Matching with Couples with Farsighted Agents

Thesis presented by
Sylvain Funck

Supervisor
Vincent Vannetelbosch

Reader
Ana Mauleon

Academic year 2017-2018

Master 120 en Sciences économiques, Orientation générale, Finalité approfondie

Ecole d'économie de Louvain/UCL • Place Montesquieu 3 • 1348 Louvain-la-Neuve
Département des Sciences économiques/UNamur • Rempart de la Vierge 8 • 5000 Namur

Acknowledgements

I would like to acknowledge my supervisor Vincent Vannetelbosch and his wife Ana Mauleon for their support on this thesis. Their enthusiasm on this literature was contagious and it has motivated me to pursue this topic in the future.

I want also to thank my parents for their support for this project and since the beginning of my studies. Even if the topic of this thesis and the language used in it was not their strength, their kind attentions and their listening to my issues have been precious.

Finally, I would like to acknowledge my friends and colleagues who have read my thesis, have given me useful advices or have just listened to my concerns. A special thank you to my roommates for all their different kinds of encouragement in the end of this year.

Abstract

I study the model of matching model with couples where all the agents are farsighted. This means that they can predict the blocking coalitions that will be created if they deviate from a matching and they join one only if the final matching make them better off. I study the composition of the von Neumann-Morgenstern (vNM) farsightedly stable set and show that the result obtained by Mauleon, Vannetelbosch and Vergote (2011) is expandable to the matching market with couples: a singleton is a vNM farsightedly stable set if and only if it is a stable matching.

1. Introduction

When couples work in the same labour market, they could have common preferences on where each member of the couple works. For example, even if the husband really likes a job in Paris, while the other likes a job in New-York, many couples would not accept to work in the same time in these very distant cities. The matching markets with couples gather all the possible ways to combine these preferences with the employer's preferences on these workers. A couple and a hospital can match together to deviate from a matching if they are both an improvement from this matching. In this paper, we assume that these agents know the preferences of others and can forecast the possible implications of a deviation from a matching and the possible coalitions they can make to improve their position.

Matching theory has been first applied by the National Resident Matching Program (NRMP). Since 1952, they have centralized the problem to assign to each medical school graduate a hospital to work for as resident. As each student and hospital have preferences on how they are matched, the problem is to find a matching respecting stability, that is a matching where no student and hospital would both strictly prefer to be matched together rather than with their actual mate. The NRMP used the algorithm provided by Gale and Shapley (1962) known as the deferred acceptance (DA) algorithm to solve this problem. They proved that when we are in a two-sided market, a stable matching always exists. As an increasing number of these students marry other medical students (Roth, 2003), this algorithm had to be changed to respect these preferences. The current algorithm, provided by Roth and Peranson (1999), allows students to apply to the program as an individual or as a couple and adapts the DA algorithm to fit to this additional possibility.

As this perfectly illustrates our problem, hospitals and couple of students are used for our model. Roth (1983) showed that the set of stable matchings could be empty in this market. However, a stable matching always exists in the market with couples when preferences are weakly responsive, i.e., when unilateral improvement on acceptable positions for one partner's job is beneficial for the couple as well (Klaus and Klijn, 2005). In their paper, they also proved that, under an unemployment aversion condition, weak responsiveness is a maximal domain for the existence of a stable matching. Furthermore, under weakly responsive preferences, starting from any matching, there always exists a path of matchings formed by respecting blocking coalitions that yields a stable matching (Klaus and Klijn, 2007). Unfortunately, Kojima et al (2010) showed that a huge proportion of couples violate weak responsiveness. They prove that a stable matching exists with high probability in large markets with couples even without this restriction.

These results have been found under the hypothesis that agents are myopic. This means that they do not forecast how other agents react to their actions. For myopic players, different matchings are compared using direct dominance. If there exists a coalition that can block a matching and where each member of this coalition is better off, then this matching will be directly dominated by the one formed by the blocking coalition. Harsanyi (1974) and Chwe (1994) introduced indirect dominance which arranges the notion of dominance to the farsighted behaviour of agents, anticipating other agents reactions to a deviation. To study these matchings in a two-sided market, Mauleon et al (2011) provided the notion of vNM farsightedly stable set which is an extension of vNM stable sets for indirect dominance. No matchings inside this set can indirectly dominate each other (internal stability) and each matching outside this set must be indirectly dominated by a matching of this set (external stability). They obtained that a set of matchings is vNM farsightedly stable if and only if it is a singleton composed of a stable matching. Klaus et al (2009) showed that, in roommate markets, singletons are vNM farsightedly stable if and only if it is a stable matching.

In this paper, I extend these notions to the market with couples and obtain this same result for singletons vNM farsightedly stable sets. This means that, for any market with couples, all matchings are indirectly dominated by a stable matching. To the best of my knowledge, this paper is the first to add farsightedness in the literature of matchings with couples. The main contribution of this paper is that it allows to obtain a stability result without the unrealistic assumption of weak responsiveness.

The paper is organized as follows. In section 2, I introduce the model of two-sided matching with couples. In section 3, I characterize farsighted agents in this model using indirect dominance and vNM farsightedly stable sets and illustrate these notions with some examples. In section 4, I provide proofs of my results. I show that the composition of singleton vNM farsightedly stable sets are stable matchings and that there exists no vNM farsightedly stable sets of 2 elements with strictly unemployment averse couples. Section 5 concludes.

2. Model

A matching problem in a couple market consists of a set of hospitals H and a set of individuals $N = \{s_1, \cdots, s_{2n}\}$ partitioned into a set of couples $C = \{c_1, \cdots, c_n\} = \{(s_1, s_2), \cdots, (s_{2n-1}, s_{2n})\}$.
Each hospital $h \in H$ has a strict, complete and transitive preference relation $\succeq_h$ represented by a strict ordering $P(h)$ of students $s \in N$ and of the prospect of having its position unfilled $\emptyset$. An individual s is *acceptable* if $s \succ_h \emptyset$. The preferences of all hospitals are denoted by P^H.
Each couple $c \in C$ has a strict, complete and transitive preference relation $\succeq_c$ over the elements in $\mathcal{H} := [H \cup \{u\} \times H \cup \{u\}] \setminus \{(h, h) | h \in H\}$ where u denote the prospect of being unemployed. We will use this set in the to describe the possible combinations of hospitals or unemployments that a couple may reach. These preferences are represented by a strict ordering $P(c)$. A pair of hospitals $(h, h') \in \mathcal{H}$ are *acceptable* if $(h, h') \succ_c (u, u)$. The preferences of all couples are denoted by P^C. A couple market is a set (P^H, P^C)
The domain of weakly responsive preferences has been introduced by Klaus

and Klijn (2005) and is defined as follows. Preferences are *responsive* if an unilateral improvement from one member of the couple $c = (s_1, s_2)$ is beneficial for the couple as well. They are *weakly responsive* if it applies only on acceptable positions. Formally, it must respect the following properties:

(i) for all $h \in H$, $(u, h) \succ_c (u, u) \Leftrightarrow h \succ_{s_2} u$, $(h, u) \succ_c (u, u) \Leftrightarrow h \succ_{s_1} u$,

(ii) for all $(h_1, h_2) \in \mathcal{H}$, and $h' \in H$, $h_1 \succeq_{s_1} u$, $h_2 \succeq_{s_2} u$, and $h_1 \succ_{s_1} h'$ imply $(h_1, h_2) \succ_c (h', h_2)$ and $h_1 \succeq_{s_1} u$, $h_2 \succeq_{s_2} u$, and $h_2 \succ_{s_2} h'$ imply $(h_1, h_2) \succ_c (h_1, h')$, and

(iii) for all $(h_1, h_2) \in \mathcal{H}$, $u \succeq_{s_1} h$, and $u \succeq_{s_2} h_2$ imply $(u, u) \succ_c (h_1, h_2)$.

Couples are *strictly unemployment averse* if they are always worse off when one member of the couple lose his/her position,i.e., for any couple $c \in C$ and any acceptable pair of hospitals for c, $(h_1, h_2) \in \mathcal{H}$, $(h_1, h_2) \succeq_c (u, h_2) \succeq_c (u, u)$ and $(h_1, h_2) \succeq_c (h_1, u) \succeq_c (u, u)$.

A *matching* is a function $\mu : N \cup H \to N \cup H \cup \{u, \emptyset\}$ satisfying the following properties :

(i) For any $s \in N$, $\mu(s) \in H \cup \{u\}$.

(ii) For any $h \in H$, $\mu(h) \in N \cup \{\emptyset\}$.

(iii) For any $c = (s, s') \in C$, $\mu(c) = \mu(s)\mu(s') \in \mathcal{H}$.

(iv) For any $s \in N$ and $h \in H$, $\mu(\mu(s)) = s$ and $\mu(\mu(h)) = h$.

The set of matchings is denoted by $\mathcal{M}$. A matching is *individually rational* if each agent is matched with an acceptable agent (or pair of agents). A student and a hospital matched together are called *mates*.

A hospital is a *one-sided blocking coalition* of $\mu \in \mathcal{M}$ if it is matched with an unacceptable student in μ. A couple is a *one-sided blocking coalition* of this matching if it is better off by unmatching one or both couple's members of her(their) mate(s) in μ, i.e., $c = (s_1, s_2) \in C$ is a one-sided blocking coalition if $\mu(s_1, s_2) \prec_c (\mu(s_1), u)$ or $\mu(s_1, s_2) \prec_c (u, \mu(s_2))$. Let $c \in C$ and $(h_1, h_2) \in \mathcal{H}$. We define $\{h_1, h_2, c = (s_1, s_2)\}$ as a *two-sided blocking coalition* of $\mu \in \mathcal{M}$ if the couple c is strictly better off with (h_1, h_2) rather than their mates in μ and h_1 and h_2 prefer respectively s_1 and s_2 rather than their mates in μ. Formally, it must respect the following properties: (i) $(h_1, h_2) \succ_c \mu(c)$, (ii) $s_1 \succ_{h_1} \mu(h_1)$ and (iii) $s_2 \succ_{h_2} \mu(h_2)$.[1] A matching μ is *stable* if there exists neither one-sided nor two-sided blocking coalitions to μ. We use the notation $\mu' = \mu - i$ for any matching μ' where $\mu'(j) = \mu(j)$ for any $j \notin \{i, \mu(i)\}$ and $\mu'(i), \mu'(mu(i) \in \{u, \emptyset\}$ for any $i, j \in I \cup H$.

Definition 1. *Given a matching μ, a coalition $S \in H \cup C$ enforces a matching μ' over μ if the following conditions hold :*

(i) For any $c = (s_1, s_2) \in C$ and $h \in H$ such that $\mu'(h) \in c$ and $\mu'(h) \neq \mu(h)$, $\{c, h\} \subset S$

(ii) For any $h \in H$ and $c = (s_1, s_2) \in C$ such that $\mu(h) = s_1$ and $\mu'(h) = \emptyset$, $\{c, h\} \cap S \neq \emptyset$.

It means that whenever there is a change between μ and μ', each member of a new match and each agent leaving a match must be in the coalition. Using this notion, we can describe the notion of dominance. A matching μ' directly dominates μ if there exists a coalition S which enforces μ' over μ and is strictly better off in μ'. This allows us to introduce the concept of Von-Neumann stable set. A set of matchings is vNM if it respects internal stability (there does not exist inside the set 2 matchings such that one dominates another) and external stability (every matching outside the set is dominated by a matching inside the set).

3. Farsighted agents

The notions above describe stability and dominance in a myopic setting where agents do not anticipate the consequences of their decisions. Harsanyi (1974) argued that coalitions could be formed to change a myopic outcome unsatisfactory for the member of the coalitions. We can see from the next example that making a myopic improvement can lead to some loss for a deviating agent.

[1] We use the convention that $\forall s, s' \in I$, $s \sim_u s'$

Table 1
Example 1

P^H			P^C	
h_1	h_2	h_3	s_1s_2	s_3s_4
s_3	s_2	s_4	h_1h_2	uh_1
s_1		s_1	h_3h_2	h_1h_3
s_4		s_3	uh_2	

Example 1. Let's take the matching $\mu_1(s_1, s_2, s_3, s_4) = (h_3, h_2, u, h_1)$. We see from table 1 that it is dominated by $\mu_2(s_1, s_2, s_3, s_4) = (h_1, h_2, u, u)$ where the coalition $\{h_1, (s_1, s_2)\}$ enforces μ_2 over μ_1 and is better off in μ_2. However, the coalition $\{h_1, h_3, (s_3, s_4)\}$ enforces μ_3 over μ_2 where $\mu_3(s_1, s_2, s_3, s_4) = (u, h_2, h_1, h_3)$. Therefore, $\mu_3 > \mu_2 > \mu_1$ and (s_1, s_3) is worse off in μ_3 rather than in μ_1. If this couple knew that deviating from μ_1 would lead to μ_3, they would not have deviated.

That is why we should study agents able to perfectly anticipate the future deviations that any decision would trigger. These agents are called farsighted and are characterized by the notion of indirect dominance.

Definition 2. *Given $\mu, \mu' \in \mathcal{M}$, μ indirectly dominates μ' if there exists a sequence of matchings $\mu_0, ...\mu_k$ and coalitions $S_0, ..., S_k \subset H \cup C$ such that the following conditions hold :*

(i) $\mu_0 = \mu'$ *and* $\mu_k = \mu$,

(ii) μ_{i+1} *is enforceable by* S_i *over* μ_i, $\forall i \in \{0, ..., k-1\}$, *and*

(iii) $\mu(j) \succ_j \mu_i(j)$, $\forall j \in S_i$.

This means that when agents are farsighted, they will deviate from a matching only if they can reach a better matching through a sequence of deviating coalitions. We use the notation $\mu \gg \mu'$ for indirect dominance. We can see trivially that if $\mu > \mu'$, then $\mu \gg \mu'$. From this concept we can derive vNM farsightedly stable set as the matchings respecting internal and external stability for indirect dominance.

Definition 3. *A set of matching $V \subset \mathcal{M}$ is a vNM farsightedly stable set if it satisfies the following properties :*

(i) Internal stability : There exists no $\mu, \nu \in V$ such that $\mu \gg \nu$, and

(ii) External stability : For every $\nu \notin V$, there exists $\mu \in V$ such that $\mu \gg \nu$.

We can first look at the impact of farsighted agents on stable matchings in the case of weakly responsive preferences. To do so, let's look at the couple market from table 2.

Table 2

Example 2

P^H									P^C			
h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	s_1s_2	s_3s_4	s_5s_6	s_7s_8
s_4	s_4	s_7	s_4	s_2	s_3	s_7	s_3	s_8	h_3h_2	h_2h_4	h_3h_6	h_3h_8
s_1	s_3	s_8		s_5	s_6	s_3	s_8		h_1h_2	h_8h_4	h_5h_6	h_3h_4
	s_2	s_5					s_4		h_6h_2	h_2h_3	h_5h_3	h_7h_8
		s_4							h_3u	h_6h_4	h_3u	h_7h_4
		s_1							h_1u	h_8h_2	h_5u	h_7h_3
									uh_2	h_6h_2	uh_6	h_3u
									h_6u	h_2h_1	uh_3	h_7u
										h_8h_3		uh_8
										h_6h_3		uh_4
										h_8h_1		uh_3
										h_6h_1		
										h_2u		
										uh_4		
										h_8u		
										h_6u		
										uh_2		
										uh_3		
										uh_1		

Example 2. The couple market (P^H, P^C) described by table 2 is the one used by Klaus and Klijn (2007). We know from this paper that the only

stable matching is $\mu(s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) = (h_1, u, h_2, h_4, h_5, h_6, h_3, h_8)$. We can prove that $\{\mu\}$ is a vNM farsighetdly stable set. To do so, we only have to show that it indirectly dominates any other matching ν. Looking at preferences, we can form a sequence of coalitions of agents who cannot be better off in their matching than in μ whatever ν is. The matching enforced by the coalition will only result from members of the coalition breaking up their match. $S_0 = \{h_3, h_4, (s_7, s_8), (s_3, s_4)\}$ is formed by agents who are in their top choice in μ. $S_1 = \{h_1, h_2, h_6, h_7, h_8, h_9, s_5s_6\}$ are agents who cannot be better off than matched with agents of S_1 or their match in μ. Proceeding the same way, we get $S_2 = \{(s_1, s_2)\}$. Therefore, in μ_3, all the agents will be unmatched. Then, $S_3 = i \in C \cup H | \mu(i) \neq \{u, \emptyset\}\}$ and $\mu_4 = \mu$.

We have the intuition from this example that all the stable matchings form singleton vNM farsightedly stable set. We still have to check under which conditions this result could be generalised. We could think that weak responsiveness is a natural constraint because it is the one needed for the existence of stable matching and of a path to stability[2]. Taking the couple market from the counter-example for the existence of a path to a stable matchings from Klaus and Klijn (2007), we see that this rule could still work without weak responsiveness.

Table 3
Example 3

P^H			P^C	
h_1	h_2	h_3	s_1s_2	s_3s_4
s_2	s_2	s_2	h_3h_1	h_2h_3
s_1	s_3	s_4	h_2h_3	
	s_1	s_1	h_1h_2	

Example 3. We see in table 3 that preferences are not weakly responsive and Klaus and Klijn demonstrated that there could exist no path to a stable matching. If we look at the matchings $\mu_1(s_1, s_2, s_3, s_4) = (h_3, h_1, u, u)$, $\mu_2(s_1, s_2, s_3, s_4) = (h_1, h_2, u, u)$ and $\mu_3(s_1, s_2, s_3, s_4) = (u, u, h_2, h_3)$. These

[2]Klaus and Klijn (2005,2007)

matchings are the only individually rational matchings respecting couples and hospitals top choices. We can see easily that $\mu_1 \gg \mu_2 \gg \mu_3 \gg \mu_1 \not\ll \mu_2 \not\ll \mu_3 \not\ll \mu_1$. Thus, none of these matchings form a singleton vNM farsightedly stable set.

$\mu_4(s_1, s_2, s_3, s_4) = (h_2, h_3, u, u)$ is the last individually rational matching and the only stable matching. We can trivially see that μ_4 indirectly dominates the 3 first matchings. The only stable matching of this market form also a vNM farsightedly stable set.

4. Main results

Lemma 1. *With strictly unemployment averse couples, if μ is individually rational, $\mu \gg \mu'$ if there doesn't exist $h \in H$ and $c \in C$ with $\mu'(h) \in c$, such that both c and h strictly prefer μ' to μ.*

Proof. Let $B(\mu', \mu)$ be the set of hospitals and couples which strictly prefer μ to μ' and $I(\mu', \mu)$ the ones who are indifferent between them. We will show that we can construct a sequence of matchings and blocking coalitions leading to μ from μ'.[3]

Let $\mu_0 = \mu'$, $\mu_1 = \mu' - B(\mu', \mu)$, and $\mu_2 = \mu$. Along with these matchings we have the coalitions $S_0 = B(\mu', \mu)$ and $S_1 = B(\mu', \mu) \cup \{i \in H \cup C \setminus I(\mu', \mu) | \mu'(i) \subset B(\mu', \mu)\}$. We only have to show that these matchings are enforceable by their preceding blocking coalition and that these coalitions are better off in μ_2 than in the matching they block. We see trivially that the coalition S_0 can enforce μ_1 because all the members of the coalition withdraw from their match and they are better off in μ_2 by definition of $B(\mu', \mu)$.

I will show that each hospitals and individuals of S_1 (except the partner of individuals who are indifferent between μ and μ') are unmatched in μ_1 and as μ is individually rational, they will be better off in μ_2. The assumption of the lemma means that if one member of the couple was unemployed in μ' but not in μ, either the couple or the hospital matched with the other member would belong to $B(\mu', \mu)$. Then, they will all be unmatched in μ_1.

[3]We take the same structure as the proof of Lemma 1 from Mauleon et al (2011).

If $c = (s_1, s_2) \in C, (h_1, h_2) \in \mathcal{H}$ are such that $\mu'(c) = (h_1, h_2) \neq \mu(c)$, we know by this assumption that either :

(i) $\mu' \succ_c \mu$ and $\mu \succeq_{h_1,h_2} \mu'$,or

(ii) $\mu \succ_c \mu'$.

In case (ii), as $c \in B(\mu', \mu)$, h_1 and h_2 will also be unmatched in μ_1. In case (i), without loss of generality, $h_1 \in B(\mu', \mu)$ and either $h_2 \in B(\mu', \mu)$ or $\mu'(h_2) = \mu(h_2) = s_2$[4]. In the first situation, c, h_1 and h_2 will be unmatched in μ_1. In the second, h_1 and s_1 will still be unmatched in μ_1. In addition, if $\mu(s_1) \neq u$, as couples are strictly unemployment averse and as μ is individually rational, $(\mu(s_1), h_2) \succ_c (u, h_2)$. [5] Thus, in all of these cases, c and h_1 (and h_2 if h_2's preference between μ and μ' is strict) will be better off in μ_2 rather than in μ_1.

Furthermore, since the cases above have considered all the agents who changed their match between μ' and μ and each of them were either included in $B(\mu', \mu)$ or matched with a member of this set in μ' (and then unmatched in μ_1), we know that μ is enforceable from μ' by S_1. As they are all better off in μ_2, we have proven that $\mu \gg \mu'$.

□

This lemma is restricted to strictly unemployment averse couples. We can see in the proof that this assumption is only used to the case where μ must be such that $(\mu(s_1), \mu(s_2)) \succ_c (u, \mu(s_2))$. However, if μ is a stable matching, this case is always true. In fact, if it was not the case c would be a one-sided blocking coalition to μ which is contradictory to the fact that μ is stable. Therefore, this result holds without the restriction on strictly unemployment averse couples for stable matchings.

Corollary 1. *If μ is a stable matching, $\mu \gg \mu'$ if there doesn't exist $h \in H$ and $c \in C$ with $\mu'(h) \in c$, such that both c and h strictly prefer μ' to μ.*

Example 4. Let's look at the couple market in table 4. We see that couples are strictly unemployment averse. However, if we take the matchings

[4]The proof would be the same if we took $h_2 \in B(\mu', \mu)$ and either $h_1 \in B(\mu', \mu)$ or $\mu'(h_1) = \mu(h_1)$.

[5]If $\mu(s_1) = u$, $\mu(c)$ will already be formed in μ_1.

$\mu(s_1, s_2, s_3, s_4) = (h_1, h_2, h_4, h_3)$ and $\mu'(s_1, s_2, s_3, s_4) = (h_3, h_4, h_1, h_2)$, we have that $\mu \gg \mu'$ even if c and h_3 prefer μ' rather than μ. Therefore, the reciprocal of Lemma 1 is not true.

Table 4
Example 4

P^H				P^C	
h_1	h_2	h_3	h_4	s_1s_2	s_3s_4
s_1	s_2	s_1	s_3	h_1h_4	h_4h_3
s_3	s_4	s_4	s_2	h_3h_4	h_4u
				h_1h_2	uh_3
				h_3h_2	
				h_1u	
				h_3u	
				uh_4	
				uh_2	

Lemma 2. *If $\mu \gg \mu'$, then there exists no blocking coalitions of μ matched in μ'.*

Proof. Let us suppose that there exists a two-sided blocking coalition of μ respected in μ'. Then, $\exists c = (s, s') \in C$ and $(h, h') \in \mathcal{H}$ such that $\mu'(c) = (h, h')$, $(h, h') \succ_c \mu(s_1, s_2)$, $s \succ_h \mu(h)$ and $s' \succeq_{h'} \mu(h')$. But if they all prefer μ' to μ, they will never join any coalition leading to μ from μ'. Then, there will never exist a sequence of enforceable matchings leading to μ from μ' by a sequence of coalitions which are strictly better off in μ. The proof for a one-sided blocking coalition is the same, as the member of this coalition will prefer to stay unmatched rather than joining any coalition leading to μ. Therefore, by definition 2 , $\mu \not\gg \mu'$.

□

This result is a necessary condition for indirect dominance. We had in the previous lemma a sufficient condition (when couples are strictly unemployment averse) for indirect dominance that was more strict than this condition. In fact, for two matchings $\mu, \mu' \in \mathcal{M}$ with μ individually rational, if there exists no $h \in H$ and $c \in C$ with $\mu'(h) \in c$, such that both c and h strictly prefer μ' to μ, then there will not exist any two-sided blocking coalition of μ matched in μ' and as μ is individually rational there exists no one-sided blocking coalition. If there exists no myopic blocking coalition, there could still exist $c \in C$ and $h, h' \in H$ such that $\mu' \succ_{c,h} \mu$ and $\mu'(c) = (h, h')$ but $\mu \succ_{h'} \mu'$. If $\mu(c) \succ_c (h, u)$, these agents will not form a blocking coalition to μ. It is for this particular case that we must extend the proof of Lemma 1 to have a necessary and sufficient condition for indirect dominance.

Lemma 3. *With strictly unemployment averse couples, if μ is individually rational, $\mu \gg \mu'$ if and only if there does not exist any blocking coalition of μ matched in μ'.*

Proof. We first prove the necessary and then the sufficient condition for indirect dominance.

$\Rightarrow$ We know by Lemma 2 that it is true.

$\Leftarrow$ Lemma 1 proves it for the case where there exists no $h \in H$ and $c \in C$ with $\mu'(h) \in c$, such that both c and h strictly prefer μ' to μ. We only have to prove it for the case described above.
In this case, we have $c \in C$, $h_1, h_2 \in H$ such that $\mu'(c) = (h_1, h_2)$, $\mu' \succ_{c,h_1} \mu$ and $\mu \succ_{h_2} \mu'$. Therefore, h_2 will join the first coalition of the sequence leading to μ. In the next step, $\mu_1(c) = (h_1, u)$. We know that c or h_1 will unmatch only if one of them is better off in μ. However, if none of them unmatch, they will form a myopic blocking coalition to μ and as they are matched in μ', it would be a contradiction to the sufficient condition of the lemma.

□

As we have to use lemma 1 in the proof, the condition of strict unemployment aversion cannot be removed as we can see in the next example.

Table 5

Example 5

P^H			P^C	
h_1	h_2	h_3	s_1s_2	s_3s_4
s_3	s_2	s_1	h_1h_2	h_1u
s_1		s_4	uh_2	uh_3
			h_3h_2	

Example 5. Let's look at the matching market with couples of table 5. We see that the couple (s_1, s_2) is not strictly unemployment averse. If we take the matchings μ, μ' such that $\mu(s_1, s_2, s_3, s_4) = (h_3, h_2, h_1, u)$ and $\mu'(s_1, s_2, s_3, s_4) = (h_1, h_2, u, h_3)$, we have that $\mu \succ_{\{h_1,h_3,(s_3,s_4)\}} \mu'$, $\mu \sim_{h_2} \mu'$ and $\mu' \succ_{(s_1,s_2)} \mu$. Therefore, there exists no blocking coalition of μ matched in μ'. If the lemma worked in this setting, we would have that $\mu \gg \mu'$. However, we see that it is not the case because, starting from μ', no deviation from one of the agent preferring μ to μ' would make (s_1, s_2) better off in μ. Thus, $\mu \not\gg \mu'$

Strict unemployment aversion has just been used in Lemma 1. Therefore, as I have shown in the Corollary 1, this restriction can be dropped for stable matchings.[6]

Corollary 2. *If μ is a stable matching, $\mu \gg \mu'$ if and only if there does not exist any blocking coalition of μ matched in μ'.*

As we know that a stable matching does not allow any blocking coalition to exist, we can derive from this a general theorem about the singleton vNM farsightedly stable sets.

Theorem 1. *Let $\mu \in \mathcal{M}$, $\{\mu\}$ is a vNM farsightedly stable set if and only if μ is a stable matching.*

Proof. If μ is a stable matching, it means that there exists no blocking coalitions of μ in any $\mu' \in \mathcal{M}$. By Corollary 2, this condition is satisfied if and only if $\mu \gg \mu'$ for any $\mu' \in \mathcal{M}$ which is the definition of a singleton vNM

[6]Note that μ is always individually rational as it is stable.

farsightedly stable set.
If $\{\mu\}$ is a vNM farsightedly stable set, by Lemma 2, there exists no blocking coalition of μ matched in any $\mu' \in \mathcal{M}$. That is, μ is a stable matching.

□

As Klaus and Klijn (2005) showed that the domain of weakly responsive preferences is a maximal domain for the existence of stable matchings, we can transpose this result to the existence of singleton vNM farsightedly stable sets.

Corollary 3. *With strictly unemployment averse couples, the maximal domain for the existence of singleton vNM farsightedly stable set is weak responsiveness.*

Furthermore, using Lemma 3, we can show that no vNM farsightedly stable set of 2 elements exists when couples are strictly unemployment averse.

Lemma 4. *Under strict unemployment aversion, there exists no vNM farsightedly stable set composed of 2 elements.*

Proof. We make a proof by contradiction, supposing that such set exists. We call it $V = \{\mu_1, \mu_2\}$. Note that μ_1 and μ_2 are individually rational[7] and not stable[8]. Thus, we can construct a matching μ_1' from μ_1 such that it satisfies the two-sided blocking coalitions of μ_1 matched in μ_2. Using Lemma 3, we have that $\mu_1 \not\gg \mu_1'$ as no blocking coalition of μ_1' are matched in μ_1. Hence, we have that $\mu_2 \gg \mu_1'$ by external stability of V.
It means that there exists no two-sided blocking coalition of μ_2 matched in μ_1. In fact, if such agents existed, they also would be matched in μ_1'. As they are both worse off in μ_2, they would never join any coalition leading to μ_2. As $\mu_2 \gg \mu_1'$, this is not possible. However, by Lemma 3, this imply that $\mu_2 \gg \mu_1$ which contradicts internal stability of V.

□

[7]If not, it would not indirectly dominates the matching respecting one-sided blocking coalitions of μ_1. If μ_2 indirectly dominates it, it also would indirectly dominates μ_1 and internal stability would not be respected.

[8]If it was stable, internal stability would not be respected as a stable matching indirectly dominates any other matching.

5. Conclusion

In this paper, I have first found a necessary and sufficient condition for indirect dominance and characterized vNM farsightedly stable sets, showing that stable matchings are the only singleton vNM farsightedly stable sets. This result is proved without any restriction on the domain of couples preferences. Finally, I show that, with strictly unemployment averse couples, there exists no vNM farsightedly stable sets with 2 matchings.

A possible extension of this paper would be to study heterogeneity among the couples such that there exists both myopic and farsighted couples. Herings et al (2018) introduced the notion of myopic-farsighted improving path for the marriage problem, catching the idea that myopic agents do not anticipate other agents actions in this path contrary to farsighted agents which look at the final matching of this path. This improvement path allows them to define a myopic-farsighted stable set which is a set respecting internal and external stability for myopic-farsighted improving path. They showed that the optimal matching for the more farsighted set is a myopic-farsighted stable set. It would be interesting to transpose these notions to the market with couples to see who would benefit from the myopia of some couples or if it could exist myopic-farsighted stable set even when no stable matching exists. This will be let for future research.

References

[1] M. S.-Y. Chwe, *Farsighted Coalitional Stability*, Journal of Economic Theory **63** (1994), 299-325. ↑

[2] J. C. Harsanyi, *An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition*, Management Science **20** (1974), 1472-1495. ↑

[3] P.J.J Herings, A. Mauleon and V. Vannetelbosch, *Matching with Myopic and Farsighted Players*, Working Paper (2018). ↑

[4] B. Klaus and F. Klijn, *Stable matchings and preferences of couples*, Journal of Economic Theory **121** (2005), 75-106. ↑

[5] ______, *Paths to stability for matching markets with couples*, Games and economic behavior **58** (2007), 154-171. ↑

[6] B. Klaus, F. Klijn and M. Walzl, *Farsighted Stability for Roommate Markets*, Journal of Public Economic Theory **13** (2009), 109-135. ↑

[7] B. Klaus, F. Klijn and Toshifumi Nakamura, *Corrigendum to "Stable matchings and preferences of couples"*, Journal of Public Economic Theory **144** (2009), 2227-2233. ↑

[8] F. Kojima, P. A. Pathak and A. E. Roth, *Matching with Couples: Stability and Incentives in Large Markets*, Quarterly Journal of Economics **128** (2010), 1585–1632. ↑

[9] A. Mauleon, V. Vannetelbosch and W. Vergote, *vNM farsightedly stable sets in two-sided matchings*, Theoretical Economics **6** (2011), 499-521. ↑

[10] A. E. Roth, *The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory*, Journal of Political Economy **92** (1984), 991-1016. ↑

[11] A. E. Roth and M. A. O. Sotomayor, *Two-sided matching, a study in game-theoretic modeling and analysis*, Cambridge University Press, 1990. ↑

[12] A. E. Roth and Elliott Peranson, *The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design*, American Economic Review **89** (1999), 748-780. ↑

[13] A. E. Roth, *The Origins, History, and Design of the Resident Match*, Journal of the American Medical Association **289** (2003), 909-912. ↑