A General Equilibrium Model for Basic Income

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Contents

Abstract

This paper implements a general equilibrium model with heterogeneous agents to determine the impact of basic income on total welfare, defined as the sum of agents' utility. Firstly, a central planner solution is described. Afterwards, a decentralized solution where the government chooses only public expenditure and taxation is analyzed. Next, a subsidy s is assigned to agents who do not work. Finally, the introduction of basic income ω is discussed. All models are calibrated using aggregate EU15 data. The results show that while the conditional subsidy s improves total welfare with respect to the basic decentralized model, basic income does not. Moreover, basic income causes a non negligible reduction in working hours and economic output.

1 - Introduction

Basic income is commonly acknowledged as an universal unconditional payment in cash, paid at the individual level on a periodic base (Widerquist et al., 2013; Van Parijs and Vanderborght, 2017).

In the last twenty years a growing literature has tried to assess its desirability. This is reflected in a wide production of papers, where founding works are Atkinson et al. (1996) and Van Parijs (2001), and in concrete policy initiatives. For example, in Finland a basic income experiment has started in January 2017 and it is programmed to last up to January 2019 (Kela, 2016).

Different authors and institutions share the idea that the current social security system is under pressure. Esping-Andersen (2002, p. 2) writes about a new welfare challenge, characterized by a revolution in demographic and family behavior, which is causing a “proliferation of new and far less stable household and family arrangements”; the technological transformation and dominance of service employment produce a “wholly new set of societal winners and losers”. Similarly, Bonoli (2007) points out that new social risks - the fact that employment income alone sometimes is not sufficient to ensure a poverty-free existence, family instability and the difficulty for two-earner couples to reconcile work and family life - “are not well covered by the welfare states that we have inherited from the postwar years”. “Existing social protection systems were, in large part, modeled on employer-employee contracts, stable careers pattern, and social contracts, which can appear outdated today” (OECD, 2017, p. 1). Many authors, such as Van Oorschot (2002) and Mkandawire (2005), point out that the current means-tested social security policies are administratively costly to implement, they are likely to stigmatize the beneficiaries causing a drop of adoptions and people that are eligible often do not receive them; means-tested benefits cause also the so called poverty trap situation, i.e. when the means-tested benefits depend on income, people could be unwilling to start a new working activity if the rise in income determines the reduction or loss of the benefits.

According to people advocating basic income introduction, basic income is the answer to these problems. Van Parijs and Vanderborght (2017) describe the basic income as “a radical proposal for a free society and a sane economy”.

The features of basic income - cash income, individual, universal, obligation free -

are radical. The consequences of the introduction of this policy however are mostly unknown. In fact, it has never been applied in the form just described. The only current basic income scheme that satisfies the above-mentioned characteristics is the state of Alaska basic income scheme (Widerquist et al., 2013). Nevertheless, this scheme is financed by profits obtained from a fund created with oil revenues. Given that in most countries basic income policies should be financed with taxation, the Alaska case fails to represent the economic consequences of financing a basic income policy (Van Parijs and Vanderborght, 2017). This problem is common to other small scale experiments that are usually financed through the general state budget and hence do not take into consideration the consequences of financing basic income. Moreover, such experiments are too small to properly show the effects of basic income on labor market conditions (Van Parijs and Vanderborght, 2017).

As written by Van Parijs and Vanderborght (2017), the amount of resources assigned to basic income should be between two extremes. It should be high enough to make a difference, but at the same time it should be sustainable. Some authors consider as minimum level of basic income the level that it would be achievable replacing all existing cash transfers (OECD, 2017). Amounts higher than this level would require an increase in taxation or a shift of resources among different public expenditures. Murray (2008, p. 2) writes: "No matter how theoretically persuasive those arguments might be, no Western nation can afford to add a significant GI [Guaranteed Income] to its existing commitments. On the contrary, all Western nations need to restructure their existing benefit systems to avoid bankruptcy. If a GI is to be financially feasible, it must replace existing programmes rather than augment them". Van Parijs and Vanderborght (2017) write about a basic income of 25% of GDP per capita which would for example correspond, corrected according to purchasing power parity, to $860 a month in UK. In UK the total government expenditures are around 41% of GDP, and the average EU 28 total government expenditures amount to 45.8% of GDP in 2017 (Eurostat, 2018). Also, assuming that part of the cost of basic income scheme could be covered by the abolition of existing money transfers, it is clear that there is a trade-off between basic income, increase in taxes, and the other public expenditure.

"Since about 1960, this literature [basic income literature] has become extremely rich, covering such diverse topics as the philosophical justification of the idea; its economic and political feasibility; or its impact on freedom, social justice, economic activity, psychological well-being, and much more" (Widerquist et al., 2013, p. ix). The recent anthology of Widerquist et al. (2013) summarizes the current state of the knowledge and it gives an overview about its complexity. Obviously, the literature production is not limited by the papers presented in this anthology, since in the last years there have been further developments. The feasibility of basic income has been further discussed (Torry, 2014) and numerous researches have assessed the consequences on income at individual and household level of introducing and financing a basic income scheme (OECD, 2017; MacDonald, 2016; Martinelli, 2017; Jones and Marinescu, 2018).

Nevertheless, much it is left to do concerning the development of a general equilibrium model that offers an overall picture of the desirability of basic income in a theoretical framework, where the different consequences of the introduction of basic income could be easily assessed and the channels through which basic income influences the overall economic output could be easily identified. Only one paper takes this approach. Yunker (2013) develops a general equilibrium model for basic income and he calibrates it for the U.S. In his paper, he looks to the effects of basic income using a one period ls model, i.e. labor - savings model, where agents differ in their labor productivity. Yunker (2013) shows that the introduction of basic income increases social welfare, specified as the sum of individual welfare, but, at the same time, it determines a reduction of output and savings.

The aim of this paper is to contribute to the literature production in this prospective. Indeed, a general equilibrium model is developed to answer the question whether basic income should be introduced in the European framework, i.e. the estimates of the model parameters are based on EU15. In detail, firstly the central planner allocation will be analyzed. Then, it will be discussed if the laissez-faire market allocation where the government decides only public expenditure and taxation decentralizes the planner's allocation. Afterwards, it will be analyzed if the market allocation with a conditional benefit, i.e. means-tested benefit, assigned to agents that do not work decentralizes the planner allocation and, if not, improves upon the laissez-faire market. Finally, the paper discusses how basic income modifies the economy output compared to the conditional benefit.

In the next section, section 2, the models are presented. In section 3, the models are calibrated. In section 4 the models are numerically solved and the results analyzed. Section 5 contains the concluding remarks.

2 - The Models

Generally speaking, the models are developed to make the main characteristics of basic income easily identifiable. Indeed, the models structure accommodate the main advantages and disadvantages that the literature attributes to basic income. In detail, basic income allows to redistribute wealth among agents. In fact, each agent receives the same monetary amount; if the tax system is not regressive, this means that rich agents receive a payment lower than the taxes they pay to finance it, compared with poorer agents. Secondly, basic income could mitigate the effects of negative income shocks on labor income, being its payment periodic and unconditional. Finally, the supporters of basic income underline its effects on labor market; implicitly, they prefer it compared to the current social security systems. Indeed, unemployment benefits or other poverty reduction subsidies could determine shame/stigma costs (Baumberg, 2016); basic income is assigned to everybody, hence it avoids these costs. Moreover, basic income, being unconditional, is considered a good instrument to exit the unemployment trap, i.e. the situation in which agents do not start a work activity to not lose the benefits (Carone et al., 2004; Carone, Salomaki et al., 2004). At the same time, basic income has to be financed by taxation; this would affect the agents' labor supply and capital supply, and hence their utility.

First Best Model

The economy is populated by n infinitely living agents identified by the subscript i, such that $i \in I = \{1, ..., n\}$. In each period of time the agent's utility depends on the level of leisure, l_i, private consumption, c_i, and public expenditure, g. With the term public expenditure we refer to what the literature calls public consumption, i.e. defence, public order, justice, health, education (Fiorito and Kollintzas, 2004).

In each period of time the output of the economy is produced according to a Cobb-Douglas production function, whose inputs are aggregate capital, K, and aggregate labor, L:

$$Y = F(K, L) = AK^{1-\phi}L^{\phi} \tag{1}$$

The central planner chooses in each period of time the amount of leisure and consumption for each agent, public expenditure and capital that maximize social welfare, specified as the sum of individuals utilities. Hence, the planner solves the following recursive maximization problem:

$$V(K) = \max_{\{c_i\}_i, \{l_i\}_i, g, K'} [\sum_{i=1}^{n} u^i(c_i, l_i, g) + \upsilon EV(K')] \tag{2}$$

subject to

$$\begin{aligned} \sum_{i=1}^{n} c_i + g + K' = AK^{1-\phi}L^{\phi} + (1-\delta)K \\ L = \sum_{i=1}^{n} \xi_i(1-l_i) \\ K \quad \text{given} \\ \left.\begin{aligned} l_i + n_i = 1 \\ \ln \xi_i' = \rho \ln \xi_i + \epsilon_i', \quad \epsilon_i \sim N(0, \sigma_\epsilon^2), \quad 0 \leq \rho \leq 1 \end{aligned}\right\} \forall i \in I \end{aligned} \tag{3}$$

The *prime* symbol identifies the variables one period ahead. A is the total factor productivity, υ is the inter-temporal discount rate chosen by the central planner, δ is the depreciation rate of capital. The agents' available time is normalized to one. It can be used for leisure, l_i, and labor, n_i. ξ_i is the agent specific labor productivity where ϵ_i is an exogenous shock to agent's productivity.

Therefore, the social planner in each time period chooses $\{c_i\}_i$, $\{l_i\}_i$, g and K', such that all agents derive the same marginal utility from private consumption and hence it is not possible to increase total utility moving one unit of consumption from one agent to another

$$u_c^i(c_i, l_i, g) = \lambda \quad \forall i \in I, \tag{4}$$

the disutility that an agent suffers from a marginal reduction of his or her leisure time is equal to the utility that the central planner can provide by making the agent working this additional time and using the resulting output increase for consumption or public expenditure,

$$u_l^i(c_i, l_i, g) = \lambda \xi_i F_L(K, L) \quad \forall i \in I, \tag{5}$$

the sum of each agent's marginal utility of public goods is equal to the marginal utility of private consumption, i.e. total utility cannot be increased allocating one unit of output from public expenditure to private consumption,

$$\sum_{i=1}^{n} u_g^i(c_i, l_i, g) = \lambda \quad \forall i \in I, \tag{6}$$

and the level of capital is set such that the loss of utility of not assigning one unit of marginal consumption or public expenditure today is equal to the expected utility of allocating tomorrow for consumption or public expenditure the additional output obtained from the today additional marginal increase of capital

$$\lambda = \upsilon E \lambda' [F_{K'}(K', L') + 1 - \delta]. \tag{7}$$

Notice that, equation (4) and (5) imply that the social planner chooses each agent private consumption and leisure time such that

$$\frac{u_l^i(c_i, l_i, g)}{\xi_i} = \frac{u_l^i(c_j, l_j, g)}{\xi_j} \quad \forall i, j \in I. \tag{8}$$

Decentralized Models

In this section, there is not a central planner anymore that decides the amount of labor and leisure time for each agent and allocates private consumption, but rather a government that decides the level of public good and taxation. From this basic specification, whose result will be analyzed in detail in section 4, two different policies are introduced. A benefit s which is assigned only to the agents that do not work, and a basic income ω, which is an unconditional benefit assigned to all agents. For conciseness, it is directly presented here the situation in which the subsidy s is in place, and afterwards the situation in which basic income is introduced.

In brief, agents choose the level of private consumption, labor and capital that maximize their utility function, given the level of taxation and public expenditure chosen by the government. The government, taking into account the optimal agents' decisions, fixes the level of taxation and public expenditure that maximize the sum of individual utilities.

As before, there are n infinitely living agents identified by the subscript i, such that $i \in I = \{1, ..., n\}$. There is a representative firm that produces output according to a Cobb-Douglas, i.e. linearly homogeneous neoclassic, production function, $Y = AK^{1-\phi}L^{\phi}$, whose inputs are aggregate capital, K, and labor, L; A is the total factor productivity. The capital and labor are supplied to the firm by the agents; in return, each agent gets a wage and the interests on capital supplied. In detail, in regard to capital, each agent maintains the ownership over the capital lent in the previous period, k_i, that is utilized for production in the current period. Hence, in the current period, he or she owns other than the interests on the capital lent, also the amount of capital that lasts after production, i.e. $(1-\delta)k_i$. As in the first best model, the *prime* symbol identifies the variables one period ahead in time.

The firm's problem

In each period of time, aggregate capital is the sum of capital provided by the agents and aggregate labor is the sum of labor supplied by the agents:

$$K = \sum_{i=1}^{n} k_i \tag{9}$$

$$L = \sum_{i=1}^{n} \xi_i(1 - l_i). \tag{10}$$

The firm chooses the demand for aggregate capital, K, and the demand of aggregate labor, L, to maximizes its profit:

$$\Pi = AK^{1-\phi}L^{\phi} - wL - rK. \tag{11}$$

Note that the firm takes the wage rate, w, and the return paid on capital supplied by the agents, r, as given and the input markets are set as perfectly competitive.

The maximization problem leads to:

$$r = (1-\phi)AK^{-\phi}L^{\phi} \tag{12}$$

$$w = \phi AK^{1-\phi}L^{\phi-1} \tag{13}$$

or equivalently

$$r = (1-\phi)A(\sum_{i=1}^{n} k_i)^{-\phi}(\sum_{i=1}^{n} \xi_i(1-l_i))^{\phi} \tag{12a}$$

$$w = \phi A(\sum_{i=1}^{n} k_i)^{1-\phi}(\sum_{i=1}^{n} \xi_i(1-l_i))^{\phi-1}. \tag{13a}$$

This means that the firm demands capital and labor up to the point that their marginal product equates their rental cost, r and w.

The agents' problem

Each consumer chooses the level of private consumption, c_i, leisure, l_i, and capital, k_i', such that

$$V^i(k_i, g, \tau) = \max_{c_i, l_i, k_i'} [u^i(c_i, l_i, g) + \chi E V^i(k_i', g', \tau')] \tag{14}$$

subject to

$$\begin{aligned} l_i + n_i &= 1 \\ k_i' + c_i = (1-\tau) r k_i + (1-\delta) k_i + (1-\tau)\xi_i w n_i &+ I_{n_i} s \\ k_i \quad &\text{given} \\ c_i, l_i, n_i, k_i' &\geq 0 \end{aligned} \tag{15}$$

where

$$I_{n_i} = \begin{cases} 0, & n_i > 0 \\ 1, & n_i = 0 \end{cases}$$

$$\ln \xi_i' = \rho \ln \xi_i + \epsilon_i', \quad \epsilon_i \sim N(0, \sigma_\epsilon^2), \quad 0 \leq \rho \leq 1$$

Note that we assume that the government can fix only a constant tax rate and public expenditure level over time. Hence,

$$\begin{aligned} \tau' &= \tau \\ g' &= g. \end{aligned} \tag{16}$$

The agents' common inter-temporal discount rate is identified by χ. τ is the tax rate. Notice that income from labor and capital are taxed at the same constant tax rate.

The value s represents an exogenous benefit that is assigned if an agent does not work. Hence, the indicator I takes value one only if n_i is equal to zero. The value s is therefore a means-tested benefit that reproduces the unemployment trap situation, i.e. individuals that stay unemployed not to lose the unemployment benefits.

Finally, notice that the capital held by each agent has to be at lest zero, i.e. the agents are borrowing constrained. This is a commonly used setup in the literature (Aiyagari, 1994; Huggett, 1996).

The first order conditions of the agents' problem are:

$$u_c^i(c_i, l_i, g) = \mu_i \tag{17}$$

$$u_l^i(c_i, l_i, g) = \mu_i(1-\tau)\xi_i w \tag{18}$$

$$\mu_i = \chi E\mu_i'((1-\tau)r' + 1 - \delta) \tag{19}$$

This means, looking at equation (17) and (18), that at optimum all agents choose in each period of time their labor participation such that the marginal utility that they obtain by enjoying one unit of leisure is equal to the utility they get by working an additional hour and consuming what they get:

$$u_l^i(c_i, l_i, g) = u_c^i(c_i, l_i, g)(1-\tau)\xi_i w \quad \forall i \in I. \tag{20}$$

In this framework, taxation reduces the agents' return to labor. Hence, this decentralized solution allows an efficient allocation of labor only if taxation is set equal to zero. Indeed, only if $\tau = 0$:

$$(1-\tau)w, \quad \text{that is} \quad (1-\tau)\phi A(\sum_{i=1}^{n} k_i)^{1-\phi}(\sum_{i=1}^{n} \xi_i(1-l_i))^{\phi-1}$$

is equal to

$$F_L(K, L), \quad \text{that is} \quad \phi A K^{1-\phi}(\sum_{i=1}^{n} \xi_i(1-l_i))^{\phi-1}.$$

Equation (19) underlines that each agent supplies capital up to the point the utility he or she loses by not consuming a marginal unit of resources today is equal to the expected present value of the utility he or she gets by supplying today an additional marginal unit of capital.

$$u_c^i(c_i, l_i, g) = \chi E u_c^i(c_i', l_i', g)((1-\tau)r' + 1 - \delta) \quad \forall i \in I. \tag{21}$$

As with labor, the tax rate on capital revenues set to finance public expenditure determines a wedge with respect to the first best situation. Consequently, the first best allocation cannot be achieved.

Note that if we remove the means-tested benefit s, the first order conditions of the agent's problem remain the same.

The government

A stationary equilibrium is given by a feasible allocation [g, c, l, k that satisfy the feasibility constraint], a price system [w, r] , and a government policy [τ, g] such that

(a) given the price system and the government policy, the [feasible] allocation solves both the firm's problem and the agents' problem; (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints and (c) the distribution of agents over the state variable k is constant.[1]

In this section, we are looking at the government policy that in stationary equilibrium maximizes the welfare of the economy, specified as the sum of the present value of each agent's utility.

In detail, given that $\tau' = \tau$ and $g' = g$, the problem of the central planner is to

$$\max_{g,\tau} \sum_{i=1}^{n} V^i(k_i, g, \tau) = \max_{g,\tau} [\sum_{i=1}^{n} u^i(c_i, l_i, g) + \chi E \sum_{i=1}^{n} EV^i(k_i', g, \tau)] \tag{22}$$

such that in each period of time the government budget constraint is satisfied,

$$\sum_{i=1}^{n} \tau(r_{(K,L)} k_i + \xi_i w_{(K,L)} n_i) = g + \sum_{i=1}^{n} I_{n_i} s \tag{23}$$

$$g, \tau \geq 0, \tag{24}$$

the feasibility constraint is satisfied,

$$\sum_{i=1}^{n} c_i + g + K' = AK^{1-\phi} L^{\phi} + (1-\delta)K, \tag{25}$$

where

$$K = \sum_{i=1}^{n} k_i \tag{26}$$

and

$$L = \sum_{i=1}^{n} \xi_i (1 - l_i), \tag{27}$$

given that the price system is

$$r = (1-\phi) A (\sum_{i=1}^{n} k_i)^{-\phi} (\sum_{i=1}^{n} \xi_i (1 - l_i))^{\phi} \tag{12a}$$

$$w = \phi A (\sum_{i=1}^{n} k_i)^{1-\phi} (\sum_{i=1}^{n} \xi_i (1 - l_i))^{\phi - 1}, \tag{13a}$$

[1]This definition is the stationary formulation of Sargent and Ljungqvist (2012, Chapter 16)'s definition of competitive equilibrium.

and $\{c_i\}_i$, $\{l_i\}_i$, $\{k'_i\}_i$ solve the agents' problem specified by the equations systems (14) and (15).

Notice that, as Yunker (2013), we set the problem such that the government maximizes the sum of the present value of agents' utility. Hence, the discount rate that the government uses to discount future utilities is the agents discount rate. We could set that the central planner evaluates future utilities of the agents differently from what the agents do. Nevertheless, as it will be further explained in section 3, in the literature the social discount rate is estimated to have the same value of our calibrated agents inter-temporal discount rate.

Note that if in a model there is only one representative agent, the knowledge that two constraints are satisfied among the feasibility constraint, the government budget constraint and the agent budget constraint, it implies that also the third one is satisfied. This it is not the case with heterogeneous agents. In fact, if the feasibility constraint and the government budget constraint are satisfied, this does not imply that the agents' budget constraints are satisfied. Indeed, from the merging of the government budget constraint and the feasibility constraint we get the sum over i, $\forall i \in I$, of the individual budget constraints. Hence, the agents budget constraints are never redundant. On the other hand, if the agents budget constraints and the government budget constraint are satisfied, then also the feasibility constraint is satisfied. Therefore, the feasibility constraint is omitted in the government maximization problem.

Finally, note that in the feasibility constraint the exogenous benefits, $I_{n_i}s$, are inside the consumption term.

In each period of time, the optimal level of g and τ, given that r and w are specified by the equations (12a) and (13a), are

$$\begin{aligned}\sum_{i=1}^{n} u^i_g(c_i, l_i, g) - \varepsilon + \sum_{i=1}^{n} \mu_i[u^i_{lg}(c_i, l_i, g) - u^i_{cg}(c_i, l_i, g)(1-\tau)\xi_i w_{(K,L)}] \\ + \sum_{i=1}^{n} \nu_i[u^i_{cg}(c_i, l_i, g) - E\chi u^i_{cg}(c'_i, l'_i, g)((1-\tau)r'_{(K',L')} + 1 - \delta)] = 0\end{aligned} \tag{28}$$

$$\begin{aligned}\varepsilon \sum_{i=1}^{n}(r_{(K,L)}k_i + \xi_i w_{(K,L)} n_i) + \sum_{i=1}^{n} \mu_i u^i_c(c_i, l_i, g)\xi_i w_{(K,L)} \\ + \sum_{i=1}^{n} \nu_i \chi E u^i_c(c'_i, l'_i, g) r'_{(K',L')} + \sum_{i=1}^{n} \varsigma_i[-r_{(K,L)}k_i - \xi_i w_{(K,L)} n_i] = 0,\end{aligned} \tag{29}$$

where ε is the multiplier of the government budget constraint, μ_i, $\forall i \in I$, is the multiplier of the agent's i labor supply equation (20), ν_i, $\forall i \in I$, is the multiplier of the agent's i capital supply equation (21) and ς_i, $\forall i \in I$, is the multiplier of agent's i budget constraint.

Notice that if we remove the benefit s the first order conditions with respect to g and τ do not change.

Moreover, shame/stigma costs associated to this type of conditional benefit could be easily introduced by using a cost parameter, e.g. d, which reduces the value of benefit available to the agents, $I_{n_i}[s-d]$, while in the government budget constraint the cost of the benefit remains $\sum_{i=1}^{n} I_{n_i} s$.

Basic income

Introducing basic income, ω, the problem of the government remains very similar to the previous case. Note that we assume that the government can fix only a constant level of basic income over time, i.e $\omega' = \omega$. In detail, the government's objective is

$$\max_{g,\tau,\omega} \sum_{i=1}^{n} V^i(k_i, g, \tau, \omega) = \max_{g,\tau,\omega} [\sum_{i=1}^{n} u^i(c_i, l_i, g) + \chi E \sum_{i=1}^{n} EV^i(k_i', g, \tau, \omega)] \tag{30}$$

where $\{c_i\}_i$, $\{l_i\}_i$, $\{k_i'\}_i$ solve the agents' problem specified by the equations systems (14) and (15), with the only difference that the agents' budget constraint is

$$k' + c_i = (1-\tau) r k_i + (1-\delta) k_i + (1-\tau) \xi_i w n_i + \omega. \tag{31}$$

This means that the first order conditions of the agent problem remain the same. Introducing basic income, the budget constraint of the government becomes

$$\sum_{i=1}^{n} \tau (r_{(K,L)} k_i + \xi_i w_{(K,L)} n_i) = g + n\omega \tag{32}$$

$$g, \tau, \omega \geq 0. \tag{33}$$

The feasibility constraint

$$\sum_{i=1}^{n} c_i + g + K' = AK^{1-\phi} L^{\phi} + (1-\delta) K, \tag{34}$$

and the price system specified by the equations (12a) and (13a) remain the same. As before, in the feasibility constraint the basic income is inside the consumption term.

Moreover, knowing that the government budget constraint and that each agent budget constraint are satisfied, the feasibility constraint can be omitted.

Given this setup, in each period of time the FOC with respect to ω is:

$$\sum_{i=1}^{n} \varsigma_i = n\varepsilon. \tag{35}$$

where, as before, ς_i, $\forall i \in I$, is the multiplier of agent's i budget constraint and ε is the multiplier of the government budget constraint. This means that basic income is expanded up to point that the gain in utility when marginally relaxing the budget constraint of each agent - the gain in utility of giving each agent one marginal unit of additional resources - is equal to the cost of financing this measure - the cost of increasing the government expenditure of n marginal units.

3 - Calibration

To solve the models, we assign values to the model parameters using estimates based on EU15 data. For the Cobb Douglas production function, the parameters to be estimated are ϕ and A. The agent's utility function is set to be a nested CES function:

$$\begin{aligned} u^i(c_i, l_i, g) &= (\alpha \tilde{c}_i^{\frac{\gamma-1}{\gamma}} + (1-\alpha) l_i^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}} \\ \tilde{c}_i &= (\beta\ c_i^{\frac{\zeta-1}{\zeta}} + (1-\beta) g^{\frac{\zeta-1}{\zeta}})^{\frac{\zeta}{\zeta-1}} \end{aligned} \tag{36}$$

This functional form is relatively simple but sufficiently sophisticated to capture the substitutability relations between public expenditure, private consumption and leisure, which constitute key elements to determine the optimal level of these three variables. Moreover, as stated by Bouakez and Rebei (2007), it clearly represents the decreasing marginal return of its arguments. To compute the agent's utility α, β, the elasticity of substitution among private consumption and public expenditure ζ, ϵ_{cg}, and the elasticity between aggregate consumption and leisure γ, $\epsilon_{\tilde{c}l}$, are estimated. Finally, the depreciation rate, δ, the parameters governing the evolution of agents' productivity, ρ, σ_ϵ, the inter-temporal social discount rate that is used in the first best model, υ, and the agents' common inter-temporal discount rate, χ, are estimated.

ϕ is set to match the empirical labor share i.e. $\phi = 0.64$ (Arpaia, Pérez and Pichelmann, 2009). As in Conesa, Kitao and Krueger (2009) and Garcia-Milà, Marcet and

Ventura (2010), A is set to one. The share parameter β, using the estimates of Coenen, Straub and Trabandt (2012), is set equal to 0.75. The share parameter α is calibrated such as each agent's working time is approximately 1/3 of his or her time. Note that the calibration is performed using the decentralized model with neither basic income, i.e. ω, nor benefits for agents that do not work, i.e. s, when the tax rate is set equal to 0.4. This tax rate is used as benchmark since it is the average tax rate with respect to GDP in EU15 (European Commission, 2018a). Hence, α is set equal to 0.58. Using the estimates of Auteri and Costantini (2010), the elasticity of substitution between private consumption and public consumption - ϵ_{cg} - is set equal to 0.65. The elasticity of substitution between aggregate consumption and leisure - $\epsilon_{\tilde{c}l}$ - is set equal to 2 (Coenen, Straub and Trabandt, 2012; Ercolani and e Azevedo, 2014). ρ is set equal to 0.95 and σ_ϵ equal to 0.21 (Storesletten, Telmer and Yaron, 2001; Chang and Kim, 2006). The discount rate of the first best problem, υ, is set equal to 0.96 (Evans, 2007). The agents' common inter-temporal discount rate, i.e.χ, is set such that the steady state net interest rate when taxation is at 40% is 4% (Ercolani and e Azevedo, 2014; Smets and Wouters, 2003).

In table 1 the parameters used in the models are reported.

ϕ	A	α	β	ϵ_{cg}	$\epsilon_{\tilde{c}l}$	δ	ρ	σ_ϵ	υ	χ
0.64	1	0.58	0.75	0.65	2	0.1	0.95	0.21	0.96	0.96

Table 1: Parameters value

4 - Results

To solve the models presented, firstly the productivity process is discretized. Then, a grid search technique is applied to compute, for all the models, the steady state of the economy and the associated total welfare, i.e.

$$\sum_{i=1}^{I} V^i(c_i, l_i, g, k_i). \tag{37}$$

The MATLAB code for each model is in the appendix.

First Best Model

As anticipated, we use a grid search technique to find what is the level of public expenditure, capital and, for each agent, the level of private consumption and labor that maximize total welfare. Note that to compute the first best model we use the results of equation (8), i.e.

$$\frac{u_l^i(c_i, l_i, g)}{\xi_i} = \frac{u_l^j(c_j, l_j, g)}{\xi_j} \quad \forall i, j \in I.$$

Hence, given the consumption and leisure time of one agent, the consumption and leisure time of all the other agents is fixed accordingly. This means that, if we choose agent 1 as reference agent, the central planner problem becomes

$$\begin{aligned} V(K) = \max_{c_1, l_1, g, K'} [u^1(c_1, l_1, g) + u^2(c_2(c_1, l_1), l_2(c_1, l_1), g) + ... \\ ... + u^n(c_n(c_1, l_1), l_n(c_1, l_1), g) + \upsilon EV(K')] \end{aligned} \tag{38}$$

subject to

$$\begin{aligned} \sum_{i=1}^{n} c_i + g + K' = AK^{1-\phi}L^{\phi} + (1-\delta)K \\ L = \sum_{i=1}^{n} \xi_i (1 - l_i | c_1, l_1) \\ K \quad \text{given} \\ \left. \begin{array}{r} l_i + n_i = 1 \\ ln\xi_i' = \rho ln\xi_i + \epsilon_i', \quad \epsilon_i \sim N(0, \sigma_\epsilon^2), \quad 0 \le \rho \le 1 \end{array} \right\} \forall i \in I \end{aligned} \tag{39}$$

In MATLAB, we implement the model fixing each time a different level of public expenditure, g, to see what level maximizes total welfare. Table 2 summarizes the results. In detail, V is the total welfare of the economy, Y is total output, K is the level of capital, K/Y, the capital output ratio, WT the average working time, C is the steady state level of average consumption, C/Y is the consumption output ratio and g/Y is the public expenditure output ratio.

We notice that, at all the different public expenditure levels, the central planner chooses a steady state level of capital such as net marginal productivity of capital is equal to 4%, which is the long run level of the net marginal productivity of capital

g	V	Y	K	K/Y	WT	C	C/Y	g/Y
0.1	15.54	1.12	3.04	2.72	0.35	0.72	0.64	0.09
0.2	16.36	1.17	3.29	2.81	0.36	0.64	0.55	0.17
0.3	15.76	1.20	3.38	2.82	0.37	0.56	0.47	0.25
0.4	14.57	1.07	2.85	2.65	0.32	0.39	0.36	0.37
0.5	12.58	0.97	2.26	2.34	0.30	0.24	0.25	0.52

Table 2: First Best Model

according to equation (7). Moreover, the capital output ratio is around 3, which is commonly acknowledged to be the ratio taking place in the economy. [2]

According to our first best model, a public expenditure of 0.2 is welfare maximizing. Of great interest it is how the central planner treats the different agents. With a public expenditure of 0.2, but the same takes place in the other situations, we observe that the labor supply is unequally divided among agents. The central planner makes the more productive agents work all their available time, while makes the working time of less productive agents equal to zero. At the same time, the central planner compensates the more productive agent with a share of consumption that is 4 times the consumption of the least productive agents. Indeed, the central planner, in the attempt to maximize total welfare, makes the more productive agents work longer hours and then compensates their reduced leisure time with higher consumption. However, note that all agents enjoy about the same level of welfare; for the less productive agents the welfare comes mostly from leisure time, whereas for the more productive agents it comes from consumption.

Basic Decentralized Model

Now, we look to the decentralized solution, and, in particular, we keep as benchmark case a decentralized model with neither the government subsidy s nor basic income ω. In table 3 we compute some aggregate statistics when tax rate is set to 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. We look at these tax rates because they are the tax rates with respect to GDP that we find in EU15 (European Commission, 2018a). Compared to the first best model, the GINI coefficient, the fraction of individual with zero net worth, i.e. NW, and the fraction of wealth to the richest one percent, i.e. R1, are computed.

[2] Notice however that Piketty (2015) states that the figure for rich countries could be in reality more near to 5-6.

τ	V	Y	K	K/Y	WT	C	C/Y	g	g/Y	GINI	NW	R1
0.1	9.66	0.64	1.93	3.02	0.23	0.38	0.60	0.06	0.10	0.60	0.18	0.04
0.2	10.92	0.79	2.02	2.57	0.30	0.43	0.54	0.16	0.20	0.61	0.19	0.04
0.3	10.93	0.79	1.74	2.21	0.33	0.38	0.48	0.24	0.30	0.62	0.19	0.04
0.4	10.36	0.73	1.39	1.90	0.33	0.30	0.41	0.29	0.40	0.62	0.19	0.04
0.5	9.44	0.63	1.00	1.57	0.32	0.22	0.34	0.32	0.50	0.62	0.19	0.07
0.6	8.37	0.52	0.66	1.27	0.28	0.14	0.27	0.31	0.60	0.62	0.29	0.12

Table 3: Decentralized model with neither ω nor u

As we can see, a tax rate of 0.3 is associated with a slightly higher total welfare than the other tax rates analyzed. Hence, according to our model, this is the optimal tax rate that the government should set. Observe however, that this tax rate performs similarly with respect to the 20% tax rate. Basically, the model suggests that the agents, starting from a low level of public expenditure, accept an increase in taxation and hence a reduction in private consumption, if this implies an increase of public expenditure. A welfare maximizing tax rate of 20%-30% could seem low compared to the tax rates we find in Europe. However, we should remember that our central planner is responsible only for the expenditure in public consumption.

Regarding the overall performance of the model, if we look at the results according to the tax rate of 30%, the model matches the economy main characteristics. The private consumption-output ratio implied by the model is around 50% and the average consumption output ratio in Euro area is 50% (European Commission, 2018b). The public consumption-output ratio in the model is 30% and the average public consumption in Europe is around 20% of GDP (Fiorito and Kollintzas, 2004). For what concerns the capital-output ratio, our figures are less than 3-4. Nevertheless, they are in line with the results in general equilibrium literature; indeed the Smets and Wouters (2003) general equilibrium model for euro area implies a 2.2 capital output ratio. Finally, the wealth GINI coefficient matches the current one, estimated to be .60-.70 (Davies et al., 2009). The model is less representative in regards to the amount of agents with zero wealth and the amount of wealth held by the richest one percent of the agents; indeed, in the economy the share of individuals with no wealth is about 5-15% and the wealth to the top one percent ranges between 20-30% (Davies et al., 2011; Atkinson and Piketty, 2010). However, this is an expected result. Indeed, the structure of our model follows Aiyagari (1994), and we know that to match the wealth distribution we should use models such as Huggett (1996) and Cagetti and De Nardi (2006), but this falls outside of the scope of this paper.

In table 4 we see the percentage change in welfare, V, consumption, C, assets holding, A, and working time, WT, for the least productive agents and more productive agents when we move from one tax rate, i.e. "starting tax rate", to the following one, i.e. "final tax rate". Note that L identifies the low productive agents and H the high productive ones. In columns V, the levels of welfare at the "final" tax rate are also reported.

τ	V-L	V-H	C-L	C-H	A-L	A-H	WT-L	WT-H
from 0.2 to 0.3	2.42, -1%	3.09, 1.5%	-17%	-12%	-16%	-13%	0.2%	9%
from 0.3 to 0.4	2.28, -6%	2.95, -5%	-24%	-20%	-22%	-20%	-5%	2%

Table 4: Welfare - Consumption - Assets - Labor distribution

Moving from one tax rate to the higher one, all agents enjoy a higher public expenditure level. However, they all experience a reduction in their available resources. In particular, the increase in taxation reduces considerably the consumption level and capital accumulation capability of less productive agents.

Compared to the first best solution, this decentralized solution implies a lower total welfare, V, which is also distributed differently among the different classes of agents. Indeed, in the first best model all agents enjoy about the same level of welfare, while in the decentralized solution the welfare of more productive agents is about 30% higher than the welfare of less productive agents.

Subsidy for Not Working Agents

Now we look to the decentralized models with a subsidy assigned when an agent does not work. In detail, we look at tax rates of 0.2, 0.3, 0.4 and 0.5 with s equal to 0.02, 0.05, 0.07, 0.08 and 0.1. The results are summarized in table 5; the column S/Y is the ratio of total expenditure sustained by the government for financing s over total output.

First thing to observe is that the results with s equal to 0.07 associated to τ equal to 0.5 and 0.2 and all the results for s equal to 0.08 and 0.1 are not reported. The reason is that, if the government commits to assign a benefit of this size, it has not enough resources to finance this policy. Indeed, many agents stop working. As a result, aggregate output reduces and with it average consumption and public expenditure. The

τ-s	V	Y	s/Y	K/Y	WT	C	C/Y	g	g/Y	GINI	NW	R1
.2-.02	10.74	0.74	0.012	2.62	0.27	0.40	0.54	0.14	0.19	0.60	0.09	0.06
.3-.02	10.96	0.76	0.012	2.28	0.30	0.38	0.49	0.22	0.29	0.59	0.09	0.04
.4-.02	10.35	0.69	0.015	1.94	0.28	0.29	0.41	0.27	0.39	0.59	0.05	0.04
.5-.02	9.48	0.58	0.020	1.61	0.24	0.21	0.36	0.28	0.48	0.59	0.15	0.10
.2-.05	10.02	0.59	0.055	2.73	0.18	0.34	0.59	0.09	0.15	0.66	0.14	0.05
.3-.05	10.64	0.65	0.047	2.31	0.22	0.34	0.52	0.16	0.25	0.59	0.16	0.05
.4-.05	10.34	0.60	0.054	1.95	0.22	0.28	0.46	0.21	0.34	0.60	0.19	0.04
.5-.05	9.48	0.44	0.086	1.64	0.16	0.19	0.43	0.18	0.41	0.64	0.32	0.07
.3-.07	10.24	0.54	0.090	2.32	0.18	0.30	0.56	0.11	0.21	0.62	0.18	0.08
.4-.07	9.84	0.44	0.123	1.98	0.14	0.23	0.52	0.12	0.28	0.69	0.29	0.04

Table 5: Decentralized model with subsidy s for not working agents

economy converges to a degenerate solution with consumption, labor supply, capital and public expenditure equal to zero.

We have seen that when τ is equal to 0.5 or 0.2 the sustainable level of s is lower. Indeed, τ equal to 0.5 determines a lower total output due to its distortive effects on labor supply and capital supply, and τ equal to 0.2, even if it does not distort the economic activity, does not allow the government to collect enough resources to finance a high benefit s.

Note that the highest levels of subsidy that are sustainable correspond to around 1/3 of the steady state average consumption.

Among the non degenerate solutions, the tax rate at 0.3%, together with a moderate small subsidy of 0.02 is associated with the highest total welfare. The subsidy s improves slightly total welfare with respect to the situation in which this subsidy is not introduced. In detail, comparing the basic model with τ at 30% and this model with τ at 30% and s at 0.02, we see that capital, output and average consumption remain about constant, public expenditure and working time are slightly lower and all the inequality indexes of our economy decrease. If we look specifically to the more productive and less productive agents, we notice that moving from the situation in which there is not subsidy to a subsidy of 0.02, the welfare of the less productive class of agents increases by around 1% and the welfare of the more productive agents decreases by 0.4%. Consumption for less productive agents decreases by 3%, and that of more productive agents increases by about 2%. The amount of assets of less productive agents increases by 9%, while for more productive ones it decreases by 2%. Finally, the working time of less productive agents is reduced approximately by around 26%, while the working hours of more productive agents is reduced approximately by 3%.

Basically, the introduction of the subsidy leaves unchanged the behaviors of more productive agents but it greatly modifies the behavior of the less productive ones. Indeed, low productive agents can decide to go out of the labor market since, even if they do not work, they can enjoy the small level of consumption guaranteed by s. Hence, these agents enjoy more leisure time and their total welfare increases.

Note that this outcome goes in the direction of the first best solution, where the working time of less productive agents is equal to zero.

From these results, one supporting statement of basic income is weakened. In detail, supporters of basic income underline that the unconditionality of the basic income is a key advantage with respect to conditional subsidies. The results show instead that a conditional subsidy assigned to not working agents could be welfare improving. Moreover, this type of subsidy affects the behavior of less productive agents but it does not change much the behavior of more productive agents.

Basic Income

Finally, we introduce basic income to the economy. In table 7 we summarize the results for tax rates at 20%, 30% 40% and 50% with a basic income of 0.02, 0.05, 0.07, 0.08 and 0.1. BI/Y are the expenditures of basic income with respect to the economy total output.

τ-ω	V	Y	BI/Y	K/Y	WT	C	C/Y	g	g/Y	GINI	NW	R1
.2-.02	10.52	0.69	0.03	2.59	0.26	0.39	0.57	0.12	0.17	0.61	0.19	0.05
.3-.02	10.87	0.73	0.03	2.23	0.29	0.37	0.51	0.20	0.27	0.62	0.19	0.04
.4-.02	10.38	0.67	0.03	1.87	0.29	0.29	0.44	0.25	0.37	0.63	0.19	0.05
.5-.02	9.60	0.57	0.04	1.58	0.26	0.21	0.37	0.27	0.47	0.63	0.19	0.09
.2-.05	9.55	0.51	0.10	2.67	0.18	0.32	0.62	0.06	0.12	0.60	0.18	0.13
.3-.05	10.48	0.61	0.08	2.24	0.23	0.35	0.57	0.13	0.22	0.64	0.19	0.08
.4-.05	10.39	0.58	0.09	1.91	0.23	0.29	0.51	0.18	0.31	0.64	0.19	0.10
.5-.05	9.64	0.45	0.11	1.60	0.18	0.20	0.44	0.18	0.39	0.70	0.35	0.14

Table 6: Decentralized model with basic income

As with the subsidy s, some levels of basic income are not sustainable, i.e. the economic activity converges to zero. In detail, this happens with all tax rates when ω is higher than 0.05.

Among the stable solutions, the taxation level of 30% with a basic income of 0.02 is the combination that maximizes total economy welfare. This policy implies a slightly lower total welfare than the situation in which there is no basic income in the economy. Moreover, basic income has non negligible consequences on the steady state of the economy. Indeed, average working time decreases by around 12%, average capital by 6%, and consequently total output decreases by approximately 8%. Average consumption and public expenditure decrease respectively by 2.5% and 17%.

If we compare less productive and more productive agents, we clearly see that total welfare of less productive agents increases by around 1%, while the welfare of more productive agents decreases by around 1.7%. Less productive agents reduce their working time by 21% and more productive by 6%. Consumption remains stable - less productive agents reduce their consumption by approximately 0.2% and more productive agents by 0.7%. Regarding assets, the asset holding of less productive agents decreases by 8% and that of more productive agents decreases by 6%.

These results show that on the one hand, the total cost borne by society to finance this type of measure is higher than in a case of a subsidy s of the same magnitude. This implies that since the government has less resources to finance public expenditure, g decreases and all agents suffer from this reduction. On the other hand, ω is assigned independently of the working condition. Hence, it is no longer necessary to work zero hours to receive the subsidy. For this reason, less productive agents reduce their labor supply but not as they do with the conditional subsidy s. Consequently, low productive agents can maintain unaltered consumption, while with the subsidy s their consumption decreases by 3%. Note that we identify the higher reduction in working hours when the benefit s is in place with the unemployment trap situation. Basic income allows also more productive agents to reduce their working time. However, their increase in welfare due to higher leisure time does not compensate for their reduction in utility due to the decrease in public expenditure.

At this point some readers may argue what follows. Since basic income is more expensive than the subsidy s and it competes with public expenditure for resources, while in the basic decentralized model taxation at 0.3 maximizes welfare, if we introduce basic income it could be optimal to adopt a higher level of taxation, but not as distortive as the 0.4 tax rate. Nevertheless, even if we consider a tax rate of 0.35 associated to a basic income of 0.02 and 0.05, we get respectively a total welfare of 10.71 and 10.51, that are lower than the welfare obtained in the basic decentralized framework.

We have seen that s slightly improves total welfare than the decentralized model without benefit, while ω causes a small reduction in total welfare. Moreover, basic income changes considerably the steady state level of the economy variables, i.e. output and working time decrease. While, in fact, a low conditional subsidy s affects mainly the behavior of less productive agents, basic income modifies the behaviors of all agents. As reported in the introduction, this result is in line with the findings of Yunker (2013) who shows that basic income implies a reduction of output and savings.

Given the results of our models, the possibility of introducing the basic income seems doubtful. In fact, a conditional benefit s could achieve higher total welfare levels compared to the basic decentralized model, maintaining the cost of the policy limited to 1.5% of total output and without modifying heavily the agents behavior.

Robustness Checks

To check the robustness of our model, we perform a sensitivity analysis on the model parameters. In detail, we modify the parameters of the decentralized solution with neither basic income nor unemployment subsidy, with tax rate fixed at 30%. We modify, one at a time, the agent's utility function weight for public and private consumption, i.e. β, the weight of aggregate consumption and leisure, α, the elasticity of substitution between private consumption and public consumption, ϵ_{cg}, and the elasticity of substitution between aggregate consumption and leisure, $\epsilon_{\tilde{c}l}$. In detail, we set β equal to 0.7 and 0.8, α equal to 0.57 and 0.58, ϵ_{cg} equal to 1 and $\epsilon_{\tilde{c}l}$ equal to 0.8 and 3.2.

Notice that small changes in our parameters leave the economy steady state basically unchanged. The only considerable change is obtained reducing the elasticity of substitution between aggregate consumption and leisure from 2 to 0.8, i.e. the substitutability among leisure and aggregate consumption is reduced. In detail, this change pushes especially the less productive agents, that before they were enjoying much free time, to increase their labor supply in order to increase consumption.

	V	Y	K	K/Y	WT	C	C/Y	g	g/Y	GINI	NW	R1
β .7	10.75	0.70	1.57	2.24	0.29	0.34	0.49	0.22	0.31	0.66	0.20	0.14
β .8	10.81	0.82	1.78	2.17	0.34	0.36	0.44	0.25	0.23	0.61	0.19	0.08
α .57	11.11	0.76	1.68	2.20	0.32	0.39	0.51	0.23	0.30	0.63	0.120	0.10
α.59	11.00	0.83	1.86	2.25	0.34	0.39	0.47	0.26	0.31	0.62	0.19	0.07
$\epsilon_{cg}1$	11.23	0.85	1.82	2.13	0.35	0.42	0.49	0.25	0.29	0.63	0.19	0.09
$\epsilon_{\tilde{c}l}$-0.8	11.21	0.92	2.11	2.30	0.44	0.48	0.52	0.27	0.30	0.66	0.41	0.05
$\epsilon_{\tilde{c}l}$-3.2	11.15	0.73	1.65	2.26	0.27	0.35	0.48	0.23	0.31	0.67	0.27	0.14

Table 7: Decentralized model with neither ω nor u

5 - Conclusions

This paper shows that in an economy ruled by a central planner, the central planner forces more productive agents to work longer hours and assigns higher consumption to them, while less productive agents have more free time and consume less; this solution implies a similar level of welfare for each agent. In a decentralized framework, the welfare of less productive agents decreases considerably since they have to work, even if with a really low productivity, to earn the means to consume. If we introduce a subsidy s conditional on not working, i.e. a means-tested benefit, the welfare of less productive agents increases since they can enjoy, also without working, a minimum consumption. At the same time, if the subsidy is low and hence the total expenditure to finance this measure is limited, the behavior of more productive agents remains unchanged.

The introduction of basic income brings a small reduction of total welfare with respect to the basic decentralized model. Moreover, it brings considerable changes to the steady state level of economy variables. Indeed, both labor supply of all agents and average output decrease. Hence, this paper shows that basic income is not necessarily an improvement over a conditional benefit and that it is fundamental in the discussion on the introduction of basic income to consider what type of changes in the economic structure we are willing to accept.

Appendix

MATLAB Code

```
%%%%%%%%%%CODES%%%%%%%%%%

%I kindly thank Dr Rigas Oikonomou, professor of UCLouvain, for making
%available to me his replication code of Aiyagari (1994), from which I
%have developed my first best model and decentralized models.

%The agents' productivity process is discretized using
%the addacouper function


%%%%ADDACOUPER%%%%

function [Z,PI] = addacooper(n,mu,rho,sigma)

%The function approximates an AR(1) process following Tauchen (1986)
%and Tauchen and Hussey (1991). The process is discretized into N
%intervals. Z is the conditional mean of each interval, PI the probability
%of transition to any of these intervals. See Adda and Cooper (2003)
%pp 57- for more details.
%
%
%Martin Flodn, 2005

sigmaUNC = sigma/sqrt(1-rho^2);
E   = zeros(n+1,1);
Z   = zeros(n,1);
PI  = zeros(n,n);
MFPI = zeros(n,n);

E(1)     = -1E6;
E(end) = 1E6;
for i = 2:n
      E(i) = sigmaUNC*norm_inv((i-1)/n) + mu;
end

```

```
for i = 1:n
    Z(i) = n*sigmaUNC*(norm_pdf((E(i)-mu)/sigmaUNC)...
           - norm_pdf((E(i+1)-mu)/sigmaUNC)) + mu;
end

for i = 1:n
    for j = 1:n
        E1 = E(j);
        E2 = E(j+1);
        th_fcn  = @(u) n/sqrt(2*pi*sigmaUNC^2)*(exp(-(u'-mu).^2 /...
        (2*sigmaUNC^2)).*(norm_cdf((E2-mu*(1-rho)-rho*u')/sigma) -...
        norm_cdf((E1-mu*(1-rho)-rho*u')/sigma)));
        PI(i,j) = quadl(th_fcn,E(i),E(i+1),1e-10);
        MFPI(i,j) = norm_cdf((E(j+1)-mu*(1-rho)-rho*Z(i))/sigma) -...
        norm_cdf((E(j)-mu*(1-rho)-rho*Z(i))/sigma);
    end
end

for i = 1:n
    PI(i,:) = PI(i,:) / sum(PI(i,:));
    MFPI(i,:) = MFPI(i,:) / sum(MFPI(i,:));
end


function c = norm_cdf(x)
    c = 0.5 * erfc(-x/sqrt(2));

function p = norm_pdf(x)
    p = 1/sqrt(2*pi) * exp(-(x)^2/2);

function y = norm_inv(x)
    f = @(x0,a) (0.5 * erfc(-x0/sqrt(2))-a);
    y = fzero(f,0,[],x);


%%%%FIRST BEST MODEL%%%%

upsilon=.96; % intertemporal discount rate
alpha= 0.58; % weight of aggregate consumption and leisure
beta= 0.75;  % weight of private consumption and public expenditure
```

```
zeta= 0.5385;% zeta=(1 -elasticity(c-g))/elasticity(c-g)
gamma= -0.5; % gamma=(1-elasticity(aggregatec-l))/elasticity(aggregatec-l)
phi= 0.64;    % weight of aggregate labor and capital
delta= 0.1;   % depreciation rate

%We run the model for g equal to 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6.
%In this example we set g = 0.3
g = 0.3;

nl=5; % we discretize the productivity process in nl intervals
rhol=.95; %expected value of the agents' productivity shock
sigmal= 0.21; %standard deviation of the productivity shock

[Z,PI] = addacooper(nl,0,rhol,sigmal);

ll=exp(Z); %conditional mean of each interval
prob=PI;    %probability of transition to any of these intervals

%Note that we have just divided our agents in "nl" productivity categories
%We set a grid of consumption and leisure points for the agents of one
%of these "nl" productivity intervals. We call the selected
%productivity interval "reference class".

consumptiongrid1 = linspace(0.05,1.5,150); %grid points for consumption
                                            %of the reference class
leisuregrid1 = linspace(0.05,1/nl,150);     %grid points for leisure
                                            %of the reference class

nc = length(consumption1);
nleisure = length(leisure1);

%Given the consumption and leisure of the reference class, we obtain the
%consumption and leisure of the other classes of agents using equation (4)
%and (5), i.e. the FOCs of the fist best problem. Hence, for each
%combination of leisure and consumption of the reference class, we look
%to the leisure and consumption combination of each other productivity
%class of agents. Since we set 150 points for consumptiongrid1 and 150
%for leisuregrid1, we have for our reference class 22500 different
%combination of leisure and consumption.

```

```
for i = 1:nc
    for h = 1:nleisure

        fun=@(x)EquationsToBeSolved(x,g,ll,i,h,nl,consumption1,leisure1);
        %The system of equation "EquationsToBeSolved" is reported at the
        %end of this section.

        options = optimoptions('lsqnonlin', 'TolFun', 1e-15,...
        'MaxIterations', 100000, 'MaxFunctionEvaluations', 120000);

        lb = zeros(2*(nl-1),1);
        lb(nl:2*(nl-1)) = 0.05;
        ub = 25*ones(2*(nl-1),1);
        ub(nl:2*(nl-1))= 1/nl;

        x0 = 0.001*ones(2*(nl-1),1);
        x = lsqnonlin(fun,x0, lb, ub, options);

        cons = reshape(x(1:nl-1),nl-1,1);
        leisure = reshape(x(nl:2*nl-2),nl-1,1);

        cons1(i,h) = cons(1);
        cons2(i,h) = cons(2);
        cons4(i,h) = cons(3);
        cons5(i,h) = cons(4);
        cons3(i,h) = consumptiongrid1(i);
        leis1(i,h) = leisure(1);
        leis2(i,h) = leisure(2);
        leis4(i,h) = leisure(3);
        leis5(i,h) = leisure(4);
        leis3(i,h) = leisuregrid1(h);

    end
end

%To visualize how the combinations of leisure and consumption for
%each agent evolve, we can reshape the results.

%leis1 = reshape(leis1,150,150); %leisure points of agents of the first
                                  %productivity interval
```

```
%leis2 = reshape(leis2,150,150); %leisure points of agents of the second
                                 %productivity interval
%leis3 = reshape(leis3,150,150);
%leis4 = reshape(leis4,150,150);
%leis5 = reshape(leis5,150,150);

%cons1 = reshape(cons1,150,150); %consumption points of agents of first
                                 %productivity interval
%cons2 = reshape(cons2,150,150);
%cons3 = reshape(cons3,150,150);
%cons4 = reshape(cons4,150,150);
%cons5 = reshape(cons5,150,150);

leis1 = leis1(:);
leis2 = leis2(:);
leis3 = leis3(:);
leis4 = leis4(:);
leis5 = leis5(:);

cons1 = cons1(:);
cons2 = cons2(:);
cons3 = cons3(:);
cons4 = cons4(:);
cons5 = cons5(:);

meanconsumption = cons1 + cons2 + cons3 + cons4 + cons5;

asset= linspace(0.4,7,40); %we set a grid of assets
na=length(asset);

V=zeros(na,nl);  % initial guess for the value function
V1=zeros(na,nl); % we preallocate the variables for speed
aprime = zeros(na,1);
diff = zeros(na,1);
totalabor = zeros(na,1);
oldlaborsupply = zeros(na,1);
indexaprime = zeros(na,nl);
totalworking = zeros(na,1);

%%%We solve our recursive problem%%%%
```

```
196
197 while dist>=.02 %As long as the value function is not stationary, we
198                 %repeat the loop
199     dist
200
201     for f = 1:na
202
203     laborsupply = ones(na,1); %we set an initial guess of mean labour
204                               %supply
205         distans = 1;
206
207         while distans > .02 %this while loop updates our initial guess
208         %for mean labor supply. To facilitate convergence, we advice to
209         %start with a less strict constraint and to update it after few
210         %iterations. Remember to take out line 186 when updating the
211         %constraint.
212
213         cc = - asset' -g + asset(f)^(1-phi)*(laborsupply(f))^phi +...
214             (1-delta)*asset(f);
215         %note that cc is (na,1) vector. It is the value of average
216         %consumption for each level of next period asset in our grid,
217         %i.e. asset', given a starting value of asset, i.e. asset(f)
218
219         indcon=find(cc<0); %we check when consumption is negative
220
221         nextV1=(V*prob(1,:)'); %continuation utility
222         nextV1(indcon)= -100;  %penalty to negative consumption
223
224         nextV2=(V*prob(2,:)');
225         nextV2(indcon)= -100;
226
227         nextV3=(V*prob(3,:)');
228         nextV3(indcon)= -100;
229
230         nextV4=(V*prob(4,:)');
231         nextV4(indcon)= -100;
232
233         nextV5=(V*prob(5,:)');
234         nextV5(indcon)= -100;
235
```

```
cc = max(cc,.000000000000000000000000000001);

    summa = zeros(na,1); %preallocation for speed
    minValue = zeros(na,1);
    closestIndex = zeros(na,1);
    util1= zeros(na,1);
    util2= zeros(na,1);
    util3= zeros(na,1);
    util4= zeros(na,1);
    util5= zeros(na,1);


    for k = 1:na  %for each element of the cc vector, we select
                  %the nearest value of meanconsumption

    [minValue(k),closestIndex(k)]= min(abs(meanconsumption-cc(k)))

    util1(k)=(alpha*(beta*cons1(closestIndex(k)).^(-zeta) +...
    (1-beta)*g.^(-zeta)).^(gamma/zeta)+...
    (1-alpha)*(leis1(closestIndex(k)).^(-gamma))).^(-1/gamma) +...
    upsilon*nextV1(k)';
    util2(k)=(alpha*(beta*cons2(closestIndex(k)).^(-zeta) +...
    (1-beta)*g.^(-zeta)).^(gamma/zeta) +...
    (1-alpha)*(leis2(closestIndex(k)).^(-gamma))).^(-1/gamma)+...
    upsilon*nextV2(k)';
    util3(k)=(alpha*(beta*cons3(closestIndex(k)).^(-zeta) +...
    (1-beta)*g.^(-zeta)).^(gamma/zeta) +...
    (1-alpha)*(leis3(closestIndex(k)).^(-gamma))).^(-1/gamma)+...
    upsilon*nextV3(k)';
    util4(k)=(alpha*(beta*cons4(closestIndex(k)).^(-zeta) +...
    (1-beta)*g.^(-zeta)).^(gamma/zeta) +...
    (1-alpha)*(leis4(closestIndex(k)).^(-gamma))).^(-1/gamma)+...
    upsilon*nextV4(k)';
    util5(k)=(alpha*(beta*cons5(closestIndex(k)).^(-zeta) +...
    (1-beta)*g.^(-zeta)).^(gamma/zeta) +...
    (1-alpha)*(leis5(closestIndex(k)).^(-gamma))).^(-1/gamma)+...
    upsilon*nextV5(k)';

    summa(k) = util1(k) + util2(k) + util3(k) +util4(k) +util5(k);

```

```
        end

    [va,ind] = max(summa);
    index = closestIndex(ind);

    totalworking(f) = (1/nl -leis1(index)) + (1/nl-leis2(index)) +...
    (1/nl-leis3(index))+(1/nl-leis4(index)) + (1/nl-leis5(index));
    %totalworking is the mean working time in the economy
    steadyconsumption(f) = cons1(index) + cons2(index) +...
    cons3(index)+ cons4(index)+ cons5(index);
    %steadyconsumption is mean consumption in the economy

    work1(f) = (1/nl-leis1(index));
    work2(f) = (1/nl-leis2(index));
    work3(f) = (1/nl-leis3(index));
    work4(f) = (1/nl-leis4(index));
    work5(f) = (1/nl-leis5(index));

    consumption1(f) = cons1(index);
    consumption2(f) = cons2(index);
    consumption3(f) = cons3(index);
    consumption4(f) = cons4(index);
    consumption5(f) = cons5(index);

    averagelabor(f) = (1/nl -leis1(index))*ll(1) +...
    (1/nl -leis2(index))*ll(2) +(1/nl - leis3(index))*ll(3) +...
    (1/nl - leis4(index))*ll(4) + (1/nl - leis5(index))*ll(5);
    %totalabor is the average labor supply in the economy
    oldlaborsupply(f) = laborsupply(f);
    diff(f) = laborsupply(f) -averagelabor(f);
    distans= abs(laborsupply(f)-averagelabor(f));

    %if averagelabor is different from our initial labor supply guess,
    %we update the guess

    if diff(f) > 0.07
        laborsupply(f) = laborsupply(f) - 0.01;
    elseif diff(f) < -0.07
        laborsupply(f) = laborsupply(f) + 0.01;
    else
```

```
                laborsupply(f) = (laborsupply(f)+ averagelabor(f))/2;
            end

            end

            %Value of continuation utility for each agent
            V1(f,1)=util1(ind);
            V1(f,2)=util2(ind);
            V1(f,3)=util3(ind);
            V1(f,4)=util4(ind);
            V1(f,5)=util5(ind);

            aprime(f)= asset(ind);

            for j = 1:nl
            indexaprime(f,j)=ind;
            end

        end

dist=max(max(abs(V1-V)));

V=V1;

end


%%%Steady State Distribution

problongrun=ones(na,1)/(na);
Q=zeros(na,na);

for i=1:na
Q(i,indexaprime(i,1))=1;
end

distprob=1;

while distprob>=.000000000000001
problongrunnew= Q'*problongrun(:);
```

```
distprob=max(max(abs(problongrunnew-problongrun)));
problongrun=problongrunnew;
end

%%%Aggerate results of the economy

%average welfare in the economy
for k = 1:nl
    partialwelfare(k) = V(:,k)'*problongrun(:,k);
end

averagewelfare = sum(partialwelfare)

%average capital
amean = asset(:)'*problongrun;

%average working time
Workingtime = totalworking(:)'*problongrun

%workingtime1 = work1(:)'*problongrun;
%workingtime2 = work2(:)'*problongrun;
%workingtime3 = work3(:)'*problongrun;
%workingtime4 = work4(:)'*problongrun;
%workingtime5 = work5(:)'*problongrun;

%average labor supply
LL = averagelabor(:)'*problongrun;

%average consumption
averageconsumption = -g -delta*amean +amean^(1-phi)*LL^(phi)

%alternatively, we can obtain the average consumption as
%averageconsumption = steadyconsumption(:)'*problongrun

%consumptionsteady1 = consumption1(:)'*problongrun
%consumptionsteady2 = consumption2(:)'*problongrun
%consumptionsteady3 = consumption3(:)'*problongrun
%consumptionsteady4 = consumption4(:)'*problongrun
%consumptionsteady5 = consumption5(:)'*problongrun
```

```
totaloutput = amean^(1-phi)*LL^(phi)

capitalOutratio = amean/totaloutput

publicexpenditureOutratio = g/totaloutput

consumptionOutratio = averageconsumption/totaloutput


%%%EquationsToBeSolved%%%

function F = EquationsToBeSolved(x,g,ll,i,h,nl,consumption1,leisure1)

%Paramenters
alpha= 0.58;
beta= 0.75;
zeta= 0.5385;
gamma= -0.5;

cons = reshape(x(1:nl-1),nl-1,1);
leisure = reshape(x(nl:2*nl-2),nl-1,1);

F = zeros(2,nl-1);

for f = 1:nl-1

F(1,f) = beta*zeta*(cons(f)).^(-zeta-1)*(alpha*gamma/zeta)*(beta*(cons(f)).^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta-1)*(1/gamma)*(alpha*(beta*(cons(f))^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta) +(1-alpha)*(leisure(f)).^(-gamma)).^(-1/gamma-1) -beta*zeta*(consumption1(i)).^(-zeta-1)*(alpha*gamma/zeta)*(beta*(consumption1(i)).^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta-1)*(1/gamma)*(alpha*(beta*(consumption1(i))^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta) +(1-alpha)*(leisure1(h)).^(-gamma)).^(-1/gamma-1);

if f == 1 || f == 2
F(2,f) = ((1-alpha)*gamma*(leisure(f))^(-gamma-1)*(1/gamma)*(alpha*(beta*(cons(f))^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta) +(1-alpha)*(leisure(f)).^(-gamma)).^(-1/gamma-1))/(ll(f)*(beta*zeta*(cons(f)).^(-
```

```
zeta-1)*(alpha*gamma/zeta)*(beta*(cons(f)).^(-zeta) + (1-beta)*g.^(-
zeta)).^(gamma/zeta-1)*(1/gamma)*(alpha*(beta*(cons(f))^(-zeta) + (1-
beta)*g.^(-zeta)).^(gamma/zeta) +(1-alpha)*(leisure(f)).^(-gamma))
.^(-1/gamma-1)))-((1-alpha)*gamma*(leisure1(h))^(-gamma-1)*(1/gamma)*(
alpha*(beta*(consumption1(i))^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/
zeta) +(1-alpha)*(leisure1(h)).^(-gamma)).^(-1/gamma-1))/(ll(3)*(beta*
zeta*(consumption1(i)).^(-zeta-1)*(alpha*gamma/zeta)*(beta*(
consumption1(i)).^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta-1)*(1/
gamma)*(alpha*(beta*(consumption1(i))^(-zeta) + (1-beta)*g.^(-zeta)).^(
gamma/zeta) +(1-alpha)*(leisure1(h)).^(-gamma)).^(-1/gamma-1)));
else
F(2,f) = ((1-alpha)*gamma*(leisure(f))^(-gamma-1)*(1/gamma)*(alpha*(beta*(
cons(f))^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta) +(1-alpha)*(
leisure(f)).^(-gamma)).^(-1/gamma-1))/(ll(1+f)*(beta*zeta*(cons(f)).^(-
zeta-1)*(alpha*gamma/zeta)*(beta*(cons(f)).^(-zeta) + (1-beta)*g.^(-
zeta)).^(gamma/zeta-1)*(1/gamma)*(alpha*(beta*(cons(f))^(-zeta) + (1-
beta)*g.^(-zeta)).^(gamma/zeta) +(1-alpha)*(leisure(f)).^(-gamma))
.^(-1/gamma-1)))-((1-alpha)*gamma*(leisure1(h))^(-gamma-1)*(1/gamma)*(
alpha*(beta*(consumption1(i))^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/
zeta) +(1-alpha)*(leisure1(h)).^(-gamma)).^(-1/gamma-1))/(ll(3)*(beta*
zeta*(consumption1(i)).^(-zeta-1)*(alpha*gamma/zeta)*(beta*(
consumption1(i)).^(-zeta) + (1-beta)*g.^(-zeta)).^(gamma/zeta-1)*(1/
gamma)*(alpha*(beta*(consumption1(i))^(-zeta) + (1-beta)*g.^(-zeta)).^(
gamma/zeta) +(1-alpha)*(leisure1(h)).^(-gamma)).^(-1/gamma-1)));
end

F = F(:);

end


%%%%DECENTRALIZED SOLUTION WITH NEITHER BASIC INCOME NOR SUBSIDY%%%%

chi=.96;       % individual intertemporal discount rate
alpha= 0.58; % weight of aggregate consumption and leisure
beta= 0.75;  % weight of private consumption and public expenditure
zeta= 0.5385;% zeta=(1 -elasticity(c-g))/elasticity(c-g)
gamma= -0.5; % gamma=(1-elasticity(aggregatec-l))/elasticity(aggregatec-l)
phi= 0.64;    % weight of aggregate labor and capital
delta= 0.1;  % depreciation rate
```

```
tau = 0.3; % we set the level of taxation

g = 0.2; % we set an initial guess for g

nl=4; % we discretize the productivity process in nl intervals

rhol=.95;
sigmal= 0.21;
[Z,PI] = addacooper(nl,0,rhol,sigmal);

ll=exp(Z);
prob=PI;

%equilibrium interest rate
req= (1/sigmaind -1 + delta)/(1-tau);

rhigh=req-.0001;
rlow=req/1.1;

borrow=.0; %borrowing constraint

asset=[linspace(-borrow,15,400)]; %grid of asset points

leisure= [linspace(0,1,60)]; %grid of leisure points

na=length(asset);
nleisure = length(leisure);

w = 1; %initial guess for wage
r=(rhigh+rlow)/2; %initial guess for interest rate

L = zeros(1,nl); %preallocation for speed
V1 = zeros(na,nl);
aprime = zeros(na,nl);
indexaprime = zeros(na,nl);
leisurechoice = zeros(na,nl);
consumptionpath = zeros(na,nl);

balance = 1;
V=zeros(na,nl); %initial guess for the value function
```

```
while balance >= .001 %we adjust public expenditure if the government
                      %budget constraint is not balanced.
distgeneral=1;

while distgeneral>=.005 %we adjust the interest rate if it differs
                        %from the initial guess.

%%%Solve the value function

    dist=1;
    tic;
    while dist>=.001

        for d = 1: nleisure

            for i =1:na

                for j =1:nl

                    cc = -asset +(1-delta)*asset(i) +...
                    (1-tau)*r*asset(i) +(1-tau)*w*ll(j)*(1-leisure(d));

                    indcon=find(cc<0);
                    cc=max(cc,.000000000000000000000000000001);

                    nextV=(V*prob(j,:)'); %continuation utility
                    nextV(indcon)= -10000;

                    util=(alpha*(beta*cc.^(-zeta) +...
                    (1-beta)*g.^(-zeta)).^(gamma/zeta) +...
                    (1-alpha)*(leisure(d).^(-gamma))).^(-1/gamma) +...
                    chi*nextV(:)';

                    [va,ind]=max(util); %maximize over the asset grid

                    %we update the value function
                    if d == 1
                    V1(i,j) = util(ind);
                    aprime(i,j)=asset(ind);
```

```
                    indexaprime(i,j)=ind;
                    leisurechoice(i,j) = leisure(d);
                    consumptionpath(i,j) = cc(ind);

                    else
                    V1(i,j)=max(V1(i,j), util(ind));

                    if util(ind) == V1(i,j)
                    aprime(i,j)=asset(ind);
                    indexaprime(i,j)=ind;
                    consumptionpath(i,j) = cc(ind);
                    leisurechoice(i,j) = leisure(d);

                    end
                    end
                end
            end
        end

    dist=max(max(abs(V1-V)));

    V=V1;

    end
    toc;


%%%Steady state

problongrun=ones(na,nl)/(na*nl);

Q1=zeros(na,na);
Q2=zeros(na,na);
Q3=zeros(na,na);
Q4=zeros(na,na);

for i=1:na

Q1(i,indexaprime(i,1))=1;
Q2(i,indexaprime(i,2))=1;
```

```
Q3(i,indexaprime(i,3))=1;
Q4(i,indexaprime(i,4))=1;

end

distprob=1;

while distprob>=.000000000000001

problongrun1= Q1'*problongrun(:,1);
problongrun2= Q2'*problongrun(:,2)
problongrun3= Q3'*problongrun(:,3);
problongrun4= Q4'*problongrun(:,4);

problongrunnew=[problongrun1,problongrun2,problongrun3,problongrun4];
problongrunnew=problongrunnew*prob;
distprob=max(max(abs(problongrunnew-problongrun)));
problongrun=problongrunnew;

end

%%%We update the interest rate and wage rate

%average capital
capital = asset(:)'*problongrun;
amean=sum(capital);

%average working time
for j = 1:nl
workingchoice(:,j) = 1-leisurechoice(:,j);
end

for k = 1:nl
working(k) = workingchoice(:,k)'*problongrun(:,k);
end

Workingtime = sum(working)

%average labor supply
for j = 1:nl
```

```
worksupply(:,j) = (1-leisurechoice(:,j))*ll(j);
end

for k = 1:nl
    L(k) = worksupply(:,k)'*problongrun(:,k);
end

LL = sum(L)

%we update the interest rate

rnew=(1-phi)*amean^(-phi)*LL^(phi);

distgeneral=abs(rnew-r)

rold = r;

[r,rnew]

%we update the wage rate

wnew = (phi/(1-phi))*rnew*amean*LL^(-1);

wold = w;

[w, wnew]

if (rnew - r) >= 0.07
    r = r + 0.01;
elseif rnew - r >= 0.01 && (rnew - r) < 0.07
    r = r + 0.005;
elseif rnew - r >= 0 && (rnew - r) < 0.01
    r = (r + rnew)/2;
elseif (rnew -r) <= -0.07
    r = r - 0.01;
elseif rnew - r <= -0.01 && (rnew -r) > -0.07
    r = r -0.005;
elseif rnew - r < 0 && (rnew -r) > -0.01
    r = (r + rnew)/2;
end
```

```
w = (phi/(1-phi))*r*amean*LL^(-1);

end

%We balance the government budget constraint

diff = g - (tau*r*amean + tau*w*LL)
balance = abs(g - (tau*r*amean + tau*w*LL))

[g, tau]

gold = g;

g = (g + tau*r*amean + tau*w*LL)/2

end

%%%Aggregate statistics

for k = 1:nl
    partialwelfare(k) = V(:,k)'*problongrun(:,k);
end

averagewelfare = sum(partialwelfare);

CapitalOutRatio = amean/(amean^(1-phi)*LL^(phi))

for k = 1:nl
    conspartial(k) = consumptionpath(:,k)'*problongrun(:,k);
end

ConsumptionSteady = sum(conspartial)
%We could also get the ConsumptionSteady from the agent budget constraint
%ConsumptionSteady = -delta*amean +(1-tau)*rold*amean +(1-tau)*LL*wold

ConsumptionOutRatio = ConsumptionSteady/(amean^(1-phi)*LL^(phi))

PublicExpOutRatio =  gold/(amean^(1-phi)*LL^(phi))
```

```
%%%Income distribution
LRdist=sum(problongrun');
indexpos=find(LRdist>0);
LRdist1=LRdist(indexpos);
asset1=asset(indexpos);

S(1)=0;
for i=2:length(asset1)+1;
S(i)=S(i-1)+asset1(i-1)*LRdist1(i-1);
end

partial=0;
for i=2:length(asset1)+1;
partial=partial+LRdist1(i-1)*(S(i-1)+S(i))/S(end);
end

GINI=1-partial %GINI coefficient

fractionzero=sum(LRdist1(asset1<=0)) %fraction of individuals with no
                                     %assets
richestonepercent= min(find(cumsum(LRdist1)>=.99)); %assets to the
                                                    %richest one percent
wealthrich=(LRdist1(richestonepercent:end)*...
            asset1(richestonepercent:end)')/amean


%%%%DECENTRALIZED MODEL WITH BENEFIT FOR AGENTS WHO STAY UNEMPLOYED%%%%

%With respect to the decentralized model previously described we set,
%at the beginning of the code, the value for s, e.g. 0.02

s = 0.02

%Then, the first part of the recursive problem becomes

    dist=1;

    tic;
    while dist>=.001;

```

```
        for d = 1: nleisure

            for i =1:na

                for j =1:nl

                if d == nleisure %the subsidy is assigned only when the
                                 %agents do not work and hence their
                                 %leisure time is 1.
                    I = 1;
                else
                    I = 0;
                end

                %Note that the consumption vector depends on I*s
                cc = I*s -asset +(1-delta)*asset(i) +...
                    (1-tau)*r*asset(i) +(1-tau)*w*ll(j)*(1-leisure(d));

                ...

%Then, also the adjustment process for g changes. Indeed, in the
%government budget constraint it is necessary to consider that the
%government assigns a benefit to agents that do not work. In detail,
%after the adjustment process of r and w, we set:

subsidirequirements = leisurechoice == 1; %number of cases in which the
                                          %subsidy is required
payment = s*subsidirequirements;

for k = 1:nl %cost to assign the subsidy to each class of agents
partialsubsidy(k) = payment(:,k)'*problongrun(:,k);
end

subsidy = sum(partialsubsidy);  %total cost of subsidy for society


%We balance the government budget constraint

diff = subsidy + g - (tau*r*amean + tau*w*LL)
balance = abs(subsidy +g - (tau*r*amean + tau*w*LL))
```

```
[g, tau]

gold = g;

g = g -diff/2

end

%%%Aggregate statistics

%As concern to the aggregate statistics, compared to the previous case we
%add:

subsidyOutRatio = subsidy/(amean^(1-phi)*LL^(phi))

for k = 1:nl
    conspartial(k) = consumptionpath(:,k)'*problongrun(:,k);
end

ConsumptionSteady = sum(conspartial)
%We could also get the ConsumptionSteady from the agent budget constraint
%ConsumptionSteady = subsidy -delta*amean +(1-tau)*rold*amean +...
%                    (1-tau)*LL*wold


%%%%DECENTRALIZED SOLUTION WITH BASIC INCOME%%%%


%With respect to the decentralized model previously described, we set at
%the beginning of the code the value for omega, e.g. 0.02

omega = 0.02

%Then, the first part of the recursive problem becomes

    dist=1;

    tic;
    while dist>=.001;
```

```
        for d = 1: nleisure

            for i =1:na

                for j =1:nl

                %Note that the consumption vector depends also on omega
                cc = omega -asset +(1-delta)*asset(i) +...
                    (1-tau)*r*asset(i) +(1-tau)*w*ll(j)*(1-leisure(d));

                ...

%Then, also the adjustment process for g changes. Indeed, in the
%government budget constraint now it is necessary to take into account
%that the government assigns omega to the agents.

...

diff = omega + g - (tau*r*amean + tau*w*LL)
balance = abs(omega +g - (tau*r*amean + tau*w*LL))

[g, tau]

gold = g;

g = g -diff/2

end


%%%Aggregate statistics

%As concern to the aggregate statistics, compared to the previous case we
%add:

omegaoutratio = omega/(amean^(1-phi)*LL^(phi))

for k = 1:nl
    conspartial(k) = consumptionpath(:,k)'*problongrun(:,k);
```

```
end

ConsumptionSteady = sum(conspartial)
%We could also get the ConsumptionSteady from the agent budget constraint
%ConsumptionSteady = omega -delta*amean +(1-tau)*rold*amean +...
%                            (1-tau)*LL*wold
```

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