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A stochastic equilibrium model for short-term electricity market design

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Abstract

The significant influx of renewable electricity has profoundly changed the electricity market. The demand for electricity generation by traditional power plants is more variable. To meet these new peaks of demand, the number of gas-fired plants, able to turn on very quickly to meet the sudden need for electricity, should increase. Unfortunately, the opposite happens. Gas power plants go bankrupt because their profitability has declined since the arrival of renewable energy. They are not paid fairly for the valuable services they provide to the electricity market. In this master thesis, one will try to better describe the short-term electricity market to ultimately be able to repair this inconsistency.
The short-term electricity market is described as a non-cooperative game involving four agents. One makes the unit commitment more realistic by describing it in a risky environment. The financial risk is evaluated using the good-deal pricing. A heuristic method to obtain the optimal decisions of these agents is presented. Finally, our method is applied to two exercises.

Contents

Notations

Sets

$\mathcal{G},\ g$	Set of generators and its index
$\mathcal{H},\ h$	Set of time steps and its index
$\Omega,\ \omega$	Set of scenarios and its index
$\mathcal{K},\ k$	Set of loads of the system operator and its index

Variables

Important note: MW is a measure of power while MWh is a measure of energy. However, since we are using an hourly resolution the conversion from one to another is done with a factor of 1.

of the producers

$y_{g,h}(\omega)$ Electric power generated by the power plant g during the time step h if the scenario ω occurs [MW].

$r_{g,h}(\omega)$ Reserves booked by the power plant g for the time step h if the scenario ω occurs [MW].

$u_{g,h}$ Status ON-OFF of the power plant g during the time step h [/].
It is equal to 0 when it is OFF and equal to 1 when it is ON.

$v_{g,h}$ Start-up of the power plant g durng the time step h [/].
It is equal to 1 when the power plant is started at the time step h.

y_g^{DA} Electricity futures contracts with a settlement the next day sold by the power plant g [MW].

$r_{g,h}^{DA}$ Reserves booked by the power plant g for the next day during the time step h [MW].

of the consumers

$d_h(\omega)$ Electric power consumed during the time step h if the scenario ω occurs [MW].

d^{DA} Electricity futures contracts with a settlement the next day bought by the consumers [MW].

of the system operator

$d_{k,h}^R(\omega)$ Reserves consumed by the load k of the system operator during the time step h if the scenario ω occurs [MW].

$d_{k,h}^{R,DA}$ Reserves booked by the load k of the system operator for the next day bought during the time step h [MW].

of the market

$\pi_h(\omega)$	Price of the electricity in real-time during the time step h if the scenario ω occurs [€/MWh].
$\lambda_h(\omega)$	Price of the reserves in real-time during the time step h if the scenario ω occurs [€/MWh].
π^{DA}	Price of the electricity futures contracts in the day-ahead market [€/MWh].
λ_h^{DA}	Price of the reserves in the day-ahead market for the time step h [€/MWh].

of the risk-aversion

$q_i(\omega)$	Risk-averse probability of the agent i for the scenario ω [/].

other notations used for the variables

$\bar{x}(\omega)$	Corresponds to $x(\omega) \cdot p(\omega)$.
$\tilde{x}_i(\omega)$	Corresponds to $x(\omega) \cdot q_i(\omega)$.
$\hat{x}(\omega)$	Corresponds to $x(\omega) \cdot \hat{q}(\omega)$.
$x^*(\omega)$	Corresponds to the value of $x(\omega)$ in the previous iteration.

Parameters

$C_g(\omega)$	Marginal cost of producing electricity by the power plant g if the scenario ω occurs [€/MWh].
K_g	No-load cost for producing electricity by the power plant g [€/MWh].
S_g	Marginal cost for producing electricity by the power plant g [€/MWh].
$P_g^{\min}$	Minimum power generation of the power plant g [MW].
$P_g^{\max}(\omega)$	Maximum power generation of the power plant g if the scenario ω occurs [MW].
R_g	Maximum power that can be booked by the power plant g for the reserves [MW].
V	Price at which the consumers value electricity [€/MWh]. Also called the value of lost load.
$D_h(\omega)$	Amount of electric power that the consumers want to consume during the time step h if the scenario ω occurs [MW].
V_k^R	Price at which the system operator values the load k of reserves [€/MWh].
D_k^R	Amount of reserves the load k of the system operator wants to have [MW].
$V_k^{R,DA}$	Price at which the system operator values the load k of reserves in the day-ahead [€/MWh].
$p(\omega)$	Probability that the scenario ω occurs [/].
$\overline{SR}$	Sharpe-ratio which is a measure of the risk-aversion [/].

1 Introduction

In Belgium, the share of electricity generated from renewable sources was only 2.4% in 2005. [1] This share gradually increased to reach 17.2% in 2017, and now, Belgium aims to raise it to more than 40% by 2030. The graph, given hereunder, shows the trend towards more renewable energy sources: the installed capacity of biomass, solar, wind and hydro power keeps going up.

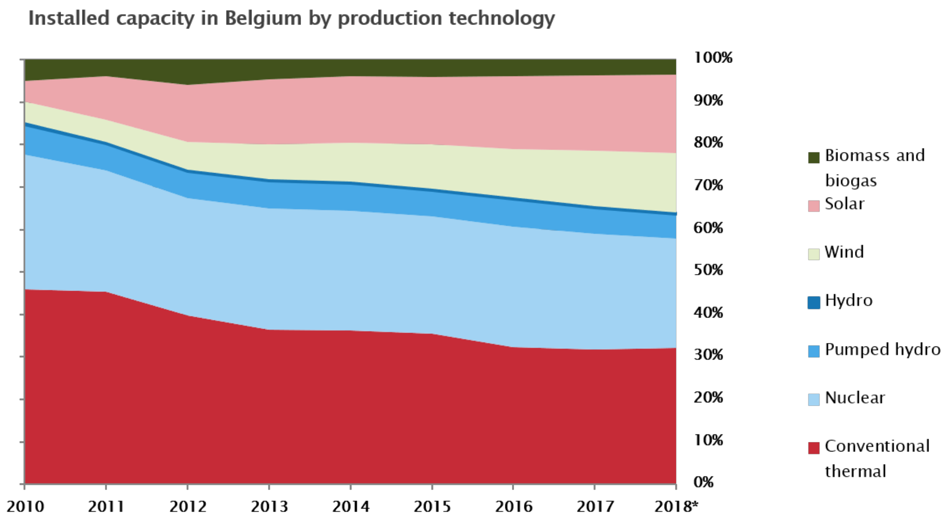

Figure 1.1: Installed capacity in Belgium by production technology from 2010 to 2018 [2]

Solar and wind power, which makes up the majority of these renewable energies, have significantly changed the electricity market. One would like to increase the flexibility of conventional power plants to compensate for the intermittency of solar and wind power. But at the same time, conventional power plants are seeing their profitability fall because of the low marginal cost of solar and wind power. From then on, to accompany the transition towards more solar and wind power, one question arises. *How should the electricity market be changed to pay flexible power plants correctly for the valuable service they provide?* In this master thesis, one would like to increase the knowledge about the short-term electricity market to eventually be able to answer this question.

Chapter 2 describes the short-term electricity market in a risky environment. Each agent involved in this market is presented and the choice it faces is formulated as an optimization problem. One will see in which case, it is possible to formulate this problem as a unique optimization problem and therefore easily obtain the Nash equilibrium.

The short-term electricity market is made more realistic in Chapter 3 because the time periods are no longer considered as independent. New constraints that link the unit commitment variables from one time period to the next are added. The domain that describes the unit commitment is given and the necessary modifications to include the unit commitment domain to the former problem is explained.

There is no simple method of solving the unit commitment problem in a risky environment. In Chapter 4 a heuristic algorithm to approach the solution with reasonable computational complexity is proposed. One does not have a proof of the convergence of the heuristic algorithm.

Finally, in the fifth chapter, the heuristic algorithm is applied to two small examples. These two deliberately simple exercises allow a better understanding of the unit commitment problem in a risky environment.

2
Short-term electricity market

2.1 Overview of the problem

In the short-term electricity market, one must deal with the existing power plants. The question of generation capacity expansion is not raised. It is necessary to determine which power plant should be turned ON, how the production of electricity should be dispatched, etc.

The modeling of the short-term electricity market used in this master thesis is composed of four agents, it takes place in two stages and three products are traded.

2.1.1 Brief description of the agents

The four agents have different needs and specific capacity for action.

- **The producers** want to maximize their profits without taking too much risk. The set of generators is depicted by $\mathcal{G}$ and is indexed by g. Each power plant can trade electricity futures contracts, sell electricity and sell reserves.
- **The consumers** want to consume a certain amount of electricity and are worried about curtailment. The demand of the consumers is aggregated into one load denoted by l. The load can trade electricity futures contracts and buy electricity.
- **The system operator** wants to reduce the curtailment as much as possible. For this purpose, the system operator buys reserves from the producers. The system operator is considered as risk-neutral.
- **The market agent** is introduced for the sake of representing the market as a Nash equilibrium. It corresponds to the competitive forces that set prices so that supply balances demand.

The short-term electricity market form a non-cooperative game involving these four agents.

2.1.2 The timeline

The model is divided into two stages: the day-ahead and the real-time.

In the **day-ahead**, the agents do not have yet a clear idea of what will happen. The agents only know the set of possible outcomes Ω

$$\Omega = \{\omega_1, \omega_2, \cdots, \omega_N\} \tag{2.1}$$

and the probability associated with each outcome

$$p(\cdot) : \Omega \to \mathbb{R}. \tag{2.2}$$

Although the exact values of some parameters, such as the demand of the consumers $D(\omega)$ or the price of the electricity $\pi(\omega)$, are not known yet some decisions have to be made. The producers must decide whether the power plant should be ON or OFF. Electricity futures contracts and reserves are also traded.

In **real-time** the outcome ω is known. The agents trade electricity and reserves while being constrained by the decisions they made in the day-ahead.

2.1.3 The products traded

Three products are traded: electricity futures contracts, electricity and reserves.

Electricity futures contracts with a settlement the next day are traded on the day-ahead market by the producers and the consumers to hedge against the financial risk. These products are purely financial, buying x [MW] of this contract gives the following payoff:

$$x \cdot \Big(\pi(\omega) - \pi^{DA}\Big). \tag{2.3}$$

where π^{DA} is the price of the futures contracts and $\pi(\omega)$ is the spot price of the underlying asset (electricity). The number of contracts exchanged must balance at all time. Therefore, the number of futures contracts sold by the power plants y_g^{DA} should be equal to the number of futures contracts bought by the consumers d^{DA}:

$$\sum_{g \in \mathcal{G}} y_g^{DA} = d^{DA}. \tag{2.4}$$

Electricity is sold by the producers to the consumers on the spot market at a price $\pi(\omega)$ if the scenario ω occurs. The sum of the electricity produced should always be greater or equal to the consumption of electricity:

$$\sum_{g \in \mathcal{G}} y_g(\omega) \geq d(\omega). \tag{2.5}$$

In this master thesis, one considers that the electricity price is non-negative. If the production is greater than the consumption, then the electricity price is 0.

The **reserves** are bought in the day-ahead and in real-time by the system operator. They form a power reserve that producers make available for the system operator to compensate for a temporary decrease of production or to handle sudden increases in demand. Their role is essential to prevent electricity shortage. And the price at which the system operator is ready to buy reserves depends on the uncertainty to meet the demand.

In the day-ahead, the price of the reserves is λ^{DA}. The amount of reserves sold by a producer in the day-ahead is denoted r_g^{DA} and the amount of reserves bought by the load k of the system operator in the day-ahead is denoted $d_k^{R,DA}$. Each of the load k of the system operator corresponds to an amount of reserves that the system operator is ready to buy if the price is lower or equal to $V_k^{R,DA}$.
At any time, the sum of the reserves sold by the generators should be equal to the quantity of reserves the system operator received.

$$\sum_{g\in\mathcal{G}} r_g^{DA} = \sum_{k\in\mathcal{K}} d_k^{R,DA} \tag{2.6}$$

Reserves sold in the day-ahead look like futures contracts, but in order to sell reserves in the day-ahead the power plants must have the physical capacity.

In real-time, the price of the reserves is $\lambda(\omega)$. The final amount of reserves made available by the power plant g to the system operator is denoted by $r_g(\omega)$. While the final amount of reserves consumed by the load k of the system operator is denoted by $d_k^R(\omega)$.

2.2 Risk measure

Every agent wants to implement a strategy to maximize its surplus. But since the surpluses are stochastic, there is usually no strategy that maximizes surpluses in each scenario. To overcome this issue and determine the ideal strategy, the agents assign a deterministic value to each lottery (i.e. a distribution of stochastic payoffs). This value is called the certainty equivalent and depends on the risk perception of the agent. **Each agent will maximize the certainty equivalent to determine its optimal strategy**.

Subsequently, the two cases will be considered. The agent could either be risk-neutral or risk-averse. If the agent is risk-neutral, it means that the agent is insensitive to financial risk and so the certainty equivalent is equal to the expected value. If, on the contrary, the agent is risk-averse then the certainty equivalent will be smaller or equal to the expected value. For the latter, the good-deal risk measure will be used. This risk-measure, which is a coherent risk-measure as described in section 6.1 of [3], is denoted by $\mathcal{R}(\cdot)$.

For the sake of clarity, let us consider a small example. Let $\Omega = \{\omega_1, \omega_2, ..., \omega_N\}$ be a finite sample space, $p(\cdot) : \Omega \to \mathbb{R}$ be a function which assigns a probability to each event and $f(\cdot) : \Omega \to \mathbb{R}$ be a function which assigns a surplus to each event. In the risk-neutral case, the certainty equivalent is given by:

$$\mathbb{E}\Big[f(\omega)\Big] = \sum_{j=1}^{N} p(\omega_j) \cdot f(\omega_j). \tag{2.7}$$

In the risk-averse case, the certainty equivalent is given by:

$$\mathcal{R}\Big[f(\omega)\Big] = \sum_{j=1}^{N} q(\omega_j) \cdot f(\omega_j) \tag{2.8}$$

where $q(\cdot) : \Omega \to \mathbb{R}$ is a function which assigns a risk-averse probability to each event.

2.2.1 Good-deal risk measure

In this master thesis, one will use the good-deal pricing. The certainty equivalent given by the good-deal risk measure for a certain Sharpe-ratio, $\overline{SR}$, is given by the following minimization problem. [4]

$$\mathcal{R}\Big[f(\Omega)\Big] = \min_{q(\omega)} \sum_{\omega\in\Omega} q(\omega) \cdot f(\omega) \tag{2.9}$$

$$\text{s.t.} \quad \sum_{\omega\in\Omega} q(\omega) = 1 \tag{2.10}$$

$$\sum_{\omega\in\Omega} \frac{q(\omega)^2}{p(\omega)} \leq 1 + \overline{SR}^2 \tag{2.11}$$

$$0 \leq q(\omega). \tag{2.12}$$

The objective function (2.9) means that the risk-averse agent apprehends the unknown with pessimism or prudence, he assigns risk-averse probabilities that reflect its vision of the future. The risk-averse probability will exceed the actual probability when an event is unfavorable ($\frac{q(\omega)}{p(\omega)} > 1$), i.e., when the surplus is low. And the opposite happens when an event is favorable ($\frac{q(\omega)}{p(\omega)} < 1$). The second constraint (2.11) restricts the distance between the actual probabilities and the risk-averse probabilities based on the Sharpe-ratio that represents the risk aversion of the agent. The more you increase the Sharpe-ratio, the more risk-averse the agent and the lower the certainty equivalent.

Small example. Consider a simple game with four possible outcomes $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. Depending on the outcome ω, the player receives a certain amount of money $f(\omega)$.

	ω_1	ω_2	ω_3	ω_4
$f(\omega)$	-20	10	20	40

Knowing that each outcome has the same probability of happening, how much is a player ready to pay to play this game?

The answer depends on its risk-aversion. If the agent is risk-neutral ($\overline{SR} = 0$), then he will be ready to pay $\mathbb{E}\Big[f(\omega)\Big] = 12.5$ to play this game. But if the agent is risk-averse ($\overline{SR} = 0.05$), then he will only be ready to pay $\mathcal{R}\Big[f(\omega)\Big] = 11.42$.

	ω_1	ω_2	ω_3	ω_4
$q(\omega)$	0.269	0.251	0.246	0.234

2.3 Agent problems: individual maximization of the surplus

2.3.1 Producers

Each generator, g, wants to maximize the certainty equivalent of its profit. To achieve this goal, the generator takes actions at two different stages. In the first stage, the generator knows the probability distribution of future scenarios but does not know which scenario will occur. However, it must already decide: whether to start-up or not its plant, which amount of electricity to trade on the day-ahead market, and which amount of reserves to sell on the day-ahead market. In the second stage, based on the decisions taken in the previous stage and the actual scenario, the generator must decide: the production of electricity, and which amount of reserves to trade.

	Stage 1	**Stage 2**
Action	Switch on power plant: u_g	
Pay-off	$-K_g \cdot u_g$	
Action	Day-ahead electricity: y_g^{DA}	Production of electricity: $y_g(\omega)$
Pay-off	$\pi^{\text{DA}} \cdot y_g^{\text{DA}}$	$\pi(\omega) \cdot \left(y_g(\omega) - y_g^{\text{DA}}\right) - C_g(\omega) \cdot y_g(\omega)$
Action	Day-ahead reserves: r_g^{DA}	Real-time reserves: $r_g(\omega)$
Pay-off	$\lambda^{\text{DA}} \cdot r_g^{\text{DA}}$	$\lambda(\omega) \cdot \left(r_g(\omega) - r_g^{\text{DA}}\right)$

The decisions taken in the first stage depend on the payoff obtained in the second stage. It is, therefore, logical to describe the second stage before describing the first.

Second stage problem. Given the scenario ω and the commitment u_g, each generator g must choose its production of electricity $y_g(\omega)$, and its volume of reserves $r_g(\omega)$ in order to maximize its profit.

$$\Pi_g(u_g; \omega) = \max_{y_g(\omega), r_g(\omega)} \Big(\pi(\omega) - C_g(\omega)\Big) \cdot y_g(\omega) + \lambda(\omega) \cdot r_g(\omega) \tag{2.13}$$

$$\Big(\mu_g^{\min}(\omega)\Big): \quad y_g(\omega) - P_g^{\min} \cdot u_g \geq 0 \tag{2.14}$$

$$\Big(\mu_g^{\max}(\omega)\Big): \quad P_g^{\max}(\omega) \cdot u_g - y_g(\omega) - r_g(\omega) \geq 0 \tag{2.15}$$

$$\Big(\xi_g(\omega)\Big): \quad R_g \cdot u_g - r_g(\omega) \geq 0 \tag{2.16}$$

$$0 \leq y_g(\omega),\ r_g(\omega) \tag{2.17}$$

The objective function (2.13) is the sum of the revenues from the sales of electricity and from the sales of the reserves minus the cost to produce the electricity.
The constraints (2.14), (2.15) and (2.16) correspond, respectively, to the minimum production of electricity, the maximum possible production, and the maximum amount of reserves.

First stage problem. Knowing the probability distribution of the scenarios (Ω, P), each generator g must choose whether to start-up its plant u_g and whether it wants to exchange electricity y_g^{DA} or reserves r_g^{DA} on the day-ahead market.

$$\begin{aligned} \max_{u_g, y_g^{\mathrm{DA}}, r_g^{\mathrm{DA}}} \ & \mathcal{R}_g\Big(\Pi_g(u_g;\omega) - \pi(\omega)\cdot y_g^{\mathrm{DA}} - \lambda(\omega)\cdot r_g^{\mathrm{DA}}\Big) \\ & \qquad - K_g \cdot u_g + \pi^{\mathrm{DA}} \cdot y_g^{\mathrm{DA}} + \lambda^{\mathrm{DA}} \cdot r_g^{\mathrm{DA}} && (2.18) \\ (\alpha_g) : \quad & 1 - u_g \geq 0 && (2.19) \\ (\xi_g^{\mathrm{DA}}) : \quad & R_g \cdot u_g - r_g^{\mathrm{DA}} \geq 0 && (2.20) \\ & 0 \leq u_g,\ r_g^{\mathrm{DA}} && (2.21) \end{aligned}$$

The objective function (2.18) is the certainty equivalent of the previous payoff plus the payoff of the day-ahead products minus the cost to start-up the plant.
The transaction of electricity on the day-ahead market is purely financial and is used by the generator to mitigate the risk. During the day-ahead, the generator sells a certain quantity of electricity at the day-ahead price $\pi^{\mathrm{DA}} \cdot y_g^{\mathrm{DA}}$; and during the real-time, the generator delivers that predetermined quantity of electricity at the real-time price $-\pi(\omega) \cdot y_g^{\mathrm{DA}}$.
The constraints (2.19) and (2.20) corresponds, respectively, to the status ON-OFF of the power plant and the maximum amount of reserves sold in the day-ahead.

Complete problem. Gathering both stages, the complete problem of the producers is given hereunder.

Definition 1: Maximization of the surplus of the producers

$$\begin{aligned} \max_{u_g, y_g^{\mathrm{DA}}, r_g^{\mathrm{DA}}} \mathcal{R}_g\Bigg(& \max_{y_g(\omega), r_g(\omega)} \Big(\pi(\omega) - C_g(\omega)\Big)\cdot y_g(\omega) + \lambda(\omega)\cdot\Big(r_g(\omega) - r_g^{\mathrm{DA}}\Big) - \pi(\omega)\cdot y_g^{\mathrm{DA}}\Bigg) \\ & - K_g \cdot u_g + \pi^{\mathrm{DA}} \cdot y_g^{\mathrm{DA}} + \lambda^{\mathrm{DA}} \cdot r_g^{\mathrm{DA}} && (2.22) \\ (\alpha_g) : \quad & 1 - u_g \geq 0 && (2.23) \\ \Big(\tilde{\mu}_g^{\min}(\omega)\Big) : \quad & y_g(\omega) - P_g^{\min} \cdot u_g \geq 0 && (2.24) \\ \Big(\tilde{\mu}_g^{\max}(\omega)\Big) : \quad & P_g^{\max}(\omega) \cdot u_g - y_g(\omega) - r_g(\omega) \geq 0 && (2.25) \\ (\xi_g^{\mathrm{DA}}) : \quad & R_g \cdot u_g - r_g^{\mathrm{DA}} \geq 0 && (2.26) \\ \Big(\tilde{\xi}_g(\omega)\Big) : \quad & R_g \cdot u_g - r_g(\omega) \geq 0 && (2.27) \\ & 0 \leq u_g,\ r_g^{\mathrm{DA}},\ y_g(\omega),\ r_g(\omega) && (2.28) \end{aligned}$$

2.3.2 Consumers

The demand for electricity of the consumers may be aggregated into several loads. Here, for simplicity, the demand will be aggregated into a single load l. The maximization of the surplus of the consumers is equivalent to the maximization of the surplus of that load. In the first stage, the load can choose whether to trade electricity on the day-ahead market. In the second stage, the load can choose its consumption of electricity.

The demand for electricity is equal to $D(\omega)$ as long as the price of the electricity $\pi(\omega)$ is smaller or equal the value of lost load V. However, if the price of electricity is higher than the value of lost load, then the demand is equal to 0.

Second stage problem. Given the scenario ω, the load must choose its consumption of electricity in order to maximize its surplus.

$$\Pi_l(\omega) = \max_{d(\omega)} \Big(V - \pi(\omega)\Big) \cdot d(\omega) \tag{2.29}$$

$$\Big(\nu(\omega)\Big): \quad D(\omega) - d(\omega) \geq 0 \tag{2.30}$$

$$0 \leq d(\omega) \tag{2.31}$$

The objective function (2.29) corresponds to the price at which the load valuates electricity minus how much it pays for it.
The constraint (2.30) is the maximum demand.

First stage problem. Knowing the probability distribution of the scenarios, the load can also choose to buy electricity on the day-ahead market.

$$\max_{d^{\mathrm{DA}}} \mathcal{R}_l \Bigg(\Pi_l(\omega) + \pi(\omega) \cdot d^{\mathrm{DA}}\Bigg) - \pi^{\mathrm{DA}} \cdot d^{\mathrm{DA}} \tag{2.32}$$

The objective function (2.32) is the certainty equivalent of the surplus and the exchange of electricity bought on the day-ahead market is used to mitigate the financial risk.

Complete problem. Gathering both stages, the complete problem of the consumers is given hereunder.

Definition 2: Maximization of the surplus of the consumers

$$\max_{d^{\mathrm{DA}}} \mathcal{R}_l \Bigg(\max_{d(\omega)} \Big(V - \pi(\omega)\Big) \cdot d(\omega) + \pi(\omega) \cdot d^{\mathrm{DA}} \Bigg) - \pi^{\mathrm{DA}} \cdot d^{\mathrm{DA}} \tag{2.33}$$

$$\Big(\tilde{\nu}(\omega)\Big): \quad D(\omega) - d(\omega) \geq 0 \tag{2.34}$$

$$0 \leq d(\omega) \tag{2.35}$$

2.3.3 System operator

The single purpose of the system operator is to ensure the security of supply, or at least to reduce the curtailment as much as possible. To achieve this, the system operator buys reserves from the producers. The income of the producers increases by selling reserves to the system operator. The sale of reserves gives an incentive to the producers to start-up more capacities. And this greater number of production capacity turned ON ensure the stability of the electricity system in case of unusually large demand and/or a problem in the production.

The valuation of the reserves is given by an Operating Reserve Demand Curve [ORDC] and the auction of reserves in real-time introduces scarcity pricing. When the the system operator holds little reserves the price goes up, on the contrary when the system operator holds enough reserves the price goes down.

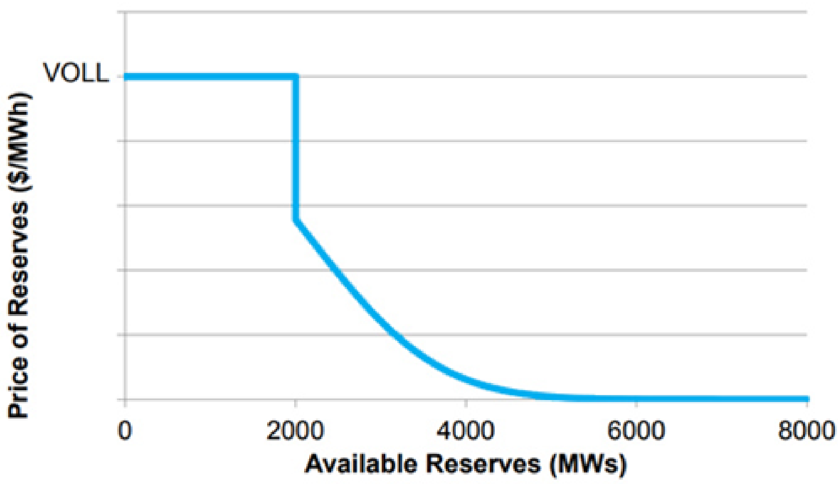

Figure 2.1: Example of ORDC in Texas. Price of the reserves depending on the available reserves. Source: ERCOT.

Second stage problem. Given the scenario ω, the system operator must decide the amount of reserves it wants to have in real-time $d_k^R(\omega)$. The amount of reserves of the system operator in real-time does not depend on the amount sold during the day-ahead, it can be increased or even be reduced if the system is tight.

$$\Pi_{SO}(\omega) = \max_{d_k^R(\omega)} \quad \sum_{k \in \mathcal{K}} \Big(V_k^R - \lambda(\omega)\Big) \cdot d_k^R(\omega) \tag{2.36}$$

$$\Big(\nu_k^R(\omega)\Big): \quad D_k^R - d_k^R(\omega) \geq 0 \tag{2.37}$$

$$0 \leq d_k^R(\omega) \tag{2.38}$$

The objective function (2.36) is the valuation of the reserves by the system operator minus how much it pays for it.
The constraints (2.37) is the maximum demand of reserves at a certain price V_k^R.

First stage problem. Knowing the probability distribution of the scenarios, the system operator must decide the amount of reserves it wants to have in the day-ahead $d_k^{R,DA}$ in order to maximize the expected value of its surplus.

$$\begin{aligned}
\max_{d_k^{R,\mathrm{DA}}} \quad & \mathbb{E}_P\left[\Pi_{SO}(\omega) - \sum_{k\in\mathcal{K}} \Big(V_k^R - \lambda(\omega)\Big)\cdot d_k^{R,\mathrm{DA}}\right] \\
& + \sum_{k\in\mathcal{K}} \Big(V_k^{R,\mathrm{DA}} - \lambda^{\mathrm{DA}}\Big)\cdot d_k^{R,\mathrm{DA}} \qquad (2.39)\\
(\nu_k^{R,\mathrm{DA}}): \quad & D_k^R - d_k^{R,\mathrm{DA}} \geq 0 \qquad (2.40)\\
& 0 \leq d_k^{R,\mathrm{DA}} \qquad (2.41)
\end{aligned}$$

The objective function (2.39) is the expected value of the surplus of the system operator, which is the sum of the surplus realized in the day-ahead and in real-time. It is important to note that the system operator only pays once for the reserves. The price at which the system operator values the reserves depends on the uncertainty of the production to meet the demand. For this reason, the system operator is ready to pay for the reserves at a higher price in the day-ahead $V_k^{R,\mathrm{DA}} > V_k^R$.
The constraint (2.40) is the maximum demand of reserves at a certain price.

Complete problem. Gathering both stages, the complete problem of the system operator is given hereunder.

Definition 3: Maximization of the surplus of the system operator

$$\begin{aligned}
\max_{d_k^{R,\mathrm{DA}}} \quad & \mathbb{E}_P\left[\max_{d_k^R(\omega)} \sum_{k\in\mathcal{K}} \left(\Big(V_k^R - \lambda(\omega)\Big)\cdot\Big(d_k^R(\omega) - d_k^{R,\mathrm{DA}}\Big)\right)\right] \\
& + \sum_{k\in\mathcal{K}} \Big(V_k^{R,\mathrm{DA}} - \lambda^{\mathrm{DA}}\Big)\cdot d_k^{R,\mathrm{DA}} \qquad (2.42)\\
(\nu_k^{R,\mathrm{DA}}): \quad & D_k^R - d_k^{R,\mathrm{DA}} \geq 0 \qquad (2.43)\\
\Big(\bar{\nu}_k^R(\omega)\Big): \quad & D_k^R - d_k^R(\omega) \geq 0 \qquad (2.44)\\
& 0 \leq d_k^{R,\mathrm{DA}},\ d_k^R(\omega) \qquad (2.45)
\end{aligned}$$

The term $-d_k^{R,DA}$ in the expected value of the objective function is essential. It shows that the system operator is not willing to pay twice for the same reserves. The system operator will not pay $V_k^{R,DA}$ for reserves in the day-ahead and then pay V_k^R for the same reserves in real-time.

2.3.4 Market agent

The last agent is the market, its role is to fix the prices so that the offer matches the demand. It fixes the price of the electricity in the real-time $\pi(\omega)$ and in the day-ahead π^{DA}. It also fixes the price of the reserves in the real-time $\lambda(\omega)$ and in the day-ahead λ^{DA}.

Definition 4: Maximization of the surplus of the market agent

$$\begin{aligned}
\max_{\pi(\omega),\lambda(\omega),\pi^{\mathrm{DA}},\lambda^{\mathrm{DA}}} \quad & \mathbb{E}_P\left[\left(d(\omega) - \sum_{g\in G} y_g(\omega)\right)\cdot \pi(\omega)\right] \\
& + \mathbb{E}_P\left[\left(\sum_{k\in\mathcal{K}}\left(d_k^R(\omega) - d_k^{R,DA}\right) - \sum_{g\in G}\left(r_g(\omega) - r_g^{DA}\right)\right)\cdot \lambda(\omega)\right] \\
& + \left(\sum_{g\in G} y_g^{\mathrm{DA}} - d^{\mathrm{DA}}\right)\cdot \pi^{\mathrm{DA}} \\
& + \left(\sum_{k\in\mathcal{K}} d_k^{R,\mathrm{DA}} - \sum_{g\in G} r_g^{\mathrm{DA}}\right)\cdot \lambda^{\mathrm{DA}} && (2.46) \\
& 0 \leq \pi(\omega),\ \lambda(\omega),\ \lambda^{DA} && (2.47)
\end{aligned}$$

2.4 Equilibrium conditions

The equilibrium conditions can be derived for each of the four problems. In this section, one will only show in detail the way to get the equilibrium conditions of the producers because the process is similar for the other agents.

2.4.1 Producers

Obtaining the equilibrium conditions The complete problem of the producer has been described earlier (Definition 1). Keeping the risk-averse probabilities q_g as a parameter, one can write the Lagrangian function.

$$\begin{aligned}
\mathcal{L}_g(\mathbf{x}, \boldsymbol{\zeta}) = &\sum_{\omega\in\Omega} q_g(\omega)\cdot\left(\Big(\pi(\omega) - C_g(\omega)\Big)\cdot y_g(\omega) + \lambda(\omega)\cdot\Big(r_g(\omega) - r_g^{\mathrm{DA}}\Big) - \pi(\omega)\cdot y_g^{\mathrm{DA}}\right) \\
&- K_g\cdot u_g + \pi^{\mathrm{DA}}\cdot y_g^{\mathrm{DA}} + \lambda^{\mathrm{DA}}\cdot r_g^{\mathrm{DA}} \\
&+ \alpha_g\cdot(1-u_g) \\
&+ \sum_{\omega\in\Omega} q_g(\omega)\cdot\mu_g^{\min}(\omega)\cdot\Big(y_g(\omega) - P_g^{\min}\cdot u_g\Big) \\
&+ \sum_{\omega\in\Omega} q_g(\omega)\cdot\mu_g^{\max}(\omega)\cdot\Big(P_g^{\max}(\omega)\cdot u_g - y_g(\omega) - r_g(\omega)\Big) \\
&+ \xi_g^{\mathrm{DA}}\cdot(R_g\cdot u_g - r_g^{\mathrm{DA}}) \\
&+ \sum_{\omega\in\Omega} q_g(\omega)\cdot\xi_g(\omega)\cdot\Big(R_g\cdot u_g - r_g(\omega)\Big) \\
&+ \sigma_{1,g}\cdot u_g + \sigma_{2,g}\cdot r_g^{DA} + \sum_{\omega\in\Omega} q_g(\omega)\cdot\Big(\sigma_{3,g}(\omega)\cdot y_g(\omega) + \sigma_{4,g}(\omega)\cdot r_g(\omega)\Big). \quad (2.48)
\end{aligned}$$

where $\mathbf{x}$ is the optimization variable

$$\mathbf{x} = \Big(y_g(\omega),\ r_g(\omega),\ u_g,\ y_g^{DA},\ r_g^{DA}\Big) \quad (2.49)$$

and $\boldsymbol{\zeta}$ is the non-negative dual variable

$$\boldsymbol{\zeta} = \Big(\alpha_g,\ \mu_g^{\min}(\omega),\ \mu_g^{\max}(\omega),\ \xi_g^{DA},\ \xi_g(\omega),\ \sigma_{1,g},\ \sigma_{2,g},\ \sigma_{3,g}(\omega),\ \sigma_{4,g}(\omega)\Big). \quad (2.50)$$

The stationarity conditions of $\mathbf{x}$ give the five equations hereunder.

$$\pi(\omega) - C_g(\omega) + \mu_g^{\min}(\omega) - \mu_g^{\max}(\omega) + \sigma_{3,g}(\omega) = 0 \quad (2.51)$$

$$\lambda(\omega) - \mu_g^{\max}(\omega) - \xi_g(\omega) + \sigma_{4,g}(\omega) = 0 \quad (2.52)$$

$$-K_g - \alpha_g - \mathbb{E}_{Q_g}\Big[\mu_g^{\min}(\omega)\cdot P_g^{\min}\Big] + \mathbb{E}_{Q_g}\Big[\mu_g^{\max}(\omega)\cdot P_g^{\max}(\omega)\Big] + \xi_g^{\mathrm{DA}}\cdot R_g + \mathbb{E}_{Q_g}\Big[\xi_g(\omega)\cdot R_g\Big] + \sigma_{1,g} = 0 \quad (2.53)$$

$$-\mathbb{E}_{Q_g}\Big[\pi(\omega)\Big] + \pi^{DA} = 0 \quad (2.54)$$

$$-\mathbb{E}_{Q_g}\Big[\lambda(\omega)\Big] + \lambda^{DA} - \xi_g^{\mathrm{DA}} + \sigma_{2,g} = 0 \quad (2.55)$$

The primal feasibility, the dual feasibility and the complementary slackness give the following equations. The notation $0 \leq a \perp b \geq 0$ is equivalent to $a \geq 0$, $b \geq 0$ and $a \cdot b = 0$.

$$\begin{aligned}
0 \leq y_g(\omega) &\perp \sigma_{3,g}(\omega) \geq 0 && (2.56)\\
0 \leq r_g(\omega) &\perp \sigma_{4,g}(\omega) \geq 0 && (2.57)\\
0 \leq \mu_g^{\min}(\omega) &\perp y_g(\omega) - P_g^{\min} \cdot u_g \geq 0 && (2.58)\\
0 \leq \mu_g^{\max}(\omega) &\perp P_g^{\max}(\omega) \cdot u_g - y_g(\omega) - r_g(\omega) \geq 0 && (2.59)\\
0 \leq \xi_g(\omega) &\perp R_g \cdot u_g - r_g(\omega) \geq 0 && (2.60)\\
0 \leq u_g &\perp \sigma_{1,g} \geq 0 && (2.61)\\
0 \leq \alpha_g &\perp 1 - u_g \geq 0 && (2.62)\\
0 \leq r_g^{\mathrm{DA}} &\perp \sigma_{2,g} \geq 0 && (2.63)\\
0 \leq \xi_g^{\mathrm{DA}} &\perp R_g \cdot u_g - r_g^{\mathrm{DA}} \geq 0 && (2.64)
\end{aligned}$$

Finally, it is possible to replace the σ present in the equations above by their values given in the stationarity conditions (2.53), (2.55), (2.51) and (2.52). Using the same process, the equilibrium conditions have been calculated for each agent and are written in the definition 5.

Analysis of the equilibrium conditions One will analyze two equilibrium conditions of the producers more closely to get a better understanding of the problem.

The equilibrium condition (2.65) associated with the production of electricity $y_g(\omega)$ is an interesting one. The condition $y_g(\omega) \geq 0$ is superfluous because the production of electricity is already constrained between the minimum production $P_g^{\min} \cdot u_g$ and the maximum production $P_g^{\max}(\omega) \cdot u_g$. If the power plant is ON then $y_g(\omega) > 0$ and $C_g(\omega) - \pi(\omega) - \mu_g^{\min}(\omega) + \mu_g^{\max}(\omega) = 0$. Therefore, one can deduce that $\mu_g^{\max}(\omega) - \mu_g^{\min}(\omega)$ represents the marginal profit and it might be different from zero in two cases:

- If the electrical production is minimal $y_g(\omega) = P_g^{\min} \cdot u_g$ then $\mu_g^{\min}(\omega)$ tells how much could have been saved marginally by reducing the available capacity.
- If the electrical production and the reserves booked are maximal $y_g(\omega) + r_g(\omega) = P_g^{\max}(\omega) \cdot u_g$ then $\mu_g^{\max}(\omega)$ tells how much could be gained by increasing the available capacity.

The equilibrium condition (2.70) associated with the status ON-OFF u_g of the power plant also deserves special attention. If the certainty equivalent of the revenues obtained by having the power plant turned ON is greater than the no-load cost:

$$\mathbb{E}_{Q_g}\Big[P_g^{\max} \cdot \mu_g^{\max}(\omega) + R_g \cdot \xi_g(\omega)\Big] + R_g \cdot \xi_g^{\mathrm{DA}} - \mathbb{E}_{Q_g}\Big[P^{\min} \cdot \mu_g^{\min}(\omega)\Big] > K_g$$

then $\alpha_g > 0$ which means that the power plant is completely started $u_g = 1$.

2.4.2 Gathering all the equilibrium conditions

While maintaining the risk-averse probability q_g and q_l as parameters, the other equilibrium conditions were calculated and are given in the definition hereunder. If the variable associated is free, it has been marked with brackets.

Definition 5: Equilibrium conditions of all the agents

Equilibrium conditions of the producers

$$0 \leq y_g(\omega) \perp C_g(\omega) - \pi(\omega) - \mu_g^{\min}(\omega) + \mu_g^{\max}(\omega) \geq 0 \tag{2.65}$$

$$0 \leq r_g(\omega) \perp -\lambda(\omega) + \mu_g^{\max}(\omega) + \xi_g(\omega) \geq 0 \tag{2.66}$$

$$0 \leq \mu_g^{\min}(\omega) \perp y_g(\omega) - P_g^{\min} \cdot u_g \geq 0 \tag{2.67}$$

$$0 \leq \mu_g^{\max}(\omega) \perp P_g^{\max}(\omega) \cdot u_g - y_g(\omega) - r_g(\omega) \geq 0 \tag{2.68}$$

$$0 \leq \xi_g(\omega) \perp R_g \cdot u_g - r_g(\omega) \geq 0 \tag{2.69}$$

$$0 \leq u_g \perp K_g + \alpha_g - R_g \cdot \xi_g^{\mathrm{DA}} + \mathbb{E}_{Q_g}\left[P^{\min} \cdot \mu_g^{\min}(\omega) - P_g^{\max} \cdot \mu_g^{\max}(\omega) - R_g \cdot \xi_g(\omega)\right] \geq 0 \tag{2.70}$$

$$0 \leq \alpha_g \perp 1 - u_g \geq 0 \tag{2.71}$$

$$0 \leq r_g^{\mathrm{DA}} \perp -\lambda^{\mathrm{DA}} + \xi_g^{\mathrm{DA}} + \mathbb{E}_{Q_g}\Big[\lambda(\omega)\Big] \geq 0 \tag{2.72}$$

$$0 \leq \xi_g^{\mathrm{DA}} \perp R_g \cdot u_g - r_g^{\mathrm{DA}} \geq 0 \tag{2.73}$$

$$(y_g^{DA}) \qquad \mathbb{E}_{Q_g}\Big[\pi(\omega)\Big] - \pi^{DA} = 0 \tag{2.74}$$

Equilibrium conditions of the consumers

$$0 \leq d(\omega) \perp \pi(\omega) - V + \nu(\omega) \geq 0 \tag{2.75}$$

$$0 \leq \nu(\omega) \perp D(\omega) - d(\omega) \geq 0 \tag{2.76}$$

$$(d^{DA}) \qquad \pi^{DA} - \mathbb{E}_{Q_l}\Big[\pi(\omega)\Big] = 0 \tag{2.77}$$

Equilibrium conditions of the system operator

$$0 \leq d_k^R(\omega) \perp \lambda(\omega) - V_k^R + \nu_k^R(\omega) \geq 0 \tag{2.78}$$

$$0 \leq \nu_k^R(\omega) \perp D_k^R - d_k^R(\omega) \geq 0 \tag{2.79}$$

$$0 \leq d_k^{R,\mathrm{DA}} \perp \lambda^{\mathrm{DA}} - \mathbb{E}_P\Big[\lambda(\omega)\Big] - V_k^{R,\mathrm{DA}} + V_k^R + \nu_k^{R,\mathrm{DA}} \geq 0 \tag{2.80}$$

$$0 \leq \nu_k^{R,\mathrm{DA}} \perp D_k^R - d_k^{R,\mathrm{DA}} \geq 0 \tag{2.81}$$

Equilibrium conditions of the market agent

$$0 \leq \pi(\omega) \perp \sum_{g \in \mathcal{G}} y_g(\omega) - d(\omega) \geq 0 \tag{2.82}$$

$$0 \leq \lambda(\omega) \perp \sum_{g \in G} \Big(r_g(\omega) - r_g^{\mathrm{DA}}\Big) - \sum_{k \in \mathcal{K}} \Big(d_k^R(\omega) - d_k^{R,\mathrm{DA}}\Big) \geq 0 \tag{2.83}$$

$$0 \leq \lambda^{\mathrm{DA}} \perp \sum_{g \in G} r_g^{\mathrm{DA}} - \sum_{k \in \mathcal{K}} d_k^{R,\mathrm{DA}}(\omega) \geq 0 \tag{2.84}$$

$$(\pi^{\mathrm{DA}}) \qquad \sum_{g \in \mathcal{G}} y_g^{DA} - d^{DA} = 0 \tag{2.85}$$

2.5 Risk-neutral case

There are solvers able to solve Generalized Complementarity Problems (GCP) but the computational time required for such a large problem would be massive. It is especially true because the equilibrium conditions given in the definition 5 do not represent the initial problem entirely. The equilibrium conditions related to the risk-averse probabilities have yet to be added. For these reasons, the problem presented initially will not be solved as currently framed.

Here, one will show that by making the producers and the consumers risk-neutral, the individual maximization problems can be turned into a single optimization problem which is easy to solve. This is done in three steps:

1. The equilibrium conditions of the risk-neutral problem will be described.
2. The equivalent Variational Inequality problem will be given.
3. The unique optimization problem will finally be found.

2.5.1 Equilibrium conditions

There are two differences between the equilibrium conditions given in the definition 5 and the equilibrium conditions of the risk-neutral problem.

Firstly, since every agent is risk-neutral, the electricity futures contracts are no longer useful. Indeed trading electricity futures will not change the expected value of the profit of the producers nor the expected value of the surplus of the producers. Therefore, the equilibrium conditions (2.74), (2.77) and (2.85) are not taken into account.

Secondly, the two equilibrium conditions (2.70) and (2.72) that depended on the risk-averse probabilities are changed into:

$$0 \leq u_g \ \perp \ K_g + \alpha_g - R_g \cdot \xi_g^{\mathrm{DA}} + \mathbb{E}_P\Big[P^{\min} \cdot \mu_g^{\min}(\omega) - P_g^{\max} \cdot \mu_g^{\max}(\omega) - R_g \cdot \xi_g(\omega)\Big] \geq 0$$

and

$$0 \leq r_g^{\mathrm{DA}} \ \perp -\lambda^{\mathrm{DA}} + \xi_g^{\mathrm{DA}} + \mathbb{E}_P\Big[\lambda(\omega)\Big] \geq 0.$$

2.5.2 Variational Inequality problem

The following definition comes from Chapter 1 of [5]. In its general form, the variational inequality is formally defined below.

Definition. Given a subset X of the Euclidean n-dimensional space $\mathbb{R}^n$ and a mapping $F : X \to \mathbb{R}^n$, the *variational inequality*, denoted $VI(X, F)$, is to find a vector $x \in X$ such that

$$(w - x)^\top F(x) \geq 0, \qquad \forall w \in X.$$

Using the proposition 2.2 of [6], the risk-averse problem forming the GCP is transformed into the following Variational Inequality (VI).

$$VI(X, F)$$

where

$$X = \left\{ \begin{pmatrix} y_g(\omega) \\ r_g(\omega) \\ u_g \\ r_g^{DA} \\ d(\omega) \\ d_k^R(\omega) \\ d_k^{R,DA} \end{pmatrix} \geq \mathbf{0} \;\middle|\; \begin{array}{c} P_g^{\min} \cdot u_g \leq y_g(\omega) \\ y_g(\omega) + r_g(\omega) \leq P_g^{\max}(\omega) \cdot u_g \\ r_g(\omega) \leq R_g \cdot u_g \\ u_g \leq 1 \\ r_g^{\mathrm{DA}} \leq R_g \cdot u_g \\ d(\omega) \leq D(\omega) \\ d_k^R(\omega) \leq D_k^R \\ d_k^{R,\mathrm{DA}} \leq D_k^R \\ d(\omega) \leq \sum_{g \in \mathcal{G}} y_g(\omega) \\ \sum_{k \in \mathcal{K}} \left(d_k^R(\omega) - d_k^{R,\mathrm{DA}} \right) \leq \sum_{g \in G} \left(r_g(\omega) - r_g^{\mathrm{DA}} \right) \\ \sum_{k \in \mathcal{K}} d_k^{R,\mathrm{DA}}(\omega) \leq \sum_{g \in G} r_g^{\mathrm{DA}} \end{array} \right\}$$

and

$$F \begin{pmatrix} y_g(\omega) \\ r_g(\omega) \\ u_g \\ r_g^{DA} \\ d(\omega) \\ d_k^R(\omega) \\ d_k^{R,DA} \end{pmatrix} = \begin{pmatrix} C_g(\omega) \\ 0 \\ K_g \\ 0 \\ -V \\ -V_k^R \\ -V_k^{R,DA} + V_k^R \end{pmatrix}.$$

Because the jacobian of F is symmetric and all the other conditions given in the Theorem 1.3.1 of [5] are respected, the variational inequality VI(X,F) can be solved as a unique optimization problem.

2.5.3 Equivalent problem

The unique optimization problem in the risk-neutral case is given hereunder.

Definition 6: Physical Problem

$$\max_{y,r,r^{DA},u,d,d^R,d^{R,DA}} \quad V\cdot\mathbb{E}\Big[d(\omega)\Big]+\sum_{k\in\mathcal{K}}\Big((V_k^{R,\mathrm{DA}}-V_k^R)\cdot d_k^{R,\mathrm{DA}}+V_k^R\cdot\mathbb{E}\Big[d_k^R(\omega)\Big]\Big)$$

$$-\sum_{g\in G}\Big(\mathbb{E}\Big[C_g(\omega)\cdot y_g(\omega)\Big]+K_g\cdot u_g\Big) \tag{2.86}$$

$$\Big(\bar{\mu}_g^{\min}(\omega)\Big):\quad y_g(\omega)-P_g^{\min}\cdot u_g\geq 0 \tag{2.87}$$

$$\Big(\bar{\mu}_g^{\max}(\omega)\Big):\quad P_g^{\max}(\omega)\cdot u_g-y_g(\omega)-r_g(\omega)\geq 0 \tag{2.88}$$

$$\Big(\bar{\xi}_g(\omega)\Big):\quad R_g\cdot u_g-r_g(\omega)\geq 0 \tag{2.89}$$

$$(\alpha_g):\quad 1-u_g\geq 0 \tag{2.90}$$

$$\Big(\xi_g^{\mathrm{DA}}\Big):\quad R_g\cdot u_g-r_g^{\mathrm{DA}}\geq 0 \tag{2.91}$$

$$\Big(\bar{\nu}(\omega)\Big):\quad D(\omega)-d(\omega)\geq 0 \tag{2.92}$$

$$\Big(\bar{\pi}(\omega)\Big):\quad y_g(\omega)-\sum_{g\in G}d(\omega)\geq 0 \tag{2.93}$$

$$\Big(\bar{\nu}_k^R(\omega)\Big):\quad D_k^R-d_k^R(\omega)\geq 0 \tag{2.94}$$

$$\Big(\nu_k^{R,\mathrm{DA}}\Big):\quad D_k^R-d_k^{R,\mathrm{DA}}\geq 0 \tag{2.95}$$

$$\Big(\bar{\lambda}(\omega)\Big):\quad \sum_{g\in G}\Big(r_g(\omega)-r_g^{\mathrm{DA}}\Big)-\sum_{k\in\mathcal{K}}\Big(d_k^R(\omega)-d_k^{R,\mathrm{DA}}\Big)\geq 0 \tag{2.96}$$

$$\Big(\lambda^{\mathrm{DA}}\Big):\quad \sum_{g\in G}r_g^{\mathrm{DA}}-\sum_{k\in\mathcal{K}}d_k^{R,\mathrm{DA}}\geq 0 \tag{2.97}$$

$$0\leq y_g(\omega),\ r_g^{\mathrm{DA}},\ r_g(\omega),\ u_g,\ d(\omega),\ d_k^{R,\mathrm{DA}},\ d_k^R(\omega) \tag{2.98}$$

Interpretation. This problem corresponds to the maximization of the total welfare. The objective function is the sum of the valuation of the electricity by the consumers and the valuation of the reserves by the system operator minus the cost to produce the electricity by the producers.

The surplus of the market does not appear in the objective function. For one simple reason: it is equal to zero. Nevertheless, the price of the different products corresponds to the dual variables of the constraints which inspect that the offer meets the demand. The notation $\overline{\pi}(\omega)$ is equivalent to $p(\omega)\cdot\pi(\omega)$.

All the constraints of the individual agents ended up in the constraints of the unique optimization problem.

3
Convex Hull representation

In the previous chapter, to avoid further complicating the problem, only one time period was considered. This simplification was good enough as long as the time periods were independent. Here, a little more realism is restored, the new problem takes place over several periods of time which are not independent. Indeed, the power plants can not be turned ON and OFF instantly, it is necessary to schedule the ON-OFF status in advance while taking into account the different constraints that link them together from one time period to the next.

The status ON-OFF of the power plants during the time periods should be described with binary variables. But adding binary variables to our problem would make it far too complicated to solve, not to say unsolvable. To circumvent this, the binary variables are relaxed and based on [7], the Convex Hull of the power plants is described.

3.1 Unit commitment

Compared to the problems presented in the second section, only the optimization problem of the producers is modified. But the production changes of the producers will impact the whole energy market. These improvements are critical to better represent the electricity market.

To describe the unit commitment of the power plant g during the time period h, three variables are used.

- $u_{g,h}$ describes the ON-OFF status: it is equal to 0 if the status is OFF and it is equal to 1 if the status is ON.
- $v_{g,h}$ indicates a start-up decision: it is equal to 1 when the power-plant is started and it is equal to 0 otherwise.
- $z_{g,h}$ indicates a shut-down decision: it is equal to 1 when the power-plant is shut-down and it is equal to 0 otherwise.

The constraints that describes the status ON-OFF and that link the power plants from one time period to the next are the following. Only the most important constraints are described, but realism can further be improved as described in Chapter 7 of [8].

- **Relaxing** the binary variables. The problem would be unsolvable with binary variables, but the side effect of this relaxation is that the results do not always have a physical justification. When $0 < u_{g,h} < 1$ it means that the power plant g is partly ON during the time period h.

$$0 \leq u_{g,h} \leq 1 \tag{3.1}$$

$$0 \leq v_{g,h} \leq 1 \tag{3.2}$$

$$0 \leq z_{g,h} \leq 1 \tag{3.3}$$

- **State transitions** between the time periods.

$$u_{g,h} = u_{g,h-1} + v_{g,h} - z_{g,h} \tag{3.4}$$

- **Minimum down-time**. The power plants need some time to cool down once they have been shut down.

$$\sum_{t=h-DT_g+1}^{h} z_{g,t} \leq 1 - u_{g,h} \tag{3.5}$$

- **Minimum up-time**. Once the power plants have been started, they must continue to operate for a minimum period of time before it can shut down.

$$\sum_{t=h-UT_g+1}^{h} v_{g,t} \leq u_{g,h} \tag{3.6}$$

In fact, one can describe the same problem with only two variables u and v. Removing z, does not change the solution. Therefore, for the rest of the master thesis, one will only work with u and v.

- The state transition constraint (3.4) can be transformed into the following inequality.

$$u_{g,h} - u_{g,h-1} \leq v_{g,h} \tag{3.7}$$

  Because starting the power plants has a cost $v_{g,h}$ will always be equal to $u_{g,h} - u_{g,h-1}$ or 0. The variables u and v obtained are the same and z could be found again if wanted.

- The constraint (3.5) can be rewritten using the equality (3.4).

$$\sum_{t=h-DT_g+1}^{h} v_{g,t} \leq 1 - u_{g,h-DT_g} \tag{3.8}$$

Finally, the convex hull set of feasible commitment and dispatch decisions for the power plant g is described by:

$$\mathcal{U}_g = \{(\mathbf{u}_g, \mathbf{v}_g) \in \mathbb{R}^{24} \times \mathbb{R}^{24} | (3.1), (3.2), (3.7), (3.8), (3.6)\} \tag{3.9}$$

All the problems presented in this master thesis can easily be adapted to the addition of those new constraints. The necessary modifications are further discussed in the appendix.

What about the transition from one day to the next? Since the algorithm solves the problem over a period of one day, how to ensure that the status ON-OFF of each power plant is the optimal one when the day ends?

Taking into account all these potential problems, the choice was made to solve the problem while considering that the same day was repeated constantly. This means that the boundary conditions are wrapped around each other.

4 Heuristic Algorithm

In the second chapter, the short-term electricity market in a risky environment was described; nonetheless, the resolution method offered so far is only valid for risk-neutral agents. This chapter addresses this problem, an algorithm to solve the problem with risk-averse agents is presented. The algorithm presented is heuristic, one has no proof of its convergence, but the results obtained are promising.

The algorithm divides the problem into two parts: the *Physical problem* and the *Financial problem*. The physical part solves the main problem for fixed probabilities and the financial part computes the risk-averse probabilities based on the payoffs. Each part is solved in turn using the results of the other part. This process is repeated as many times as necessary until convergence. Finally, the financial part should converge towards the risk-averse probabilities and the physical part should give the much-awaited answers.

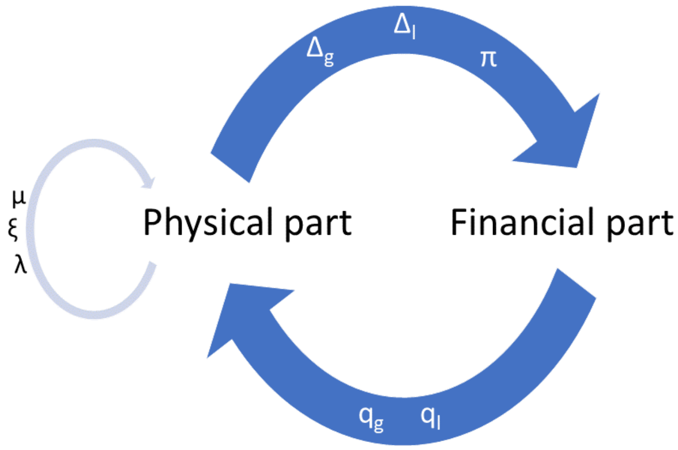

Figure 4.1: Diagram explaining how the heuristic algorithm works

In the diagram above, one can see three arrows. The two big arrows represent the results send from one problem to another. The physical part send the stochastic payoffs Δ_g, Δ_l and the price of the electricity in real-time $\pi(\omega)$; and the financial part send the risk-averse probabilities $q_g(\omega)$ and $q_l(\omega)$. The finest arrow represents the iteration to solve the physical problem for some given probabilities. The variables memorized from one iteration to the next are $\mu_g^{\min}(\omega), \mu_g^{\max}(\omega), \xi_g(\omega)$ and $\lambda(\omega)$.

4.1 Physical problem

The physical problem aims to obtain the solution to the equilibrium conditions given in the definition 5 for some fixed probabilities. One can first observe that the equilibrium conditions (2.74), (2.77) and (2.85) are useless for fixed probabilities, consequently these equations will not be considered in the physical problem. These three equilibrium conditions are related to electricity futures and are used to mitigate the financial risk. It is, therefore, logical to consider these equations in the financial problem.

4.1.1 Equilibrium conditions

As described in section 2.5, it is only possible to transform the equilibrium conditions (given in the definition 5) into a unique optimization problem (such as the one given in the definition 6) if the probabilities are the same for each producer and for the system operator because of three problematic equilibrium conditions (2.70), (2.72) and (2.80). To thwart this issue, one will use the same probabilities for each agent.

The probabilities used to solve the physical problem are denoted $\hat{q}(\cdot) : \mathbb{R} \to \Omega$. It is the mean risk-averse probability of the producers, its value has been chosen heuristically to improve the convergence of the physical problem.

$$\hat{q}(\omega) = \frac{1}{\#\mathcal{G}} \sum_{g \in \mathcal{G}} q_g(\omega) \tag{4.1}$$

The three equilibrium conditions (2.70), (2.72) and (2.80) must undergo a modification to comply with the new probabilities used.

$$0 \leq u_g \;\perp\; K_g + \alpha_g - R_g \cdot \xi_g^{\mathrm{DA}} + \mathbb{E}_{\hat{Q}}\Big[P^{\min} \cdot \mu_g^{\min}(\omega) - P_g^{\max} \cdot \mu_g^{\max}(\omega) - R_g \cdot \xi_g(\omega)\Big] + \theta_g \geq 0 \tag{4.2}$$

$$0 \leq r_g^{\mathrm{DA}} \;\perp -\lambda^{\mathrm{DA}} + \xi_g^{\mathrm{DA}} + \mathbb{E}_{\hat{Q}}\Big[\lambda(\omega)\Big] + \eta_g \geq 0 \tag{4.3}$$

$$0 \leq d_k^{R,\mathrm{DA}} \;\perp\; \lambda^{\mathrm{DA}} - \mathbb{E}_{\hat{Q}}\Big[\lambda(\omega)\Big] - \iota - (V_k^{R,\mathrm{DA}} - V_k^{R,\mathrm{DA}}) + \nu_k^{R,\mathrm{DA}} \geq 0 \tag{4.4}$$

with

$$\theta_g = \mathbb{E}_{Q_g^* - \hat{Q}}\Big[P^{\min} \cdot \mu_g^{*,\min}(\omega) - P_g^{\max} \cdot \mu_g^{*,\max}(\omega) - R_g \cdot \xi_g^*(\omega)\Big] \tag{4.5}$$

$$\eta_g = \mathbb{E}_{Q_g^* - \hat{Q}}\Big[\lambda^*(\omega)\Big] \tag{4.6}$$

$$\iota = \mathbb{E}_{P - \hat{Q}}\Big[\lambda^*(\omega)\Big] \tag{4.7}$$

and where a variable with an asterisk as superscript corresponds to the value of that variable at the previous iteration.

The values of θ_g, η_g and ι change after each iteration, but during an iteration they are constant. Those values are added to allow us solve the physical problem for the given risk-averse probabilities q^* and not for $\hat{q}$.

4.1.2 Variational Inequality problem

Using the proposition 2.2 of [6], the problem forming the GCP is transformed into the following Variational Inequality (VI). This time around, one will consider the dual problem because one would like to control some of these variables from one iteration to the next.

$$VI(Y, G)$$

where

$$Y = \left\{ \begin{pmatrix} \mu_g^{\min}(\omega) \\ \mu_g^{\max}(\omega) \\ \alpha_g \\ \xi_g(\omega) \\ \xi_g^{DA}(\omega) \\ \nu(\omega) \\ \nu_k^R(\omega) \\ \nu_k^{R,DA} \\ \pi(\omega) \\ \lambda(\omega) \\ \lambda^{DA} \end{pmatrix} \geq \mathbf{0} \;\middle|\; \begin{array}{c} -\pi(\omega) - \mu_g^{\min}(\omega) + \mu_g^{\max}(\omega) \geq -C_g(\omega) \\ -\lambda(\omega) + \mu_g^{\max}(\omega) + \xi_g(\omega) \geq 0 \\ \alpha_g - R_g \cdot \xi_g^{DA} + \mathbb{E}_{\hat{Q}}[\,\cdots] \geq -K_g - \theta_g \\ -\lambda^{\mathrm{DA}} + \xi_g^{\mathrm{DA}} + \mathbb{E}_{\hat{Q}}\big[\lambda(\omega)\big] \geq -\eta_g \\ \pi(\omega) + \nu(\omega) \geq V \\ \lambda(\omega) + \nu_k^R(\omega) \geq V_k^R \\ \lambda^{\mathrm{DA}} - \mathbb{E}_{\hat{Q}}\big[\lambda(\omega)\big] + \nu_k^{R,\mathrm{DA}} \geq V_k^{R,\mathrm{DA}} - V_k^R + \iota \end{array} \right\}$$

and

$$G \begin{pmatrix} \mu_g^{\min}(\omega) \\ \mu_g^{\max}(\omega) \\ \alpha_g \\ \xi_g(\omega) \\ \xi_g^{DA}(\omega) \\ \nu(\omega) \\ \nu_k^R(\omega) \\ \nu_k^{R,DA} \\ \pi(\omega) \\ \lambda(\omega) \\ \lambda^{DA} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ D(\omega) \\ D_k^R \\ D_k^R \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Because the jacobian of G is symmetric and all the other conditions given in the Theorem 1.3.1 of [5] are respected, the variational inequality VI(Y,G) can be solved as a unique optimization problem.

4.1.3 Equivalent problem

To this unique optimization problem, one will add one term in the objective function to slow down the speed of the modifications from one iteration to the next. The additional term is the following:

$$\kappa_{phys} \cdot \sum_{\omega\in\Omega} \Bigg\{ \sum_{g\in\mathcal{G}} \bigg[\Big(\mu_g^{\min}(\omega) - \mu_g^{*,\min}(\omega)\Big)^2 + \Big(\mu_g^{\max}(\omega) - \mu_g^{*,\max}(\omega)\Big)^2 + \Big(\xi_g(\omega) - \xi_g^*(\omega)\Big)^2 \bigg] + \Big(\lambda(\omega) - \lambda^*(\omega)\Big)^2 \Bigg\} \tag{4.8}$$

If μ, ξ and λ converge then our heuristic algorithm gives the much waited solutions for the given risk-averse probabilities q^*.

Definition 7: Dual formulation of the physical problem

$$\min_{\alpha,\mu,\pi,\nu,\lambda,\xi} \quad \mathbb{E}_{\hat{Q}}\Big[D(\omega)\cdot\nu(\omega)\Big] + \sum_{k\in\mathcal{K}} \Big\{ D_k^R \cdot \nu_k^{R,\mathrm{DA}} + \mathbb{E}_{\hat{Q}}\Big[D_k^R \cdot \nu_k^R(\omega)\Big]\Big\} + \sum_{g\in G} \alpha_g$$
$$+ \kappa_{phys} \cdot \sum_{\omega\in\Omega} \Bigg\{ \sum_{g\in\mathcal{G}} \bigg[\Big(\mu_g^{\min}(\omega) - \mu_g^{*,\min}(\omega)\Big)^2 + \Big(\mu_g^{\max}(\omega) - \mu_g^{*,\max}(\omega)\Big)^2 + \Big(\xi_g(\omega) - \xi_g^*(\omega)\Big)^2 \bigg] + \Big(\lambda(\omega) - \lambda^*(\omega)\Big)^2 \Bigg\} \tag{4.9}$$

$$\Big(\hat{y}_g(\omega)\Big): \quad -\pi(\omega) - \mu_g^{\min}(\omega) + \mu_g^{\max}(\omega) \geq -C_g(\omega) \tag{4.10}$$
$$\Big(\hat{r}_g(\omega)\Big): \quad \xi_g(\omega) - \lambda(\omega) + \mu_g^{\max}(\omega) \geq 0 \tag{4.11}$$
$$(u_g): \quad \mathbb{E}_{\hat{Q}}[\,\cdots] - R_g \cdot \xi_g^{\mathrm{DA}} + \alpha_g \geq -K_g - \theta_g \tag{4.12}$$
$$\Big(r_g^{\mathrm{DA}}\Big): \quad \xi_g^{\mathrm{DA}} + \mathbb{E}_{\hat{Q}}\Big[\lambda(\omega)\Big] - \lambda^{\mathrm{DA}} \geq -\eta_g \tag{4.13}$$
$$\Big(\hat{d}(\omega)\Big): \quad \nu(\omega) + \pi(\omega) \geq V \tag{4.14}$$
$$\Big(\hat{d}_k^R(\omega)\Big): \quad \nu_k^R(\omega) + \lambda(\omega) \geq V_k^R \tag{4.15}$$
$$\Big(d_k^{R,\mathrm{DA}}\Big): \quad \nu_k^{R,\mathrm{DA}} - \mathbb{E}_{\hat{Q}}\Big[\lambda(\omega)\Big] + \lambda^{\mathrm{DA}} \geq V_k^{R,\mathrm{DA}} - V_k^R + \iota \tag{4.16}$$
$$0 \leq \alpha_g, \mu_g^{\min}(\omega), \mu_g^{\max}(\omega), \xi_g(\omega), \xi_g^{\mathrm{DA}}, \nu(\omega), \nu_k^R(\omega), \nu_k^{R,\mathrm{DA}}, \pi(\omega), \lambda(\omega), \lambda^{DA} \tag{4.17}$$

where

$$\theta_g = \mathbb{E}_{Q_g^* - \hat{Q}}\Big[P_g^{\min} \cdot \mu_g^{*,\min}(\omega) - P_g^{\max}(\omega)\cdot\mu_g^{*,\max}(\omega) - R_g \cdot \xi_g^*(\omega)\Big] \tag{4.18}$$
$$\eta_g = \mathbb{E}_{Q_g^* - \hat{Q}}\Big[\lambda^*(\omega)\Big] \tag{4.19}$$
$$\iota = \mathbb{E}_{P - \hat{Q}}\Big[\lambda^*(\omega)\Big]. \tag{4.20}$$

4.2 Financial problem

While the physical problem solves the main problem for some probabilities given, the financial problem calculates the risk-averse probabilities and the quantity of electricity futures traded. As described in section 2.2, the risk-measure used in this master thesis is the good-deal pricing. To hedge against the financial risk the producers and the consumers can to trade electricity futures on the day-ahead market.

The risk-averse probabilities and the amount of electricity futures traded by the two risk-averse agents, namely the producers and the consumers, are computed based on the payoffs obtained in the physical problem. Their payoffs can be divided into two parts: one part is deterministic and the other is stochastic. For the financial problem, only the stochastic part of the payoffs matters because of the translation invariance of a coherent risk-measure:

$$\mathcal{R}\Big(a + b(\omega)\Big) = a + \mathcal{R}\Big(b(\omega)\Big). \tag{4.21}$$

The stochastic payoffs denoted Δ_g for the producers and Δ_l for the producers are

$$\Delta_g(\omega) = \Big(\pi^*(\omega) - C_g(\omega)\Big) \cdot y_g^*(\omega) + \lambda^*(\omega) \cdot \Big(r_g^*(\omega) - r_g^{*,\mathrm{DA}}\Big), \tag{4.22}$$

$$\Delta_l(\omega) = \Big(V - \pi^*(\omega)\Big) \cdot d^*(\omega). \tag{4.23}$$

The problem of the producers

$$\max_{y_g^{DA}} \ \mathcal{R}_g\Big(\Delta_g(\omega) - \pi^*(\omega) \cdot y_g^{DA}\Big) + \pi^{DA} \cdot y_g^{DA}, \tag{4.24}$$

the problem of the consumers

$$\max_{d^{DA}} \ \mathcal{R}_g\Big(\Delta_l(\omega) + \pi^*(\omega) \cdot d^{DA}\Big) - \pi^{DA} \cdot d^{DA}, \tag{4.25}$$

and the problem of the market agent

$$\max_{\pi^{DA}} \ \Big(\sum_{g \in \mathcal{G}} y_g^{DA} - d^{DA}\Big) \cdot \pi^{DA} \tag{4.26}$$

can be transformed into a single optimization problem using [9].

This financial problem concludes the resolution of our entire problem if it was calculated for the right stochastic payoffs. And the stochastic payoffs given by the physical part were the good one if they were computed for the right risk-averse probabilities. In conclusion, if the probabilities computed in this iteration are close enough to the probabilities computed in the previous iteration, then one is done and otherwise one would need to calculate the payoffs again.

The same trick was used in the physical part. To slow down the speed of the modifications from one iteration to the next, one additional term was added. The additional term is the following:

$$\kappa_{fin} \cdot \sum \Big(q_i(\omega) - q_i^*(\omega)\Big)^2. \tag{4.27}$$

Finally, the heuristic financial problem is obtained.

Definition 8: Formulation of the financial problem

$$\min_{q_i(\omega),\pi^{\text{DA}}} \quad \sum_{i\in\{G\cup L\}} \mathbb{E}_{Q_i}\Big[\Delta_i\Big] + \kappa_{fin}\cdot \sum_{i\in\{G\cup L\}} \Big(q_i(\omega) - q_i^*(\omega)\Big)^2 \tag{4.28}$$

$$\text{s.t.} \quad \sum_{\omega\in\Omega} q_i(\omega) = 1 \tag{4.29}$$

$$(d^{\text{DA}}): \quad -\mathbb{E}_{Q_l}\Big[\pi^*(\omega)\Big] + \pi^{\text{DA}} = 0 \tag{4.30}$$

$$(y_g^{\text{DA}}): \quad \mathbb{E}_{Q_g}\Big[\pi^*(\omega)\Big] - \pi^{\text{DA}} = 0 \tag{4.31}$$

$$\sum_{\omega\in\Omega} \frac{q_i(\omega)^2}{p(\omega)} \leq 1 + \overline{SR}^2 \tag{4.32}$$

$$0 \leq q_i(\omega) \tag{4.33}$$

The dual variables of the constraint (4.31) give the electricity traded on the day-ahead market.

4.3 Algorithm

The algorithm is heuristic and one has no proof of its convergence. An algorithm of the same nature was used in the master thesis [10] to solve the Risky Competitive Capacity Expansion Problem described in [11].

First step. The risk-neutral problem presented in the definition 6 is solved. The payoff of the producers Δ_g, the payoff of the consumers Δ_l and the price of electricity $\pi^*(\omega)$ are recorded.

Second step. Based on the results obtained in the first step and with $q_i^*(\omega) = p(\omega)$, the financial problem is solved. Once resolved, the probabilities are updated:

$$q_i^*(\omega) = q_i(\omega), \tag{4.34}$$

$$\hat{q}(\omega) = \frac{1}{\#\mathcal{G}} \sum_{g\in\mathcal{G}} q_g(\omega). \tag{4.35}$$

Third step. The physical problem presented in the definition 7 is solved. For the first iteration $k_{phy} = 0$. Afterwards $k_{phy} > 0$ and after each iteration some variables are recorded:

$$\Big(\mu_g^{*,\min}(\omega), \mu_g^{*,\max}(\omega), \xi_g^*(\omega), \lambda^*(\omega)\Big) \leftarrow \Big(\mu_g^{\min}(\omega), \mu_g^{\max}(\omega), \xi_g(\omega), \lambda(\omega)\Big). \tag{4.36}$$

This physical problem is solved over and again until one obtains convergence. One considers that the physical problem has converged when the additional term is significantly lower than the objective value of the problem.

$$f_{phy} = \frac{1}{obj_{phy}} \cdot \kappa_{phy} \cdot \sum_{\omega\in\Omega} \left\{ \sum_{g\in\mathcal{G}} \left[\Big(\mu_g^{\min}(\omega) - \mu_g^{*,\min}(\omega)\Big)^2 + \Big(\mu_g^{\max}(\omega) - \mu_g^{*,\max}(\omega)\Big)^2 \right.\right.$$
$$\left.\left. + \Big(\xi_g(\omega) - \xi_g^*(\omega)\Big)^2 \right] + \Big(\lambda(\omega) - \lambda^*(\omega)\Big)^2 \right\} \tag{4.37}$$

Once the physical problem has converged, Δ_g, Δ_l and $\pi^*(\omega)$ are updated.

Fourth step. The financial problem is solved. If the additional term is significantly lower than the objective value then it is the end.

$$f_{fin} = \frac{1}{obj_{fin}} \cdot \kappa_{fin} \cdot \sum_{\omega\in\Omega} \Big(q_i(\omega) - q_i^*(\omega)\Big)^2 \tag{4.38}$$

However, if the additional term is not significantly lower than the objective value, then the probabilities are updated.

$$q_i^*(\omega) = q_i(\omega), \tag{4.39}$$
$$\hat{q}(\omega) = \frac{1}{\#\mathcal{G}} \sum_{g\in\mathcal{G}} q_g(\omega). \tag{4.40}$$

And one starts again from the third step.

To recapitulate, the pseudo-code of the heuristic algorithm is given hereunder.

Algorithm 1: Dual of the physical problem + Financial problem

initialization: $q_i(\omega) = p(\omega)$
while $f_{fin} > \epsilon$ **do**
  while $f_{phy} > \epsilon_{phy}$ **do**
    solve Dual of the physical problem (definition 7)
      with $\kappa_{phy} = 0$ for the first iteration
    $\Big(\mu_{g,*}^{\min}(\omega), \mu_{g,*}^{\max}(\omega), \xi_{g,*}(\omega), \lambda_*(\omega)\Big) \leftarrow \Big(\mu_g^{\min}(\omega), \mu_g^{\max}(\omega), \xi_g(\omega), \lambda(\omega)\Big)$
    $\leftarrow$ update of f_{phy}
  end
  $\leftarrow$ update of Δ_g, Δ_l and $\pi^*(\omega)$
  solve Financial problem (definition 8)
  $\leftarrow$ update of the risk-averse probabilities $q_i^*(\omega)$
  $\leftarrow$ update of f_{fin}
end

5 Application

Our heuristic method of resolution is applied to two exercises. For the first exercise, the time periods are considered as independent. For the second exercise, the time periods are dependent. These exercises were created to observe the influence of risk-aversion on the unit commitment problem and have been intentionally simplified to facilitate the observation.

5.1 Single time period

5.1.1 Overview of the problem

Description of the consumption of electricity

The demand for electricity is stochastic and is aggregated into one load, $D(\omega)$. The consumption of electricity, $d(\omega)$, is equal to demand as long as the price of electricity, $\pi(\omega)$, is smaller than the value of lost load **VOLL** $= 200$ [€/MWh].

In fact, in this exercise, the demand for electricity corresponds to the demand of the consumers minus a random production of the wind and solar power. Indeed, thanks to their marginal costs close to zero, the wind and solar power will almost always produce electricity at full capacity.

One has selected 100 scenarios for the demand $\Omega = \{1, 2, \cdots, 100\}$ and each scenario has an equal probability $p(\omega) = 0.01$. The demand follows a normal distribution with a mean of $\mu = 10$ [MW] and a variance of $\sigma^2 = 0.4$. For $j = 1, 2, ..., 100$, one has:

$$D(\omega = j) = F_d^{-1}\left(\frac{j}{N}\right), \tag{5.1}$$

where F_d is the cumulative distribution function of $D \sim \mathcal{N}(10, 0.4)$. The probability density function and the 101-quantiles are given on the next page.

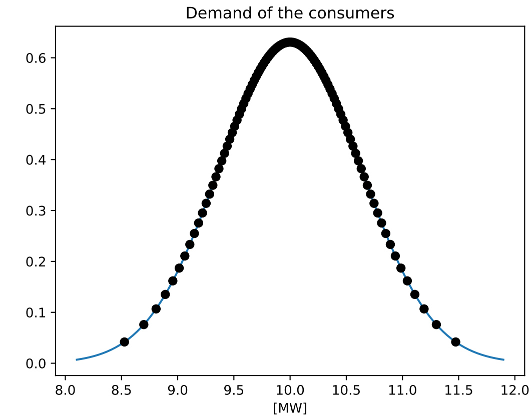

Figure 5.1: The maximum demand for electricity. The line in blue is the probability density function of the production $\mathcal{N}(10, 0.4)$ and the black dots are the 101-quantiles.

Description of the consumption of reserves

In the table hereunder, the demand of the system operator D_k^R, the valuation of reserves in real-time V_k^R and the valuation of reserves in the day-ahead $V_k^{R,DA}$ are given for every load k. One chose the valuation of the reserves in real-time such that every technology prefers to sell electricity rather than reserves when the system is tight ($V_3^R > \mathbf{VOLL} - C_g$).

k	D_k^R	V_k^R	$V_k^{R,\mathrm{DA}}$
1	1	100	120
2	0.75	75	90
3	0.5	50	60

The plot below clarifies the meaning of the loads. This plot shows the offer of the system operator for reserves in the day-ahead.

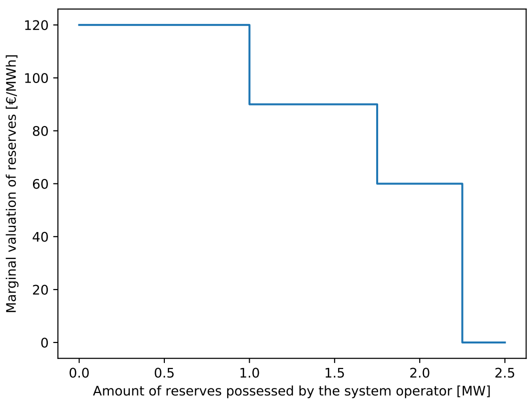

Figure 5.2: The valuation of the reserves in the day-ahead by the system operator.

Description of the production

Three technologies are considered: baseload, midload and peakload. The characteristics of those technologies are described in the table below. The marginal cost of production C_g, the no-load cost K_g, the minimum production P_g^{min}, the maximum production P_g^{max}, and the maximum amount of reserves R_g are given.

Technology	C_g [€/MWh]	K_g [€/MW]	P_g^{min} [MW]	P_g^{max} [MW]	R_g [MW]
baseload	30	200	4	6	1.0
midload	60	100	3	6	1.5
peakload	90	50	0.5	2	1.5

The graphic below explains why the technologies are named: baseload, midload and peakload.

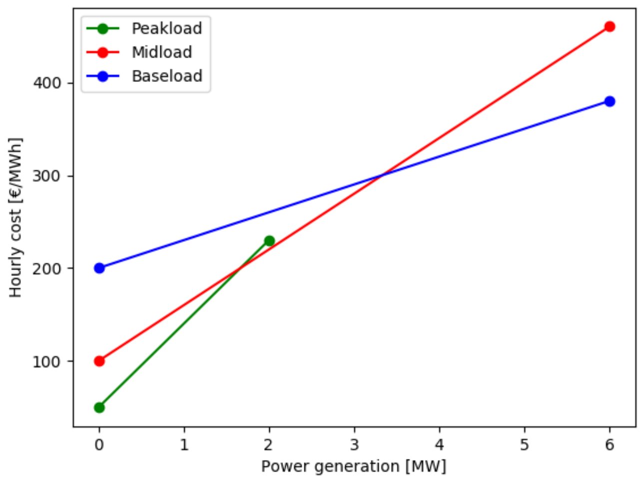

Figure 5.3: Hourly cost depending on the power generation of the three technologies.

Description of the risk-aversion

On the one hand, the agent wants to maximize its profit and on the other hand, it is afraid of financial losses. These two goals are contradictory because by being risk-averse an agent reduces its expected profit.

The risk-aversion considered in this problem is rather low because the operation is repeated every day. Due to this repetition, the agent accepts a greater risk to not compromise too much its expected profit. The agent knows that even if he loses money during one day, he can regain it the next day. In this master thesis, the value of the Sharpe-ratio is equal to 0.03 and the risk-aversion is the same for the consumers and the producers.

5.1.2 Analysis of the results

Day-ahead results

Unit commitment The status ON-OFF of power plants is influenced by the risk-aversion. As explained in the small example in subsection 2.2.1., for the same payoff a risk-averse agent could prefer not to start-up its power plant. In this example, the peakload decides not to start as much capacity as in the risk-neutral case because it would not increase the certainty equivalent of its profit anymore.

α_g, the dual variable associated with the constraint $u_g \leq 1$, demonstrate this. It is equal to 0 for the peakload because increasing the peakload capacities would not increase the certainty equivalent of its profit. It also shows that one would prefer to start-up a new baseload capacity over a midload capacity.

	Risk-Neutral		SR = 0.03	
Technology	u_g	α_g	u_g	α_g
baseload	1	61.6	1	63.35
midload	1	20.3	1	21.47
peakload	0.258	0	0.242	0

However, one can note that the difference between the amount of capacity started in the risk-neutral case and the risk-averse case is small. The trade of electricity futures contracts and reserves in the day-ahead is the reason for this small gap. Thanks to these products, the agents can obtain a more stable payoff and the decision to switch ON or OFF the power station is only slightly affected.

Day-ahead reserves Although the producers meet the entire demand of the system operator for day-ahead reserves in both cases (the risk-neutral and the risk-averse), these two situations are different. In the risk-neutral case, the reserves sold in the day-ahead and the reserves sold in real-time have the same value $\lambda^{DA} = \mathbb{E}\Big[\lambda(\omega)\Big] = 38.8$ [€/MWh]. While in the risk-averse case, some producers prefer to sell reserves in the day-ahead rather than in real-time to mitigate the financial risk $38.7 = \lambda^{DA} \leq \mathbb{E}\Big[\lambda(\omega)\Big] = 39.4$ [€/MWh].

	Risk-Neutral		SR = 0.03	
Technology	r_g^{DA} [MW]	ξ^{DA}	r_g^{DA} [MW]	ξ^{DA}
baseload	1	0	0.387	0
midload	0.862	0	1.5	0.041
peakload	0.388	0	0.363	0.097

Electricity futures contracts In the risk-averse case, the agents can decide to trade electricity futures contracts to mitigate the financial risk.

	SR = 0.03
Agent	y_g^{DA} and (d^{DA}) [MW]
baseload	5.705
midload	4.207
peakload	0.116
consumer	(10.028)

The results transcribed in the table above show that the power plants sell approximately their mean production and the consumers buy approximately their mean demand on the day-ahead market. The price of the futures contracts in the day-ahead $\pi^{DA} = 73.9$ [€/MWh] is lower than the expected price of electricity in real-time $\mathbb{E}\Big[\pi(\omega)\Big] = 74.2$ [€/MWh]. Meaning that the producers accept a lower price in the day-ahead and thus a lower expected payoff to get a less variable payoff.
One could think that the consumers are the winners in the risk-averse case. That, however, would mean forgetting the modifications in the unit commitment decisions. In the risk-neutral case, thanks to the greater amount of capacity turned on, the expected price of electricity is lower $\mathbb{E}\Big[\pi(\omega)\Big] = 73.6$ [€/MWh].

Real-time results

One will explain the three following graphs: (i) the production of electricity, (ii) the reserves and (iii) the price of these products at the same time because they are interrelated. Explanations are given by grouping similar scenarios together.

- <u>In the scenarios 1-2</u> the demand for electricity of the consumers is very low. Since the amount of capacity turned ON was chosen before knowing the scenario, one ends up with too much capacities. Both the midload and the peakload are producing electricity at minimum capacity and their marginal costs of production are higher than the price of electricity. The price of electricity is equal to the marginal cost of production of the baseload $\pi(\omega) = C_{\text{baseload}} = 30$ [€/MWh]. The price of reserves is equal to zero $\lambda(\omega) = 0$ [€/MWh] because of the abundance of supply and because the marginal cost is zero.

- <u>In the scenarios 3-66</u> there is no more an abundance of reserves. To meet the full demand for electricity and reserves, the midload must increase its production. The price of electricity is now fixed by the marginal cost of production of the midload $\pi(\omega) = 60$ [€/MWh]. The price of reserves increases to $\lambda(\omega) = 30$ [€/MWh] because the profit of the baseload for selling more electricity or more reserves must be the same. The price difference is equal to the marginal cost of production of the baseload $\pi(\omega) - \lambda(\omega) = C_{\text{baseload}}$.

- The scenario 67 is a bit particular. The sum of the demand for electricity and reserves exactly matches the maximum available capacity of the baseload and the midload while the peakload is producing at minimum capacity but is selling reserves at maximum capacity.
- In the scenarios 68-84 one can not meet the full demand for electricity and reserves if the peakload does not increase its production. However, to maximize the total welfare, a shortage of the third load of reserves is preferred over an increase in the production of the peakload. The price of reserves is fixed by the valuation of the third load $\lambda(\omega) = V_3^R = 50$ [€/MWh] and the price of electricity is $\pi(\omega) = \lambda(\omega) + C_{\text{baseload}} = 80$ [€/MWh]. The baseload decreases its sale of reserves and produces electricity instead.
- In the scenarios 85 - 89 the baseload produces electricity at full capacity and stops selling reserves. The midload must now decide between selling electricity or reserves. Consequently, the price difference between the electricity and the reserves is equal to the marginal cost of production of the midload $\pi(\omega) - \lambda(\omega) = C_{\text{midload}}$. $\pi(\omega) = 110$ [€/MWh] and $\lambda(\omega) = 60$ [€/MWh].
- In the scenarios 90 - 100 one starts to have a shortage in the second load of reserves $D_2^{R,DA} > d_2^{R,DA}$. The price of reserves increases to the valuation of the second load $\lambda(\omega) = V_2^R = 75$ [€/MWh] and consequently the price of the electricity increases $\pi(\omega) = 135$ [€/MWh].

Production of electricity On the graph below one can observe the production of electricity of the different technologies depending on the scenario. The baseload is depicted in blue, the midload in red and the peakload in green. This graph corresponds to the risk-neutral case, however, the risk-averse case is similar.

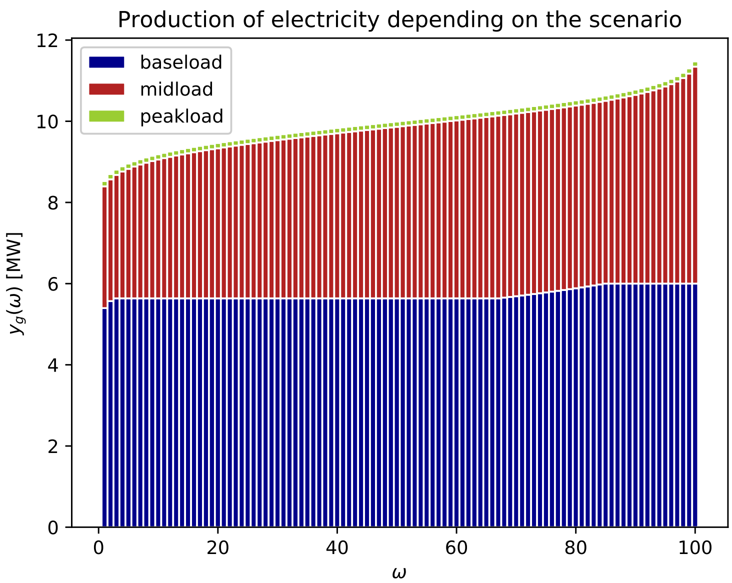

Figure 5.4

Sale of reserves On the graph below one can observe the sale of reserves of the different power plants depending on the scenario. This graph also corresponds to the risk-neutral case, however, the risk-averse case is similar.

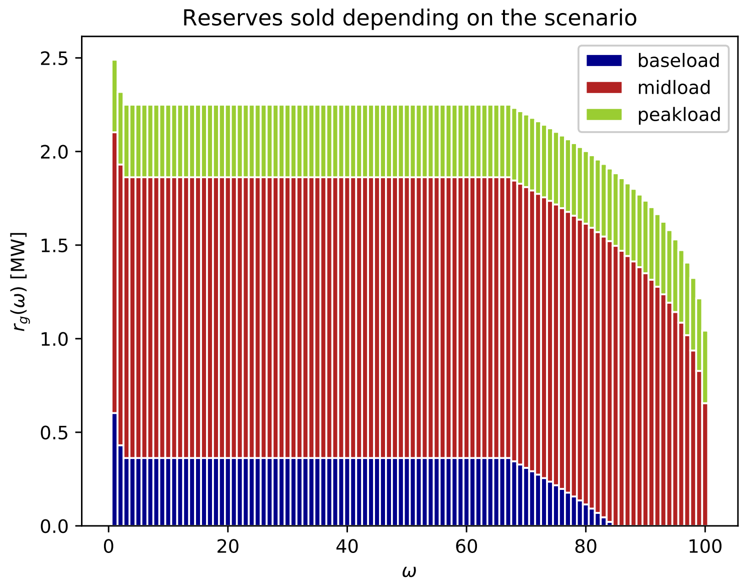

Figure 5.5

Prices On the graph below one can observe the price of the electricity and the price of reserves in real-time. This graph corresponds to the risk-neutral case

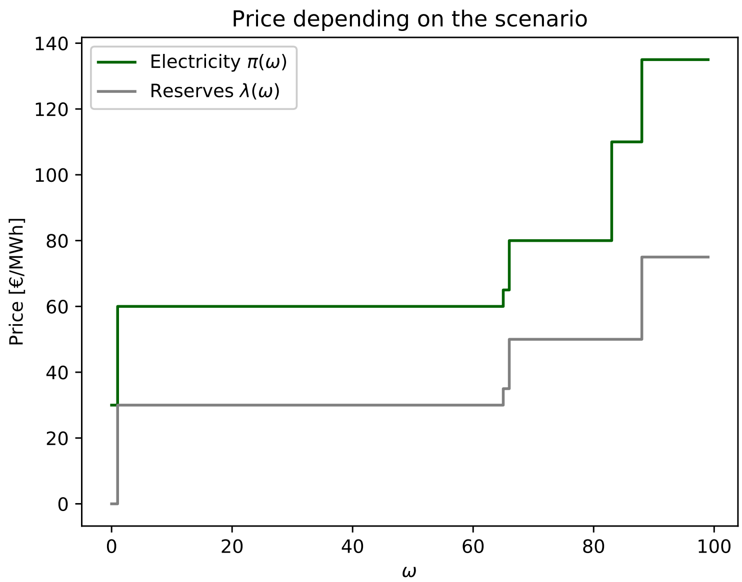

Figure 5.6

Total surplus The difference in the payoffs between the risk-neutral case and the risk-averse case is particularly interesting. One thing is sure: the total welfare in the risk-averse case is lower than the one in the risk-neutral case. Indeed, the risk-neutral problem is equivalent to the maximization of the total welfare.

In the table below, one can observe that the losers of the risk-aversion are the consumers. By reducing the amount of capacity, the price of electricity increases which reduces the surplus of the consumers but increases the surplus of the producers. Hopefully, electricity futures are traded and day-ahead reserves are sold. These products guarantee a certain income for the producers and incite them to turn ON their power plants.

	Risk-Neutral		SR = 0.03	
Agent	Surplus in the day-ahead [€]	Expectation of the total surplus [€]	Surplus in the day-ahead [€]	Expectation of the total surplus [€]
baseload	-161.2	61.6	236.6	63.2
midload	-66.5	20.3	268.9	21.6
peakload	2.1	0	10.5	0.001
consumer	0	1251.4	-741.0	1248.2

The graphic on the left represents the distribution of the surplus of the consumers in the risk-neutral case. It ranges from 700.7 to 1478.7 [€]. The surplus of the consumers increases slowly with the quantity of electricity consumed but decreases steeply when the price of electricity is increasing. On the right, the distribution of the surplus in the risk-averse case is represented. Through the procurement of electricity futures contracts, the variability of the surplus decreases. The surplus in the risk-averse case ranges from 1009.3 to 1358.5 [€].

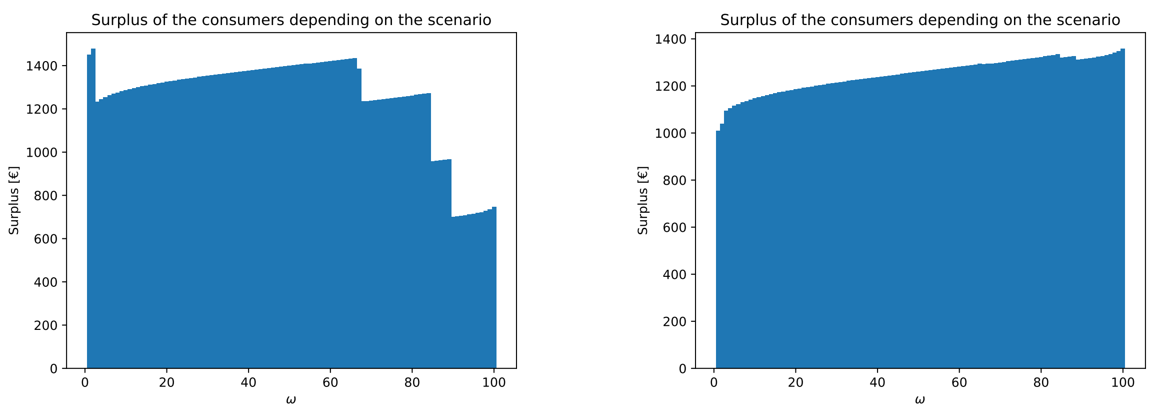

Figure 5.7

The graphics below show how the distribution of the profits of the producers have changed from the risk-neutral (left) to the risk-averse (right) case. For instance, the profit of the baseload (blue), which ranged from -161.2 to 393.8 [€] in the risk-neutral case, lies between 60.8 [€] and 69.6 in the risk-averse case. The majority of the benefit is determined the day before.

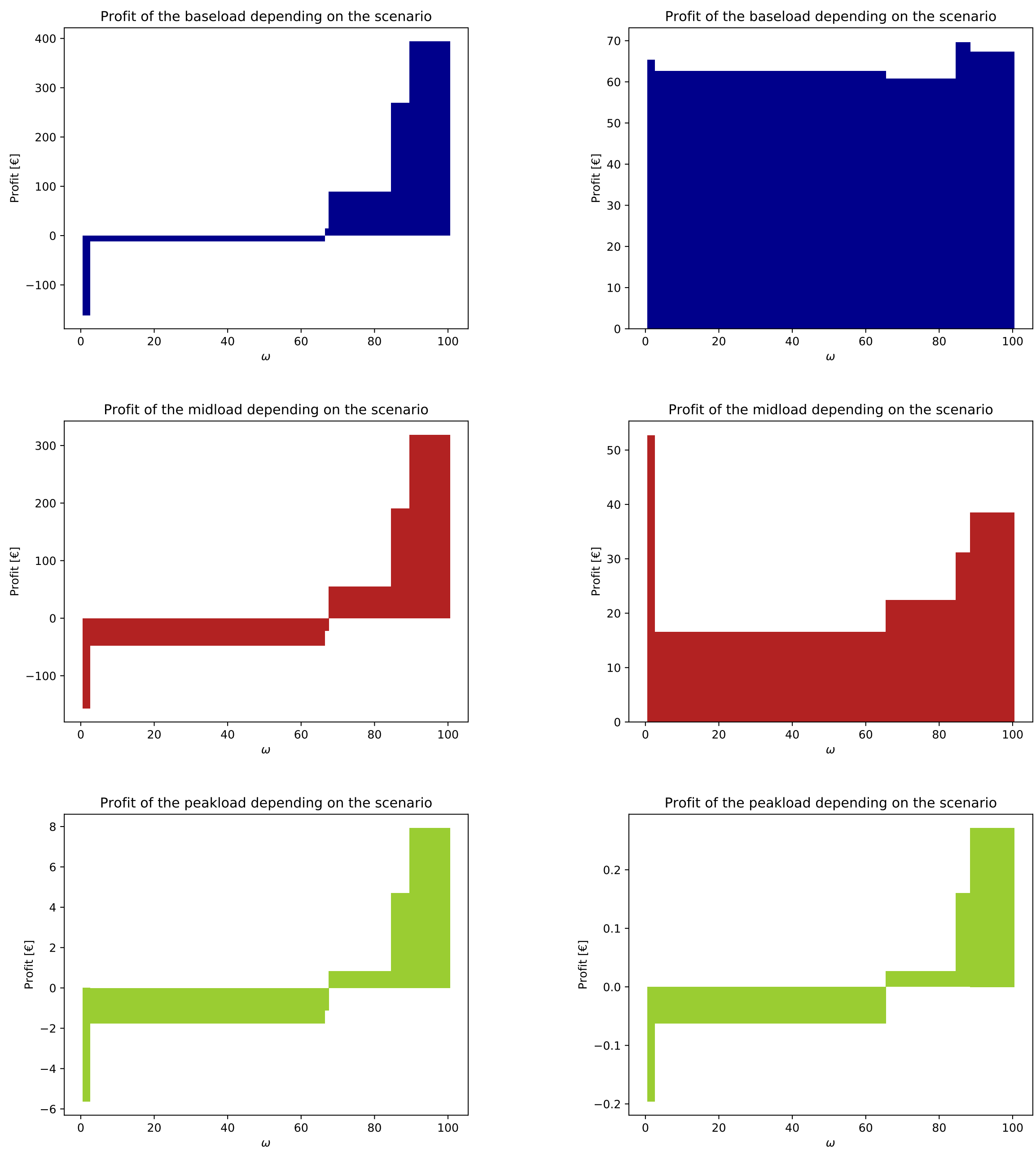

Figure 5.8: Profits of the producers depending on the scenario. The graphics on the left correspond to the risk-neutral case and the graphics on the right corresponds to the risk-averse case. The blue color is used for the baseload, the red color is used for the midload and the green color is used for the peakload.

5.2 Multiple time periods

5.2.1 Overview of the problem

Description of the consumption of electricity

The demand for electricity is stochastic and aggregated into one load every hour, $D_h(\omega)$. The consumption of electricity, $d_h(\omega)$, is equal to the demand during the time period h as long as the price of electricity, $\pi_h(\omega)$ is smaller than the value of lost load $\mathbf{VOLL} = 200$[€/MWh].

The maximum demand of the consumers is simulated for an entire day. Each hour the maximum demand follows a normal distribution $D_h \sim \mathcal{N}(\mu_h, \sigma_h^2) = a_h \cdot \mathcal{N}(10, 0.5) = \mathcal{N}(a_h \cdot 10, a_h^2 \cdot 0.5)$. In which a_h is a positive parameter that gives the distribution of the demand depending on the hour. During period of low demand a_h has a small value, for example at 4 a.m. $a_4 = 0.5$. On the contrary during period of high demand a_h has a high value, for example at 7 p.m. $a_{19} = 1.2$.

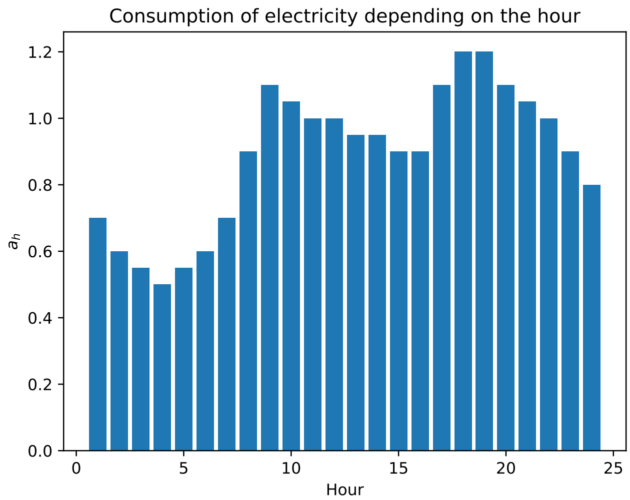

Figure 5.9: Value of the parameter a_h depending on the hour.

One has selected 20 scenarios for the demand $\Omega = \{1, 2, \cdots, 20\}$ and each scenario has an equal probability $p(\omega) = 0.05$. The demand divided by a_h follows a normal distribution with a mean of $\mu = 10$ [MW] and a variance of $\sigma^2 = 0.5$. For $j = 1, 2, ..., 20$, one have:

$$\frac{D(\omega = j)}{a_h} = F_d^{-1}\left(\frac{j}{N}\right), \tag{5.2}$$

where F_d is the cumulative distribution function of $\mathcal{N}(10, 0.5)$.

It is important to note that in this exercise the scenario ω is the same during the whole day. The demand for electricity corresponds to a different normal distribution during every hour, but the quantile is always the same.

Description of the consumption of reserves

The consumption of reserves of the system operator is the same as in the previous exercise. Although it would have been logical to adapt the demand of reserves depending on the hour and the mean consumption of electricity at that time.

Description of the production

The technologies and their features are the same as in the previous example. Unlike the previous exercise, the hours are no longer considered as independent. The unit commitment constraints that link one hour to the next are added: the state transitions, the minimum up-time UT_g and the minimum down-time DT_g.

The minimum up-time and down-time given in the table below are high to make sure that the optimal solution obtained in this problem is different from the optimal solution obtained without these additional constraints.

Technology	UT_g [# hours]	DT_g [# hours]
baseload	12	8
midload	6	4
peakload	3	2

Finally, the producers would have to pay a start-up cost S_g to turn their power plants ON.

Technology	S_g [€/MWh]
baseload	100
midload	50
peakload	25

Description of the risk-aversion

In this exercise, the consumers and the producers analyze the payoff of the whole day and not hour by hour. There are twenty different outcomes possible for each agent. Each payoff correspond to one scenario.

The risk-aversion of the consumers and the producers is the same. The Sharpe-Ratio for the daily stochastic payoff is $\overline{SR} = 0.03$. Since the Sharpe-Ratio depends on the square root of the time, the corresponding annualized Sharpe-Ratio is $\overline{SR} = 0.573$.

5.2.2 Analysis of the results

Day-ahead results

The graphic below gives the unit commitment decisions in the risk-neutral case.

- The baseload is always ON. In this way, it must not pay the start-up cost while respecting all the constraints.
  The rest of the production is divided between the midload and the peakload.
- The midload is mainly turned ON during the day between 8 a.m. and 10 p.m. when the demand is high. It is not started during the night because it has a high minimum production capacity, the demand for electricity is low and the baseload is already ON.
- The peakload is started when the demand is at its highest point and during the night because the midload is turned OFF.

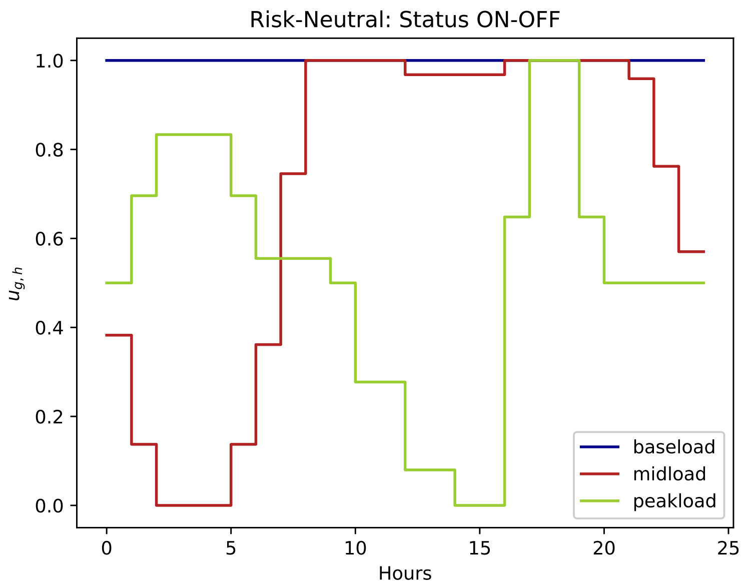

Figure 5.10: Status ON-OFF

On the next page, a graphic shows the difference in the unit commitment decisions between the risk-averse and the risk-neutral problem. From this graph one can extract two pieces of information. It seems that in a risky environment, (i) the peakload is turned OFF more often and (ii) this benefits the midload which is turned ON more often.

It makes sense. The payoff of the peakload is more variable, therefore, in a risky environment the peakload will prefer to reduce the amount of capacity turned ON. As regards the midload, it can more easily reduce the variance of its payoff and it will take advantage of the withdrawal of the peakload to increase its profit.

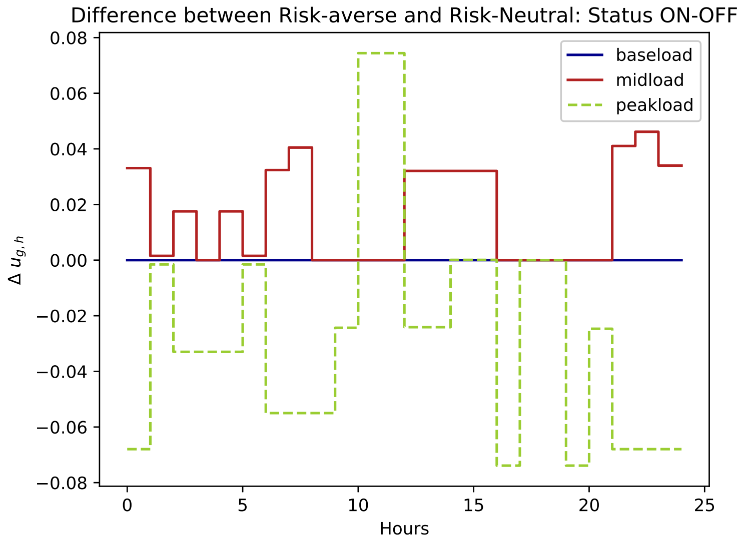

Figure 5.11: Difference between $u_{g,h}$ obtained in the risk-averse case and $u_{g,h}$ obtained in the risk-neutral case.

Electricity futures contracts In the risk-averse case, the agents can decide to trade electricity futures contracts to mitigate the financial risk. Given that the agents want to balance their payoffs over the whole day (as opposed to hour by hour), they will trade electricity futures contracts that last during the entire day.

Two equations given previously must be adapted:

- The payoff for an agent buying x [MW] of electricity futures contract given in (2.3) is changed into:

$$x \cdot \Big(\sum_{h=1}^{24} \pi_h(\omega) - 24 \cdot \pi^{DA} \Big). \tag{5.3}$$

- The constraint of the financial problem (4.31) is changed into:

$$\sum_{h=1}^{24} \mathbb{E}_{Q_i} \Big[\pi_h(\omega) \Big] = 24 \cdot \pi^{DA} \tag{5.4}$$

The number of contracts traded by the agent i is the dual variable of this constraint.

In this exercise, the price of electricity futures contracts is $\pi^{DA} = 75.28$ [€/MWh] and the mean expected price is $\frac{1}{24} \sum_{h=1}^{24} \mathbb{E}\Big[\pi_h(\omega)\Big] = 73.31$ [€/MWh]. It means that to maximize its certainty equivalent the consumers are ready to pay more in the day-ahead.

Agent	baseload	midload	peakload	consumer
y_g^{DA} and (d^{DA}) [MW]	5.410	3.151	0.347	(8.908)

Real-time results

Total surplus As in the previous exercise, the total welfare decreases from the risk-neutral case to the risk-averse case and once again the losers are the consumers which pay more to obtain the same amount of electricity.

	Risk-Neutral		SR = 0.03	
Agent	Surplus in the day-ahead [€]	Expectation of the total surplus [€]	Surplus in the day-ahead [€]	Expectation of the total surplus [€]
consumer	0	21461	-16094	21267
baseload	-3922	1415	5657	1525
midload	-960	635	4728	714
peakload	-214	19	596	18

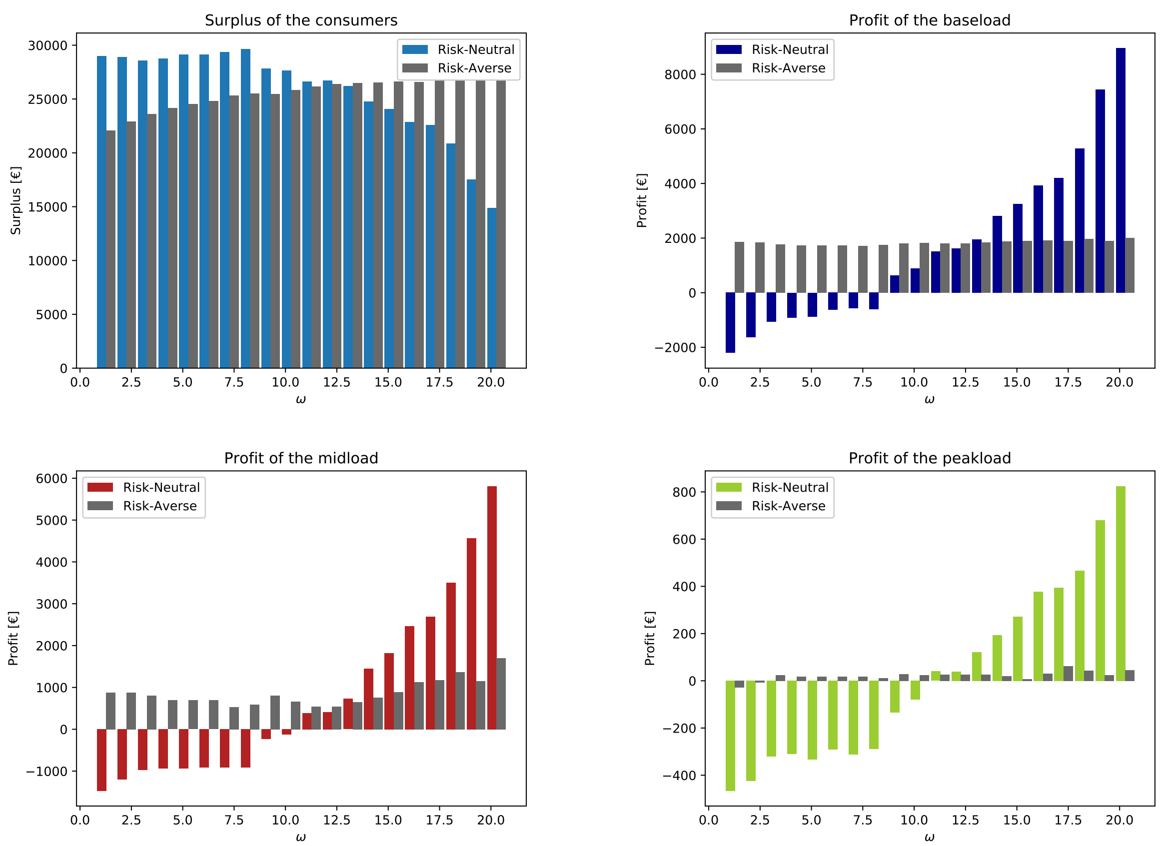

Figure 5.12: Comparison of the surplus of the four agents in the risk-neutral and in the risk-averse case.

5.3 Convergence of the algorithm

The method used to solve the unit commitment problem in a risky environment is heuristic. And although it is not known if the decisions obtained with our method are the optimal one, the heuristic method converged for the two exercises and the results obtained are coherent.

The exercises were solved on an Intel® Core^{TM} i5-2320 CPU @ 3.00GHz × 4 using the solver **Artelys Knitro** on AMPL and the computation time for the second exercise was about 5 minutes.

Nevertheless, the algorithm can still be greatly optimized. For the moment, after each financial iteration, one repeats the physical iteration until convergence. However, one could accelerate the process by stopping to wait for the perfect payoffs before solving the financial iteration again. With this new method the number of physical iterations should decrease and, as a result, the computation time should greatly decrease. The number of financial iterations should increase in return, but this is only a minor problem because the financial iteration is very little computationally intensive.

The graph of f_{fin} for the first exercise is given hereunder.

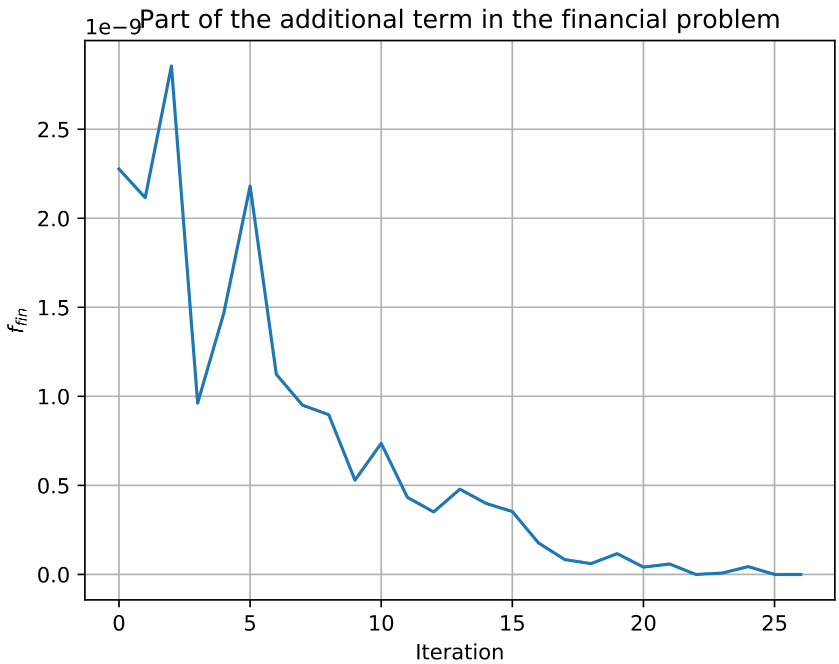

Figure 5.13: Value of f_{fin} depending on the iteration of the financial problem.

As a reminder

$$f_{fin} = \frac{1}{obj_{fin}} \cdot \kappa_{fin} \cdot \sum_{\omega \in \Omega} \Big(q_i(\omega) - q_i^*(\omega) \Big)^2.$$

6 Conclusion

The electricity market is quickly changing, and the way electricity is sold must be adapted accordingly. The flexibility of the power plants should be better paid. To meet this new needs, one needs to get a better understanding of the unit commitment problem in a risky environment. In this master thesis, one tried to increase the knowledge on this subject to help make a recommendation in the future.

After having described the short-term electricity market as a non-cooperative game involving four agents, a heuristic method to obtain the optimal decisions of these agents has been presented. This method works with the resolution in turn of a physical part and a financial part. In Chapter 5 our heuristic method has been used to solve two problems. The first exercise highlighted the interest of considering a risky environment. And in the second exercise, the unit commitment problem is made more realistic by making the time periods dependent. One is not completely sure that the results obtained are the optimal decisions, but the results are coherent and promising.

Finally, although many improvements are still needed before a recommendation can be made to change the design of the electricity market. For example: one could apply our method on a more realistic example, other way of measuring the financial risk could be used, etc. One learned useful pieces of information with our two exercises. Based on the two exercises, one could see the interest of working in a risky environment: the power plants that concern us (the peakload) are the most negatively impacted by the financial risk. One also observed that the consumers are great losers in a risky environment. It is their surplus that decreases the most because of increasing prices.

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Appendix

Producers - Problem with multiple time periods

Maximization problem

$$\max_{\boldsymbol{u}_g, \boldsymbol{y}_g^{\mathrm{DA}}, \boldsymbol{r}_g^{\mathrm{DA}}} \mathcal{R}_g \left(\max_{\boldsymbol{y}_g(\omega), \boldsymbol{r}_g(\omega)} \Big(\boldsymbol{\pi}(\omega) - \boldsymbol{C}_g(\omega) \Big)^\top \cdot \boldsymbol{y}_g(\omega) + \boldsymbol{\lambda}(\omega)^\top \cdot \Big(\boldsymbol{r}_g(\omega) - \boldsymbol{r}_g^{\mathrm{DA}} \Big) - \boldsymbol{\pi}(\omega)^\top \cdot \boldsymbol{y}_g^{\mathrm{DA}} \right)$$
$$- K_g \cdot \mathbf{1}^\top \cdot \boldsymbol{u}_g - S_g \cdot \mathbf{1}^\top \cdot \boldsymbol{v}_g + (\boldsymbol{\pi}^{\mathrm{DA}})^\top \cdot \boldsymbol{y}_g^{\mathrm{DA}} + (\boldsymbol{\lambda}^{\mathrm{DA}})^\top \cdot \boldsymbol{r}_g^{\mathrm{DA}} \tag{6.1}$$

$$(\alpha_{g,h}): \quad 1 - u_{g,h} \geq 0 \tag{6.2}$$

$$(\beta_{g,h}): \quad 1 - v_{g,h} \geq 0 \tag{6.3}$$

$$(\gamma_{g,h}): \quad v_{g,h} - u_{g,h} + u_{g,h-1} \geq 0 \tag{6.4}$$

$$(\delta_{g,h}): \quad u_{g,h} - \sum_{t=h-UT_g+1}^{h} v_{g,t} \geq 0 \tag{6.5}$$

$$(\epsilon_{g,h}): \quad 1 - u_{g,h-DT_g} - \sum_{t=h-DT_g+1}^{h} v_{g,t} \geq 0 \tag{6.6}$$

$$\Big(\mu_g^{\min}(\omega)\Big): \quad y_g(\omega) - P_g^{\min} \cdot u_g \geq 0 \tag{6.7}$$

$$\Big(\mu_g^{\max}(\omega)\Big): \quad P_g^{\max}(\omega) \cdot u_g - y_g(\omega) - r_g(\omega) \geq 0 \tag{6.8}$$

$$(\xi_g^{\mathrm{DA}}): \quad R_g \cdot u_g - r_g^{\mathrm{DA}} \geq 0 \tag{6.9}$$

$$\Big(\xi_g(\omega)\Big): \quad R_g \cdot u_g - r_g(\omega) \geq 0 \tag{6.10}$$

$$0 \leq \boldsymbol{u}_g, \ \boldsymbol{r}_g^{\mathrm{DA}}, \ \boldsymbol{y}_g(\omega), \ \boldsymbol{r}_g(\omega) \tag{6.11}$$

Equilibrium conditions

$$0 \leq u_g \;\perp\; K_g + \alpha_g - R_g \cdot \xi_g^{\mathrm{DA}} + \mathbb{E}_{Q_g}\Big[\cdots\Big] + \gamma_{g,h} - \gamma_{g,h+1} + \epsilon_{g,h+DT_g} \geq 0 \tag{6.12}$$

$$0 \leq v_{g,h} \;\perp\; S_g + \beta_{g,h} - \gamma_{g,h} + \sum_{t=h}^{h+UT_g-1} \delta_{g,t} + \sum_{t=h}^{h+DT_g-1} \epsilon_{g,t} \geq 0 \tag{6.13}$$

$$0 \leq \alpha_{g,h} \;\perp\; 1 - u_{g,h} \geq 0 \tag{6.14}$$

$$0 \leq \beta_{g,h} \;\perp\; 1 - v_{g,h} \geq 0 \tag{6.15}$$

$$0 \leq \gamma_{g,h} \;\perp\; v_{g,h} - u_{g,h} + u_{g,h-1} \geq 0 \tag{6.16}$$

$$0 \leq \delta_{g,h} \;\perp\; u_{g,h} - \sum_{t=h-UT_g+1}^{h} v_{g,t} \geq 0 \tag{6.17}$$

$$0 \leq \epsilon_{g,h} \;\perp\; 1 - u_{g,h-DT_g} - \sum_{t=h-DT_g+1}^{h} v_{g,t} \geq 0 \tag{6.18}$$

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