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Loss Aversion and Social Preferences

A Theoretical Approach to Examine the Impact of
Social Preferences on Individual Reference Points

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Abstract

Economic research explored reference-dependent decision-making in many ways. Yet, one of the main questions linked to this field still remains unanswered: what influences an individual's reference point? Many authors contributed to the debate of plausible candidates for the question on what individuals may refer while forming their decisions (e.g. status quo ([Kahneman and Tversky \[1979\]](#), [Kahneman et al. \[1991\]](#)) or rational expectations (e.g. [Kőszegi and Rabin \[2006\]](#), [Abeler et al. \[2011\]](#))). However, there is no clear finding why the reference point varies. Based on results supporting the idea of individuals comparing themselves with others to form a decision, we try to investigate whether peer outcomes may influence an individual's reference point and her risk attitudes, given she observes their outcomes. In particular, we study social reference points by combining models of social preferences with models of reference-dependent preferences and loss aversion in order to address this question and to find a plausible answer on how social preferences may drive risk-taking behavior.

Acknowledgments

"Enlightenment or awakening is not the creation of a new state of affairs but the recognition of what already is"

- Alan Watts

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Thanks to the creation of this work, I realized once again what a comprehensive and funny social environment I have. Without these persons, you would most likely find another work in front of you. Given that we attempt to rationalize a decision-making process under risk by introducing social preferences, I experienced the importance of social aspects in how we make decisions even further, and hope that this work will be sufficient to convince you as well.

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1 Introduction

Reference-dependent preferences and loss aversion have been used in a wide range of topics to explain consumer behavior. In contrast to classic economic theories, a reference-dependent decision maker evaluates personal outcomes relative to a reference point, and loss aversion implies that losses relative to the reference point loom larger than same-sized gains. Applied to classic economic settings such as industrial organization ([Heidhues and Kőszegi \[2008\]](#) or [Rosato \[2016\]](#)) and labor contracts ([Herweg et al. \[2010\]](#)), as well as in financial research on asset pricing ([Abel \[1990\]](#)) and financial markets ([Gottlieb \[2012\]](#)), these two features have greatly enhanced our understanding of individuals' attitudes towards risk. For instance, they predict first-order risk aversion: i.e. decision makers show risk aversion already for small stakes - a finding that is hard to reconcile with classic economic models ([Rabin \[2013\]](#)). As the examples given above show, this has crucial implications in many economic settings.

A key question when studying reference-dependent decision makers is: what influences an individual's reference point? Candidates for such reference points include the status quo or rational expectations. Findings of individuals comparing themselves with others when forming a decision also suggest that the social context affects an individual's reference point (see [Festinger \[1954\]](#), [Clark and Oswald \[1996\]](#), and [Neumark and Postlewaite \[1995\]](#)). If we interpret the social context as an individual's exogenous peer group in a particular economic setting, this raises questions about how reference-dependent preferences link to social preferences, and ultimately, how the social context influences an individual's risk preferences. This has crucial implications for many current economic issues. For example, do inequality-reducing policies (e.g. redistributive tax systems) enhance or reduce a person's willingness to take on risks? In this way, do social reference points boost or setback the effectiveness of such policies? Existing approaches of reference-dependent utility focus on selfish decision makers and, hence, do not consider distributive aspects.

Our principal objective is to gain a better understanding for the social dimension in economic decision-making under risk. This work explores the social environment's impact on a person's reference-dependent preferences, loss aversion and her personal risk attitudes - an issue that has not been explicitly considered yet in existing theories on reference point formation, despite its accumulating empirical evidence.¹

¹Exceptions are [Schwerter \[2016\]](#) and [Gamba et al. \[2017\]](#), but they assume reference points as exoge-

We give an overview of the literature on reference dependent preferences and social preferences, together with empirical findings on how social contexts affect individual risk-taking, in section 2. Beginning with reference dependent preferences, we elaborate different notions of social preferences and empirical findings, before highlighting the intersection of these research branches and identify how social reference points act as their mutual connection.

We proceed in section 3 by establishing a formal framework by combining theories of reference-dependent preferences with social preferences, allowing us to observe the impact of a peer's outcome on an individual's utility (including the individual's reference point). Applying it to simple lottery choices, we use this novel approach to study reference-dependent decision-making in social contexts more precisely in section 4, and construct formal hypotheses which we subsequently test by comparing it to a related study. As the main result, we find that decision makers accept more risk when their referents earn higher payoffs. Given that a person's desire to be ahead of her peers is sufficiently weak, she is willing to accept even more risks and, hence, ultimately overtakes them. We conclude in section 5 by encouraging possible extensions and briefly summarize our most important insights.

2 Related literature

Most decisions under risk are not only evaluated by their potential counterfactuals, but also relative to how other people chose (Schwelter [2016]). It is thus natural to consider social preferences and risk-taking behavior simultaneously. But despite the closeness of these two behavioral determinants, their respective literature streams evolved on different paths. In order to bring both of them together, we first discuss the literature branches on reference-dependent preferences and social preferences separately, followed by the literature on behavioral insights from the field. This allows us to stress shortcomings in both branches and visualizes what is required to identify an overlap in their research objectives.

nous and hence do not study reference point *formation*.

2.1 Reference-dependent preferences and loss aversion

The presumably greatest share of literature studying reference-dependent preferences relies on exogenous reference points (e.g. [Kahneman and Tversky \[1979\]](#), [Tversky and Kahneman \[1991\]](#), [Schwerter \[2016\]](#) and [Gamba et al. \[2017\]](#)). Although this type of reference point helps in understanding the underlying characteristics of reference dependence in the decision-making process (e.g. loss aversion or status quo bias) and thus identifies expected utility theory as a special case ([Gul et al. \[1991\]](#) and [Masatlioglu and Ok \[2005\]](#)), it fails to explain how reference points endogenously adjust to the economic environment.

Furthermore, after [Kahneman and Tversky's Prospect Theory \[1979\]](#), many authors supported a status quo-based reference point, as in most cases the level on which a person evaluates an outcome usually coincides with the current asset position she holds (see e.g. [Kahneman and Tversky \[1979\]](#) and [Gaertner and Xu \[2011\]](#)). This notion, however, does not find reasonable explanations for disposition effects in the absence of physical endowments: a 5-percent wage gain, for instance, is for a person who expected an increase of 10 percent more likely to be perceived as loss rather than a gain. Status quo-based theories fail to explain these events of sensations because the wage gain has not been realized yet. Therefore, various authors –most notably [Kőszegi and Rabin \[2006\]](#) and [Abeler et al. \[2011\]](#)– argued in favor of a purely expectations-based reference point which is consistent with previous literature and subsequent experiments.²

In order to overcome issues with one reference point in particular, a recent literature stream studies the effects of multiple coexisting reference points ([Koop and Johnson \[2012\]](#), [March and Shapira \[1992\]](#), [Sullivan and Kida \[1995\]](#), [Ordóñez et al. \[2000\]](#) and [Markle et al. \[2018\]](#)). But because observing one or more reference points in a single individual is persistently difficult and complex, it remains problematic to find plausible arguments that favor one particular number of reference points over others (see p. 142, [García-Gallego et al. \[2017\]](#)).

Beyond the question of appropriate candidates for reference points, however, all these theories of reference-dependent preferences assume almost exclusively selfish utility-maxi-

²Note that the results from [Markle et al. \[2018\]](#), who explore goals as reference points in marathon runners, can be interpreted in a sense in which goals parallel a runner's expectation. Indeed if we interpret negative departs from the goal as a form of disappointment, then disappointment aversion makes the same predictions as rational expectations. Besides that –and for further empirical results– see [Farber \[2008\]](#), [Ericson and Fuster \[2011\]](#) and [Pope and Schweitzer \[2011\]](#).

mizing decision makers and, hence, do not take any distributive aspects into account. Most importantly, these reference points are independent of outcomes in social contexts, despite the existence of the largely established literature on social preferences.

2.2 Literature on social preferences

Social preferences have been widely studied in theoretic models and empirically. In its simplest sense, social preferences mark a depart from pure self-interested utility maximization –as classically studied in economics– by extending economic problems to many individuals who additionally take distributive considerations into account when evaluating outcomes (Fehr and Fischbacher [2002]). Since the introduction of distributive concerns raises an interdependence of individual preferences, social preferences have given rise to several normative concepts. Besides individual utility maximization, the most widely studied models of social preferences are models on difference aversion and on social welfare preferences, whereas both are often combined with reciprocity concerns.

Difference aversion, or *inequity aversion*, refers to the hypothesis that any allocation characterized by distinct outcomes among individuals is less preferred than an allocation that provides each agent with a "fair share" (see Loewenstein et al. [1989], Bolton and Ockenfels [2000], Fehr and Schmidt [1999] and Englmaier and Wambach [2010]). This notion, however, may result in inefficiency as it prevents individuals with potential to obtain substantially higher outcomes without harming others from realizing them. Although Bolton and Ockenfels [2000] and Fehr and Schmidt [1999] show that difference aversion can match behavior in a number of controlled experimental games, a subsequent study by Levitt and List [2007] reveals that difference-averse behavior vanishes as soon as participants get the chance to enrich themselves at the expense of their counterparts.

More interestingly, further empirical evidence seems to point towards *social welfare preferences* (following the terminology of Charness and Rabin [2002]). Yaari and Bar-Hillel [1984], for instance, conduct an experiment in which participants are asked to form hypothetical allocation rules, suggesting that agents strive to offer social support to all individuals but particularly to those who are worst off. Another study by Andreoni and Miller [2002], in which participants allocate an amount of money between themselves and a peer given a varying exchange rate, leads to similar results.³ In addition, an extensive

³Although somewhat more difficult to interpret, results from the experimental bargaining literature

lab study by [Charness and Rabin \[2002\]](#) seems to undermine this behavior, giving further a reasonable explanation for the increased willingness to sacrifice (observed in subjects that are better off), provided these transactions are inexpensive and lead to efficiency.⁴ Based on their model, they conclude that subjects prefer efficient over egalitarian allocations. Another experimental study by [Frey and Meier \[2004\]](#) replicates these findings. Combining motives of the Rawlsian maximin criterion and a utilitarian welfare function, their model implies –similarly to the one proposed by [Fehr and Schmidt \[1999\]](#)– a constant marginal rate of substitution between individuals. One crucial drawback, however, is that these preferences do not explain attempts to hurt others, as seen in form of responder rejections in ultimatum games (see e.g. [Güth et al. \[1982\]](#), [Ochs and Roth \[1989\]](#), [Bolton \[1991\]](#), [Camerer and Thaler \[1995\]](#), and [Kagel et al. \[1996\]](#)) or punishments in public good games (see e.g. [Fehr and Gächter \[2000\]](#), [Andreoni et al. \[2003\]](#), and [Rand et al. \[2009\]](#)). Therefore, [Charness and Rabin \[2002\]](#) augment these distributive preferences with a motive for reciprocity.

In the context of social preferences, *reciprocity* refers to the desire to treat those who are fair better than those who are not ([Charness and Rabin \[2002\]](#)). Thus, an individual evaluates her peer’s action not only by how it affects the distribution ”but also by its underlying intention” ([Falk and Fischbacher \[2006\]](#)). Driven by findings indicating that distributive preferences are not sufficient to explain motives of reciprocity (see e.g. [Kahneman et al. \[1986\]](#), [Blount \[1995\]](#), [Offerman \[2002\]](#), [Andreoni et al. \[2002\]](#), and [Charness \[2004\]](#)), several theories try to combine these motives. For instance, [Rabin \[1993\]](#) and [Dufwenberg and Kirchsteiger \[2004\]](#), study reciprocity together with distributional preferences in its most general form. Other, more sophisticated modelling attempts are [Falk and Fischbacher \[2006\]](#), who link difference aversion with reciprocity in a way in which individuals are seemingly more sensitive to refusals of equal splits rather than by peers who do not want to fall behind. [Charness and Rabin \[2002\]](#) combine reciprocity with distributive preferences in general.

seem to replicate these findings (see [Roth and Murnighan \[1982\]](#) and [Kagel et al. \[1996\]](#)). Moreover, see [Rabin \[1998\]](#) for an overview.

⁴As more thoroughly discussed in section 3, [Charness and Rabin \[2002\]](#) also develop a utility function that can represent the aforementioned types of social preferences, depending on the choice of parameters.

2.3 Social dimension of risk-taking behavior

Apart from the literature of reference dependent preferences and social preferences under certainty, a great number of empirical and experimental studies provide sound evidence for the far-reaching interplay between social determinants and individual risk preferences. In an experiment by [Bolton and Ockenfels \[2010\]](#), for instance, participants make decisions in an individual and a social context, facing safe and uncertain payoffs in both conditions. They find that subjects are more willing to accept risks when safe outcome results in inequality. Interpreting this as a motive for difference aversion, the observed effects are limited, however, when individual risk-taking implies increasing uncertainty for another subject.

Findings by [Rohde and Rohde \[2011\]](#), [Linde and Sonnemans \[2012, 2015\]](#), [Schwerter \[2016\]](#) and [Fafchamps et al. \[2015\]](#) show how individual attitudes towards risk vary, depending on the social context. By varying peer's outcomes, [Rohde and Rohde \[2011\]](#) show that individual risk attitudes are significantly affected by the level of uncertainty borne by other persons. In line with the findings by [Bolton and Ockenfels \[2010\]](#), subjects show concerns for inequality in choices involving certainty, whereas effects seem to be limited when uncertainty is added.

[Linde and Sonnemans \[2012\]](#) explore the impact of social comparisons on individual risk-taking. In their set-up, subjects face multiple decisions between two lotteries while their peer earns a fixed and independent payoff. Their results suggest that subjects exhibit more risk averse behavior in situations in which they can earn at most as their peer, than in situations in which they can obtain higher outcomes. Surprisingly, given that subjects' choices affect only their own outcomes, another study by [Linde and Sonnemans \[2015\]](#) suggests that social reference points do not significantly affect individual risk attitudes.

This finding is in large contrast to [Schwerter \[2016\]](#), who employs a one-round experiment. In his setting, subjects make lottery choices in which they face a mean-variance trade-off: the potential for higher outcomes is attributed to a higher level of risk. Fixing peer outcomes at a low and a high level which serve as a social reference point, subjects deliberately choose higher outcomes when peer earnings are high (compared to when peer earnings are low). Although lottery choices do not center around referents' payoffs, [Schwerter \[2016\]](#) finds evidence for subjects' increased willingness to take on risk when they are informed that their opponent earns a higher predetermined, fixed outcome. This

finding is in line with studies that explore effects of social comparison on risk attitudes (see [Fafchamps et al. \[2015\]](#) on asset integration, and [Gamba et al. \[2017\]](#) on bank executives). A subsequent robustness analysis rules out that the observed behavior is caused purely by expectations or anchoring effects alone.

All these findings indicate a strong causal link between risk attitudes and the social context in which risky decisions are made. While results point out that individuals are willing to accept more risks to decrease inequality across the social dimension in cases in which they are ahead ([Bolton and Ockenfels \[2010\]](#)), being behind their social referents makes them willing to accept more risk ([Schwerter \[2016\]](#) and [Fafchamps et al. \[2015\]](#)).

2.4 Overlaps of the research on reference-dependent preferences and social preferences

Both the literature branches of reference-dependent preferences and social preferences grew rapidly in the past decades but were not jointly studied in a formal manner, yet. As a result, the overlap of these theories remains still an understudied research field, and existing theories of reference-dependent utility cannot fully explain how economic outcomes of peers can change an individual's risk attitudes, despite growing empirical evidence. Only a few, most notably [Schwerter \[2016\]](#) and [Gamba et al. \[2017\]](#), conducted experiments on the impact of social comparison on reference-dependent risk attitudes. But both of these works use an exogenous, rather ad-hoc formulation of a social reference points without incorporating social preferences explicitly. Our aim is to combine both literature streams in order to study how social preferences affect individual risk-taking behavior in more detail.

3 Theoretical Framework

As the previous section shows, two main features are indispensable for our model to study social reference points and their effect on individual risk attitudes: reference-dependent decision-making and social preferences. In its simplest form, reference-dependent preferences relate outcomes to a comparison standard and hence define positive departs from the comparison standard as gains and negative departs as losses. Together with loss aversion

–which enhances the sensation of losses in comparison to same-sized gains– reference-dependent preferences predict first order risk aversion.

We first present a model of reference-dependent decision-making which forms the base of our theory. We proceed by enhancing this model in a way in which it captures distributive aspects as well: instead of classic outcome-based utility as used in [Kőszegi and Rabin \[2006, 2007\]](#), we use a utility function that allows for different social preferences (see [Charness and Rabin \[2002\]](#)).

3.1 Reference-dependent decision-making

Our model builds mainly on the framework by [Kőszegi and Rabin \[2006, 2007\]](#), which, in its essence, adds a reference-dependent decision making component to classical outcome-based utility. Formally, preferences can be characterized by the utility function

$$u(x|r) = m(x) + \mu[m(x) - m(r)], \quad (1)$$

where outcomes $x \in \mathbb{R}^K$ are evaluated with respect to a (social) reference level $r \in \mathbb{R}^K$ and $K \in \mathbb{N}$. Note that while the first term $m(\cdot)$ in equation (1) represents outcome-based utility as regularly stressed in economics, $\mu(\cdot)$ depicts gain-loss utility. For simplicity, it is assumed that both $m(\cdot)$ and $\mu(\cdot)$ are separable across K ,

$$m(x) = \sum_{k=1}^K m_k(x_k) \quad \text{and} \quad \mu(x|r) = \sum_{k=1}^K \mu_k(x_k|r_k), \quad (2)$$

making it applicable to the case of multidimensional outcomes. It is assumed that $m(\cdot)$ is differentiable and strictly increasing, and that $\mu(\cdot)$ satisfies the following properties:

- A1. $\mu(x)$ strictly increasing and continuous $\forall x$, twice differentiable
 $\forall x \neq 0$, and $\mu(0) = 0$.
- A2. $y > x > 0 \Rightarrow \mu(-y) + \mu(y) < \mu(-x) + \mu(x) \forall x, y$.
- A3. $\mu''(x) \leq 0 \forall x > 0$ and $\mu''(x) \geq 0 \forall x < 0$.
- A3' $\mu''(x) = 0 \forall x \neq 0$.
- A4. $\frac{\lim_{x \rightarrow 0} \mu'(-|x|)}{\lim_{x \rightarrow 0} \mu'(|x|)} \equiv \lambda > 1$.

While assumptions A2 and A4 yield loss aversion (respectively for large and for small stakes), A3 is diminishing sensitivity. Loss aversion implies that perceived losses loom

larger than same-sized gains, and diminishing sensitivity says that differences between outcomes in the same domain remote from the reference point are perceived with less importance. In order to isolate direct consequences of loss aversion, we use $A3'$ as an approximation for slight changes in $\mu(\cdot)$. Similarly to $m(\cdot)$, we impose $A1$ on $\mu(\cdot)$ for analytic purposes in order to represent rational preferences. Thus, this model features all the assumptions that [Kahneman and Tversky \[1979\]](#) imposed on the value function defined on $x - r$.

In contrast to models with *exogenous* reference points (e.g. [Tversky and Kahneman \[1991\]](#), [Schwerter \[2016\]](#) and [Gamba et al. \[2017\]](#)) which permit the evaluation of stochastic outcomes x drawn from the probability measure F) as

$$U(F|r) = \int m(x) + \mu[m(x) - m(r)]dF(x), \quad (3)$$

the model proposed by [Kőszegi and Rabin \[2006, 2007\]](#) also allows for stochastic reference points: given a reference level r drawn from a probability measure G , the (stochastic) outcome x drawn from F is evaluated as

$$U(F|G) = \iint m(x) + \mu[m(x) - m(r)]dG(r)dF(x). \quad (4)$$

This particular feature allows us to study endogenous reference points evoked by e.g. rational expectations.

3.2 Incorporating social preferences

We further borrow a simplified version of the utility function from [Charness and Rabin \[2002\]](#) to embed social preferences in our model.⁵ Outcome-based utility is described by

$$m(x) = (a\rho + b\sigma)s + (1 - a\rho - b\sigma)x, \quad (5)$$

where x is the reference-dependent decision maker's outcome and s her peer's outcome – acting as the social reference point (hence the former r equals s). The variables a and b

⁵Although various experiments indicate that reciprocity concerns play an important role in explaining social preferences (see the discussion in section 2.2 above), we deliberately choose not to embed reciprocity in our analysis. This has two reasons. First, incorporating reciprocity would complicate both our analysis and the interpretation of the findings in section 4. Second, since the application in section 4 describes a single decision-making process where the social referent point is exogenously given, any actions from peers' side are anyway absent.

are indicator variables such that $a = 1$ if $x > s$ (else $a = 0$) and $b = 1$ if $x < s$ (else $b = 0$). As ρ represents the weight for a person's concern for when she is ahead of her opponent, the weight σ reflects the concern for when she is behind. And while $\sigma \leq 1/2$ prevents individuals from being stronger concerned with other persons' payoffs than their own (for the case in which others get more than them), the statement $\rho \geq \sigma$ is equivalent to say that an individual's preference for more is at least as high when she gets less than her peer (i.e., $x < s$) as when she gets more (i.e., $x > s$). Thus, the difference $\rho - \sigma$ captures a person's willingness to catch up with her referents.

Depending on how ρ and σ are parametrized, the utility function in expression (5) can represent different kinds of social preferences. For example, for $\sigma \leq \rho \leq 0$, $m(x)$ can be understood as a representation of competitive preferences: the decision maker prefers to perform at least as good as her peer when she is ahead as well as when she is behind. Given $\sigma < 0 < \rho < 1$, $m(x)$ decreases in the distance between x and s - a hypothesis that is generally known as difference aversion (see e.g. Bolton and Ockenfels [2000], Fehr and Schmidt [1999] and Loewenstein et al. [1989]). Another type of social preferences that can be represented with this function are social welfare preferences, under which individuals generally prefer higher outcomes for all players (including themselves, but also their peers), but prefer to obtain more than their peers when they are behind (i.e., in the case of $x < s$). Social welfare preferences, as understood in Yaari and Bar-Hillel [1984] and Andreoni and Miller [2002], imply $0 < \sigma \leq \rho \leq 1$. Finally, the absence of social preferences, which we call *pure self-interest*, can be represented by $\rho = \sigma = 0$.

4 Application to choices under uncertainty

To study reference-dependent decision-making with social preferences, we use monetary lotteries as seen in Andreoni and Harbaugh [2009] or Schwerter [2016]. The reason why we pick these lotteries is because they have been empirically tested and hence allow us to compare our insights with other studies. In particular, we first present these lotteries and the underlying decision-making process before comparing our findings to Schwerter [2016]. Interestingly, the model by Schwerter [2016], who also studies social reference points, makes numerous predictions that are in line with the approach we present. As we see later, however, the model presented here suggests behavior that is more conform with empirical results and hence provides a better fit for them.

4.1 Binary lotteries over monetary outcomes

Suppose that lottery choices are designed in such a way that monetary higher outcomes involve more risk. While a decision maker can earn the outcome x with probability $q(x)$, with probability $1 - q(x)$ she earns nothing. Given its specification, $q(x)$ is decreasing in x . Outcome choices can be made between the range of $\bar{x} - r$ and \bar{x} , where $x = \bar{x} - r$ has probability $q(\bar{x} - r) = 1$ and $q(\bar{x}) = 0$. Specifically, subjects choose a binary lottery from a set of lotteries $(x(q), q)$, with $x(q) = x - rq$, $q = i/100$ for integers $i \in [0, 100]$ (therefore $q(x) = (\bar{x} - x)/r$). Ultimately, subjects faced a trade-off between the size of an outcome and its likelihood. Following [Schwerter \[2016\]](#), a person makes a binary lottery choice after observing the predetermined earnings s of a peer which can be either high (s_{HI}) or low (s_{LO}) such that

$$\frac{1}{2}\bar{x} > s_{HI} > \underline{x} = \bar{x} - r > s_{LO} > 0. \quad (6)$$

4.2 Applying the model

Now, we apply the model proposed in section 3. Given decision makers have social preferences that are approximately linear in x and s , their preferences can be represented by $m(x, s) = \rho s + (1 - \rho)x$ when they are ahead of their peer (i.e. $x > s$), and $m(x, s) = \sigma s + (1 - \sigma)x$ when peers earn relatively more than them (i.e. $x < s$). In order to highlight the effects of loss aversion, assume that $A3'$ holds and hence $\mu(z) = \eta z$ for $z > 0$ and $\mu(z) = \eta \lambda z$ for $z < 0$, where $\eta > 0$ is the weight attached to gain-loss utility and $\lambda > 1$ the *loss aversion index* as labeled by [Köbberling and Wakker \[2005, p. 121\]](#). Suppose first a person is seeking for outcomes higher than those of her peer. If that person has rational expectations about outcomes, her overall preferences are given by

$$\begin{aligned} U(x|s) = & \frac{\bar{x} - x}{r} [\rho s + (1 - \rho)x] + \left(1 - \frac{\bar{x} - x}{r}\right) \sigma s \\ & + \eta \left[\left(\frac{\bar{x} - x}{r}\right)^2 \left([\rho s + (1 - \rho)x] - [\rho s + (1 - \rho)x]\right) \right. \\ & \quad + \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r}\right) \left([\rho s + (1 - \rho)x] - \sigma s\right) \\ & \quad + \lambda \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r}\right) \left(\sigma s - [\rho s + (1 - \rho)x]\right) \\ & \quad \left. + \left(1 - \frac{\bar{x} - x}{r}\right)^2 \left(\sigma s - \sigma s\right) \right]. \end{aligned} \quad (7)$$

The first line is outcome-based utility. Subjects obtain with probability $q(x) = (\bar{x} - x)/r$ more than their peers and nothing with $1 - q(x)$. Note that although a person gets with probability $1 - q(x)$ nothing, social preferences enable her to benefit (or suffer) from peer earnings s . The remaining lines are gain-loss utility. In particular, the second line compares the case of getting more than the peer with the expectation to get more than the peer. Since expectations parallel the hypothetical outcome, this feels neither like a gain nor a loss. The third line depicts a gain as it compares getting more than the peer with the expectation to get nothing. The third line compares getting nothing with the expectations to get more than the peer, which is evaluated as a loss. The last line shows the comparison between getting nothing and expecting to get nothing, which, similarly to the second line, is neither a gain nor a loss.

Since anticipated outcomes lead to no gain-loss sensation, we can simplify the expression above as

$$\begin{aligned}
 U(x|s) = & \sigma s + \frac{\bar{x} - x}{r} [\rho s + (1 - \rho)x - \sigma s] \\
 & - \eta(\lambda - 1) \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r} \right) [\rho s + (1 - \rho)x - \sigma s].
 \end{aligned} \tag{8}$$

Note that the first term in the first line, σs , represents the minimal result the decision maker will obtain no matter what state of the world is realized. The term in brackets compares the favorable outcome of $x > s$ with the worst outcome of getting nothing. Following the definition of social risk in [Brainard and Dolbear \[1971\]](#), this term can be interpreted as the excess utility of each risky outcome higher than the worst outcome. The second line is gain-loss utility, which captures a mixed feeling of getting more than the peer and obtaining nothing. Since losses are more pronounced than same-sized gains, $(\lambda - 1)$ is positive.

Suppose now the decision maker chooses an outcome x below s . By applying the same reasoning as above, her overall utility is given by:

$$\begin{aligned}
 U(x|s) = & \sigma s + \frac{\bar{x} - x}{r} (1 - \sigma)x \\
 & - \eta(\lambda - 1) \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r} \right) (1 - \sigma)x.
 \end{aligned} \tag{9}$$

Again, due to social preferences, she obtains σs for sure, and gets x with probability $(\bar{x} - x)/r$. Her gain-loss utility is a mixed feeling of getting x and getting nothing. [Figure 1](#) depicts $U(x|s)$ for the case of $\rho > \sigma$ and $s = 8$.

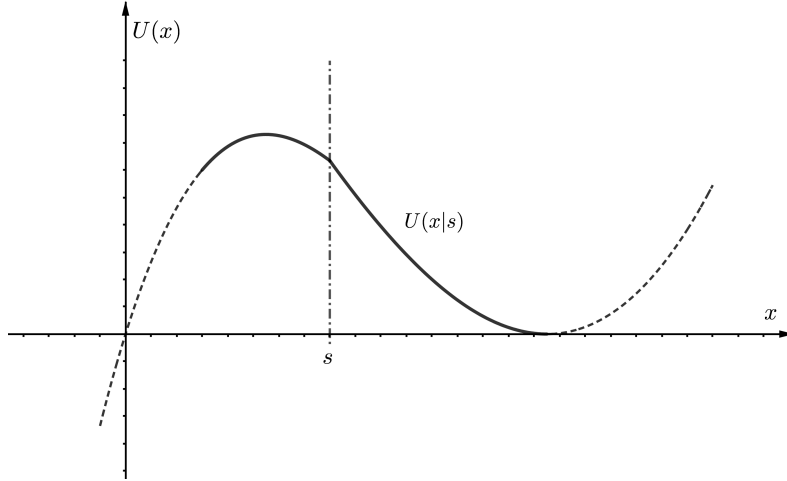


Figure 1: Graphic representation of $U(x|s)$.

The solid line graphs $U(x|s)$ in the range of the selectable x , i.e. $x \in [\bar{x} - r, \bar{x}]$, while the dotted line plots $U(x|s)$ beyond this domain. Note that since $U(x|s)$ is a cubic polynomial, beyond its local extrema $U(x|s)$ tends toward $-\infty$ on the left and $+\infty$ on the right.

First, define $\alpha \equiv \eta(\lambda - 1)$ as a person's gain-loss sensitivity and $\beta \equiv (\rho - \sigma)/(1 - \rho)$ as a normalized version of her willingness to catch up with her peers.⁶ Note that if we let $\alpha = 0$, our theory makes the same predictions as expected utility theory with social preferences. Moreover, letting $\rho = \sigma = 0$ reduces social preferences to the case of self-interest driven preferences. The Lemmas 1 and 2 and Proposition 1 highlight these reflections. More specifically, they provide a simplified insight on how individual risk attitudes change under each theory. Their proofs, together with the proof of Proposition 2, can be found in appendix at the end of this work.

Lemma 1. *For $\rho = \sigma = 0$ and $\alpha > 0$, the optimal lottery choice x is $x < \frac{\bar{x}}{2}$.*

Lemma 2. *For $\alpha = 0$ and $\rho, \sigma \neq 0$, the optimal lottery choice x is $x \leq x^*$.*

Lemma 1 says that under reference dependence, the optimal lottery choice x will be smaller than $x^* = \bar{x}/2$ implied by expected utility theory - leading to risk averse behavior. The reason therefore is loss aversion: because outcome-based utility is maximized at x^* , gain-loss utility rapidly decreases for any $x > s$ as higher lottery choices are linked to more risk. This predicts risk aversion which makes choosing a lottery with an outcome of $x < x^*$ more desirable.

⁶An alternative interpretation of β is the "desire not to fall behind." But as those interpretations imply the same -namely that a person does not want to be worse off than her opponent- we remain with the first version.

Lemma 2 establishes that under social preferences, individuals cannot exhibit stronger risk-taking behavior as compared to how they would behave under expected utility theory alone. The reason therefore lies in the convexity of $m(x, s)$: Because $m(x, s)$ is a positive-linear combination of s and x and $s < x^*$, the solution x^* is the supremum of this combination.⁷ In fact, x is bounded above by x^* when $\rho = \sigma$. This means that when a person cares as much about being behind as when ahead, she picks the lottery that minimizes this trade-off. Alternatively, when she cares less about being behind (i.e. $\rho > \sigma$), she can accept lotteries that involve less uncertainty and thus seeks for smaller outcomes with higher probability.

The next proposition makes a statement relative to expected utility theory about the case in which we combine reference-dependent preferences with social preferences.

Proposition 1. *For $\alpha > 0$ and $\rho, \sigma \neq 0$, the optimal lottery choice x is $x < x^*$.*

Based on the results of Lemma 1 and Lemma 2, Proposition 1 manifests that a person cannot exhibit more risk-taking behavior when she has both social preferences and reference-dependent preference compared to how she would choose in the absence of both. Therefore, desirable lottery choices can only be found in the interval $[\bar{x} - r, \bar{x}/2)$. Another striking insight from Proposition 1 is that loss aversion yields risk aversion even when individuals have distributive concerns.

But how does the decision-making process exactly look like? That is, how will a person choose who is both loss averse and has social preferences? Based on Proposition 1, under which such a person would seek for lower outcomes as expected utility theory predicts, the following proposition demonstrates how social preferences and loss aversion interact.

Proposition 2. *The optimal lottery choice x is*

1. $x = \bar{x} - r$ for $\alpha \geq \bar{\alpha}$,

2. $x = \frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha}$ for $\bar{\alpha} < \alpha < \underline{\alpha}$,

3. $x = s$ for $\alpha \leq \underline{\alpha}$ and $\beta \geq \tilde{\beta}$,

4. $x = \frac{r + \alpha(2\bar{x} - r - s\beta) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r) + \alpha s\beta[r + \alpha(2\bar{x} - r + s\beta)]}}{3\alpha}$ for $\alpha \leq \underline{\alpha}$ and $\beta < \tilde{\beta}$,

where $\bar{\alpha} = \frac{2r - \bar{x}}{\bar{x} - r}$, $\underline{\alpha} = \frac{r(\bar{x} - 2s)}{s(4\bar{x} - 2r - 3s) - \bar{x}(\bar{x} - r)}$, and $\tilde{\beta} = \frac{r(\bar{x} - 2s) + \alpha[\bar{x}(\bar{x} - r) + s(2r + 3s - 4\bar{x})]}{s[r + \alpha(2\bar{x} - r - 2s)]}$.

⁷Because $s < x^*$, for any $\gamma \in [0, 1]$, the term $\gamma s + (1 - \gamma)x^*$ is maximized at $\gamma = 0$.

Proposition 2 establishes that for sufficiently high loss aversion, i.e. $\alpha > \bar{\alpha}$, the optimal lottery choice x is $(\bar{x} - r)$. Because loss aversion is very high, the person tries to avoid any risk of potential losses. Therefore, no matter her social preferences, she will seek for the safest outcome possible. Because $(\bar{x} - r)$ is completely certain, her utility will be maximized at this choice. This can be seen in expression (9): while consumption utility is maximized at $x = \bar{x}/2$ (as implied by expected utility), utility from gains and loss enters negatively – making accepting risks less favorable. For increasing α , she thus reduces her exposure to potential losses up to $\alpha = \bar{\alpha}$, where it is minimized at the certain outcome $(\bar{x} - r)$. Her behavior is therefore fully governed by loss aversion. The following hypothesis reflects this reasoning.

Hypothesis 1. *Decision makers who are highly loss averse choose safer outcomes.*

When the agent becomes less loss averse, however, accepting risks becomes more affordable. Given $\alpha < \bar{\alpha}$, she hence chooses $x > (\bar{x} - r)$. At the limit $\alpha = 0$, overall utility consists exclusively of consumption utility. She hence chooses at most the outcome $\bar{x}/2$. Outcomes above $\bar{x}/2$ are less preferred because every additional unit of x above $\bar{x}/2$ involves more risk and hence makes its realization less likely. As a result, marginal utility for $x > \bar{x}/2$ is negative. If the person is, in addition, loss averse, Proposition 1 says that marginal utility will become negative even earlier, suggesting a lower level of risk to be optimal (see Figure 1). This reasoning resonates in the second statement of Proposition 2: given a social reference point s greater than x , a loss averse person makes her lottery choice only based on her own sensitivity for gains and losses. Because $x < s$, she knows that her optimal lottery choice will be below her peer’s earnings and hence remains unaffected by s . As a result, she chooses independently of her peers, with the optimal lottery choice being

$$x = \frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha}. \quad (10)$$

Now consider the case in which a decision maker has a very small, but still persistent gain-loss sensitivity. Reducing her sensation of gains and –most importantly– of losses makes her willing to accept higher levels of risk as the effect of risks of potential losses vanishes. In particular, if her gain-loss sensitivity falls below a certain threshold, her decision-making is not governed by mere loss aversion anymore. Thus, given $\alpha \leq \underline{\alpha}$, she

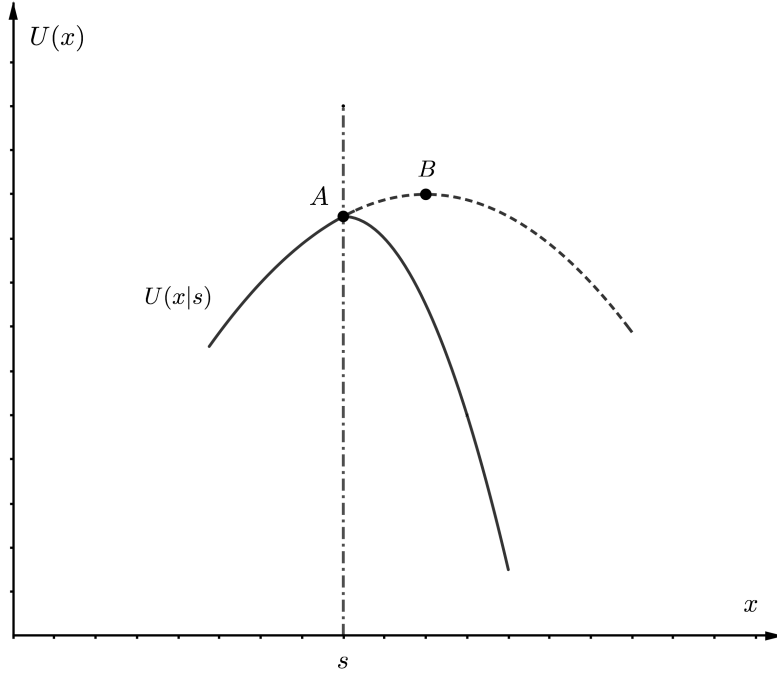


Figure 2: Local maximum depending on β .

While the solid line left from s depicts $U(x|s)$ for $x < s$, right from s it graphs $U(x|s)$ for $x > s$ for the case in which $\alpha \leq \underline{\alpha}$ and $\beta \geq \tilde{\beta}$ - yielding a local maximum at $x = s$ (see point A). The dotted line right from s plots $U(x|s)$ for the case of $x > s$ and $\alpha < \underline{\alpha}$ with $\beta < \tilde{\beta}$ - implying a maximum in the interval (s, x^*) (see point B).

is hence able to catch up with her peers by accepting more risks. Interestingly, $\alpha \leq \underline{\alpha}$ is furthermore a necessary condition for individuals to seek for higher outcomes than their peers, but, as we see below, it is not sufficient (see Figure 2). As β measures an individual's willingness to catch up, a high β reflects the motivation to be at least as good as the peer. Given that a person is weakly loss averse, she settles at $x = s$ if she has a strong motive to perform as good as her peer. This behavior arises because social preferences interact with loss aversion. Specifically, under social preferences, when a person wants to perform at least as good as her peer, she wants to choose outcomes that are at least as high as s . This urge is even stronger under loss aversion: because losses are evaluated in relation to what peers earn, the desire to be ahead becomes more pronounced. However, choosing higher outcomes involves also more risks. Therefore, even under social preferences, she faces a trade-off between higher outcomes and their associated likelihood (see Lemma 2). Since loss aversion implies a distaste for risks because these involve potential losses (see Lemma 1), she is tempted to minimize her risk exposure even further – trading it off

against her desire to be ahead. As a result, she chooses the lottery that maximizes her preferences for not falling behind while taking no more risk than strictly necessary. These preferences are hence maximized at $x = s$.

Hypothesis 2. *Lottery choices bunch at the amount of peer's earnings if decision makers are weakly loss averse and have a strong desire to be at least as good as their peer.*

However, this specification of preferences depicts a special case for loss aversion and social preferences, and is therefore unlikely to be observed in a heterogeneous sample decision-making subjects with different levels of loss aversion and social preferences. We thus hypothesize:

Hypothesis 3. *Given a heterogeneous sample of decision-making subjects, the lottery choices of these subjects do not bunch at peers' earnings.*

How would a decision maker act if she does not have a strong desire to catch up, but is still weakly loss averse? Surprisingly, she would choose an outcome that is above of what her peer gets. In particular, given $\beta < \tilde{\beta}$ and $\alpha < \underline{\alpha}$, she chooses $x > s$ as long as $x < \bar{x}/2$ (see Proposition 1). This is so because for $x > s$, losses are proportional to β . In fact, this becomes more apparent when we rewrite equation (8) as

$$U(x|s) = \sigma s + \frac{\bar{x} - x}{r} [s(\rho - \sigma) + (1 - \rho)x] - \alpha \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r}\right) [s(\rho - \sigma) + (1 - \rho)x]. \quad (11)$$

Note that $s(\rho - \sigma)$ is proportional to β , and that since $\rho \geq \sigma$, it is positive for any $\rho > \sigma$ and goes to zero as soon as σ approaches ρ . Thus, a decreasing β implies that $s(\rho - \sigma)$ decreases and vice versa. Note further that while outcome-based utility increases with greater difference between ρ and σ , gain-loss utility on the other hand decreases. Hence, if a person cares less about being ahead (in other words, being behind becomes more acceptable), losses relative to the peer are perceived as weaker, so the sensation of losses decreases even further. Because, in turns, the implications of loss aversion on decision-making become less pronounced, the agent can accept more risk, and thus seeks for higher outcomes. But due to the fact that marginal outcome-based utility is negative for any $x > \bar{x}/2$ and loss aversion still prevails (even weakly), the optimal lottery choice will be below $\bar{x}/2$, namely

$$x = \frac{r + \alpha(2\bar{x} - r - s\beta) - \sqrt{r^2(1 - \alpha)^2\alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r) + \alpha s\beta[r + \alpha(2\bar{x} - r + s\beta)]}}{3\alpha}. \quad (12)$$

Further, note the expression above increases in s for s in the interval of desirable outcomes, i.e. $s \in [\bar{x} - r, \bar{x}/2)$. Therefore, raising peers' earning leads to a *keeping-up-with-the-winners effect* (Gamba et al. [2017]): since x moves with s in the interval $[\bar{x} - r, \bar{x}/2)$, a loss averse decision maker seeks for higher outcomes while simultaneously accepting higher levels of risk.

Hypothesis 4. *Decision makers choose higher outcomes when peers earnings are more than the amount of the safe outcome, but within the set of desirable outcomes.*

Finally, by looking at equation (9), it becomes apparent that the optimization of for $x < s$ is independent of s , and that for $s = 0$, the optimal lottery choice for the case $x > s$ coincides with the optimal lottery choice for $x < s$. We thus conjecture that in the case in which social comparisons are absent or irrelevant, a highly risk averse person will seek for safer outcomes. Adding a salient possibility for social comparisons into the decision-making context, however, should make this person willing to accept more risks (see previous hypothesis).⁸

Hypothesis 5. *For peer earnings that are outside the set of possible lottery choices, decision makers choose independently of what their peers obtain.*

Having derived some testable hypotheses, we now compare this model with an alternative version by evaluating its consistency with some empirical findings. Note, however, that in order to make these statements, we had to make no further specifications about whether these preferences reflect difference aversion or other social preferences but merely that distributive preferences exist.

4.3 Comparison between models

The presented model displays an alternative to the one proposed in Schwerter [2016]. One crucial difference is the conceptualization of the reference point: while the framework proposed in this work allows for an endogenous reference point formation with peer earnings being simply a "cue", Schwerter [2016] uses an exogenous reference point fully represented

⁸By *salient* we mean peers' earnings that are in the range of desirable outcomes. For instance, due to the fact that the range in which a decision maker can choose an outcome is bounded below by $\bar{x} - r > 0$, peer's earnings below $\bar{x} - r$ might not be sufficiently salient to raise a social comparison motive because the decision maker will receive in any case more than her peer if she accepts no risk at all.

by peer earnings.⁹ As a result, reference points in the presented approach arise naturally from the social context as individuals form rational expectation over outcomes. Another distinct feature is the explicit extension to social contexts: whereas the social reference point by [Schwerter \[2016\]](#) is induced only by peer outcomes, our approach embeds preferences over the distribution of outcomes directly in the utility function - attributing a measurable parameter β to a person's willingness to "catch up" with her referent.

With his model, [Schwerter \[2016\]](#) explores the impact of social reference points on individual behavior when a person's decision affects only her own outcomes. It forecasts that lottery choices center around the social reference point s when s is sufficiently large. This conjecture is mainly based on model specifications: while marginal utility is strictly positive for any $x < s$, it is strictly negative for any $x > s$ for a sufficiently large s . This prediction, however, is in large contrast to the here-proposed model. Introducing social preferences shows that lottery choices paralleling peer outcomes are a special case. In fact, given a heterogeneous sample with different levels of loss aversion and social preferences, bunching around s would be very unlikely. This finding is also confirmed by the data. [Schwerter \[2016\]](#) reports only a very small fraction of subjects chose this outcome: only four out of 264 subjects selected an outcome equal to the one of their peer – providing supportive ground for hypothesis 3.

Although [Schwerter \[2016\]](#) finds that more risk averse subjects choose on average less risky lotteries, it remains unclear whether this risk aversion arises solely from loss aversion or from the outcome-based preferences (or both). We are hence unable to make final statements about Hypotheses 1 and 2.

Furthermore, his analysis of the data shows that subjects chose on average riskier lotteries when peer earnings are high as compared to the case when peer earnings are low. This finding, which is significant at the one-percent level, is a strong argument in favor of Hypothesis 4 and was predicted by both models. It is moreover in accordance with the findings by [Bolton and Ockenfels \[2010\]](#): catching up to one's peer does not only result in more risk-taking, but also in attempts to decrease inequality among individuals.

In addition, Hypothesis 5 generated by our model states that for peer earnings outside the set of possible lottery choices, decision-making subjects choose independently from s .

⁹Given the specification above, our model still posits rational expectations as the reference point. A sharp distinction, however, is that these rational expectations are not solely based on outcomes per se, but build on the distribution of outcomes.

Schwerter [2016] reports no significant difference in lottery choices between the treatments with high and low reference points in the nonsocial experiment. And although no explicit analysis was conducted, Figure 6 in Schwerter [2016] indicates no substantial difference in lottery choices in the treatments of low reference points between the main and the nonsocial experiment, either. Even though we cannot take this as statistical evidence, together with the previous finding in which a salient reference point affected risk-taking behavior in a positive manner, it provides little support for Hypothesis 5.

Finally, Schwerter [2016] finds in his experimental analysis that lotteries with outcomes above peer earnings are chosen more often when peers' earnings are high. In contrast to his framework, also this finding can be explained with our model. Since the optimal lottery choice x increases in peer earnings s for the case $x > s$, more subjects will choose higher outcomes when s is high. Therefore, given that these subjects' desire to catch up is relatively small ($\beta < \tilde{\beta}$), they will seek for outcomes higher than those of their referents.

In line with a variety of empirical findings, Schwerter's [2016] model falls back when it comes to explain why people seek for outcomes that are above those of their opponents. Although it correctly predicts the rough direction in which risk attitudes shift when others earnings change, it does not fully capture determinants of social behavior. In particular, it generates misleading insights in cases in which opponents obtain an outcome that otherwise requires a fair amount of risk. While the model in Schwerter [2016] predicts bunching for sufficiently large peer earning, the alternative framework above allows for a heterogeneity of social preferences and forecasts less homogeneous behavior, resulting in insights more consistent with empirical findings.

5 Conclusions

While we intended to rationalize risk-taking behavior in a social context, our analysis remains extendable into various directions. One pursuable way would be to introduce reciprocity concerns. Although we used distributive preferences and social preferences in an interchangeable way, we almost exclusively focused on distributive concerns. One way to incorporate reciprocity would be in terms of changes in preferences: under the specification of social welfare preferences, for example, individuals face their opponents neutrally while being more concerned about less concerned about being behind. But when others

pursue mere self-interest or reject offers (as in ultimatum games), their concerns about distributional aspects decrease by the same proportionality. As explained in [Charness and Rabin \[2002\]](#), this change of preferences could be captured by an additional parameter that adjusts preferences as soon as a person feels unfairly treated.

Moreover, we applied our model in a social context in which peers acted in the most passive way. The analysis becomes more complicated, however, if we allow for peer activity as well. Linked to the discussion about reciprocity, how are individual risk attitudes affected when peers behave badly? But in an even broader sense, how does peer behavior interfere with personal decision-making under uncertainty? Not only did we make simplifying assumptions about behavior, but also the rigidity of the social context limits the applicability of this example. To find answers, a more general approach is desirable.

Finally, although the behavioral implications of our model seem to be consistent with findings from the lab, some of the underlying parameters remain difficult to measure. While the reference-dependent framework incorporates diminishing sensitivity which can be estimated directly from choice data, measuring precisely the "degree" of a person's loss aversion remains more difficult. One way out, in fact, is the method for a parameter-free estimation of loss aversion in a given choice problem provided by [Abdellaoui et al. \[2007\]](#), but this approach requires a very large sample size. Alternatively, [Abdellaoui et al. \[2008\]](#) developed a way to estimate loss aversion at the cost of further assumptions. Nevertheless, the estimation for a person's willingness to catch up with her peers posits another obstacle to fully verify this theory empirically. [Charness and Rabin \[2002\]](#) estimate parameters of social preferences in a variety of tests and games. Although their extensive analysis seems to be promising in terms of accuracy, it still remains confined to the lab. However, in comparison to alternative theories of social reference points, the presented approach makes further-reaching predictions at the cost of one additional parameter.

The presented model provides theoretical ground that rationalizes individual decision-making under uncertainty in social contexts. By combining reference-dependent utility with social preferences, it replicates numerous behavioral traits that have been observed empirically. While loss aversion makes individuals more risk averse, distributional preferences yield sufficient motives to catch up with one's peers and ultimately to overtake them. Implicitly assuming that reference points are formed by rational expectations, this framework incorporates determinants from social contexts that create further insights about economic behavior under uncertainty.

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Appendix

Proof of Lemma 1.

Proof. In order to determine the range in which the optimal lottery choice x can be found, and due to the fact that $U(x|s)$ is a piece-wise defined function, we have to distinguish between two different cases: $x < s$ and $x > s$. We first demonstrate the prediction by expected utility, whose solution we denote by x^* . We further show that the statement above holds for the case of $x < s$ in Step 2, proceeded by examining the case of $x > s$ in Step 3. Denote $\alpha \equiv \eta(\lambda - 1)$ as gain-loss sensitivity, where $\eta > 0$ and $\lambda > 1$.

Step 1. Consider first only expected utility theory. Because this implies $\alpha = 0$, both expressions (9) and (8) collapse to

$$U(x|s) = \frac{\bar{x} - x}{r}x. \quad (13)$$

Note that $U(x|s)$ does only depend on x . From the first order condition, it becomes immediately apparent that

$$x^* \equiv x = \frac{\bar{x}}{2}. \quad (14)$$

Step 2. For $x < s$, Lemma 1 is satisfied because it holds $s < x^*$ by definition, and since it is assumed that $x < s$, also $x < x^*$ must hold.

Step 3. For the case of $x > s$, the optimal lottery choice is $x < x^*$. In the absence of social preferences, overall utility collapses from (8) to

$$U(x|s) = \frac{\bar{x} - x}{r}x - \alpha \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r}\right)x. \quad (15)$$

From its first order condition,

$$\frac{1}{r} \left[\bar{x} - 2x - \alpha \left(\bar{x} - 2x + \frac{4\bar{x}x - 3x^2 - \bar{x}^2}{r} \right) \right] = 0, \quad (16)$$

it becomes apparent that for $\rho = \sigma = 0$, the optimal lottery choice has to be $x < x^*$. To show this, assume $x \geq x^*$. While the difference $(\bar{x} - 2x) = 0$ if and only if $x = x^*$, the term $(4\bar{x}x - 3x^2 - \bar{x}^2)$ remains positive for any $x < \bar{x}$ - yielding $U'(x|s) < 0$. And while for any x above x^* converging to \bar{x} this term tend to zero, $(\bar{x} - 2x)$ becomes negative for that particular x - implying $U'(x|s) < 0$ as well. Thus, for $\alpha > 0$ and $\rho = \sigma = 0$, the lottery optimal choice is $x < x^*$. \square

Proof of Lemma 2.

Proof. As for the proof of Lemma 1, we start by showing that Lemma 2 holds for $x < s$, followed by the proof for $x > s$ in a second step.

Step 1. Consider first $x < s$. Because $s < x^*$ by definition and $x < s$ by assumption, these two conditions imply the optimal $x < x^*$ - satisfying Lemma 2 with strict inequality.

Step 2. We now show that the result from Step 1 holds for the case of $x > s$ as weak inequality. Indeed, in the limit case of $\alpha = 0$ with social preferences $(\rho, \sigma \neq 0)$, the optimal lottery choice is $x \leq x^*$. Recall that it holds $1 \geq \rho \geq \sigma$ by definition. Taking $\alpha = 0$ translates equation (8) to

$$U(x|s) = \frac{\bar{x} - x}{r}(\rho s + (1 - \rho)x) + \left(1 - \frac{\bar{x} - x}{r}\right)\sigma s \quad (17)$$

with its first order condition

$$\frac{1}{r} \left[(1 - \rho)(\bar{x} - 2x) - s(\rho - \sigma) \right] = 0. \quad (18)$$

Since $1 \geq \rho \geq \sigma$, the latter term in the first order condition is negative for $\rho > \sigma$ and zero for $\rho = \sigma$. Because the absolute value of $(\bar{x} - 2x)$ is first decreasing in x for x small enough and increasing for $x > x^*$, it is minimized at $x = x^*$. Therefore, the first order conditions reaches its root at $x = x^*$ if $\rho = \sigma$ and $x < x^*$ if $\rho > \sigma$. As a result, the optimal x cannot exceed x^* . This proves Lemma 2. \square

Proof of Proposition 1.

Proof. To be able to determine the range of the optimal x , we have to distinguish between $x < s$ and $x > s$. As for the previous proofs, we begin with the former case, and end this proof by verifying that this statement also remains true for the latter one.

First, however, note that since $\lambda > 1$ and $\eta > 0$, $\alpha = \eta(\lambda - 1) > 0$ by definition.

Step 1. For $x < s$, Proposition 1 holds because $s < x^*$ is fulfilled by definition, and due to the assumption of $x < s$, $x < x^*$ must thus be satisfied as well.

Step 2. For $x > s$, the optimization problem is

$$\begin{aligned} \max_{x \in [\bar{x}-r, \bar{x})} U(x|s) &= \sigma s + \frac{\bar{x} - x}{r} [\rho s + (1 - \rho)x - \sigma s] \\ &\quad - \alpha \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r}\right) [\rho s + (1 - \rho)x - \sigma s], \end{aligned} \quad (19)$$

yielding as first order condition

$$\begin{aligned} & \frac{1}{r} \left[(1 - \alpha) \left[(1 - \rho)(\bar{x} - 2x) - s(\rho - \sigma) \right] \right. \\ & \left. + \frac{\alpha}{r} \left[s(\rho - \sigma)(2x - 2\bar{x}) - (1 - \rho)(4\bar{x}x - \bar{x}^2 - 3x^2) \right] \right] = 0. \end{aligned} \quad (20)$$

Note that for any $x < \bar{x}$, the last term is always negative. Consider first $\rho = \sigma$. Because the second and the third term cancel out, the first term has to become positive in order for x to become optimal, but this is only the case if $x < x^*$.

Now consider $1 \geq \rho > \sigma$. Suppose further that the optimal $x = x^*$. Thus, the term multiplied by α/r in the first order condition then becomes negative. Note that x^* being an optimizer implies that $U'(x^*|s) = 0$, therefore, the first term must be greater zero and of same magnitude as the second term. We can rewrite the term in the first line - and dropping $(1 - \alpha)$ - as

$$\bar{x} - 2x - \rho(\bar{x} - 2x) - s(\rho - \sigma), \quad (21)$$

where $x = x^*$ yields $\bar{x} - 2x^* = 0$, and hence the expression above collapses to $-s(\rho - \sigma)$, which, given $\rho > \sigma$, is negative and therefore contradicts x^* being an optimizer. Consider now $x > x^*$. Because this implies $\bar{x} - 2x$ to be negative, and because of $\rho \leq 1$, expression (21) becomes negative, too - contradicting $x > x^*$ being an optimizer once again. As a result, $U(x|s)$ is optimized at $x < x^*$. \square

Proof of Proposition 2.

Proof. We start by deriving first the local optima for each case of x (i.e., first for $x < s$ and then for $x > s$), followed directly by establishing their boundary conditions. We finally prove this proposition by discussing the nature of the global optimum. Denote $\beta = (\rho - \sigma)/(1 - \rho)$ and x^\dagger as the optimal x .

(1) *Local optimum for $x < s$.*

Step 1. For $x < s$, the reference-dependent decision maker maximizes

$$\begin{aligned} U(x|s) = & \sigma s + \frac{\bar{x} - x}{r} (1 - \sigma)x \\ & - \alpha \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r} \right) (1 - \sigma)x \end{aligned} \quad (22)$$

with respect to x , leading to the following first order condition:

$$\frac{1-\sigma}{r} \left[3\frac{\alpha}{r}x^2 - 2 \left(1 + \alpha \left(\frac{\bar{x}}{r} - 1 \right) \right) x + \bar{x} \left(1 + \left(\frac{\bar{x}}{r} - 1 \right) \right) \right] = 0. \quad (23)$$

Note that $(1-\sigma)$ is a mere common factor and thus irrelevant for the x^\dagger . Note further that the strictly positive first coefficient of the quadratic polynomial $U'(x|s)$ - hence $U(x|s)$ is a cubic polynomial with a strictly positive first coefficient - is sufficient to guarantee for $U(x|s)$ being first increasing up to the point where $U'(x|s) = 0$, decreasing between the local extrema (if they exist) and then increasing (see Figure 1).¹⁰

At optimum, the first order condition has to be equal to zero, and due to its quadratic form, it has either two local extrema or none. Because $U(x|s)$ is first increasing up to the first local extreme point, the lower solution must correspond to x^\dagger (as long as it is distinct from the other critical x). By denoting each polynomial coefficient of the first order condition in (23) by A, B, C respectively, the extreme points of $U(x|s)$ correspond to

$$x_{1/2} = \frac{-B^2 \pm \sqrt{B^2 - 4AC}}{2A}, \quad (24)$$

yielding as a local maximum

$$x^\dagger \equiv \frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1-\alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha}, \quad (25)$$

since $U''(x^\dagger|s) < 0$.

Step 2. Since the domain is restricted to $[\bar{x} - r, \bar{x})$ by definition and $x < s$, x^\dagger may be a solution only on $[\bar{x} - r, s)$.¹¹ We can hence locate corner solutions by solving

$$\bar{x} - r > \frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1-\alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha} \quad (26)$$

and

$$s < \frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1-\alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha}, \quad (27)$$

which yield

$$\bar{\alpha} \equiv \frac{2r - \bar{x}}{\bar{x} - r} \quad (28)$$

¹⁰In other words, given the first coefficient of $U(x|s)$ is strictly positive and $U(x|s)$ is a cubic polynomial, $U(x|s)$ is concave for $x < x^\dagger = \arg \max U(x|s)$, and convex for $x > x^\dagger$.

¹¹Proposition 1 establishes that although the domain is limited above by \bar{x} , agents will never choose $x > x^*$. Therefore, we can narrow the upper boundary even closer to x^* without loss of generality.

and

$$\underline{\alpha} \equiv \frac{r(\bar{x} - 2s)}{s(4\bar{x} - 2r - 3s) - \bar{x}(\bar{x} - r)}, \quad (29)$$

respectively. While $\alpha \geq \bar{\alpha}$ implies $x^\dagger = \bar{x} - r$, $\alpha \leq \underline{\alpha}$ suggests x^\dagger to be in the domain $[s, x^*)$ (see the discussion about the global optimum below).

(2) *Local optimum for $x > s$.*

Step 3. As for the case $x < s$, individuals maximize

$$\begin{aligned} U(x|s) = & \sigma s + \frac{\bar{x} - x}{r} [\rho s + (1 - \rho)x - \sigma s] \\ & - \alpha \frac{\bar{x} - x}{r} \left(1 - \frac{\bar{x} - x}{r} \right) [\rho s + (1 - \rho)x - \sigma s] \end{aligned} \quad (30)$$

for $x > s$, leading to the following first order condition:

$$\begin{aligned} \frac{1}{r} \left[3 \frac{\alpha}{r} (1 - \rho)x^2 - 2 \left[(1 - \alpha)(1 - \rho) + \frac{\alpha}{r} [2(1 - \rho)\bar{x} - s(\rho - \sigma)] \right] x \right. \\ \left. + \frac{\alpha}{r} \bar{x} [(1 - \rho)\bar{x} - 2s(\rho - \sigma)] + (1 - \alpha) [(1 - \rho)\bar{x} - s(\rho - \sigma)] \right] = 0. \end{aligned} \quad (31)$$

Similarly as for $x < s$, the first order condition is a quadratic polynomial with strictly positive first coefficient, indicating that between extrema (if existent), $U(x|s)$ is increasing (e.g., see Figure 1). Since for $x > s$, $U(x|s)$ is a third degree polynomial as well, and thus has either no or two local extrema. Furthermore, by dropping the multiplier $1/r$ and denoting the coefficients in the first order condition above by A' , B' and C' , respectively, local extrema are attained at

$$x_{1/2} = \frac{-B' \pm \sqrt{(B')^2 - 4A'C'}}{2A'}, \quad (32)$$

yielding as a local maximizer

$$x^\dagger = \frac{r + \alpha(2\bar{x} - r - s\beta) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r) + \alpha s\beta[r + \alpha(2\bar{x} - r + s\beta)]}}{3\alpha}, \quad (33)$$

as the second derivative of $U(x|s)$ at x^\dagger is negative (i.e., $U''(x^\dagger|s) < 0$).

Step 4. In a similar fashion as for $x < s$, x^\dagger is defined on (s, \bar{x}) for $x > s$.¹² It can be shown - by solving $s > x^\dagger$ for the case of $x > s$ - that for $\alpha < \underline{\alpha}$, x^\dagger will be in $[s, x^*)$.

¹²As for the case of $x < s$, and as Lemmas 1 and 2 and Proposition 1 show, the upper boundary of \bar{x} can be lowered to x^* (see previous footnote).

However, as we show in the discussion for the global optimum, $\alpha < \underline{\alpha}$ is not sufficient to guarantee for $x^\dagger > s$: for the case of $x > s$, x^\dagger does not only depend on α but also on β (see equation (33) and Figure 2).

In order to determine for which critical value of β the maximum of $U(x|s)$ will be above s , we need to solve

$$s < \frac{r + \alpha(2\bar{x} - r - s\beta) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r) + \alpha s\beta[r + \alpha(2\bar{x} - r + s\beta)]}}{3\alpha} \quad (34)$$

for β . As a result, we obtain

$$\beta < \tilde{\beta} \equiv \frac{r(\bar{x} - 2s) + \alpha[\bar{x}(\bar{x} - r) + s(2r + 3s - 4\bar{x})]}{s[r + \alpha(2\bar{x} - r - 2s)]}. \quad (35)$$

Note that $\tilde{\beta}$ is monotonically decreasing in α .

(3) Global optimum.

Step 5. Due to the fact that $U(x|s)$ for $\alpha > \bar{\alpha}$ is strictly decreasing in the range of $[\bar{x} - r, x^*]$, x^\dagger has beyond the boundary on the left. But since x is bounded below by $\bar{x} - r$, the the optimal x^\dagger cannot decrease this threshold (e.g. see Figure 1). As a result, for $\alpha > \bar{\alpha}$, $x^\dagger = \bar{x} - r$. Moreover, for $\alpha = \bar{\alpha}$, it holds $x^\dagger = \bar{x} - r$. Together with the preceding result, this proves the first statement in Proposition 2.

Step 6. Besides that for $x < s$ and $\alpha < \bar{\alpha}$ the optimal x^\dagger is above the lower bound $\bar{x} - r$, it is bounded above by s for $\alpha < \underline{\alpha}$. In order to illustrate this, suppose this it not the case and that for $\alpha < \underline{\alpha}$, it holds $x^\dagger < s$ – yielding a contradiction by its definition. The statement $x^\dagger < s$ can be rewritten as

$$s > \frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha} \quad (36)$$

for $\alpha \geq \underline{\alpha}$, whose solution yields $s < s$. Thus, for $\alpha < \underline{\alpha}$, it must be such that $x^\dagger \in [s, x^*)$, and as a result, for $\alpha \in (\underline{\alpha}, \bar{\alpha})$, the optimal lottery choice x^\dagger is

$$\frac{r + \alpha(2\bar{x} - r) - \sqrt{r^2(1 - \alpha)^2 + \alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r)}}{3\alpha}, \quad (37)$$

which parallels the second statement of Proposition 2.

Step 7. We now prove the third claim of Proposition 2. For $\beta \geq \tilde{\beta}$, $U(x|s)$ will be strictly decreasing on $[s, x^*)$, suggesting any $x > s$ not to be optimal.¹³ On the other hand, for $\alpha < \underline{\alpha}$, $U(x|s)$ will be strictly increasing in the interval $[\bar{x} - r, s]$, suggesting any $x < s$ not being optimal as well. As a result, given $\alpha < \underline{\alpha}$ and $\beta \geq \tilde{\beta}$, the global maximum has to be attained at $x = s$ (see Figure 2, point A).

Step 8. Suppose still that $\alpha < \underline{\alpha}$, but - in contrast to Step 7 - that $\beta < \tilde{\beta}$. Since $\alpha < \underline{\alpha}$ implies $U(x|s)$ being strictly increasing in the range of $[\bar{x} - r, s]$, the optimal lottery choice $x^\dagger \geq s$. Because for $\beta < \tilde{\beta}$, the marginal utility in the case of $x > s$ at s is positive (i.e. $U'(s|s) > 0$ for $x > s$), the optimal x has to be above s , implying $x^\dagger > s$ (see Figure 2, point B). Thus, for $\alpha < \underline{\alpha}$ and $\beta < \tilde{\beta}$, it is optimal to choose

$$x = \frac{r + \alpha(2\bar{x} - r - s\beta) - \sqrt{r^2(1 - \alpha)^2\alpha\bar{x}r + \alpha^2\bar{x}(\bar{x} - r) + \alpha s\beta[r + \alpha(2\bar{x} - r + s\beta)]}}{3\alpha}, \quad (38)$$

which is greater than s . This proves Proposition 2's last statement. \square

¹³In other words, any additional unit of x leads to a stronger disutility which can be avoided by decreasing x .