

Analog/digital hybrid precoding for SWIPT systems

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Abstract

The purpose of this research is to understand the trade-off between the communication rate and the energy harvested in a multiple-input and multiple-output simultaneous wireless information and power transfer system. This will help the conception of accurate simultaneous communications for many users and the supply of batteryless devices.

In this work, the rate and the energy harvested will be maximized, ensuring a certain level of harvested energy such that batteryless devices can be activated.

Some simulations have been launched with an isotropic normalized Rayleigh channel on three different, previously built, models of optimization namely signal coordination, multi-service coordination of a fully digital system and multi-service coordination of a hybrid precoding with an antenna array system.

The outcomes are that the rate as well as the energy harvested are increasing with the input power of the system for the three models, and that the multi-service coordination of a hybrid precoding system achieves the best performance in terms of rate (14.4 bit/s/Hz for an array of two antennas and 16.3 bit/s/Hz for an array of 4 antennas) and energy harvested (234 mJ for two antennas per array and 441 mJ for 4 antennas per array) in comparison with signal coordination (14.3 bit/s/Hz and 42.7 mJ) and multi-service coordination of a fully digital system (12.4 bit/s/Hz and 116 mJ) at an input power of 20 dBm and any threshold of activation of the harvester devices (from 1 mJ to 2.5 mJ).

This comparison shows that the multi-service coordination of a hybrid precoding system is advantageous, compared to 3G or 4G systems, in terms of communication accuracy and harvested energy for femtocell base stations in a static environment.

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1 | Symbols and acronyms

In this chapter, we will define the notations that will be used in the whole report.

1.1 Acronyms

- AR: average rate
- BS: base station
- CPPS: circularly polarized phase shifter
- DB: digital precoding
- EE: energy efficiency
- EH: energy harvester
- ER: energy receiver
- ET: energy transfer
- HB: multi-service coordination of a hybrid precoding system
- IR: information receiver
- MIMO: multiple-input and multiple-output
- MISO: multiple-input and single-output
- MS: multi-service coordination of a full digital system
- MU: multi-user
- PS: phase shifter
- RF: radio-frequency
- SC: signal coordination
- SINR: signal-to-interference and noise ratio
- SIMO: single-input and multiple-output
- SISO: single-input and single-output
- SNR: signal-to-noise ratio
- SWIPT: simultaneous wireless information and power transfer
- UE: user equipment

1.2 Symbols

1.2.1 Parameters

- N_T : number of transmitters at the base station
- N_A : number of antennas per antenna array
- N_I : number of information receivers
- N_R : number of energy harvesting receivers
- σ_I^2 : noise introduced on the information channels [W]
- $\mathbf{Id}(N)$: identity matrix of size N
- $0_{N_1, N_2, \dots}$: zero matrix of size $N_1 \times N_2 \times \dots$
- $1_{N_1, N_2, \dots}$: unitary matrix of size $N_1 \times N_2 \times \dots$
- $R_{(i,j)}$: rate arriving at $\text{IR}i$, $\forall i \in \{1, \dots, N_I\}$, where $j = \{SC, MS, HB\}$ represents the signal coordination model, multi-service coordination model of a full digital system or the multi-service coordination model of a hybrid precoding system
- $R'_{(i,j)}$: rate arriving at $\text{IR}i$ with the interferences of the previous iteration (ν) for the model j : $j = \{SC, MS, HB\}$ representing the signal coordination model, multi-service coordination model of a full digital system or the multi-service coordination model of a hybrid precoding system
- $\mathbb{T}_{(i,j)}(f(x))$: i^{th} order of Taylor's development of $f(x)$ where $j = \{SC, MS, HB\}$ represents the signal coordination model, multi-service coordination model of a full digital system or the multi-service coordination model of a hybrid precoding system
- $\mathbb{E}(\mathbf{x})$ stands for the mean of the vector $\mathbf{x}$
- $\mathbf{A} \times \mathbf{B}$ represent the vectorial product of $\mathbf{A}$ and $\mathbf{B}$
- $\mathbf{A} \cdot \mathbf{B}$ represent the scalar product of $\mathbf{A}$ and $\mathbf{B}$
- $\mathbf{A} \succeq 0$ indicates that $\mathbf{A}$ is a positive semidefinite matrix
- $\Re(a)$ is the real part of a
- $\|\cdot\|_F$ is the Frobenius norm
- $\angle \mathbf{A}$ is the phase matrix of $\mathbf{A}$

1.2.2 Vectors and matrices

Each vector will be represented by a bold lowercase letter ($\mathbf{a}$), and each matrix by a bold uppercase letter ($\mathbf{A}$).

When the same letter is used for a vector and a matrix, this means that they are linked by the following relation : $\mathbf{A} = \mathbf{a}\mathbf{a}^H$, where the vector $\mathbf{a}$ is represented by a $N \times 1$ vector, and $\mathbf{a}^H$ denotes the conjugate transpose of $\mathbf{a}$.

When a vector is complex, it belongs to the $\mathbb{C}$ class of vectors. When it is hermitian, it belongs to $\mathbb{H}$.

Mostly the following vector and matrix are used :

- $\mathbf{w}_I \in \mathbb{C}^{N_T \times 1}$ is the information vector

- $\mathbf{w}_i \in \mathbb{C}^{N_I \times 1}$ is the information vector intended for IRi, $i \in [1, \dots, N_I]$
- $\mathbf{w}_E \in \mathbb{C}^{N_R \times 1}$ is the energy vector
- $\mathbf{h}_i = [h_{i1} \ h_{i2} \ \dots \ h_{iN_T}]$ is the channel vector pointing toward Rx*i* with $i \in \{1, 2, 3\}$
- $\mathbf{h}_{i,inf} = [h_{i1} \ h_{i2} \ \dots \ h_{iN_I}]$ is the channel vector pointing toward Rx*i* and carrying information, with $i \in \{1, \dots, N_T\}$
- $\mathbf{h}_{i,en} = [h_{i1} \ \dots \ h_{iN_R}]$ is the channel vector pointing toward Rx*i* and carrying energy, with $i \in \{1, \dots, N_T\}$
- $\mathbf{a}_{hi} \in \mathbb{C}^{N_I \times N_I \times N_A}$ is the phase shifter vector related to the information vector, with $i \in \{1, \dots, N_I\}$
- $\mathbf{a}_g \in \mathbb{C}^{N_A \times N_A}$ is the phase shifter vector related to the energy vector

2 | Introduction

For a few decades, research in the field of mobile telecommunications has led to many commercialized applications. It all started with the GSM system, followed by the WCDMA (wideband code division multiple access) and the LTE (long term evolution) systems. The last two are mostly known as the 3G and 4G technical systems. Nevertheless, customers wish to be able to connect more and more devices to the network. Indeed, in addition to their mobile phone and smart watch, they wish to connect their cars, security system, home sensors and other low power electrical devices. The inter-networking of the devices constituting this emerging wireless ecosystem is often referred to as the Internet of Things.

As a matter of fact, many devices, such as sensors, are energy-constrained and need to be powered by batteries. These have a limited operation time, and therefore need to be replaced or recharged. Such operations are costly and may be hazardous (toxic environment) or impossible (sensor embedded in human body or building structures). In this scenario, energy harvesting will provide constant energy supplies in a *greener*, safer and more practical way [2]. Among the different energy harvesting technologies, this work will focus on radio frequency (RF) energy harvesting that is founded on the ability of converting the energy carried by an electromagnetic wave through the air into electrical energy at the point of use. In particular, we will propose a technology that will not only increase the network capacity and speed, but will also deliver energy wirelessly to low energy devices equipped with harvesting capabilities. Indeed, many harvesting devices cannot be activated only by using ambient electromagnetic waves, so by sending energy, this system will help to harvest more energy. Unfortunately, sending energy will add interferences to the communication system, so the incoming model will need coordination between its RF chains to reduce the interferences. The coordinated transmission of information and energy over the same wireless resources is also known as simultaneous wireless information and power transfer (SWIPT).

The major objective of this thesis is to shed a light on the following fundamental questions:

- How to increase the number of users without reducing the rate quality?
- How to exploit a multi-antenna wireless system to improve the harvested energy and reduce the interferences?
- How to improve the energy transfer efficiency by reducing the number of RF chains?
- How to reduce the complexity of the algorithm of the multi-service coordination system?

An already implemented solution for rate improvement in *information-only* wireless systems is the joint use of antenna arrays and digital precoding at the transmitter side. However, this solution cannot be exploited alone for SWIPT systems that might require a relatively large number of antennas. Indeed, digital precoding requires one RF chain per antenna which would increase the size and the total power consumption of base stations and access points, thus raising concerns about the costs and the practicality of a fully digital SWIPT solution. A second solution would be to use SWIPT systems combined with *multi-service coordination of hybrid precoding* models. Actually, SWIPT system would bring the possibility to simultaneously send information

and harvest energy, and the *hybrid precoding* model would enable to allocate smartly the power between all transmitters and to direct the energy and information signals while reducing the number of needed RF chains.

This technique raises the following questions : how to implement such a precoding? In which situation/configuration of the antennas is energy harvesting feasible? What is the trade-off between information rate and energy harvested? Would all users be well served?

Previous work to answer these questions has been published. Reference [1] defines the concept of energy harvesting, and in which kind of environment we may harvest energy. Then, reference [2] gives an overview of what is a SWIPT system and how to maximize the transfer of energy with MIMO systems.

Next, reference [3] investigates the number of RF chains needed and reference [4] introduces some models of hybrid precoding. Besides, a first model combining both rate and energy maximization is proposed in reference [5].

Starting from those pieces of information, we have built updated models to improve the rate at the information receiver and the energy harvested at the batteryless devices. The first one is based on reference [5], and takes into account the interference between the users and between the users and the energy harvester. We have called it the *signal coordination* model. Then, we have modeled a *multi-service coordination for a full digital system*. In this, we separate the system conveying the information and the one conveying energy. Finally, we have added phase shifters to the previous model in order to have a *multi-service coordination for hybrid precoding systems* considering both interferences and energy harvesting.

This thesis presents first a list of all the notations used, thereafter a state of the art of the above cited concepts and models. After, the developed models and the corresponding results will be discussed. Finally, we highlight the principal conclusions drawn from our research.

3 | State of the art

In this chapter, we will first introduce the concept of energy harvesting and then make the link with SWIPT systems.

Second, we will use this concept to define what is a **signal coordination**, a **multi-service coordination** in both **full digital** and **hybrid precoding** system.

Then, for the specific case of the hybrid precoding system, we will discuss several architectures, and the number of RF chains needed to achieve the same performances, in terms of rate, as for the signal coordination model.

3.1 Energy harvesting and SWIPT systems

Energy harvesting has become a hot topic in the field of telecommunication. Indeed, it allows to collect ambient energy to supply batteryless devices such as indoor/outdoor sensors, RFID's and tags. It hence reduce significantly the number of batteries needed to be recharged or replaced. To understand fully this principle, let's define the term energy harvesting:

Definition 3.1.1. Energy Harvesting [6]: the process of capturing minute amounts of energy from one or more energy sources (e.g., thermal, light, wind,...), accumulating them and storing them for later use.

An energy harvester device will hence be composed of a rectifier with a diode, allowing to store energy. This nonlinear system will store energy until there is enough to activate it. The minimal required amount of energy to be transferred to the device such that it can perform its task is called the sensitivity, and is noted T [1].

In our work, the targeted devices cannot be activated only by collecting ambient energy; hence, we will build a system that will send energy to reach the threshold of activation T . We will thus focus on radiatively-harvested energy in SWIPT (simultaneous wireless information and power transfer) systems.

Definition 3.1.2. SWIPT systems [2], combine a *WPT* (Wireless Power Transfer) system and a *WIT* (Wireless Information Transfer) system. The first one is used to transfer wirelessly energy from one power source to one or several electrical devices. The second refers to the transmission of information through a given channel.

In such a system, we will thus send simultaneously information and energy signals.

We can directly draw that the benefit of using SWIPT systems is the maximization of both the power transfer and the communication rate of each user.

3.2 Multi-antenna system

According to the literature [2], multi-antenna systems is a paramount technique to both have high-performances in term of energy harvested and transmission rate. Hence, combining a multi-antenna system with a SWIPT one could lead to huge increases of rate and energy harvested.

First, we will describe what is a multi-antenna system, and the links that characterize it. Then, we will study the benefits of multi-antenna systems.

3.2.1 Communication links

We will define 4 types of communication links. The first three are used to understand the meaning of the last one (multi point-to-point), that will be used in the coming considered systems to send information.

First let's define the simplest communication link, named the point-to-point link.

Definition 3.2.1. Point-to-point communication (SISO) [9]: A point-to-point link is a dedicated link between one transmitter and one receiver (see FIGURE 3.1a)

Since a multi point-to-point communication links is the combination of multipoint-to-one links and one-to-multipoint links, we will define those two in terms of the point-to-point communication link.

Definition 3.2.2. Multipoint-to-one links (MISO): Several transmitters have a one-to-one communication link to the **same** receiver (see FIGURE 3.1b).

Definition 3.2.3. One-to-multipoint links (SIMO): Several receivers have a one-to-one communication link to the **same** transmitter (see FIGURE 3.1c).

The three above concepts are shown in FIGURE 3.1.

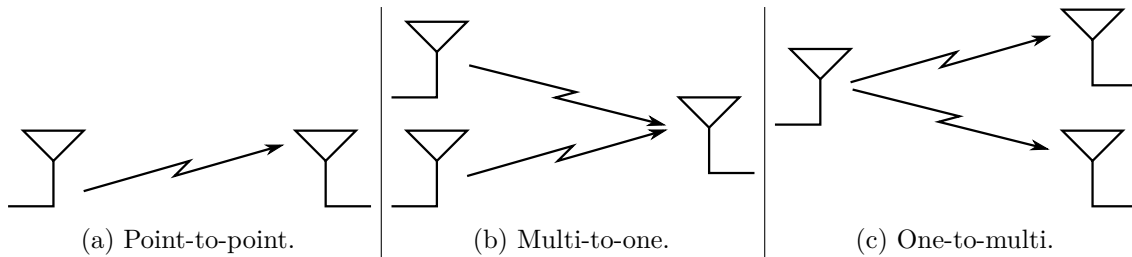


Figure 3.1: Communication links of a SISO, MISO and SIMO system.

The meaning of the **multi point-to-point** communication system (MIMO) is hence the following: the transmission points are coordinated meaning that the service provided to one user equipment comes from several transmission points, and the harvested energy comes from all the points of the system [10].

As highlighted by the authors of paper [2], MIMO systems benefit of high performances in term of rate and energy. Indeed, the use of antenna arrays allows to benefit from diversity. As a matter of fact, such system can benefit from both array gain and diversity gain.

3.2.2 Diversity

First let's introduce some specific terms related to the use of diversity.

Definition 3.2.4. Diversity branch [14]: One diversity branch represents, in communication terms, one transmission link. We will have multiple diversity branches if and only if the different links are **independent**.

From the concept of diversity branches, we can define the diversity.

Definition 3.2.5. Diversity gain [14]: Suppose we transmit the same signal through several diversity branches; the diversity gain is related to the probability that all diversity branches fade during the transmission of one signal (assuming the channel to be constant during the transmission of one sequence of symbols). If the probability that one branch fades is P_{out} and the one that all branches fade is P_{out}^n , then the diversity order is n . This means that the bit error rate drops n times as fast for an increasing SNR with respect to the one branch system (the one that has no diversity). The diversity gain is defined as:

$$g_d(\rho) \triangleq -\frac{\log(P_{\text{out}}^n)}{\log(\rho)},$$

ρ being the SNR of a single-branch. The diversity gain will be named, in the following, *space diversity* since the independent branches are provided by spatially separated antennas.

Definition 3.2.6. Array gain [15]: It evaluates the increase in average output SNR relative to the single-branch average SNR ρ .

$$g_a \triangleq \frac{\rho_{\text{out}}}{\rho}$$

In the following, we will bound the values of the two gains assuming:

- N_T antennas at the transmitter side.
- N_R antennas at the receiving side.
- The information sent is the same for all transmitters.
- The channel vector ($\mathbf{h} \in \mathbb{C}^{1 \times N_R}$) has zero-mean, circularly-symmetric complex Gaussian random variables entries. Besides, we also assume $\mathbf{h}$ to be quasi-stationary [14].
- The channel is constant over the transmission of the whole sequence of symbols, but may vary over a longer time.

With the previous assumption, the transmission can be written as:

$$\mathbf{y} = \sqrt{E_s} \mathbf{H} \mathbf{w}_t s + \mathbf{n},$$

where E_s is the symbol energy ($\sqrt{E_s}$ used to normalize the signal), $\mathbf{H} = \mathbf{h} \mathbf{h}^H$ the channel covariance matrix, $\mathbf{w}_t$ the weighting vector, s the signal and $\mathbf{n}$ the noise vector.

At the receiver side, the signals arriving are combined with a weighted sum, $\mathbf{w}_r$ being the weighting vector.

$$\begin{aligned} z &= \mathbf{w}_r^H \\ &= \sqrt{E_s} \mathbf{w}_r^H \mathbf{H} \mathbf{w}_t s + \mathbf{w}_r^H \mathbf{n} \end{aligned}$$

Array gain

In order to achieve the best performances in term of array gain, we want to maximize the SNR, in other words, we want to maximize:

$$\frac{\|\mathbf{w}_r^H \mathbf{H} \mathbf{w}_t\|_F^2}{\|\mathbf{w}_r\|_F^2},$$

$\|\cdot\|_F$ being the Frobenius norm.

We will perform the maximization with the use of the singular value decomposition of $\mathbf{H}$ [16].

$$\mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$$

$\mathbf{\Sigma}$ is a diagonal matrix enclosing the singular values of $\mathbf{H}$, written $\{\lambda_1, \lambda_2, \dots, \lambda_N\}$, N being the dimension of the channel matrix.

It can be shown, with this notation, that the SNR is maximized when $\mathbf{w}_t$ and $\mathbf{w}_r$ are the singular vectors corresponding to the maximum singular value of $\mathbf{H}$ [17] (this combining method is called *Maximum Ratio Combining* [14]). This resolution of the maximization problem is called the *dominant eigenmode transmission* and leads, in the case of an independent and identically distributed Rayleigh channel, to an array gain bounded by:

$$\boxed{\max\{N_T, N_R\} \leq g_a \leq N_T N_R}$$

Actually, it represent the minimum and maximum number of independent branches in the MIMO system.

Spatial diversity

Using the Maximum Ratio Combining technique at the receiver, we are able to bound the probability of error as a function of the SNR [17]:

$$\bar{N}_e \left(\frac{\rho d_{\min}^2}{4 \min\{N_T, N_R\}} \right) \geq P_{\text{out}}^n \geq \bar{N}_e \left(\frac{\rho d_{\min}^2}{4} \right)^{-N_R N_T},$$

$\bar{N}_e$ and $d_{\min}$ being respectively the number of nearest neighbors and minimum distance of separation of the considered constellation.

Hence, every multi-antenna systems will increase the SNR and reduce the probability of detection errors thanks to the array of antennas.

3.3 Energy-efficient SWIPT systems

We will now investigate a multi-antenna system maximizing both the energy transferred and the communication rate in a SWIPT system [5]. More precisely, we will analyze the trade-off between the maximization of the communication rate and the energy harvested.

Such a system (FIGURE 3.2) is composed of one base station, several information receivers and several energy harvesting receivers.

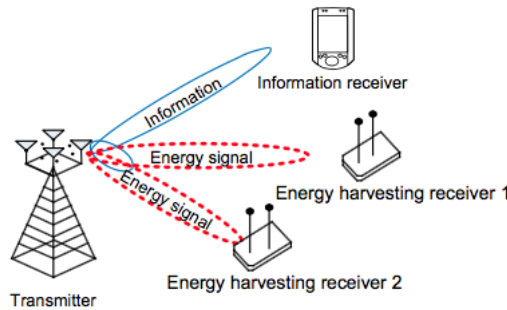


Figure 3.2: A downlink communication system with an information receiver and $J = 2$ energy harvesting receivers [5].

Besides, it sends two different types of signal:

- $\mathbf{w}_I \in \mathbb{C}^{N_T \times 1}$: the information signal.

- $\mathbf{w}_E \in \mathbb{C}^{N_T \times 1}$: the energy signal facilitating the energy transfer to the energy harvesters (EHs).

In order to set up a model for the rate and energy maximization, the authors in [5] make the hypothesis that the energy signal is known at the information receiver (IR) and the interferences it causes can be completely canceled. However, it will require a successive interference cancellation technique.

In addition, to be able to maximize both the rate and the energy harvested by a system, we need to understand what these notions are and how they behave independently and together.

3.3.1 Rate maximization

When we talk about rate maximization, we are talking of maximizing the sum rate of all users (or information receivers) in a cell of a network.

Let's begin by considering only one user (case of article [5]).

The rate can be expressed as:

$$R = B \log_2 \left(1 + \frac{\mathbf{w}_I^H \mathbf{H} \mathbf{w}_I}{\sigma_I^2} \right),$$

B being the system bandwidth and $\mathbf{H}$ the covariance channel matrix; H indicates the conjugate transpose, and σ_I^2 is the noise introduced on the information link.

In the article, the authors have chosen to analyze the achievable rate energy efficiency (AR-EE), meaning the achievable rate by the system compared to the total power allocated to it.

The total power can be expressed as:

$$P_{\text{tot}} = \frac{\text{Tr}(\mathbf{W}_I) + \text{Tr}(\mathbf{W}_E)}{\xi} + N_T P_{\text{ant}} + P_c,$$

with P_{tot} being the power dissipation of the system, P_{ant} the power dissipation at each transmit antennas, considering that each antenna relates to one RF chain [5], P_c denoting the fixed circuit power consumption, $\xi \in]0, 1]$ being the constant power amplifier efficiency, $\mathbf{W}_E = \mathbf{w}_E \mathbf{w}_E^H$ and $\mathbf{W}_I = \mathbf{w}_I \mathbf{w}_I^H$ as noted in CHAPTER 1.

The AR-EE maximization problem can thus be written as:

$$\begin{aligned} & \underset{\mathbf{W}_E \in \mathbb{H}^{N_T}, \mathbf{w}_I}{\text{maximize}} \quad \frac{B \log_2 \left(1 + \frac{\mathbf{w}_I^H \mathbf{H} \mathbf{w}_I}{\sigma_I^2} \right)}{P_{\text{tot}}} \\ & \text{subject to} \quad \text{C1 : } \text{Tr}(\mathbf{W}_I) + \text{Tr}(\mathbf{W}_E) \leq P_{\text{max}} \\ & \quad \quad \quad \text{C2 : } \mathbf{W}_E \succeq 0, \end{aligned} \tag{3.1}$$

with P_{max} being the the maximal input power allocated to the system.

The result obtained by the authors is shown in FIGURE 3.3.

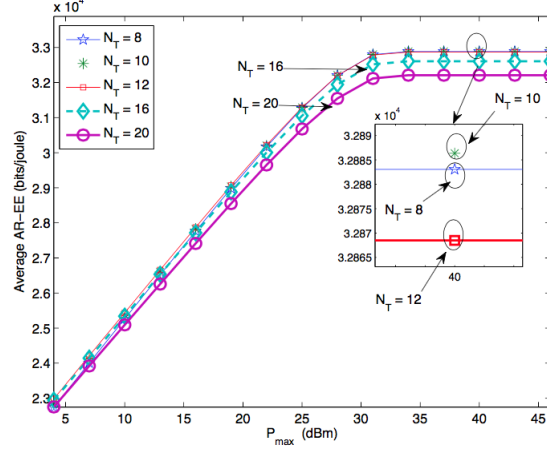


Figure 3.3: Average rate-energy efficiency (bit/joule) versus maximum transmit power budget $P_{\max}$ (dBm) for 10 energy harvesters [5].

We can see that up to 30 dBm, the average rate energy efficiency increases almost linearly with respect to $P_{\max}$. Above this threshold, it reaches a plateau since the gain in rate is compensated by the increase of the total power consumption. Besides, we also see that increasing the number of RF chains does not increase the rate. Indeed, the extra gain obtained by using more RF chains is not sufficient to compensate for the total increased energy cost (term $N_T P_{\text{ant}}$ in P_{tot}).

3.3.2 Energy maximization

Energy can be harvested from both the information and the energy signals due to the broadcast nature of the channels. The total harvested energy can thus be written as:

$$\frac{\sum_j P_j^{\text{harv}}}{P_{\text{tot}}} = \frac{\sum_j \eta_j \mathbf{G}_j^H \text{Tr}(\mathbf{W}_I + \mathbf{W}_E) \mathbf{G}_j}{P_{\text{tot}}},$$

P_j^{harv} being the total harvested energy at the energy receiver j (ER j), η_j being the energy conversion efficiency of ER j and $\mathbf{G}_j \in \mathbb{C}^{N_T \times N_R}$ being the channel matrix between the transmitter and ER j .

The maximization problem of the energy transfer energy efficiency is:

$$\begin{aligned} & \underset{\mathbf{W}_E \in \mathbb{H}^{N_T}, \mathbf{W}_I}{\text{maximize}} \quad \frac{\sum_j P_j^{\text{harv}}}{P_{\text{tot}}} \\ & \text{subject to} \quad \text{C1 : } \text{Tr}(\mathbf{W}_I) + \text{Tr}(\mathbf{W}_E) \leq P_{\max} \\ & \quad \quad \quad \text{C2 : } \mathbf{W}_E \succeq 0 \end{aligned} \tag{3.2}$$

The result is shown in FIGURE 3.4.

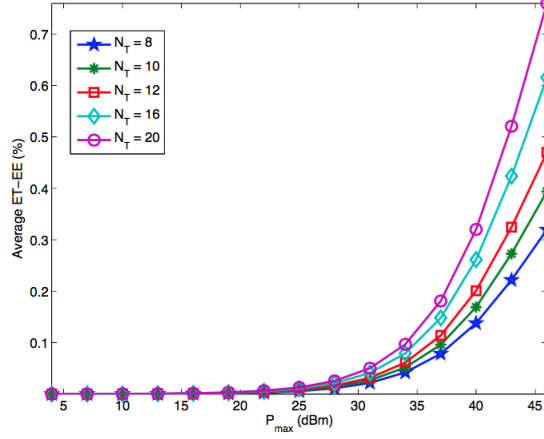


Figure 3.4: Average energy transfer efficiency versus maximum transmit power budget $P_{\max}$ (dBm) for 10 energy harvesters [5].

As can be observed, the increase is small for small values of $P_{\max}$. This is due to the dominance of the fixed circuit power consumption ($N_T P_{\text{ant}} + P_c$). Nevertheless, for higher values of $P_{\max}$, it is the total power consumption that dominates. Besides, the energy transfer increases with the number of transmit antennas because it is a quasi linear function with respect to $\mathbf{W}_I$ and $\mathbf{W}_E$.

3.3.3 Rate-energy trade-off

The concept of maximizing both the rate and the energy is introduced, ideally, to power a device, and to make use of multi-antenna processing to transmit information [7].

To solve such a problem, the authors of [8] weight the objective functions of the two previous models (equations 3.1 and 3.2).

Doing so, they obtain a multi-objective optimization problem (MOOP) :

$$\begin{aligned}
 & \underset{\mathbf{W}_E \in \mathbb{H}^{N_t}, \mathbf{w}_I}{\text{maximize}} \max_{i=1,2} \{ \omega_i (F_i^* - F_i) \} \\
 & \text{subject to C1 : } \|\mathbf{w}_I\|^2 + \text{Tr}(\mathbf{W}_E) \leq P_{\max} \\
 & \text{C2 : } \mathbf{W}_E \succeq 0
 \end{aligned}$$

with F_i^* being the optimal objective value of the problem i (in our situation of problems 3.1 and 3.2), F_i being the objective value of the problem i and ω_i being imposed on the objective function i such that $\sum_i \omega_i = 1$ and $0 \leq \omega_i \leq 1$.

This problem being non-convex, the authors modify it by the use of the Charnes-Cooper transformation in order to get a convex problem. The joint convex optimization problem can be written as:

$$\begin{aligned}
 & \underset{\overline{\mathbf{W}}_I, \overline{\mathbf{W}}_E \in \mathbb{N}^{N_T}, \theta, \tau, \gamma_j, \rho_j}{\text{maximize}} \quad \tau \\
 & \text{subject to} \quad \text{Tr}(\overline{\mathbf{W}}_I + \overline{\mathbf{W}}_E) \leq \theta P_{\max} \\
 & \quad \overline{\mathbf{W}}_I \succeq 0, \overline{\mathbf{W}}_E \succeq 0 \\
 & \quad \frac{\text{Tr}(\overline{\mathbf{W}}_I + \overline{\mathbf{W}}_E)}{\xi} + \theta P_B \leq 1 \\
 & \quad \theta \geq 0
 \end{aligned}$$

$$\begin{aligned}
& \omega_i (F_i^* - F_i) \leq \tau \\
& \rho_j \geq 0 \quad \forall j \\
& \begin{bmatrix} \rho_j \mathbf{I}_{N_T} + \widetilde{\mathbf{W}}_E & \widetilde{\mathbf{W}}_E \hat{\mathbf{g}}_j \\ \hat{\mathbf{g}}_j^H \widetilde{\mathbf{W}}_E & -\rho_j \epsilon_j - \frac{\gamma_j}{\eta_j} + \hat{\mathbf{g}}_j^H \widetilde{\mathbf{W}}_E \hat{\mathbf{g}}_j \end{bmatrix} + \mathbf{U}_{\mathbf{g}_j}^H \widetilde{\mathbf{W}}_I \mathbf{U}_{\mathbf{g}_j} \succeq 0 \quad \forall k, \forall j
\end{aligned}$$

with $\overline{\mathbf{W}}_I = \theta \mathbf{w}_I \mathbf{w}_I^H = \theta \mathbf{W}_I$, $\overline{\mathbf{W}}_E = \theta \mathbf{W}_E$, $\theta = \frac{1}{P_{\text{tot}}}$, ξ being the power amplifier efficiency, $P_B = N_T P_{\text{ant}} + P_c$, P_{ant} being the single antenna power consumption, P_c being the static circuit power consumption, $\widetilde{\mathbf{W}}_I = \mathbf{I}_{N_R} \otimes \overline{\mathbf{W}}_I$, $\widetilde{\mathbf{W}}_E = \mathbf{I}_{N_R} \otimes \overline{\mathbf{W}}_E$, $\hat{\mathbf{g}}_j = \text{vec}(\hat{\mathbf{G}}_j)$, $\hat{\mathbf{G}}_j = \frac{\mathbf{G}_j}{\|\mathbf{G}_j\|}$, $\epsilon_j \geq \|\Delta \mathbf{G}_j\|_F^2$, $\gamma_j \leq \min P_j^{\text{harv}} \quad \forall j$, and $\mathbf{U}_{\mathbf{g}_j} = [\mathbf{I}_{N_R N_T}, \hat{\mathbf{g}}_j]$.

The result of this MOOP problem is shown in FIGURE 3.5.

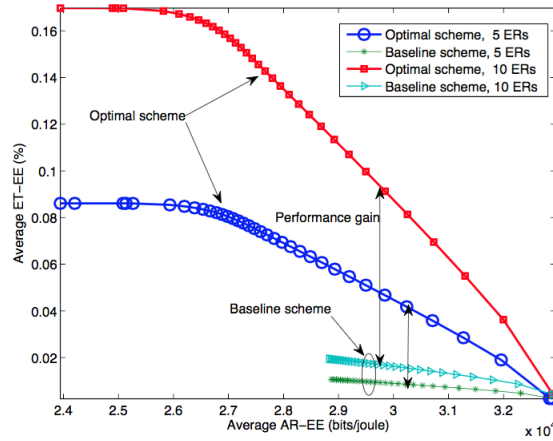


Figure 3.5: System performance trade-off region between the average energy transfer efficiency (ET-EE) and the average rate efficiency (AR-EE) for $P_{\text{max}} = 40$ dBm. The double-sided arrows indicate the performance gain achieved by the proposed optimal scheme over the baseline scheme [5].

The conclusion that can be drawn from FIGURE 3.5 is that the ET-EE is monotonically decreasing with respect to the AR-EE. The two subproblems are thus in conflict with each other. We cannot maximize both the rate and the energy harvested. Nevertheless, the trade-off region grows with the number of energy receivers (ERs). This comes simply by the fact that more energy receiver are able to harvest more energy (a new ER located into the direction of a side lobe will obviously increase the total amount of harvested energy).

3.4 Considered systems

3.4.1 Signal coordination

This transmission system is a MIMO system that has the same number of transmit and receive antennas. The technique used to send information and harvest energy is the multi point-to-point communication system.

FIGURE 3.6 shows a representation of such a system. In this one, we have represented three base stations in order to see better what is happening. Each base station may have several

antennas, but is associated to only one RF chain.

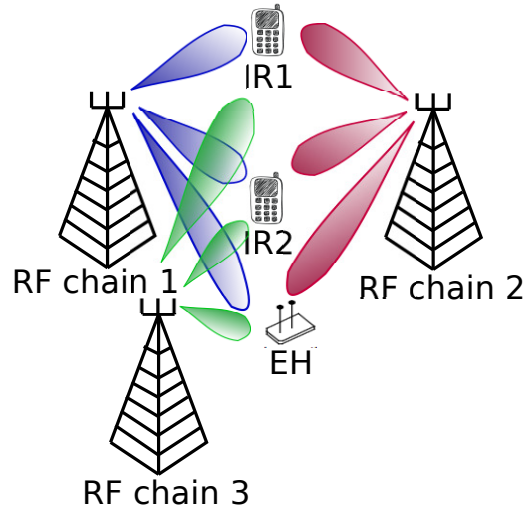


Figure 3.6: Scheme of a signal coordination system with three radio-frequency chains, two information receivers (IRs) and one energy harvester (EH). On it, the lobes of the radiation pattern going to the IRs to convey information and the one going to the EH to harvest energy are drawn.

The purpose of signal coordination is to maximize the sum rate of all IRs using a **linear precoding** and a **power control** on the transmitting side, and to transmit a minimum amount of energy (called T) to the EH such that it can be activated.

With linear precoding, all the base stations are aware of the signal that the other base stations are going to send. Hence, the base stations linearly adjust their signals to simultaneously harvest enough energy and maximize the information rate.

On top of it, power control is used to allocate the power on the different antennas of a RF chain such that the channel link condition are taken into account.

Definition 3.4.1. Power control [11]: consists of weighting the digital signals of an antenna array by adjustment of their amplitudes, such that we allocate more power to the one having the highest SNR at the reception (the one that introduces less noise and fading effects).

In this thesis, our interest will be in linear antenna arrays. As an example, let's analyze FIGURE 3.7.

Starting with a linear array, a feeding power (P), and knowing the channel state information (CSI), we are able to distribute power wisely to maximize the accuracy of the received signal. As can be seen, more power is allocated to the signal that is the less degraded, while propagating through the channel.

On the one hand, it may seem counterintuitive to allocate more power to the best channel state, but on the other hand, it makes sense not to waste energy on channels that have a poor SNR [12].

Besides, since the channel is time varying, an user badly served at time t , may be well served at $t + 1$, so the power allocation is constantly adjusted to adapt with the variations of the environment with respect to the users.

In addition to the power control principle, the system harvests energy. Indeed, each RF chains sends both information and energy signal. Hence, the dimension of such vectors are $N_T \times 1$.

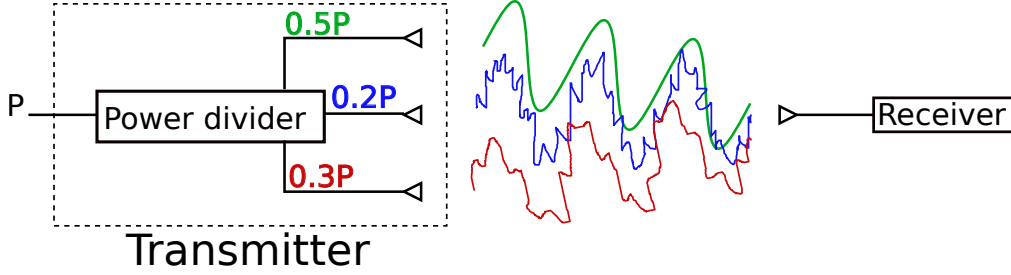


Figure 3.7: Scheme of a linear array with three antennas with power control. The transmitter sends a signal through all the antennas of its array. The allocation of power for each antenna depends on the channel condition, assuming that we have a perfect knowledge of the channel state information. As it can be seen, the signals that are the most degraded during the transmission receive less power.

In FIGURE 3.6, we consider that we must serve two information receivers and one energy harvester simultaneously. Such a coordination will introduce the following problems:

- The information decoder and the energy harvesters do not have the same sensitivity. As a consequence, the information signal and the energy signal may be associated to very different level of transmit power, thus making problematic to linearly combine them. Therefore, signal coordination may be extremely sensitive to imperfect channel state information and more robust techniques are required, especially when the channel state information is not well known.
- When we deal with legacy systems that does not handle wireless energy, no signal coordination is possible since we cannot modify such systems.

3.4.2 Multi-service coordination of full digital systems

Multi-service coordination of full digital systems is a combination of several signal coordination schemes. More precisely, it is the combination of a coordinated signal for the RF chains sending information, and point-to-point systems for each RF chain sending only an energy vector. Indeed, the different energy vectors sent are independent from each other.

It comes that the dimension of the information and energy vectors are not the same. Indeed, the dimension of $\mathbf{w}_I$ is $N_I \times 1$, and the dimension of $\mathbf{w}_E$ will always be 1×1 . The scheme can be seen in FIGURE 3.8, with only the *useful* paths drawn (the one conveying either information or energy to their corresponding devices).

In this scheme, the interferences are not drawn, but we consider that both the information vectors and the energy one interfere with each other. Besides, the dashed lines represent energy transfer.

3.4.3 Multi-service coordination of hybrid precoding systems

The term hybrid precoding refers to the combination of a digital power allocation and an analog precoding. Such system is used to control both the amplitudes and phases of the elemental signals in order to get the desired beams [13].

The digital part will subdivide the input power into the different RF chains (SECTION 3.4.1), and the analog one will direct the information/energy beams.

To be able to direct the beams, the system needs several antennas at each RF chain. It thus has arrays of antennas.

Each antenna of the array will send the same information and/or energy signal, but with different

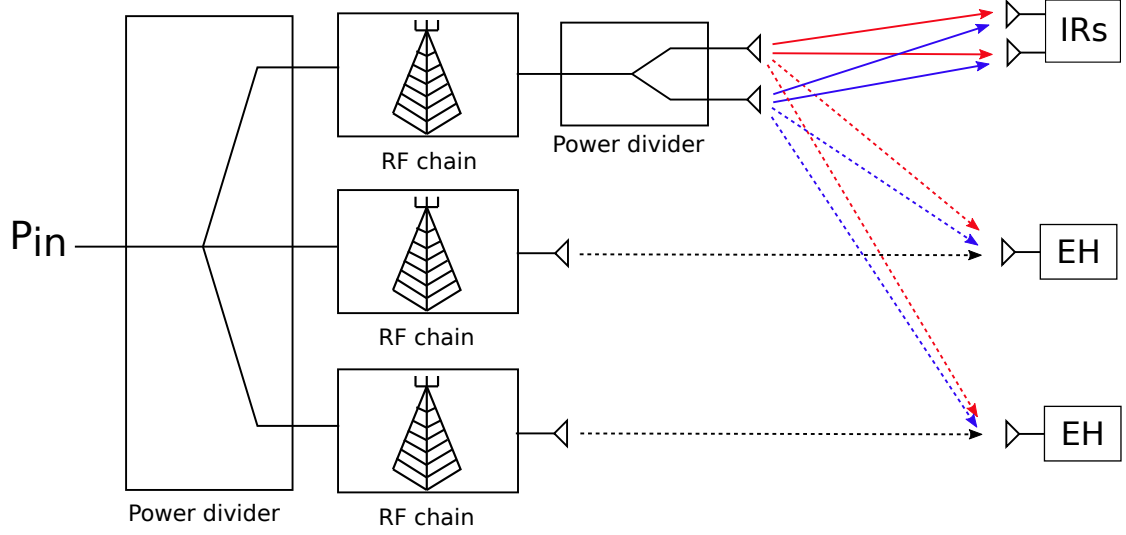


Figure 3.8: Scheme of the multi-service coordination of full digital systems with three RF chains, two information receivers (IRs) and two energy harvesters (EHs). The straight lines represent the links used to convey information through the IRs. The dashed ones are the ones used to harvest energy at the EHs. The interferences due to the energy vectors coming at the IRs are not drawn, but are nevertheless existing.

phases. The purpose of such an operation is to reduce the interferences between user equipments when sending information, and to diminish the interferences on the information receivers caused by the energy signals. Besides, it will help the system to achieve a certain level of harvested energy. As a matter of fact, it will direct some information signals towards the energy harvesters to ensure the minimal amount of energy to be harvested. Moreover, it overcomes the problem of limited achievable rate of article [5]. Indeed, in this system the number of RF chains is not increasing, hence the total energy cost ($N_T P_{ant}$) stays the same, but the rate and the energy increases thanks to the array gain (section 3.2.2).

An hybrid precoding system will thus be equipped with a power divider and as many phase shifters as antennas on the RF chains.

The scheme of this model is represented in FIGURE 3.9. In which, the beams are directed respectively through the information receivers (IRs) and the energy harvesters (EHs). Moreover, the information vector going through the EHs are not drawn.

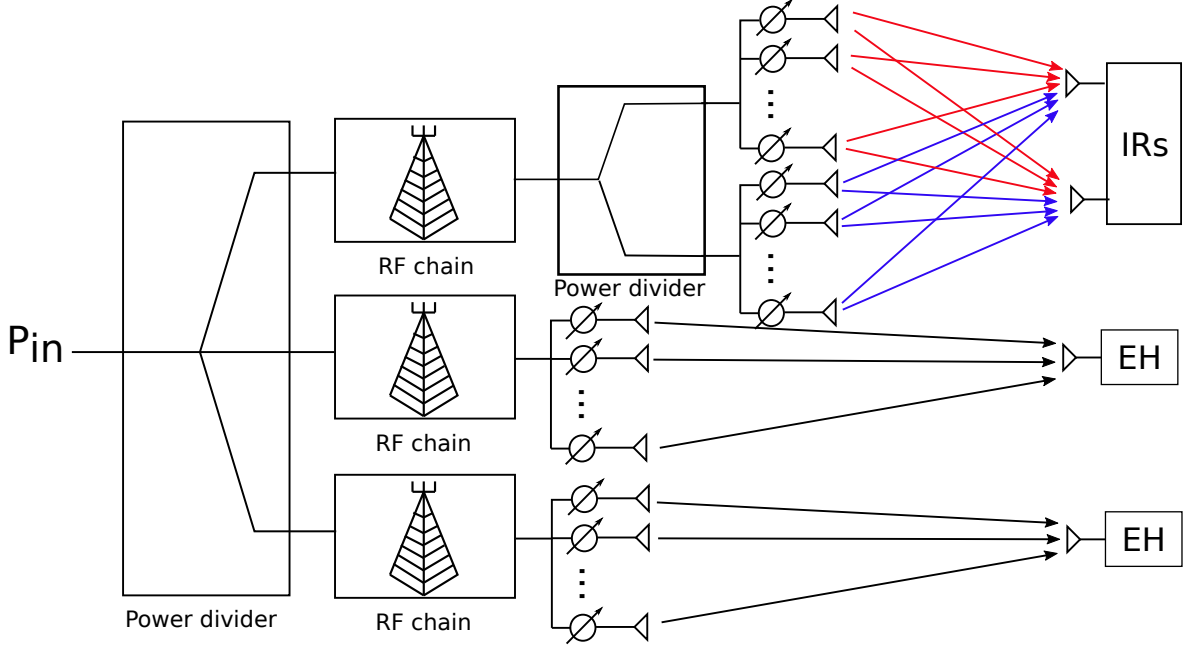


Figure 3.9: Scheme of the multi-service coordination of hybrid precoding systems with three RF chains, two information receivers (IRs) and two energy harvesters (EHs). The red and blue links represent the information signals transmitted to the IRs. The black one represent the energy signal arriving at the EHs. In addition, but not drawn, there are information signals going to the EHs, and energy signals going to the IRs for each antenna of each array.

3.4.3.1 Analog precoding

In order to understand the principle the hybrid precoding, let's define the term **analog precoding** in the field of telecommunications.

Definition 3.4.2. Analog precoding [13]: The term phase precoding relates to the function performed by a device or apparatus in which energy radiated by an aperture antenna is focused along a specific direction in space. The objective is either to preferentially receive a signal from that direction or to preferentially transmit a signal in that direction.

To illustrate the concept, let's analyze FIGURE 3.10.

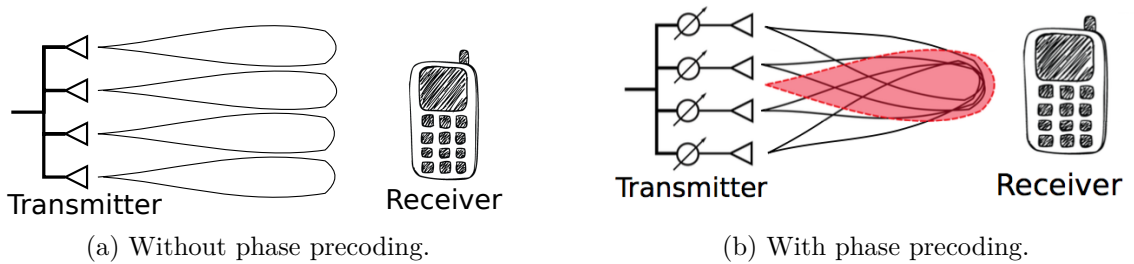


Figure 3.10: Phase precoding example of an array with 4 antennas. On the left, we can see that the main lobes of the radiation pattern are not in phase at the receiver. On the right, when applying phases shifters, we see that the signal of all antennas arrives in phase at the receiver.

As can be seen in FIGURE 3.10a, each antenna is directional and sends the same information in the direction given by its point-to-point communication link. The phase precoding (or analog precoding) will phase shift every communication link such that at the location of the receptor the information adds in phase and the interference adds non

coherently. We will hence have a higher amplitude of the signal and less interference (FIGURE 3.10b).

3.5 Analysis of the hybrid precoding system

We will first investigate several combinations of digital and analog precoding for the hybrid model for massive MIMO systems. Indeed, at this time, research on hybrid architecture have been studied only for massive MIMO systems. We will hence study them but apply them on power transfer systems. Thus, the same types of architectures will be used to improve SWIPT systems performances.

Then, we will discuss the number of phase shifters needed in massive MIMO systems such that we achieve the same performances as the signal coordination case. Once again, no literature exist on the number of RF chains needed in SWIPT systems to reach, with a hybrid precoding system, the performances of a signal coordination system. Hence, we will study this number, and compares it with the results given by our model.

3.5.1 Architectures

The article about the hybrid precoding for Massive MIMO [3] points out the taxonomy and algorithms of several transceiver architectures. Besides, it highlights the questions that still have to be answered in order to have a commercialized solution for the fifth-generation cellular systems.

Model with CSI knowledge

The models shown in FIGURE 3.11 are set for a downlink transmission and for both cm- and mm-waves.

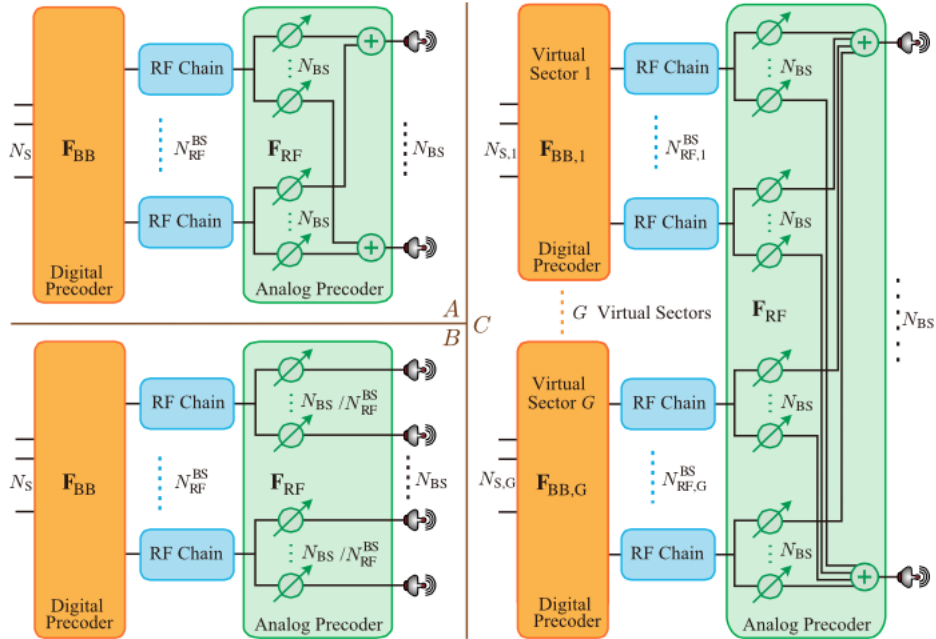


Figure 3.11: Typical block diagrams of hybrid precoding structure at base station for a downlink transmission, where structures A, B, and C denote the full-complexity, reduced-complexity, and the virtual-sectorization structures, respectively [3].

The basic principles for the three models are the following [3]:

- At the transmission side: first a digital precoder $\mathbf{F}_{\text{BB}}$ processes N_S data streams and produce N_{RF}^{BS} outputs. Then, the upconverted RF signals are transformed into N_{BS} antenna elements for transmission with the help of an analog precoder $\mathbf{F}_{\text{RF}}$.
- At the receiver side: the process is the same but applied in reverse order. First, an analog precoder ($\mathbf{W}_{\text{RF}}$) is applied to obtain N_{RF}^{UE} outputs. Those ones are downconverted, then processed by a digital precoder ($\mathbf{W}_{\text{BB}}$) to finally obtain the output signal $\mathbf{y}$.

The distinction between the three models relies on the way the analog precoder is designed.

- Model A: each output at the TX is a linear combination of all RF signals.
- Model B: each RF chain is only connected to a subset of antenna elements. It reduces the complexity but degrades the performances.
- Model C: it uses the analog precoder to create multiple *virtual sectors*, enabling separate baseband processing.

In order to technically implement those solutions, we need to approximate the optimal precoder, to decouple the design of the analog and digital precoders and to consider the practical array size limitation of the structure.

First let's start with the approximation of the precoders. The main difficulty is that the digital and analog precoders are linked together, leading to a non-convex optimization problem. One solution is to expand the problem into a series of convex sub-problems.

The second issue is to ensure the decoupling between the analog and digital precoders, and between the precoders at the TX and RX. A way to do this is to solve sequentially the rate maximization problem. An other solution is to set the digital precoder to a unitary matrix.

The last huge problem of the hybrid precoding lies in the allowed number of antennas per array and so in the number of phase shifters. In order to respect the size limitation constraint, we need to design, in addition, a frequency-domain scheduling. This implies that different user equipments are served on different subcarriers, making the narrowband precoder no longer applicable.

With this, we have a pretty good survey of the possible hybrid transceiver models with CSI knowledge and their drawbacks.

We can thus conclude that there are no perfect model, but rather a bunch of them having each a different complexity/performance trade-off. The best solution will thus be to adapt the model depending on the channel characteristics.

In our modelization, we have chosen to use the architecture B of FIGURE 3.11. Indeed, this architecture allow us to consider separately two systems; one for the transmission of information and the other one for the transmission of energy.

3.5.2 Number of phase shifters

In digital precoding, one RF chain is needed for each antenna element at the base station (BS) and user equipment (UE). When the number of RF chains, N , is very large, the separation of the RF chains for each antenna is highly inefficient.

Some people have thus introduced the concept of hybrid precoding. It means that we deploy precoding in both digital and analog domains [4].

Some authors have examined the number of phase shifter needed (analog precoding), in order to reach the same performances as for the ideal full digital transmission mode.

In order to find it, the authors have asked themselves the two following questions:

- **P1:** How to design the hybrid precoding such that it ensures the same performances as the digital one for any transceiver optimization criteria and channel matrix? To do so, they performed a study on the number of required RF chains and phase shifters (PSs), assuming the PSs have an infinite resolution (continuous angle).
- **P2:** Working with MIMO system implies that each of the users may use only part of the available spectrum. The question raised is thus how to schedule the users? To do so, they use the architecture developed based on the use of practical constant phase PSs.

The two previous problems (**P1** and **P2**) have been analyzed with the following assumptions/hypothesis:

- We consider downlink channels.
- Each user is equipped with a single antenna (MU-MISO).
- We consider frequency selective channels due to the large bandwidth used and the propagation phenomena.
- Perfect channel state information is available at the base station.

In order to solve those problems, the authors of [4] define a design model for the hybrid precoding (FIGURE 3.12).

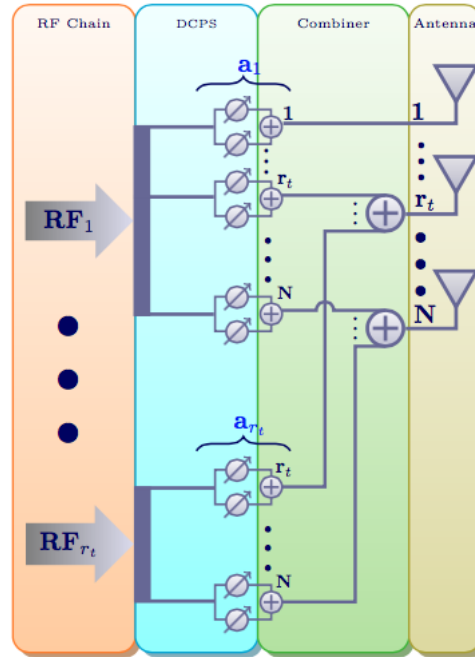


Figure 3.12: Detailed hybrid precoding design with direct current phase shifters [4].

The results found for this two problems are valid only for microwave and millimeter wave massive MIMO applications, and are the following:

1. The authors set the number of RF chains to r_t (rank of the combined digital precoder matrices of all sub-carriers) and the one of the phase shifters to $N_p = 2r_t(N - r_t + 1)$ (N being the number of antennas at the BS) so that the hybrid precoding fulfills the same performances as the digital precoding. Then, they provide an approach to realize the hybrid precoding, using r_t RF chains and $N_{cp} \setminus \|N_p\|$ circularly polarized phase shifters (CPPSs). It results that only 20-40 CPPSs are required per RF chain.

2. The number of RF chains depends on the number of user equipments, their channel matrices, and the precoder design criteria. So, it is possible that $N_a \leq r_t$ (N_a being the number of RF chains deployed at the base station). In this situation, the digital precoding cannot be realized by their hybrid precoding. They thus analyze the problem **P2** and provide performance analysis by considering a sum rate maximization problem.

Under the uniform linear array (ULA) channel model and ZF precoding, it is shown that the performances of the digital precoding and the hybrid precoding are the same when the angle of departure (AOD) of the channels of the users have some specific structures.

The goal of the thesis being to understand the trade-off between the communication rate and the energy harvested as a function of the system power, we are going to model the optimization problem of signal coordination, multi-service coordination in both full digital and hybrid precoding systems in CHAPTER 4. In this chapter, we will consider that each system will sent simultaneously information and power transfer.

Then, in CHAPTER 5, we will analyze the average rate-energy efficiency trade-off region for the three systems. Besides, for the case of the multi-service coordination of hybrid precoding systems, we will analyze the gain in diversity and the array gain for a specific architecture. This will allow us to draw some conclusions on the number of phase shifter needed in a multi-service coordination of a hybrid precoding system to achieve the same performances as a signal coordination system.

4 | Methods

4.1 Multi-service coordination of hybrid precoding SWIPT systems

In this section, we will develop all the steps to go from the digital model proposed in [5] to a model for the multi-service coordination of hybrid precoding system with interferences and any number of antennas per RF chains.

4.1.1 Model

First, let's describe the model considered for our implementation.

We consider three RF chains that can be coordinated assuming a total power constraint. In the case of a multi-service coordination system, two are used to convey information, and the third one to send energy. For a signal coordination system, the three send both information and energy vectors. We thus consider two information receivers and one energy harvesting receiver. This situation can be seen in FIGURE 4.1.

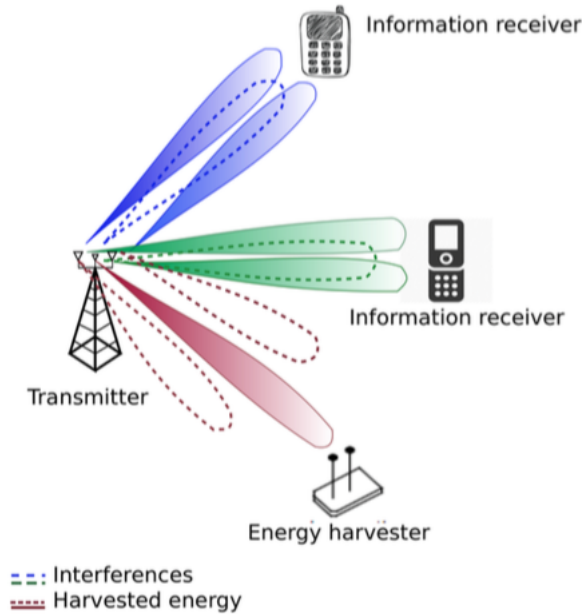


Figure 4.1: Modelization of the situation: three RF chains, two information receivers (IRs) and one energy harvester (EH). Only the lobes of the radiation pattern pointing toward an information receiver or an energy harvester are drawn.

The goal of my work is to find the optimal precoder matrices for the digital and analog part of the hybrid precoding. Those optimal precoder matrices will maximize the sum of rates (the sum of the rates arriving at all the information receivers).

To do so, we start to analyze a *signal coordination model*, then we move to a model for a *multi-service coordination of full digital system* and finally to a model for a *multi-service coordination of hybrid precoding systems* with N_A antennas per transmitter.

4.1.2 Signal coordination

The first step is to add the interferences to the model given in [5], as schematized in FIGURE 4.2.

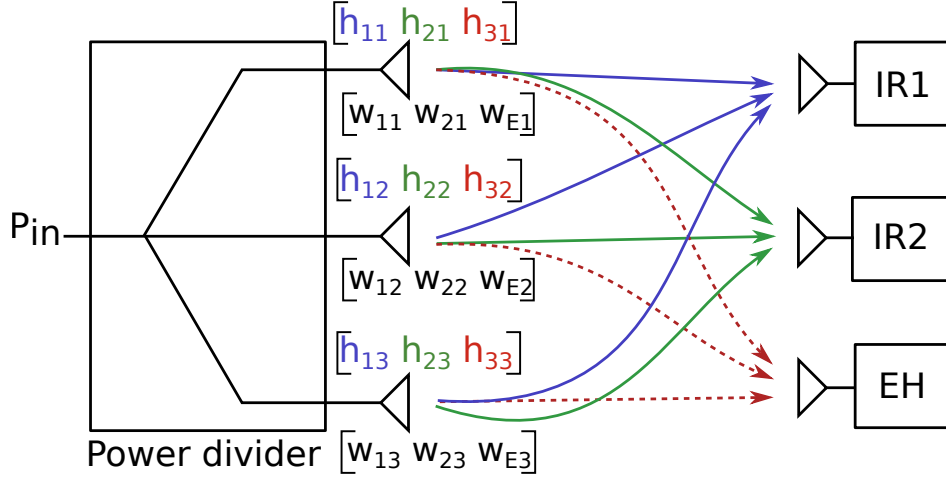


Figure 4.2: Signal coordination scheme with three RF chains, two information receivers and one energy harvester. Each antenna has three communication links (one for each receptor), noted $\mathbf{h}_{ij}$; $i \in \{1, 2, 3\}$ representing the receiver and $j \in \{1, 2, 3\}$ representing the transmitter of the link. Each antenna conveys two information signals (one for each information receiver), and one energy signal (for the energy harvester), respectively noted $\mathbf{w}_{1i}$, $\mathbf{w}_{2i}$ and $\mathbf{w}_{Ei}$; $i \in \{1, 2, 3\}$ representing the RF chains from which the signal is coming from.

The vector $\mathbf{w}_1 = [w_{11} \ w_{12} \ w_{13}]$ represents the information vector received by IR1, $\mathbf{w}_2 = [w_{21} \ w_{22} \ w_{23}]$ represents the information vector received by IR2 and $\mathbf{w}_E = [w_{E1} \ w_{E2} \ w_{E3}]$ the energy sent toward the EH.

4.1.2.1 Problem formulation

The problem can be written as:

$$\begin{aligned}
 & \underset{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_E \in \mathbb{H}^{N_T}}{\text{maximization}} \quad R_{(1,SC)} + R_{(2,SC)} \\
 & \text{s.t.} \quad \text{Tr}(\mathbf{W}_I + \mathbf{W}_E) \leq P_{\text{in}} \\
 & \quad \text{Tr}(\mathbf{H}_3(\mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_E)) \geq T \\
 & \quad \mathbf{W}_1 \succeq 0 \\
 & \quad \mathbf{W}_2 \succeq 0 \\
 & \quad \mathbf{W}_E \succeq 0,
 \end{aligned} \tag{4.1}$$

where $T = \gamma P_{\text{in}}$, γ being the minimal transferred energy efficiency, with $\gamma \in \mathbb{R}^+$ and

$$\begin{aligned}
 R_{(1,SC)} &= \log_2 \left(1 + \frac{\mathbf{H}_1 \mathbf{W}_1}{\sigma_I^2 + \mathbf{H}_1 \mathbf{W}_2 + \mathbf{H}_1 \mathbf{W}_E} \right) \\
 R_{(2,SC)} &= \log_2 \left(1 + \frac{\mathbf{H}_2 \mathbf{W}_2}{\sigma_I^2 + \mathbf{H}_2 \mathbf{W}_1 + \mathbf{H}_2 \mathbf{W}_E} \right)
 \end{aligned}$$

We remind that $\mathbf{H}_i$ is the covariance matrix of channel i , and $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H$ is the information matrix received at IR i ; σ_I^2 is the noise introduced in the channel.

Since the objective function is concave, the *cvx* solver of *MatLab* will not be able to find any solution. We will thus add some artifacts in order to make the problem convex.

The main idea is to split the information (convex function) and interference terms (concave function), and to assume the last one known [8]. Then, we solve iteratively the problem until the difference between the preset interferences and the computed ones are below a certain threshold (Δ) or that we have reach the maximal number of iterations ($\text{iteration}_{\text{maximal}}$).

In order to not modify the problem, the first and second Taylor series terms have to be added to the objective function.

The problem 4.1 thus becomes:

$$\begin{aligned}
& \underset{\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_E \in \mathbb{H}^{N_T}}{\text{maximization}} \quad f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu) - \mathbb{T}_{(1,SC)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu)) \\
& \quad - \mathbb{T}_{(2,SC)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu)) \tag{4.2} \\
& \text{Tr}(\mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_E) \leq P_{\text{in}} \tag{C1} \\
& \text{Tr}(\mathbf{H}_3(\mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_E)) \geq T \tag{C2} \\
& \mathbf{W}_1 \succeq 0 \tag{C3} \\
& \mathbf{W}_2 \succeq 0 \tag{C4} \\
& \mathbf{W}_E \succeq 0, \tag{C5}
\end{aligned}$$

where $\mathbf{W}_1^\nu$, $\mathbf{W}_2^\nu$ and $\mathbf{W}_E^\nu$ are the solutions of the problem at the iteration ν , $\mathbf{W}_1^0$, $\mathbf{W}_2^0$ and $\mathbf{W}_E^0 = \frac{1}{3}P_{\text{in}}\mathbf{Id}(N_T)$, and

$$\begin{aligned}
f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu) &= R_1^\nu + R_2^\nu \\
R_1^\nu &= \log \det \left(\frac{\mathbf{H}_1 \mathbf{W}_1}{\sigma_I^2 + \mathbf{H}_1(\mathbf{W}_2^\nu + \mathbf{W}_E^\nu)} \right) \\
R_2^\nu &= \log \det \left(\frac{\mathbf{H}_2 \mathbf{W}_2}{\sigma_I^2 + \mathbf{H}_2(\mathbf{W}_1^\nu + \mathbf{W}_E^\nu)} \right) \\
\mathbb{T}_{(1,SC)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu)) &= \Re \left(\text{Tr} \left[\left(\mathbf{H}_2^H \left(\frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_2(\mathbf{W}_2^\nu + \mathbf{W}_E^\nu))} - \frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_2(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu + \mathbf{W}_E^\nu))} \right) \right. \right. \right. \\
&\quad \left. \left. + \mathbf{H}_3^H \left(\frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_3(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu))} - \frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_3(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu + \mathbf{W}_E^\nu))} \right) \right) \right]^H \\
&\quad \left. (\mathbf{W}_1 - \mathbf{W}_1^\nu) \right] \right) \\
&\quad + \Re \left(\text{Tr} \left[\left(\mathbf{H}_1^H \left(\frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_1(\mathbf{W}_1^\nu + \mathbf{W}_E^\nu))} - \frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_1(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu + \mathbf{W}_E^\nu))} \right) \right. \right. \right. \\
&\quad \left. \left. + \mathbf{H}_3^H \left(\frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_3(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu))} - \frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_3(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu + \mathbf{W}_E^\nu))} \right) \right) \right]^H \\
&\quad \left. (\mathbf{W}_2 - \mathbf{W}_2^\nu) \right] \right) \\
\mathbb{T}_{(2,SC)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu)) &= \tau_1 \text{Tr} \left[(\mathbf{W}_1 - \mathbf{W}_1^\nu)(\mathbf{W}_1 - \mathbf{W}_1^\nu)^H \right] + \tau_2 \text{Tr} \left[(\mathbf{W}_2 - \mathbf{W}_2^\nu)(\mathbf{W}_2 - \mathbf{W}_2^\nu)^H \right],
\end{aligned}$$

$\tau_1 \geq 0$ and $\tau_2 \geq 0$ are proximal-like regularization terms [18].

This problem needs to be slightly modified since the constraint (C2) is not accepted by the *cvx* solver. We hence transform it into its standard form [19]:

$$\begin{aligned} \text{Tr}(\mathbf{H}_3 \mathbf{W}_E + \mathbf{W}_1 + \mathbf{W}_2) - T &= s1 \\ s1 &\geq 0, \end{aligned}$$

with $s1$ a new variable that has to be positive.

4.1.2.2 Algorithm

Algorithm 1 gives an overview of the implementation for such a non convex optimization problem.

Algorithm 1 Signal coordination scheme.

```

1: function IDEAL_FULL_DIGITAL( $N_T, H_1, H_2, H_3, \sigma_I^2, P_{\text{in}}, T, \Delta, \text{iteration}_{\text{maximal}}$ )
2:    $\tau_1, \tau_2 = 0$ 
3:    $\mathbf{W}_1 = \frac{1}{3N_T} P_{\text{in}} \mathbf{Id}(N_T); \mathbf{W}_2 = \frac{1}{3N_T} P_{\text{in}} \mathbf{Id}(N_T); \mathbf{W}_E = \frac{1}{3N_T} P_{\text{in}} \mathbf{Id}(N_T)$ 
4:    $\mathbf{W}_1^0 = 0.5 \mathbf{W}_1; \mathbf{W}_2^0 = 0.5 \mathbf{W}_2; \mathbf{W}_E^0 = 0.5 \mathbf{W}_E$ 
5:    $\text{maxit} = 0;$ 
6:    $\nu = 0$ 
7:   while ( $\|\mathbf{W}_1 - \mathbf{W}_1^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_2 - \mathbf{W}_2^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_E - \mathbf{W}_E^\nu\|_F \leq \Delta$ ) and  $\text{maxit} \leq$ 
       $\text{iteration}_{\text{maximal}}$  do
8:      $\mathbf{W}_1^\nu = \mathbf{W}_1, \mathbf{W}_2^\nu = \mathbf{W}_2, \mathbf{W}_E^\nu = \mathbf{W}_E$ 
9:     Solve the problem 4.2  $\Rightarrow \mathbf{W}_1, \mathbf{W}_2$  and  $\mathbf{W}_E$ 
10:    if  $\text{isnan}(\mathbf{W}_E)$  then
11:       $\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_E = 0_{N_T, N_T}$ 
12:       $\text{maxit} = \text{iteration}_{\text{maximal}} + 1$ 
13:    end if
14:    Set  $\nu \leftarrow \nu + 1$ 
15:    Set  $\text{maxit} \leftarrow \text{maxit} + 1$ 
16:  end while
17: end function

```

As can be read in algorithm 1, we have added a case condition. Indeed, when the input power is too low and T too high, we are not able to fulfill the constraint C2. Thus, for those values of power, we set all vectors to zero and go out of the while loop, such that we will generate neither any rate, nor any energy harvested.

4.1.3 Multi-service coordination of a full digital system

The second step is to consider the information vector and the energy vector as distinct from each other [3]. This means that we have spectrum coordination between the information and the energy covariance matrices, and signal coordination between the different information coordination matrices ($\mathbf{W}_1$ and $\mathbf{W}_2$).

The dimensions of the information matrices are $N_I \times N_I$ and the one for the energy is 1×1 since there is no coordination between the energy harvesters.

We thus have two different systems, one for the transfer of information, and the other one for the transfer of energy.

Those two can be optimized together or separately. The power will hence be allocated differently and in the following way:

1. We consider one unique problem where the total power is divided optimally between the three transmitters by the solver. This is the **collocated transmitters problem**.

2. We consider 2 separate problems. One for the information receivers and the second one for the energy harvester. The program will optimally choose α such that it allocates $(1 - \alpha)P_{\text{in}}$ to TX1 and TX2 and αP_{in} to TX3. This is the **separate transmitters with power reallocation problem**.

The two formulations and results will be analyzed in the following sections.

4.1.3.1 Collocated transmitters problem

The model is represented in FIGURE 4.3.

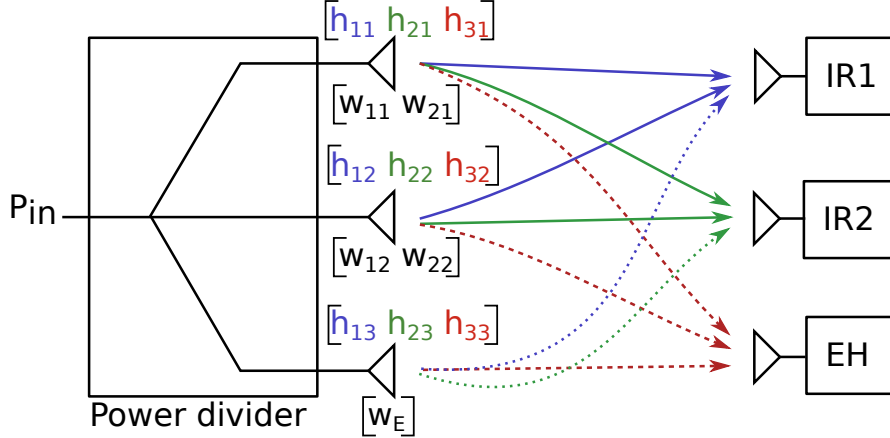


Figure 4.3: Scheme of a multi-service coordination of a full digital system with two information receivers (IRs) and one energy harvester (EH) for collocated transmitters. Each antenna has a channel link going to every receiver, noted $\mathbf{h}_{ij}$; $i \in \{1, 2, 3\}$ representing the receiver and $j \in \{1, 2, 3\}$ representing the transmitter of the link. Among the three RF chains, only two are used to send information signals, respectively to IR1 and IR2. Those two have a radiation pattern such that the information is also radiated toward the EH, harvesting hence energy. The third antenna is only used to send energy to the EH, and causes interferences to the IRs.

In this figure, the straight lines represent the channel vector conveying the useful information to the IRs, the dashed ones represent the information or energy being sent to the EH in order to harvest energy, and the dotted ones represent the interferences caused by the energy signal on the information receivers.

The convexify [8] optimization problem for the collocated transmitters multi-service coordination of a full digital system with collocated transmitters is:

$$\begin{aligned} \underset{\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu}{\text{maximization}} \quad & f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu) - \mathbb{T}_{(1,MS)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu)) \\ & - \mathbb{T}_{(2,MS)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu)) \end{aligned} \quad (4.3)$$

$$\text{such that } \text{Tr}(\mathbf{W}_1 + \mathbf{W}_2) + \text{Tr}(\mathbf{W}_E) \leq P_{\text{in}} \quad (C1)$$

$$\text{Tr}(\mathbf{H}_{3,en} \mathbf{W}_E + \mathbf{H}_{3,inf} \mathbf{W}_1 + \mathbf{H}_{3,inf} \mathbf{W}_2) \geq T \quad (C2)$$

$$\mathbf{W}_1 \succeq 0$$

$$\mathbf{W}_2 \succeq 0$$

$$\mathbf{W}_E \succeq 0,$$

where:

$$f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu) = R_{(1,MS)}^\nu + R_{(2,MS)}^\nu$$

$$\begin{aligned}
R_{(1,MS)}^\nu &= \log \det \left(\frac{\mathbf{H}_{1,inf} \mathbf{W}_1}{\sigma_I^2 + \mathbf{H}_{1,inf} \mathbf{W}_2^\nu + \mathbf{H}_{1,en} \mathbf{W}_E^\nu} \right) \\
R_{(2,MS)}^\nu &= \log \det \left(\frac{\mathbf{H}_{2,inf} \mathbf{W}_2}{\sigma_I^2 + \mathbf{H}_{2,inf} \mathbf{W}_1^\nu + \mathbf{H}_{2,en} \mathbf{W}_E^\nu} \right) \\
\mathbb{T}_{(1,MS)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu, \mathbf{W}_E^\nu)) &= \Re \left(\text{Tr} \left[\left(\mathbf{H}_{2,inf}^H \left(\frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_{2,inf} \mathbf{W}_2^\nu + \mathbf{H}_{2,en} \mathbf{W}_E^\nu)} \right. \right. \right. \right. \\
&\quad \left. \left. \left. - \frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_{2,inf}(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu) + \mathbf{H}_{2,en} \mathbf{W}_E^\nu)} \right) \right)^H (\mathbf{W}_1 - \mathbf{W}_1^\nu) \right] \right) \\
&\quad + \Re \left(\text{Tr} \left[\left(\mathbf{H}_{1,inf}^H \left(\frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_{1,inf} \mathbf{W}_1^\nu + \mathbf{H}_{1,en} \mathbf{W}_E^\nu)} \right. \right. \right. \right. \\
&\quad \left. \left. \left. - \frac{1}{\sigma_I^2 + \text{Tr}(\mathbf{H}_{1,inf}(\mathbf{W}_1^\nu + \mathbf{W}_2^\nu) + \mathbf{H}_{1,en} \mathbf{W}_E^\nu)} \right) \right)^H (\mathbf{W}_2 - \mathbf{W}_2^\nu) \right] \right) \\
\mathbb{T}_{(2,MS)}(f(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1^\nu, \mathbf{W}_2^\nu)) &= \tau_1 \text{Tr} [(\mathbf{W}_1 - \mathbf{W}_1^\nu)(\mathbf{W}_1 - \mathbf{W}_1^\nu)^H] + \tau_2 \text{Tr} [(\mathbf{W}_2 - \mathbf{W}_2^\nu)(\mathbf{W}_2 - \mathbf{W}_2^\nu)^H]
\end{aligned}$$

We remind that $\mathbf{H}_{i,inf}$ is a channel matrix conveying information to the receiver i (either an information receiver or an energy receiver) and that $\mathbf{H}_{i,en}$ is a channel matrix conveying energy to the receiver i . This problem needs to be slightly modified since the constraint (C2) is not accepted by the *cvx* solver. We hence transform it into its standard form [19]:

$$\begin{aligned}
\text{Tr}(\mathbf{H}_{3,en} \mathbf{W}_E + \mathbf{H}_{3,inf} \mathbf{W}_1 + \mathbf{H}_{3,inf} \mathbf{W}_2) - \text{T} &= \text{s1} \\
\text{s1} &\geq 0,
\end{aligned}$$

with s1 a new variable that has to be positive.

Besides, we will add a case condition in our problem depending on the fulfillment of the constraint (C2). When it is not tight, the problem is infeasible, meaning that we do not inject enough power (P_{in}) compared to the desired harvested energy (T). In this situation, we will set $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_E$ to zero. Moreover, we go out of the while loop. This will automatically give a null rate and energy harvested.

Algorithm Algorithm 2 summarizes the steps of the *MatLab* function that solves the collocated transmitters problem for a multi-service coordination of a full digital system with collocated transmitters.

Algorithm 2 Multi-service coordination of a full digital system: collocated transmitters problem.

```

1: function SEMI_HYBRID( $N_I, N_R, H_{1,inf}, H_{1,en}, H_{2,inf}, H_{2,en}, H_{3,inf}, H_{3,en}, P_{in}, \sigma_I^2, \text{iteration}_{\text{maximal}}, \Delta, T$ )
2:    $\tau_1, \tau_2 = 0$ 
3:    $\mathbf{W}_1 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_2 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_E = \frac{1}{3N_R} P_{in} \mathbf{Id}(N_R)$ 
4:    $\mathbf{W}_1^0 = 0.5 \mathbf{W}_1; \mathbf{W}_2^0 = 0.5 \mathbf{W}_2; \mathbf{W}_E^0 = 0.5 \mathbf{W}_E$ 
5:    $\text{maxit} = 0;$ 
6:    $\nu = 0$ 
7:   while ( $\|\mathbf{W}_1 - \mathbf{W}_1^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_2 - \mathbf{W}_2^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_E - \mathbf{W}_E^\nu\|_F \leq \Delta$ ) and  $\text{maxit} \leq$ 
iterationmaximal do
8:      $\mathbf{W}_1^\nu = \mathbf{W}_1, \mathbf{W}_2^\nu = \mathbf{W}_2, \mathbf{W}_E^\nu = \mathbf{W}_E$ 
9:     Solve the problem 4.3  $\Rightarrow \mathbf{W}_1, \mathbf{W}_2$  and  $\mathbf{W}_E$ 
10:    if isnan( $\mathbf{W}_E$ ) then
11:       $\mathbf{W}_1, \mathbf{W}_2 = 0_{N_I, N_I}$ 
12:       $\mathbf{W}_E = 0_{N_R, N_R}$ 
13:       $\text{maxit} = \text{iteration}_{\text{maximal}} + 1$ 
14:    else if isinf( $\mathbf{W}_E$ ) then
15:       $\mathbf{W}_1, \mathbf{W}_2 = 0_{N_I, N_I}$ 
16:       $\mathbf{W}_E = 0_{N_R, N_R}$ 
17:       $\text{maxit} = \text{iteration}_{\text{maximal}} + 1$ 
18:    end if
19:    Set  $\nu \leftarrow \nu + 1$ 
20:    Set  $\text{maxit} \leftarrow \text{maxit} + 1$ 
21:  end while
22: end function

```

4.1.3.2 Separate transmitters problem with power reallocation

We will separate the model for a multi-service coordination of a full digital system into 2 sub-problems.

Such a resolution can be schematized as in FIGURE 4.4.

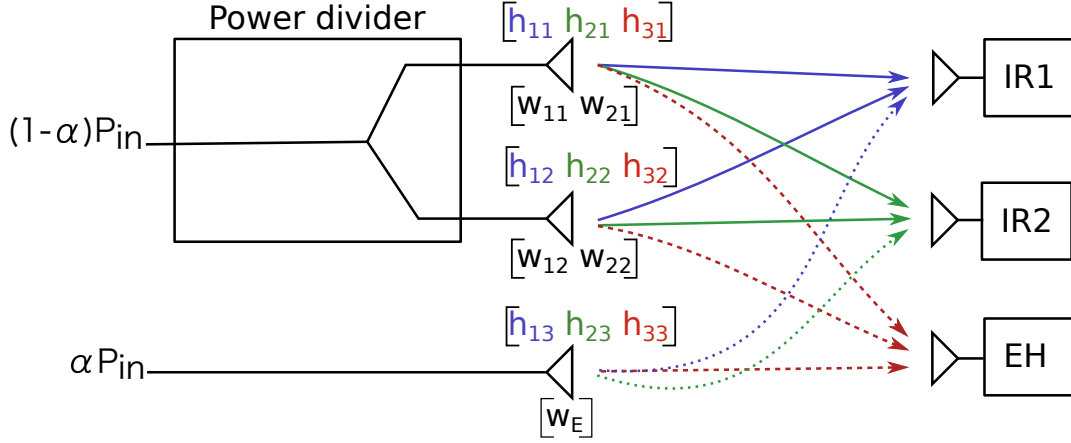


Figure 4.4: Scheme of a multi-service coordination of a full digital system with two information receivers (IRs) and one energy harvester (EH) for separate transmitters with power reallocation. Each antenna has a channel link going to every receiver, noted $\mathbf{h}_{ij}$; $i \in \{1, 2, 3\}$ representing the receiver and $j \in \{1, 2, 3\}$ representing the transmitter of the link. Among the three RF chains, only two are used to send information signals, respectively to IR1 and IR2. Those two have a radiation pattern such that the information is also radiated toward the EH, harvesting hence energy. The third antenna is only used to send energy to the EH, and causes interferences to the IRs. Besides, the power allocated to the third RF chain is αP_{in} , and the one allocated to the first two RF chains is $(1 - \alpha)P_{\text{in}}$.

The first problem consists of maximizing the rate, knowing the value of the energy beamvector.

$$\begin{aligned} \underset{\mathbf{W}_1, \mathbf{W}_2, \in \mathbb{H}^{N_I}}{\text{maximization}} &= R'_{(1,MS)} + R'_{(2,MS)} - \mathbb{T}_{(1,MS)}(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1', \mathbf{W}_2', \mathbf{W}_E') \\ &\quad - \mathbb{T}_{(2,MS)}(\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_1', \mathbf{W}_2') \end{aligned} \quad (4.4)$$

$$\text{such that } \text{Tr}(\mathbf{W}_1 + \mathbf{W}_2) \leq (1 - \alpha)P_{\text{in}} \quad (C1)$$

$$\begin{aligned} \mathbf{W}_1 &\succeq 0 \\ \mathbf{W}_2 &\succeq 0 \end{aligned} \quad (4.5)$$

The second problem consists in minimizing $\mathbf{W}_E$, knowing its maximal allocated power.

$$\underset{\mathbf{W}_E \in \mathbb{H}^1, s1}{\text{minimize}} \mathbf{W}_E \quad (4.6)$$

$$\text{s.t. } \text{Tr}(\mathbf{W}_E) \leq \alpha P_{\text{in}} \quad (C2)$$

$$\text{Tr}(\mathbf{H}_{3,en} \mathbf{W}_E + \mathbf{H}_{3,inf} \mathbf{W}_1 + \mathbf{H}_{3,inf} \mathbf{W}_2) - s1 = T \quad (C3)$$

$$s1 \geq 0,$$

$$\mathbf{W}_E \succeq 0,$$

The solver has, in addition, a case loop that *optimizes* the value of α at each iteration. Indeed, the constraint (C3) will always be fulfilled, but the value of $\mathbf{W}_E$ may exceed the allocated power (αP_{in}). So, when $\mathbf{W}_E > \alpha P_{\text{in}}$, we increase α such that $\alpha = \frac{\mathbf{W}_E}{P_{\text{in}}}$. If α is bigger than one, we set $\mathbf{W}_E$ and α to 0 since C3 will never be tight for $P = P_{\text{in}}$ with $\mathbf{W}_E > 0_{N_R, N_R}$. Furthermore, even by setting $\mathbf{W}_E$ to zero, the problem might still not be feasible (we cannot reach T). For this situation, we set all vectors to zero so that the rate and energy harvested are null and we go out of the while loop.

Algorithm Algorithm 3 draws the basic operations to perform in *MatLab* to solve these two problems:

Algorithm 3 Multi-service coordination of a full digital system for separate transmitters with power reallocation.

```

1: function SEMI_HYBRID( $N_I, N_R, H_{1,inf}, H_{1,en}, H_{2,inf}, H_{2,en}, H_{3,inf}, H_{3,en}, P_{in}, \sigma_I^2, \text{iteration}_{\text{maximal}}, \Delta, T, \alpha$ )
2:    $\tau_1, \tau_2 = 0$ 
3:    $\mathbf{W}_1 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_2 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_E = \frac{1}{3N_R} P_{in} \mathbf{Id}(N_R)$ 
4:    $\mathbf{W}_1^0 = 0.5\mathbf{W}_1; \mathbf{W}_2^0 = 0.5\mathbf{W}_2; \mathbf{W}_E^0 = 0.5\mathbf{W}_E$ 
5:    $\text{maxit} = 0;$ 
6:    $\nu = 0$ 
7:   while ( $\|\mathbf{W}_1 - \mathbf{W}_1^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_2 - \mathbf{W}_2^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_E - \mathbf{W}_E^\nu\|_F \leq \Delta$ ) and  $\text{maxit} \leq$ 
iterationmaximal do
8:      $\mathbf{W}_1^\nu = \mathbf{W}_1, \mathbf{W}_2^\nu = \mathbf{W}_2, \mathbf{W}_E^\nu = \mathbf{W}_E$ 
9:     Solve the problem 4.4  $\Rightarrow \mathbf{W}_1, \mathbf{W}_2$ 
10:    Solve the problem 4.6  $\Rightarrow \mathbf{W}_E$ 
11:    if isnan( $\mathbf{W}_E$ ) then
12:       $\mathbf{W}_1, \mathbf{W}_2 = 0_{N_I, N_I}$ 
13:       $\mathbf{W}_E = 0_{N_R, N_R}$ 
14:       $\text{maxit} = \text{iteration}_{\text{maximal}} + 1$ 
15:    else if isinf( $\mathbf{W}_E$ ) or  $\mathbf{W}_E > P_{in}$  then
16:       $\mathbf{W}_E = 0_{N_R, N_R}, \alpha = 0$ 
17:    else if  $\mathbf{W}_E \geq \alpha P_{in}$  or  $\mathbf{W}_E \leq \alpha P_{in}$  then
18:       $\alpha = \frac{\mathbf{W}_E}{P_{in}}$ 
19:       $\mathbf{W}_E = \alpha P_{in}$ 
20:    end if
21:    Set  $\nu \leftarrow \nu + 1$ 
22:    Set  $\text{maxit} \leftarrow \text{maxit} + 1$ 
23:  end while
24: end function

```

The particularity of this code is that we modify the value of α depending on the solution obtained at each iteration. We thereby kind of find the optimal α for the considered problem.

4.1.4 Multi-service coordination of a hybrid precoding system

The third and last step is to replace at the base station each antenna with an array of antennas, and to introduce analog precoding. Each array will have a phase precoder matrix, giving the phase shift applied to all information and energy vectors toward all received antennas ([3] and [4]).

The purpose of having an array of antennas is to achieve a better performance since the beamform of the phase precoder and information precoder will lead to less interferences.

There are several possibilities to implement the phase matrix precoder:

- All antennas of an array transmit through the same channel (as is the case in figure 3.11A of section 3).
- All antennas of an array transmit their information through different channels (as is the case of figure 3.11B of section 3).
- The information and the energy vectors can have different structures since they are not linked to each other.

For our implementation, we have chosen to transmit every information through different channel realizations for both the information and energy vectors to reduce the complexity of the problem

[3]. Besides, to find the phase shifter of the energy vector, we have chosen to minimize the interferences caused by $\mathbf{W}_E$ rather than maximizing the energy transferred. Indeed, we only want to fulfill the constraint on the minimal amount of energy harvested while maximizing the rate.

The scheme of the hybrid precoding model with different channel realizations can be seen in FIGURE 4.5.

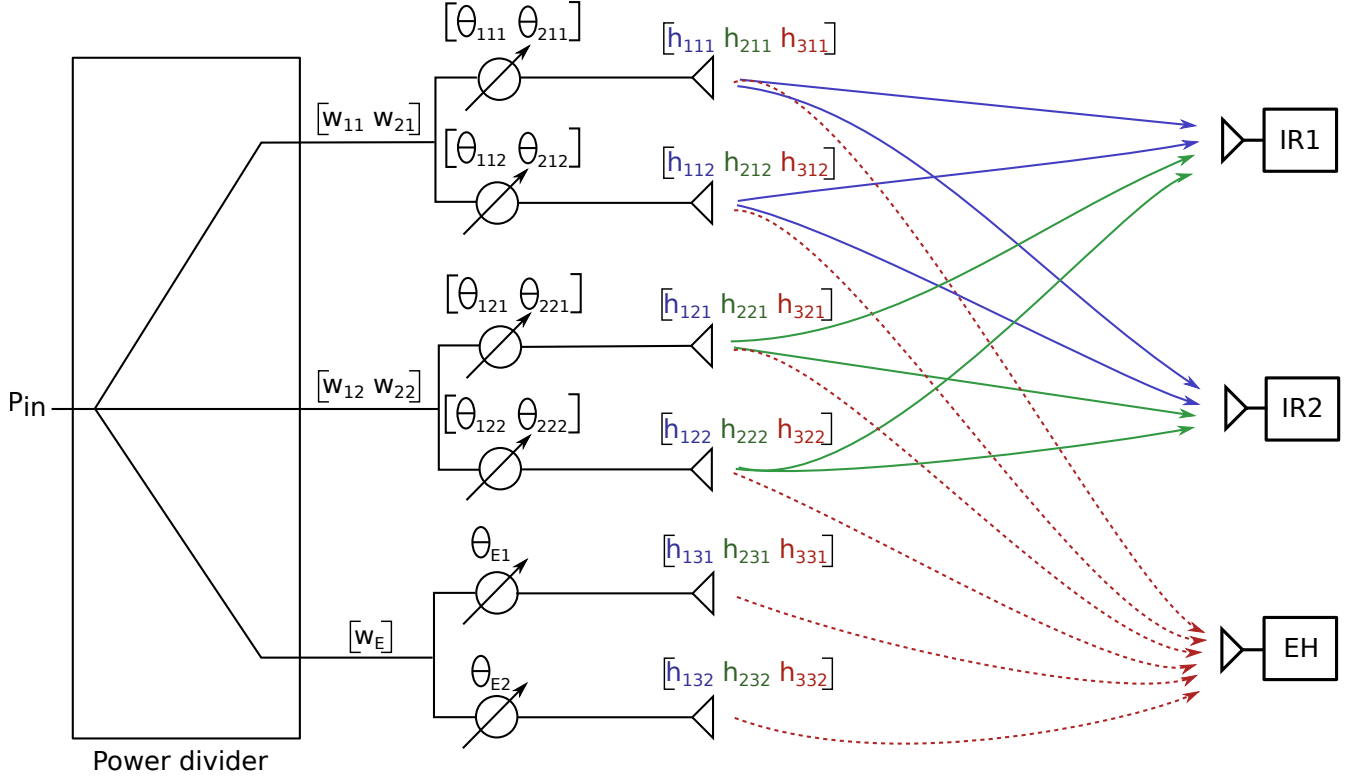


Figure 4.5: Model of a multi-service coordination of hybrid precoding system with two antennas per transmitter for the colocated transmitters problem. The scheme has three RF chains, two information receivers (IRs) and one energy harvester (EH). Each phase shifter has a phase angle noted θ_{ijk} , and each antenna has a channel link going to every receiver, noted $\mathbf{h}_{ijk}$; $i \in \{1, 2, 3\}$ representing the receiver and $j \in \{1, 2, 3\}$ representing the transmitter of the link; $k \in \{1, 2\}$ represents the antennas of the array. Among the three RF chains, only two are used to send information signals, respectively to IR1 and IR2 (straight lines). Those two have a radiation pattern such that the information is also radiated toward the EH, harvesting hence energy (dashed lines coming from the first two RF chains). The third FR chain is only used to send energy to the EH (dashed lines coming from the third RF chain), and causes interferences to the IRs (not drawn on the scheme).

Example of analog precoding Let's consider the model of the multi-service coordination of a full digital system, and let's add to it the phase precoder.

The information vectors are given by:

$$\mathbf{w}_1 = \begin{bmatrix} \theta_{111}w_{11} & \theta_{211}w_{21} \end{bmatrix}$$

$$\mathbf{w}_2 = \begin{bmatrix} \theta_{112}w_{12} & \theta_{212}w_{22} \end{bmatrix}$$

where $\theta_{ij} = \exp(j\theta_{ij})$.

The information covariance matrices can thus be decomposed as the element-wise product of the

phase shifters matrix and the power splitter matrix.

$$\mathbf{W}_1 = \begin{bmatrix} (\theta_{11}w_{11})(\theta_{11}w_{11})' & (\theta_{11}w_{11})(\theta_{21}w_{21})' \\ (\theta_{21}w_{21})(\theta_{11}w_{11})' & (\theta_{21}w_{21})(\theta_{21}w_{21})' \end{bmatrix} = \begin{bmatrix} 1 & \theta_{11}\theta'_{21} \\ \theta_{21}\theta'_{11} & 1 \end{bmatrix} \cdot \begin{bmatrix} |w_{11}|^2 & w_{11}w'_{21} \\ w_{12}w'_{11} & |w_{21}|^2 \end{bmatrix}$$

$$\mathbf{W}_2 = \begin{bmatrix} (\theta_{12}w_{12})(\theta_{12}w_{12})' & (\theta_{12}w_{12})(\theta_{22}w_{22})' \\ (\theta_{22}w_{22})(\theta_{12}w_{12})' & (\theta_{22}w_{22})(\theta_{22}w_{22})' \end{bmatrix} = \begin{bmatrix} 1 & \theta_{12}\theta'_{22} \\ \theta_{22}\theta'_{12} & 1 \end{bmatrix} \cdot \begin{bmatrix} |w_{12}|^2 & w_{12}w'_{22} \\ w_{22}w'_{12} & |w_{22}|^2 \end{bmatrix}$$

On the other hand, the energy vector is given by:

$$\mathbf{w}_E = [\theta_E w_E]$$

From this simple example, we have to introduce new notations to expand the model with arrays of antennas.

- Information matrices: we will create three-dimensional matrices $\in \mathbb{H}^{N_I \times N_I \times N_A}$. The third dimension is used to store the matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ relative to each antenna of one array. The new matrix notation for the information matrices is the following:

- θ_{ijk} is the phase precoder of each information vector, $(i, j, k) \in \{1, \dots, N_I\} \times \{1, \dots, N_I\} \times \{1, \dots, N_A\}$.
- $\mathbf{a}_{hi} \in \mathbb{C}^{N_I \times N_I \times N_A}$ is the phase precoder matrix for TXi:

- $\mathbf{H}_{i,inf} \in \mathbb{C}^{N_I \times N_I \times N_A}$ is the channel matrix between TX1 and TX2 to the receptor i .
- Both channel matrices and phase precoder matrices are 3 dimensional matrices, the third dimension used to store the matrices of the different antennas of an array.
- The notation Σ will be used to sum the third dimension of the matrices.
- Energy matrices: we create a two dimensional matrix of size $N_A \times N_A$ for the phase precoder and the channel conveying energy. Indeed, for the transfer of energy we consider all the interactions between all the beams conveying $\mathbf{w}_E$.
- $\mathbf{a}_g \in \mathbb{C}^{N_A \times N_A}$ is the precoder matrix for TX3:

$$\mathbf{a}_g = \begin{bmatrix} 1 & \cdots & \theta_{E,1}\theta'_{E,N_A} \\ \vdots & \ddots & \vdots \\ \theta_{E,N_A}\theta'_{E,1} & \cdots & 1 \end{bmatrix}$$

- $\mathbf{H}_{i,en} \in \mathbb{C}^{N_A \times N_A}$ is the channel matrix between TX3 and the receptor i .

With those notations, the total transmitted rate to both IR1 and IR2 can be written as :

$$\log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_2) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} \right) +$$

$$\log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_1) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} \right)$$

We will now solve the optimization problem by two different ways as it has been done for the multi-service coordination scheme of a full digital system.

4.1.4.1 Collocated transmitters problem

With this resolution, we will optimize simultaneously the energy and information vectors. Then, we will optimize the phase shifters separately.

A. Maximization of the rate with respect to $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_E$

$$\begin{aligned}
& \underset{\mathbf{W}_1, \mathbf{W}_2 \in \mathbb{H}^{N_I \times N_I}, \mathbf{W}_E \in \mathbb{H}^1}{\text{maximize}} \quad \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} \right) + \\
& \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} \right) - \\
& \Re \left(\text{Tr} \left[\left(\sum \mathbf{H}_{2,inf} \times \left(\frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} - \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1) + \sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E} \right) \right) \right] \right) \times \\
& \left(\mathbf{a}_{h1} \cdot \mathbf{W}_1 - \mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu \right) \Big] \Big) - \\
& \Re \left(\text{Tr} \left[\left(\sum \mathbf{H}_{1,inf} \times \left(\frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} - \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1) + \sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E} \right) \right) \right] \right) \times \\
& \left(\mathbf{a}_{h2} \cdot \mathbf{W}_2 - \mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu \right) \Big] \Big) \quad (4.7)
\end{aligned}$$

$$\text{subject to } \text{Tr}(\mathbf{W}_1 + \mathbf{W}_2) + \text{Tr}(\mathbf{W}_E) \leq P_{\text{in}} \quad (\text{C1})$$

$$\text{Tr} \left(\sum \mathbf{H}_{3,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1) + \sum \mathbf{H}_{3,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2) \right) + \text{Tr}((\mathbf{H}_{3,en} \times \mathbf{a}_g) \times \mathbf{W}_E) \geq T \quad (\text{C2})$$

B. Maximization of the rate with respect to the phase precoder matrices

$$\begin{aligned}
& \underset{\mathbf{a}_{h1}, \mathbf{a}_{h2} \in \mathbb{C}^{N_I \times N_I \times N_A}}{\text{maximize}} \quad \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} \right) + \\
& \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E)} \right) \\
& \text{subject to } \mathbf{a}_{h1}, \mathbf{a}_{h2} \text{ being hermitian toeplitz matrices} \\
& \text{diag}(\mathbf{a}_{h1}) = 1_{N_I, 1, N_A} \\
& \text{diag}(\mathbf{a}_{h2}) = 1_{N_I, 1, N_A} \quad (4.8)
\end{aligned}$$

C. Minimization of the interferences with respect to the phase precoder matrices

$$\begin{aligned}
& \underset{\mathbf{a}_g \in \mathbb{C}^{N_A \times N_A}}{\text{minimize}} \quad (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E \\
& \text{subject to } \mathbf{a}_g \text{ being a hermitian toeplitz matrix} \\
& \text{diag}(\mathbf{a}_g) = 1_{N_A, 1} \quad (4.9)
\end{aligned}$$

The pseudo-code is the following:

Algorithm 4 Multi-service coordination of a hybrid precoding system: collocated transmitters problem.

```

1: function HYBRID_V2_ALL(Input)
2:   Input:  $(N_I, N_A, H_{1,inf}, H_{1,en}, H_{2,inf}, H_{2,en}, H_{3,inf}, H_{3,en}, P_{in}, \sigma_I^2,$ 
3:    $\text{iteration}_{\text{maximal}}, \text{iteration}_{\text{maximal}_a}, \Delta, \Delta_\phi, T)$ 
4:    $\tau_1, \tau_2 = 0$ 
5:    $\mathbf{a}_{h1} = \exp(-1i\angle \mathbf{H}_{1,inf}), \mathbf{a}_{h2} = \exp(-1i\angle \mathbf{H}_{1,inf})$ 
6:    $\mathbf{a}_{h1}^\mu = 0.5\mathbf{a}_{h1}, \mathbf{a}_{h2}^\mu = 0.5\mathbf{a}_{h2}$ 
7:    $\mathbf{a}_g = \exp(-1i\angle \mathbf{H}_{3,en})$ 
8:    $\mathbf{a}_g^\mu = 0.5\mathbf{a}_g$ 
9:    $\text{maxit}_a = 0$ 
10:   $\mu = 0;$ 
11:  while  $(\|\mathbf{a}_{h1} - \mathbf{a}_{h1}^\mu\|_F \leq \Delta_\phi \text{ or } \|\mathbf{a}_{h2} - \mathbf{a}_{h2}^\mu\|_F \leq \Delta_\phi \text{ or } \|\mathbf{a}_g - \mathbf{a}_g^\mu\|_F \leq \Delta_\phi)$  and  $\text{maxit}_a \leq$ 
    $\text{iteration}_{\text{maximal}_a}$  do
12:     $\mathbf{a}_{h1}^\mu = \mathbf{a}_{h1}, \mathbf{a}_{h2}^\mu = \mathbf{a}_{h2}, \mathbf{a}_g^\mu = \mathbf{a}_g$ 
13:     $\mathbf{W}_1 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_2 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_E = \frac{1}{3} P_{in} \mathbf{Id}(N_A)$ 
14:     $\mathbf{W}_1^0 = 0.5\mathbf{W}_1; \mathbf{W}_2^0 = 0.5\mathbf{W}_2; \mathbf{W}_E^0 = 0.5\mathbf{W}_E$ 
15:     $\text{maxit} = 0;$ 
16:     $\nu = 0$ 
17:    while  $(\|\mathbf{W}_1 - \mathbf{W}_1^\nu\|_F \leq \Delta \text{ or } \|\mathbf{W}_2 - \mathbf{W}_2^\nu\|_F \leq \Delta \text{ or } \|\mathbf{W}_E - \mathbf{W}_E^\nu\|_F \leq \Delta)$  and  $\text{maxit}$ 
    $\leq \text{iteration}_{\text{maximal}}$  do
18:       $\mathbf{W}_1^\nu = \mathbf{W}_1, \mathbf{W}_2^\nu = \mathbf{W}_2, \mathbf{W}_E^\nu = \mathbf{W}_E$ 
19:      Solve the problem 4.7  $\Rightarrow \mathbf{W}_1, \mathbf{W}_2$  and  $\mathbf{W}_E$ 
20:      if  $\text{isnan}(\mathbf{W}_E)$  then
21:         $\mathbf{W}_1, \mathbf{W}_2 = \mathbf{Id}(N_I)$ 
22:         $\mathbf{W}_E = \mathbf{Id}(N_A)$ 
23:         $\text{maxit} = \text{iteration}_{\text{maximal}} + 1$ 
24:      else if  $\text{isinf}(\mathbf{W}_E)$  then
25:         $\mathbf{W}_1, \mathbf{W}_2 = \mathbf{Id}(N_I)$ 
26:         $\mathbf{W}_E = \mathbf{Id}(N_A)$ 
27:         $\text{maxit} = \text{iteration}_{\text{maximal}} + 1$ 
28:      end if
29:      Set  $\nu \leftarrow \nu + 1$ 
30:      Set  $\text{maxit} \leftarrow \text{maxit} + 1$ 
31:    end while
32:    Solve the problem 4.8  $\Rightarrow \mathbf{a}_{h1}$  and  $\mathbf{a}_{h2}$ 
33:    Solve the problem 4.9  $\Rightarrow \mathbf{a}_g$ 
34:    Set  $\mu \leftarrow \mu + 1$ 
35:    Set  $\text{maxit}_a \leftarrow \text{maxit}_a + 1$ 
36:  end while
37: end function

```

In this algorithm, we have two interleaved while loops. As a matter of fact, the algorithm solves for given phase precoders (the power precoder problem, equation 4.7). Then, for given power precoder matrices, it optimizes the phase precoders. Finally, it repeats this procedure $\text{iteration}_{\text{maximal}_a}$ times or stops when the Frobenius norm between the old and current phase precoder matrices is below Δ_ϕ .

This procedure is thus quite time-consuming. In fact, it is maxit_a times longer than for a multi-service coordination of a full digital system, but leads to better performances (see CHAPTER 5).

4.1.4.2 Separate transmitters problem with power reallocation

This resolution will solve iteratively first the information vectors, then the energy vector and finally their respective phase precoders.

A. Maximization of the rate with respect to $\mathbf{W}_1$ and $\mathbf{W}_2$

$$\begin{aligned}
& \underset{\mathbf{W}_1, \mathbf{W}_2 \in \mathbb{H}^{N_I \times N_I}}{\text{maximize}} \quad \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E^\nu)} \right) + \\
& \quad \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2))}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E^\nu)} \right) - \\
& \quad \Re \left(\text{Tr} \left[\left(\sum \mathbf{H}_{2,inf} \times \left(\frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E^\nu)} - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu) + \sum \mathbf{H}_{2,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu) + (\mathbf{H}_{2,en} \times \mathbf{a}_g) \times \mathbf{W}_E^\nu)} \right) \right) \right]^T \times \\
& \quad \left(\mathbf{a}_{h1} \cdot \mathbf{W}_1 - \mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu \right) \Big] \Big) - \\
& \quad \Re \left(\text{Tr} \left[\left(\sum \mathbf{H}_{1,inf} \times \left(\frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu) + \mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E^\nu)} - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h1} \cdot \mathbf{W}_1^\nu) + \sum \mathbf{H}_{1,inf} \times (\mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu) + (\mathbf{H}_{1,en} \times \mathbf{a}_g) \times \mathbf{W}_E^\nu)} \right) \right) \right]^T \times \\
& \quad \left(\mathbf{a}_{h2} \cdot \mathbf{W}_2 - \mathbf{a}_{h2} \cdot \mathbf{W}_2^\nu \right) \Big] \Big) \\
& \text{subject to } \text{Tr}(\mathbf{W}_1 + \mathbf{W}_2) \leq (1 - \alpha)P_{in} \tag{4.10}
\end{aligned}$$

B. Minimization of the energy beamvector

$$\mathbf{W}_E = \max \left(0, (T - \text{Tr}(\sum \mathbf{H}_{3,inf} * \mathbf{a}_{h1} \cdot * \mathbf{W}_1 + \sum \mathbf{H}_{3,inf} * \mathbf{a}_{h2} \cdot * \mathbf{W}_2)) \setminus (\mathbf{H}_{3,en} \cdot * \mathbf{a}_g) \right) \tag{4.11}$$

C. Maximization of the rate with respect to the phase precoder matrices

$$\begin{aligned}
& \underset{\mathbf{a}_{h1}, \mathbf{a}_{h2} \in \mathbb{C}^{N_I \times N_I \times N_A}}{\text{maximize}} \quad \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{1,inf} * \mathbf{a}_{h1} \cdot * \mathbf{W}_1^\nu)}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{1,inf} * \mathbf{a}_{h2} \cdot * \mathbf{W}_2^\nu + (\mathbf{H}_{1,en} \cdot * \mathbf{a}_g) * \mathbf{W}_E^\nu)} \right) + \\
& \quad \log_2 \left(1 + \frac{\text{Tr}(\sum \mathbf{H}_{2,inf} * \mathbf{a}_{h2} \cdot * \mathbf{W}_2^\nu)}{\sigma_I^2 + \text{Tr}(\sum \mathbf{H}_{2,inf} * \mathbf{a}_{h1} \cdot * \mathbf{W}_1^\nu + (\mathbf{H}_{2,en} \cdot * \mathbf{a}_g) * \mathbf{W}_E^\nu)} \right) \\
& \text{subject to } \mathbf{a}_{h1}, \mathbf{a}_{h2} \text{ being hermitian toeplitz matrices} \\
& \quad \text{diag}(\mathbf{a}_{h1}) = 1_{N_I, 1, N_A} \tag{4.12} \\
& \quad \text{diag}(\mathbf{a}_{h2}) = 1_{N_I, 1, N_A} \tag{4.13}
\end{aligned}$$

D. Minimization of the interferences with respect to the phase precoder matrices

$$\underset{\mathbf{a}_g, \mathbf{a}_g \in \mathbb{C}^{N_A \times N_A}}{\text{minimize}} \quad \mathbf{H}_{1,en} * \mathbf{a}_g \cdot * \mathbf{W}_E + \mathbf{H}_{2,en} * \mathbf{a}_g \cdot * \mathbf{W}_E$$

$$\begin{aligned} &\text{subject to } \mathbf{a}_g \text{ being a hermitian toeplitz matrix} \\ &\text{diag}(\mathbf{a}_g) = \mathbf{1}_{N_A,1} \end{aligned} \quad (4.14)$$

The algorithm solving iteratively those equations is:

Algorithm 5 Multi-service coordination of a hybrid precoding system for separate transmitters with power reallocation.

```

1: function HYBRID_v2(Input)
2:   Input: ( $N_I, N_A, H_{1,inf}, H_{1,en}, H_{2,inf}, H_{2,en}, H_{3,inf}, H_{3,en}, P_{in}, \sigma_I^2, \text{iteration}_{\max}, \text{iteration}_{\max,a}$ ,
3:    $\Delta, \Delta_\phi, T, \alpha$ )
4:    $\tau_1, \tau_2 = 0$ 
5:    $\mathbf{a}_{h1} = \exp(-1i\angle \mathbf{H}_{1,inf}), \mathbf{a}_{h2} = \exp(-1i\angle \mathbf{H}_{1,inf})$ 
6:    $\mathbf{a}_{h1}^\mu = 0.5\mathbf{a}_{h1}, \mathbf{a}_{h2}^\mu = 0.5\mathbf{a}_{h2}$ 
7:    $\mathbf{a}_g, \mathbf{a}_g = \exp(-1i\angle \mathbf{H}_{3,en})$ 
8:    $\mathbf{a}_g^\mu = 0.5\mathbf{a}_g$ 
9:    $\text{maxit}_a = 0$ 
10:   $\mu = 0$ ;
11:  while ( $\|\mathbf{a}_{h1} - \mathbf{a}_{h1}^\mu\|_F \leq \Delta_\phi$  or  $\|\mathbf{a}_{h2} - \mathbf{a}_{h2}^\mu\|_F \leq \Delta_\phi$  or  $\|\mathbf{a}_g - \mathbf{a}_g^\mu\|_F \leq \Delta_\phi$ ) and  $\text{maxit}_a \leq$ 
     $\text{iteration}_{\max,a}$  do
12:     $\mathbf{a}_{h1}^\mu = \mathbf{a}_{h1}, \mathbf{a}_{h2}^\mu = \mathbf{a}_{h2}, \mathbf{a}_g^\mu = \mathbf{a}_g$ 
13:     $\mathbf{W}_1 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_2 = \frac{1}{3N_I} P_{in} \mathbf{Id}(N_I); \mathbf{W}_E = \frac{1}{3} P_{in} \mathbf{Id}(N_A)$ 
14:     $\mathbf{W}_1^0 = 0.5\mathbf{W}_1; \mathbf{W}_2^0 = 0.5\mathbf{W}_2; \mathbf{W}_E^0 = 0.5\mathbf{W}_E$ 
15:     $\text{maxit} = 0$ ;
16:     $\nu = 0$ 
17:    while ( $\|\mathbf{W}_1 - \mathbf{W}_1^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_2 - \mathbf{W}_2^\nu\|_F \leq \Delta$  or  $\|\mathbf{W}_E - \mathbf{W}_E^\nu\|_F \leq \Delta$ ) and  $\text{maxit}$ 
     $\leq \text{iteration}_{\max}$  do
18:       $\mathbf{W}_1^\nu = \mathbf{W}_1, \mathbf{W}_2^\nu = \mathbf{W}_2, \mathbf{W}_E^\nu = \mathbf{W}_E$ 
19:      Solve the problem 4.10  $\Rightarrow \mathbf{W}_1, \mathbf{W}_2$ 
20:      Solve the problem 4.11  $\Rightarrow \mathbf{W}_E$ 
21:      if  $\text{isnan}(\mathbf{W}_E)$  then
22:         $\mathbf{W}_1, \mathbf{W}_2 = \mathbf{Id}(N_I)$ 
23:         $\mathbf{W}_E = \mathbf{Id}(N_A)$ 
24:         $\text{maxit} = \text{iteration}_{\max} + 1$ 
25:      else if  $\text{isinf}(\mathbf{W}_E)$  or  $\mathbf{W}_E > P_{in}$  then
26:         $\mathbf{W}_E = \mathbf{Id}(N_A), \alpha = 0$ 
27:      else if  $\mathbf{W}_E \geq \alpha P_{in}$  or  $\mathbf{W}_E \leq \alpha P_{in}$  then
28:         $\alpha = \frac{\mathbf{W}_E}{P_{in}}$ 
29:         $\mathbf{W}_E = \alpha P_{in}$ 
30:      end if
31:      Set  $\nu \leftarrow \nu + 1$ 
32:      Set  $\text{maxit} \leftarrow \text{maxit} + 1$ 
33:    end while
34:    Solve the problem 4.13  $\Rightarrow \mathbf{a}_{h1}, \mathbf{a}_{h2}$ 
35:    Solve the problem 4.14  $\Rightarrow \mathbf{a}_g$ 
36:    Set  $\mu \leftarrow \mu + 1$ 
37:    Set  $\text{maxit}_a \leftarrow \text{maxit}_a + 1$ 
38:  end while
39: end function

```

5 | Results

This chapter will analyze the signal-to-interference and noise ratio, rate, energy and Pareto curves of each model detailed in CHAPTER 4.

Since the goal of the thesis is to understand and compare the trade-off between the rate and energy for the aforementioned models, we have chosen to work with normalized parameters. In particular, the channel vector is random Rayleigh distributed with a zero mean and a variance of 1. This means that we do not consider any path loss or distance dependent receivers.

Hence, the initialization parameters chosen for the algorithms are the following:

- System setup:
 - Number of radio-frequency chains: $N_T = 3$.
 - Number of information receivers: $N_I = 2$.
 - Number of energy harvesters: $N_R = 1$.
 - Number of antenna par antenna array: $N_A = \{2, 4\}$.
 - Channel covariance matrices: Random Rayleigh normalized.
 - Noise introduced by the channel on all information vectors: $\sigma_I^2 = 10^{-3}$.
 - P_{in} : total isotropic power injected in the system; also called input power.
- Iterative process: stopping criterion.
 - Tolerance between the Frobenius norm of the old and current power allocation precoder: $\Delta = 10^{-6}$.
 - Tolerance between the Frobenius norm of the old and current phase shifter precoder: $\Delta_\phi = 10^{-6}$.
 - Maximal number of iterations for the power allocation precoder: $\text{iteration}_{\text{maximal}} = 20$.
 - Maximal number of iterations for the phase shifter precoder: $\text{iteration}_{\text{maximal}_a} = 10$.
- Initialization of the power precoder (equal distribution) for $N = \{N_T, N_I, N_I\}$ and $N_E = \{N_T, 1, 1\}$ respectively for the signal coordination model, the model of multi-service coordination of full digital systems and the model of multi-service coordination of hybrid precoding systems:
 - $\mathbf{W}_1^0 = \frac{1}{3N} P_{\text{in}} \mathbf{Id}(N)$.
 - $\mathbf{W}_2^0 = \frac{1}{3N} P_{\text{in}} \mathbf{Id}(N)$.
 - $\mathbf{W}_E^0 = \frac{1}{3N_E} P_{\text{in}} \mathbf{Id}(N_E)$.
- Initialization of the phase shifter precoders:

- $\mathbf{a}_{h1}(:, :, j) = \exp(-1i\angle\mathbf{H}_{1,inf}(:, :, j))$, with $j \in \{1, \dots, N_A\}$.
- $\mathbf{a}_{h2}(:, :, j) = \exp(-1i\angle\mathbf{H}_{2,inf}(:, :, j))$, with $j \in \{1, \dots, N_A\}$.
- $\mathbf{a}_g = \exp(-1i\angle\mathbf{H}_{3,en})$.

- Range of power and energy transfer threshold:

- Power: $P_{in} \in [10, \dots, 20]$ dBm.
- $P_0 = 10$ dBm.
- Energy efficiency threshold: $\gamma = [0.1, \dots, 2.5]$.
- Energy efficiency threshold for the Pareto curves (to have a high dynamic): $\gamma = [1, \dots, 25]$.

5.1 Signal coordination

5.1.1 SINR

First let's analyze the signal to interference noise ratio for both information receivers. From FIGURE 5.1, two main observations emerge.

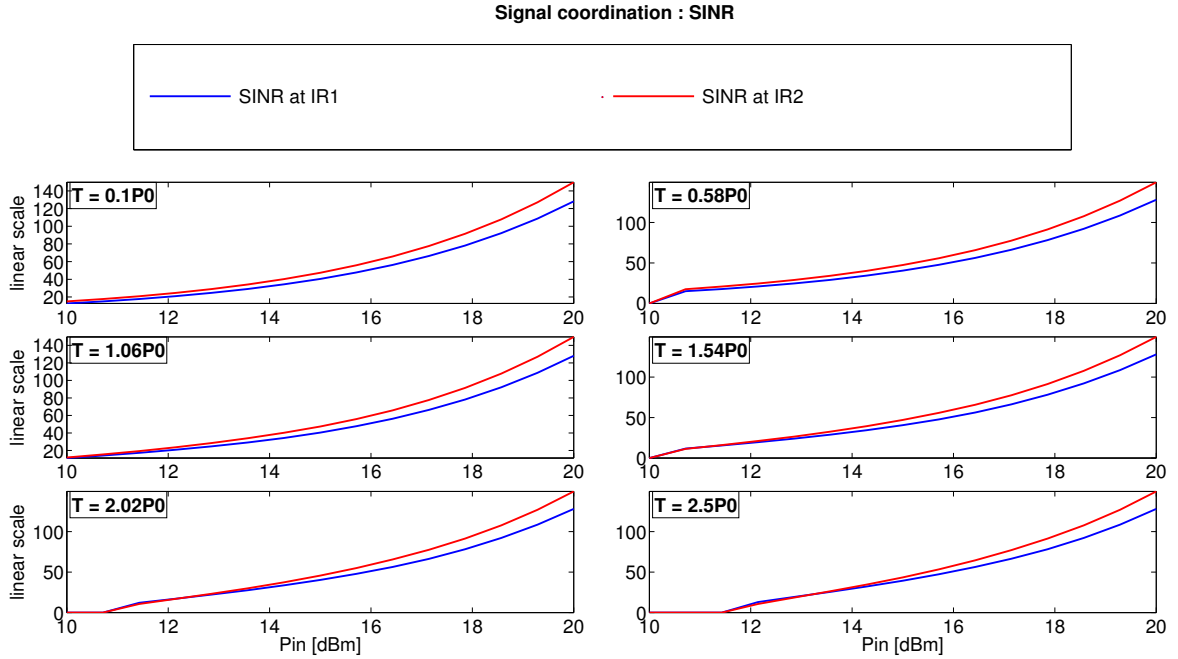


Figure 5.1: SINR as a function of P_{in} in the signal coordination scheme: starting from the upper left subfigure, to the lowest right, the threshold T is increasing. There are small differences in SINR at IR1 and IR2. This is due to the power control precoder that allocates more power to the channel being in a better condition. Besides, for T increasing, the SINR is set to zero every time the constraint on the minimal amount of harvested energy is not fulfilled.

First, the SINR at both IRs are practically the same. This phenomenon is expected thanks to the coordination of signals. Indeed, maximizing the sum of rates is the same as maximizing the SINR since the relation between these two functions is:

$$\text{Rate} = \log_2(1 + \text{SINR}) \quad (5.1)$$

By the principle of the power control [11] and the signal coordination, the power allocated to $\mathbf{W}_1$ and $\mathbf{W}_2$ is proportional to the channels quality, the received signals and the interferences. In this situation, the channel covariance matrices are the following:

$$\mathbf{H}_1 = \begin{bmatrix} 1.346 & -0.168 - 0.38i & -1.151 - 0.408i \\ -0.168 + 0.38i & 0.128 & 0.259 - 0.274i \\ -1.151 + 0.408i & 0.259 + 0.274i & 1.108 \end{bmatrix}$$

$$\mathbf{H}_2 = \begin{bmatrix} 0.589 & -0.719 + 0.689i & 0.59 - 0.294i \\ -0.719 - 0.689i & 1.687 & -1.066 - 0.332i \\ 0.59 + 0.294i & -1.066 + 0.332i & 0.739 \end{bmatrix}$$

Hence, to maximize the sum of rates, the algorithm allocates more power to $\mathbf{W}_1$ which has a worse covariance channel matrix than $\mathbf{W}_2$ ($\text{Tr}(\mathbf{W}_2) > \text{Tr}(\mathbf{W}_1)$); for further explanation, refer to ANNEXE A.1).

The second noticeable fact is that the SINR is exponentially increasing with P_{in} . This behavior is expected for low values of P_{in} . For high values of P_{in} , we expect to reach a plateau when the increase in signal power is compensated by the increase in interferences. FIGURE 5.1 only shows the SINR in its increasing regime.

To prove this, let's analyze the different components of the SINR arriving at IR1 (FIGURE 5.2).

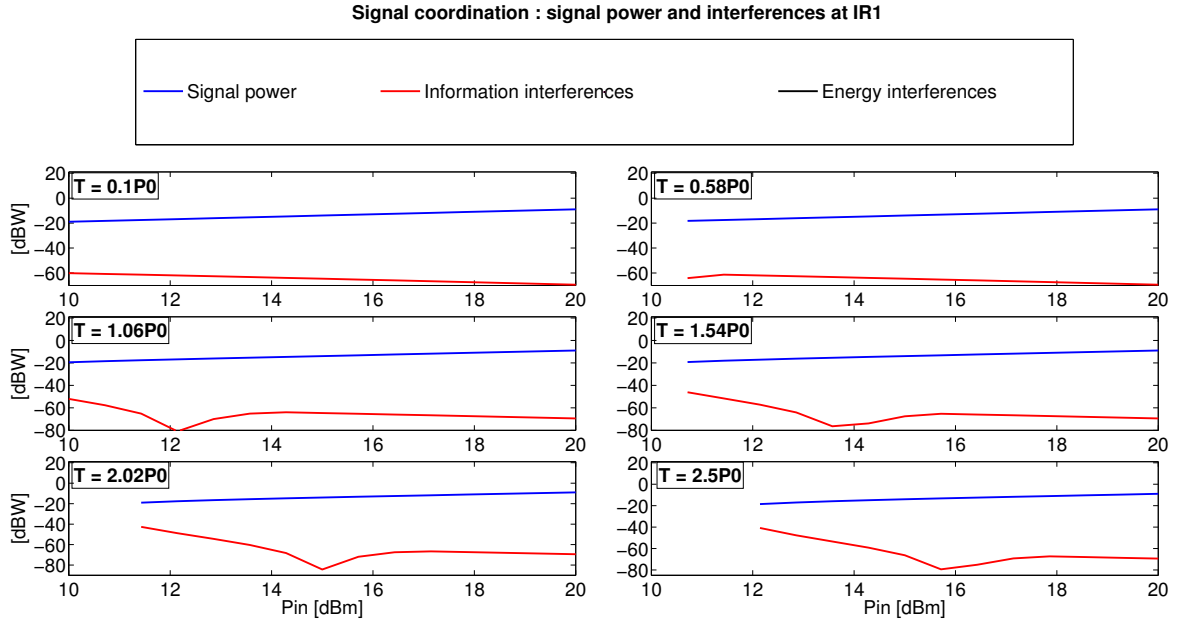


Figure 5.2: Signal power and interferences in the signal coordination scheme at IR1 as a function of P_{in} . The linear energy signal is null for all input power, hence not in the dynamic of the figure, and the signal power is higher than all interferences. Besides, the signal power increases a little bit with P_{in} , while the information interferences decrease.

First, we notice that the interferences due to the energy vector are closed to $-\infty$ dBW, so not in the range of FIGURE 5.2. It means that no power is allocated to $\mathbf{w}_E$. Moreover, the signal power increases with P_{in} , while the interferences decrease. In this situation, the interferences decrease because the power allocation is fully coordinated, and hence, with more power, a better adjustment can be made to both reduce the interferences and increase the rate. Besides, up to $T = 1.06P_0$, we can met the constraint on the minimal amount of harvested energy. From $T =$

1.54P₀, the constraint is not met but $\mathbf{w}_E$ is not activated. Hence, a further analysis with higher values of energy threshold is run to analyze the behavior of $\mathbf{w}_E$.

FIGURE 5.3 shows the power allocation between $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_E$ for $T \in \{7, 8, 9, 10\}$.

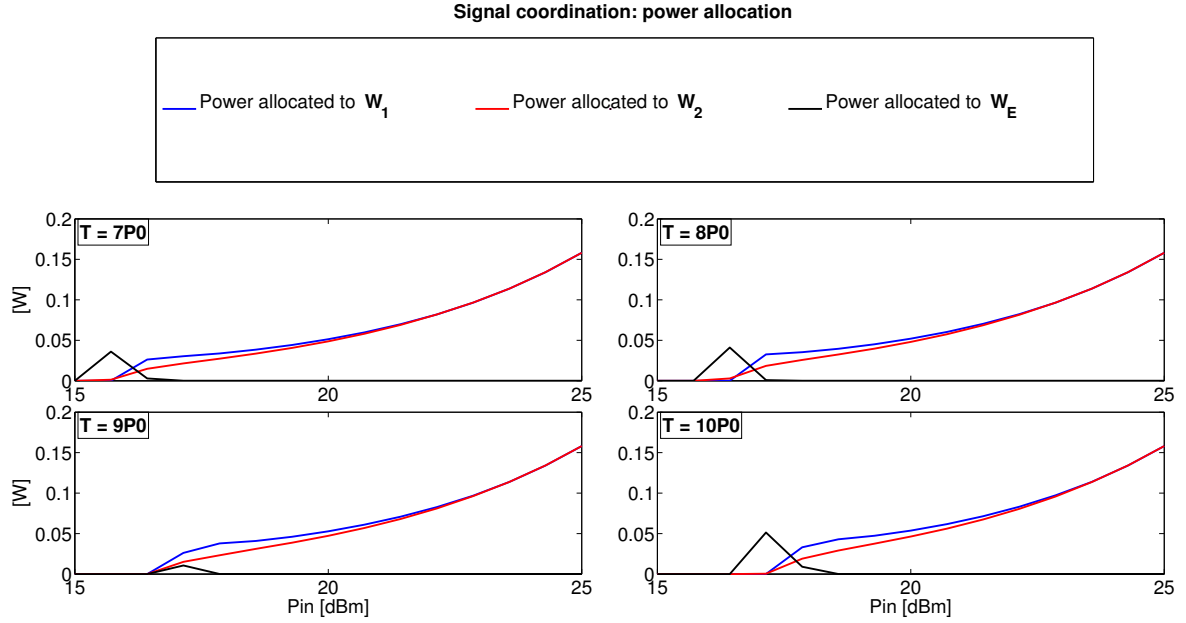


Figure 5.3: Power allocation in the signal coordination scheme as a function of P_{in} . For small values of P_{in} , where the constraint on the minimal harvested energy is not met, there is no power send by the information system ($\mathbf{W}_1$ and $\mathbf{W}_2$). At the transition between a feasible problem and a non-feasible one, the transmitter sending energy sends a peak of power. Besides, for high values of P_{in} , $\mathbf{W}_1$ and $\mathbf{W}_2$ have the same allocation of power while no power is allocated to $\mathbf{W}_E$.

In FIGURE 5.3, we see the transition between the non feasible region of our problem to the feasible one. As a matter of fact, when the problem is not feasible because there is not enough power allocated to the system to met the constraint on the minimal amount of harvested energy, no power is allocated to either the information vectors, either the energy one. Then, for higher values of P_{in} , allocating all the power to $\mathbf{W}_E$ gives a feasible solution. Hence, for those values of input power, we have a peak in power for $\mathbf{W}_E$. Finally, for even higher P_{in} , the problem becomes feasible by allocating the power only on the information vectors ($\mathbf{W}_1$ and $\mathbf{W}_2$). The impact on the interferences for $T \in \{7, \dots, 10\}$ can be seen in FIGURE 5.4.

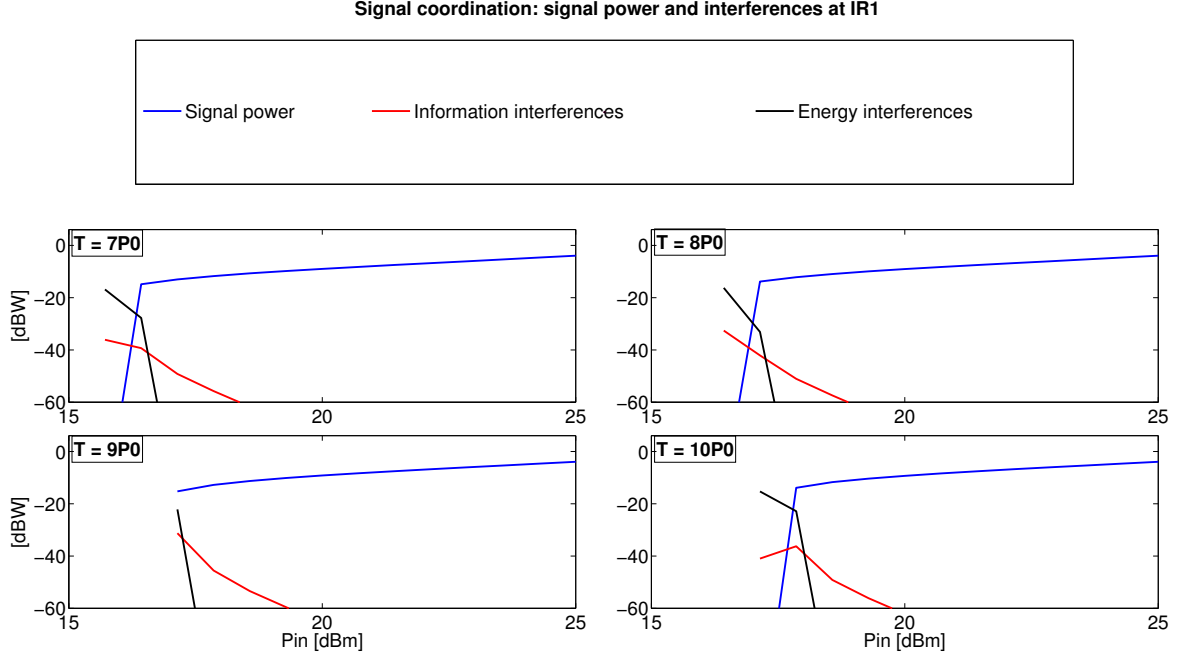


Figure 5.4: Interferences in the signal coordination scheme as a function of P_{in} for $T \in \{7, 8, 9, 10\}$. When the constraint on the minimal amount of harvested energy is met by allocating all the power to $\mathbf{W}_E$, the energy interferences are dominant. Then, for an increasing input power, the signal power increases and goes above the interferences, while the information interferences keep decreasing.

As expected, when the constraint on the minimal amount of harvested energy is met for the first time, the energy interferences term is dominant. Indeed, for the corresponding values of input power, there are no information power transmitted since all the power is allocated to $\mathbf{W}_E$. Then, in the transition zone (feasible problem with a null rate to a feasible problem with a transmission rate), the signal power increase and goes above the energy interferences that are decreasing. Above the transition, the signal power is constantly increasing, while both interferences are decreasing.

The same behavior is observed at IR2 for $T \in \{0.1, \dots, 2.5\}$ (FIGURE 5.5) and $T \in \{7, \dots, 10\}$ (FIGURE 5.6). The only difference is that the signal power is slightly lower and the information interferences slightly higher.

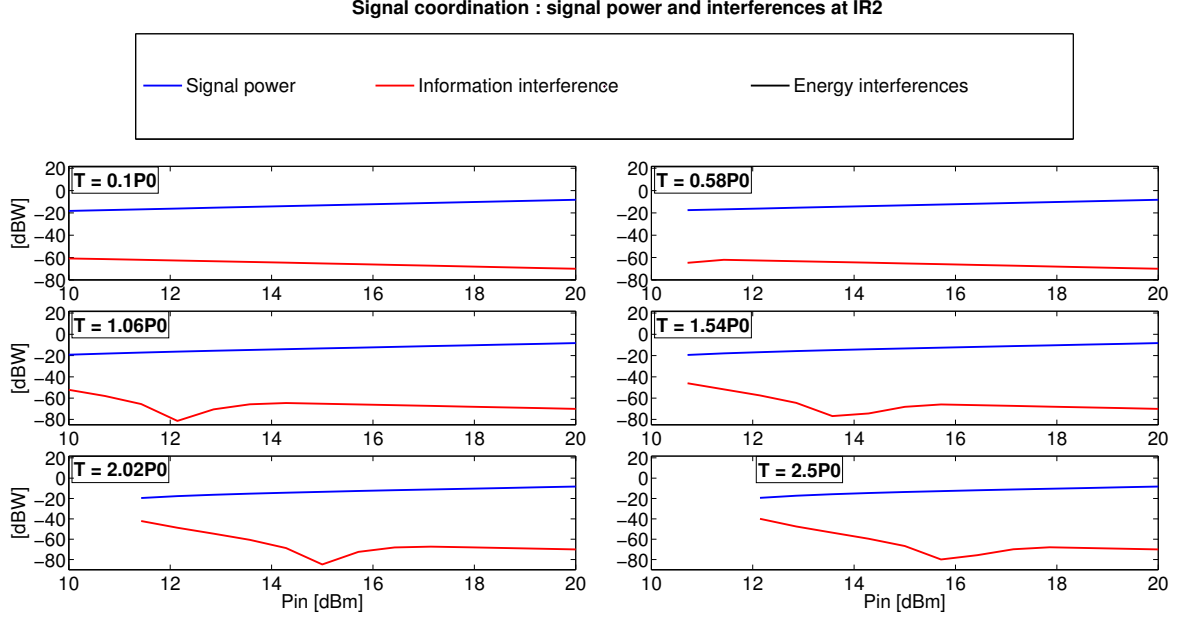


Figure 5.5: Signal power and interferences in the signal coordination scheme at IR2 as a function of P_{in} . The signal power is higher than the interferences, and the interferences due to the energy vector are null (around $-\infty$ dBW). Besides, the signal power increases a little bit with P_{in} , while the information interferences decrease.

For higher values of T ($T \in \{7, \dots, 10\}$), the interference terms can be seen in FIGURE 5.6.

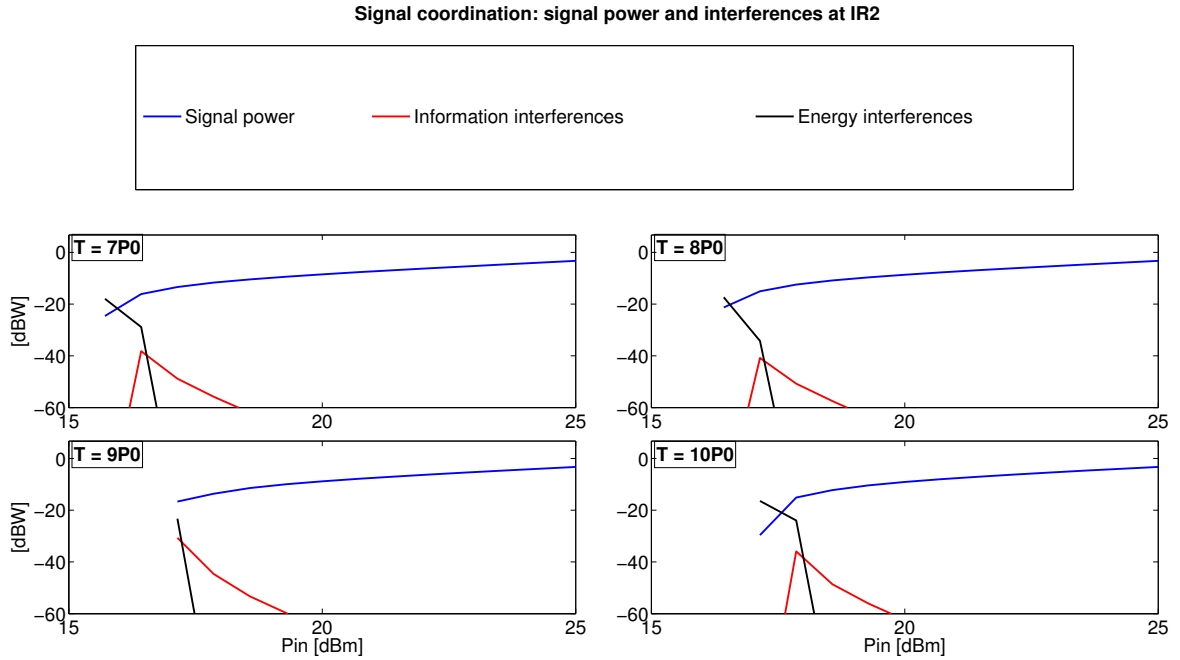


Figure 5.6: Interferences in the signal coordination scheme as a function of P_{in} for $T \in \{7, 8, 9, 10\}$. When the constraint on the minimal amount of harvested energy is just met, the energy interferences are dominant. Then, for an increasing input power, the signal power increases and go above the interferences, while the information interferences keep decreasing.

The behavior is slightly different than the interferences coming at IR1. Indeed, the SINR at IR2 is higher than the one at IR1 so the signal power is higher and the interferences lower. As a

matter of fact, the information interferences start by increasing when $\mathbf{w}_E$ decrease to compensate the loss of harvested energy such that the constraint on the minimal amount of harvested energy is still met. Then, they decrease while the signal power is increasing.

5.1.2 Rate and energy

Now that we have proven that the algorithm has an increasing SINR with respect to the input power, we analyze in FIGURE 5.7 with more details the rate and energy received, for different thresholds of minimal transferred energy.

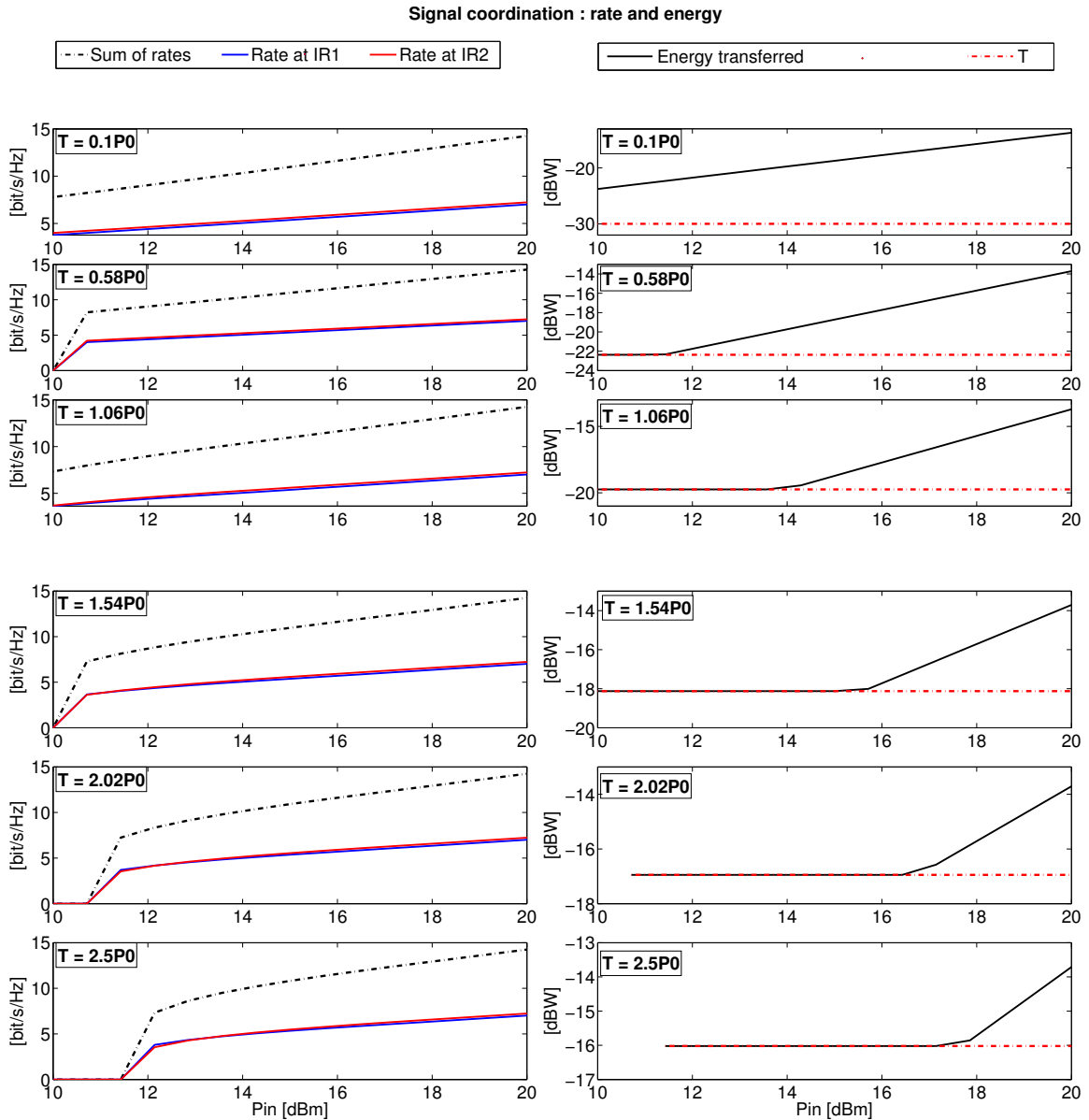


Figure 5.7: Rate and energy harvested in the signal coordination scheme as a function of P_{in} . From top left to bottom left, the rate is decreasing for low P_{in} since the constraint on the minimal amount of harvested energy starts to not be fulfilled anymore. In the right column, the harvested energy increases linearly for $P_{in} > T$. For lower value of P_{in} , either the constraint is tight, either it is not met (from $T \geq 2.02P_0$).

First, we can see that the rate is directly linked to the constraint on the minimal amount of energy transferred. Indeed, when the constraint is not fulfilled, the rate is automatically set to zero since the batteryless devices targeted for an application cannot be activated. Besides, there are no values drawn for the energy since we do not succeed to harvest T dBW of energy harvested.

Second, the difference in rate between the two information receivers is quite small. This is expected and again explained by the relation 5.1.

Next, we can see that for $T = 0.58P_0$, the rate is null although the constraint on the minimal energy harvested is fulfilled. This comes from the iterative process which stops before having found the optimal allocation of power.

Moreover, one can observe that the higher the threshold, the longer the constraint is tight. As a matter of fact, the system can only harvest more than T dBW when the input power is slightly lower than the threshold power. The slight difference between the input power and the minimal amount of energy to be transmitted is due to the antenna gains. Indeed, we are using an isotropic power and normalized channels without considering any path loss, so the total received power is higher than the transmitted one.

Finally, in view of understanding fully how the energy is harvested, we analyze the different signals arriving at the energy harvester (FIGURE 5.8).

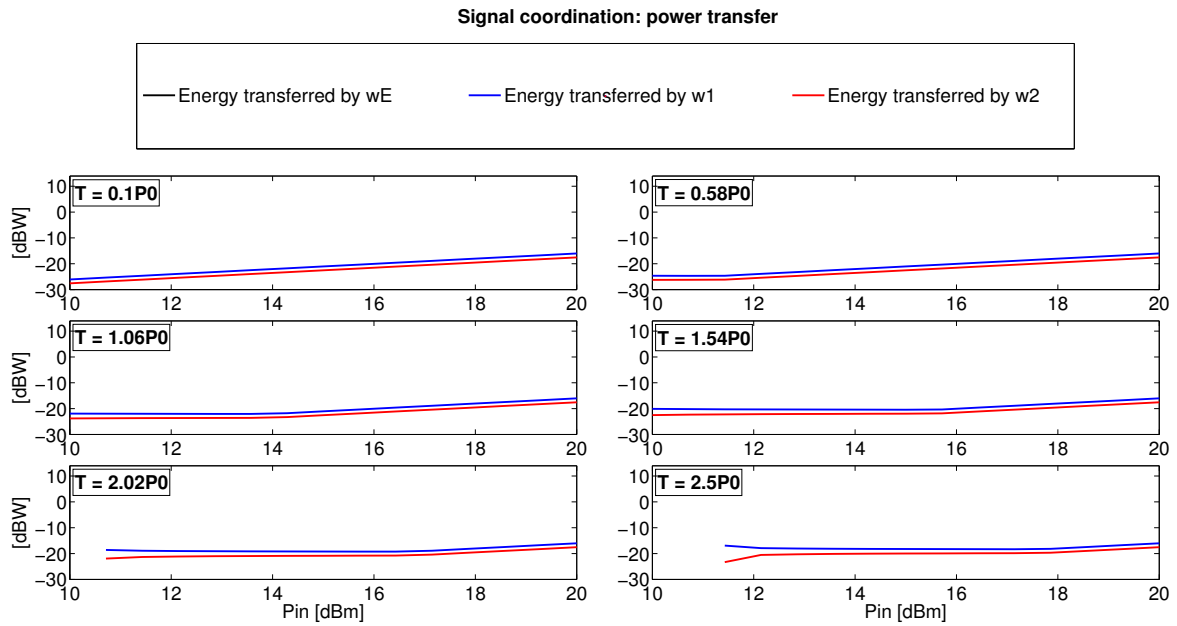


Figure 5.8: Power transferred at the energy harvester as a function of P_{in} . The harvested energy is the same for each T , when the constraint on T is met. Besides, there is a slight difference in the energy transferred by w_1 and w_2 due to the difference in the communication links. The energy transferred by w_E is not drawn since closed to $-\infty$ dBW and hence outside of the dynamic.

As can be seen, the contribution of the energy vector is null since no power is given to w_E . We can hence say that the coordination between the information and energy vectors manages to send efficiently the information both toward the information receivers and the energy harvester without creating significant energy interferences.

Moreover, we see that the contribution of the information vectors are practically the same. This is again due to the power control that gives a little bit more of power to w_1 since it has a worse channel covariance matrix.

5.1.3 Pareto curve

Finally, we will analyze the Pareto curves for $P_{\text{in}} = 15$ dBm. Those give information about the trade-off between the maximization of the rate and the maximization of the harvested energy.

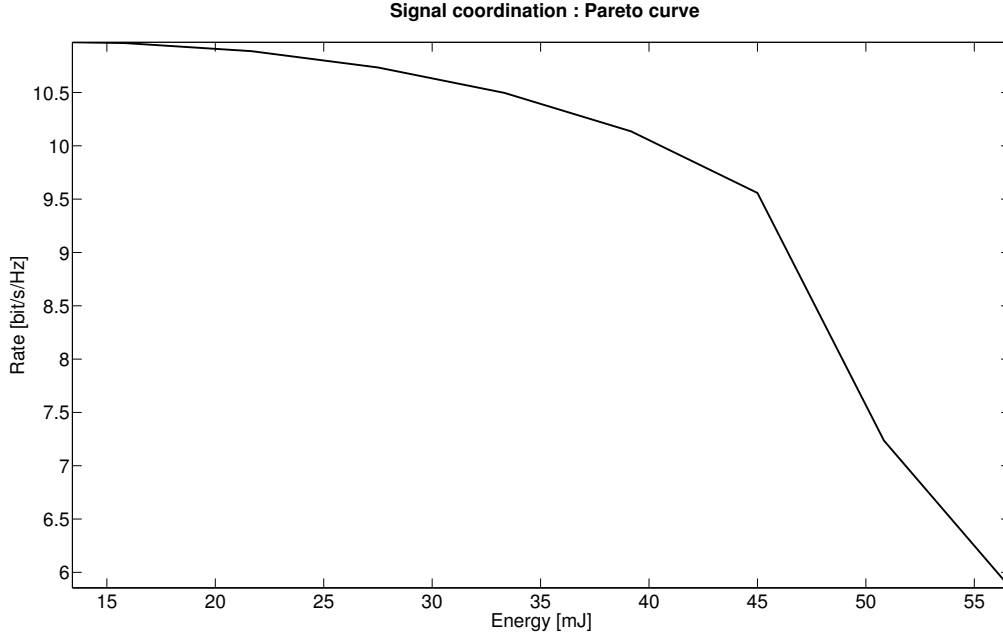


Figure 5.9: Pareto frontier: rate as a function of energy. Those two resources are in conflict with each other, since the first one is maximal when the second is minimal and reciprocally.

In FIGURE 5.9, we see that it is impossible to maximize both the rate and the energy simultaneously. This matches the results of [5]. Besides, the maximal harvested energy is 56.7 mJ for a total rate of 5.85 bit/s/Hz. In addition, we see that up to 45 mJ we are in the convex region of the problem, meaning that T is small enough to fulfill the constraint on the minimal amount of harvested energy. Beyond 45 mJ, the convexity of the curve changes because the problem is not feasible anymore (not convex).

5.1.4 Conclusion

In order to supply a batteryless device wirelessly, using a signal coordination scheme, we need to feed the system with pretty much the same amount of minimal energy as we wish to harvest. Besides, the maximal rate achieved will not exceed 14 bit/s/Hz, and the energy harvested will not exceed 56.7 mJ for an input power of 15 dBm. Meaning that for a reasonable injected power in a femtocell base station, we cannot activate devices requiring more than 56.7 mJ of input power. In addition, when the value of T is high ($T \in \{7, \dots, 10\}$), the system is infeasible for low values of P_{in} since the constraint on the minimal amount of energy harvested is not met. Increasing P_{in} will make the problem feasible by according all the power to $\mathbf{W}_E$. Increasing further P_{in} will allow to allocate all the power to only $\mathbf{W}_1$ and $\mathbf{W}_2$ so that the constraint is met and the rate is maximized.

5.2 Multi-service coordination of a full digital system

As for the **signal coordination** model, we will analyze the SINR, rate, energy and Pareto curves for several values of minimal harvested energy. Since we had two different algorithms for this

model, we will analyze both of them starting by the collocated transmitters one.

5.2.1 Collocated transmitters problem

SINR

FIGURE 5.10 shows the curve of SINR at both IR1 and IR2, versus the injected power.

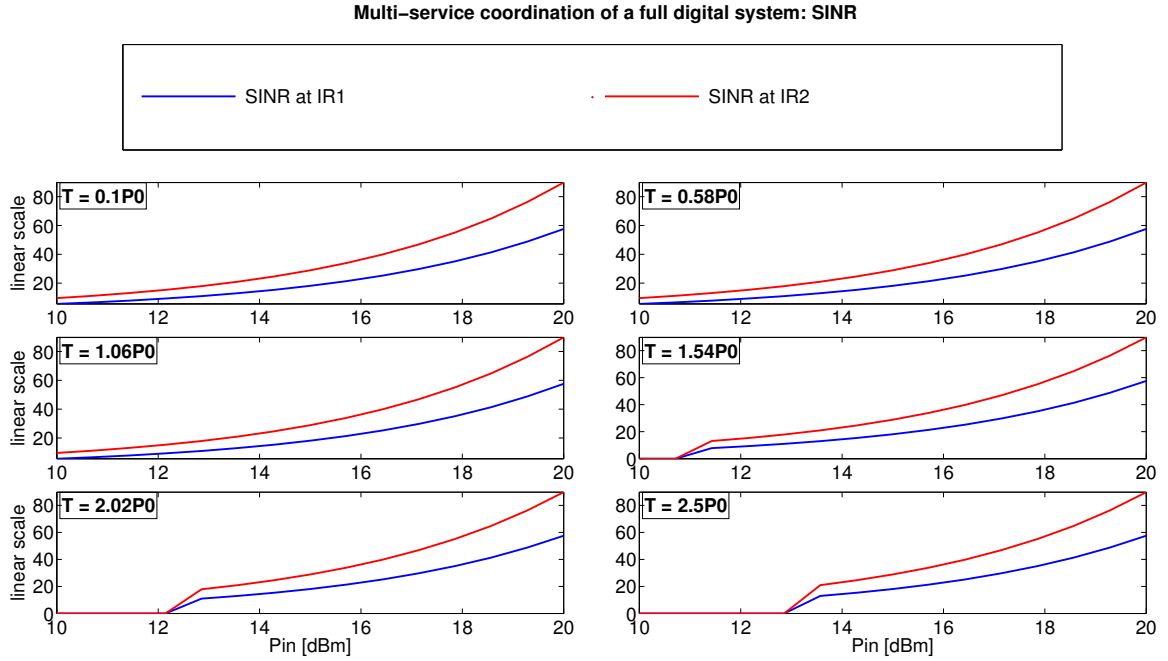


Figure 5.10: SINR as a function of P_{in} in the multi-service coordination scheme of a full digital system: starting from the upper left subfigure, to the lowest right, the threshold T is increasing. There are small differences in SINR at the IR1 and IR2. This is due to the power control precoder that allocates more power to the channel being in a worse condition.

There is no noticeable difference compared to the SINRs of the signal coordination scheme, except that the values are lower. This is normal since we only have coordination between the transmitter sending information and no more with the one sending energy. This diminution in coordination leads directly to increased information and energy interferences. In FIGURE 5.11 and FIGURE 5.12, we can see even better this increase.

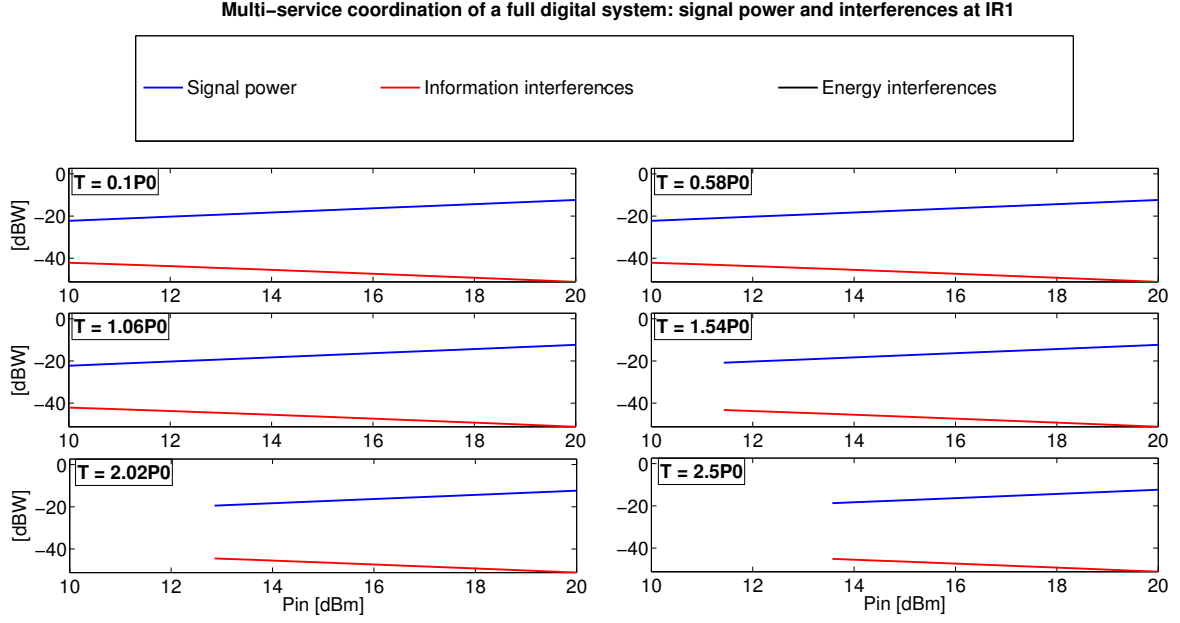


Figure 5.11: Signal power and interferences in the multi-service coordination scheme of a full digital system at IR1 as a function of P_{in} . There is no energy signal because no power is allocated to $\mathbf{w}_E$ by the algorithm. Indeed, it adds interferences for the rate and does not improve significantly the amount of energy harvested in comparison to $\mathbf{w}_1$ and $\mathbf{w}_2$. The signal power is higher than all interferences. Besides, the signal power increases a little bit with P_{in} , while the information interferences decrease.

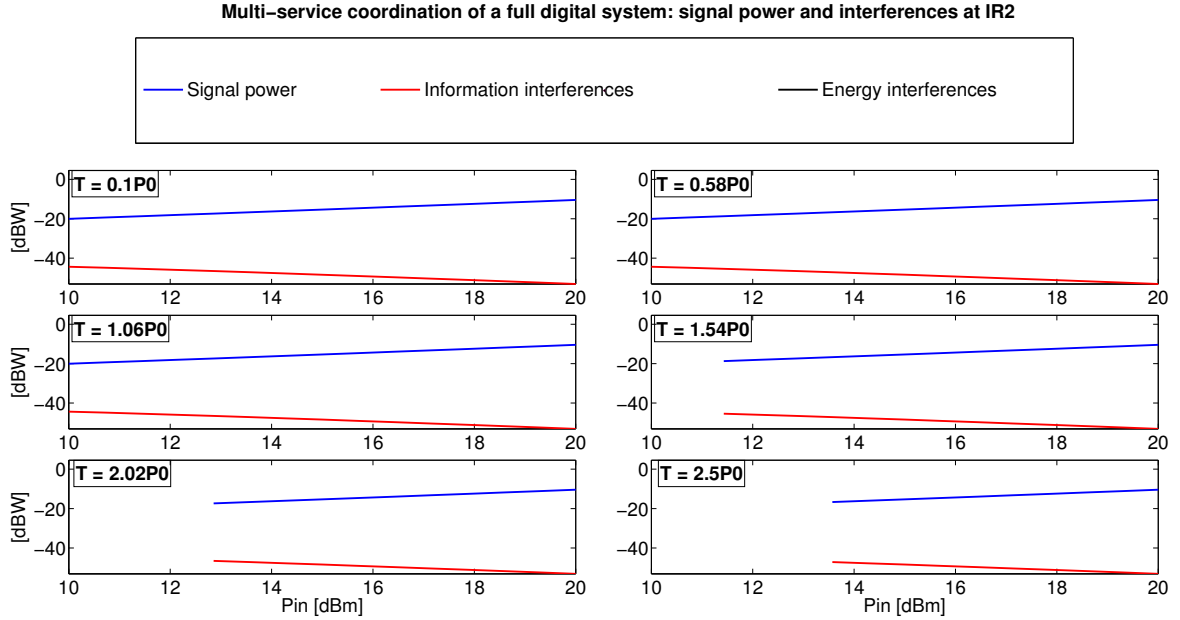


Figure 5.12: Signal power and interferences in the multi-service coordination scheme of a full digital system at IR2 as a function of P_{in} . There is no energy signal because no power is allocated to $\mathbf{w}_E$ by the algorithm. Indeed, it adds interferences for the rate and does not improve significantly the amount of energy harvested in comparison to $\mathbf{w}_1$ and $\mathbf{w}_2$. The signal power is higher than all interferences. Besides, the signal power increases a little bit with P_{in} , while the information interferences decrease.

Making a comparison between the pairs of figures 5.2-5.11 and 5.5-5.12, we see that the

signal power is the same, but that the interferences caused by the information are higher for the multi-coordination scheme of a full digital system by more or less 10 dBW.

Besides, since we do not have coordination with the energy vector, the algorithm does not allocate any power to the energy to reduce most interferences and thereby increase the rate. A direct consequence will be that the minimal energy transferred threshold will be fulfilled less frequently than in the signal coordination scheme (see subsection 5.2.1).

To understand fully the power allocation of such a model, let's analyze the allocation of power between $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_E$ for $T \in \{7, \dots, 10\}$ (FIGURE 5.13).

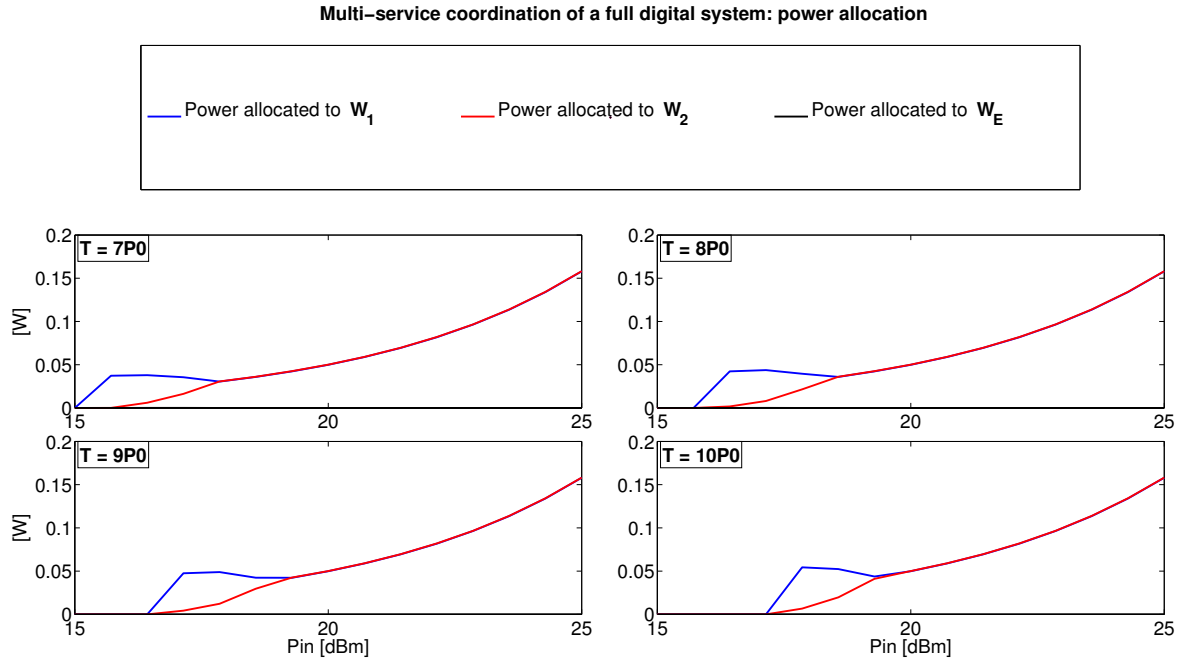


Figure 5.13: Power allocation in the multi-service coordination scheme of a full digital system as a function of P_{in} . No power is allocated to $\mathbf{W}_E$, and more power is allocated to $\mathbf{W}_1$ than for $\mathbf{W}_2$ at low value of P_{in} .

For low values of P_{in} , the constraint on the minimal amount of harvested energy is not met, and hence no power is allocated to the three signals. For higher values of P_{in} , the constraint is met, and the algorithm always choose to allocate the power between $\mathbf{W}_1$ and $\mathbf{W}_2$. As a matter of fact, for the multi-service coordination scheme of a full digital system, $\mathbf{W}_E$ will never be able to reach the constraint by its own. It will always need the help of at least an information vector. Moreover, more power is allocated to $\mathbf{W}_1$ than to $\mathbf{W}_2$ for low values of P_{in} in order to met the constraint.

Besides, the behavior of the interferences do not change for an increasing value of T since the algorithm always allocates the power between $\mathbf{W}_1$ and $\mathbf{W}_2$.

Rate and energy

In FIGURE 5.14, we can see that the performances achieved have the same behavior as for the signal coordination scheme. The differences are that the achieved rate and the energy harvested are lower, hence leading to a bigger range of power unsuitable for a given situation where we have batteryless devices.

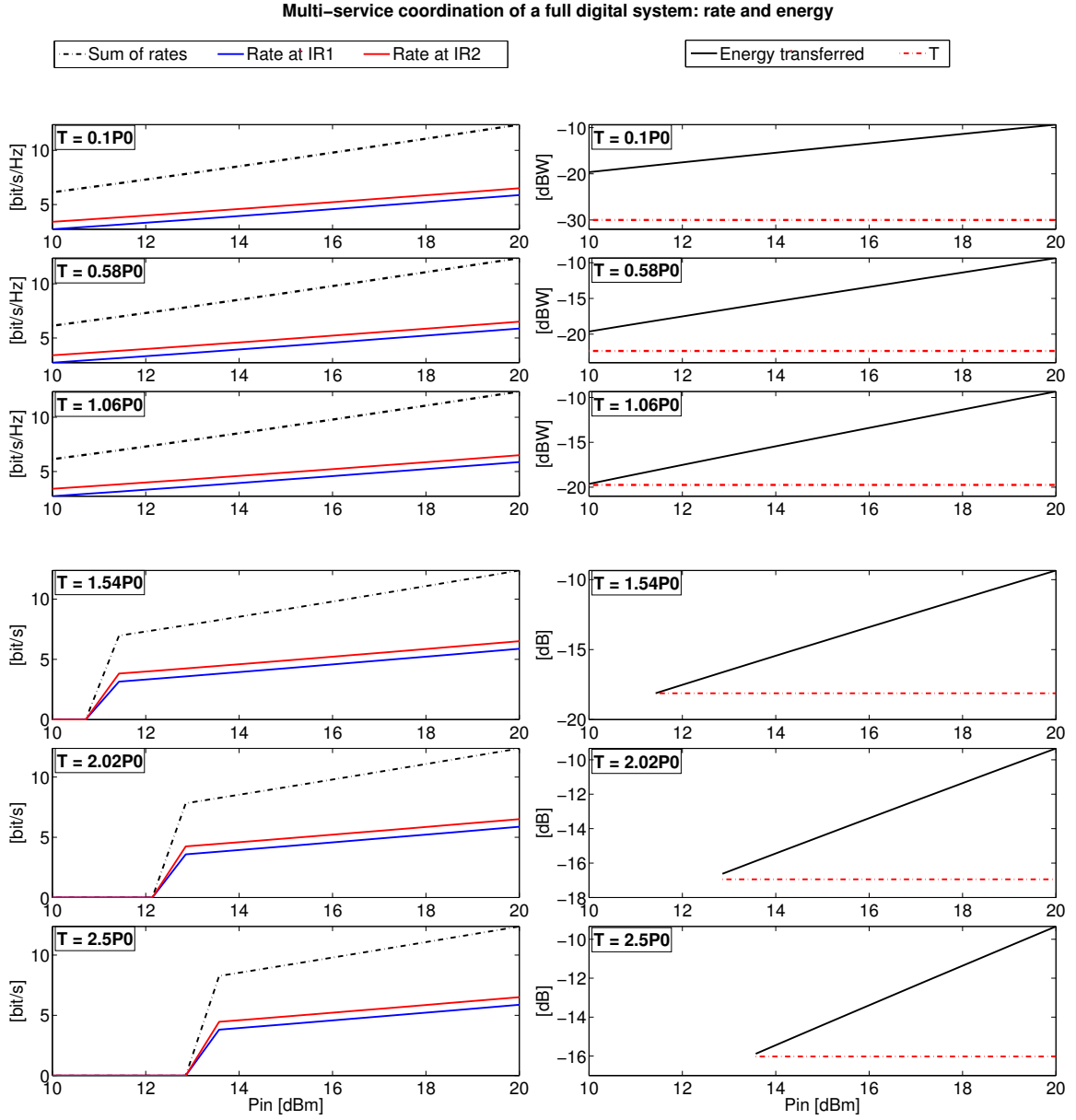


Figure 5.14: Rate and energy in the multi-service coordination scheme of a full digital system as a function of P_{in} . From top to bottom T is increasing. For increasing values of T , the rate and energy are lower, and set to zero when the constraint on the minimal amount of harvested energy is not fulfilled. Besides, there is more power allocated to the information vector which is in the worst channel state.

To understand why we have lower values in more detail, we analyzed the different components of the energy harvested (FIGURE 5.15).

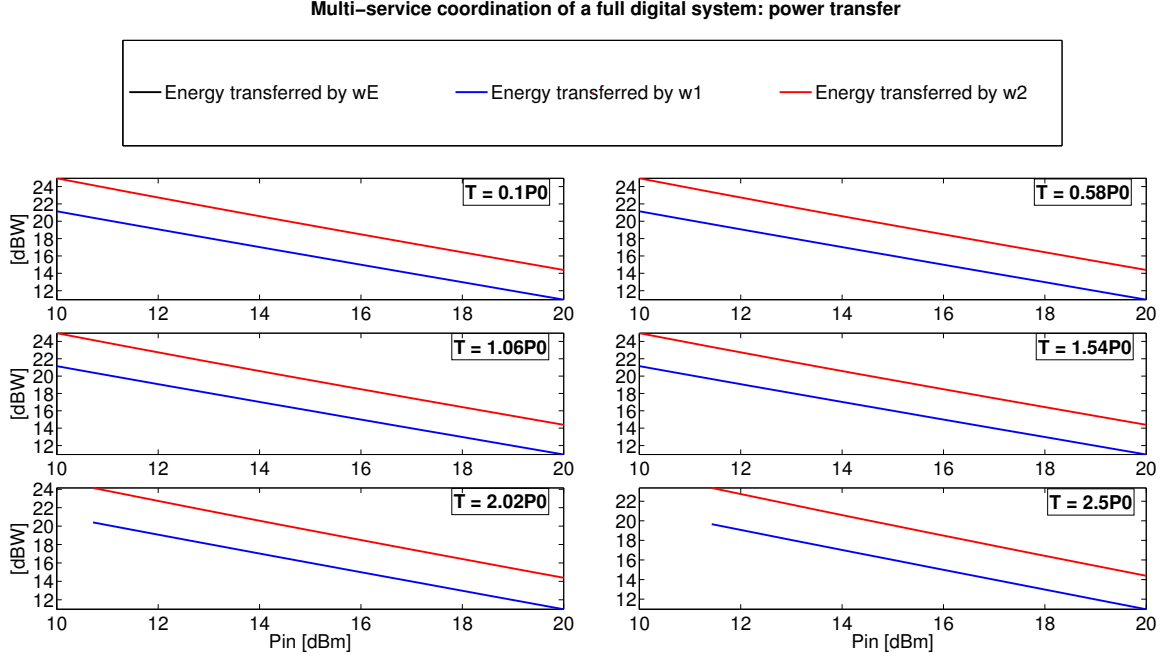


Figure 5.15: Power transfer in the multi-service coordination scheme of a full digital system with respect to P_{in} . The value of the harvested energy is the same for all T , except when the constraint on T is not fulfilled. Then, the power transferred is set to $-\infty$. Moreover, due to the differences in channel condition, one information vector harvest more energy than the other one.

In FIGURE 5.15, we directly see that no power is allocated to $\mathbf{W}_E$; hence, no energy can be harvested from it. Consequently, the total energy harvested is lower than in the previous scheme since the information vectors are designed to maximize the rate rather than to maximize the energy transfer.

Pareto curve

The Pareto curve is finally analyzed to observe the maximal rate/energy achieved and the trade-off between them for an input power of 15 dBm.

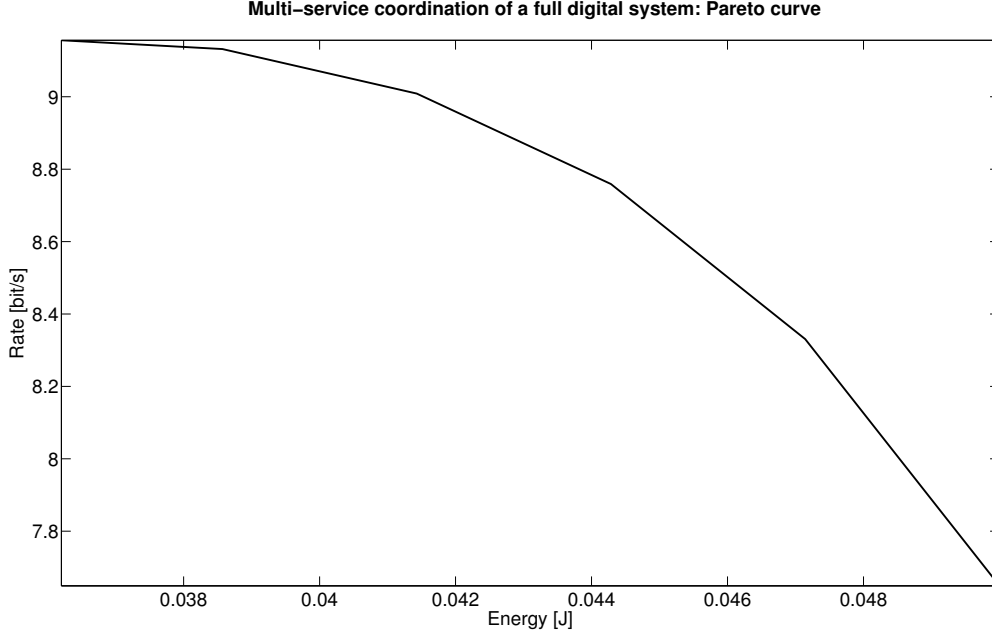


Figure 5.16: Pareto curve for the multi-service coordination scheme of a full digital system. It is the rate as a function of the energy. Those two resources are in conflict with each other since one is at its maximum when the other one is at its minimum, and reciprocally.

As can be seen in figure 5.16, we manage to harvest as much energy as in the signal coordination scheme, but at the cost of a lower rate. Indeed, the rate has a lower dynamic range in the multi-service coordination scheme of a full digital system for the above-cited reasons (no coordination with the energy).

5.2.2 Separate transmitters problem with power reallocation

We will now analyze the differences between the collocated transmitter problem and the separate transmitters problem with power reallocation.

SINR

In FIGURE 5.17 we can see that there are no differences with respect to FIGURE 5.10.

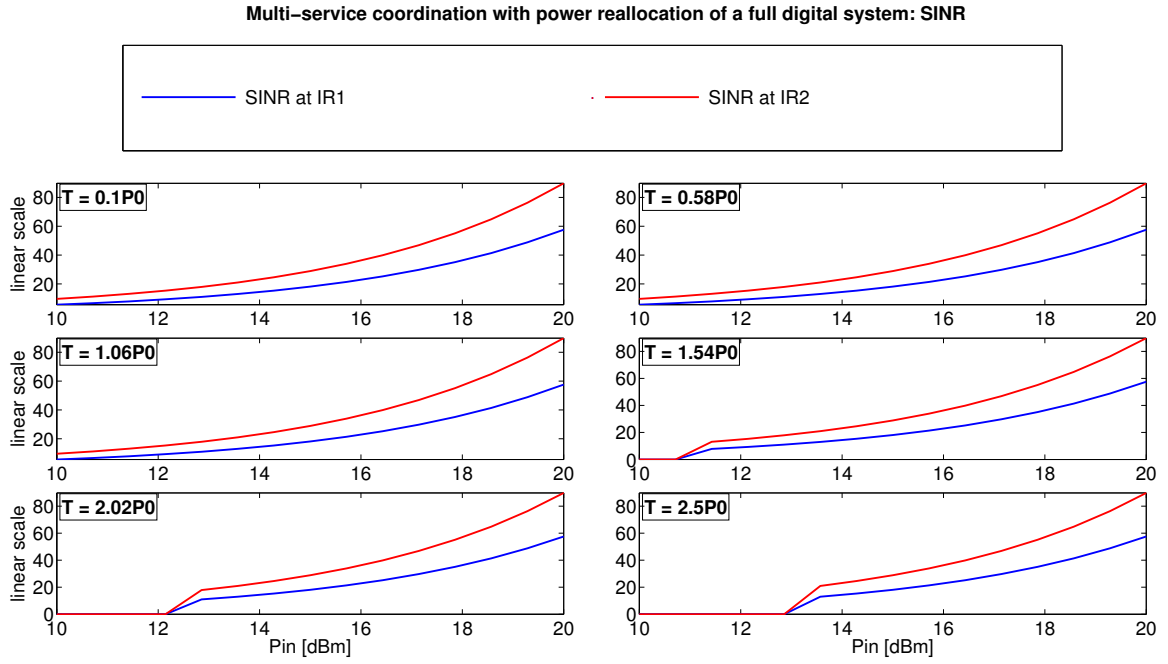


Figure 5.17: SINR as a function of P_{in} in the multi-service coordination scheme of a full digital system with power reallocation: starting from the upper left subfigure, to the lowest right, the threshold T is increasing. There are small differences in SINR at the IR1 and IR2. This is due to the power control precoder that allocates less power to the channel which is in better condition.

Rate and energy

In FIGURE 5.18, we plot both the rate and the energy harvested for several values of T . There are no differences in term of performances between the two algorithms of the multi-service coordination scheme of a digital system.

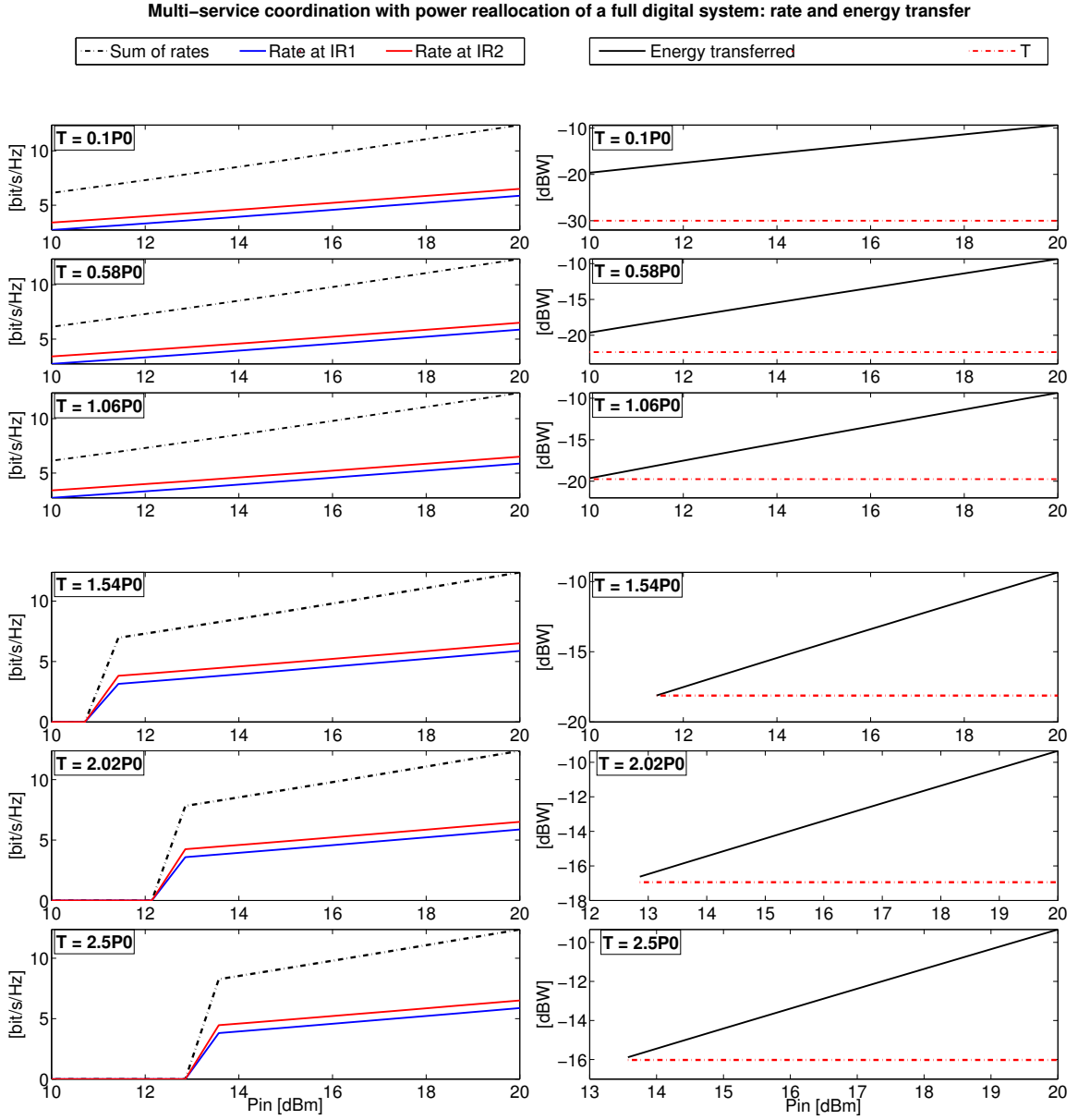


Figure 5.18: Rate and energy in the multi-service coordination scheme of a full digital system with power reallocation as a function of P_{in} . From top to bottom T is increasing. For increasing values of T , the rate and energy are lower, and set to zero when the constraint on the minimal amount of harvested energy is not fulfilled. Besides, there is less power allocated to the information vector which is in the best channel state.

Since there are no differences in rate and energy harvested, the detailed power transferred will exactly be the same as for the collocated transmitters problem and is thus not analyzed here.

Pareto curve

The Pareto curve is exactly the same as for the collocated transmitters case, which makes sense since both the rate and energy are the same for any injected power.

Conclusion

We can use the multi-service coordination scheme of a full digital system only when we power the femtocell base station with more or less the same power as the minimal harvested energy we desire. For lower values, no power is allocated to both energy and information vectors.

Besides, the average rate will not exceed 12 bit/s/Hz.

Finally, for a given power where the constraint on the minimal energy harvested is tight, the multi-service coordination scheme of a full digital system achieves the same amount of harvested energy as the signal coordination scheme, but at the cost of a reduction in rate by more or less 2 bit/s/Hz.

5.3 Multi-service coordination of a hybrid precoding system

As for the **signal coordination** model, we will analyze the SINR, rate, energy and Pareto curves for several values of minimal harvested energy and several numbers of antennas per array. Since we have two different algorithms for this model, we will analyze both of them starting with the colocated transmitters problem.

5.3.1 Collocated transmitters problem

SINR

FIGURE 5.19 shows the curve of the SINR at both IR1 and IR2.

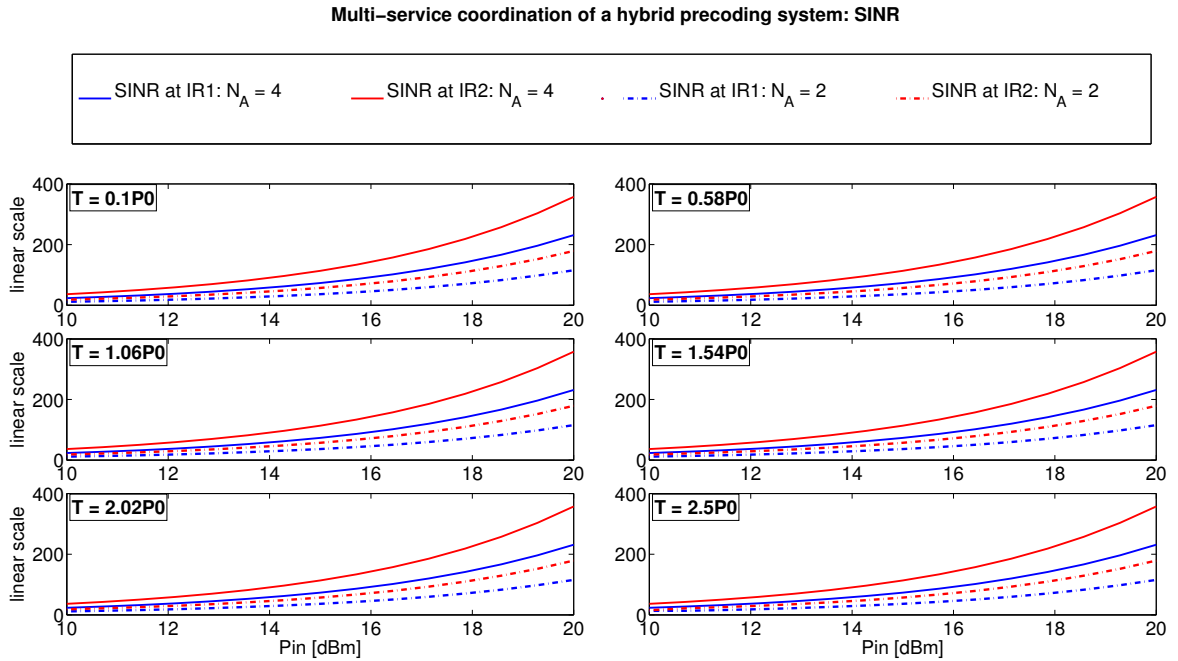


Figure 5.19: SINR in the multi-service coordination scheme of a hybrid precoding system as a function of P_{in} . The SINR at IR2 is better than the one at IR1 since its channel is in a better condition. Besides, adding antennas to the antenna arrays increases the SINR thanks to the array gain. Those observations are true for every values of T .

There is no noticeable difference compared to the SINRs of the signal coordination scheme, except notably that the values are higher. This is normal since we have an array and a diversity gain that compensates for the loss of coordination with the energy vector.

Besides, we can see that we achieve better performances when the number of antennas increases.

This is directly linked to the fact that the array gain is linearly proportional to the number of transmit antennas (subsection 3.2.2).

We will now investigate deeper the received SINR by considering the different signals composing it.

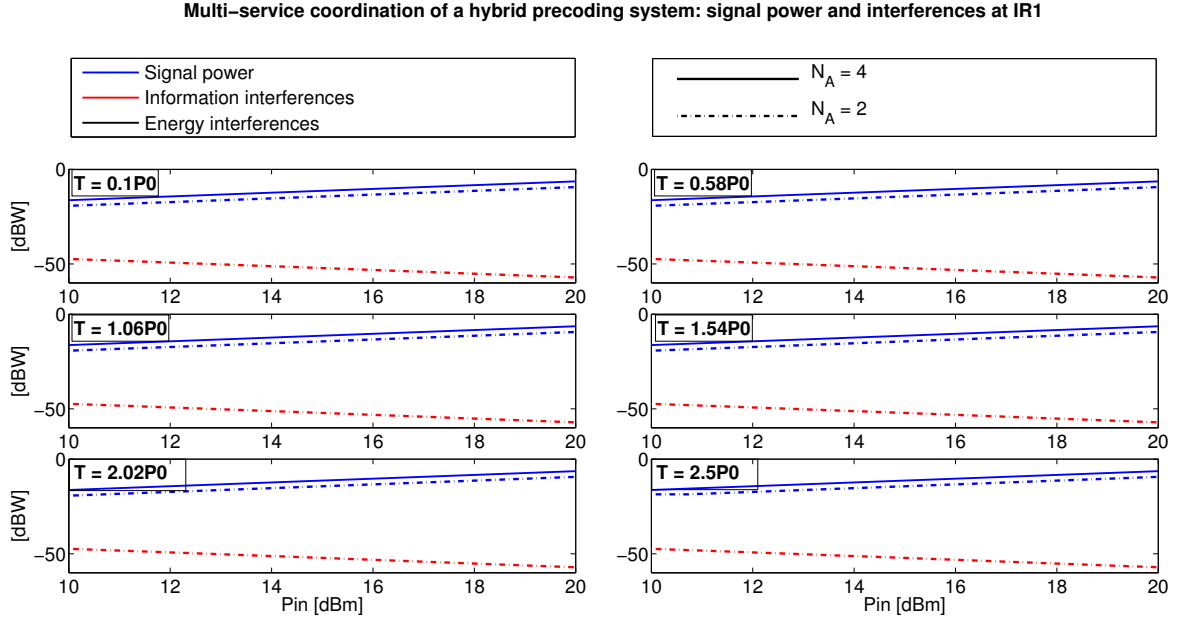


Figure 5.20: Signal power and interferences at IR1 in the multi-service coordination scheme of a hybrid precoding system as a function of P_{in} . For $N_A = 2$, we have energy and information interferences. Besides, the signal power is higher than both interferences. Moreover, increasing P_{in} rises the signal power and diminishes the information interferences. This is due to a better allocation of power. For $N_A = 4$, we do not have any interferences, meaning that the more phase shifters we have, more the interferences are reduced.

Multi-service coordination of a hybrid precoding system: signal power and interferences at IR2

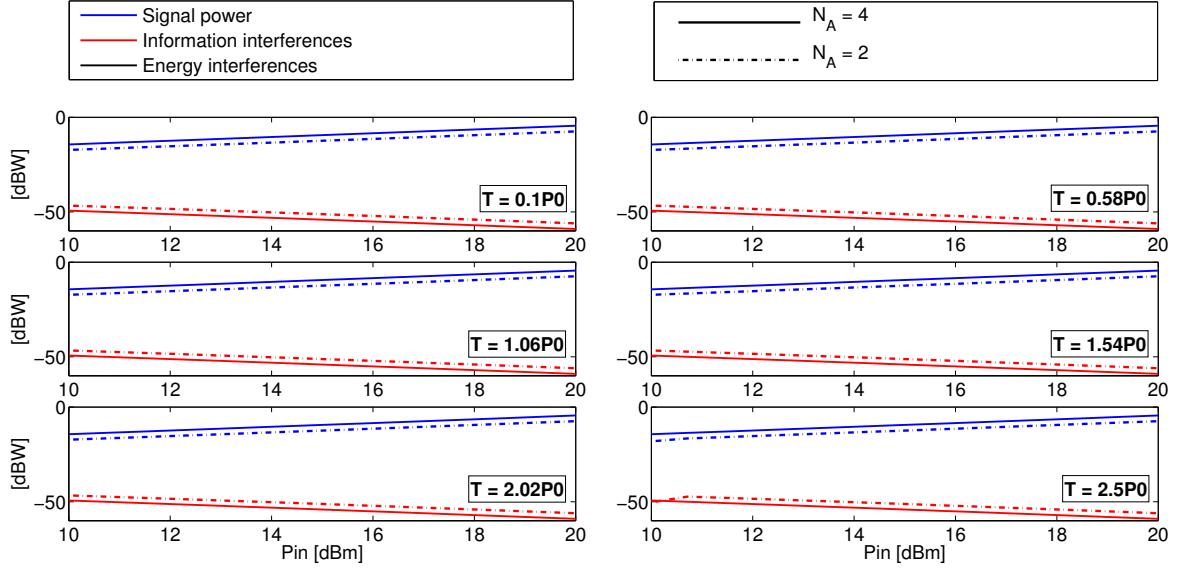


Figure 5.21: Signal power and interferences at IR2 in the multi-service coordination scheme of a hybrid precoding system as a function of P_{in} . For both $N_A = 2$ and $N_A = 4$, the signal power is higher than the interferences, and it increases with P_{in} , while the information interferences decrease with P_{in} .

In FIGURE 5.21 we see that for 4 antennas, we have a higher signal power and lower interferences than for 2 antennas. This is logical since for the power signal we have a higher array gain, and for the interferences, we have more phase shifters, so a better precoding that leads to less interferences.

In top of it, in FIGURE 5.20, we do not see any curves of interference for $N_A = 4$. The reason is that the phase shifters are set optimally such that all the information vector $\mathbf{w}_2$ is oriented towards IR2 and EH. The same holds true for $\mathbf{w}_E$. Hence, there are no interferences due to both information and energy vectors arriving at IR1. This is confirmed by the SINR curves in FIGURE 5.19: the SINR at IR2 is much larger (around 26 units higher) than the one at IR1 for $N_A = 4$. We can directly conclude that 4 antennas per array are enough to direct one information signal exclusively to the desired user equipment.

Power allocation

In FIGURE 5.22, we see the allocation of power between the three signals.

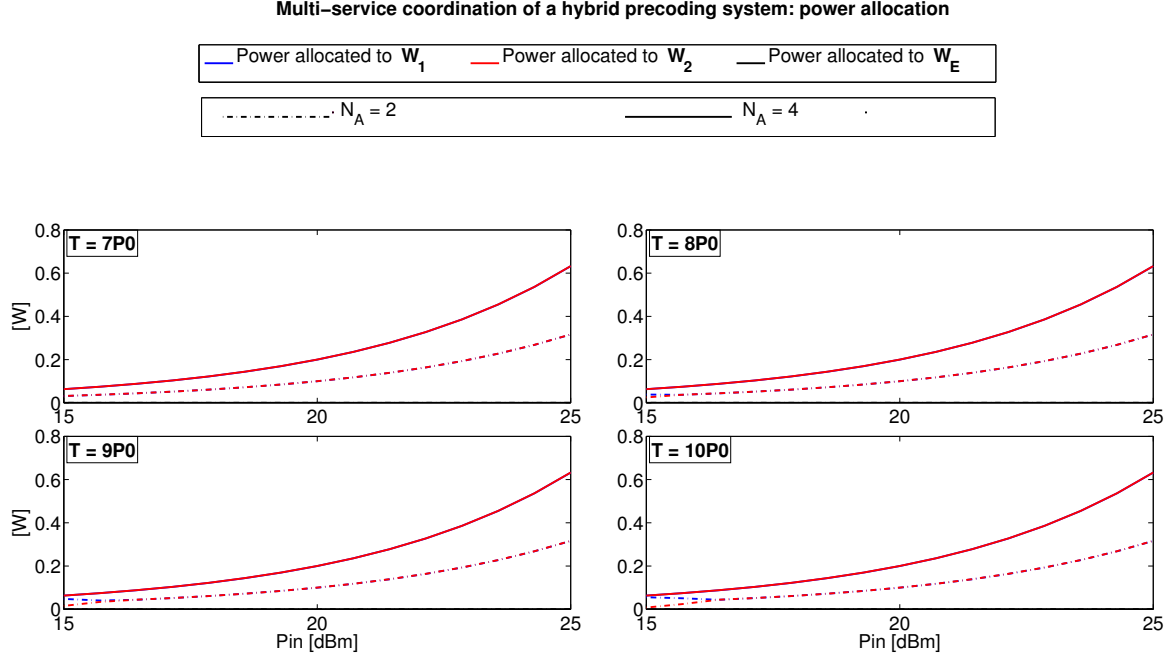


Figure 5.22: Power allocation in the multi-service coordination scheme of a hybrid precoding system as a function of P_{in} . The power allocated to $\mathbf{W}_1$ and $\mathbf{W}_2$ is the same, except for $P_{\text{in}} < 16$ dBm, while no power is allocated to $\mathbf{W}_E$ for all T .

We notice that there are no power allocated to $\mathbf{W}_E$ and that the power is allocated equally between $\mathbf{W}_1$ and $\mathbf{W}_2$. We can hence say that a multi-service coordination scheme will never allocate power to $\mathbf{W}_E$ because it is more beneficial to allocate power to $\mathbf{W}_1$ and $\mathbf{W}_2$ to have an information rate. Besides, for low values of P_{in} and 2 antennas per array, $\mathbf{W}_1$ receives more power than $\mathbf{W}_2$. The reason is that allocate more power to $\mathbf{W}_1$ allows to met the constraint on the minimal amount of harvested energy and hence make the problem feasible.

Rate and energy

In FIGURE 5.23, we can see that the performances achieved have the same behavior as for the signal coordination scheme. Nevertheless, two observations are in order.

First, the model with a higher number of antennas per array will always achieve the best performances. Indeed, the signal power is increased thanks to the array gain, and the interferences reduced thanks to the optimal precoding phase shifter matrices.

Second, the energy harvested is higher for an increasing number of antennas. It is explained by the fact that we have an array gain increasing on the energy and information vectors for an increasing number of antennas.

Multi-service coordination scheme of a hybrid precoding system: rate and energy

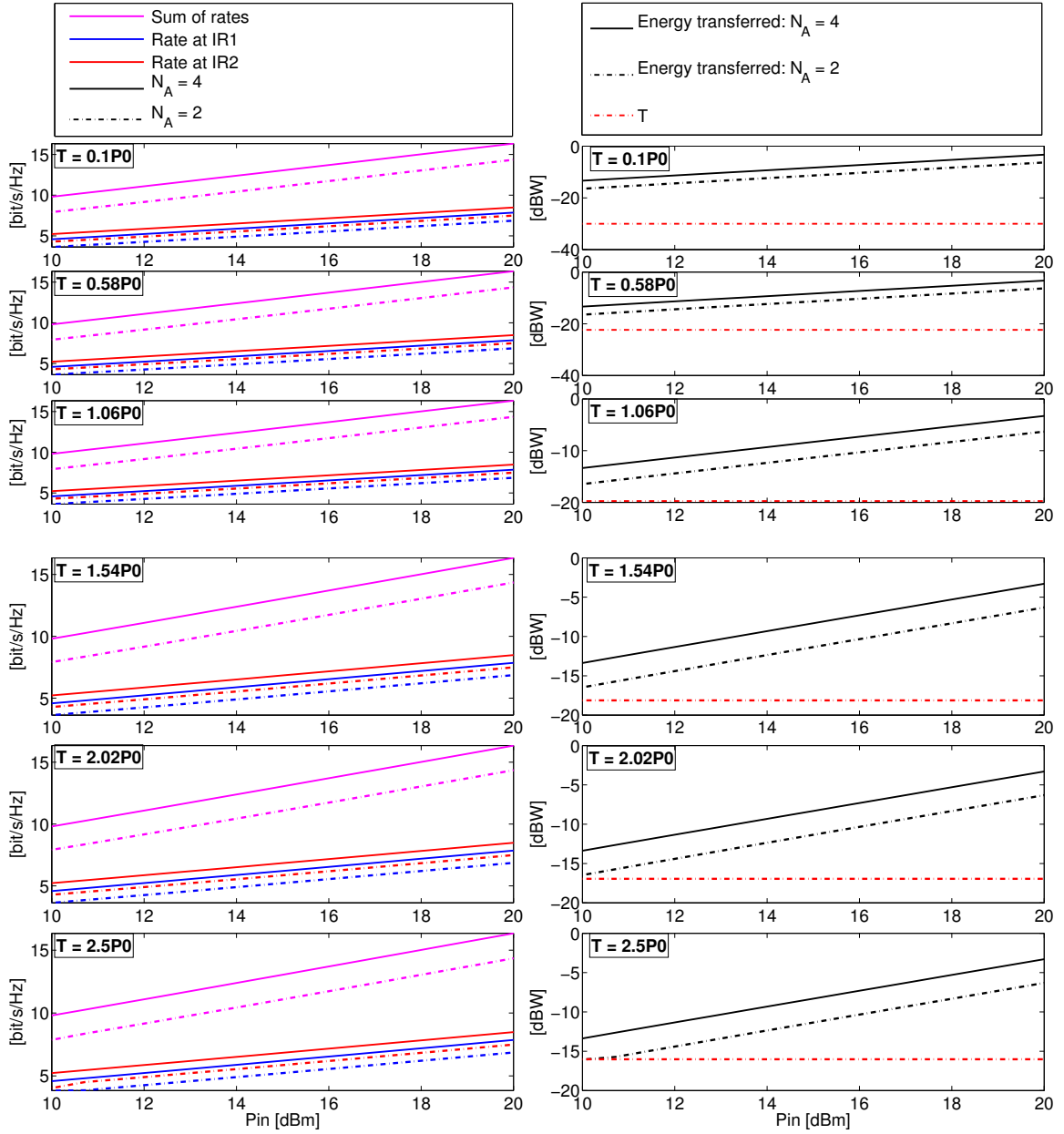


Figure 5.23: Rate and energy in the multi-service coordination scheme of a hybrid precoding system as a function of P_{in} . From top to bottom, T is increasing. The rate is increasing with P_{in} and N_A . Besides, the constraint on the minimal amount of harvested energy is fulfilled for any T .

Pareto curve

Finally, we analyze in FIGURE 5.24 the Pareto frontier for $N_A = 2$ and $N_A = 4$.

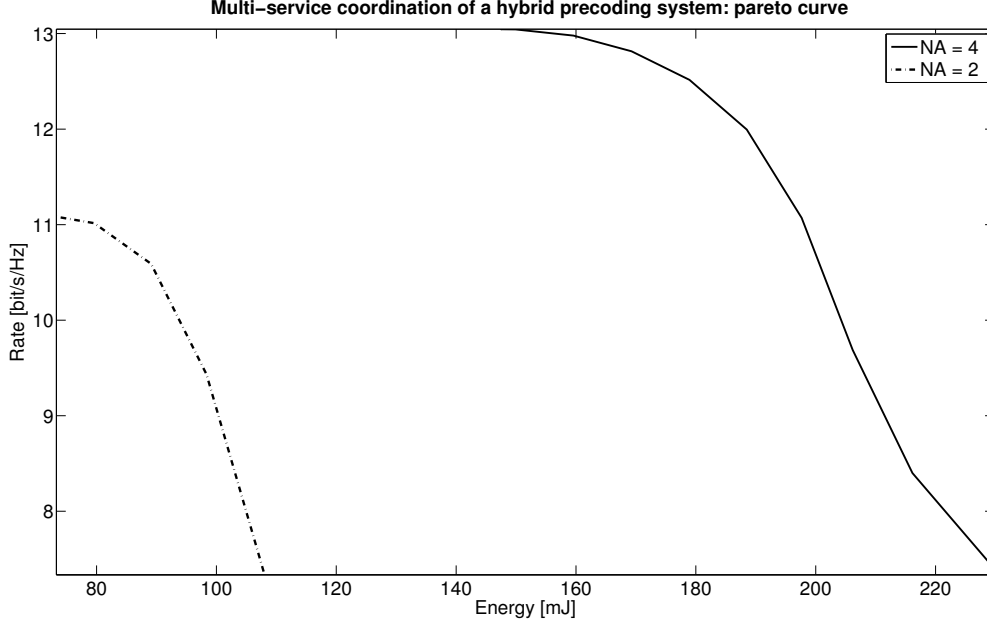


Figure 5.24: Pareto frontier for the multi-service coordination scheme of a hybrid precoding system (rate versus harvested energy). There is a trade-off between the rate performances and the energy one. Besides, this trade-off is highly reduced when increasing N_A .

We can notice that as predicted and explained before, the rate and energy harvested are higher when N_A increases.

5.3.2 Separate transmitters problem with power reallocation

The results found for the separate transmitters with power reallocation problem are exactly the same as for the collocated transmitters problem and will then not be disarmed here.

5.3.3 Conclusion

The first huge difference with the other models, is that we have an array gain and a focusing of the wave to targeted information receivers and energy harvester. It results in an increase of both rate and energy harvested. This increase is high for bigger arrays, but from $N_A = 4$, we already achieve to focus one information toward one unique user equipment.

Moreover, we are able to supply batteryless devices more often and with an higher power. This solution is thus more adapted to environments where we have a lot of batteryless sensors.

Finally, all multi-service coordination scheme will choose to allocated all the power to $\mathbf{W}_1$ and $\mathbf{W}_2$ because when the problem is feasible (the constraint on the minimal amount of harvested energy is met), it chooses to maximize the rate rather than the energy harvested as set in the ALGORITHMS 4 and 5.

5.4 Comparison between the different schemes

This section aims at comparing the different schemes in term of SINR, rate, energy harvested and Pareto curves. It will hence allow to evaluate the quality of each scheme in comparison to the others.

5.4.1 SINR

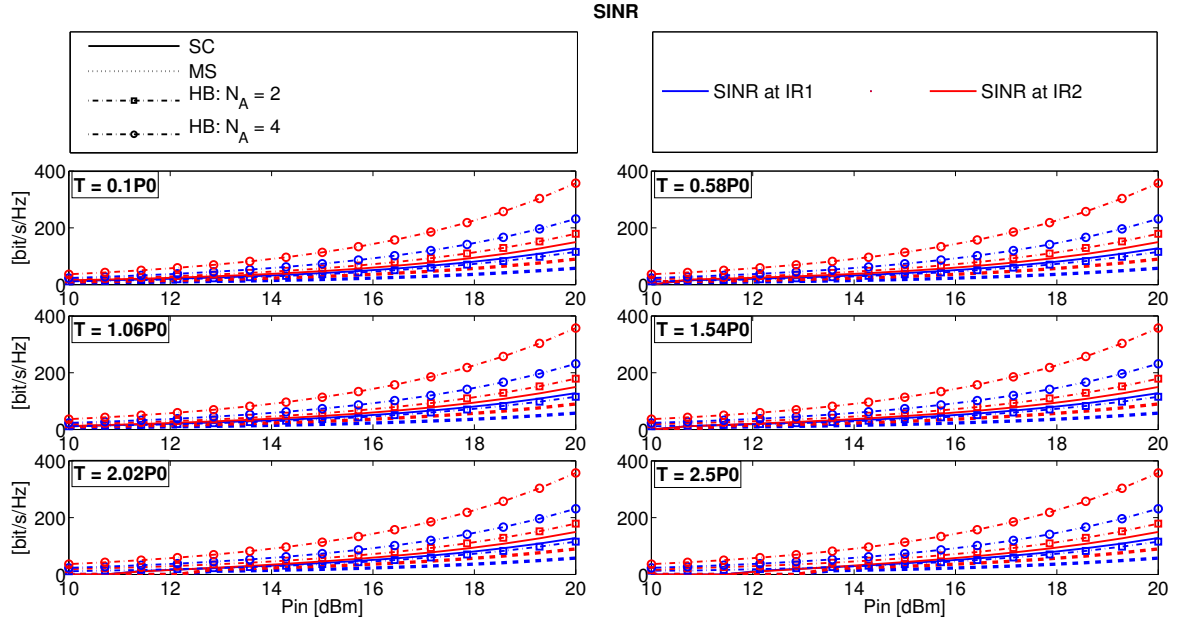


Figure 5.25: SINR for all the models as a function of P_{in} . The performances from the worst to the best scheme are: multi-service coordination of a full digital system (MS), signal coordination (SC), multi-service coordination of a hybrid precoding system (HB) with $N_A = 2$ and $N_A = 4$. The multi-service coordination of a full digital system is the worst model since there is no coordination between the information and energy vector. Hence, the algorithm never allocates any power to $\mathbf{w}_E$. The second worst scheme is the signal coordination since it achieves coordination between the information and the energy signal. Then comes the multi-service coordination of a hybrid precoding system. It is the best model since it benefits from diversity, and this spatial diversity increases with the number of antennas per antenna array. Those remarks are correct for each value of T .

In FIGURE 5.25, one can see that the SINR is increasing from the multi-service coordination scheme of a full digital system model to the multi-service coordination of a hybrid precoding system one with the maximal number of antennas per array. In between these two extreme models, signal coordination achieves a slightly lower performance than the hybrid one with 2 antennas. It means that for the considered channel, the multi-service coordination scheme of a hybrid precoding system is always better than the signal coordination scheme.

Nevertheless, this will not always be the case. Indeed, if we have poorer channels, the signal coordination might achieve a better performance.

Moreover, the SINR at IR2 is always larger the one at IR1 since the channel conveying $\mathbf{w}_2$ is always in a better condition than the one conveying $\mathbf{w}_1$. Indeed, from one model to another one, the channel realizations stay the same in order to allow a proper comparison between the models.

5.4.2 Sum of rates

As predicted with the SINR, the lowest performance as regards the sum of rates is achieved by the multi-service coordination scheme of a full digital system, and the best one by the multi-service coordination scheme of a hybrid precoding system. In FIGURE 5.26, we better see that the multi-service coordination scheme of a hybrid precoding model with two antennas achieves pretty much the same performance as the signal coordination scheme when the constraint on the minimal energy harvested is fulfilled for both models. When the constraint on the minimal amount of

energy transferred is not met for the signal coordination scheme, it is met for the multi-service coordination scheme of a hybrid precoding system. Hence, the multi-service coordination scheme of a hybrid precoding system achieves a higher rate for the corresponding values of P_{in} .

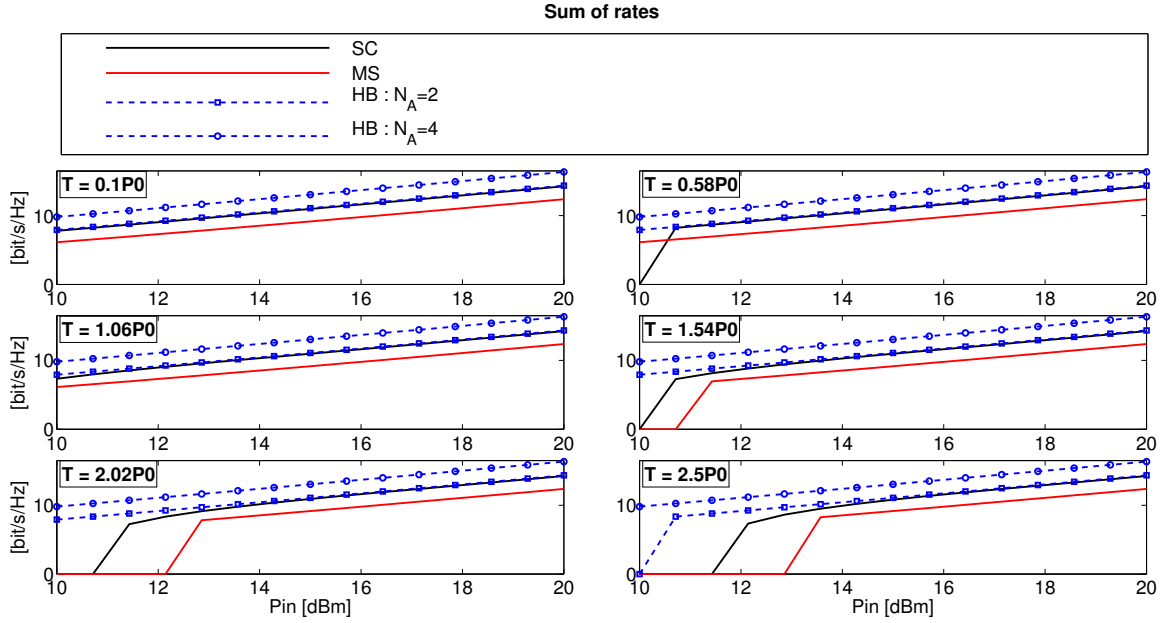


Figure 5.26: Sum of rates for all the models as a function of P_{in} . For the rate, the worst model is the multi-service coordination of a full digital system (MS). Indeed, it has no coordination between the energy and information signal and thus $\mathbf{w}_E$ is always set to zero. Then comes the signal coordination (SC) and finally the multi-service coordination scheme of a hybrid precoding system (HB) because it exploits spatial diversity.

5.4.3 Energy harvested

As said before, more energy will be harvested for the multi-service coordination scheme of a hybrid system, and less for the signal coordination scheme, since it is able to maximize the rate while keeping the energy harvested equal to T (FIGURE 5.27).

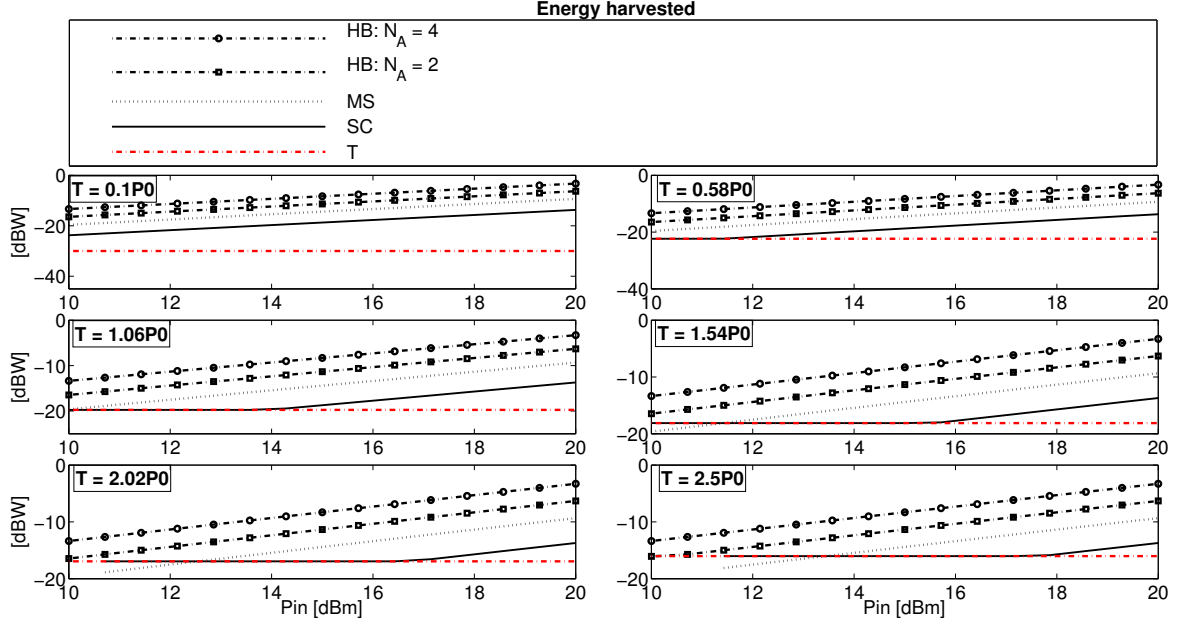


Figure 5.27: Energy harvested for all the models as a function of P_{in} . From the worst to the best achievement in term of harvested energy: the signal coordination (SC), the multi-service coordination of a full digital system (MS) and the multi-service coordination scheme of a hybrid precoding system (HB). The first one met the constraint (T), but it is sufficient in terms of activation of the harvester device. The second has a higher energy harvested but does not fulfill the constraint for higher values of T in comparison with the signal coordination scheme. Finally, the multi-service coordination scheme of a hybrid precoding system always fulfills the constraint.

5.4.4 Pareto curves

Finally, in FIGURE 5.28, we can see that multi-service coordination of a full digital system is the worst model in terms of energy/rate trade-off, and that the multi-service coordination of a hybrid system is the best one.

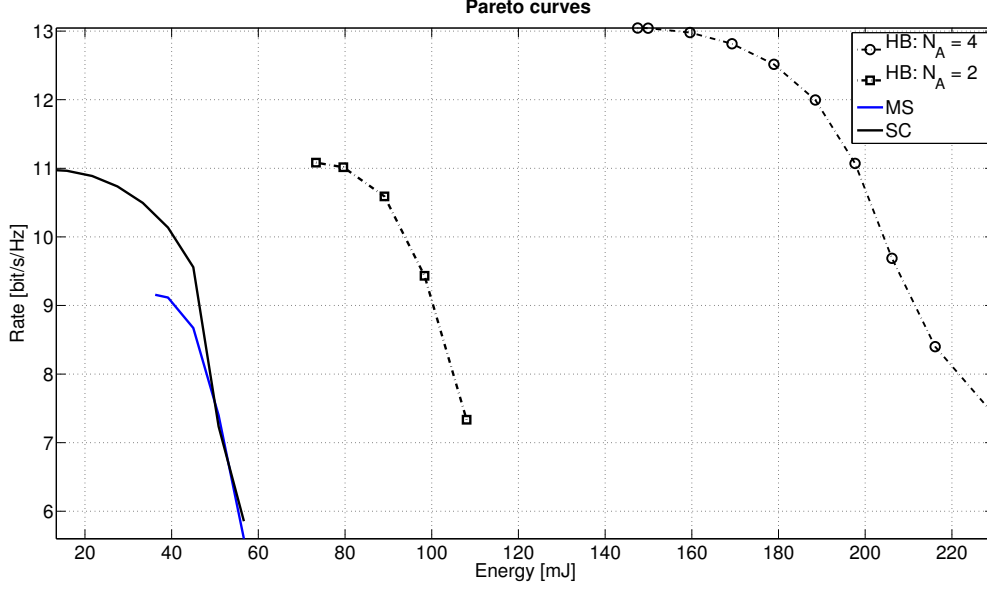


Figure 5.28: Pareto curves for all the models (rate as a function of the harvested energy). The trade-off between those resources is, from the strongest to the weakest, the multi-service coordination scheme of a full digital system (MS), the signal coordination scheme (SC) and finally the multi-service coordination scheme of a hybrid precoding system (HB).

5.4.5 Conclusion

To conclude, we will make the comparison of the different models in term of precoder matrices, allocation of power and array size.

The power control part of the precoder gives a reverse result compared to a precoder build on the waterfilling principle. Indeed, regarding the multi-service coordination scheme of a full digital system, our precoder allocates more power to the users being in the worst channel condition. As for the energy harvested, it was predictable that it will be lower than T for some values of P_{in} . Actually, we only play on the allocation of power with an isotropic channel non logarithmically decreasing with the distance, and thus we cannot modify either the interferences caused by the energy vector, nor the energy harvested thanks to the information vectors. It directly results that no power is allocated to the RF chain sending an energy vector since it creates interferences and provides no benefits in comparison to the other RF chains, except a slightly higher antenna gain that is relative to the channel link. Since this link is randomly generated by a zero-mean Rayleigh normalized function, as well as the other communication links, it will never be dominant in comparison to the other communication links of the system. Hence, the algorithm will choose not to use it.

Regarding the phase shifter precoder, it has a huge impact on the behavior of both the rate and the energy harvested. Genuinely, we do not see anymore a waterfilling behavior for the allocation of power at low injected power. Indeed, when the constraint on the minimal harvested energy is not fulfilled for the multi-service coordination scheme of a full digital system, it is for the multi-service coordination scheme of a hybrid precoding system. For such a precoding, the power is allocated such that we always fulfill all the constraints, meaning that for a small injected power, one information vector is essentially directed towards the EH. As an outcome, the allocation of power does not respect the waterfilling principle anymore, and one user equipment is free of interferences and noise when $N_A \geq 4$. Nevertheless, when the injected power increases such that we do not need to feed the energy vector to fulfill all the constraints, the behavior of

the power control goes back to a waterfilling allocation.

Besides, looking at the allocation of power, we notice that the behavior is different when comparing a multi-service coordination problem with a signal coordination one. Indeed, in the multi-service coordination scheme, the algorithm never allocates power to $\mathbf{W}_E$. There are two reasons to that. The first one is that when the problem is feasible, the best allocation is to set $\mathbf{W}_E$ to zero to maximize the rate. The second one, that we draw from the results, is that when there is enough power injected in the system so that $\mathbf{W}_E$ can harvest enough energy to meet T , there is enough power to meet T by allocating the power only between $\mathbf{W}_1$ and $\mathbf{W}_2$. For the signal coordination, there is a region where the problem is feasible only by allocating all the power to $\mathbf{W}_E$, hence making the problem feasible for smaller values of P_{in} .

Moreover, with respect to the size of the antenna array, we can address the following remarks. First, the more antennas we have, the higher is the array and diversity gain, and so, the higher are the rate and energy harvested. Second, we see that with only two antennas per array, we manage to harvest 19.95 mJ for an input power of 10 mW, meaning that the algorithm has a better achievement in term of harvested energy in comparison with the signal coordination scheme of a full digital system. Next, as regards the Pareto curves, the bigger the array is, the better the Pareto frontier. In fact, the system can achieve simultaneously a higher rate and harvest more energy.

Finally, we can say that our multi-service coordination scheme of a hybrid precoding system achieves good performances for a femtocell base station. However, if we consider the path loss, the energy harvesting will be much lower and the constraint on the minimal harvested energy will not be fulfilled beyond few meters. Indeed, taking the multi-service coordination scheme of a hybrid precoding system with 4 antennas per base station leads to a flat array (2×2 antennas) of $5 \times 5 \text{ cm}^2$ for a carrier frequency of 6 GHz. Such an array of antennas is feasible and could be put on a lot of batteryless sensor/devices. As a matter of fact, we could build arrays going up to 16 antennas using a carrier frequency of 6 GHz. It ensues arrays of size $10 \times 10 \text{ cm}^2$, assuming a spacing of $\frac{\lambda}{2}$ between all antennas.

Moreover, looking at the health norm regulation, we can say that our design does not exceed the maximal amount of power density allowed ([20] and [21]).

We can thus conclude that our multi-service coordination scheme of a hybrid precoding system could find useful applications.

6 | Conclusion

After an analysis about the main rate-energy trade-off of different schemes in SWIPT systems, our results confirm expectations based on previous scientific work.

The comparison of all schemes allows us to tell that the multi-service coordination scheme of a hybrid precoding system technique is more efficient both in term of rate and energy harvested than the other methods. As a matter of fact, the multi-service coordination scheme of a full digital system gives the poorer results since there is coordination only between the users and no phase shifter precoding.

Then comes the signal coordination because it coordinates both the information and energy signals, allowing to reach better performances in terms of rate (due to the diminution of interferences from the other users information and energy vectors). Besides, in term of energy harvested, the constraint on the minimal amount of energy to be transferred is tight as long as we transfer power to the energy vector. When there is enough input power to set $\mathbf{w}_E$ to zero and still fulfill the constraints, the energy harvested increases with P_{in} . It results in a lower value in term of energy harvested for low injected power, but a smarter one since the targeted device receives enough energy to be in its operational mode.

Finally comes the multi-service coordination scheme of a hybrid precoding system, which gives better performances with an increasing number of antennas per array. This is both due to the array gain that is linearly increasing with N_A , and to the reduction of interferences for all receivers due to a better beamforming of the signals (for an increasing N_A , the signal arrives more and more in phase at the location of the receivers). Nevertheless, continuously increasing the number of antennas makes no sense because, on the one hand, the current technology wants to build small devices (the size of the antenna array is limited). On the other hand, the user equipment and harvester do not necessarily need a higher rate or level of harvested energy in order to be functional.

With those remarks, a direct application of the multi-service coordination scheme of a hybrid precoding SWIPT system will be to harvest energy from femtocell base stations in indoor facilities; this will require to pay attention to the health norms of the targeted environment in order to set properly the input power of the system.

This master thesis has, not treated the case of Rayleigh fading communication links, nor the case of more than 2 users. We cannot assure that all users will be served in a multi-user situation, we can assume that the bigger the antenna array will be, the less interferences and noise we will have, and hence, the better the users will be served.

Further research has hence to be led, from this theoretical report, to implement a practical application that could be used, for example, in smart cities.

Hence, more realistic simulations still need to be done. Nevertheless, we thought that the results for the three schemes will not be severely modified. As a matter of fact, we thought that they will be logarithmically decreased with the distance from the transmitter to the receiver. Besides, due to higher attenuation of the signal, we might not serve all user equipment. In this case, the

hypothesis is that the users farthest away from the transmitter will receive less or no information signal. Regarding the energy receiver, they could not all received enough energy to be activated. We though that the most affected energy harvesters will be the farthest away from the femtocell base station.

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A | Annexes

A.1 Development of the power allocation between $\mathbf{W}_1$ and $\mathbf{W}_2$

This annexe aims at understanding the power allocation between $\mathbf{W}_1$ and $\mathbf{W}_2$ given the channel covariance matrices.

Let's assume $\mathbf{H}_1$ and $\mathbf{H}_2$ be the covariance channel matrices of $\mathbf{W}_1$ and $\mathbf{W}_2$, σ_I^2 the noise introduced by the channel and $\mathbf{W}_E$ the energy covariance matrix.

The SINR at IR1 and IR2 are respectively given by :

$$\begin{aligned} \text{SINR at IR1: } SINR_1 &= \frac{\mathbf{H}_1 \mathbf{W}_1}{\sigma_I^2 + \mathbf{H}_1(\mathbf{W}_2 + \mathbf{W}_E)} \\ \text{SINR at IR2: } SINR_2 &= \frac{\mathbf{H}_2 \mathbf{W}_2}{\sigma_I^2 + \mathbf{H}_2(\mathbf{W}_1 + \mathbf{W}_E)} \end{aligned}$$

Since the optimization problems maximize the sum of rates, they maximize:

$$\begin{aligned} \max \log(1 + SINR_1) + \log(1 + SINR_2) &= \max 10^{\log(1+SINR_1) \log(1+SINR_2)} \\ &= \max 10^{\log(1+SINR_1)} 10^{\log(1+SINR_2)} \\ &= \max (1 + SINR_1)(1 + SINR_2) \\ &= \max 1 + SINR_1 + SINR_2 + SINR_1 SINR_2 \end{aligned}$$

The term $1 + SINR_1 + SINR_2 + SINR_1 SINR_2$ will be maximized when $SINR_1 \cong SINR_2$, since the sum of SINRs is upper bounded by the constraint on the input power ($\text{Tr}(\mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_E) \leq P_{\text{in}}$), and the product of SINRs is maximal when $SINR_1 = SINR_2$.

We conclude that the sum of rates will be maximized when the ratio $\frac{\text{Transmitted signal}}{\text{Interferences}}$ is the same for both information receivers.

Lets analyze mathematically what it means:

$$\begin{aligned} 1 + \frac{\mathbf{H}_1 \mathbf{W}_1}{\sigma_I^2 + \mathbf{H}_1(\mathbf{W}_2 + \mathbf{W}_E)} &= 1 + \frac{\mathbf{H}_2 \mathbf{W}_2}{\sigma_I^2 + \mathbf{H}_2(\mathbf{W}_1 + \mathbf{W}_E)} \\ \Rightarrow \frac{\mathbf{H}_1 \mathbf{W}_1}{\sigma_I^2 + \mathbf{H}_1(\mathbf{W}_2 + \mathbf{W}_E)} &= \frac{\mathbf{H}_2 \mathbf{W}_2}{\sigma_I^2 + \mathbf{H}_2(\mathbf{W}_1 + \mathbf{W}_E)} \\ \Rightarrow \begin{cases} \mathbf{H}_1 \mathbf{W}_1 = \delta \mathbf{H}_2 \mathbf{W}_2 \\ \sigma_I^2 + \mathbf{H}_1(\mathbf{W}_2 + \mathbf{W}_E) = \delta (\sigma_I^2 + \mathbf{H}_2(\mathbf{W}_1 + \mathbf{W}_E)) \end{cases} &, \quad (\text{E1,E2}) \\ \delta &\geq 0 \end{aligned}$$

From equalities E1 and E2, we will make the comparison between $\mathbf{H}_1$ and $\mathbf{H}_2$ in term of quality based on the following situations :

1. $\mathbf{W}_1 = \mathbf{W}_2$
2. $\text{Tr}(\mathbf{W}_1) > \text{Tr}(\mathbf{W}_2)$

3. $\text{Tr}(\mathbf{W}_1) < \text{Tr}(\mathbf{W}_2)$

The first case is trivial : $\mathbf{H}_1 = \mathbf{H}_2$. Nevertheless, this situation is rarely to occur since the channel covariance matrices are randomly generated.

For the two other situations, let's define $\mathbf{W}_1 = \mathbf{W}_2 + \boldsymbol{\epsilon}$ and $\mathbf{W}_1 = \mathbf{W}_2 - \boldsymbol{\epsilon}$ respectively, with $\boldsymbol{\epsilon}$ being a semidefinite diagonal matrix. The equalities E1 and E2 are transformed as followed :

$$\begin{cases} \mathbf{H}_1 (\mathbf{W}_2 \pm \boldsymbol{\epsilon}) = \delta \mathbf{H}_2 \mathbf{W}_2 \\ \sigma_I^2 + \mathbf{H}_1 (\mathbf{W}_2 + \mathbf{W}_E) = \delta (\sigma_I^2 + \mathbf{H}_2 (\mathbf{W}_2 \pm \boldsymbol{\epsilon} + \mathbf{W}_E)) \end{cases} \quad (\text{E1}', \text{E2}')$$

With those 2 modified equalities, we express $\mathbf{H}_1$ as a function of $\mathbf{H}_2$, and then express $\mathbf{H}_2$ as a function of $\mathbf{W}_2$ and $\boldsymbol{\epsilon}$.

$$\delta \mathbf{H}_2 (\boldsymbol{\epsilon}^2 \pm 2\mathbf{W}_2 \boldsymbol{\epsilon} \pm \mathbf{W}_E \boldsymbol{\epsilon}) = \sigma_I^2 (1 - \delta) (\mathbf{W}_2 \pm \boldsymbol{\epsilon})$$

Since $\mathbf{W}_1 \succeq \mathbf{0}$, $\mathbf{W}_2 \succeq \mathbf{0}$ and $\mathbf{W}_E \succeq \mathbf{0}$ (equation 4.2 in section 4.1.2), $(\mathbf{W}_2 \pm \boldsymbol{\epsilon}) \succeq \mathbf{0}$. Besides, since the product of two semi-definite matrices is a semi-definite matrix, $\mathbf{W}_E \boldsymbol{\epsilon} \succeq \mathbf{0}$ and $\mathbf{W}_2 \boldsymbol{\epsilon} \succeq \mathbf{0}$. Moreover, the addition or subtraction of semidefinite matrices is a semi-definite matrix. Hence, it is clear that:

$$\begin{cases} \boldsymbol{\epsilon}^2 + 2\mathbf{W}_2 \boldsymbol{\epsilon} + \mathbf{W}_E \boldsymbol{\epsilon} \succeq \mathbf{0} \\ \boldsymbol{\epsilon}^2 - 2\mathbf{W}_2 \boldsymbol{\epsilon} - \mathbf{W}_E \boldsymbol{\epsilon} \preceq \mathbf{0} \end{cases}$$

Finally, since $\mathbf{H}_2$ is a hermitian matrix, we can draw the following conclusions from (E1') :

2. $\text{Tr}(\mathbf{W}_1) > \text{Tr}(\mathbf{W}_2)$

- $1 - \delta \in [0, \dots, 1[\Rightarrow \text{Tr}(\mathbf{H}_2 \mathbf{W}_2) > \text{Tr}(\mathbf{H}_1 (\mathbf{W}_2 + \boldsymbol{\epsilon}))$

We deduce then that $\mathbf{H}_2$ is in better condition than $\mathbf{H}_1$.

3. $\text{Tr}(\mathbf{W}_1) \preceq \text{Tr}(\mathbf{W}_2)$

- $1 - \delta \in [1, \dots, \infty[\Rightarrow \text{Tr}(\mathbf{H}_2 \mathbf{W}_2) < \text{Tr}(\mathbf{H}_1 (\mathbf{W}_2 + \boldsymbol{\epsilon}))$

We deduce then that $\mathbf{H}_1$ is in better condition than $\mathbf{H}_2$.

To conclude, we can affirm that the algorithm will gives more power to the information vector being in the worst channel condition to maximize the sum of rates.

