

Monitoring of Arctic sea ice volume variability by an optimized observation system

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Abstract

Current observation systems of Arctic sea ice thickness and volume come with great uncertainty due to limited quality satellite and in situ observations. Using the Community Earth System Model Large Ensemble Project dataset, we investigate a statistical model based on local ice thickness measurements, able to reproduce the Arctic sea ice volume variability.

A statistical model is proposed, computed by iterative correlation method. This observation system is able to reproduce the Arctic sea ice volume variability in pre and post-industrial conditions and also in future climate conditions.

The Arctic sea ice thickness variability is both natural and anthropogenic. Until now, less attention has been given to the natural variability of the Arctic sea ice thickness. A principal component analysis over Arctic sea ice thickness time series shows the major modes of natural sea ice thickness variability. The fifth first modes account for more than 45% of the variability. The first mode is link to the Arctic Oscillation, which is a dipole spatial pattern of sea ice thickness variability with opposite signs of polarity between the East Siberian Sea and near the Fram Strait.

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Introduction

The cryosphere is the collective term used for the components of the Earth system that contain a substantial fraction of water in the frozen state. The cryosphere comprises several components: snow, river and lake ice, sea ice, ice sheets, ice shelves, glaciers and ice caps, and frozen grounds which exist, both on land and beneath the oceans. The lifespan of each component is very different. Nevertheless, all components of the cryosphere are inherently sensitive to changes in air temperature and precipitation, and hence to a changing climate.

Changes in the longer-lived components of the cryosphere are the result of an integrated response to climate, and the cryosphere is therefore often referred to as a ‘natural thermometer’ for our planet. But, as our understanding of the complexity of this response has grown, it is increasingly clear that elements of the cryosphere should rather be considered as a ‘natural climate-meter’, responsive not only to temperature but also to other climate variables. However, it remains the case that the conspicuous and widespread nature of changes in the cryosphere means are frequently used emblems of the impact of changing climate. It is thus imperative that we understand the context of current change within the framework of past changes and natural variability (IPCC, 2014 [13]).

In the past years, the Arctic sea ice has diminished way over its natural variability. Its extent has decreased with an overall trend of $-51.4 \pm 1.9 \times 10^3 \text{ km}^2 \text{ yr}^{-1}$ over the 1979-2012 period (Stocker *et al.*, 2013 [25]). The decrease in the mean ice thickness over the Arctic Ocean from 1980 to 2008 is between -1.33 and -2.3 m (Stocker *et al.*, 2013 [25]).

In the context of the recent climate changes, direct observations of the ice thickness distribution are essential to document the true evolution of the air–ice–ocean system. Some measurement devices already exist, such as satellite imagery since 1979, satellite altimetry, buoys measurements, upward looking sonar, etc. Arctic variables such as ice concentration or ice extent are well represented but direct observations of the ice thickness and volume are few and far between and are far too sparse to permit an analysis of the modes of variability. Models alone are not sufficient because of uncertainties in their representations of the ice thickness. A long-term observing system could help to verify what the interannual variability of the total ice volume is and how the regional distribution of the ice mass changes from year to year. One of the goals of this Master’s thesis is to propose an observation system measuring ice thickness at some local station, able to reproduce the variability of the ice.

In order to build that model, we must have a good understanding of the variability of the Arctic sea ice. This variability is both natural and anthropogenic. Furthermore, the natural variability can be divided into the internal variability (which comes from the

the coupled interactions of the ice, ocean, atmosphere, etc) and the external variability (which comes from external climate factors as volcano or solar parameters). A better characterisation of the internal variability of the Arctic sea ice will help us to better understand the true impact of the anthropogenic forcing in the recent decline of the ice, but also help us to improve and calibrate our observation system of Arctic sea ice. Indeed, if we are able to reproduce that internal variability, it may improve climate models and future climate predictions.

The aim of this Master's thesis is to characterise the internal variability of the Arctic sea ice both spatially and temporally, build a statistic model able to predict the Arctic sea ice internal variability and finally, find the position of a monitoring measurement device over the Arctic Ocean able to reproduce the Arctic sea ice variability for future years. We will work with a database that gives the thickness of the Arctic sea ice for several hundred years coming from a simulation of the CESM1-CAM5-BGC model (Kay *et al.*, 2015 [15]). It is a global coupled climate model, that consists of coupled atmosphere, ocean, land and sea ice models.

The first chapter of the thesis is the state of the art, dedicated to the description of sea ice characteristics and variability. The second chapter describes the data, introduce the scientific question and presents a first statistic model to reproduce the Arctic sea ice volume variability. The third chapter describes the natural variability of the Arctic sea ice through a principal component analysis and links some of the results with climatic phenomena of natural variability. The fourth chapter uses an optimisation problem to build a statistic model of Arctic sea ice volume prediction. And finally, the last chapter investigates the robustness of our statistic model under post-industrial and future climate conditions.

Chapter 1

State of the art

1.1 Physical characteristics of sea ice

Sea ice (Figure 1.1) is an important component of the climate system. The presence of sea ice cover on the ocean modifies the exchanges between ocean and atmosphere in multiple ways. It changes the surface albedo, insulates the ocean from the atmosphere, and provides a barrier to the exchange of momentum and gases such as water vapour and CO_2 between the ocean and atmosphere. Salts ejected by sea ice alters the density structure and modifies the circulation of the ocean. Regional climate changes affect the sea ice characteristics and these changes can feed back on the climate system, both regionally and globally. Sea ice is also a major component of polar ecosystems. Plants and animals at all tropic levels find a habitat in, or are associated with, sea ice.

Sea ice motion create areas of open water where, during cold months, new ice can quickly form and grow. On the other hand, convergent ice motion causes the ice cover to thicken by deformation.



Figure 1.1: Arctic sea ice (CarbonBrief, 2018 [32]).

Sea ice thickness can also increase by basal freezing during winter months. But the thicker the ice becomes, the more it insulates the ocean from the atmosphere and the slower the

basal growth is. There is an equilibrium thickness for basal ice growth that is dependent on the surface energy balance and heat from the deep ocean below. Snow cover lying in the surface of sea ice provides additional insulation, and alters the surface albedo and aerodynamic roughness.

Because sea ice is formed from seawater, it contains sea salt, mostly in small pockets of concentrated brine. The total salt content in newly formed sea ice is only 25 to 50 % of that in the parent seawater, and the residual salt rejected as the sea ice forms alters the ocean water density and the water column stability. The salinity of the ice decreases as it ages, and particularly during the Arctic summer when melt water drains through and flushes the ice. The salinity of the ice affects its mechanical strength, its thermal and electrical properties.

Geographical constraints play a dominant but not exclusive role in determining the quite different characteristics of sea ice in the Arctic and Antarctic. This is one of the reasons why changes in sea ice extent and thickness are very different in the north and the south.

Multi-year ice is ice that survived at least one summer. Generally, multi-year ice is less saline and has a distinct microwave signature that differs from the seasonal ice, and thus can be discriminated and monitored with satellite microwave radiometers (AR5, IPCC 2013 [13], Vaughan *et al.*, 2013 [28]).

1.2 Geography of the Arctic

The Arctic is composed of many seas. A simple way to represent it is to divide it into 9 zones, which are depicted in Figure 1.2.

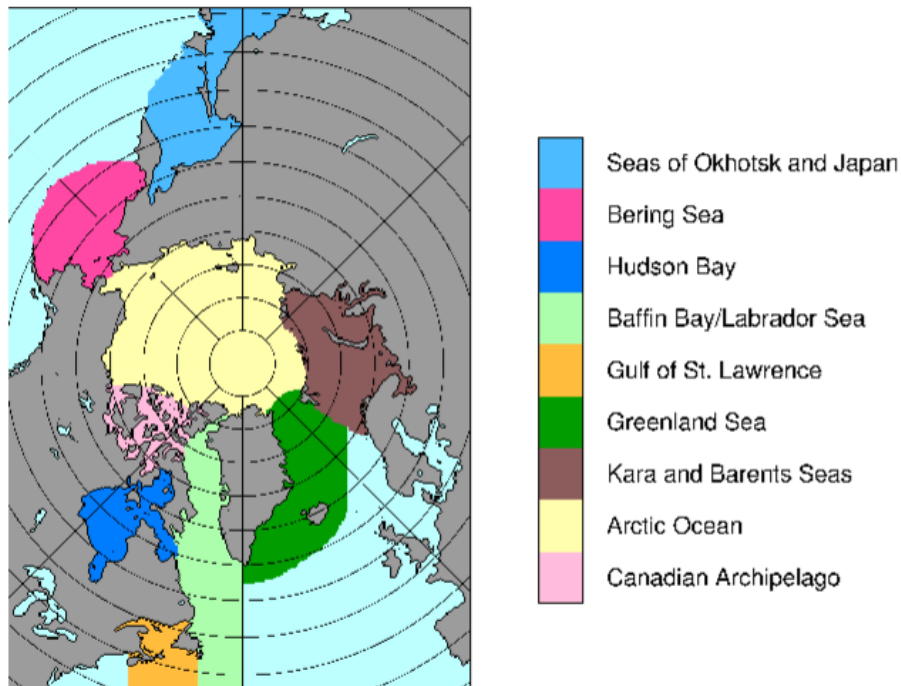


Figure 1.2: Arctic sea ice principal geographical areas (Cavalieri and Parkinson, 2012 [4]).

1.3 Arctic sea ice : mean state

Mean state : ice extent

Sea ice extent is defined as the sum of oceanic areas with ice concentration of at least 15%, while ice area is the product of the ice concentration and area of each data element within the ice extent. Arctic sea ice cover varies seasonally, with average extent varying between about $6 \times 10^6 \text{ km}^2$ in summer and about $15 \times 10^6 \text{ km}^2$ in winter (Figure 1.3). The summer ice cover is confined to mainly the central Arctic Ocean and the Canadian Arctic Archipelago, while winter sea ice reaches as far as 44° N , into the peripheral seas. At the end of summer, the Arctic sea ice cover consists primarily of the previously thick, old and ridged ice types that survived the melt period.

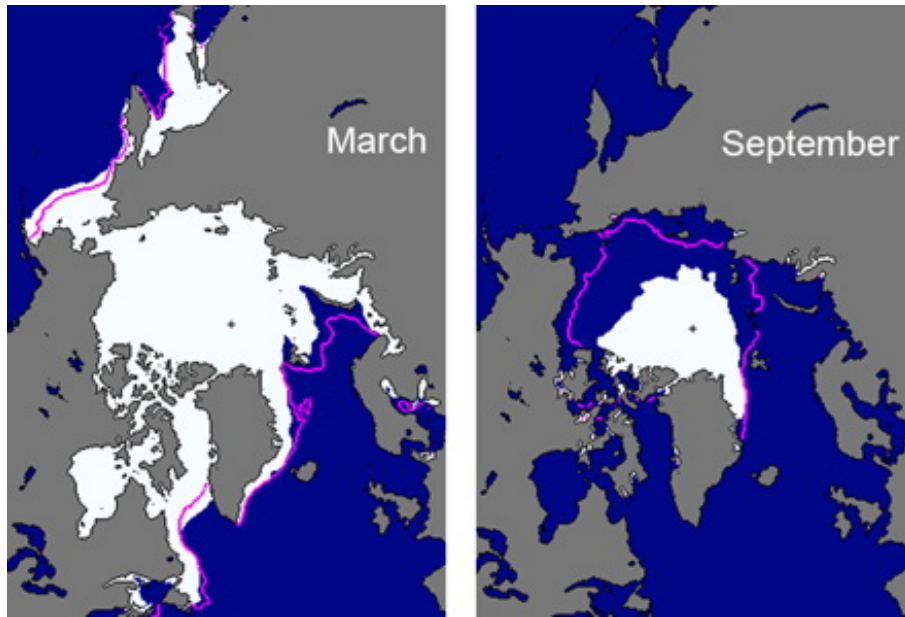


Figure 1.3: The white area shows the Arctic ice extent after the growing and the melting seasons (2012). It is compared with the average for 1981-2010, shown with a pink border, (EarthLabs, 2018 [35]).

Typically, Arctic sea ice reaches its maximum seasonal extent in February or March, whereas the minimum occurs in September, at the end of the summer melt.

Mean state : motion of the ice

Arctic sea ice is mainly caused by surface winds and ocean currents. It is season-dependent. During winter, the circulation is dominated by the anticyclonic Beaufort Gyre in the Pacific sector of the basin and the transpolar drift stream, which transports ice out of the gyre and into Fram Strait (the passage between Greenland and Svalbard, which is the primary location for ice export from the basin). In summer, the circulation is much weaker, the Beaufort Gyre is smaller and is constrained to mostly the Beaufort Sea. The transpolar drift stream is much weaker and a small cyclonic gyre develops in the eastern part of the basin. (See both circulations in Figure 5.8 in the annex.)

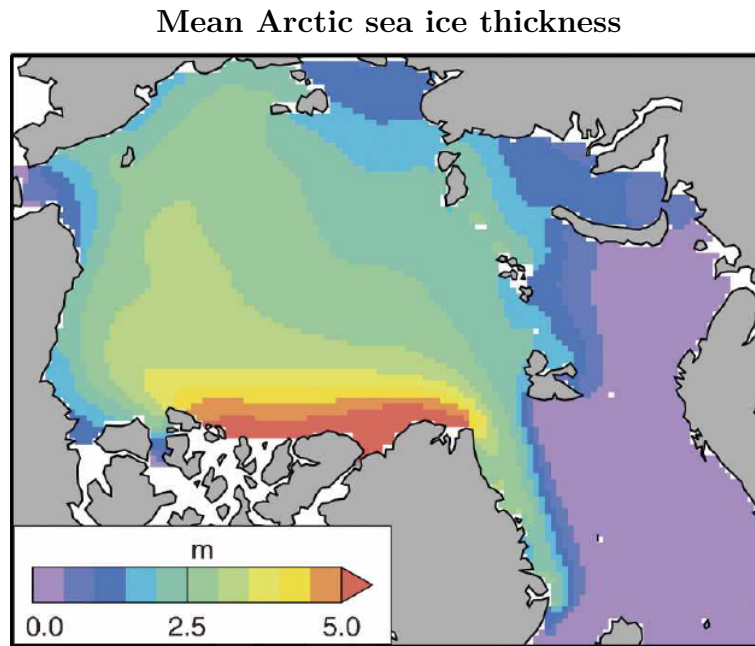


Figure 1.5: Mean Arctic sea ice thickness over the 51 yr period 1953-2003 (Lindsay and Zhang, 2005 [18]).

1.4 Arctic sea ice : variability

We describe in the previous section the mean state of the Arctic sea ice, we will now look at its variability and also the more recent variability of the last 30 years. But before that, we must bring more information about the distinction between natural and anthropogenic variability.

Attribution of Anthropogenic Forcing

It is well recognised that the recent Arctic sea ice decline has both natural and anthropogenic drivers, but their relative importance is poorly known. This uncertainty arises from the fact that the contribution of atmospheric internal variability in the Arctic climate system is inadequately understood, and it is unclear how well models reproduce these processes (Ding *et al.*, 2017 [10]).

Early work indicated that the decline of the sea ice before the 1990s was in part due to an upward trend in the North Atlantic Oscillation¹ (NAO) index. However, since the early 1990s, the apparent link between the NAO and sea ice has largely disappeared, with Arctic sea ice declining further despite the reversal in the NAO trend. On the other hand, several studies have argued that the recent NAO trend may still play a key role in the sea ice retreat. Alternatively, the Arctic dipole² has been suggested as a driver of sea ice

¹Weather phenomenon in the North Atlantic Ocean of fluctuations in the difference of atmospheric pressure at sea level between the Icelandic low and the Azores high. See section 1.5 on page 18

²Pressure pattern characterized by high pressure on the Arctic regions of North America and low pressure on those of Eurasia, observed for the first time in the first decade of 2000s and perhaps linked to recent climate change

change in some regions of the Arctic (Ding *et al.*, 2017 [10]).

In the case of the Arctic we have high confidence in the observations since 1979, from models and from simulations comparing with and without anthropogenic forcing, and from physical understanding of the dominant processes; taking these three factors together, it is very likely that anthropogenic forcing has contributed to the observed decreases in Arctic sea ice since 1979 (AR5, IPCC 2013 [13]).

Attribution of anthropogenic forcing is investigated by comparing September sea ice extent as projected by seven models from the set of CMIP5 models hincasts to control runs without anthropogenic forcing (Figure 1.6) (Wang and Overland, 2009 [30]). Another study by Ding *et al.* [10] estimates that about 42% of the sea ice decline observed since 1979 in September is due to internal variability based on the simulation of the impact of atmospheric circulation on sea ice.

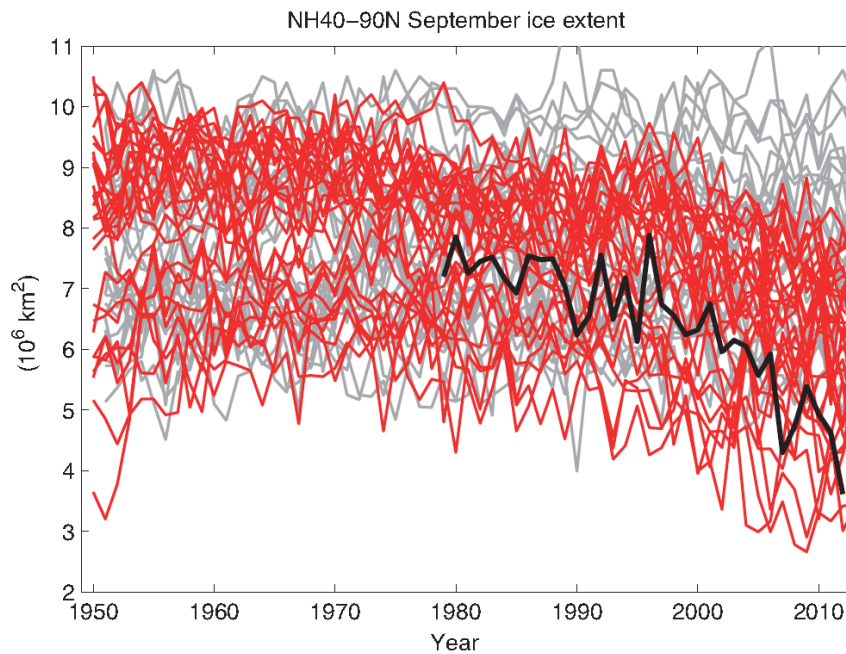


Figure 1.6: September Arctic sea ice extent adapted from Wang and Overland 2012 [31]. Only CMIP5 models (Coupled Model Intercomparison Project Phase 5) that simulated seasonal mean and magnitude of seasonal cycle in reasonable agreement with observations are included in the plot. The grey lines are the runs from the pre-industrial control runs, and the red lines are from historical simulations runs patched with RCP 8.5⁴ runs for the period of 2005-2012. The black line is based on data from National Snow and Ice Data Center (NSIDC, 2018 [34]).

⁴The RCPs (representative concentration pathways) are four greenhouse gas concentration trajectories adopted by the IPCC for its fifth Assessment report (AR5) in 2004. The RCP 8.5 accounts for radiative forcing of 8.5 W/m^2 in 2100 relative to pre-industrial values.

Variability : seasonal modes of variability

From monthly mean sea ice concentration data (conventional estimates with microwave measurements from polar-orbiting satellites since 1972), Deser *et al* [9] analysed the leading pattern in the Arctic sea ice variability during both winter and summer.

During winter, the dominant structure of sea ice variability (which accounts for 35% of the variance) exhibits out-of-phase fluctuations between the Labrador and Greenland/Barents Seas. A smaller amplitude seesaw occurs in the Pacific sector between the Bering Sea and the Sea of Okhotsk.

Differently, during summer, the dominant mode of sea ice variability is more spatially uniform than that in winter. Summer ice extent for the Arctic as a whole has exhibited a nearly monotonic decline ($-4\% \text{decade}^{-1}$) during the past 40 years.

Variability : ice extent

Interannual variability is largely determined by extent of the ice cover in the peripheral seas in winter and by the ice cover that survives the summer melt in the Arctic Basin (Cavalieri *et al.*, 2003 [5], AR5 IPCC 2013 [13]). Changes in decadal averages in Arctic ice extent are more pronounced in summer than in winter (AR5, IPCC 2013 [13]).

The monthly averaged Arctic sea ice extent time series over the period 1978-2015 is presented in Figure 1.7. The three years with the largest March sea ice extents were 1979, 1983, 1982, with values of 16.1, 15.9, 15.8 million square kilometers, respectively. The four years with the smallest September sea ice extents were 2007, 2008, 2010, 2012 with values of 4.3, 4.7, 4.9, 4.8 million square of kilometers, respectively.

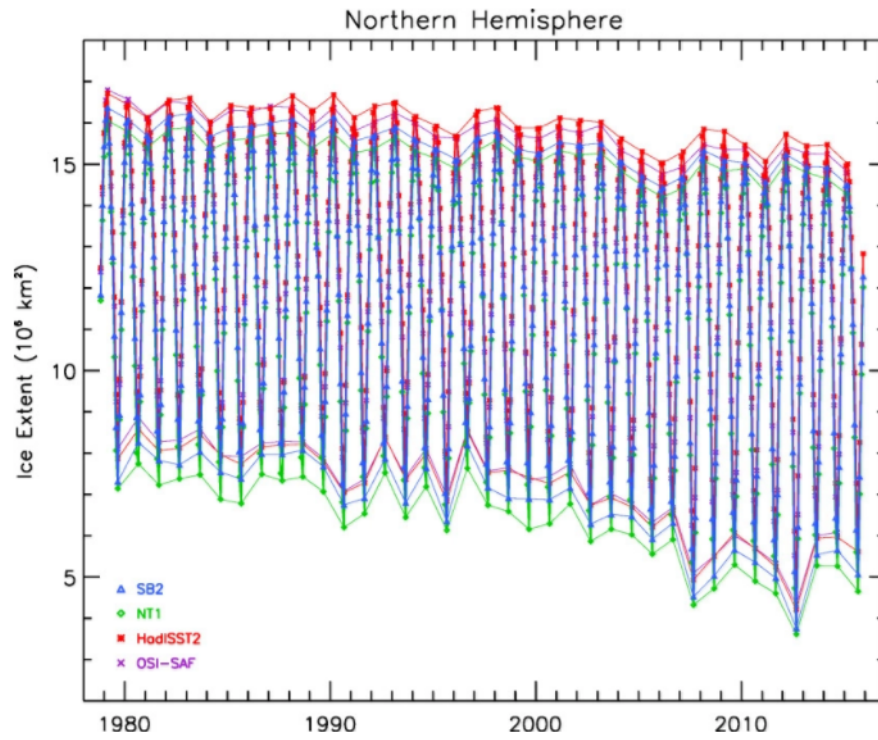


Figure 1.7: Monthly averaged Arctic sea ice extents over the 1978-2015 period, derived using SB2, NT1, HadISST2 and OSI-SAF data (Comiso *et al.*, 2017 [7]).

Figure 1.8 shows the monthly anomalies of sea ice extent for the November 1978-December 2015 period with an overall trend of $-4.35 \pm 0.14\% \text{decade}^{-1}$. The largest negative excursions from the trend line occur in September 2007 and September 2012, which had two of the smallest sea ice extent of this 37 yr period.

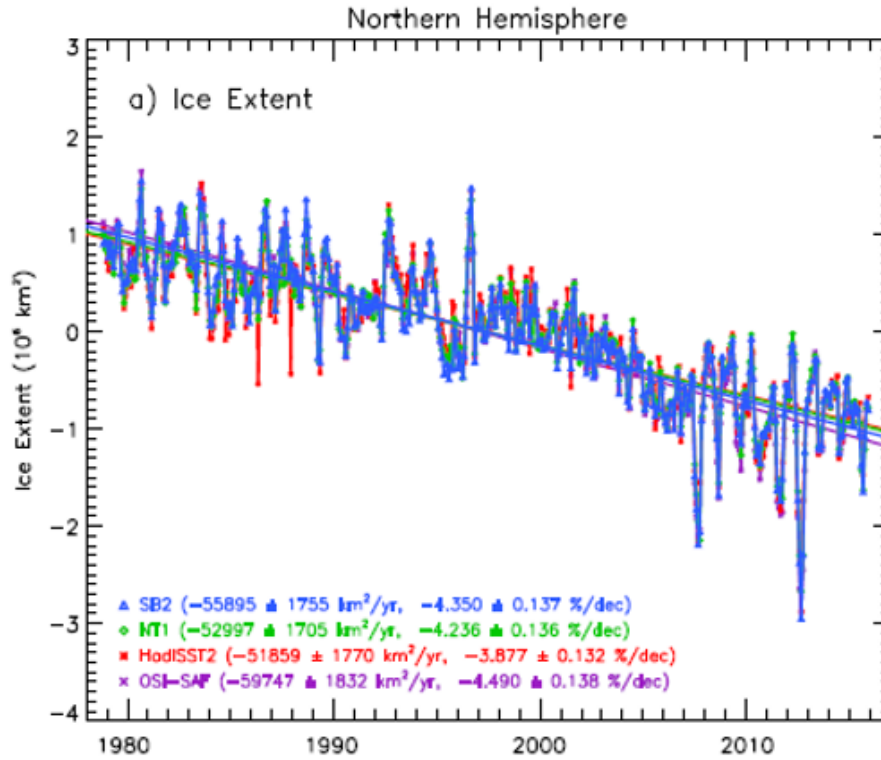


Figure 1.8: Monthly anomalies in Arctic sea ice extent for NT1, HadISST2 and OSI-SAF data sets using passive microwave data from November 1978 to December 2015 (Comiso *et al.*, 2017 [7]).

Summer ice extent for the Arctic as a whole has exhibited a nearly monotonic decline ($24\% \text{ decade}^{-1}$) during the past 40 yr. The spatial patterns of ice retreat during the early (1957–78) and late (1979–96) portions of the record differ slightly, with the maximum ice declines located in the Barents, Kara, and Laptev Seas during the early period and in the East Siberian Sea during the later period. Both patterns of summer ice retreat appear to be initiated by atmospheric circulation anomalies over the high Arctic in late spring (Deser *et al.*, 2000 [9]).

Variability : motion of the ice

Using the International Arctic Buoy Programme (IABP data), Rampal *et al.* [27] found an increase in average ice drift speed between 1978 and 2007 of $17 \pm 4.5\%$ per decade in winter and $8.5 \pm 2.0\%$ per decade in summer.

Sea ice export through Fram Strait is a major component of the Arctic Ocean ice mass balance. Approximately 10% of the area of Arctic sea ice is exported annually through this strait. Over a 32 year satellite record (1979-2010), the annual mean outflow of ice area through Fram Strait was $699 \pm 112 \times 10^3 \text{ km}^2$. Between 2005 and 2008, more than a third of the thicker and older sea ice loss occurred by transport of thick, multi-year ice, typically found west of the Canadian Archipelago, into the southern Beaufort Sea, where it melted in summer (Kwok *et al.*, 2009 [16]).

Uncertainties remain in the relative contributions of in-basin melt and export to observed changes in Arctic ice volume loss, and it has also been shown that export of thick ice

through Nares Strait (waterway between Greenland and Ellesmere Island) could account for a small fraction of the loss (AR5, IPCC 2013 [13]).

Variability : ice thickness

As for the ice thickness mean state, we investigate the variability based on models simulations. Figure 1.9 depicts the standard deviation of the annual mean ice thickness over the period 1953-2003. The interannual variability is the lowest in the central part of the basin, about 0.4 m. It is large both near the Canadian coast and in the east Siberian Sea, with maxima near 1.0 m.

Standard deviation of the annual mean ice thickness

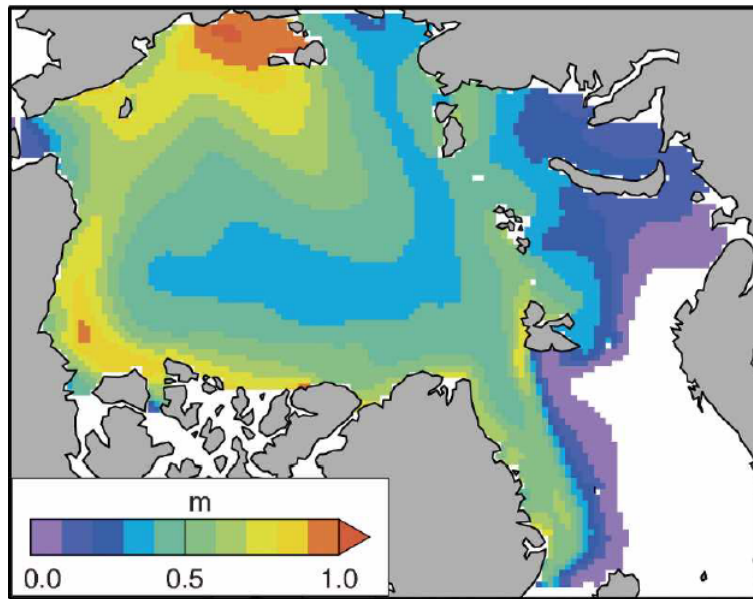


Figure 1.9: Standard deviation of the annual mean ice thickness over the 51 year period 1953-2003 (Lindsay and Zhang, 2005 [18]).

The variability of the Arctic sea ice thickness is greater in the Atlantic sector (values exceeding 25% – 30% in the Greenland and Barents Seas and the Davis Strait) than in the Pacific (values exceeding 15% – 20% in the Bering Sea and the Sea of Okhotsk). The region of maximum variability in the Greenland Sea coincides with the sea ice feature known as the "Odden". The Odden is a tongue of sea ice that episodically advances rapidly northeastward into the Greenland Sea from the edge of the main ice pack near 10°W between 72° and 74°N. At least one such advance (and retreat) occurs during most winters; the maximum area of the Odden ranges from close to zero in some winters to nearly $3 \times 10^5 \text{ km}^2$ in other years. The episodic nature of the Odden is undoubtedly primary reason why the eastern Greenland Sea contains a variance maximum (Deser *et al.*, 2000 [9]).

Overall, variance in sea ice thickness is greatest along Arctic Ocean coastlines. Sea ice thickness anomalies have a typical time scale of about 6-20 months, a time scale that lengthens about a season when accounting for ice transport, and a typical length scale of about 500-1000 km. Regionally, ice thickness tends to be most variable along the Arctic coastlines and in regions of thickest ice. The length and time scales of these anomalies

are also related, albeit weakly, to the mean ice thickness, with smaller yet longer-lived anomalies in models with thicker ice (Blanchard-Wrigglesworth and Bitz, 2014 [2]).

The annual mean Arctic ice thickness time series shown in Figure 1.10 allows to investigate the more recent variability of the ice thickness. The thickness is the greatest in 1991 and there is a secondary maximum in 1970.

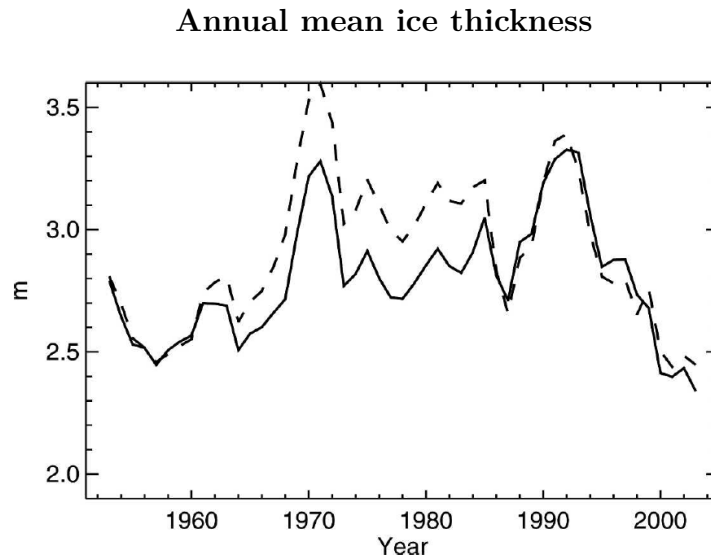


Figure 1.10: Time series of annual mean ice thickness averaged over the Arctic Ocean. The dashed line is from a model-only simulation, and the solid line is from a simulation that includes assimilation of ice concentration and, starting in 1979, ice velocity (Lindsay and Zhang, 2005 [18]).

There is a little change in the mean thickness in the year after the 1991 maximum, in the following year, the ice begins the thinning trend that persists until 2003. The decrease in the mean thickness over the Arctic Ocean from 1991 to 2003 is -1.33 m or 40%, and the ice in 2003 was at a minimum mean thickness for this 51-year period (Stocker *et al.*, 2013 [25]).

1.5 The North Atlantic Oscillation and how it impacts the Arctic sea ice

The North Atlantic Oscillation (NAO) consists of two pressure centers in the North Atlantic : one is an area of low pressure typically located near Iceland, and the other an area of high pressure over the Azores. Fluctuations in the strength of these features significantly alters the alignment of the jet stream, especially over the eastern U.S., and ultimately affects temperature and precipitation distributions in this area. The fluctuation is characterised by a positive and a negative index.

The positive phase of the NAO reflects a strengthening of the Icelandic low and Azores high. This strengthening results in an increased pressure gradient over the North Atlantic, which causes the westerlies to increase in strength. The increased westerlies allow cold air to drain off the North American continent rather than letting it build up and move south. At the opposite, a negative NAO indicates weakening of both the Icelandic low and Azores high, which decreases the pressure gradient across the North Atlantic. This decreased pressure gradient results in a slackening of the westerlies (North Carolina Climate Office, 2018 [33]).

The NAO impact on the Arctic is called the Arctic Oscillation (AO). It is characterised by some unusually high pressures over the northern Beaufort Sea and Greenland, and unusually low pressures over northeastern Eurasia. This pattern brings in warm southerly winds along the shores of the East Siberian and Chukchi Seas. It favours strong ice melt in these sectors and pushes the ice away from the coast, leaving open water. The pressure pattern also favours the transport of ice out of the Arctic Ocean and into the North Atlantic through Fram Strait. The Arctic dipole anomaly was very well developed throughout the summer of 2007 and was in part responsible for the very low September ice extent recorded that year (National Snow and Ice Data Center).

Figure 1.11 resumes how the AO impacts the Arctic sea ice motion on average during low and high index.

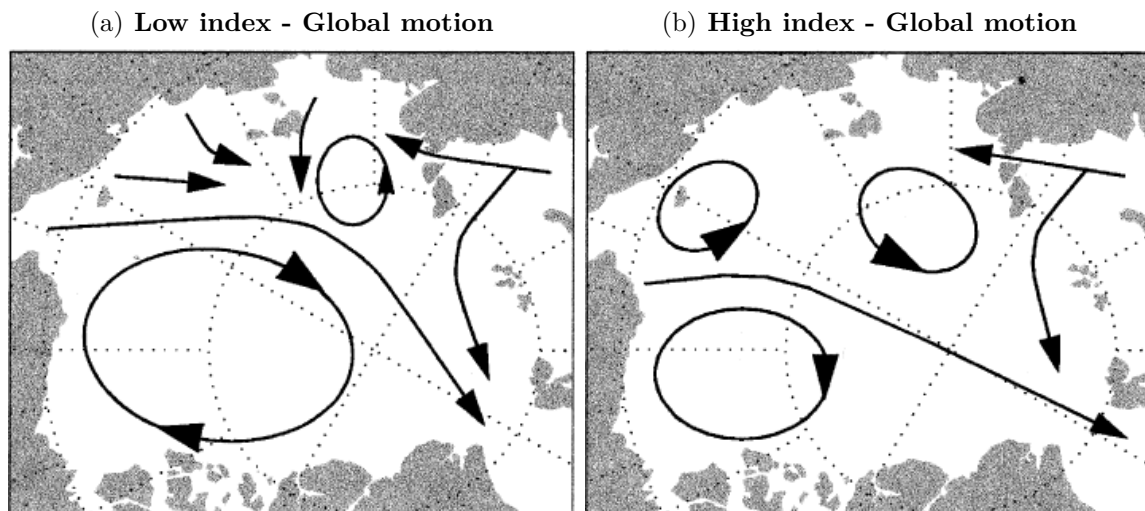


Figure 1.11: Regimes of ice motion during low (a) and high (b) index phases of the Arctic Oscillation (Rigor *et al.*, 2002 [20]).

1.6 Observation system and how to find the best locations to monitor the variability ?

In order to predict the ice variability, we need good and well placed observation system. Some observation systems already exist. As the North Pole Environmental Observatory (Morison *et al.*, 2002 [19]), which is located very close to the Pole and was first established in the spring of 2001. There is a second one established in the Chukchi Sea in 2003. Other mooring programs also exist, such as those in the Fram Strait or in the eastern Beaufort Sea.

A study by Lindsay and Zhang [18] estimated that just three optimally placed stations were needed to monitor 90% of the variance of the basin-wide mean thickness. They used principal component analysis to determine the locations of the major modes of variability.

The major modes of spatial variability of the simulated annual mean ice thickness are illustrated with the first three principal components in Figure 1.12. The spatial domain used to determine the principal component patterns included only the Arctic Ocean. The map shows the correlation on the annual mean ice thickness for all ice covered locations with each of the first three principal component time series.

The first mode, representing 30% of the Arctic Ocean variance, shows much of the basin increases and decreases in ice thickness in a coordinated fashion. The maximum of the pattern is near the center of the basin, in part because points near the edges gave fewer neighbours with which to build high covariability. This pattern likely reflects the basin-wide inter-annual variability in the thermal forcing. It is not strongly related to the winter AO index.

The second and third principal component patterns, explaining 18% and 15% of the variance, respectively, together represent modes of lateral variability, meaning points on opposite sides of the basin have an element of anti-correlation. The second component shows a mode with opposite signs centered near the Beaufort and Laptev Seas (it represents an oscillation between the Beaufort Sea–Canadian coast and the East Siberian–Laptev Seas). It is very similar to the east–west Arctic anomaly pattern and is well correlated with the winter AO index. It likely has a strong dynamic dependence because of this correlation. The third pattern has centers in the East Siberian Sea and off Svalbard in an orientation orthogonal to the second component (it represents an oscillation between the Chukchi–East Siberian Seas and the Svalbard region) and is also correlated with the winter AO index, although not as strongly (Lindsay and Zhang, 2005 [18]).

The model results were also used to determine the optimal locations to monitor both the spatial and temporal variabilities of the annual mean ice thickness. Lindsay and Zhang used the first 10 principal components in multi-linear regression models to represent the variability and determine the locations that best explain the variability. They tested for the best single location, the best additional location given a measurement at the North Pole, and the best additional location given measurements at both the North Pole and in the Chukchi Sea. With the three stations, 57% of the spatial and temporal variability of the annual mean ice thickness in the Arctic Ocean is monitored. According to this analysis the optimal new location to monitor the basin-wide spatial variability of the annual mean

thickness is in the East Siberian Sea (Lindsay and Zhang, 2005 [18]).

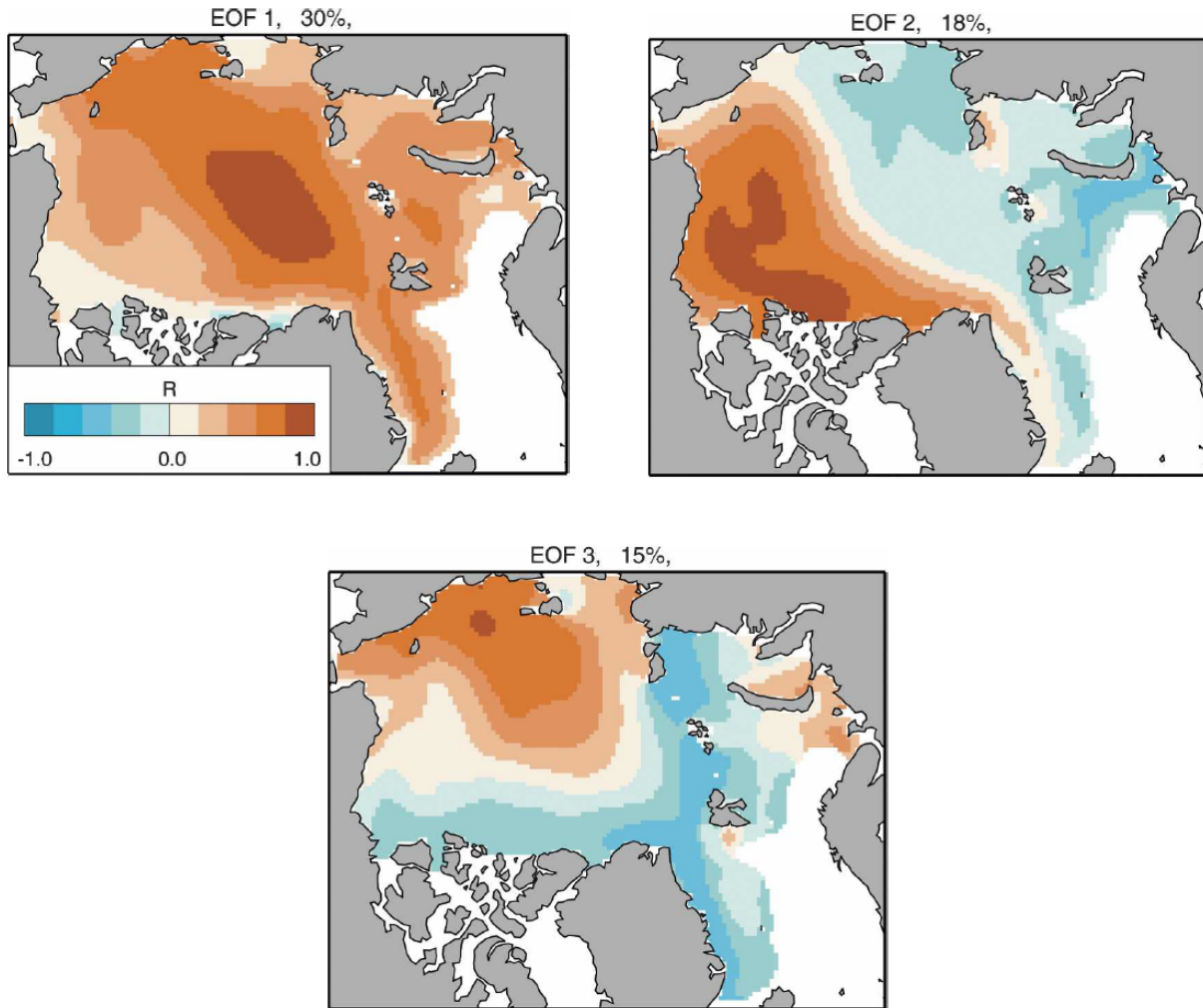


Figure 1.12: Correlation of the annual mean ice thickness at each location with the first three principal components of the annual mean ice thickness for the Arctic Ocean computed from 51 years of a model simulation, 1953–2003. The percentages indicate the fraction of the total variance of the annual mean ice thickness explained by each of the EOFs (Lindsay and Zhang, 2005 [18]).

Other factors might also be important for selecting ice thickness monitoring locations. Among these is the monitoring of locations that show large differences in the simulations from different ice models so as to provide information about which models do well. Also, there is little information about the mean and variability of the ice thickness in the region north of the Canadian Archipelago, a region that may be particularly important if it is one of the last bastions of multiyear ice in an ice-diminished Arctic Ocean. Of course, logistics requirements will always play an important role in selecting monitoring sites (Lindsay and Zhang, 2005 [18]).

Chapter 2

Initial exploration of sea ice thickness variability and scientific question

In order to propose an observation system able to reproduce the variability of the Arctic sea ice, we must look at the predictability of the ice and investigate the dataset available.

In this second chapter we will explore and analyse the dataset that will be used in the following chapters and we will investigate the stationary character of the variability and mean state of the ice thickness. We will first present the database, then we will look at the mean, the standard deviation of the mean, the correlation between thickness of the ice and volume of the entire Arctic basin. Finally, we will look at a first method to reproduce the ice volume anomaly based on some stations measuring the thickness anomaly over the Arctic basin.

2.1 The dataset

The dataset is the CESM1-CAM5-BGC-LE set (Kay *et al.*, 2015 [15]). CESM-LE stands for the Community Earth System Model Large Ensemble. Its goal is to enable assessment of recent past and near future climate change in the presence of internal climate variability. It is a hundred years long run simulation with preindustrial, historical and near future conditions. Controlled simulation enables assessment of internal climate variability in the absence of climate change.

The CESM1(CAM5) is a global coupled climate model, that consists of coupled atmosphere, ocean, land and sea ice models. The model also includes land carbon cycle calculation, diagnostic biogeochemistry (BGC) calculations for the ocean ecosystem and the atmospheric carbon dioxide cycle.

The initial conditions are multicentury 1850 control simulation with constant preindustrial forcing. The ocean model was initialised from observations, and the atmosphere, land and sea ice models were initialised with previous CESM1-CAM5 simulations (because of their short time scale memory that are negligible on a multicentury simulation).

More practically, the data set is composed of 18 NetCDF files of 100 years time series at a rate of one data every month. Each NetCDF file will be described in the following chapters as a matrix h of dimension $(1200 \times 104 \times 320)$. The $((104 \times 320))$ matrix represents the Arctic Basin divided in 33280 grid points. Each station is paired with a specific area. So the matrix h represents the monthly thickness time series. Each point located at (i, j) over the matrix (104×320) has 1200 months of thickness time series.

After analysing the data, we saw that inside the Canadian Arctic Archipelago, there were some values of ice thickness reaching more than 60 meters. So in order to see how it impact our results in this chapter, we decided to filter these regions out (blue area in Figure 2.1) and compute the ice volume anomaly without them. More precisely, it is the region between 240° - 270° of longitude and 70° - 80° of latitude.

Filter without Canadian Arctic Archipelago

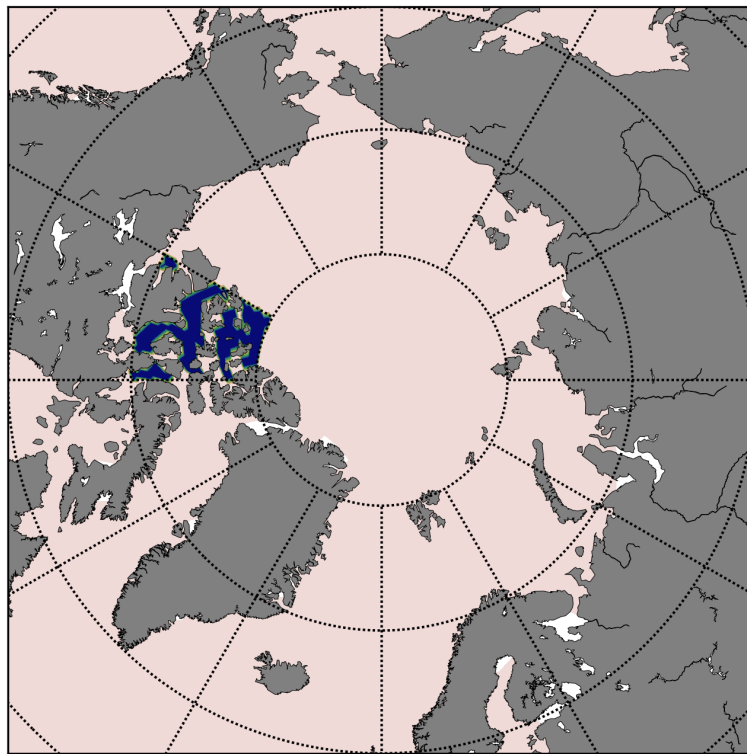


Figure 2.1: Filter of the Canadian Arctic Archipelago, the blue area is filtered out.

For more information about the model and about all the implementation hypothesis, see Kay *et al.*, 2015 [15].

Sea ice volume over the entire Arctic

Sea ice volume is an important climatology variable, it is computed as the sum of the multiplication of each area by the mean thickness over that area.

The following section's computations are based on the data anomaly, a common way to represent a variable in climatology. Indeed, it allows to show the variation of a variable beyond its trivial seasonality. We compute the mean of the variable and build a new

variable called "anomaly_variable" which is the variable minus its mean.

As an example, Figure 2.2 presents a 10 yr Arctic sea ice volume time series (a) and the corresponding ice volume anomaly time series (b). Working with the ice volume anomaly allows to get rid of the seasonal variability.

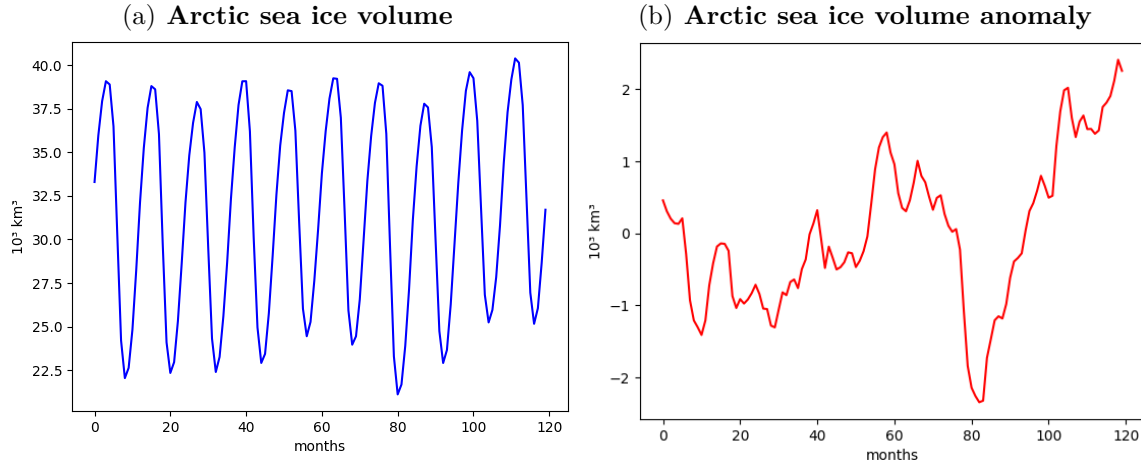


Figure 2.2: Arctic sea ice volume (a) and ice volume anomaly (b) over a 10 yr period.

Bitz *et al.* [3] try to assess the potential natural interannual, decadal, and longer timescale variability in the Arctic sea ice that results because of the special thermodynamics associated with the coupled Arctic atmosphere, sea ice, and upper-ocean system. They employed a single-column model and concluded that sea ice thickness varies predominantly at interdecadal timescales with a characteristic timescale of 15 years.

Arctic mean sea ice circulation

Figure 2.3 gives the mean Arctic sea ice circulation during 100 years. The vectors are unitary and represent the direction of the ice flow and the field plotted behind is the intensity of the flow. We recognise the main structures of the Arctic sea ice circulation, the Beaufort gyre and the transpolar drift.

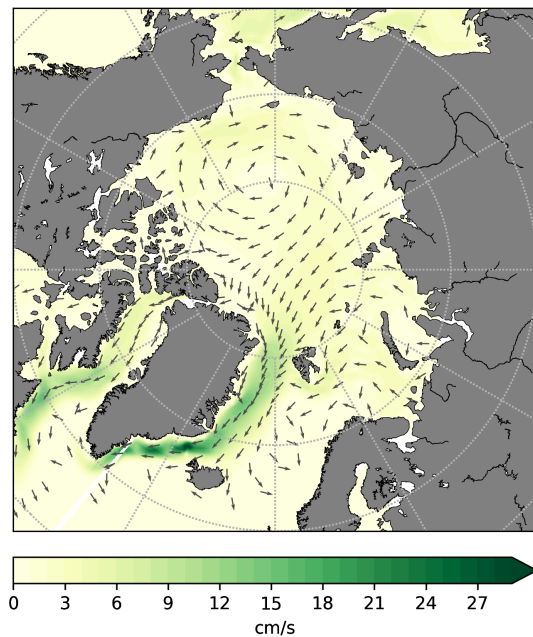


Figure 2.3: Mean Arctic sea ice circulation over a 100 yr period.

Dataset validation

One of the goal of this Master's thesis is to propose an observation system able to reproduce the Arctic sea ice variability. We will build that system based on our dataset, but the model that produced the dataset did not use reanalysis. In other words, it has not been forced by observations. So, in order to give credit to our observation system, we must know if the dataset we will use is close enough to the reality.

We compare our data base historic run with the PIOMAS data base over the 1979-2005 period. PIOMAS is a coupled ice-ocean model used to derive Arctic sea ice volume. It is capable of assimilating satellite ice concentration data and observations of sea surface temperature (Schweiger *et al.*, 2011 [24]).

The mean Arctic sea ice thickness over the 1979-2005 period is 0.55 meter for PIOMAS and 0.6 meter for our database. We show the monthly ice volume time series and monthly ice volume anomaly time series in Figures 2.4 and 2.5, respectively.

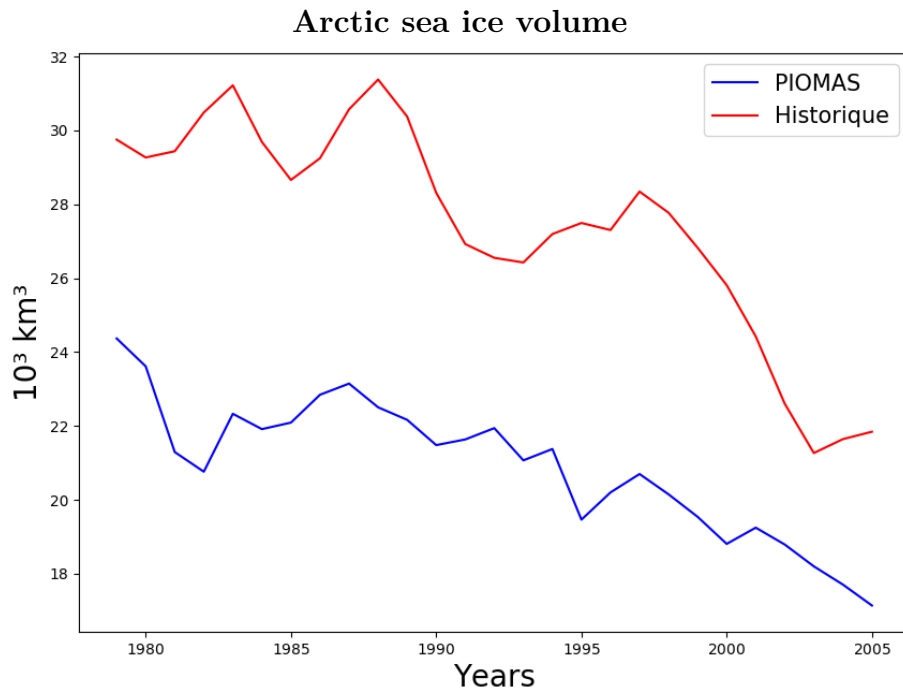


Figure 2.4: Arctic sea ice volume for PIOMAS (blue curve) and our data base (red curve) for the 1979-2005 period.

We note that our dataset ice volume time series presents a systematic overestimation of approximately $5 \cdot 10^3 \text{ km}^3$ that stays constant over the 1979-2005 period. The presence of an overestimation corroborates the difference in the mean ice thickness over the period. Figure 2.5 validates the fact that the overestimation stays constant over the period.

Because our system is based on ice thickness and ice volume anomalies, it allows not to take into account the overestimation. Indeed, applying our system in true condition with real Arctic sea ice thickness measurement will delete the overestimation and predict the correct variability of the Arctic sea ice.

The same validation as been made by Labe *et al.* [17] and they reach the same conclusion.

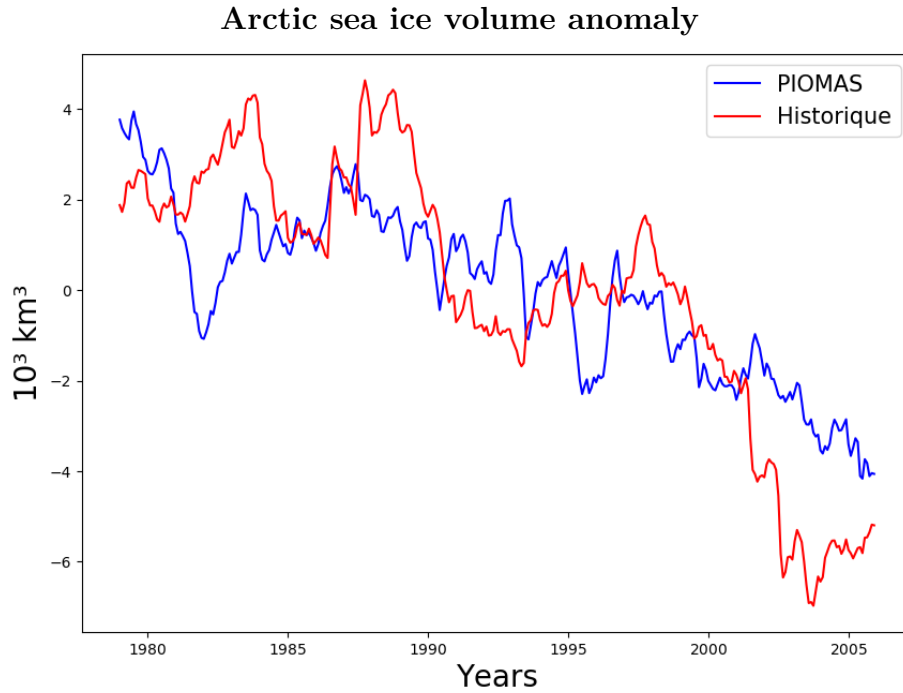


Figure 2.5: Arctic sea ice volume anomaly for PIOMAS (blue curve) and our data base (red curve) for the 1979-2005 period. The anomalies are computed over the 1979-2005 period.

2.2 Master's thesis question

The aim of this Master's thesis is to characterise the internal variability of the Arctic sea ice thickness and to propose a monitoring system of the sea ice volume variability. Finding the appropriate monitoring system of the sea ice volume variability can be viewed as answering the question :

« For N , a positive integer, where are the N station locations that have the best representation of the Arctic sea ice thickness variability ? »

We write this problem in a mathematical shape :

With $\bar{V}(t)$ the ice volume anomaly of the entire Arctic sea ice, $\bar{h}_i(t)$ the Arctic sea ice thickness at location i over time, we want to minimise the cost function $\bar{F}_N$ at time t as :

$$\bar{F}_N(t) = \|\bar{V}(t) - \omega_N \bar{h}_N(t)\|_2^2 \quad (2.1)$$

where $\omega_N = (\omega_1 \dots \omega_N)$ is a vector parameter, corresponding to the N station weights, $\bar{h}_N(t) = (h_1(t) \dots h_N(t))^T$ are the N stations with sea ice thickness measurements and $\|\cdot\|_2$ is the norm 2 operator.

The problem 2.1 has some constraints. Indeed, we have more than 33000 stations available

measuring the Arctic sea ice thickness in our database. Do we have to constrain ourselves on the number of stations, taking a very small number, or do we constrain ourselves with the quality of the fit taking as many stations as needed ? And also, what period do we consider ? Indeed, we want to represent the internal variability of the Arctic sea ice, so we must know at what timescales that variability is present.

We will answer those questions in the following chapters. In the present chapter we investigate which time period is long enough to represent the changes in the Arctic sea ice. Then, we investigate how to reproduce the Arctic sea ice volume by measuring the ice thickness at some locations and how many stations are needed. In the third chapter, we will characterise the ice thickness variability in order to select by ourselves the station locations. In the fourth chapter, we will look at an optimisation method to find the best station locations. Finally in the last chapter, we will investigate the robustness of our model under post industrial and future climate conditions.

2.3 Arctic sea ice thickness variability

In order to characterise the ice thickness variability and propose a system of observation, we first need to know the period of time that is long enough to capture the ice thickness variability of our data base. In other words, how long is the time window for which the mean thickness and the standard deviation of the mean do not change to much when we slide the time window.

Mean Arctic sea ice thickness over different periods

We looked at the Arctic mean sea ice thickness over 100, 200 and 300 yr periods on two different NetCDF files (see Figure 2.6 and Figure 5.4 in the annex). As expected, the mean shows the same pattern over the different periods, with high values along the Greenland north coast and decreasing over the Arctic basin. The same pattern is found by Blanchard *et al.* [2], when they used fully coupled GCMs (general circulation models) and sea ice-ocean-only models forced with atmospheric reanalysis and satellite measurements.

We conclude that there are no significant differences in mean ice thickness over the different NetCDF files.

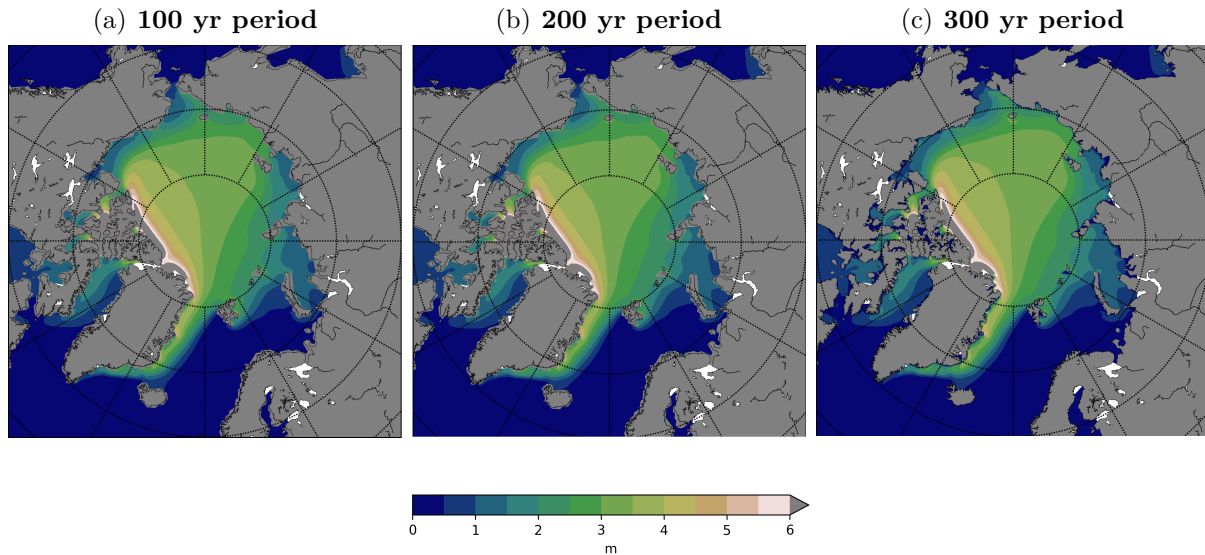


Figure 2.6: Mean Arctic sea ice thickness over 100 (a), 200 (b) and 300 (c) yr periods.

Standard deviation of mean Arctic sea ice thickness over different periods

A first way to represent the ice thickness variability is to explore the standard deviation of the annual mean ice thickness. Figure 2.7 and Figure 5.4 in the annex present the standard deviation of the mean Arctic sea ice thickness over 100, 200 and 300 yr period on two different NetCDF files.

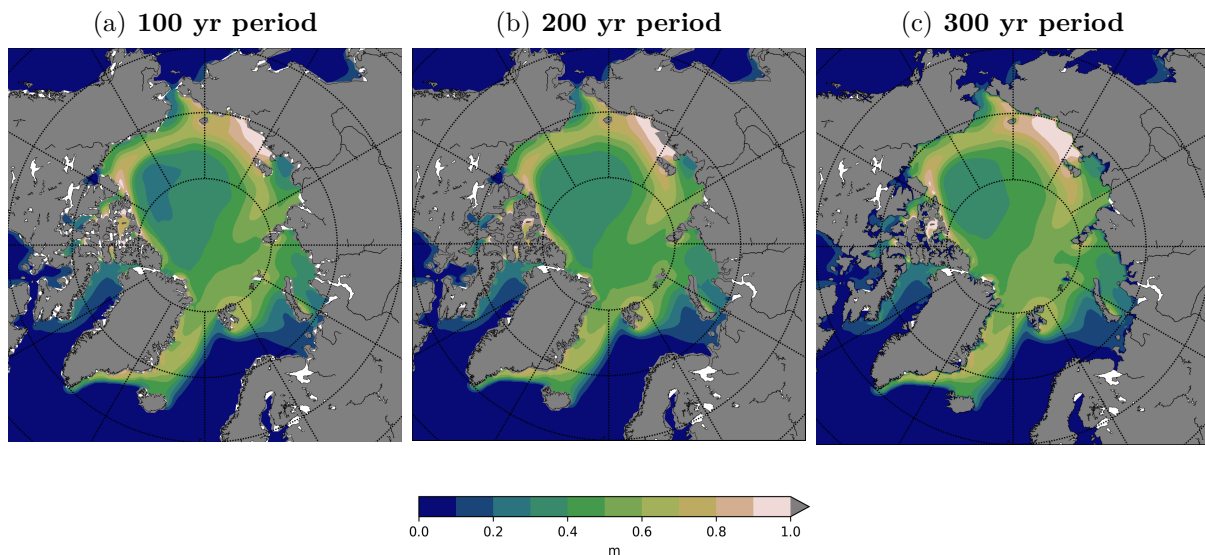


Figure 2.7: Standard deviation of the mean Arctic sea ice thickness over 100 (a), 200 (b) and 300 (c) yr periods.

As for the mean, the standard deviation does not present important changes when we enlarge the range of reference. For each window, the standard deviation exhibits the same pattern. It presents a minimum at the center of the Arctic basin and increases around

that point symmetrically towards the coasts. There are maximum values in the East Siberian Sea and high values in the Beaufort Sea and on the east coast of Greenland, near the Fram Strait (at the Odden's location) and in the Canadian Archipelago.

We find the same pattern of standard deviation in Blanchard *et al.* [2], especially for the CCSM4 models. Finding similitude between our data set and the models of the Blanchard article is again a good way of validating our database. Indeed, those models are forced with atmospheric reanalysis and satellite measurements.

Ice thickness and ice volume anomalies correlation over different periods

Another useful quantity is the correlation map between the global ice volume anomaly over the entire Arctic with the local ice thickness anomaly. Figure 2.8 and Figure 5.4 in the annex show the correlation map over 100, 200 and 300 yr periods. The correlation map presents some strong correlation between thickness and volume anomalies in the center of the Arctic basin, decreasing towards the coasts.

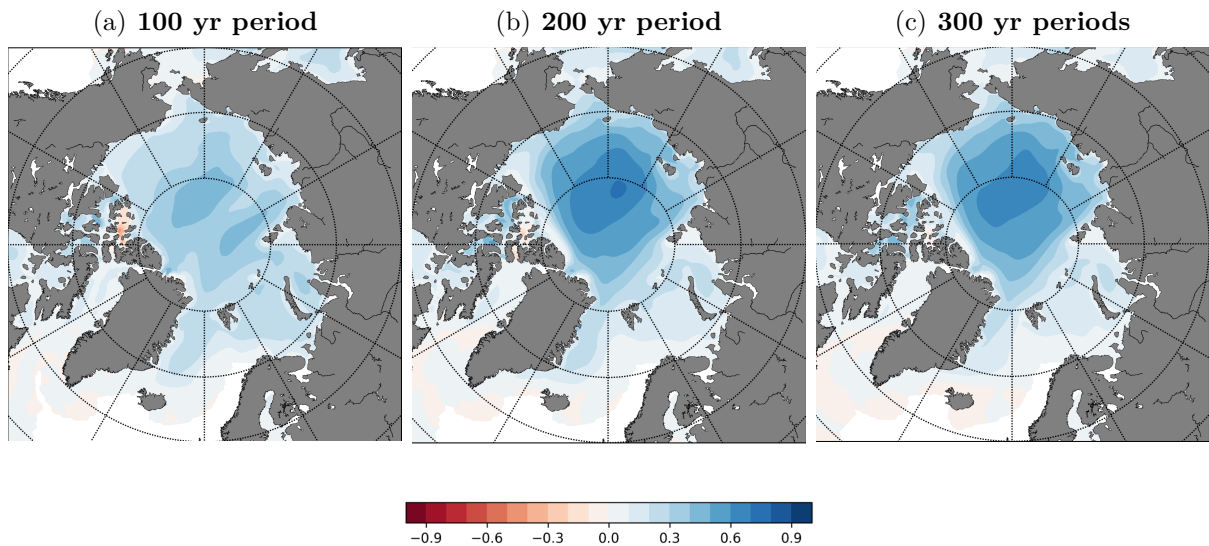


Figure 2.8: Correlation of the Arctic sea ice volume anomaly with the local anomalies of ice thickness over 100 (a), 200 (b) and 300 (c) yr periods.

We conclude that we need at least 200 years in order to reproduce a similar pattern for different periods. Once 200 years is reached, each map presents the same pattern, an important correlation at the center of the Arctic basin. The less but still significant pole of positive correlation on the Canadian Archipelago is a drawback of the model and will be removed (see Figure 2.1) in the following parts of the Master's thesis.

2.4 Multilinear regression

This section is a first trial to analyse how we are able to describe the ice volume anomaly of the entire Arctic based only on a few stations measuring the evolution of the ice thickness anomaly. The first method used is a multilinear regression. It is a linear approach to model the relationship between a variable Y and n other variables $X = (x_1, x_2, \dots, x_n)$

$$Y = a_1x_1 + a_2x_2 \quad \dots \quad + a_nx_n + b \quad (2.2)$$

Where b is the residual and a_i are the weights given to each station.

In our case, Y is the ice volume anomaly that we want to predict and x_i are the thickness measurements at the stations.

Selection of stations

Based on the previous section about the correlation between the ice volume and ice thickness, we selected 10 stations over the Arctic (see Figure 2.9) that have a good correlation with the ice volume and that cover the main regions of the Arctic.

The first station is placed in the middle of the Arctic basin, the second is located in the Fram Strait, the third in the East Siberian Sea, the fourth in the Beaufort Bay, the fifth in the North Coast of Greenland, the sixth in the Laptev Sea, the seventh in the Kara Sea, the eighth in the Hudson Bay, the ninth in the Chukchi Sea and the tenth in the Bering Sea.



Figure 2.9: Stations selected in the Arctic.

Quality of the model

Figure 2.10 presents the quality of the prediction with different numbers of stations. Each graph shows the true ice volume anomaly (in blue) from the 500 to 700 yr period, the predicted ice volume anomaly trained on the same period (in red) and the predicted ice volume anomaly trained on another period (in this case the 700 to 900 yr period - in green in the graph). By trained we mean that the weights are computed on that specific period.

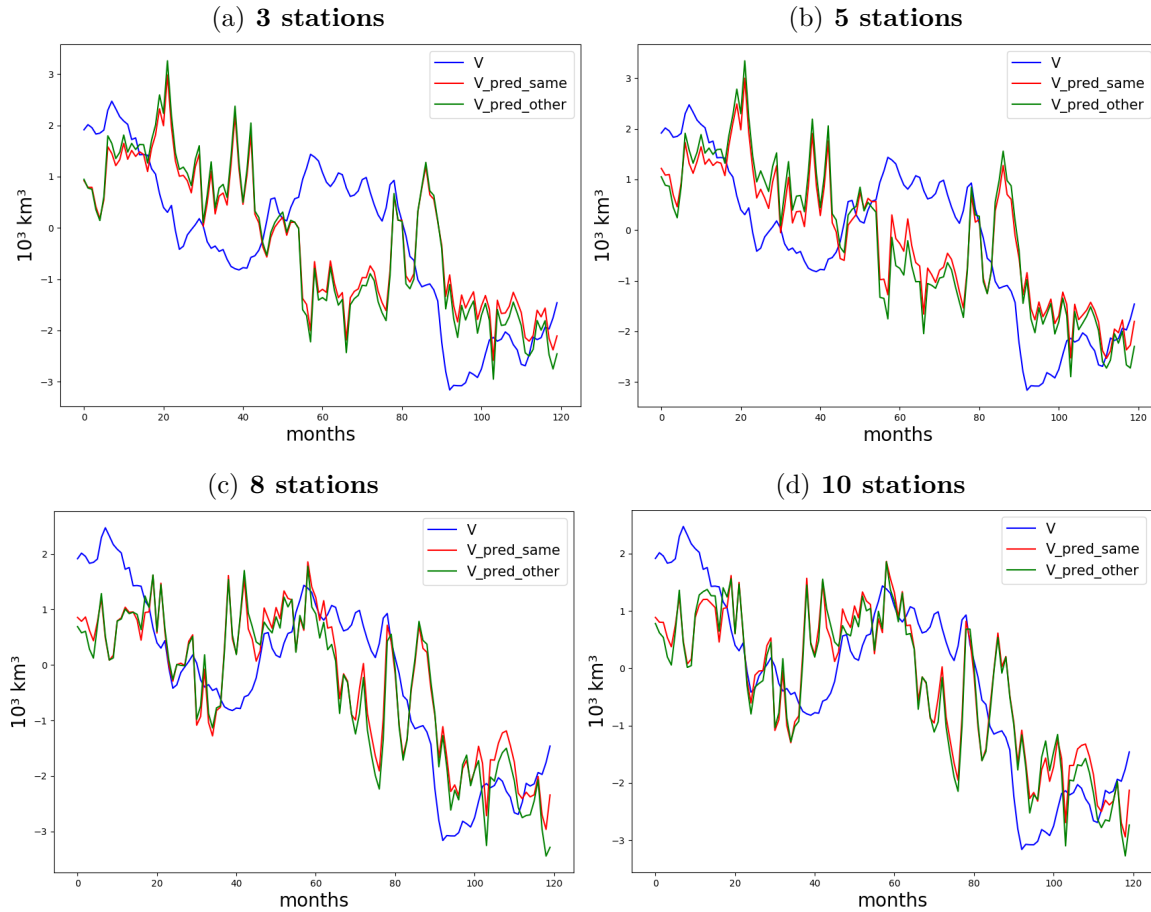


Figure 2.10: True ice volume anomaly over the Arctic and predicted ice volume anomaly trained on the same period and on another one - 3, 5, 8 and 10 stations.

As expected, the prediction of the ice volume anomaly is more accurate when the number of stations increases. Indeed, the error between the true ice volume anomaly and the predicted one decreases when the number of stations increases. The way it decreases is relevant. It shows the quality and the good representation of the volume anomaly by each station. Figure 2.11 (a) shows the evolution of the error when the number of stations increases. The weights are computed on a 200 yr ice thickness time series and the error is the difference between the initial volume anomaly of the 10 first years of those 200 years and the predicted one. The error is computed as the root mean square (rms) of the difference between the true ice volume anomaly and the predicted one. We also show, in Figure 2.11 (b), the evolution of the correlation between the two quantities.

Furthermore, the differences between both predictions are negligible. This reveals that the period of time chosen is long enough. In other words, the quality of the model will

be the same for any NetCDF file, as soon as we use the model on a 200 yr period.

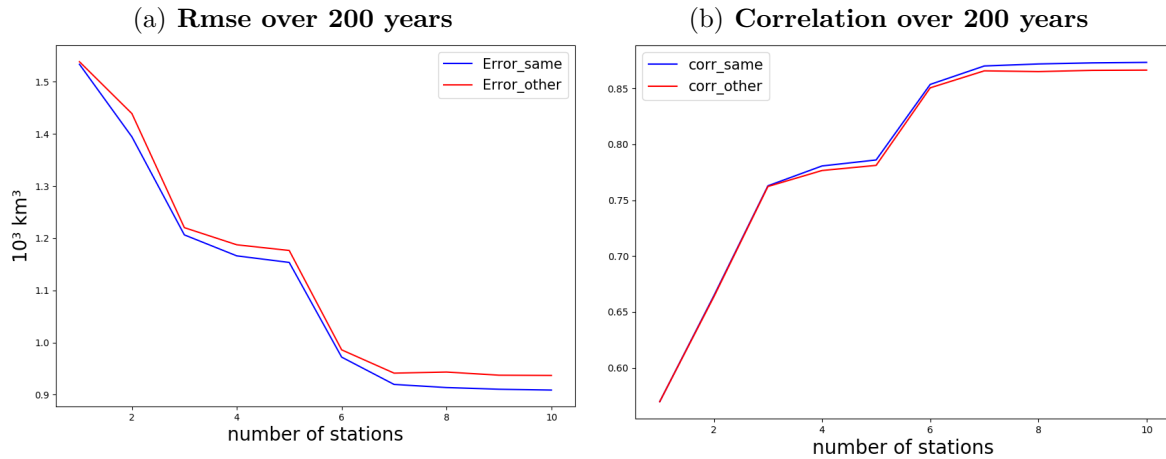


Figure 2.11: Error (a) and correlation (b) of the prediction in function of the number of stations for both predictions, the one trained on the same period (blue curve) and the one that is not (red curve).

As for the prediction trained on the same period, the error of the prediction trained on another period decreases as well and the correlation increases when the number of stations increases (see Figure 2.11). The fact that the error does not decrease at each station we add can be explained by the fact that the coefficients are computed on the 200 year time series and that the error is computed on the differences of the first 10 years on both time series. In other words, the coefficients are computed for the best fit of the 200 year time series and it may not be the best for the first 10 years.

Furthermore, we see that rmse seems to reach a plateau. It allows to conclude that the number of 10 stations should not be exceeded. We will use 10 stations in our observation system in chapter 4.

2.5 Summary

This second chapter allowed to introduce the dataset, to learn about the variability in space and time of the Arctic sea ice thickness. It gave us the opportunity to build a first method. Globally this first method presents some good results, we are able to reproduce with a sufficient level of accuracy the ice volume anomaly over the entire Arctic basin only with a few stations selected by hand. We reach a root mean squared error of $0.9 \cdot 10^3 \text{ km}^3$, we will keep that value to see how we can improve our model in the next chapters.

We conclude that a period of 200 years and 10 stations are enough to predict the variability of the ice with accuracy.

Chapter 3

Characterisation of the Arctic sea ice internal variability

In this third chapter we will analyse the spatial and temporal inter-annual variability of the ice thickness anomaly. To achieve that, we will investigate the unknown seasonal-dependence of the variability using the principal component analysis. Then, we will explicit and describe the major modes of variability and compare them with the scientific literature. And finally we will explore the physics explanations of the modes of variability.

3.1 Principal component analysis

A principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of linearly uncorrelated variables called principal components. This transformation is defined in such a way that each principal component, starting with the first one, accounts for as much of the variability in the data as possible, and each succeeding component is orthogonal to the preceding components.

Let's say that all the data are stored in a $N \times M$ matrix X , with N the number of samples and M the number of parameters needed to define the spatial structure (in our case, values at individual grid points). In other words, each row of X is a "snap shot" of the data at an instant in time.

A PCA requires centered data, so we will work with the monthly mean sea ice thickness anomaly time series, noted here X'_{nm} .

Our matrix X'_{nm} is too big to be easily understood : we have 33200 variables over 1200 samples for a 100 yr period of ice thickness time series. We want to characterise the variability by a small number of spatial structures. But, how can we find the best spatial structures ? The basic idea of a PCA is to use the data to choose the components. If space and time are discretized, and only a single component is considered, then the data are approximated in the form :

$$X'_{mn} = s[u]_n[v]_m$$

where the N dimensional vector u defines the time variability, the M dimensional vector v defines the spatial structure and s defines the amplitude of the component.

The last equation can be rewritten as

$$X' = suv^T \quad \text{with } T \text{ the transpose operator}$$

But again, how to find the best way of choosing s , u and v .

We want to minimise the sum of square of the difference between X' and the components.

$$\|X' - suv^T\|_F^2 = \sum \sum (X'_{nm} - s[u]_n[v]_m)^2$$

(the F is for the Frobenius norm and it can be interpreted as a distance between the data and the components). The minimisation problem produces an (almost) unique solution by imposing constraints. Those constraints can be written as :

$$\sum [u]_n^2 = 1, \quad \sum [v]_m^2 = 1, \quad s > 0$$

These constraints fix the length of u and v , but not their signs, $-u$ can be multiplied by negative one provided v also is multiplied by negative one, since the two sign changes cancel in the product. Then, the minimisation problem can be rewritten as :

$$\text{Minimise } \|X' - suv^T\|_F^2 \quad \text{subject to } u^T u = 1, \quad v^T v = 1 \quad \text{and } s > 0$$

The solution we seek is unique up to a sign.

Solution based on singular value decomposition

The above optimisation problem is solved by the singular value decomposition (SVD). The SVD is a matrix decomposition as :

$$X'_{[N \times M]} = U_{[N \times N]} S_{[N \times M]} V_{[M \times M]}^T$$

where U and V are unitary matrices, S is a diagonal matrix, with non negative diagonal elements. The columns of U are called the left singular vectors, the columns of V are called the right singular vectors, and the diagonal elements of S are called the singular values. We also have :

$$U^T U = U U^T = I \quad \text{and} \quad V^T V = V V^T = I$$

(We suppose that the singular values are in decreasing order)

It can now be shown that the truncated sum :

$$\dot{X}' = s_1 u_1 v_1^T + s_2 u_2 v_2^T + \dots + s_t u_t v_t^T$$

where the sum stops at t terms, minimises $\|X' - \dot{X}'\|_F$ out of all matrices $\dot{X}'$ of rank t . This means that the first two singular values and vectors can be used to construct the best rank approximation of X' with respect to the Frobenius norm.

We define the empirical orthogonal function component as the right singular vector v_i of the data matrix X' . The component defines the spatial structure of the component.

We also define the principal components (PC) as the left singular vector times the singular value $s_i u_i$ of the data matrix X' . The principal components define the time series of the components.

A useful tool is the fraction of variance explained by the i th component. It is explained by the variance of the PC divided by the total variance :

$$\frac{s_i^2}{s_1^2 + \dots + s_R^2}$$

where R is the number of components (Delsoll and Tippet, 2018 [8]).

3.2 Results

In this section, we apply a PCA on different matrices of ice thickness anomaly. For each PCA, we present the first five principal components with the percentage of explained variability.

In order to apply the PCA, we must compute the matrix of ice thickness anomaly. We used a 200 yr period. Then we had to transform our anomaly matrix from a $(104 \times 320 \times 2400)$ matrix to a (2400×33200) matrix. Next, we performed the PCA on that specific matrix. And finally, in order to represent our empirical orthogonal functions on a map, we had to reshape the (2400×33200) matrix into a $(104 \times 320 \times 2400)$ matrix.

PCA on Arctic sea ice thickness anomalies

The first PCA analysis we made is an analysis of the entire Arctic sea ice thickness monthly anomaly time series matrix. Figure 3.1 presents the first five principal components. The first component (a) presents a dipole between the Fram Strait and the East Siberian Sea. (The sign of the dipole here has no importance). The second one (b) represents a pole of variation with the highest values in the East Siberian Sea and going into the center of the Arctic basin. The third one (c) exhibits another dipole perpendicular to the first one. The fourth one (d) shows a dipole which presents the characteristic of the last two. The fifth component (e) presents a complicated dipole.

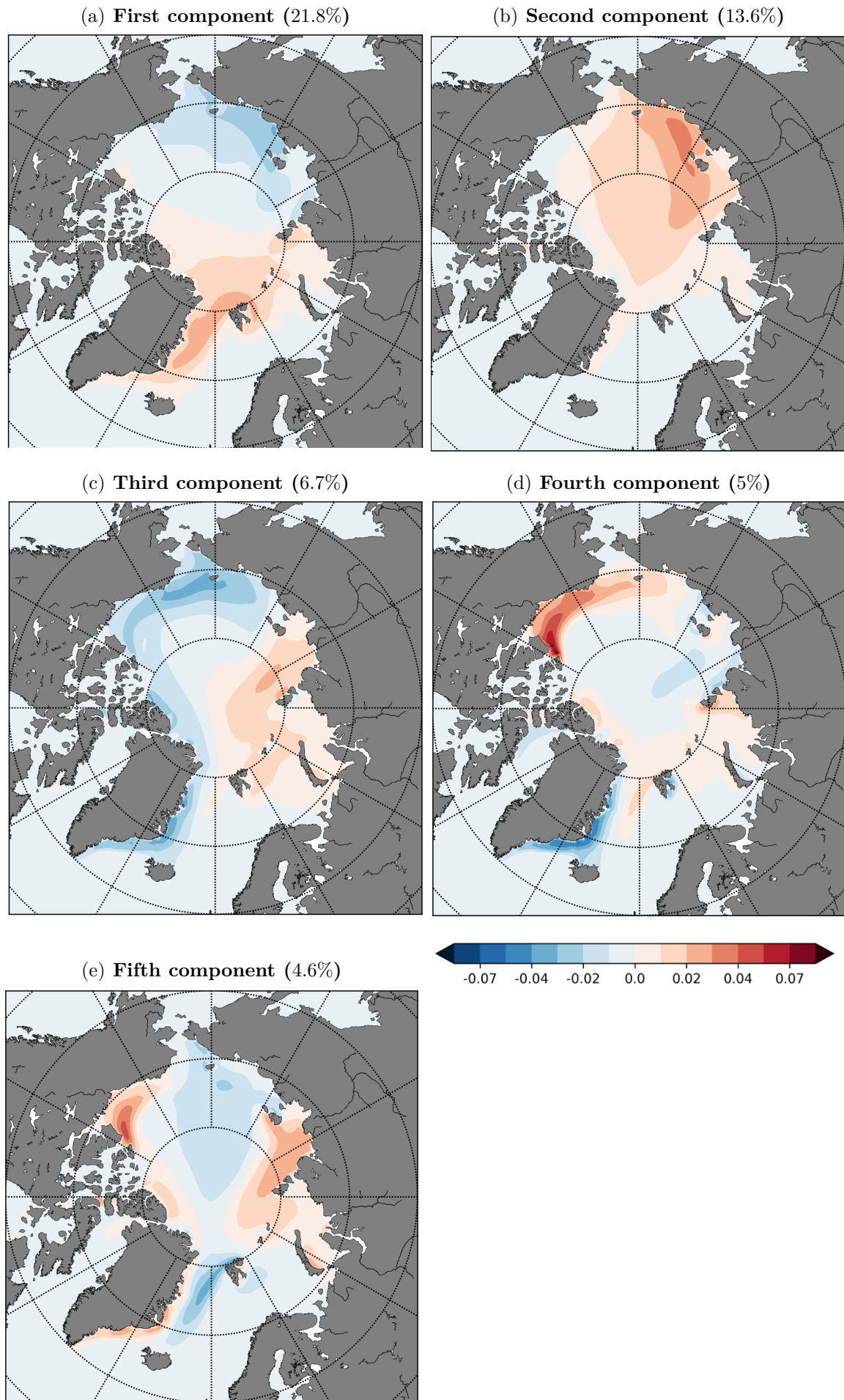


Figure 3.1: First five components of the PCA on the Arctic sea ice thickness anomaly time series over a 200 yr period. And the percentage of variability of each component.

PCA on mean over summer months Arctic sea ice thickness anomaly

In order to analyse the possible seasonal dependence of the variability, we apply this second principal component analysis on the mean over summer months (July, August, September) of the Arctic sea ice thickness anomaly time series. That allows us to study specifically the summer variation and also to get rid of the seasonal cycle. The first five components are presented in Figure 3.2. The first component (a) shows a dipole similar to the first component of the PCA on monthly thickness anomaly time series; it is a dipole between the East Coast of Greenland (and Fram Strait) and the East Siberian Sea. The second component (b) exhibits a pole of variability centered on the East Siberian Sea, going to the center of the Arctic basin; this pole is very similar to the second component of the PCA on the monthly thickness anomaly time series. The other components present some complicated dipoles.

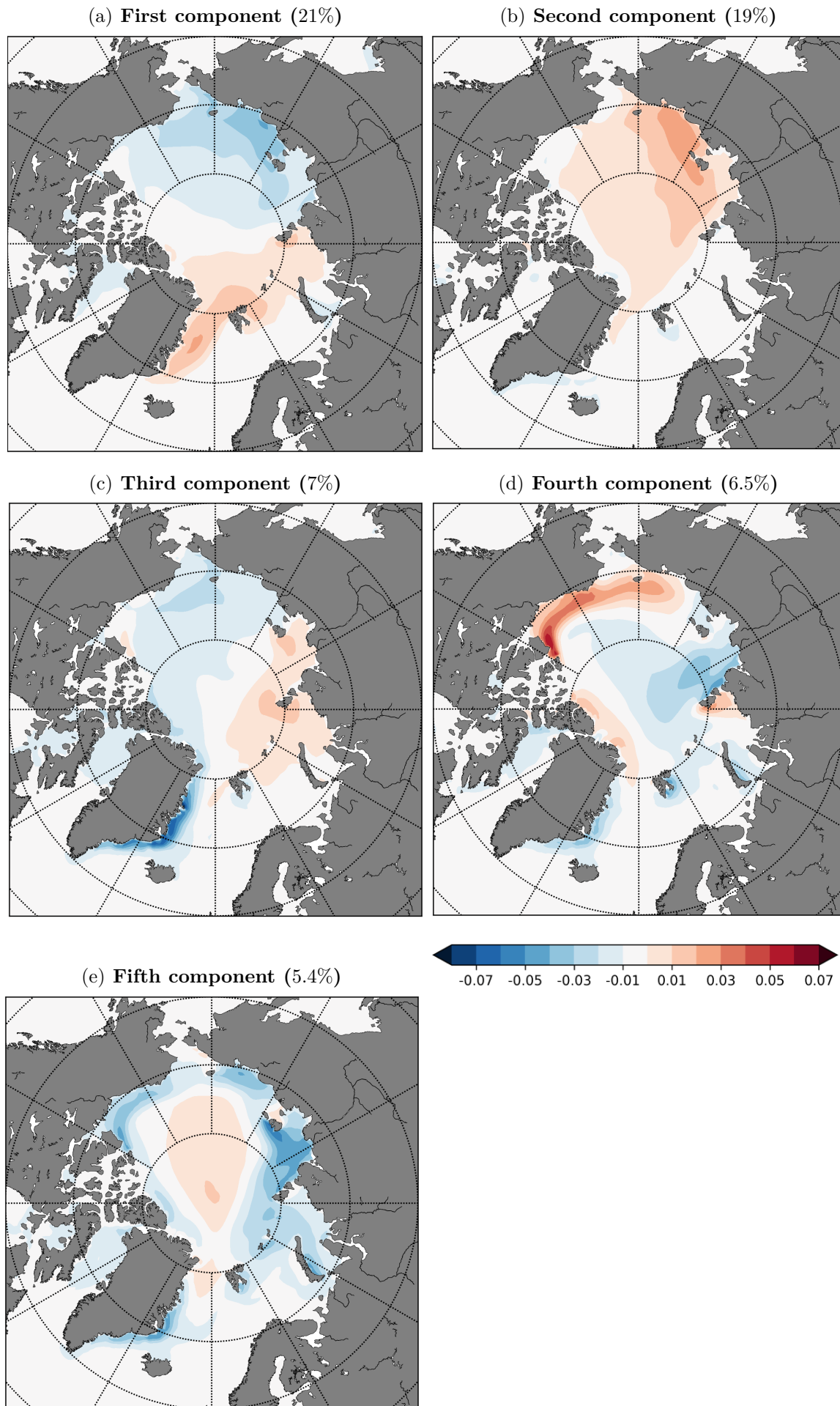


Figure 3.2: Components of mean over summer months Arctic sea ice thickness anomaly time series over a 200 yr period. And the percentage of variability of each component.

PCA on mean over winter months Arctic sea ice thickness anomaly

We now look at the possible winter dependence of the variability by applying the third principal component analysis on the mean over winter months (January, February, March) of Arctic sea ice thickness anomaly time series. The first five components are presented in Figure 3.3. The first component presents a dipole between the East Siberian Sea and the Fram Strait, it looks very similar to the first component of the PCA on the monthly thickness anomalies time series. The other modes show pattern of variability close to the summer mean modes with few differences in shape and amplitude.

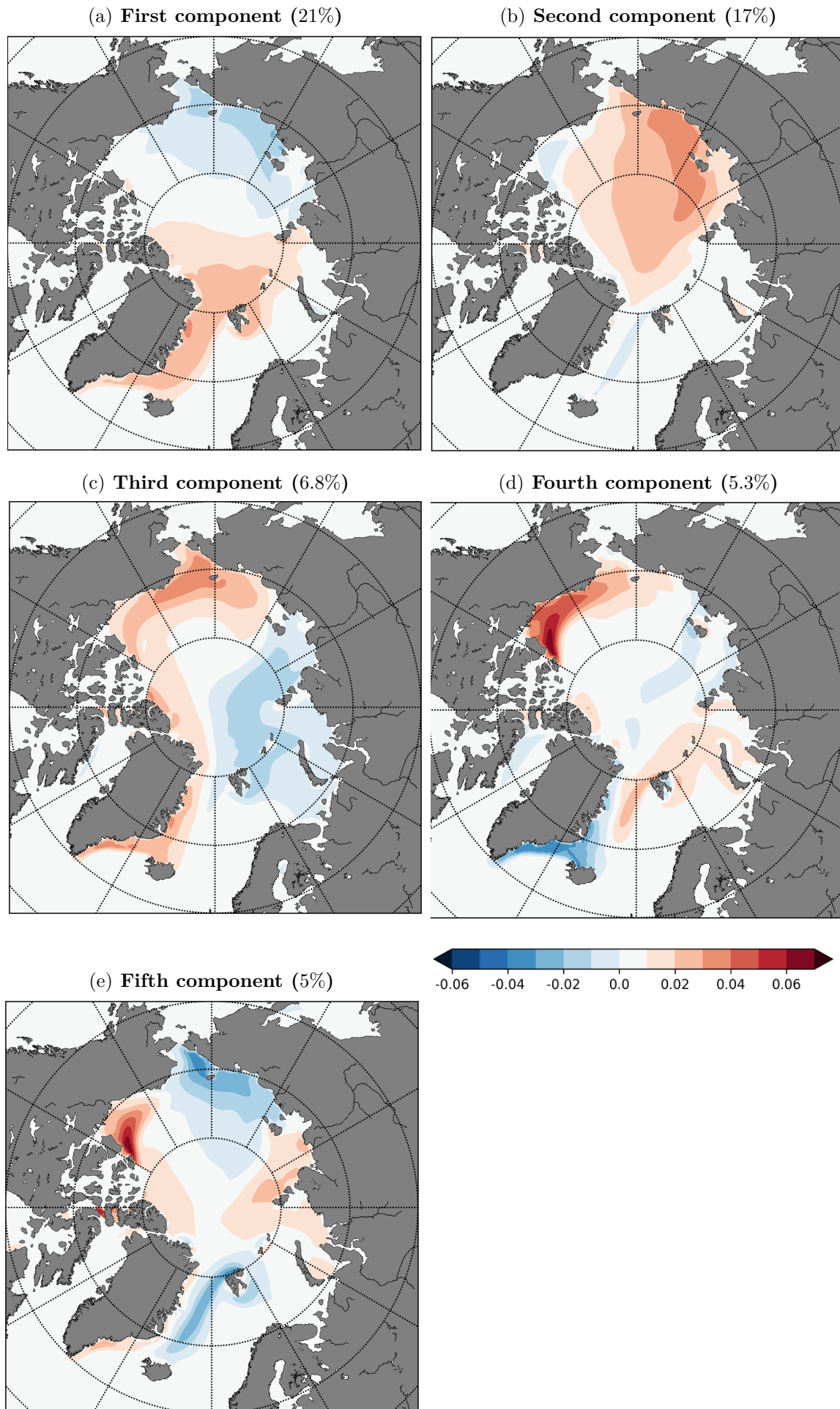


Figure 3.3: Components of mean over winter months Arctic sea ice thickness anomaly time series over a 200 yr period. And the percentage of variability of each component.

PCA on annual mean Arctic sea ice thickness anomaly

The last principal component analysis we have made is on the annual mean of the Arctic sea ice thickness anomaly time series. This analysis does not take into account the summer or winter variability, it gives a very rough and global overview of the Arctic sea ice thickness variability. The first five components are presented in Figure 3.4. The first component presents a large dipole between the East Siberian Sea and the Fram Strait, very similar to the first component of the three previous PCA. The second component shows a pole on the center of the Arctic basin. It can be seen as a mean of the summer and winter pole over the Arctic basin and East Siberian Sea. The third one exhibits a dipole similar to the third component of the monthly thickness anomaly time series PCA and to the third component of winter mean thickness anomaly time series PCA, a dipole between the Chukchi Sea, Beaufort Sea, the north of the Canadian Archipelago and the Greenland and the east of the Arctic basin, centered on the Laptev Sea.

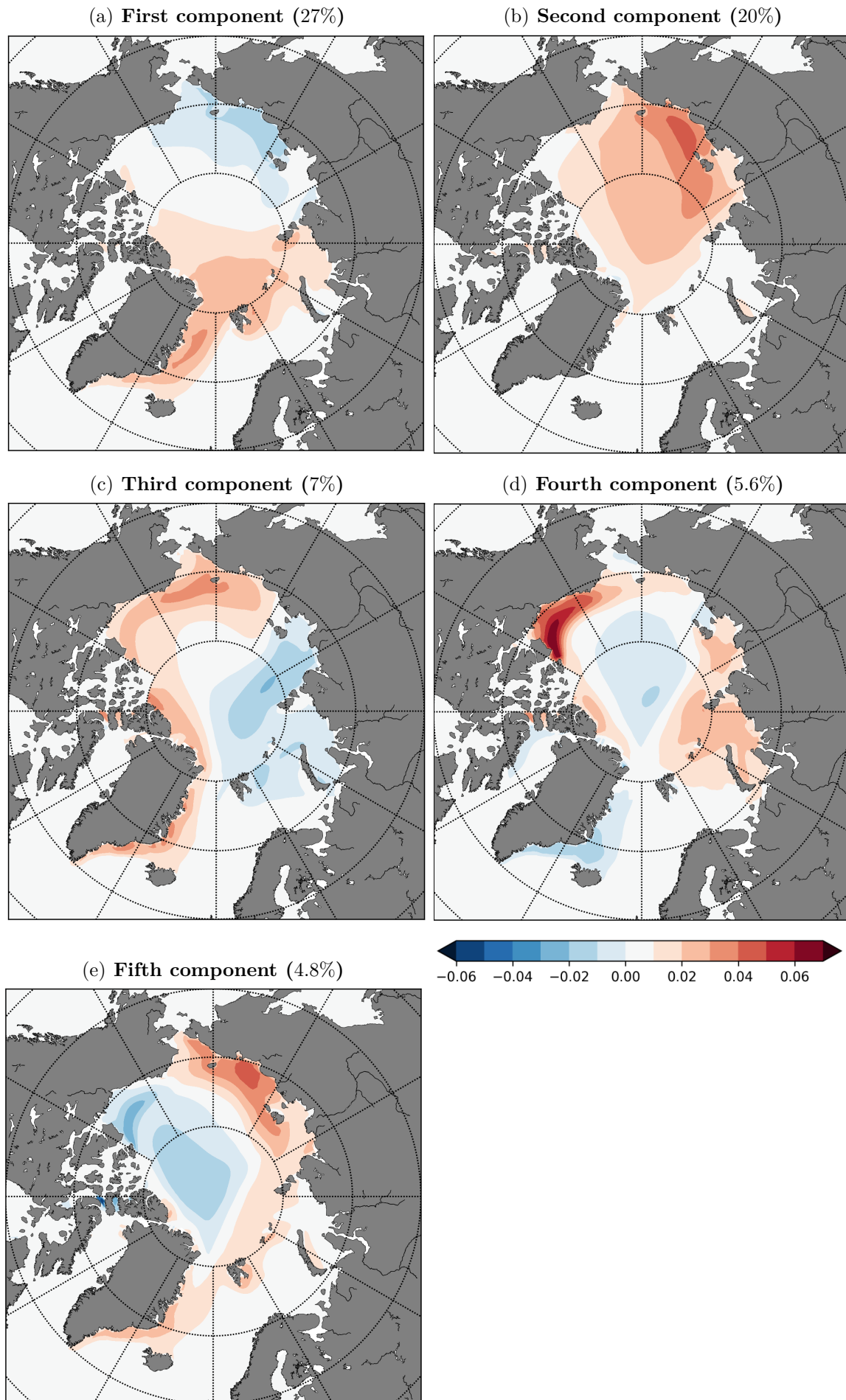


Figure 3.4: Components of annual mean Arctic sea ice thickness anomaly time series over a 200 yr period. And the percentage of variability of each component.

Figure 3.5 presents the cumulated variability explained by each components. As expected, the cumulated variability increases with the number of components. For the same number of components, the annual mean ice thickness anomaly components are the one that account for the most variability, followed by the mean over winter and summer months PCA and finally the monthly mean PCA. Indeed, taking a mean over summer, winter months or over the entire year decreases the variability of the dataset.

The first five components of the monthly mean Arctic sea ice thickness anomaly PCA explain respectively 21.8%, 13.6%, 6.7%, 5%, 4.6% of the variability. Respectively 21%, 19%, 7%, 6.5%, 5.4% of the variability for the PCA over summer months. Respectively 21%, 17%, 6.8%, 5.3%, 5% of the variability over the winter months, and finally, 27%, 20%, 7%, 5.6%, 4.8% (cumulated 64%) of the variability for the PCA over annual mean.

Figure 3.5 indicates than more that 80% of the variability is explained with the ten first components. All the variability can only be explained with all the components (there are 2400 of them in this case). So explaining 50% of the variability with only 5 components is very efficient.

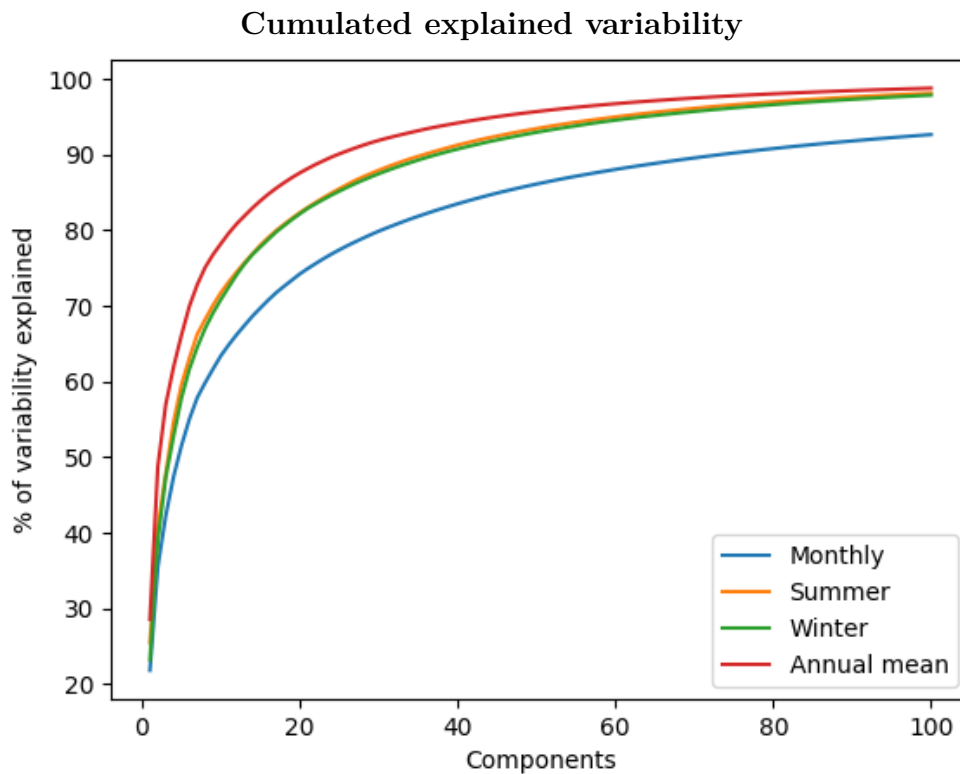


Figure 3.5: Percentage of the variability explained in function of the number of components, for the monthly, summer, winter and annual mean ice thickness anomaly time series PCA.

3.3 Validation of the modes of variability

In Lindsay and Zhang [18], a PCA has been applied to annual mean ice thickness derived from a model simulation over a period of 50 years. The particularity is that they applied

the PCA over the time and not over the stations, calling their results EOF (empirical orthogonal function). Then, they computed the correlation between the annual mean ice thickness and the first 3 EOF. So, even if they did not applied the PCA on the spatial dimension, like we did, their results present the first 3 spatial patterns that represent most of the variability of the ice.

Keeping in mind that their results take into account the anthropogenic forcing of the last century, comparing our results with theirs is a first attempt of validation. We will try to find our poles of variation into their results.

Figure 3.7 shows the 3 first modes of variability of the ice based on annual mean ice thickness (our results on the left, Lindsay and Zhang on the right). The first mode of Lindsay and Zhang shows a pole of variability on the center of the Arctic basin that looks very similar to our second component. On the other hand, our first component presents the same pattern as the third mode found by Lindsay and Zhang, a large dipole between the Fram Strait and the East Siberian Sea. Finally, our third component is similar the their second mode; it exhibits a dipole between the Barents Sea and the Beaufort Sea.

We also look at the scientific paper by Fuckar *et al.* [11]. They used K-means cluster analysis to determine robust modes of the Arctic sea ice variability based on sea ice thickness reconstructed with an ocean-sea-ice general circulation model. They present the first 3 winter and summer sea ice thickness clusters pattern over the 1958-2013 period (like the Lindsay and Zhang results, they take the anthropogenic forcing into account). Figure 3.6 presents the first 3 clusters pattern for the summer mean of 1958-2013.

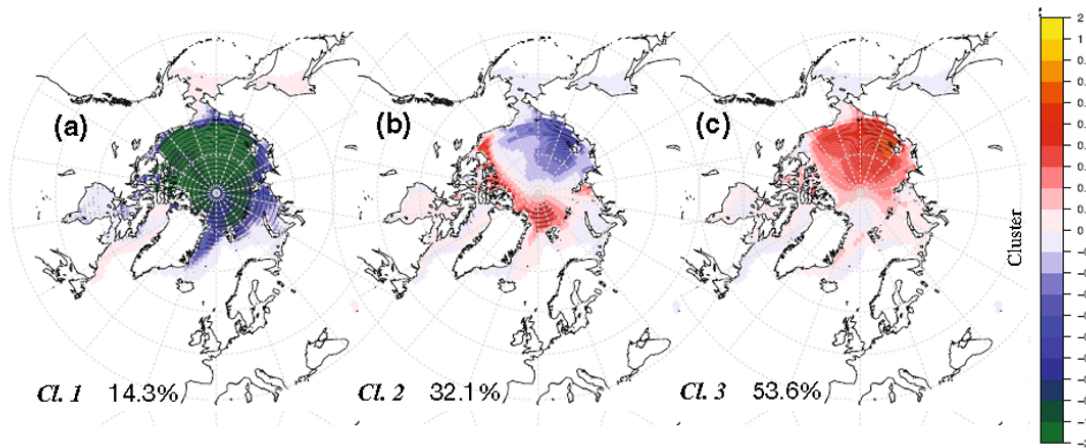


Figure 3.6: 3 first summer Arctic sea ice thickness clusters pattern over 1958-2013 period (Fuckar *et al.*, 2015 [11]).

We find a similitude between our modes of variability and those presented in Figure 3.6. The first mode is predominantly a negative or thinning mode called the central Arctic thinning (CAT) cluster. Cluster 2, which looks like our first pole of summer mean variability, is an Atlantic-Pacific dipole (APD) mode characterized by a large area of positive sea ice thickness anomalies along the coast of Canadian Archipelago and reaching the Greenland, Barents and Kara Seas and negative anomalies extending from the Beaufort Sea to the Laptev Sea. The third cluster is primarily a positive mode and exhibits similitude with our second summer mean mode of variability (Fuckar *et al.*, 2015 [11]).

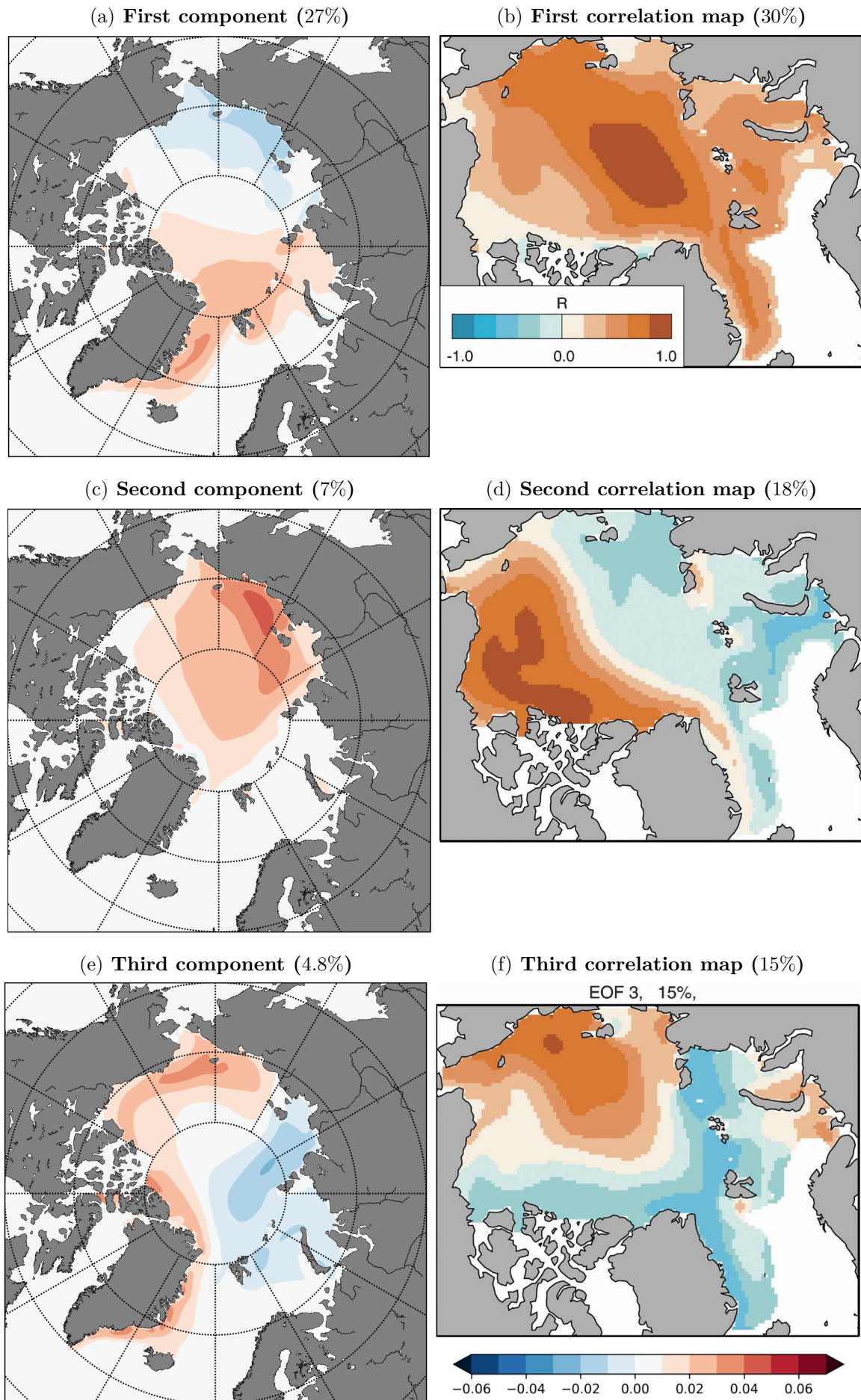


Figure 3.7: Comparison between correlation map between EOF on annual mean thickness anomaly and the annual mean thickness (b, d, f) by Lindsay and Zhang [18] and our annual mean PCA 3 first modes of ice thickness variability (a, c, e).

As we said above, Lindsay and Zhang obtained their results on a 50 yr period. In the previous chapter, we concluded that a 200 yr period is needed to have a variability pattern that stays constant over the entire dataset. So we decided to check how representative are our components if we compute them on a shorter period. Figure 3.8 presents the first principal component of two PCA computed on two distinct 50 yr period.

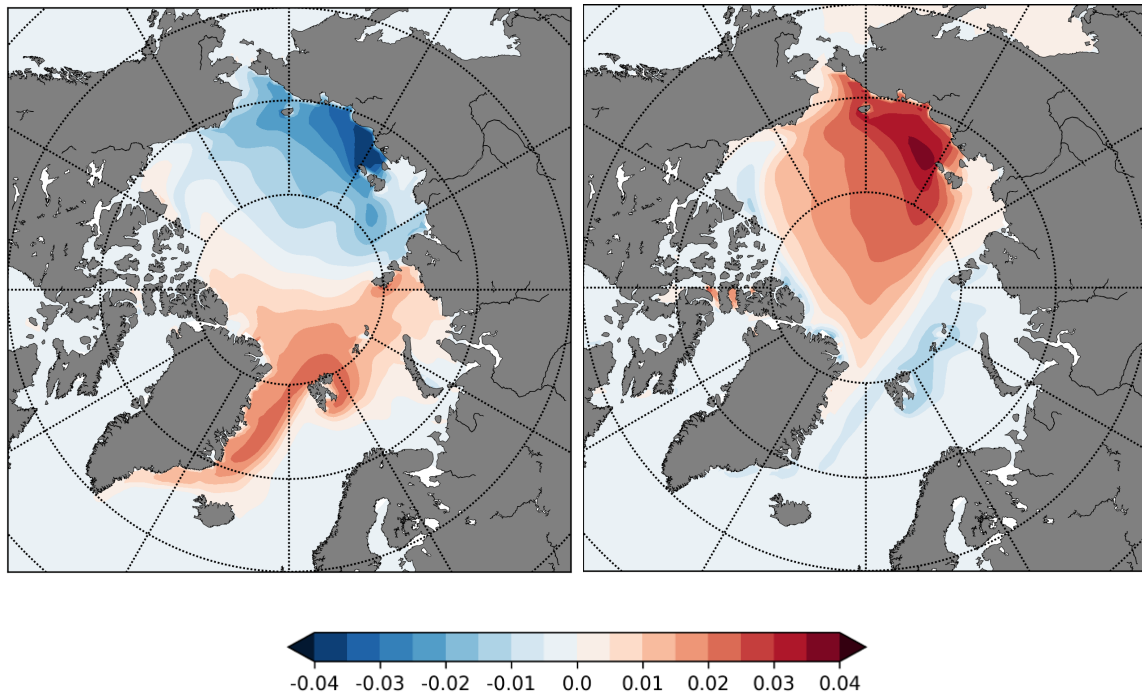


Figure 3.8: First components of PCA over annual mean ice thickness over two different 50 yr period.

The first components are different in amplitude and in shape (the sign here has no importance). Even if both present a maximum pole of variability in the East Siberian Sea, the rest of the pattern looks very different. As a conclusion, the fact that we find our poles of variability in the results of Lindsay and Zhang and Fuckar *et al.* is a first way of validation but we must keep in mind that the comparison is unreliable taking into account that the period is not representative and that they used the anthropogenic forcing.

3.4 Characterisation of the poles of variability

The three major poles of variability that account for more than 50% of the variability of the dataset are described here :

- The East Siberian Sea - Fram Strait Dipole : this first mode of variability represents 21.8 % of the total variability. It corresponds to a dipole between the East Siberian Sea and the Fram Strait. This mode shows relevant differences in shape between PCA over summer and winter months. The East Siberian Sea region of the dipole is larger in shape and amplitude while summer months. On the other hand, the Fram Strait region of the dipole is larger in shape and amplitude while winter months.

- The East Siberian Pole : the second mode of variability corresponds to a pole of variability centered on the East Siberian Sea and reaching the center of the Arctic basin. It accounts for 13.6 % of the variability. This pole is well represented on the second component of the summer and winter mean PCA both in shape and intensity, but accounts for 19% for the summer and 17 % for the winter PCA.
- The third mode : the third mode of variability that counts for 6.7% of the variability corresponds to a dipole between the Laptev and Kara Seas on the one hand and the east coast of Greenland, the Chukchi and Beaufort Seas on the other hand. It is well represented on the summer and winter mean PCA but with some differences. In the summer third component, the dipole in the east coast of Greenland is far more important in intensity than in the Beaufort and Chukchi seas. For both the summer and winter PCA, this third mode accounts for 7% of the variability.

3.5 Physical meaning of the poles

In order to explain the Arctic sea ice natural variability, we must look at the Arctic Oscillation¹ (AO). Indeed, we saw that the principal seas that are concerned with the modes of variability are the Beaufort Sea, the East Siberian Sea, the Fram Strait and the Laptev Sea. And we know that the ice activity in those regions is strongly related with the AO's impact on the ice circulation, depending if the AO index is high or low.

These zones are the regions where the Beaufort Gyre, the Transpolar Drift Stream, the Laptev Sea Gyre and the East Siberian Circulation are located². When the index of the AO is high (Figure 3.9 (b)), the cyclonic anomaly implies increased divergence of sea ice due to Ekman transport at the sea surface to the right of the wind forcing. This divergence should enhance the formation of new ice in the open leads during winter, and thus enhance the flux of heat from the ocean. The decreased convergence in the reduced anticyclonic Beaufort gyre implies less ridging and less recirculation of ice. Both processes should contribute to the thinning of sea ice over the Arctic Ocean (Rigor *et al.*, 2002 [20]).

In other words, when the AO index is high (Figure 3.9 (b)), ice in the Beaufort Gyre takes a year longer to be advected from the west to the east Arctic than during the low index (Figure 3.9 (a)), and the ice passing over the North Pole reaches Fram Strait more quickly during high index conditions. Figure 3.9 shows how the AO can affect the drift of the ice and thus how it can apply a variability on the Arctic sea ice thickness depending on whether his index is high or low.

¹Section 1.5 page 18

²Figure 1.4 page 10

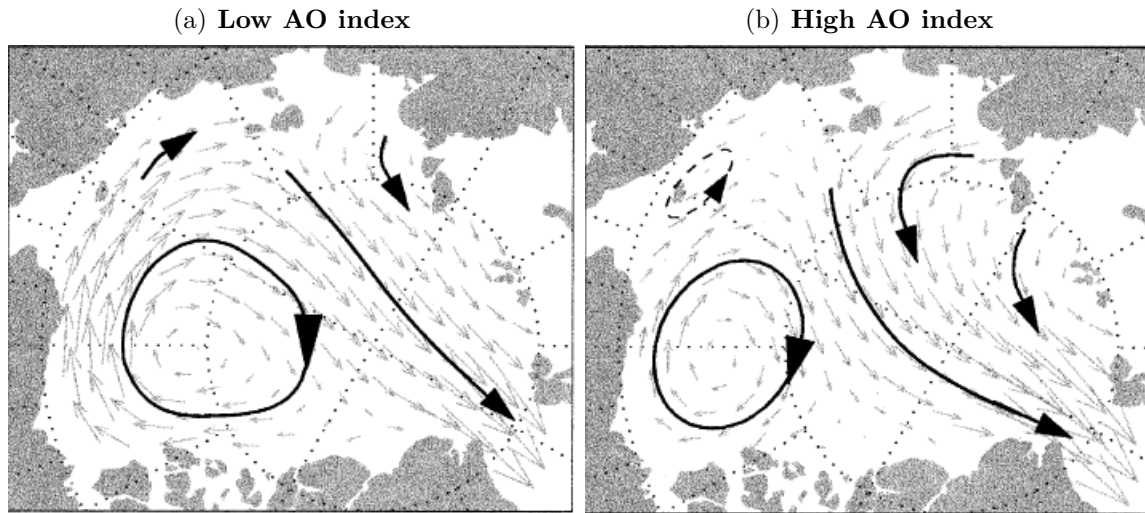


Figure 3.9: Mean sea ice circulation for low (a) and high (b) index of the Arctic Oscillation (Rigor *et al.*, 2002 [20]).

In order to find the potential link between the AO and our modes of variability, we first need to compute the temporal variation in our first mode high phase and low phase³. As for important phenomena of Earth climate variability, we will build a characterisation of our mode using high/low index theory. This can be done by computing the score of the first mode (see Figure 3.10). The score is the vector $T = X * C$, where X is the PCA matrix and C the first principal component. In other words, T is a vector that links the matrix of ice thickness anomaly and the component. It is the temporal variation of the pole. When T is positive, the pole is in high phase and, when T is negative, the pole is in low phase.

The score T fluctuates between positive and negative values, corresponding to high and low index of the mode. It is now possible to characterise the low index and the high index by looking at the ice circulation map at times corresponding to low and high values of the score. We computed the mean circulations of ice at low and high index using the values larger than twice the standard deviation. They are presented in Figure 3.11. We recognise the same difference between both anticyclonic Beaufort Gyre in low and high index as the one presented by the article by Rigor *et al.* [20] (see Figure 3.9). The low index presents a large Beaufort Gyre, centered between the Beaufort Sea and the East Siberian Sea. On the other hand, the high index circulation presents a weaker anticyclonic gyre centered on the Beaufort Sea. Both circulations exhibit the characteristics of the AO. In high index, the ice takes a year longer to cross the Arctic basin because of the weaker Beaufort Gyre, but cross the Fram Strait at higher velocity.

³Reminder : our first mode is a dipole which oscillates between positive and negative phase over time.

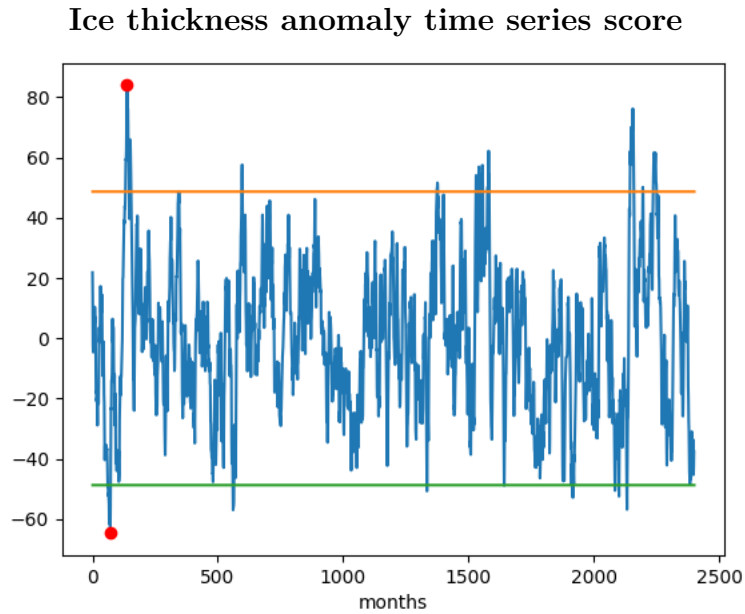


Figure 3.10: Score for the first component of the PCA on ice thickness anomaly over 200 yr period. The straight lines represent twice the standard deviation and the blue dots represent the global maxima and minima.

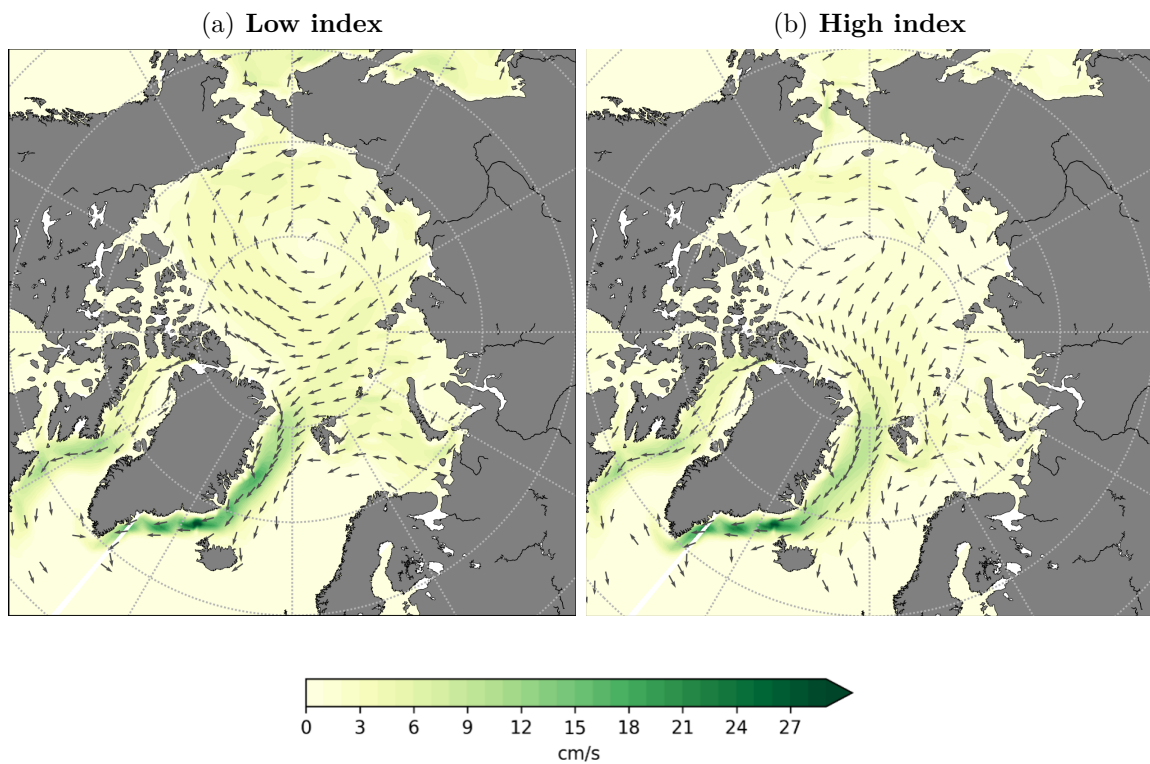


Figure 3.11: Regimes of ice motion during low (a) and high (b) index of our first mode of variability.

We conclude that the circulation of the ice at each phase of the first pole of variability corresponds to the ice circulation induced by the AO as described in the paper by Rigor *et al.* [20].

3.6 Summary

The most accurate principal component analysis we have made, the one on the monthly ice thickness anomaly time series, presents three principal modes of variability that account respectively for 21, 13 and 7% of the explained variability. The first principal component presents a dipole between the Fram Strait and the East Siberian Sea. This dipole is found in both summer and winter analysis of the Arctic sea ice variability (both 21% of variability explained). The East Siberian Sea region of the dipole is larger in shape and amplitude while summer months. On the other hand, the Fram Strait region of the dipole is larger in shape and amplitude while winter months.

The second mode of variability detected is a pole centered in the East Siberian Sea and spreading into the Arctic Basin. This pole is found in the second component of the summer and winter variability analysis with respectively 19% and 17% of variability explained. The third mode of variability corresponds to a dipole between the Laptev and Kara Seas on the one hand and the east coast of Greenland, the Chukchi and Beaufort Seas on the other hand.

Our modes of internal variability have similitude with modes of anthropogenic and natural variability from Lindsay and Zhang [18] or Fuckar *et al.* [11]. We also manage to link our first mode of variability to the Arctic Oscillation, showing how its low and high index can impact the ice circulation and so induce a variability in the Arctic sea ice thickness.

Chapter 4

Observation system

The present chapter is dedicated to solving the optimisation problem of this Master's thesis. Indeed, in the second chapter we investigated a first way to represent the entire volume of the Arctic sea ice based on a few stations, randomly selected, measuring the thickness anomaly of the ice. In the third chapter, we investigated the internal variability by looking at the major modes of variability of the Arctic sea ice thickness with some PCA analysis. We will now look for a way to select the N station locations that represent the internal variability of the Arctic sea ice as much as possible.

The optimisation problem has been introduced in the second chapter. We want to minimise :

$$\bar{F}_N(t) = ||\bar{V}(t) - \omega_N \bar{h}_N(t)||_2^2 \quad (4.1)$$

where $\bar{V}(t)$ is the ice volume anomaly time series, $\bar{h}_N(t)$ is the ice thickness anomaly time series at each station of the Arctic and ω_N is the vector of weights of each of the N stations.

4.1 The LASSO optimisation problem

The problem 4.1 can be modified in a certain way in order to put a constraint on the number of stations, more specifically on the sum of all the weights ω . Such a problem is called a LASSO problem (Least Absolute Shrinkage and Selection Operator) (Tibshirani, 1996 [26]).

There are two reasons why we prefer to use LASSO either than ordinary least square estimates (OLS). The first reason is prediction accuracy : the OLS estimates often have low bias but large variance; prediction accuracy can sometimes be improved by shrinking or setting to 0 some coefficients. By doing so we sacrifice a little bias to reduce the variance of the predicted values and hence may improve the overall prediction accuracy. The second reason is interpretation. With a large number of predictors, we often would like to determine a smaller subset that exhibit the strongest effects.

The LASSO was originally proposed for linear regression models, it becomes a popular model selection and shrinkage estimation method. LASSO shrinks some weights and sets others to 0, and hence tries to retain the good features of both subset selection. We have a response $\bar{V} \in \mathbb{R}^n$, an $n \times p$ design matrix $\bar{h}_N(t)$ and a parameter vector $\omega \in \mathbb{R}^p$.

$$\text{Minimise}_{\omega} \quad \bar{F}_{\lambda}(t) = \underbrace{\frac{1}{2} \|\bar{V}(t) - \omega \bar{h}_N(t)\|_2^2}_{(1)} + \lambda \underbrace{\|\omega\|_1}_{(2)}$$

The parameter $\lambda \geq 0$ controls the amount of regularization applied to the estimate ω . Setting $\lambda = 0$ reverses the LASSO problem to Ordinary Least Squares which minimizes the unregularized empirical loss. On the other hand, a very large λ will completely shrink β to 0 thus leading to the empty or null model. In general, moderate values of λ will cause shrinkage of the solutions towards 0, and some coefficients may end up being exactly 0 (Tibshirani, 1996 [26]).

In other words, by increasing λ , we decrease the non null entry of the vector ω , a.k.a. the number of stations. Intuitively, we know that a large λ will give a less accurate fit with a few stations and a small λ will give a better fit with a large number of stations.

A first improvement of the LASSO method was to recompute the coefficients with a multilinear regression. Indeed, the coefficient given by the LASSO optimisation minimise (1) + (2) and we wish to find the coefficients that minimise (1). So, we simply reinjected the stations selected by the LASSO method in a linear regression, allowing to easily improve the quality of the fit.

Once we look at the station locations given by the optimisation problem (for example, see Figure 4.1 (a)), we find that some of them are very close to each other. Indeed, the constraint is modeled as the norm 1 of the coefficient ω (the norm 1 is $\|\mathbf{x}\|_1 = |x_1| + \dots + |x_n|$). So, two stations close to each other with coefficient $\frac{C}{2}$ is the same constraint as only one station with coefficient C . Therefore, as a second improvement, we decided to use a strategy of optimisation that filters some stations out by choosing a weighted mean location for the stations too close to each other. Using the stations coefficient as the weight, the weighted mean allows to use the minimum distance between each station as a new constraint.

More specifically, we compute the straight line between two stations knowing their latitude and longitude. Then, using their coefficient from the regression to compute the weighted mean, we select the nearest station to the weighted mean point along the line.

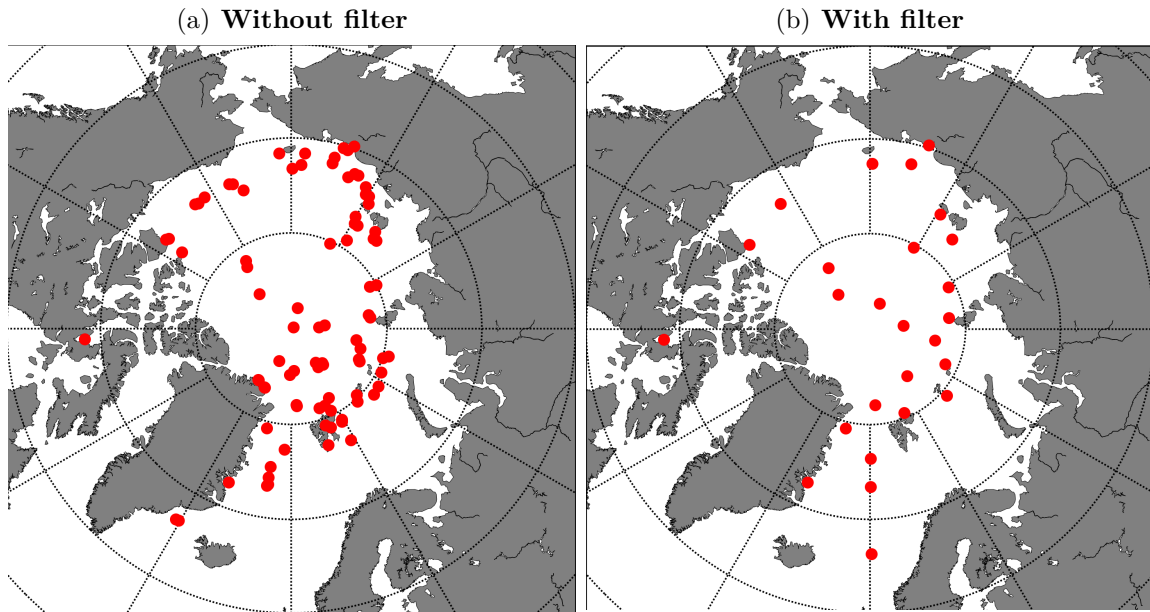


Figure 4.1: Station locations with (b) and without (a) filter of 300 km minimum distance, 100 yr period with $\lambda = 0.1$.

In the following section, we solve our modified LASSO problem to select the station locations. Then, in order to visualise and evaluate the fit, we compare the predictions with the predictions of chapter 2 model.

4.2 Results

Figure 4.2 shows how the λ parameter and the period on which we computed the modified LASSO optimisation can impact the results. It depicts the station locations obtained by the modified LASSO method for different λ parameters and over different 100 yr period with a distance constraint of 300 km between each station.

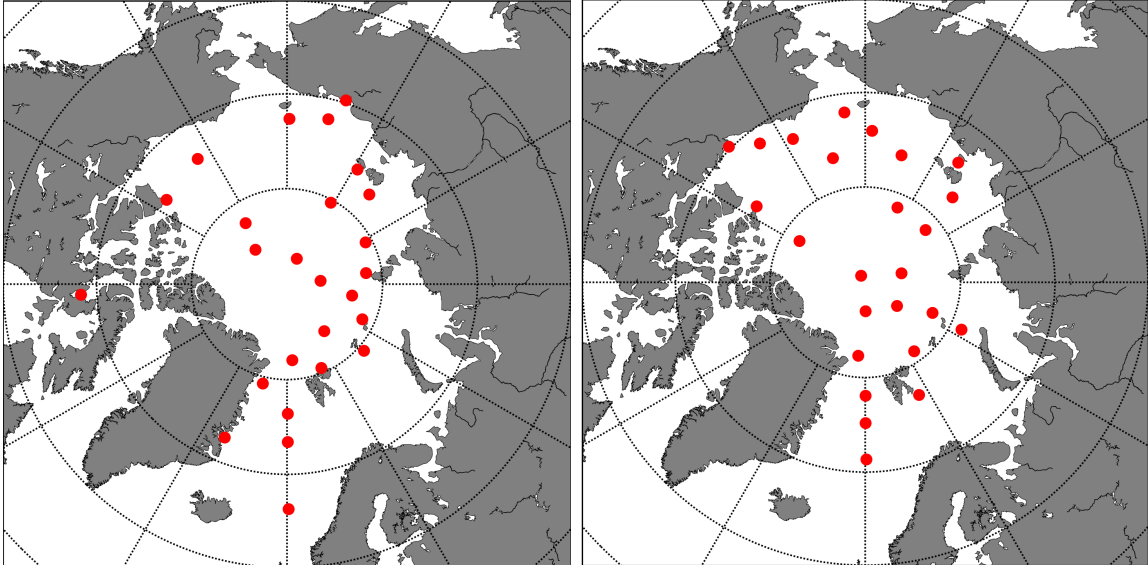
First of all, we see that for different periods of time of 100 years each, the same λ parameter can give different number of stations and different station locations. But, even if the number of stations may change, the stations cover the same area. Globally, we find stations in the center of the Arctic basin, near the Fram Strait, the East Siberian Sea and in the Beaufort Sea, i.e. in areas corresponding to the main poles of variability.

Repeating that result for different time periods, we conclude that the station locations change with the period but all the results present the same accuracy of prediction at constant number of stations, even if their locations are not the same. Indeed, the shorter the period of time, the more accurate the prediction, but the quality of the prediction stays the same even if the station locations change.

About the station locations and their weights (weights in the linear regression), we observe that there are only a few stations located in the center of the Arctic basin. Indeed, in the second chapter, we found that the ice thickness anomaly in the center of the Arctic basin is highly correlated with the global ice volume anomaly. So, we expect to have stations with higher weights at the center of the Arctic basin. Furthermore, we see that only a

few stations are needed to represent the center of the basin, which is related to the spatial length scale of the ice thickness anomaly. This was pointed out by Blanchard *et al.* [2] which investigates the spatial length scale of the ice thickness anomaly.

(a) $\lambda = 0.1$, rmse = 0.54, cor = 0.96, 23 stations (b) $\lambda = 0.1$, rmse = 0.49, cor = 0.96, 25 stations



(c) $\lambda = 0.3$, rmse = 0.79, cor = 0.92, 11 stations (d) $\lambda = 0.3$, rmse = 0.75, cor = 0.92, 13 stations

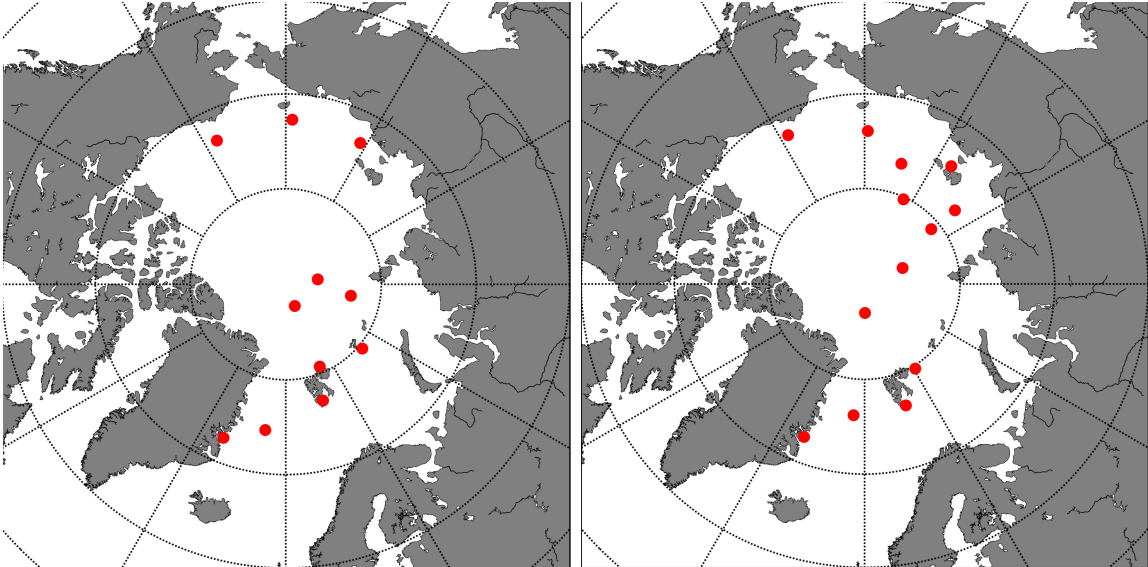


Figure 4.2: Station locations with $\lambda = 0.1$ (top) and $\lambda = 0.3$ (bottom) for different 100 yr period (left : 600-700 years, right : 700-800 years). Rmse expressed in 10^3 km^3 .

Figure 4.3 shows the spatial length scale map of the ice thickness anomaly. The map's computation is based on the e-folding distance, defined as the region centered around a point P , inside which all the points have a correlation of ice thickness anomaly with the point P higher than $\frac{1}{e}$. It confirms that the spatial length scale of the ice thickness anomaly is greater in the center of the Arctic basin, and that only a few stations are needed to represent that specific area.

Spatial length scale of ice thickness variability

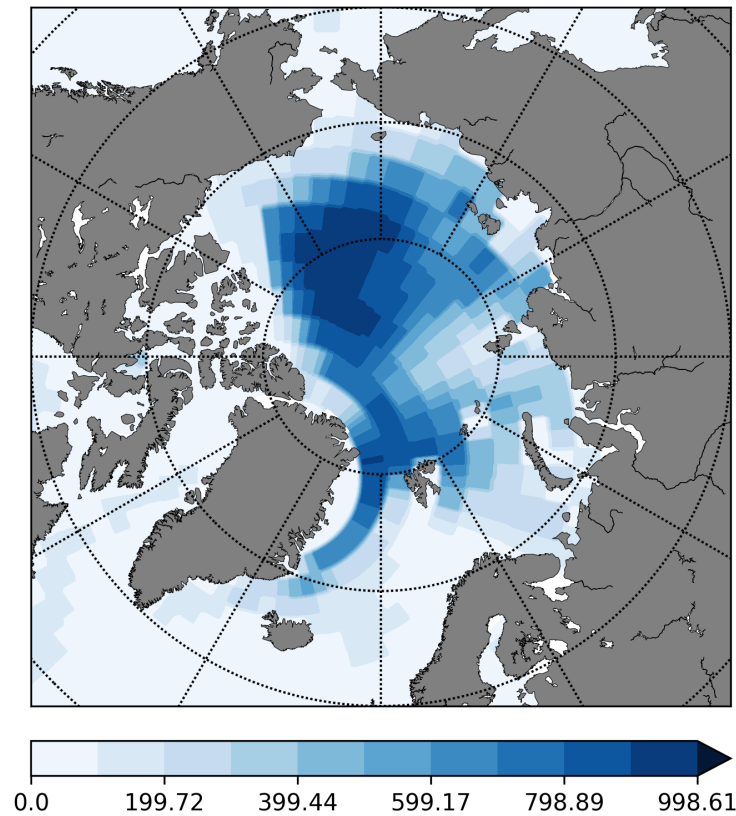


Figure 4.3: Map of spatial length scale of ice thickness anomaly, the units are the sum of the correlation grid point inside a circle of an e-folding distance radius.

4.3 Best 10 stations

At the end of the second chapter, we choose to build a 10 stations observation system. In order to select the best 10 station locations, we decided to select the best 10 stations for several periods of 200 years. With a 200 yr period, we assume that we are able to capture the entire variability of the Arctic sea ice. As for the last section, we find that the optimum 10 stations are not the same for different periods of time, but all of them reach the same rmse around $0.9 \cdot 10^3 \text{ km}^3$ and the same correlation, around 0.85.

To choose the best stations, we divided our dataset into two parts, a training set and a testing set. The training set is used to build the best coefficient for the stations. For each period of 200 years in the training set, we use a linear regression to find the stations coefficient, then for every station we compute the mean of all of their coefficients. The second part is used to choose the best stations. We build an ice volume anomaly prediction based on the station locations and the station coefficients, and compute the rmse and the correlation between each prediction and the true ice volume anomaly. The best 10 stations (shown in Figure 4.4) correspond to the lowest mean rmse ($1.49 \cdot 10^3 \text{ km}^3$) and the highest mean correlation (0.73) over all the 200 yr periods of the testing set.

$$\text{rmse} = 1.49, \text{cor} = 0.73$$

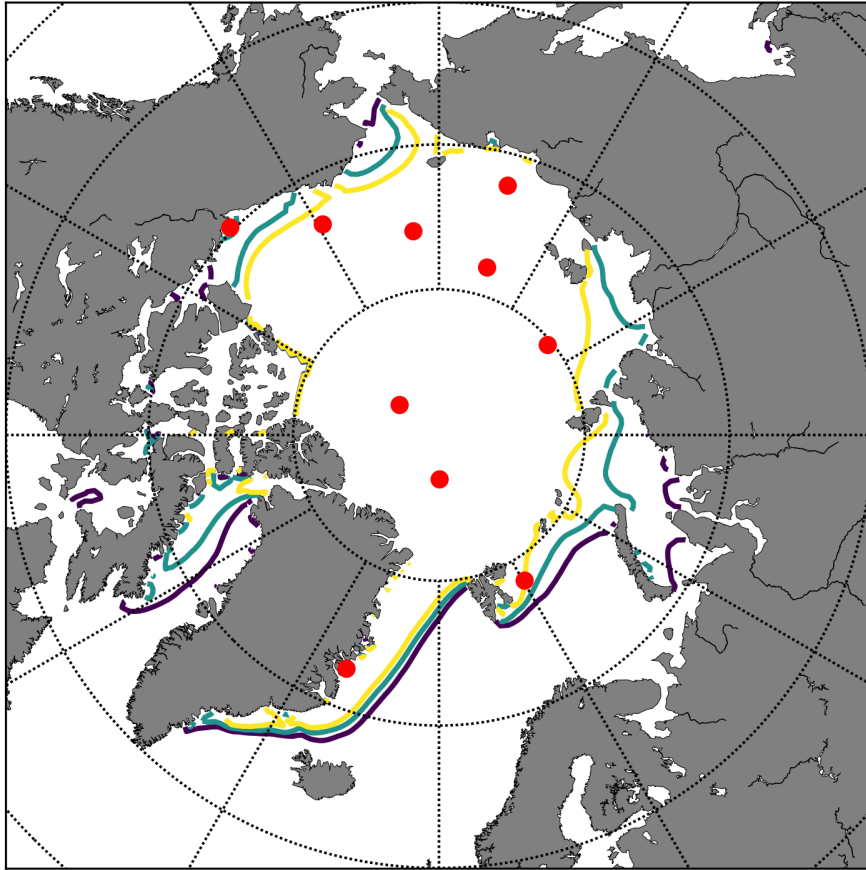


Figure 4.4: 10 station from the LASSO method, with the mean ice thickness front of 1, 1.5 and 2 meter. Rmse expressed in 10^3 km^3 .

As said above, only a few stations are needed to represent the spatial length scale of ice thickness anomaly in the center of the Arctic basin. In order to verify that and to assert the position of our stations, we computed the e-folding distance of each station (see Figure 4.5).

The stations near the center of the Arctic Basin (*b*, *c*, *f*, *g* and *i*) cover a larger area of correlation. All together, the 10 stations cover well the entire Arctic basin and its peripheral seas. We also note that the stations with the largest coefficient are the stations near the center of the Arctic basin. Indeed, that is where the ice is the thickest and has the lowest variability on average. It is also where the correlation between the total sea ice volume anomaly and the ice thickness anomaly is high.

We also see that the coastal stations are located where the standard deviation of the mean ice thickness is high and where the mean ice thickness is greater than 1 meter, so we can expect to find ice at each station most of the time (see Figure 2.7 in chapter two).

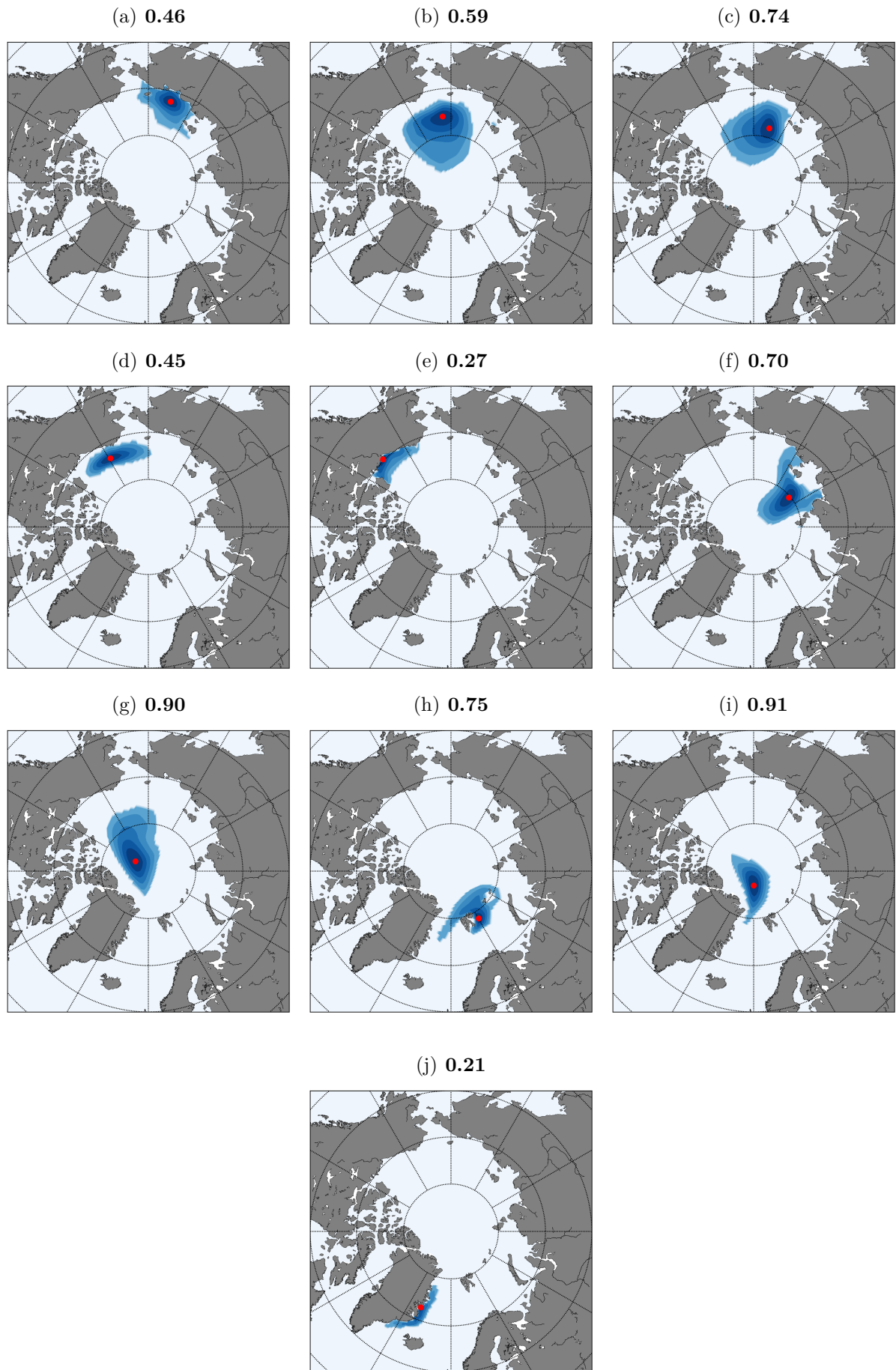


Figure 4.5: Spatial length scale of ice thickness anomaly map for each station, with each station's coefficient, for an e-folding distance over 100 yr period (800-900 years).

Figure 4.6 shows the ice volume anomaly prediction made with our 10 best station locations. We also show the prediction made with the chapter two's method. The root mean squared error between the true ice volume anomaly and the predicted one is $0.85 \cdot 10^3 \text{ km}^3$ for the 10 best stations and $0.88 \cdot 10^3 \text{ km}^3$ for the chapter two's method, and the correlation is respectively 0.91 and 0.89.

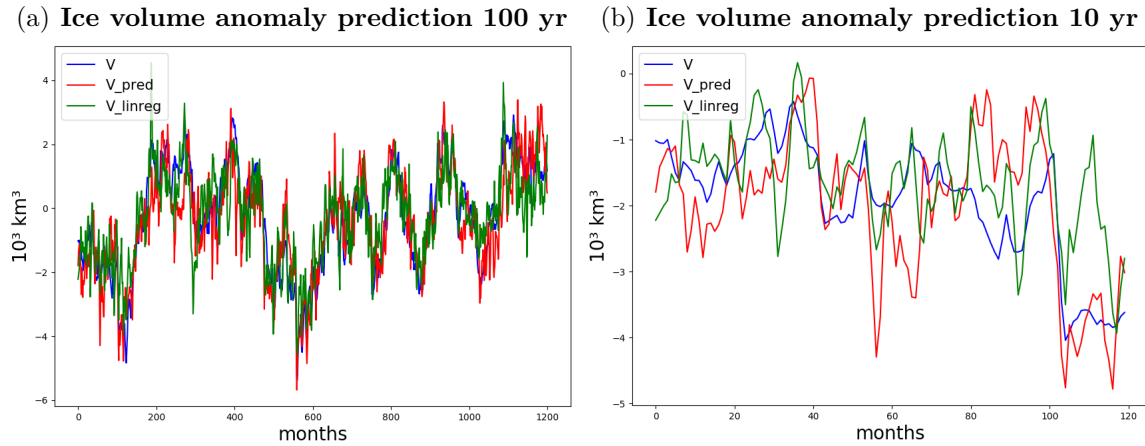


Figure 4.6: Ice volume anomaly prediction over a 100 (a) and 10 (b) year period data set included in the testing set. The original ice volume anomaly (blue), the predicted ice volume anomaly based on the best 10 stations (red) and the predicted ice volume anomaly with chapter two's method (green).

Even if, at first glance, the improvement of the 10 best stations seems negligible, the quality of the modified LASSO method is much better than the linear regression proposed in chapter two. Indeed, the 10 best station prediction is not train on that specific period while the linear regression of chapter two is.

We notice that we are able to represent the low frequency variability of the ice.

4.4 Method robustness

The modified LASSO method presented above has proved itself useful to reproduce the ice volume anomaly with a high level of prediction. But is it the unique and best solution ? And is it practical to use it in some observation systems ?

The optimisation problem is convex and then the solution is unique. But, as said above, the LASSO problem puts a constraint on the absolute sum of the stations coefficient. So, this is not exactly what we wanted. We wanted to optimise the prediction under the constraint on the number of stations, not the sum of their coefficients. In order to counteract that and build a constraint on the number of stations, we had to use a filter that puts a constraint on the minimal distance between stations. Thenceforth, we cannot prove that our solution is the best one. Indeed, the use of a filter is a strategic choice that could have been done in multiple different ways.

There is also the fact that the solution is data dependent; we did not find the same 10 stations applying our modified LASSO problem on different time periods of 200 years.

But, that problem has been taking care by the validation of the model (by building a training and testing part).

Another gap in our results comes the fact that we predict the internal variability of the Arctic sea ice volume based on some measurement of the Arctic sea ice thickness. But in a practical application, if we measure the Arctic sea ice thickness at the given station location, it will not be the same ice thickness anomaly time series because there will be some variability resulting from the anthropogenic forcing. Does the model still work in post industrial and future climate conditions ?

4.5 Other methods

Several other methods could have been used besides the LASSO method. Two of them are described here. The first one is an iterative method (see Algorithm 1) computed on a 200 yr time series that finds each station by selecting the one with the highest correlation with the ice volume anomaly time series, then subtracts its local ice thickness anomaly time series to the ice volume anomaly and finds the next station with that new ice volume anomaly time series. Such a method gives some good results. We found 10 stations that give a final mean rmse of $1.44 \cdot 10^3 \text{ km}^3$ and a mean correlation of 0.75 on average over the testing set. That represents an improvement compared to the modified LASSO method both on rmse and correlation. We show in Figure 4.7 our new ice volume anomaly prediction.

Algorithm 1 Correlation method

Initialisation : choose the number of stations N

Iter = 0

Compute ice volume_anomaly

Compute ice thickness_anomaly

while $iter < N$ **do**

 Compute the correlation between the ice thickness anomaly and the ice volume anomaly

 Station_i = max(correlation)

 Volume_anomaly = Volume_anomaly - thickness_anomaly[station_i]

 Iter = iter +1

end

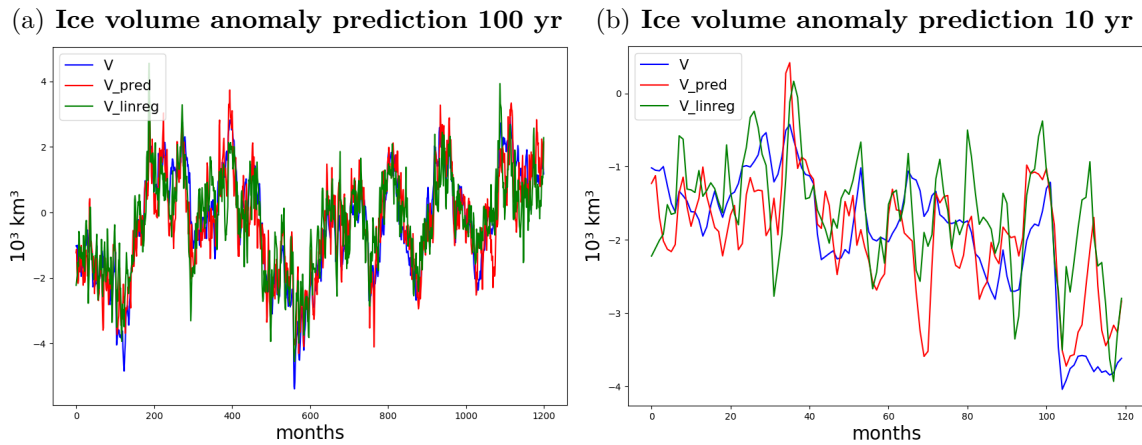


Figure 4.7: Ice volume anomaly prediction over a 100 (a) and 10 (b) yr period dataset included in the testing set. The original ice volume anomaly (blue), the predicted ice volume based on the best 10 stations (red) and the predicted ice volume anomaly with chapter two's method (green).

The station locations for this new correlation method are displayed in Figure 4.8.

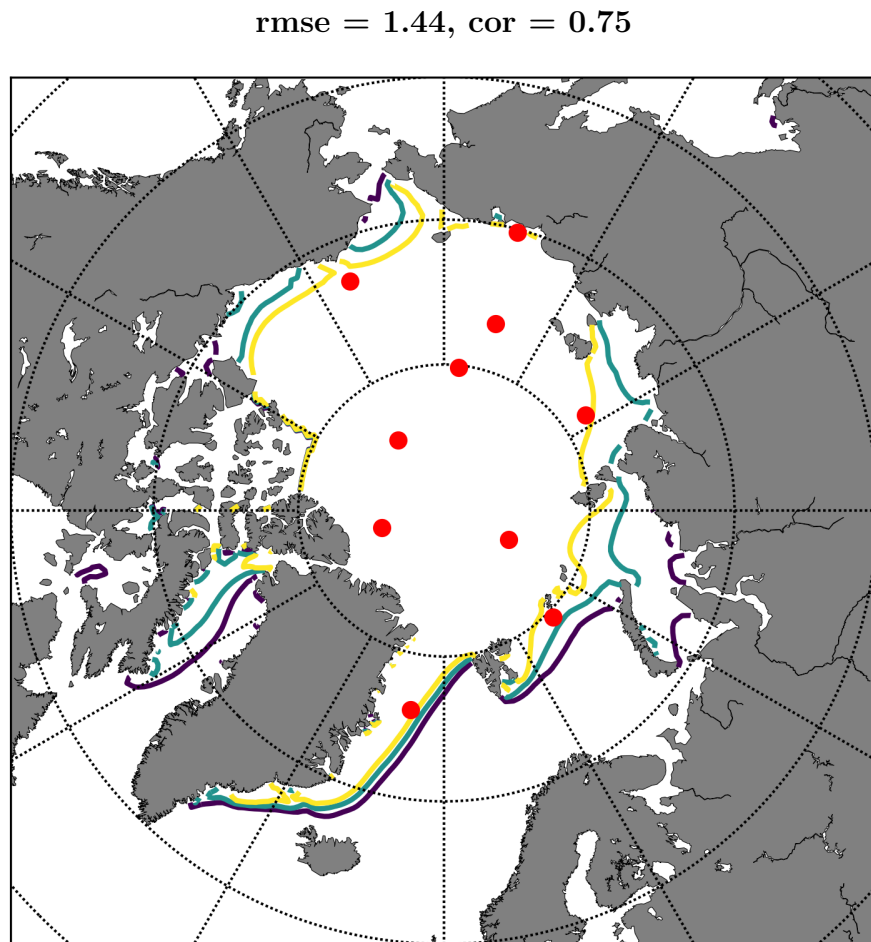


Figure 4.8: 10 station selection from the correlation method, with the mean ice thickness front of 1, 1.5 and 2 meter. Rmse expressed in 10^3 km^3 .

The second method, that we did not implement in this Master's thesis, is the method used by Lindsay and Zhang [18]. It uses the correlation between each local thickness time series with the 10 first principal components multiplied by the total variance of each principal component. And, as for the previous method, once one finds the station with the highest correlation, one subtracts the linear regression of each the principal component (pc) based on the station ice thickness time series found to each of the 10 pc, then repeat those steps until one has found all the stations needed.

4.6 Summary

In this chapter, we have built a statistical model of Arctic sea ice volume anomaly prediction, based on 10 stations, that measures the thickness anomaly of the ice. The station locations have been chosen by different methods. The first one uses the LASSO optimisation problem. It gives a rmse of $1.49 \cdot 10^3 \text{ km}^3$ and a correlation of 0.73 on average on the testing set. The second method uses the correlation between the ice volume anomaly and the ice thickness anomaly to choose the station. It gives a rmse of $1.44 \cdot 10^3 \text{ km}^3$ and a correlation of 0.75 on average on the testing set.

The best 10 stations chosen are those given by the second method. The stations are well placed in the Arctic Ocean, they are able to reproduce easily the Arctic sea ice volume anomaly time series of 200 years with a rmse of $1.44 \cdot 10^3 \text{ km}^3$ and a correlation of 0.75. This is a huge improvement compared to the statistical model built in the second chapter, taking into account that the $1.44 \cdot 10^3 \text{ km}^3$ rmse is reached without training the model on the testing period.

The coefficients, latitudes and longitudes of the stations are given in Table 4.1.

Coefficient	Latitude	Longitude
0.93	80.1	174.0
1.01	84.2	213.1
0.73	76.7	164.5
0.80	85.1	65.6
0.96	85.5	285.7
0.58	72.9	202.3
0.65	79.5	45.7
0.49	78.2	124.0
0.43	76.1	350.3
0.28	70.2	165.2

Table 4.1: Coefficients, latitudes and longitudes of the 10 best stations selected.

Chapter 5

Post-industrial and future climate conditions

The purpose of this fifth chapter is to validate our observation system of ice volume anomaly prediction under post-industrial and future climate conditions. Indeed, until now, we have worked on the natural variability of the Arctic sea ice without the anthropogenic forcing, building a statistical model capable of reproducing the variability of the Arctic sea ice volume anomaly. But how does the model behave under anthropogenic forcing conditions ?

Firstly we will analyse the quality of our statistical model prediction on those new dataset, then we will investigate a new selection of stations position.

The dataset used in this chapter, as the dataset used in the previous chapters of this Master's thesis, comes from the CESM1-CAM5-BGC-LE set (Kay *et al.*, 2015 [15]). It covers the post-industrial time from 1850 to 2006 and a climate projection from 2006 to 2100 following the RCP 8.5 scenario. RCP 8.5 is one of the four RCPs (representative concentration pathways) selected by the IPCC¹ for its Fifth Assessment report (AR5) in 2004, and defined by their total radiative forcing² pathway and level by 2100. The RCPs were chosen to represent a broad range of climate outcomes, based on a literature review, and are neither forecasts nor policy recommendations. The RCP 8.5 accounts for a total radiative forcing of 8.5 W/m^2 in 2100 relative to pre-industrial values (IPCC, Data Distribution Center, 2018 [37]).

5.1 Post-industrial conditions

The statistical model prediction over the period 1850-2006 is presented in Figure 5.1. The rmse is $0.92 \cdot 10^3 \text{ km}^3$ and the correlation is 0.94.

¹Intergovernmental Panel on Climate Change.

²Cumulative measure of human emissions of GHGs (greenhouse gases) from all sources expressed in Watts per square meter.

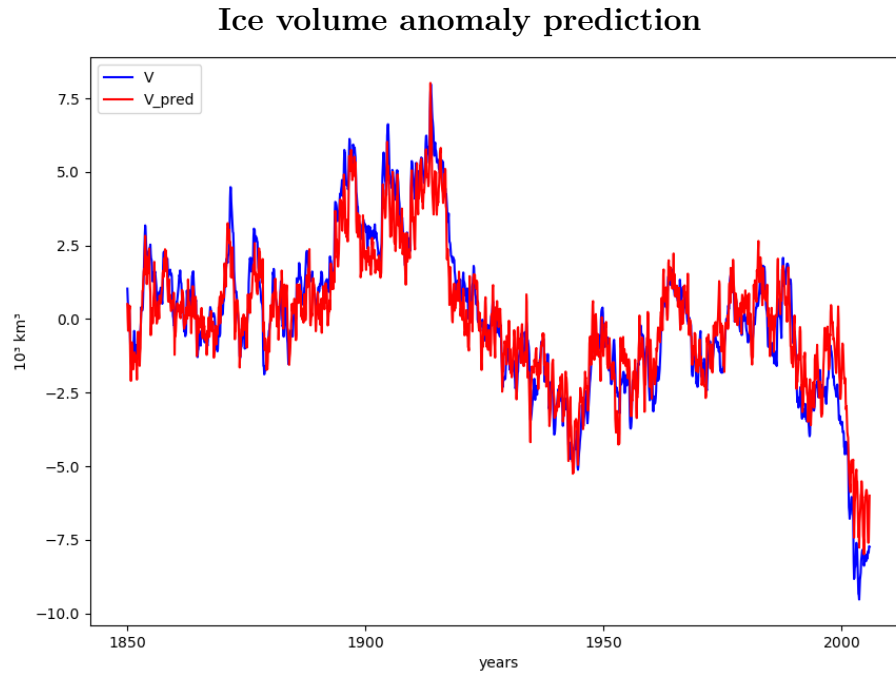


Figure 5.1: Ice volume anomaly prediction under post-industrial conditions, ice thickness and volume anomalies are relative to the pre-industrial conditions. The blue curve is the true ice volume anomaly, the red curve is the ice volume anomaly predicted by the 10 stations selected in chapter fourth (see Table 4.1).

We see that the post-industrial conditions impacts the ice as a negative volume anomaly along the end of the 20th century. More precisely, the prediction is quite accurate for the first part of the time period, even when the negative but weak trend appears, and the quality of the fit drops at the end of the prediction, when the negative trend gets stronger.

The fact that the model is able to reproduce the ice volume anomaly under post-industrial conditions with a weak negative trend, comes from several factors. Firstly, even if the mean ice thickness decreases over time, all the stations stay inside the mean ice thickness front of 1 meter (see Figure 5.2). So, the stations are still used. Secondly, in the first part of the ice decline, the impact of the anthropogenic forcing is very light. Indeed, study about previous ice extent evolution shows that until 1960 no decline was detected in the ice extent (Walsh and Chapman, 2001 [29]).

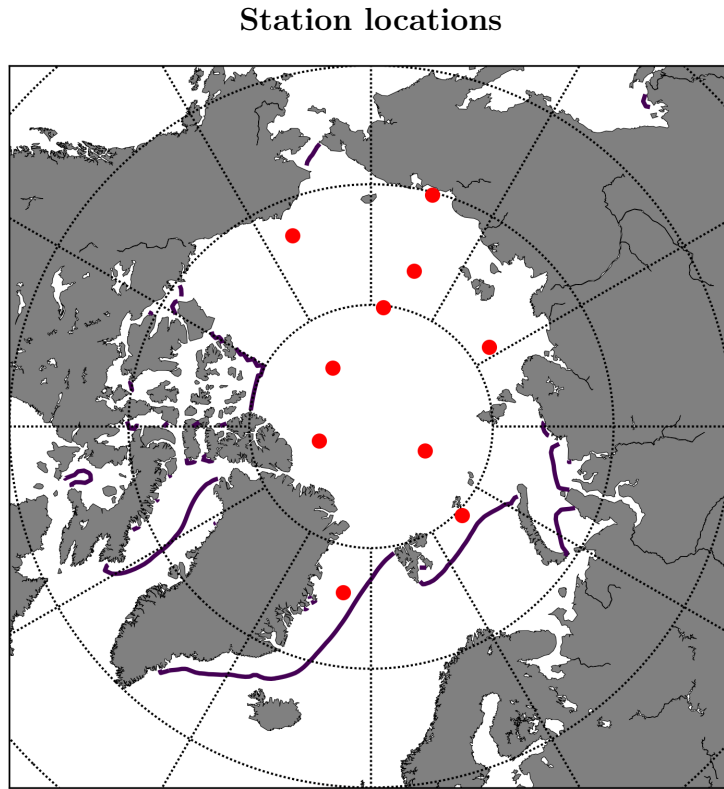


Figure 5.2: Station locations and the mean ice thickness front of 1 meter.

5.2 Future climate RCP 8.5 condition

The model prediction over the 2005-2100 period is presented in Figure 5.3. The rmse is $5.68 \cdot 10^3 \text{ km}^3$ and the correlation is 0.93. As expected, the model is not able to reproduce the ice volume anomaly correctly. Before 2050, the model seems to follow the true ice volume anomaly (good correlation) but the quality drops after 2050. Around that time, the anomalies reach a constant value, representing a constant summer ice volume, equal to zero (see Figure 5.9 in the annex).

Figure 5.4 shows the station locations and the mean ice thickness front of 1 meter. Only 2 of the 10 stations are in the area. So, in most of the year, some of the stations are not used. This explains the shift between the true ice volume anomaly and the predicted one (see Figure 5.3). We conclude that the station locations and weights are not suited for the future climate projection.

Repeating the correlation method on the RCP 8.5 scenario, we selected 5 stations to reproduce the ice volume anomaly (see Figure 5.5). We considered only the 2006-2050 period. Indeed, as shown in Figure 5.3, the dynamics of the Arctic sea ice variability is completely modified around the year 2050. There is a mean of 0.59 m ice during the 50 first years and 0.16 m during the second part of the century (almost a factor 4). In addition, while there is still some summer ice during the first part of the century, after the 2050 years, there is no more summer ice. Those two conditions combined together prevent the model to be accurate over the entire time series.

Ice volume anomaly prediction

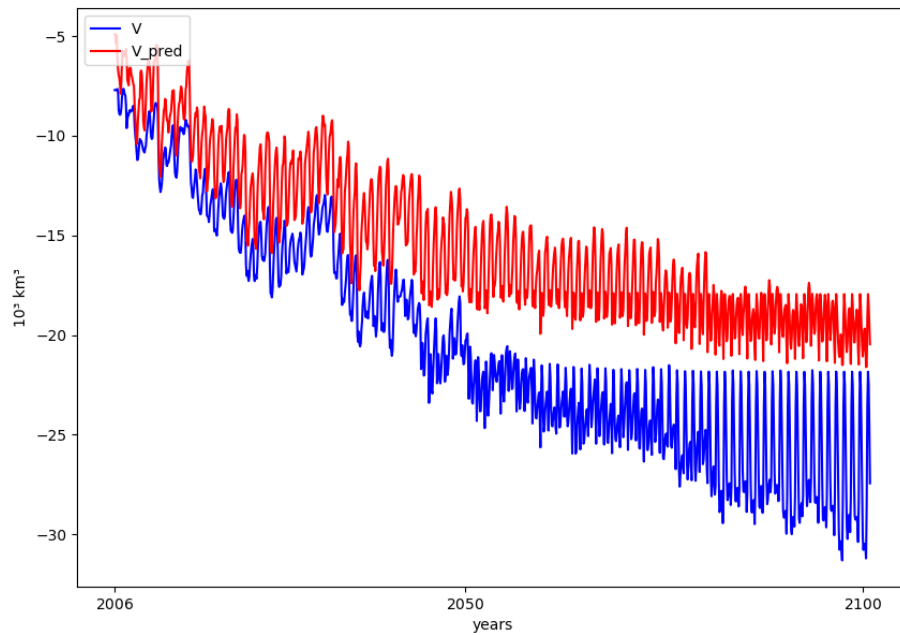


Figure 5.3: Ice volume anomaly prediction under future climate (RCP 8.5) conditions. Ice thickness and volume anomalies are relative to the pre-industrial conditions. The blue curve is the true ice volume anomaly, the red curve is the ice volume anomaly predicted by the 10 stations selected in chapter fourth (see Table 4.1).

Station locations

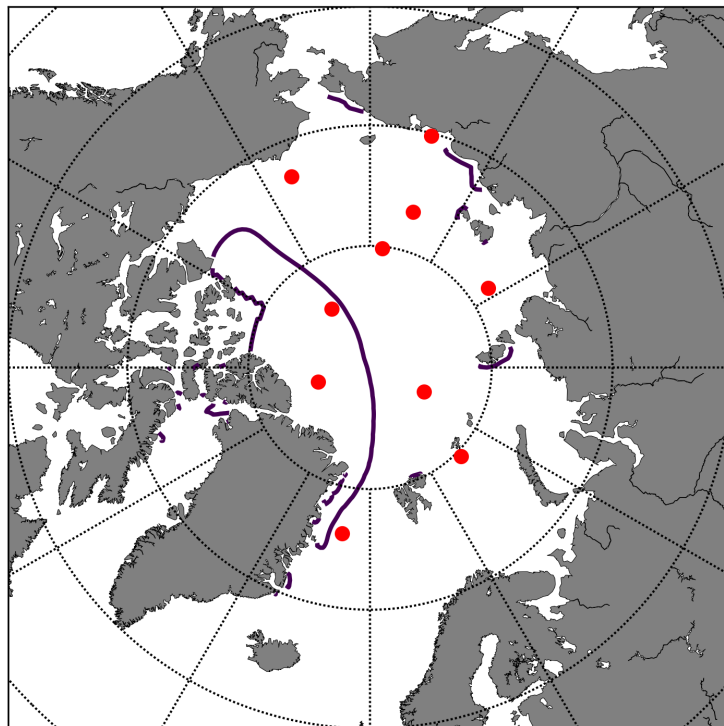


Figure 5.4: Station locations and the mean ice thickness front of 1 meter.

The future climate model's stations are presented in Figure 5.5. The new prediction is shown in Figure 5.6. The rmse is $0.72 \cdot 10^3 \text{ km}^3$ and the correlation is 0.99.

$$\text{rmse} = 0.73, \text{corr} = 0.98$$

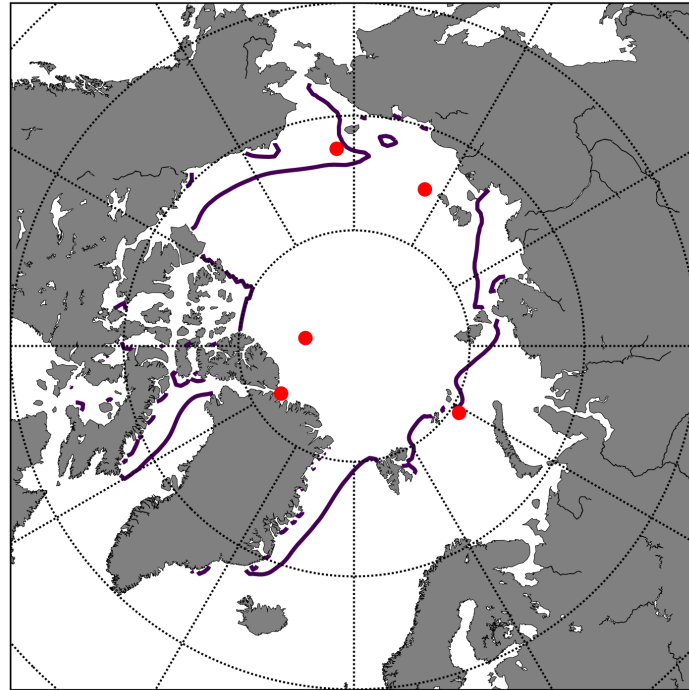


Figure 5.5: Station locations and the mean ice thickness front of 1 meter. Rmse expressed in 10^3 km^3 .

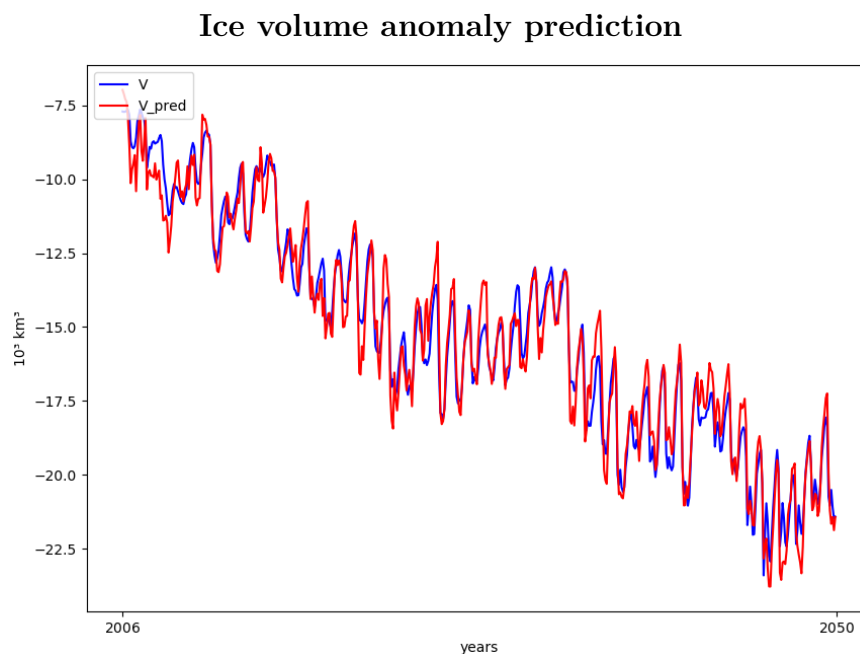


Figure 5.6: Ice volume anomaly predicted by the new stations under future climate (RCP 8.5) conditions. The anomalies are relative to the pre-industrial conditions. The blue curve is the true ice volume anomaly, the red curve is the ice volume anomaly predicted by the new statistical model (see Table 5.1).

5.3 Overall prediction

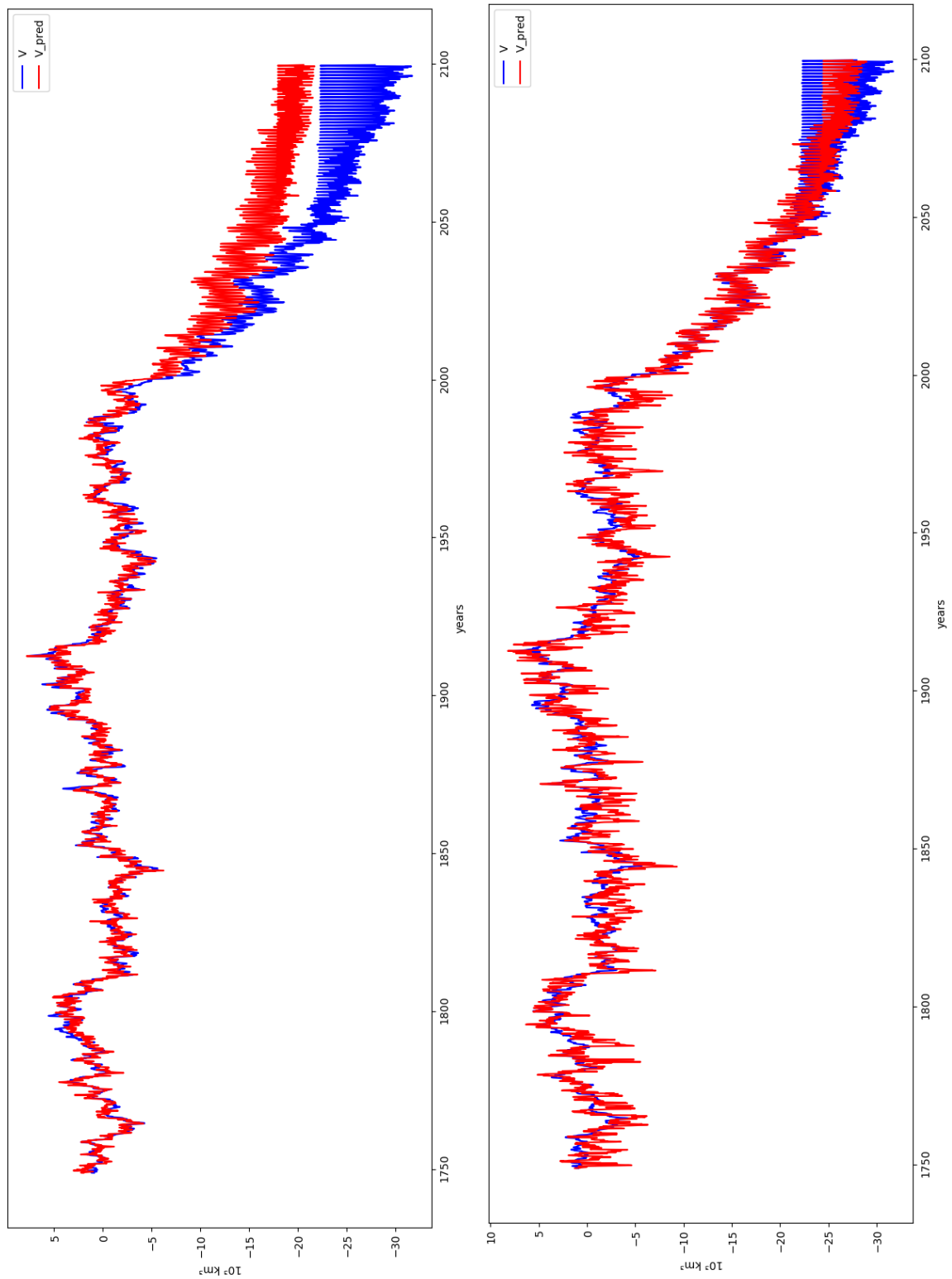


Figure 5.7: Overall prediction of the Arctic sea ice volume anomaly. The anomalies are relative to the pre-industrial conditions. First model (Table 4.1) prediction on the top, second model (Table 5.1) prediction on the bottom. The blue curve is the true ice volume anomaly, the red curve is the predicted ice volume anomaly.

5.4 Summary

We have investigated in this last chapter the robustness of our observation system. We have tested the quality of the prediction over a new dataset containing a post-industrial 1850-2006 period and a future climate RCP 8.5 projection 2006-2100.

For the post-industrial conditions, the model gives a good retrospective prediction. Until the end of the 20th century, the anthropogenic forcing did not affect the prediction of the variability in the Arctic sea ice. Indeed, the impacts of the anthropogenic forcing on the Arctic sea ice have been detected for the first time in the 1970s and 1980s (NSIDC, 2018 [36]). Looking at Figure 5.1, it is clear that the prediction quality drops around this time.

The model prediction for the 2006-2100 period is less good. There is a clear gap between the prediction and the true ice volume anomaly. Before 2050, the model seems to follow the true ice volume anomaly (good correlation) but the quality drops after 2050 because the dynamics of the ice is completely changed. So, we used the correlation method to compute a new statistical model of Arctic sea ice variability prediction for the specific period of 2006-2050 that gave a better prediction.

The coefficients, latitudes and longitudes of the second model stations are given in Table 5.1.

Coefficient	Latitude	Longitude
3.18	85.7	260.2
1.79	72.8	185
1.26	79.2	57.5
0.49	82.4	303.1
0.85	75.1	155.8

Table 5.1: Coefficients, latitudes and longitudes of the best stations selected for the future climate RCP 8.5 conditions.

Conclusion

In the context of recent climate changes, good monitoring of the Arctic sea ice variability has become a key factor in climate models calibration and climate prediction. Observation systems that take into account the spatial and temporal variability of the Arctic sea ice, will allow to improve meteorological and climate predictions.

The first goal of this Master's thesis was the characterise the internal variability of the Arctic sea ice. In the third chapter, we have been able to characterise the principal modes of the Arctic sea ice variability with some principal component analysis. The three more important poles that account for more than 45% of the variability have been fully described. The first mode is a dipole spatial pattern of sea ice thickness variability with opposite signs of polarity between the East Siberian Sea and near the Fram Strait.

Until now, less attention has been given to the modes of internal variability of the Arctic sea ice thickness in the scientific literature, but we compared our modes with modes of variability under anthropogenic forcing from scientific paper such as Fuckar *et al.* ([11]) or Lindsay and Zhang ([18]). We also linked the first mode of variability with the Arctic Oscillation and its impact on the Arctic sea ice motion.

The second goal of this Master's thesis was to propose an observation system measuring the Arctic sea ice thickness at some local stations, capable of reproducing the Arctic sea ice volume variability. In the second chapter, we investigated the predictability of the ice volume variability, we found that 10 measurement stations were enough to reproduce the variability with a sufficient level of accuracy.

In the fourth chapter, we tested several method to select the 10 best measurement stations. We firstly tried a modified LASSO optimisation method. We chose to use a distance constraint between the stations by the use of a filter and found relatively accurate results. Then, we applied the correlation method to select the stations, a method of our own making inspired from Lindsay and Zhang [18]. Such a method gave greater accuracy in the prediction of Arctic sea ice volume variability.

Finally in the last chapter, we evaluated the robustness of our method under post-industrial (1850-2006) and future climate RCP 8.5 (2006-2100) conditions. Our statistical model is able to reproduce reasonably well the Arctic sea ice volume variability over the 1850-2006 period, except for the last decade, when the trend has strongly decreased. For the 2006-2100 period, the prediction was less good, the anthropogenic forcing impacts the dynamics of the ice and changes its variability in a way that the model is not capable of following. We applied the correlation method on the 2006-2050 period in order to find a new model that fitted the actual and near future Arctic sea ice conditions. Our

new model performs well on that period and is also capable of predicting the variability on past climate conditions with of course a lower level of accuracy than our first model.

The observation system propose in this Master's thesis is a statistical model. At the opposite of climate models, our statistical model produce the Arctic sea ice variability only based on Arctic ice thickness measurement. It will bring information about Arctic sea ice thickness and volume with less uncertainty than usual climate models. Nevertheless, a big limitation of our model comes from the dataset on which we computed the model. Indeed, it derives from climate model simulations that did not use any observation or reanalysis. And despite an acceptable dataset validation, it is still a source of uncertainty.

Bibliography

- [1] AMAP. *AMAP Assessment Report : Arctic Pollution Issues. Arctic Monitoring and Assessment Programme (AMAP)*. Oslo, Norway, 859 pp, 1998.
- [2] BLANCHARD-WRIGGLESWORTH, E. and BITZ, C.M. *Characteristics of Arctic Sea-Ice Thickness Variability in GCMs*. Journal of Climate, Department of Atmospheric Sciences, University of Washington, Seattle, Washington, August 2014.
- [3] BITZ, C.M., BATTISTI, D.S., MORITZ, R.E. and BEESLEY, J.A. *Low-Frequency Variability in the Arctic Atmosphere, Sea Ice and Upper-Ocean Climate System*. Journal of climate, volume 9, 1996.
- [4] CAVALIERI, D.J. and PARKINSON, C.L. *Arctic sea ice variability and trends, 1979–2010*. The Cryosphere, 6, 881–889, 2012, Cryospheric Sciences Laboratory/Code 615, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA, 15 August 2012.
- [5] CAVALIERI, D.J., PARKINSON, C.L. and VINNIKOV, K.Y. *30-Year satellite record reveals contrasting Arctic and Antarctic decadal sea ice variability*. Geophysical Research Letters, Volume 30, Issue 18, 2003.
- [6] COMISO, J. and HALL, D.K. *Climate trends in the Arctic as observed from space* Advanced review, WIREs Clim Change 2014, 5:389-409. doi: 10.1002/wcc.277
- [7] COMISO, J., MEISER, W. and GERSTEN, R. *Variability and trends in the Arctic sea ice cover: Results from different techniques*. Cryospheric Sciences Laboratory, NASA Goddard space flight center, Greenbelt, Maryland, USA, Journal of geophysical research, 2017
- [8] DELSOLE and TIPPETT. *Chapter 9 : Principal Component Analysis*. Statistical Methods in Climate Science
- [9] DESER, C., WALSH, J.E. and TIMLIN, M.S. *Arctic Sea Ice Variability in the Context of Recent Atmospheric Circulation Trends* 2000, American Meteorological Society
- [10] DING, Q., SCHWEIGER, A., L'HEUREUX, M., BATTISTI, D.S., PO-CHEDLEY, S., JOHNSON, N.C., BLANCHARD-WRIGGLESWORTH, E., HARNOS, K., ZHANG, Q., EASTMAN, R. and STEIG, E.J. *Influence of high-latitude atmospheric circulation changes on summertime Arctic sea ice* Nature Climate Change, vol 7, April 2017.
- [11] FUCKAR, N.S., GUEMAS, V., JOHNSON, N.C., MASSONNET, F. and DOBLAS-REYES, F.J. *Clusters of interannual sea ice variability in the northern hemisphere* November 2015

- [12] GOOSSE, H., ARZEL, O., BITZ, C.M., de MONTETY, A. and VANCOPPEN-
NOLLE, M. *Increased variability of the Arctic summer ice extent in a warmer climate* Geophysical research letters, vol. 36, L23702, 2009
- [13] IPCC *CLIMATE CHANGE 2013, The Physical Science Basis*
- [14] KAY, J.E., HOLLAND, M., and JAHN, A. *Inter-Annual to multi-decadal Arctic sea
ice extent trends in a warming world* Geophysical Research Letters, Vol 38, 2011.
- [15] KAY, J.E., DESER, C., PHILLIPS, A., MAI, A., HANNAY, C., STRAND, G.,
ARBLASTER, J.M., BATES, S.C., DANABASASOGLU, D., EDWARDS, J., HOL-
LAND, M., KUSHNER, P., LAMARQUE, J.-F., LAWRENCE, D. , LINDSAY, K.,
MIDD LETON, A., MUNOZ, E., NEALE, E., OLESON, K., POLVANI, L., and
VERTENSTEIN, M. *The Community Earth System Model (CESM) Large Ensem-
ble Project, A Community Resource for Studying Climate Change in the Presence of
Internal Climate Variability.* American Meteorological society, August 2015.
- [16] KWOK, R., CUNNINGHAM, G.F., WENSNAHAN, M., RIGOR, I., ZWALLY, H.J.
and YI, D. *Thinning and volume loss of the Arctic Ocean sea ice cover.* Geophys.,
2009.
- [17] LABE, Z., MAGNUSDOTTIR, G. and STERN, H. *Variability of Arctic sea-ice thick-
ness using PIOMAS and the CESM Large Ensemble* Depth of Earth System Science,
University of California, Irvine, 2018
- [18] LINDSAY, R.W. and ZHANG, J. *Arctic Ocean Ice Thickness: Modes of Variability
and the Best Locations from Which to Monitor Them* Journal of physical oceanogra-
phy, Polar science center, University of Washington, Seatle, Washington, june 2005
- [19] MORISON, J. H., McPHEE, M. G., KIKUCHI, T. and STANTON, T. P. *Ocean-
to-ice heat flux at the North Pole environmental observatory.* Geophysical research
lettres, 2003.
- [20] RIGOR, I.G., WALLACE, J.M. and COLONY, R.L. *Response of Sea Ice to the Arctic
Oscillation* Journal of Climate, volume 15, p. 2648, 2002
- [21] PENDSE, G.V. *A tutorial on the LASSO and the "shooting algorithm".* P.A.I.N
Group, Imaging and Analysis Group (IMAG), McLean Hospital, Harvard Mdecial
School, February 8, 2011.
- [22] ROTHROCK, D.A., YU, Y. and MAYKUT, G.A. *Thinning of the Arctic Sea-Ice
Cover.* Geophysical Research Letters, VOL. 26, No. 23, Pages 3469-3472, December
1, 1999.
- [23] SERREZE, M.C., MASLANIK, J.A., SCAMBOS, T.A., FETTERER, F.,
STROEVE, J., KNOWLES, K., FOWLER, C., DROBOT, S., BARRY, R.G. and
HARAN, T.M. *A record minimum arctic sea ice extent and area in 2002.* Geophysical
Research Letters, Vol. 30, No. 3, 1110, 2003.
- [24] SCHWEIGER, A., LINDSAY, R., ZHANG, J., STEELE, M., STERN, H. and
KWOK, R. *Uncertainty in modeled Arctic sea ice volume* Journal of Geophysical re-
search, vol 116, 2011.

- [25] STOCKER, T.F., QIN, D., PLATTNER, G.K., ALEXAMDER, L.V., ALLEN, S.K., BINDOFF, N.L., BRÉON, F.M, CHURCH, J.A, CUBASCH, U., EMORI, S. P., FORSTER FRIEDLINGSTEIN, P., GILLET, N., GREGORY, J.M., HARTMANN, D.L., JANSEN, E., KIRTMAN, B., KNUTTI, R., KRISHNA KUMAR, K., LEMKE, P., MAROTZKE, J., MASSON-DELMOTTE, V., MEEHL, G.A., MOKHOV, I.I., PIAO, S., RAMASWAMY, V., RANDALL, D., RHEIN, M., ROJAS, M., SABINE, C., SHINDELL, D., TALLEY, L.D., VAUGHAN, D.G., XIE, S.P. *Technical Summary. In : Climate Change 2013 : The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 2013.
- [26] TIBSHIRANI, R. *Regression shrinkage and selection via the lasso.* J. Roy, Statist, Soc, 1996.
- [27] RAMPAL, P., WEISS, J. and MARSAN, D. *Positive trend in the mean speed and deformation rate of Arctic sea ice, 1979–2007* Journal of Geophysical Research, Vol. 114,
- [28] VAUGHAN, D.G., COMISO, J.C., ALLISON, I., CARRASCO, J., KASER, G., KWOK, R., MOTE, P., MURRAY, T., PAUL, F., REN, J., RIGNOT, E., SOLOMINA, O., STEFFEN, K. and ZHANG, T. *Observations : Cryosphere. In : Climate Change 2013 : The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 2013
- [29] WALSH, J. E., and CHAPMAN, W. L. *20th century sea ice variations from observational data.* Ann. Glaciol, 33, 2001.
- [30] WANG, M. and OVERLAND, J.E. *A sea ice free summer Arctic within 30 years ?* University of Washington, Seattle, Washington USA, 2009.
- [31] WANG, M. and OVERLAND, J.E.
A sea ice free summer Arctic within 30 years: An update from CMIP5 models
Geophysical research letters, vol. 39, 2012.
- [32] <https://www.carbonbrief.org/arctic-sea-ice-could-shrink-to-record-low-this-summer>
- [33] <http://sco-web1.meas.ncsu.edu/climate/patterns/NAO.html>
- [34] <http://nsidc.org/arcticseaicenews/>
- [35] <https://serc.carleton.edu/eslabs/cryosphere/2c.html>
- [36] https://nsidc.org/cryosphere/arctic-meteorology/climate_change.html
- [37] http://sedac.ipcc-data.org/ddc/ar5_scenario_process/RCPs.html

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Annex

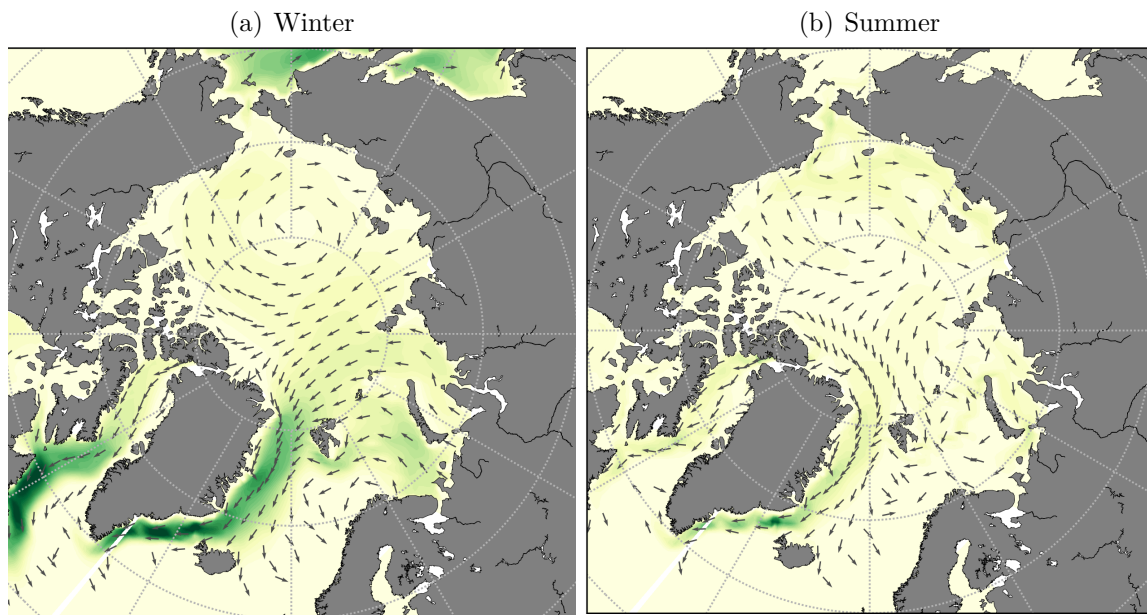


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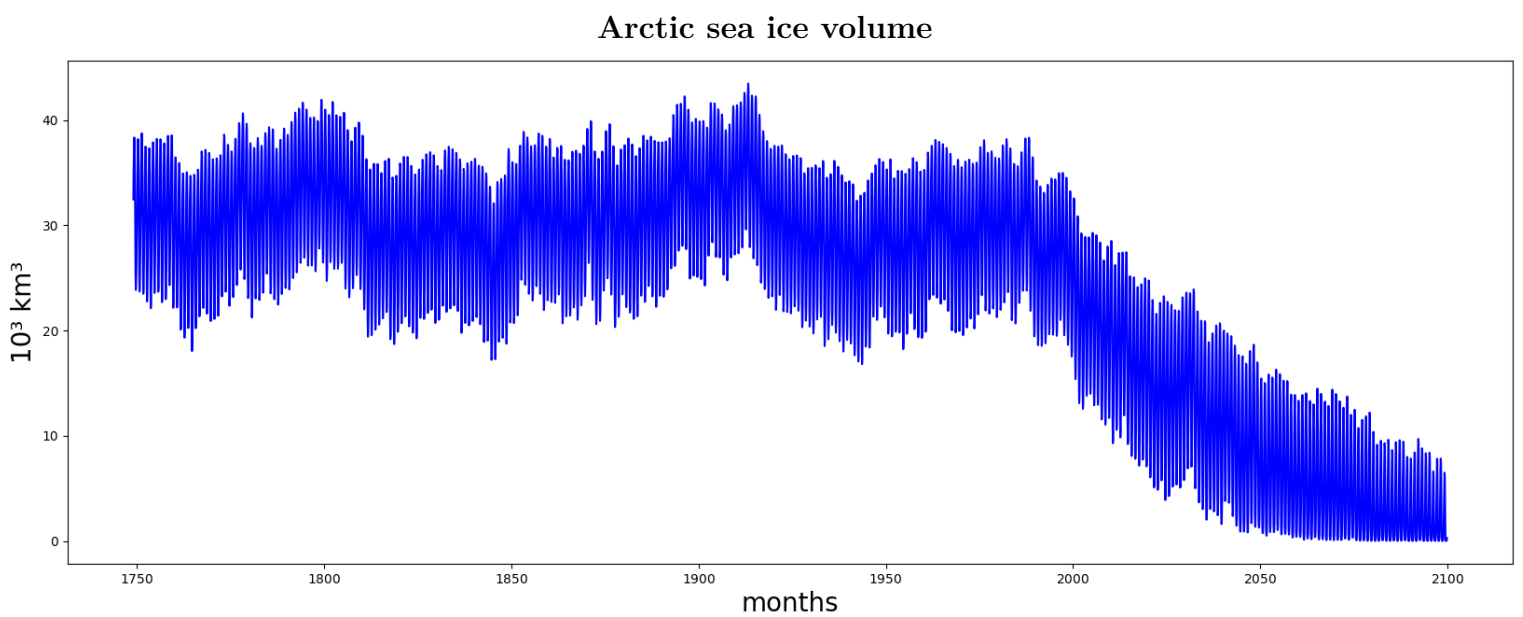


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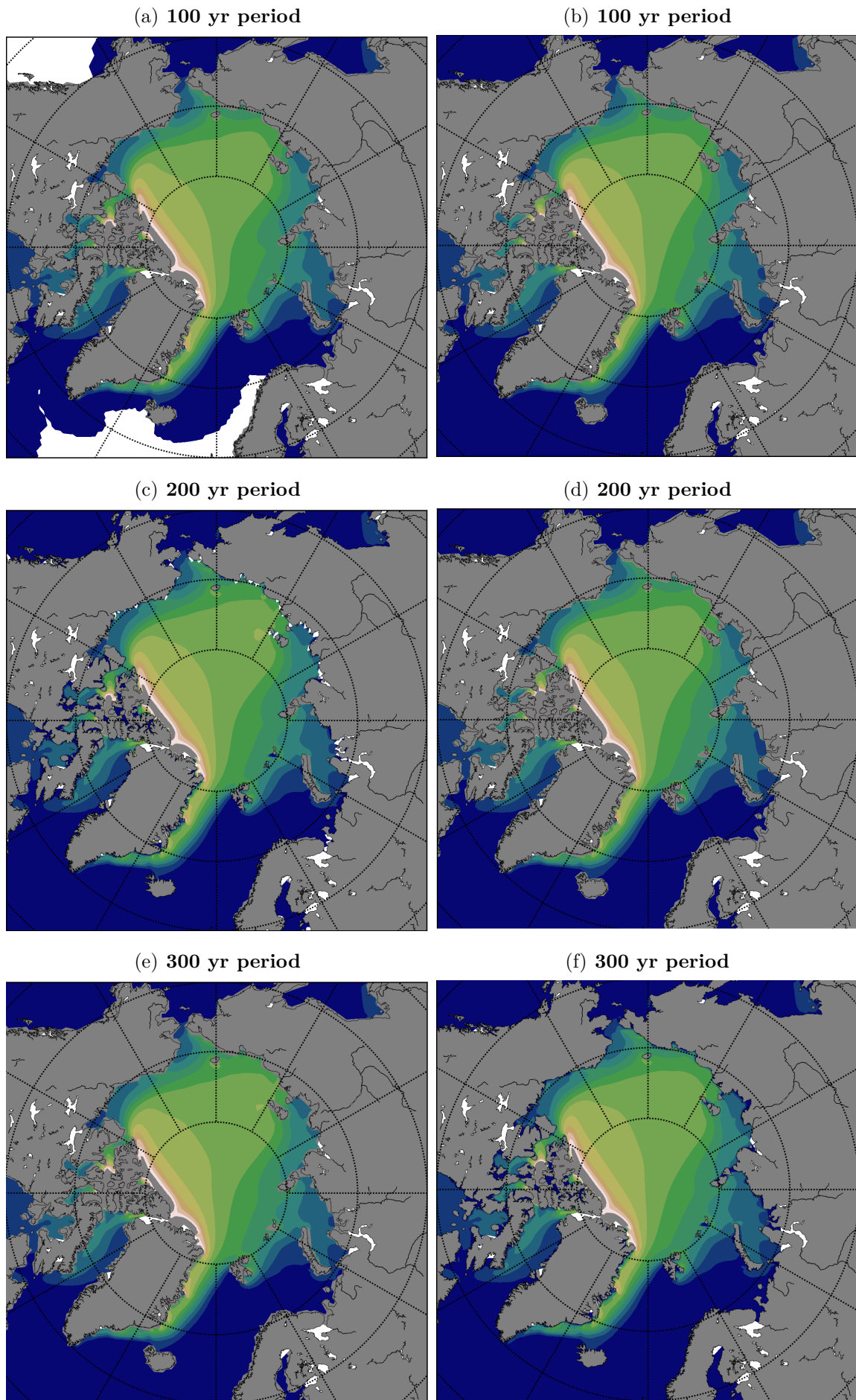


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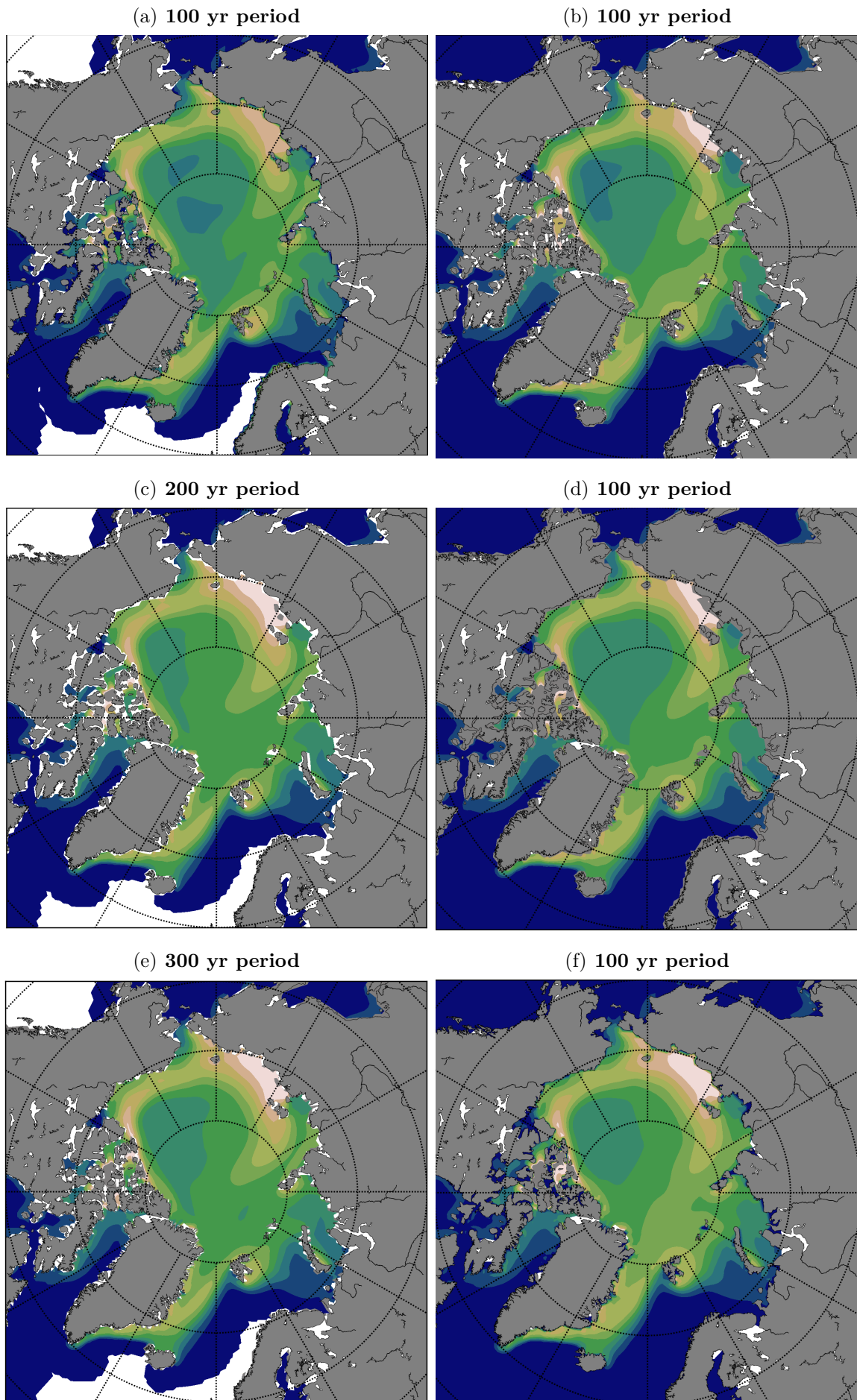


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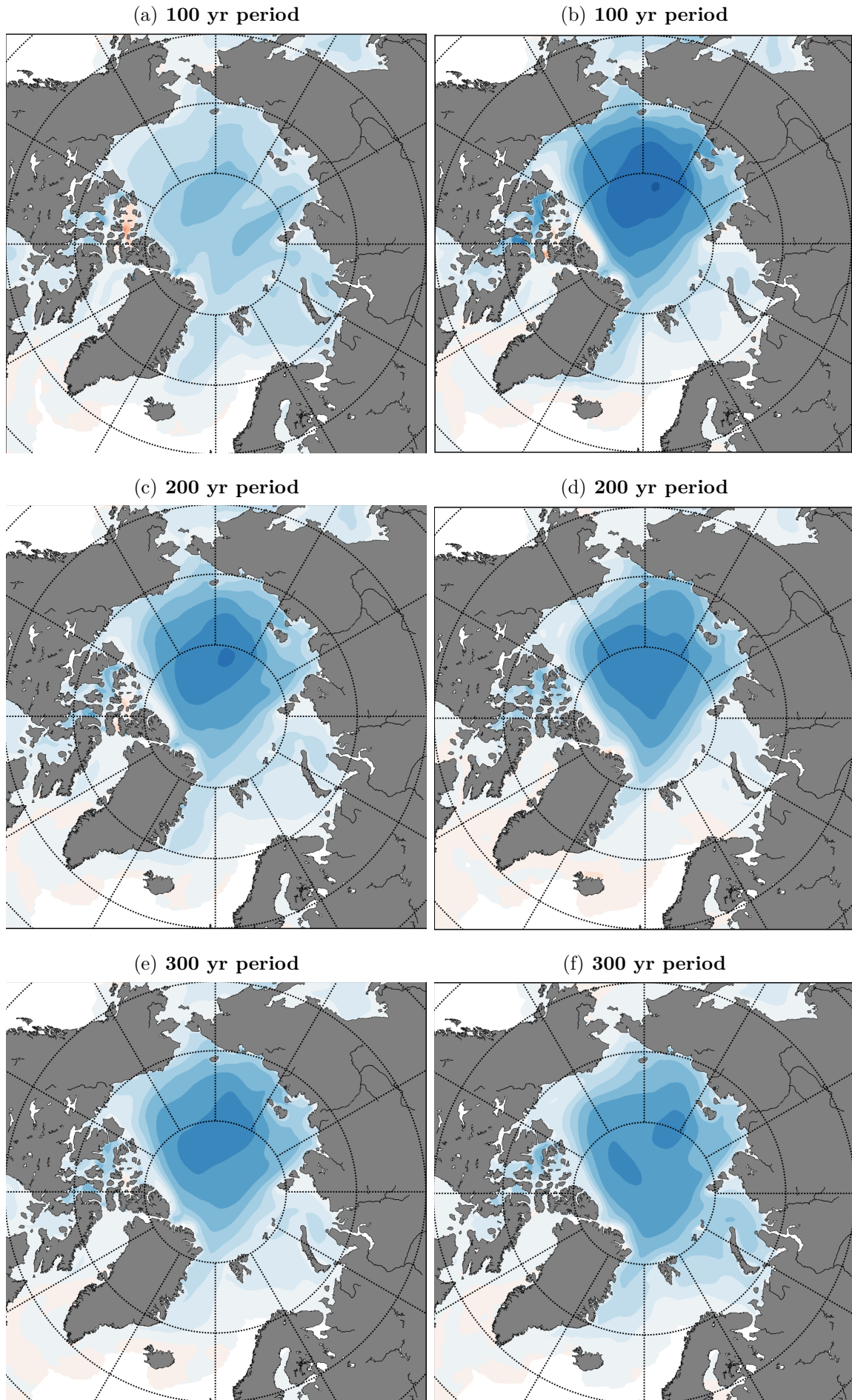


Figure 5.12: Correlation of the volume anomaly with the locals anomalies of thickness over 100, 200 and 300 years over different periods.

