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A TAXONOMY OF MINI FLASH CRASHES

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1. INTRODUCTION

The recent years have seen the emergence of a new type of trading called high-frequency trading (HFT), defined as “professional traders acting in a proprietary capacity that engage in strategies that generate a large number of trades on a daily basis” by the U.S. Securities and Exchange Commission (SEC (A), 2010). With its emergence, a number of new issues have been raised and a lot of research started to investigate them, the main topics being the impacts of HFT on market quality measures such as liquidity, price discovery or volatility.

Another topic around HFT that was thoroughly studied is its importance in the Flash Crash of the 6th May 2010. On that day, the prices of many U.S. equity products declined and then bounced back in a matter of a few minutes. During the afternoon, major equity indices in the futures and securities markets dropped by 5-6% in a few minutes before recovering to their previous levels. At the same time, nearly all the individual equity securities and exchange traded funds (ETFs) traded experienced decreases in price ranging from 5% to 15% before recovering almost or all their losses in a short period of time. The day finally ended with the futures and securities markets only suffering a drop of 3% in comparison to the previous day's close. This Flash Crash raised a lot of questions, especially about how such an event could happen. As a result, many studies focused on the subject and accused HFT to be the prime cause of the disaster. Even if Kirilenko et al. (2011) or other researchers proved that HFT did not trigger the Flash Crash but exacerbated it, the question of the impacts caused by the high frequency at which some market participants are trading remains highly discussed.

Nevertheless, in November 2010, Nanex Llc., a data analytics company, published a paper that had the effect of a bomb in the financial industry, revealing that so called Mini Flash Crashes were occurring by the thousands each year and on all kind of assets. They can be defined as sudden and extreme price changes that happen over a particularly short period. These are really interesting because, like the Flash Crash, the triggering and effects of those events are not obvious. Unfortunately, literature on the subject is still sparse because it requires super computers and access to large volumes of data in order fully analyze the phenomenon. However, some authors, like Golub et al. (2011) or Johnson et al. (2012), already proved that the exercise was possible to realize and we will try in this paper to add our little piece to the puzzle of Mini Flash Crashes.

To do so, we will try to answer the research question: “Is it possible to determine a taxonomy of Mini Flash Crashes?” and if yes, which categories should be included. In order to answer

this question, we will deliver two analyses, one qualitative analysis first and then a quantitative one. The qualitative part will give background information as well as a broad literature review around the subjects of the May, 6 Flash Crash and Mini Flash Crashes. Concerning the quantitative part, we will build a database of Mini Flash Crashes from May 2011 to September 2014 that we will analyze using different types of measures and statistics. It will give us insights about which categories should be added to our taxonomy if one is possible.

In the end, we will answer our research question and propose a taxonomy for Mini Flash Crashes if the answer is positive. We think it might help future researchers in determining the key factors to which they must turn for their analysis. Furthermore, it might give an insight to regulators on how they could stop these events, by identifying the factors having an influence on the price change or the crash time.

The paper is organized as followed: Section 2 presents the background information that is an important foundation for the rest of the paper. Section 3 proposes a broad literature review. Section 4 introduces the data sample and the methodology used for the quantitative part. Section 5 describes the results concerning the measures and statistics. Section 6 answers the research question. Section 7 concludes the thesis and lastly, Section 8 and 9 present respectively the references and the appendixes.

2. BACKGROUND INFORMATION

In this section, we will provide all the information required to fully understand the content of this thesis. We will start by developing the evolution of the financial markets during the past decade. Then, we will define high-frequency trading and all the related strategies. We will finish with a clear explanation of the event of the 6th May 2010 Flash crash.

2.1 EVOLUTION OF MARKETS

2.1.1 MARKETS DEVELOPMENT

The history of financial markets has always been linked to the speed of communication and of data processing. It all started after the creation of the first stock exchange in London in 1801. At that time, traders were using horse-riding messengers and the one who possessed the best animal could get the information of, for example, victory in war before the others and could profit from this speed advantage to increase his profit. Moreover, since long-distance communications technologies were not reliable, major trading institutions located their offices as close as possible to the exchanges, creating clusters of financial institutions. After that,

communications technologies never ceased to evolve, horses being substituted by pigeons, then the telegraph and finally the telephone, which remained the dominant way of communicating for the financial institutions during the first 70 years of the twentieth century alongside printed paper (Cliff et al., 2011).

At that time, we were right in the midst of floor-based trading. People had to meet physically to trade and so, buyers and sellers were gathering at the exchanges to identify their counterparts. However, individual investors could not gain access to the stock markets. Therefore, they had to ask licensed brokers to trade for them. (McGowan, 2010) It is only in the mid 70's that the computerization of the order flow began. It happened when the New York Stock Exchange (NYSE) introduced the "Designated Order Turnaround" (DOT) system, which was later upgraded to "Super-DOT" in 1984. It enabled an investor to submit buy and sell orders electronically to specialists. As a consequence, as exchanges started the implementation of computerized communications systems, traders were given the possibility to choose to be connected to a trading platform instead of meeting physically on a trading floor (McGowan 2010).

Furthermore, in 1971, NASDAQ became the world's first electronic market. It allowed for dealers to compete in market making activity rather than employing a specialist auction system.

This era finally ended in the 1980's when fully electronic financial markets and a specific trading strategy called 'program trading' appeared. It consists of buying a particular stock index futures and at the same time, selling the corresponding equity. Traders in the 80's used it a lot on the trading between S&P500 equity and future markets. The reason is that if the prices of stocks and corresponding futures moved too much, they could make a profit by pre-programming a computer to enter this double order in the NYSE's order routing system (McGowan, 2010) Nowadays, people still think that this practice, called index arbitrage, was the main cause of the Black Monday Crash that took place in October 1987 (Cliff et al., 2011).

After this event, many people started raising doubts about the computerization of the financial markets. The enthusiasm of the beginning remained dampened during some years. Nevertheless, the price of computers steadily declined along the years, as predicted by Moore's law¹ and so, people were able to get a computer 100 times more powerful without paying more. As computers had now become part of the investment funds management, opportunities for

¹ Moore's law stipulates that, across the history of computing hardware, the transistors number in an integrated circuit has multiplied by two more or less every two years.

more sophisticated programs helping investment decisions appeared and in the late 90's, we saw the birth of a new practice 'hedging' and of the related 'hedge funds'. (Cliff, 2011)

Another important development that occurred in the late 90's is the appearance of automated trading systems. As technology steadily improved since the events of 1987, people deployed more intelligent systems based on rigorous mathematical approaches with the purpose of 'executing' trades. It means people were still involved in the process of decisions but that the execution in itself was deployed by the automated execution system (AES) alone. It helped the human traders to deal with complicated trades by choosing the optimal timing and portions of the order to be submitted. As time advanced, major financial institutions began experimenting new algorithms, new methods as AES proved to be efficient. Algorithmic trading was born (Cliff, 2011).

Another kind of programs was developed at the same time. While AES systems purpose was to reduce market impact, some teams of traders upgraded methods of statistical arbitrage for identifying trading opportunities (Cliff et al., 2011). It consisted of the analysis and cross-comparison of thousands of assets data (volume and price) in order to detect trading opportunities. This new evolution was made possible by the recent development of computer based trading infrastructures and the powerful computers used by the traders. Concerning the development of the infrastructure, it brought two important new features. The first one to be introduced in the trading venues was the *Straight-Through Processing* (STP). It allowed a single electronic flow of transaction-processing steps to encompass the entire trading process. Its advantage was that no humans were involved anymore, reducing the latency of order execution. Secondly, *Direct Market Access* (DMA) appeared. It permitted traders and investors to interact with exchanges directly without having to contact brokers or investments banks. Finally, we can conclude that, at the end of the 90's, it had become quite common to execute the whole trading process electronically and within a small number of seconds.

2.1.2 MARKET FRAGMENTATION

At the same time, the whole financial industry became fragmented. New exchanges appeared as well as new entities involved in the securities trading which lead to a financial market fragmentation. People were now able to trade on different venues and this development created incentives for traders to profit from price inconsistencies and arbitrage opportunities. In addition, as the market was divided between multiple exchanges, competition got fiercer and

so, tightened bid-ask spreads². Moreover, it facilitated the creation of new financial entities involved in trading (Cliff, 2011).

It all started in the late 90's with the birth of new electronic trading venues called electronic communication networks (ECNs). As Liebenberg (2002) defines, it is "a type of computer systems that facilitates trading of financial products, such as stocks and currencies, outside of the traditional stock exchanges". The real advantage this system offers is that an individual can enter orders electronically in the program and it will automatically match and execute contra-side orders. If it cannot, then it will check on the regular stock exchanges, such as NASDAQ or NYSE, only if they propose the best prices. (McGowan 2010) It provided many advantages compared to past trading venues such as a reductions of costs and trading errors, improvement of operational efficiencies, a better risk management amongst others. Meanwhile, financial institutions which were usually trading on NASDAQ began creating ECNs. Therefore, 1997 saw the use of ECNs catapulted to new heights which announced the arrival of new dominant types of trading: algorithmic and high-frequency trading.

The next milestone took place in 1998, when the U.S. Securities and Exchange commission (SEC) issued a brand new regulation, the Regulation Alternative Trading System (Reg. ATS), to officially recognize ECNs as new trading venues. This allowed them to directly compete with the more traditional financial exchanges (Markham et al., 2008). On the other hand, the SEC introduced in 2005 the Regulation National Market System. (Reg. NMS) Its first purpose was to strengthen and modernize the national equity markets but it also tightened the scope of actions of all ECNs. Indeed, the regulation introduced three new particularly constraining rules. For example, the Trade-Through rule decreed "that market orders be posted electronically and immediately executed at the best price nationally" (McGowan, 2010). Nevertheless, ECNs continued competing in the financial world. In fact, BATS for example, realized 10% of market share in the US equities the year (2007) it became a licensed exchange (Gyurko, 2011).

Nowadays, we find various type of ECNs. They differ in many ways: the targeted clientele, the order routing strategies, speed, quality and certainty of execution, and the type of information they provide for investors (Gyurko, 2011). Some have been successful. Let's take the example of the Chi-X. It was launched in March 2007 and was the first multilateral trading facility in Europe. Considered as one of the fastest exchanges in the European financial

² The bid-ask spreads corresponds to the amount between the ask and the bid. The ask always exceeding the bid.

industry, it has been competing profitably with the more traditional trading venues since its inception and in February 2011, BATS Global Markets decided to buy it for \$ 300 million. The BATS Chi-X Europe ended 2011 with a European market share of 25%.

The end of the 20th century also saw the introduction of another type of venue, referred to as dark pools. Large financial institutions willing to sell or buy large block of orders, for example on ECNs, were most of the time annoyed because it had a an enormous impact on the asset price. Furthermore, they could suffer from predatory trading practices if the intention of a large trade execution had leaked. Fortunately, dark pools provided the solution. First, they protect the anonymity of traders and prevent information leakage and secondly, they offer direct trading opportunities without price impact (Gyurko, 2011). These being possible because of the main two features of dark pools. Liquidity is not quoted, execution is thus uncertain and unavailable and finally, prices are not determined by the dark pools but instead, follow the reference prices determined by primary exchanges (Gyurko, 2011). In other words, all of this contributes to the success of dark pools which financial institutions used regularly when they wanted to buy or sell large blocks of financial instruments, reducing the market impact of the trades.

All these evolutions, from the use of the horse to the ECNs and dark pools, created the current financial industry. Multiple factors such as cheap computer power, rapid development of trading programs, direct access to trading venues, favorable regulations and fragmentation of the financial industry led to new trading opportunities. Markets participants now compete on short-term asset values where the key competitive advantages rely on data analysis and speed of execution. Algorithms do not only execute like before but are now capable of taking their own decisions based on observed order flows. The race between the investors has begun and new strategies have arisen as we reach the microseconds trades used by the so-called high-frequency trader (Cliff, 2011).

2.2 HIGH-FREQUENCY TRADING

At the beginning of the 2000s, HFT only accounted for 10% of all equity trades but it rapidly increased. Indeed, in 2012, it represented over 50% of all the U.S. equity-exchange trading volume and approximately 40 to 60 percent of trading activity in the U.S. financial markets for stocks, options and currencies (Goldstein et al., 2014). Its growth was also significant in Europe and Asia. Stock trading volume accounted to HFT reached, in 2012, 45 % in Europe, 40% in Japan and 12% in the rest of Asia (Proper, 2012). These numbers are quite impressive when we know that high-frequency trading firms represents only 2% of the 20,000 companies involved

in the U.S. financial markets. However, the concept of HFT is still unclear on many points. This is because it did not exist until recently and that data have not been readily available or required too much work to gather (Jones, 2013). Therefore, it is difficult to discuss the impact of it on the financial markets or even define the concept of HFT. Yet, we will try defining HFT, explain its characteristics and develop its strategies in this section.

2.2.1 DEFINITIONS

As mentioned before, it has been difficult to establish a clear definition for HFT and for all the terms in rapid trading and in computer controlled trading (Brogaard, 2010). Even the SEC admits it and declares when talking about high-frequency trading that it “does not have a settled definition and may encompass a variety of strategies in addition to passive market making” (SEC (A), 2010). As a result, many authors, to remain consistent, refer to each other and try using the same definition. We will do the same in this paper and use the definition proposed by the SEC and adopted, for example, in the works of Brogaard (2010) or Jones (2013). According to the SEC: HFT refers to “professional traders acting in a proprietary capacity that engage in strategies that generate a large number of trades on a daily basis” (SEC (A), 2010).

Two other terms are always mentioned when talking about HFT: “pinging” and “algorithmic trading”. According to the SEC’s definition, pinging is “an immediate-or-cancel order that can be used to search for and access all types of undisplayed liquidity, including dark pools and undisplayed order types at exchanges and ECNs. The trading center that receives an immediate-or-cancel order will execute the order immediately if it has available liquidity at or better than the limit price of the order and otherwise will immediately respond to the order with a cancelation” and clarifies “There is an important distinction between using tools such as pinging orders as part of a normal search for liquidity with which to trade and using such tools to detect and trade in front of large trading interest as part of an ‘order anticipation’ trading strategy”

Concerning the second term, algorithmic trading, Hendershott (2011) defined it as “the use of computer algorithms to automatically make trading decisions, submit orders, and manage those orders after submission”. HFT and AT look pretty similar since they both use automated decision making technology but they differ in the sense that AT investment horizon can go from seconds to years when HFT targets very short horizon for their positions. In addition, HFT tries to close the trading day in a neutral position. In the end, we can declare that HFT must be a type of algorithmic trading but that AT can differ from HFT.

2.2.2 HFT CHARACTERISTICS

High-frequency trading's simplest form consists of the collection of tiny gains (even some cents) on short-term fluctuations of asset prices. The firms using HFT scan the financial markets for distortions and trade as fast as possible to profit from the brief inefficiencies before they disappear. This is a part of the first main characteristic of HFT. It depends on multiple market components to make profits and is thus characterized by a high turnover capital. The remaining characteristics include a dependence on ultra-low latency, a limited shelf-life of algorithms and participation on various trading venues and asset classes (Iati, 2009).

Firstly, let's take a look at latency. It is seen as the speed factor in HFT and is, of course, an important factor for all the strategies as it describes the delay experienced in a system. A low or ultra-low latency means that the execution of trades can be done in no time. It is a matter of less than 1 microsecond when we are dealing with ultra-low latency. We can then say that it is vital for HFT traders to lower their latency as much as possible as they make profits if they can process information through their algorithms microseconds faster than their competitors. Therefore, all the hardware and programs of HFT firms must constantly be upgraded if they want to remain on top of the competition. (McGowan, 2010) Aside from this "technology arms race", HFT traders began searching for other ways to diminish their latency and one of them is co-location. As Iati (2009) mentioned "To realize any real benefit from implementing HFT strategies, a trading firm must have a real-time, colocated, high-frequency trading platform, one where data is collected, and orders are created and routed to execution venues in sub-millisecond times." Firms using HFT started renting or buying spaces in data centers in which the exchanges computer servers are located in order to get closer to 0 latency. Therefore, real estate prices skyrocketed around the exchanges. Yet, HFT firms continued paying which proves the importance and impact of co-location (Rogow, 2012). Nevertheless, profits made by HFT traders are usually only measured in fraction of dollars or even pennies. Consequently, thousands of this kind of trades must be executed before any investors understand and realize the profits they actually made.

The second main characteristics of high-frequency trading lies in its shelf-life. Indeed, if they want to remain at the top of the pack, HFT firms must upgrade their algorithms continuously or else their competitive advantage will diminish over time. Fortunately, thanks to technology innovation, constantly upgrading has become cheap this past decade which aided the success of HFT (McGowan, 2010).

There are two reasons why HFT bears a shelf-life if we look at the micro-level. To begin with, high-frequency trading is highly related to all the interactions in the financial markets and the correlations between the securities, so the traders must always adjust their algorithms to match new situations. It is especially the case in highly volatile and frequently changing markets where the codes of algorithms have to be changed sometimes within minutes and this volatility impacts the HFT environment. In fact, Popper (2012) reports that HFT volume in the U.S. from 2009 to 2012 dropped from 6 billion shares to 3 billion shares and that he profits declined of 74%. One of the reason mentioned is that during those years, the U.S stock indices had risen steadily, interest rates were low and some commodities were less volatile than previously which is an adverse situation for HFT which tend to capture profits from inefficiencies in the financial markets (Goldstein et al., 2014).

Then, the second reason firms have had to alter their trading strategies is because of what we call “reverse engineering” by rival firms. This phenomenon takes place because of the environment surrounding HFT. Since all HFT firms are constantly upgrading and changing their algorithms to surpass competition, the susceptibility of their strategies being reverse engineered by their rivals is extremely high and with it, the chance their most profitable ideas will turn into their riskiest. This technology arms-race had led to an uprising of all the prices for the best algorithms, hardware and employees and is the second reason mentioned by Goldstein et al. (2014) to explain the decline in profit and volume of HFT firms in the U.S between 2009 and 2012, declaring that most of them had to cut jobs or go bankrupt because they could not afford such a level of competition.

The last main characteristic of high-frequency trading is participation in multiple asset classes and exchanges. HFT firms trades various types of asset such as stocks, options, currencies, commodities, etc. Furthermore, they trade the same asset on numerous exchanges simultaneously (McGowan, 2010). And lastly, we can also include two characteristics that corresponds to HFT: the submission of multiple orders, cancelled shortly after it and ending the trading day as close to a neutral position as possible (Jones, 2013).

2.2.3 HFT STRATEGIES

In this section, we will present the principal strategies applied by high-frequency traders and we count three of them: (1) acting as a formal or informal market-maker, (2) high-frequency relative-value trading, and (3) directional trading on order flow, new releases, or other high-frequency signals (Jones, 2013).

Market-making

This strategy, called “passive market making” in SEC (A) (2010), involves market-makers placing limit orders on both sides of the electronic order book in order to derive profits from the bid-ask spreads by buying at the bid price and selling at the ask price. In fact, they are providing liquidity to participants that want to trade directly. In this strategy, market makers might lose money by trading with an informed counterparty. Therefore, they must make sure their limit orders integrate as much information as possible to avoid losing money to informed counterparties. As a consequence, HFT market-makers must constantly adjust their quotes depending of other order submissions and cancellations. Quotes might also be updated if the price of a related exchange traded fund (ETF)³ or futures contract moves. That is the reason why HFT markets-makers submit and cancel numerous orders per transaction (Jones, 2013).

We also find another type of market-making strategy known as “rebate trading”. In the current financial industry, multiple markets and exchanges offer rebates or fees if you make or take liquidity (Goldstein et al., 2014). For example, on the U.S. equity markets, exchanges and ECNs, liquidity rebates of up to 30 cents per share for each share sold or purchased from each posted bid or offer can be received by market-makers. Sometimes, they can even trade for free because they are considered to be adding liquidity. As a result, ECNs and exchanges cover their exchange fees and commission costs (SEC (A), 2010). The advantage of HFT traders in this strategy relies on the fact that many market centers offer their rebate based on trade volume. In this case, many buy-side investors cannot catch the maximum liquidity rebates from ECNs and exchanges compared to HFT market-makers whose trading volume is enormous.

Arbitrage and relative value trading

We will start by explaining arbitrage trading with a short and clear example. In Chicago, on the Chicago Mercantile Exchange, S&P500 futures are traded and the largest exchange-traded fund (ETF), which the ticker symbol is SPY, tracks the S&P 500 index. It is possible to trade SYP on every financial market in the U.S. but also in some foreign exchanges. As the two instruments are almost similar, their prices should move in the same direction and magnitude and HFT trades use this correlation to gather profits. If the futures price moves up due to an arrival of buy orders, but the ETF price does not goes up simultaneously, HFT traders would in

³ ETFs are a particular type of marketable security following an index, a commodity, a bond, etc. They are traded like common stocks on a stock exchange.

very rapidly buy SPY and sell S&P 500 futures contracts, to ensure a small profit is made on the difference in prices between the two instruments (Jones, 2013).

In this situation, the distances between the markets and speed of light remain constraining factors and again shows the importance of speed in this strategy. If a HFT arbitrageur is systematically quicker than his competitors, he will have the opportunity to constantly buy the mispriced shares of SPY and sell the mispriced S&P500 futures which we will bring the prices of the two instruments to their normal level. Therefore, in the situation where a trader possesses the fastest technology, nothing will be left for slower traders that would like to profit from any price discrepancies.

This technique not only works with futures but with many different classes of asset. It is also used “to exploit covered interest rate disparities in the foreign exchange market, price discrepancies between highly correlated stocks and also between derivatives and their underlying asset.” (Goldstein et al., 2014). Starting from the same example as before for stocks, if the price of the S&P500 futures increases but that the prices of the related stock do not move, HFT traders will grasp this arbitrage opportunity by buying components stocks in right proportions until the prices are brought back in line.

The other form of the strategy is known as relative value trading. It takes place when individual securities are traded in relation to another. In the case in which the prices of two closely related securities would temporarily diverge, HFT programs would grasp the opportunity to generate a profit. For example, if the prices of oil futures go up quickly, it might imply a rapid sale of airline stocks. In other words, a price move in a distinct listed equity option could imply a profitable trading opportunity in the underlying stock (Jones, 2013).

Directional Trading

Some specific events such as company announcements of earnings or other economic figures lead to price movements among affected securities. These opportunities are captured by HFT firms to make short-term profits. Again, speed is a crucial factor in this strategy. For example, Scholtus et al. (2014) showed the extreme importance of high-speed responses in HFT strategies based on U.S. macroeconomics news releases.

Other HFT traders even analyze news text using algorithms to determine if they bear trading opportunities (Goldstein et al., 2014). They search for words like “raise” or “increase” often linked with “earnings forecasts”, then they determine the subject company and finally, decide

whether or not to place orders. Nowadays, News providers even sell summary measures concerning the news to HFT firms which are ready to pay to avoid losing time performing the analysis (Jones, 2013). Moreover, some pay to get specific reports on measures such as consumer sentiment index in order to maybe catch the future market trends. However, all these practices have led to increased attention from the regulators regarding issues like early releases of reports and news.

Another type of directional trading is based on order flow signals. For example, one submitter might execute a large buy order at the prevailing ask price which could suggest it owns important positive information. Therefore, an HFT strategy might start buying the same shares to generate a gain.

It is a huge concern for large institutional traders. Indeed, if one starts gradually buying shares of PepsiCo, for example, an HFT firm might identify the sequence of large buy orders and replicates its. In reaction, it might purchase shares of PepsiCo which will drive the price up and increase the amount the large institutional trader has to pay for the desired shares. The HFT firm can even make a profit if it sells its purchased shares to the institutional trader. Because of this kind of anticipation strategies, institutional traders put in place processes to disguise their general trading intentions. Multiple options exist. Firstly, they can break up their order into numerous pieces. The goal is to dress up their order into uninformed ones made by usual retail investors. Secondly, they can use dark pools. Indeed, they allow institutional traders to trade anonymously, covering their trading intentions. Finally, they can work with algorithms. This leads to a game of hide-and-seek between computer programs (Jones, 2013).

This last issue is not new. Already within manual markets, large institutional traders were worried about information leakages about their trading intentions, undertaking great efforts to cover their large order sequences. For example, they were stationing a broker at the relevant post on the NYSE floor. However, all this game has changed over the years with the appearance of automated markets and the evolution of the trading strategies. Nowadays, many institutional traders complain about the rise of HFT strategies, declaring their costs increased because of the negative effect of the game they have to play with the HFT traders. In fact, overall institutional trading costs have decreased even if HFT became more prominent, suggesting that this effect is rather minor as shown by Jones (2013).

2.3 THE 6TH MAY 2010 FLASH CRASH

On the 6th May 2010, the prices of many U.S. equity products declined and then rebounded in the matter of a few minutes. During the afternoon, major equity indices in the futures and securities markets, which were already down of 4% relative to the previous day close, dropped again a further 5-6% in a couple of minutes before recovering to their previous levels.

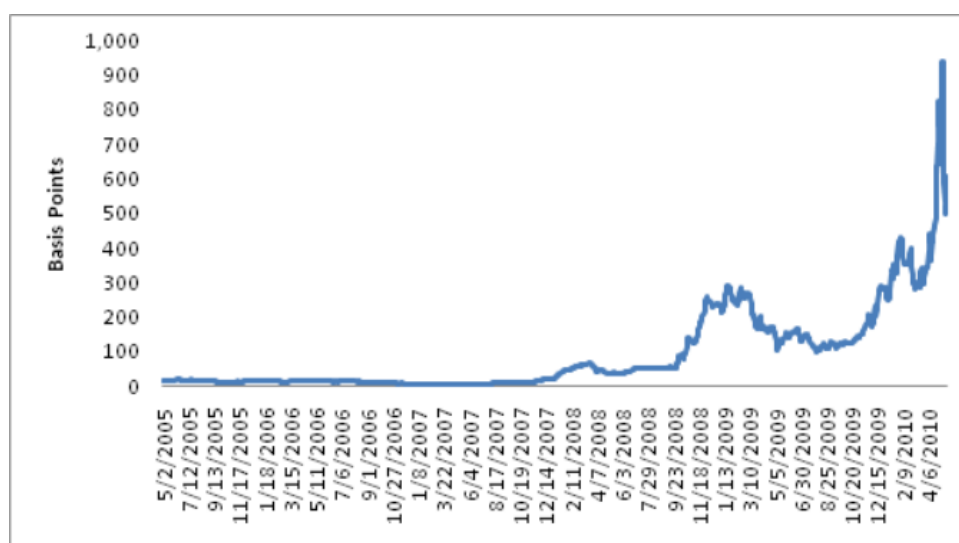
On the same day, almost all the individual equity securities and exchange traded funds (ETFs) traded experienced declines in price ranging from 5% to 15% before rebounding and recovering almost or all their losses in a short period of time. In fact, many other equities traded at levels far distant from the usual ones in both ways. For example, more than 20,000 trades concerning over 300 securities were executed at prices 60% away from their values a few minutes before. In addition, these trades were executed at extreme prices, some for pennies and others for 100,000\$ or even more, before bouncing back to their original levels.

Finally, the day reached its end with the futures and securities markets suffering a loss of only 3% from their prior day close (CFTC & SEC (A), 2010).

2.3.1 STARTING EVENTS

The day began in a particularly unstable way for the markets due to political and economic news from overseas about the European debt crisis. As a consequence, we saw a rise of the premiums for buying protection against default by the Greek government on their sovereign debt. For example, spreads for Credit Default Swap (CDS) protecting against the default of debt securities issued by Greece skyrocketed. In fact, it moved from a previous close at 844.2 basis points to 937.9 on May 6. The sudden increase was triggered by an announcement of the European Central Bank at 8:30 a.m. forbidding people to purchase Greek Government Bonds. (See Figure 1)

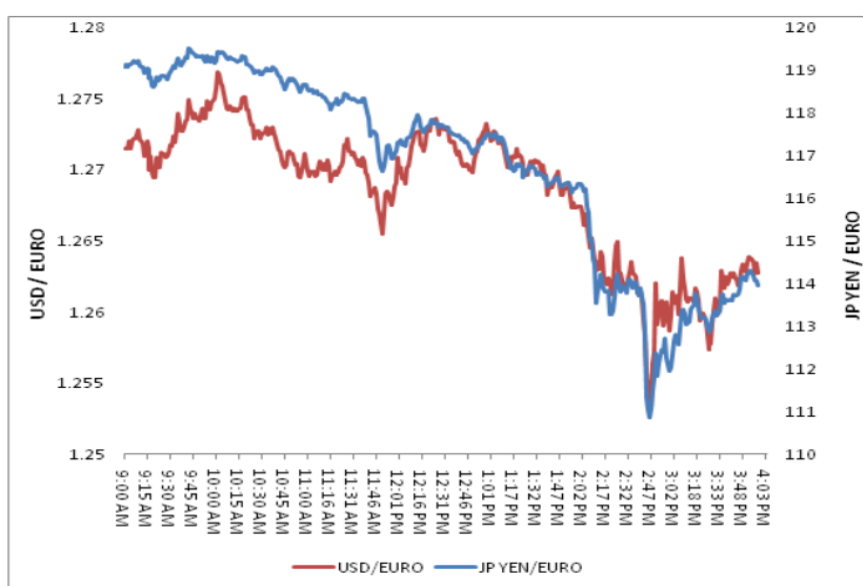
Figure 1: Credit Default Spreads on Greek Sovereign Debt



Source: CFTC & SEC (B) (2010). P.15

Furthermore, around 1p.m., the Euro started decreasing against the U.S. Dollars and Japanese Yen (See Figure 2).

Figure 2: Euro-Dollar and Euro-Yen Exchange Rates on May 6, 2010

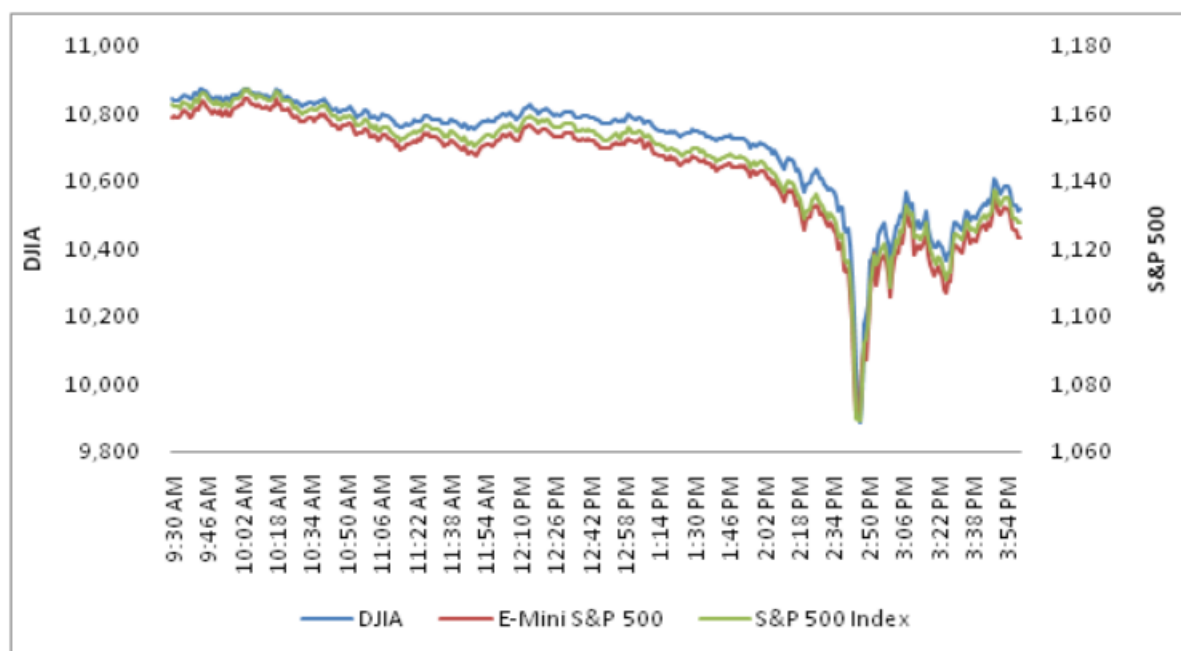


Source: CFTC & SEC (B) (2010). P.16

At the same moment, the price volatility of some individual securities were already affected by negative sentiments concerning the markets. Liquidity Replenishment Points (LRPs) or simply volatility pauses triggered and their number reached levels above the common ones on the NYSE (CFTC & SEC (A), 2010).

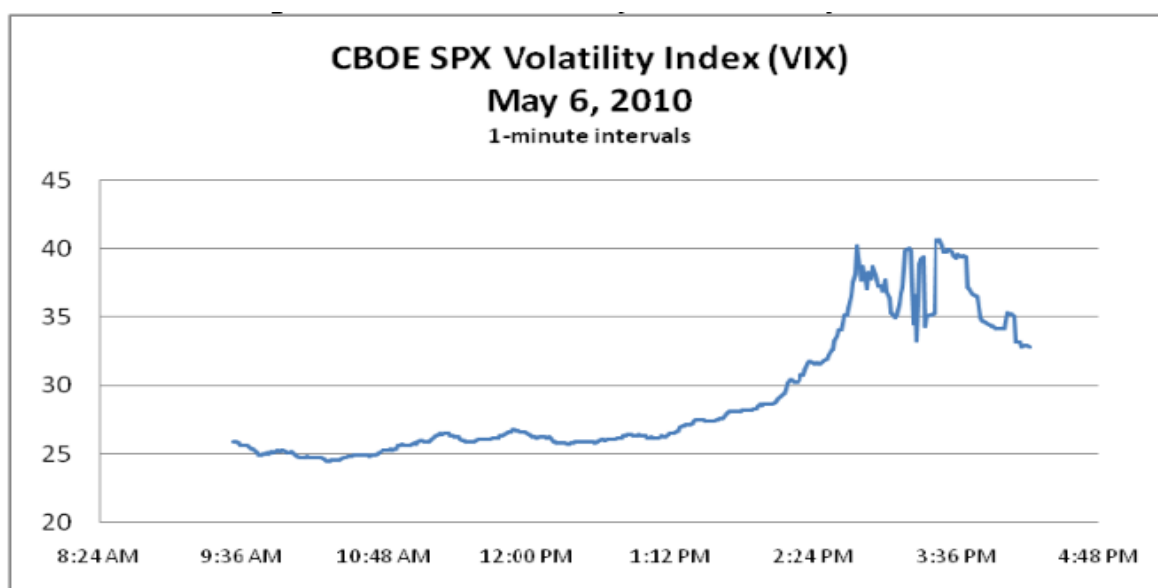
At 2:30 p.m., major equity indices and equity index futures had already declined. For example, from 9:30 a.m. and 2:00 p.m., the Dow Jones Industrial average (DJIA) experienced a decline of 161 points to 10,712 (-1.5%), the S&P500 Index declined of 33 points to 1,145 (-2.9%) and the E-mini S&P 500 Index June Futures suffered a decline of 15 points to 1,143 (-1.3%) as we can see in the following graph (See Figure 3).

Figure 3: Select Equity Indices and Equity Index Futures, May 6, 2010



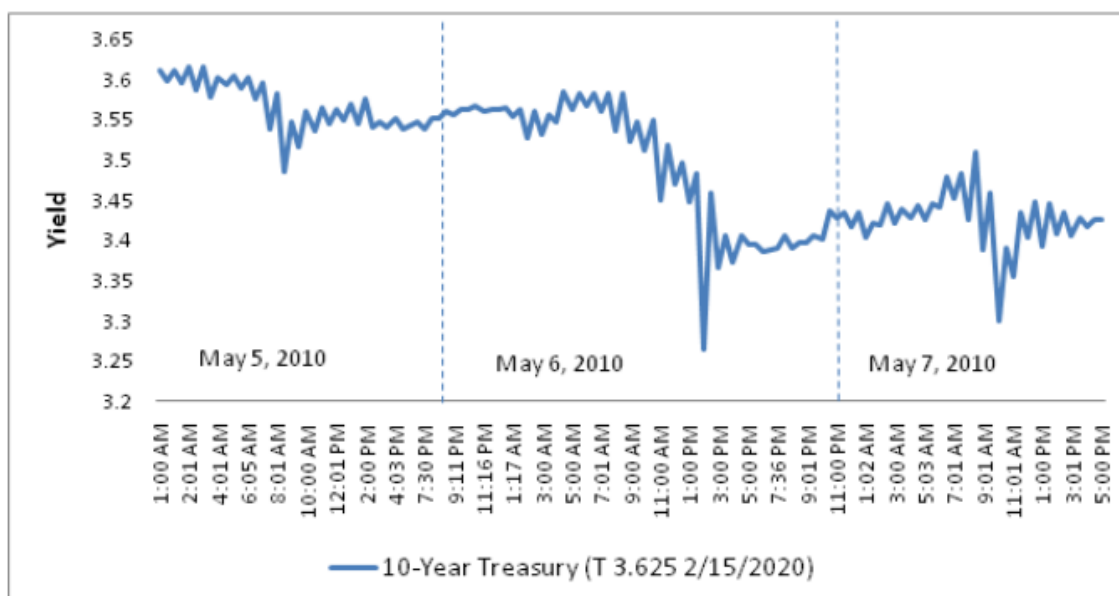
Source: CFTC & SEC (B) (2010). P.11

Moreover, general indicators of market uncertainty such as the Chicago Board Exchange SPX Volatility Index (VIX), known as the “fear index” because a high value corresponds to a greater range of returns, already went up by 22, 5% from the opening, reaching 31.1 points at 2:30 p.m. Then, the rise gained momentum and the VIX closed at 32.80, recording a 31.7% rise from the previous day’s close (See Figure 4).

Figure 4: CBOE SPX Volatility Index Intraday Levels

Source: CFTC & SEC (B) (2010). P.13

Finally, investors started a “flight-to-quality”⁴ resulting in a reduction of the ten-year U.S. Treasury yield. As shown on the following figure, it declined from 3.58% on May 5 to an intraday low of 3.26% before ending at 3.41% on May 7 (See Figure 5) (CFTC & SEC (B), 2010).

Figure 5: Ten-Year U.S. Treasury Note Yield

Source: CFTC & SEC (B) (2010). P.14

⁴ The term « flight-to-quality » describes a situation in which investments are made into asset classes that are viewed as less risky in times of financial uncertainty.

At 2:32 p.m., a major event happened. A mutual fund, Waddell & Reed Financial of Overland Park in Kansas, U.S., engaged in a program to sell 75,000 E-mini contracts with a combined value of 4.1 billion, employing a computer program sell algorithm as a hedge to an equity position in the unstable situation of the financial markets.

There are numerous ways how an individual can execute a trade. To begin with, he can engage an intermediary that would execute the trade and then manage the position. A second option is to manually enter the orders into the market. Lastly, he may employ a computer execution algorithm to fulfill his needs. In the end, the choice between the three options depends on how much human judgement in the execution of the trade the customer wants.

In the case of the 6th May Flash Crash, the mutual fund used a sell Algorithm. The program was set to feed orders into the June 2010 E-Mini market with a target execution rate of 9% of the trading volume computed over the last minute, without taking into account price or time.

As result of the execution of this program, the largest net change in daily position since the 1st January 2010 for a trader in the E-Mini market occurred. A case of equal or larger single-day sell program in term of size had only happened twice during the last 12 months before the crash including one by the same fundamental trader⁵. The execution of the sell program used by the large fundamental trader was composed of, on one hand, trades entered manually over the course of a day and, on the other hand, multiple automated execution algorithms depending of price, time and volume. For information, 5 hours were needed by the mutual fund to execute the initial 75,000 E-mini contracts of the large sell program this first time.

However, the difficult situation of the financial markets and the choice of the fundamental seller changed the time required by the sell algorithm to execute the program on the 6th May. Indeed, markets were affected by a high volatility and thinning liquidity, and the sell algorithm was programmed to only target volume and not price or time which lead to an execution time of only 20 minutes.

Three types of buyer absorbed all this sell pressure: HFT firms and intermediaries in the future markets, fundamental buyers in the same markets and cross-market arbitrageur's in the equities markets by purchasing the E-mini contracts and selling products like SPY or individual equities in the S&P500.

⁵ It corresponds to markets participant that trade to accumulate or diminish a net long or short position. It allows fundamental buyer or seller to gain long-term exposure to a market or to hedge already-existing exposures in related markets

The first batch of orders submitted by the Algorithms were bought by the HFT firms and the intermediaries and therefore, they built a temporary net long position of 3,300 contracts in general. Temporary because HFT firms traded huge number of contracts without acquiring an aggregate inventory of over three to four thousand contracts in both direction in their common strategies. Indeed, between 2:40 and 2:44 p.m., 2,000 E-mini contracts were sold aggressively by HFT traders to diminish their temporary inventory of long positions. Simultaneously, 140,000 E-mini contracts were traded by HFT traders representing more than 33% of the total trading volume at that moment.

As a consequence, the Sell Algorithm used by the large fundamental trader increased the rate at which it was feeding the orders into the market, even if orders already sent to the market were not yet fully absorbed by fundamental buyers or cross-market arbitrageurs. In reality, particularly in times where volatility is high, a high trading volume cannot be considered as a relevant indicator for market liquidity (CFTC & SEC (A), 2010).

2.3.2 LIQUIDITY CRISIS IN THE E-MINI

Following those events, a liquidity crisis at the broad index level in the E-mini market as well as one with individual stocks took place simultaneously, worsening the situation. Firstly, the Sell Algorithm and HFT traders were now competing for liquidity and thus, their selling pressures pushed down the price of E-mini by more or less 3% between 2:41 and 2:44 p.m. At the same, the price of SPY dived by 3% because of cross-market arbitrageurs that bought E-mini contracts now selling equivalent amounts in the equities markets.

As a result of the decline of the E-mini's price and as multiple traders were unwilling or unable to submit orders, HFT traders began selling and buying from each other, generating a "hot-potato" effect (Kirilenko et al., 2011). 27,000 contracts were traded by HFT traders between 2:45:13 and 2:45:27 which represented 49% of the total trading volume. By that time, E-mini prices had already fallen by 5% and the ones of SPY by 6%.

At 2:45:28 p.m., only 1,050 E-mini contracts of buy-side resting orders were left accounting for less than 1% of the number observed at the start of the day. Simultaneously in SPY, buy-side resting orders decreased to 600,000 shares (equal to 1,200 E-mini contract) which accounts for more or less 25% of its depth at the start of the day.

In total, the Sell Algorithm sold 35,000 E-mini contracts of the 75,000 intended between 2:32 and 2:45 p.m. for a value of \$1.9 billion. At the same time, fundamental sellers sold 80,000

contracts net and fundamental buyers purchased 50,000 contracts net. In the end, the fundamental imbalance reached a value of 30,000 contracts. If we compare the same thirteen minutes during the three last days, the level of net selling of fundamental sellers is fifteen times bigger and the level of net buying of fundamental buyers, ten times bigger.

Finally, at 2:45:28 p.m., a circuit breaker called the Chicago Mercantile Exchange (CME) Stop Logic Functionality launched and stopped trading on the E-mini for five seconds to pause the rapid decline. This short period of time allowed the sell-pressures to disappear and thus, buy-side interest rose again. It helped to stabilize the prices and at 2:45:53, trading resumed and the E-mini as well as the SPY started recovering. The Sell-Algorithm program finally ended at 2:51 p.m. as the prices on the E-mini and SPY were increasing again.

2.3.3 LIQUIDITY CRISIS IN THE EQUITIES MARKETS

The second liquidity crisis took place in the equities markets around 2:45 p.m. Many interviews conducted by the CFTC & SEC ((A), 2010) with a large pool of market participants reported that automated trading systems used by liquidity providers stopped in reaction to the rapid decline of price in the E-mini and SPY during the first liquidity crisis. These pauses are triggered in order to permit traders and risks managers to fully understand the market situations before trading again. They launch when particular thresholds are crossed by price movements.

After these pauses, liquidity providers had to think whether or not to continue trading depending on their assessment of the risks they would be exposed to. Various factors were studied to assess risks by the market participants: whether noticed serious price moves could be an indicator of erroneous data; the impact of such moves on risk and position limits; impacts on intraday profit and loss ("P&L"); the potential for trades to be broken, leaving their firms inadvertently long or short on one side of the market; and the ability of their systems to handle the very high volume of trades and orders they were processing that day. Furthermore, the interviews report that participant were afraid of an enormous and chaotic event for which their strategies were not suited because of all the declines taking place in various securities (CFTC & SEC (A), 2010).

In reaction to their assessments, market participants applied several different strategies. Some diminished the liquidity on offer, others widened their quote spreads and most of them just left the markets fearing the current conditions. During the same time, some went back to manual trading but were constrained to restricting their focus on a bunch of individual securities

as they could not keep up with the steady increase in volume occurring while securities prices suffered a rapid decline.

In the end, HFT firms, which typically provide and take liquidity as part of their strategies, increased their trading volume and became net sellers with other usual liquidity providers in the decreasing markets. After markets started recovering, some firms decided to stop trading or diminished their activity, but many others continued which ultimately caused severe price dislocation for individual securities.

Furthermore, several “over-the-counter” (OTC) market makers, whose jobs is to execute the buy and sell orders received by retail customers, started routing almost all of them on the public markets instead of executing them internally as they generally do. Yet, they entered markets with a declining liquidity and a fierce competition to get access to it.

As prices in the E-mini and SPY were recovering from their declines after 2:45 p.m., one might think the situation would get better but in fact, sell orders for individual securities and ETFs (especially stop-orders triggered by the severe prices declines of certain securities) had trouble finding counterparts, leading to a further decrease in securities prices.

If we look at the twenty minutes between 2:40 and 3:00 p.m., it is interesting to note the number of shares traded and their values when traded. Indeed, more or less 2 billion shares traded with a total volume surpassing \$56 billion. 98% of them traded with a value ranging in the 10% away from their value at 2:40 p.m. On the other hand, some traded at irrational prices such as one penny or, on the opposite end of the spectrum, at \$100,000. The reason is that participant willing to buy or sell individual securities or ETFs could not find counterparts as liquidity vanished from the markets. All of this was the result of the so-called stub quotes. These are quotes generated by market makers at distant levels from the actual market to fulfill continuous two-sided quoting obligations even when a market maker has retired from active trading.

After some time, buy-side and sell-side interest came back and an orderly price discovery process started functioning as a reaction from market participants who verified their systems and data. Around 3:00 p.m. most securities had returned to trading prices reflecting their true value. Yet, 20,000 trades concerning over 300 securities and ETFs executed between 2:40 p.m. and 3:00 p.m. traded at prices over 60% away from their prices at the beginning of the twenty minutes. Following the markets closure, FINRA and the exchanges met and agreed together to

cancel all this kind of trades under the so-called “clearly erroneous” trade rules (CFTC & SEC (A), 2010).

2.4 CURRENT REGULATORY ENVIRONMENT

In this section, we will provide an overview of all the regulations implemented by the regulators in order to prevent another Flash Crash and to reduce the risks imposed in the financial markets by the use of HFT practices. To do so, we will analyze which regulations have been introduced starting from the events of the 6th May until now.

2.4.1 CIRCUIT BREAKERS

Single-Stock Circuit Breakers

Before the events of May 6th, market-wide circuit breakers already existed. However, they failed to trigger during the crash and thus, U.S. exchanges and the Financial Industry Regulatory Authority (FINRA) proposed a new type of regulation to respond to the unusually volatile trading that affected many individual securities. They decided to launch a pilot program of new single-stock circuit-breakers triggering trading pauses of five minutes if a stock price experiences a price moves of at least 10% in a five minute range. At the beginning, only the S&P500 index's stocks were affected by this rule but in June 2011, it was extended on a pilot basis to all the National Market System securities (Carroll, 2012).

Limit Up/Limit Down (LULD)

In May 2012, the SEC approved a new NMS regulation plan in order to address extreme market volatility by creating a market-wide limit up/limit down mechanism for NMS Stocks to replace the single-stock circuit breaker rules. The reason is that circuit-breakers had been triggered by erroneous trades. The aim of the plan is to avoid trades in individual NMS stocks to be executed outside of a certain price range (Babel, 2013).

The price range is set at a percentage level above and below the average price of the individual stock in the last five minutes. Different types of range can be applied depending on the price of the stock. These are usually described in percentage with four different levels: 5%, 10%, 20% or 75%. Furthermore, these price ranges double during the periods of opening and closing in the trading day and a five minutes halt happens if the stock price does not return inside the price range in less than 15 minutes.

The first phase of implementation happened in April 2013 with the application of the LULD procedures to the S&P500, the RUSSEL1000 and selected exchange traded products. Then in October 2013, the LULD mechanism was finally applied to the rest of the National Market System securities.

Revised Market-Wide Circuit Breakers

On February 2013, revisions concerning the regulation on market wide-circuit breakers was implemented. These procedures may pause trading or even close the markets in case, for example, a market price decreases so much it may remove all liquidity from a market. It is important to note that these trading halts happen simultaneously on multiple markets. Securities and futures exchanges employ that type of procedure to minimize risks in extreme circumstances.

Under these revisions of the trading rule implemented by the SEC, market-wide circuit breakers will trigger and cause coordinated cross-market trading pauses if a rapid decline is occurring on the SP&500 Index . Three trigger levels exist: 7% (level 1), 13% (level 2) and 20% (level 3). They are calculated based on the previous day's closing price of the S&P 500 Index. If a level 1 or 2 circuit breaker triggers due, for example, to a rapid decline in prices, the market will be affected by a market-wide trading halt of 15 minutes. In the case of a level 3 circuit breaker, the triggering will stop market-wide trading for the rest of the trading day (SEC, 2013).

2.4.2 STUB QUOTES

In November 2010, the SEC approved new regulations proposed by FINRA and exchanges to ban “stub quoting” in the U.S equities markets and reinforce minimum quoting norms for market makers. Stub quoting refers to a practice in which market makers try selling or buying stock at prices far away from the prevail market with no intention of them to be truly executed. Market makers make use of this procedure to comply with their two-sided quoting obligation in times when they do not wish to continuously provide liquidity. During the Flash Crash, due to the rapidly declining level of liquidity in the markets caused by the withdrawal of market participants quotes, market makers started using stub quotes since they had no other alternatives. As a result, most of the trades executed at extreme prices on May 6 were simply executions against stub quotes which aggravated the market crisis (Carroll, 2012).

"By prohibiting stub quotes, we are reducing the risk that trades will be executed at irrational prices, and then need to be broken, if the markets become volatile," said former SEC Chairman Mary L. Schapiro. "While we continue to look at other potential obligations for market participants, this is an important step in our effort to improve the functioning of the U.S. markets, and restore investor confidence following the events of May 6."

The new regulations concerning stub-quotes required market makers in exchange-listed equities to provide an active two-sided quotations in regular market hours that are within a particular percentage range of the national best bid and offer (NBBO). The range is based on four criteria. First, market makers have to enter quotes a maximum of 8% away from the NBBO for securities concerned by the pilot program of a circuit breaker. Secondly, they cannot enter quotes more than 20% away from the NBBO before 9:45 a.m. and after 3:30 p.m. because circuit breakers cannot be triggered in these periods. Thirdly, for exchange-listed equities not subject to the circuit breaker pilot program, market makers have to enter quotes a maximum of 30% away from the NBBO. Finally, a quote of a market maker will be permitted to "drift" an extra 1.5% away from the NBBO before a new quote with the applicable range has to be entered. These new regulations were completely implemented in December 2010 (SEC (B), 2010).

2.4.3 CONSOLIDATED AUDIT TRAIL

In May 2010, the SEC proposed a new regulation that would require the self-regulatory organizations (SROs) to create a consolidated audit trail system in order to allow regulators to track information linked to trading orders received and executed on the securities markets.

At this time, no database including orders and executions data that was accessible existed. In fact, each SRO was using its own audit-trail system to gather information regarding orders in its markets. Another problem was that each audit-trail system possessed its respective requirements and procedures. Therefore, if regulators wanted to analyze orders and executions in the markets, they must adapt different large volumes of data before even starting the analysis. In this way, former SEC Chairman Mary L. Schapiro said "It is like trying to put together a jigsaw puzzle, but only being able to see a small part of the final picture. To see the complete picture, regulators must have access to a robust and effective consolidated order and transaction tracking system." That is why the SEC proposed the new regulation: to get a new, uniform and consolidated cross-market order and execution tracking system.

The objective of the proposal is three-fold. Firstly, it will grant access to regulators to timely and comprehensible data on orders and executions in the National Market Systems (NMS) securities for all markets participant involved in the markets. Secondly, it will permit SROs to have an improved view of all their members and markets in order to better fulfill their regulatory duties. And finally, the SEC will be able to rapidly and accurately realize cross-markets analysis due to its new oversight on the NMS for securities. This new regulation was completely implemented in May 2012 (SEC (C), 2010).

2.4.4 SECURITIES EXCHANGE ACT RULE 15c3-5

In November 2010, the SEC introduced the Securities Exchange Act Rule 15c3-5 named Risk Management Controls for Brokers or Dealers with Market Access. The SEC stated that the rule's goal is to "reduce the risks faced by broker-dealers, as well as the markets and the financial system as a whole, as a result of various market access arrangements, by requiring effective financial and regulatory risk management controls reasonably designed to limit financial exposure and ensure compliance with applicable regulatory requirements to be implemented on a market-wide basis." Furthermore, the SEC explained that the rule is "designed to ensure that broker-dealers appropriately control the risks associated with market access, so as not to jeopardize their own financial condition, that of other market participants, the integrity of trading on the securities markets, and the stability of the financial system." Under this rule, broker-dealers who have an access to the market are obliged to create, document and maintain, on the first hand, a risk management control system, and on the other, supervisory procedures effectively constructed to manage all the risks related to this business activity.

Specifically, it means the risk management control system and supervisory procedures must be effectively designed to limit financial exposure that could be brought by the market-access to the broker-dealer and to allow him to comply with regulations requirements linked to market-access. Moreover, according to rule 15c3-5, risk management controls systems and supervisory procedures have to be under the complete control of the concerned broker-dealer and constantly reviewed to ensure effectiveness. Finally, the broker-dealer CEO must verify annually that the broker-dealer controls and procedures observe rule 15c3-5 requirements.

It is interesting to note that risk management controls system apply to every kind of order, with no differentiation on whether it was manually entered by a trader or automatically generated by a program following a set of instructions. Especially in the current situation of the financial markets, where most orders are generated and processed by market participants at high

speed, any error in an automated program can have terrible consequences. In addition, many market participants possess interconnections and all trade on multiple trading venues. Therefore, if an error occurred in any system or firm, it could easily propagate in the market place and generate a cascade of error-related activities (SEC (A), 2014).

2.4.5 REG SCI

This last important rule was adopted by the Securities and Exchange Commission in November 2014 and implemented at the beginning of December. Its aim is to strengthen the technology infrastructure of the U.S. securities markets. The rules, including the Regulation Systems Compliance and Integrity (Reg SCI), incorporates requirements for key market participants are intended to diminish the frequency of systems issues and improve resiliency in the case of systems problems.

As SEC Chair Mary Jo White stated “The rules adopted today mark an historic shift in the Commission’s regulation of the U.S. securities markets that will better protect investors by requiring comprehensive new controls for the technological systems that form the core of our current markets,” In addition, she said “The rules provide greater accountability for those responsible for our critical market systems, helping ensure that such systems operate effectively and that any issues are promptly corrected and communicated to market participants and the Commission.”

Particularly, Reg SCI replaces the voluntary Automated Review Policy (ARP) program adopted in 1989. It does not only render it mandatory but also adds requirements as a reaction to the current situation in the securities markets where technology and automated systems became essential and bear some dangerous risks. Indeed, several technology issues in the markets impacted investors heavily and generated huge losses, clearly demonstrating all the risks related to systems issues (Ryan et al., 2014).

Under REG SCI, particular ATs, self-regulatory organizations, plan processors and certain exempt clearing agencies must possess comprehensive procedures and policies set for their technological systems. Moreover, the rules provide a framework for these entities in order to, for example, apply pertinent corrections to systems issues; notify participants and members about systems issues; provide reports and info concerning systems problems and changes to the SEC; conduct continuity testing and review annually their automated systems (SEC (B), 2014).

3. LITERATURE REVIEW

The aim of this part is to present and review in more details than previously the work done by different authors on topics directly related to our subject. We will have three parts. To begin with, we will analyze what has been undertaken concerning the causes of the May 6 Flash Crash. Then, we will have a look at Mini Flash Crashes, their definition and their impact on the financial markets. Finally, we will describe some examples of famous Mini Flash Crashes.

3.1 CAUSES OF THE FLASH CRASH

In this section, we will provide a summary of all the possible causes found by researchers for the Flash Crash of the 6th May 2010. More especially, we will present their main results and findings as well as how they got them.

3.1.1 HFT PROCESSES

Kirilenko et al.'s (2011) goal was to analyze the structure of the U.S. Financial Markets during the events of the May 6 Flash Crash. They used audit-trail data to describe the structure of the E-Mini S&P500 stock index futures market on the 6th May. Furthermore, they wanted to find possible answers to three particular questions: “How did High Frequency Traders (HFTs) trade on May 6?”, “What may have triggered the Flash Crash?” and “What role did HFTs play in the Flash Crash?”. In the end, they concluded that HFT processes did not trigger or cause the Flash Crash but that their response to the large selling pressure exacerbated market volatility.

Indeed, they found that HFT practices during the crash were inconsistent with the market making definition. HFT traders were trading in the direction of price changes, accounting for a large part of the total trading volume without accumulating a significant inventory which is typical for this type of trader. They are always willing to keep the smallest inventory position possible without taking into account market conditions, like the period of high volatility experienced during the crash. In addition, as they were heavily selling due to a high pressure, their contribution to the total volume might have been taken as liquidity by fundamental sellers. Lastly, as they wanted to rebalance their positions, they steadily searched to acquire liquidity which exacerbated price volatility.

Therefore, it shows how fragile the financial markets can become when an imbalance occurs, for example, in the case a large trader is willing to buy or sell quantities higher than what intermediaries want to hold and, at the same time, long-term suppliers of liquidity are not quick to enter the market on the buy side even if significant price concessions are proposed.

Technological innovations are crucial for markets development but requires fitted regulations to avoid problems related to new trading practices (Kirilenko et al., 2011).

In this second study, Nanex Llc. has brought into lights a new potential cause of the Flash Crash. At exactly 14:45:5 on May 6, latency between the Consolidated Quote System (CQS) and Openbook pricing for NBBO reached a peak of 24 seconds. This implies that subscribers to Openbook, which had premium access, were all well aware that 99% of the remaining investing public could only see pricing data a quarter minute after them, and thus, they were able to trade accordingly on secondary “dark” venues. This means that some people could take advantage of a “NBBO arbitrage opportunity” while others could not. However, Nanex proved that, in fact, something or someone could trigger a quote saturation resulting in a latency arbitrage between CQS and Openbook.

Specifically, they showed that HFT firms can create latency arbitrage whenever they want between the NYSE pricing data dissemination to CQS, but not to NYSE’s own products, like Openbook, by pushing the consolidated NYSE quote rate beyond a threshold of 20,000/second. This raised some questions like “Was the Flash Crash caused on demand by some kind of conspiracy theory?”, “Did the NYSE’s liquidity Replenishment Points failed only as a result of HFT quote bombardment?” and “Can most of the investing public still have any faith in the public exchanges as it seems so easy for HFT firms to cause latency arbitrage by bombarding the exchanges with thousands quotes not intended to being traded on?” (Zero Hedge, 2010).

3.1.2 HIGH LEVELS OF MARKET FRAGMENTATION

In the study of Madahavan (2011), he explains how market fragmentation might have been the cause of the May 6 Flash Crash. More specifically, he shows that the impact of the Flash Crash across stocks was systematically linked to prior fragmentation. Basing his analysis on fragmentation measured by quote competition (typical of high-frequency activity) instead of volume allowed him to draw better explanations.

He found that fragmentation was at its highest levels when he performed his analysis (October 2011), employing data from January 1994 to September 2011. Furthermore, he demonstrated that trade and quote fragmentation behaved differently on the 6th May 2010. He also showed that the link between high frequency quotation activity and the high levels of fragmentation that day could prove why the Flash Crash happened and offered a counterpoint to the theory that the Flash Crash was caused by a sum of unlikely events. Finally, by controlling

fragmentation, he found that exchange-traded products were differentially affected which reflects how difficult it is to put a price on component securities (Madhavan, 2011).

3.1.3 ORDER FLOW TOXICITY

Easley, López de Prado and O'Hara created in 2010 a measure capable of estimating order flow toxicity based on trade intensity and volume imbalance. Order flow is considered as toxic when it selects market makers, not aware that they may be providing liquidity at loss. They named their new procedure the *Volume-Synchronized Probability of Informed Trading (VPIN)* toxicity metric. What is great about it is that it is updated in volume-time which renders it applicable to the high frequency world. However, it requires trades to be classified as buys or sells so the authors developed a new bulk volume procedure better suited to high frequency markets (Easley et al., 2012).

In 2010, the same authors applied their metric to the May 6 Flash Crash events. When order flow toxicity is too high, market makers make losses and so, mitigate their risk by diminishing or even liquidating their positions which can create market illiquidity and leads to terrible consequences such as the ones encountered during the Flash Crash. They showed that order flow became highly toxic on that day and that movements of the *VPIN* toxicity metric foreshadowed what happened, acting like an early warning. Therefore, they proposed the introduction of a “*VPIN* contract” that would have a dual goal: offering market makers a tool to measure flow toxicity and a risk management measure to mitigate the risk of being adversely selected. In the end, it would give some kind of “yellow light” sign to high-frequency market makers to know if they can still provide liquidity in the marketplace (Easley et al., 2010).

In September 2011, Bethel, Leinweber, Rübel and Wu performed thorough analysis to find indicators that could foretell unusual market conditions and examine options to confirm these predictions. All their tests confirmed the use of two particular metrics to measure market fragmentation and to be able to deliver signals before the Flash Crash: the *VPIN* and a volume version of the *Herfindahl-Hirschman Index (HHI)*⁶ (Bethel et al., 2011).

However, in March 2015, Abad, Massot and Pascual analyzed if *VPIN* could act as a potential trigger for trading halts instead of price limits. They showed that in toxic periods, *VPIN* is ineffective if changes occur in the key parameters. Therefore, they concluded that a

⁶ The Herfindahl-Hirschman Index (HHI) is usually used to calculate market concentration by summing the squares of the market shares of all individual firms implicated. (U.S. Department of Justice and the Federal Trade Commission 2010)

properly calibrated version of *VPIN* could work as a warning signal but it may never replace price limits (circuit breakers) due to its defaults (Abad, 2015).

3.1.4 OVERUSE OF INTERMARKET SWEEP ORDERS (ISOs)

Through their analysis of the Flash Crash, Chakravarty, Upson and Wood put into lights an excessive use of ISOs, especially by informed institutional traders. They allow to trade through the best prices in the markets. Using the information shares method of Hasbrouck (1995), they demonstrated that ISO trades were containing more information than Non-Sweep Order (NSOs) trades on the 6th of May 2010. Moreover, they showed that the ISO ability to trade through at best prices was much more employed, especially during the 30 minutes before the Flash Crash. Their findings point out that market returns were heavily impacted by the ISO volume imbalance but not at all by NSO volume imbalance. Furthermore, people moved to ISO trades because market liquidity was becoming thinner and they allowed them to catch more easily liquidity from counterparts. At the end, they recommended the creation of ISO labeled trading usage halts to avoid the previously mentioned problems occurring in periods of high volatility in the markets (Chakravarty, 2011).

3.2 MINI FLASH CRASHES

After having reviewed the flash crash and its consequences, we will focus on the main point of our study: Mini Flash Crashes. They are defined as sudden and extreme price changes that happen in a particularly short period. The next figure (See Figure 6) presents a type of flash crash called a down crash. During the 1st of May in 2011, the stock of Rare Element Resources Ltd. (NYSE ticker: REE) experienced a sudden drop of 1.5% in less than 1 millisecond.⁷ In fact, the stock price declined by 0.25 before reaching a bottom and then, returned rapidly to its previous levels.

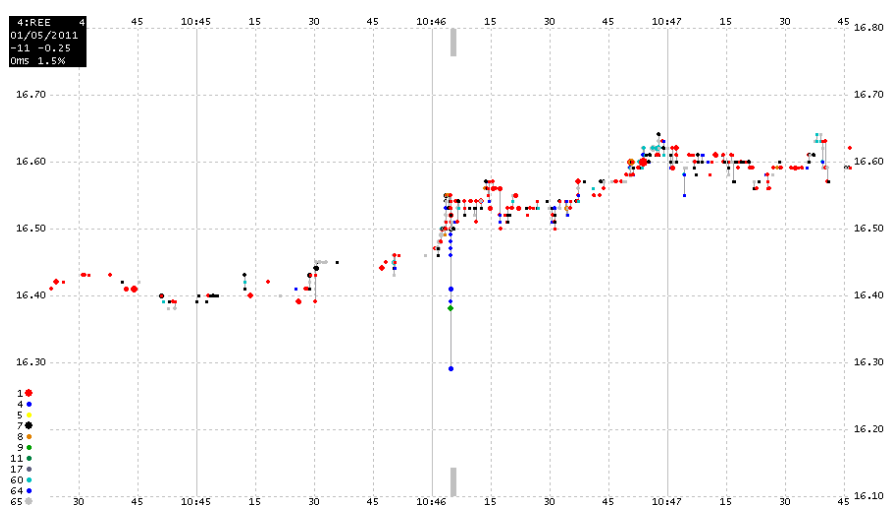
It is only in November 2010 that Mini Flash Crashes became known by most people after Nanex, Llc, a data analytics company, published a thorough analysis of flash equity failures between 2006 and 2010 in the U.S. stock markets (Nanex Llc., 2010).

The main cause of this phenomenon was attributed to a new type of trader that we already described in the first part, HFT traders. However, as we saw in the part of the 6th May 2010 Flash Crash, different authors suggested that one-sidedness of the market can cause flash crashes. Kirilenko et al. (2011), showed by performing thorough analysis of the May 6 events,

⁷ 1 millisecond corresponds to 1/1000 of a second.

that HFT did not trigger the flash crash but exacerbated market volatility. Easley and al. (2011) developed a measure to predict unstable periods in which a flash crash may occur named Volume-Synchronized Probability of Informed trading (VPIN). Chakravarty et al. (2011) found that there was an extensive use of Intermarket Sweep Orders (ISOs) during the Flash Crash and determined ISO as a reliable proxy for aggressive liquidity taking. Finally, Nanex Llc. (2010) found that Quote Stuffing and delays in NYSE Consolidated Quotation System were responsible for the May 6 events.

Figure 6: Example of a Down Crash



Source: Nanex Llc. (2011) (http://www.nanex.net/FlashCrashEquities/FlashCrashAnalysis_Equities.html)

3.2.1 DEFINITION

The most used definition for a Mini Flash Crash was given by Nanex Llc. in November 2010. They identified two different types of crash and we are going to present both of them.

First take a look at **down crashes**. To be considered as a candidate, the stock price change must satisfy three conditions:

1. It must tick *down* at least 10 times before ticking up.
2. It must occur within 1.5 seconds.
3. Price change has to exceed 0.8%.

Usually, this type of crash is called “flash crashes”, however, we will refer to them as “down crashes” for our study to avoid confusion with the 6th May 2010 Flash Crash.

The second type is **up crashes**. Again, to be considered as a candidate, the stock price change must satisfy three conditions:

1. It must tick *up* at least 10 times before ticking down.
2. It must occur within 1.5 seconds.
3. Price change has to exceed 0.8%.

Generally referred as “flash dashes”, we will instead use the term of “up crashes” in our study. (Nanex Llc. 2010)

3.2.2 EVOLUTION FROM 2006 TO 2011

In this section, we will provide an overview of the analysis already done on Mini Flash Crashes. We will focus on three particular studies, one conducted by Nanex Llc. (2010), one by Golub et al. (2011), both based on data from 2006 to 2010, and then, we will share the results of Golub et al. (2012). We chose those because they are the most complete analysis in the small literature about Mini Flash Crashes.

Flash Equity Failures in 2006, 2007, 2008, 2009, 2010 by Nanex Llc. (2010)

In their study, Nanex Llc. analyzed all listed equities from 2006 to 2010 to determine potential Mini Flash Crashes in individual stocks. To do so, they used the exact same definition as the one previously mentioned in this paper. In the end, they were quite surprised by the number of incidents they found. (See Table 1)

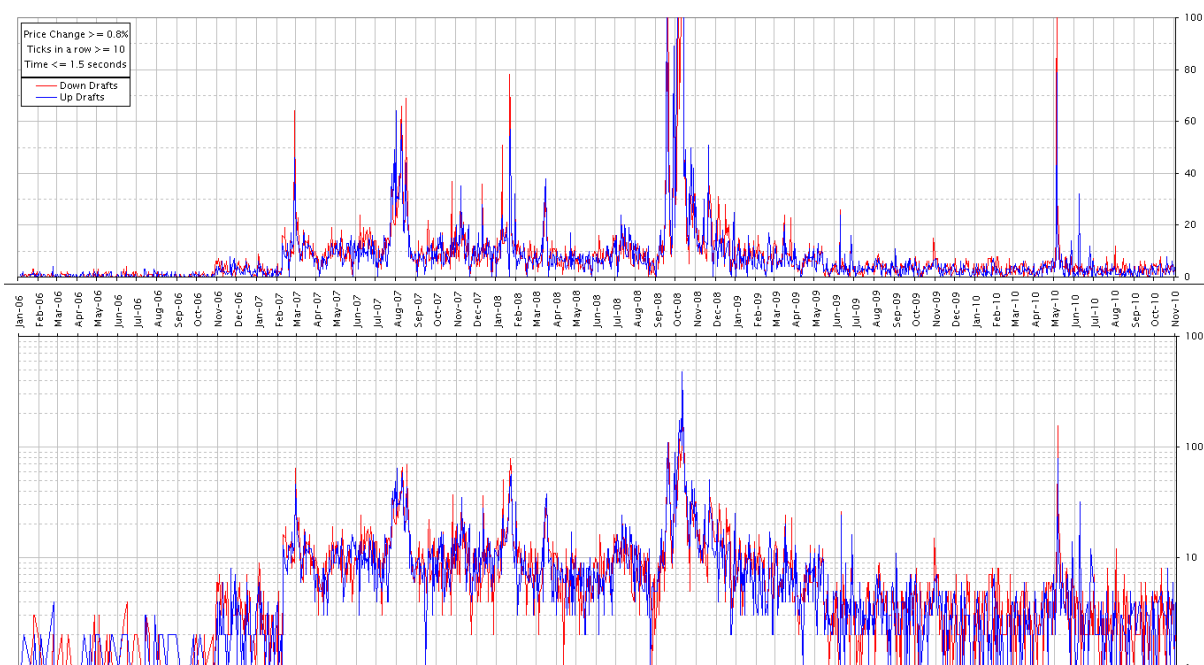
Table 1: Number of Mini Flash Crashes from 2006 to November 2010

Year	Down Crashes	Up Crashes	Total
(November) 2010	1041	777	1818
2009	1462	1253	2715
2008	4065	4354	8419
2007	2576	2456	5032
2006	254	208	462
Total	9398	9048	18446

Source: Nanex Llc. (2011) (http://www.nanex.net/FlashCrashEquities/FlashCrashAnalysis_Equities.html)

And more precisely, we can see on the next figure the daily occurrence of these events. (Second chart uses a logarithmic scale)

Figure 7: Daily Occurrence of Up and Down Crashes between 2006 and November 2010



Source: Nanex Llc. (2011) (http://www.nanex.net/FlashCrashEquities/FlashCrashAnalysis_Equities.html)

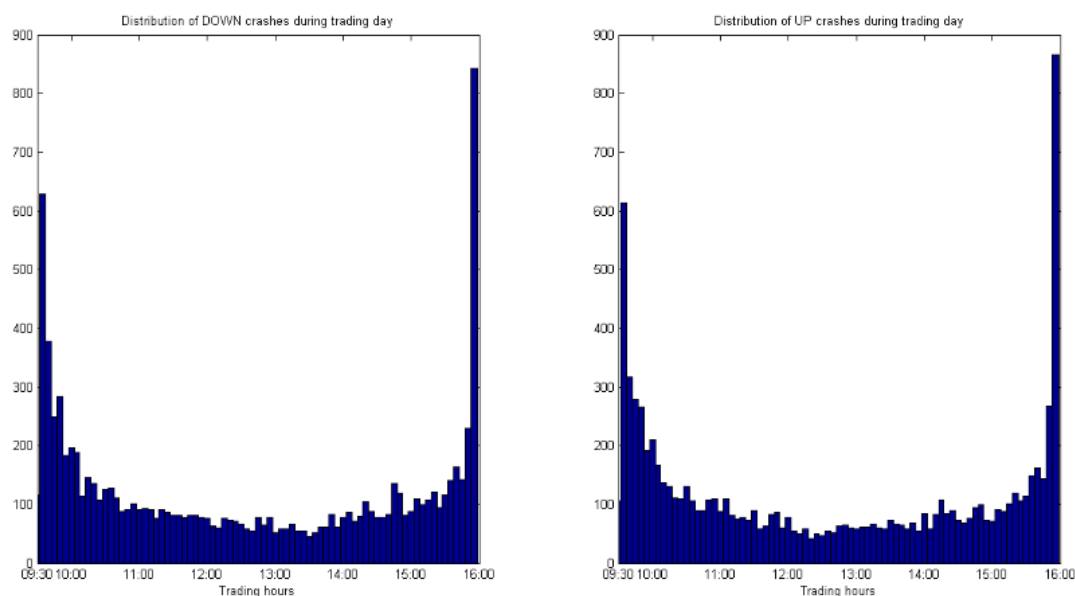
It is interesting to note that this period includes two major events, the Lehmann Brothers collapse and the May 6th Flash Crash, but that there is no bias towards down or up crashes. Moreover, more up than down crashes occurred in 2008 which is a bit ironic knowing that this year is considered as the most devastating one for the U.S. equities markets in recent history.

Mini Flash Crashes by Golub et al. (2011)

The aim of their paper is to study the statistical properties of Mini Flash Crashes using simple data mining practices on tick-by-tick transaction data. They employed data from the Wharton Research Data Services (WRDS) for some information: time stamp of trade, price at which the trade occurred volume of the trade, exchange from which the trade originated and type of order. The remaining data used came from Nanex Llc.: date of the crash, ticker of the stock in which the crash occurred and hour, minute of the crash. However, they verified every crash announced by Nanex Llc. and found some errors. Indeed, only 5001 on 9048 up crashes and only 4765 on 9839 down crashes satisfied all the conditions set by the definition, especially the one that requires 10 price changes.

They first completed simple analysis of their data. To begin with, they analyzed the occurrence of Mini Flash Crashes during the trading day as you can see on the following figure, divided into up and down crashes.

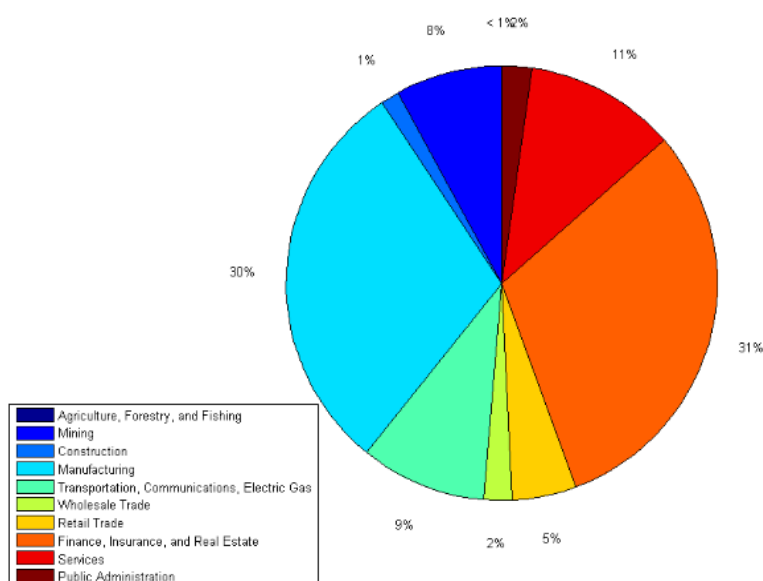
Figure 8: Distribution of Up and Down Crashes during the Trading Day



Source: Golub et al. (2011). P. 6

These graphs show two interesting things. Firstly, they both look the same. There is no difference in terms of occurrence during the trading day between up and down crashes. Secondly, peaks happen at the beginning and end of the trading day. This can be explained by the fact that these periods correspond to the most volatile of all trading days.

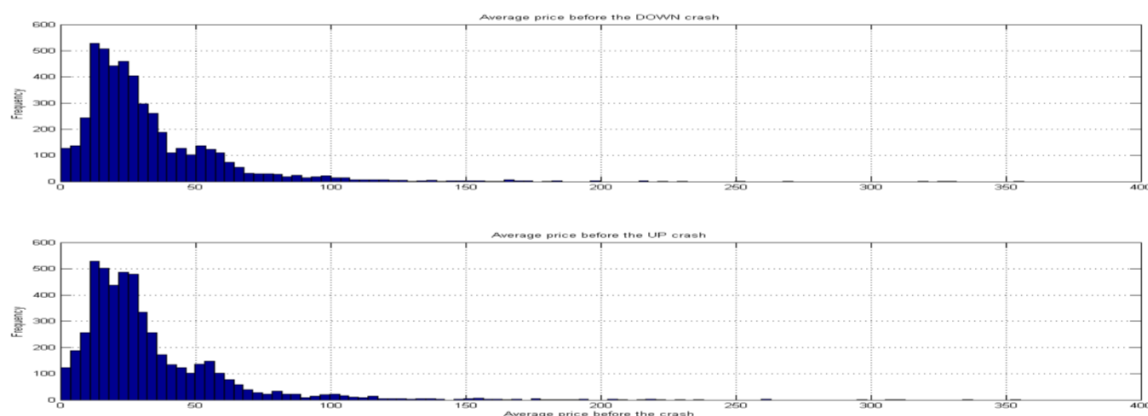
Then, they focused their analysis on the most targeted sector by Mini Flash Crashes, with the Finance, Insurance and Real Estate sector and the Manufacturing sector being the winners.

Figure 9: Most Targeted Sectors

Source: Golub et al. (2011). P. 7

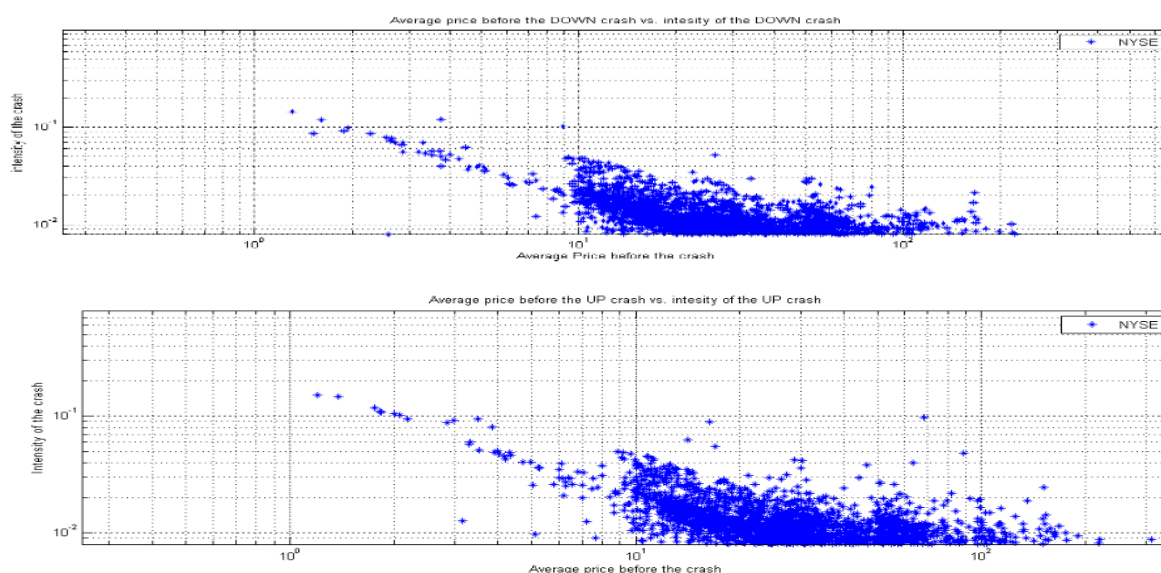
Finally, they discovered that 39% of trades in the Mini Flash Crashes occurred on the NYSE. NASDAQ and ARCA were close with 28% of the trades followed by American Stock Exchange, BATS, Boston Stock Exchange, Chicago Board Options Exchange, Chicago Stock Exchange, International Securities Exchange and National Stock Exchange accounting for the last 5%.

In the second part of their analysis, Golub et al. (2011) described the different characteristics of Mini Flash Crashes and the relationships between different aspects of them. At the beginning, they concentrated on the average price before up and down crashes. They denoted two important peaks: one between \$10 and \$25, and one around \$50 with most of the stocks involved in the crashes priced under \$100 (See Figure 10).

Figure 10: Average Stock Price before a Crash

Source: Golub et al. (2011). P. 8

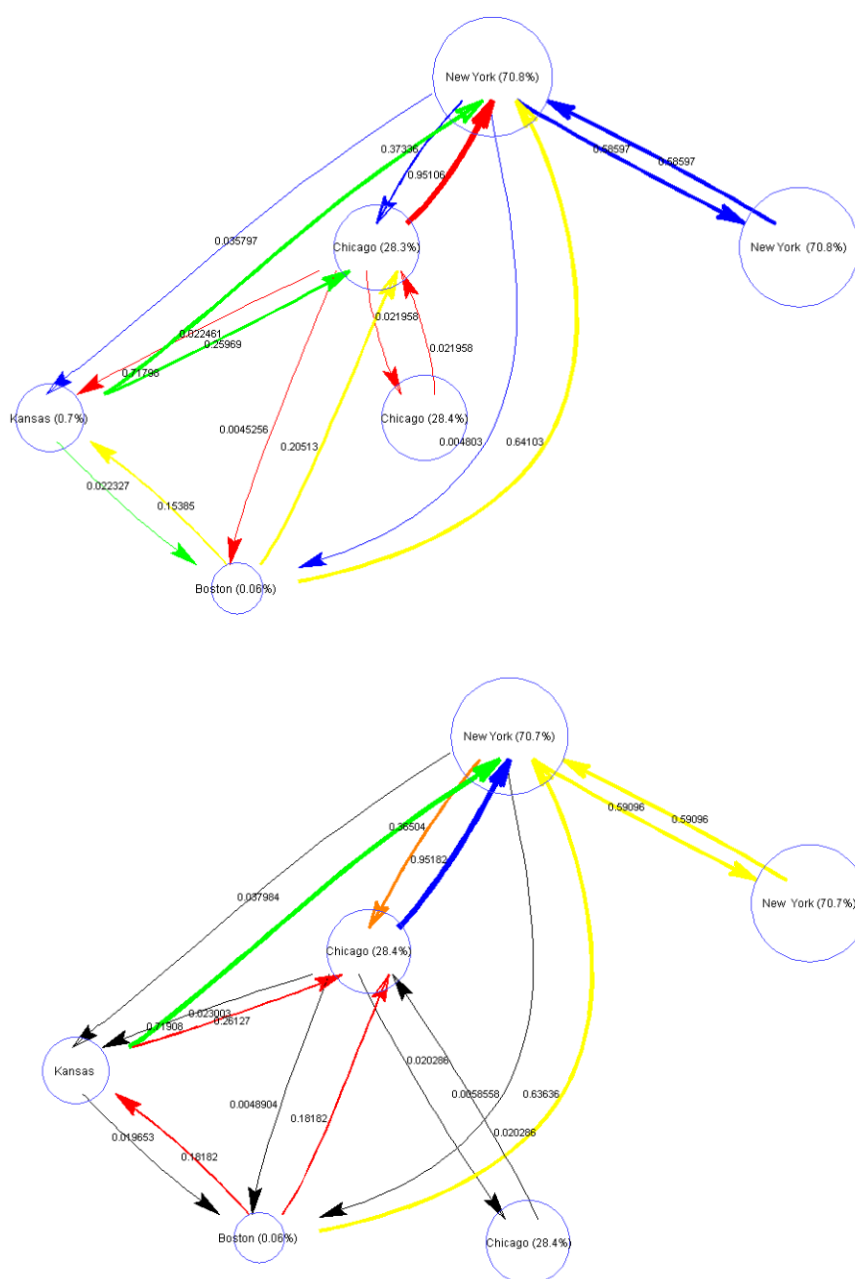
Next, they analyzed the relationship between the average price before a crash and the intensity of it, defined as the total percentage price change of the crash. They used graphs plotted on a log-log scale and separated them based on which exchange the Mini Flash Crash occurred. They found a negative correlation for crashes coming from NYSE. It means that Mini Flash Crashes originating from NYSE in low priced stocks happen with a bigger intensity that in the highly priced stocks (See Figure 11). However, all the crashes happening on other exchanges did not show this kind of relationship. (See Appendix A for the other exchanges)

Figure 11: Average Price/Intensity of the Crash – Up and Down Crash on NYSE

Source: Golub et al. (2011). P. 9

To end this part, they analyzed the geographical movements of trading in crashes between different trading places including Boston, Chicago, Kansas, New York and the greater area of New York. The reason is because trading happens on more than one exchange in the same geographical region. They ended up with two graphs representing the trading movements during crashes throughout the five previously mentioned geographical locations (See Figure 12). For example, if we look at the first graph which represents a down crash: In the case a trade occurs in Chicago, the next trade will have a probability of 95,106% to happen in New York.

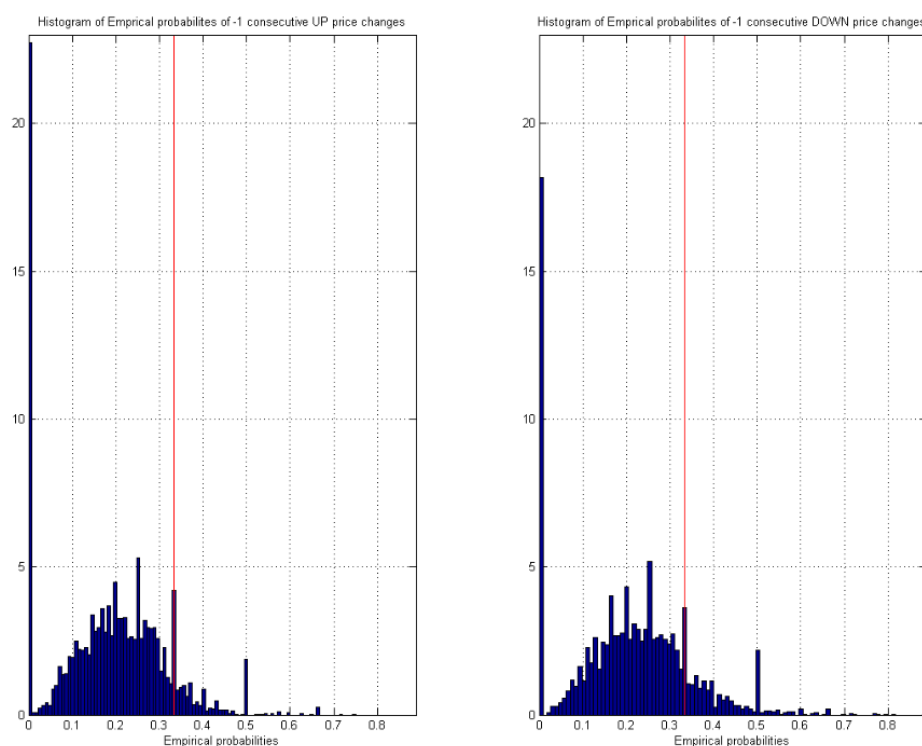
Figure 12: Geographical Movement of Trading in a Down and an Up Crash



Source: Golub et al. (2011). P. 10 & 11

In the third part of this paper, the aim was to analyze the number of consecutive price movements before the crashes. To begin with, they computed empirical probabilities of consecutive price movements for a crash. As you can see on Figure 13, the empirical probabilities for down movements are higher than the ones for up movements which means that more down price movements occur than up price movements before a down crash (See Appendix A for the Up Crash).

Figure 13: Empirical Probabilities for Up and Down Price Movements before a Down Crash



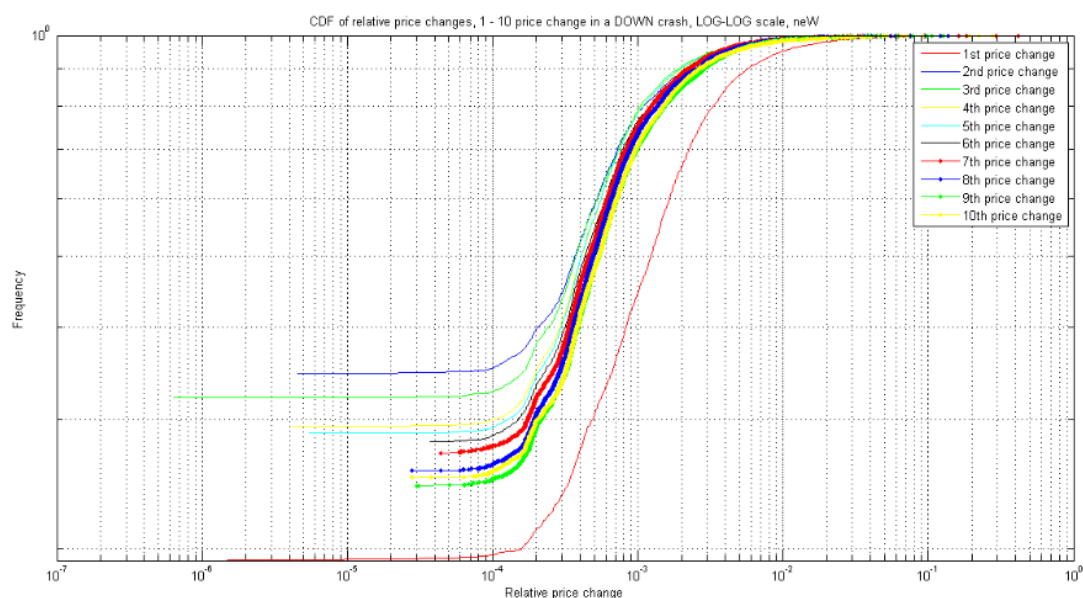
Source: Golub et al. (2011). P. 12

Then, they analyzed whether an up or down crash is preceded by consecutive price movements in the same direction as the Mini Flash Crash. Their findings allowed them to draw a hypothesis concerning the cause of Mini Flash Crashes. In the financial markets, HFT traders behave as market makers, meaning they provide liquidity by buying and selling a lot rapidly, making profit from the difference between the bid and the ask, and maintaining a low inventory of a thousand shares. In the situation of an important price movement for a stock, market makers will supply all the orders in the direction of the movement. For example, in an up price movement, markets makers will act as sellers to all the buy orders following the upward trend. However, in the case the price movement continues its ascent, the inventory exposures of market makers will steadily increase and then, will obtain less advantageous prices. As a result,

risk management thresholds can be breached and thus, market makers must start taking liquidity instead of providing it by buying aggressively all the shares bought a moment before because HFT traders cannot afford waiting for the price to revert considering their trading speed. In the end, this kind of action exacerbates price movements and creates a sharp upward spike.

In the fourth part, their main finding is that the first price change in a Mini Flash Crash is always the largest and strongest one included in the at least 10 price changes required by the Nanex Llc. definition. In addition, they found that it does not dictate whether it is an up or a down crash. In the following figure, you will see the distribution of percentage price changes of the first 10 movements in down crashes, with the red line representing the first price change (See Appendix A for the Up Crash version of the figure).

Figure 14: Cumulative Distribution of the First 10 Price Changes in a Down Crash

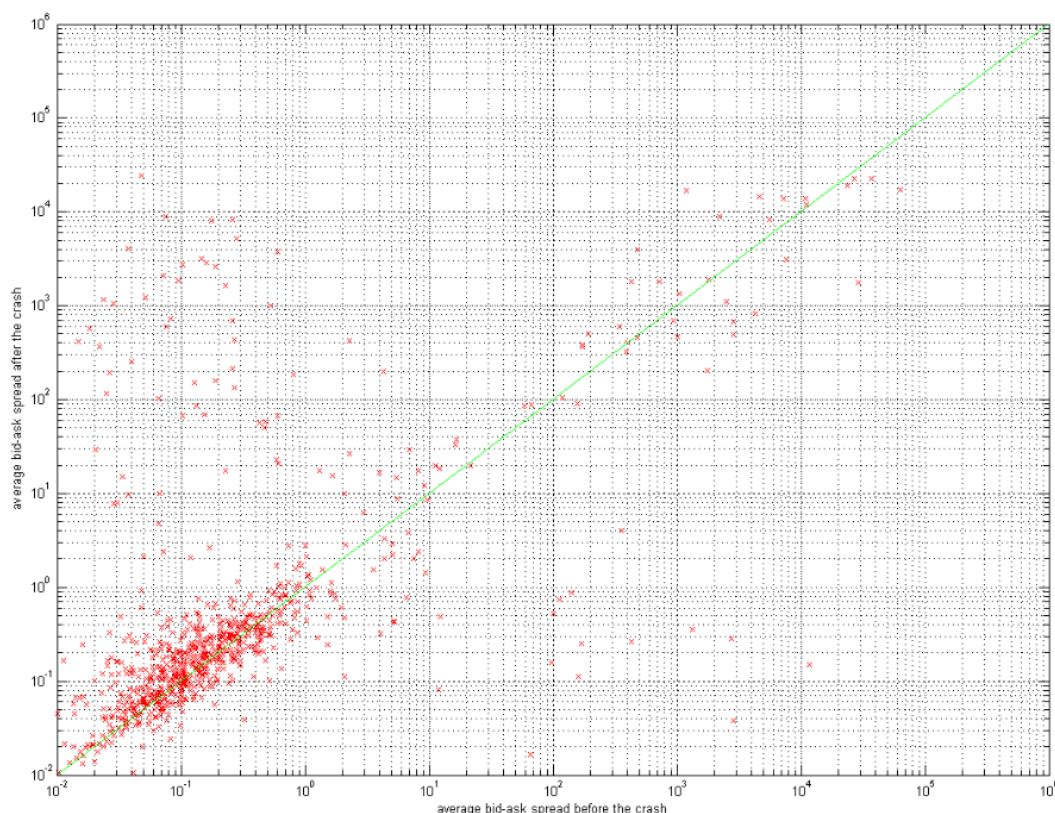


Source: Golub et al. (2011). P. 13

In this fifth part, Golub et al. (2011) analyzed the impact on liquidity and more especially, the fluctuations of the bid-ask spread during a Mini Flash Crash. To do so, they computed the average bid-ask spread before and after the crashes on a sub-sample of 1000 Mini Flash Crashes. The results showed that no fluctuation seems to happen for the spread after a Mini Flash Crash. One might think that the bid-ask spread should widen after the crash occurred as traders might stop their activity in reaction to the recent events but apparently, they do not. We can see the results on Figure 15, showing in log-log scale the scatter plot of average bid-ask spread before and after Mini Flash Crashes.

However, Golub et al. stated well that no conclusions could be drawn at this time as an average was used and it might not represent deviations regarding the amount of data employed.

Figure 15: Average Bid-Ask Spread Before and After the Mini Flash Crashes



Source: Golub et al. (2011). P. 15

Lastly, they looked at regulation issues and more precisely at the “top of the book” protection. Its use is to prevent “trade-throughs” which might be a possible cause for Mini Flash Crashes. These are trades executed at a price inferior to the NBBO⁸. Otherwise, the rule is referred to as the SEC Rule 611 and its problem is that it only protects the top of the book. Consider this example:

Security XYZ and order book on Ask:

INET		ARCA	
Price	Size	Price	Size
32.01	500	32.02	500
32.00	500		

⁸ The National Best Bid and Offer (NBBO) is the lowest available ask price and the highest available bid price for investors when they sell and buy securities.

A buyer is willing to buy 1000 share of XYZ on ARCA. ARCA receives the order; however, the current national best offer is on INET at 32.00 for 500 shares. As a result, ARCA has to send 500 shares to INET in order to respect SEC Rule 611 protecting the top of the book. Then, the remaining 500 shares can be filled on the original exchange, in our case ARCA, without taking into account quotations on all the other trading centers. Therefore, 500 shares will be bought on INET at 32.00 and 500 on ARCA at 32.02 but none will be bought at 32.01 on INET. Thus, our buyer paid one additional cent for its last 500 shares because the only protected quotation is the top of book, which in our case is the 500 shares at 32.00 on INET (Zero Hedge, 2011).

This example demonstrates well how a large sell or buy order routed to an exchange might cause big issues. The “domestic” exchange would route a part of the order to the exchange with the NBBO, respecting the “top of the book” protection rule, and would then fulfill the rest with its own quotes. In the end, if the domestic exchange does not have much liquidity, a severe price change might occur.

In this case, it is not the fault of the “domestic” exchange but how the rule is designed. It only followed the regulation that stipulates it must route as much as possible to the exchange with the NBBO. The example proves the lack of protection for depth of the book quotations. A part of the order can be executed at an inferior price even if a huge amount of additional liquidity lies in other trading centers. SEC should rethink its rule and expand it to protect depth of the book quotations.

In addition, many traders do not understand how their orders are routed between all the exchanges. The best way to avoid the problem in this example would be to use a “smart” order router. It would have focus on researching liquidity on multiple exchanges but also on ECNs and dark pools. But currently, many traders are not aware of all this and if SEC do not expand its order protection rule to depth of the book quotations, trade-through and its associated issues might become common.

High Frequency Trading and Mini Flash Crashes by Golub et al. (2012)

In November 2012, Anton Golub and John Kean now accompanied by Ser-Huang Poon decided to further their investigation of the Mini Flash Crashes phenomenon. This time, they focused on the four most volatile months between 2006 and 2011. In contrast with their last study, they found that Mini Flash Crashes are caused by regulation framework and market

fragmentation, and more especially, by the aggressive use of ISOs and exploitation of the SEC Rule 611. In addition, they discovered that Mini Flash Crashes impact negatively market liquidity resulting in a wider spread, an increased number of crossed and locked NBBO quotes and a decline in quote volume. Finally, they also determined an association with Fleeting Liquidity.

Firstly, they computed descriptive statistics of Mini Flash Crashes occurring in September to November 2008 and May 2010 (See Table 2). It reports:

- Total Crashes: Total number of crashes.
- Up/Down Crashes: Total number of up/Down crashes.
- ISO/auto-routing initiated: Total number of crashes initiated by an ISO/auto-routing.
- Unclassified: Total number of crashes not initiated by an ISO/auto-routing.
- Avg % Change: Average total percentage price change in a crash. They excluded two types of Mini Flash Crashes. The ones occurring in penny-stocks and the ones with percentage price changes surpassing 100%.
- Avg Time: Average crash time in milliseconds.
- Avg Trade Vol: Average total volume of the trades.
- Avg No of Trades: Average number of trades.
- ISO Trades: Average number of ISO marked trades as a percentage of total number of trades.
- Exchange: Percentage of crashes happening at this exchange.

Table 2: Descriptive Statistics of Mini Flash Crashes

	Sep 2008	Oct 2008	Nov 2008	May 2010	Total
Total Crashes	1058	2919	672	491	5140
Up Crashes	527	1709	337	187	2760
Down Crashes	531	1210	335	304	2380
ISO initiated	719 (67,96%)	1922 (65,84%)	519 (77,23%)	328 (66,80 %)	3488 (67,85%)
auto-routing initiated	29 (2,74%)	171 (5,86%)	11 (1,63%)	27 (4,02%)	238 (4,64%)
unclassified	310 (29,30%)	826 (28,30%)	142 (21,13%)	136 (27,70%)	1414 (27,51%)
Avg % Change	2,69%	2,14%	2,17%	4,12%	2,45%
Avg Time	437	432	374	319	413
Avg Trade Vol	12465,97	13900,13	18390,18	14810,52	14352,39
Avg No of Trades	21,15	17,98	19,08	19,72	18,94
ISO Trades	89,53%	85,80%	89,95%	84,30%	86,98%
by Exchanges					
NYSE	37,09%	67,56%	58,14%	58,41%	59,29%
NASDAQ	32,58%	18,75%	23,94%	19,39%	22,28%
ARCA	29,03%	13,15%	17,43%	19,39%	17,52%
AMEX	1,30%	0,46%	0,16%	1,17%	0,65%
BATS	0,00%	0,00%	0,33%	1,40%	0,19%
ISE	0,00%	0,08%	0,00%	0,23%	0,07%

Source: Golub et al. (2012). P. 8

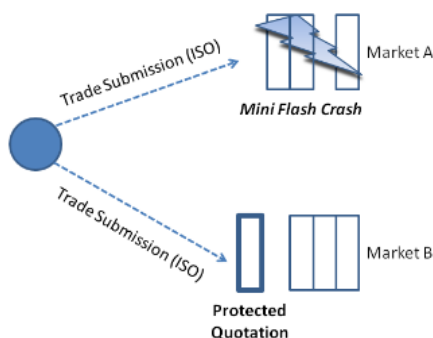
We can denote two interesting things. NYSE, NASDAQ and ARCA possess the highest percentage of crashes which means most Mini Flash Crashes occurred there. Furthermore, an extremely high number of trades correspond to ISO trades. It proves that there was an excessive use of ISOs.

In order for a crash to be considered as *ISO Initiated*, it must fulfill:

- (1) The conditions defining a Mini Flash Crash set by Nanex Llc.
- (2) The trades composing the Mini Flash Crash needs to be marked as ISO, except the first k trades that can be marked as non-ISO if executed within the least market-wide available best quotes in the last one second.

An ISO initiated Mini Flash Crash happens this way. A trader submits a large package of ISO-marked orders to exchange A and does the same on all the trading venues quoting at NBBO to match the size of protected quotes. As a result, ISO-marked orders arriving at exchange A trade through the order book resulting in a Mini Flash Crash even if additional liquidity exists deeper in the order book of other trading venues. Figure 16 represents how this type of Mini Flash Crash occurs.

Figure 16: Illustration of the mechanics of ISO Initiated Mini Flash Crashes



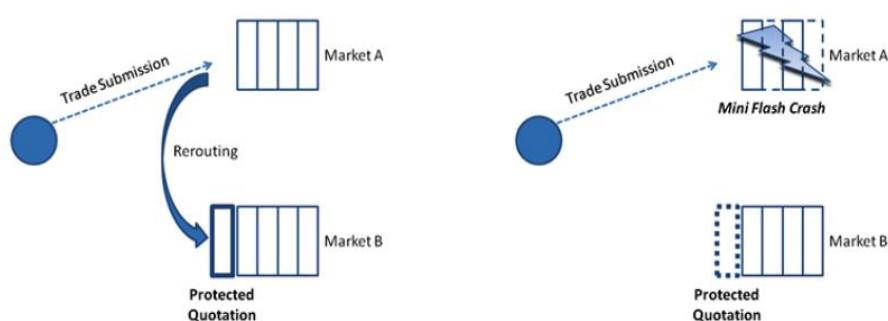
Source: Golub et al. (2012). P. 9

Next, for a crash to be considered as *auto-routing Initiated*, it must fulfill:

- (1) The conditions defining a Mini Flash Crash set by Nanex Llc.
- (2) The trades composing the Mini Flash Crash needs to be marked as regular, except the first k trades that can be marked as non-regular if executed within the least market-wide available best quotes in the last one second.
- (3) For all trading venues quoting at NBBO, the Top of the Book must be cleared before the execution of regular marked trades composing the Mini Flash Crash.

An auto-routing initiated Mini Flash Crash happens this way. A trader submits a large order to an exchange A but it does not display the NBBO. Therefore, a part of the order of the protected quotation size at exchange B is routed from A to respect the Order Protection Rule. Once it is done, the remaining part of the initial order is filled on exchange A without taking into account prevailing liquidity on other trading venues, causing a Mini Flash Crash. Figure 17 represents how this type of Mini Flash Crash occurs.

Figure 17: Illustration of the mechanics of Auto-Routing Initiated Mini Flash Crashes



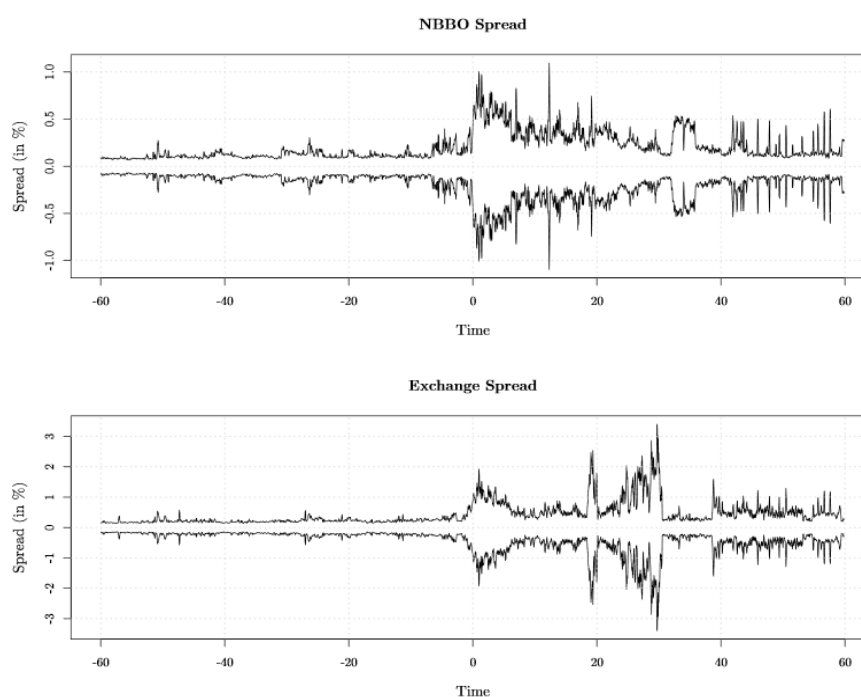
Source: Golub et al. (2012). P. 10

With these definitions, Golub et al. (2012) were able to classify the crashes: 3488 (67.85%) as ISO initiated, 238 (4.64%) as auto-routing initiated and 1414 (27.51%) as unclassified. Despite the fact both type of Mini Flash Crash differ in term of mechanics, they are both the results of under protection for Depth of the Book quotations. Additional liquidity is available deeper in the order book of other trading venues but because of the Order Protection Rule, which applies only to the Top of the Book, trades can occur at inferior prices on a distinct exchange, causing Mini Flash Crashes.

They finished this part by speculating on who might be the cause of Mini Flash Crashes. As mentioned before, most of the crashes occurred because of an aggressive use of ISOs. In fact, retail and institutional investors cannot employ such mechanisms, only broker-dealers or traders with sponsored access can. Moreover, if we look at the magnitude and speed of the Mini Flash Crashes, just one type of trader fit these conditions: HFT traders.

Secondly, they analyzed the impact of a Mini Flash Crash on the NBBO spread and Exchange spread. They found that it was detrimental for both of them. On the following figure (Figure 18), a NBBO spread and Exchange spread are represented on a period of 2 minutes, one before the occurrence of the Mini Flash Crash and one after. Mid-price is scaled to 0 on the Y-axis.

Figure 18: NBBO Spread and Exchange Spread

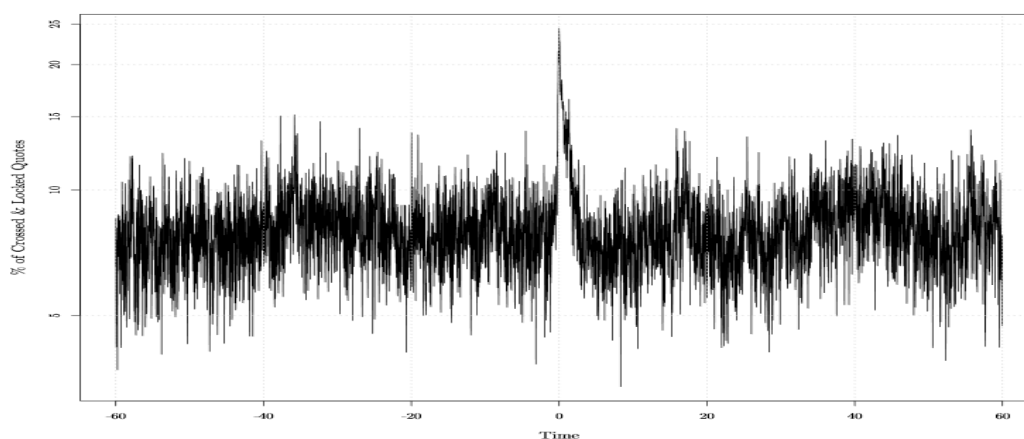


Source: Golub et al. (2012). P. 13

We can deduce two things from these figures. To begin with, both spreads are stable in the minute before the crash with values on average at 0.24% and 0.46% respectively. Then, they increase rapidly to reach values of 0.58% and 1.32% which corresponds to an increase of 141.60% and 188.05%. Second interesting fact, despite the short time period for the Mini Flash Crash (approximately 1.5 seconds), its impact on quote spread continues for the next whole minute.

Thirdly, they looked at the percentage of crossed and locked NBBO quotes included in all quotes before and after a Mini Flash Crash. We find them when the National Best Bid for a particular security is higher or equal to the National Best Offer. Fast traders use them as they can buy the security at the crossed offer quote and then sell it at the crossed bid quote which represents a risk-less profit for them. The authors used a time frame of two minutes again, with one before and one after the crash (See Figure 19).

Figure 19: Percentage of Crossed and Locked Quotes



Source: Golub et al. (2012). P. 14

Contrary to spread difference, the percentage of crossed and locked NBBO only rises a few seconds after the crash to then returns to its previous levels. Indeed, the percentage is equal to an average of 8.01% in the minute before the crash, compared to 24.40%, right after the crash, and finally, declines to an average of 8.23% in the minute after the crash.

Fourthly, they conducted an analysis of the average quoted volume at the NBBO and at all the exchanges involved, before and after a Mini Flash Crash. In both cases, the average quoted volume decreased with a greater decline on the Bid side. This proves the existence of sell-side pressure in the markets right after the crash occurred (See Table 3 & Appendix A for the related graphs).

Table 3: Quoted Liquidity at NBBO and Exchanges where Mini Flash Crashes Occur

	Average Quoted Volume		% Difference
	Before the Crash	After the Crash	
National Best Bid Liquidity (in shares)	1575.05	1272.05	-19.23%
National Best Offer Liquidity (in shares)	1323.49	1210.30	-9.35%
Exchange Bid liquidity (in shares)	1196.9	870.42	-37.51%
Exchange Offer Liquidity (in shares)	991.58	990.99	-0.06%

Source: Golub et al. (2012)

Lastly, they also discovered an association with Fleeting Liquidity. This phenomenon is quite recent as it is linked with the emergence of high speed trading. Indeed, traders with speed advantages are now able to submit and then cancel orders in the matter of a few milliseconds. The problem with this practice is that it creates a false impression of demand for a stock and overpriced supply. Therefore, some people are tricked by this “artificial” liquidity that they think real and trade based on false information. We call this the fleeting liquidity.

In the case of Mini Flash Crashes, Fleeting Liquidity appears if the quotes disseminated by the SIP⁹ are not reached while the Mini Flash Crashes occur. It means the best displayed quotation was cancelled before the SIP could even disseminate the removal of these resting limit orders. The following Table 4 summarize the statistics of this phenomenon.

The first observation to note, 37.99% of the Mini Flash Crashes studied present signs of Fleeting Liquidity. For the variables in the table from “Time” to “Type”, the authors used a particular logit-regression to discover associations between Fleeting Liquidity and Mini Flash Crashes. Three variables come out as significant: “Time” and “Vol” at the 1% level and “NoTrades” at the 5% level, and they all bear a negative coefficient. All this means that highly rapid Mini Flash Crashes with small volume and number of trades are linked with Fleeting Liquidity.

⁹ The Securities Information Processor (SIP) is the institution responsible to determine the NBBO and disseminate it to all its subscribers.

Table 4: Statistics of Mini Flash Crashes with Fleeting Liquidity

	Sep 2008	Oct 2008	Nov 2008	May 2010	Total
Total Crashes	1058	2919	672	491	5140
Up Crashes	527	1709	337	187	2760
Down Crashes	531	1210	335	304	2380
Fleeting Liquidity	377 (35,63%)	1114 (38,16%)	289 (43,00%)	173 (35,23%)	1953 (37,99%)
					FleetLiq
Intercept					0.1444 (1,4678)
Time					-0.2065** (-2,7481)
%PriceChange					0.1098 (0,7346)
Exch					0.0102 (0,2474)
Vol					-4.6876E-6** (-2,7527)
UpDown					0.0022 (0,0738)
NoTrades					-0.0067* (-2,2836)
Type					-0.0464 (-1,0201)
Classification Precision					54,80%
**Statistically Significant at 1% Level					
*Statistically Significant at 5% Level					

Source: Golub et al. (2012). P. 19

3.3 EXAMPLES OF MINI FLASH CRASHES

In this section, we will present five Mini Flash Crashes that occurred between 2011 and 2014. We chose them according to the impact they had on the markets or simply because they show particular aspects of Mini Flash Crashes.

The IBM Flash Dash

On January 25, 2011, IBM was trading at 160.71 around 3:15pm but suddenly skyrocketed to 164.35 at 3:18:15pm and then, returned immediately to its previous levels. All of that happened in the space of one second. We will present this Flash Dash based on the work of Dennis Dick and Zero Hedge who summarized the event.

Their main finding concerning the cause of the dash is that it was caused by the fragmented liquidity in the current market structure which proved again the problem linked to the famous SEC rule 611. On January 25, a trader submitted a buy order of more or less 60,000 shares and routed it to NYSE. Following the order protection rule, the NYSE first submitted the order to exchanges proposing the NBBO (160.71). Next, the remaining shares were fulfilled on NYSE. You can have a look at all the trades which occurred at 15:18:15pm on Table 5.

Table 5: Trades which occurred at 15:18:15pm (IBM Stock)

Time	Last	Size(00s)	Time	Last	Size(00s)	Time	Last	Size(00s)
15:18:15	160.76	8	15:18:15	160.98	8	15:18:15	162.84	48
15:18:15	160.76	8	15:18:15	160.99	1	15:18:15	160.78	1
15:18:15	160.77	2	15:18:15	161.00	4	15:18:15	160.78	1
15:18:15	160.78	1	15:18:15	161.02	1	15:18:15	162.86	1
15:18:15	160.79	5	15:18:15	161.07	1	15:18:15	162.87	2
15:18:15	160.80	8	15:18:15	161.09	5	15:18:15	162.88	4
15:18:15	160.81	3	15:18:15	161.10	1	15:18:15	162.89	5
15:18:15	160.82	8	15:18:15	161.13	2	15:18:15	162.92	2
15:18:15	160.83	19	15:18:15	161.21	1	15:18:15	162.93	4
15:18:15	160.84	6	15:18:15	161.22	4	15:18:15	162.94	2
15:18:15	160.85	4	15:18:15	161.30	1	15:18:15	162.95	3
15:18:15	160.76	1	15:18:15	161.31	8	15:18:15	162.96	15
15:18:15	160.86	21	15:18:15	161.33	2	15:18:15	160.79	1
15:18:15	160.87	14	15:18:15	161.36	2	15:18:15	160.89	2
15:18:15	160.88	7	15:18:15	161.53	10	15:18:15	160.89	2
15:18:15	160.89	15	15:18:15	161.65	22	15:18:15	163.22	1
15:18:15	160.78	1	15:18:15	161.72	10	15:18:15	163.77	122
15:18:15	160.90	15	15:18:15	162.00	80	15:18:15	164.12	1
15:18:15	160.91	3	15:18:15	162.20	10	15:18:15	164.26	70
15:18:15	160.92	23	15:18:15	160.79	1	15:18:15	164.35	2
15:18:15	160.81	2	15:18:15	162.22	2	15:18:15	160.78	1
15:18:15	160.93	9	15:18:15	162.24	1	15:18:15	160.76	1
15:18:15	160.94	8	15:18:15	162.38	1	15:18:15	160.76	3
15:18:15	160.95	2	15:18:15	162.48	1	15:18:15	*63 trades	at 160,76-160,86
15:18:15	160.97	4	15:18:15	162.69	2			

Source: Zero Hedge (2011) (<http://www.zerohedge.com/article/sec-rule-611-key-unlocking-mystery-behind-ibm-flash-dash>)

As we can see on these trades series, depth of book quotations were not well protected. Other exchanges possessed quantities of additional liquidity as the consolidated offer never surpassed 160.86 during the underlying time frame. On the other hand, since the SEC rule 611 only protected the top the book, 160.71 in this case, trades occurred at superior prices even if there was liquidity on other trading centers (Zero Hedge, 2011).

The Bats IPO

On the 23rd March 2012, the IPO of BATS completely failed. BATS is one of the largest market operators in the U.S. with four equities exchanges. At the beginning, the stock was trading at a price of \$15.2 but only after 900ms, the stock price had already declined to \$0.2848 and after 1.5 seconds, it had reached the value of \$0.0002. It was finally halted after 567 trades were executed.

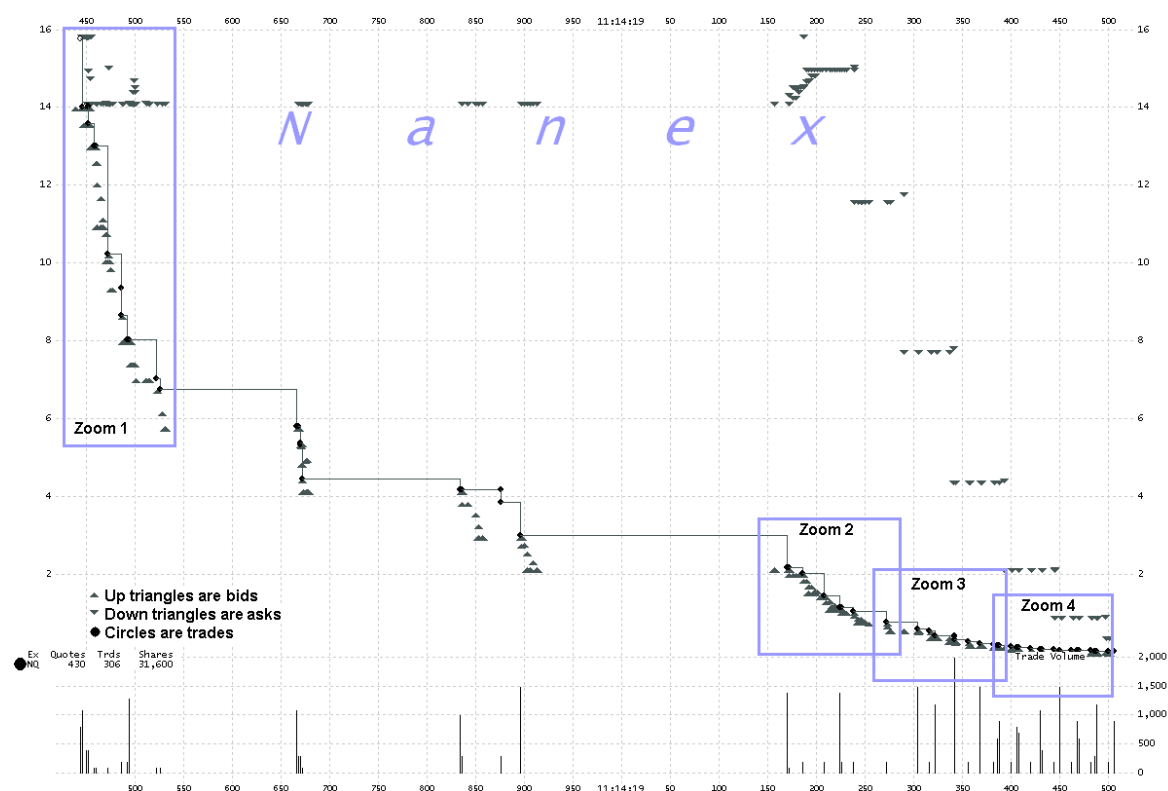
The following figure presents all the trades in BATS after the “opening trade report” from the BATS Exchange. Except the trades for the opening coming from BATS, all the trades were executed on NASDAQ and only one was executed on BAT-Y. All of those that matched the

bid prices were market *ISO* and only 24 coming from NASDAQ were marked regular, matching the ask prices. (See Figure 20)

As reported by Nanex Llc., we are clearly in an *ISO* orders abuse that may have even been intentional. An *ISO* order is usually used to sweep the top of the book at all the exchanges in order to profit from a particular exchange price even if others have a better one. The problem in this case, is that there were no other exchanges with bids to sweep according to the SIP, responsible for identifying the NBBO from all exchanges. That is why the use of *ISO* orders in that case was highly suspicious (Nanex Llc. (D), 2012).

In the opinion of Zero Hedge, based on the Nanex Llc. study, it was a 100% intentional NASDAQ algorithm that dragged BATS stock price to zero in order to make a mockery of BATS and ruin its chances of one day going public. He argues that exchanges like NASDAQ or even NYSE would benefit from this situation as BATS is one of their competitor and less competition means more revenues. Moreover, the algorithm acted like a “master watch maker” with quotes highly accurate and precisely updated. Nevertheless, this opinion can only be a guess as the only thing 100% sure is that the algorithm executed on NASDAQ and was directly linked to it because the algorithm used *ISO* orders (Zero Hedge, 2012).

Figure 20: Trades in BATS



Source: Nanex Llc (D) (2012) (<http://www.nanex.net/aqck/2970.html>)

The Facebook IPO

On the 18th of May 2012, trading on Facebook shares was supposed to start at 11:00am after its IPO was priced the day before but failed up until 11:30:34am. The process used in this kind of situation is called NASDAQ's "IPO Cross" opening process. It has three purposes: collect the buy and sell orders until the cross time, match the highest number of buyers and sellers at a specific market-clearing opening price and finally, permit continuous trading of the stock's shares. The process usually takes a few milliseconds to run and then, double checks if the order book is cleared or if a new order appeared or was cancelled. In the case of a change occurring, the whole process restarts. That is why from 11:00am to 11:30:34am, trading failed to begin three times because of the interest in the Facebook IPO but also, because algorithms were constantly submitting and cancelling orders, resulting in an infinite loop in the process (Jones 2013). At the same time, spikes appeared on multiple stocks such as AAPL, SPY, VXX, NFLX, INTU, QCOM, etc. as well as on the Facebook stock. Most of those occurred on three exchanges: CBOE, Chicago and AMEX and almost all price spikes executed at quotes far away from the NBBO (Nanex Llc. (A), 2012).

After that, NASDAQ was not able to provide an eligible Best Bid/Offer quote during the first 3 hours (11:30:34 to 13:50:01) in which 272 million shares of Facebook traded. At that moment, NASDAQ opened another matching engine based on orders placed up until 11:11am which means orders entered or cancelled between that time and 11:30am were not taken into account. It is only at 13:50:01 that opening cross execution reports were finally disseminated (Nanex Llc. (B), 2012).

The real cause of the opening delay was found by Telis Demos in an article published in the Financial Times. According to him, excessive quote cancellation lead to a painful 20 minutes delay by fitting in the five milliseconds window taken by the process to determine a price. Only one type of trader can cancel quotes at this rate and this is HFT traders. Moreover, they were also the only ones profiting from the NASDAQ two hours quotes outage from the SIP as they had direct premium feeds unlike common retail customers (Demos, 2012).

Knight Capital Nightmare

On the 1st of August 2012, Nanex Llc. was alarmed at the opening of the financial markets as many NYSE stocks were experiencing unusually high trade rates and opened sharply up or

down. According to them, Knight Capital Group, one of the largest market-makers in the U.S. equities, accidentally released the testing system of their new market-making software on the NYSE live system.

This testing system was first used by Knight Capital in their laboratories to ensure its new Retail Liquidity Provider (RLP) market-making software was working by sending buy and sell orders patterns to it, the results being analyzed and recorded. Since everything was fine, they decided to deploy their new software on the NYSE live system.

The deployment of the software was done by a different group than Knight Capital and they included by mistake the testing system in the release package. Thus, it started working on the NYSE live system. This means it had to test every market-making software running with real orders and dollars. In addition, no one knew it was functioning because the testing system was not communicating its results to anyone, this function being not part of its design.

Two phenomena happened during this situation. Firstly, for stocks where Knight Capital was the only market-maker and the testing system was the only trading algorithm crossing the bid/ask spread, buy and sell patterns of trade executions were seen at prices just over the bid or below the ask, on the NYSE and all marked regular. This case is equivalent to the one in the laboratories. The market-making software caught the orders and executed them, resulting in multiple wash sales.

Secondly, for stocks where Knight Capital was not the only one running market-making software or where the testing system was not the only trading algorithm active, orders sent by it were intercepted and executed by other people than Knight Capital which was now holding positions on these stocks and could make or lose money.

Some stocks moved aggressively at the opening because the testing system was buying at the bid and selling at the ask without “thinking”, and because the bid/ask spread is always wide during the opening period.

The reason the testing system was not stopped by Knight Capital is simply because they did not know it was even running. To begin with, it was not supposed to be included in the package and then deployed on the NYSE live system. Furthermore, it did not include a feedback functionality. When they finally understood they were losing money, Knight Capital put a stop to their market-making software which they thought to be guilty. They did that two times (at

9:48am and 9:52am) as trading suddenly dropped but finally, someone found and destroyed the testing system right before the release of the economic news at 10:00am (Nanex Llc. (E), 2012).

The Form-T Trades Problem

January 27, 2014, was the day with the highest number of Mini Flash Crashes since the beginning of the year. Other than that, we decided to present this event because of its particular cause that had not been discussed in this paper yet: Form-T trades.

Form-T trades are specifically called like that because they are executed outside the normal market hours, meaning before 9:30am and after 4:00pm. In addition, if they appear sooner than the markets opening, they must possess a time-stamp before 9:30:00am. However, multiple Form-T trades on the 27th January 2014 executed with time-stamps exceeding 9:30am by almost one second. An unusual number of such trades occurred on two exchanges out of 14: NASDAQ and ARCA. In its research, Nanex Llc. concluded that either these Form-T trades or the conditions behind them lead to a large price swings in over 264 stocks the second after the markets opened. As a result, the number of Mini Flash Crashes reached a new yearly high (Nanex Llc., 2014).

4. METHODOLOGY

In this part, we will explain how we further investigated the “Mini Flash Crashes” phenomenon. In the previous sections, we computed a thorough analysis of all theories and studies that could help us to determine a taxonomy for Mini Flash Crashes. Now, we will show how we analyzed a self-created database by using statistical methods in order to derive new insights for our taxonomy.

4.1 THE DATA SAMPLE

What was really difficult when we started this study is that we had no access to any specific database or program to get tick-by-tick transaction data. As a consequence, we decided to build our own database on information coming from the Nanex Llc. website. Indeed, they present Mini Flash Crashes they consider as important or news worthy since May 2011. We will thus explain in this section how we created our data sample and how we computed the different variables. Then, we will show you an example of a Mini Flash Crash and its corresponding line in our database. Finally, we will discuss the different issues we had creating the database.

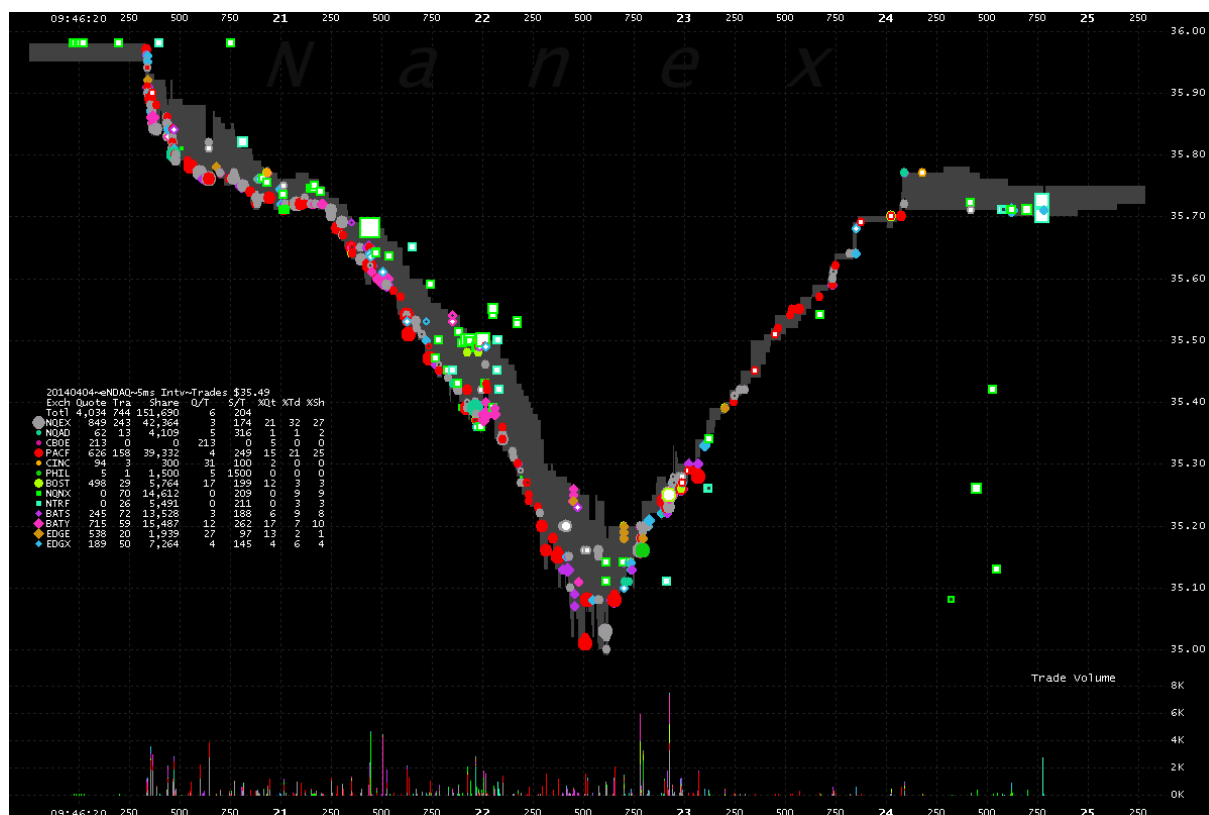
The data sample incorporates 192 Mini Flash Crashes from May 2011 to September 2014 and 11 variables. Here are the variables name and description:

- SYMBOL: Ticker symbol of the Stock, Mutual Fund, ETF, Index or Future.
- TYPE OF CRASH: Up or Down Crash.
- DATE: The date of the Mini Flash Crash.
- YEAR: The year during which the Flash Crash occurred.
- TIME OF THE DAY: (1) Opening period (until 9:50am) (2) Mid period (3) Closing period (from 3:30pm) (4) Extended hours (<9:30am, >16:00pm).
- CRASH TIME: Total time of the Mini Flash Crash in milliseconds (only the drop/increase).
- DOMINANT EXCHANGE: Exchange where most of the trades occurred.
- PRICE AT THE BEGINNING: Price at the beginning of the Mini Flash Crash.
- PRICE CHANGE: Price change in percentage (%).
- SECTOR CONCERNED: (1) Agriculture, Forestry and Fishing (2) Mining (3) Construction (4) Manufacturing (5) Transportation, Communications and Electric Gas (6) Wholesale Trade (7) Retail Trade (8) Finance, Insurance and Real Estate (9) Services (10) Public Administration
- ASSET TYPE: (1) Stock (2) ETF (3) Future.

The values for each variable were derived from different methods. The simplest, which was applied to most of them, consists of looking at the graph given by Nanex Llc. This method was used for SYMBOL, TYPE OF CRASH, DATE, YEAR, TIME OF THE DAY and DOMINANT EXCHANGE. For the CRASH TIME, we had to look at when the Mini Flash Crash started until the price reached a peak or a bottom. Related to that, the values for PRICE AT THE BEGINNING were taken from the price at which the first trade executed when the Mini Flash Crashes began. Then, the PRICE CHANGE, which is a percentage, was computed using PRICE AT THE BEGINNING and the price of the last trade executed at the end of CRASH TIME. Finally, some researches on the internet allowed us to fulfill the columns for SECTOR CONCERNED and ASSET TYPE.

We will now show you an example of a Mini Flash Crash and the values we included in our database (See Figure 21).

Figure 21: NASDAQ Stock Mini Flash Crash



Source: Nanex Llc. (B) (2014) (<http://www.nanex.net/aqck2/4598.html>)

On April 4, 2014, the price of the NASDAQ OMX Group, Inc. suddenly dropped by 2.72% from 35.98 to 35 in only 2300 milliseconds. Here are the corresponding values we encoded in the database for this Mini Flash Crash:

SYMBOL	TYPE OF CRASH	DATE
NDAQ	Down	04-04-14
YEAR	TIME OF THE DAY	CRASH TIME
2014	Opening period	2300
DOMINANT EXCHANGE	PRICE AT THE BEGINNING	PRICE CHANGE
NASDAQ	\$ 35,98	2,72%
SECTOR CONCERNED	ASSET TYPE	
Finance, Insurance and Real Estate	Stock	

For example, we have a CRASH TIME of approximately 2300 because if we look at the graph, the drop seems to start at 20:46:20:300 and stop around 20:46:22:600 which makes 2300 milliseconds. The PRICE AT THE BEGINNING was easy to find as it was given by Nanex

Llc. in their description of the crash. As a result, we simply computed the PRICE CHANGE with their numbers (35.98 to 35) which gave us 2.72%. The rest of the variables values were found by looking at the graph or by searching on the net, principally on Yahoo Finance. With that, we ended up with a database composed of 192 supposed Mini Flash Crashes. However, we had some issues during the database's building process.

In fact, when we first referenced all the news concerning Mini Flash Crashes from the Nanex Llc. website, we counted 480 of them. Quickly, we deleted several from our list because there were about researches or other stuff which will not help us building our database. Then, we deleted others because the drop or the increase was simply under the 0.8% required by the definition of a Mini Flash Crash. Taking into account all of that, our first database was composed of 332 Mini Flash Crashes, or more or less 70% of the original news, but we forgot something. The definition also mentions that the crash must occur within 2.5 seconds. As a result, we ended up with a data sample of 192 Mini Flash Crashes which accounts for 57,83% of our previous database as you can see on the Table 6. It is interesting to note that some crashes occur at more than 2,5 seconds but our study focuses only on Mini Flash Crashes corresponding to the Nanex Llc. definition, which is why we cutted some much data. Furthermore, we did not look at the number of tick *down* or *up* as we could not do it simply by looking at the graphs. We thus hypothesized that all Mini Flash Crashes were satisfying this condition. Moreover, we had at the beginning three other variables that we unfortunately dropped because we could not fill them as we wanted. These were TOTAL TRADES, TOTAL QUOTES and TYPE OF TRADES INVOLVED. For the first two, we could not find the data for the CRASH TIME period but for the entire yes, and for the last one, which was separated in "Regular", "Regular and ISO" and "Form-T", Nanex Llc. was not systematically precising the type of trade involved in the Mini Flash Crashes. Finally, this data sample represents only a tiny part of the big picture, as thousands of Mini Flash Crashes occur every year.

Table 6: Number of Crashes per Time Interval.

Interval in Sec.	N° of Crashes	Ratio
0-2,5	192	57,83%
2,5-30	86	25,90%
30-150	41	12,35%
>150	13	3,92%
Total général	332	100%

4.2 RESEARCH DESIGN AND METHODS

Our quantitative analysis is divided in two parts. The first one includes general measures while the second focuses on statistical methods. Their aim is to give us insights at which category could be included in our taxonomy of Mini Flash Crashes.

In the general measures, we will mostly look at the frequencies of our different variables. To do so, we created two new variables: PRICE AT THE BEG MOD and PRICE CHANGE MOD, based on two previous variables (PRICE AT THE BEGINNING and PRICE CHANGE). For these, we simply approximated the values of the variables so that our graphs will look clearer. In addition, we took out some extreme values that occur in really specific cases. For PRICE CHANGE, all the values over 100% were deleted and for PRICE AT THE BEGINNING, one value was removed because it was 1000 times higher than all the others. Finally, we separated the DOMINANT EXCHANGE and PRICE AT THE BEGINNING analysis in two categories of ASSET TYPE (Stock and ETF/Future) because they present highly different values.

In the statistical part, our aim is to determine the relationships between the different variables in term of dependence. Firstly, we will look at the descriptive statistics of the different quantitative variables: PRICE CHANGE, PRICE AT THE BEGINNING and CRASH TIME. Then, we will explore the relationship between our three most important qualitative variables: TIME OF THE DAY, CRASH TYPE and ASSET TYPE, and the two crucial quantitative variables for Mini Flash Crashes: PRICE CHANGE and CRASH TIME. After it, we will test the relationship of dependence between, first, the qualitative variables with a Chi-squared test, and then, the quantitative variables, using the “Pearson Correlation”. Next, we will analyze the impacts of independent variables on a dependent variable. We will take as dependent variables: PRICE CHANGE and CRASH TIME. As they are quantitative variables, we will use an ANOVA, a MANOVA and a multiple linear regression. The first two, to study the impact of qualitative variables on each dependent variable, and the last one, to analyze if the quantitative variables have a significant effect on each dependent variable. In fact, we created dummy variables for TIME OF THE DAY, CRASH TYPE and ASSET TYPE in order for them to be included in the multiple linear regression.

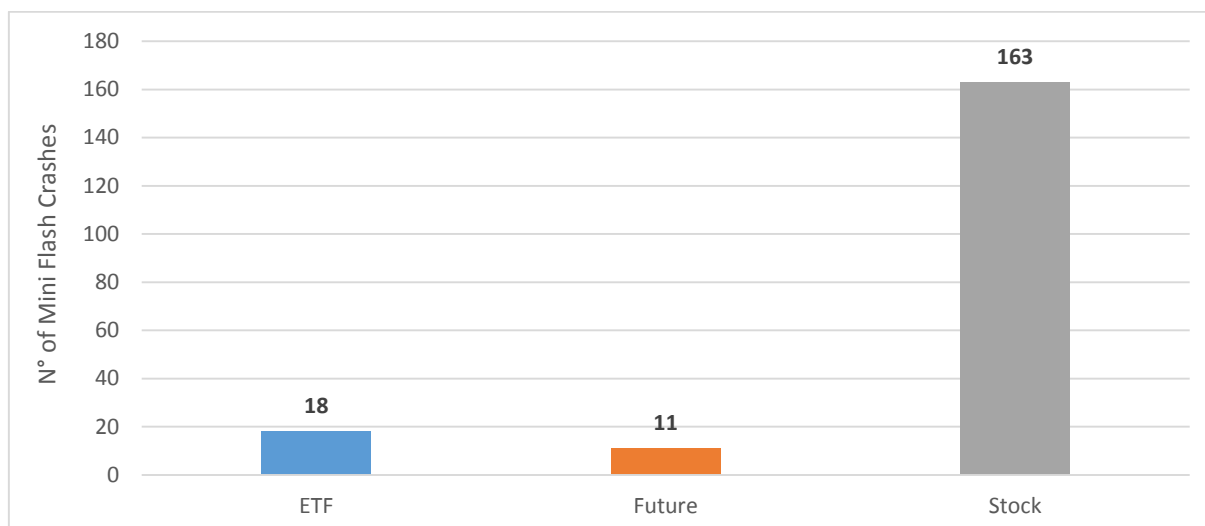
5. RESULTS

In this section, we will present the results of the two parts we just discussed.

5.1 SUMMARY OF MEASURES

For this part, we first analyzed the occurrence of Mini Flash Crashes for the different ASSET TYPE which are Stock, ETF and Future as you can see on the next figure.

Figure 22: Number of Mini Flash Crashes per Asset Type



This graph shows us that Stock is the most involved ASSET TYPE in the Mini Flash Crashes accounting for 163 on the 192 studied. ETF follows with 18 while Future only counts 11 Mini Flash Crashes.

Next, we compared the frequencies of Mini Flash Crashes on the different period of the trading day. To do so, we conducted an analysis of the variable TIME OF THE DAY which is divided in 4 periods of time: Opening period (9:30-9:50), Mid period (9:50-15:30), Closing period (15:30-16:00) and Extended hours (<9:30 and >16:00) and computed a ratio to compare the periods on the same scale. We also added a filter for the ASSET TYPE to look at their occurrence during the trading day as you can see on the next table.

Table 7: Frequencies of Mini Flash Crashes per Time Period

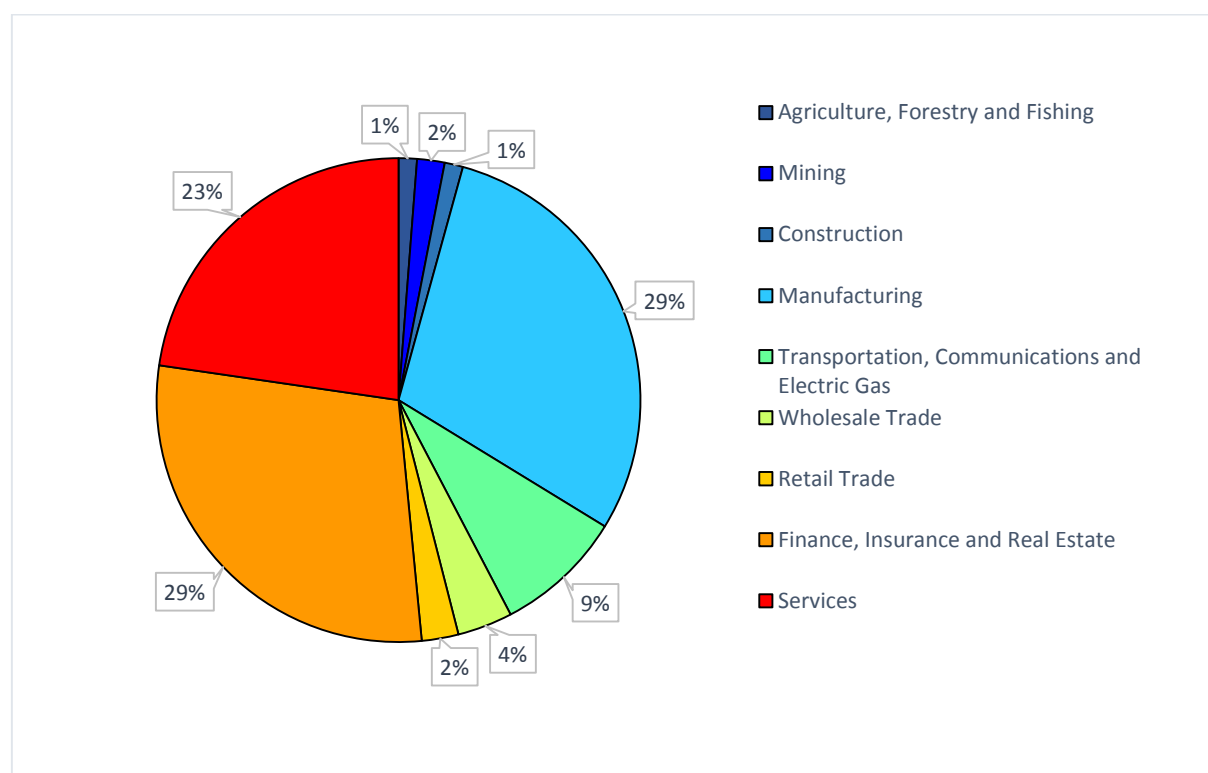
	Closing period	Opening period	Mid period	Extended hours
Future			4	7
ETF	1	3	11	3
Stock	26	38	85	14
Total	27	41	100	24
Number of hours	0,5	0,3333	6	17,5
Ratio per hour	54	112	16,66	1,37

Some interesting points can be deduced from this table. First of all, we can see that the Opening and Closing period possess the highest ratio per hour. This is in line with what Golub et al. discovered in 2011. These high ratios can be explained by the fact that these periods are known to be the most volatile of the trading day. Secondly, we can denote that Mini Flash Crashes in Future only occur in the Mid period or the Extended hours.

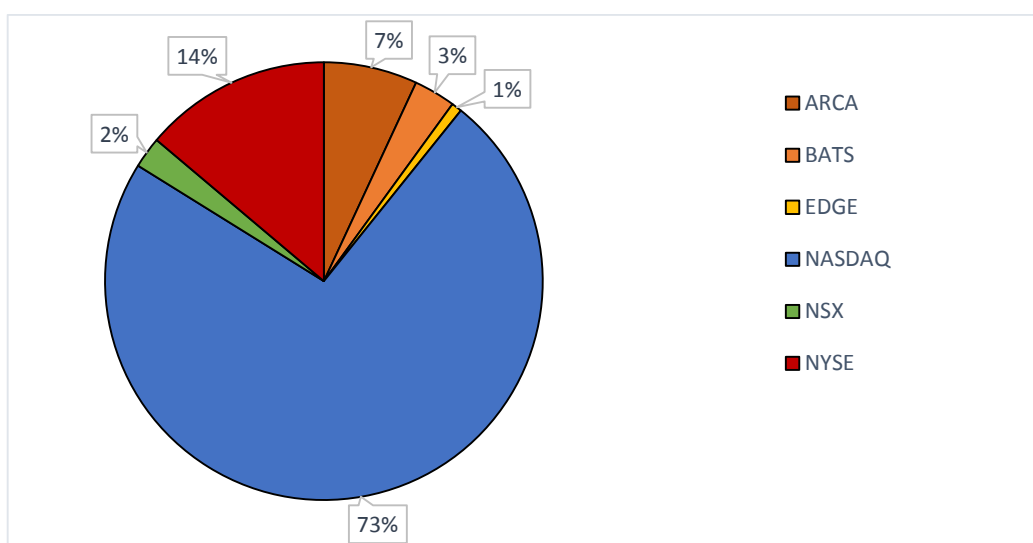
After it, we took a look at the division in SECTOR CONCERNED for Mini Flash Crashes happening in Stocks (See Figure 23).

The most targeted sectors are the Manufacturing sector, the Finance, Insurance and Real Estate sector and the Services sector. If we compare it to the findings of Golub et al. (2011), the Services sector records a sharp increase from 11% to 23%, approaching the two others which remain at the top.

Figure 23: Division in Sector Concerned for Mini Flash Crashes in Stocks

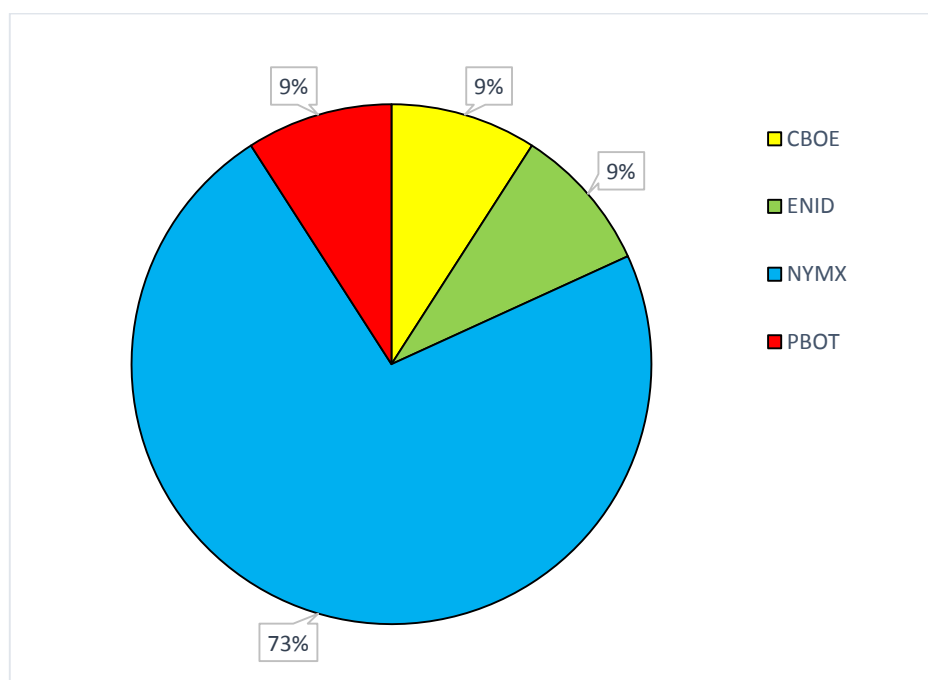


Then, we focused our analysis on the DOMINANT EXCHANGE in the Mini Flash Crashes. For the reminder, an exchange is considered as dominant in a crash if most of the trades are executed on it. For this part, we separated the analysis in two categories of ASSET TYPE: Stock and ETF/ Future, because they are traded on different type of exchanges. The next Figure presents the division for Stock and ETF.

Figure 24: Division of Dominant Exchange for Stock and ETF

Most of the trades seem to occur on NASDAQ accounting for 73%. NYSE and ARCA are the two following exchanges with respectively 14% and 7%. The rest of the exchanges, BATS, NSX and EDGE, are under the bar of 3% and account for the last 6%. We have here a huge difference with the findings of Golub et al. (2011). Indeed, NASDAQ has a much higher percentage, it was previously at 28%, which results in a drop of percentage for NYSE and ARCA.

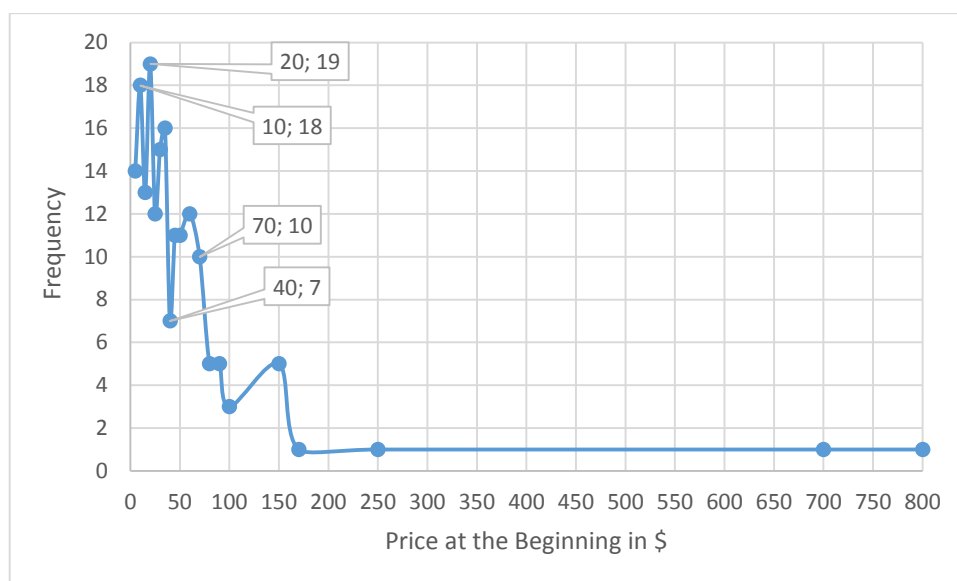
The following figure presents the division in DOMINANT EXCHANGE for Future.

Figure 25: Division in Dominant Exchange for Future

For the future contracts, Mini Flash Crashes occurred on four different exchanges. NYMX happens to be the most represented with 73% while CBOE, ENID and PBOT account each for 9%.

Next, we looked at the frequencies of the PRICE AT THE BEGINNING to discover if there was any trend. Again, we conducted an analysis for Stock and ETF, and one for Future, as this last category incorporates values 10 times higher than the previous one. Next figure presents the results for Stock and ETF.

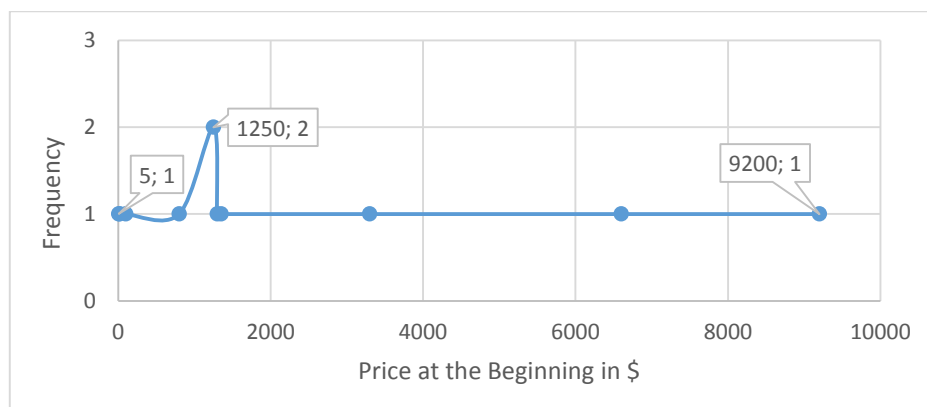
Figure 26: Frequencies of Price at the Beginning for Stock and ETF



Two notable peaks can be seen at values of 20\$ and 10\$. Moreover, most of the Mini Flash Crashes seem to occur before 70\$ with the exception of 40\$ which is less frequent than its neighboring values.

Concerning the Future, take a look at the next figure.

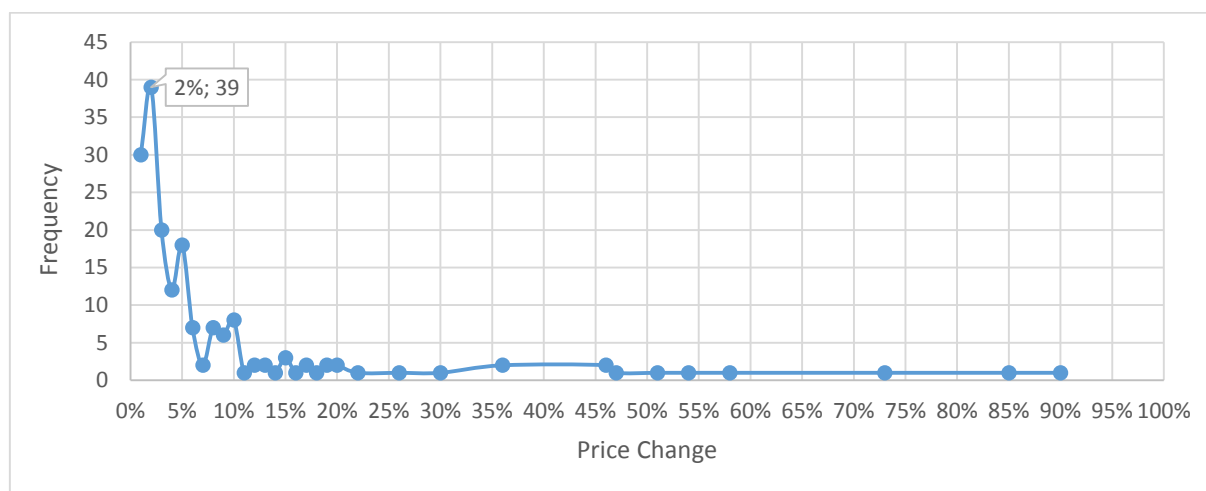
Figure 27: Frequencies of Price at the Beginning for Stock and ETF



We can see that the scale of prices is different from the previous graph where the maximum was at 800\$. Here, it goes from 5\$ to 9200\$ and that is why we separated our data sample in two categories based on ASSET TYPE. In addition, we can denote a peak at 1250\$.

Finally, we analyzed the frequencies of the different price changes for all ASSET TYPE (See Figure 28).

Figure 28: Frequencies of Price Changes



This graph shows a peak at 2% with a frequency of 39 and that most price changes for the studied Mini Flash Crashes possess a value of 5% or less.

5.2 SUMMARY STATISTICS

For this part, we started with the descriptive statistics of our quantitative variables which are CRASH TIME, PRICE AT THE BEGINNING and PRICE CHANGE. The following table presents the results:

Table 8: Descriptive Statistics of the Quantitative Variables

	N	Minimum	Maximum	Sum	Mean	Std. Deviation	Variance
CRASH TIME	192	1,0	2500,0	155575,0	810,286	679,5495	461787,577
PRICE AT THE BEGINNING	191	3,2	9194,0	33953,9	177,769	865,0848	748371,763
PRICE CHANGE	184	,80%	99,99%	1915,13%	10,4083%	19,28067%	371,744
N valide (liste)	183						

Nothing much is to say about this table. We can see that, as the definition implies, the minimum PRICE CHANGE is 0.8% and the maximum CRASH TIME is 2.5 seconds. In addition, you can see that only a part of the full data sample was taken into account for PRICE

AT THE BEGINNING and PRICE CHANGE. It is because these values were considered as extreme, for example sub-penny priced assets or PRICE CHANGE over 100%.

We then explored the relationship between our three most important qualitative variables: TIME OF THE DAY, CRASH TYPE and ASSET TYPE, and the two crucial quantitative variables for Mini Flash Crashes: PRICE CHANGE and CRASH TIME. It is particularly interesting to look at the mean for each variable's groups to discover if there are differences among them. This analysis only showed us that different means exist between groups of the same qualitative variable. You can find summary tables in Appendix B.

After that, we tried finding the existence of a relationship of dependence between our qualitative variables by evaluating the statistical significance (Chi-squared test). We thus tested two hypothesis:

$H_0: O_i = E_i \rightarrow$ The observed frequencies are the same as the ones expected (No relationship of dependence) or $H_a: O_i \neq E_i \rightarrow$ At least one frequency is different (Existence of a relationship of dependence).

The next tables present the results for each pair of qualitative variables:

Table 9: Chi-squared Tests between the Qualitative Variables.

CRASH TYPE * TIME OF THE DAY	Value	df	Asymp. Sig. (2-sided)
Pearson Chi- Square	1,219 ^a	3	,748
Likelihood Ratio	1,231	3	,746
N. of Valid Cases	192		

CRASH TYPE * ASSET TYPE	Value	df	Asymp. Sig. (2-sided)
Pearson Chi- Square	2,275 ^a	2	,321
Likelihood Ratio	2,214	2	,331
N. of Valid Cases	192		

TIME OF THE DAY * ASSET TYPE	Value	df	Asymp. Sig. (2-sided)
Pearson Chi- Square	31,773 ^a	6	,000
Likelihood Ratio	24,732	6	,000
N. of Valid Cases	192		

As we can see on the first two tables, the p-value is over our threshold of 0.05. Therefore we cannot reject H_0 which means that there is no relationship of dependence between CRASH TYPE and TIME OF THE DAY, and between CRASH TYPE and ASSET TYPE. However, the third table shows a p-value of 0.000 and it is clearly under 0.05. Thus, a dependence's relationship exists between TIME OF THE DAY and ASSET TYPE. The number of objects that falls in those two categories do not differ significantly from the expected number.

Then, we tested the relationship of dependence between the quantitative variables by analyzing the degree of linear association. For that, we used the "Pearson Correlation" and our two hypothesis were the following:

$H_a: \rho = 0 \rightarrow$ No linear correlation. (No relationship of dependence) or $H_a: \rho \neq 0 \rightarrow$ The linear correlation is different than 0. (Existence of a relationship of dependence).

The next table presents our findings:

Table 10: Correlations between the Quantitative Variables

		CRASH TIME	PRICE AT THE BEGINNING	PRICE CHANGE
CRASH TIME	Pearson Correlation	1	-,064	-,067
	Sig. (2-tailed)		,377	,365
	N	192	191	184
PRICE AT THE BEGINNING	Pearson Correlation	-,064	1	-,070
	Sig. (2-tailed)	,377		,346
	N	191	191	183
PRICE CHANGE	Pearson Correlation	-,067	-,070	1
	Sig. (2-tailed)	,365	,346	
	N	184	183	184

In this case, all the p-values (colored in blue) are higher than our threshold of 0.05. As a result, we can keep H_0 which means no relationship of dependence exists between our quantitative variables.

Next, we proceeded to an ANOVA to help evaluating the deviation of the mean value of a dependent variable under the effect of multiple independent variables. Our hypothesis were the following:

H0: the mean of all groups is the same or Ha: there is at least one mean which is different.

We also computed a test of homogeneity of variances to see if we were using an ANOVA or the test of WELCH (same purpose) and a multiple comparison analysis when H0 was rejected to determine the significant differences between the groups.

In this section, we will not present the results on tables as there is too much to show. However, we will discuss them.

We will divide the discussion in two parts, one per independent variable (PRICE CHANGE and CRASH TIME). We took those and not PRICE AT THE BEGINNING because they are the most important when talking about Mini Flash Crashes.

Firstly, we will analyze the results for PRICE CHANGE. At the beginning, we tested the homogeneity of variances to determine the type of test we would have to use. For TIME OF THE DAY and TYPE OF CRASH, the hypothesis is not respected. We then had to use the WELCH test for those and an ANOVA for ASSET TYPE. The p-values in the case of TIME OF THE DAY and TYPE OF CRASH are under 0.05 which means that at least one mean value for one group in these variables was different from the other. On the opposite, ASSET TYPE possesses a p-value of 0.266 which implied that H0 cannot be rejected. Finally, we computed a multiple comparison analysis for the TIME OF THE DAY mean values and not for TYPE OF CRASH because it has only two different groups. We found a significant difference in mean values for the groups “Opening period” and “Mid period”. As a result, we can affirm that those two groups and the groups “Up” and “Down” differ highly in term of PRICE CHANGE.

Secondly, we will analyze the results for CRASH TIME. In this case, all the p-values for the test of homogeneity of variances are over our threshold. As a result, we could use an ANOVA for all the dependent variables. We only had to reject H0 in the case of TYPE OF CRASH because it has a p-value of 0.016, the other dependent variables satisfying the hypothesis that the means of their groups are similar. As a result, we did not have to conduct a multiple comparison analysis and we can conclude that the groups “Up” and “Down” differ highly in term of CRASH TIME.

Following the ANOVA, we decided to do a MANOVA. The difference lies in the fact that we are analyzing all the simultaneous effects of the independent variables on the dependent variable rather than doing it one by one. Again, we divided our analysis in two for PRICE CHANGE and CRASH TIME.

We will start with the results concerning PRICE CHANGE. Unfortunately for this one, we were not able to realize the MANOVA as H_0 was rejected in the test of homogeneity of variances. On the other hand, H_0 was not rejected in the case of CRASH TIME. Indeed, the p-value is equal to 0.246 which means the hypothesis of homogeneity of variances is respected. Therefore, we conducted a MANOVA.

The results allowed us to affirm that the independent, qualitative variables have no significant effect, even simultaneously, on the dependent variable. Indeed, no p-value is under the threshold of 0.05 and so, we cannot reject H_0 which implies a significant effect on CRASH TIME. Furthermore, we deduced that the variation of these independent variables can explain 9,7% of the total variance of the dependent variable (See Appendix B for a summary of the results).

Finally, we conducted a multiple linear regression. Its purpose is to determine the relation of causality between a quantitative dependent variable and quantitative independent variables. Again, we did it two times for PRICE CHANGE and CRASH TIME. In addition, we created dummy variables from the qualitative variables: TIME OF THE DAY, CRASH TYPE and ASSET TYPE, in order to incorporate them. One was created for CRASH TYPE (1 for “Up” and 0 for “Down”), four for TIME OF THE DAY (one per group) and three for ASSET TYPE (one per group).

For this part, three steps needed to be done. Firstly, we had to look at R^2 to see if the model was explaining at least a bit of the total variance of the independent variable ($R^2 > 0$). Secondly, we had to verify the existence of a linear regression’s model. For this, we conducted an ANOVA and the p-value had to be under 0.05, rejecting H_0 and confirming the existence of a model. Lastly, we had to verify these hypothesis:

$H_0: \beta = 0 \rightarrow$ the coefficient is equal to 0 (No significant impact from the related independent variable) or $H_a: \beta \neq 0 \rightarrow$ the coefficient is different than 0 (The related independent variable has a significant impact).

We will begin with the results for PRICE CHANGE. The first two tests were a success. Indeed, the R^2 is equal to 0.117 and the p-value, for the ANOVA, to 0. We then looked at the coefficients and the significance of the independent variables as you can see on the next table.

Table 11: Test of the Coefficients of the Multiple Linear Regression for PRICE CHANGE

Model		Non standardized coefficients		Standardized coefficients	t	Sig.
		B	Standard error	Bêta		
1	(Constante)	10,528	2,523		4,174	,000
	PRICE AT THE BEGINNING	,000	,002	-,016	-,175	,861
	CRASH TIME	-,002	,002	-,079	-1,065	,288
	TYPEOFCRASH_10	-5,270	2,800	-,138	-1,882	,061
	Openingperiod_1	9,073	3,510	,197	2,585	,011
	Closingperiod_1	5,568	3,876	,108	1,437	,153
	Extendedhours_1	14,233	4,665	,247	3,051	,003
	ETF_1	-2,483	4,592	-,039	-,541	,589
	Future_1	-13,741	7,440	-,178	-1,847	,066

Firstly, we can see that Midperiod_1 and Stock_1 were excluded from the model. The reason is that it would have caused a problem of perfect collinearity in the case they were included. Secondly, we can deduce that two independent variables (Openinperiod_1 and Extendedhours_1) seem to have a significant impact as they both possess a p-value under 0.05.

After it, we excluded all the variables with a p-value over 5%. By doing so, we got this model:

Table 12: Adjusted Test of the Coefficients of the Multiple Linear Regression for PRICE CHANGE

Model		Non standardized coefficients		Standardized coefficients	t	Sig.
		B	Standard error	Bêta		
1	(Constant)	10,389	1,998		5,200	,000
	Futures_1	-13,817	6,297	-,170	-2,194	,030
	TYPEOFCRASH_10	-6,664	2,835	-,167	-2,350	,020
	Openingperiod_1	9,248	3,465	,193	2,669	,008
	Extendedhours_1	12,374	4,739	,205	2,611	,010

If we compare it to the previous model, we now have four independent variables possessing a significant impact on the dependent variable: PRICE CHANGE, the two previously mentioned as well as Typeofcrash_10 and Futures_1.

Concerning the multiple linear regression for CRASH TIME, we had to stop it at the ANOVA level. In fact, the model has a R^2 equal to 0.076 but the p-value in the ANOVA is over the threshold of 0.05. Thus, we could not continue our analysis.

6. A TAXONOMY OF MINI FLASH CRASHES

This is the final section of the paper before the conclusion. Therefore, we will answer the research question, which is: “Is it possible to determine a taxonomy of Mini Flash Crashes?”, and discuss the findings in accordance to the qualitative and quantitative parts of this thesis.

6.1 ANSWERING THE RESEARCH QUESTION

In response to the research question: “Is it possible to determine a taxonomy of Mini Flash Crashes”, we would say yes and can support the affirmative response. To do so, we will present all the categories of the classification and explain why we decided to include them in our taxonomy based on the qualitative and quantitative parts of this paper.

As a first category, we decided to take the one that seems the most obvious: the type of crash. For this category, two groups exist: Up Crash or Down Crash. To begin with, the definition of Nanex Llc. which is used by all the researches about Mini Flash Crashes, already highlighted the difference between the two groups. Furthermore, it was proven in our quantitative part with the ANOVA analysis that the “Up” and “Down” groups possess highly different means if we look at PRICE CHANGE and CRASH TIME. In conclusion, this category is worthy to be part of the taxonomy as the two groups show big differences.

For the second category, we took ‘time of the day’. This category is separated in four groups: the Opening period (9:30-9:50), the Mid period (9:50-15:30), the Closing period (15:30-16:00) and the Extended hours (<9:30 & 16:00>). Firstly, the different periods show great differences in term of frequencies. As shown by Golub et al. (2011) and our quantitative analysis, the Opening period and the Closing period possess high frequencies regarding Mini Flash Crashes in comparison to the Mid period. This can easily be explained by the well-known high volatility of these two periods, starting and ending the trading day. Secondly, Extended hours needs to be added as a group because it represents a particular period with a specific type of trades called Form-T trades as discussed in the last example of Mini Flash Crashes in part 3.3. Thirdly, we

found with the ANOVA that the groups of “Opening period” and “Mid period” were highly different for their means in term of PRICE CHANGE. Lastly, the multiple linear regression showed us that the groups of “Opening period” and “Extended hours”, taken as independent variables, have a significant impact on the PRICE CHANGE dependent variable. These are the reasons why we included the category ‘time of the day’ in our taxonomy.

Our third category is price change. This one is separated in two groups: Simple change and Big change. The ‘Simple change’ group includes all the Mini Flash Crashes with a price change inferior to 100% and therefore, the ‘Big change’ group incorporates the others. We used this separation because in our analysis and in the one of Golub et al. (2012), we both excluded these Mini Flash Crashes due to their extreme values that are specific to certain kind of events.

We decided to take as a fourth category the dominant exchange. We think all Mini Flash Crashes should be classified depending on the exchange on which most of the trades are executed. Indeed, differences seem to happen if we analyze the situation of Mini Flash Crashes occurring on different exchanges. For example, Golub et al (2011) found a negative correlation between the average price before a crash and the intensity of it only on the NYSE. That is why we put this category in our taxonomy.

In 2012, Golub et al. brought to light a new kind of classification for Mini Flash Crashes based on the type of trades present in a crash. They proposed three groups: ISO initiated, a crash with ISO marked trades, auto-routing initiated, a crash with only regular marked trades and unclassified (The exact definitions for this these categories were detailed in part 3.2.2.). We decided to include this category in our taxonomy, but we also added another group: Form-T-initiated. This group will include all Mini Flash Crashes with trades marked as Form-T, except the first k trades that can be marked as non-Form-T if executed within the least market-wide available best quotes in the last one second. We will thus have, as our fifth category, the type of initiation with four groups: ISO initiated, Auto routing initiated, Form-T initiated and Unclassified.

Lastly, we decided to take as our last category the asset type. The category is divided in three groups: Stock, ETF and Future. The reason for this choice is that the ‘Future’ group seems to possess a significant impact on PRICE CHANGE, if we take it as the dependent variable, while the other groups do not show this.

6.2 DISCUSSION OF RESEARCH FINDINGS

In the end, only six categories compose our taxonomy of Mini Flash Crashes. It might seem small but the current studies and researches only allow the inclusion of the ones exposed here. Indeed, no quantitative studies have ever given any arguments in favor of the inclusion of other categories. We might think of the price at the beginning of the crash or the year, etc. but those did not show enough reasons to create an entire classification based on them. The thing is that analyzing Mini Flash Crashes is an extremely difficult exercise to realize, not only because thousands of them happen every year, but also because it would require extremely well developed technologies, such as super computers, to analyze the millions of lines of data needed to do a proper analysis, as well as access to all of these data.

7. CONCLUSION

We tried through this entire paper to determine if a taxonomy of Mini Flash Crashes was possible and if yes, which categories should be included in it. To begin with, we researched background information to understand fully the basis of our topic. We started by revising the entire history of the financial markets, starting with the creation of the first exchange in London to the current days with the rise of computers and market fragmentation. It allowed us to discover that improvements in that field have always been linked with the speed of communication and data processing. We then defined High-Frequency trading and described its strategies and characteristics. It gave us a better understanding of that phenomenon which is crucial in the current state of the financial markets as it represents more than 50% of all the U.S. equity-exchange trading volume. In addition, we understood that HFT is not a strategy in itself but rather a way to implement a trading strategy. After it, we took a look at the most famous of all the crashes: the 6th May 2010 Flash Crash. We tried describing it as much as we could since this event was the trigger for all the researches about HFT, anomalies and crashes in the financial markets. Finally, we looked at the current regulatory environment and what regulations have been implemented in response to the Flash Crash events.

In the subsequent section of this paper, we undertook an extensive literature review of our topic. We mainly focused on two parts: the causes of the Flash Crash and Mini Flash Crashes. For the first one, we discovered that the Flash Crash was not caused by a simple factor but by many different factors. For example, HFT processes, firstly accused of being the only cause, were found to have exacerbated the crash rather than being a trigger. As other causes, we can quote high levels of market fragmentation, order flow toxicity or an over-use of ISOs. Next, we

looked at Mini Flash Crashes. We first explained the definition of Nanex Llc. which we used in our paper. We then reviewed three different studies concerning the evolution of Mini Flash Crashes from 2006 to 2011. The first, from Nanex Llc., focuses on the increasing number of them along these years. The second one, from Golub et al. (2011), delivers primary measures on Mini Flash Crashes, from the average price change to their distribution into sectors. The last one, from Golub et al. (2012), proposes interesting findings. They found that Mini Flash Crashes are caused by regulation framework and market fragmentation, and more especially, by the aggressive use of ISOs and exploitation of the SEC Rule 611. Lastly, we presented five typical cases of Mini Flash Crashes.

The qualitative part of our paper being finished, we moved on to the quantitative part. Firstly, we described the way we constructed our data sample based on Mini Flash Crashes data from May 2011 to September 2014, available on the Nanex Llc. website. Secondly, we presented our research design and methods. We explained the analysis we wanted to do and how we modified our data sample in order to conduct them. Thirdly, we showed our results from a statistical and metric perspective. The metric part allowed us to draw frequencies for most of our variables and the statistical part permitted us to determine the relationships between the different variables in terms of dependence. We found that the groups “Up” and “Down” were highly different in terms of mean for CRASH TIME and PRICE CHANGE. Furthermore, the groups “Opening period” and “Mid period” showed big differences in means for PRICE CHANGE. Finally, we found that the independent variables “Openingperiod_1” and “Extendedhours_1” have a significant impact on the dependent variable PRICE CHANGE.

In the last section, we presented our answer to the research question: “Is it possible to determine a taxonomy for Mini Flash Crashes?” and the answer is yes. We decided to include six categories of classification, each divided into groups. The first category is the type of crash with two groups: Up crash and Down crash. The second one is the time of the day. It possesses four groups: Opening period (9:30-9:50), Mid period (9:50-15:30), Closing period (15:30-16:00), Extended hours (<9:30 & >16:00). The third category concerns the price change with two groups: Simple Change (<100%) and Big Change (>100%). As fourth category, we have the dominant exchange or the exchange on which most trades were executed during the Mini Flash Crash. Finally, the fifth category is the type of initiation, separated in four groups: ISO initiated, Auto routing initiated, Form-T initiated and Unclassified and the last category is the asset type, composed of three groups: Stock, ETF and Future.

Concerning future research, we will advise people with access to advanced technologies, such as supercomputers, and that can construct a more furnished database than the one used in this paper, to further the analysis to determine the relationships in term of dependence between the variables in a Mini Flash Crash in order to improve the taxonomy started in this thesis. Moreover, since the literature around Mini Flash Crashes is still not well furnished, any study on the impacts or causes of them would be warmly welcomed by the community of persons interested in the subject.

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