Louvain School of Management

Leveraged Long ETFs Tracking France's CAC 40 index: Quantitative Analysis of Rebalancing, Price Deviations and Tracking Errors

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Année académique 2018-2019

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Leveraged ETFs Are A Loser's Game

-US News and World Report (April 18th 2018),

The Father Of ETFs Warns About The Danger Of Leveraged Funds

-Bloomberg (April 29th 2018),

The Evils Of Leveraged ETFs And Those That Suggested Them

-Gold Seek (May 2nd 2018).

These are a few of the first titles one can come across when googling Leveraged Exchange Traded Funds[1] in the news tab. While Exchange Traded Funds [2] have been widely documented since mutual funds, their derivative counterparts, the misunderstood LETFs, are being feared and considered dangerous, as shown by these headlines of financial journals. Are these LETFs truly a loser's game though? Are they as dangerous and as hazardous as the media portrays them?

This thesis will try to answer these questions with an in-depth analysis and explanation of the inner mechanisms of LETFs. It is a comprehensive report describing these funds and analysing the mechanisms behind the functioning of the positively leveraged ETFs.

The report will begin by describing what these LETFs are, and how they work from a theoretical point of view. It aims at summarizing all the important mechanisms of this special type of funds, as well as reminding what traditional ETFs are.

[1] Thereafter referred to as 'LETFs'

[2] Thereafter referred to as 'ETFs'

Afterwards, the literature review will be presented, explaining briefly each previous study on LETFs and their conclusion.

Finally, our own quantitative analysis will be described. This analysis uses data from Bloomberg on various LETFs tracking a French index, the CAC 40. It is divided in three parts: firstly the analysis of the rebalancing mechanisms of our sample, then the work on the pricing efficiency of our LETFs, and lastly the long term returns and tracking errors linked to our sample.

Chapter 1

Leveraged Exchange-Traded-Funds

The purpose of this chapter is to reassess the framework and give more insights on what LETFs are. As a relatively new financial product, it is still quite unknown of the public. In order to shed light on the topic, this chapter will therefore establish a summary of the theoretical framework on ETFs and more specifically on LETFs, and describe the scope of analysis of this paper.

Starting with a concise definition of ETFs and LETFs, we will then take a look at the evolution and growing importance of ETFs and LETFs, from their introduction to the present day. Afterwards, we will move on to the assessment of the specific characteristics of LETFs, which make them so incredibly complex, and which justify the writing of this thesis. We will then investigate the risks of LETFs and how they differ from Inverse ETFs. We will conclude this chapter by listing the shortcomings and advantages of this type of funds.

1.1 Definition of Exchange Traded Funds

The definition of ETFs given by [Chen, 2019] is the following "an exchange-traded fund (ETF) is an investment fund that tracks an index, specific asset or basket of assets – including stocks, bonds, currencies, real estate and commodities". ETFs can be described as a pooled portfolio, a group of pooled assets sold as shares that track the yield and performances of these underlying assets. It has the advantage of offering diversification in a single investment. In this respect, ETFs are quite similar to other kind of pooled investments, and inherit a lot of characteristics from open-end mutual funds and close-end investment funds.

On the one hand, they are similar to the former in their open-end structure, the structure can expand or shrink by issuing new shares or selling existing ones following the demand on the market. However, instead of shares being directly sold by the unit to retail investors, blocks of thousands of shares are created for ETFs, *creation units*, and sold to large deal-brokers, *authorized participants*, who in turn sell them individually to investors on the exchange markets. This is illustrated by Figure 1.1 coming from the Investment Company Factbook [Ried et al., 2017]

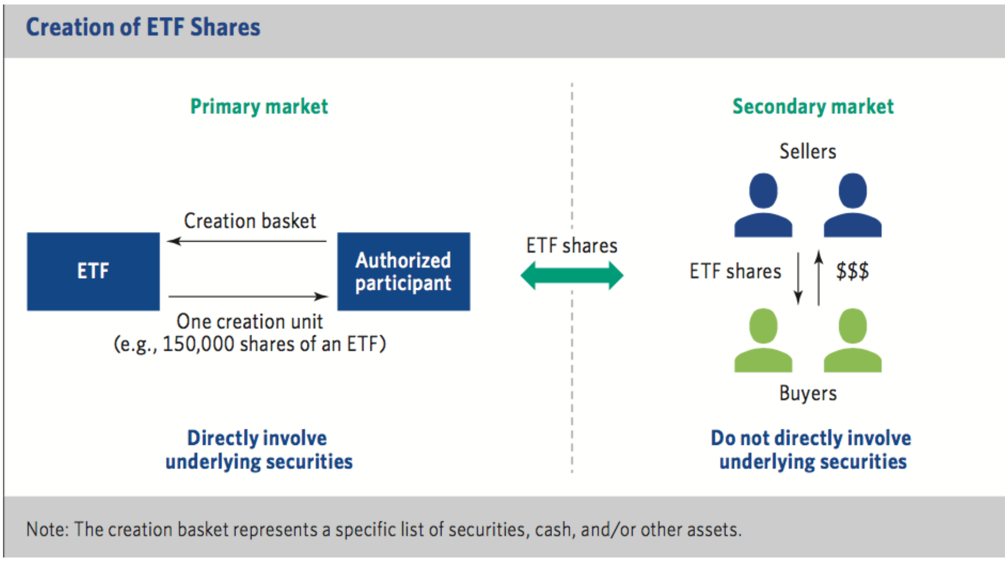

Figure 1.1: Creation of ETFs shares [Ried et al., 2017]

The role of the authorized participant is very important in ETFs trading. They can request the issuance of new creation units by providing the necessary amount of assets and receiving the equivalent in ETFs shares. Moreover, they can request the destruction of ETFs units and receive the amount in assets securities in exchange. In that transaction, the ETF receives (or gives up) the assets at the Net Asset Value, while the authorized participant sells (or buys) the ETFs shares at their market price, which can differ from the NAV. The authorized participant then needs to act on supply and demand to try and keep the ETFs price in line with the NAV.

On the other hand, ETFs inherit from close-end investments funds the possibility of being traded on the stock exchange between investors throughout the whole day at a price that can differ from its NAV (Net Asset Value). Unlike mutual funds who are only bought or sold once a day at the NAV via the manager of the fund. They are bought and sold on this secondary market between investors. This does not involve the trading of the underlying securities themselves. The secondary trading represents the great majority of ETFs trades, tradings between the ETFs themselves and authorized participants are considerably more scarce. ETFs are therefore a combination of the best of both worlds, being exchange-traded and having an open-end structure.

Furthermore, most ETFs are passively managed and are similar to index funds in that respect. In practical, that means that no fund manager tries to 'beat' the market by actively creating an optimal basket of assets that would yield the highest returns. Instead, the fund tracks the performance of an index (or commodity, or basket of assets), returning the same yields as that index. Even though some rare exceptions of actively managed ETFs exist, the majority is passively managed as it reduces the management costs considerably.

ETFs have gained in popularity through the years, they are now present in many sectors and in many forms. Some types are: ETFs of ETFs which are funds that tracks the performances of other Exchange-Traded-Funds, IPO ETFs, Inverse ETF, etc.. However, we will be focusing on LETFs in this paper. This specific kind of funds are ETFs that use leverage to enhance the returns on their underlying assets. In order to do so, debt and/or financial derivatives are used. The leverage mechanisms and their consequences will be addressed further in this paper.

1.2 Evolution and growing importance of (Leveraged) ETF

ETFs have been available in the USA for 25 years. They were first introduced in 1993 as a broad domestic equity fund tracking the S&P 500. This ETF was known as the famous SPDR (Standard Poor's depositary receipt) fund or "Spider" and it quickly became the biggest ETF in the world. Over they years, several other ETFs tracking the biggest indices were created, mostly in the US. To name a few, The "Dow Diamonds" tracking the Dow Jones index was created in 1998, the "cubes" tracking the NASDAQ-100 in 1999. Until 2008, ETFs were only referred to as index ETFs, because they were only legally authorized to track indices. But that year, regulations changed and the SEC (Securities of Exchange Commission) approved several funds that were actively managed, completely transparent, and meeting specific requirements. They had to publicly disclose the weightings, the composition of the assets and the components they held.

According to the graph below (1.2) coming from Statista [Szmigiera, 2019], the total amount of assets under management of ETFs amounted to almost 4.7 trillion US$ in investments worldwide in 2018. This represents an important sum. It can further be noticed that since 2008 the amount of underlying assets has more than quadrupled, and is expected to continue rising steadily in the next few years.

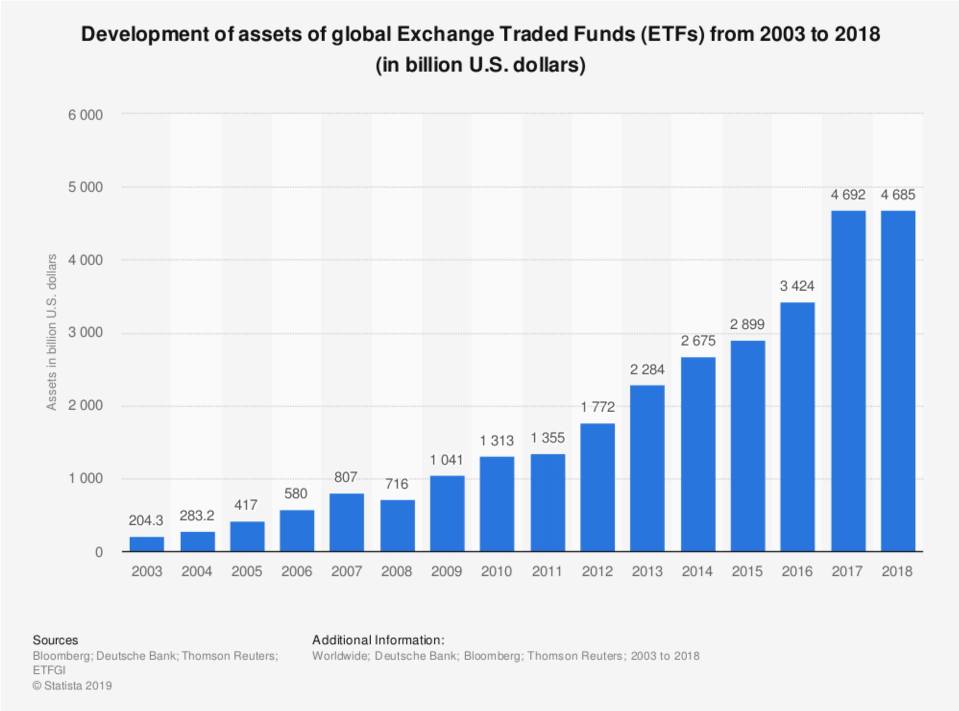

Figure 1.2: Evolution of ETFs AUM in US$ trillion [Szmigiera, 2019]

The proportion of ETFs is not equally distributed worldwide, the United States obviously hosts the biggest market for ETFs trading. American ETFs accounted for almost three quarters of the total amount of assets under management by ETFs in 2016, about 2.5 trillion US$. Europe's ETFs, on which this paper will set its focus, accounted for only 16% percent of the total ETF assets, a "mere" 560 billion US$ according to the Investment Institute Factbook.

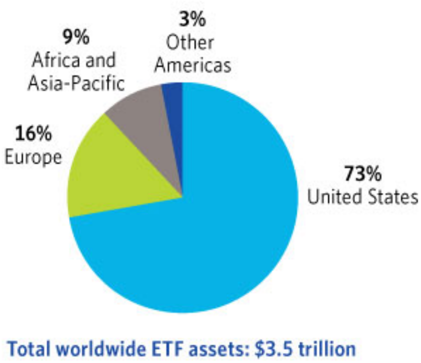

Figure 1.3: Worldwide Repartition of Exchange-Traded-Funds [Ried et al., 2017]

Leveraged ETFs appeared in the US some time after ETFs rose to the spotlight. They were first introduced in 2006 in the US, and most of the main LETFs were created in between 2006 and 2008. The biggest of them is the ProShare UltraPro (TQQQ) that offers a 3x daily long leveraged on the NASDAQ-100 index. It currently has over $3,620$ million USD in AuM.

1.3 Specific Characteristics of Leveraged ETFs

LETFs are Exchange-Traded-Funds that use debt or derivatives like future, swaps, and forwards agreement in order to increase and lever their returns. These funds are synthetic products based on derivatives, rather than holding the actual assets that compose the underlying index (or commodity, etc.). They can either be positively leveraged if they try to magnify the returns of the underlying index 2 or 3 folds (2x or 3x) or negatively leveraged (Inverse ETF) if they try to get the inverse return or an inverse multiple of the return of the underlying index. This paper will concentrate on positive LETFs.

Both mechanisms provide an interesting structure to the fund. Let us take a closer look at the case of positive LETFs, for each dollar of the LETFs net asset value, the managers of the fund "borrows" an additional dollar (or two) and invest both (or the three) dollars in a basket of stocks tracking the index. In order to keep the same leverage ratio everyday, LETFs need to rebalance, which means that they need to borrow more money or sell some of their assets daily.

In order to fully grasp the mechanism behind such structure, we will first discuss the functioning of the daily rebalancing system. Then, the path dependency and other consequences that it implies will be discussed. Further, we will examine the relationship between the LETFs evolution and the volatility of the underlying securities.

1.3.1 Rebalancing

Rebalancing acts as a key feature of LETFs. However, it can cause various problems and this financial instrument being already quite complex, it further complexifies its working. Rebalancing is necessary to keep the same leverage ratio daily overtime, which in turn is important in order to reach daily targets promised to investors. These rebalancings are often carried out through a derivative transaction by the manager with a counterparty, typically a swap agreement [Shum et al., 2015]. This would be similar to the manager borrowing more funds from the bank, or selling assets to this counterparty.

If the price of the underlying index goes up, then the manager needs to get more funds to reach the same leverage ratio and exposure the next day; and if the price goes down, the manager needs to give up some of the assets he bought with leverage in order to keep the exposure constant. In other words, LETFs would buy more securities when prices are high, and sell them when prices are low. Here is a numerical example of these mechanisms, coming from the work of [Shum et al., 2015]

Exposure:	2x					
Time	Index Price	Price Change	ETF NAV	ETF Desired Exposure	ETF Actual Exposure After Price Change	Rebalancing Amount
0	100	0	100	200		
1	110	10%	120	240	220	20
2	99	-10%	96	192	216	-24

Exposure:	3x					
Time	Index Price	Price Change	ETF NAV	ETF Desired Exposure	ETF Actual Exposure After Price Change	Rebalancing Amount
0	100	0	100	300		
1	110	10%	130	390	330	60
2	99	-10%	91	273	351	-78

Figure 1.4: Rebalancing Numerical Example [Shum et al., 2015]

An important consequence of this mechanism is that the returns of the LETFs are not equal to the returns one might expect, because of the compounding effects after a holding period longer than one day. Indeed it is not equal to the total progression of the funds times the leverage ratio. Let us take an example, suppose the index went +20% over the course of two days, it does not necessarily mean that a x2 LETF tracking will have a 40% return, it is also dependent on the path this index took to get to +20%. According to [Trainor and Baryla, 2008], these LETFs are "A risky double that doesn't multiply by two". A further section will cover what is called the path-dependency of LETFs returns.

Before diving into that section devoted to the explanation of the path-dependency and the volatility decay, the work of [Cheng and Madhavan, 2009] is worth mentioning. This work established the framework and model that was widely used in the literature. The detailed derivation of their model on the rebalancing mechanism can be found in the appendix A. The equation of the model is as follow:

$$\Delta_{tn+1} = L_{tn+1} - E_{tn+1} = A_{tn}(x^2 - x)r_{tn,tn+1}$$

With Δ_{tn+1} being the amount of exposure that the LETFs management needs to adapt and rebalance at the time tn+1,
L_{tn+1} being the notional amount of capital needed before the market opening the following day to keep the leverage ratio constant,
E_{tn+1} being the exposure of the total return swap on time n+1,
A_{tn} being the LETF's net asset value on time n,
x being the leveraged factor,
And $r_{tn,tn+1}$ being the returns of the underlying index from time n to n+1.

This equation is very useful as it allows to calculate the rebalancing needs of LETFs with publicly available information. We will see later that this equation also has many other implications, e.g. the path-dependency of LETFs returns.

1.3.2 Path-dependency and Volatility Decay

In order to explain these two concepts clearly, we will start with an example. Suppose two indices achieve the same growth of +30% over a period of 3 days, but they took different paths to achieve such growth. The first one has grown at a constant rate of +10% each day while the other one is more volatile going from +40% to -20% to +10%. Because of the rebalancing mechanism, an LETF tracking the first index would considerably yield better returns than an LETF tracking the volatile index. This principle is called the path-dependency of LETFs' returns.

Theoretically, we could intuitively infer from such an example that the more volatile the index, the lower the returns of LETFs. Higher volatility means more rebalancing in order to keep the leverage ratio constant, and since the rebalancing mechanism implies selling when the price is low and buying when the price is high, higher volatility on longer term would mean smaller returns. This phenomenon appears clearly on the following plots 1.5, representing the natural decay of a x2 and a x3 LETF, from Dvega/Dtime (2013) [Roche, 2013], a financial web blog:

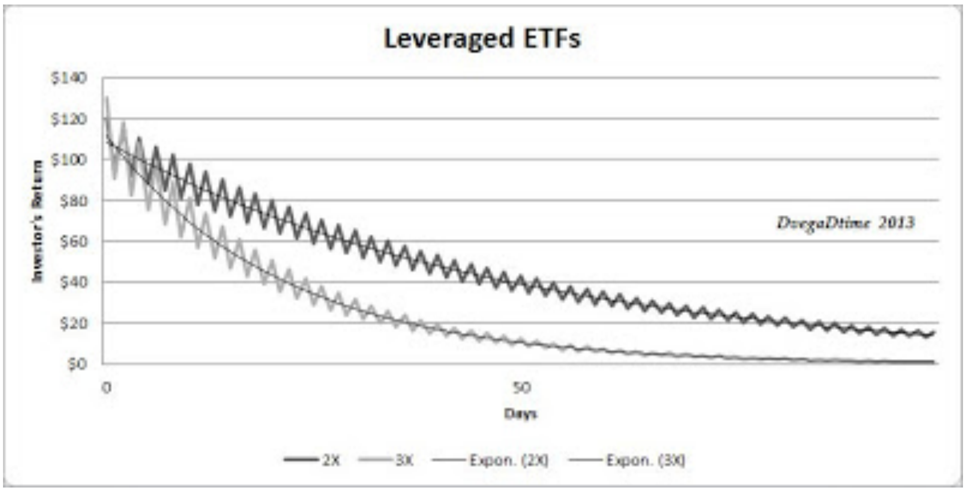

Figure 1.5: 2x and 3x Leveraged ETF returns decay (10% volatility) [Roche, 2013]

This theoretical example clearly shows that the returns tend towards 0 on the long term, and that this effect is reinforced when the leverage is increased. The path of the index is not plotted on the graph but it is assumed to grow and recess by the same amount day after day with a 10% level of volatility.

Now if we double the volatility level of the index the graph becomes as follows:

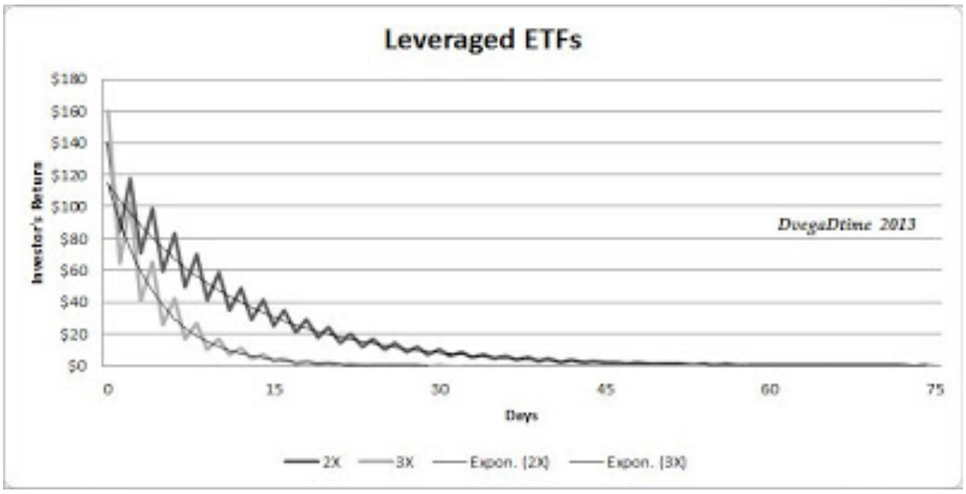

Figure 1.6: 2x and 3x Leveraged ETF returns decay (20% volatility) [Roche, 2013]

It can be noticed that the decay of wealth for investors is theoretically decreasing towards 0 at an even faster pace than on the previous example.

This path-dependency feature of LETFs has been widely documented and empirically verified in the literature, among others [Cheng and Madhavan, 2009], [Avellaneda and Zhang, 2010], [Charupat and Miu, 2011],[Carver, 2009]. Most studies claim that LETFs do not represent a long-term investment tool, other papers refer to this phenomenon as "the volatility decay" of returns [Guo and Leung, 2015], [Leung and Santoli, 2012], [Abdou, 2017] due to its sensitivity to the volatility of the underlying index. Moreover, most of them acknowledge that this financial vehicle is not a good tool for buy-and-hold investors but rather a short term instrument meant to be exchanged intraday and held for maximum a day or two.

However, if some specific conditions are met, i.e. a low level of volatility of the underlying index and a clear growth trend, LETFs could outperform other investment alternatives, even in the long run [Trainor and Carroll, 2013]. The figure (1.7) coming from the work of [Trainor and Carroll, 2013] shows "the decay for a 3x leveraged fund under the assumption of a 2% non-annualized monthly return and annualized standard deviations ranging from 10% to 30%."

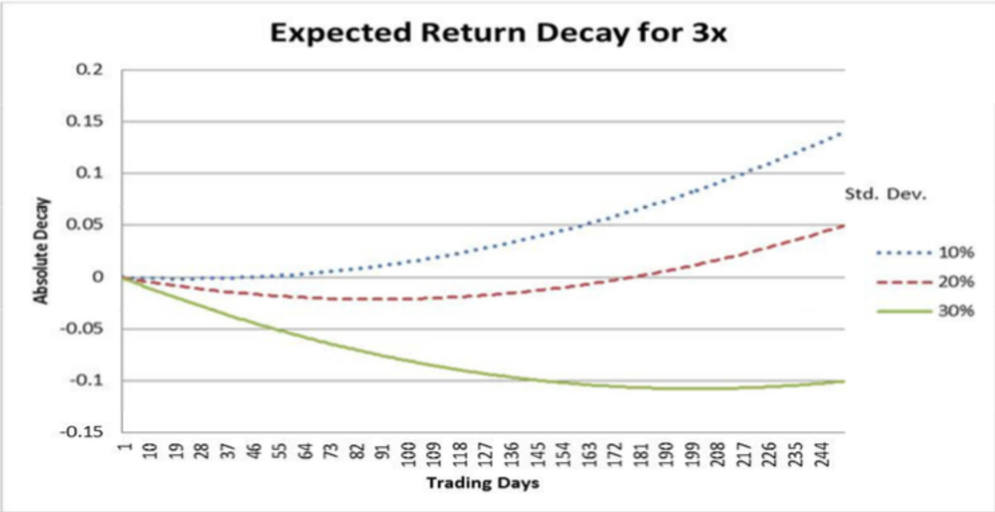

Figure 1.7: Expected return decay for 3x LETF [Trainor and Carroll, 2013]

This figure illustrates that if the conditions of low volatility and a constant positive trend of the returns of the underlying index are fulfilled, then the leverage effect on a positive trend can outweigh the negative volatility effect. In other words, over a few indices, LETFs could turn out to be a profitable long-term investment. Nevertheless, most real world examples show that it is typically not the case. The volatility decay is present in almost every LETF, as shown by the majority of the empirical studies on the subject.

1.3.3 Impact on the volatility of the underlying assets

Rebalancing is performed at the end of the trading day in order to readjust the amount of index exposure of the LETF before the next trading day. These rebalancings go in the same direction as the returns of the underlying index. If the return increases, the LETF will buy more of its assets at the end of day while if the return of the index decreases, the LETF will seek to sell part of its assets.

Some research have been conducted on the hypothesis that the trading caused by the need for rebalancing is responsible for an increase in the volatility during this time period. Indeed, according to a great deal of experts, [Cheng and Madhavan, 2009],[Charupat and Miu, 2011],[Bai et al., 2012], [Shum et al., 2015], end-of-day volatility is definitely linked with LETF rebalancing trading. The buying and selling of all these derivatives necessary to readjust the exposure of LETFs can cause an artificially high volatility on the market at the end of the day. LETFs are often held responsible for part of the high volatility that is observed in late-day trading. This repercussion can in turn be further increased by predatory investors trying to capitalize on this volatility increase of the end of the day.

A few academics go further by saying that LETFs have a significant impact on the volatility of their underlying assets, not only at the closing of the day [Curcio et al., 2012], [Yagi and Mizuta, 2016]. They claim that the volatility of the underlying index of the LETFs has been indeniably higher since such funds have been introduced on the market. The simulation conducted by [Yagi and Mizuta, 2016] gave significant results on the effect of LETFs on volatility.

This effect on volatility is considered rather important and could be an argument for future policy implications to protect the markets [Cheng and Madhavan, 2009].

Nevertheless, there is some disagreement among the researchers as [Trainor, 2010] claims that the increase in volatility observed empirically in some studies is nothing but a "spurious coincidence" and is not caused by LETFs trades.

1.4 Inverse ETFs

Even though this study does not focus on Inverse LETFs, we ought to give a brief explanation on the functioning of such funds. Inverse ETFs are funds that seek to deliver the inverse return on an underlying index. In order to reach that goal, financial derivatives are used, similarly to the case of positive LETFS. Swap agreements, forwards, futures contracts and options are traded to track and provide the inverse return on an underlying index. In other words, Inverse ETFs work in the exact same way as positive LETFs and are thus entailed with the same risks. The need of rebalancing to readjust exposure as well as its induced path-dependency of returns is also inherent to IETFs. Furthermore, these inverse funds can have a multiple of -1, -2 or -3, seeking respectively to deliver the inverse, twice the inverse or three times the inverse of the returns of the underlying index.

Most reseachers that investigated LETFs included IETS in their analysis.[Cheng and Madhavan, 2009],[Dulaney et al., 2012], [Chen and Diaz, 2012], [Bai et al., 2012]. In some studies, the funds are referred to as bull and bear ETFs, bull for normally leveraged and bear for inversely leveraged. In addition, they could also be referred to as long and short ETFs, long ETFs for LETFs and short ETFs for IETFs. Moreover, IETFs are not necessarily leveraged, even though they entail the same risks and mechanisms as LETFs.

The scope of this paper is limited, we will therefore only examine positively (bull/long) LETFs in our empirical analysis.

1.5 Risks of LETFs

LETFs are complicated tools that can be profitable for the investors when used right. However, they definitely entail some risks.

Investors must take these risks into consideration when investing in this kind of funds. From their point of view, the most important concerns are the impact of market volatility on the performance of LETFs and their efficiency in reaching their daily targets. [Rompotis, 2013]. As mentioned earlier, the volatility of the market strongly impacts the returns of LETFs. The more volatile the market is, the more likely that the funds do not meet the daily target of returns. Tracking errors increase in highly volatile environment and the investors have higher probability of never receiving the expected returns. This effect is exacerbated on longer holding periods, and in a (even moderate) volatile environment, investors will automatically see their returns decline over time due to this volatility decay.

Another big risk when investing in LETFs is a risk common to ETFs in general. It is related to the difference between the market price and LETFs prices on the secondary markets. The price of (L)ETFs shares can differ from their Net Asset Value. Authorized Participants try to keep these prices the closest possible to the NAV, but do not succeed each time. There is usually a gap between there prices. Furthermore, while these prices fluctuate mostly according to the NAV, they also fluctuate following supply and demand movements, and investors might not know whether they buy the shares at a discount or at a premium rate. This represents another risk for investors that do not have all the information in hand to take the investment decision [Rompotis, 2013].

The third risk threatening both the investors and the fund itself is a risk common to a lot of synthetic financial products. It is the so-called 'counterparty risk'. This risk is caused by the use of financial derivatives, like swap agreement, that are often used for the rebalancing mechanism of LETFs. This is the risk that the counterparty of the swap agreement fails to meet his obligations. [Bush, 2009] studies this counterparty risk and gives three solutions to mitigate such risk. The first solution would be to secure the counterparty agent itself. LETFs should try to diversify their agents acting as counterparties as well as assess their credit review. The second solution the study mentions is to enter short-term swap agreements as opposed to one-year swaps, or alternatively to use futures instead of swaps to mitigate this counterparty risk. The third way would be to enter a tri-party agreement between a swap provider, the LETF and a custodian; that way, the collateral will be held by the custodian instead of the swap provider. The collateral will therefore not be at risk, if the swap provider default.

A last risk of the LETFs is worth mentioning, it is related to the impact the fund can have on the volatility of the underlying market. This does not directly concern the fund nor the investors, but it can lead to market failures if the number of LETFs - and therefore the amount of rebalancing - increases by much. Indeed, [Yagi and

Mizuta, 2016] claims that if the impact of LETF on volatility is high enough, the market of the underlying may be destroyed. This risk of LETFs should probably be addressed by regulators.

1.6 Advantages and criticisms

The use of LETFs entails some risks and are often heavily criticized by the media. But it is undeniable that LETF can offer consequent gains. We will here look at the various advantages that LETFs can bring investors.

First and foremost, investors buying LETFs shares have the possibility of obtaining leverage without investing directly in financial derivatives. They also avoid to get involved into multiple costly trades to obtain a set level of leverage. That means that LETFs investors do not need to maintain a margin account that has to be refinanced when losses reduce the margin amount significantly. [Rompotis, 2013].

Furthermore, the costs related to the management of ETFs are usually quite low. Indeed, ETFs are typically passively managed and that represents a good chunk of the costs of managing the fund. Passive ETFs or index funds have thus lower management fees than actively managed mutual funds. This is also the case for LETF, although the costs of management for LETFs are significantly higher than for normal ETFs. This is due to more frequent transactions, and thus management, needed to rebalance the LETF and keep a constant leverage mutliple. [Rompotis, 2013]

Moreover, LETFs can easily be used to make short-term gains. As a flexible and exchange-traded vehicle, it enables investors to capitalize on the trend of the current price and can deliver some significant short-term profit that are multiplied by the leverage. [Rompotis, 2013]

A last advantage that ought to be highlighted is the arbitrage possibility. A great number of investors and researchers have developed some strategies that could offer arbitrage possibilities to LETFs shareholders. Indeed, there are possibilities to capitalize on the prediction of the decay of LETFs on longer horizons. This involves the short trading of LETFs. In practical, an investor could 'borrow' some LETFs shares and sell them right away at the present price, and pay back them later at the later price. Since LETFs prices are expected to go down, short selling them could profit the investor. In order to gain from this volatility decay whether the market is going up or down, researchers [Dobi and Avellaneda, 2012], [Jiang and Peterburgsky, 2013], [Guo and Leung, 2015], [Hessel et al., 2018] came up with the idea of short selling pairs of leveraged (bull) and inverse (bear) ETFs. This method allows investors to hedge their portfolio against volatility and mitigate the risks [Rompotis, 2013]. In addition to that, when short selling ordinary stocks, the amount of loss is potentially infinite whereas when investors sell LETFs short, the amount lost cannot exceed the initial investment [Rompotis, 2013].

We will now turn to the criticisms of LETFs. Some weaknesses of this type of funds have already been mentioned in a previous section devoted to the risks. However, we consider that there is a distinction to be made between risks and criticisms. Among others, one criticism is that investors are not well informed of the existence of the risks of LETFs.

One of the most recurrent criticism on LETFs is definitely its inadequacy for buy and hold investors. Its inability to deliver the leverage multiple times the index returns on period longer than one day is inherent to its design. Volatility decay is inevitable and LETFs are bound to undeperform static leverage in most cases. Such a decay in wealth exacerbated by high volatility and combined with misinformation can easily lead to a disaster in financial media. There are several studies showing that LETFs can cause important losses in the long term, and some other studies revealing that the investors are not informed well enough [Cheng and Madhavan, 2009], [Guedj et al., 2010], [Dulaney et al., 2012].

Furthermore, the tax advantages of LETFs are not as important as for simple ETFs. Normal ETFs are considered tax efficient because the redemption of their shares is performed in-kind, simply in the form of exchange of their underlying securities. This leaves the seller of such a share with limited taxable capital gains, and give therefore an important advantage to ETFs in comparison with other types of pooled funds. However, in the case of LETFs, the redemption of shares cannot simply be handled with the exchange of the underlying securities. Indeed, portfolios of LETF contain different types of derivatives, e.g. futures and options, that are necessary to perfom the rebalancing mechanism. That forces the management of the LETF to liquidate part of these options, swaps or futures in order to have the cash necessary for the redemption. This new liquidity is considered as a taxable gain for the investor, and he therefore has to follow the normal tax regime. In conclusion, LETFs are not tax efficient for investors, and this is considered a sizable weakness. [Rompotis, 2013]

A last weakness of the LETF, related to their trading strategies rather than to their design, is the risk associated to short selling stocks. Indeed, while short selling LETFs can profit the investors, they need to be able to find the necessary shares to sell first. Moreover, they do not choose when the shorting position is over as the broker can ask for a reimbursement any time, and that could put the investor in a delicate position. In addition to that, the accounting for selling short is really complex, and can be costly. This is why the short selling of LETF is sophisticated and should be left to well informed financial players. [Rompotis, 2013].

1.7 Conclusion on the theory

Exchange traded-funds definitely are a hot topic on the financial market. Their spectacular rise in popularity, as well as the various advantages they offer to investors could only cause their proliferation, diversification and complexification. LETFs, synthetic products using financial derivatives, were then bound to appear on the stock-exchange. Even though these funds seem promising, they can be quite dangerous when you are not well informed of their functioning and the risks they entail.

After explaining the working and evolution of 'traditional' ETFs, we focused on the main subject of our analysis, the leveraged version of ETFs - LETFs. We have seen that the mechanism behind these funds is associated with needs for "rebalancing". The rebalancing mechanism causes the returns of the LETFs to be path-dependent. This in turn is causing the "volatility decay", a wealth decay for longer holding period related to the volatility of the underlying. Afterwards, we explained the relationship between rebalancing trades and the volatility typically observed at the end of the day, and how LETFs can be responsible for an increase of the volatility of the underlying assets. We dug further by clarifying the difference and similarities between LETFs and IETFs, as well as their nomenclature in the literature. Further, we mentioned the risks of LETFs and we finished this section by listing and explaining the various advantages and criticisms of our object of study.

Chapter 2

Literature Review

In regards to the success and continuous rise in popularity of ETFs, many researchers have investigated their functioning and mechanisms in the last years. They have had a considerable interest in discovering the trends, tendencies and general aspects of these (relatively new) financial instruments. ETFs represent a very extensive research field for the finance academics worldwide, however, we will here focus on the main subject of our study, LETFs and their reciprocal: Inverse LETFs.

Most early researches tended to establish the baseline on which these LETF reach their daily targets, their rebalancing and the mathematical and statistic formulas, including various implications of these processes. This subject of the literature will be adressed in the below section: Rebalancing and Achieving Target Goals.

Subsequent researches then analyze the actual short- and long-term returns of LETFs as well as the suitability for either short- or long-term gains through these financial vehicles. These studies tend to compare these returns with the return of more static, traditional leveraged tools, with their underlying indices or with ETFs.

Special attention has also been given to the types of investors of those specific synthetic ETFs. Who are they, what is there return goal, and are they using this tool in an efficient way?

Lastly, we will discuss some studies that were conducted on the European market, since it is the scope of our work.

2.1 Rebalancing and achieving target goals

The structure and rebalancing of LETFs is necessary to reach the target results that were promised to investors. It can however be responsible for certain consequences on the expected return of the LETF when the investor holds it more than one day. This rose the interest of many researchers.

The first article on the subject highlighted that, due to the compounding nature of the return and the daily rebalancing of these types of funds, the long term returns were not equal to the leveraged multiple times the return of the benchmark. [Trainor and Baryla, 2008]. According to them, the daily target can be attained; however, when investor are misinformed about the rebalancing mechanism and the coumpounding factors, they might expect a higher long-term return than the actual return.

[Cheng and Madhavan, 2009] is the first to publish a paper on the subject that modelled the rebalancing demands of these LETFs. This work brought out the path-dependence pattern of the returns of LETFs. This pattern can be shown to negatively impact the buy-and-hold investors and treat LETF as a long term investment. An explicit model was developed on the impact of the needs for rebalancing on the dynamics of the underlying assets and on the return of the fund. This study served as the base model for a high number of further studies.

Another paper focusing on the need for rebalancing in order to reach the long-term targets by [Hill and Teller, 2009]. The authors mostly highlight the ideal frequency of rebalancing that is needed by comparing static leveraged with rebalanced leveraged funds tracking the same indices. They found out that this frequency is dependant on the volatility of the underlying index and of the structure of the fund, i.e. whether it is a bear or a bull ETF).

[Avellaneda and Zhang, 2010] further documents the necessity of rebalancing and empirically demonstrates the path-dependence of LETFs returns. It tries to explain the underperformance of LETF compared to their respective indices during the year 2008 and 2009 and links it to the phenomenon of path-dependency of return typical to LETF.

A later study also proves the path-dependency of LETF [Jarrow, 2010]. Building on the works of [Cheng and Madhavan, 2009], and [Avellaneda and Zhang, 2010], he derives formulas proving that a k-time leveraged ETF does not mean that the investor would earn k-time the return on this ETF and that LETF are, in fact, path-dependent.

Furthermore,[Dulaney et al., 2012] published a paper stating that the main causes for the tracking error in LETFS and Inverse ETFs is the high-frequency of rebalancing. It shows that a decrease in this frequency enables the fund to match more closely the return of the underlying index and concludes that the high frequency of rebalancing of these funds makes them unsuitable for passive, long-term investors.

2.2 Long-term returns of LETF versus traditional instruments

The daily rebalancing of LETF is reponsible for their unexpected return and for its difference with the leverage ratio of the fund that one might expect. Researchers have therefore tried to compare and understand these returns, and tried to see whether an investor would be better off by buying and holding stocks in these funds or by keeping them only in the short run.

[Trainor and Baryla, 2008] were the first to study the return structure of LETFs. They conducted a Monte-Carlo simulation on a period of ten years, and they found out that a typical x2 leveraged stock ETF is likely to only magnifiy returns by 1.4 times on an annual basis with the standard deviation measure of their risk being doubled and the magnitude of extreme negative returns quadrupled. They conclude that, because of extreme swings in value being inherent to these types of funds, they seem to be more suitable for short-term investors.

[Carver, 2009] considers this subject and points out that, generally speaking, LETFs under-perform what might be expected in the long-run. He even points out that the returns of ETF with high leverage (3x) tends to converge to 0 over a long time span. He links this to the volatility of the underlying assets and states that this effect is increased with extreme market conditions. He claims, however, that under better market conditions, moderately leveraged (2x) adaptive ETFs could become an attractive investment.

As previously mentioned, the work of [Cheng and Madhavan, 2009] sets up the baseline for the model of LETFs studies and is among the most cited papers on the subject. In this research, it is concluded that, even though LETFs are able to track the returns of their underlying indices at a daily level, they cannot over a longer period due to the path-dependency of their returns. It then leads to a loss of value for buy-and-hold investors. It claims that this effect is reinforced by "the drag on returns from high transaction costs and tax inefficiency".

[Avellaneda and Zhang, 2010] confirms this by an empirical study on 56 leveraged ETFs over an horizon divided in quarters. The author finds out that, as expected, there is a decay of the wealth for buy-and-hold LETFs investors. Nevertheless, they claim that viable long-term LETFs that replicate well their underlying ETFs is possible, but that would require dynamic rebalancing. The "optimal" rebalancing frequency would then be in the order of once a week for the funds they studied.

According to them, the need for a dynamic rebalancing shows that LETFs are unsuitable for buy-and-hold investors.

In the same line of work, [Guedj et al., 2010] follows the argument stating that LETFs and Inverse ETFs do not deliver the returns investors may expect for periods longer than a day or two [Cheng and Madhavan, 2009]. They went further by calculating the shortfalls of the holding of LETFs for an excessive amount of time. They found out that in a period of several weeks, investors engaging in such behaviour would lose about 3% of their initial investment. They also searched the viability of long term investment in monthly rebalanced LETFs instead of daily rebalanced, and claim that even though the shortfall is reduced in the former, it remains significant, with an added risk due to the daily fluctuating leverage ratio.

The study of the tracking errors of LETFs is pursued by [Charupat and Miu, 2011], with an empirical study on Canadian LETFs. The findings of the study point out that tracking errors stay relatively small on a period up to a week, but become increasingly large the longer the time horizon studied. They conclude that concerning the tracking errors, LETFs succeed in delivering the promised results for holding periods up to one week, but their returns start to deteriorate after this holding period, and beyond one month, the actual returns are wildly different from the initial promises.

[Shum, 2011] studies the returns of LETFs during the recent financial crisis and tries to disentangle the effect of compounding on long term returns caused by the frequent rebalancing, from both the effect of the management of the fund and the effect of the trading premiums/discounts. The author argues that tracking errors are not caused solely by the compounding effect and that, depending on the fund, the management effect can outweigh the compounding one. Furthermore, according to her, the performances were further distorted during the financial crisis due to substantial premiums/discounts caused in turn by reduced liquidity.

Quite similarly, [Leung and Santoli, 2012] conducted an empirical analysis on the returns of major LETFs tracking the S&P 500 index. The study aimed to determine the admissible horizon during which an investor could hold these ETFs, and therefore build a strategy for investing in LETFs. They found out that the performance of LETFs declines as the horizon increases when you compare it with the performance of their underlying ETFs. They also state that the higher the leverage, the higher the erosion and decay of wealth for the investors.

In a posterior study, [Trainor and Carroll, 2013] aim to quantify and measure the different variables that influence the amount of decay as well as the trade-off between these variables. Decay is the negative difference between an LETFs return over a timeframe, minus its leverage ratio and its unleveraged counterpart. They found out that the amount of decay is influenced by the daily leverage ratio, time, index return, and volatility. After quantifying and calculating the maximum holding period for investors under various volatility levels for the market, they argue that "for decay thresholds of only -2% and low volatility levels, holding periods beyond 6 months can be justified for some leveraged funds". This goes against most of the prior literature showing that holding LETFs can only be profitable in the short term.

[Leung and Ward, 2015] also study the performance of LETFs compared to their leveraged benchmark. More specifically, they examine the case of gold-tracking LETFs and their price relationship with gold spots and futures. They find out that optimized portfolios with short-term gold future are rather effective in tracking gold prices and delivering promised returns. Nevertheless, they show that over longer holding periods, gold LETFs tend to underperform their static leveraged alter-ego. Furthermore, it is illustrated in their work that the effect of this decay worsens as the the holding period increases.

In opposition with the previous literature, [Cheng and Miller, 2016] suggest in a study conducted on the analysis of tracking errors for LETFs in the oil sector, that there is a tendency for the returns of LETFs to converge towards a low tracking error, and that while the convergence is not instantaneous, these errors tend to reverse over time. They claim that over the course of several days the discrepancy between the LETF and its underlying tends to gradually diminish.

2.3 Types of investors in Leveraged ETF

Misinformation, new added complexity and misguiding promises of high return can mislead the investors on the subject of LETFs. It has been established that the return of an LETF is rarely equal to its leveraged ratio times the return of the underlying on periods longer than a day. But do investors know that? Researches that address this subject are presented in this subsection.

[Cheng and Madhavan, 2009] first reports that initially the investors that were interested in trading LETFs were solely short-term traders. However, over the years these funds started also to raise the interest of private investors willing to leverage their portfolio.

Subsequently, [Guedj et al., 2010] investigates the "distributions of holding periods for investors in leveraged and inverse ETFs". Using empirical data on the DPK fund, they found out that more than 16% of these shares are held longer than one week, more than 6% longer than a month, and more than 1% longer than a quarter. This implies that a lot of investors do not understand that these LETFs are only interesting as short-term investments. They then computed the potential shortfall for those investors, as mentioned previously.

Looking at Canadian data and ETF market, the study of [Charupat and Miu, 2011] points out that the most likely type of investor to hold shares in LETFs are "retail traders with an average holding period of under 15 days". These traders would also be more actively trading than traditional traders and they would have as a goal to implement specific trading strategies.

Another paper covering this subject is [Dulaney et al., 2012]. It argues that while ETFs offer simplicity and enable retail investors to take sophisticated positions, these products might mislead investors in thinking that, even when leveraged, the ETFs will track and follow the price of its underlying and have the same returns times the leverage ratio. It admits that this reasoning might seem sound, but that its almost never the case and therefore that these synthetic ETFs are not suitable for typically uninformed retail investors.

2.4 European studies

Most of the first studies conducted on the subject were based on the S&P 500, the Dow Jones or on the American market. These results however could be USA biased and need back-testing in other areas. The scope of the present paper covers the behaviour of European LETFs. We will therefore document in this section the different studies that have been conducted on European markets. European LETFs would theoretically not be different than LETFs found in the US; hence, the studies should not differ very much from the literature review presented in the previous subsections. The literature on LETFs based on the European market is quite scarce, however, and is dwarfed by the number of studies conducted on American and Asian markets.

The study of [Giese, 2010] discusses the risk-profile of leveraged and inverse ETFs. As mentioned previously, it tries to "develop a very general model for the long-term performance of a dynamic leveraged and inverse fund strategy". This would provide help to investors and/or regulators by giving them insights on the long-term behaviour and returns of these funds. The study then gives a numerical example by taking as example an European index, the EUROSTOXX 50, which gives an insight on how the LETFs market in Europe behaves.

[Rompotis, 2015] conducts an empirical analysis on the trading behaviour of the market for LETFs in the UK. The author analyses the performance and trading trends of LETFs, the influence of the volatility of the underlying on the performance of these funds, as well as the pricing efficiency of LETFs. He found results that are in line with the existing literature, that the daily targets of LETFs on the UK market are usually met, but tracking errors can be significant, especially in the long-term. He also confirms that the volatility of the underlying has a significant impact on returns, and is related with higher tracking errors for bull ETFs and lower tracking errors for bear ETFs. He finally proves that for the UK market, the higher the market volatility, the higher the deviation between trading prices and the NAV (Net Asset Value).

[Flores, 2015] studies the German market and more specifically the LETFs tracking the DAX index. The author tries to assess the impact of this type of fund on the underlying assets, and to isolate the microstructure effects potentially caused by LETFs. Following his analysis of the DAX index and the related LETFs from 2006 to the end of 2014, he found out that the volatility of the DAX index stocks increased by 10% after the introduction of LETFs on the German market. Moreover, he confirms what a lot of previous American literature stated about the increase of the volatility of the underlying at the end of the day, stating precisely that in the time interval 17:00:00 and 17:29:59 the highest level of volatility can be observed, single-handedly accounting for 4% of the average daily volatility. These results corroborate mostly with the previous literature.

2.5 Conclusion

The subject definitely is widely documented in the literature. While the topic is rather technical and the whole subject relatively new, many researchers, academics and investors have been willing to investigate and study the mechanisms behind this type of funds. Nevertheless, the topic of LETFs still has some grey areas, and further research would be needed to enable experts to fully grasp the functioning and financial effect of it.

Many researches have attempted to set up a framework or model in order to explain the structure of returns and the rebalancing mechanism of LETFs. Some of these models have become the reference in the field. We have noticed that additional studies usually build on the existing models in order to create the most accurate, comprehensive and understandable explanation of LETFs characteristics, path-dependency and poor long term performances. The misinformation of investors on these funds lacks research and is therefore still debated.

Furthermore, it appears clearly that most studies focus on American markets and indices, further research might therefore focus their attention on the cases of European countries in order to extent the results for the US to what can be observed worldwide. Analysis have been conducted on the German and the UK markets as well as on the Eurozone. More specific analysis on other countries such as France or Luxemburg could be a good way to extend research on the matter.

Many other studies on LETFs have been conducted and would deserve our interest. However, these are beyond the scope of this thesis. To name a few, studies concerning the returns of dynamically leveraged versus statically leveraged ETFs, studies about the impact of LETFs on the volatility of their underlying assets and studies related to the optimization, strategies, and predictive models for trading LETFs. This part of the literature review can be found in the appendix B.

Chapter 3

Empirical Analysis

The two previous chapters were devoted to the theoretical explanation of LETFs, and to a review of the research works conducted on the field. We now turn to our own empirical analysis. In this section, we ought to uncover the dynamics of LETFs and therefore confirm or disprove the theory stated previously. Indeed, this paper has as goal to verify and illustrate the assumptions made in the theoretical part and/or proven by researchers. In order to do so, we will be using data on the French market. Even though studies have been conducted on the German and the UK LETFs, none were focused on the French market before. This is rather surprising as the French market is considerable. It thus appeared as a natural market to analyse. Moreover, the French economy being one of the biggest in Europe, we deemed interesting to focus our investigation on this country.

Our empirical investigation will be divided in three parts. In the first one, we will take a look at the rebalancing mechanisms, try to quantify and plot it. The second part of the analysis will tackle the problem of pricing efficiency of LETFs compared to the NAV at the close of each day. Finally, in the third part, the topic of the returns of LETFs, and their profitability on time frames longer than a day will be addressed.

3.1 Sample Analysis : The CAC 40 and its LETFs

We will examine certain French LETFs we deemed more interesting to analyze because of their longevity on the market, the size of their total asset value, or their high leverage ratio. We specifically chose 6 of them, namely "Lyxor CAC 40 Daily (2x) Leveraged UCITS ETF (LVC)", "Lyxor/SGAM ETF Leveraged CAC 40 (L40)", "ComStage ETF CAC 40 Leverage UCITS ETF (Z4L)", "ComStage CAC 40 Leverage UCITS(PCC2)", "ETFX CAC 40 2x Long Fund(FRL2)", "ETFS 3x Daily Long CAC 40(FR3L)", that we compare with two regular ETFs: "LYXOR ETF CAC 40" and "AMUNDI ETF CAC 40 (C40)". We chose these 6 LETFs because of their size of at least 1 million euro of assets under management. This panel represents the major part of LETFs that follow the CAC40 and were the only LETFs we could find on the Bloomberg terminal with reliable information. We therefore assume that it represents the majority of the existing LETFs that tracks the CAC 40.

[h] Name	Symbol	Underlying benchmark	Multiple	Listing date	Delisting date
Lyxor CAC 40 Daily Leveraged UCITS ETF	LVC	CAC40	x2	June 13, 2008	Still in use
Lyxor/SGAM ETF Leveraged CAC 40	L40	CAC40	x2	October 21, 2005	July 9, 2010
ComStage CAC 40 Leverage UCITS	PCC2	CAC40	x2	October 8, 2014	June 30, 2017
ComStage ETF CAC 40 Leverage UCITS ETF	Z4L	CAC40	x2	January 12, 2010	June 30, 2017
ETFX CAC 40 2x Long Fund	FRL2	CAC40	x2	July 2, 2009	June 11, 2012
ETFS 3x Daily Long CAC 40	FR3L	CAC40	x3	May 22, 2014	Still in use
LYXOR ETF CAC 40	CAC	CAC40	x1	December 13, 2000	Still in use
AMUNDI ETF CAC 40	C40	CAC40	x1	February 3, 2005	Still in use

Table 3.1: Descriptive data of the panel of ETFs

Lyxor LVC is the biggest of our sample of LETFs, with more than 243 million EUR in total assets in 2018 (according to Bloomberg). It is based in France on Paris stock exchange and promises twice the return depending on the underlying index CAC40. It was created relatively early, in 2008, and is the earliest LETF still in use. The data available is very compelling, it seems to be the reference of an LETF following the CAC40.

Lyxor L40 is the oldest leveraged fund on the CAC40. It was created in 2005. This fund has been delisted in 2010 because it was acquired and changed by another fund, GLE:FP. It was a large fund as well and amounted 120.862 million euros in assets at the time they were delisted. Even though it is not a single fund anymore, we decided to include it in some of our analysis when we look at a larger timeframe.

ComStage PCC2 is an LETF from the Lisbon stock exchange that also follows the CAC40 with x2 leverage ratio. It was created in October 2014 but was delisted on June 14, 2017 by ComStage.

ComStage Z4L is based in Luxemburg and is also a x2 leveraged ETF that tracks the CAC 40 index. Similarly to PCC2, Z4L is managed by ComStage, their prices and NAV are therefore very much alike. It is a rather small fund with only 3.43 million euros under management. Also delisted in June 2017 by ComStage, it was created earlier than PCC2, in 2010.

FRL2 is an even smaller fund that we decided not to use in all parts of our analysis, as the Bloomberg terminal does not provide any data on its price, but only on its NAV. It only had 1.859 millions in total assets in November 2012 when it was delisted. Its inception date is the 29th June 2009, it lasted only three years. Including data on this fund might still be interesting as that would increase the sample size and it represents a significant part of the volume of LETFs traded.

The last LETF of the sample is different from the others as it is a three times LETF. Indeed, FR3L aims to "enable investors to gain a three times daily leveraged 'long' exposure to CAC 40 Total Return Index (CAC 40) by tracking the CAC 40 Daily 3x Leverage Net Return Index" according to Bloomberg. it was created later than most of the other funds in our sample, only in 2014, but it is still in use today. It is to our knowledge the only positive LETF following the CAC40 with a multiple higher than 2. Including data of this fund is very interesting in order to see if the leverage ratio has any effects on the parameters we will analyze.

We presented the six LETFs from which we drew the data. We focused solely on positive "long" ETFs tracking the same index, whatever the stock exchange they are traded in. However, this does not constitute the whole sample, as we also need a control group. This group is represented by two traditional ETFs, "Lyxor ETF CAC 40" and "Amundi ETF CAC 40". We will compare the data from the LETF panel with data from these two ETFs, and try to draw conclusions.

Our first traditional ETF, Lyxor CAC 40 is a sizable ETF compared to our LETFs total assets. It has more than 4 billion euros in assets, and is the largest ETF tracking the CAC 40 index. It is significantly older than the other funds as it was created in December 2000. It is still traded today on the EURONEXT Paris exchange. This fund provides good data for our comparison as it is considered as a standard ETF. We will not be using the data available for the period 2000-2005, since it does not make sense to use data prior to the introduction of the first LETF of the sample.

The second "regular" ETF is Amundi C40. This fund is also larger than the LETF of the sample, with currently approximately 1.4 billion euros in total assets. It was created later than CAC, in February 2005 and is still traded today.

[h] Name	Symbol	Total Assets (M EUR)	Price on 16/08/2018 or at delisting (EUR)
Lyxor CAC 40 Daily Leveraged UCITS ETF	LVC	243,283	19,74
Lyxor/SGAM ETF Leveraged CAC 40	L40	120,862	19,39
ComStage CAC 40 Leverage UCITS	PCC2	Unknown	18,06
ComStage ETF CAC 40 Leverage UCITS ETF	Z4L	3.431	17,93
ETFX CAC 40 2x Long Fund	FRL2	1,859	68,91 (NAV and not price)
ETFS 3x Daily Long CAC 40	FR3L	1.943	16,91
LYXOR ETF CAC 40	CAC	4 143,97	54,35
AMUNDI ETF CAC 40	C40	1 482,121	76,93

Table 3.2: Price and Total Assets of the sample ETFs

3.2 Rebalancing mechanisms

The rebalancing mechanism is inherent to LETFs, as it allows to provide investors the same returns with a constant leverage multiple each day. In this section we will quantify the rebalancing volumes needed per day for our sample of 6 LETFs and we will draw conclusions on the trend of these rebalancing needs.

3.2.1 Methodology

In order to do so, we will use the conceptual framework developed by [Cheng and Madhavan, 2009] that we briefly introduced in the theoretical part of the present work. The equation that allows us to compute the rebalancing volumes could be rewritten as follows:

$$RA_t = NAV_{t-1}(x^2 - x)r_t$$

The development of that equation can be found in appendix A. This method to calculate the rebalancing amount uses the NAV of the fund at the closure of the previous day, the return of the index and the leverage multiple as in many previous studies. We will mostly follow the methodology of [Flores, 2015] and [Shum et al., 2015], they conducted similar investigations to our work, respectively on the German and American markets. We will then plot our findings against the CAC 40 evolution, try to uncover any significant trends in order to either confirm or disprove the previous literature and draw the appropriate conclusions.

As the dates of inception and liquidation of the various funds of our sample are very different, we use two sample periods. The first one encompasses the whole time span of each LETF and calculates the needs for rebalancing from 2005 until 2018, to illustrate the evolution of rebalancing needs. The second period we use is a shorter sample period. It will enable us to see whether the relationship between volatility and rebalancing needs indeed exists. In that case, the data would be skewed if we analyzed the whole period between 2005 and 2018, as some funds have been delisted and some have been created during that time period. We need a sample period where the number of LETFs of our sample is consistent, this is why we decided to analyze the data from 20/10/14 to 14/06/2017. These dates respectively correspond to the creation of the PCC2 fund and to the delisting of both Z4L and PCC2 funds. By analyzing this time period, we get a sample of 4 LETFs (LVC, Z4L, PCC2 and

FR3L) used over the whole period. This makes the data more compelling to compare the volatility of the underlying with rebalancing needs.

3.2.2 Data

In order to conduct our analysis, we need data of the NAV of our funds as well as the performances of the CAC 40. These can be found on the Bloomberg terminal. The historical data we use is the daily NAV of the shares of the 6 LETFs tracking the CAC40 as well as the daily historical price of the CAC40. These 6 funds were all at some point used between 2006 and 2018, but some have been created later than 2006 and some have been delisted before 2018. These are the only data we could find on CAC40 LETFs on the Bloomberg terminal, we therefore make the assumption that this sample represents the whole volume of long LETF tracking the CAC40. We make our analysis period start a bit before the inception of the first LETFs, as to show the evolution of the impact of LETFs rebalancing flows from a period without any LETF to the present day (6th of June 2018).

3.2.3 Results

3.2.3.1 Evolution of rebalancing needs from 2005 to 2018

This subsection will investigate the rebalancing needs related to the tracking of the CAC40 index from 2005 to 2018. The purpose is to observe the evolution of the rebalancing needs over time. We assume that these figures are a good representation of the evolution of positive LETFs and their rebalancing needs through time as no other funds following the CAC40 could be found on the Bloomberg terminal. In brief, our analysis will show that the main factor of influence is the rebalancing needs associated to LVC the largest fund of our sample. LVC has been created in 2008 and is still traded now. Another result is that this set of data seems to represent well all LETFs trading that track the CAC40.

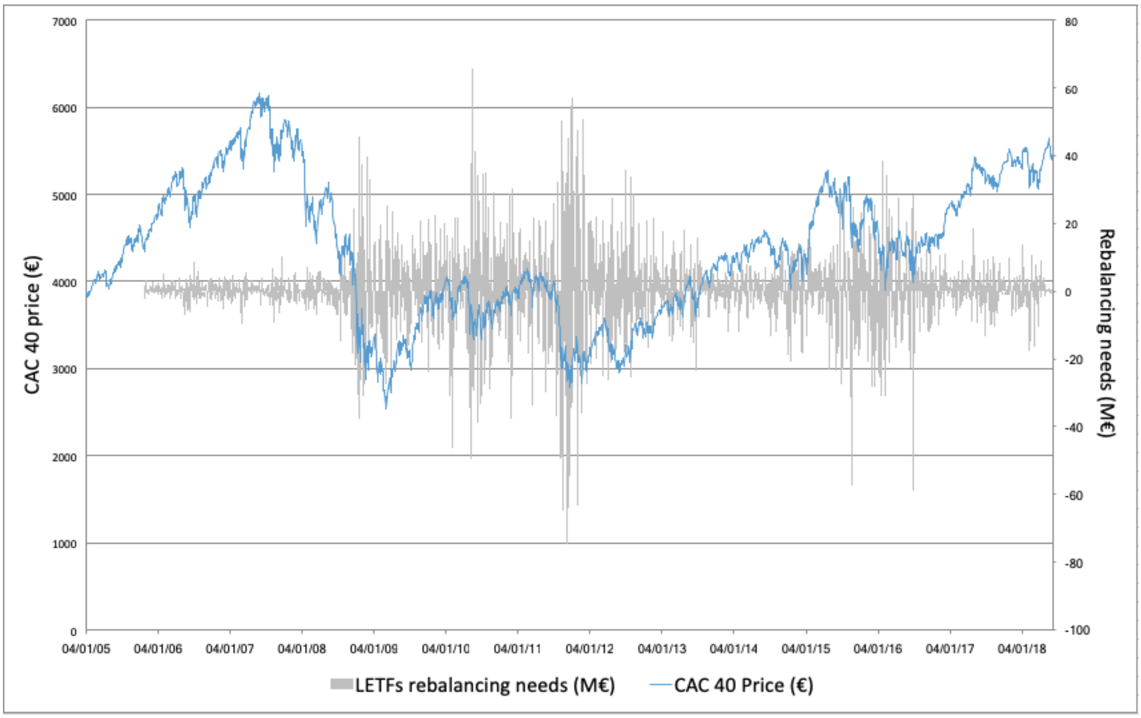

Figure 3.1: Daily LETFs Rebalancing Flows vs CAC40 prices

The first figure (3.1) shows the daily rebalancing needs during the last 13 years. We start our analysis in 2005 in order to show the progressive inception of the LETFs of our sample tracking the same index. The rebalancing needs are plotted against the CAC40 returns to show that the rebalancing needs are dependent of the direction in which the index moves, positive if it goes upwards and negative if it goes downwards. This theory is clearly verified on this plot and confirms the previous literature ([Cheng and Madhavan, 2009], [Flores, 2015]) on rebalancing needs. This graph also hints at the fact that rebalancing needs go up during periods of high volatility of the index, for instance, in 2008 and between 2011 and 2012. However, we cannot confirm that from this figure. Indeed, the graph could be misleading since two funds were delisted around these periode; L40 in 2010 and FRL2 in 2012. This may inflate the amount in 2008 and 2011, relative to the volatility.

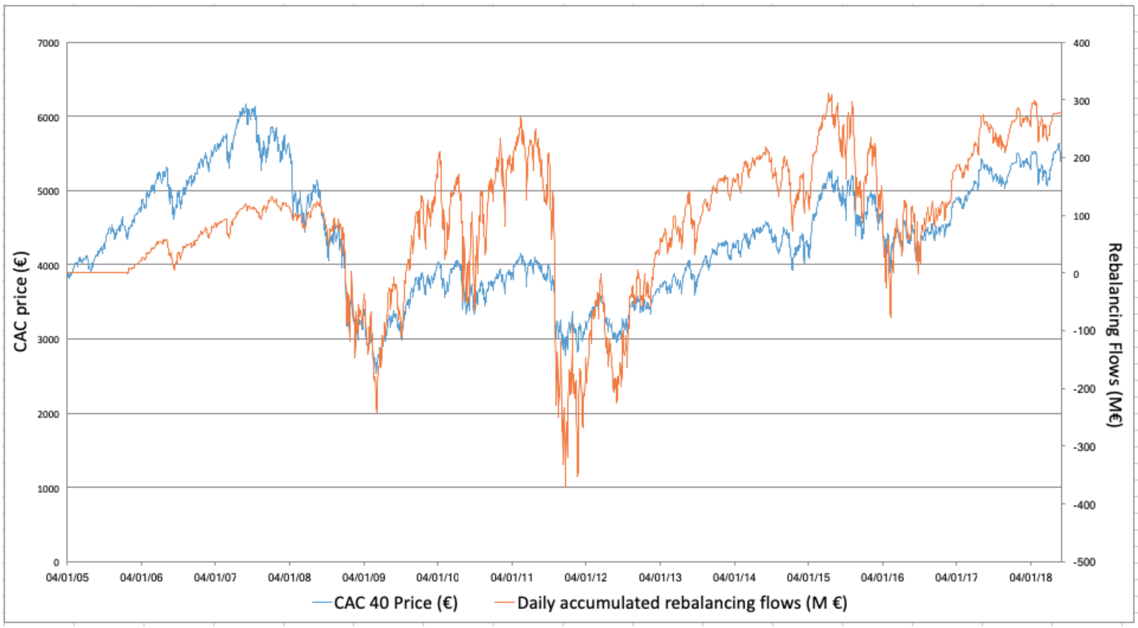

Figure 3.2: Daily Accumulated LETFs Rebalancing Flows vs CAC40 prices

Figure 3.2 represents the rebalancings flows accumulated daily for our sample. The evolution of the accumulated rebalancing needs is plotted, in other words the incremental addition of all the rebalancing needs. This figure shows that rebalancing flows naturally follow the path of the index. As we can see, the amount of the rebalancings is now at 300 million euros in the positive, meaning that the LETFs is on average more often potively rebalanced. This is probably due to the fact that the CAC 40 returns move upwards more often than they do downwards.

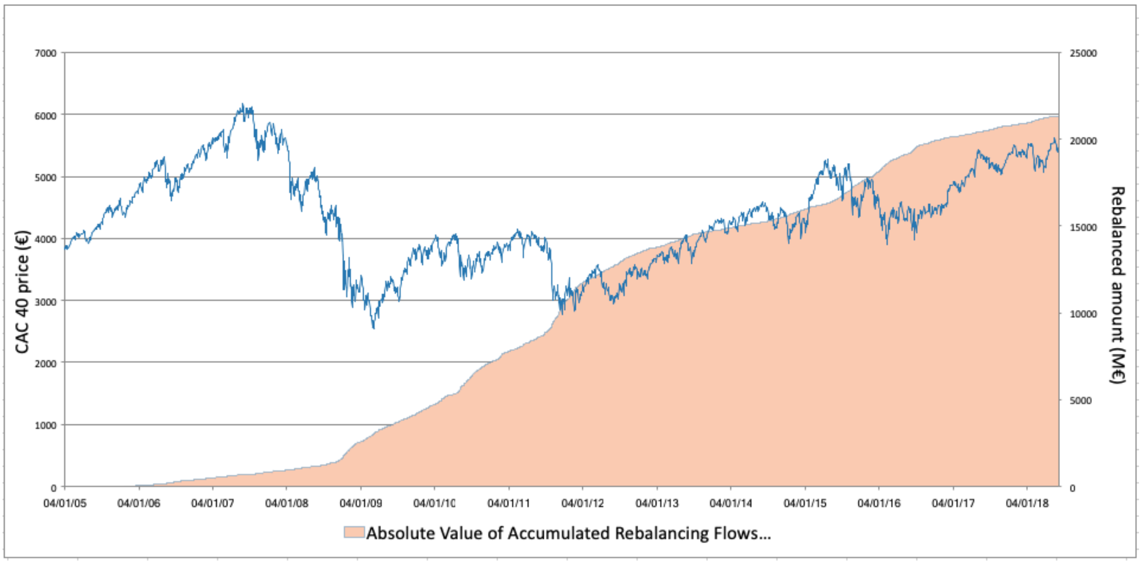

Figure 3.3: Daily Accumulated Absolute LETFs Rebalancing Flows vs CAC40 prices

The last interesting figure over the evolution of the rebalancing amount over time depicts the level of accumulated rebalancing needs in absolute value. Figure 3.3 covers the period from 2005 to June 2018. We notice a strong, almost exponential increase of rebalancing from 2008 to 2012, but the increase slows down right after 2012, growing almost linearly in that second period. It should also be reminded that LVC is responsible for the majority of these rebalancing needs so event though the sample size is not totally exhaustive, it is considered to be an accurate estimation of the whole LETFs market on the CAC40.

The previous literature stated that rebalancing was responsible for a major part of the volatility at the end of the day of LETFs [Cheng and Madhavan, 2009], the exponential increase of these leveraged funds we observe would thus cause distress on the market. From our analysis, we conclude that the evolution of the rebalancing needs linked to CAC40 is not alarming. These graphs show a linear increase of the rebalancing on the major part of the period, and an exponential rise in early years. A linear trend means that the amount of rebalancing needed for these CAC40 LETFs is actually not expanding. Furthermore, we observe that the magnitude of these amounts are quite small in comparison to the size of the whole market, therefore considered too small to have a real impact on the volatility of the CAC40. This conclusion cannot be generalized to all LETFs however, and should be tested on other indices, as there are only a few LETFs that track the CAC40.

3.2.3.2 Rebalancing needs versus Volatility from 20/10/14 to 14/06/2017

In order to confirm the conclusion drawn from the previous analysis, we will construct a graph with a different sample period, while being consistent with the number of LETFs in the previous sample. Figure 3.4 shows the results and is key for our comparison between rebalancing needs and volatility.

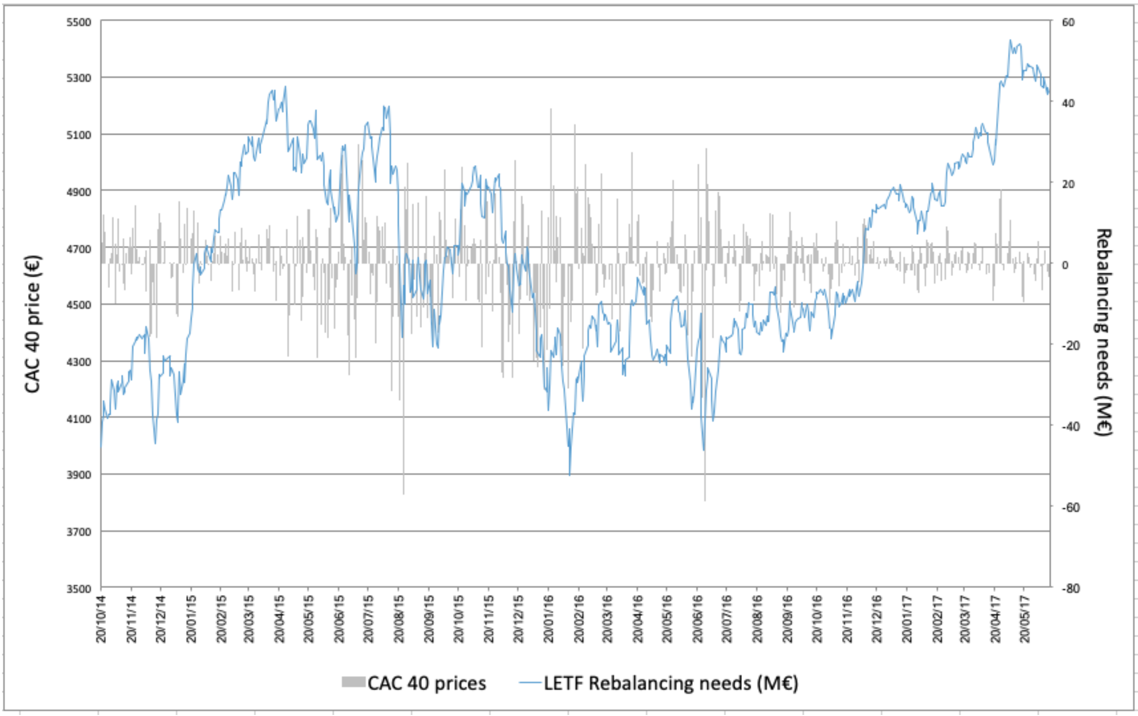

Figure 3.4: Daily LETFs Rebalancing Flows vs CAC40 prices

As we can see on the graph, the theory seems to be confirmed. There is an important drop in the returns of the index between 09/08/2015 and 09/09/2015, and that corresponds to the period during which the largest amount of negative rebalancing funds was needed. Looking at the whole picture, there seem to be more positive or negative rebalancing needed between the 09/05/2015 and the 09/08/2016 when the volatility of the underlying index is higher than after August 2016, period when the index return consistently increases.

We could thus conclude that, in line with the theory established by [Cheng and Madhavan, 2009], [Flores, 2015] and [Shum et al., 2015], the rebalancing needs are indeed linked with the volatility of the underlying. This would necessarily cause low performance of the LETFs in the long-term, the more volatile the underlying index is.

3.3 End of day price deviation with NAV

One of the main feature of ETFs and their leverages counterparts is that they encompass mechanisms that keep their price close to their NAV [Charupat and Miu, 2011]. These mechanisms consist in the the creation and redemption of provisions, creation units. Authorized participants can sell or buy large blocks of these ETFs at their NAV. In the way it works, this mechanism can close the gap between the price and the NAV. Indeed, the pariticipants can capture the price difference when they issue or redeem new shares of ETFs. We will empirically verify the efficiency of this mechanism on LETFs compared to normal ETFs in the next section of our analysis.

3.3.1 Methodology

In order to perform this analysis, we will be using the same two sample periods as in the previous section. Practically, we will compute the price deviation from its NAV at the end of day. Following the work of [Charupat and Miu, 2011], we calculate the price deviation using this equation:

$$\pi_t^1 = \frac{P_t^i - NAV_t^j}{NAV_t^j} \times 100$$

π_t^i being the percentage deviation, P_t^i being the price of the share at the end of day and NAV_t^j the Net Asset Value per share of the fund on the same day. When π_t^i is positive, the funds trades at premium, while when the deviation is negative, it trades at discount.

The same equation will be used in order to compute the price deviation for the two regular ETFs of our sample. Moreover, the same time frames will be used.

We will use Excel to calculate the average of the price deviations for each fund, as well as the 5th and 95th percentile, the standard deviation and the percentage of premium that are positive and negative.

Afterwards, we will have a look at the annualized volatility of each of these funds, computed as follows :

$$Vol_A = \sigma_i * \sqrt{252}$$

It simply represents the daily volatility (equal to the standard deviation) times the square root of 252, because there are 252 trading days in a year.

The results are divided into two subsections, referring to the two sample periods.

3.3.2 Data

Following the equations, the data we need to compute a price deviation are the price of each LETF at the end of the day and the NAV per share at the end of the same day. These data can be found on the Bloomberg terminal. These data points will be compared between the inception and the delisting, or between the inception and the date 16/08/2018 - date at which we retrieved the data. Moreover, we all compare these data between the 09/10/2014 and the 14/06/2017 for the relevant funds; LVC, PCC2, Z4L, FR3L and two ETFs: CAC and C40, in order to reduce the bias linked to time and get a consistent time frame.

3.3.3 Results

3.3.3.1 Price deviation between inception and delisting or between inception and 16/08/2018

First, we will take a look at the results of the price deviations for the entire existence of each LETF of our sample. To do so, we computed the deviation for each end-of-day for the whole existence of each LETF, and we then reported their average in table 3.3. We also computed the 5th and 95th percentile, the standard deviation, as well as the proportion of negative and positive premium. Lastly, the annualized volatility was reported as well.

	N	Average	5th Percentile	95th percentile	Standard deviation	Percentage positive	Percentage Negative	Annualized volatility
LVC	2562	-0,037	-0,408	0,323	1,535	46,547	52,595	46,256
L40	513	-0,012	-0,618	0,709	0,627	40,272	52,724	48,650
PCC2	651	0,005	-0,144	0,130	0,118	64,308	35,538	40,131
Z4L	1802	-0,007	-0,257	0,212	0,349	43,594	55,907	41,769
FR3L	1032	0,047	-0,629	0,761	0,608	51,113	48,790	53,596
CAC	2562	0,018	-0,195	0,292	0,794	56,691	41,904	22,229
C40	2553	0,032	-0,184	0,292	0,171	58,089	38,856	22,094

Table 3.3: Price deviation from NAV between inception and delisting or 16/08/2018 (in %)

For LETFs, this table 3.3 shows that price deviations are on average rather small, going from -0.037% in the case of LVC to 0.047% for FR3L, with 2 of them positive and 3 negative. However, this does not necessarily mean that premium or discount are always small, but rather that on average, the amount of premium and/or discount counterbalance each other. To have more information on the real pricing efficiency, we take a look at the gap between the 5th and 95th percentile and at the standard deviation. These appear to be quite large, for instance for FR3L it ranges from - 0.629 to 0.761, which is predictable for a x3 LETFs. L40 also reveals a relatively wide gap between its centiles, from -0,618% to 0,709%. Furthermore, the standard deviation of these LETFs also suggests that the funds are prone to significant discounts and premiums, even though their average is close to zero. Only PCC2 has a small standard deviation at 0.118, efficiently pricing its share. In addition, we ran an hypothesis testing on the average and we found out that it was significantly different from 0 at the 1% level only for FR3L.

In the literature, arbitrage opportunities were proved to be more complicated for LETFs than for ETFs. In this holds true, we should expect the two regular ETFs following the CAC40 of our sample to be more efficiently priced, with less discrepancy between the price and the NAV. Also, the price deviation should be larger to make it worthwile ([Charupat and Miu, 2011]). From our analysis, we observe that they do indeed have a smaller gap between centile and an average deviation close to 0%. However, in the case of CAC, the standard deviation is significant, and the pricing therefore not much more efficient than for LETFs. This is solely an indicative result. We cannot infer conclusion solely from these tables. Other factors, e.g. the fund size, should be taken into account in order to fully capture the dynamics at stake, but this goes beyond the scope of this thesis. Moreover, the data is slighly biased as the sample period is not the same for every fund. Therefore, the second sample period analysis might be more insightful.

We notice that even though the annualized volatility of each LETF is almost double the percentage of the annualized volatility of traditional ETFs (table 3.3), the price deviations from NAV are not significantly different between the ETFs and the LETFs of our sample. In other words, we cannot confirm nor disprove what the previous literature states. After running an hypothesis testing, we find out that the average price deviation for C40 is also statistically significant different from 0 at the 1% level.

Furthermore, we observe in table 3.3 that the LETFs tend to trade at a discount (except for PCC2 in the positive) while ETFs usually trade at a premium.

In brief, we cannot draw any conclusion from this figure except that traditional ETFs tend to trade at a premium more often than their leveraged counterparts. Using data from the same time period for each fund might reduce the bias caused by any specific event, for example an increase in volatility and price deviation due to the financial crisis of 2008, period during which only the funds LVC and the regular ETF existed. This might give more compelling results.

The next figure depicts the average premiums or discounts when the benchmark increases or decreases from the inception of the fund until the date of 16/08/2018.

		Average premium when	
	N	Benchmark return increases	Benchmark return decreases
LVC	2562	0,238	-0,328
L40	513	-0,030	0,005
PCC2	651	0,004	0,005
Z4L	1802	-0,057	0,042
FR3L	1032	0,012	0,083
CAC	2562	0,170	-0,142
C40	2553	0,021	0,044

Table 3.4: Average premium whether the underlying benchmark increases or decreases from inception to delisting or 16/08/2018

In the work of [Charupat and Miu, 2011], positive premium were observed for LETFs when the underlying benchmark increased and negative when it decreased, and vice-versa for inverse ETFs. However, we do not observe clear trends concerning the link between the increase (or decrease) of the benchmark and the premium (or discount) of the LETFs in table 3.4. The same holds true for the two traditional ETFs that show no clear trend. Following our analysis, it therefore seems that the daily increasing or decreasing movements on the market do not influence the direction of the pricing of the ETF on the CAC40 index, unlike for ETFs on the Canadian market as proven in the work of [Charupat and Miu, 2011].

The average premium of LVC in table 3.4 is quite astounding and its high value, both when the benchmark increases and decreases, could be explained by its longer sample size that encompasses higher volatility periods. This will be verified in our second sample period.

3.3.3.2 Price deviations from 20/10/14 to 14/06/2017

The results for the first sample period were not very conclusive. We will focus in this subsection on the analysis of the price deviation using the second sample period, the same for every fund. We hope to get more compelling results and to reduce the bias linked to the increased volatility during specific events such as the financial crisis of 2008.

The first table illustrates the average price deviation of the same ETFs except L40 (it was delisted in 2010).

	N	Average	5th Percentile	95th percentile	Standard Deviation	Percentage positive	Percentage Negative	Annualized volatility
LVC	650	0,002	-0,130	0,146	0,103	47,489	51,937	40,389
PCC2	650	0,005	-0,144	0,130	0,118	64,308	35,538	40,161
Z4L	650	-0,027	-0,195	0,097	0,247	38,392	61,002	39,863
FR3L	650	0,067	-0,650	0,840	0,700	51,804	32,364	60,784
CAC	650	0,085	-0,061	0,279	0,112	73,601	26,255	19,653
C40	650	0,090	-0,047	0,302	0,109	75,434	22,977	19,816

Table 3.5: Price deviations from NAV from 20/10/14 to 14/06/2017 (in %)

The results show that the averages are even closer to 0 in 3.5 than in 3.3. The standard deviation and the gap between the 5th and 95th percentiles are similar for each LETF, meaning that their pricing was more efficient during this time period. This may be due to the fact that the CAC40 was less volatile between 20/10/14 and 14/06/2017 than before that period. However, we notice that the average of Z4L is significantly different from 0 at the 1% level while FR3L is now significant only at the 5% level. FR3L has the highest standard deviation (0.7%) and gap between its centiles (-0.65 to 0.840). This is probably due to the fact that FR3L is triple leveraged, while the rest of the funds are 2x.

We see in table 3.5 that traditional ETFs also have a low average and percentile gap. However, the results for LETFs are lower. Again, we are not able to confirm the previous literature about pricing efficiency of LETFs. The average of the price deviation of the two regular ETFs from our panel are significantly different from 0 at the 1% level. This could suggest that there is no real pricing efficiency difference between the ETFs and the LETFs following the CAC40. But as a lot of other factors that can influence the pricing efficiency are not taken into account in the analysis, we cannot draw a clearcut conclusion on the difference of pricing efficiency between LETFs and ETFs. Nevertheless, it appears that the method used by [Charupat and Miu, 2011] is not sufficient to show a lower pricing efficiency for LETFs of our sample. Even though we notice that LETFs are on average twice as volatile as the traditional ETFs of our control group.

The table 3.5 shows that ETFs are traded at premium even more often in this time frame than the previous one, while the results for LETFs stay relatively similar to our first analysis.

The next figure details the average price deviation when the movement of the return of the underlying index (CAC40) increases or decreases.

		Average premium when	
	N	Benchmark return increases	Benchmark return decreases
LVC	650	-0,007	0,038
PCC2	650	0,004	0,005
Z4L	650	-0,045	-0,007
FR3L	650	0,015	0,123
CAC	650	0,080	0,091
C40	650	0,081	0,101

Table 3.6: Average premium whether the underlying benchmark increases or decreases from 20/10/14 to 14/06/2017

Table 3.6 shows that similarly to table 3.4, we do not observe any clear trend on the link between the increase (or decrease) of the benchmark and the premium (or discount) of the LETFs. Using another sample of data did not provide additional answer in that respect. LVC has now a relatively normal average premium in both cases, due to its sample not incorporating volatile periods.

3.3.3.3 Cross sectional behaviours of premiums and discount

We now turn to the analysis of the cross-sectional behaviour of our sample. This methodology follows the work of [Charupat and Miu, 2011]. In order to do so, we compute the correlation between the premium of each ETFs and look for an underlying trend on the pricing mechanisms of these funds. The next graph shows the correlation table between the price deviation of each ETF on the period going from 20/10/14 to 14/06/2017.

	LVC	PCC2	Z4L	FR3L	CAC	C40
LVC	1,000	0,213	0,234	0,261	0,330	0,286
PCC2		1,000	0,883	0,111	0,198	0,118
Z4L			1,000	0,141	0,212	0,193
FR3L				1,000	0,197	0,116
CAC					1,000	0,426
C40						1,000

Table 3.7: Correlation of the price deviations of ETFs (premiums/discounts)

Unsurprisingly, we observe in 3.7 that the price deviations are all positively correlated to each other. However, the correlations are not very strong, all below 0.5. One exception is the correlation between the funds Z4L and PCC2 at 0.883, this is probably because these two funds are both managed by ComStage and just based on different countries. These correlations are all significantly different from 0 at the 1% level, but they are rather weak in comparison to the results in the previous literature.

Such low correlations mean that the movements of premium and discount between LETFs following the same index are less homogeneous than in other markets. Even though there seems to be a common factor that influences the values of the premiums and discounts, it is not as strong as in the Canadian market for instance ([Charupat and Miu, 2011]). The correlation between the two regular ETFs amounting 0.426 is among the highest, suggesting that the factors that influence premium and discount values are stronger for ETFs than for LETFs.

To go further in the cross-sectional behaviour of these ETFs we need to determine whether the returns of the CAC40 are the influencing factor for the values of the premium. Although the figures on tables 3.4 and 3.6 suggest that it is not the case, we verify this intuition by computing the correlations between the CAC40 returns and the price deviation of each ETF of the sample. The results are presented in table 3.8 for the period between 20/10/14 and 14/06/2017 and in table 3.9 for the whole life span of the ETFs.

	LVC	PCC2	Z4L	FR3L	CAC	C40
CAC 40 returns	-0,44	-0,06	-0,12	-0,05	-0,10	-0,10

Table 3.8: Price deviation correlation with benchmark index return from 20/10/14 to 14/06/2017

	LVC	L40	PCC2	Z4L	FR3L	CAC	C40
CAC 40 returns	0,52	-0,15	-0,06	-0,25	-0,04	0,54	-0,11

Table 3.9: Price deviation correlation with benchmark index return

For both samples, we confirm that the correlations between the price deviation of the ETFs and the CAC40 returns are rather small. Only two of them are positively correlated in table 3.9 above the 0.5 threshold. In table 3.8, they are all negatively and weakly correlated with the benchmark returns. These results confirm that the premium and discount values are not highly influenced by the return of the benchmark.

3.3.4 Conclusion on price efficiency

We tried to replicate the analysis of [Charupat and Miu, 2011] on another market and by reducing the scope to a single benchmark, CAC40. Our analysis proved interesting in regards to the notion of the pricing efficiency of LETFs. From the previous literature, we were expecting the pricing to be less efficient for LETFs than for ETFs. However, in our analysis on the French CAC40, we found out that the values of the price deviation are not directly correlated with the benchmark returns nor with each other. This is a major difference from the results of [Charupat and Miu, 2011] on the Canadian market and from the other previous work on pricing efficiency. Moreover, we observed that the method used by [Charupat and Miu, 2011] is not sufficient to draw a clear conclusion on the pricing efficiency of the LETFs in our sample. Analyses on the difference of the pricing efficiency between the treatment group and the control group are not very conclusive either, and hint at no real difference of efficiency. However, as other parameters that could potentially influence the pricing efficiency were not included in our study, we cannot clearly state that a difference does not exist.

There are several possible reasons for these poor results. First, the French market is fundamentally different from the North-American markets. Second, the poor quality data we used cannot fully capture the correlations. A last possible reason is that the authorized participant does not always behave as the literature says it should, by using the arbitrage opportunities and correct the price deviations on the market.

3.4 Long-term returns and estimations of tracking errors

We should remind the reader that the primary objective of an LETF is to deliver the return of the underlying index times a leverage ratio on a daily basis. The term **daily** in the previous sentence is key, indeed LETFs do not have any long-term constraint.

It is widely documented in the literature that the returns of the LETFs tend to underperform. It is shown that after a holding period of more than a day, the LETF would yield worse returns than expected in most cases. This section aims at analysing the long-term returns of the LETFs tracking the CAC 40 index. Estimating the tracking errors would enable us to see whether the existing literature applies on the CAC40 index for the long-term returns of LETFs. In fact, the tracking errors represents the disparity between the promised return and the actual return. An LETF would thus try to minimize the tracking errors by all means.

3.4.1 Methodology

In order to analyse the long-term returns, we decide to follow the methodology applied by [Charupat and Miu, 2011]. The returns are calculated as follows:

$$r^i_{t-1,t} = \frac{NAV^i_t}{NAV^i_t{-1}} - 1$$

where $r^i_{t-1,t}$ is the return of the fund i from $t-1$ to t, NAV^i_t is the Net Asset Value at time t and $NAV^i_t - 1$ is the Net Asset Value at time $t-1$. We compute this return on each day from the inception of the LETF until its delisting or until the 18th of June 2018 (date of data extraction). Using this equation, we then run a regression for each of our funds compared to the underlying benchmark returns, the CAC40 index return. To compute the return of the index, the equation is the same, except that we take the points of the index into consideration rather than the NAV.

Theoretically, the results of the regression we would expect for a 2x LETF delivering the promised returns on the underlying is a value for the intercept close to 0 and a slope close to 2. These would be the results of the statistical analysis if they were no tracking error. However, the theory dictates that the tracking errors increase with the length of the holding period [Avellaneda and Zhang, 2010] [Leung and Santoli, 2012], [Rompotis, 2013].

Therefore, we decided to test our regression for each fund on different timescales. We use overlapping data for holding period of a day, 7 days (5 trading days), 30 days, 90 days, and a year. This means that in our equation, $t-1$ would be equal to either the day prior, the week prior, the month prior, the semester prior, or the year prior to the date at which we compute the return. We expect return to significantly deviate from the promised return, the longer the holding period is. In practical, the slope should differ more significantly from 2 for 2x LETFs, and 3 for 3x LETFs, as the holding period increases; and the intercept should gradually differ from 0.

We draw the same regressions on our two regular ETFs, in order to analyse whether what we observe is specific to LETFs.

3.4.2 Data

We use the same data as in the previous analysis (Bloomberg). Again, we only need the evolution of the CAC40 index and the NAV per share for all of our funds. We compute these regression for the life span of each fund, from their inception to their delisting (or the 18th of June 2018).

3.4.3 Results

The figure 3.10 shows the results of the Ordinary Last Squares (OLS) analysis for the return of each ETFs against the benchmark returns. The alpha in each column represents the intercept while the beta is the slope of the regression. The results for the regular ETFs are presented at the end of the table and will be used in comparison with the results of the LETFs.

	1 day		7 days		30 days		90 days		365 days	
	Alpha	Beta	Alpha	Beta	Alpha	Beta	Alpha	Beta	Alpha	Beta
LVC	0,0164	1,9721	0,0598	1,9537	0,2429	1,8558	0,5898	1,9054	1,2882	2,0866
p-values	*0,0004*	*0,0000*	0,2218	*0,0000*	*0,0009*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*
L40	-0,0433	1,7045	-0,1471	1,7913	-0,3408	1,9125	-0,4573	2,0277	-0,9314	1,8804
p-values	*0,1751*	*0,0000*	*0,0012*	*0,0000*	*0,0000*	*0,0000*	*0,0002*	*0,0000*	*0,0007*	*0,0000*
Z4L	0,0232	1,9438	0,1018	1,9450	0,3551	1,9344	0,9305	1,9292	2,7939	2,0885
p-values	*0,0413*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*
PCC2	0,0557	1,9370	0,1449	1,9602	0,4535	1,9698	1,0562	2,0254	3,1050	2,1442
p-values	*0,0012*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*
FR2L	0,0075	2,0444	-0,0091	2,0583	-0,1338	2,0571	-0,7287	2,0315	-3,8578	1,9437
p-values	*0,7428*	*0,0000*	*0,8338*	*0,0000*	*0,1235*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*
FR3L	0,0299	2,9372	0,1099	2,9888	0,2501	3,0389	0,0060	3,1818	-1,8157	3,4110
p-values	*0,1357*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,9505*	*0,0000*	*0,0000*	*0,0000*
C40	0,0096	0,9964	0,0488	0,9937	0,2136	0,9901	0,6490	1,0025	2,7636	1,0425
p-values	*0,0000*	*0,0071*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,0000*	*0,3132*	*0,0000*	*0,0000*
CAC	-0,0003	0,9924	5,0673	0,8206	4,9236	0,8972	4,6149	0,9431	3,0725	0,9823
p-values	*0,9527*	*0,0000*	*0,0000*	*0,0013*	*0,0000*	*0,0005*	*0,0000*	*0,0012*	*0,0000*	*0,0075*

Table 3.10: OLS regression results of CAC 40 LETFs on various time frames

Firstly, we take a look at the beta coefficient, representing the slope of the line between our data points. In theory, the beta of x2 LETFs should be 2, x3 should be 3 and x1 for traditional ETFs. This would be the case if they were no tracking errors, and the ETFs would track perfectly the underlying index, the CAC 40. However, the literature dictates that we should expect the tracking error to increase with the holding period for LETFs. Table 3.10 does not confirm that result. There is no clear trend of the beta deviating from 2 (or 3 in the case of FR3L) more and more as the holding periods increases. The coefficient relatively stays close to the expected number. Except for L40 that is tracking the CAC40 in a very poor way, with a slope of 1.7 for daily returns. This figure 1.7 means that the returns have been 1.7 times the returns of the index rather than twice the returns that the funds promised. These results for the beta coefficient do not coincide with the work of [Charupat and Miu, 2011] and other studies in the literature. Indeed, the worsening of the tracking efficiency the longer the holding period is not confirmed by the data.

Let us now turn to the analysis of the alpha coefficient, the intercept. The first thing we notice is that the values of the intercepts are very high. Indeed, we would expect the intercepts to stay close to zero. However, as shown in the results they seem to increase the longer the holding period. This is probably related to the increasing tracking errors of LETFs on longer holdings. A visual representation of these regressions can be found in the appendix, it illustrates the unpredictable returns of LETFs compared to the benchmark returns, and gets worse as the holding period increases.

We can compare the results of our analysis to the figures for the regular ETFs, we see that the beta coefficient stays rather close to the expected result. Both slopes stay close to 1 with no clear trend of decreasing return with increasing holding periods. These are the results we would expected, and it shows that our control group stays relatively close to what the theory might predict. Moreover, we observe that the value of the intercept increases when the holding period increases as well, and that illustrates that even for the regular ETFs of our sample, the predictability of returns decreases with the increase of the holding periods.

To go further in the analysis, we compute a hypothesis testing for each coefficient to see if they are statistically significant at the 1% or 5% level. The H_0 hypothesis for alpha is 0 in every case and we test the beta coefficient to be either equal to 2 in the case of x2 LETFs, to 3 for FRL3 and to 1 for our two regular ETFs. The test we run is a simple Student Test on their values and standard errors. The p-values can be found in the table 3.10. However, results show that almost all the coefficients are different from their expected value at the 1% level. There are two possible reasons for this. Either our test is not accurate enough because it does not take autocorrelation into account, even though we believe this would only slightly change our results. Either the quality of the data we used is insufficient and the tracking of all these LETFs is poor, and that would explain small p-values.

A visualization of the tracking errors for LVC and for FR3L on each period of time can be found in the appendix C. Those two graphs allow us to better visualize the scattering of returns on longer holding periods for a x2 LETF and a x3 LETF.

In conclusion, the results of our regression analysis are not very satisfactory as we can neither confirm nor disprove the statements of the previous literature on the subject. Indeed, the slope of the beta coefficient does not deviate gradually from its expected value as the holding period increases, but the alpha coefficient does. The same results hold for the regular ETFs, and that does not provide any clarification. The p-values allow us to reject the null hypothesis for most of the cases, meaning that the fund in our sample does not track the underlying as we would have expected.

3.5 Summary of the analysis

This section will briefly summarize the conclusions we can draw from the three parts of the analysis we conducted on our sample. Firstly, we observed that the rebalancing needs are not steeply increasing for LETFs that track the CAC40. We showed that the rebalancing needs are dependent on the volatility of the underlying, and that coincide with the results in the literature.

Secondly, the part of the analysis devoted to the pricing efficiency delivered more surprising results. Indeed, we showed that the pricing efficiency of the LETFs tracking the CAC40 is not worse than for the traditional ETFs. The price of the LETFs in the sample are closer to their NAV than what we could have expected. Moreover, the price deviation of the sample LETFs are not strongly correlated with each other, nor to the benchmark return or magnitude. The data we used does not behave as expected, and turned out to be more unpredictable than what the previous literature was able to prove.

Lastly, the results of the OLS regression used to estimate the tracking errors were not very conclusive either. The data suggests that a longer holding period does indeed cause less predictable results for LETFs, but the poor quality of the data does not allow to draw any clear cut conclusion. Except that our sample seems to track the CAC40 in an inefficient way. These unsatisfactory results can have various reasons: the size of our sample is too small to provide relevant results, our test analysis was not strong enough, the LETFs of our sample are not efficient in tracking the CAC40 consistently. Since previous studies have shown that investors tend to hold their LETFs for too long, the unpredictability might prove risky when investing in LETFs.

The contribution of this analysis to the literature can be summarized in three sentences: CAC40 LETFs returns seem not to behave following what the previous literature would predict; CAC40 LETFs are for the moment not a threat for the index itself; CAC40 LETFs can be harmful to investors if they hold them too long.

Conclusion

This report studying LETFs tracking the French index CAC 40 tried to grasp all the important aspects composing this type of funds. We started with a theoretical part explaining what LETFs are and how they work. Then, we presented a literature review of the relevant work published on the subject. And lastly, we performed our own quantitative analysis in three parts.

The data was not easily available and of poor quality but we could nevertheless come to some interesting results. None of our three different analyses confirmed the conclusions of previous studies: the rebalancing needs are not increasing as much as expected, the price does not deviate as much from the NAV as we would have thought and the estimations of the tracking errors were not really conclusive and rather unpredictable.

However, some results did confirm the statements of the previous literature. The rebalancing needs are linked to the volatility of the underlying, and the longer the holding period of an LETFs, the more unpredictable the returns would be.

This thesis aimed to cover the major parts in order to understand the functioning of LETFs. However, it is not exhaustive. Among others, we did not tackle the concern related to the volatility on the market at the end-of-day, nor did we compare LETFs with inverse ETFs which would probably behave differently[1]. These interesting topics could be the subject of further studies.

[1] Additional literature review on these subject can be found in appendix B)

In conclusion, my personal view is that LETFs are considered to be complex and dangerous products, however I do believe that this is an exaggeration of the reality. In my opinion, the volume that LETFs represent in the market is too small to possibly have an impact on the stability or volatility of the whole market. They are gaining importance on the market and if handled with caution by informed investors, I am convinced that they can be profitable.

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Appendices

Appendix A

Madhavan model of rebalancing needs

- Notation:
  - S_n = index level on day n = 0, 1,…, N
  - $r_{n\text{-}1,n}$ = return on underlying index from date n-1 to n
  - A_n = Leveraged ETF's NAV on day n
  - L_n = Notional amount of total return swaps required before n+1
  - x = Leverage factor (x = -3, -2, -1, 2, 3)
  - E_{n+1} = Exposure of total return swaps on day n+1
- Derivation

$$L_n = xA_n$$ Notional amount of swaps

$$E_{n+1} = L_n(1+r_{n,n+1}) = xA_n(1+r_{n,n+1})$$ Exposure increases with returns

$$A_{n+1} = A_n\ (1 + x\, r_{n,n+1})$$ AUM increases with returns

$$L_{n+1} = xA_n\ (1 + x\, r_{n,n+1})$$

$$\Delta_{n+1} = L_{n+1} - E_{n+1} = A_n\ (x^2 - x)\, r_{n,n+1}$$ Hedge is change in exposure needed

Appendix B

Addition to the Literature Review

B.1 Relation with the volatility of the underlying

As researchers have often stated, volatility is a key element on the subject of LETFs. A volatile underlying index or ETF would result in poorer performance of the fund due to their path-dependency and their rebalancing need. This is the so-called volatility decay and is directly linked with the path-dependency of LETFs' returns. Furthermore, LETF trading has, according to certain academics, a non negligible impact on the volatility of the underlying assets themselves. A lot of studies have also been conducted on this subject and try to demonstrate whether or not LETFs increase the volatility of their underlying assets/ETF/index. This is linked to the fact that, as the fund need to rebalance everyday at the end of the day to keep the same leverage ratio, LETFs' rebalancing may well be responsible for a substantial increase in the end-of-the-day volatility that's been observed on various financial markets.

An early paper that emitted the theory of LETFs having a dangerous impact on the volatility of the financial market was [Cheng and Madhavan, 2009]. This study claims that in addition to the returns of LETFs being directly dependent on the volatility of their underlying index, these funds also would induce the volatility of these indices toward the close of the trading day. This would be due to rebalancing need. To find that out, they based their analysis on market-on-close volumes.

A further study [Trainor, 2010], claims the opposite and argues that the increased volatility of the S&P 500 in those days (of the financial crisis) were not caused by leveraged ETFs trading. The researcher studied whether or not the momentum of prices at the end of the day were linked with ETF trading and found out that the volatility spikes at the end of the day related to rebalancing of LETFs were consistent and did not differ from the one unrelated to the issue (the one 30 minute prior). Moreover, he argues that, since the financial crisis, the number of LETFs continued to grow, while volatility decreased, and that this level of volatility during the crisis was expected and weren't linked to LETFs, even though their number had risen tremendously just previously. He calls the issue a "spurious coincidence".

Subsequent work conducted on the Canadian LETFs market [Charupat and Miu, 2011] shows as its results that the premiums of bull (bear) ETFs all negatively (positively) correlate with the returns on their own benchmarks. From this observation, they conclude that the end-of-day rebalancing exposure linked with LETFs trading is (at least in part) responsible for the increase in volatility towards the closure of the trading day.

[Chen and Diaz, 2012] publish about the spillover effect on return and volatility of LETFs on their underlying indices and vice-versa. It is a study of the bilateral effects of volatility on both the index and its related LETFs. Using complicated mathematical models, they find a strong relationship between the lagged returns of LETFs and the current returns of their indices as well as strong evidence on the claim of leveraged ETFs causing higher volatility.

[Curcio et al., 2012] tests the impact that the introduction of (leveraged) ETFs on the real estate market has on the volatility of real estate stock prices. After analyzing the data on 10 real estate (related) ETFs and comparing the volatility of stocks over 64 trading days before and after the inception of these ETFs, they came to the conclusion that not only traditional ETFs caused a statistically significant increase on the volatility of real estate stock prices since their introduction, but leveraged ETFs were the ones causing the highest increase in volatility, even approximately tripling the volatility in the underlying real estate securities in the case of LETFs tied to the Dow Jones US Real Estate and Financial Indices.

Also collecting data on the financial market, the study of [Bai et al., 2012] examines the impact of 6 such leveraged ETFs on 63 real estate sectors stocks. The hypothesized theory that LETFs increase late-day volatility (due to their rebalancing and due to predatory market participants that increase this effect by trying to capitalize on this predictable volatility and momentum increase each day) is verified by their empirical results. Indeed, they find a strong relationship between the rebalancing demand and the component stock returns which implies that absolute rebalancing demand explains late-day component volatility. They find out that this effect is stronger on smaller stocks that are less actively traded. They conclude by stating that the positive relationship between the rebalancing of leveraged ETFs and stocks' late returns is reversed in the first two hours of the next day.

[Shum et al., 2015] study the same hypothesized theory. They sample a balanced panel of individual stocks that closely track the S&P 500 index as well as 52 large-cap leveraged ETFs, for the period surrounding the financial crisis. They then conducted an empirical analysis (2006-2011) to see whether or not there is an actual relation with end of day volatility and leveraged ETFs trading. They came to the realization that "end-of-day volatility was positively and statistically significantly correlated with the ratio of potential rebalancing trades to total trading volume". They state that this effect is larger on more volatile days.

According to [Yagi and Mizuta, 2016], empirical results cannot be trusted concerning the impact of leveraged Exchange-Traded-Funds on the markets, as there is various phenomena and factors that influence pricing, and it would therefore be realistically impossible to isolate a single factor causing that price momentum and volatility. That's why they conducted an artificial financial market simulation on a computer, to try and assess the actual single-handed impact of LETFs on their underlying assets. The results are rather concerning, as they found out through this simulation that not only LETF trading leads to higher market volatility, but that "if the leveraged ETF trading impact on the (underlying) market is greater than that of ordinary volatility, then the market may be destroyed". They have not yet found what level of LETF trading would be enough to snuff out a market.

B.2 Optimization, Strategies, and predictive models for trading LETFs

While the previous studies and work seen in this paper were more focused on empirical evidence on how the LETFs worked and what implication they would have for investors and financial markets, this section focuses more on studies that attempt to optimize investments in LETFs. As previously mentioned, those funds are more suitable for short-terms investments and daily holding periods. It has been showed that these funds would typically underperform on longer holding period. However, many researchers tried to come up with a strategy that would let investors get away with holding these shares longer, and with combination that would outperformed the index these LETFs are based on, whereas holding a single LETFs would lose investors money due to the volatility decay.

[Giese, 2010], mathematically demonstrates that "dynamic leveraged long or short trading strategies shows a clear trade-off between exploiting the potential of higher returns,(...), and adverse losses owing to the volatility of the underlying, which is proportional to the leverage squared". This to them proves that there exists an optimal leverage ratio. They compute these ratios with various market parameters and on two different timeframes by simulating the results of long and short trading strategies on the EUROSTOXX 50. They conclude that the optimal leverage strongly depends on market conditions and would be higher in bullish market, and lower in bearish market.

Subsequent work about strategic shorting trade of LETFs was conducted by [Dobi and Avellaneda, 2012]. After reassessing the conclusion that LETFs fail to perform as expected on most cases, on holding period longer than one day, they got the idea of constructing shorting trades intended to capture this slippage or decay of LETFs. In order to compute the real potential of this strategy, they had to take into account the costs of borrowing rates needed to short these LETFs and compare it to the slippage. They made this calculation on 21 trades and found out that in 16 of these cases the mean value of the difference was strictly positive with a 95 confidence interval, meaning that the slippage amount exceeded the costs of borrowing in most cases. They conclude that the shorting of LETFs yields an arbitrage possibility, due to the predictable negative expected returns that LETFs have compared to their benchmark indices.

[Leung and Santoli, 2012] first introduce the concept of admissible leverage ratio for leveraged Exchange-Traded-Funds, as well as the concept of admissible risk horizon. Using complex statistical and mathematical formulas, they give guidelines on risk control and proper selection of LETFs and associated strategies under various risk measures, namely the Value at Risk (VaR) and the conditional Value at Risk (CVaR). They propose a stop-loss exit strategy that they deem the best suited when holding LETFs. They also state that holding short LETFs would be a winning strategy under certain circumstances (High realized variance and no strong trends for the underlying). This is a strategy that plays on the expected value erosion of the LETFs, this volatility decay.

On a slightly different note, a simulation conducted by [Jiang and Peterburgsky, 2013] analyzes trading strategies involving (3x) leveraged bull and bear ETFs tracking the S&P 500. They simulated daily returns over a 48 year period and came up with a strategy involving "shorting the bear triple-leveraged ETF and the bull triple-leveraged ETF in a 2:1 proportion, and simultaneously holding Treasuries long." which presumably outperforms in an outstanding manner the underlying index, and therefore showing that a profitable long-term strategy for LETF is possible.

A further study on the subject conducted by [Guo and Leung, 2015] constructs a benchmark process accounting for volatility decay of LETFs. They use it to analyze the performance of several commodity LETFs and unsurprisingly find that these LETFs underperform compared to their benchmark. They introduce the idea of "realized effective fee" which would be the discrepancy between the benchmark's- and the LETFs' returns. With that in mind, they propose some strategies that could benefit from this predictable volatility decay. These researchers mostly extended and analyzed more in depth the known strategy of statically shorting both long and short LETFs, by applying it to the case of LETF pairs with asymmetric leverage ratios. They find out that while this strategy is probably profitable, it could be negatively impacted by tracking errors and/or big movement of the underlying index. It also depend on the holding horizons.

[Trainor and Gregory, 2015] were the first to discuss option strategies on leveraged ETFs. In their paper, they investigated the "interchangeability of S&P 500 ETF options with leveraged S&P 500 ETF options and to what extent these options allow investors to manage their risk exposure". By deriving and comparing call-call and put-put parity for option on LETFs and on their underlying index, this study's main finding is that these options are indeed interchangeable. Their empirical data suggests that they yield similar results. This finding however, does not hold true on longer time period, where result can differ from one another a bit more. They thus states that both a covered call and a protective put strategy on LETFs would reduce risk.

[Leung and Park, 2016] later produce a paper on a rather technical basis, developing an analysis based on the 'Martingale Extraction Approach". Thanks to that tool and their statistical and mathematical development, they are able to model the long-term growth rate of expected utility of LETFs. Deriving from this formulas, they are also able to determine an optimal leveraged ratio as well as to determine the effects of various parameters on the model. According to them, this model and these results are quite compelling, not only for retail investors or professional traders that look to know the long term performances of LETFs on the market, but also for regulators and ETF providers that have the same vested interest in knowing these predictions.

Another study trying to set up a model in order to predict future returns of LETFs is the one of [Abdou, 2017]. He tries to construct an ARIMA (Autoregressive Integrated Moving Average) that applies to the case of LETFs in order to account for volatility decay. He verifies his theoretical hypothesis empirically and conclude that his model can indeed account for volatility decay and that it can capture the intensity of this decay and its direction.

[Hessel et al., 2018] recently published a paper examining the strategy of shorting a pair of leveraged and inverse ETF of the same index together. This the strategy that bets on the mean reversion of the underlying. Theoretically it would profit when the underlying benchmark moves in a mean reverting way, and would be at a loss when the underlying index moves in a trending fashion. The higher the volatility, the higher the profit for this strategy. Profit for shorting LETFs pair would not depend on the direction of the movement of the benchmark, only on its movement fashion. They empirically tested their strategy on 6 ETFs pair each related to a different underlying benchmark index and found out that on 4 of these 6 markets, the shorting strategy generates in average monthly returns of more than 1%. They go further in their study and claim that these results could be enhanced by starting the strategy after a month of high volatility for the benchmark.

Appendix C

ETF tracking errors

These graphs use a logarithmic scale, meaning they compare logarithmic returns rather than raw returns. This gives us a better visual representation of the scattering of return on longer holding periods without altering the correctness of the graph nor the scale of the relation between the CAC40 and the LETF.

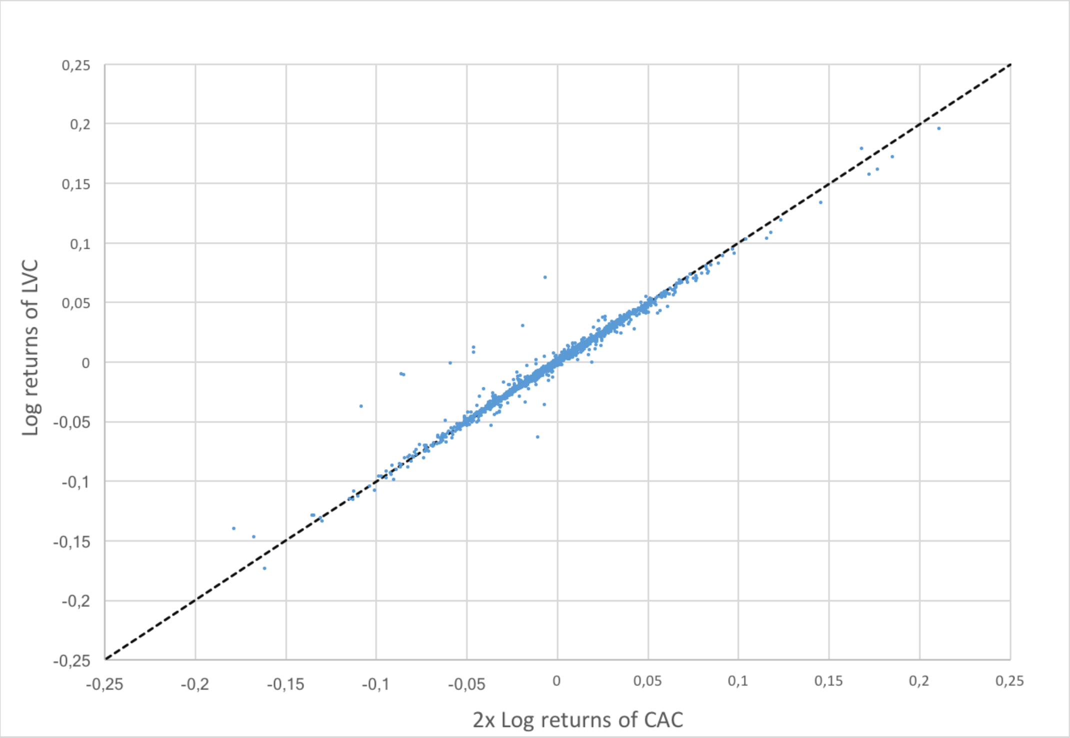

Figure C.1: Returns of LVC against CAC on a daily basis (logarithmic scale)

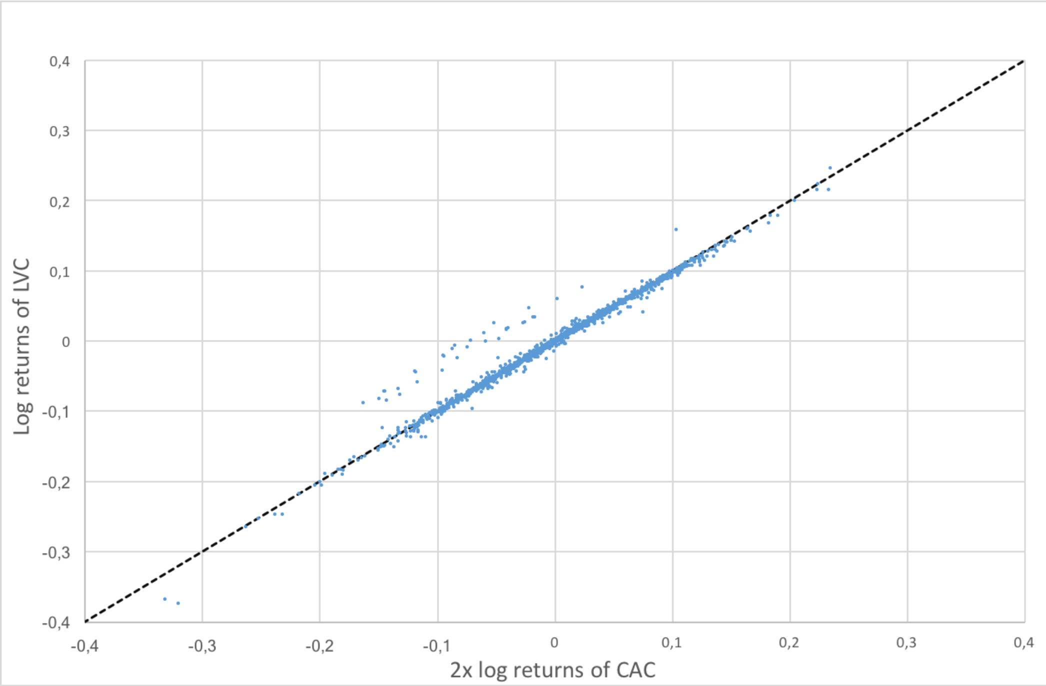

Figure C.2: Returns of LVC against CAC on a weekly basis (logarithmic scale)

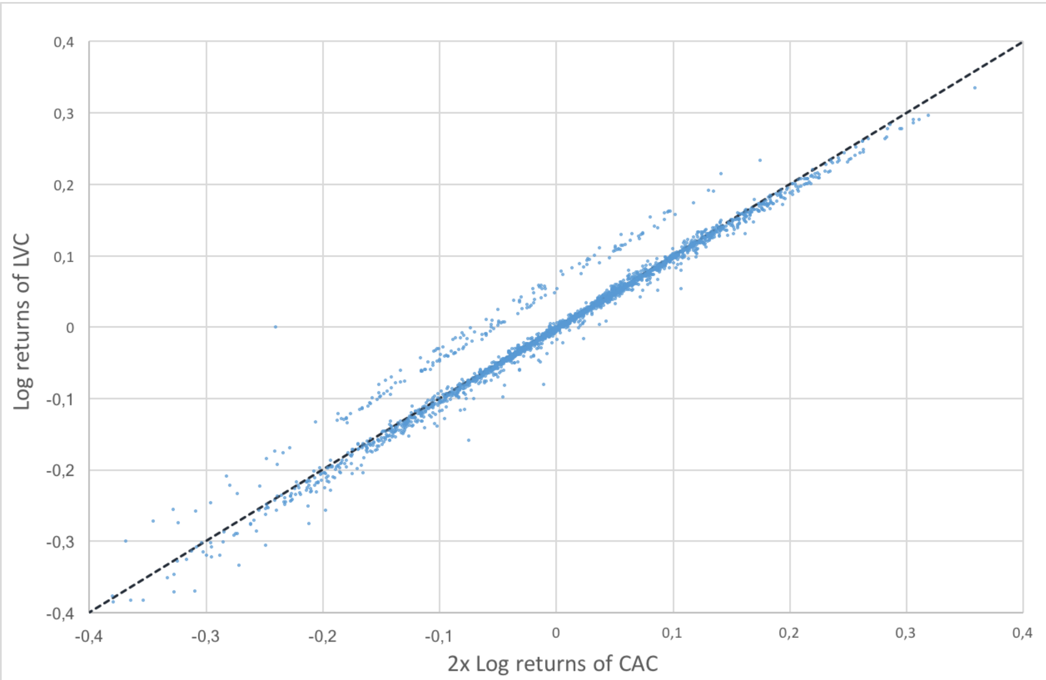

Figure C.3: Returns of LVC against CAC on a monthly basis (logarithmic scale)

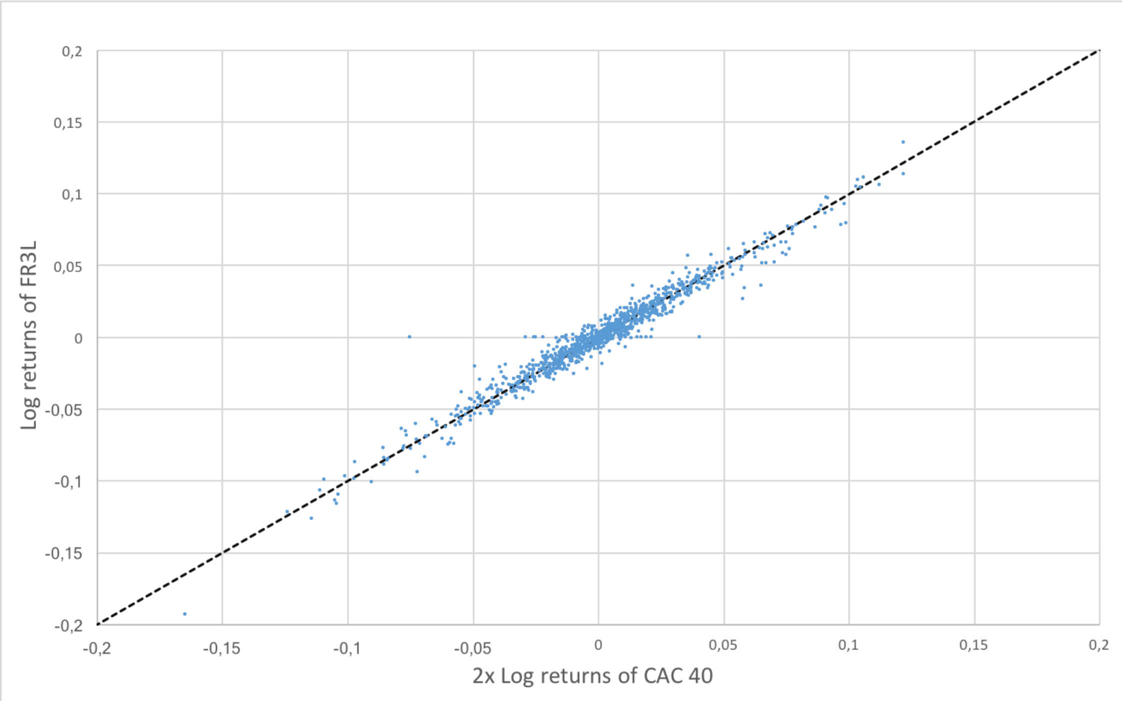

Figure C.4: Returns of FR3L against CAC 40 on a daily basis (logarithmic scale)

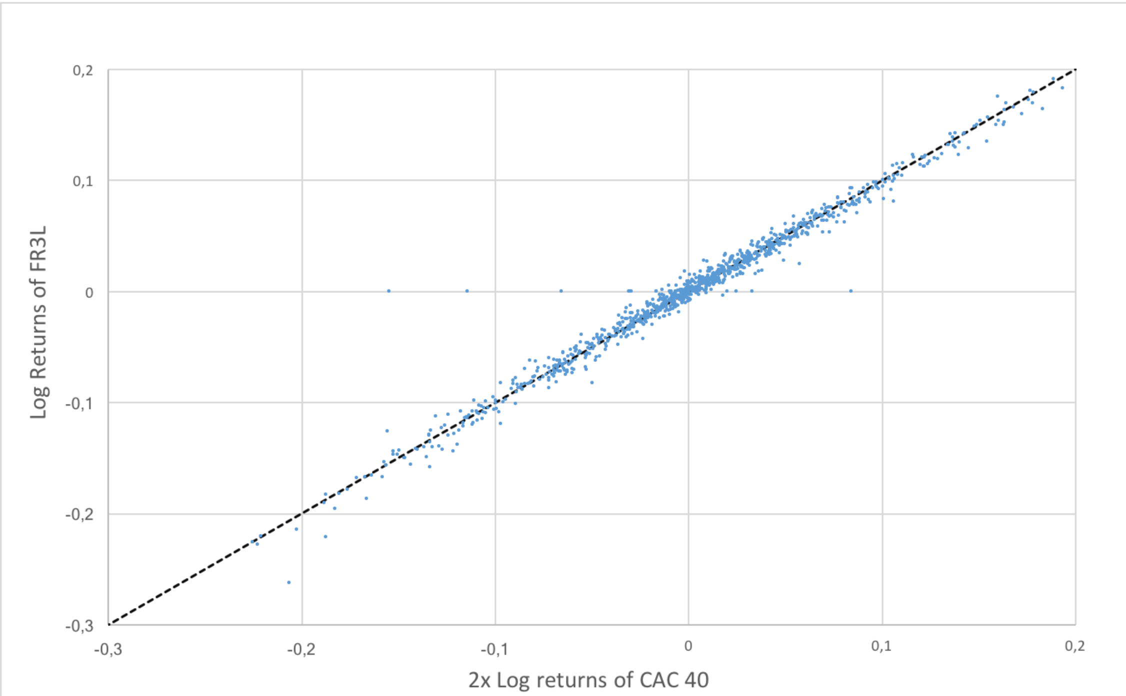

Figure C.5: Returns of FR3L against CAC 40 on a weekly basis (logarithmic scale)

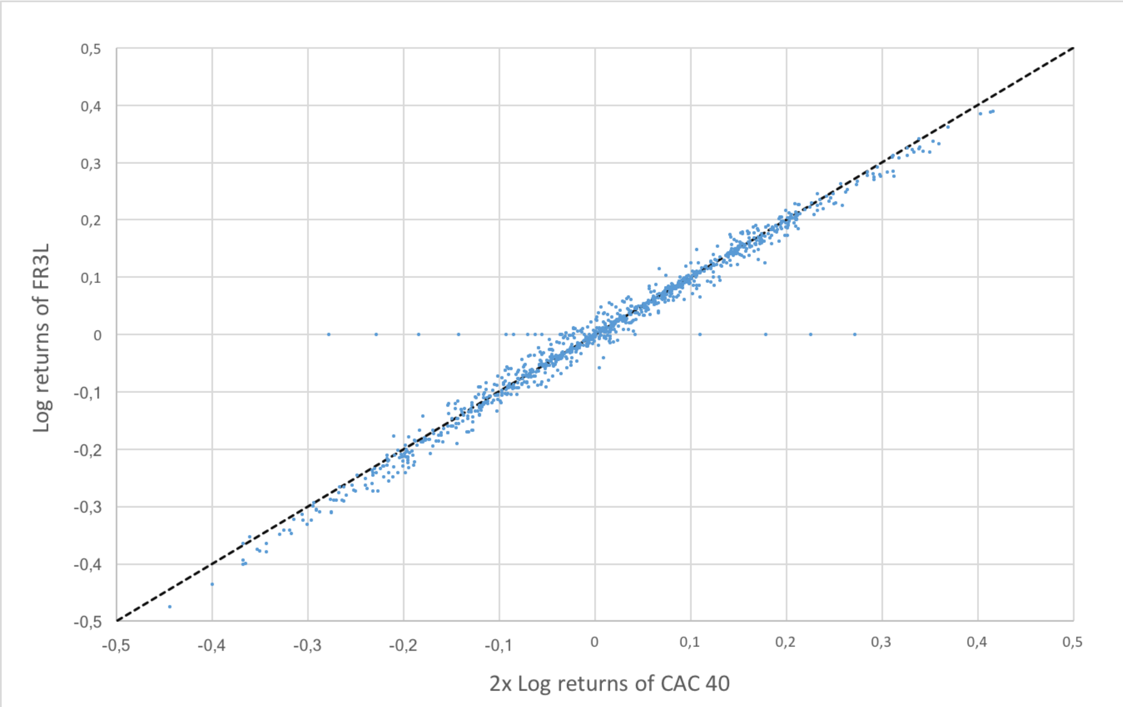

Figure C.6: Returns of FR3L against CAC 40 on a monthly basis (logarithmic scale)

Appendix D

Leveraged ETFs vs static leverage

D.1 Theory

The work of [Rompotis, 2013] theoretically compare the returns of leveraged ETFs against static leverage of the same index. It uses an example that assumes a two days timeframe. It assumes that the investors has 1 dollars and borrows $M-1$ dollars that he invests in either a statically leveraged fund or an LETF.

The returns of the statically leveraged fund would then be described by this equation in [Rompotis, 2013] :

$$\text{Value with Static Leverage} = M(1+r_1)\times(1+r_2)-(M-1)$$

With r_1 and r_2 being the returns of the index respectively on the first and second day.$(M-1)$ represent the amount borrowed and is to be repaid at the end of the second day.

If the same investor would invest his dollar in an LETF that has a leverage multiple of M, his returns would be described by this equation :

$$\text{Value with Leverage Rebalanced Daily} = 1\times(1+M\times r_1)\times(1+M\times r_2)$$

By substracting the first equation to the second, [Rompotis, 2013] finds the difference in returns between static leveraged and LETFs:

$$\delta = (M^2 - M) \times (r_1 \times r_2)$$

This equation is dependent on the leverage multiple (which are always positive in the equation), and on the returns of the index on the first or second day. There is therefore 4 different scenarios that can arise. Either both days have positive returns, or negative returns, or they can have nil returns and the last possibility would be that one of the return is positive and the other is negative.

In the first or in the second scenarios, meaning if r_1 and r_2 both have the same sign, the result of the equation is positive. That implies that LETFs perform better that their static leverage counterpart. In the third scenario then the result of the equation is nil and both of our funds have a nil return as well. Nevertheless, in the fourth scenario where the returns of the index is positive on day 1 and negative on day 2 or vice versa, then the result of the equation is negative, meaning that the LETF underperforms compared to its statically leveraged peer.

This is linked with the rebalancing mechanisms and compounding themselves, but also to the volatility of the market. In scenario 1 and 2, there is a clear trends of returns for the index either a growing market with low volatility or a recessing one with low volatility as well. The third scenario implies that the index does not change at all, thus is extremely nonvolatile while the fourth one implies no clear trend of growing or recessing and a higher volatility as the index grew on the first day and receded on the second (or vice versa). In that case the LETF will return less than the M leverage multiple promised and underperfoms against statically leveraged funds.

This theoretical analysis of LETFs vs Static leverage is consistent with the previous explanation of volatility decay and rebalancing. Even though LETFs can outperform the benchmark when the market has a clear trend and low volatility, it is rarely the case in real-world example and most empirical studies show that they underperform in the long run [Trainor and Baryla, 2008], [Avellaneda and Zhang, 2010], [Charupat and Miu, 2011], [Carver, 2009], [Guedj et al., 2010], [Leung and Santoli, 2012], [Leung and Ward, 2015].

D.2 Static vs Dynamic Leverage for our sample

These plots are merely a thought experiment showcasing the theory developed above in our sample. They show that static leverage would indeed yield better result in most cases. This however, does not take the cost of borrowing in order to implement static leverage into account, which discredits these results to some extends, and which is why this part is only an appendix to our main study.

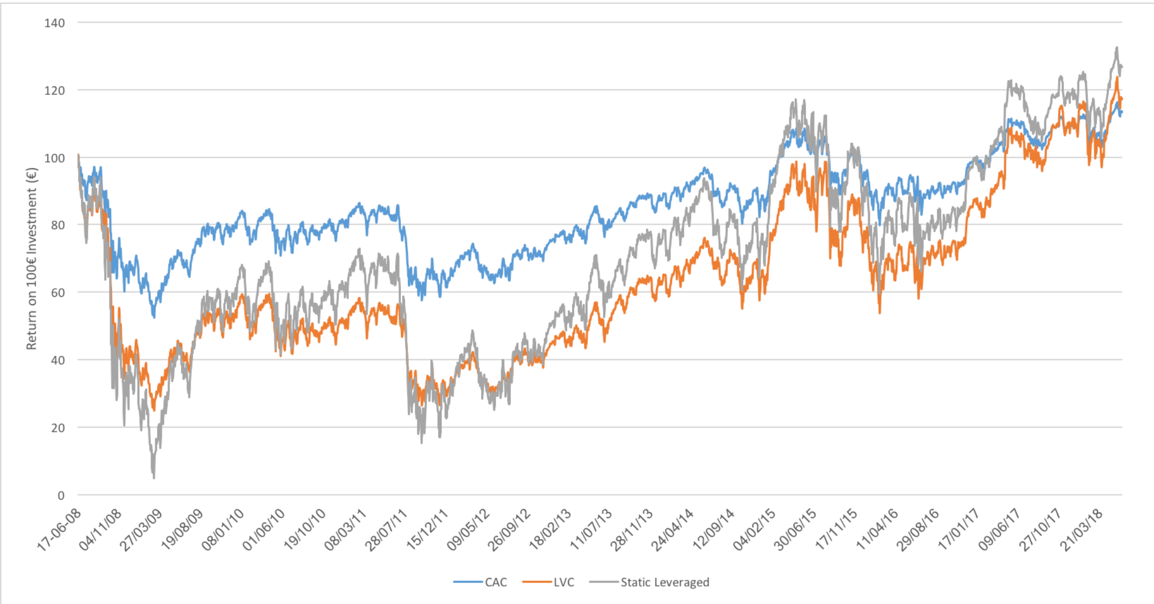

Figure D.1: Theoretical Static Leveraged returns and ETF's returns vs LVC's returns on a 100€ investment

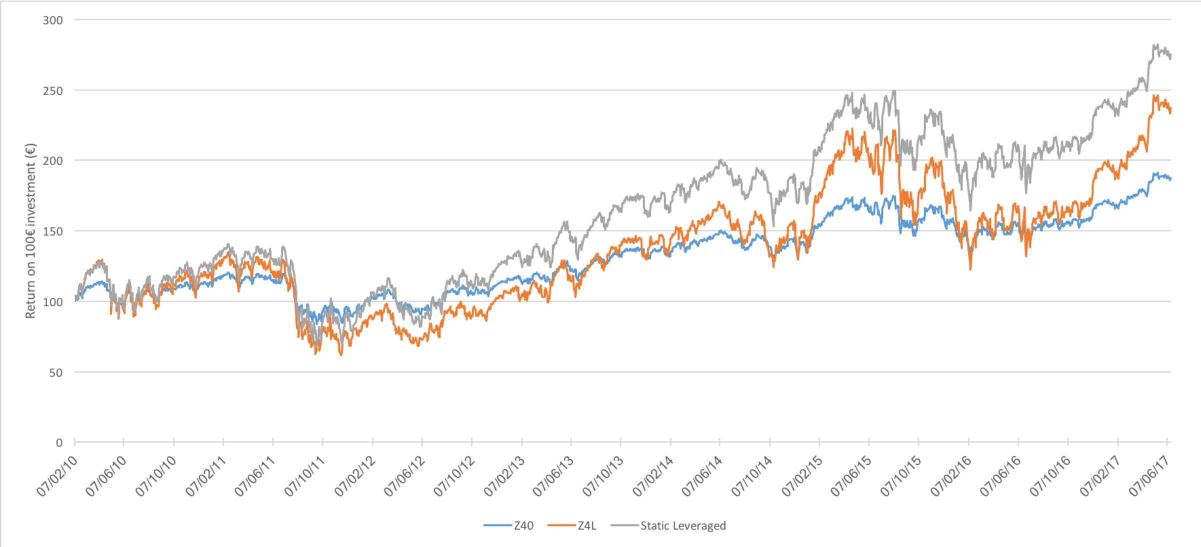

Figure D.2: Theoretical Static Leveraged returns and ETF's returns vs Z4L's returns on a 100€ investment

Figure D.3: Theoretical Static Leveraged returns and ETF's returns vs L40's returns on a 100€ investment

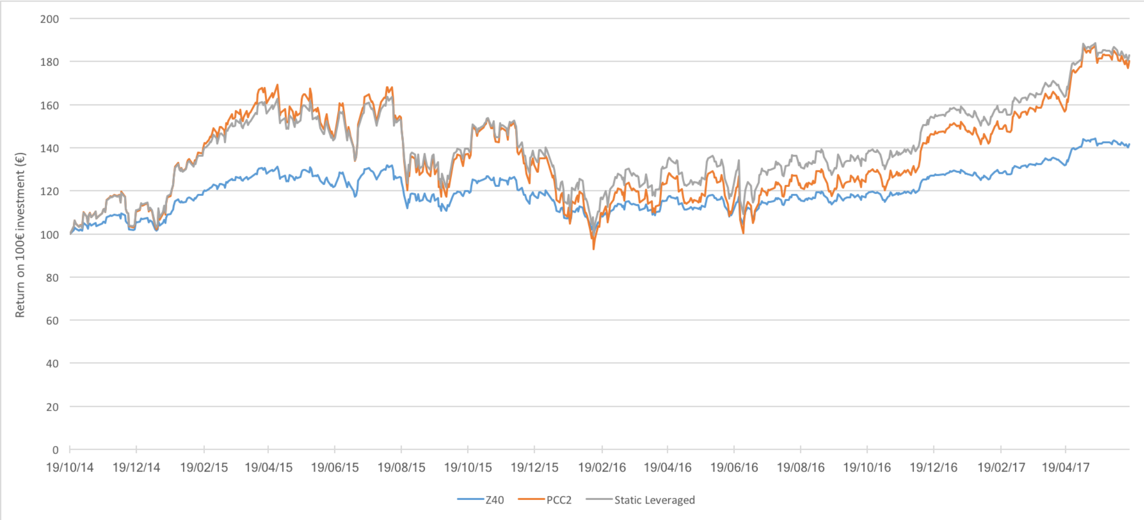

Figure D.4: Theoretical Static Leveraged returns and ETF's returns vs PCC2's returns on a 100€ investment

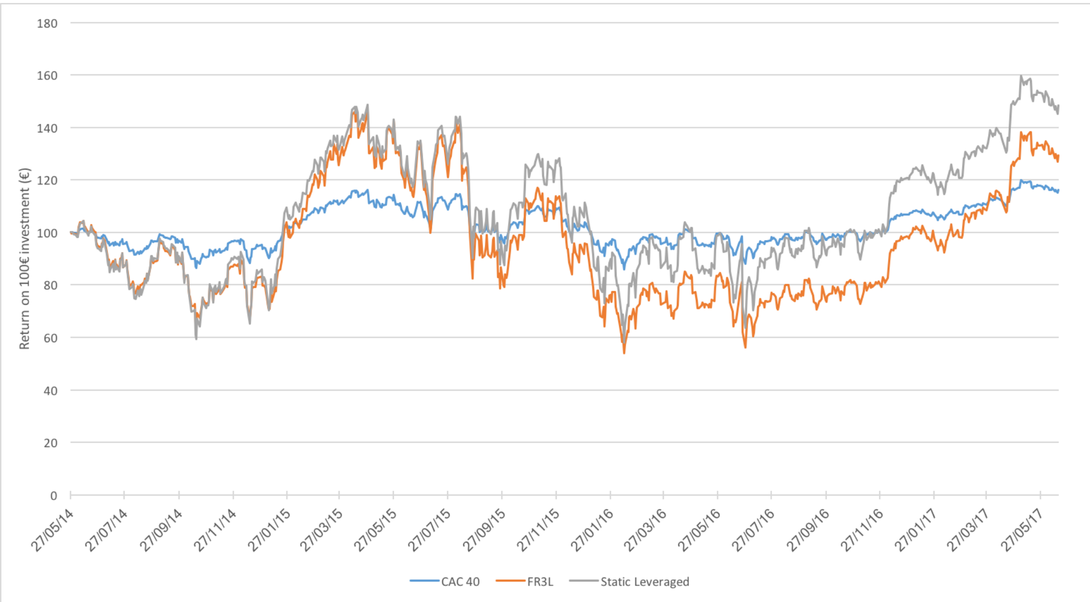

Figure D.5: Theoretical Static Leveraged returns and ETF's returns vs FR3L's returns on a 100€ investment

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