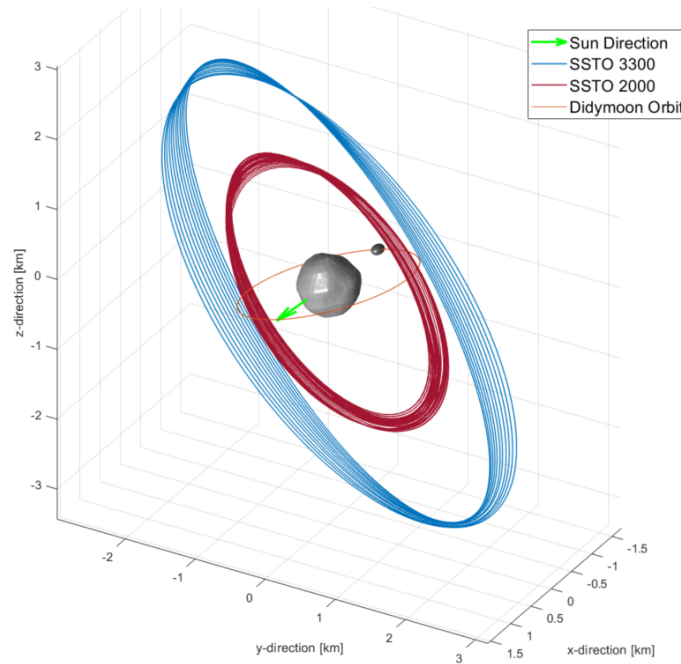
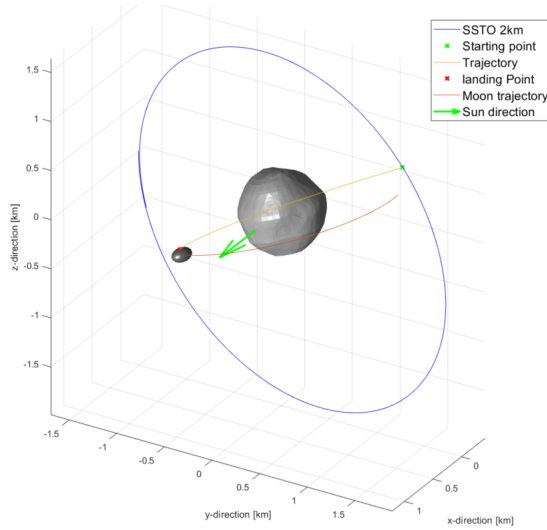


Figure 2.5: SSTO orbits.

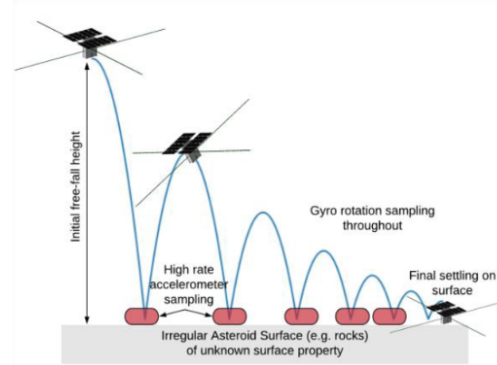
While on the SSTO, Juventas will perform its radio science experiments using the ISL and exercise the low-frequency radar. They will contribute to the primary objectives of the mission of Juventas. The low-frequency nature LFR allows it to have large penetration depth and to determine the internal structure of

Didymoon. The radio science experiments will allow to characterize the gravity field of Didymoon. To have more than 95% of coverage, 60 days of operation are needed, see [29].

Once enough data have been gathered by the nanosatellite, the most risky mission phase will start: Juventas will attempt to land on the surface of Didymoon. As Juventas has not been designed to be a lander, no landing site is preferable. This phase is set to last only a few hours. During the descent, the on-board visible camera will allow to obtain pictures of the Didymoon surface and the DART crater. The gyros, accelerometers and 3-axis gravimeter will record the dynamics at the impact and during the bounces, allowing to reconstruct the surface Coefficient of Restitution (CoR). Once Juventas has settled on Didymoon and assuming it is still operational, Juventas will have to stay working for one more day (two orbits of Didymoon around Didymain) to contribute to the secondary science objective: study the dynamical properties of the asteroid. Figure 2.6 illustrates a landing approach as well as the bouncing of the surface of Didymoon.



(a) Landing approach. Only the last period of the SSTO 2000 is represented.



(b) Bouncing, from [38].

Figure 2.6: Descent & Bouncing of Juventas.

2.3.4 Landing constraints

To adequately design the trajectories, some constraints have to be taken into account. [30][29]. The main ones are the following:

- Juventas shall leave the SSTO 2000 (POP) after 30 days
- Juventas shall attempt to touch down on the Didymoon surface at low speed < 10 cm/s [30]. This constraints is important, as it decreases the risk of Juventas bouncing out of the gravitational attraction of Didymoon
- The Didymoon phase angle should be outside the eclipse zone
- The phase angle of Juventas with respect to Didymain during the landing should be less than 90°
- The phase angle of Juventas with respect to Didymoon during the landing should be less than 90°
- The DeltaV should not be over 19.9 cm/s

A complete explanation of the phase angles can be found in *Appendix A*. As a conclusion, the science objectives, investigation, measurements, payload and mission phases are summarized in Table 2.5:

Science objective	Investigation	Measurements	Instruments	Mission Phase
Determine gravity field of Didymoon	Gravity field characterization outside Brillouion sphere at least up to degree and order 2	Deflection during orbit measured with ranging, line-of-sight to Hera	ISL radio link	Proximity operations
	Surface gravity	Surface acceleration	Gravimeter	Surface operations
	CubeSat touchdown/ bouncing	Dynamic recording of each event	IMU	Landing / bouncing
Determine interior structure of Didymoon	Reconstruction of material density and largest monolithic object	Properties of (back-) reflection of transmitted signal	Low frequency radar	Proximity operations
Surface properties of Didymoon	Visible imaging	Inspection of Didymoon surface features and impact crater	Visible camera	Proximity operations, surface operations
	Surface strength measurement	Rebounds from the surface	IMU	Landing / bouncing
(Secondary) Determine dynamical properties of Didymoon	Variable surface acceleration	Surface acceleration over >1 orbit	Gravimeter	Surface operations
	Attitude and time dependent surface illumination	Attitude and time dependent surface illumination	Star tracker, sun sensors	Surface operations

Table 2.5: Science objectives and payloads, from [38].

Chapter 3

Orbital mechanics

This third chapter is entirely dedicated to the dynamical environment of Didymos. First, the equations of motion of Juventas are developed in the Circular Restricted Three Body Problem. The characteristics of such a complex and low gravity environment are discussed. Then, orbital perturbations are added to the model in order to have a more realistic behavior of Juventas in the vicinity of Didymos.

3.1 The Circular-Restricted Three Body Problem

In the case of multi-body environments, different techniques are available to compute interplanetary trajectories of a spacecraft.

The most famous method is the patched-conics method¹. Nevertheless, it has a main drawback: it is not valid when the spheres of influence (SoI) of multiple bodies overlap. The SoI is defined as the spheroid region around a body where the gravitational influence of that body overwhelms that of the Sun. Outside this region, the Sun gravity influence is considered as the only source. The radius of the SoI is calculated as in [1]:

$$r_{SoI} = R_{Sun-body} \left(\frac{m_{body}}{m_{Sun}} \right)^{\frac{2}{5}} \quad (3.1)$$

Where $R_{Sun-body}$ is the distance between the Sun and the body analyzed, m_{body} and m_{Sun} are the mass of the respective celestial bodies.

¹Method where space is divided into parts, where each one of them corresponds to the sphere of influence of a body [49].

Here, the SoI of Didymain and Didymoon are calculated:

$$r_{SoI-Main} = 9099.49 [m] \qquad r_{SoI-Moon} = 1400.89 [m]$$

As the sum of the radii of these two spheres is greater than the distance between Didymain and Didymoon, these SoI overlap and the method of patched conics cannot be used. The overlap is presented in Figure 3.1.

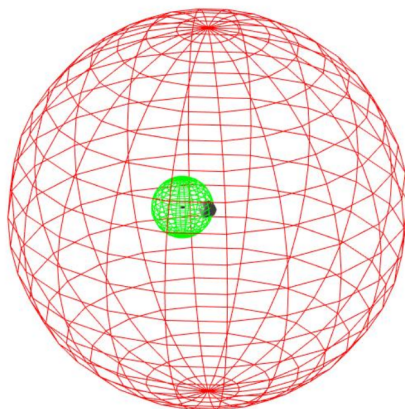


Figure 3.1: Red sphere: SoI of Didymain. Green sphere: SoI of Didymoon.

Due to the presence of two gravitational sources, the most accurate way to represent the motion of the third body, Juventas, in the Didymos environment is the use of the **Perturbed Circular-Restricted Three Body problem** (PCR3BP). Here, **perturbed** means that additional perturbations acting on the spacecraft are considered. This is done in *section 3.2*. For now, the CR3BP is developed.

3.1.1 Main assumptions

The assumptions of the classical CR3BP are the following;

- **Circular:** The two main bodies follow a circular path around their barycenter.
- **Restricted:** The third body (smallest body, here Juventas, considered as a point-mass) has no impact on the dynamics of the two main bodies, because his mass is considered negligible compared to the two main bodies.
- **Three Body Problem:** The third body navigates in the vicinity of the two main bodies; each one is modeled as a point-mass body.

To correctly simulate the dynamical environment of Didymos, the adequacy of the assumptions of the problem are checked:

- The eccentricity of Didymoon with respect to Didymain being very small ($e_{cc} = 0.03$, see Table 2.2), and the barycenter of the system being 10.8 meters close of the center of mass of Didymain, as a first approximation, it is correct to consider a circular trajectory for Didymoon around the barycenter of Didymos.
- Juventas has a mass of the order of $1e-12$ compared to the mass of Didymos. It is therefore correct to assume that Juventas has no impact on the dynamics of the main bodies.
- In a first approach, the primary and secondary asteroids will be considered as point-mass bodies. For the sake of consistency with the mass distribution of the real system, gravitational perturbations will be further added in *Section 3.2*.

Proceeding from the underlying assumptions stated above and analyzing the dynamical proprieties of Didymos, the use of a Circular Restricted Three Body Problem is appropriate.

3.1.2 Equations of motion

First, the reference frames are presented, then the gravitational forces and the inertial acceleration of Juventas are calculated so as to apply Newton's second law and obtain the equations of motion (EoM). In what is next, bold symbols denote vectors, while the rest are scalar quantities.

Reference frames

The Restricted-Circular Three Body Problem theory can be found in [1; 11; 4]. Studying Didymos using the CR3BP implies to work in a frame specific to the Three Body Problem: the rotating frame² centered on the barycenter of the system (see Figure 3.2). The position vector $\mathbf{r}$ of Juventas can be expressed in the rotating reference frame $\mathcal{F} : (\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z)$. The latter always respects the following proprieties, where m_1 is used to describe the main body, m_2 the moon and m Juventas:

- The $\hat{\mathbf{e}}_x$ direction always points towards m_2
- The $\hat{\mathbf{e}}_z$ direction is always parallel to the angular velocity of m_1 and m_2 and $\hat{\mathbf{e}}_y$ completes the right-handed system

²The rotating frame is also commonly referred as the synodic frame.

- The frame is centered on the barycenter of the system $m_1 - m_2$
- m_1 and m_2 are always aligned on the axis $\hat{\mathbf{e}}_x$
- m_1 is at a fixed distance r_1 from the barycenter
- m_2 is at a fixed distance r_2 from the barycenter
- m_1 and m_2 are at a fixed distance r_{12} from each other
- The frame rotates with constant angular velocity $\boldsymbol{\omega} = \omega \hat{\mathbf{e}}_z$

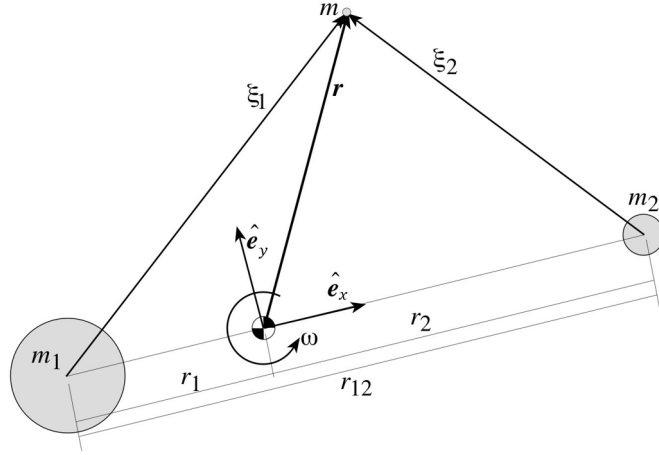


Figure 3.2: Illustration of the CR3BP rotating frame, from [1].

A secondary inertial frame $\mathcal{I}$, also referred as the fixed frame, is defined to be coincident with the synodic frame at $t = 0$. A third frame, the Equatorial J2000 is also used in calculations. More details on the usual frames and transformations can be found in *Appendix B*.

While m_1 and m_2 only perform planar motion in the $(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y)$ plane, m can freely move in the three dimensions.

The position vector is expressed in the rotating frame $\mathcal{F}$ as:

$$\mathbf{r} = r_x \hat{\mathbf{e}}_x + r_y \hat{\mathbf{e}}_y + r_z \hat{\mathbf{e}}_z \quad (3.2)$$

To compute the motion of Juventas in the vicinity of Didymos, Newton's second law is applied:

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = m \mathbf{a} \quad (3.3)$$

Where $\mathbf{F}_1$ and $\mathbf{F}_2$ are the gravitational forces exerted by m_1 and m_2 on Juventas and $\mathbf{a}$ is the absolute acceleration of $\mathbf{r}$ expressed in terms of coordinates in the rotating frame, $\mathcal{F}$.

Gravitational forces

The gravitational force $\mathbf{F}$ exerted by the two main bodies on Juventas can be expressed in terms of frame $\mathcal{F}$ components as:

$$\mathbf{F} = -G \mathbf{m} \begin{bmatrix} \frac{m_1}{\xi_1^3}(r_x - r_1) + \frac{m_2}{\xi_2^3}(r_x - r_2) \\ \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3}\right) r_y \\ \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3}\right) r_z \end{bmatrix} \quad (3.4)$$

Where G is the gravitational constant³ and ξ_i are the relative distances between Juventas and body m_i :

$$\xi_i = \sqrt{(r_x - r_i)^2 + r_y^2 + r_z^2} \quad (3.5)$$

Absolute acceleration

The absolute velocity of Juventas can be computed by taking the time derivative of equation (3.2). The relation, commonly known as the 'Transport Theorem' is:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} + \dot{\mathbf{r}} \quad (3.6)$$

Where $\dot{\mathbf{r}}$ is the velocity of Juventas in the rotating frame (here expressed in terms of frame $\mathcal{F}$ components):

$$\dot{\mathbf{r}} = \begin{bmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{r}_z \end{bmatrix} \quad (3.7)$$

In the fixed frame, the absolute acceleration of Juventas is expressed as:

$$\mathbf{a} = \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \dot{\mathbf{r}} + \ddot{\mathbf{r}} \quad (3.8)$$

³The value of the Gravitational Constant is equal to $6.67430 \cdot 10^{-11} [m^3 / kg \cdot s^2]$

In equation (3.8), $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ represents the Euler acceleration, $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ the centrifugal acceleration, $2\boldsymbol{\omega} \times \dot{\mathbf{r}}$ is the Coriolis acceleration and $\ddot{\mathbf{r}}$ is the relative acceleration of Juventas in the rotating frame.

In the CR3BP, the velocity of the circular orbit is constant, so that $\dot{\boldsymbol{\omega}} = 0$. The term $\ddot{\mathbf{r}}$ expressed in the rotating frame $\mathcal{F}$ is:

$$\ddot{\mathbf{r}} = \begin{bmatrix} \ddot{r}_x \\ \ddot{r}_y \\ \ddot{r}_z \end{bmatrix} \quad (3.9)$$

The absolute acceleration of Juventas can be simplified using the CR3BP assumption that $\dot{\boldsymbol{\omega}} = 0$ and re-written as:

$$\mathbf{a} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \dot{\mathbf{r}} + \ddot{\mathbf{r}} \quad (3.10)$$

Finally, by rearranging all the terms, the absolute acceleration vector of $\mathbf{r}$ expressed in terms of coordinates in the rotating frame $\mathcal{F}$ is:

$$\mathbf{a} = (\ddot{r}_x - 2\dot{r}_y \omega - r_x \omega^2) \hat{\mathbf{e}}_x + (\ddot{r}_y + 2\dot{r}_x \omega - r_y \omega^2) \hat{\mathbf{e}}_y + \ddot{r}_z \hat{\mathbf{e}}_z \quad (3.11)$$

Equations of Motion

By injecting equations (3.4) and (3.11) into Newton's second law (3.3) and expanding the equations in the frame $\mathcal{F}$, the EoM of Juventas become:

$$\begin{cases} \ddot{r}_x - 2\omega \dot{r}_y - \omega^2 r_x + G \left(\frac{m_1}{\xi_1^3} (r_x - r_1) + \frac{m_2}{\xi_2^3} (r_x - r_2) \right) & = 0 \\ \ddot{r}_y + 2\omega \dot{r}_x - \omega^2 r_y + G \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3} \right) r_y & = 0 \\ \ddot{r}_z + G \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3} \right) r_z & = 0 \end{cases} \quad (3.12)$$

Equation 3.12 can be re-written in a more convenient form, that will later be useful when adding orbital perturbations to the model:

$$\begin{cases} \ddot{r}_x & = 2\omega \dot{r}_y + \omega^2 r_x - U_{1x} - U_{2x} \\ \ddot{r}_y & = -2\omega \dot{r}_x + \omega^2 r_y - U_{1y} - U_{2y} \\ \ddot{r}_z & = -U_{1z} - U_{2z} \end{cases} \quad (3.13)$$

Where the sub-subscripts $(\cdot)_{\cdot_x}, (\cdot)_{\cdot_y}, (\cdot)_{\cdot_z}$ denote the partial derivatives in the corresponding direction. U_1 and U_2 are the point-mass gravitational potentials due to the primary and secondary bodies, Didymain and Didymoon, respectively.

3.1.3 Jacobi integral

After obtaining the EoM of Juventas, the Jacobi integral can be calculated. It is an integral of motion, meaning it is constant along the trajectory of the considered body. The method is derived from [4; 11]. Let the specific potential energy function $U(r_x, r_y, r_z)$ be defined as:

$$U(r_x, r_y, r_z) = -\frac{\omega^2}{2} (r_x^2 + r_y^2) - \frac{G m_1}{\xi_1} - \frac{G m_2}{\xi_2} \quad (3.14)$$

In equation (3.14), the first term is a rotating contribution, the second and third are gravitational contributions. Using the vector notation, the EoM can be re-written in a compact form:

$$\ddot{\mathbf{r}} + 2\boldsymbol{\omega} \times \dot{\mathbf{r}} = -\nabla_{\mathbf{r}} U = \begin{bmatrix} \frac{-\partial U}{\partial r_x} \\ \frac{-\partial U}{\partial r_y} \\ \frac{-\partial U}{\partial r_z} \end{bmatrix} \quad (3.15)$$

By taking the dot product of equation (3.15) by $\dot{\mathbf{r}}$, a perfect differential equation is obtained:

$$(\ddot{\mathbf{r}} + 2\boldsymbol{\omega} \times \dot{\mathbf{r}}) \cdot \dot{\mathbf{r}} = \ddot{\mathbf{r}} \cdot \dot{\mathbf{r}} = \frac{1}{2} \frac{d}{dt} (\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}) \quad (3.16)$$

$$\nabla_{\mathbf{r}} U \cdot \dot{\mathbf{r}} = \frac{\partial U}{\partial \mathbf{r}} \cdot \dot{\mathbf{r}} = \frac{dU}{dt} \quad (3.17)$$

By combining (3.16) and (3.17):

$$\frac{1}{2} \frac{d}{dt} (\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}) = -\frac{dU}{dt} \implies \frac{1}{2} \frac{d}{dt} (\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}) + \frac{dU}{dt} = 0 \quad (3.18)$$

The simplifications were made as follows:

- $(2\boldsymbol{\omega} \times \dot{\mathbf{r}}) \cdot \dot{\mathbf{r}} = 0$ because $\dot{\mathbf{r}}$ is perpendicular to $(2\boldsymbol{\omega} \times \dot{\mathbf{r}})$
- $\nabla_{\mathbf{r}} U \cdot \dot{\mathbf{r}} = \frac{\partial U}{\partial r_x} \cdot \frac{\partial r_x}{\partial t} + \frac{\partial U}{\partial r_y} \cdot \frac{\partial r_y}{\partial t} + \frac{\partial U}{\partial r_z} \cdot \frac{\partial r_z}{\partial t} = \frac{dU}{dt}$

Then, integrating equation (3.18) with respect to time:

$$\dot{\mathbf{r}} \cdot \dot{\mathbf{r}} = v^2 = -C - 2U \implies v^2 + 2U = -C \quad (3.19)$$

Where the constant resulting of the integration, C is an integral of motion. It can also be written in the form:

$$\frac{1}{2} v^2 + U = E \quad (3.20)$$

Where the term $\frac{1}{2} v^2$ is the specific kinetic energy, U is the specific potential energy and E is the specific total energy, such as $-\frac{1}{2} C = E$. In this form, C is defined as the Jacobi Constant, or Jacobi Integral. The definition of the potential $U(r_x, r_y, r_z)$ can be substituted in (3.19). The full Jacobi integral becomes:

$$v^2 = \omega^2 (r_x^2 + r_y^2) + 2 \frac{G m_1}{\xi_1} + 2 \frac{G m_2}{\xi_2} - C \quad (3.21)$$

The non dimensional Jacobi constant can be obtained by dividing expression (3.21) by $(\omega r_{12})^2$. For the sake of clarity, the non-dimensional Jacobi constant is used throughout the work.

The scalar constant C , simply called the Jacobi constant, provides a negative relative energy measure of Juventas. It also allows us to study the regions in space where Juventas can travel, given a certain value of C , determined by the initial conditions of the problem. At any time, equation (3.21) must be satisfied. Finally, the Jacobi constant C is very useful in numerical simulations, as it allows the user to verify the accuracy of the integration of the EoM by making sure that C remains unchanged. This verification is done in *Chapter 5*.

3.1.4 Zero velocity surfaces

Another use of the Jacobi integral very useful in the analysis of the dynamical environment of a binary system are the zero relative velocity surfaces, or Hill curves. The Jacobi integral can be used to investigate in which regions Juventas can travel in the vicinity of the two bodies given its initial state. Indeed, the position and the velocity of Juventas evolve in the CR3BP but the Jacobi constant remains unchanged according to equation (3.21). The more important the orbit's energy, the farther the nanosatellite can go from the attractive body and the more regions are allowed to the spacecraft. The boundaries between the allowed and the

forbidden regions are found by setting $v = 0$ in the Jacobi integral. It represents the geometric location of points where the kinetic energy is zero. Then, for a given value of the Jacobi constant, the zero relative velocity curves are determined by the following equation:

$$C = \omega^2 (r_x^2 + r_y^2) + 2 \frac{G m_1}{\xi_1} + 2 \frac{G m_2}{\xi_2} \quad (3.22)$$

Using the parameters of the Didymos system (see Table 2.2), the zero velocity surfaces of the Didymos system were computed and are represented on Figure 3.3.

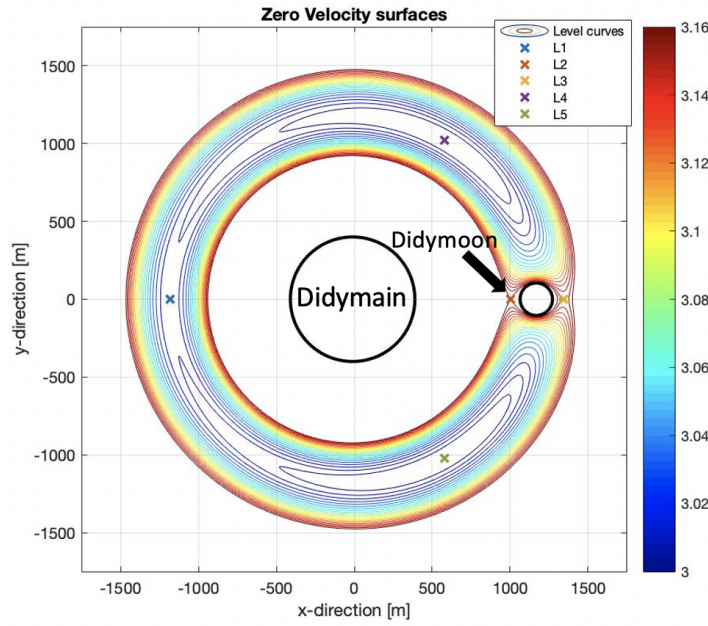


Figure 3.3: Zero velocity surfaces of Didymos, using the non dimensional Jacobi constant.

Figure 3.3 allows a better understanding of the regions allowed taken the energy of the spacecraft. For example, if Juventas is close to Didymain and has a Jacobi constant equal to 3.16 [-] (non dimensional value), it will always move inside the red circle and ricochet on the circle without never going trough it. Ensuring that the spacecraft never overpasses the boundaries is a second approach to verifying that the implementation works correctly. This energetic verification is done in *Chapter 5*. On Figure 3.3, The L_i correspond to Lagrange Points, which are now discussed.

3.1.5 Lagrange points

A last interesting particularity of the CR3BP are the Lagrange Libration Points, L_i . These fixed points correspond to positions in the rotating frame where the gravitational forces from Didymain and Didymoon are balanced by the centrifugal forces. It is very important to understand that the Lagrangian point are fixed in a rotating frame (non-inertial frame), they rotate as Moon and Main in a inertial frame. On these libration points, Juventas would then remain in a stable or near-stable position in the rotating frame $\mathcal{F}$. There are a total of 5 libration points, fixed in the rotating frame $\mathcal{F}$. Their calculation is done by setting to zero the relative velocities and accelerations in equation (3.12):

$$\begin{cases} \ddot{r}_x = \dot{r}_x = 0 \\ \ddot{r}_y = \dot{r}_y = 0 \\ \ddot{r}_z = \dot{r}_z = 0 \end{cases} \quad (3.23)$$

The set of three differential equations obtained is then:

$$\begin{cases} G \left(\frac{m_1}{\xi_1^3} (r_x - r_1) + \frac{m_2}{\xi_2^3} (r_x - r_2) \right) = \omega^2 r_x & (3.24a) \\ G \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3} \right) r_y = \omega^2 r_y & (3.24b) \\ G \left(\frac{m_1}{\xi_1^3} + \frac{m_2}{\xi_2^3} \right) r_z = 0 & (3.24c) \end{cases}$$

From (3.24c), it can be seen that all stationary points have $r_z = 0$ and therefore must lie in the rotating plane made by m_1 and m_2 . Then, from (3.24b), two solutions exist:

- Case $r_y = 0$: corresponds to the *collinear* solution where the Lagrange points are aligned with the rotating axis $\hat{\mathbf{e}}_x$. It allows to compute the positions of L_1 , L_2 and L_3 by solving (3.24a).
- Case $r_y \neq 0$: corresponds to the *equilateral triangle* solution where $\xi_1 = \xi_2$ must be satisfied. Using (3.24a) and (3.24b) allows to compute the positions of L_4 and L_5 .

It is important to note that the *collinear* Lagrange points are unstable (sensitive to the perturbations) and the *equilateral* Lagrange points are stable (non-sensitive to the perturbations). Indeed, if the spacecraft is placed in L_4 or L_5 , it would

maintain its position over time (with respect to the synodic frame), which is not the case with L_1 , L_2 and L_3 . In these cases, correction maneuvers are needed for the spacecraft to remain in its configuration [4].

Using the parameters of the Didymos system (see Table 2.2), the position of the five libration points are computed. The result can be appreciated in Figure 3.4. Table 3.1 summarizes the positions.

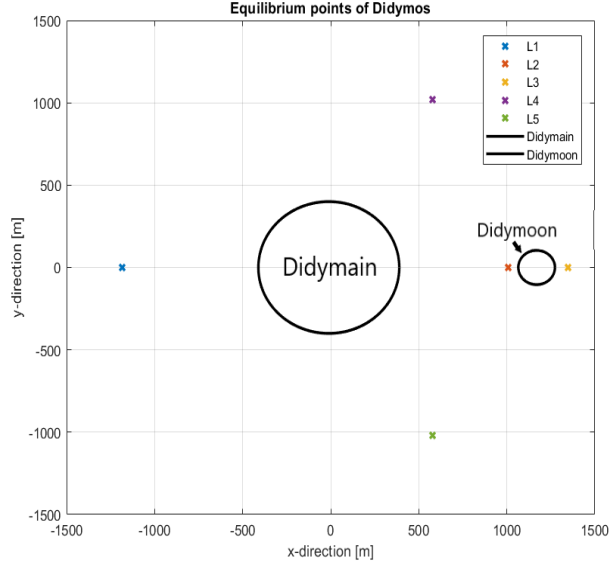


Figure 3.4: Libration points of the Didymos system

Libration Point	L_1	L_2	L_3	L_4	L_5
r_x [m]	-1184,53	1005,94	1348,89	579,12	579,12
r_y [m]	0	0	0	1021,90	-1021,90

Table 3.1: Position of L_i in the Didymos system

3.2 Orbital perturbations

The equations developed in *Section 3.1* describe the motion of Juventas in the unperturbed Didymos system where Didymoon and Didymain are considered as point-mass asteroids. To have a model that truly represents the dynamical environment of Didymos, perturbations must be added. Table 3.2 lists the order of magnitude of orbital perturbations 1 km from the primary body, courtesy of [6]:

Perturbations	Order of magnitude [m/s ²]
Didymos gravity	10^{-5}
Solar radiation pressure	10^{-8}
Sun's gravity (4th body)	10^{-11}
Earth's gravity (4th body)	10^{-13}

Table 3.2: Dynamical effects acting on Juventas in Didymos

In the light of the order of magnitude of the gravity of Didymos (10^{-5}), gravitational perturbations (GP) and the solar radiation pressure (SRP) must be considered in the model. The gravity perturbations are caused by the particular shape of the Didymos asteroids. Solar radiation pressure is due to the photons emitted by the Sun that impact the spacecraft.

3.2.1 Gravity perturbations

In the CR3BP, the two asteroids are considered as point-mass bodies with each one having a spherical gravitational field. Didymain and Didymoon having more complicated shapes, the gravitational attraction of the bodies has to be adapted. Gravitational perturbations are considered as conservative, meaning that they do not vary the energy level of the CubeSat with time. In the literature, several methods are available to specify the gravitational potential of a non-spherical body, see [12]. Two of them, that will be used to describe respectively Didymain and Didymoon gravitational potentials are:

- Addition of a perturbation term in the Equations of Movement to adapt the gravitational potential of a point-mass body. The perturbation term is better known as **spherical harmonics**.
- Replace the potential of the point-mass body by a **closed-form gravitational potential** of a body with a specific shape (e.g. constant density ellipsoid, constant density polyhedron).

3.2.1.1 Spherical harmonics

The disturbing potential can be distinguished in two contributions (see Figure 3.5):

- Zonal harmonics: function of the geocentric⁴ distance ξ_i and the latitude, ϕ .
- Sectorial harmonics: function of the geocentric distance ξ_i and the longitude, λ .



Figure 3.5: Harmonics contributions, from [2].

The sum of both contributions results in Tesseral Harmonics, function of the geocentric distance ξ_i , the longitude and latitude. In the current shape models, only zonal harmonics are considered for Didymain, see [29].

The gravitational potential shall satisfy Laplace's equation outside of the body, $\nabla^2 U = 0$. The spherical harmonics method consists in constructing a set of orthogonal solutions to Laplace's equation and adequately choosing the coefficients to match the actual potential function of the body. The spherical harmonic expansion uses the spherical coordinates (ξ_i, ϕ, λ) . The transformation between Cartesian and spherical coordinates in the synodic frame can be done with:

$$\begin{cases} \xi_i = \sqrt{(r_x - r_i)^2 + r_y^2 + r_z^2} \\ \phi = \arcsin\left(\frac{r_z}{\xi_i}\right) \\ \lambda = \arctan\left(\frac{r_y}{r_x}\right) \end{cases}$$

⁴As measured from the body center of mass

In spherical coordinates, the form for the zonal spherical harmonic potential of the gravity field of body i is given by, from [2]:

$$U(\xi_i, \phi) = U_{point-mass} - U_{harmonics} \quad (3.1)$$

$$= \frac{\mu_i^*}{\xi_i} - \frac{\mu_i^*}{\xi_i} \sum_{n=2}^{\infty} J_n \left(\frac{R_i}{\xi_i} \right)^n P_n(\sin(\phi)) \quad (3.2)$$

Where in equation (3.2), $\mu_i^* = G m_i$ is the standard gravitational parameter of body i , P_n are the Legendre polynomials of degree n (see *Appendix D* for more information on those polynomials), J_n are the harmonic coefficients of degree n used to match the actual shape of the body, R_i is the radius of the circumscribing sphere that contains body i . The different degrees of the spherical harmonics are used to describe different shapes, depicted in Figure 3.6.

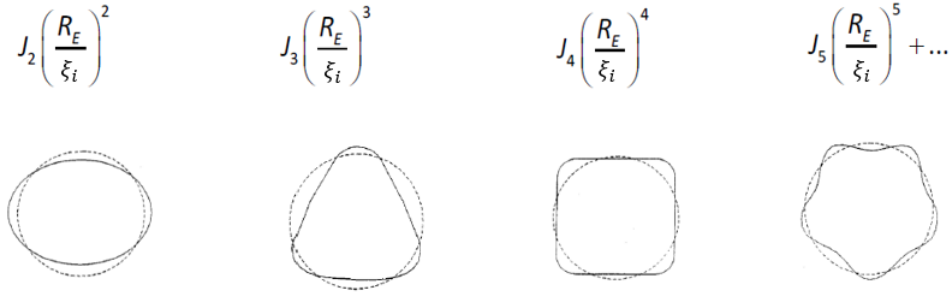


Figure 3.6: Dotted line: Spherical shape. Solid line: shape where the spherical harmonic is added. Illustration of the oscillation of the Legendre function in the latitude direction. These figures should be viewed as a cross-section of the Legendre polynomials rotated around the north-south axis.

Spherical harmonics limitations

The spherical harmonics approach does not work in all situations. A fundamental assumption is that the series (3.2) converges to the actual gravity field anywhere in space. It means that ξ_i must always be bigger than R_i in order for the ratio to respect the inequality $\left(\frac{R_i}{\xi_i}\right)^n < 1$. Having $\xi_i > R_i$ means that the particle m is outside the circumscribing sphere. Indeed, if $\xi_i < R_i$, the series will diverge. Therefore, once Juventas is inside the Brillouin⁵ sphere of radius R_i , this method is not valid anymore as the spherical harmonic method does not work properly. The Brillouin sphere around Didymoon is depicted in Figure 3.7.

⁵Name given to the circumscribing sphere, after Léon Brillouin.

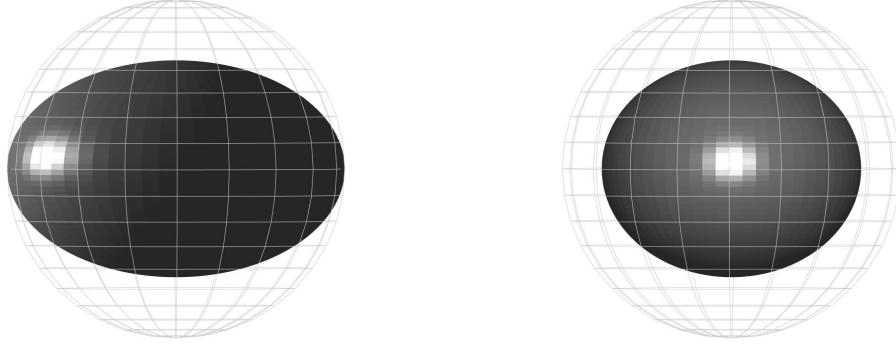


Figure 3.7: Representation of the Brillouin sphere around Didymoon.

However, an exception exists for triaxial ellipsoids. If all three eccentricities are lower than $1/\sqrt{2}$, the gravitational potential using spherical harmonics converges even inside the Brillouin sphere [12]. It is therefore necessary to check the values of the three eccentricities (one ellipse per plane) before rejecting the spherical harmonics method.

3.2.1.2 Main's gravity perturbation representation

As it can be seen on Figure 3.8, the Brillouin sphere encloses perfectly Didymain. Also, Juventas is designed to land on Didymoon and will therefore always remain outside the circumscribing sphere of Didymain. For both these reasons, a spherical harmonics approach is chosen to model the shape of Didymain.

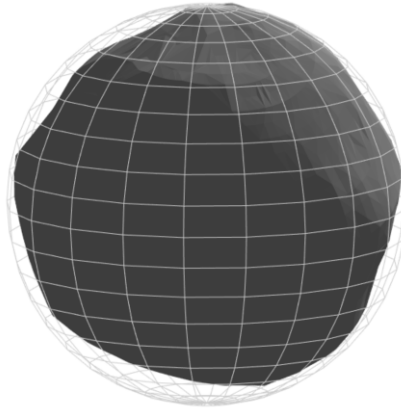


Figure 3.8: Representation of the Brillouin sphere around Didymain.

Legendre polynomials and J_n coefficients of order 2 and 4 are used to model the mass distribution [29]. The values are retrieved from GMV reports and can be found in Table 3.3. Currently, those values are the most accurate ones available to represent the gravitational field of Didymain:

J2	-0.011432722
J4	0.004583058

Table 3.3: Didymain spherical harmonics coefficients.

Using Equation (3.2) and replacing with the Didymain parameters, the full gravitational potential expression of Didymain, using spherical harmonics is obtained:

$$U_{1,sh}(\xi_1, \phi) = \frac{G m_1}{\xi_1} \left(1 - J_2 \left(\frac{R_1}{\xi_1} \right)^2 P_2(\sin(\phi)) - J_4 \left(\frac{R_1}{\xi_1} \right)^4 P_4(\sin(\phi)) \right) \quad (3.3)$$

$$\begin{cases} P_2(\sin(\phi)) &= \frac{1}{2} \left(3 \left(\frac{r_z}{\xi_1} \right)^2 - 1 \right) \\ P_4(\sin(\phi)) &= \frac{1}{8} \left(35 \left(\frac{r_z}{\xi_1} \right)^4 - 30 \left(\frac{r_z}{\xi_1} \right)^2 + 3 \right) \end{cases} \quad (3.4)$$

With $R_1 = 399$ m (from Table 2.2) and $\xi_1 = \sqrt{(r_x - r_1)^2 + r_y^2 + r_z^2}$. The effects of the spherical harmonics on the shape of Didymain are depicted in Figure 3.9.

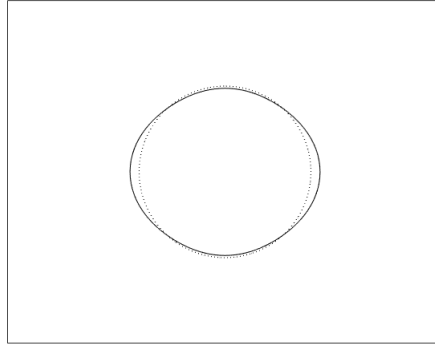


Figure 3.9: Didymain's body shape. Dotted line: Spherical shape. Solid line: modeled shape. Should be viewed as a cross-section around the north-south axis.

From the potential of Equation (3.3), the acceleration exerted on Juventas in the synodic frame in the vicinity of Didymain can be obtained by performing a partial derivative in each direction:

$$\begin{cases} U_{1,sh_x} = \frac{\partial}{\partial r_x} U_1(\xi_1, \phi) \end{cases} \quad (3.5a)$$

$$\begin{cases} U_{1,sh_y} = \frac{\partial}{\partial r_y} U_1(\xi_1, \phi) \end{cases} \quad (3.5b)$$

$$\begin{cases} U_{1,sh_z} = \frac{\partial}{\partial r_z} U_1(\xi_1, \phi) \end{cases} \quad (3.5c)$$

3.2.1.3 Moon's gravity perturbation representation

The use of spherical harmonics is not adequate as the spherical harmonics expansion diverges inside the Brillouin sphere of Didymoon, depicted in Figure 3.7. Also, using the information from Table 2.2, one can calculate the eccentricities of the ellipsoid:

$$\begin{cases} e_{xy} &= \sqrt{1 - \frac{\beta^2}{\alpha^2}} = \sqrt{1 - \frac{79^2}{103^2}} = 0.64 < 1/\sqrt{2} \\ e_{xz} &= \sqrt{1 - \frac{\gamma^2}{\alpha^2}} = \sqrt{1 - \frac{66^2}{103^2}} = 0.76 > 1/\sqrt{2} \\ e_{zy} &= \sqrt{1 - \frac{\beta^2}{\gamma^2}} = \sqrt{1 - \frac{66^2}{79^2}} = 0.54 < 1/\sqrt{2} \end{cases}$$

One of the eccentricities is greater than $1/\sqrt{2}$, confirming that the spherical harmonics expansion diverges inside the Brillouin sphere. In the case of Didymoon, it is important to have an accurate gravitational model down to the surface, since the final objective of Juventas is to land on the asteroid. The closed-form gravitational potential of a constant density ellipsoid is studied, as in [12]. The constant density hypothesis is validated by the current model of Didymos, where Didymoon is considered to have a constant density. The ellipsoid (Figure 3.10) has semi-major axis $\alpha \geq \beta \geq \gamma$ and the shape is represented by the equation $(x/\alpha)^2 + (y/\beta)^2 + (z/\gamma)^2 \leq 1$.

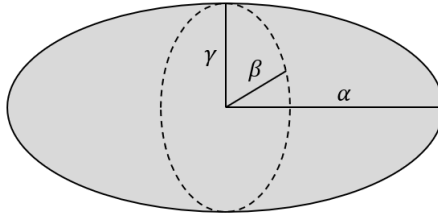


Figure 3.10: Triaxial ellipsoid.

In the Didymos system, Didymoon center of mass is at a distance r_2 from the barycenter of the system (see Figure 3.2). The semi-major axis α is always along

$\hat{\mathbf{e}}_x$, β is parallel to $\hat{\mathbf{e}}_y$, and γ is parallel to $\hat{\mathbf{e}}_z$. The position vector of Juventas $\mathbf{r}_m$ is defined from the center of mass of the ellipsoid:

$$\mathbf{r}_m = (r_x - r_2)\hat{\mathbf{e}}_x + r_y\hat{\mathbf{e}}_y + r_z\hat{\mathbf{e}}_z \quad (3.6)$$

The gravitational potential of the exterior of the moon, up to its surface is then defined by the equations:

$$\left\{ \begin{array}{l} U_{2,ell}(\mathbf{r}_m) = -\frac{3\mu}{4} \int_{\lambda(\mathbf{r}_m)}^{\infty} \phi(\mathbf{r}_m, u) \frac{du}{\Delta(u)} \\ \phi(\mathbf{r}_m, u) = \frac{(r_x - r_2)^2}{u + \alpha^2} + \frac{r_y^2}{u + \beta^2} + \frac{r_z^2}{u + \gamma^2} - 1 \\ \Delta(u) = \sqrt{(u + \alpha^2)(u + \beta^2)(u + \gamma^2)} \end{array} \right. \quad (3.7a)$$

$$\phi(\mathbf{r}_m, u) = \frac{(r_x - r_2)^2}{u + \alpha^2} + \frac{r_y^2}{u + \beta^2} + \frac{r_z^2}{u + \gamma^2} - 1 \quad (3.7b)$$

$$\Delta(u) = \sqrt{(u + \alpha^2)(u + \beta^2)(u + \gamma^2)} \quad (3.7c)$$

Where $\lambda(\mathbf{r}_m)$ is obtained by solving the equation $\phi(\mathbf{r}_m, \lambda) = 0$, $\lambda(\mathbf{r}_m)$ being the maximum real root, which always exists. The partial derivatives of the potential can be obtained by applying Leibniz's rule⁶. They express the acceleration exerted on Juventas by Didymoon in the synodic frame $\mathcal{F}$:

$$\left\{ \begin{array}{l} U_{2,ell_x} = -\frac{3\mu(r_x - r_2)}{2} \int_{\lambda(\mathbf{r}_m)}^{\infty} \frac{du}{(\alpha^2 + u)\Delta(u)} \\ U_{2,ell_y} = -\frac{3\mu r_y}{2} \int_{\lambda(\mathbf{r}_m)}^{\infty} \frac{du}{(\beta^2 + u)\Delta(u)} \\ U_{2,ell_z} = -\frac{3\mu r_z}{2} \int_{\lambda(\mathbf{r}_m)}^{\infty} \frac{du}{(\gamma^2 + u)\Delta(u)} \end{array} \right. \quad (3.8a)$$

$$U_{2,ell_y} = -\frac{3\mu r_y}{2} \int_{\lambda(\mathbf{r}_m)}^{\infty} \frac{du}{(\beta^2 + u)\Delta(u)} \quad (3.8b)$$

$$U_{2,ell_z} = -\frac{3\mu r_z}{2} \int_{\lambda(\mathbf{r}_m)}^{\infty} \frac{du}{(\gamma^2 + u)\Delta(u)} \quad (3.8c)$$

Where the sub-subscripts $(\cdot)_{.x}$, $(\cdot)_{.y}$, $(\cdot)_{.z}$ denote the direction of derivation.

3.2.2 Solar radiation pressure

The last orbital perturbation added to the model is the Solar radiation pressure (SRP). This perturbation is due to the Sun, which constantly emits electromagnetic radiations, or in other terms photons that carry momentum. When impacting the small orbiting body, they modify its trajectory by exerting pressure on it. From a momentum budget point of view, there are four contributions to the SRP's:

⁶ $\frac{d}{dt} \int_a^b f(x, t) dx = \int_a^b \partial_t f(x, t) dx$

- Momentum of photons impacting on the surface of the spacecraft. This component is in the direction of the photons; along the line from the Sun to the surface of the spacecraft.
- Recoil momentum of the photons reflected from the surface. The direction of the reflected light depends on the local orientation of the surface (surface normal plays a role of bisection the angle formed by the incident and reflected light).
- Recoil momentum of the photons diffusely scattered from the surface. The photons leave normal to the surface on average, providing an additional momentum.
- Thermal contribution coming from the photons absorbed by the body and raising the body temperature.

A simple model for computing the acceleration due to solar radiations it to assume that the spacecraft presents a constant area perpendicular to the sun-line. The acceleration will act away from the sun-line and its general expression is, from [2]:

$$\mathbf{a}_{SRP} = -p_{SR@1AU} \frac{\mathbf{r}_{sc-sun}}{|\mathbf{r}_{sc-sun}|^3} d_{AU}^2 c_R \frac{A_{sun}}{m} \quad (3.9)$$

Where $\mathbf{r}_{sc-sun}$ is the sun-spacecraft distance in the synodic frame, $p_{SR@1AU} = 4.5e-6$ N/m² is the pressure exerted by the photons on the surface of the spacecraft at a distance of 1 AU⁷, d_{AU} is the distance between the spacecraft and the sun, expressed in Astronomical Units, here being equal to 1.645 (see Table 2.2), c_R is the reflectively coefficient of the spacecraft and $\frac{A_{sun}}{m}$ is its area-to-mass relative to the sun. In the case of Juventas, the nominal parameters are:

reflectively coefficient	area-to-mass
1.5 [-]	30 [kg/m ²]

Table 3.4: Nominal parameters of Juventas.

Finally, it is important to mention that this non-gravitational perturbation is non-conservative, varying the energy level of the body with time.

⁷An Astronomical Unit (AU) is the average distance between Earth and the Sun, which is equal to approximately 150 million kilometers.

3.3 Perturbed equations of motion

The equations of motion (EoM) of Juventas have been developed in *Section 3.1*. Then, a more precise gravitational model was developed for Didymain using spherical harmonics of the 2nd and 4th order and for Didymoon, where the closed-form gravitational potential of a constant density ellipsoid was used. Finally, the SRP was modeled.

The equations of motion of the CR3BP (3.13) can be modified as follows:

- The point-mass gravitational potential of Didymain U_1 is replaced by equation (3.5), where the spherical harmonics were added, denoted by $U_{1,sh}$.
- The point-mass gravitational potential of Didymoon U_2 is replaced by the closed-form gravitational potential of the triaxial ellipsoid, equation (3.7a), denoted by $U_{2,ell}$.
- The SRP is added to the model, modeled by equation (3.9), denoted by $\mathbf{a}_{SRP}$, the acceleration vector exerted on Juventas.

The Perturbed Circular Restricted Three Body Problem (PCR3BP) equations are then obtained:

$$\begin{cases} \ddot{r}_x &= 2 \omega \dot{r}_y + \omega^2 r_x - U_{1,sh_x} - U_{2,ell_x} + a_{SRP_x} \\ \ddot{r}_y &= -2 \omega \dot{r}_x + \omega^2 r_y - U_{1,sh_y} - U_{2,ell_y} + a_{SRP_y} \\ \ddot{r}_z &= -U_{1,sh_z} - U_{2,ell_z} + a_{SRP_z} \end{cases} \quad (3.10)$$

Where the sub-subscript $(\cdot)_{\cdot_x}, (\cdot)_{\cdot_y}, (\cdot)_{\cdot_z}$ denote the direction of each acceleration term in the case of the gravitational potentials. To highlight the importance of the perturbations added to the model, a qualitative comparison can be done using a landing trajectory. The trajectory has the following initial conditions:

x [m]	y [m]	z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]
1369.810	1336.772	-18.969	0.098	-0.126	-0.07

The landing trajectory computed using the PCR3BP is compared to trajectories where the perturbations have been removed one by one and to the CR3BP. The landing trajectories are shown in Figures 3.11 and 3.12. Table 3.5 contains a comparison of the landing velocities and impact position in each case.

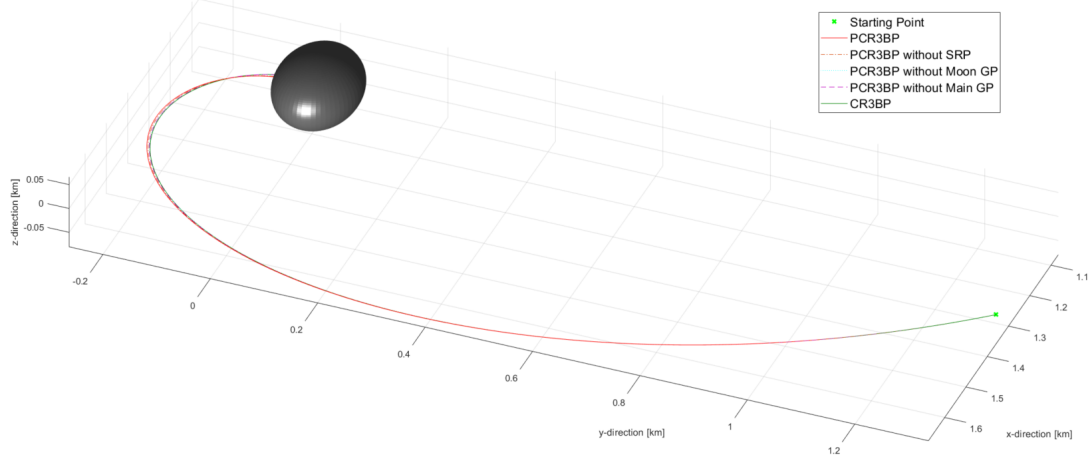
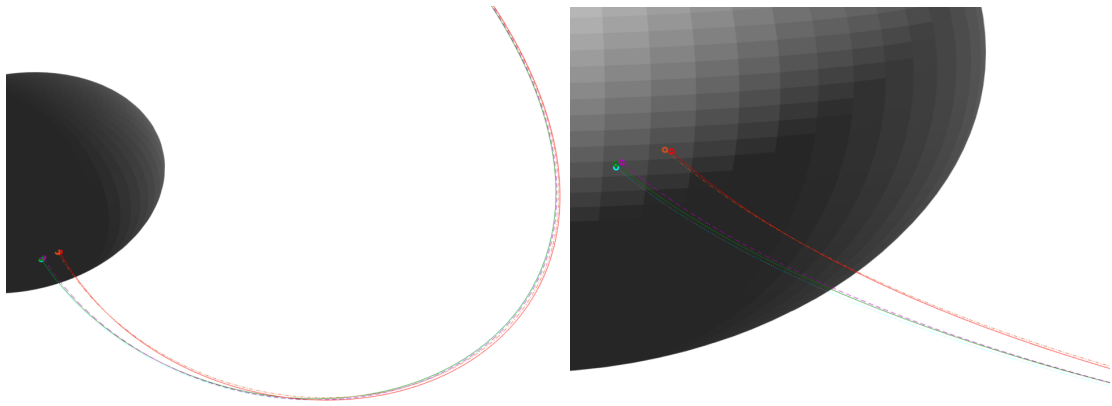


Figure 3.11: Landing trajectory comparison perturbations comparison.
Trajectories in the SYNODIC frame.



(a) Zoom on the trajectory.

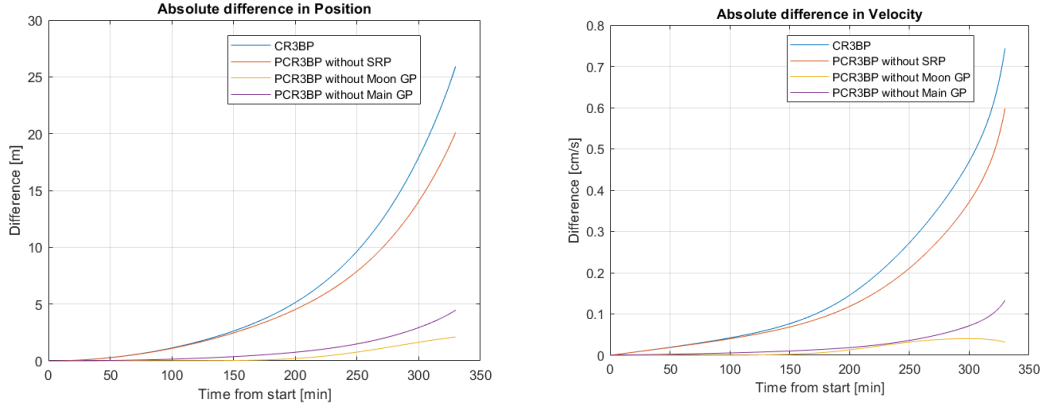
(b) Zoom on the impact zones on Didymos.

Figure 3.12: Zoom on the landing trajectory comparison.

Case	Velocity at impact [cm/s]	Impact position [lat; long] [°]	Transfer duration [min]
PCR3BP	8.83	[-10.22;-110.93]	345.23
PCR3BP, no SRP	8.69	[-8.45;-102.83]	349.80
PCR3BP, no Moon GP	8.90	[-10.80;-110.82]	345.65
PCR3BP, no Main GP	8.81	[-10.04;-109.97]	346.35
CR3BP	8.79	[-8.75;-101.66]	351.47

Table 3.5: Velocity and position at impact on Didymoon comparison.

The norm in velocity and position are also compared, by taking as reference the PRC3BP. The absolute difference in each case is shown in Figure 3.13.



(a) Position difference w.r.t the PCR3BP (b) Velocity difference w.r.t the PCR3BP

Figure 3.13: Absolute differences in position and velocity along the trajectory.

As it can be seen through Figures 3.11 to 3.13 and in Table 3.5, the perturbations added to the model have a non-negligible impact on the position and velocity along the trajectory. The position deviation can reach up to 25.92 [m] compared to the simplest model, the CR3BP. The velocity deviation can reach up to 0.74 [cm/s], the worst case being also in the CR3BP. It is worth mentioning that the deviations from the PRC3BP increase over time, as the perturbations add up with time. The perturbations also have an impact on the transfer time to the moon. A simple comparison in Table 3.5 allows us to calculate a difference of up to 6.24 minutes between the CR3BP and the PRC3BP. From Figures 3.13a and 3.13, the perturbation that has the most influence on the landing trajectory is the SRP, followed by the Main gravity perturbation and finally the Moon gravity perturbation. Adding the perturbations to the model was therefore mandatory.

Chapter 4

MATLAB® propagator

In this chapter, all the numerical ingredients used to develop the Propagator are presented. First, the chosen MATLAB® solver is discussed. Then, in order to retrieve the position of several celestial bodies, the SPICE TOOLKIT and SPICE kernels are used in the propagator. A short introduction is given on these tools. Finally, the considered dynamics in the MATLAB® propagator are summarized.

4.1 ODE solver

The set of Equations of Motion of Juventas, see equation (3.10), is numerically integrated using ODE113 in MATLAB®. The integration precision were set to:

Relative precision	Global precision
10^{-16}	10^{-17}

Table 4.1: Integration precision

ODE113 is a multi-step, variable step and variable order (VSVO) Adams-Bashforth-Moulton solver. The solutions at several preceding time points are necessary to compute the current solution. More information on the method can be found in *Appendix E*. The order¹ of the solver can vary between order 1 to maximum 13. This solver was chosen over ODE45 which uses a Runge-Kutta method, a single-step solver which only needs the solution at the preceding time point [53]. Indeed, ODE113 is more efficient at solving problems with high tolerance and is recommended for orbital mechanics problems, where high accuracy and smoothness is required in the solution [52].

¹The order of the method corresponds to the number of previously computed results used for the new approximation. For example, order 1 corresponds to an Euler implicit and Euler explicit formula.

4.2 SPICE TOOLKIT & SPICE Kernels

SPICE is a toolkit developed by the Navigation and Ancillary Information Facility (NAIF) team of the Jet Propulsion Laboratory (JPL) under contract with NASA and assists scientists in space mission design [45].

In the propagator, the position of the Sun relative to Juventas, the position of Didymoon or of the barycenter of Didymos are needed in order to make calculations. For example: SRP evaluation or frame transformation. Such information can be found in SPICE kernels and used with The SPICE TOOLKIT for MATLAB®. The kernels are a type of data file containing all the ephemerides of various celestial bodies and spacecrafts. For example, ESA kernels containing the information of various space missions can be obtained from [26]. Specific files to the HERA mission contain for instance the ephemeridis of the Didymos system, as well as HERA trajectories. Kernel files containing preliminary Juventas trajectories were obtained from GMV and GOM Space. In the toolkit, many functions allow to perform calculations specific to space mission design and to use the data contained in the kernels. Finally, the SPICE kernel data can be visualized in a SPICE enhanced version of Cosmographia; an interactive tool used to produce 3D rendering of planet ephemerides, spacecraft trajectories and instruments field of view [21]. It can be used to visualize the HERA mission, such as in Figure 4.1, where the HERA spacecraft and Didymos can be seen.

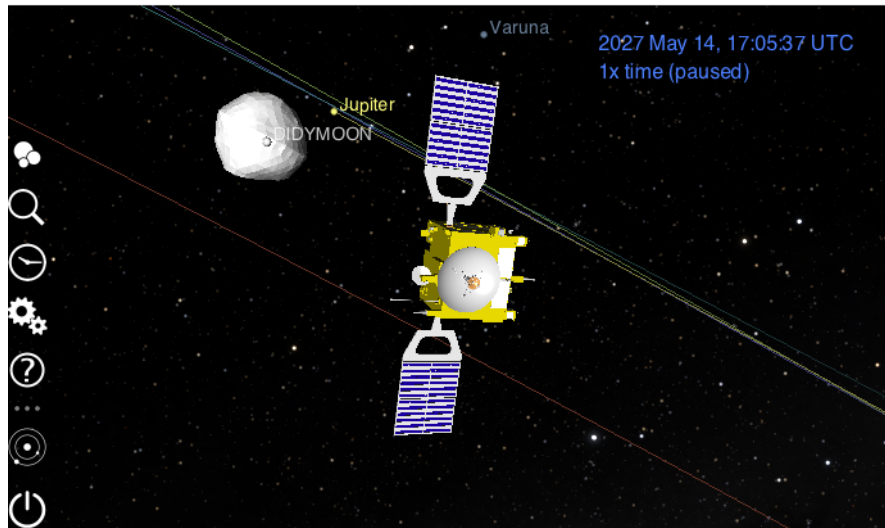


Figure 4.1: Spice-enhanced Cosmographia Tool.

4.3 Considered dynamics

As a conclusion, the following dynamics are considered in the MATLAB® propagator:

- Perturbed-Circular-Restricted Three Body Problem used to simulate the motion of Juventas in the Didymos system
- Didymos barycenter ephemerides is computed using the SPICE kernels provided by ESA
- Juventas is modeled as a point-mass body
- Didymain point-mass influence and 2^{nd} and 4^{th} order spherical harmonics for Didymain
- Didymoon constant density triaxial ellipsoid gravitational attraction
- Solar radiation pressure perturbation, assuming a constant section area for Juventas

All the physical parameters used to describe the Didymos system are summarized in Table 4.2.

Physical parameters	Designation	Value	Units
<i>Didymos System</i>			
Mass parameter	μ	9.214 e-3	-
Total mass	m_{tot}	5.278 e11	kg
Main CoM offset	r_1	-10.974	m
Moon CoM offset	r_2	1169.026	m
Main-Moon CoM distance	r_{12}	1180	m
Rotating frame angular velocity	ω	1.464 e-4	rad/s
<i>SRP</i>			
Reflectivity coefficient	c_R	1.5	-
Area-to-mass	A_{sun}/m	30	kg/m ²
<i>Gravity anomalies</i>			
Main non-spherical harmonic, order 2	J_2	-0.011432722	-
Main non-spherical harmonic, order 4	J_4	0.004583058	-
Secondary semi-axes	α, β, γ	103, 79, 66	m

Table 4.2: Parameters used to model the dynamical environment of Didymos.

Chapter 5

Verification & validation

After deriving the perturbed equations of motion of Juventas in the vicinity of Didymos and developing the MATLAB® propagator, a verification of the results and a complete validation can be made.

In this chapter, an energetic verification is performed on the CR3BP and on the CR3BP where gravity perturbations are added. Then, the full PCR3BP is validated using a landing trajectory and comparing the obtained results with data from companies developing Juventas and a space mission design software developed by NASA.

5.1 Energetic verification

As a reminder, the Jacobi constant C is a particularity of the CR3BP. It gives an indication of the energy of a particle in the rotating frame. It is an integral of motion, meaning C remains invariant with time. Making sure that C remains constant verifies from an energetic point of view the validity of the propagator. The Jacobi constant, expressed in Cartesian coordinates in the rotating frame $\mathcal{F}$ is:

$$-C = -\omega^2(r_x^2 + r_y^2) - 2U_2 - 2U_1 + v^2 \quad (5.1)$$

Where the first term is a rotating contribution, the second and third are gravitational contributions, respectively from Didymoon and Didymain and v is the velocity of Juventas.

The Jacobi constant verification is done on an SSTO of 2 km. It allows to have a long duration since the POP lasts 30 days. In what is next, first, the verification is done in the case of the CR3BP. Then, conservative gravity perturbations are added and the Jacobi constant as well as the zero velocity surfaces are analyzed.

5.1.1 CR3BP

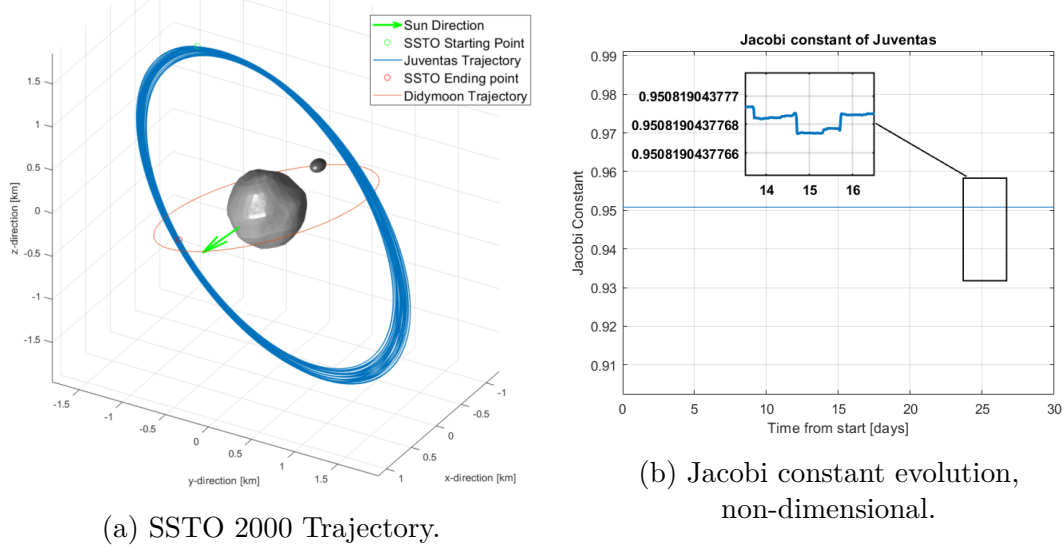
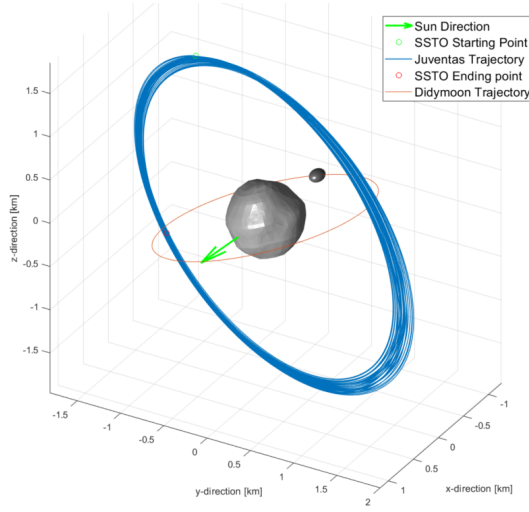


Figure 5.1: Propagated SSTO 2000.

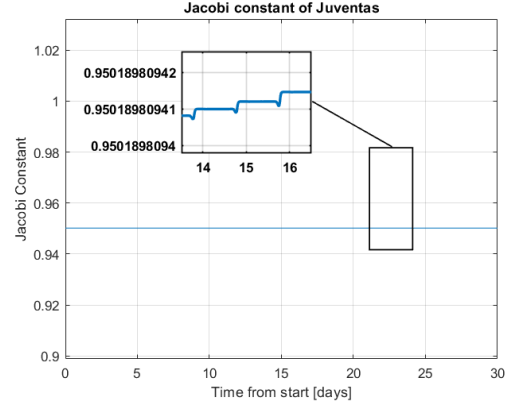
As it can be seen on Figure 5.1b, on a large time scale, the Jacobi constant of Juventas on the SSTO2000 (Figure 5.1a) is invariant. Nevertheless, when zooming on the Jacobi constant, it can be seen that variations in the latter appear at the order $1e - 12$. It is due to the precision of the integrator used to solve the ODE in MATLAB®. The variations being relatively small and the energy of Juventas staying constant on long time scales, from an energetic point of view, the CR3BP model implemented is correct.

5.1.2 CR3BP and gravity perturbations

Here, the conservative gravity perturbations due to Didymoon and Didymain have been added to the model. As they are conservative, the energy of the orbit of Juventas should not change. It is confirmed, as on Figure 5.2b, on large time scales, the Jacobi constant keeps a constant value. As in the previous case, when zooming, variations appear, here of the order $1e - 11$. These variations being very small, the gravity perturbations added to the model are verified from an energetic point of view.



(a) SSTS 2000 Trajectory.



(b) Jacobi constant evolution, non-dimensional.

Figure 5.2: Propagated SSTS 2000.

It is also interesting to compare the Jacobi constant in the case where there are no gravity perturbations, see Table 5.1:

Case	CR3BP	CR3BP + GP
Jacobi constant	0.950819	0.950189

Table 5.1: Jacobi constant comparison, non-dimensional.

Over the 30 days of the SSTS, a difference of 0.00063 [–] can be observed in the relative energy of Juventas. This difference is due to the different potential energies (because of the presence of gravity perturbations in the second case) and therefore accelerations exerted on Juventas in the vicinity of Didymos, hence also slightly modifying the Jacobi constant.

5.1.3 Zero velocity surfaces with GP

A further energetic verification concerning the gravity perturbations can be made by looking at the zero velocity surfaces of the Didymos system when adding those perturbations. As a reminder, the zero velocity surfaces are used to determine the regions accessible by the nanosatellite for a given Jacobi constant. These curves are characterized by a null Juventas velocity. If the Jacobi constant is indeed constant, Juventas should respect the zones in which it can move. A trajectory with the following initial conditions is propagated:

x [m]	y [m]	z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]
1305	54.3	0	-0.0028	0	0

The trajectory was propagated for one day and is shown in Figure 5.3. The non-dimensional Jacobi constant corresponding to these initial condition is equal to 3.15655 [-].

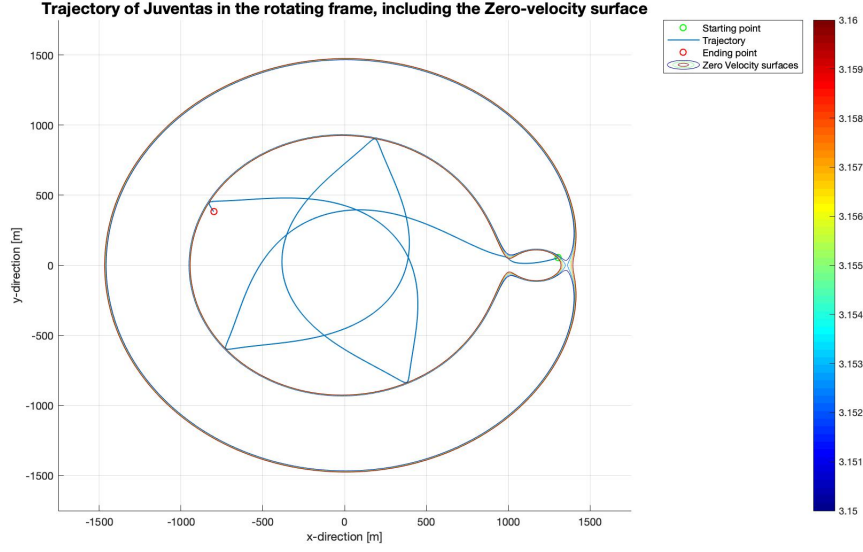


Figure 5.3: Juventas trajectory and zero velocity surfaces.

These initials conditions have also been chosen in order to have an energy level allowing Juventas to leave the realm around Didymoon and join the realm around Didymain by going through the open neck. Indeed, Juventas ricochets on the zero velocity surface corresponding to its energy level and stays in the allowed region of the Didymos system.

As a conclusion, it has been seen that the Jacobi constant keeps a constant value on the studied trajectories. Also, the zones where the spacecraft is allowed are respected, showing that the implementation of the CR3BP and the gravity perturbations is verified from an energetic point of view. An optional study of the impact of the SRP's on the Jacobi constant can be found in *Appendix C*.

5.2 PCR3BP validation

After the energetic verification, the PCR3BP equations implemented in the MATLAB® propagator are now going to be validated. To do so, a landing trajectory provided by GMV¹ is propagated, see [29], and compared with different validated results: GMV and MONTE. MONTE is an astrodynamic Python library developed by NASA’s Jet Propulsion Laboratory and can be used, as explained on their website, to support all the phases of space mission development from early space design and analysis through flight navigation services. More information can be found at [44].

With MONTE, the parameters of Didymos can be modified so as to compare trajectories in the exact same dynamical environment: same gravity perturbation definitions and same SRP equation. Concerning the results obtained by GMV, a particular attention has to be brought to the fact that the implementation of the dynamical model of Didymos is different than the one in the present work, summarized in *Section 4.4*. Table 5.2 shows the dynamics that diverge with GMV’s model:

	GMV model	PCR3BP	MONTE
Sun gravity influence	yes	no	no
Eclipses in SRP calculation	yes	no	no
Didymoon GP	Non-spherical potential of 4 th order	Constant density ellipsoid	Constant density ellipsoid
Didymain CoM offset	no	yes	yes
Didymoon ephemerides	Two Body Problem	Circular Restricted Three Body problem	Circular Restricted Three Body problem

Table 5.2: Model differences, [29].

Despite the differences in implementation with GMV, the models should give similar results as they are used to represent the motion of a body in the same dynamical environment. The results obtained using the PCR3BP model are compared to the results from MONTE and GMV. Only 24 epochs are used for the comparison².

¹GMV is a private company working on the development of the Juventas CubeSat, more precisely the GNC.

²Only 24 epochs were provided by GMV. So as to be consistent, only 24 epochs were evaluated with MONTE and the developed propagator.

The initial conditions of the trajectory, given in the synodic frame centered on the barycenter are:

x [m]	y [m]	z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]
1369.810	1336.772	-18.969	0.098	-0.126	-0.07

Figures 5.4 and 5.5 show the trajectory at the 24 epochs for the 3 considered cases.

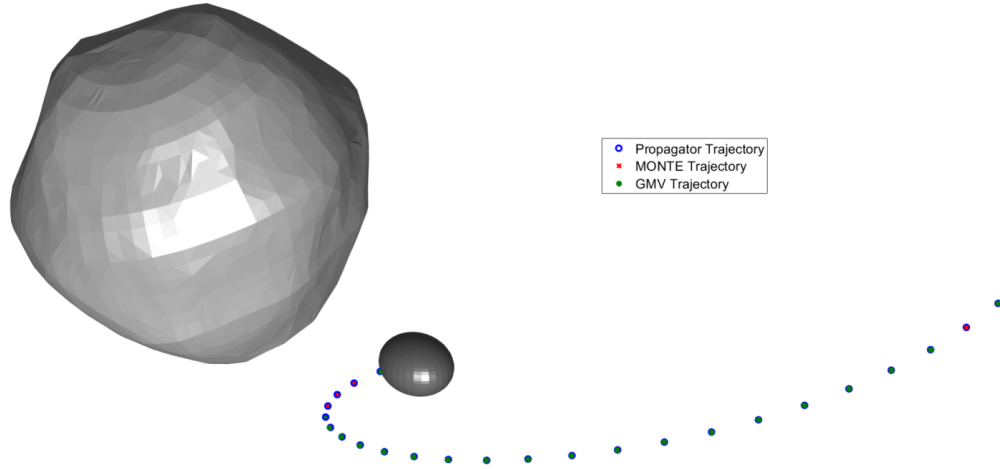


Figure 5.4: Juventas trajectory in the SYNODIC frame

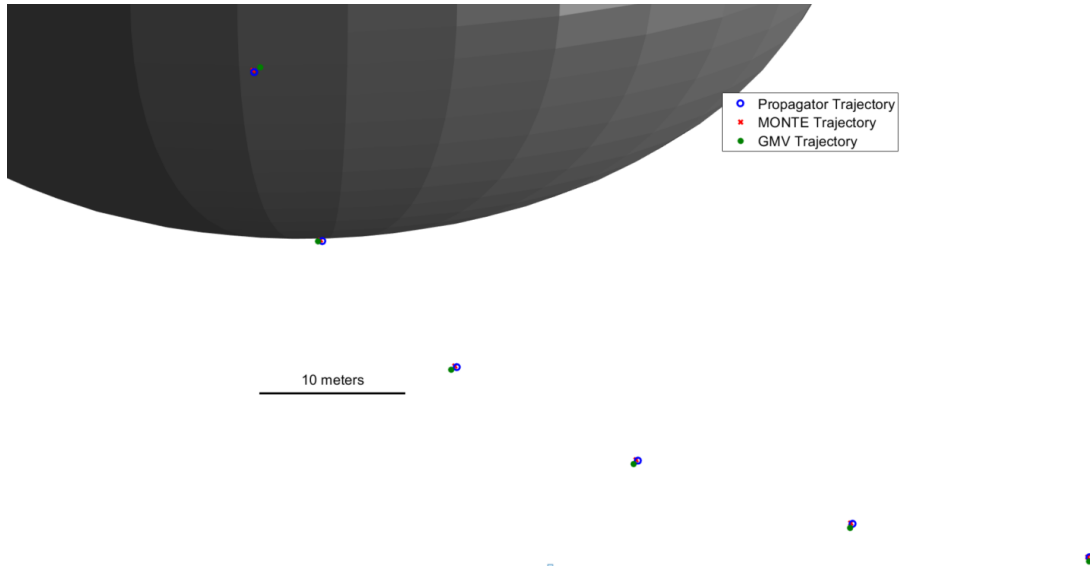


Figure 5.5: Zoon on the Juventas trajectory in the SYNODIC frame

The evolution of the relative errors in position and velocity along the trajectory is shown in Figure 5.6. Table 5.3 contains the relative and absolute errors for the landing position and the impact velocity. The errors were calculated by comparing the position and velocity deviation at each epoch.

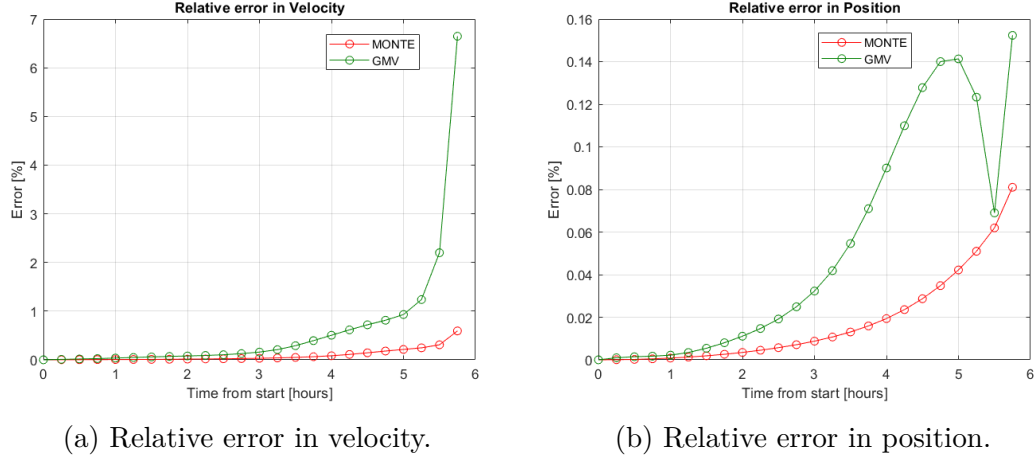


Figure 5.6: Relative errors in position and velocity between the propagator & Monte and the propagator & GMV.

	Impact Velocity		Landing Position	
Error	Absolute error	Relative error	Absolute error	Relative error
GMV	5.981 [mm/s]	6.646 %	1.743 [m]	0.152 %
MONTE	0.523 [mm/s]	0.593 %	0.925 [m]	0.0810 %

Table 5.3: Absolute and relative errors, comparison of the propagator with GMV & MONTE results.

Looking at Figures 5.6 and Table 5.3, it can be seen that the maximum relative errors with MONTE are very low: less than 1 % in position and velocity. The maximum discrepancy increases over time and is found to be at the last evaluated epoch. Indeed, the errors add-up over time during the propagation.

When comparing the results of the developed propagator with GMV, the results are less good: although it only reaches a maximum value of 0.152 % of relative error in position, a deviation of 6.466 % can be observed in terms of velocity. This last important error is mainly due to the divergences that exist in the models.

By looking at the results, even though the errors increase over time, they are very low. This is not a problem, since the propagator is only used to simulate

landing trajectories and the bouncing on Didymos, having simulation times of approximately 6 hours. Such low simulation times allow to keep low relative errors, like in the present case, see Figure 5.6. As a conclusion, the propagator developed in MATLAB® is validated.

Chapter 6

Additional numerical tools

Additionally to the MATLAB® propagator, numerical tools were developed, allowing to perform multiple studies of the landing phase. First, the numerical implementation of the bouncing phase is presented. Then, a Monte Carlo tool is developed, allowing to take into account multiple uncertainties related to the Didymos system and Juventas. Finally, the Covariance Matrix Adaptation - Evolution Strategy optimization tool is provided, allowing to optimize multiple parameters of the landing phase.

6.1 Bouncing phase implementation

After the observations phases, Juventas will try to land on the surface of Didymoon to perform its final science objectives. By the end of the landing trajectory, the nanosatellite will bounce on the surface of the asteroid, before stabilizing and reaching its final rest position. The interaction between Juventas and Didymoon is a crucial part of the mission as it will assess whether or not the mission is feasible. In this section, the different aspects related to the interaction between both bodies is studied.

In the present Thesis, Juventas is considered as a point-mass body during the descent as well as for the bouncing phase. First, the bouncing theory is presented. Then, the MATLAB® implementation is discussed: an event was implemented, allowing to detect when Juventas touches the surface of Didymoon.

6.1.1 Point mass body bouncing

At the impact, the velocity of Juventas has a specific direction and magnitude. This velocity can be decomposed into normal and tangential components with respect to the surface, as shown on Figure 6.1. In the Figure, the subscript $(\cdot)_0$ and $(\cdot)_1$ respectively denote the velocity before and after the bounce. The notation

$\perp$ stands for the perpendicular velocity component and $\parallel$ for the parallel velocity component to the local surface.

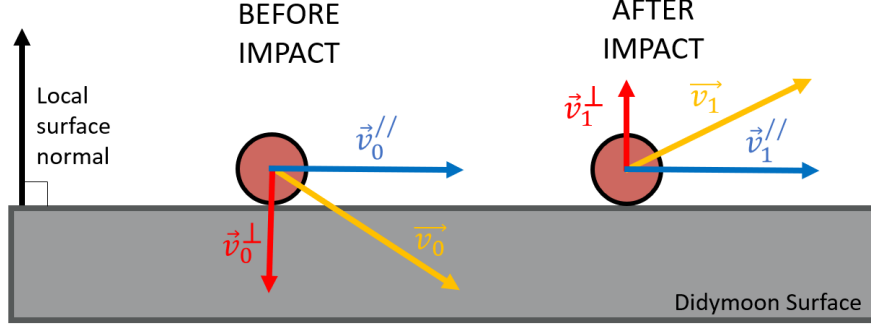


Figure 6.1: Velocity decomposition.

The physical interaction between the spacecraft, considered as a point-mass body and the surface is described by two coefficients: the restitution coefficient of the normal velocity component, e and the restitution coefficient of the tangential velocity component, f . The restitution coefficients e allows to assess the quantity of energy dissipated at each bounce:

$$e = \sqrt{\frac{K_{after}}{K_{before}}} \quad (6.1)$$

Where K is the kinetic energy. The normal and tangential restitution coefficients can be defined as:

$$e = \frac{v_1^\perp}{v_0^\perp} \quad f = \frac{v_1^\parallel}{v_0^\parallel} \quad (6.2)$$

Both restitution coefficients range from 0 (fully inelastic collision) to 1 (fully elastic collision). However, so has to obtain results representative of the actual shape of Juventas, numerically, the tangential restitution coefficient f has values ranging from -1 (direction opposed to velocity before touchdown) to 1 (same direction as velocity before touchdown), thus allowing Juventas to bounce in the opposite direction:

$$e \subseteq [0, 1] [-] \quad f \subseteq [-1, 1] [-] \quad (6.3)$$

As it can be seen from the restitution coefficients, the dissipated energy results in a decrease in the norm of the velocity vector after bouncing.

In the real case, different effects are to be considered as dissipative terms. At touchdown, part of the energy will be absorbed by the structure of Juventas and part of it will be absorbed by Didymoon. Moreover, the attitude of the spacecraft at arrival will also have an impact on both restitution coefficients. Indeed, Juventas has deployed solar arrays and radar antennas.

Both restitution coefficients are actually very difficult to evaluate and they are usually estimated using the expected surface properties as well as the Juventas structure and attitude. On an asteroid, the speeds are very small and so very elastic shocks should be expected [14]. However, the regolith material¹ may have properties that damp the energy at each bounce.

The boundary values of the coefficient of restitution (CoR) e used in the present study are based on JAXA's Hayabusa mission to Itokawa; which is currently a good approximation to Didymoon's surface, see [27]. The spacecraft Hayabusa bounced with a CoR of approximately $e = 0.8$ on Itokawa, see [14]. The studies on the 433 Eros asteroid performed by the Near Earth Asteroid Rendezvous – Shoemaker (NEAR Shoemaker), led to the determination of a very low CoR of $e = 0.1$ [46]. As recommended by the ROB, these two values are used as the boundaries of the normal restitution coefficient. Finally, the tangential restitution coefficient has a range of values that goes from -0.5 to 0.5, recommended by the ROB. Such ranges for the restitution coefficients allow to take into account multiple cases of surface properties, Juventas attitude and energy absorption by the structure of the CubeSat. Both restitution coefficients are summarized here under:

$$e \subseteq [0.1, 0.8] [-] \qquad f \subseteq [-0.5, 0.5] [-] \qquad (6.4)$$

A uniform probability distribution is assumed for the tangential and normal restitution coefficients.

The CoR e and f only have an impact on the magnitude of the velocity vector and can numerically represent a bounce that goes either in the opposed or same direction as before the touchdown. In order to have a more representative model of the real phenomenon, divergence is added to the direction of the bouncing. For this purpose, a parameter is added to the model, allowing to rotate the tangent velocity component and therefore adding randomness to the direction of bouncing.

¹Layer of unconsolidated material covering the surface of a body. It can be made of rock, dust, and can be found on asteroids.

This parameter has a normal truncated distribution characterized by a mean of 0° , a standard deviation of 30° and truncated at 45° . These values were recommended by the ROB so as to simulate the bouncing. Figure 6.2 shows an example of ground track with and without divergence in bouncing.

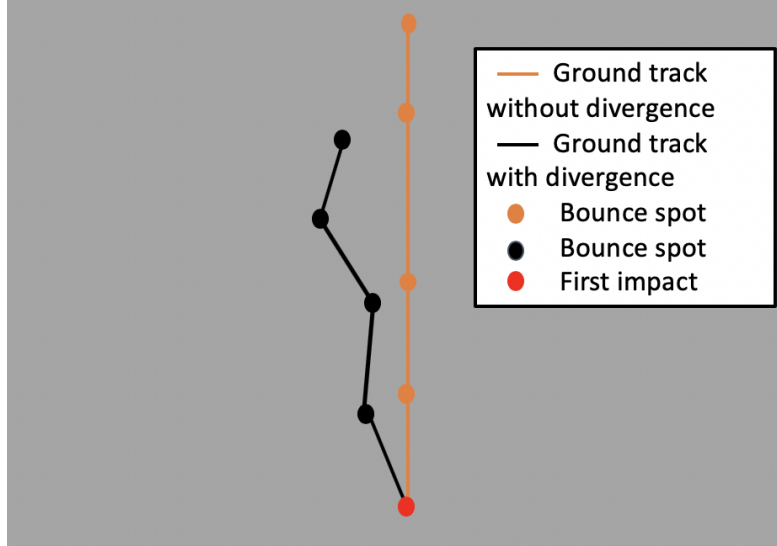


Figure 6.2: Representation of the divergence in the bouncing direction.

6.1.2 Landing EVENT

The implementation of the bouncing is based on a rigorous "Event" detection function. It is a MATLAB® function that stops the ODE integration when a stop condition is reached. In the case of the Didymos system, three situations were identified:

- Juventas lands on Didymoon
- Juventas lands on Didymain
- Juventas escapes the system and goes into deep space

In the first case, the solution after the bounce is taken as the new Initial Conditions of a new propagation. In the last two cases, the propagation is definitely stopped as both situations only happen in the case of a mission fail. Figure 6.3 shows an example of bouncing on Didymoon for multiple trajectories.

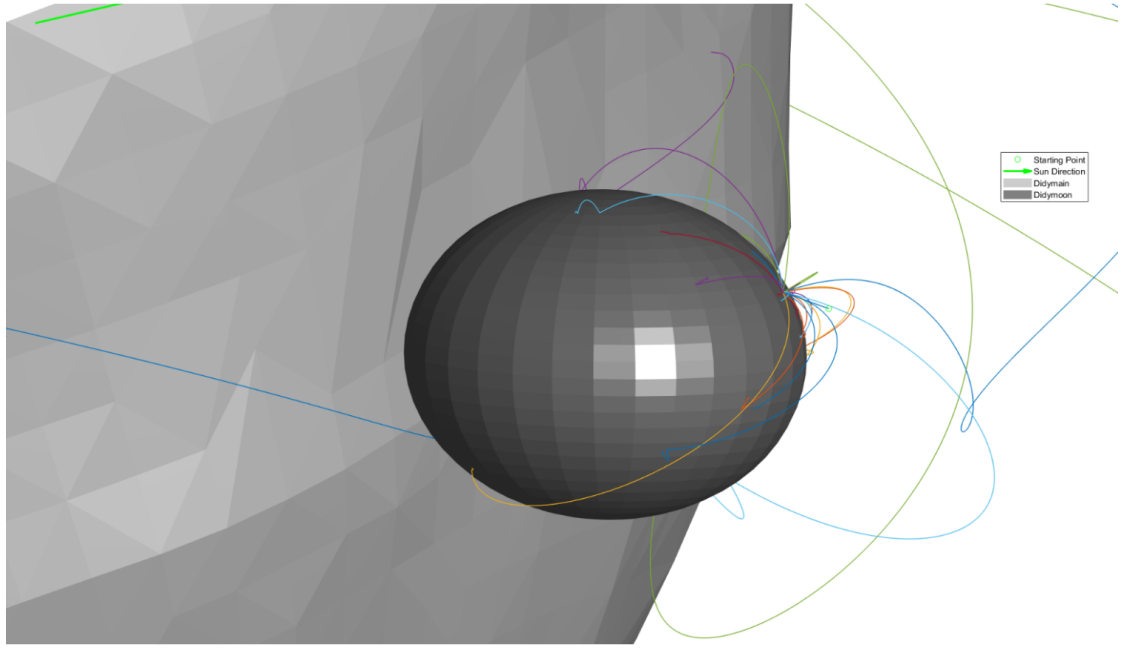


Figure 6.3: Example of bouncing on Didymoon for 25 trajectories.

Besides the landing EVENT, Juventas is considered at rest on the surface of the moon if its relative **velocity** to Didymoon is less than 1 cm/s and the **maximum height** during the last bounce is less than 3 cm. These conditions are depicted in Figure 6.4.

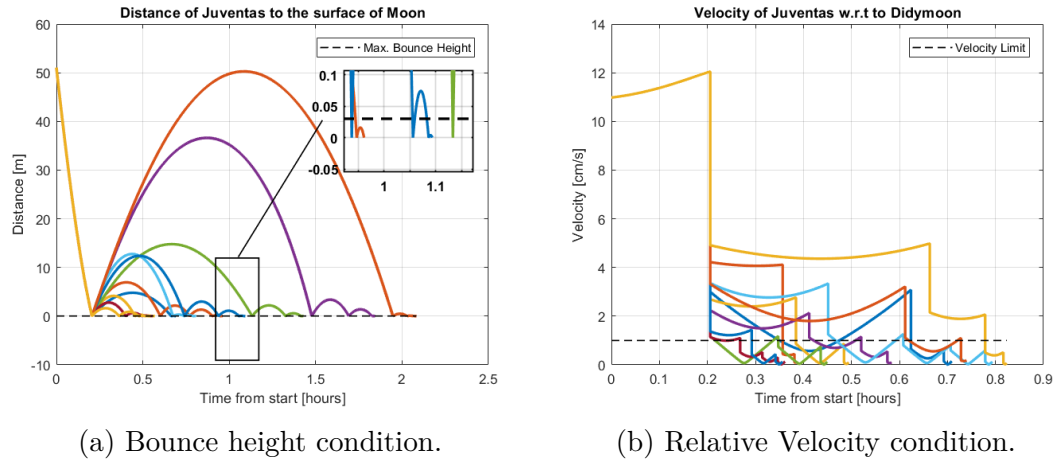


Figure 6.4: Illustration of the rest conditions for different bounces.

Looking at Figure 6.4a and 6.4b, it is interesting to note that the height condition

is stiffer than the velocity condition. Indeed, once the minimum bounce height is reached, the propagation stops. However, multiple bounces can still occur even if the 1 cm/s limit has been reached.

6.2 Didymoon topography

Particular attention must be paid to the topography of Didymoon since Juventas will land on this asteroid. In this section, the focus is on the terrain elevation of Didymoon. To represent the surface topography, two models have been implemented: the **smooth model** and the **"Fractal Brownian Motion" model**. Both models are reviewed.

6.2.1 Smooth model

This model considers Didymoon as a perfect triaxial ellipsoid with a smooth surface. In this case, no addition of rock nor any terrain elevation is considered, see Figure 6.5.

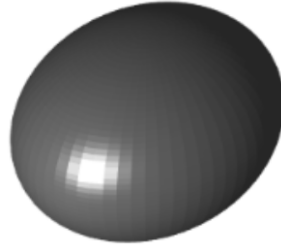


Figure 6.5: Didymoon smooth surface model.

The distance $d(\mathbf{r}_m)$ from Juventas to the surface of the ellipsoid is calculated as in [35]:

$$\left\{ \begin{array}{l} k_1 = \left\| \frac{\mathbf{r}_m}{\mathbf{P}_{ell}} \right\| \\ k_2 = \left\| \frac{\mathbf{r}_m}{\mathbf{P}_{ell}^2} \right\| \end{array} \right. \quad (6.5a)$$

$$\left\{ \begin{array}{l} k_1 = \left\| \frac{\mathbf{r}_m}{\mathbf{P}_{ell}} \right\| \\ k_2 = \left\| \frac{\mathbf{r}_m}{\mathbf{P}_{ell}^2} \right\| \end{array} \right. \quad (6.5b)$$

$$d(\mathbf{r}_m) = k_1 \frac{k_1 - 1}{k_2} \quad (6.6)$$

Where in equations (6.5a) and (6.5b), $\mathbf{r}_m$ is the vector position of Juventas relative

to the CoM of Didymoon and $\mathbf{P}_{ell} = [\alpha \quad \beta \quad \gamma]$, is a vector containing the semi-principal axes of the asteroid.

6.2.2 "Fractal Brownian Motion" model

A second considered approach to modeling the surface of Didymoon is the use of a procedural technique based on fractals. A procedural technique allows the generation of a variety of digital surface features such as real terrain. In the case of a two-dimensional terrain features, the technique takes the form of a height field. Starting from a base height $z = 0$, the procedure will modify the height field by adding terrain features and generate a $z(x, y)$.

The use of fractals is of interest, since most features on Earth display a kind of self-similarity across a wide range of scales, meaning that the same pattern is repeated at different scales, such as in mountains [57].

Fractal Brownian motion

The fractal Brownian motion (fBm) creates an appearance of randomness in the features by summing up several scaled versions of sinus waves:

$$N(A, f, \mathbf{x}) = \sum_{i=0}^n A_i \cdot \sin(f_i \cdot \mathbf{x}) \quad (6.7)$$

By adding different iterations of noise (octaves), where for each sine wave the frequency is successively incremented in regular steps (lacunarity) and the amplitude is decreased (gain), fractal noise can be obtained. The amplitude A_i and the frequency f_i of the i -th octave of noise are given by:

$$\begin{aligned} A_i &= A_0 \cdot L^{-Hi} \\ f_i &= f_0 \cdot L^i \end{aligned}$$

Where L is the lacunarity and H is the fractal increment. From [17], $H \sim 0.8$ is a good choice for representing natural phenomena and a lacunarity $L \sim 2$ works well for most purposes. A lacunarity of $L = 2\pi/3 \approx 2.0744$ is chosen to avoid overlapping the frequencies and a repetition of the pattern [17]. From these equations, only the initial frequency f_0 , amplitude A_0 and number of octaves are free parameters. With only these three numbers, all types of surface can be obtained. As illustrated in Figure 6.6, the result when adding together multiple octaves is a random-looking 1D surface that never repeats.

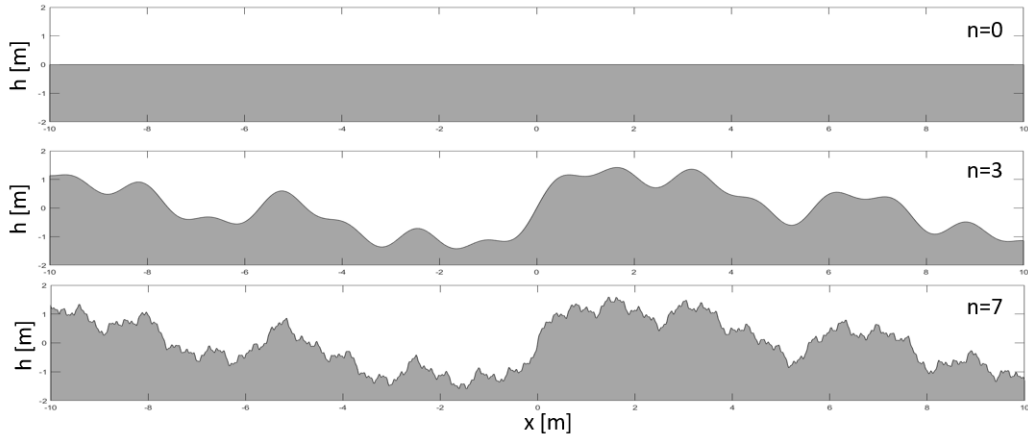


Figure 6.6: One-dimensional noise field for increasing number of octaves.

Extension to 3D

The fBm concept can be extended to 3D. Starting from a smooth surface, a grid of random numbers can be modulated and interpolated to generate a height field. Figure 6.7 illustrates the application of the fBm to a surface measuring 20 meters per side.

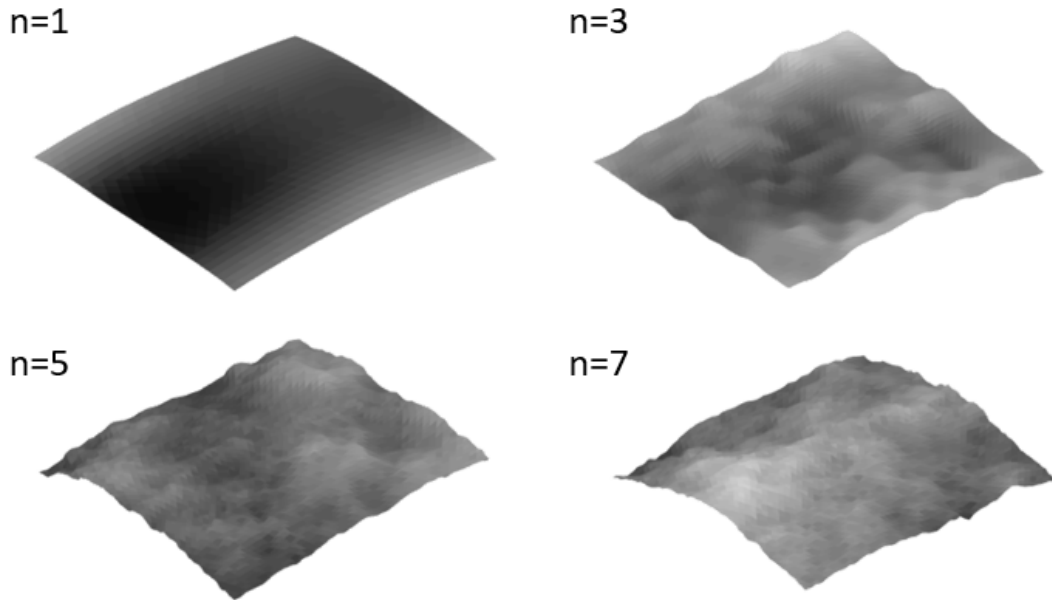


Figure 6.7: Three-dimensional noise field for increasing number of octaves.

Noise Resolution

The resolution (i.e. the size and number of facets constituting the surface) of the fBm can be adjusted with a resolution parameter. Figure 6.8 shows a 20 meters per side surface, where the facets can be observed. A low resolution allows to obtain a surface that can be handled by computers having an average computational capacity.

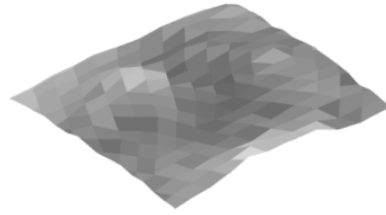


Figure 6.8: 20x20 meters surface. Facets can be observed due to low resolution.

Application of the fBm to Didymoon

The fBm concept can be applied to Didymoon. Starting from the smooth ellipsoid surface of the asteroid, the fBm algorithm can be applied in order to obtain a noisy surface. As no Earth-Didymos observation allow to characterize the surface of Didymoon, a qualitative approach is used to obtain the parameters to obtain the parameters. A qualitative comparison with the surface of the Itokawa asteroid led to the selection of the parameters in equation (6.8). The model obtained for Didymoon is illustrated in Figure 6.9.

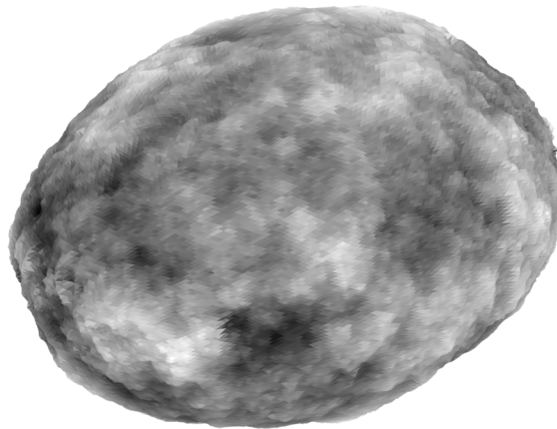


Figure 6.9: Didymoon model.

$$A_0 = 4 [m] \quad f_0 = 0.035 [Hz] \quad n = 7 [-] \quad (6.8)$$

If fBm is used to model Didymoon, the distance $d(\mathbf{r}_m)$ of Juventas to the noisy surface of Didymoon is computed as:

$$d(\mathbf{r}_m) = d_{ell}(\mathbf{r}_m) - d_{fBm}(\mathbf{r}_m) \quad (6.9)$$

Where $d_{ell}(\mathbf{r}_m)$ is the distance from Juventas to the surface of the smooth ellipsoid, $d_{fBm}(\mathbf{r}_m) = \sum_{i=0}^n A_i \cdot u(f_i \cdot \mathbf{r}_m)$ and $u(f_i \cdot \mathbf{r}_m)$ is a function that samples the fBm surface and linearly interpolates the grid. The distance $d_{fBm}(\mathbf{r}_m)$ is calculated as the normal distance between the fBm surface and the corresponding point of the smooth model. Figure 6.10 illustrates this calculation. $d_{fBm}(\mathbf{r}_m) < 0$ if the noisy surface is "below" the smooth surface and $d_{fBm}(\mathbf{r}_m) > 0$ if the noisy surface is "above" the smooth surface.

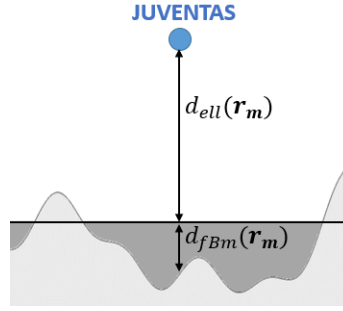


Figure 6.10: Calculation method of the distance between Juventas and Didymoon noisy surface. Dark gray: smooth surface. Light gray: noisy surface.

The MATLAB® code for the fBm was provided by Dr. Stefaan Van Waal and is fully implemented in the MATLAB® propagator.

6.3 Monte-Carlo

In space mission design, the study of uncertainty propagation is essential in order to assess the feasibility of a mission. Indeed, a nominal trajectory might land, but if the uncertainties related to the systems of the spacecraft are taken into account, the propagation might fail. A tool that is commonly used to respond to such a request is a Monte Carlo analysis. It is a computational method that allows the

user to simulate a phenomenon including multiple degrees of freedom containing uncertainties and test multiple randomized samples so as to study the effect of the uncertainties on the output.

In the context of the HERA mission, the dynamical model developed for Didymos is actually full of uncertainties due to the lack of information of the binary asteroid system. The Juventas CubeSat systems also contain uncertainties. Of them, three main sources of dispersion are of particular interest in the descent phase. First, the final structure of the spacecraft is yet unknown. Then, a second source comes from the Guidance, Navigation and Control system (GNC). The third source of errors arises from the thrusters.

Performing a Monte Carlo analysis on nominal landing trajectories will allow to assess the robustness of the proposed solution. In what is next, the different uncertainties related to Didymos and the spacecraft are presented. Then, the main data that can be obtained from the post-process of the Monte Carlo results are presented. Finally, the developed algorithm is validated using results from GMV.

6.3.1 Didymos system uncertainties

In the developed dynamical model of the system, nominal values were used, found in Table 4.2. However, due to the lack of precision in the Earth-Didymos observations, the parameters of the system are not known with certainty. Studying the effect of these uncertainties on landing trajectories is worth analyzing. Such an analysis is performed in *Chapter 8*. A full list of the Didymos uncertainties can be found in the AIM report, see [27]. A study of the sensitivity of the dependent parameters on the independent parameters of the system can also be found in [59]. The main uncertainties that are going to be studied are summarized in Table 6.1.

Parameter	Value	Unit	Reference
Total mass	5.278 ± 0.54 10^{11}	kg	GMV
Distance between the centre of Main and Moon	$1.18 + 0.04 / - 0.02$	km	[27]

Table 6.1: Main uncertainties related to the Didymos system.

6.3.2 Juventas uncertainties

The final design of Juventas has not yet been released. Among others, the final surface coating is still unknown. For now, it is a SurTec650 coated aluminum. Anodized areas could later be added in order to balance out the thermal properties

of the exterior surface of the nanosatellite [38]. The surface coating is important, as the emissivity and absorptivity coefficients will have an impact on the momentum transferred from the photons to the spacecraft through the solar radiation pressure effect. The uncertainties related to the structure of Juventas are summarized in Table 6.2. The parameters follow a normal distribution on the range of acceptable values.

Parameter	Mean Value	Range	Unit	Reference/Note
Coefficient of reflectivity	1.5	[1 2]	-	[5]
Mass to effective area ratio	30	[20 40]	kg/m ²	[5]

Table 6.2: Spacecraft design uncertainties.

Concerning the GNC, two modes of operations are under study: ground orbit determination or on-board estimation, although the second one is said to be the most likely scenario [29]. Uncertainties in position and velocity determination arise from the GNC. Table 6.3 summarizes all the information.

Parameter	1 σ error	Unit	Reference/Note
<i>Ground orbit determination</i>			
Position uncertainty	100	m	GMV
Velocity uncertainty	1	mm/s	GMV
<i>On-board estimation</i>			
Position uncertainty	10	m	GMV
Velocity uncertainty	0.2	mm/s	GMV

Table 6.3: GNC Uncertainties.

Finally, the maneuver execution error contemplates several scenarios as well: single impulse and 1s impulse maneuvers. The uncertainties are summarized in table 6.4.

Parameters	1 σ error	Unit	Reference/Note
<i>Single impulse</i>			
Impulse magnitude	5	%	GMV
Impulse orientation	1.5	degree	GMV
<i>Impulses of 1s</i>			
Impulse magnitude	0.5	%	GMV
Impulse orientation	0.2	degree	GMV

Table 6.4: Thruster Uncertainties.

6.3.3 Acquired data

From the Monte Carlo analysis, many relevant information can be obtained from the post data processing. Such information can be used to characterize the studied trajectories. Of them, three important quantities can be calculated:

- **Success rate:** Out of the total tested number of trajectories tested, the percentage of trajectories that impact Didymoon.
- **Escaped trajectories:** After impacting Didymoon, the percentage of trajectories that bounce out of the gravitational influence of Didymoon.
- **Global success rate:** Out of the total of trajectories tested, the percentage of trajectories that impact and rest on Didymoon.

The distribution of several parameters of the descent and the bouncing can also be obtained:

- **Settling time:** Time between the first bounce and when Juventas is at rest on Didymoon.
- **Bounces:** Number of bounces of Juventas on Didymoon.
- **Covered distance:** Distance between the impact location and rest location. Calculated as the sum of the distance between each bounce.
- **Latitude - Longitude:** First impact position, geocentric coordinates.
- **DeltaV:** Magnitude of the maneuver applied for leaving the SSTO.
- **Impact velocity:** Magnitude of the velocity just before the impact on Didymoon.
- **Transfer time:** Time taken by Juventas between leaving the SSTO and the first impact on the moon.
- **Impact angle:** Angle of incidence at the first impact position. An angle of 0° is defined normal to the surface and 90° tangent to it.

6.3.4 Validation

For the validation of the Monte Carlo algorithm, GMV provided five landing trajectories and the MC results using 100 samples², where only the uncertainties related to the thrusters in the case of a single impulse are studied.

²Only 100 samples were provided by GMV so as to have an indication on the results that should be obtained with the MC algorithm.

Three quantities are compared in order to validate the MC code: the success rate, the mean transfer time and the mean impact velocity. First, a convergence analysis is performed in order to determine the number of samples needed to ensure the convergence of the results. 5000 samples in total are used in this first analysis. Then, the results are compared.

Convergence analysis

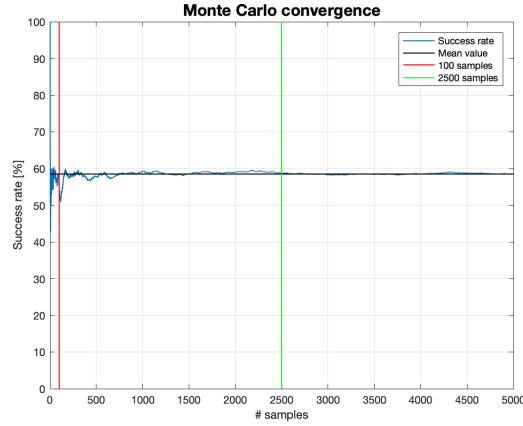
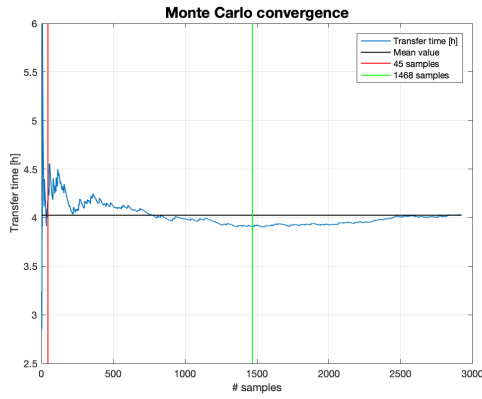
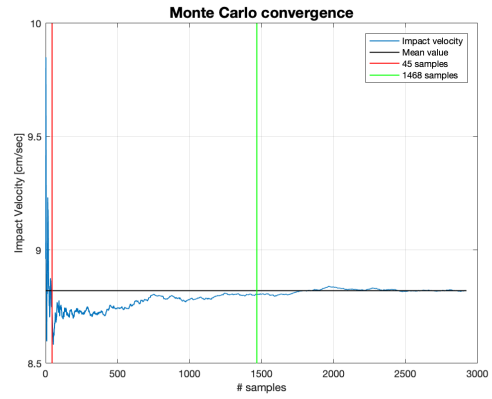


Figure 6.11: Convergence results for the success rate.



(a) Mean transfer time.



(b) Mean impact velocity.

Figure 6.12: Convergence results.

As it can be seen in Figure 6.11, about 1000 samples per MC simulation are sufficient to converge and obtain a stationary approximation of the success rate. However, while the success rate is calculated based on all the samples of the simulation, the

mean transfer time and mean impact velocity are computed by only using the samples that did impact Didymoon.

Looking at Figures 6.12a and 6.12b, approximately 2500 samples that land are needed for convergence of the results, thus approximately 50% of the total number of samples used in one simulation. Running a simulation of ~ 5000 samples can take up to 2 hours to run. Given the important quantity of analysis that will be carried out in *Chapter 7* and *Chapter 8*, the MC analysis will be performed using a total of 2500 samples, providing results that are accurate enough to do the surveys.

Monte Carlo validation

The comparison with the GMV results is now performed to validate the Monte Carlo algorithm. The number of samples per simulation is fixed at 2500. As it can be seen on Figures 6.11 and 6.12, using 100 samples per simulation (red line) is not sufficient as loads of fluctuations still happen at the beginning of the simulation. However, it gives an approximation of what should be obtained.

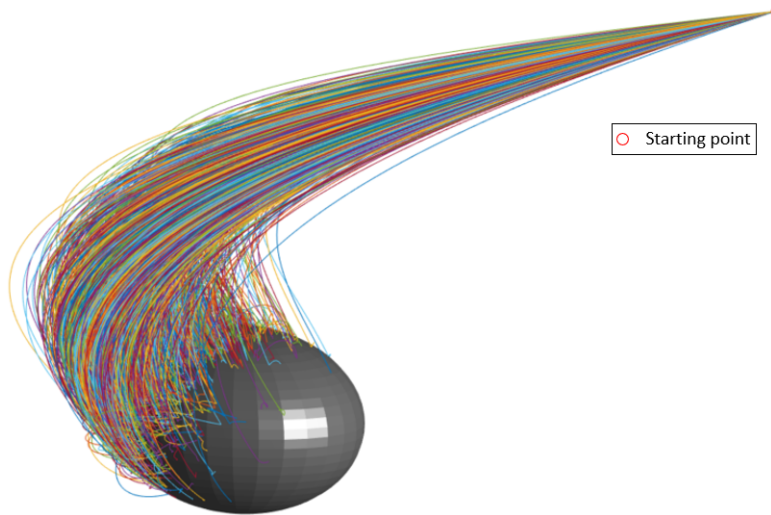


Figure 6.13: MC simulation using 2500 samples.

Figure 6.13 shows an example of MC simulation where thruster uncertainties are taken into account. In Table 6.5, the success rate, the mean landing velocity and the mean transfer duration are compared.

Trajectory	Type	Success Rate [%]	Mean impact velocity [cm/s]	Mean transfer duration [hours]
Traj 1	Propagator	83.00	8.25	2.70
	GMV	70.00	8.62	2.73
Traj 2	Propagator	66.28	8.17	3.11
	GMV	58.00	8.80	2.98
Traj 3	Propagator	56.80	8.90	3.66
	GMV	39.00	8.84	3.46
Traj 4	Propagator	58.72	8.80	3.90
	GMV	45.00	8.97	3.67
Traj 5	Propagator	58.12	8.82	3.97
	GMV	54.00	8.99	4.47

Table 6.5: Comparison of MC results.

The comparison highlights some important points:

1. **Success rate:** Although the values obtained are not exactly the same, they follow the same trend: when GMV gets a high (respectively low) success rate, the same is obtained with the propagator. A difference of $\sim 10\%$ is observed for each result.
2. **Mean impact velocity³:** In terms of norm, the values obtained are really close, a maximum error of 7% in the norm is obtained in trajectory 2.
3. **Mean transfer duration:** The values obtained are once again really close, with relative errors of 1.11% for trajectory 1, 4.18% for trajectory 2, 5.46% for trajectory 3, 5.89% for trajectory 4 and 12.59% for trajectory 5.

As a conclusion, when comparing the results obtained with the MC propagator and the ones of GMV, a certain difference can be observed, mainly due to the fact that only 100 MC samples were used in the case of GMV. Despite those differences, the values obtained follow a similar trend and the MC algorithm is therefore validated.

³The values were provided by GMV in terms of norm.

6.4 Optimization tool

In space mission design, it can be convenient to optimize certain parameters of a trajectory. In this chapter, an optimization method based on evolution strategies, the Covariance Matrix Evolution Strategy (CMA-ES), developed by A. Gawelczyk, N. Hansen and A. Ostermeier in the 1990's is presented.

The developed algorithm is implemented in the case of the HERA Mission so as to optimize several design parameters that arise from a landing trajectory of Juventas. As a reminder, this Master Thesis aims at finding robust landing trajectories while respecting a set of mission requirements. In what is next, the setup of the optimization is discussed and then the algorithm is presented.

6.4.1 Setup

An optimization problem can always be represented under a standard form:

$$\begin{aligned} & \underset{x}{\text{max/minimize}} && f(x) \\ & \text{subject to} && g_i(x) \leq 0, \quad i = 1, \dots, m. \\ & && h_j(x) = 0, \quad j = 1, \dots, m. \\ & && x \geq 0, m \geq 0, n \geq 0 \end{aligned}$$

Where $f(x)$ is the objective function to maximize/minimize, x is the vector containing the decision variables and $g_i(x) \leq 0$ and $h_j(x) = 0$ are the inequality and equality constraints to respect.

Objective functions

In the case of a landing trajectory of Juventas, many objective functions are possible. A non-exhaustive list of objective functions to optimize is given:

- The transfer time between the SSTO and the surface of Didymoon
- The DeltaV applied to leave the SSTO
- The impact velocity on Didymoon
- The angle of arrival at the surface of Didymoon

Decision variables

Taking a nominal landing trajectory of Juventas, many decision variables can be identified. They should allow to explore a wide range of landing possibilities that exist in the neighborhood of a first obtained solution. A few examples are:

1. **Magnitude of the departure velocity vector:** the norm of the nominal velocity leaving the SSTO varies during the optimization
2. **Orientation of the departure velocity vector:** the pitch and yaw angles vary during the optimization
3. **Initial epoch:** the departure time is directly related to the initial position of Juventas on the SSTO. It can be varied to find a better departure position

Constraints

First, a set of constraints has to be respected regarding Juventas. They consist of all the landing constraints, from *Section 2.3.4*:

- Juventas shall leave the SSTO 2000 on the 30th day of observations
- Impact velocity at landing < 10 cm/s
- DeltaV budget for the landing < 19.9 cm/s
- All illumination constraints shall be respected

Additionally, one has to set constraints on the decision variables. Here, two choices can be made:

- The optimum is calculated in the neighborhood of the nominal trajectory, a **local optimum** is found. The range of values that can take the decision can be limited. This method is interesting in the case where the computational power of the user is limited.
- The optimum is calculated on the entire SSTO, a **global optimum** is found. The range of values that can take the decision variables can be very large (i.e. 24 hours around the initial epoch).

Based on these considerations, the lower and upper bounds of the decision variables can be chosen by the user. Finally, the existing constraints can be changed, or more can be added to the optimization, depending on the needs of the particular landing trajectory that is being optimized.

6.4.2 The Covariance Matrix Adaptation - Evolution Strategy

Generally, the optimization algorithm is chosen regarding the type of objective function. Indeed, if it is a first order function, a simplex algorithm is preferable, or if the function is non-linear, non-linear methods such as the Newton method are more appropriate. Concerning Juventas landing trajectories, the objective functions can be considered as black boxes, being highly non-linear, non-convex⁴. A non-deterministic⁵ algorithm is therefore needed to solve such objective functions. The choice was made to use a Covariance Matrix Adaptation Evolution Strategy (CMA-ES) algorithm. As mentioned on their website, the CMA-ES is *"an evolutionary algorithm for difficult non-linear non-convex black-box optimization problems in continuous domain. The CMA-ES is considered as state-of-the-art in evolutionary computation and has been adopted as one of the standard tools for continuous optimization in many (probably hundreds of) research labs and industrial environments around the world."*, [22].

The algorithm is based on two main very complex statistical principles: a covariance matrix adaptation (CMA) and an evolution strategy (ES).

The Covariance Matrix contains the covariance (measure of the joint variability of two random variables) between each pair of decision variables (multivariate distribution) of the problem to optimize. At each iteration, the matrix is updated using a maximum likelihood estimator.

Evolution strategies are a class of nature-inspired optimization methods that use mutation, recombination and selection applied to a population of individuals of potential solutions in order to evolve iteratively towards a better solution [28]. At each step, part of the population (called the parents) is selected and mutated, giving birth to the new offspring. These new individuals are used as the new population.

The combination of both methods allows to have an optimization tool that converges rapidly towards the optimal solution.

Illustration

To illustrate the model, an iteration works as follows [7]:

- Starting from an initial population of size N , only the μ best decision variables minimizing the DeltaV are kept. Using this μ elements, their mean is calculated and will be the mean of the next iteration.

⁴Problem that might present multiple local optimum within each one [48].

⁵A non-deterministic algorithm will exhibit different behaviors for the same input.

- The population of size μ (namely the parent population) and the covariance matrix of the previous iterations are used to calculate a new approximation of the covariance matrix. The covariance matrix is then adapted using an evolution path that computes the direction in which to minimize the objective function.
- From the adapted covariance matrix and the mean (previously calculated), a new population of size N characterized by a multivariate distribution is generated.

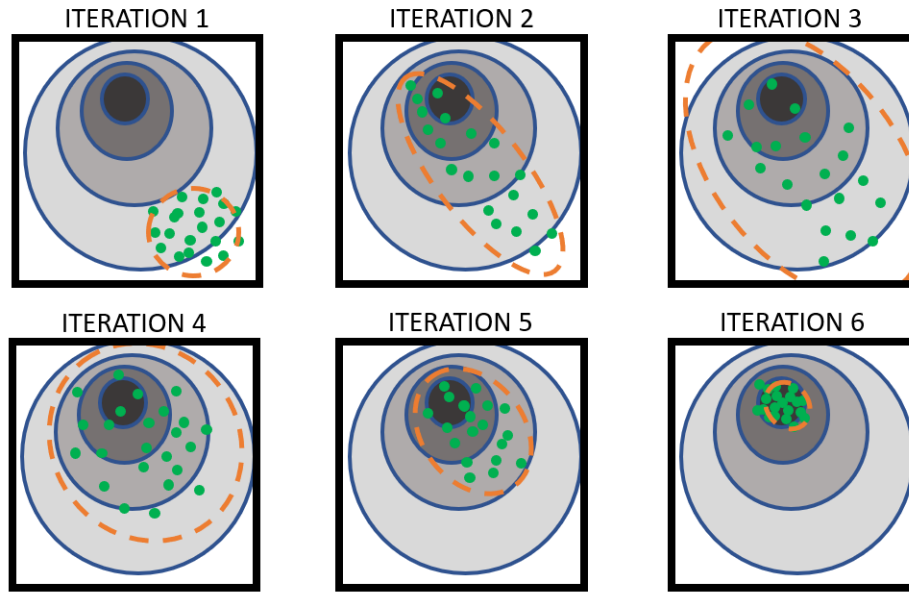


Figure 6.14: Illustration of an optimization using CMA-ES.

Figure 6.14 shows an example of optimization of a two-dimensional problem. As it can be seen, the distribution of the population (green dots) evolves and moves towards the optimum. At each generation, the CMA-ES algorithm produces a new set of population and orients the shape of the distribution (orange dotted line) in the right direction. At the end, the population of decision variables is concentrated around this optimal solution.

Application

In conclusion, the CMA-ES algorithm can be used to optimize certain parameters of a landing trajectory. The tool has been implemented in the MATLAB® but will not be used in the next chapters. However, its usage is strongly encouraged in future studies.

Chapter 7

Landing phase analysis

After developing the dynamical environment of Didymos, setting up a MATLAB® propagator and validating it, the study of the landing phase can begin. This phase of the Juventas mission is divided into two parts: the landing trajectory and the bouncing of Juventas on the surface of Didymoon. It is considered as the most crucial part of the mission, since the achievement of many science objectives depend on the success of this last part of the mission of Juventas.

The landing takes place after the Proximity Observation Phase (POP) on an SSTO at an altitude of 2 km and allows Juventas to land on Didymoon so that the nanosatellite can perform its surface measurements. The bouncing phase starts right after the first touchdown on Didymoon and lasts up to the point where Juventas comes to rest.

This chapter is made out of three main sections. First, a set of possible landing trajectories are calculated and proposed. Then, in order to assess the robustness of the solutions, a Monte Carlo analysis is performed, taking into account several uncertainties. Thanks to the analysis of the results in the parameter space, the most crucial parameters of the descent and bouncing are also identified. Finally, based on the analysis of the error propagation, the three best landing scenarios are selected.

7.1 Landing trajectory

In this section, the different landing strategies able to reach Didymoon are presented, as well as their main characteristics. Then, the constraints that Juventas has to respect to comply with the mission requirements and the criteria used to select the possible landing trajectories are discussed. Next, to obtain landing trajectories, a backward propagation¹ is performed from the surface of the moon up to the SSTO.

¹Propagation that goes backwards in time.

The methodology is presented and the results are exposed.

7.1.1 Landing strategies

The landing strategies are divided in two categories: direct landing and indirect landing.

Indirect landing strategies take advantage of the complex three body dynamics, using the Lagrange Points (see *Section 3.1.5*) to land on Didymoon: from the SSTO, Juventas passes either through the L2 point or the L4/L5 before reaching the surface of the moon.

Direct landing strategies are a simple case of ballistic landing² from the SSTO. Two types of direct landing strategies can be considered:

- One DeltaV; where only one maneuver is performed while leaving the SSTO
- Two DeltaV; where a second braking maneuver is performed close to the surface of Didymoon to reduce the touchdown velocity

In the case of Juventas, the direct landing approach is preferred over the indirect. This option allows us to simplify the transfer of Juventas to Didymoon since direct strategies are quicker and simpler than indirect ones, see [29].

In the next section, only the case of direct landing where only one DeltaV maneuver is applied while leaving the SSTO is considered. Only instant impulses are considered.

7.1.2 Selection criteria

To comply with the mission requirements (see *Chapter 2* for full mission requirements), the proposed landing approaches shall respect many constraints. The first one is the departure date: Juventas has to leave the SSTO as early as the 30th day of observation, that is on the 09 AUG 2027. Moreover, a maximum landing velocity of 10 cm/s is recommended to reduce the probability of leaving Didymoon's gravitational attraction after the first bounce on the surface.

Additionally to the mission constraints, a first study of direct landing trajectories was carried out by the GMV team and a set of recommendations was published for the selection of the optimal landing trajectories, see [29]. Based on their conclusions, the selection criteria for the study are summarized in Table 7.1. The criteria concern: the landing velocity, the transfer duration, the phase angles with respect to Didymain and Didymoon and the DeltaV burn for leaving the SSTO.

²Type of landing where no maneuver is performed during the descent.

Criteria	Threshold	Reason
Landing Velocity	≥ 8 cm/s	Larger feasible area available
Duration of trajectory	<6 hours	Reduce propagated errors
Phase Angle w.r.t. Didymain	<90°	Good illumination conditions
Phase Angle w.r.t. Didymoon	<90°	Good illumination conditions
DeltaV	<19.9 cm/s	Reduce the maneuver errors

Table 7.1: Criteria to select the trajectory.

Special attention should be paid to the landing velocity threshold of 8 cm/s. In the study performed by GMV, landing velocities of 5 cm/s to 10 cm/s with a step of 1cm/s as well as with a direction normal to the surface were considered. The main conclusions of the report were that for low landing velocities (below 8 cm/s), the feasibility to reach a large landing area on the surface of Didymoon was very low (minimum $\sim 1\%$ for 5 cm/s and maximum $\sim 12\%$ for 7 cm/s). For larger velocities, the reachable surface can go up to $\sim 75\%$. The low feasibility to reach a large landing area can be justified by the fact that in the case of low velocities, the kinetic energy of Juventas is also low, therefore limiting the reachable zones on the moon surface by the nanosatellite. Therefore, the limit of 8 cm/s is kept.

Having the feasibility to reach a large landing area is essential to the mission success. Indeed, if it is large, in the case of an error during the burns or in the positioning of Juventas in the Didymos system, the landing will still be feasible. Whereas if this value is very low, a small deviation might compromise the landing and the entire mission.

Next, an important criterion to take into account during the landing phase is the illumination of Juventas and its phase angle³ with respect to Didymain and Didymoon. Indeed, Juventas scientific payload needs the illumination of the Sun to work properly. Both these parameters have to be lower than the limit of 90°. Further information on phase angles can be found in *Appendix A*. The departure point on the SSTO is excluded from the analysis, since by definition, the SSTO plane is perpendicular to the Sun.

In order to respect the allocated DeltaV for the landing phase and to reduce the maneuver errors, it is chosen to have a threshold of 19.9 cm/s.

Finally, a transfer time lower than 6 hours is chosen to minimize the propagation

³In astronomy, the phase angle is defined as the angle formed by the Sun, the observed body and the observer.

errors during the approach. The error propagation topic is further discussed in *Section 2* of this chapter.

7.1.3 Backward propagation

In order to study the **One DeltaV Direct Landing** trajectory to Didymoon, a backward propagation is performed from the surface of the moon up to the SSTO. It allows obtaining trajectories that link the SSTO and Didymoon's surface. The complete methodology is divided in two points: the surface conditions and the stop conditions. They are now presented.

Surface conditions

First, a grid cell of landing sites is generated on the surface of Didymoon. Each initial position is spaced by 10° in latitude and longitude. Figure 7.3 shows the surface discretization in an equal-area Mollweide projection. This type of projection is explained in more details in *Appendix F*.

The 0° latitude is chosen to be coincident with Didymoon's equator, i.e. perpendicular to its rotation axis $\hat{\mathbf{e}}_z$. The positive latitudes are chosen along Didymoon's pole direction (i.e. positive $\hat{\mathbf{e}}_z$). The positive longitudes are chosen along Didymoon's velocity direction (i.e. positive $\hat{\mathbf{e}}_y$). Figure 7.2 depicts the latitude and longitude reference.

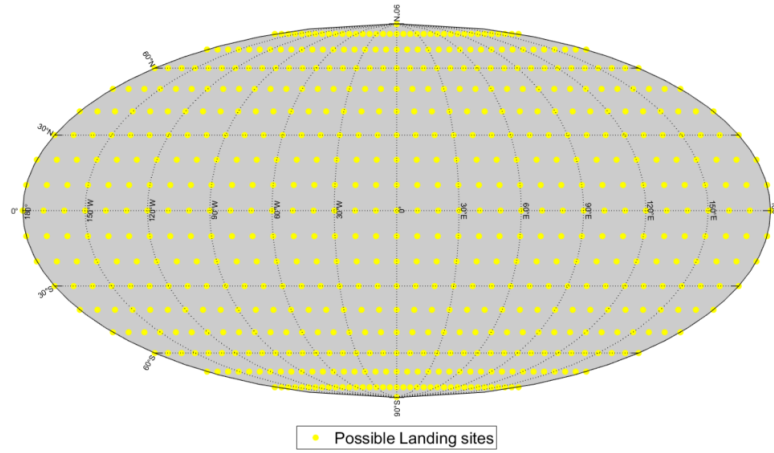


Figure 7.1: Didymoon's surface discretization.

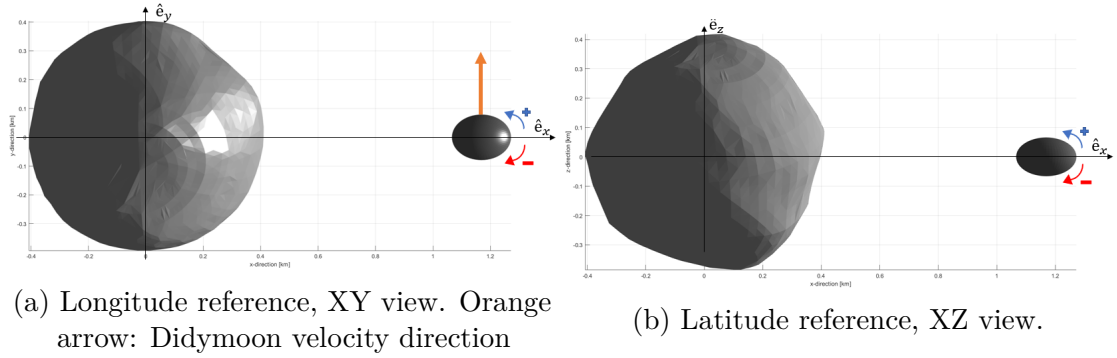


Figure 7.2: Longitude and latitude representation.

The following parameters were varied in terms of initial epoch and landing velocity for the propagation:

- **Landing velocity:** velocities of 8 cm/s, 9 cm/s and 10 cm/s are considered, and their direction is chosen to be normal to the surface. The upper velocity of 10 cm/s is selected to reduce the probability of bouncing out of the attraction of Didymoon, as recommended by the ROB (see *Section 2.3.4*). The lower velocity of 8 cm/s is selected based on the results from the GMV study, discussed here above.
- **Landing epoch:** during the last day of the SSTO, landing epochs separated by 15 min have been studied between **09 AUG 2027 14:15:00** and **10 AUG 2027 02:15:00**, so a total of 49 Didymoon positions are considered. Didymoon's period being 11.92 hours, the epochs are chosen so as to cover one complete orbit of the asteroid.

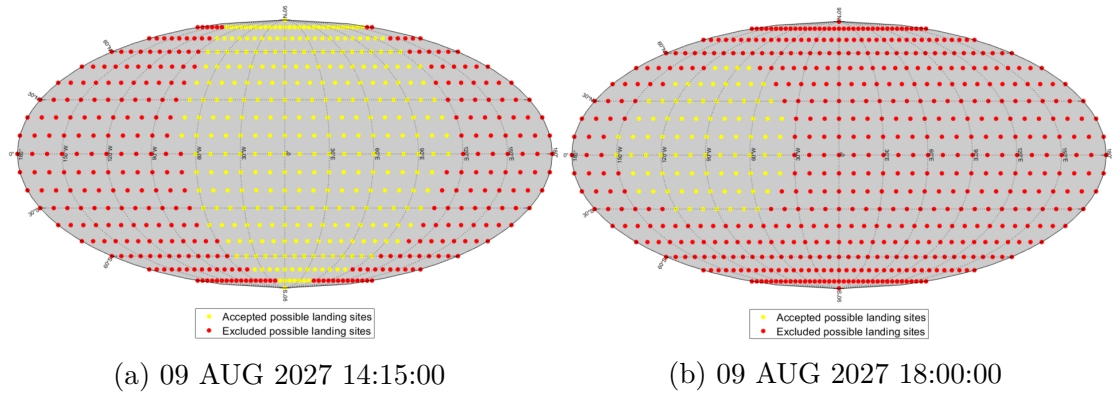


Figure 7.3: Landing sites exclusion due to illumination conditions.

Due to the illumination conditions, many landing sites are excluded from the study

at each epoch. Figure 7.3 shows two examples of epochs containing excluded landing sites from the analysis.

The phase angle of Didymoon with respect to Didymain at landing is also important, as an eclipse might occur. Assuming parallel rays from the Sun, the eclipse zone is situated from 160.83° to 180° of Didymoon phase angle, see Figure 7.4.

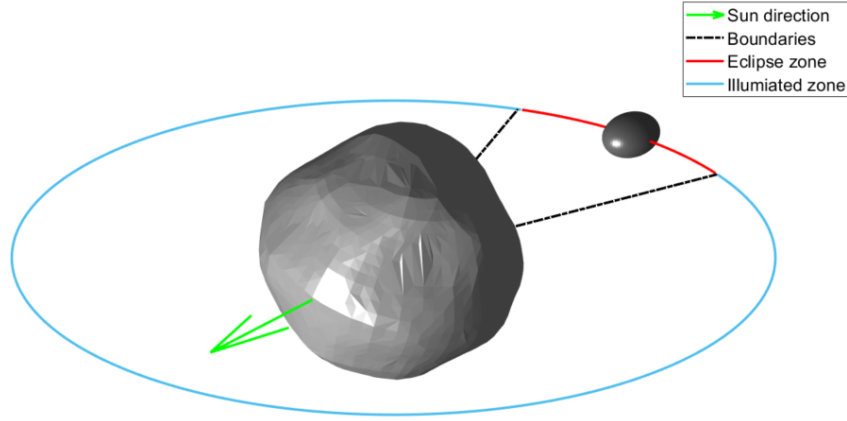


Figure 7.4: Eclipse zone.

For the considered landing epochs, the eclipse starts on 2027 AUG 09 at 20:19:23 and ends on 2027 AUG 09 at 21:28:52. The eclipse lasts a total of 1.158 hours.

Due to the combined illumination conditions, the landing epochs between **09 AUG 2027 18:15:00** and **09 AUG 2027 23:45:00** are discarded from the study. The trajectories were then backward propagated using the MATLAB® propagator.

Stop conditions

The backward propagation algorithm is stopped when either of the two criteria is met:

- the propagation reaches minus 12 hours: the trajectory is rejected, as it does not reach the SSTO.
- the landing trajectory comes within 4 meters of the SSTO orbit: the trajectory is accepted. A maximum error of 4 meters from the SSTO orbit is acceptable and allows to limit the propagation of errors due to the release from the SSTO.

7.1.4 Results

A total of 56,943 possibilities were tested. Figure 7.5 shows an example of backward propagation. From all the possibilities, 31 trajectories respecting the selection criteria of Table 7.1 were identified. For each case, the main characteristics are: the landing date, the impact velocity, the transfer duration, the geocentric coordinates of the landing position, the minimum distance from the SSTO and the DeltaV applied while leaving the SSTO. The full set of trajectories can be found in *Appendix G*. Table 7.2 contains the example of three trajectories. Figures 7.6, 7.7 and 7.8 show these three trajectories in the Equatorial J2000 frame.

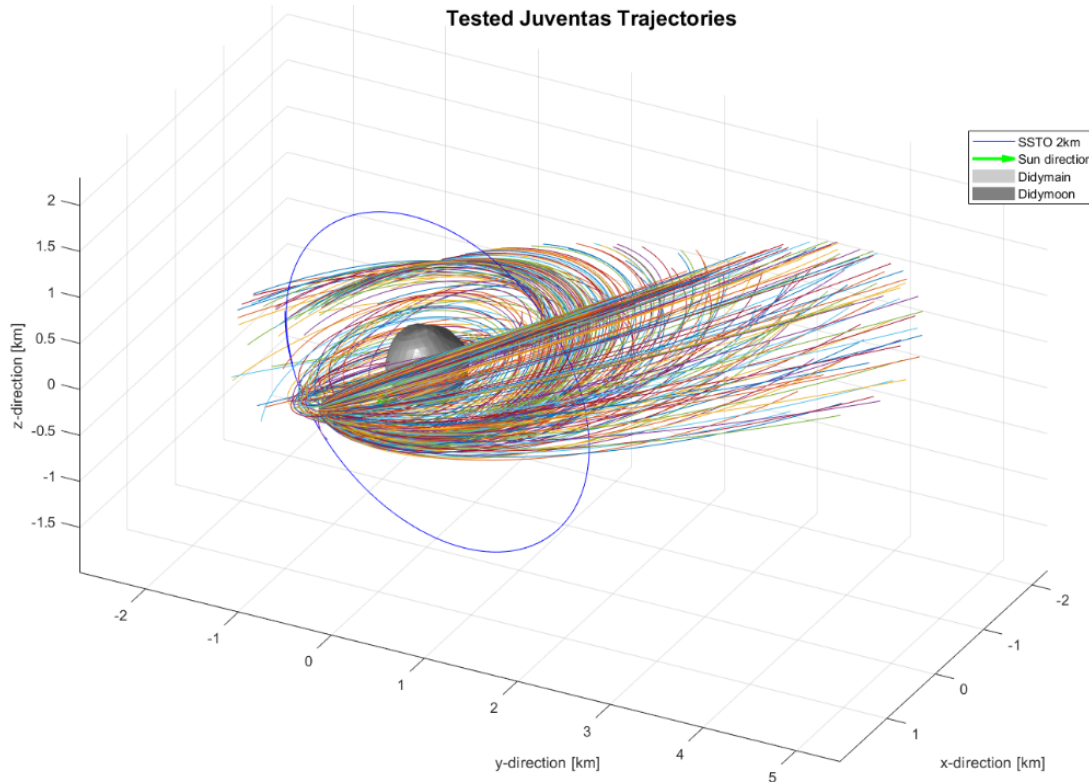


Figure 7.5: Backward propagation on the 2027 AUG 09 15:30:00.

Landing Date	Impact Velocity [cm/s]	Duration [hours]	Didymoon Coordinates [Lat; Long] [°]	Minimum distance from SSTO [m]	DeltaV [m/s]
09 AUG 2027 16h00	0.08	5.348	[10;-80]	3.477	0.1840
10 AUG 2027 01h15	0.1	1.959	[0;0]	1.228	0.1911
10 AUG 2027 02h15	0.1	3.458	[30;0]	1.206	0.1780

Table 7.2: 3 Possible landing trajectories.

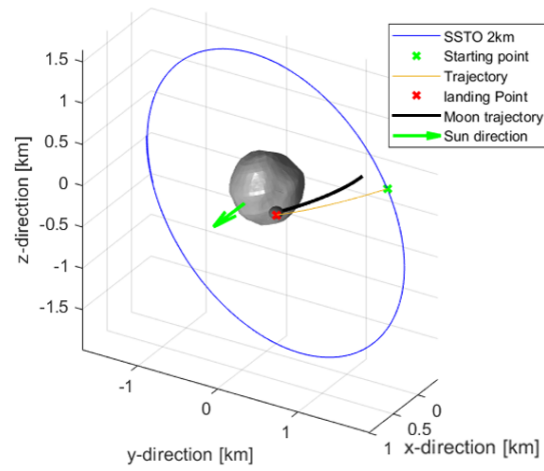


Figure 7.6: Landing approach on 09 AUG 2027 16:00:00.

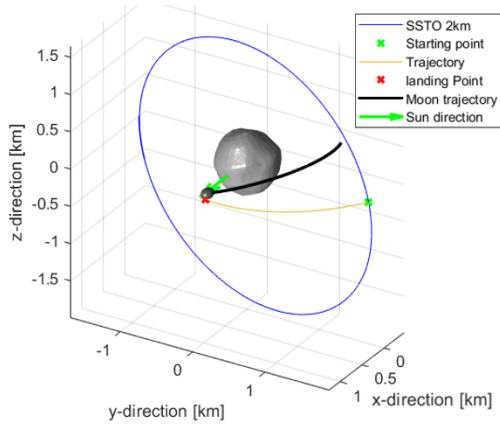


Figure 7.7: Landing approach on 10 AUG 2027 01:15:00.

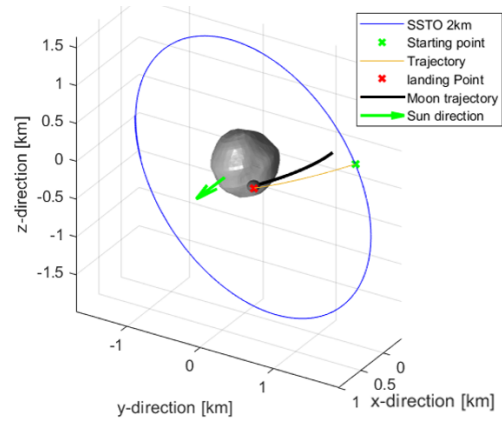


Figure 7.8: Landing approach on 10 AUG 2027 02:15:00.

A first important result from the backward propagation is that all the trajectories have different departure dates from the SSTO. In the case of a problem on board of Juventas, the nanosatellite would have sufficient time to solve its issues and choose an approach to Didymoon from the set of trajectories. In space mission design, having multiple contingency plans is primordial for the mission success.

Secondly, as it can be seen on Figure 7.9, a wide range of landing site position are possible. Figures 7.6 to 7.8 also illustrate this observation. In the future, when further information on Didymos and the surface of Didymoon will be made available, the engineers might select the landing site according to the mission requirements, as well as down-selecting a dedicated trajectory.

For example, once DART will have impacted Didymoon, a landing site inside or next to the newly formed crater might be chosen among all the possibilities. Finally, although most of the latitudes are covered, the longitude of the landing sites have a particularity: they are positioned between -100° and 10° . As a reminder, negative longitudes are defined opposed to the velocity direction of the moon (see Figure 7.12a).

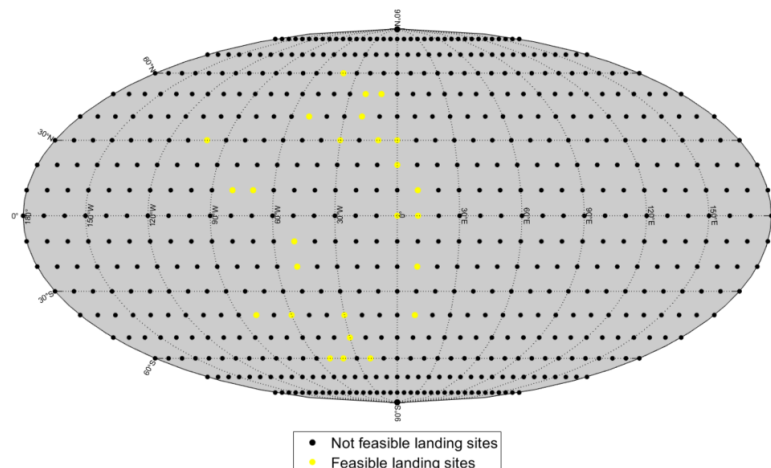


Figure 7.9: Feasible Landing Sites.

Only 31 possibilities might look a small amount. However, it is important to mention that the backward propagated trajectories were forced to obey several criteria:

- Only velocities with a direction normal to the surface of Didymoon are considered;

- Each landing epoch is spaced by 15 minutes;
- The possible landing sites considered in the simulation are spaced by 10° in latitude and longitude.

A more detailed analysis, taking into account the following new conditions might lead to new results:

- Landing velocities not only normal to the surface, but having multiple possible incidence angles;
- Landing epochs spaced by less than 15 minutes;
- A more refined grid cell of landing sites on the surface of Didymoon.

As a conclusion, in this section, the one DeltaV direct landing strategy was studied. A total of 31 possibilities for Juventas were obtained. In the next sections, a Monte Carlo taking into account uncertainties related to the spacecraft is performed on these trajectories.

7.2 Monte Carlo analysis

In space mission design, the propagation of uncertainties are a topic of high importance, since it allows to assess the robustness of the trajectories. In the case of the Juventas landing, after obtaining a set of possible approaches, a closer look is taken at the behavior of the nanosatellite when adding uncertainties during the landing and the bouncing. To do so, a Monte Carlo (MC) analysis is performed, where the maneuver errors related to the thrusters are taken into account.

For the maneuver dispersion, the error due to a single impulse is considered, as the **One DeltaV Direct Landing** strategy is studied. As a reminder, the 1σ error are:

Parameter	1σ error	Unit	Reference/Note
Impulse magnitude	5	%	GMV
Impulse orientation	1.5	degree	GMV

Table 7.3: Thruster Uncertainties.

The smooth topography model is used to represent the surface of Didymoon and to study the bouncing. The algorithm is performed using 2500 samples, in order to have a convergence and obtain accurate results. The full MC analysis results of the 31 trajectories can be found in *Appendix H*.

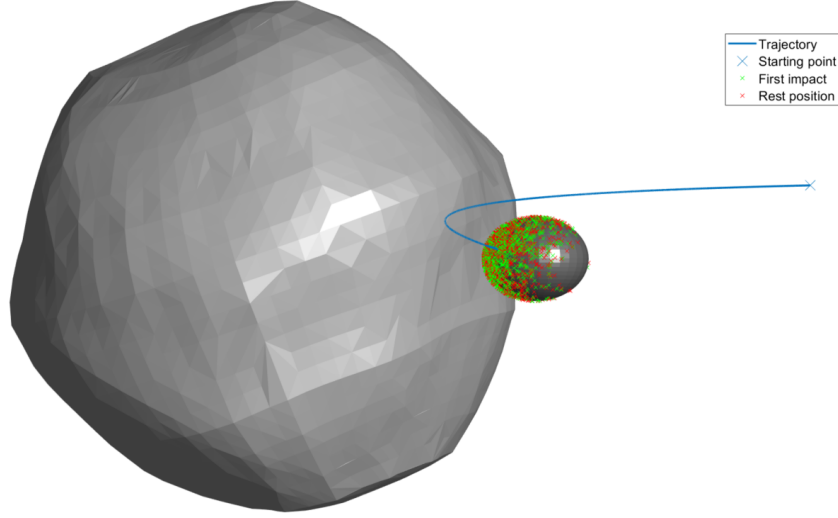


Figure 7.10: MC simulation on a trajectory landing on 09 AUG 2027, 14:30:00.

Figure 7.10 shows an example of MC propagation for 260 samples. It is also important to remind that in the current landing strategy, although available, the use of the GNC is not considered (i.e. no correction maneuvers are applied during the descent). In what is next, the results obtained are discussed for the landing and then for the bouncing. Finally, recommendations are given for future studies.

7.2.1 Error propagation during the approach

From the MC analysis performed on the 31 trajectories, two important results can be observed concerning the landing: the transfer time of the trajectories have an important impact on the success rate and on the landing site dispersion. Both points are now discussed.

Each data point corresponds to the mean result from applying the MC algorithm to one trajectory. For each trajectory, the mean transfer time is calculated as the mean of the samples the samples that impact Didymos. In each analysis, the linear fit is obtained by applying a least square regression to the data. The 95% prediction interval is also determined.

7.2.1.1 Success rate

The success rate of a trajectory is important, as it assesses the robustness of the chosen solution. Figure 7.11 shows the success rate vs. the mean transfer time.

On Figure 7.11, a general trend can be observed: the largest the transfer time, the

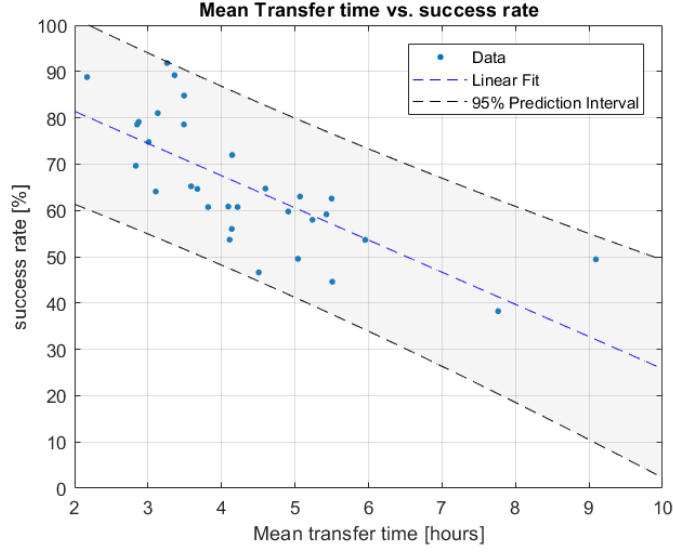


Figure 7.11: Success rate vs. transfer time.

lower the success rate. For low transfer time (i.e. <4 hours), the success rate can be as high as 91.84% in the best case and as low as 35.40% for important transfer times (i.e. >7 hours).

The uncertainties related to the thrusters induce an error on the trajectory which accumulates in time. If the transfer time to Didymoon is low, these errors do not have time to accumulate significantly, thus leading to a landing trajectory that does not diverge importantly from the nominal case and a high success rate. However, a high transfer time means that the errors can accumulate significantly, thus leading to trajectories that do not land on Didymoon and to a low success rate.

It is also noticed that the trajectories having an important success rate (i.e. low transfer time) are trajectories where at the landing epoch, Didymoon is close to the SSTO. The distance covered by Juventas in these cases is generally lower than for trajectories having an important transfer time.

7.2.1.2 Landing site dispersion

The landing site dispersion is now studied. For now, no landing site is preferable for Juventas. However, if the mission requirements are updated and require the spacecraft to land on a particular site (for example the crater created by the DART impact), one would try to reduce the probability of deviating from the selected landing site. This can be done by reducing the transfer time.

For increasing transfer times, an increasing dispersion around the nominal impact

location can be expected. It is due to the fact that for increasing descent times, the errors on the trajectory due to the burns can accumulate more and more. The trajectories thus deviate more significantly than for low transfer times.

It can be illustrated by two trajectories: #5 and #23. Table 7.4 contains information on the landing site of those trajectories.

Trajectory	Mean duration [hours]	Impact Location [Lat; Long][°]	1 σ Latitude [°]	1 σ Longitude [°]
5	2.170	[0;0]	28.356	44.245
23	5.497	[0;10]	24.837	73.779

Table 7.4: Main results from MC analysis for trajectories 5 and 23.

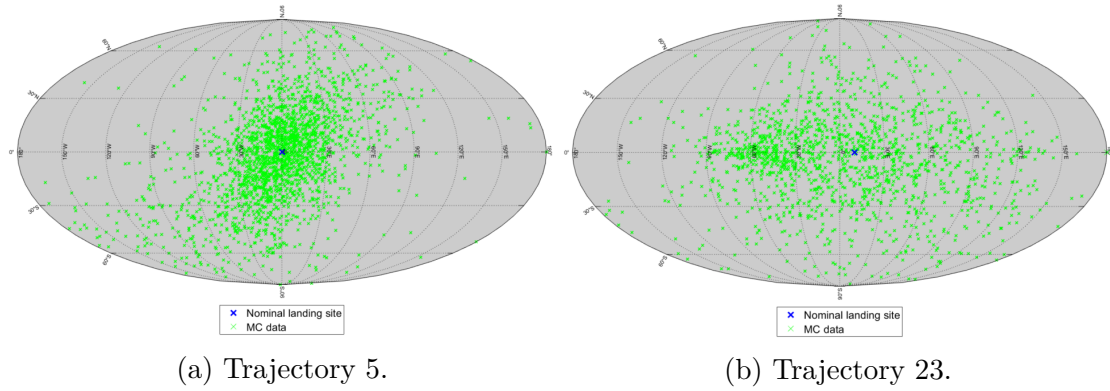


Figure 7.12: Mollweide projections of the dispersion around the nominal landing site for trajectories #5 and #23.

Figure 7.12 shows the dispersion of the landing sites over the surface of Didymoon. Figure 7.13 shows the distribution of the landing sites. It is clearly notable that for a short transfer time (i.e. trajectory #5), the distribution of the longitude of the impact sites are concentrated around the nominal case, here with 1σ of 44.245 °, while for a longer descent time (i.e trajectory #23), the landing sites are more spread out, here with a 1σ of 73.779 °. Concerning the latitudes, a small difference of 3.519 ° is observed between the two cases. Finally, as it can be seen on Figure 7.13, the nominal case is always close to the mean result of the Monte Carlo.

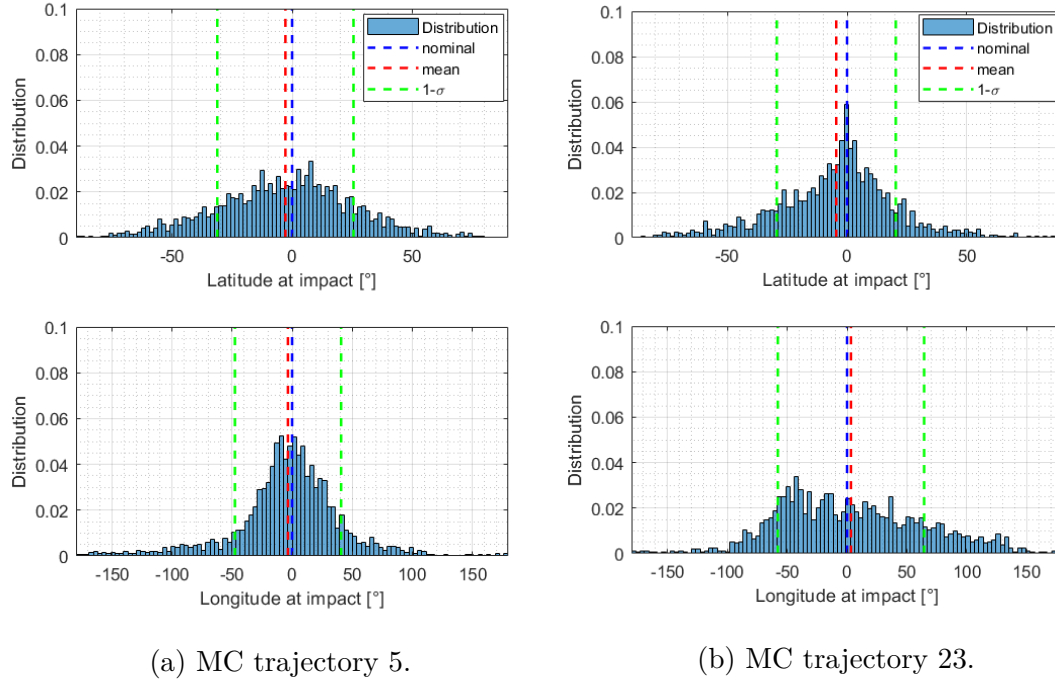


Figure 7.13: Results from MC Analysis.

7.2.2 Error propagation during the bouncing

In this section, the propagation of the error due to the burn for leaving the SSTO during the bouncing is analyzed. In each case, a least squares regression is applied to the data to obtain a linear fit. The 95% prediction interval is also shown. Two main parameters have been observed as having an influence on the behavior of Juventas on the surface of Didymoon: the impact velocity and the incidence angle at the impact⁴. Their influence on the percentage of escaped trajectories from Didymoon, the settling time and the number of bounces is analyzed.

7.2.2.1 Trajectories escaping Didymoon

In some cases, after impacting Didymoon, Juventas will still have a quantity of energy such that the nanosatellite will leave the gravitational attraction of Didymoon and will not come back, leading to the failure of the descent phase. When bouncing out of the Didymoon attraction, two cases are possible; either Juventas crashes on Didymain or it goes into deep space. The probability of escaping Didymoon is calculated as:

⁴An angle of 0° is defined normal to the surface and 90° tangent to it.

$$P_{escape} = \left(1 - \frac{N_{stay}}{N_{impact}}\right) \quad (7.1)$$

where N_{stay} is the number of Juventas trajectories reaching a rest position on Didymoon and N_{impact} the number of trajectories that impact Didymoon. On Figure 7.14, the percentage of cases leaving Didymoon after bouncing at least once on its surface as a function of the mean impact velocity and the angle of incidence are shown.

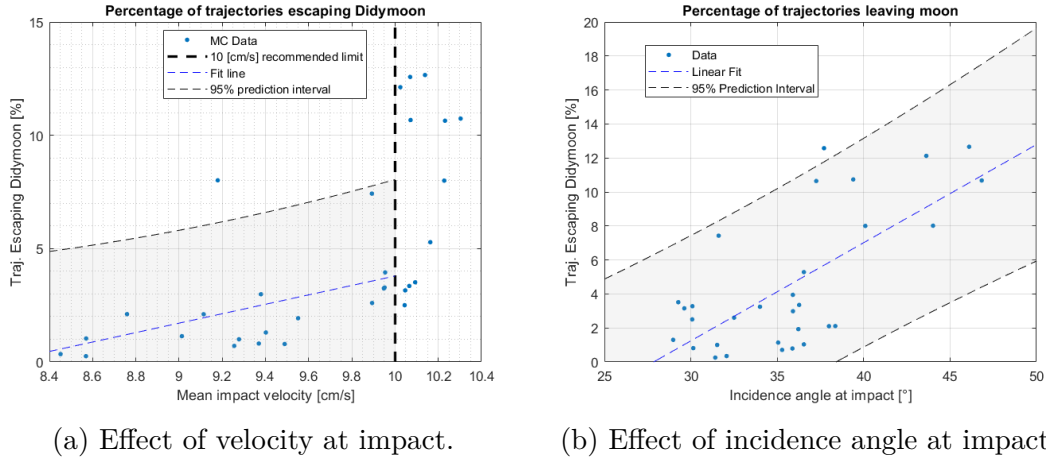


Figure 7.14: Parameters influencing the percentage of trajectories escaping Didymoon.

Up to 10 cm/s, the trend is linear: the more important the mean impact velocity, the more chance Juventas has to bounce out of Didymoon. The mean trend increases slowly up to 4%, but in some cases, the probability can be of up to 8.01%. For each trajectory, it is therefore crucial to study the escape from Didymoon. Figure 7.14a also confirms the 10 cm/s recommended limit: for trajectories with mean impact velocities over 10 cm/s, the probability of escaping Didymoon soars. Up to 12.66 % of the trajectories can leave Didymoon.

Having an important velocity at touchdown means that Juventas has an important quantity of energy to dissipate before coming to rest. This energy is either absorbed by the structure or by the asteroid. In some cases, the nanosatellite has so much energy that the combination of an important coefficient of restitution (CoR) and an important touchdown velocity leads to Juventas escaping Didymoon. In order not to damage the spacecraft when impacting the surface and prevent it from leaving moon, low impact velocities are recommended.

An interesting observation from Figure 7.14b is that if the incidence angle increases (i.e. the velocity is more and more tangent to the local surface), the percentage of trajectories escaping Didymoon increases.

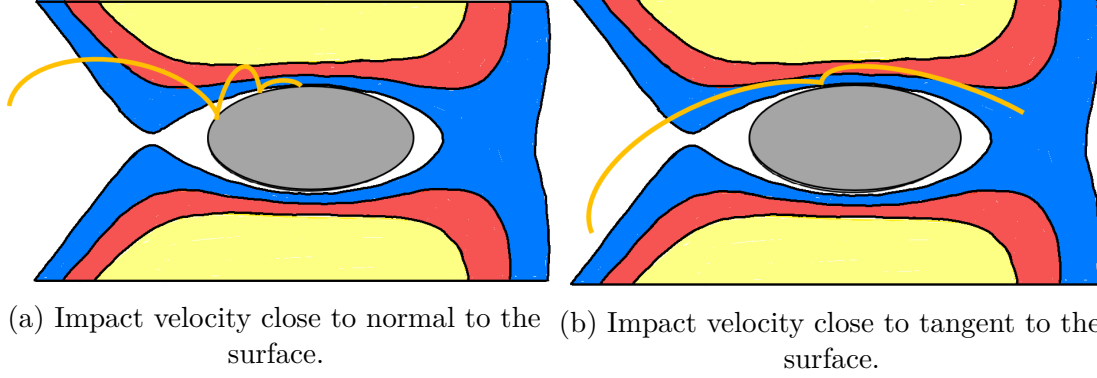


Figure 7.15: XY plane view of bounces on the surface of Didymoon. Orange line: Juventas trajectory, gray: Didymoon, different colors correspond to the regions between zero velocity curves.

Figure 7.16 represents two bouncing situations: (a) the impact angle is close to 0° and (b) the impact angle is close to 90° . Additionally, three zero velocity curves are shown (black lines), allowing to better understand the reachable regions by Juventas for a given Jacobi constant. It is obvious that as Juventas loses energy, less and less regions are reachable in space (the yellow zone is the forbidden area after the first impact, red after the second impact and blue after the third).

If the impact angle is close to 0° , Juventas bounces high and the ground track is small. Hence, many bounces are needed for Juventas to reach the other side of Didymoon and leave the attraction. However, after each bounce, Juventas dissipates a quantity of energy such that the zero velocity surfaces close around Didymoon and prevent Juventas from leaving the moon.

If the impact angle is close to 90° , Juventas does not bounce high but far from the impact point. In some cases, it reaches the other side of Didymoon and Juventas still has an energy level such that it can escape from the moon attraction.

This illustrations proves that it is easier for Juventas to leave the attraction of Didymoon with a velocity direction tangential to the surface. In this case, it is therefore recommended to have an incidence angle close to normal to the surface.

7.2.2.2 Settling time

Here the effect of the first impact velocity and the incidence angle at impact on the settling time are studied. The results from the 31 MC are shown in Figure 7.16:

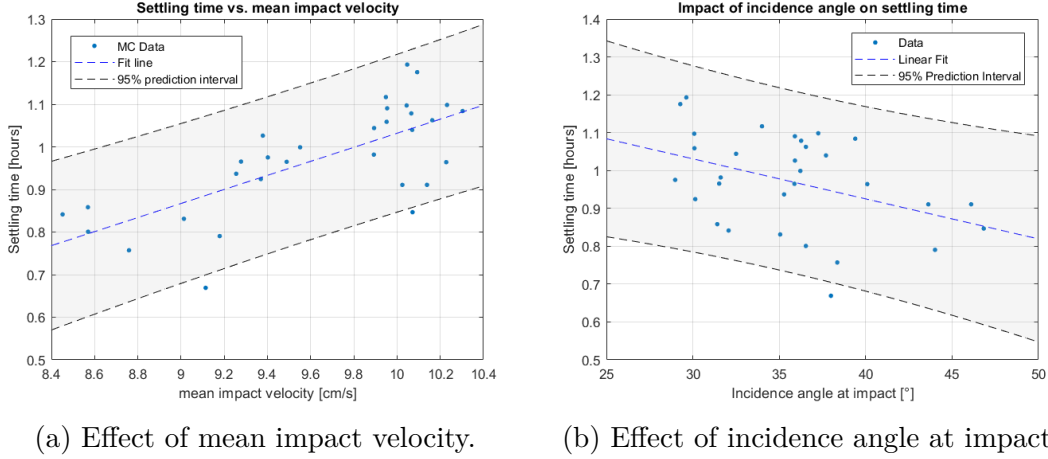


Figure 7.16: Parameters influencing the settling time.

When the velocity at which Juventas arrives at the surface of Didymoon increases, it is expected that the settling time of the latter also increases. Indeed, a more important velocity means more energy, which takes more time to dissipate. This is confirmed by the results shown on Figure 7.16a. From the results, it is expected that Juventas will take 40 minutes to 70 minutes before coming to rest.

Next, an interesting observation from Figure 7.16b is that if the incidence angle increases (i.e. the velocity is more and more tangent to the local surface), the settling time decreases.

As a reminder, the CoR normal to the surface e follows a uniform distribution on $[0.1, 0.8]$ and the CoR tangent to the surface f follows a uniform distribution on $[-0.5, 0.5]$. If Juventas arrives more and more tangentially to the surface, the tangent velocity component will be larger than the normal one. As f takes smaller values than e , the energy is going to be dissipated much faster in the tangent direction. Having a quasi-normal velocity to the surface is a disadvantage to a faster dissipation of the energy.

Also, when arriving tangentially to the surface, Juventas is expected to bounce less higher but farther, staying close to the surface and feeling more the gravitational pull of the asteroid.

For Juventas, reducing the settling time is important. Indeed, it is yet not known

how the spacecraft will behave on the surface of the asteroid and if it will have enough resources to be active for a long time. Reducing the settling time allows to start the science objectives of the mission faster. To do so, the mean impact velocity must be decreased and the incidence angle increased.

7.2.2.3 Number of bounces

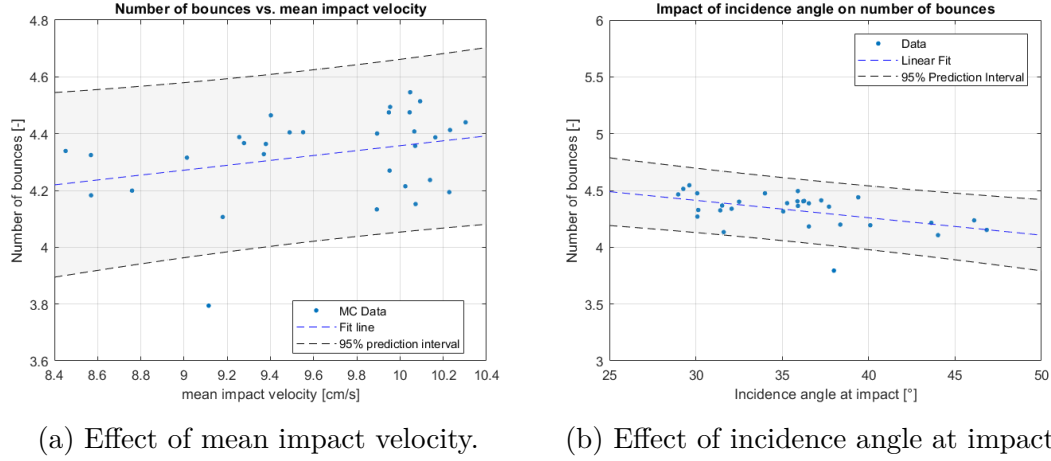


Figure 7.17: Parameters influencing the number of bounces.

The same analysis can be done for the number of bounces of Juventas. The number of bounces vs. the mean impact velocity and the incidence angle is shown respectively in Figure 7.17a and 7.17b. For increasing mean impact velocity, the number of bounces increases. However, the increase is very low: in the best cases, Juventas is expected to bounce less than 4 times (mean of 3.794 on the MC trajectory analyzed) and in the worse, more than 4 times (mean of 4.546 on the MC trajectory analyzed). Nevertheless, minimizing the number of bounces is important, as each bounce can cause damage to the structure of the spacecraft and thus compromise the mission. As before, an important impact velocity means that more energy has to be dissipated, thus leading to a more important mean number of bounces.

Figure 7.17b shows that the mean number of bounce decreases if the mean incident angle increases. This results is intrinsically linked to the previous observation: if the incidence angle decreases, the settling time increases. And a high settling time induces a higher number of bounces than in a low settling time.

7.2.3 Discussion

From the MC analysis, many trajectory design parameters have been identified as being crucial in the landing phase of the Juventas mission. First, when designing landing trajectories, the transfer duration to the moon should be minimized. Minimizing the latter has two main consequences:

- High landing success rate and a robust landing trajectory. It is primordial, since it is the most risky phase of the mission.
- In the case where a particular landing site is selected, for example the crater resulting from the DART impact, having a landing site dispersion concentrated around the nominal case is interesting for the mission since it increases the chances of impacting the targeted landing site.

Regarding the bouncing of Juventas, the velocity at impact should be minimized during the design of the mission. Minimizing the velocity at impact has the following consequences:

- Low percentage of trajectories that escape the gravitational attraction of Didymoon after bouncing. It is important to reduce the risk of escaping the asteroid. Indeed, arriving at the surface of Didymoon is necessary, but staying on it is crucial to achieve all the mission goals.
- Low settling time. It allows Juventas to start its surface measurements as fast as possible. A consequence of low settling time is a reduced number of bounces. Reducing the number of bounces as much as possible is important, in order to reduce the damage of Juventas when impacting the surface.

When designing the mission, attention should also be paid to the angle of arrival at the surface. A trade-off has to be found, so as to minimize the risk of escaping Didymoon but also to reduce the settling time.

Adjusting the three design parameters should allow Juventas to have an optimal trajectory, where the success rate is the highest and the settling time is the lowest. Thus ensuring a landing of the nanosatellite on Didymoon and a rapid start of the surface measurements.

A method for drastically reducing the impact velocity would be to apply a braking maneuver close to the surface of Didymoon. The norm and direction of the braking maneuver should be carefully calculated so as not to miss the landing on Didymoon. Such a method consists in studying a two DeltaV direct landing trajectory.

Finally, even if the results of the MC analysis allow to obtain main trends, one should remain critical when analyzing such results. Only 31 data points were

obtained and studied. A study containing more data points (thus more trajectories from the backward propagation) would allow to confirm the results.

7.3 Results

After studying the results from the MC analysis, a set of optimal trajectories is proposed for Juventas. This Master Thesis aims at proposing reliable direct landing trajectories, based on a single impulse strategy from the SSTO, ensuring Juventas a landing on Didymoon, as well as a low settling time, allowing the spacecraft to start the surface measurements as quickly as possible.

Criteria for choosing the best trajectories are now selected. From the MC analysis, the selection is based on a combination of a **high global success rate** and a **low settling time**. Also, no landing site is preferred over other ones. To help in the selection, two zones have been determined: the *optimal zone*, containing the best trajectories and the *contingency zone*, containing additional possible trajectories. The limits of both zones are presented in Table 7.5.

Zone	Global success rate [%]	Settling time [hours]
Contingency Zone	$75 < x < 80$	$1 > x > 0.9$
Optimal Zone	$x > 80$	$x < 0.9$

Table 7.5: Zone limits.

The criteria are based on a mission design choice. The contingency cases allow to have a set of trajectories that are still reasonably good, in case Juventas cannot use the optimal solutions, for instance if there is a problem on the systems on board of the nanosatellite. Based on the selection criteria, Figure 7.18 allows us to make the selection:

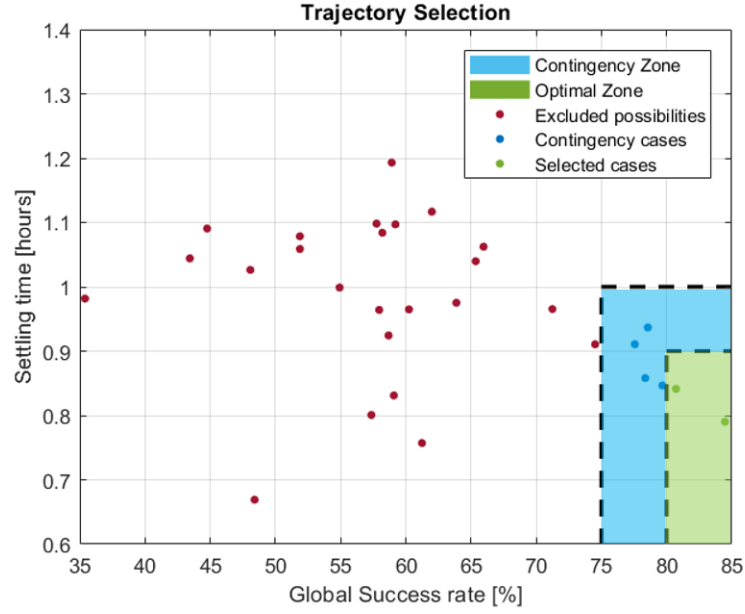


Figure 7.18: Possible trajectories selection.

In view of Figure 7.18, two trajectories are qualified as optimal. Out of the 31 possibilities, they correspond to trajectories #7 and #17. Their main characteristics are summarized in Table 7.6. The contingency trajectories are trajectories #5, #8, #18, #19. Their characteristics are summarized in *Appendix H*.

Trajectory	Landing Date	Global Success Rate [%]	Mean Transfer time [hours]	Mean Settling time [hours]	Mean Distance Covered [m]	Mean Number of bounces [-]
#7	09 AUG 2027 14h30	80.720	3.132	0.841	18.459	4.338
#17	10 AUG 2027 01h30	84.480	3.258	0.790	22.443	4.106

Table 7.6: Proposed trajectories for Juventus.

Figure 7.19 depicts the two trajectories. Both trajectories are very different, in the way that at the landing epoch, Didymoon is much closer to the SSTO plane and departure point in the case of trajectory #17, thus allowing Juventus to cover less distance. However, both trajectories have a roughly equal mean transfer time.

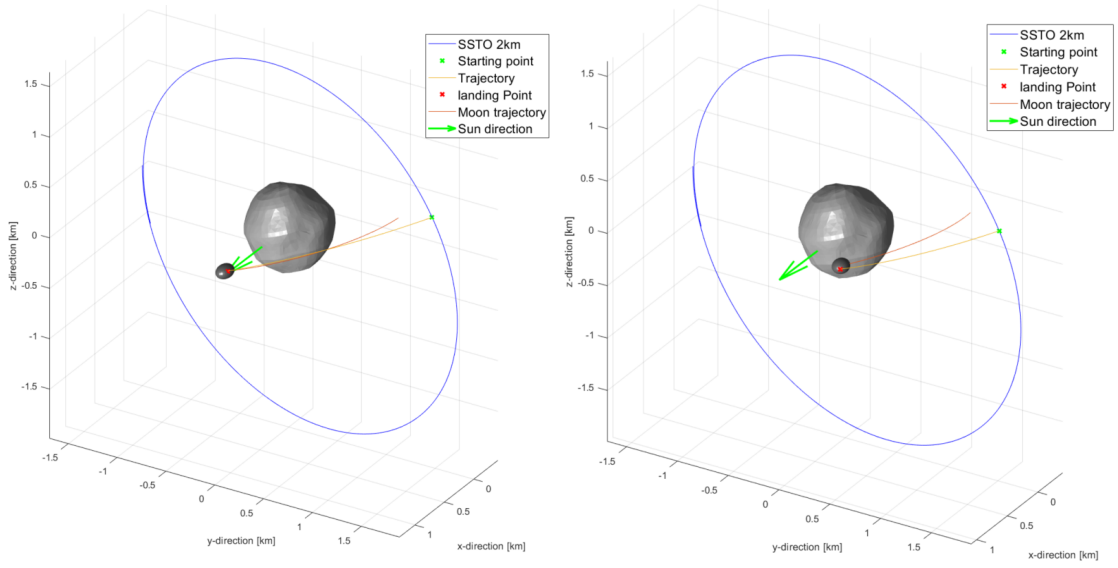


Figure 7.19: Simulated Didymoon approaches. Trajectory #7 (left) and trajectory #17 (right).

A closer look is made at the obtained information from the MC simulations for the descent and the bouncing of the two selected trajectories. It allows to study the expected behavior of Juventas.

7.3.1 Descent to Didymoon

In this section, the distribution of the transfer time and impact velocity are discussed.

Transfer time

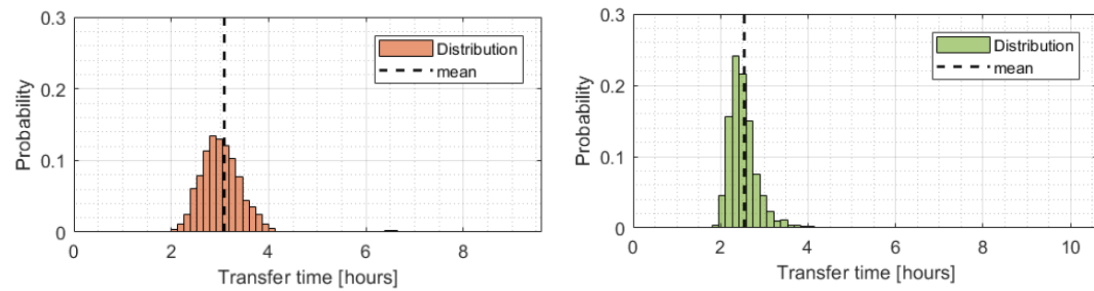


Figure 7.20: Simulated transfer time. Trajectory #7 (left) and trajectory #17 (right).

From Figure 7.20, the MC analysis yielded to mean transfer times of 3.132 hours and 3.258 hours respectively for trajectory #7 and #17. This result must be compared to the nominal case: 2.90 hours for trajectory #7 and 2.32 hours for trajectory #17.

A zoom on Figure 7.21 for large transfer times allows to observe that even though the probability is very low, some trajectories can have transfer times of up to 10 hours in the worst case. Due to the accumulation of errors along the trajectory, such trajectories go in orbit around Didymoon and/or Didymain before impacting the moon of the binary system. Although counted as successful cases, such trajectories should be rejected, thus leading to a slightly lower global success rate. These trajectories also explain the difference that exists between the nominal case and the mean from the Monte Carlo.

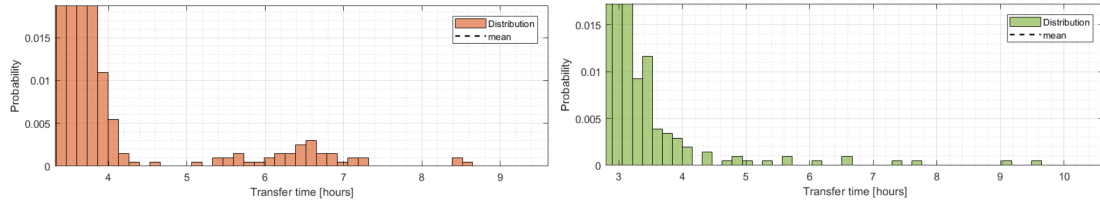


Figure 7.21: Simulated transfer time. Trajectory #7 (left) and trajectory #17 (right).

Impact velocity

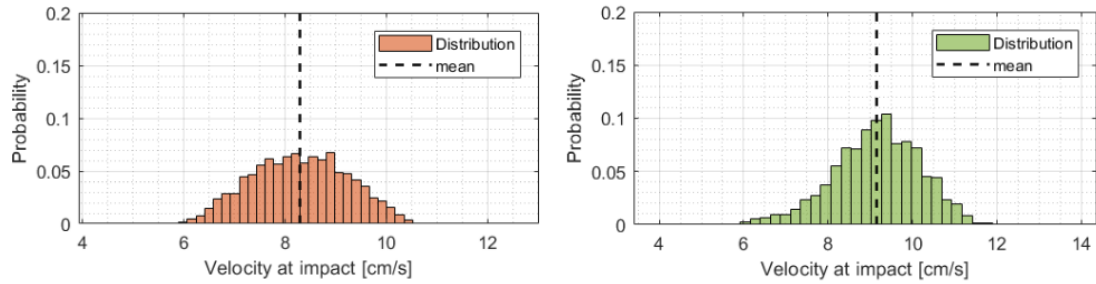


Figure 7.22: Simulated impact velocity. Trajectory #7 (left) and trajectory #17 (right).

Figure 7.22 shows the velocity of Juventas with respect to Didymoon at the moment of impact. Trajectory #7 was designed to have a nominal impact velocity of 8 cm/s, while 9 cm/s for trajectory #17. The MC results yield to mean velocities of

8.292 cm/s and 9.152 cm/s, respectively. An important information obtained is the probability of having an impact velocity larger than 10 cm/s. For trajectory #7, there is 3.022 % chance while there is 21.354 % chance for trajectory #17. Having a trajectory with smaller impact velocities is more interesting. The latter means less kinetic energy has to be dissipated, leading to less risks of having Juventas out of use.

7.3.2 Bouncing on Didymoon

In this section, the distribution of the settling time, distance covered on Didymoon and number of bounces are discussed.

Settling time

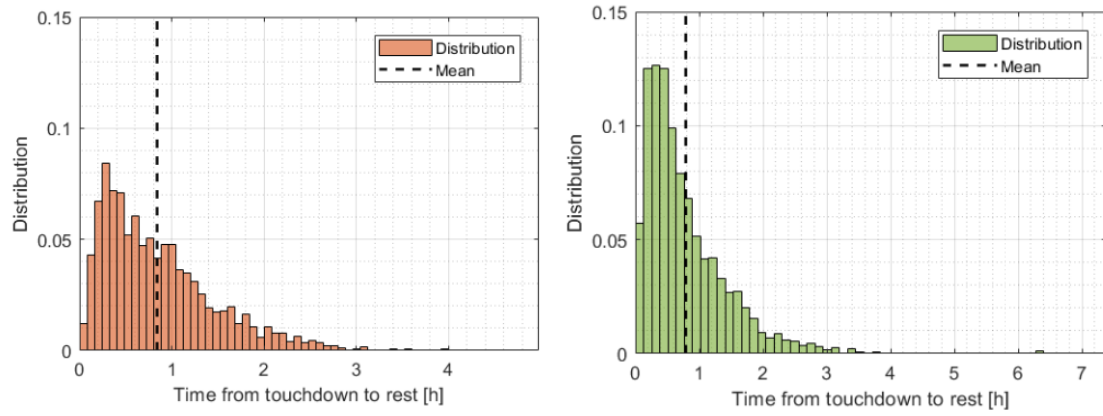


Figure 7.23: Simulated time from touchdown to rest. Trajectory #7 (left) and trajectory #17 (right).

From Figure 7.23, the simulations yielded to a mean value of less than 54 minutes for the total time from touchdown to rest (as chosen in the selection criteria). In both cases, the probability of having a settling time lower than the mentioned value is important. However, in the worst case, the bouncing can go up to almost 7 hours. Such an important settling time corresponds to cases where the CubeSat leaves the surface of Didymoon, orbits around it and/or Didymain and then comes back on Didymoon. Figure 7.24 shows two examples of such trajectories.

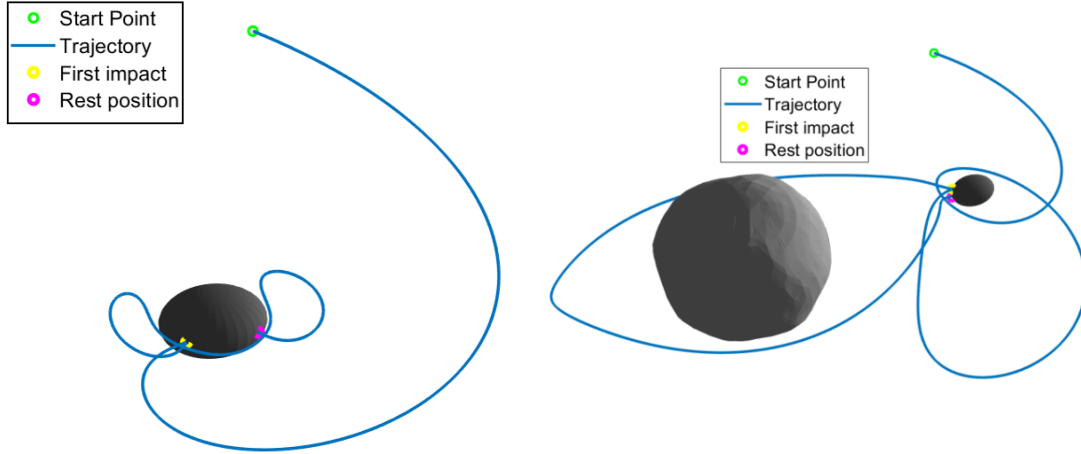


Figure 7.24: Example of trajectories that orbit around Didymoon (left) and Didymain (right) before rest, in the synodic frame.

Distance covered on Didymoon from the touchdown location

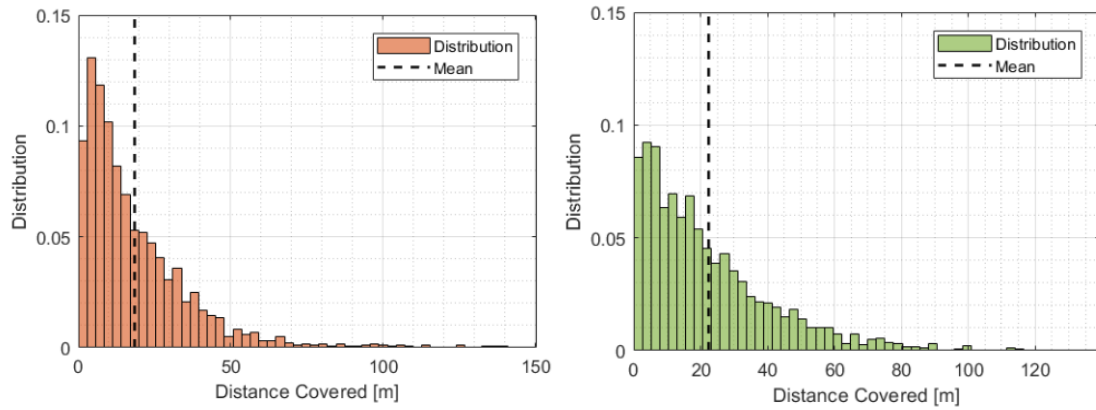


Figure 7.25: Simulated distance covered. Trajectory #7 (left) and trajectory #17 (right).

As shown by the location of the vertical dashed line on Figure 7.25, it has been determined that the CubeSat is expected to cover a mean distance of 18.459 meters and 22.443 meters from the touchdown location, respectively for trajectories #7 and #17. From the Figure, it can also be seen that the CubeSat can travel further away, up to 145 meters for trajectory #7 and 118 meters for trajectory #17. Knowing that the semi-major axis of Didymoon measures 103 meters and semi-

minor axis measures 66 meters, these distances covered after touchdown correspond to approximately one fourth of the circumference of Didymoon⁵.

Number of bounces

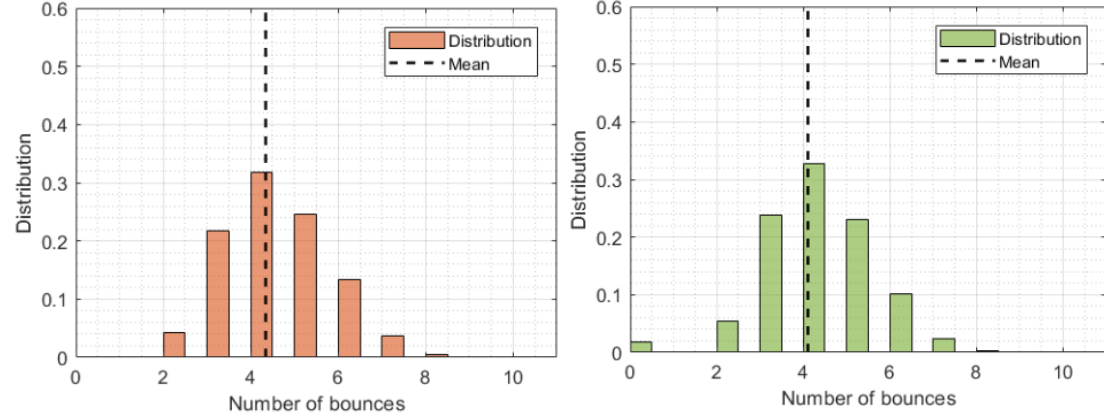


Figure 7.26: Simulated number of bounces. Trajectory #7 (left) and trajectory #17 (right).

From Figure 7.26, Juventas will bounce around 4 times on average. A maximum of 8 bounces can be expected for the CubeSat before it comes to rest and in the best cases, 3 or even 2 bounces can be observed. A particular case with 0 bounces can be seen on Figure 7.26 for trajectory #17. The probability of having such a behavior is very low, less than 2%. Such cases happen when enough energy has been dissipated at the impact of Juventas on Didymoon, therefore leading to a bounce with a height of less than 3 cm and a relative velocity of less than 1 cm/s with respect to the asteroid after the impact (from *Section 6.1.2*, stop criteria for the bouncing).

In conclusion, in this key section of the Master Thesis, two landing trajectories were selected for Juventas. Its behavior was studied by analyzing and comparing the Monte Carlo results of both trajectories. This comparison allowed to acquire a better understanding of the expected behavior of Juventas during the landing phase.

⁵The maximum circumference is $2 * \pi * \alpha = (2) (\pi) (103) \sim 650$ meters. $\frac{145}{650} \sim 1/5$. The minimum circumference is $2 * \pi * \gamma = (2) (\pi) (66) \sim 450$ meters. $\frac{145}{450} \sim 1/3$

Chapter 8

Sensitivity to Didymos parameters

As presented in *Chapter 6*, due to a lack of Earth based observations of Didymos, the parameters determining the system are known with a certain degree of uncertainty. *Chapter 7* aimed at proposing a framework allowing to determine direct landing trajectories to Didymoon, as well as presenting the influence of the uncertainties related to the burns on the descent and the bouncing on Didymoon. In these studies, the nominal parameters of Didymos were considered. This chapter is about the sensitivity of a landing trajectory to two crucial parameters of Didymos: the total mass of the system and the distance between the CoM of both asteroids. The uncertainties related to those parameters are reminded in Table 8.1:

Parameter	Value	Unit	Reference
Total mass	$5.278 \pm 0.54 \cdot 10^{11}$	kg	GMV
Distance between the center of Main and Moon	$1.18 + 0.04 / - 0.02$	km	[27]

Table 8.1: Uncertainties related to Didymos system.

The selected trajectory is the same as the one used to validate the propagator. The initial conditions in the synodic frame are:

x [m]	y [m]	z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]
1369.810	1336.772	-18.969	0.098	-0.126	-0.07

8.1 Total mass of Didymos

Here, the total mass m_{tot} of Didymos is varied and all other parameters have constant nominal values. A Monte Carlo simulation of 2500 samples was performed for five different cases. Remark: varying the mass has no impact on the position of the center of mass of the system. In each case, the success rate obtained for

the trajectory is computed. Table 8.2 summarizes the obtained values. Figure 8.1 shows the effect of varying the mass of the system on the success rate.

Total mass $\times 10^{11} [kg]$	4.738	4.873	5.008	5.143	5.278	5.413	5.548	5.683	5.818
Success rate [%]	35.72	51.36	63.68	66.28	62.52	54.00	42.20	29.68	19.28

Table 8.2: Results from the MC analysis when varying m_{tot} .

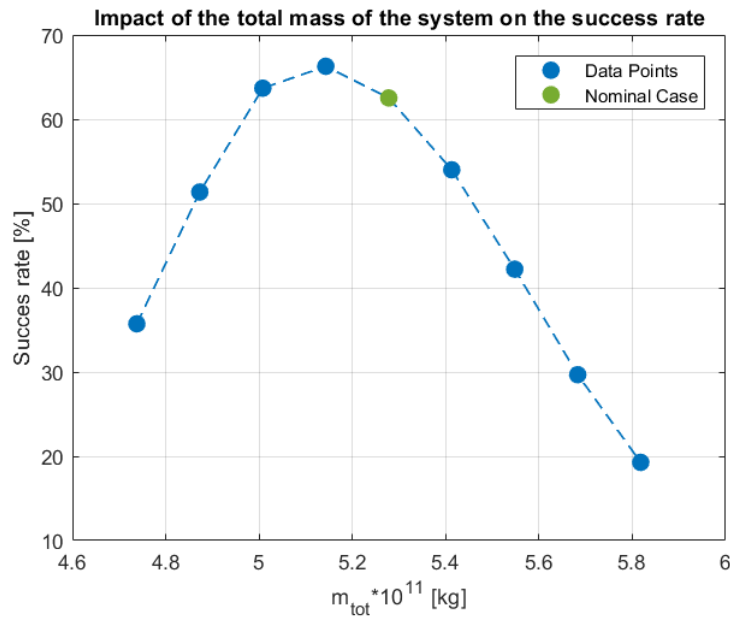


Figure 8.1: Impact of varying m_{tot} on the success rate.

Varying the mass of Didymos yielded to interesting results. First, when the mass of the system is varied, the success rate of the landing trajectory can drop to values as low as 19.28%. The success rate peaks for values of the mass between $5 \times 10^{11} [kg]$ and $5.3 \times 10^{11} [kg]$ and stays at success rates above 60%. Also, varying the total mass of the system of 10% around the optimal case lead to a loss of approximately 30% of success rate.

The optimal success rate is found for $m_{tot} = 5.143e11 [kg]$. The maximum success rate achievable for this landing trajectory is 66.28%, 3.78% more than the nominal case. For landing trajectories, there might therefore exist a combination of Didymos

parameters that lead to higher success rates than the one obtained in the nominal case.

Figure 8.2 shows the trajectory of Juventas in the nominal case as well as for the extreme ones. For the trajectories that do not land, propagation is stopped after 8 hours.

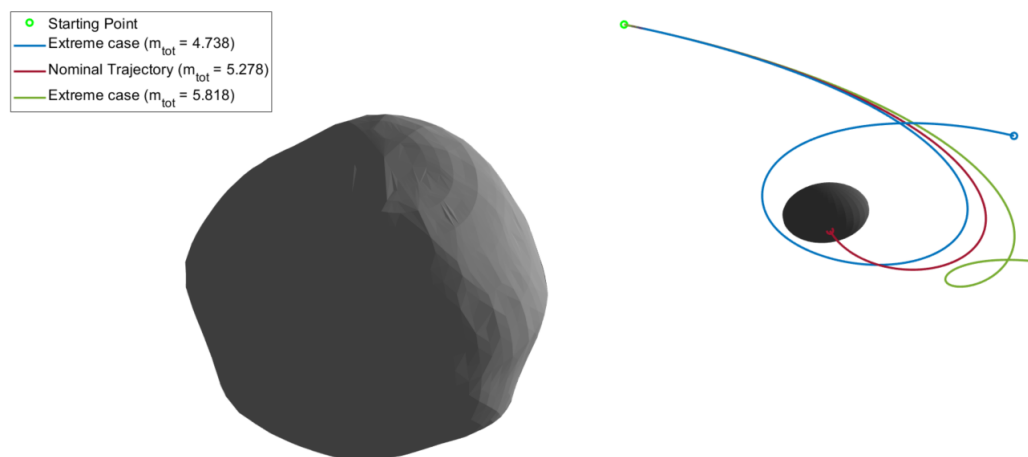


Figure 8.2: Trajectories with varying m_{tot} .

As shown on Figure 8.2, varying the total mass of the system has a huge impact on the landing trajectory. In the case of the boundary values, the trajectories fail to land on Didymoon. Figure 8.3 helps understanding this result. The gravitational force exerted by a body on Juventas can be calculated using Newton's gravitational formula:

$$F_{i-j} = G \frac{m_i m_j}{\xi_i^2} \quad (8.1)$$

where F_{i-j} is the force exerted by body i on body j , ξ_i is the distance between the nanosatellite and body i , m is the mass of Juventas, m_i is the mass of body i and G is the gravitational constant. Here $i = \{1; 2; 3\}$ corresponds to $\{\text{Main}, \text{Moon}, \text{Juventas}\}$. The force exerted by body i on Juventas is in the direction that links the two bodies. The mass of Didymoon represents only 0.9% of the mass of the system of asteroids.

- If m_{tot} is decreased, the mass of the moon also decreases, leading to a lower force F_2 applied on Juventas. F_1 decreases, but remains much more important than F_2 . Juventas will therefore strongly feel the gravitational pull of Didymain and get closer to it (blue trajectory on Figure 8.2).

- If m_{tot} is increased, the mass of the moon increases, leading to a larger force F_2 on Juventas. F_1 also increases, but the direction of the forces is such that Juventas is more likely to drift away (green trajectory on Figure 8.2).

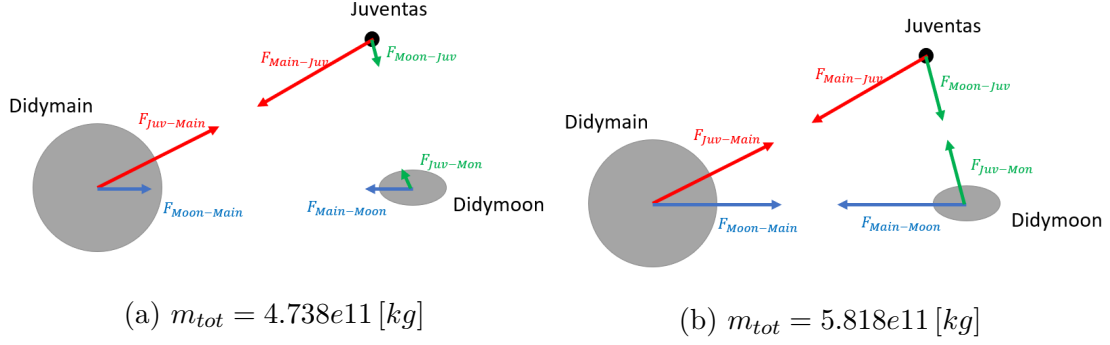


Figure 8.3: Impact of varying m_{tot} on the force exerted by the asteroids on Juventas. The vectors are not to scale.

8.2 Distance between Didymain and Didymoon

Here, the distance r_{12} between the center of mass (CoM) of Didymain and Didymoon is varied and all other parameters of the system have nominal values. A Monte Carlo simulation of 2500 samples was performed for five different cases. In each case, the success rate obtained for the trajectory is computed. Table 8.3 summarizes the obtained values. Figure 8.4 shows the effect of varying the distance between the asteroids on the success rate.

CoM distance [m]	1160	1165	1170	1175	1180	1190	1200	1210	1220
Success Rate [%]	13.28	24.52	38.48	51.88	62.52	68.00	56.00	44.88	38.48

Table 8.3: Results from the MC analysis when varying r_{12} .

From Figure 8.4, it is obvious that the distance between the asteroids has an important impact on the success rate of the mission.

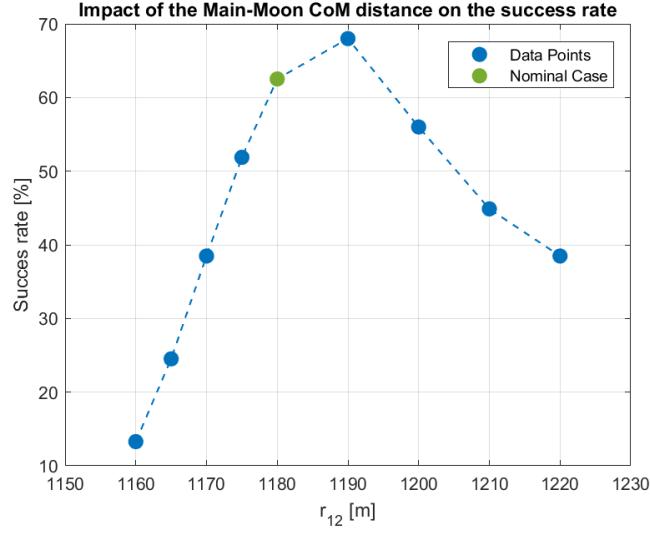


Figure 8.4: Impact of varying r_{12} on the success rate.

As the distance is varied, the success rate drops and can reach values as low as 13.28%. Such a low success rate value is not acceptable, since it means that the likelihood of failing the landing is very high. The optimal success rate is for $r_{12} = 1190$ [m]. Figure 8.5 shows the trajectory of Juventas in the nominal case as well as for the boundary ones. For the trajectories that do not land, propagation is stopped after 8 hours.

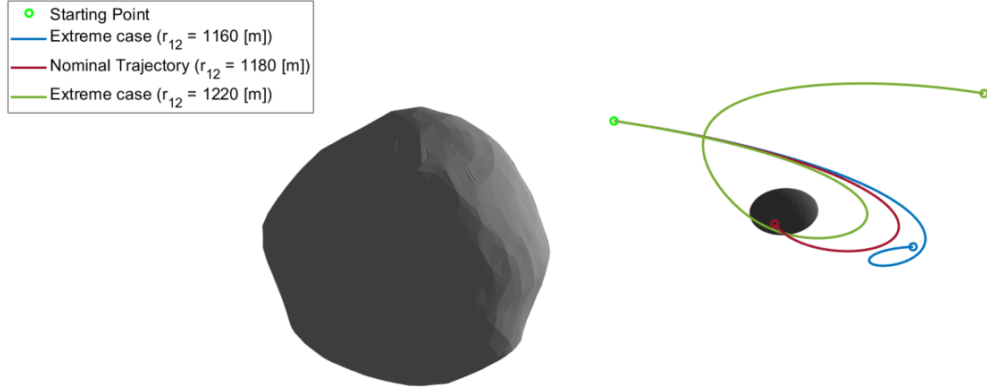


Figure 8.5: Trajectories with varying r_{12} .

As shown on Figure 8.5, varying the distance between the asteroids has a huge impact even on the nominal trajectory. Here, in the two limit cases, both trajectories

do not land on Didymoon. The force exerted by Didymoon on Juventas, expressed in terms of r_{12} is:

$$F_{moon-juv} = G \frac{m_{moon} m_{juv}}{\sqrt{(r_x - (1 - \mu)r_{12})^2 + r_y^2 + r_z^2}} \quad (8.2)$$

where only the r_{12} distance varies and the initial position of Juventas in the synodic frame remains unchanged (i.e. r_x , r_y and r_z unchanged). μ is the mass parameter of the system.

- When $r_{12} = 1160$ [m], Didymoon is closer to Didymain and Juventas (its position with respect to the barycenter is unchanged) is farther from Didymoon. In equation (8.2), r_{12} decreases and the denominator increases. The force applied on Juventas is thus smaller than in the nominal case and the nanosatellite drifts away (blue trajectory on Figure 8.5).
- When $r_{12} = 1220$ [m], Didymoon is farther to Didymain. In this case, Juventas flies closer to Didymoon. In equation (8.2), r_{12} increases and the denominator decreases. The force applied by Didymoon on Juventas is bigger than in the nominal case. Juventas feels more the gravitational pull of this asteroid (green trajectory on Figure 8.5).

8.3 Discussions

As it can be seen, the Didymos parameters have a very important impact on the success of the landing of Juventas. Prior to arrival of Juventas, the LICIA CubeSat and Earth observations will refine the knowledge of the dynamical environment of Didymos. The exact determination of the parameters is crucial and further studies should focus on developing systems that would allow to know them more accurately.

Moreover, as the success rate of the landing trajectory highly depends on the two analyzed parameters of the system, the results are extrapolated to be the same for the two selected trajectories of Juventas in *Chapter 7*. The trajectories obtained are therefore considered to be only reliable in the case of the nominal Didymos system as their success rate is expected to drop when varying the parameters of the system.

A suggestion would be to perform additional studies on landing trajectories, where more uncertainties related to Didymos are taken into account. It would allow to obtain landing trajectories that are also robust for a range of different Didymos systems.

Chapter 9

Conclusions

This last chapter concludes this Master Thesis and some suggestions are given for future work.

As a reminder, the Royal Observatory of Belgium is focused on studying the approach of Juventas to Didymoon and its behavior on the surface of the asteroid. This Master Thesis provides them with a numerical tool allowing the modeling of the behavior of a nanosatellite in the vicinity of Didymos, a first landing and bouncing analysis and also a set of reliable landing trajectories.

Numerical tools

The Circular Restricted Three Body problem was implemented in MATLAB®, allowing the simulation of the motion of a nanosatellite in Didymos. Orbital perturbations were added in order to have a model that truly represents the dynamical environment of the binary system of asteroids. The latter are the gravity perturbations, caused by the irregular shape of the asteroids and the solar radiation pressure. The model was then validated, by comparing a landing trajectory with MONTE and GMV.

The bouncing of Juventas on Didymoon was also modeled, by considering the CubeSat as a point-mass body. Two models for the topography of Didymoon were also implemented: a smooth model and a fractal Brownian motion model.

Finally, a Monte Carlo algorithm was developed, allowing to perform error propagation analysis where uncertainties related to Didymos and Juventas have been taken into account. The CMA-ES optimization tool was also implemented in the code.

Error propagation

A backward propagation allowed to obtain 31 landing trajectories for Juventas. A Monte Carlo analysis was then performed on those trajectories, where the uncertainties related to the thrusters were taken into account and the smooth model for Didymoon was used. The application of the Monte Carlo algorithm led to some very important results concerning the landing and the bouncing. The key mission design parameters and their influence on the trajectories have also been identified.

Regarding the landing trajectory, the conclusion is that the transfer duration to Didymoon should be minimized in order to obtain:

- A high success rate: it is crucial, as it confirms the reliability of the solution;
- A landing site dispersion concentrated around the nominal touchdown location.

About the bouncing, the results have shown that the velocity at impact should be minimized, as it allows to:

- Reduce the chances of escaping from the attraction of Didymoon after the first bounce;
- Obtain a low settling time, important to start the surface measurements as quickly as possible;
- Reduce the number of bounces, therefore limiting the potential damage that could be caused to Juventas.

Concerning the incidence angle of the direction of the velocity at arrival, a trade-off must be found:

- The more tangentially Juventas impacts the surface, the higher the risk is of escaping from Didymoon;
- The more normally Juventas impacts the surface, the more the settling time increases.

Selected trajectories and bouncing behavior

Out of the 31 trajectories and based on the conclusions of the error propagation study, two trajectories were identified as being robust. They are proposed as solutions for the landing of Juventas on Didymoon. The two trajectories allowed to study the behavior of Juventas on Didymoon. The following is expected for Juventas:

- A transfer time to Didymoon of about 3 hours;
- An impact velocity of approximately 8 cm/s;
- A mean settling time of less than 54 minutes;
- A mean distance covered on Didymoon from the touchdown location of about 20 meters, though distances of up to 140 meters can also be expected;
- Around 4 bounces of Juventas and up to 9 in the worst case scenario.

Sensitivity to Didymos parameters

A sensitivity analysis of two parameters of Didymos was performed on a landing trajectory. It was shown that a strong dependency exists between the success rate and the parameters of the system of asteroids, which led to the following conclusions:

- Prior to the arrival of Juventas, Earth observations and the DART mission should better characterize the asteroids, allowing a more accurate knowledge of the system;
- The two final selected trajectories are highly sensitive to the parameters of Didymos and can be considered as reliable only in the case of nominal values of Didymos.

Suggested future work

The analysis and the researches can be extended in several directions. Different suggestions are proposed:

- *Advanced surface interaction.* In this Thesis, point-mass body bouncing was considered for Juventas. The interaction of the nanosatellite and Didymoon can be improved in many ways. A more complex shape considering a spherical or rectangular paralepipiped mass distribution for Juventas would be more representative of the actual 6U CubeSat. The addition of rocks on the surface of Didymoon could also be implemented.
- *Advanced studies.* Further studies could perform a backward propagation with a more refined grid cell on Didymoon, consider more velocities and more landing epochs, in order to obtain more landing possibilities for Juventas. With additional data, a Monte Carlo analysis could verify that the obtained trends are correct. Another study could also include uncertainties related to Didymos. It would allow to obtain landings that are reliable for a range of parameters of Didymos.

- *Advanced Landing strategy.* Studies of a two DeltaV strategy where a braking maneuver is performed above the surface would lead to new promising results. It would indeed allow to drastically lower the impact velocity.
- *Attitude dynamics.* Juventas is equipped with scientific payload such as visible cameras and multiple radar antennas. On the SSTO and during the landing, Juventas has to be oriented in a way allowing measurements to be performed. In the work, Juventas was considered as a point-mass body during the landing. A suggestion is to study the attitude dynamics of Juventas.
- *Trajectory optimization.* One of the main conclusions of this Thesis is that minimizing the descent duration increases the success rate. Using the CMA-ES to minimize the transfer time could also be considered.

Appendix A

Note on Phase Angles

In the Didymos system, three phase angles can be calculated. Their calculation is performed in order to check that the illumination criteria are respected during the landing.

The first two angles can be seen in Figure A.1:

1. α_1 : Angle formed between the Sun, Didymoon and Juventas.
2. α_2 : Angle formed between the Sun, Didymain and Juventas.

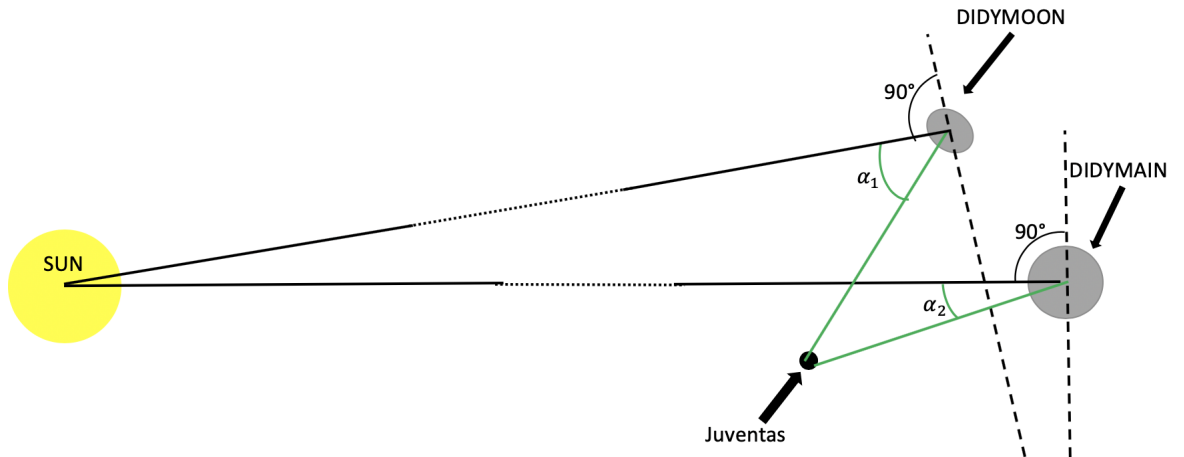


Figure A.1: Phase angles of Juventas with respect to the two asteroids.

These two angles are limited to maximum 90° , thus ensuring that the asteroids are always visible by Juventas.

Another phase angle, α_3 , must not be neglected: the angle formed by the Sun, Didymain and Didymoon. It highlights the case of an eclipse. It is limited to $[-160.83; 160.83]$ (see Figure A.2):

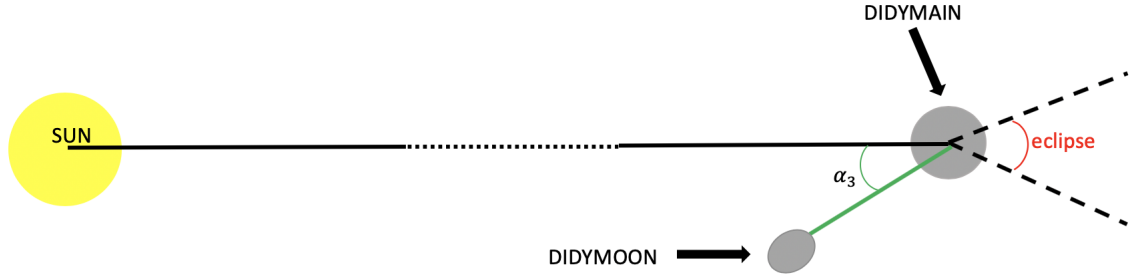


Figure A.2: Phase angle of Didymoon.

The dashed lines represent the boundaries of the eclipse zone. Figure A.3 presents the evolution of the phase angles during a landing trajectory of Juventas. As it can be observed, all the phase angles and thus illumination conditions are respected. At the moment of landing, all the phase angles converge to the same value. Indeed, as Juventas is on Didymoon, Juventas is very close to the CoM of the asteroid. It implies that all the phase angles converge to the same value.

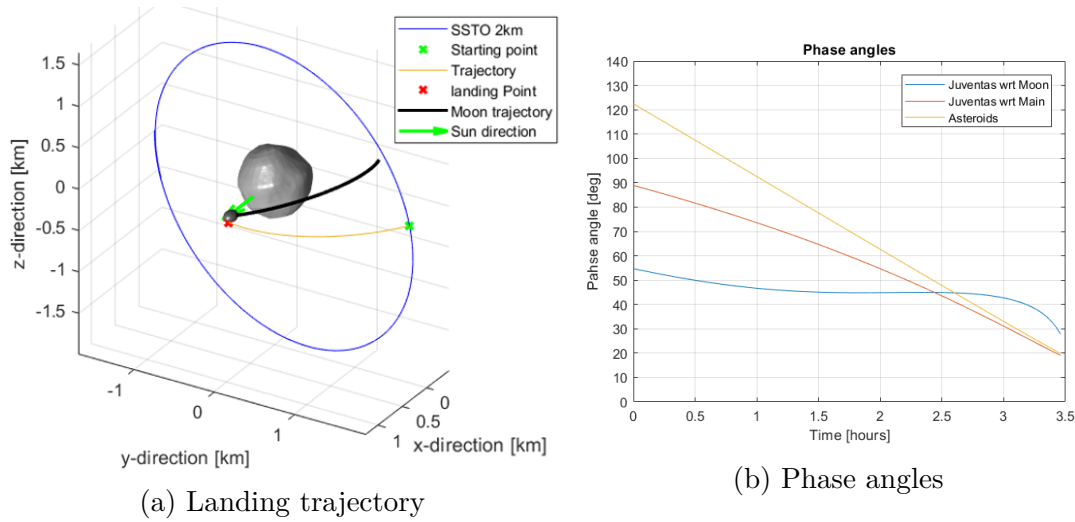


Figure A.3: Landing approach on 10 AUG 2027 01h15.

Appendix B

Reference frames & frames transformation

In the Master Thesis, a total of three reference frames are defined. The synodic frame $\mathcal{F}$, specific to the CR3BP, the inertial frame $\mathcal{I}$, coincident with $\mathcal{F}$ at $t = 0$ and the Equatorial J2000 frame.

The position of various celestial bodies such as Didymoon or the Sun can be retrieved from the SPICE TOOLKIT and the SPICE Kernels in the Equatorial J2000 Frame. Also, the equations of the CR3BP are solved in the synodic frame. Finally, it can be more adequate in some cases to show trajectories in either the Equatorial J2000 frame or inertial frame, instead of the synodic one. It is thus necessary to establish rotation matrix to perform frame changes in the computations. These frame changes can be represented as follows:

$$\text{J200 frame} \xrightarrow{A} \text{Inertial frame} \xrightarrow{B} \text{Synodic frame}$$

Where A and B are the transfer matrix allowing the frame change. The method to compute each matrix is presented here below.

Matrix A: From Equatorial J2000 to the inertial frame

The transfer matrix is obtained by using the ephemerides of Didymoon in the equatorial J2000 frame. The procedure is the following, where $\mathbf{r}_{moon}$ and $\mathbf{v}_{moon}$ are respectively the position and the velocity of Didymoon centered on the barycenter:

$$\mathbf{col1} = \frac{\mathbf{r}_{moon}}{\|\mathbf{r}_{moon}\|} \quad \mathbf{col3} = \frac{\mathbf{r}_{moon} \times \mathbf{v}_{moon}}{\|\mathbf{r}_{moon} \times \mathbf{v}_{moon}\|} \quad \mathbf{col2} = \frac{\mathbf{col1} \times \mathbf{col3}}{\|\mathbf{col1} \times \mathbf{col3}\|} \quad (\text{B.1})$$

$$A = \begin{pmatrix} \text{col1}_x & \text{col2}_x & \text{col3}_x \\ \text{col1}_y & \text{col2}_y & \text{col3}_y \\ \text{col1}_z & \text{col2}_z & \text{col3}_z \end{pmatrix} \quad (\text{B.2})$$

Matrix B

The transfer matrix from the inertial frame $\mathcal{I}$ to the synodic frame $\mathcal{F}$ is simply the rotation matrix about the $\hat{e}_z$ axis:

$$B = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{B.3})$$

where ω is the rotating frame angular velocity. Therefore, we can apply the following transformation to go from one frame to another:

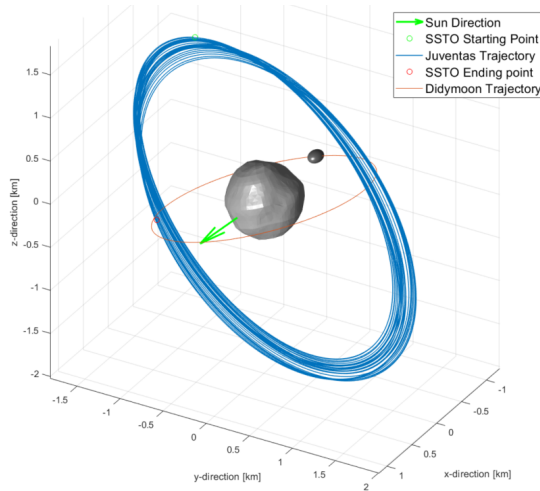
$$\mathbf{r}_{\text{rot}} = BA^T \mathbf{r}_{\text{J200}} \quad \mathbf{v}_{\text{rot}} = A^T \mathbf{v}_{\text{J200}} - \omega \times \mathbf{r}_{\text{rot}} \quad (\text{B.4})$$

Appendix C

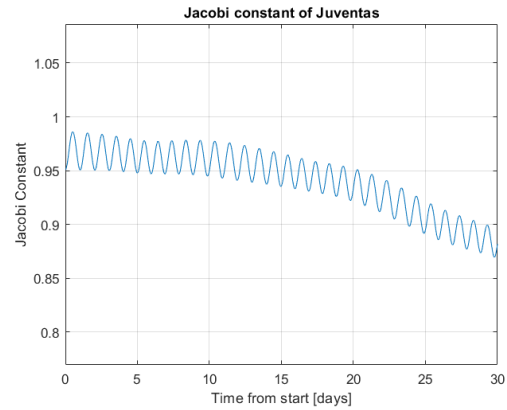
Energetic analysis of the impact of SRP's on the SSTO2000

The impact of SRP's on the Jacobi constant is different than the gravitational perturbations, since it is a non-conservative perturbation. Strictly speaking, it is not an energetic verification but just an analysis of the impact on the Jacobi Constant.

The SRP is a non-conservative perturbation, having an impact on the Jacobi constant as it can be sen on Figure C.1b. An important characteristics is to be highlighted: the overall Jacobi constant value decreases over time and smaller fluctuations also appear on a smaller time scale. The overall decrease in the value of the relative energy impacts the position and velocity of Juventas along the SSTO.



(a) SSTO 2000 Trajectory.



(b) Jacobi Constant evolution.

Figure C.1: Propagated SSTO 2000.

Indeed, as noted in [3], orbital perturbations affect the orbital elements at different time scales and each perturbation can have several characteristic frequencies, linked to physical properties, as it can be seen on Figure C.2. The different time scales are known as the complete, the long period and the secular (or short-period) evolution.

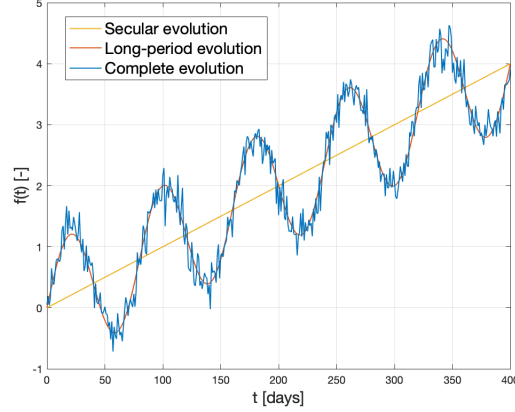


Figure C.2: Representation of the different timescales.

On Figure C.1b, the secular evolution and long-period evolution can be easily distinguished. The secular evolution has a period equal to the period of the heliocentric orbit of Didymos system, which lasts 770.294 days - approximately 2.11 years. Indeed, the magnitude of the SRP depends on the distance between the sun and the system and, as this orbit is an ellipse (eccentricity equal to 0.383 [-], see Table 2.2), the distance and so the magnitude evolve as a function of time. Therefore, the decrease of the Jacobi constant is intrinsically linked to the heliocentric period of Didymos system. The long-period evolution corresponds to the period of Juventas on the SSTO, which period lasts 1.07 [days].

Appendix D

Legendre polynomials

The simplest orthogonal polynomials that can be found are the Legendre Polynomials. They are orthogonal on the interval $[-1; 1]$:

$$\int_{-1}^1 P_m(x) P_n(x) dx = 0 \quad \text{for } m \neq n$$

They can be evaluated using the formula, where n is the order of the polynomial:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad (\text{D.1})$$

The Legendre polynomials of order 1 to 4 are shown in Figure D.1.

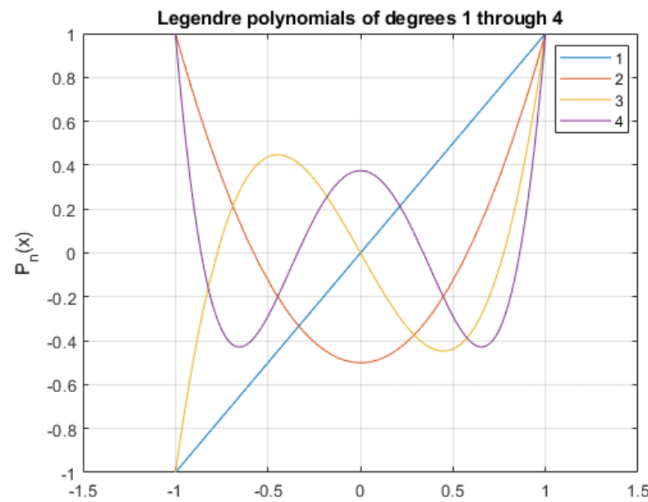


Figure D.1: Legendre Polynomials.

Appendix E

Adams-Bashforth-Moulton Method

The Adams-Bashforth-Moulton method has the form:

$$U_{i+1} = U_i + \int_{X_i}^{X_{i+1}} f(x, u(x)) dx \quad (\text{E.1})$$

The basic idea is to obtain an approximation of the integral of f on the interval $[X_i; X_{i+1}]$ using the results $U_{i-n}, U_{i-n+1}, \dots, U_i$ previously obtained. An *implicit method*, here Adams-Moulton is obtained by integrating on $[X_i; X_{i+1}]$ the interpolation polynomial of order n passing through $(X_{i-n+1}, F_{i-n+1}), \dots, (X_{i+1}, F_{i+1})$. An *explicit method*, here Adams-Bashforth is obtained by integrating on $[X_i; X_{i+1}]$ the interpolation polynomial of order n passing through $(X_{i-n}, F_{i-n}), \dots, (X_i, F_i)$. The methods are depicted in Figure E.1.

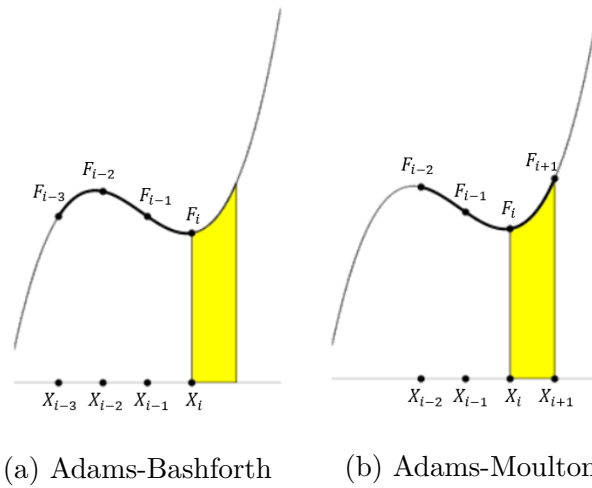


Figure E.1: Adams-Bashforth-Moulton of order 4, from [9]

ODE113 uses the predictor-corrector principle: an *explicit method*, here Adams-Bashforth is used to obtain a predicted value P_{i+1} at the point $i+1$ of the integration, which will be corrected using an *implicit method*, here Adams-Moulton to obtain an approximated of the solution U_{i+1} of the equation. A new iteration could be started, by taking U_{i+1} as the new prediction P_{i+1} and correct it using the implicit method to obtain a new approximation. Nevertheless, such a strategy is not efficient, and using a variable step method where only one correction is performed at each time stop is more interesting. [9]

Appendix F

Mollweide Projection

At first, the landing spots were presented on a "standard latitude and longitude grid":

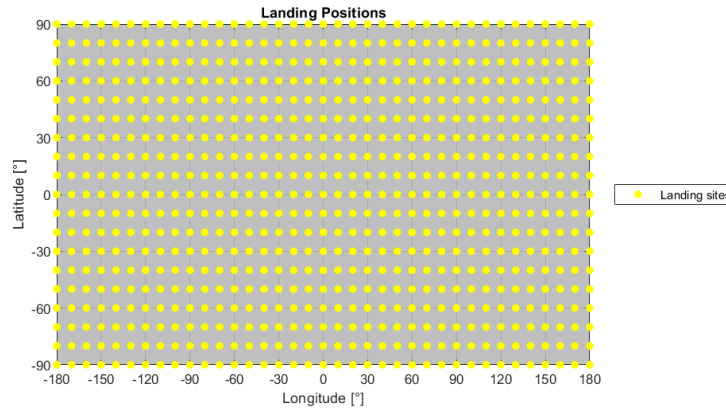


Figure F.1: Standard latitude-longitude representation.

This representation is quite poor, especially at the poles. On the poles, only one position exists. Indeed, in the first representation the poles contained a position at each longitude, which is not correct. Therefore, an adaptation of this map was interesting.

The type of projection that is then used is a Mollweide projection. It is an equal area, pseudo cylindrical projection, meaning that the map area proportions are conserved in the mapping. On the map, the equator of the body is represented as a straight line from west to east, perpendicular to the central meridian. While the parallels to the equator compress near the poles, the meridians at 90° form a perfect circle and the meridians at 180° form an ellipsoid whose semi-axes have a 2:1 ratio, [43].

The projection takes as inputs the latitude and the longitude of a point on the

map and gives as output the map coordinates (x,y) of the Mollweide projection. Figure F.2 shows the new map projection.

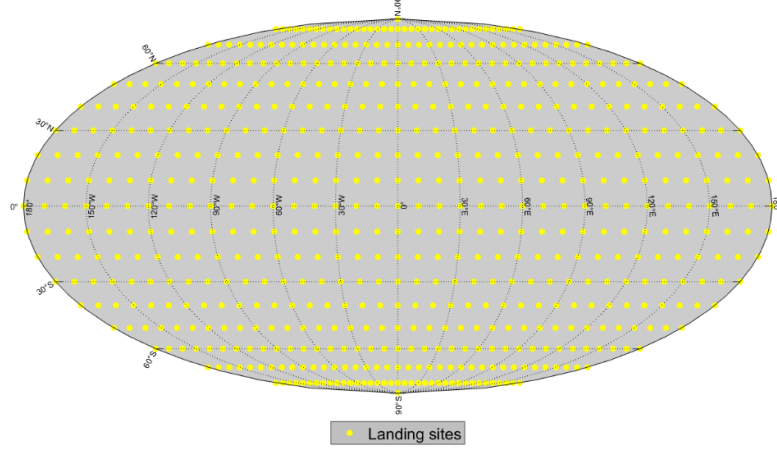


Figure F.2: Landing sites on the Mollweide projection.

The equations allowing to do the mapping are:

$$\begin{cases} x = R \frac{2\sqrt{2}}{\pi} (\lambda - \lambda_0) \cos \theta & (F.1a) \\ y = R \sqrt{2} \sin \theta & (F.1b) \\ 2\theta + \sin 2\theta = \pi \sin \phi & (F.1c) \end{cases}$$

Where λ is the latitude, ϕ is the longitude, R is the radius of the planet and θ is an auxiliary angle defined by the third equation. This last equation must be solved iterative, using for example a Newton-Raphson method:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

In this case, the third equation is solved as:

$$\theta_{k+1} = \theta_k - \frac{2\theta + \sin(2\theta) - \pi \sin(\phi)}{2 + 2\cos(2\phi)}$$

Appendix G

Backward Propagation Results

Trajectory	Landing Date	Impact Velocity [m/s]	Duration [$hours$]	Didymoon Coordinates [$Lat; Long$] [$^{\circ}$]	Minimum distance from SSTO [m]	DeltaV [m/s]
1	09 AUG 2027 15h15	0.08	4.535	[30;-10]	0.337	0.1929
2	09 AUG 2027 15h15	0.1	4.577	[50;-10]	0.813	0.1907
3	09 AUG 2027 15h15	0.1	3.973	[-60;-50]	2.433	0.1725
4	09 AUG 2027 15h15	0.1	4.024	[-60;-40]	2.524	0.1723
5	10 AUG 2027 01h15	0.1	1.959	[0;0]	1.228	0.1911
6	10 AUG 2027 02h15	0.1	3.458	[30;0]	1.206	0.1780
7	09 AUG 2027 14h30	0.08	2.900	[-10;-50]	0.541	0.1933
8	09 AUG 2027 14h30	0.09	2.737	[10;-70]	1.674	0.1986
9	09 AUG 2027 14h30	0.1	3.009	[40;-50]	3.091	0.1939
10	09 AUG 2027 14h30	0.1	2.739	[-40;-60]	0.471	0.1989
11	09 AUG 2027 15h30	0.08	5.776	[10;10]	2.730	0.1825
12	09 AUG 2027 15h30	0.09	4.316	[-40;-80]	3.764	0.1689

Table G.1: Possible landing trajectories

Trajectory	Landing Date	Impact Velocity [m/s]	Duration [hours]	Didymoon Coordinates [Lat; Long] [°]	Minimum distance from SSTO [m]	DeltaV [m/s]
13	09 AUG 2027 15h30	0.09	4.561	[-50;-30]	3.849	0.1682
14	09 AUG 2027 15h30	0.1	4.701	[60;-40]	1.538	0.1945
15	09 AUG 2027 15h30	0.1	5.036	[-40;10]	3.716	0.1660
16	09 AUG 2027 15h30	0.1	4.536	[-60;-20]	0.698	0.1658
17	10 AUG 2027 01h30	0.09	2.32	[0;0]	2.563	0.1878
18	10 AUG 2027 01h30	0.1	2.737	[0;10]	3.556	0.1751
19	09 AUG 2027 14h45	0.08	3.295	[-20;-50]	3.514	0.1863
20	09 AUG 2027 14h45	0.09	3.386	[-40;-30]	2.788	0.1846
21	10 AUG 2027 01h45	0.1	2.751	[20;0]	2.493	0.1748
22	10 AUG 2027 01h45	0.1	3.195	[10;10]	2.806	0.1672
23	09 AUG 2027 15h00	0.08	4.956	[0;10]	0.559	0.1778
24	09 AUG 2027 15h00	0.09	3.815	[40;-50]	0.418	0.1927
25	09 AUG 2027 15h00	0.09	4.076	[40;-20]	3.057	0.1907
26	09 AUG 2027 15h00	0.1	4.078	[50;-20]	1.067	0.1897
27	09 AUG 2027 16h00	0.08	5.348	[10;-80]	3.477	0.1840
28	09 AUG 2027 16h00	0.09	5.167	[30;-100]	2.552	0.1908
29	10 AUG 2027 02h00	0.1	2.547	[30;-30]	3.017	0.1932
30	10 AUG 2027 02h00	0.1	3.189	[-20;10]	3.185	0.1889
31	10 AUG 2027 02h15	0.1	3.458	[30;0]	1.206	0.1780

Table G.2: Possible landing trajectories

Appendix H

Monte Carlo Analysis Results

Trajectory	Landing Date	Success Rate [%]	Mean Duration [hours]	Impact velocity > 10 [cm/s] [%]	Illumination condition respected [%]	Mean Incidence angle [deg]
1	09 AUG 2027 15h15	57.960	5.237	5.788	83.403	36.541
2	09 AUG 2027 15h15	46.600	4.505	70.777	66.041	35.904
3	09 AUG 2027 15h15	60.840	4.092	57.230	54.582	29.624
4	09 AUG 2027 15h15	60.720	4.217	57.770	56.148	30.081
5	10 AUG 2027 01h15	88.800	2.170	56.369	86.333	46.104
6	10 AUG 2027 02h15	65.200	3.587	67.079	90.309	39.396
7	09 AUG 2027 14h30	81.000	3.132	3.022	70.019	32.071
8	09 AUG 2027 14h30	79.120	2.873	17.260	56.466	35.281
9	09 AUG 2027 14h30	64.080	3.105	59.354	67.935	34.002
10	09 AUG 2027 14h30	78.560	2.852	58.258	54.195	29.274
11	09 AUG 2027 15h30	49.440	9.093	3.057	80.578	37.992
12	09 AUG 2027 15h30	64.720	4.597	11.020	53.600	28.979

Table H.1: MC results for Landing, trajectory 1 to 12

Trajectory	Landing Date	Success Rate [%]	Mean Duration [hours]	Impact velocity > 10 [cm/s] [%]	Illumination condition respected [%]	Mean Incidence angle [deg]
13	09 AUG 2027 15h30	59.160	5.427	11.451	61.077	30.141
14	09 AUG 2027 15h30	44.600	5.506	66.482	55.064	32.504
15	09 AUG 2027 15h30	38.240	7.763	64.067	62.372	31.611
16	09 AUG 2027 15h30	53.640	5.955	59.6762	55.5898	30.090
17	10 AUG 2027 01h30	91.840	3.258	21.354	86.268	44.013
18	10 AUG 2027 01h30	89.200	3.361	54.769	86.646	46.831
19	09 AUG 2027 14h45	78.560	3.488	2.705	71.107	31.414
20	09 AUG 2027 14h45	71.960	4.145	18.753	66.198	31.524
21	10 AUG 2027 01h45	74.760	3.008	56.976	94.186	37.705
22	10 AUG 2027 01h45	84.800	3.490	49.812	91.733	43.625
23	09 AUG 2027 15h00	62.560	5.497	1.698	86.348	38.360
24	09 AUG 2027 15h00	60.720	3.817	17.197	65.338	35.879
25	09 AUG 2027 15h00	56.000	4.139	22.214	75.455	36.218
26	09 AUG 2027 15h00	53.680	4.110	70.007	67.000	36.267
27	09 AUG 2027 16h00	59.760	4.907	2.234	72.579	35.054
28	09 AUG 2027 16h00	49.560	5.040	4.159	48.336	35.908
29	10 AUG 2027 02h00	70.320	2.833	61.676	79.461	36.5419
30	10 AUG 2027 02h00	63.000	5.067	68.323	75.155	40.096
31	10 AUG 2027 02h15	64.640	3.671	64.819	90.581	37.257

Table H.2: MC results for Landing, trajectory 13 to 31

Trajectory	Landing Date	Escaped Trajectories [%]	Mean Settling time [hours]	Mean Distance from touch down [m]	Mean number bounces [–]	Global Success Rate [%]
1	09 AUG 2027 15h15	1.035	0.800	25.275	4.182	57.360
2	09 AUG 2027 15h15	3.948	1.090	28.566	4.494	44.760
3	09 AUG 2027 15h15	3.155	1.193	25.075	4.545	58.920
4	09 AUG 2027 15h15	2.503	1.097	23.327	4.475	59.200
5	10 AUG 2027 01h15	12.657	0.910	28.148	4.236	77.560
6	10 AUG 2027 02h15	10.736	1.084	28.147	4.439	58.200
7	09 AUG 2027 14h30	0.345	0.841	18.459	4.338	80.720
8	09 AUG 2027 14h30	0.707	0.936	25.565	4.388	78.560
9	09 AUG 2027 14h30	3.245	1.116	27.269	4.474	62.000
10	09 AUG 2027 14h30	3.513	1.175	25.260	4.514	75.800
11	09 AUG 2027 15h30	2.103	0.669	17.997	3.794	48.400
12	09 AUG 2027 15h30	1.297	0.975	21.707	4.4646	63.880

Table H.3: MC results for Bouncing, trajectory 1 to 12

Trajectory	Landing Date	Escaped Trajectories [%]	Mean Settling time [hours]	Mean Distance from touch down [m]	Mean number bounces [–]	Global Success Rate [%]
13	09 AUG 2027 15h30	0.811	0.924	19.000	4.327	58.680
14	09 AUG 2027 15h30	2.600	1.044	26.525	4.400	43.440
15	09 AUG 2027 15h30	7.426	0.891	23.159	4.133	35.400
16	09 AUG 2027 15h30	3.281	1.058	23.515	4.269	51.880
17	10 AUG 2027 01h30	8.013	0.790	22.443	4.106	84.480
18	10 AUG 2027 01h30	10.672	0.846	27.754	4.151	79.680
19	09 AUG 2027 14h45	0.254	0.858	17.232	4.324	78.360
20	09 AUG 2027 14h45	1.000	0.965	20.137	4.366	71.240
21	10 AUG 2027 01h45	12.573	1.039	26.546	4.356	65.360
22	10 AUG 2027 01h45	12.122	0.910	27.429	4.214	74.520
23	09 AUG 2027 15h00	2.110	0.757	18.290	4.199	61.240
24	09 AUG 2027 15h00	0.790	0.964	22.200	4.404	60.240
25	09 AUG 2027 15h00	1.928	0.998	23.519	4.405	54.920
26	09 AUG 2027 15h00	3.353	1.078	27.260	4.407	51.880
27	09 AUG 2027 16h00	1.137	0.831	21.087	4.315	59.080
28	09 AUG 2027 16h00	2.986	1.026	25.014	4.363	48.080
29	10 AUG 2027 02h00	5.284	1.087	26.599	4.440	65.960
30	10 AUG 2027 02h00	8.000	0.964	28.791	4.193	57.960
31	10 AUG 2027 02h15	10.643	1.098	28.339	4.412	57.760

Table H.4: MC results for Bouncing, trajectory 13 to 31

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