UCL
Université catholique de Louvain

École polytechnique de Louvain (EPL)

Real-time optimisation of a wastewater treatment network

Dissertation presented by
Ayoub Ben Ayed

for obtaining the Master's degree in
Electrical Engineering

Supervisor(s)
Denis Dochain, Vincent Rocher

Reader(s)
Julien Hendrickx, Carlos Eduardo Robles Rodriguez

Academic year 2017-2018

In the preamble of this master thesis, I wish to express my most sincere gratitude for the people who have helped me and contributed to the elaboration of this work as well as the success of this year study.
I would like to thank sincerely M. Denis Dochain, who as promoter, has always showed responsivity and availability all the year long, and for his inspiration, time and help.
I thank the SIAAP and more particularly M. Vincent Rocher and M. Jean Bernier for the proposition of this interesting and motivating research topic and the logistic support.
I also express my gratitude to M. Carlos Eduardo Robles Rodriguez who offered me his support, time and mentored me all along the elaboration of this work.
Finally, I address my most heartfelt greetings to all my relatives and friends who always supported and cheered me up during the realisation of this master thesis.
Thanks to all.

Contents

Chapter 1

Context

Wastewater treatment is a public health issue; every day 4000 children below 5 years die from diarrhea due to the drinking of non-treated water. Moreover, every year, 2 millions of tons of sewage are discharged in the river; coming from sewers, industrial or agricultural waste [2].

Wastewater treatment is also an environmental issue; the presence of excess in phosphorous and nitrate nutrients in water induces the growth of aquatic plants and algae. This process known as eutrophication results in oxygen depletion, ecosystem deterioration and aquatic fauna extinction.

Pharmaceutical residues are also found in the effluents through human dejection. It has been demonstrated that oestrogens feminises the fishes [3] and the anxiolytic increases their aggressivity [4]. In France, fourth worldwide greatest pharmaceuticals consumer, more than $100\mu g/L$ of residues are found in wastewater and concentration of the order of ng/L in the drinking water [5] although the risks and effects are still not well evaluated on the human body.

Different policies have been put into place to regulate the transport and treatment of the wastewater as well as the quality of the water. Early European water legislation began with standards for rivers and lakes used for drinking water in 1975, and culminated in 1980 in setting binding quality targets for drinking water. It also included quality objective legislation on fish waters, shellfish waters, bathing waters and groundwaters [6].

The Council Directive 91/271/EEC concerning urban wastewater treatment was adopted on 21 May 1991 to protect the water environment from the adverse effects of discharges of urban wastewater and from certain industrial discharges [7]. On 27 February 1998 the Commission issued Directive 98/15/EC amending Directive 91/271/EEC to clarify the requirements of the Directive in relation to discharges from urban wastewater treatment plants to sensitive areas which are subject to eutrophication [8].

This directive has been transposed into French law in 1994 and depending on the agglomeration size requests an abatment of 70% of the nitrates. Regarding Belgium, the situation is more complex because each level of government is responsible for the transposition of European directive in its jurisdiction [9]. As such, Belgium was sentenced two times regarding this directive, once in 2004 because he didn't transposed it and second time in 2013 for impairments [10].

In 2000, the Directive 2000/60/EC, also known as the Water Framework Directive (WFD), imposes to the European state members the return of the good chemical and ecological state (see table 1.1) of the surface and ground water within 2015. The state is calculated from the 90 percentile of the concentrations present in the river on two consecutive years. Transposed in 2004 in the French law, it also requests a non-deterioration of the actual water quality and the

Quality parameters by element	Classes state limits				
	High	Good	Moderate	Poor	Bad
Oxygen Balance					
Dissolved oxygen (mg O_2 / L)	8	6	4	3	
Saturation level in dissolved O_2 (%)	90	70	50	30	
DBO5 (mg O_2 / L)	3	6	10	25	
Dissolved organic carbon (mg C / L)	5	7	10	15	
Temperature					
Cyprinid waters	24	25.5	27	28	
Nutrients					
PO_4^{3-} (mg PO_4^{3-} / L)	0,1	0,5	1	2	
Total phosphorus (mg P / L)	0,05	0,2	0,5	1	
$NH4^+$ (mg NH_4^+ / L)	0.1	0.5	2	5	
NO_2^- (mg NO_2^- / L)	0.1	0.3	0.5	1	
NO_3^- (mg NO_3^- / L)	10	50	-	-	
Acidification					
Minimum pH	6.5	6	5.5	4.5	
Maximum pH	8.2	9	9.5	10	
Salinity					
Conductivity (mS / m)	-	-	-	-	
Chlorides (mg Cl^- / L)	-	-	-	-	
Sulphates (mg SO_4^{2-} / L)	-	-	-	-	

Table 1.1: Classes states of the Seine quality in function of the concentrations of the different quantities in an observation point

interruption of the rejection of dangerous substance before 2020.

In Paris the organism in charge of address those issues is the SIAAP (Syndicat Interdépartemental pour l'Assainissement de l'Agglomération Parisienne). This syndicate is in charge of the treatment of waste, pluvial and industrial water for 9 millions people in the Paris agglomeration. In addition, the SIAAP investigates and develops different solutions to improve the performance of their network.

In this context, the master thesis was interested on developing a control strategy to optimise the partitioning of sewage between the wastewater treatment plants (WWTP) of the network. Based on a simplified model of the network and real measurements of the Seine river, we have shown that model predictive control (MPC) provides a suitable solution.

The master thesis is organised as follow. First a state of the art on model predictive control is presented. Its most common formulation is reviewed and is expressed as quadratic program optimisation problem. Some feasibility and stability property will be formulated. Specifities of Matlab model predictive toolbox used in this thesis for simulation will be presented before describing the wastewater treatment network (WWTN) and its modelling. Plant model performance will be assessed against real measures, before describing the systematic approach used for MPC parameters tuning. Finally, the control law efficiency will be evaluated on different scenarios and compared against real measurements.

Chapter 2

Model Predictive Control

2.1 Overview

Model Predictive Control (MPC) is a class of control techniques first derived from Internal Model Control, or IMC, and is widely applied in the process industries due to its capability to deal with constraints in an optimal fashion. As its name suggests, MPC is based on predictions of setpoint tracking behaviour or disturbance rejection over both past controlled and manipulated variables measurements, in which each prediction is followed by an optimisation routine to find the optimal input for the closed loop response imposed by a certain criteria, such as maximising a profit function or production rate [11].

The success of the MPC relies on its ability to easy handle multivariate systems, complicated dynamics (e.g. non-linear, time varying plant, inverse response, important delays,...) and constraints on controlled and manipulated variables. It also presents a systematic approach, thus applicable to a wide range of problems and easier design maintenance. In fact, changing model or specifications does not require complete redesign and can sometimes be done on the fly (Adaptive MPC) [12]. Originally reserved for very slow bio-chemical process, MPC is now successfully used to solve problems in automotive and aeronautical industry thanks to the development of fast and low complexity algorithms (Explicit MPC) [13][14].

However, MPC requires a (simplified) prediction model as every model-based technique, full state estimation (observers) and may suffer from large computation issues in its classical formulation. Moreover, calibration of MPC can be very complex and often requires additional level of expertise due to the high degree of freedom (weights, horizon, constraints,...)[15].

2.2 History of the MPC

The development of predictive control model takes inspiration of the formalism and methodology developed by Kalman et al. in the early 1960s [16][17]. He demonstrated that the optimal control law for linear time-invariant (LTI) system was the one minimising a quadratic objective function of the states and inputs. Although the resulting control law has many nice properties including that of closed loop stability, it had very low impact in the development in the industry process. The reason for that, is that there were no constraints in its formulation and the non-linearities of real system [18].

In the late 1970's, taking the advantage of digital computers, model predictive control developped and reached successful applications in the industry. The most important one were by Richalet et al. [19] which proposed and heuristic approach (MPHC), later known as Model Algorthmic Control (MAC); and by Cutler and Remaker [20] which intrduced Dynamic Control

Matrix (DMC).

Their strategies relied on the use of a dynamic model of the process (impulse response for the former and step response for the latter) to predict the effect of future control actions. They were determined repeatedly by minimising at each sampling period, the predicted error subject to operational constraints. However, their initial version of MPC were not automatically stabilising and required stable plant and large horizon compared to settling time to achieve it [21].

As a second wave, QDMC algorithm was presented in 1983 by Cutler et al [22]. Its approach is based on quadratic programming to solve the constrained open-loop optimal control problem. In its formulation the system is linear, the objective function quadratic and the constraints are described by linear inequalities. Although its systematic approach to integrate constraints, QDMC presents no clear way to handle unfeasible solution[18].

Later on, IDCOM-M (Identification and Command) and SMOC (Shell Multivariate Optimizing controller) emerged as a third generation of MPC. The first, successor of MPHC, is presented in a paper by Grodidier Froisy, and Hamman in 1988 [23]. It uses two separate objectives functions, one for the output and another for the input in case of extra freedom degree. Each output is driven as closely as possible to a desired value at a point in time known as the coincidence point.

On the other hand SMOC was the first to use a process model in the form of state equations, an approach which is currently dominant in research studies concerning the MPC algorithms. Moreover, it also used Kalman filter to estimate the states to measure disturbances form measured outputs [24].

Over the last years a fourth generation of MPC was developed and are still in research presently to address non-linear process (DMC+) and robustness to model uncertainty (Robust MPCT). Decentralized MPC is also a rather common trend used in the industry for large scale application. In place of maintaining an operating extremely large MPCs, they are often split into smaller MPCs that are installed on subsystems of the original process.

However, distributed MPC with cooperating controllers, is a new concept that is yet to take in the process industry (Camponogara et al 2002; Christofides et al 2013)[25]. Finally, Econonmic MPC (EMPC) raise lots of interest recently; its central idea is to use a single MPC objective function to control and optimise economic conditions (Ellis et al. 2014)[26] [25]. The evoltion and different generations of MPC are shown on the figure 2.1.

2.3 Formulation of the MPC problem

MPC is based on iterative, finite-horizon optimisation of a plant model. At time k, the current plant state is sampled and based on the process model, estimated over p sampling period called Prediction Horizon. The MPC elaborate the control strategies by minimising a defined objective function so that the predicted output reaches the reference trajectory as we can see on the figure2.2. The control horizon m corresponds to the number of control moves considered during the prediction horizon. At each iteration, only the first control move is kept, the others are discarded and the operation is repeated on the next sampling period. The prediction horizon keeps being shifted forward and for this reason MPC is also called receding horizon control[27].

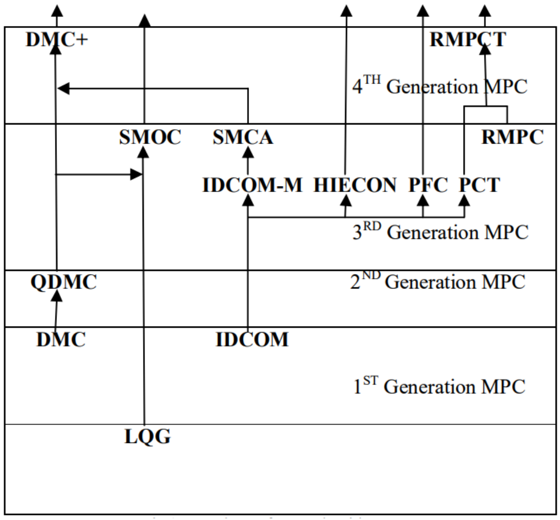

Figure 2.1: Evolution of the MPC [18]

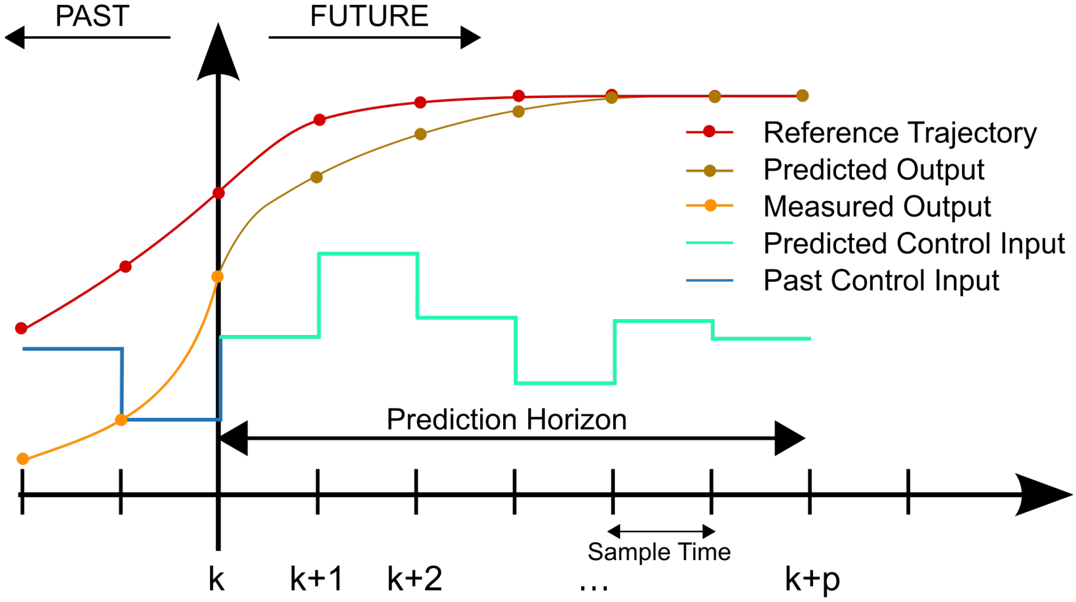

Figure 2.2: MPC basic scheme [27]

Objective function

Depending on the MPC formulation, the cost function J may differ, but generally it depends quadratically on the error between the reference r and the predicted outputs y as well as the manipulated variable variations Δu [14],[28]. Assuming a plant with n_u manipulated variables, n_y measured outputs and a prediction horizon p; the cost function to minimise at instant k is equal to;

$$J(z_k) = \sum_{j=1}^{n_y} \sum_{i=1}^{p} w_{i,j}^{y} [r_j(k+i) - y_j(k+i|k)]^2 + \sum_{j=1}^{n_u} \sum_{i=0}^{p-1} w_{i,j}^{\Delta u} [\Delta u_j(k+i|k)]^2$$

where z_k is the decision, given by;

$$z_k = [\, u(k|k)\, u(k+1|k) \dots u(k+p-1|k)\,]$$

$\Delta u_j(k+i|k) = u_j(k+i|k) - u_j(k+i-1|k)$ is the control move at the ith prediction horizon step and w_j^y and $w_j^{\Delta u}$ are the tuning weight that are constant with respect to the index i=1:p.

Note that alternatively, one may rewrite the cost function in matrix form;

$$J(z_k) = (Y_k - R_k)^T Q(Y_k - R_k) + U_k^T R_{\Delta u} U_k$$

where Q (n_y-by-n_y) and $R_{\Delta u}$ (n_u-by-n_u) are positive semi-definite weight matrix, and;

$Y_k = [\,y(k+1|k) \ldots y(k+p|k)]$ is the matrix of the predicted outputs.

$R_k = [\,r(k+1) \ldots r(k+p)]$ is the matrix of reference trajectory.

$U_k = [\,\Delta u(k|k) \ldots \Delta u(k+p|k)]$ is the matrix of control moves.

Constraints
The cost function is also generally subject to constraints on the manipulated variables, on manipulated variables rate, and on the outputs. At instant k,

$$\begin{aligned} u_{min}(k+j) \leq &\quad u(k+j|k) &\leq u_{max}(k+j) &\qquad \text{j=0,..., N-1} \\ \Delta u_{min}(k+j|k) \leq &\quad \Delta u(k+j|k) &\leq \Delta u_{max}(k+j) &\qquad \text{j=0,..., N-1} \\ y_{min}(k+j) \leq &\quad y(k) &\leq y_{max}(k+j) &\qquad \text{j=1,..., N} \end{aligned}$$

Note that some constraints are implicit; the control horizon m forces some manipulated variable increment to be zero and the state observer used for plant prediction is a set of implicit equality constraints.

2.3.1 Resolution of the optimisation problem

The optimisation problem is generally solved by convex quadratic programming (QP). The quadratic problem with n variables and m constraints is formulated as follows [29],[30]:

$$\text{Minimize } \tfrac{1}{2}x^T P x + c^T x$$
$$\text{Subject to } Ax \leq b$$

where b and c are respectively real m- and n-dimensional vector. P is n-by-n symmetric matrix while A is mxn dimensional real matrix.

It has been proved that the optimisation problem is convex if P is a positive semi-definite matrix. This means that if a local optimum is found, it corresponds to the global optimum. Also it ensures the problem is tractable and many algorithms are able to solve it polynomial time (ellipsoid method, active set, augmented Lagrangian,...). However, if P is indefinite, then the problem is known to be NP-hard [15].

Expression of the MPC as a QP problem
Let's first assume we are provided with a linear plant discrete state space representation. For non-linear system, the state space representation is most of the time obtained by linearisation of the plant around its nominal value;

$$\begin{cases} x[k+1] & = Ax[k] + Bu[k] \\ y[k] & = Cx[k] \end{cases}$$

Assume we minimise the cost function at instant k, let's express the predicted output in function of the control moves. From the state space representation,

$$y(k+j|k) = Cx(k+j|k)$$

Therefore,

$$y(k+1|k) = Cx(k+1|k) = Ax(k|k) + Bu(k|k)$$

If we express the input based on the previous input and the control move;

$$u(k|k) = u(k-1) + \Delta u(k|k)$$

We obtain,

$$y(k+1|k) = C\left[Ax(k) + B\left(u(k-1) + \Delta u(k|k)\right)\right]$$

Computing the states recursively we can show,

$$y(k+j|k) = C\left[A^j x(k) + \sum_{h=0}^{j-1} A^{j-h-1}B\left(u(k-1) + \sum_{i=0}^{h} \Delta u(k+i|k)\right)\right]$$

Alternatively in the matrix form, we obtain;

$$Y_k = S_x x(k) + S_{u1}u(k-1) + S_u U_k$$

with,

$$S_x = \begin{bmatrix} CA \\ CA^2 \\ ... \\ CA^p \end{bmatrix}, S_{u1} = \begin{bmatrix} CB \\ CB + CAB \\ ... \\ \sum_{h=0}^{p-1} CA^{p-h-1}B \end{bmatrix},$$

$$S_u = \begin{bmatrix} CB & 0 & ... & 0 \\ CB + CAB & CB & ... & 0 \\ ... & ... & ... & ... \\ \sum_{h=0}^{p-1} CA^{p-h-1}B & \sum_{h=0}^{p-2} CA^{p-h-1}B & ... & CB \end{bmatrix}$$

By replacing Y_k in the objective function and letting $G_k = S_x x(k) + S_{u1}u(k-1) - R_k$, we obtain;

$$J(z_k) = (G_k + S_u U_k)^T Q(G_k + S_u U_k) + U_k^T R_{\Delta u} U_k$$

After distributing and regrouping the terms we obtain,

$$J(z_k) = U_k^T[S_u^T Q S_u + R_{\Delta u}]U_k + 2G_k^T Q S_u U_k + G_k^T Q G_k$$

Since G_k doesn't depend of the control moves and minimising z_k and U_k are equivalent, the cost function to minimise can be reduced to,

$$J(U_k) = U_k^T[S_u^T Q S_u + R_{\Delta u}]U_k + 2G_k^T Q S_u U_k$$

We observe that the cost function is of the form of the one in the quadratic programming problem with Hessian symmetric matrix $P = 2S_u^T Q S_u + R_{\Delta u}$ and $c^T = 2G_k^T Q S_u$.

Regarding the constraints they can also be rewritten in the form of QP problem. Double inequalities can be split in two inequalities and expressed in the sense of "less or equal";

$$g_1 \leq g_2 \leq g_3 \quad \rightarrow \quad g_1 \leq g_2 \quad \& \quad g_2 \leq g_3 \quad \rightarrow \quad g_1 \leq g_2 \quad \& \quad -g_3 \leq -g_2$$

After expressing in the matrix format, we obtain;

$$\begin{bmatrix} u_{min}(k) \\ ... \\ u_{min}(k+p) \\ -u_{max}(k) \\ ... \\ -u_{max}(k+p) \\ \Delta u_{min}(k) \\ ... \\ \Delta u_{min}(k+p) \\ -\Delta u_{max}(k) \\ ... \\ -\Delta u_{max}(k+p) \\ y_{min}(k+1) \\ ... \\ y_{min}(k+p+1) \\ -y_{max}(k+1) \\ ... \\ -y_{max}(k+p+1) \end{bmatrix} \leq \begin{bmatrix} u(k|k) \\ ... \\ u(k+p|k) \\ -u(k|k) \\ ... \\ -u(k+p|k) \\ \Delta u(k|k) \\ ... \\ \Delta u(k+p|k) \\ -\Delta u(k|k) \\ ... \\ -\Delta u(k+p|k) \\ y(k+1|k) \\ ... \\ y(k+p_1|k) \\ -y(k+1|k) \\ ... \\ -y(k+p+1|k) \end{bmatrix}$$

Again we can express $u(k+j|k)$ and $y(k+j+1|k)$ in terms of $u(k+j|k)$ and regroup the terms so that;

$$\begin{bmatrix} -T_r \\ T_r \\ -I \\ I \\ -S_u \\ S_u \end{bmatrix} U_k \leq \begin{bmatrix} u(k-1) - u_{min} \\ u_{max} - u(k-1) \\ -\Delta u_{min} \\ \Delta u_{max} \\ S_x x(k|k) - S_{u1}u(k-1) - y_{min} \\ y_{max} - S_x x(k|k) - S_{u1}u(k-1) \end{bmatrix}$$

where T_r is a lower triangular matrix with only ones and I the identity matrix.

Remarks

- We have considered previously an ideal case where the plant is perfectly estimated by the model and no perturbation is considered. In practice, modelling error and perturbations influence the dynamic response of the controller. Therefore state observer, generally Kalman filter, is used to improve the dynamic response to apparent disturbance;i.e when the measured plant output deviate from its predicted trajectory. It also enables asymptotic rejection of sustained perturbations by emulating classical integral feedback controller [31], [1].

- Unconstrained linear MPC results in a state-feedback law. This can be proved by dynamic programming using Riccati iterations or more simply by zeroing the gradient of the cost function;

  $$\nabla_U J(U_k) = 2[S_u^T Q S_u + R_{\Delta u}]U_k + 2S_u^T Q G_k = 0$$

  By isolating U_k and replacing G_k bu its expression, we obtain;

  $$U_k = -[S_u^T Q S_u + R_{\Delta u}]^{-1} S_u^T Q \{S_x x(k) + S_{u1}u(k-1) - R_k\}$$

  Keeping only the first control move and regrouping term we fin finally;

  $$\Delta u(k|k) = Gr(k+1) - Hu(k-1) - Kx(k)$$

The control move to apply depends on the reference trajectory, the previous move and effectively of the actual state x(k) (=state-feedback).

- General shortcut that is made about MPC is to consider that finding optimal solution ensures stability. In general additional constraints have to be added to the MPC problem to ensure stability. Kwon et al. (1983) and Meadows et al. (1995) proved that stability can be achieved by the use of terminal constraints. The introduction of dual-mode designs (Mayne and Michalska, 1993) and the use of infinite prediction horizons (Rawlings and Muske, 1993)[32] was also proposed. Clarke and Scattolini (1991) and Mosca et al. (1990) independently developed stable predictive controllers by imposing end-point equality constraints on the output after a finite horizon. The base idea of their proofs use the cost function as a Lyapunov function [21].

2.3.2 MPC design in Matlab [1]

To address the objective of this master thesis, we decided to rely on the model predictive control toolbox from Matlab. It offers a very handy Graphical User Interface (Simulink) as well as the possibility to execute code at the command line. Several linear MPC design features are available;

- Explicit, Economic, Adaptive, Mulitple, Gain-Scheduled MPC, ...
- Time varying/adaptive models, weights, constraints, reference
- Mixed input/output constraints
- Stability and feasibility design review
- Versatile simulation options (scenarios)
- ...

Prediction models can also be generated by the Identification Toolbox or automatically linearized from Simulink diagrams. Finally, the toolbox support C-code and IEC 61131-3 Structured Text generation for rapid prototyping and embedded system implementation.

Implementation of the objective function in Matlab
In comparison to the classical MPC formulation presented before, Matlab add quadratic terms in the cost function for manipulated variable tracking and allow soft constraints by the use of a non-negative slack variable ϵ_k;

$$J^*(z_k^*) = J(z_k) + \sum_{j=1}^{n_u}\sum_{i=0}^{p-1} w_{i,j}^u [u_j(k+i|k) - u_{j,target}(k+i|k)]^2 + \rho_\epsilon \epsilon_k^2$$

with the decision;

$$z_k^* = [\, z_k \; \epsilon_k \,]$$

$u_{j,target}(k+i|k)$ is the target for manipulated variables at instant k, while $w_{i,j}$ and ρ_ϵ are respectively the weights on manipulated variable tracking and constraint violation.

The constraints can also be rewritten as follows;

$$\begin{aligned} u_{min}(k+j) - \epsilon_k V_{j,min}^u(k+j) \leq \quad & u(k+j|k) && \leq u_{max}(k+j) + \epsilon_k V_{j,max}^u(k+j) \\ \Delta u_{min}(k+j|k) - \epsilon_k V_{j,min}^{\Delta u}(k+j) \leq \quad & \Delta u(k+j|k) && \leq \Delta u_{max}(k+j) + \epsilon_k V_{j,max}^{\Delta u}(k+j) \\ y_{min}(k+j) - \epsilon_k V_{j,min}^y(k+j) \leq \quad & y(k) && \leq y_{max}(k+j) + \epsilon_k V_{j,max}^y(k+j) \end{aligned}$$

where V_j correspond to the constraint softening constants and is equal to zero for hard constraint. Note that Matlab cost function also introduce scaling parameters for manipulated variables and measured outputs scaling. In fact, although humans think infinite precision, computers do not and are limited by the finite bit number representations. To avoid numerical difficulties, variables should ideally lie between -1 and 1.

Chapter 3

Model of the wastewater treatment network

3.1 Description of the network

The SIAAP is responsible of the collecting and the treatment of wastewater of the Paris agglomeration and the quality control of the Seine river and two of its affluents; the Marne and the Oise. Depending of the weather conditions, the Seine flow can vary from 100 - 1000 m^3/s. The Seine cover an area of 79000 km^2, representing approximately 13% of the French territory and concern 30% of the its population.

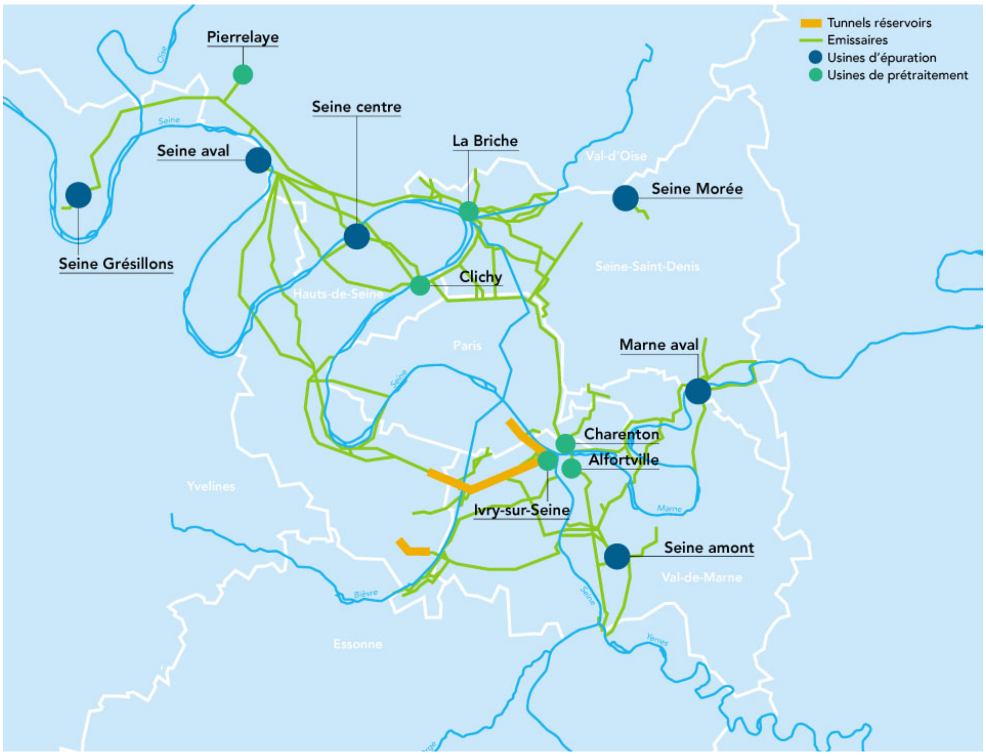

Figure 3.1: wastewater treatment network of the SIAAP [33]

To transport the wastewater the SIAAP manages 440km of conducts dispatched on 1800 km^2 of territory (figure 3.1). Those conducts of diameter between 2.5 and 4m and situated between 10 and 100m deep pipes million of m^3 of wastewater to the 6 treatment plants of the network;Seine Amont, Seine Morée, Marne Aval, Seine centre (SEC), Seine Aval (SAV) and Seine Grésillons (SEG). The first three treat and discharge the water into the Seine downstream from Paris while the last three do it upstream.

The network is also composed of 4 tunnel tanks and 8 storage bays with a total capacity of 900 000m^3 to store temporary the water excess and relax the treatment plants in case of heavy rain. Finally, the SIAAP also has pre-treatment facilities whose role as implied is to pre-treat the wastewater but also to dispatch the flow on the treatment plants of the network. Some of those facility as Clichy-la-Garenne can work as pumping station to avoid flooding by evacuating the rainwater excess in the Seine. The network finally disposes of different measurement points across the river to monitor its quality (see figure 3.2).

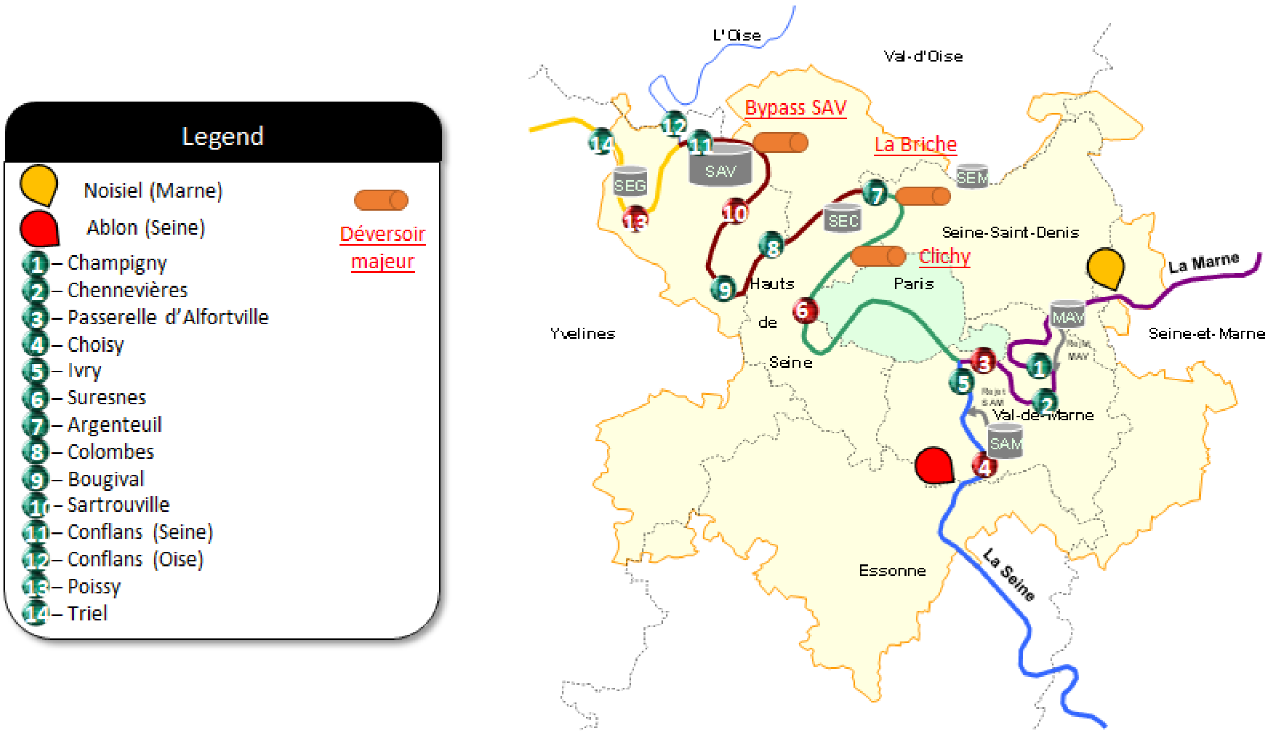

Figure 3.2: Measurement points in wastewater treatment network of the SIAAP [33]

In this thesis, we concentrated our study area to the west part of the network from Suresnes to Poissy which includes, the Oise confluent, the main pre-treatment facility of Clichy and LaBriche, and the treatment plants of SEC, SAV and SEG. The four main conducts piping the wastewater from Clichy to the WWTPs can be seen on the figure 3.3, they are referenced with their maximal capacity in the table 3.1. The EGN conduct deliver SEC and SEG treatment plant, while CAA and CAB conducts deserve relatively straightforward SAV.

Conducts	**Maximal flow** $[m^3/d]$
Emissaire général (EGN) between Clichy and SEG	780000
Emissaire général (EGN) between SEC and SEG	780000
Clichy-Archères-Argenteuil (CAA)	1040000
Clichy-Archères-Bezons (CAB)	1200000

Table 3.1: Major conducts and their capacity piping water between Clichy and the WWTPS

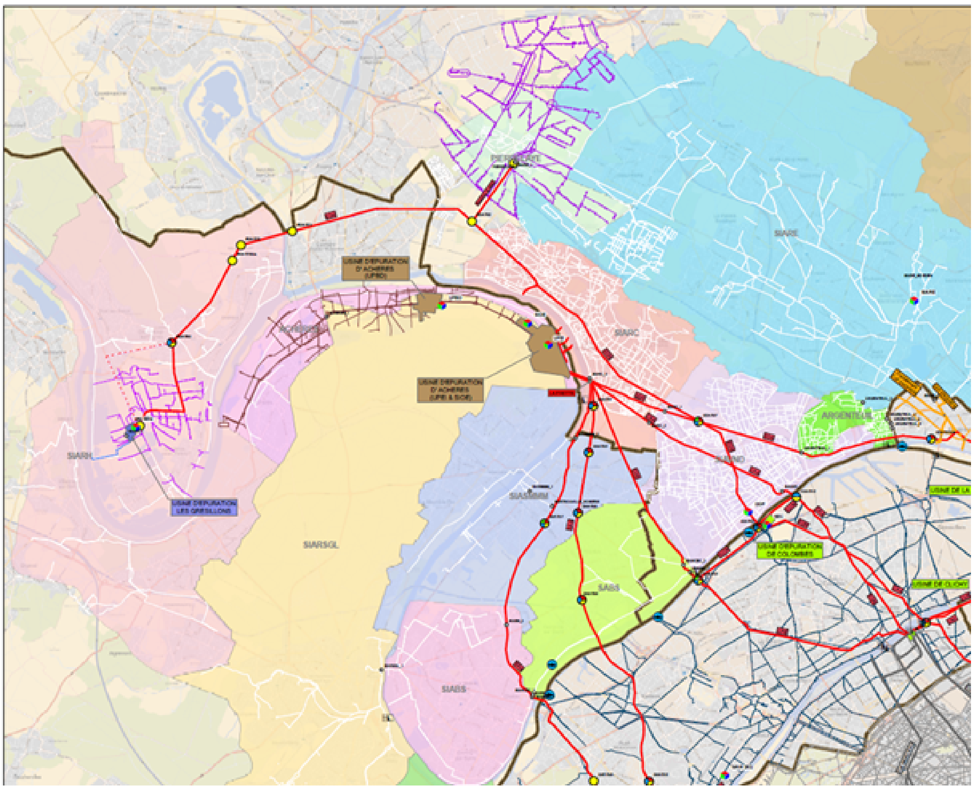

Figure 3.3: West part of the wastewater treatment network of the SIAAP [33]

3.2 Model description and assumptions

The model, visible on the figure 3.4, is composed of a unique input node regrouping Clichy-Labriche pre-treatment facility where all the wastewater arrive. The flow is then split in four; three parts going respectively to each of the WWTPs and another part that is directly rejected in the Seine. We make the assumption that no pre-treatment is done at this point. Storage tanks and bays as the major spilway of SAV are not considered in the model. The amounts of water that surpass the maximum capacity is assumed to be directly rejected in the Seine at Clichy.

We consider the following measurement points; Suresnes (1) for the state of Seine before the WWTPs, Sartrouville (2) after SEC, Conflans (3) after SAV, Poissy (4) after the confluence of the Oise and Triel (5) after SEG.

We also neglect the degradation of water quality in the Seine and in the conducts and consider instantaneous mixing of the treated water and the Seine at the output of the WWTPs. In fact, the mixing time is negligible compared to the travelling time and bio-chemical time process taking place in the WWTPs.

In the first versions of the network model we considered travelling time between Clichy-LaBriche node and the WWTPs, and in the Seine. However we finally neglected it, as it increased the complexity for only very small improvement of the model as presented in the section 3.4. This is mainly comes from the fact that travelling time is negligible compared to the bio-chemical time process taking place in the WWTPs and the weekly measurement we were provided at the different measurement point. To compute the travelling time we first determined the evolution of the water speed v in function of time. The speed is linked to the flow Q and the section S of

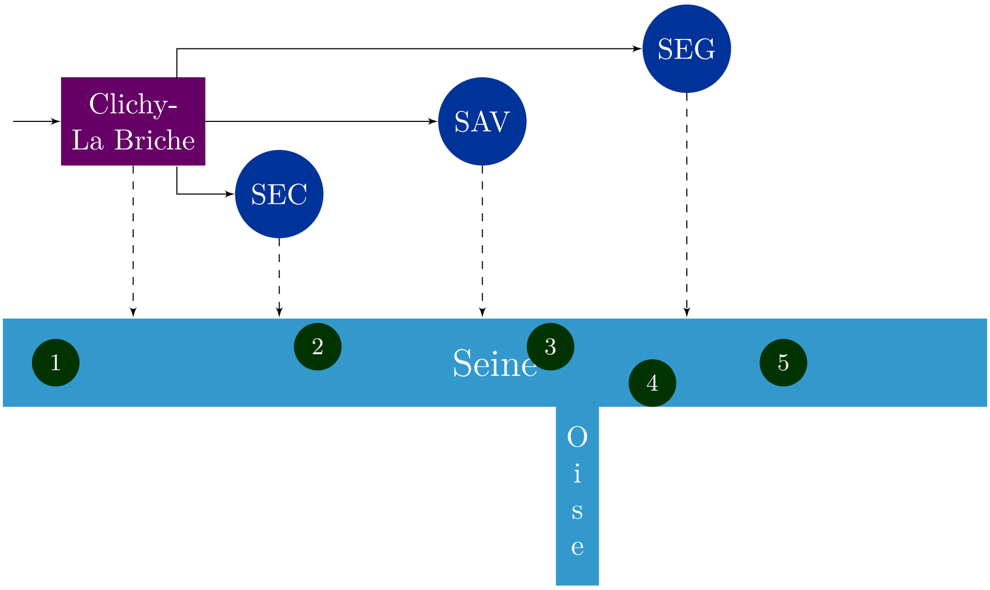

Figure 3.4: Model of the wastewater treatment network

the river or the conduct since by the following relation,

$$Q(t) = v(t).S(x)$$

In all generalities, the section can vary with the position whereas the flow is time dependant. The evolution of the position in function of time is therefore obtained by solving the following differential equation

$$\frac{dx}{dt} = \frac{Q(t)}{S(x)}$$

Finally, the travelling time t_{tr} is obtained by looking at which time the position is equal at the distance to travel assuming the referential has been put at the origin in time t=0;

$$x(t_{tr}) = d$$

Assuming constant section and flow, the travelling time is given by the following relation;

$$t_{tr} = \frac{d.S}{Q}$$

Distance were computed using Google Maps measurement tool. For the conducts, the diameter has been considered to be 4 m and the flow estimated by their maximal allowable flow. On the other hand, nominal flow and speed were considered for the Seine river.The travelling time in the conducts and between the different rejection points are finally summed up in the table 3.2

3.3 Model of the wastewater treatment plants

3.3.1 Description of the wastewater treatment plants

The plant of SEC, SAV and SEG treat nominally 240 000, 1 300 000 and 300 000 m^3/d of waste water respectively. Their treatment capacity goes respectively up to 400 000 , 2 300 000 and 315 000 m^3/d in wet weather. SEC receives wastewater pre-treated by the facility of Clichy-La-Garenne orignated from the Paris sewers while SAV treat more than 70% coming from the Paris Agglomeration. On the other hand, SEG lightens the flow treated by SAV, and collect

Path	Distance $[km]$	Flow $[m^3/s]$	Section $[m^2]$	t_{tr} [h]
Clichy to SEC	4.5	9	12.6	1.75
Clichy to SAV	12	26	12.6	1.61
Clichy to SEG	30	9	12.6	11.67
Suresnes to Clichy-La Briche	10	400	720	5
Clichy-La Briche to Sartouville	31	400	720	15.5
Sartouville to Conflans	11	400	720	5.5
Conflans to Poissy	9	400	720	4.5
Poissy to Triel	8.5	400	720	4.25

Table 3.2: Travelling time in the model in function of the path

and clean the residual water of 18 localities from the Yvelines and the Val-d'Oise [33].

The treatment process can be summed up in 3 steps; first a pre-treatment is applied which consist of screening, grid removal and deoiling to remove sand, oil and waste that may deteriorate the facility. Next, the wastewater undergoes physico-chemical decantation to reduce the total suspended solid (TSS), the carbon and phosphorus by clari-foculation. Biofiltration is finally applied to reduce the concentrations of nitrogenous nutrients.

More specifically in SEG, the drawdown of phosphorus nutrients is done by a tertiary treatment unit activated during summer (June-September) when discharge standards at the output of the WWTP are more strict. On his side SAV also include a classical AS biological treatment stage before physico-chemical decantation to clarify the water from carbon and suspended solids (see figure 3.5). Note that in rainy condition some treatments steps may be bypassed to handle larger input flow, changing therefore the dynamic of the WWTP.

3.3.2 Model description and assumptions

Robles-Rodriguez (et al,2018) [34] showed that a simple model based on ASM1 (Active Sludge Model 1) with one microbial population was effective to predict the concentrations of effluent at the output of the WWTPs.
The proposed model first reduces the hole treatment plant as an aerobic membrane bioreactor (MBR) (figure 3.6) where Q_{in} represent the influent rate, Q_w is the waste flow rate while Q_{out} corresponds to the effluent flow going to the Seine. The air supply has been assumed to be sufficient for microbial growth and alkalinity was neglected because the pH of the plant is maintained at neutral pH.

The model considers the conversion of the carbon by heterotrophic bacteria and the autotrophic conversion of the ammonia nitrogen first into nitrites (NO_2^-) and then in nitrates (NO_3^-). This latest is then converted into nitrous oxide (N_2O) before turning into nitrogen gas N_2. This process called denitrification occurs under oxygen depletion and in presence of a carbon source as the methanol (CH_3OH). Note that during this operation the primary oxygen source for the microorganisms is turned into nitrates.

$$\begin{aligned} NH_4 + 3O_2 &\rightarrow 2NO_2^- + 4H_2O + 4H^+ \\ 4NO_2^- + O_2 &\rightarrow 2NO_3^- \\ 6NO_3^- + 5CH_3OH &\rightarrow 2N_2 + 5CO_2 + 7H_2O + 6OH^- \end{aligned}$$

The model considers only one microbial population which merges heterotrophic and autotrophic microorganisms into one (X). Therefore its dynamic is reduced to 4 reactions; (i) Growth of microorganisms; (ii) Decay of microorganisms; (iii) Hydrolysis of entrapped organics; and (iv)

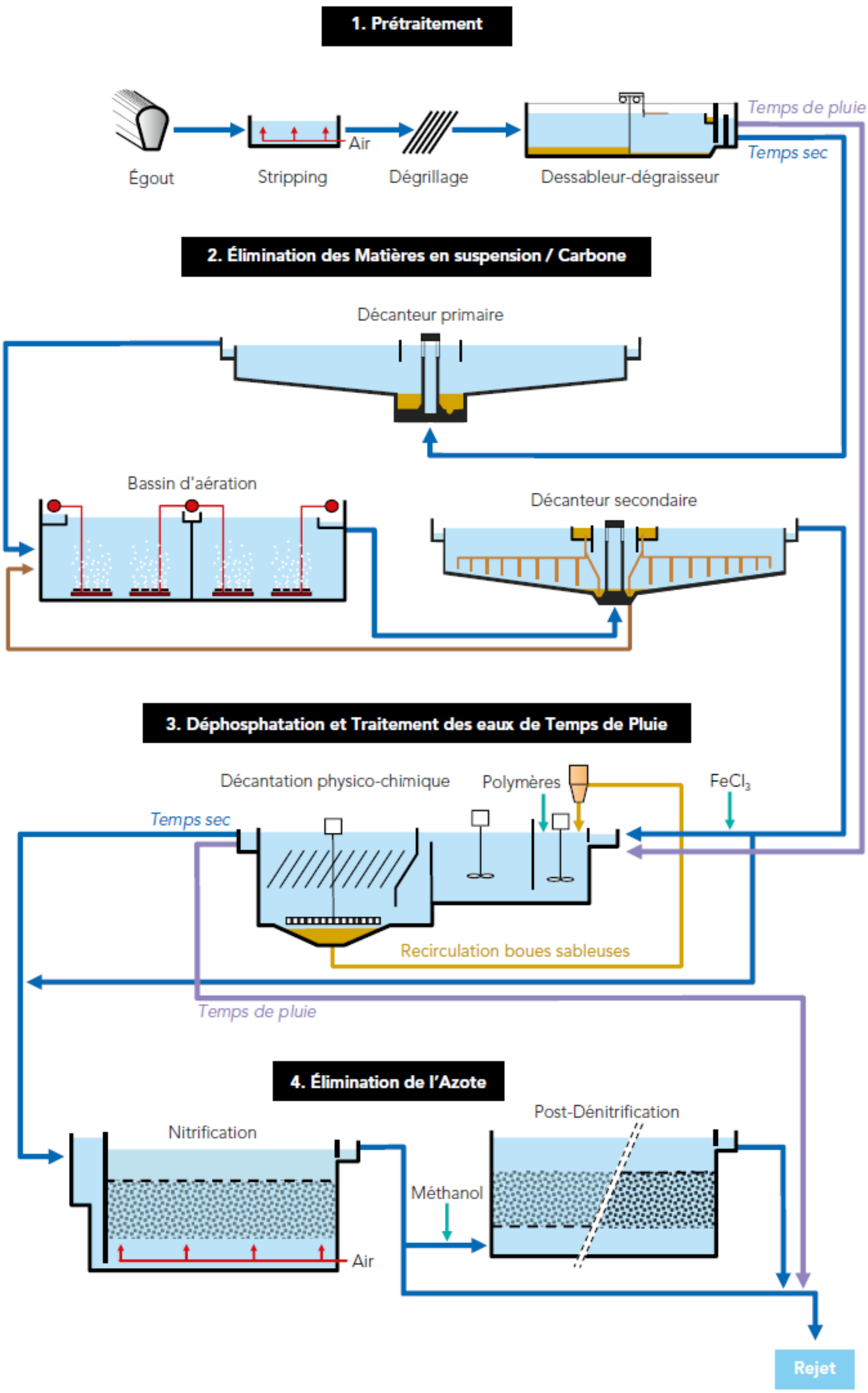

Figure 3.5: Schematic of the SAV facility for wastewater treatment

Hydrolysis of entrapped organic nitrogen.

The high sludge retention rate (SRT) is included in the model as a constant (s_d) to account for the long degradation of the Total Suspended solids (TSS). The fraction of TSS present in the influent was accounted as an active biomass, while no biomass was considered for the effluent

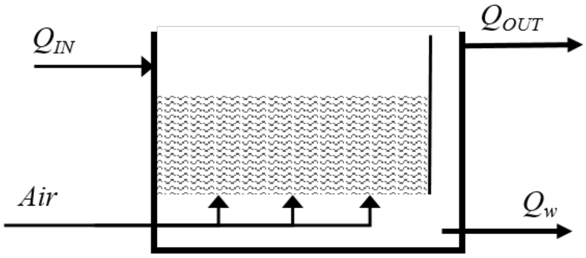

Figure 3.6: System representation of the reduced WWTP [34]

due to the membrane utilisation. Finally phosphorous treatment has not been considered in the model, and rainy and wet regime are not dissociated.

Regarding those assumptions the dynamic of the biological oxygen demand (BOD), the ammonia nitrogen (NH_4^+), nitrite (NO_2^-), nitrate (NO_3-), and total suspended solids (TSS), can be described by mass balances equations as follows,

$$\frac{dBOD_5}{dt} = BOD_{5,IN}\frac{Q_{IN}}{V} - BOD_5\frac{Q_{OUT}}{V} - BOD_5\frac{Q_W}{V} - r_1 X \tag{3.1}$$

$$\frac{dNH_4}{dt} = NH_{4,IN}\frac{Q_{IN}}{V} - NH_4\frac{Q_{OUT}}{V} - NH_4\frac{Q_W}{V} - r_2 X \tag{3.2}$$

$$\frac{dNO_2}{dt} = NO_{2,IN}\frac{Q_{IN}}{V} - NO_2\frac{Q_{OUT}}{V} - NO_2\frac{Q_W}{V} - r_3 X + \frac{1}{Y_{NH_4/NO2}} r_2 X \tag{3.3}$$

$$\frac{dNO_3}{dt} = NO_{3,IN}\frac{Q_{IN}}{V} - NO_3\frac{Q_{OUT}}{V} - NO_3\frac{Q_W}{V} - r_4 X + \frac{1}{Y_{NO_2/NO3}} r_3 X \tag{3.4}$$

$$\frac{dX}{dt} = f_X TSS_{IN}\frac{Q_{IN}}{V} - X\frac{Q_W}{V} + Y_{X/BOD} r_1 X + Y_{X/NH_4} r_2 X - bX \tag{3.5}$$

$$\frac{dTSS}{dt} = (1 - f_X)\, TSS_{IN}\frac{Q_{IN}}{V} - TSS\frac{Q_{OUT}}{V} s_d - TSS\frac{Q_W}{V} \tag{3.6}$$

In those equations X (mgX/L) is the biomass, V (m^3) the Volume of the reactor. Q_{in} is the influent flow rate while Q_{out} is the effluent flow rate. On the other hand, Q_w account for the waste flow and is assumed to be 10 percent of the influent flow rate.

$Y_{X/i}$ is the Yield coefficients for biomass production with respect to the different i compounds while the kinetics coefficient are given by Monod equations for the consumption of carbon (r_1), the nitrification of ammonia into nitrites (r_2), the nitrification of nitrites to nitrates (r_3), and the denitrification of nitrates into N_2 gas (r_4).

$$r_i = \rho_{1,\max}\frac{BOD_5}{K_{BOD} + BOD_5} \tag{3.7}$$

$$r_2 = \rho_{2,\max}\frac{NH_4}{K_{NH_4} + NH_4} \tag{3.8}$$

$$r_3 = \rho_{3,\max}\frac{NO_2}{K_{NO2} + NO_2} \tag{3.9}$$

$$r_4 = \rho_{4,\max}\frac{NO_3}{K_{NO3} + NO_3} \tag{3.10}$$

$\rho_{j,\max}$ $j = 1 - 4$ represent the maximum specific rates of the 4 reactions. The parameters K_{BOD}, K_{NH_4}, K_{NO2}, and K_{NO3} are the half saturation coefficients for BOD_5, NH_4, NO_2, and NO_3, respectively. Finally f_x is the fraction of active biomass and b represents its decay.

3.3.3 Model Evaluation

Based on daily measurements during 4 years (2009-2012) at the input and the output of the WWTPs of SEC, SAV and SEG, the model has been first calibrated on tree datasets of 150 days; beginning of 2009, middle of 2010 and late 2012. The parameter estimation has been performed by pattern search algorithm expect for four of them which were taken from the literature; f_X, b, $Y_{X/BOD}$ and Y_{X/NH_4^+}. They are reported in the table 4.

The model has then been evaluated on the rest of the data excluding period with missing and inaccurate measurements. We observed that the dynamics of TSS and BOD concentrations were well modelled. However, flow variation impact strongly nitrogenous concentrations and nitrates nutrients are most of the time underestimated by the model. The performance of each of the WWTPs has been assessed by computing the root mean square of the normalized error for each substance concentration;

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_{dat}(i) - y(i)}{\max(y_{dat})}\right)^2} \tag{3.11}$$

where y_{dat} is the experimental data, y is the model output, and n is the total number days considered in the evaluation. We observe from the table 3.3 that the model of SEG is less effective than the others and that the nitrogenous compounds dynamics are less well predicted.

Variables	SEC	SAV	SEG
TSS	0.058	0.092	0.191
BOD	0.076	0.103	0.172
NH4	0.115	0.110	0.077
NO2	0.152	0.112	0.144
NO3	0.182	0.133	0.168

Table 3.3: Statistical performance of the WWTPs

3.4 Network model evaluation

Based on the same previous set of data and a picewise linear interpolation of weekly measurement at each observation point, we evaluated our model with and without considering the travelling time. We observed that the dynamic is well respected although we still observe an underestimation of nitrates as we observed for the WWTPs model. We also computed the performance of the model using RMSE criteria as previously detailed. We observe on the table 3.4 and 3.5 that the performance are better the one obtained for the plant evaluation at the exception of the ammonia nitrogen. We can also notice that the performance are getting better as we are approaching from Triel. This comes from the dilution of the effluent concentrations in the Seine which lead to a masking of the WWTP model imperfections. We see a very small gain considering travelling time; however we can't expect more than that since we are consolidating weekly measurement that have been interpolated linearly, and thus unable to to describe hours fluctuations.

Variables	Sartrouville	Conflans	Poissy	Triel
TSS	0.081	0.081	0.075	0.054
BOD	0.168	0.158	0.162	0.141
NH4	0.135	0.079	0.143	0.105
NO2	0.119	0.14	0.167	0.135
NO3	0.049	0.119	0.123	0.119

Table 3.4: Statistical performance of the model without delay

Variables	Sartrouville	Conflans	Poissy	Triel
TSS	0.079	0.078	0.073	0.053
BOD	0.162	0.154	0.157	0.134
NH4	0.131	0.078	0.142	0.105
NO2	0.118	0.14	0.168	0.135
NO3	0.047	0.119	0.121	0.116

Table 3.5: Statistical performance of the model with delay

Chapter 4

Cases study

4.1 Choices of data and cases study

Based on the same 4 years data from 2009 to 2012, presented in section 2, we defined 3 scenarios to evaluate the control law, each one on periods of 90 days.

- (1) During Spring from March to May 2009
- (2) During Winter from December 2009 to February 2011
- (3) At the end of Autumn and the beginning of Winter from November 2010 to January 2011

Wet periods (2,3) were chosen to study the behaviour of the MPC law under extreme situations; i.e. for high input flow and variation at the input of Clichy. On the other hand, dry situation was selected to study the behaviour of the MPC law when constrains are more strict at the end of the WWTP since the Seine flow is lower and the thus dilution effect less present.

Large time period were considered to study steady period but also to account for punctual event as heavy rainfall on small time period. The scenarios have been carefully selected in order to consider period where concentration excesses above the good state limit appear to study if MPC law could have improved the situation.

The data for these periods is based on daily measures of the flow rate, and weekly measures of the nutrients concentrations at the different measurement points of the WWTN. Based on the daily data at the input of each of the WWTP, we reconstructed the flow and the concentration at the Clichy-LaBriche node. The points considered, as presented in the section 2, are Clichy and Labriche as input of the wastewater to treat, Surness for the Seine state before the different rejections points, and Sartrouville, Conflans and Triel for the Seine state respectively after the three WWTPs; SEC, SAV and SEG.

Relatively to our model and the data provided the different nutrients considered were the Total Suspend solids (TSS), the Biochemical Oxygen Demand (BOD) the ammonia nitrogen (NH_4^+), the nitrites (NO_2^-) and the nitrates (NO_3^-).

Chapter 5

MPC parametrisation

Traditionally done by trial and error procedure, MPC tuning remains a challenging problem due to its huge number of parameters even for system of modest size[35]. Although parameters influence on the plant can be intuitively understood, quantifying their effect is more complex. Moreover, badly tuned predictive controllers tend to take the plant to extreme and sometimes unstable conditions, frequently unnoticed because of physical constraints such as valve opening, maximum flow rate, etc., masking the instability problem and fooling operational groups into concluding that the plant has been optimised [21].

In the literature there are many works dealing with automatic tuning of MPC, but due to their complexity and the difficulty to perform analytically studies, the development of a general method is still a challenge. Some researchers have proposed tuning methods for specific types of MPC, but put apart some particular aspects of the problem, for instance not considering the horizons (Li and Du, 2002)[36] or focusing on single input/ single output plant (Sridhar and Cooper 1997)[37],[35].

In an alternative approach, frequency domain methods for tuning linear optimal controllers have been studied since the beginning of 1980's (Doyle et al., 1992)[38], and they are a good alternative to speed up MPC automatic tuning procedures avoiding dynamical simulations. However, some problems of stability and robustness were detected in the presence of non-linearities and load disturbances acting on the plant (Vega et al. 2007)[39],[35].

Finally, another class of tuning methods based on solving optimisation problem has also raised some interest. Their main idea is to compare the efficiency of the control with a performance function and optimise a reduced or whole set of MPC parameter. Nevertheless, those methods suffer from the high number of dynamic simulations required, making the design process very slow [35].

Tuning Method
For the aforementioned reasons, we decided to develop a systematic approach for MPC design. A first restriction is made on the parameters based on physical constraints / limitations of the plant; e.g. a valve could only be operate between its minimal and maximal flow capacity.

We then separate the remaining parameters and ranked them based on their relative influence on the process. For example, sampling time is an important parameter which will influence the time response of the controller and should be optimised first regarding the weights on measured output which will allow fine tuning. Note that set of parameters may be recombined after being optimised independently as for weights on measured output and on control moves.

Similarly to optimisation based method, the performance of the control law with a given set of parameters is addressed by a quality function while ensuring robustness and stability of the controller.

Quality function
In our case, the quality function is computed by the root mean square error (RMSE) of all nutrient concentrations n_1 on the hole time period considered n_2. An error is observed only if the concentration measured $y_{i,j}$ is above good state limit $y_{j,gsl}$ and is computed as a relative error. The use of RMSE allow us to penalise more large deviation compared to classical mean relative error.

$$RMSE = \sqrt{\frac{1}{n_2}\sum_{i=1}^{n2}\sum_{j=1}^{n1} e_{i,j}^2}$$

$$e_{i,j} = \begin{cases} \frac{y_{i,j}-y_{j,gsl}}{y_{j,gsl}} \, if \; y_{j,gsl} \leq y_{i,j} \\ 0 \, otherwise \end{cases}$$

5.1 Reduction of the number of parameters

Regarding the MPC problem, we have for each manipulated variable 10 parameters we can tune; 2 weights for tracking and control moves. We can also set 4 constraints for minimum and maximum manipulated variable (/rate), and their respective 4 softening constraints. In the other hand, for each measured output, we have 5 parameters; 1 weight for reference tracking and 4 parameters for minimum and maximum constraints with their softening parameter. Independently we have three other parameters; sample time, prediction and control horizon. In total, for n_u mv and n_y mo; this corresponds to $3 + 10n_u + 5n_y$. Note that additional mixed input and output constraints can be considered.

Without any special considerations in our model, we have 4 manipulated variables and 25 measured outputs corresponding to the effluent concentrations at each measurement point. This leads to 168 parameters to set and reduction is thus essential!

Manipulated variables
As presented in the modelling section, the flow rate at Clichy-Labrichy is divided in four; three parts that are going respectively to the three WWTPs SEC, SAV and SEG, and the fourth part that is directly rejected to the Seine at Clichy. The objective of the MPC law is to determine the optimal partitioning of the input water-waste at Clichy ensuring the good state limits are respected at the output of each wastewater treatment plant.

As a result three manipulated variables were considered in the design of the MPC, one for each of the flow going to each of the WWTP. The latest flow directly rejected Q_{rej} to the Seine is computed as the difference between the input flow Q_{in} and the flow going to the three WWTPs Q_{SEC}, Q_{SAV} and Q_{SEG};

$$Q_{rej} = Q_{in} - Q_{SEC} - Q_{SAV} - Q_{SEG}$$

In order to obtain normalised manipulated variables mv_i, each of the flow Q_i is respectively divided by a flow of 3 000 000 m^3/d which correspond approximately to the maximal Q_{max} flow treatable by the 3 WWTPs;

$$mv_i = \frac{Q_i}{Q_{max}}$$

Measured outputs

Regarding the measured outputs, only the nutrients concentrations at the output of Triel are considered. This choice is based on the following observations of the provided data. First, over the last 15 years, excess above good state limits were mostly observed on the Seine after the three WWTPs of SEC, SAV and SEG for NO_2^- and NH_4^+ (table 5.1).

Year	Suresnes	Sartrouville	Poissy	Triel
2001	0.39	0.51	3.09	-
2002	0.26	0.51	5.27	-
2003	0.35	0.63	6.60	-
2004	0.42	0.56	5.70	-
2005	0.34	0.34	5.00	-
2006	0.23	0.45	4.40	-
2007	0.18	0.33	1.60	1.80
2008	0.23	0.25	1.60	0.95
2009	0.15	0.44	1.70	1.70
2010	0.19	0.28	1.70	1.40
2011	0.14	0.48	3.80	3.10
2012	0.16	0.17	0.87	0.78
2013	0.15	0.19	1.40	0.89
2014	0.14	0.22	1.60	0.70
2015	0.16	0.2	4.10	2.90
2016	0.18	0.17	1.10	0.61

Table 5.1: Mean concentration of NH_4^+ from 2001 to 2016 at each point of measure [33]

Secondly, the concentrations of nutrients at the output of the WWTPs before dilution in the Seine are most of the time above the good state limit for the BOD, the ammonia nitrogen, the nitrite and the nitrate. (fig.5.1). This means that the realisation of concentrations below good state limit is ensured by the dilution of the concentrations in the Seine. Therefore concentrations below good state limit at Triel is necessary to respect the DCE at each point of the Seine. Or in other words, concentrations at the output of Triel below good state limit ensures good state limit at the other points of measure.

Nevertheless, we may think scenarios where the concentrations at Conflans after SAV is above the good state limit and the influence of the Oise can dilute those concentrations below good state limits. Note that, on the 4 year data provided this happened only once and Oise flow is most of the time ten order of magnitude below Seine flow.

Moreover, concentrations above good state limits at the output of the WWTP also implies that if the concentrations are already above the good state limits in the Seine, the WWTPs will not be able to improve the quality of the Seine. In fact, the concentration resulting C_r in the mixing of two flows, Q_1 and Q_2, at different concentrations , C_1 and C_2, is equal to;

$$C_r = \frac{Q_1 C_1 + Q_2 C_2}{Q_1 + Q_2}$$

Letting $x_1 = \frac{Q_1}{Q_1+Q_2}$ the flow fraction, the relation become;

$$C_r = C_1 x_1 + C_2(1 - x_1)$$

or,

$$C_r = (C_1 - C_2)x_1 + C_2$$

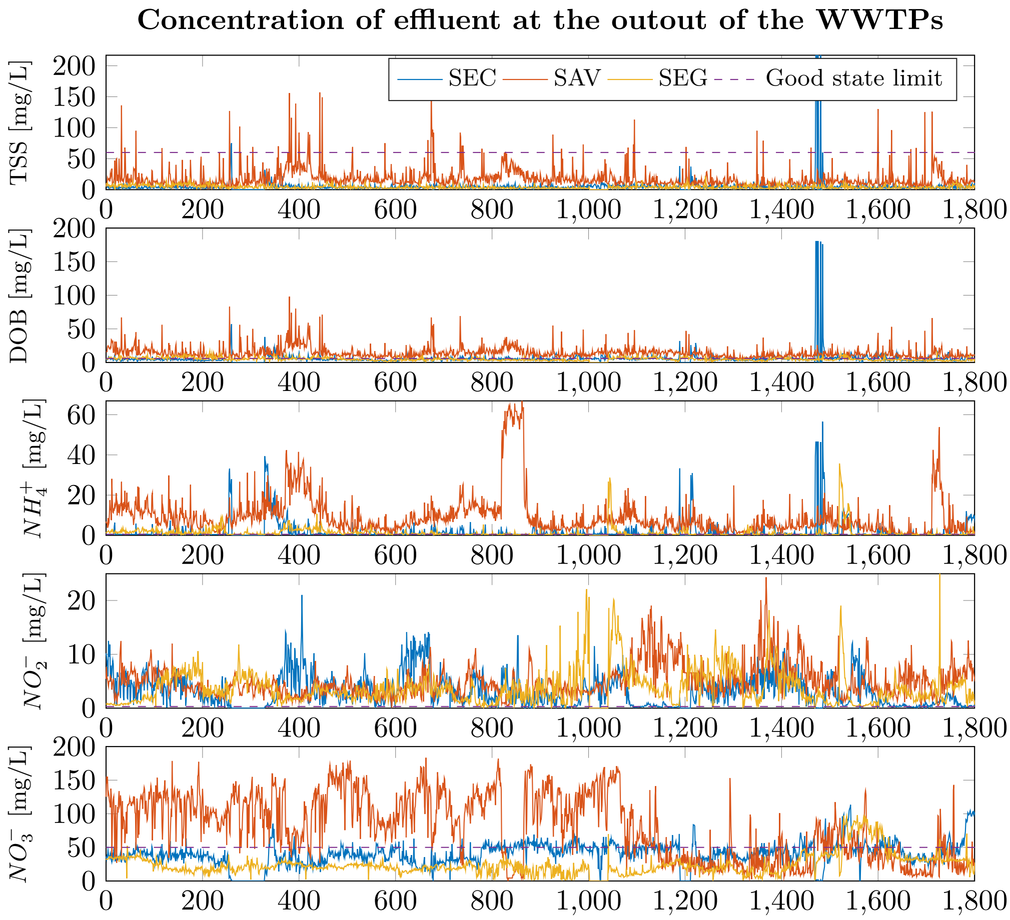

Figure 5.1: Concentrations of nutrients at the output of the wastewater treatment plants before dilution in the Seine

It is the equation of a straight line of slope $C_1 - C_2$ and intercept C_2. Since x_1 can vary from 0 to 1 C_r is bounded by the minimum and maximum of C_1 and C_2

Next, based on the data and yearly studies of the SIAAP, we observed that the concentrations of BOD, ammonia and nitrite are the most sensitive to overtaking above the good state limits. Therefore TSS and nitrate can be put apart and not considered as parameters to optimise by the MPC.

During first simulations, we also put in evidence that optimising BOD and Ammonia leaded to the same choice of partitioning leading to the conclusion that only one of this parameter have to be considered. This dependency can be observed in the equations modelling the WWTPs (2.1 - 2.2); we see that for reducing the concentration of nutrients, meaning a negative derivative, we have to increase in both cases the biomass.

As a summary, ensuring concentration below good state limit at Triel is a suffisient condition to ensure good state limit in all the network. The measured output can be reduced to two variables; the concentrations of Ammonia nitrogen and nitrites. The measured output is finally defined as a normalised concentration against good state limit.

$$mo_i = \frac{X_i}{X_{i,gsl}}$$

where X_i is the concentration at Triel of compound i and $X_{i,gsl}$ its respective good state limit. Defined this way, the corresponding reference for tracking, has been fixed to zero.

Constraints
Each of the WWTP is limited by its maximal flow it can treat. Theoretically the WWTPs should only be stopped for maintenance or configuration change as SAV WWTP in 2011. A WWTP shutdown may deteriorate microbial population and induce a negative economical impact. As a result, we imposed minimum and maximum value on the manipulated so that the flow going to each of the WWTP remains between 10% and 100% of their maximal flow.

The partitioning of the flow is mainly performed through valves which physically present a limited flow rate. We estimated this flow by analysing the daily variations at the input of each WWTPs and imposed it as constraints on minimum and maximum flow rate on manipulated variables.

Finally, we must ensure at each time the conservation of flow at Clichy-La Briche node and therefore, the sum of the flow going to each of the station should always be lower or equal to the input flow. This constraint is ensured by a time varying mixed input constraints. All those constraints are physically inviolable and are therefore considered as hard constraints.

For the output, we may think using constraints to ensure concentrations below good state limit at the output; however, it will be redundant with the output reference tracking term in the cost function. Moreover, in our plant if input are bounded, output will remain bounded. Therefore no constraints are required at the output. The constraints are summed up in the table 5.2;

	Min	Max	Min rate	Max rate
mv_{SEC}	0.013	0.133	-0.013	0.013
mv_{SAV}	0.077	0.767	-0.367	0.367
mv_{SEG}	0.003	0.033	-0.008	0.008
$(mv_{SEC} + mv_{SAV} + mv_{SEG})\, Q_{Max} \leq Q_{in}(t)$				

Table 5.2: Constraints on manipulated variables

5.2 Parameters optimisation

We are left with 11 parameters; 2 weights for output tracking, 6 weights for manipulated variable tracking and control moves and 3 parameters for sampling time, prediction horizon and control horizon. Since we are not interested in input variable tracking we can set the 3 weights to zero.

We decided to separate the optimisation of the remaining parameters as follows; first sampling time, then prediction horizon and control horizon. We then focus on weights for output tracking and weights on control moves independently, before optimising their relative weight.

Sampling time
The sampling time is generally the first parameter chosen in the design of the MPC and is kept constant while tuning the other parameters. In fact, the sampling time will influence the plant dynamic; kept too high, MPC won't be able to measures rapid disturbances, while set very low; the performance will reach a limit depending of the model dynamic and it will increase computation effort.

In fact, if we consider the extreme case where $T_s = 0$, we know perfectly the output however we will need an infinite amount of memory to keep track of the information. On the other hand when T_s become infinite we are not provided with output measure and thus not able to control the network properly.

It's common practice to set T_s between 10 and 25% of the desired closed loop response time which in our case correspond to approximately one day. Our first guess is 1/10 day and in order to find the best sampling time, we tried different Ts while keeping the same time frame of 10 days. We then compared the choices based on root mean square error criteria.

Regarding the other parameters we considered identical weights and control horizon of 1. The prediction horizon p is fixed for a given time frame T_f and sampling time since;

$$T_f = pT_s$$

We observe on the graph 5.2 a parabolic curve with a minimum at sample time of 1 day and saturation trend when T_s increases. We see as expect that small T_s are preferred, however we observe an increasing for values lower 1 day. In fact, this is due to the inherent to the length of stay in the wastewater treatment plant. This latter depends of the flow and treated volume in the WWTP and the sludge retention time (SRT).

It can be approximated by the ratio between the volume and the input flow for each of

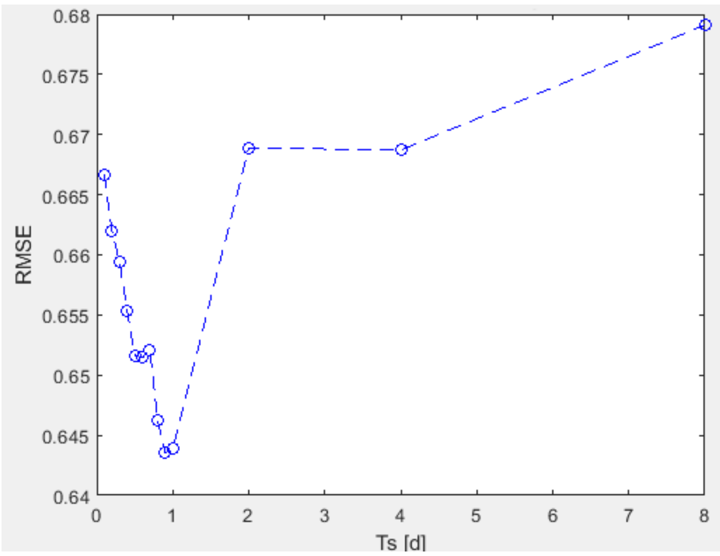

Figure 5.2: Root mean square error in function of the sampling time

the WWTP. It is minimal for SEG due to the low input flow and is equal to approximately 1 day. Considering sampling time below 1 day will actually make the MPC more sensitive to model and external disturbance (flow and concentration variation of the Seine and the Oise) and fool the effective impact of the control move on the plant.

The saturation trend is explained by the reducing impact of the control move on the dynamic compared to the flow variations at the input of Clichy and in the Seine.

Prediction horizon
The prediction horizon, p, is the number of future control intervals the MPC controller must evaluate by prediction when optimising its MVs at control interval k. Traditional tuning rule

recommends to set it early in the design so that pT_s, the time frame is equal to the desired closed loop response. Particular care has to be considered when setting p so that the controller is internally stable and anticipates constraint violations early enough to allow corrective action.

We observe on the figure 5.3 a reduction of the RMSE as p increases; however the difference is not very significant. Actually the model is not able to predict daily variations at the input of Clichy or in the Seine and therefore sampling below one day is ineffective. It may though have an impact if we want to increase the horizon prediction since it limits this latter.

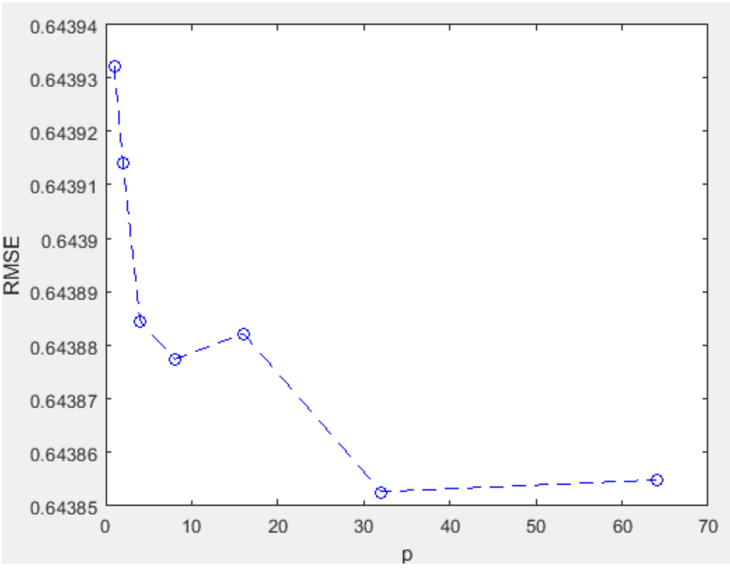

Figure 5.3: Root mean square error in function of the prediction horizon

Control horizon
The control horizon, m, is the number of manipulated variables moves to be optimised at control interval k. It must be lower or equal to p and promotes internal stable controller and robustness to disturbance if small. Large m increases the computation complexity; however controller tends to be more 'optimal'.

We observe on the figure 5.4 that the RMSE slightly increases with the control horizon before saturating. This results again from the daily variations at the input of Clichy and in the Seine. In our situation, robustness is important and Control horizon should be equal to 1.

Weights on the measured output
Playing on the measured output weights will allow us to give less or more influence on the ammonia nitrogen or the nitrite. To reduce concentrations of NH_4^+, we would intuitively treat all the water however at the cost of higher concentration of $N0_2^-$ and NO_3^- due to the conversion. Therefore, to reduce the concentration of $N0_2^-$, we would reject directly the water in the Seine. This trade-off is observable on the figures 5.6 and 5.5 obtained by considering one weight at a time on the third scenario.
When only the concentration of NO_2^- is considered (figure 5.5), all the WWTPs works at 10% of their maximal flow. The concentration of NO_2^- is well below good state limit, however, resulting from the important wastewater discharge, NH_4^+ and BOD concentrations are above their good state limit. The RMSE obtained is equal to 1.86.

On the other hand, when only NH_4^+ is considered (figure 5.5), all the WWTPs tend to work at

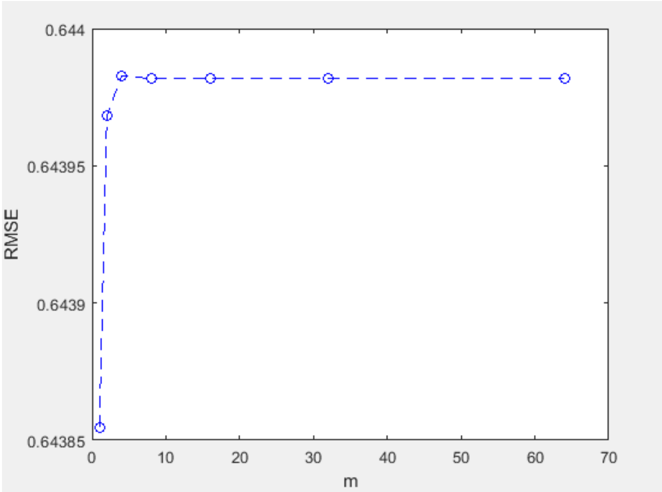

Figure 5.4: Root mean square error in function of the prediction horizon

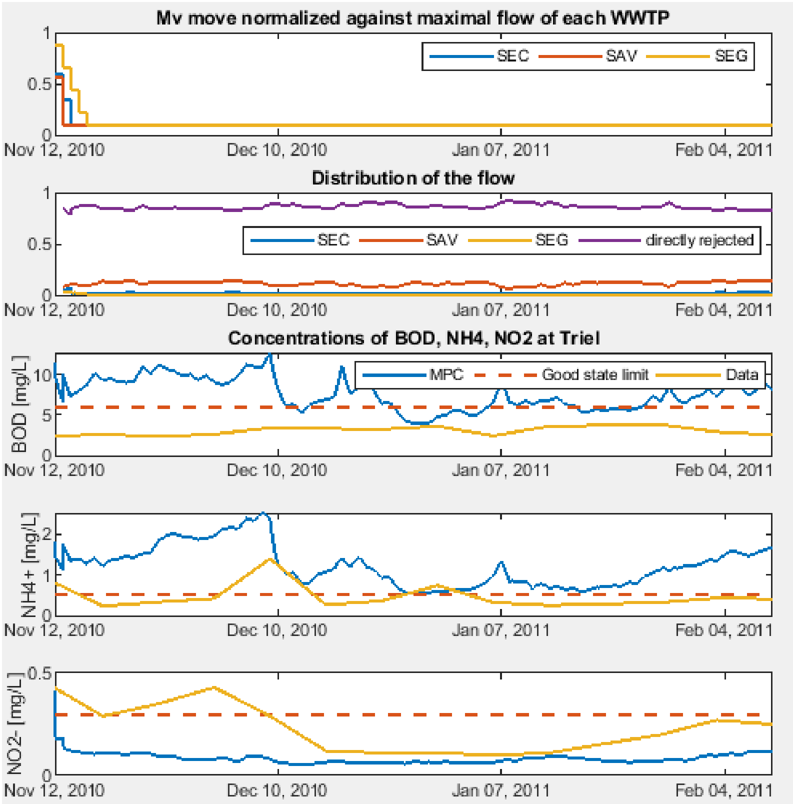

Figure 5.5: Control moves, distribution and measured concentrations at Triel for $[W_{NH_4^+}\ W_{NO_2^-}]$ = [0 1] - Scenario 3

their maximal flow. Wastewater discharged is observed due to the flow variation at the input of Clichy and the assessment of new control move every day (sampling period). This time, the concentration in NH_4^+ and BOD is below the good state limit at the cost of concentration exceeding in nitrite at the beginning and the end of the period considered. The RMSE obtained is equal to 0.33.

This tends to indicate that weight on NH_4^+ should be larger that the weight on NO_2^-. This is corroborated for the three scenarios by the evolution of RMSE against increasing weight ratio on

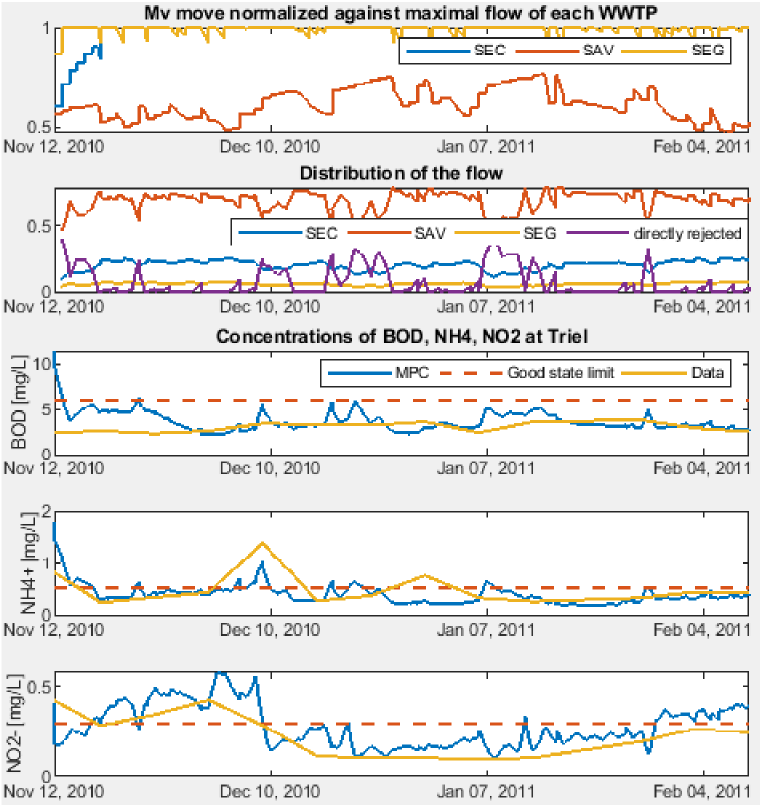

Figure 5.6: Control moves, distribution and measured concentrations at Triel for $[W_{NH_4^+}\ W_{NO_2^-}]$ = [1 0] - Scenario 3

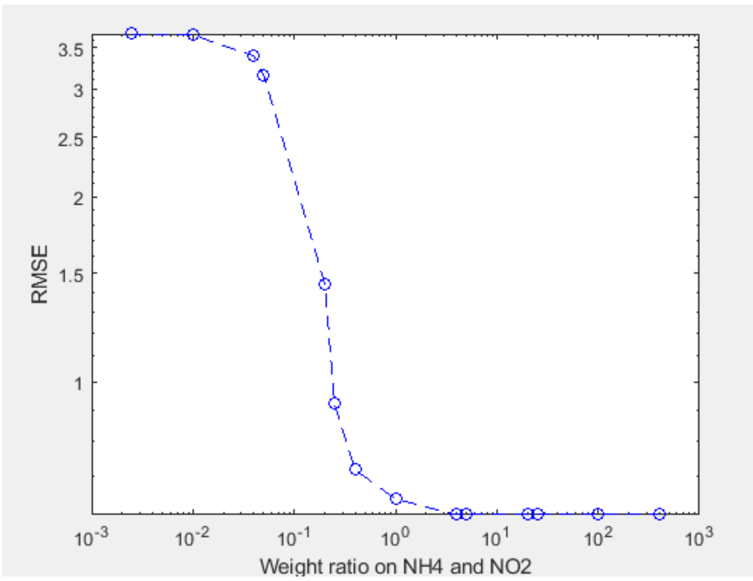

Figure 5.7: Root mean square error in function of the weight ratio between NH_4^+ and NO_2^- in logarithmic scale

NH_4^+ and NO_2^- (figure 5.7). As a result we decided to set the weight on NH_4^+ concentration five times greater than NO_2^- one.

Weights on the manipulated variables rate
Traditionally set to identical value of 0.1, large manipulated variable rate are used to avoid

large and unnecessary increments. More conservative increments provides more robust controller performance but poorer reference tracking. We considered identical weight of 1 for NH_4^+ and NO_2^- and optimised the ratio between the manipulated variables rate while keeping their relative importance with the measured output constant. The optimal ratios found are;

$$W_{\Delta mv_{SEC}} = W_{\Delta mv_{SEG}} = 0.04 W_{\Delta mv_{SAV}}$$

Wee see that SAV control move should be more penalised. Actually SAV can treat flow approximately 6 times larger that SEC and 20 times larger than SEG. When large flow variation occur at Clichy, SAV input flow will more likely fluctuate. This may keep SAV away of its nominal working condition impacting therefore its performance.

Increasing the weight on SAV manipulated variable rate, will allow to dispatch more the flow variation on SEC and SEG, keeping SAV close to its nominal working condition.

Relative weight between measured output and manipulated variable rate

After optimising independently the weight ratio on the measured output and on the manipulated variable rate; we can tune their relative importance. We observed that optimum ratio is different regarding the season. During rainy season large flow variations occurs at Clichy. Moreover, the Seine flow is important and the effluent are more diluted leading to less constraints on the output. In this condition more weight have to be put on manipulated variable rate to keep them close to their nominal working condition.

At the opposite during dry season, the constraints on effluent concentrations are harder and more weight have to be put on the output. The optimal relative weight is equal to 0.04 in rainy season (figure 5.8) and 5 in dry season (figure 5.9).

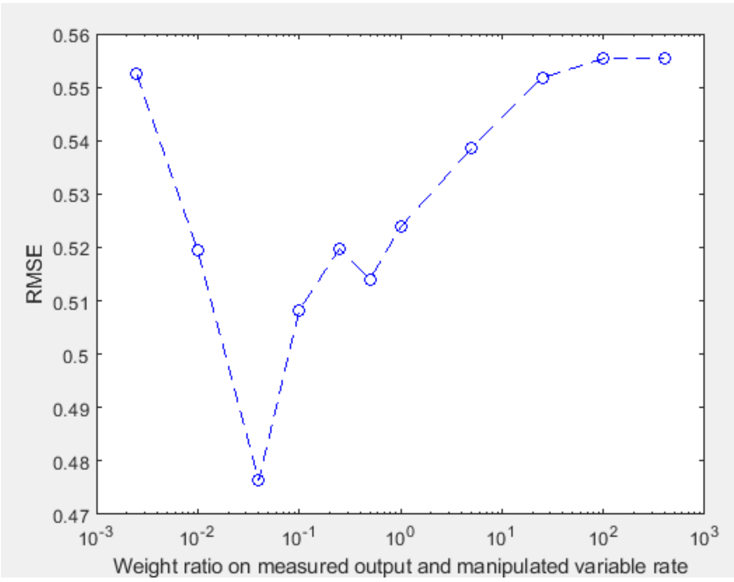

Figure 5.8: Root mean square error in function of the relative weight between the output and the manipulated variables rate rate in rainy season

Summary and evaluation of the parameter optimality

All the optimised parameters are summed up in the table 5.3. To evaluate the optimality of the parameters, we compared it to solution obtained using fuzzing methodology. This software-based technique used to find vulnerabilities in a program. [40].

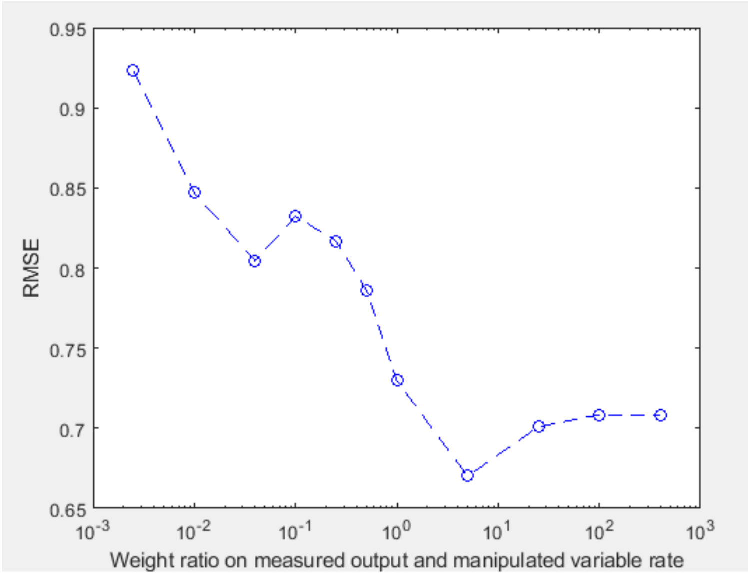

Figure 5.9: Root mean square error in function of the relative weight between the output and the manipulated variables rate in dry season

The idea is to generate randomly a huge number of set of parameters (=generation-based fuzzer) and compared the computed RMSE value against our optimal one. We also mutated random parameters from our optimal solution multiple times (=mutation-based fuzzer) and again compared it to our optimal solution. We observe that some parameter variations led to slightly lower RMSE however only for some scenario. For example, setting $W_{\Delta mv_{SAV}} = 5$ will decrease the RMSE of the first scenario by 0.0001 but will increase the RMSE of the second scenario by 0.01.

T_s	p	m	$W_{NH_4^+}$	$W_{NO_2^-}$	$W_{\Delta mv_{SEC}}$	$W_{\Delta mv_{SAV}}$	$W_{\Delta mv_{SEG}}$
1d	1	1	0.04 | 5	0.008 | 1	1	25	1

Table 5.3: Optimised parameters of the MPC; on the left for rainy season and on the right for dry season

Chapter 6

Results

6.1 Performance criteria

To measure the performance of the MPC control law we address two new criteria. The first one is computed relatively to the good state limit for each effluent concentration;

$$Perf = \frac{X_{gsl} - X}{X_{gsl}}$$

where X is the effluent concentration at one of the measurement point and X_{gsl} is the corresponding good state limit, both expressed in $[mg/L]$.

The second criterion is computed as a gain relatively to the real measurement and control that has been applied to the WWTN;

$$Gain = \frac{X_{real} - X}{X_{real}}$$

where X is the effluent concentration at one of the measurement point and X_{real} is the corresponding real concentration at the same measurement point, both expressed in $[mg/L]$.
Note that both criteria can be negative and will correspond to poorer performance; i.e respectively, concentrations above good state limit or above real measurement.

6.2 Result for rainy periods

We will in this part the results, observations and conclusion for the third scenario. Note that same conclusion may be drawn from the second scenario. Let's first observe that the concentration of NO_3^- at Triel is, as expected, below good state limit (figure 6.1). This is also the case for TSS

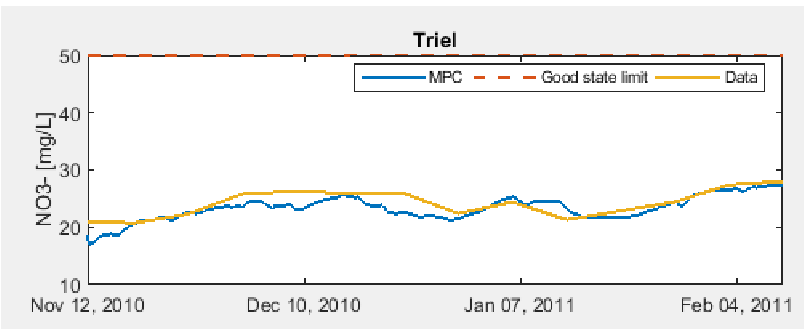

Figure 6.1: Concentration of NO_3^- at Suresnes, in the Oise and at Triel for scenario 3

expect for the periods when the Seine and the Oise are above good state limit (figure 6.2). As explained in the section 5.1, concentration resulting from dilution of two flows is bounded by the minimum concentration, which in this situation, is above good state limit. Regarding the

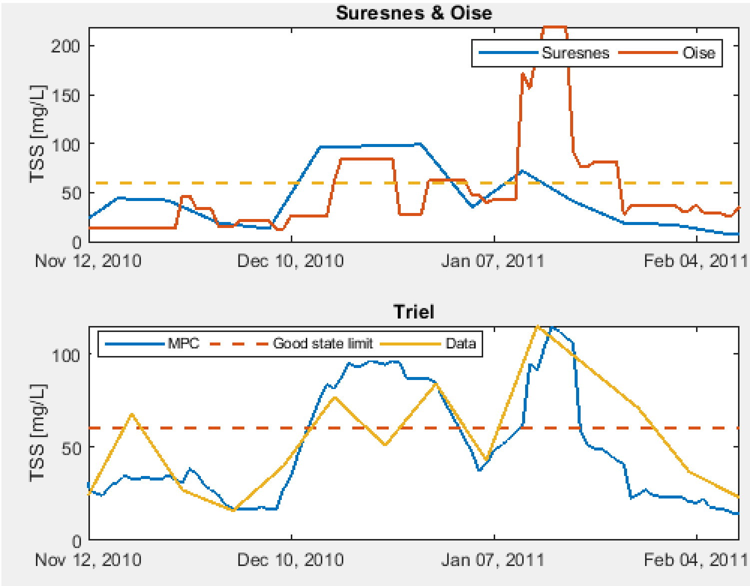

Figure 6.2: Concentration of TSS at Suresnes, in the Oise and at Triel for scenario 3

concentrations of BOD, NH_4^+ and NO_2^- at Triel, we see on the figure 6.3 that they are most of the time below good state limit. We see on the first day exceeding in BOD and NH_4^+ above good state limit which comes from the initialisation of the model. At the beginning the measured outputs are at zero and manipulated variables initialised at their nominal value. It's really on the second day that the MPC control law is synthesised and that a resorption of the error is observed.

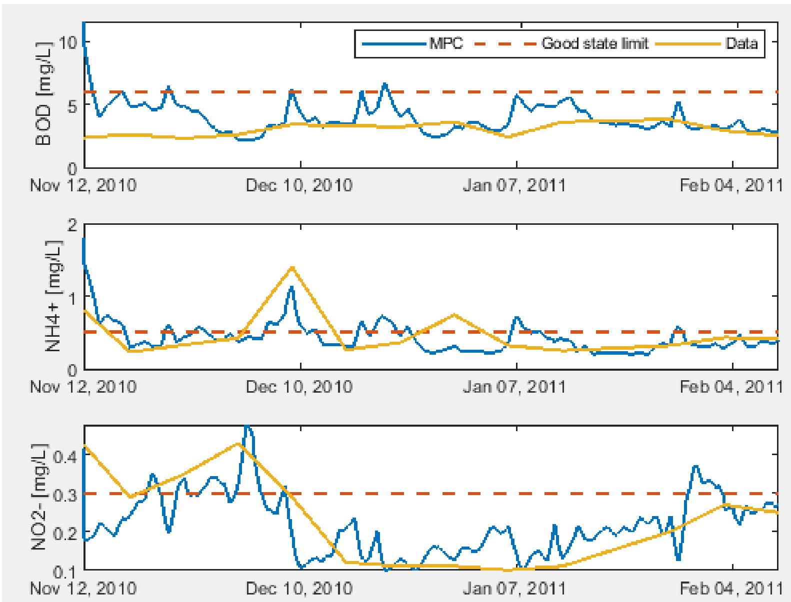

Figure 6.3: Concentrations of BOD, NH_4^+ and NO_2^- at Triel for scenario 3

Some exceeding above good state limit can be observed on the real measurements at the beginning of the period. Those excess are also present in the MPC control law, however they are less important and localised on smaller periods. This excess result actually from a combination of

Nutrients	Perf [%]	Gain [%]
TSS	11.59	11.34
BOD	35.03	-28.89
NH_4^+	18.3	-4.79
NO_2^-	29.35	-15.17
NO_3^-	53.52	3.68

Table 6.1: Mean performance criteria for each of the concentration - scenario 3

high concentrations of nitrite and Ammonia nitrogen at the input of Clichy as we can see on the figure 6.4.

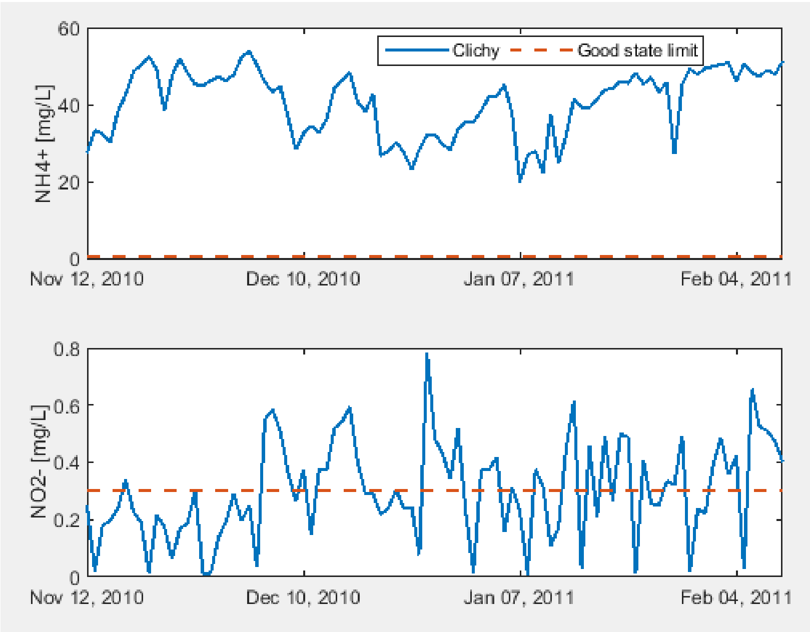

Figure 6.4: Concentrations of NH_4^+ and NO_2^- at the input of Clichy for scenario 3

The mean performance criteria for each of the concentrations are reported in the table 6.1. We observe that in average the concentrations are between 11 and 53% below the good state limit. For the BOD, NH_4^+ and NO_3^-, we are in average between 5 and 30% less good than the real measures. This may come from poorer control moves due to model approximations; no storage bays, travelling time, WWTP model,... However, this should most likely come from the real measurement itself which are done once a week, and are therefore unable to describe daily or hours fluctuations.

Finally regarding the control moves, we observe as expected from the choice of parameters that the manipulated variable on SAV is keeping SAV close to its nominal working point while the flow variation is dispatched on the two other WWTPs. The distribution of the flow is relatively steady for the real measurements; SEC, SAV, and SEG treat respectively 13.3%, 85.6% and 1% of the flow. Also very small percentage is directly rejected in the Seine. At the opposite, The MPC plays with the flow treated and discharged to minimise the objective function on the measured output and manipulated variable rate.

6.3 Result for dry period

As observed for the rainy periods and as expected, concentrations of TSS and NO_3^- are well below their good state limits (figure 6.6). Regarding the concentration of BOD, we observes that

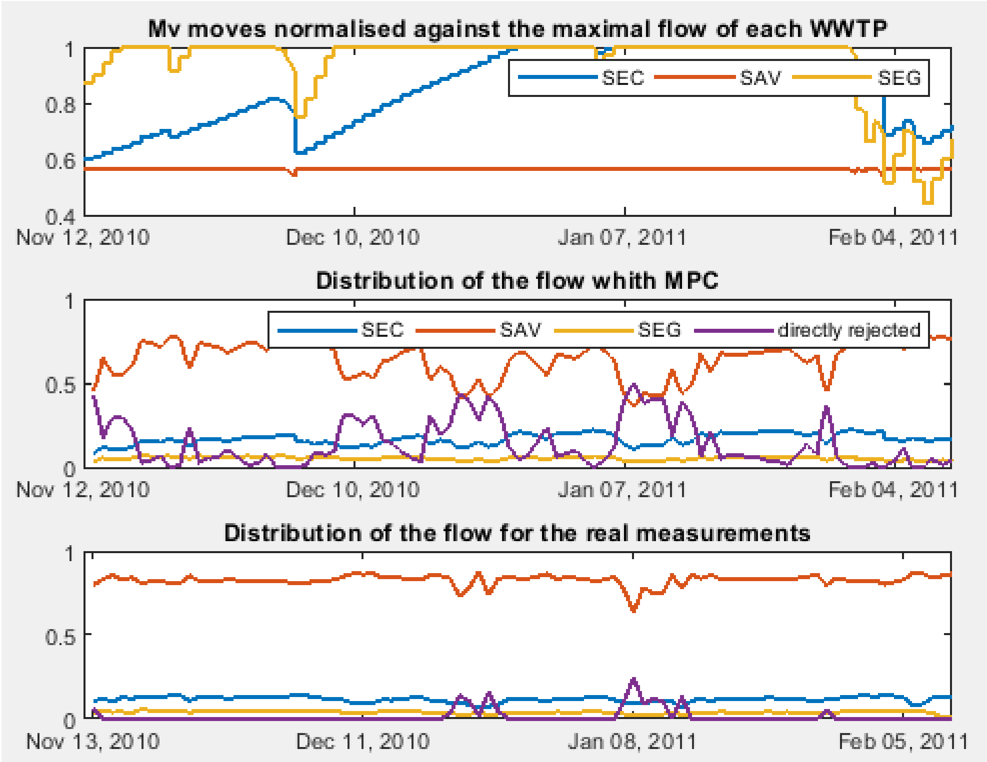

Figure 6.5: Manipulated variable moves normalised against each of the WWTP, and distribution of the flow with MPC and for real measurements - scenario 3

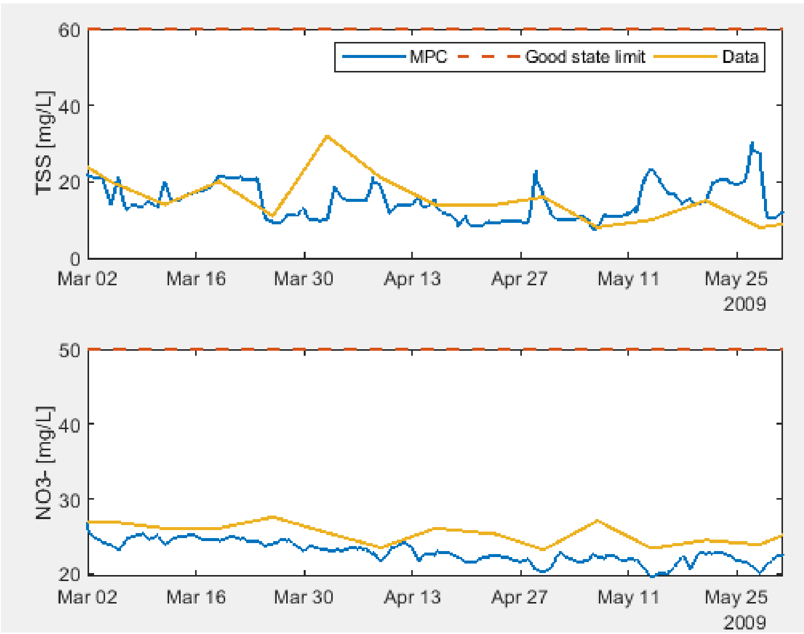

Figure 6.6: Concentrations of TSS and NO_3^- at Triel for scenario 1

good state limit is well respected expect for periodic peaks. On the other hand, the concentrations of NH_4^+ and NO_2^- are most of the time below the real measurements; however MPC law is not able to achieve concentrations below good state limit 6.7. These excesses actually result from a combination of high concentration nitrite and Ammonia nitrogen at the input of Clichy as we can see on the figure 6.8. Moreover, during the dry season the Seine flow is much lower that during rainy periods, reducing therefore the dilution capacity of the Seine. Note that peaks results

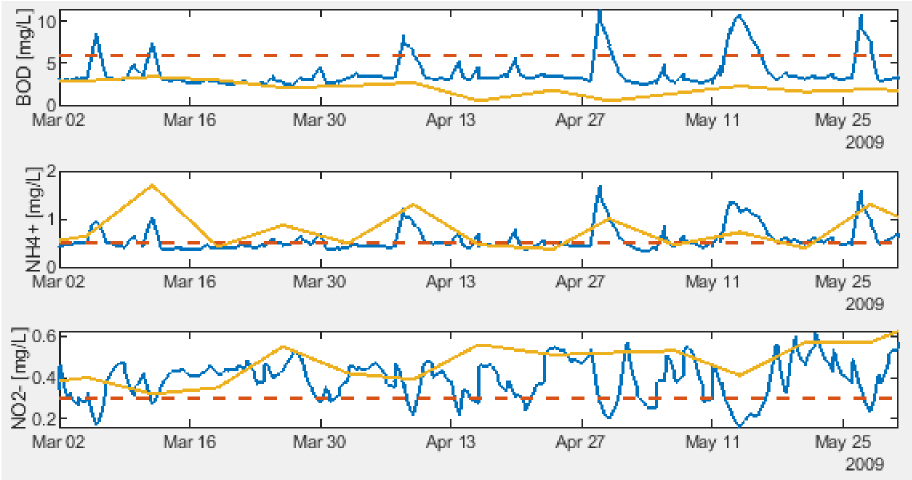

Figure 6.7: Concentrations of BOD, NH_4^+ and NO_2^- at Triel for scenario 1

actually from the overtaking of NO_2^- concentration above good state limit. Those observations

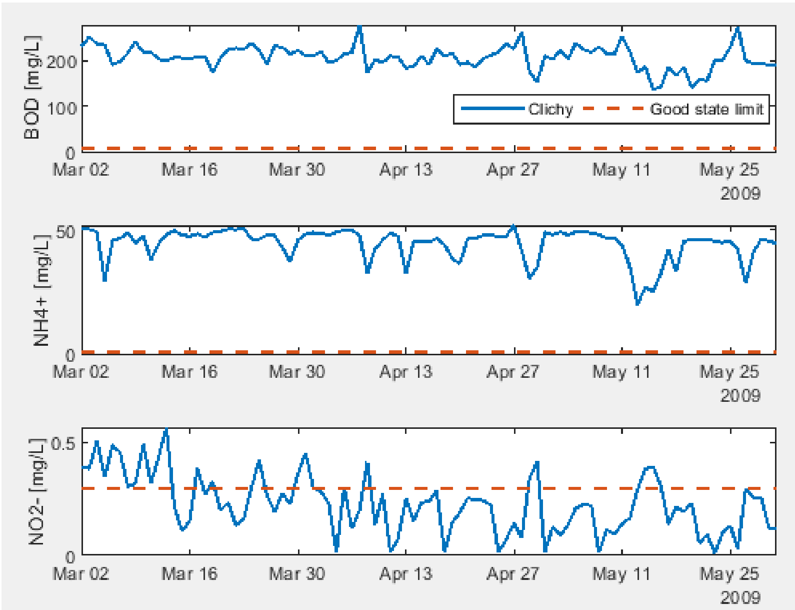

Figure 6.8: Concentrations of NH_4^+ and NO_2^- at the input of Clichy for scenario 1

are confirmed by the performance criteria reported in the table 6.2. BOD, TSS and NO_3^- are in average between 33.65 and 75.42 % below good state limit. However, due to periodic peaks BOD concentration increases in average of 145% compared to real measurements. On the other hand, MPC law improves the results on NH_4 and NO_2 by approximately 15%.

Finally regarding the control moves, we observe, that more moves are allowed on SAV manipulated variable; compared to the situation in rainy conditions. This comes from the chose of MPC parameter. As a result SEC and SEG are less sensitive to flow variation and are more likely to remain at their maximal working condition.
The distribution of the flow for the real measurements does not seem to change from the rainy periods; SEC, SAV, and SEG treat respectively 13.3%, 85.6% and 1% of the flow. On the other hand the MPC law is now less likely to reject wastewater directly in the Seine as output tracking have more weight in the objective function and dilution capacity of the Seine decreases. Again,

Nutrients	Perf [%]	Gain [%]
TSS	75.42	-3.92
BOD	33.65	-145.2
NH_4^+	-16.54	17.27
NO_2^-	-29.46	14.91
NO_3^-	54.26	9.85

Table 6.2: Mean performance criteria for each of the concentration - scenario 1

the periodic peaks can be correlated to the overtaking of NO_2^- above good state limit at the input of Clichy.

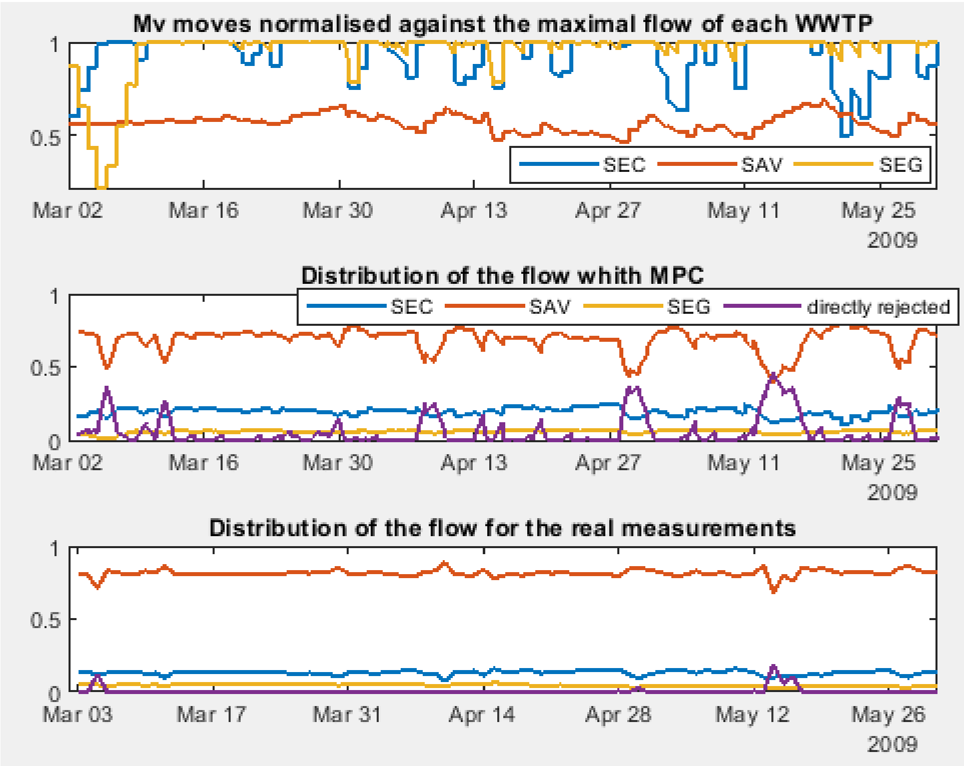

Figure 6.9: Manipulated variable moves normalised against each of the WWTP, and distribution of the flow with MPC and for real measurements - scenario 1

Chapter 7

Conclusion

This master thesis had for ambition to find a control strategy to optimise the partitioning of the sewage between wastewater treatment plant controlled by the SIAAP in the Paris agglomeration. We first started by looking in the literature for control strategy candidate and finally selected Model predictive control which, widely applied to the industry, demonstrated high capability to deal with constraints in an optimal fashion and complex plant. We then reviewed the main MPC subclass techniques and presented its most common formulation.

We then expressed the MPC in the form of quadratic programming optimisation problem and studied feasibility and stability properties related to it. We also looked at the specificities of the model predictive control toolbox of Matlab upon which we relied for this work.

Requiring an internal model of the plan for the MPC, we decided next to examine more closely the wastewater treatment network of the SIAAP. We concentrated our study area to the west part of the network which contained the most important treatment facilities and reduced it to a simple model with one input node, three wastewater treatment plant and 5 measurement points.

Based on a simple model of WWTP developed by Robles-Rodriguez et al in 2018, we demonstrated that our model reproduces very well the dynamics of total suspended solid, biological demand in oxygen, ammonia nitrogen nitrites and nitrates. The efficiency of model was evaluated on real measurements by the use root mean square criterion.

We then selected 3 scenarios to parameter the MPC and evaluate its control law. A systematic approach has been proposed for MPC parameters tuning; a first restriction is made on the parameters based on physical constraints and limitations of the plant. The remaining parameters separated and ranked based on their relative influence on the plant dynamic. Each parameter are then optimised independently by dynamical simulation. The performance of the control law with a given set of parameters is addressed by a quality function while ensuring robustness and stability of the controller.

The efficiency of the obtained parameters were evaluated using fuzzy methodology with both generation-based fuzzer and mutation-based one. The MPC control law has finally been evaluated on both dry and wet conditions and compared to real data provided by the SIAAP. The control law has showed performance above 18% regarding the good state limit under rainy conditions and an increase of 15% of the performance under dry conditions for.

At this point we demonstrated that the MPC law tends in average to improve performance of the wastewater treatment network. However it must be remembered that we wasn't able to really compare on daily basis the effect of the control law since real measurement are made only once a week at each of the measurement points. Moreover, experimental test should be carried to address real performance of the control law and measures its robustness against model disturbance.

Bibliography

[1] N. lawrence Ricker Alberto Bemporad, Manfred Morari. *Model Predictive Control Toolbox for Use with Matlab*. The MathWorks, 2005.

[2] Delphine Bossy. Journée mondiale de l'eau : les chiffres étonnants de l'or bleu, mar 2013. https://www.futura-sciences.com/planete/actualites/developpement-durable-journee-mondiale-eau-chiffres-etonnants-or-bleu-45364/.

[3] Claus Wedekind. Fish populations surviving estrogen pollution. *BMC Biology*, 12(1):10, Feb 2014.

[4] Daiane L. Gebauer, Natália Pagnussat, Ângelo L. Piato, Isabel C. Schaefer, Carla D. Bonan, and Diogo R. Lara. Effects of anxiolytics in zebrafish: Similarities and differences between benzodiazepines, buspirone and ethanol. *Pharmacology Biochemistry and Behavior*, 99(3):480 – 486, 2011.

[5] Rémi GUILLET Michel BOUVIER, François DURAND. Médicament et environnement, la régulation du médicament vis-à-vis du risque environnemental. *Conseil Général de l'Environnement et du Développement Durable*, 2010.

[6] European Commission. Introduction to the new eu water framework directive, jun 2016. http://ec.europa.eu/environment/water/water-framework/info/intro$_e$*n.htm*.

[7] Council directive 91/271/eec of 21 may 1991 concerning urban waste-water treatment. *Official Journal L135*, pages 0040–0052, may 1991.

[8] Commission directive 98/15/ec of 27 february 1998 amending council directive 91/271/eec with respect to certain requirements established in annex i thereof (text with eea relevance). *Official Journal L067*, pages 0029–0030, mar 1998.

[9] Commerce extérieur et Coopération au Développement Royaume de Belgique, Affaires étrangères. Transposition, 2016. https://diplomatie.belgium.be/fr.

[10] Nicolas de Sadeleer. La cour de justice de l'union européenne condamne financièrement la belgique pour ses insuffisances en matière d'eaux usées : comment répartir la condamnation entre les trois régions ?, feb 2014. http://www.justice-en-ligne.be/article607.html.

[11] Stefano Ciannella. *Applied Model Predictive Control, A brief guide to Matlab/ Simulink MPC toolbox*. Chemical Engineering Applied Chemistry, University of Toronto, 2004.

[12] Matthias Lorenzen, Frank Allgöwer, and Mark Cannon. Adaptive model predictive control with robust constraint satisfaction. *IFAC-PapersOnLine*, 50(1):3313 – 3318, 2017. 20th IFAC World Congress.

[13] Gorinevsky. Lecture 14, model predictive control, part 1:the concept, 2005.

[14] Alberto Bemporad. Computing the linear mpc control law: Qp solvers and explicit mpc, 2018.

[15] Alberto Bemporad. Model predictive control.

[16] R. E. Kalman. Contributions to the theory of optimal control. *Bulletin de la Societe Mathematique de Mexicana*, 5:102–119, 1960.

[17] R. E. Kalman. A new approach to linear filtering and prediction problems. *Transactions of ASME, Journal of Basic Engineering*, 87:35–45, 1960.

[18] Neha Raghu Ruchika. Model predictive control: History and development. *Transactions of ASME, Journal of Basic Engineering*, 4(6), 2013.

[19] Rault A. Testud J.L. Papon J. Richalet, J. Model predictive heuristic control: Application to industrial processes. *Automatica*, 14(2), 1978.

[20] Ramaker B.C. Cutler, C.R. Dynamic matrix control - a computer control algorithm. *Automatic Control Conference, San Francisco, CA*, 1980.

[21] Giovani Cavalcanti Nunes. *Design and analysis of multivariate predictive control applied to an oil-water-gas separator: A polynomial approach.* PhD thesis, University of Florida, 2001.

[22] Morshedi A. Haydel J. Cutler, C. An industrial perspective on advanced control. *In AICHE annual meeting, Washington, DC*, 1983.

[23] Froisy B. Hammann M. Grosdidier, P. The idcom-m controller. *In T. J. McAvoy, Y. Arkun, E. Zafiriou (Eds.), Proceedings of the 1988 IFAC workshop on model based process control, Oxford: Pergamon Press*, 1988.

[24] Piotr Tatjewski. *Advanced Control of Industrial Processes: Structures and Algorithms.* Springer, 2007.

[25] Hamza Hamadah R. Bhushan Gopaluni Michael G. Forbes, Rohit S. Patwardhan. Model predictive control in industry: Challenges and opportunities. *The International Federation of Automatic Control*, 2015.

[26] Scattolini R. Muñoz de la Peña D. Liu J. Christofides, P. D. Distributed model predictive control: A tutorial review and future research directions. *Computers Chemical Engineering*, 51, 2013.

[27] Michael Nikolaou. Model predictive controllers: A critical synthesis of theory and industrial needs. *Advances in Chemical Engineering, Academic Press*, 26, 2001.

[28] Liupig Wang. *Model Predictive Control System Design and Implementation using Matlab.* Springer, 2009.

[29] Daniel Simon. *Model Predictive Control in Flight Control Design, Stability and Reference Tracking.* PhD thesis, Linköping University, 2014.

[30] Morten Hovd. *A brief introduction to Model Predictive Control.* PhD thesis, Engineering Cybernetics Department, NTNU, 2004.

[31] Gabriele Pannocchia. Course on model predictive control, part i - introduction, 2012. Departmnet of Chemical Engineering, University of Pisa.

[32] J.B. Rawlings and K.R. Muske. The stability of constrained receding horizon control. *IEEE transactions on automatic control*, 38, 1993.

[33] Data, information and pictures provided by the SIAAP.

[34] V.Rocher D.Dochain C.E. Robles-Rodriguez, J.Bernier. A simple model pf wastewater treatment plants for managing the quality of the seine river. 2018.

[35] P.Vega M. Francisco. Automatic tuning of model predictive controllers based on multiobjective optimization. *Latin American applied research*, 2010.

[36] S. Li and G. Du. On-line tuning scheme for generalized predictive control via simulation-optimization. *IEEE International Conference on Fuzzy Systems*, 2002.

[37] R. Sridhar and D. Cooper. A tuning strategy for unconstrained siso model predictive control. *Ind. Eng Chem Res*, 36, 1997.

[38] B.A. Francis Doyle, J.C. and A.R. Tannenbaum. Feedback control theory. *MacMillan*, 1992.

[39] M. Francisco Vega, P. and E. Sanz. Norm based approach for automatic tuning of model predicitive controllers. *Proceedings of ECCE-6, Copenhague*, 2007.

[40] Kaiser G Dai H, Murphy C. Configuration fuzzing for software vulnerability detection. *International Conference on Availability, Reliability and Security International Conference on Availability, Reliability and Security*, 2010.

[41] M Henze, C.P.L.Jr Grady, W Gujer, G Marais, and T Matsuo. Activated sludge model no.1. *IAWPRC Scientific and Technical Reports No.1*, London, UK, 1987.

Tables

Parameter	Unit	SEC	SAV	SEG
$\rho_{1,\max}$	$\text{mg}O_2/(\text{mg}X-d)$	3.99	2.56	1.93
$\rho_{2,\max}$	$\text{mg}NH_4/(\text{mg}X-d)$	0.84	0.83	0.89
$\rho_{3,\max}$	$\text{mg}NO_2/(\text{mg}X-d)$	1.68	1.27	0.92
$\rho_{4,\max}$	$\text{mg}NO_3/(\text{mg}X-d)$	1.21	1.38	0.85
K_{BOD}	mgO_2/L	13.67	11.65	14.26
K_{NH}	$\text{mg}NH_4/L$	6.59	14.98	8.53
K_{NO2}	$\text{mg}NO_2/L$	2.46	1.15	2.55
K_{NO3}	$\text{mg}NO_3/L$	1.40	2.69	4.20
$Y_{X/BOD}$	$\text{mg}X/\text{mg}O_2$	0.67*	0.67*	0.67*
$Y_{X/NH}$	$\text{mg}X/\text{mg}NH_4$	0.24*	0.24*	0.24*
$Y_{NH4/NO2}$	$\text{mg}NH_4/\text{mg}NO_2$	0.28	0.25	0.27
$Y_{NO2/NO3}$	$\text{mg}NO_2/\text{mg}NO_3$	0.68	0.64	0.70
s_d	-	35.8	14.8	38.2
b	1/d	0.10*	0.10*	0.10*
f_X	-	0.08*	0.08*	0.08*

*Parameters taken from [41]

Table 1: Calibrated values for kinetic and stoichiometric parameters

Rue Archimède, 1 bte L6.11.01, 1348 Louvain-la-Neuve **www.uclouvain.be/epl**