Université catholique de Louvain

École polytechnique de Louvain (EPL)

Contract design to support renewable investment : an equilibrium approach

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for obtaining the Master's degree in
Mathematical Engineering

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Academic year 2017-2018

Disclaimer: “The views set out in this study are those of the author and do not necessarily reflect the opinion of ENGIE.”

Acknowledgements

Before jumping into the subject, I would like to take these few lines to express my gratitude to the different people that have helped me with my master thesis.

First, this work would not have been possible without the help of my two supervisors Professor Anthony Papavasiliou and Mr Gauthier de Maere d'Aertrycke. I am very grateful for all the time both of you have taken to guide me.

I would like to thank Gauthier for opening ENGIE's doors each week. I appreciated the time you took for the numerous explanations about risk averse agents and the help you provided to interpret the results in a refined way. The detailed input about the GAMS specifics was also very useful.

I would also like to thank Anthony for his availability. I was very happy to follow last year your excellent courses. I enjoyed the way you gave intuition to the optimization problems we studied. The precise comments and feedback you gave on this thesis were very helpful.

I thank my readers Professor Yves Smeers and Professor Julien Hendrickx for the time they are taking reviewing my thesis.

Thank you to my Dad as my fellow students Quentin Lété and Gauthier Muguerza for the proof reading.

Finally, I also would like to use this opportunity to thank my parents, friends and roommates for their continued support over my study years.

Abstract

Considering the risky environment in which the electricity markets evolve, financing renewable energy projects is a serious challenge. The goal of this work is to analyse financial contracts that could improve the financing of renewables. Capital budgeting is modelled with the good-deal risk measure. The problem tackled is a partial equilibrium model with incomplete markets representing the electricity sector. A new iterative approach is proposed to solve the Nash equilibrium problem, by decomposing the problem in a sequence of QCP problems. Different types of contracts are analysed to find the market tools that are the best suited for financing renewable projects. The method is then applied to a larger scale including 88 yearly scenarios with an hourly resolution.

CONTENTS

LIST OF SYMBOLS

Sets

i, I	an agent and the set of agents
k, K	a producer and the set of producers each producer represents one technology
ω, Ω	a scenario and the set of different scenarios
ℓ, L	a time segment and the set of time segments

Variables

x_k	capacity installed of technology k
$y_{k,\ell}(\omega)$	electric production of technology k during time segment ℓ
$\mu_{k,\ell}(\omega)$	marginal value of the plant's capacity
$\pi_\ell(\omega)$	price of electricity
$z_\ell(\omega)$	shortfall or curtailment of electricity during time segment ℓ for scenario ω
$q_i(\omega)$	risk averse probability of agent i for scenario ω
t_i	average hedged costs with the risk-free rate
$u_i(\omega)$	under performance of risk measure due to the unhedged risk
s_i	measure on the dispersion of the $u_i(\omega)$
W_i	quantity of contracts bought by agent i
π_c^f	price of the forward contract
$\pi_c(\omega)$	payoff of the forward contract
$\lambda(\omega)$	price of a CO2 allowance
$Storage\_cap$	total storage capacity of the battery
$Charge\_cap$	maximal charging capacity of the battery in one time segment ℓ
$e_\ell(\omega)$	storage of the battery at the end of time period ℓ
$\gamma_\ell(\omega)$	opportunity cost of the battery of producing or consuming electricity ℓ
$\kappa_\ell(\omega)$	consumption value of the battery during time period ℓ
$\sigma_\ell(\omega)$	supply value of the battery during time period ℓ
$c_\ell(\omega)$	consumption of the battery during time period ℓ
ν	marginal value of the capacity in the forward market

Parameters

$C_k(\omega)$	variable cost of producer k for scenario ω
I_k	annual investment cost of technology k
D_ℓ	demand during time segment ℓ
PC	price cap for the electricity
$p(\omega)$	probability that event ω happens
$\overline{SR}$	upper bound for the Sharpe ratio
SR_i	Sharpe ratio of agent i
$R^i(\omega)$	return rate of the portfolio of agent i
γ	$\gamma^2 = 1 + \overline{SR}^2$ simplification term for the Hansen–Jagannathan bound
$cost_i(\omega)$	cost of agent i in scenario ω
$m_i(\omega)$	$m_i(\omega) = \frac{q_i(\omega)}{p(\omega)}$ discount factor of agent i for scenario ω
$\bar{q}(\omega)$	average of the risk averse probabilities of all the producers for scenario ω
$Dissipation\_factor$	percentage of electricity remaining after the dissipation during one time segment
η	roundtrip efficiency of the battery: losses due to charging electricity and discharging the battery
$fact_{k,\ell}(\omega)$	fraction of available capacity of technology k during time segment ℓ in scenario ω. It has a value of 1 for most of the technologies but a value between 0 and 1 for renewables
NAP	cap on the CO2 emissions in *ton*
E_k	amount of CO2 emissions emitted to produced 1 unit of electricity
$MC_{k,fuel}(\omega)$	marginal cost of technology k with fuel *fuel* in scenario ω

Other

$\mathcal{G}_k$	problem of producer k
$\mathcal{C}$	consumer problem
ρ_i	risk measure of agent i
Γ_i	cost function of agent i
$\mathcal{M}_i$	space of risk averse probabilities $q_i(\omega)$
$\mathbb{E}_Q(\cdot)$	expectation with probability measure q for example $\mathbb{E}_Q(\pi(\omega)) = \sum_{\omega\in\Omega} q(\omega)\pi(\omega)$

Abbreviations

RCCEP	Risky Competitive Capacity Expansion Problem
RA	Risk Averse
RN	Risk Neutral
RAIC	Risk Adjusted Investment Cost

CHAPTER

1

INTRODUCTION

The energy sector is undergoing an important transition. Due to political commitment on environmental decisions, electricity production is shifting to highly decarbonised electricity generation. The European Union has fixed an ambitious long-term goal that is to reduce the CO2 emissions by 80 to 95 % by 2050 compared to the 1990 emission level [18]. For this reason, renewable electricity generation is essential.

Renewable technologies inherently carry a lot of risk. Whereas conventional technologies, historically used to generate the electricity, are controllable, renewable electricity production is intermittent. Meteorological conditions have a high impact on the production of renewable technologies. The question that needs to be answered is how to promote the needed investment.

On top of this, the investment cost in these renewable technologies is capital intensive, which increases the risk exposure of renewables. For conventional plants, the operating cost is high (but the investment cost is low), so the risk exposure of the investment is limited.

An example of this meteorological risk is the German 'Dunkelflaute' (dark doldrums) [16], which refers to periods with no sunshine and barely some wind. Hence the renewable electricity production is low during a Dunkelflaute. Periods of Dunkelflaute are of high risk for the electricity generation if a lot of renewable electric capacity is installed. Part of the solution is to install more electric connections between neighbouring countries to increase importing capacities. However this does not work all the time, for instance when neighbouring countries also have a Dunkelflaute. This is what happened February 3rd 2006 (Figure 1.1).

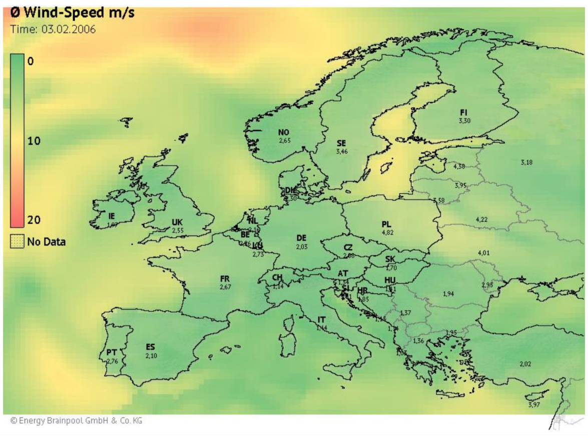

Figure 1.1: February 3rd 2006 whole of Europe was in a Dunkelflaute (source: [11])

Due to variable meteorological conditions, electricity generation cannot always be predicted at the time when the investment decisions must be made. It is crucial to find a solution for the electric production during Dunkelflaute periods. Gas plants can operate during Dunkelflaute periods but are not competitive when renewable energy is producing at full speed because of their high operating costs. Therefore, when deciding about the investment in a new gas plant, it is essential to have the correct financial tool at disposal. Aydin et al. insist on the negative correlation between renewable production and electricity price [13]. This negative correlation exists when a renewable technology has a high market share. Because this renewable technology cannot control its production, if a lot of wind or sun is generated at a certain time it must be sold at that same time. This high market share and lack of flexibility results in a cannibalisation effect meaning that renewables sell their electricity at a lower average price than the average market price.

Other types of risk also exist in the market such as risk on the growth of demand, policy risks, risk on the fuel price, ... These market risks are also important but they are not inherent to renewables, this is why this work focuses more on meteorological risk.

Which technology is at the end of the day the most subject to risk? Is there a particular problem with financing renewables? Which contract could solve this financial issue for risk averse investors? These are the main questions that will be analysed. The goal of this master thesis is to find the best possible financial instrument to incite investment in renewables.

A market in general is defined by different actors who can take decisions in this market. These different actors are represented through different agents who can take decisions in the market. The latter will be modelled as a Nash equilibrium. The problem takes place at a high level in the energy market. The goal is to find the optimal long-term investment in the different technologies that can produce electricity by taking the risk into account.

The risk sensitivity of the different agents in the market is not the same. Taking the example of the Dunkelflaute, some agents have a great benefit of this profusion of expensive electricity, others not. Conventional generators sell their electricity production at a higher price. Renewable producers cannot sell any electric production. Finally, consumers have to pay a higher price for their electric consumption. Each agent thus needs its own risk measure to evaluate its risk due to uncertainty.

Each agent will use his risk measure to make his decisions. The aim of this master thesis is to find the optimal investment with different risk averse agents. Due to this risk aversion there is a loss of welfare in the market. By this is meant that the collective surplus of all the agents is lower due to their risk aversion.

Contracts can partly solve this issue as they enable risk transfer. Taking the Dunkelflaute example again, the argument that risk was not the same for all the agents has been made. Since the risk positions of different agents are opposed, the idea is to transfer risk of one agent to another by the mean of a contract. This contract leads to less risk averse agents and to higher welfare in the market.

The problem is explained in chapter 2. Basic agent formulations are detailed and the structure of the Nash equilibrium problem is detailed. Hereafter the risk measure and contracts are introduced. It should also be stressed that the equilibrium problem at this stage is nonlinear, in some parts there are multiplications of different variables. Thus solving big instances of this problem takes a lot of time.

In chapter 3 a decomposition is developed to tackle this issue. The problem is reformulated in two parts: one physical part and one financial part. Inspiration is taken from the paper "Risk adjusted discounted cash flows in capacity expansion models" [7]. In this paper the authors decide to change the price perceived by different agents. The idea described in this thesis is to use this concept in order to slightly modify the investment costs of the different agents. This will be done by playing around and scaling adequately the different variables and equations of the problem.

A detailed formulation for each agent is given in Chapter 4. Different concepts such as CO2 emission allowances are explained and added to the model.

Chapter 5 focuses on the different financial tools that can be added to the problem in order to hedge the risk the best possible way. These cases are bound by the complete market where all the hedgeable risk is hedged.

In Chapter 6 the dataset, on which the numerical case is based, is detailed. Thereafter the problem is solved in different settings by applying the algorithm described in chapter 2. The efficiency of different contracts is then analysed. The analysis takes inspiration from what was done in the paper "Investment with incomplete markets for risk: The need for long-term contracts" [12].

CHAPTER

2

RISKY COMPETITIVE CAPACITY EXPANSION PROBLEM

In this chapter the Risky Competitive Capacity Expansion Problem is explained. This type problem has already been defined in different papers ([5], [14] and [15]). The approach that will be used is closer to the paper "Generation capacity expansion in a risky environment: A stochastic equilibrium analysis" (2011) written by Andreas Ehrenmann and Yves Smeers [5].

2.1 Definition of the problem

The problem, which is dealt with in this thesis, depends on decisions made by several agents. There are two main types of agents : producers and a consumer. The producers decide about their electricity production and sell this production to the consumer who decides about his consumption. The problem is modelled through a Nash equilibrium. The different conditions of the Nash equilibrium can be divided in these major categories:

1. $|K|$ problems $\mathcal{G}_k$ faced by a set K of producers representing the problem of each producer having to decide his investment and operating variables.
2. a problem $\mathcal{C}$ representing the aggregate consumption faced by the consumer. Here the consumer decides how much electricity he wants to consume given an electricity price.
3. on top of the agents, market clearing conditions need to be added to ensure that supply meets demand.

It should be noted that in this document "consumer" means an aggregation of all the small consumers of the real world in one consumer agent. The "producer" will sometimes be called "investor" as he has to decide the electric capacity to install. Throughout the different cases that will be discussed, one will always consider each technology k as a different producer agent. It should be observed that there is no loss of generality using this reasoning. The producer agents in our case are considered as an aggregation of all the different real-world producers that want to invest in this technology. One shall define I as the set of agents (the consumer and all the producers) and i a specific agent.

Since a Nash equilibrium problem is dealt with, each agent wants to maximize his own surplus, this is the same as minimizing its costs. The cost minimization approach will be used.

Hence, the different agents have to make choices to reach their goal that is minimizing their costs. In general there are two types of decisions that can be made by an agent: on the one hand, first stage decisions are made before uncertainty Ω happens, on the other hand, second stage decisions are made after that uncertainty Ω has happened. Therefore second stage decisions depend on Ω and are thus labelled with this uncertainty subscript. The probability of an event ω to occur is $p(\omega)$. This probability will be called the Risk Neutral probability (RN). Physical costs of the different agents are defined as Γ_i. By physical cost one means a cost related to the production or consumption of electricity and not the costs related to the contracts.

2.2 Risk measure

Risk aversion must be introduced in the problem. Risk aversion can be modelled in different ways. In this master thesis, the choice is made to model the risk by changing the risk neutral probability set P, this is the real probability of uncertainty Ω to happen, to a new probability set Q_i which is different for each agent. Since the agents are risk averse, they will see the world from a pessimistic point of view. Each agent will, from his angle, lower his probabilities for the scenarios with high surplus (or low costs), while he will increase his probabilities for the worst scenarios, scenarios he sees as harmful for his surplus (high costs). The set Q_i will contain the risk averse probabilities $q_i(\omega)$ representing the different risk aversions of each agent.

A risk measure $\rho(\cdot)$ is a function that maps uncertain outcomes, defined as random functions $Z(\omega)$, to the extended real line $\bar{\mathbb{R}} = \mathbb{R} \cup \{+\infty\} \cup \{-\infty\}$. We define $\mathcal{Z}$ as the space to which the random functions $Z(\omega)$ belong. The reader can consult [3] for formal details on the assumptions made on this space.

The cost function of an agent is taken as argument of the risk measure. This cost function Γ_i depends on decisions made by different agents and hence is specific for each agent. Agent decisions can depend on uncertainty ω. Hence the cost function will also depend on this uncertainty. The cost function will be detailed further on.

One can now define the risk measure $\rho_i(.)$ as the maximum on an uncertainty expectation $\mathbb{E}_{Q_i}(.)$ on the cost function Γ_i. The decision is made to follow the conventions of "Lectures on Stochastic Programming: Modelling and Theory" by Shapiro [3]. This gives the risk measure ρ_i as:

$$\rho_i(\Gamma_i) = \max_{q_i(\omega)\in\mathcal{M}_i} \mathbb{E}_{Q_i}(\Gamma_i) = \max_{q_i(\omega)\in\mathcal{M}_i} \sum_{\omega\in\Omega} q_i(\omega)\Gamma_i$$

$\mathcal{M}_i$ being the space where the RA probabilities $q_i(\omega)$ may belong to.

One will work with coherent risk measures for this problem. The definition of a coherent risk measure is detailed in Appendix A. From the book of Shapiro [3] section 6.3.1, the sub-differential of a coherent risk measure $\rho_i(.)$ is defined by:

$$\partial\rho_i\Big(\Gamma_i\Big) = \widetilde{q}_i(\omega) = \arg \max_{q_i(\omega)\in\mathcal{M}_i} \mathbb{E}_{Q_i}(\Gamma_i)$$

The partial derivative of risk measure ρ_i with respect to a variable a can be obtained using the chain rule:

$$\frac{\partial \rho_i\big(\Gamma_i\big)}{\partial a} = \sum_{\omega\in\Omega} \frac{\partial \rho_i}{\partial \Gamma_i(\omega)} \frac{\partial \Gamma_i(\omega)}{\partial a}$$
$$= \mathbb{E}_{\widetilde{Q}_i}(\frac{\partial \Gamma_i}{\partial a_i})$$

It should be noted that in the first line one has stressed that Γ_i depended on uncertainty ω with the notation $\Gamma_i(\omega)$. Notice also the $\widetilde{Q}_i$ in the expectation that comes from the sub-differential. For the sake of simplicity the notation $q_i(\omega)$ will be used to represent both $\widetilde{q}_i(\omega)$ and $q_i(\omega)$ for the rest of the thesis. These probabilities will be referred to as the Risk Averse probabilities (RA).

2.3 Good-Deal pricing

In this section the Good-Deal pricing risk measure is explained. To do so, inspiration was taken from the book "Asset Pricing" of John H. Cochrane [1] and the PhD thesis "Financial engineering models for electricity market : futures pricing, liquidity risks and investment" written by Gauthier de Maere d'Aertrycke [6].

Modelling risk averse probabilities of the different agents will be done using the Good-Deal pricing. It has the advantage of being a coherent risk measure (respecting convexity, monotonicity, translation equivariance and positive homogeneity) [3] (these axioms are detailed in appendix, section A). Another asset of the Good-Deal pricing is that it is explainable from a financial point of view. Indeed this risk measure rules out portfolios having a too high Sharpe ratio. The Sharpe ratio of each agent i denoted by SR_i, is a tool to measure the performance of a portfolio i by taking into account the risk on the return rate $R^i(\omega)$ of this portfolio. It is defined as:

$$SR_i = \frac{\mathbb{E}\left[R^i(\omega)\right]}{\sigma\left[R^i(\omega)\right]} \tag{2.1}$$

where $\mathbb{E}\left[R^i(\omega)\right]$ is the expected return rate of the portfolio and $\sigma\left[R^i(\omega)\right]$ the standard deviation of the portfolio. The Sharpe ratio is a ratio of the return rate on the volatility. If one increases the average return of a portfolio $\mathbb{E}\left[R^i(\omega)\right]$ by buying riskier assets, then standard deviation $\sigma\left[R^i(\omega)\right]$ of that portfolio also increases and thus the portfolio's Sharpe ratio remains unchanged. One can therefore understand that the Sharpe ratio is bounded, there is no free lunch in life. One cannot construct a portfolio having a very high expected return with no volatility or risk on it. There is thus an upper bound $\overline{SR}$ and it has to be the same for all the portfolios. Historical data shows that a value of $\overline{SR} = 0.7$ is realistic, this value is taken for the model. Considering C_1^i as a first stage cost of agent i, $C_2^i(\omega)$ as a second stage cost of agent i and R^f as the risk-free rate, the return rate $R^i(\omega)$ of agent i is typically defined as:

$$R^i(\omega) = \frac{-C_2^i(\omega) - R^f \cdot C_1^i}{C_1^i} \tag{2.2}$$

The risk-free rate R^f is fixed to 1 for the sake of simplicity.

One can introduce $m_i(\omega) = \frac{q_i(\omega)}{p(\omega)}$ a discount factor. If $m_i(\omega) > 1$ then the agent sees scenario ω as more important than it is in reality. If $m_i(\omega) < 1$ then scenario ω is seen as less important.

If an absence of arbitrage is imposed on for each agent, this means that the portfolio i priced by Q_i must have a zero expected return rate. Then one can show that:

$$
\begin{aligned}
0 &= \mathbb{E}_{Q_i}\left[R^i(\omega)\right] && (2.3)\\
\Leftrightarrow 0 &= \mathbb{E}_P\left[\frac{q_i(\omega)}{p(\omega)}R^i(\omega)\right] && (2.4)\\
\Leftrightarrow 0 &= \text{Cov}\left[\frac{q_i(\omega)}{p(\omega)}, R^i(\omega)\right] + \mathbb{E}_P\left[\frac{q_i(\omega)}{p(\omega)}\right]\mathbb{E}_P\left[R^i(\omega)\right] && (2.5)\\
\Leftrightarrow 0 &= \text{Cov}\left[\frac{q_i(\omega)}{p(\omega)}, R^i(\omega)\right] + \mathbb{E}_P\left[R^i(\omega)\right] && (2.6)\\
\Leftrightarrow 0 &= \rho_{m,R^i}\sigma\left[m_i(\omega)\right]\sigma\left[R^i(\omega)\right] + \mathbb{E}_P\left[R^i(\omega)\right] && (2.7)\\
\Leftrightarrow SR_i &= \frac{\mathbb{E}_P\left[R^i(\omega)\right]}{\sigma\left[R^i(\omega)\right]} = -\rho_{m,R^i}\sigma\left[m_i(\omega)\right] && (2.8)
\end{aligned}
$$

where the first equation states that the portfolio i priced by Q_i must have an expected return rate of 0. Thereafter the definition of the covariance and correlation coefficient are used[1]. In the last equation the definition of the Sharpe ratio of portfolio i is used. Hereby it has been shown that the Sharpe ratio of a portfolio i was equal to the following expression $SR_i = -\rho_{m,R^i}\sigma\left[m_i(\omega)\right]$, since $|\rho_{m,R^i}| \leq 1$ one has that $\sigma\left[m_i(\omega)\right]$ is an upper bound for SR_i. If one now bounds $\sigma\left[m_i(\omega)\right]$ in its turn. One obtains the Hansen–Jagannathan bound: $\sigma\left[m_i(\omega)\right] \leq \overline{SR}$. Synthesising this in an equation yields:

$$SR_i \leq \sigma\left[m_i(\omega)\right] \leq \overline{SR} \tag{2.9}$$

The Hansen–Jagannathan bound can be squared and rewritten as:

$$
\begin{aligned}
\sigma\left[\frac{q_i(\omega)}{p(\omega)}\right]^2 &\leq \overline{SR}^2\\
\Leftrightarrow \text{Var}_P\left[\frac{q_i(\omega)}{p(\omega)}\right] &\leq \overline{SR}^2\\
\Leftrightarrow \left(\sum_{\omega\in\Omega}\frac{q_i^2(\omega)}{p(\omega)}\right) - 1 &\leq \overline{SR}^2
\end{aligned}
$$

The Good-Deal pricing probabilities are defined as probabilities having to satisfy the following constraints:

$$
\begin{aligned}
\sum_{\omega\in\Omega}\frac{q_i^2(\omega)}{p(\omega)} &\leq 1 + \overline{SR}^2 && (2.10)\\
1 &= \sum_{\omega\in\Omega} q_i(\omega) && (2.11)\\
0 &\leq q_i(\omega) && (2.12)
\end{aligned}
$$

Constraint (2.10) is the Hansen–Jagannathan bound, described earlier. It thus imposes that the Sharpe ratio of the portfolio priced by $q_i(\omega)$ cannot be greater than $\overline{SR}$.

[1] Recall the property of the covariance: $\text{Cov}\left[X,Y\right] = \mathbb{E}\left[XY\right] - \mathbb{E}\left[X\right]\mathbb{E}\left[Y\right]$ and the definition of correlation coefficient: $\rho_{X,Y} = \frac{\text{Cov}[X,Y]}{\sigma[X]\sigma[Y]}$

Constraints (2.11) and (2.12) impose that $q_i(\omega)$ is a probability and thus sums to 1. This also means that there is an absence of arbitrage. Indeed no combination of financial products exists (in this market) that would enable an agent to be sure to make some profit. This is also why the good-deal is a coherent risk measure.

A $q_i(\omega)$ respecting conditions (2.10), (2.11) and (2.12) is a good-deal risk measure. In the case that is discussed, agents are risk averse, they see the world with the worst $q_i(\omega)$ possible. So writing the problem with an objective that is trying to find the worst $q_i(\omega)$ means finding the RA probabilities that maximize the costs:

$$\rho_i\Big(\Gamma_i\Big) = \max_{q_i(\omega)} \mathbb{E}_{Q_i}\Big(\Gamma_i\Big) \tag{2.13}$$

$$\text{s.t.} \quad \Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \in \mathbb{L}^{|\Omega|+1} \qquad (\sqrt{p(\omega)}u_i(\omega), s_i) \tag{2.14}$$

$$1 = \sum_{\omega \in \Omega} q_i(\omega) \qquad (t_i) \tag{2.15}$$

$$0 \leq q_i(\omega) \tag{2.16}$$

where Constraint (2.10) has been rewritten to fit a Lorentz cone $\mathbb{L}^{|\Omega|+1}$:

$$\Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \in \mathbb{L}^{|\Omega|+1} \tag{2.17}$$

where $\mathbb{L}^{n+1} = \{x \in \mathbb{R}^n, t \in \mathbb{R} : ||x||_2 \leq t\}$ and $\gamma^2 = 1 + \overline{SR}^2$.

The dual of the good-deal pricing would be:

$$\min\ t_i + \gamma s_i \tag{2.18}$$

$$\text{s.t.} \quad u_i(\omega) \geq 0 \tag{2.19}$$

$$u_i(\omega) + t_i \geq \Gamma_i \qquad (q_i(\omega)) \tag{2.20}$$

$$(\sqrt{p(\omega)}u_i(\omega), s_i) \in \mathbb{L}^{|\Omega|+1} \qquad \Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \tag{2.21}$$

where t_i is the average hedged costs with the risk-free rate. The variable $u_i(\omega)$ is interpreted as an under performance of risk measure due to the unhedged risk. If $u_i(\omega)$ is positive, the costs in scenario ω are bigger than t_i. The dispersion of $u_i(\omega)$ is measured by s_i.

Note that since the same bound on the Sharpe ratio $\overline{SR}$ is taken for all the agents, the space $\mathcal{M}_i$ in which these RA probabilities may lie is the same. It will be denoted as $\mathcal{M}$.

Please note that the constraint (2.19) is not derived from the primal, but is added to the dual. Because at the optimum $u_i(\omega) \geq 0$ is always satisfied. Indeed if $u_i(\omega)$ would be negative then this would induce that the s_i would be a little bigger than if $u_i(\omega) = 0$ due to the conic constraint. A negative $u_i(\omega)$ could also induce a higher t_i than if $u_i(\omega) = 0$ but certainly not a lower t_i. Since one wants to set t_i and s_i as small as possible $u_i(\omega) = 0$ is always at least even good or better than a negative $u_i(\omega)$. Hence, there exists an $u_i(\omega)$ that is non negative at the optimum.

In Figure 2.1 one can see an example of the probability density function (pdf) of the profit. The true probability $p(\omega)$ is indicated in blue. The good-deal probabilities $q_i(\omega)$ are indicated in green. One can observe that the good-deal deforms the probability density function by putting more weight on less favourable events. This is the key of a risk averse probability measure.

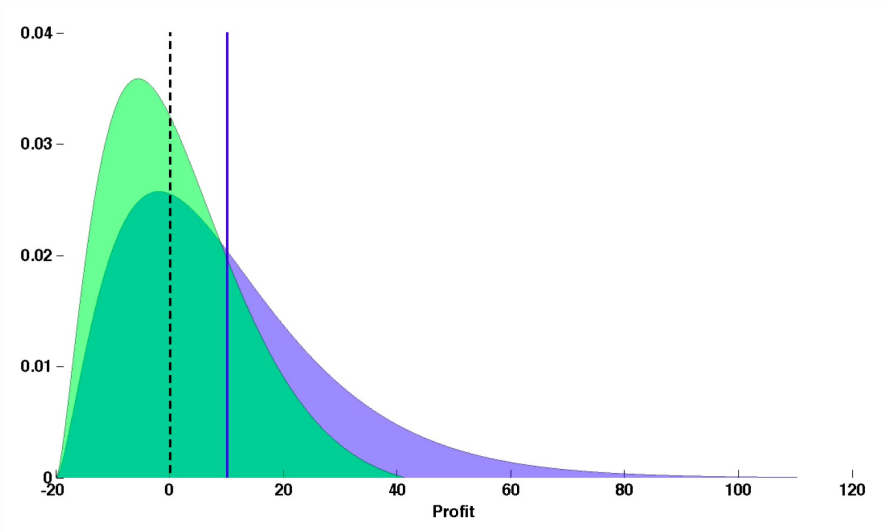

Figure 2.1: example of the probability density function of the profit. The true probability $p(\omega)$ is indicated in blue. The good-deal probabilities $q_i(\omega)$ are indicated in green. (source: [6])

2.4 Agent problems

The following sections will define the problems $\mathcal{G}_k(\cdot)$ and $\mathcal{C}(\cdot)$. The arguments of these problems can be variables for other agents, but this agent sees them as a parameter. Thereafter the equilibrium conditions will be examined.

2.4.1 Producer

The producer of each technology k wants to maximize his profit which boils down to minimize his costs, this is captured in problem $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$. The producer takes the price $\pi(\omega)$ as fixed to minimize his costs. The producer thus faces problem $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$. It is defined by:

Definition 1 (producer problem). The producer minimizes his first and second stage costs given a price $\pi(\omega)$ for the electricity. This gives the following problem:

$$\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega})) \equiv \arg\min_{x_k} \rho_k\Big(\min_{y_k(\omega)} (C_k(\omega) - \pi(\omega))y_k(\omega)\Big) + I_k x_k \tag{2.22}$$

$$\text{s.t.} \quad 0 \leq y_k(\omega) \leq x_k \qquad (\mu_k(\omega)) \tag{2.23}$$

The producer decides in a first stage to invest in a fixed amount x_k in electricity capacity and pays an Investment cost I_k for it. In a second stage he decides to produce $y_k(\omega)$. He pays the variable cost $C_k(\omega)$ for this, but also gets the electricity price $\pi(\omega)$ in return. $y_k(\omega)$ should always be smaller or equal to x_k because one cannot produce more electricity than there is capacity to do so.

The physical costs for the producer are defined by his problem $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$. For scenario ω the cost function is:

$$\Gamma_k \equiv (C_k(\omega) - \pi(\omega))y_k(\omega) + I_k x_k \tag{2.24}$$

2.4.2 Consumer

The consumer on his side wants to consume electricity to meet his demand D but at the lowest price possible. He wants to maximize his surplus which is the same as minimizing the money he has to pay for his electricity. The consumer, as the producer, considers the price $\pi(\omega)$ as a parameter. Another constraint will be added in the next section to make sure that the electricity market clears.

One can also observe that the consumer does not have to take any first stage decisions. This is not like the producer who has to invest in capacity x_k in a first stage. The risk measure $\rho_{cons}(\cdot)$ is therefore not compulsory in the consumer problem formulation. The consumer faces problem $\mathcal{C}(\boldsymbol{\pi}(\boldsymbol{\omega}))$. It is defined by :

Definition 2 (consumer problem). The consumer takes the price of electricity $\pi(\omega)$ as given. He minimizes his costs in the following problem:

$$\mathcal{C}(\boldsymbol{\pi}(\boldsymbol{\omega})) \equiv \arg\min_{z(\omega)} \rho_{cons}\Big((\pi(\omega) - PC)(D - z(\omega))\Big) \tag{2.25}$$

$$\text{s.t.} \quad 0 \leq z(\omega) \tag{2.26}$$

The consumer who receives the price signal $\pi(\omega)$ decides the curtailment $z(\omega)$ (this is the amount of energy he does not consume) and the energy he will consume $D - z(\omega)$. PC stands for the price cap, the consumer does not want to consume any energy above this threshold, if $PC < \pi(\omega)$ then the curtailment would be infinite. This would of course not be possible physically. It is captured in the equilibrium conditions of next section.

The physical cost function of the consumer is defined by his problem $\mathcal{C}(\boldsymbol{\pi}(\boldsymbol{\omega}))$:

$$\Gamma_{cons} \equiv (\pi(\omega) - PC)(D - z(\omega)) \tag{2.27}$$

This could be understood as the price $\pi(\omega)$ paid above the price cap PC by the consumer. But since the consumer always pays a price that is less or equal to the price cap his cost will be negative or equal to zero.

2.4.3 Equilibrium conditions

The problems of the different agents that were just discussed, $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$ for the producers and $\mathcal{C}(\boldsymbol{\pi}(\boldsymbol{\omega}))$ for the consumer, will now be concatenated in an equilibrium problem. This is done to give an overview of the current market structure and explain how each agent makes his decisions once he has evaluated his risk. On the one hand, taking the KKT conditions of the producer problem $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$ (section 2.4.1) gives conditions (2.28), (2.29) and (2.30). On the other hand the KKT conditions of the consumer problem $\mathcal{C}(\boldsymbol{\pi}(\boldsymbol{\omega}))$ (section 2.4.2)) gives (2.31). Finally, condition (2.32) is added so that the market clears and the problem is well defined.

$$0 \leq C_k(\omega) + \mu_k(\omega) - \pi(\omega) \perp y_k(\omega) \geq 0 \tag{2.28}$$
$$0 \leq x_k - y_k(\omega) \perp \mu_k(\omega) \geq 0 \tag{2.29}$$
$$0 \leq I_k - \mathbb{E}_{Q_k}\Big(\mu_k(\omega)\Big) \perp x_k \geq 0 \tag{2.30}$$
$$0 \leq PC - \pi(\omega) \perp z(\omega) \geq 0 \tag{2.31}$$
$$0 \leq \sum_{k \in K} y_k(\omega) + z(\omega) - D \perp \pi(\omega) \geq 0 \tag{2.32}$$

Condition (2.28) ensures that one shall produce (y positive) if one makes a non-negative marginal profit.

Condition (2.29) states that one cannot produce more electricity (y) than what is installed (x). If $y = x$ then the marginal value of the plant (μ) can take a positive value.

Condition (2.30) makes sure that one invests in technology k until the marginal profit for investing in one more unit of capacity is equal to zero.

Condition (2.31) ensures the respect of the price cap (PC). If there is a positive curtailment (z), then this would mean that the price cap is reached. One could also see this as an interpretation of the consumer agent optimization problem (section 2.4.2). Indeed if:

- $PC - \pi(\omega) > 0$ then $z(\omega) = 0$ is optimal for the consumer since for each unit of electricity consumed by the consumer will decrease his cost.
- $PC - \pi(\omega) = 0$ then the objective of the consumer problem takes the value of zero and the consumer does not care how much energy it will consume. Adding complementarity condition (2.32) with a positive price $\pi(\omega) = PC$ means that $\sum_{k \in K} y_k(\omega) + z(\omega) - D = 0$. Since all $y_k(\omega)$ are non negative, $z(\omega)$ can take any non negative value smaller or equal to D.

Condition (2.32) is added to make sure that the market clears. It states that if the price (π) is positive then the market clears i.e. the electricity produced (y) and the electricity curtailed (z) sums to the demand (D). If the market does not clear ($y > D - z$), this would be a case of overproduction of electricity, then the price is equal to zero.

2.5 Contracts

Contracts enable the different agents, consumer and producers, to hedge their risk. As mentioned in previous sections the consumer was facing risk, but could not react against it. He could only take second stage decisions, therefore he had to undergo the market.

This risk can be hedged through a forward contract, which enables to pay a fixed price for an asset whose price is going to vary in the future.

A forward contract c is introduced at price π_c^f. The payoff for the owner of the contract is $\pi_c(\omega)$. This payoff can be linked to different values, different kinds of contracts will be treated later. Each agent i can take a certain quantity W_i of this forward contract. This quantity of contracts bought by each agent is a first stage variable enables all the agents to make decisions before uncertainty happens.

2.5.1 Risk hedging problem

A general agent, thus looks at his physical costs and wants to solve the following problem:

$$\max_{W_i} \rho_i\Big(\min_{\Gamma_i} \Gamma_i - W_i\pi_c(\omega)\Big) + W_i\pi_c^f \tag{2.33}$$

$$\text{s.t.} \quad \Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \in \mathbb{L}^{|\Omega|+1} \qquad (\sqrt{p(\omega)}u_i(\omega), s_i) \tag{2.34}$$

$$1 = \sum_{\omega\in\Omega} q_i(\omega) \qquad (t_i) \tag{2.35}$$

$$0 \leq q_i(\omega) \tag{2.36}$$

This problem is an adaptation of the good-deal. Only now contracts have been added so the objective changes accordingly. Here the agent would receive $W_i\pi_c^f$ in the first stage for selling the contract, but would have to pay back $W_i\pi_c(\omega)$ in a second stage.

In the objective the term $W_i\pi_c^f$ is moved out of the risk measure because the good-deal is a coherent risk measure. Since the good-deal (ρ) is a coherent risk measure, it respects the translation equivariance which states: $\rho(Z + a) = \rho(Z) + a$ where $a \in \mathbb{R}$ and Z an outcome of Ω, see the axioms of a coherent risk measure in Appendix (section A).

Taking the dual of risk measure ρ_i gives the following problem:

$$\max_{W_i} \min_{t_i, s_i} t_i + \gamma s_i + W_i\pi_c^f \tag{2.37}$$

$$\text{s.t.} \quad u_i(\omega) \geq 0 \tag{2.38}$$

$$u_i(\omega) + t_i \geq \Gamma_i - W_i\pi_c(\omega) \qquad (q_i(\omega)) \tag{2.39}$$

$$(\sqrt{p(\omega)}u_i(\omega), s_i) \in \mathbb{L}^{|\Omega|+1} \qquad \Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \tag{2.40}$$

2.5.2 Equilibrium conditions of risk hedging problem

Now the equilibrium conditions of the risk hedging problem will be computed. Taking the KKT conditions of the risk hedging problems of the different agents give the equilibrium conditions (2.41), (2.42) and (2.43).

Since the case of a Nash equilibrium is treated, the contract market needs to clear. This means that the total quantity of the contracts bought and sold by each agent must cancel out (condition (2.44)). One has to make sure that all agents are also at equilibrium. Being at equilibrium means that none of the agents want to change their hedging position i.e. they do not want to buy neither more nor less contracts than the equilibrium quantity W_i (condition (2.45)).

Hence, the equilibrium conditions are the following:

$$0 \leq u_i(\omega) + t_i - \Gamma_i + W_i \pi_c(\omega) \perp q_i(\omega) \geq 0 \tag{2.41}$$

$$\mathbb{L}^{|\Omega|+1} \ni \begin{pmatrix} \sqrt{p(\omega)} u_i(\omega) \\ s_i \end{pmatrix} \perp \begin{pmatrix} \frac{q_i(\omega)}{\sqrt{p(\omega)}} \\ \gamma \end{pmatrix} \in \mathbb{L}^{|\Omega|+1} \tag{2.42}$$

$$1 = \sum_{\omega \in \Omega} q_i(\omega) \tag{2.43}$$

$$\sum_i W_i = 0 \tag{2.44}$$

$$\mathbb{E}_{Q_i}(\pi_c(\omega)) = \pi_c^f \tag{2.45}$$

Conditions (2.41)), (2.42) and (2.43) come from the individual contract problem described in previous section. Note that condition (2.42) is a conic complementarity condition. The notation was taken from [10]. It states that:

$$\mathcal{K} \ni a \perp b \in \mathcal{K}^* \Leftrightarrow \; a \in \mathcal{K}, \; b \in \mathcal{K}^* \text{ and } a^T \cdot b = 0$$

The notation $\mathcal{K}^*$ represents the dual cone of cone $\mathcal{K}$. The Lorentz cone $\mathbb{L}^n$ has the property of being self-dual: $\mathbb{L}^{n,*} = \mathbb{L}^n$.

Condition (2.44) states that for each contract there is a corresponding agent who has the opposite position. This constraint makes sure that the contract market clears.

Condition (2.45) states that each agent values the payoff of the contract $\pi_c(\omega)$ at the same value as the price of of the contract π_c^f.

A note on the usefulness of contracts is that a contract can only be interesting if it has a payoff $\pi_c(\omega)$ that varies in function of the scenarios. If it is not the case the price of the contract is fixed at $\pi_c^f = \pi_c(\omega)$ and there thus is no interest for any agent to buy this contract since it is the same as the risk-free rate $R^f = 1$.

2.6 Risky Competitive Capacity Expansion Problem: equilibrium conditions

Now all the elements have been introduced, the Risky Competitive Capacity Expansion Problem (RCCEP) can be defined as an equilibrium problem. The conditions are a concatenation of sections 2.4.3 and 2.5.2. These are stated below for completeness:

Definition 3 (risky competitive capacity expansion problem (RCCEP)). The RCCEP is an equilibrium problem. The producer decides his capacity investment and capacity operations. The consumer decides his consumption. Both hedge their stochastic payoff. This gives the following complementarity conditions that need to be satisfied to have in the RCCEP:

$$
\begin{aligned}
&0 \leq C_k(\omega) + \mu_k(\omega) - \pi(\omega) \perp y_k(\omega) \geq 0 \\
&0 \leq x_k - y_k(\omega) \perp \mu_k(\omega) \geq 0 \\
&0 \leq I_k - \mathbb{E}_{Q_k}\left(\mu_k(\omega)\right) \perp x_k \geq 0 \\
&0 \leq PC - \pi(\omega) \perp z(\omega) \geq 0 \\
&0 \leq \sum_{k \in K} y_k(\omega) + z(\omega) - D \perp \pi(\omega) \geq 0 \\
&0 \leq u_i(\omega) + t_i - \Gamma_i + W_i \pi_c(\omega) \perp q_i(\omega) \geq 0 \\
&\mathbb{L}^{|\Omega|+1} \ni \begin{pmatrix} \sqrt{p(\omega)} u_i(\omega) \\ s_i \end{pmatrix} \perp \begin{pmatrix} \frac{q_i(\omega)}{\sqrt{p(\omega)}} \\ \gamma \end{pmatrix} \in \mathbb{L}^{|\Omega|+1} \\
&1 = \sum_{\omega \in \Omega} q_i(\omega) \\
&\sum_i W_i = 0 \\
&\mathbb{E}_{Q_i}(\pi_c(\omega)) = \pi_c^f \\
&\Gamma_{cons} = (\pi(\omega) - PC)(D - z(\omega)) \\
&\Gamma_k = (C_k(\omega) - \pi(\omega)) y_k(\omega) + I_k x_k
\end{aligned}
$$

2.7 Conclusion

In this first chapter, the RCCEP has been defined. It represents the capacity expansion problem in a risky environment. This risky environment is modelled with risk averse probabilities that are specific to each agent. The RA probabilities are derived from the good deal pricing method. The different agents can hedge their risk through contracts.

This type of problem has already been defined in different papers. However it has never to our knowledge been solved at a large scale. This is because the RCCEP is nonlinear. Both the RA probabilities and the variables representing the agent decisions have to be decided. Hence, this problem is complex to solve at a large scale for a standard solver. The RCCEP is most commonly solved as an MCP problem with the PATH solver. Only small problems with a limited number of scenarios can be solved in a reasonable time with this solver. This is what is used in [15]. In [5] the problem was solved on a small illustrative case. In [14] an alternative approach is proposed to solve the problem. It solves bigger problems, but only with 5 or 10 days representative for the whole year.

Our goal is to solve the RCCEP at a yearly scale including multiple scenarios with an hourly resolution. The RCCEP will be further analysed in the next chapters. As in [14] the RCCEP will be reformulated in different optimization problems. The approach used to solve this problem is a heuristic and will be explained in the next chapter.

CHAPTER

3 DECOMPOSITION OF THE PROBLEM

In this chapter the RCCEP is decomposed in 2 sub-problems:

- physical part : in this part different producers define what capacity is actually built as well as when and to what extent the plants are producing electricity. The consumer decides the curtailment in this part. One can see this part as a concatenation of the problems $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$ and $\mathcal{C}(\boldsymbol{\pi}(\boldsymbol{\omega}))$ with a fixed risk measure.
- financial part : this part defines the amount of contracts that different agents will buy. The risk aversion of the different agents will also be computed in this problem. Here the costs of the agents are fixed and the risk measure can vary.

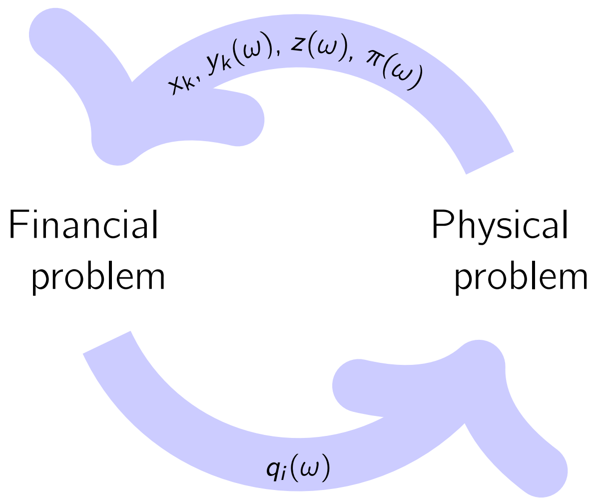

Figure 3.1: update of the decomposition

Both parts will have to update their results at each iteration. Thereafter they have to send it to the other part of the problem to make sure that it can compute the next iteration. This is what is shown in Figure 3.1. This approach is a heuristic and we have no proof on its convergence.

Q_i is the set of $q_i(\omega)$ and represents the risk aversion of each agent. $\bar{Q}$ is now defined as the mean risk aversion of the producers ($\bar{q}(\omega) = \frac{\sum_{k \in K} q_k(\omega)}{|K|}$) .

3.1 Physical part of the problem

The physical part of the problem focuses on the cost of the different agents, once the risk averse probabilities $q_i(\omega)$ and $\bar{q}(\omega)$ are fixed. The problem that one wants to solve in the physical part is the following:

Definition 4 (Nash Equilibrium problem on operating, capacity and consumption decisions). Fixing the risk averse probabilities $q_i(\omega)$ and $\bar{q}(\omega)$, gives the following problem for producer k :

$$\begin{aligned} \min_{x_k, y_k(\omega)} \ & \mathbb{E}_{Q_k}\Big[\big(C_k(\omega) - \pi(\omega)\big)y_k(\omega)\Big] + I_k x_k \\ \text{s.t.} \ & 0 \leq y_k(\omega) \leq x_k \end{aligned}$$

The consumer faces the following problem:

$$\begin{aligned} \min_{z(\omega)} \ & \mathbb{E}_{Q_{cons}}\Big[\big(\pi(\omega) - PC\big)\big(D - z(\omega)\big)\Big] \\ \text{s.t.} \ & 0 \leq z(\omega) \end{aligned}$$

The electricity market clearing condition must also be considered:

$$\sum_k y_k(\omega) + z(\omega) \geq D$$

These equations are the conditions from section 2.4.3 that have been rewritten with fixed probabilities.

The idea behind the formulation that is presented below is first to do an aggregation of the problems and clearing conditions just described above. The formulation is an attempt to solve the Nash equilibrium as a sequence of optimisation problems. Hence, this formulation will be solved several times (iterations are indexed with subscript it). it refers to the iteration that is being processed and $it-1$ to the previous iteration.

Different agents have different risk aversions. The market clearing condition (4) binds all the problems of the separate agents together. The dual variable of this market clearing condition can only be scaled by one factor. One chooses to scale it by risk probability $\bar{q}$. This unique risk aversion $\bar{q}$ will be taken to solve the whole physical part. It is thus also used to scale the capacity value $\mu_k(\omega)$.

However one still needs to include the risk aversion of the different producers. With this aim in mind the investment cost I_k is modified. It is subtracted by $\mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)] - \mathbb{E}_{\bar{Q}}[\mu_k^{it-1}(\omega)]$, where the parameter $\mu_k^{it-1}(\omega)$ has been introduced and will represent the value found for $\mu_k(\omega)$ at the previous iteration. This trick enables each producer to have his own risk aversion q_k but to solve the problem with the same global risk aversion $\bar{q}$ for all the agents. The proof should complete convincing the reader of this claim.

Proposition 1. If the $\mu_k(\omega)$ converges, the equilibrium conditions of section 2.4.3 is equivalent to solving the following problem:

$$\begin{aligned}
\min_{y,x,z} \quad & \mathbb{E}_{\bar{Q}}[\sum_{k\in K} C_k(\omega)y_k(\omega) + PC \cdot z(\omega)] + \sum_{k\in K}\Big(I_k - (\mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)] - \mathbb{E}_{\bar{Q}}[\mu_k^{it-1}(\omega)])\Big)x_k \\
\text{s.t.} \quad & 0 \leq y_k(\omega) \leq x_k && (\mu_k(\omega)\bar{q}(\omega)) \\
& \sum_{k\in K} y_k(\omega) + z(\omega) \geq D && (\pi(\omega)\bar{q}(\omega)) \\
& 0 \leq z(\omega)
\end{aligned}$$

Proof. The KKT conditions yield:

$$0 \leq C_k(\omega) + \mu_k(\omega) - \pi(\omega) \perp y_k(\omega) \geq 0 \tag{3.1}$$

$$0 \leq x_k - y_k(\omega) \perp \mu_k(\omega) \geq 0 \tag{3.2}$$

$$0 \leq I_k - \mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)] + \mathbb{E}_{\bar{Q}}[\mu_k^{it-1}(\omega) - \mu_k(\omega)] \perp x_k \geq 0 \tag{3.3}$$

$$0 \leq PC - \pi(\omega) \perp z(\omega) \geq 0 \tag{3.4}$$

$$0 \leq \sum_{k\in K} y_k(\omega) + z(\omega) - D \perp \pi(\omega) \geq 0 \tag{3.5}$$

Conditions (3.1), (3.2), (3.4) and (3.5) correspond to the equilibrium conditions (2.28), (2.29), (2.31) and (2.32) of section 2.4.3.

The only condition missing from the equilibrium conditions of section 2.4.3 is (2.30). One can notice that when $\mu_k(\omega)$ has converged (this means $\mu_k(\omega) = \mu_k^{it-1}(\omega)$), constraint (3.3) simplifies to a condition of the form:

$$0 \leq I_k - \mathbb{E}_{Q_k}[\mu_k(\omega)] \perp x_k \geq 0 \tag{3.6}$$

This is the same condition as (2.30). This concludes the proof. ■

Remark. Note that the risk aversion of the consumer is not taken into account in this problem. This is because as stated before the consumer makes no physical first stage decisions. Hence he can consume the energy he wants in each scenario, while the producer is constrained in his electricity production by x_k.

As stated in Proposition 1, a convergence on the $\mu_k(\omega)$ is required so that the problem corresponds to definition 4. The formulation of Proposition 1 only yielded convergence on the primal variables but not on the dual variables. Therefore, the dual is taken to stabilise these dual variables that do not converge.

Indeed, taking the dual gives the possibility to put a penalization on the dual variables. This is what is done with the μ. Putting a penalization on μ also constrains the modified investment cost of each producer, defined in (3.7), to not move too quickly between two iterations.

$$I_k - (\mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)] - \mathbb{E}_{\bar{Q}}[\mu_k^{it-1}(\omega)]) \tag{3.7}$$

The Physical problem defined as the dual of problem of Proposition 1 is:

Definition 5 (Physical optimisation problem). The risk averse probabilities of the agents are fixed in a global risk aversion $\bar{q}$. However each producer keeps his own risk aversion through a modified investment cost (described in equation 3.7). The capacity value $\mu_k(\omega)$ of the different producers and the electricity price $\pi(\omega)$ can be computed by solving the Physical problem that is detailed below:

$$\begin{aligned}
\min_{\pi,\mu} \quad & -\mathbb{E}_{\bar{Q}}[D\pi(\omega)] + \kappa \sum_{k\in K,\omega\in\Omega} (\mu_k(\omega) - \mu_k^{it-1}(\omega))^2 \\
\text{s.t.} \quad & \pi(\omega), \mu_k(\omega) \geq 0 \\
& 0 \leq PC - \pi(\omega) && (z(\omega)\bar{q}(\omega)) \\
& 0 \leq C_k(\omega) + \mu_k(\omega) - \pi(\omega) && (y_k(\omega)\bar{q}(\omega)) \\
& 0 \leq I_k - (\mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)] - \mathbb{E}_{\bar{Q}}[\mu_k^{it-1}(\omega)]) - \mathbb{E}_{\bar{Q}}[\mu_k(\omega)] && (x_k)
\end{aligned}$$

The Physical optimisation problem will be referred to as Physical problem. Adding a penalty term in the problem modifies the problem defined in Proposition 1. With Proposition 2 it will be shown under which circumstances the Physical problem is equivalent to Proposition 1.

Proposition 2. If the penalty term $\kappa \sum_{k\in K,\omega\in\Omega}(\mu_k(\omega) - \mu_k^{it-1}(\omega))^2$ in the Physical problem (defintion 5) has a value of zero then the Physical problem is equivalent to the equilibrium conditions of section 2.4.3.

Proof. The dual of the Physical problem is:

$$\begin{aligned}
\min_{y,x,z} \quad & \mathbb{E}_{\bar{Q}}\Big[\sum_{k\in K} C_k(\omega)y_k(\omega) + PC \cdot z(\omega)\Big] + \sum_{k\in K} \Big(I_k - (\mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)] - \mathbb{E}_{\bar{Q}}[\mu_k^{it-1}(\omega)])\Big)x_k \\
& + \sum_{k\in K,\omega\in\Omega} \mu_k^{it-1} v_k(\omega) + \frac{v_k(\omega)^2}{4\kappa} \\
\text{s.t.} \quad & 0 \leq y_k(\omega) \leq x_k + \frac{v_k(\omega)}{\bar{q}(\omega)} && (\mu_k(\omega)\bar{q}(\omega)) \\
& \sum_{k\in K} y_k(\omega) + z(\omega) \geq D && (\pi(\omega)\bar{q}(\omega)) \\
& 0 \leq z(\omega)
\end{aligned}$$

where $v_k(\omega)$ is the dual variable associated to the penalty term. One can notice that this variable allows the production constraint of the producer to be violated. This means that $y_k(\omega) > x_k$ could be a possible outcome if the $\mu_k(\omega)$ did not converge. This term cannot get too big because it is squared in the objective. The complementarity condition associated to the penalty term is:

$$2\kappa(\mu_k(\omega) - \mu_k^{it-1}(\omega)) = v_k(\omega) \tag{3.8}$$

So if the $\mu_k(\omega)$ did converge, this means that $\mu_k(\omega) = \mu_k^{it-1}(\omega)$ and so the complementarity condition implies that $v_k(\omega) = 0$ then the Physical problem is equivalent to proposition 1 and therefore to the equilibrium conditions of section 2.4.3.

■

Complementarity condition 3.8 will now be used to see what happens when the penalty term is sufficiently small. The following Lemmas can be derived from Proposition 2:

Lemma 1. If the capacity value has converged to a tolerance ϵ i.e. $\left|\mu_k(\omega) - \mu_k^{it-1}(\omega)\right| \leq \epsilon$ then the production constraint will be violated to at most $\frac{2\kappa\epsilon}{\bar{q}(\omega)}$ i.e. $|y_k(\omega) - x_k| \leq \frac{2\kappa\epsilon}{\bar{q}(\omega)}$

Proof. The proof is straightforward from complementarity condition 3.8 and the production constraint. ∎

Lemma 1 states that if the algorithm is stopped at a tolerance ϵ, the solution might be infeasible regarding the production, but this infeasibility is limited to $\frac{2\kappa\epsilon}{\bar{q}(\omega)}$.

Lemma 2. If for each scenario $\omega \in \Omega$, the capacity value has converged to a tolerance ϵ, i.e. $\left|\mu_k(\omega) - \mu_k^{it-1}(\omega)\right| \leq \epsilon$ then the investment perception of the producer is at most biased by this same ϵ i.e. $\left|I_k - \mathbb{E}_{Q_k}[\mu_k^{it-1}(\omega)]\right| \leq \epsilon$.

Proof. Inserting the tolerance in condition 3.3 yields the bound. ∎

Lemma 2 is good news because it means that if the $\mu_k(\omega)$ have nearly converged the investment value seen by the producer is nearly the good one.

3.2 Financial part of the problem

In this section x_k, $y_k(\omega)$, $\pi(\omega)$ and $z(\omega)$ are extracted from the physical part of the problem. These are used to compute the value of the cost function Γ_i of the different agents. This value is stored in $\boldsymbol{cost}_i(\omega)$ which is a parameter representing the physical cost of each agent in the financial part.

Note that as the price of the contract $\pi_c(\omega)$ is also a parameter in the financial part of the problem, it could depend on variables of the physical part of the problem or external data. Different types of contracts will be explained in a section dedicated to this purpose.

Definition 6 (Nash Equilibrium problem on risk averse probabilities and contracts). For the different agents the variables are now the risk averse probabilities $q_i(\omega)$, contract quantities W_i and the contract price π_c^f. The contract problem of each agent has been defined as:

$$
\begin{aligned}
& \max_{W_i} \max_{q_i(\omega)} \mathbb{E}_{Q_i}\left[\boldsymbol{cost}_i(\omega) - W_i \pi_c(\omega)\right] + W_i \pi_c^f \\
\text{s.t.} \quad & \left(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\right) \in \mathbb{L}^{|\Omega|+1} \\
& 1 = \sum_{\omega \in \Omega} q_i(\omega) \\
& 0 \leq q_i(\omega)
\end{aligned}
$$

Because W_i is a free variable the agent deciding this quantity will buy or sell this contract until this agent prices the contract at price π_c^f:

$$
\mathbb{E}_{Q_i}(\pi_c(\omega)) = \pi_c^f \tag{3.9}
$$

With this knowledge one can construct the following optimization problem, a concatenation of the contract problems of the different agents:

Proposition 3. The following problem is equivalent to the risk hedging problem of all the agents:

$$\max_{\pi_c^f, q_i(\omega)} \sum_i \mathbb{E}_{Q_i}\left[cost_i(\omega)\right] \tag{3.10}$$

$$\text{s.t.} \quad \mathbb{E}_{Q_i}(\pi_c(\omega)) - \pi_c^f = 0 \qquad (W_i) \tag{3.11}$$

$$1 = \sum_{\omega\in\Omega} q_i(\omega) \qquad (t_i) \tag{3.12}$$

$$\Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \in \mathbb{L}^{|\Omega|+1} \qquad (\sqrt{p(\omega)}u_i(\omega), s_i) \tag{3.13}$$

$$0 \leq q_i(\omega) \tag{3.14}$$

Proof. The KKT conditions are identical to the equilibrium conditions of the risk hedging problem described in section 2.5.2. ∎

An attentive reader will have noticed that the objective has changed. This is because the objective simplifies with equilibrium condition 3.9.

Finally one will add the penalization term $\kappa \sum_{i\in I,\omega\in\Omega}\Big(q_i(\omega) - q_i^{it-1}(\omega)\Big)^2$ to make sure that the $q_i(\omega)$ do not change too much between each iteration. This because if the $q_i(\omega)$ change a lot, then the $cost_i(\omega)$ will also change a lot and no convergence would be achieved. This yields the following problem:

Definition 7 (Financial optimisation problem). The risk averse probabilities $q_i(\omega)$ used by the agents to measure their costs are computed in the Financial optimisation problem. The risk measure used to compute the risk aversion of the agents is the good deal. Each agent has the opportunity to hedge his risk by buying contract c at price π_c^f The Financial problem is:

$$\max_{\pi_c^f, q_i(\omega)} \sum_i \Big(\mathbb{E}_{Q_i}\left[cost_i(\omega)\right]\Big) - \kappa \sum_{i\in I,\omega\in\Omega} \Big(q_i(\omega) - q_i^{it-1}(\omega)\Big)^2$$

$$\text{s.t.} \quad \mathbb{E}_{Q_i}(\pi_c(\omega)) - \pi_c^f = 0 \qquad (W_i)$$

$$1 = \sum_{\omega\in\Omega} q_i(\omega) \qquad (t_i)$$

$$\Big(\frac{q_i(\omega)}{\sqrt{p(\omega)}}, \gamma\Big) \in \mathbb{L}^{|\Omega|+1} \qquad (\sqrt{p(\omega)}u_i(\omega), s_i)$$

$$0 \leq q_i(\omega)$$

The Financial optimisation problem will be referred to as Financial problem. Adding the penalty term in the Financial problem modifies the dual problem, in a similar way the physical penalty did with the Physical problem. The following Proposition analyses the implications of this penalty term:

Proposition 4. If the penalty term $\kappa \sum_{i\in I,\omega\in\Omega}\Big(q_i(\omega) - q_i^{it-1}(\omega)\Big)^2$ in the Financial problem (definition 7) has a value of zero then the Financial problem is equivalent to the problem described in Proposition 3 and hence is also equivalent to the risk hedging problem of all the agents.

Proof. The dual of the Financial problem is given by:

$$
\begin{aligned}
&\min \sum_{i\in I}(t_i+\gamma s_i)+\sum_{i\in I,\omega\in\Omega}\left(\frac{h_i(\omega)^2}{4\kappa}-q_i^{it-1}(\omega)h_i(\omega)\right)\\
\text{s.t.}\quad & u_i(\omega)\geq 0\\
& u_i(\omega)+t_i-h_i(\omega)\geq cost_i(\omega)-W_i\pi_c(\omega) && (q_i(\omega))\\
& (\sqrt{p(\omega)}u_i(\omega),s_i)\in\mathbb{L}^{|\Omega|+1} && \left(\frac{q_i(\omega)}{\sqrt{p(\omega)}},\gamma\right)\\
& \sum_i W_i=0
\end{aligned}
$$

If $h_i(\omega)$ is positive this could impact the different variables $u_i(\omega)$, t_i, W_i. The conic complementarity constraint associated with this penalty is:

$$2\kappa(q_i(\omega)-q_i^{it-1}(\omega))=-h_i(\omega) \tag{3.15}$$

so if the change in $q_i(\omega)$ is limited then the perturbation $h_i(\omega)$ in the primal problem is limited. If the $q_i(\omega)$ have converged then $h_i(\omega)=0$. This concludes the proof. ■

The following Lemma can be introduced:

Lemma 3. If the RA probabilities have converged to a tolerance ϵ i.e. $\left|q_i(\omega)-q_i^{it-1}(\omega)\right|\leq\epsilon$ then the perturbation should be at most $\kappa\epsilon$ i.e. $|h_i(\omega)|\leq 2\kappa\epsilon$.

Proof. The proof is straight forward using complementarity condition (3.15). ■

Lemma 3 states that the contract quantity and other variables should not be too much affected if the financial penalty is sufficiently small. Having a penalty term means the risk measure thinks that it can hedge the portfolio in a certain sense but in fact it cannot hedge it as well. If the penalty is limited, the gap between what the risk measure thinks and can do is also limited.

3.3 Decomposition algorithm

The goal is to solve iteratively the Physical and Financial problem. The Physical problem was defined in Definition 5 and the Financial problem in Definition 7. The Physical problem needs to have converged on the $\mu_k(\omega)$ to have an modified investment cost (3.7) of each producer that makes sense. One will thus solve the Physical problem iteratively until the $\mu_k(\omega)$ have converged on the Physical problem. Convergence is measured with f_{phys} which is the physical penalty divide by the objective:

$$f_{phys}=\frac{\kappa_{phys}\sum_{k\in K,\omega\in\Omega}\left(\mu_k^{it_{phys}}(\omega)-\mu_k^{it_{phys}-1}(\omega)\right)^2}{obj_{phys}^{it_{phys}}}$$

Once the physical penalty is sufficiently small, the cost of the different agents and the payoff of the contract can be updated. All the information is now available to update the RA probabilities by solving the Financial problem. Thereafter one can start a new iteration by again solving the Physical problem. The algorithm stops when both physical and financial penalties are sufficiently small regarding the physical and financial objective values. This is when $f\leq\epsilon$. This is what is depicted in algorithm 1:

Algorithm 1: first algorithm

initialize: $q_i(\omega) = p(\omega)$, $\bar{q}(\omega) = p(\omega)$
while $f > \epsilon$ **do**
  while $f_{phys} > \epsilon_{phys}$ **do**
    solve Physical problem (Definition 5)
    $\leftarrow$ update of the capacity values $\mu_k^{it_{phys}}(\omega)$
    $f_{phys} \leftarrow \frac{\kappa_{phys} \sum_{k \in K, \omega \in \Omega} \left(\mu_k^{it_{phys}}(\omega) - \mu_k^{it_{phys}-1}(\omega) \right)^2}{obj_{phys}^{it_{phys}}}$
  end
  $\leftarrow$ update of the agent costs $cost_i(\omega)$
    and contract payoffs $\pi_c(\omega)$
  solve Financial problem (Definition 7)
  $\leftarrow$ update of the RA probabilities $q_i(\omega)$ and $\bar{q}(\omega)$
  $f_{fin} \leftarrow \frac{\kappa_{fin} \sum_{i \in I, \omega \in \Omega} \left(q_i^{it}(\omega) - q_i^{it-1}(\omega) \right)^2}{obj_{fin}^{it}}$
  $f = f_{fin} + f_{phys}$
end

In practice the Physical problem is much larger than the Financial problem. Indeed the Physical problem has to find the price and capacity value over all the hours of a year for each scenario, while the Financial problem only needs to find the RA probabilities and the contract price. Hence we will not require convergence on the $\mu_k(\omega)$ at each iteration and drop the physical loop. Not requiring convergence is not a problem as long as the $\mu_k(\omega)$ converge at the end of the day.

Convergence requires that the penalty terms of the physical and Financial problems have to go to zero to solve the RCCEP. It is important to note that the physical penalty is inducing a non feasible solution. This is a solution where the production of a producer is bigger than the installed capacity ($y_k(\omega) > x_k$). While the financial penalty means that the risk averse probabilities cannot hedge as well as they think they can do. Hence it is more important to converge on the Physical problem. To avoid getting stuck in a point where the physical penalty does not go to zero, one shall remove the physical penalty at some point during one iteration. One will only remove this physical penalty once the $q_i(\omega)$ and $\mu_k(\omega)$ have already more or less converged. By this is meant that the physical and financial penalties divided by their objective values are smaller than a threshold δ ($f < \delta$).

Removing this physical penalty allows the $\mu_k(\omega)$ to move freely. A couple of iterations are thereafter needed to update the modified investment cost and the $q_i(\omega)$ correctly. Each time the physical penalty is removed, δ is divided by a value of 10 in order that the next time the physical penalty is removed the problem penalty is smaller. The algorithm is detailed in Algorithm 2.

Algorithm 2: decomposition algorithm

initialize: $q_i(\omega) = p(\omega)$, $\bar{q}(\omega) = p(\omega)$
while $f > \epsilon$ **do**
  if $f < \delta$ **then**
    solve Physical problem without physical penalty (Definition 5)
    $\delta \leftarrow \frac{\delta}{10}$
  else
    solve Physical problem (Definition 5)
  end
  $\leftarrow$ update of the capacity values $\mu_k^{it}(\omega)$
  $f_{phys} \leftarrow \frac{\kappa_{phys} \sum_{k \in K, \omega \in \Omega} \left(\mu_k^{it}(\omega) - \mu_k^{it-1}(\omega)\right)^2}{obj_{phys}^{it}}$
  $\leftarrow$ update of the agent costs $cost_i(\omega)$
    and contract payoffs $\pi_c(\omega)$
  solve Financial problem (Definition 7)
  $\leftarrow$ update of the RA probabilities $q_i(\omega)$ and $\bar{q}(\omega)$
  $f_{fin} \leftarrow \frac{\kappa_{fin} \sum_{i \in I, \omega \in \Omega} \left(q_i^{it}(\omega) - q_i^{it-1}(\omega)\right)^2}{obj_{fin}^{it}}$
  $f \leftarrow f_{fin} + f_{phys}$
end

At each iteration one first checks if the sum of the physical and financial penalty terms are smaller than δ if this is the case one can solve the Physical problem without penalty this iteration and reduce the value of δ otherwise the Physical problem is solved with a penalty. The value of the capacity value $\mu_k(\omega)$ can thereafter be updated.

Once the Physical problem has been solved, nothing changes compared to the previous version of the algorithm. The physical information is used to update the costs of the different agents and the contract payoffs. Then the Financial problem can be solved and the risk averse probabilities $q_i(\omega)$ can in turn be updated. When this is done, the next iteration can be computed. The algorithm ends when $f < \epsilon$ this would mean that the $q_i(\omega)$ and $\mu_i(\omega)$ would have converged and that the algorithm would have found a solution to the problem.

3.4 Conclusion

This chapter explained a method to solve the Risky Competitive Capacity Expansion Problem. An algorithm decomposing the problem in 2 parts has been proposed : a physical part and a financial part. The decomposition algorithm is detailed in Algorithm 2. An important condition to have a feasible optimal solution is that convergence is required on all the RA probabilities $q_i(\omega)$ and the capacity value $\mu_k(\omega)$. There is no convergence guarantee of this algorithm.

CHAPTER

4 DETAILED DESCRIPTION OF EACH AGENT

In this section the different agents are explained in detail. The problem $\mathcal{G}_k(\cdot)$ faced by each type of producer is defined. As mentioned earlier, the arguments of $\mathcal{G}_k$ are variables for other agents but this agent sees them as a parameter.

The different types of producers introduced are: battery agents, renewable technology agents and thermal technology agents. A CO2 allowance market is introduced. Total amount of CO2 emissions is restrained to a CO2 cap through a market clearing condition. Finally, the electricity market clearing condition has to be slightly modified in order to remain consistent with the different modifications made. Different chronological time segments ℓ are introduced. One ℓ typically lasts one hour. The decomposition of previous section will be modified with these new agent formulations.

4.1 Modelling a battery

The variable $e_\ell(\omega)$ represents the amount of electricity stored in the battery at the end of time segment ℓ. The variable $c_\ell(\omega)$ represents the electricity charged in the battery or also the battery consumption. The electric production of the battery $y_{k,\ell}(\omega)$ is the amount of energy pumped in the grid by the battery during time segment ℓ. Each time the battery injects electricity in the grid it receives the electricity price $\pi_\ell(\omega)$ for the injected quantity and each time electricity is pumped from the grid it has to pay the electricity price $\pi_\ell(\omega)$ to buy the electricity. The battery also has to pay an operating cost $C_k(\omega)$ per unit of electricity injected or withdrawn from the electricity grid.

In order to operate the battery, two investments have to be made. On the one hand the storage capacity *Storage_cap* is the maximal amount of electricity that can be stored in the battery. On the other hand the charging capacity *Charge_cap* is the maximal amount of electricity that can be charged in the battery or pumped out of the battery during time segment ℓ. Both capacities are modelled as variables. η depicts the roundtrip efficiency of the battery. This represents the losses of electricity due to charging and discharging the battery. A way of seeing battery agent problem $\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}))$ is:

$$\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega})) \equiv \arg\min_{x_k} \rho_k \Big[\sum_{\ell \in L} \Big(C_k(\omega) - \pi_\ell(\omega)\Big) y_\ell(\omega) + \Big(C_k(\omega) + \pi_\ell(\omega)\Big) c_\ell(\omega)\Big] \tag{4.1}$$

$$+ I_{Storage\_cap} Storage\_cap + I_{charge\_cap} Charge\_cap \tag{4.2}$$

$$\text{s.t.} \quad 0 \leq e_\ell(\omega) \leq Storage\_cap \qquad (\mu_{k,\ell}(\omega)\bar{q}(\omega)) \tag{4.3}$$

$$e_\ell(\omega) = Dissipation\_factor \cdot e_{\ell-1}(\omega) - \frac{y_\ell(\omega)}{\sqrt{\eta}} + \sqrt{\eta} \cdot c_\ell(\omega) \qquad (\gamma_\ell(\omega)\bar{q}(\omega)) \tag{4.4}$$

$$0 \leq y_\ell(\omega) \leq Charge\_cap \qquad (\sigma_\ell(\omega)\bar{q}(\omega)) \tag{4.5}$$

$$0 \leq c_\ell(\omega) \leq Charge\_cap \qquad (\kappa_\ell(\omega)\bar{q}(\omega)) \tag{4.6}$$

Constraint (4.4) models the storage between time periods. The electricity that is present in the battery at the end of each time period is the electricity present at the end of the previous time period discounted by the dissipation. The electricity injected by the battery in the electricity grid should be subtracted and the electricity pumped by the battery from the electricity grid should be added. Pumping and injecting electricity is also subject to losses of electricity. This is represented by $\sqrt{\eta}$. The losses of the battery could also be modelled with a different efficiency for injecting and pumping electricity but this is not done here. The dual variable of this constraint $\gamma_\ell(\omega)$ can be interpreted as an opportunity cost of producing or consuming electricity during that time segment ℓ.

The value created by holding a battery can be defined in 3 parts:

- ◇ capacity value: $\mu_{k,\ell}(\omega)$. This value can only be generated when the battery is fully charged $Storage\_cap = e_\ell(\omega)$. There is no remaining space in the battery at that time slot.
- ◇ production value: $\sigma_\ell(\omega)$. This value is generated when the battery produces at its maximal rate during a time slot $y_\ell(\omega) = Charge\_cap$.
- ◇ consumption value: $\kappa_\ell(\omega)$. This value is generated when the consumption of the battery (energy taken off the grid by the battery) is maximal $c_\ell(\omega) = Charge\_cap$.

The KKT conditions and dual of this problem can be found in appendix.

4.2 Modelling renewables

In contrast to batteries, the approach to model renewables does not vary a lot from a general producer defined in Definition 1. However due to the varying meteorological conditions, a constraint should be added meaning, that not all the capacity, but only a fraction of it, is available at each time segment. $fact_{k,\ell}(\omega)$ represents this fraction of available capacity. It should be noted that if $fact_{k,\ell}(\omega) = 1$ at all time segments, this represents a producer that has all of its capacity available 24/7 or in other words a conventional producer.

$$\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega})) \equiv \arg\min_{x_k} \rho_k \Big[\min_{y_{k,\ell}(\omega)} \sum_{\ell \in L} (-\pi_\ell(\omega)) y_{k,\ell}(\omega) \Big] + I_k x_k \quad (4.7)$$

$$\text{s.t.} \quad 0 \leq x_k, y_{k,\ell}(\omega) \quad (4.8)$$

$$y_{k,\ell}(\omega) \leq fact_{k,\ell}(\omega) \cdot x_k \qquad (\mu_{k,\ell}(\omega)) \quad (4.9)$$

The KKT conditions of the renewable producer agent can be found in Appendix.

4.3 Modelling thermal generators

The marginal cost of thermal generators can be divided into two main parts: on the one hand there is the fuel and operating costs of the plant $C_{k,fuel}(\omega)$, on the other hand there is the price of emitting CO2 emissions $E_k \lambda(\omega)$. E_k represents the amount of emissions emitted by producer k to produce one unit of electricity and $\lambda(\omega)$ is the price of a CO2 emission. The thermal generator takes these two inputs as parameters and therefore cannot influence them. This yields an equation of marginal costs:

$$MC_{k,fuel}(\omega) = C_{k,fuel}(\omega) + E_k \lambda(\omega) \quad (4.10)$$

The subscript *fuel*, represents the case where a thermal generator operates with several types of fuels is taken into account. This yields the following maximisation problem for a thermal generator agent:

$$\mathcal{G}_k(\boldsymbol{\pi}(\boldsymbol{\omega}), \boldsymbol{\lambda}(\boldsymbol{\omega})) \equiv \arg \min_{\substack{x_k \\ y_{k,\ell,fuel}(\omega)}} \rho_k \Big[\sum_{\substack{\ell \in L \\ fuel}} \Big(MC_{k,fuel}(\omega) - \pi_\ell(\omega) \Big) y_{k,\ell,fuel}(\omega) \Big] + I_k x_k \quad (4.11)$$

$$\text{s.t.} \quad 0 \leq y_{k,\ell,fuel}(\omega) \quad (4.12)$$

$$\sum_{fuel} y_{k,\ell,fuel}(\omega) \leq x_k \qquad (\mu_{k,\ell}(\omega) q_k(\omega)) \quad (4.13)$$

4.4 CO2 emission clearing condition

One can also introduce a limit on how much CO2 emissions can be produced. E_k would be the amount of emissions that are created per unit of electricity produced by technology k. Summing on the different technologies and time segments gives the total amount of CO2 emissions produced $\sum_{k \in K, \ell \in L} E_k y_{k,\ell}(\omega)$.

To make sure that the threshold of emissions is not exceeded, the price $\lambda(\omega)$, of a CO2 emission in euro per ton, would be decided as the dual variable of the following constraint :

$$0 \leq NAP - \sum_{k \in K, \ell \in L} E_k y_{k,\ell}(\omega) \perp \lambda(\omega) \geq 0 \quad (4.14)$$

Condition (4.14) states that the total CO2 emissions cannot be greater than what is allowed (NAP).

Communicating this price to the agents can be done by modifying constraint (2.28) by (4.15):

$$0 \leq C_k(\omega) + \mu_k(\omega) + E_k \lambda(\omega) - \pi(\omega) \perp y_k(\omega) \geq 0 \tag{4.15}$$

The term $E_k \lambda(\omega)$ has been added to incorporate the price of the CO2 emissions for the different producer agents.

4.5 Market clearing condition

Due to the introduction of batteries, there is also consumption $c_\ell(\omega)$ of the battery at certain time segments. This has to be dealt with in the market clearing condition:

$$0 \leq \sum_{k \in K} y_{k,\ell}(\omega) + z_\ell(\omega) - c_\ell(\omega) - D_\ell(\omega) \tag{4.16}$$

One can also note that the Demand $D_\ell(\omega)$ is different in each time segment ℓ can vary in function of the different scenarios ω.

CHAPTER 5

RISK HEDGING TOOLS

In this chapter different types of tools that can be used to hedge risk are discussed. The goal of introducing contracts is to reduce the risk taken by the different agents. Hence a contract is a tool for these agents to hedge their risk. For the different contracts the only part that has to be defined is the contract's second stage payoff $\pi_c(\omega)$. Recall that the price received/paid in the first stage for having a short or long position on the contract is a variable that is fixed in the Financial problem.

There also exists other tools than contracts enabling risk reduction of different agents. In our case, different types of markets that have a positive effect on the risk reduction of the agents will be discussed. The electricity market and the CO2 market are already incorporated in the model. The capacity market and the complete market will be added.

5.1 Complete market

Definition 8 (complete market). A complete market is a market where the same amount of contracts are present as there are scenarios. The risk-free rate is implied as a contract. So there are $|\Omega| - 1$ contracts, other than the risk-free rate, present in the complete market.

It has been shown in [15] that in a complete market all the risk is perfectly traded. If the risk measure that is used is coherent, it has been proven that the risk aversion of the different agents coincides with each other. Hence, one same risk averse probability $q(\omega)$ will be taken for all agents. This does not mean that $q(\omega)$ boils down to $p(\omega)$. Indeed the market risk remains for the different agents. The risk averse probabilities belong to a space that is the intersection of the spaces of the different agents. In our case, all agents had the same space $\mathcal{M}$ for their RA probabilities. The RA probability $q(\omega)$ thus lies in this space $\mathcal{M}$.

An important assumption made is that all the agents are trading. This will be an issue for comparing the contracts with the complete market in the next chapter. Indeed, because of the

CO2 market, a new agent will be introduced to represent the regulator who receives the money from the CO2 allowances. Since this agent is a regulator, he will not trade his risk. But here in the complete market the assumption is made that he can trade his risk.

Definition 9 (Complete market optimization problem). The complete market can be solved with the following optimization problem :

$$
\begin{aligned}
& \min t + \gamma s \\
\text{s.t.}\quad & u(\omega),\ z(\omega),\ y_k(\omega) \geq 0 \\
& u(\omega) + t \geq \sum_{k \in K} \left(C_k(\omega) y_k(\omega) + I_k x_k\right) + PC(z(\omega) - D) && (q(\omega)) \\
& (\sqrt{p(\omega)}u(\omega), s) \in \mathbb{L}^{|\Omega|+1} && \left(\frac{q(\omega)}{\sqrt{p(\omega)}}, \gamma\right) \\
& y_k(\omega) \leq x_k && (\mu_k(\omega)) \\
& \sum_k y_k(\omega) + z(\omega) \geq D && (\pi(\omega))
\end{aligned}
$$

This problem can be solved as a single QCP, this is an interesting result compared to the incomplete case discussed in chapter 3 where a sequence of optimisation problems had to be solved.

When the costs of the different agents are summed, the price paid by the consumer cancels out with the price received by the different producers. This is the reason why a single QCP problem can solve the complete market.

5.2 Baseload contract

The baseload contract, which is defined in (5.1), has as payoff the average price of electricity. This is the same as what is done at EEX[1].

$$\pi_c(\omega) \equiv \frac{\sum_{\ell \in L} \pi_\ell(\omega)}{|L|} \tag{5.1}$$

This contract can be very useful if in some scenarios the electricity price is high on average. In that case the consumer could take a long position on the baseload contract to hedge himself against high electricity prices.

5.3 Seasonal load contract

A seasonal load contract has as payoff the mean electricity price of different periods of the year. The year will be split in 2 different parts. A spring and summer part versus an autumn and winter part. Both parts are referred to as s_1 and s_2. Let's now define the binary parameter $period_{s,\ell}$ representing if time segment ℓ is in s:

$$period_{s,\ell} = \begin{cases} 1 & \text{if } \ell \text{ is in period } s \\ 0 & \text{otherwise} \end{cases}$$

So the seasonal load contract will be defined as:

[1] European Energy Exchange, market to trade energy goods in Europe, https://www.eex.com/en/

$$\pi_{c,s}(\omega) \equiv \frac{\sum_{\ell \in L} period_{s,\ell} \cdot \pi_\ell(\omega)}{\sum_{\ell \in L} period_{s,\ell}} \tag{5.2}$$

This is the average electricity price over the part of the year s.

5.4 Volatility index on renewables

The volatility index on different renewable technologies measures how efficient each technology is in a given scenario. In a certain sense this is the average efficiency. The price of the contract would then be:

$$\pi_c(\omega) \equiv \frac{\sum_{\ell \in L} fact_{k,\ell}(\omega)}{|L|} \tag{5.3}$$

5.5 Weather derivatives

Weather derivatives are contracts obviously depending on the weather. Temperature is important in energy markets because it influences demand. In very cold days the demand will peak, hence the electricity price will increase. A contract based on temperature in the same the spirit as in [2] and [4] is introduced. More specifically temperature will be modelled using the heating degree days (HDD).

Data from Eurostat will be used to capture the HDD. The value of a heating degree day HDD_d representing the temperature of day d is defined by Eurostat [19] as :

$$HDD_d = \begin{cases} 18°\text{C} - T_{d,m} & \text{if } T_{d,m} \leq 15°\text{C} \\ 0 & \text{otherwise} \end{cases} \tag{5.4}$$

Where $T_{d,m}$ is the mean outdoor temperature of day d:

$$T_{d,m} = \frac{T_{d,min} + T_{d,max}}{2} \tag{5.5}$$

$T_{d,min}$ being the minimum temperature at day d and $T_{d,max}$ being the maximum temperature at day d. The HDD are then summed over a year, in our case one considers ω referring to a year, and this gives the payoff of a weather derivative contract:

$$\pi_c(\omega) \equiv \sum_{d \in \omega} HDD_d \tag{5.6}$$

5.6 CO2 contract

A contract having as payoff the price of a CO2 allowance will also be analysed:

$$\pi_c(\omega) \equiv \lambda(\omega) \tag{5.7}$$

Note that this contract is useless if $\lambda(\omega)$ has the same value for all scenarios ω. There would be two cases. Either the CO2 cap is very low and then $\lambda(\omega)$ would reach an upper limit. Either the CO2 cap is not constraining at all because it is set too high, then $\lambda(\omega) = 0$.

5.7 Capacity market

A capacity market is, as it states, a market to trade capacity. The idea behind this concept is to create a forward market (in the first stage) where existing capacity, that is flexible enough to guarantee production (set $K_{flex} \subset K$), is compensated by some monetary reward ν. This concept can be summarised in the following equation:

$$0 \leq \sum_{k \in K_{flex}} x_k - \max_{\omega \in \Omega, \ell \in L} D_\ell(\omega) \perp \nu \geq 0 \tag{5.8}$$

With the capacity market the regulator can impose a target on capacity. Here the regulator wishes to have enough available capacity at all time ($\max_{\omega \in \Omega, \ell \in L} D_\ell(\omega)$) and thus to have a curtailment $z_\ell(\omega) = 0$ as small as possible. Only capacity that can guarantee production is valid, this is captured in the set K_{flex}. This set contains the battery agent and the different thermal technologies. Indeed renewables cannot guarantee available production and this is the goal of the capacity market. The different producers in K_{flex} are encouraged to build this capacity by receiving ν for holding capacity. The value ν will be fixed to reach exactly the regulatory target. Therefore the general equation (2.30) can be modified to:

$$0 \leq I_k - \nu - \mathbb{E}_{Q_k}[\mu_k(\omega)] \perp x_k \geq 0 \tag{5.9}$$

One understands that, since this monetary reward ν is a first stage variable, it decreases the investment cost and thus reduces the risk for all the producers in K_{flex}.

For the battery agent, the quantity of capacity that can be offered in the capacity market is fixed to $min(\frac{x_k}{4}, Charge\_cap)$. This because getting paid the price ν for all the capacity of the battery if it is not fully charged, does not make sense, the decision is thus made to only take 25% of the battery capacity into account. *Charge_cap* is the maximal amount of electricity that can flow in or out the battery during one hour. Hence this is also the maximal electricity production for the battery. Note that if the battery is discharged curtailment could still occur.

CHAPTER

6
RESULTS

In this chapter, the decomposition algorithm, detailed in chapter 3, is used to compute the results. First the dataset is explained, including the different types of producers and the types of uncertainties that will be considered. Then numerical results are commented.

6.1 Dataset

6.1.1 Agents

As mentioned previously, one consumer agent represents the aggregated consumption. On the side of the producers, 5 different technologies, each represented by a producer agent, are modelled:

- 2 different renewable technologies: wind onshore (windon) and photovoltaics (pv).
- 1 battery agent (battery): that can store electricity from one time segment until the next time segment
- 2 different thermal generators: Open Cycle Gas Turbine (OCGT) and Combined Cycle Gas Turbine (CCGT). These thermal generators can operate on two different types of fuel: natural gas and biogas. Natural gas is cheaper but generates CO2 emissions when a plant operates with this fuel. Biogas is more expensive but does not emit CO2 emissions. This is summarised in Table 6.1. Fuel prices are exposed in the next section in table 6.5. Each thermal plant also has a different efficiency. This captures the amount of energy lost due to the transformation of energy into electricity. The efficiency of each thermal generator can be found in Table 6.2.

Table 6.3 illustrates the annualised investment costs of the different technologies. For the battery, the annualised investment cost in storage capacity is set to $27\frac{€}{kW}$. The dissipation factor of the battery is set to 0.99996. The round-trip efficiency η is set to the value of 0.95.

Table 6.1: CO2 emission rate

	emission ($\frac{ton}{MWh}$)
natural gas	0.2
biogas	0

Table 6.2: Efficiency thermal generator

	Efficiency (%)
CCGT	65
OCGT	54

Table 6.3: annualised investment costs of the technologies

	cost ($\frac{€}{kW}$)
pv	45
windon	97
battery	8
ccgt	55
ocgt	40

Having a CO2 emission market and a capacity market means in both cases that costs of producers are affected. Indeed, in a capacity market, producers who can offer their capacity receive a monetary reward ν for this, while on the CO2 emission market, producers would have to pay $\lambda(\omega)$ for their CO2 emissions.

For the capacity market the monetary reward ν is paid by the consumer, adding up to his cost function. At the end of day, the consumer pays the electricity bill and integrates these different regulatory measures (and thus the capacity price) in his choice consideration.

For the CO2 emission market, the money resulting from the CO2 emissions is typically collected by the regulator. So, a new agent representing the regulator is added. The regulator agent is risk averse but does not have access to any tool to hedge his risk. His cost is only defined by the money received for the CO2 emission allowances. Note that in the complete market described in section 5.1 the regulator agent could trade his risk. So, imposing that the regulator cannot trade his risk is an additional incompleteness of the market.

6.1.2 Uncertainty

Data from the German electricity market is used to perform a case study. The demand data comes from Open Power System Data [20], which is a database compiled from ENTSO-E, the European Network of Transmission System Operators [17]. The data for modelling the renewable production in Germany was given by [21]. The method used to create this data is based on satellite data as explained by Pfenninger and Staffell in [9] and [8]. Concerning the scenarios ω, three different types of scenarios are used:

1. yearly scenarios captured in set Ω_1: This affects the electricity demand and the renewable production. Data from 11 years (2006 to 2016) is used. Each year is considered to be equally likely to occur. The demand is normalized such that $\overline{D} = \sum_\ell D_\ell(\omega_1)$ for all $\omega_1 \in \Omega_1$
2. demand scenarios captured in set Ω_2: Here the total demand would slightly change by a factor $var(\omega_2)$ affecting the demand of each time segment ℓ. $D_\ell(\omega) = var(\omega_2) \cdot D_\ell(\omega_1)$ This affects the demand. 4 different scenarios are detailed in Table 6.4.
3. fuel costs scenarios Ω_3: This would be scenarios with different fuel prices. This affects the operating costs of the thermal technologies. These prices are detailed in Table 6.5.

So the set of total uncertainty would be Ω where $\Omega = \Omega_1 \times \Omega_2 \times \Omega_3$.

Table 6.4: demand variation uncertainty (Ω_2)

	variation (%)
scenario d1	100%
scenario d2	101%
scenario d3	103%
scenario d4	99.5%

Table 6.5: fuel price uncertainty (Ω_3)

price ($\frac{€}{MWh}$)	natural gas	biogas
scenario f1	40	75
scenario f2	30	75

6.2 Numerical results

In this first part, uncertainty is taken as follows : $\Omega = \Omega_1$, leading to 11 scenarios, corresponding to yearly data from 2006 to 2016.

Different markets are computed with the different financial tools presented in chapter 5. One last type of market is added, an incomplete market. This is the basic market with risk averse agents having no contract to hedge the risk.

For each market the problem solution will be computed in different settings: 100% of CO2 emissions[1], 50% of CO2 emissions allowed and no CO2 emission restrictions at all.

6.2.1 Share of capacity and production of the different technologies

The left part of Figure 6.1 shows the available capacity of the producers. For conventional thermal plants and the battery, capacity installed is the same as capacity available. However for renewables, capacity available is not the same because of meteorological conditions. The average available capacity of renewable producers is:

$$\mathbb{E}_P\left[\frac{\sum_{\ell\in L} fact_{k,\ell}(\omega)}{|L|}\right] x_k$$

Average production of the different technologies is showed in the right part of Figure 6.1. The production shifts from a majority of gas production in the case of 100% of CO2 emissions to a bigger share of renewable production in the case of no CO2 emissions.

The capacity invested is smaller in the incomplete market compared to the complete market. Hence, there is under-investment in the incomplete market. Technologies that seem most at risk are gas plants and in some matters battery and wind. This is because the capacity built varies between the complete market and the incomplete market.

Comparing capacity and production yields the conclusion that renewable technologies are producing more electricity compared to the capacity installed. This is a logical condition since the marginal cost of producing one unit of electricity for renewables is zero. When a battery is present, they can produce more electricity because the battery can consume some of the renewable production surplus and reinject it in the grid when it is needed.

The production also slightly increases as CO2 restrictions arise. This is because only electricity production is shown in the graph. The battery consumption is not shown to be able to see the impact of the battery on the production. However, this means that the electricity produced

[1] By this is meant 100% of the CO2 emissions emitted of the RN probability case when no restrictions are imposed on CO2 emissions

by the battery is shown twice. Firstly, when the battery is charged by another producer. Secondly, when the battery produces the electricity for the consumer. This and the losses of the battery during the charging, storage and discharging explain the increase in electricity production.

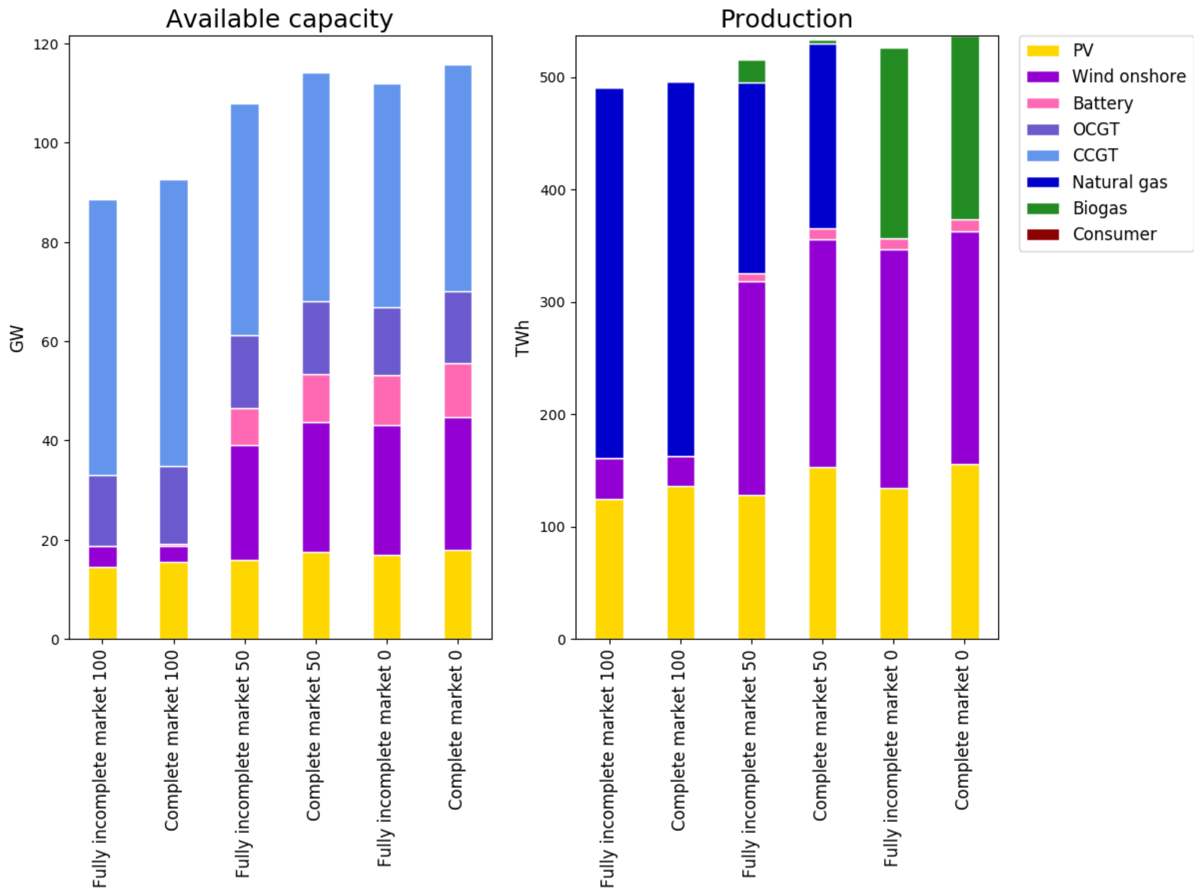

Figure 6.1: Capacity and average production of the different technologies

6.2.2 Welfare

To compute the complete market, costs of different agents are assumed to have the same value. By this is meant that 1 € in the pocket of the consumer has the same value as 1 € in the pocket of any producer. This implies that the total cost of the complete market is obtained by summing the costs of the different agents. To remain coherent with the complete market the same assumption is made for the other markets. The welfare is defined as the total costs with a negative sign. The total cost of the agents is given by:

$$\sum_{k\in K}\mathbb{E}_{Q_k}\left[\sum_{\ell\in L}\left(C_k(\omega)+E_k\lambda(\omega)-\pi_\ell(\omega)\right)y_{k,\ell}(\omega)+I_kx_k\right]+\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}\left(\pi_\ell(\omega)-PC\right)\left(D_\ell(\omega)-z_\ell(\omega)\right)\right]$$
$$+\mathbb{E}_{Q_{reg}}\left[-\sum_{k\in K,\ell\in L}E_k\lambda(\omega)y_{k,\ell}(\omega)\right]$$

The first term represents the costs of the different producers. It has a value of zero because of implicit assumptions made for each producer. These assumptions are that the costs are linear and no limit is present on the capacity installed (KKT conditions (2.28) and (2.30)). These two assumptions impose for each producer a zero cost. The second term represents the consumer cost. The last term represents the costs of the regulator agent. This cost is the money he receives from the CO2 emission allowances. Let's remind this agent is not allowed to buy or sell any contracts.

Figure 6.2 illustrates the evolution of the welfare in function of the percentage of C02 emissions allowed. It is interesting to note the difference in welfare spread between different markets in function of CO2 emissions. The riskiest case is the case where 50% of the total CO2 emissions needed by the market can be emitted. The spread in welfare between the different markets studied is bigger than when CO2 emissions are not constrained or completely prohibited.

The explanation lies with the risk aversion of the regulator agent. In the 100% and the 0% case the regulator has less risk on this cost function. Indeed the price of a CO2 allowance is nearly always the same. Therefore the cost function of the state agent does not vary a lot in the 100% and 0% case. While in the 50% case the regulator agent has an uncertain cost function. As mentioned earlier, the regulator cannot trade his risk. This is why there is a loss of welfare.

One can notice the effect of the CO2 contract. The contract has a maximal effect when the CO2 emissions are constrained at a 50% level, because at that level the CO2 price varies a lot. This has as effect of varying the operational costs of the thermal producers (OCGT and CCGT) and results in more volatile electricity prices which is bad for the different agents. A CO2 contract gives the possibility to hedge the risk. The case with 100% of CO2 emissions is a bit better than the risk averse case with no contracts. This is because the CO2 cap was fixed at the emissions in the case of the RN probabilities. However in a risk averse case one will tend to emit more CO2 emissions and thus reach the CO2 cap, having a slightly positive price for CO2 and a differentiated payoff for the CO2 contract. The case with 0% of CO2 emissions is the same as the case without CO2 contract which makes sense.

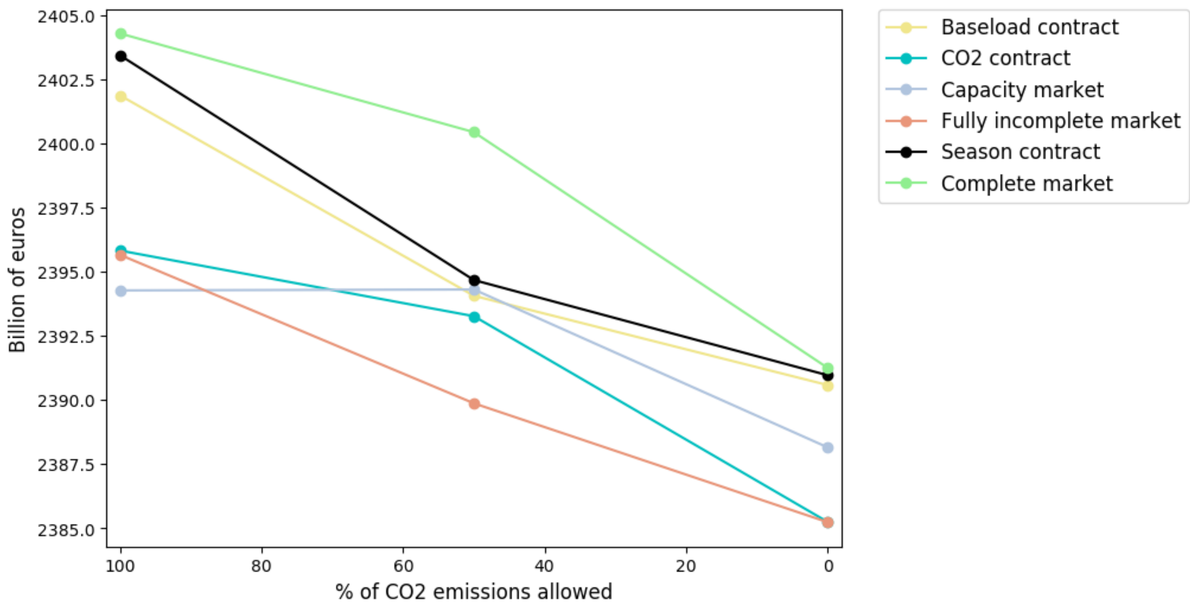

Figure 6.2: The welfare in function of the different CO2 restrictions

6.2.3 Welfare technological split

The welfare computed in the previous section gives a good idea of how different contracts affect welfare, but does not deal with the share of the different technologies in the welfare. The argument was made that the welfare of the different producers had a zero value. Hence it is not interpretable. An alternative welfare split $\widetilde{W}_i$ needs to be taken to see the share of welfare of the different technologies. The total costs of the market, which is the sum of the costs of different agents, corresponds to minus the welfare. The regulator agent *reg* has also costs that have to be taken into account. The costs of the different agents can be written as:

$$
\begin{aligned}
&\sum_{k\in K}\mathbb{E}_{Q_k}\left[\sum_{\ell\in L}\left(C_k(\omega)+E_k\lambda(\omega)-\pi_\ell(\omega)\right)y_{k,\ell}(\omega)+I_kx_k\right]+\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}\left(\pi_\ell(\omega)-PC\right)\left(D_\ell(\omega)-z_\ell(\omega)\right)\right]\\
&+\mathbb{E}_{Q_{reg}}\left[-\sum_{k\in K,\ell\in L}E_k\lambda(\omega)y_{k,\ell}(\omega)\right]\\
=&\sum_{k\in K}\mathbb{E}_{Q_k}\left[\sum_{\ell\in L}C_k(\omega)y_{k,\ell}(\omega)+I_kx_k\right]+\sum_{k\in K}\mathbb{E}_{[Q_{cons}-Q_k]}\left[\sum_{\ell\in L}\pi_\ell(\omega)y_{k,\ell}(\omega)\right]\\
&+\sum_{k\in K}\mathbb{E}_{[Q_k-Q_{reg}]}\left[\sum_{\ell\in L}E_k\lambda(\omega)y_{k,\ell}(\omega)\right]+\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}PCz_\ell(\omega)\right]-\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}PCD_\ell(\omega)\right]
\end{aligned}
$$

The second part is just reordering the terms of the expression on the first part. In the third and fourth line, the first 3 terms will be defined the welfare technological split of the different producers. The fourth term represents the curtailment of the consumer. This term is defined as the consumer's welfare split. The last term $\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}PCD_\ell(\omega)\right]$ is neglected, because this term is really big and it is the same for all the different scenarios. Remember that $\sum_{\ell\in L}D_\ell(\omega)$ is the same for all scenarios. It does not give any information on how the welfare is created.

Hence the new measure $\widetilde{W}_i$ is defined as:

Definition 10 (Welfare technological split). The welfare technological split of the producer is defined as:

$$
\begin{aligned}
\widetilde{W}_k=&\mathbb{E}_{Q_k}\left[\sum_{\ell\in L}C_k(\omega)y_{k,\ell}(\omega)+I_kx_k\right]+\mathbb{E}_{[Q_{cons}-Q_k]}\left[\sum_{\ell\in L}\pi_\ell(\omega)y_{k,\ell}(\omega)\right]\\
&+\mathbb{E}_{[Q_k-Q_{reg}]}\left[\sum_{\ell\in L}E_k\lambda(\omega)y_{k,\ell}(\omega)\right]
\end{aligned}
$$

For the consumer the welfare technological split is defined as his cost of electricity curtailed:

$$\widetilde{W}_{cons}=\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}PCz_\ell(\omega)\right]$$

Finally the last part will be neglected as it does not vary:

$$\widetilde{W}_{endowment}=-\mathbb{E}_{Q_{cons}}\left[\sum_{\ell\in L}PCD_\ell(\omega)\right]$$

This measure is shown in Figure 6.3 for the complete and incomplete market. For the producer the first term of $\widetilde{W}_k$, $q_k(\omega)(\sum_{\ell \in L} C_k(\omega) y_{k,\ell}(\omega) + I_k x_k)$, represents the cost of the producer, i.e. all the money that the producer has to pay to produce electricity, it is indicated in plain color in Figure 6.3.

The second and third terms are risk adjusted terms due to non-perfect hedging. It measures how useful the electricity produced is for the consumer and how desperate the producer wants to buy the CO2 emission from the regulator. It is represented in the hatched part of the bar in Figure 6.3. In the complete market the risk adjusted terms are zero since all the agents have the same risk measure.

The consumer values the electricity which was curtailed. And the endowment term represents the term in the consumer cost that is fixed for the different scenarios. It is not shown in the graph as it is very big. Summing the different terms of the welfare technological split recovers the total cost.

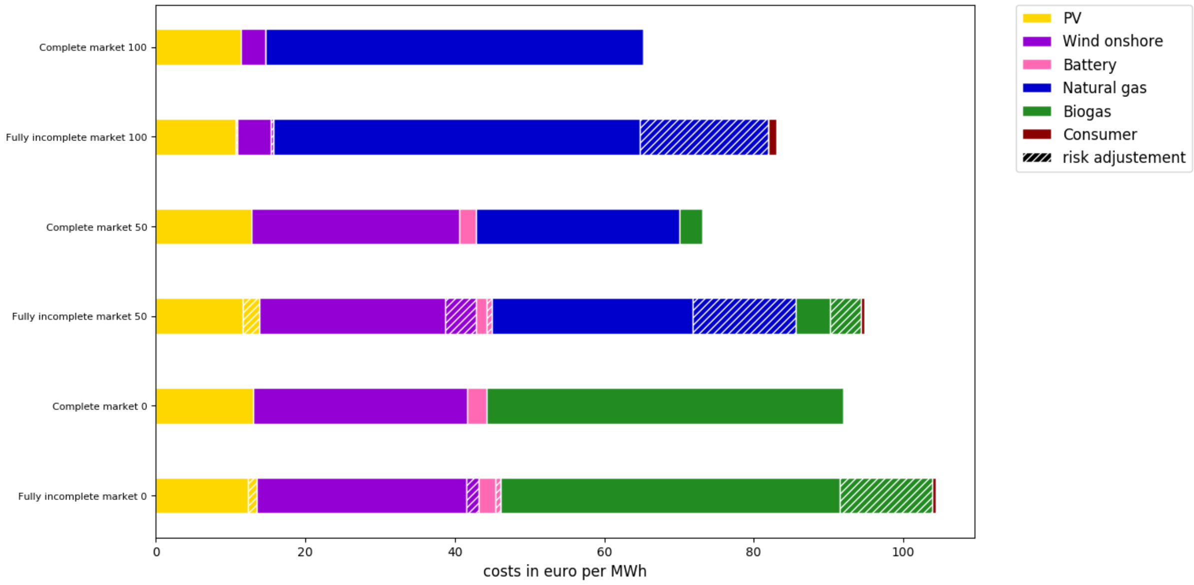

Figure 6.3: The welfare technological split of the different agents

In Figure 6.3 the cost of the different technologies can be seen. A first observation is that battery, natural gas and biogas technologies are technologies having the biggest hatched part. This is because these technologies are the most flexible. In the 0% case, the need for flexible units of production incites the transition from natural gas to biogas production.

An important remark is that inherent risk remains in the complete market. If all the agents can trade freely, they will have the same risk position. This is the inherent market risk. Therefore, it is important to bear in mind that the risk adjusted term due to non perfect hedging does not capture all the risk.

Finally the complete market with 0% CO2 yields values that are not so far from the case with 100% CO2. So solving the problem with risk on the market is as important as finding solutions to decarbonising the energy industry.

6.2.4 Mean electricity price

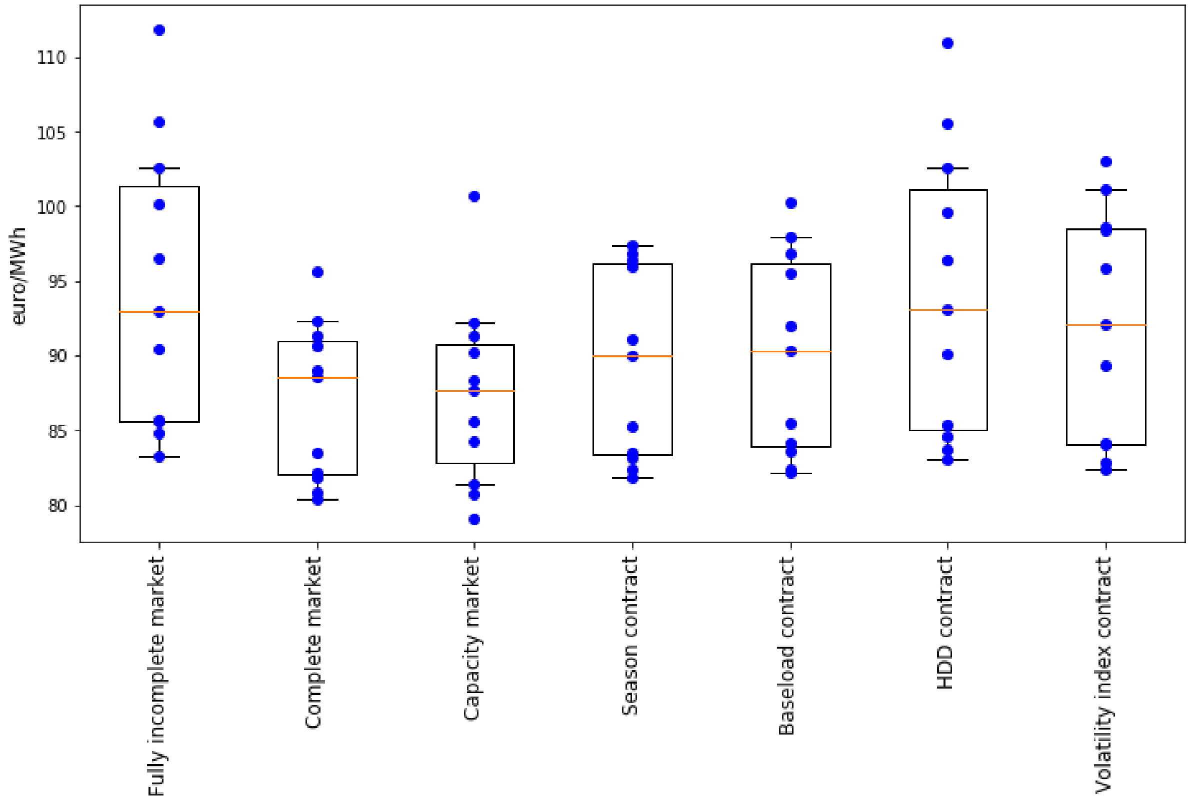

Figure 6.4: Average electricity price with 0% CO2 emissions

One can see the average price of electricity in Figure 6.4. Each dot stands for the average electricity price in one scenario. The boxplot is drawn on top of this.

This Figure shows there is a difference in electricity price between the incomplete market without any contracts and the different markets where risk can be hedged.

One can observe that the capacity market has as effect of decreasing the average electricity price. This is because capacity market leads to building more capacity. So, the price will spike less to the price cap and this will overall result in a cheaper average electricity price.

The seasonal contract has as effect of separating the average price in two categories. The average price looks a lot like the baseload contract.

The baseload contract can be analysed. Remember that the payoff of the baseload contract $\pi_c(\omega)$ is exactly the average electricity price. One can see that having this payoff guaranteed, enables the electricity price to drop to a level closer to the complete market.

The Heating degree contract does not seem of any use.

The volatility index is a bit better than the incomplete market but not much.

6.2.5 Curtailed renewable production

Curtailed renewable electricity refers to energy that could have been produced but was finally not, since there was over-production and electricity prices dropped to zero. Some renewable producers decided to not produce at full available potential $y_{k,\ell}(\omega) < fact_{k,\ell}(\omega)x_k$.

In Figure 6.5 one can see the average percentage of renewable energy that was curtailed in the different scenarios. Hence the value in the graph is :

$$\mathbb{E}_P\left[\frac{\sum_{\ell\in L} fact_{k,\ell}(\omega)x_k - y_{k,\ell}(\omega))}{\sum_{\ell\in L} fact_{k,\ell}(\omega)x_k}\right]$$

When CO2 restrictions are increasing, the share of renewable energy becomes more important, resulting in higher curtailment values.

The higher curtailment in wind for the complete market does not mean a lot, because the pv curtailment values are much lower. Indeed at most hours when curtailment occurs one can decide to curtail either wind or pv.

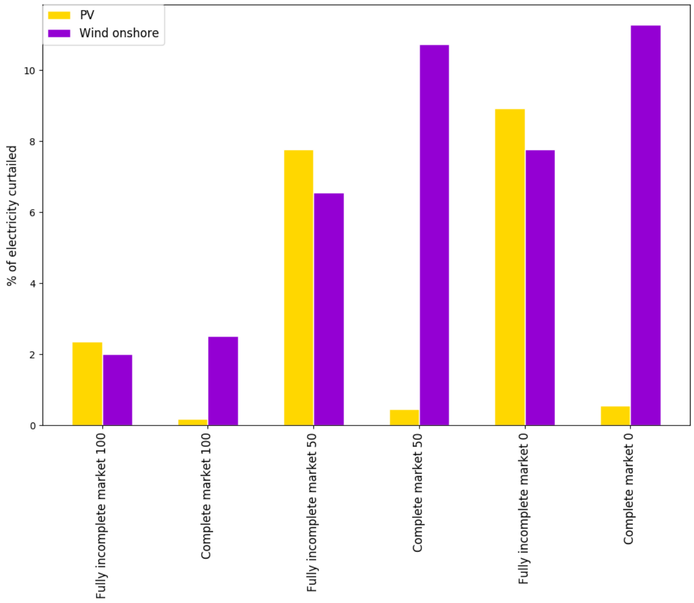

Figure 6.5: Average curtailed renewable production

Looking at the global renewable curtailment in Figure 6.6, does not show a big difference between the complete and incomplete market. There is still a bit less curtailment in the complete market, it can be explained by the higher presence of battery.

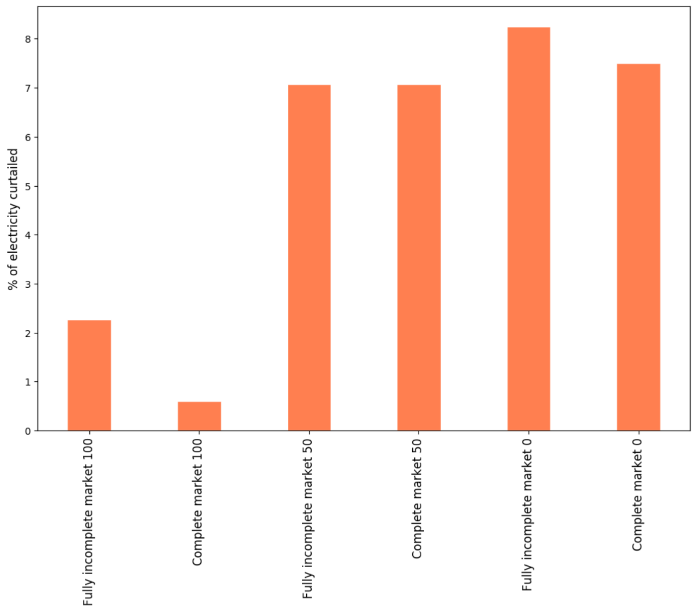

Figure 6.6: Global average curtailed renewable production

6.2.6 Market factors of renewables

The market factor of a technology is the ratio of the average price received by a technology on the average electricity price. It gives an idea about whether a technology is producing at hours where electricity is expensive or cheap. If the market factor is smaller than 1 this means the technology produces more in off-peak hours. If the market factor is greater than 1, this would mean that the technology produces more in peak hours. A market factor of 0.5 would mean that on average the producer receives half of the average electricity price for his production.

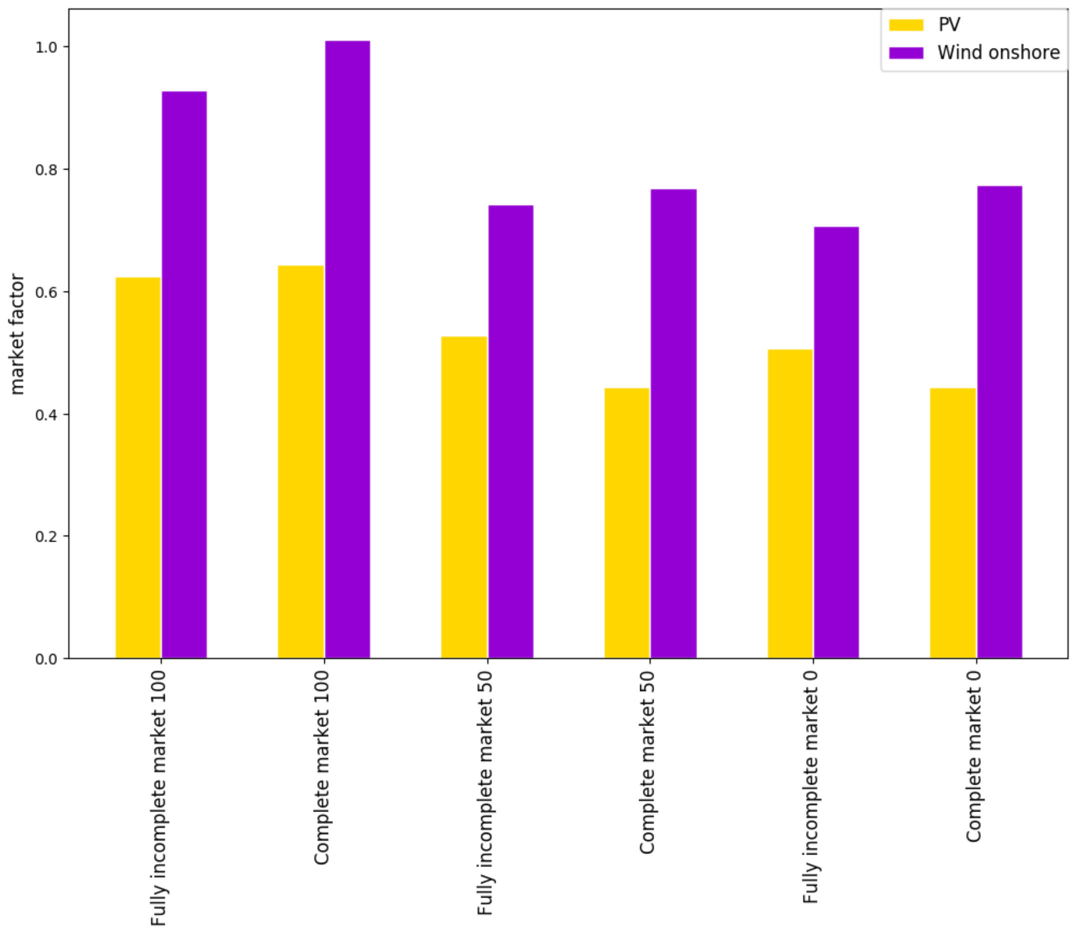

Figure 6.7: Average market factor

Figure 6.7 illustrates the market factors with different settings of CO_2 emissions. One can observe that in the case of 100% of CO_2 emissions the market factor of pv is higher than in the cases with CO_2 emission reductions. This is because the installed capacity of pv is smaller. If one installs more pv this means that more capacity will be producing electricity at the same time resulting in cheaper electricity price at these times.

One can notice that the market factor of onshore wind is significantly bigger than pv. This is due to the fact that wind blows 24/7 while photovoltaics only produce energy during the day.

6.2.7 Running hours thermal plants

The running hours of a thermal plant is a measure on the number of hours this thermal plant is actually producing during a year. The average running hours RH_k is defined as:

$$RH_k = \mathbb{E}_P \left[\frac{\sum_{\ell \in L} y_{k,\ell}(\omega)}{8760 \cdot x_k} \right]$$

Figure 6.8 shows the evolution of the average running hours of the thermal plants. As the CO2 emissions are more and more restricted, thermal plants have higher operating costs this leads to thermal plants producing less electricity. So these plants rely on a fewer amount of hours to make profit.

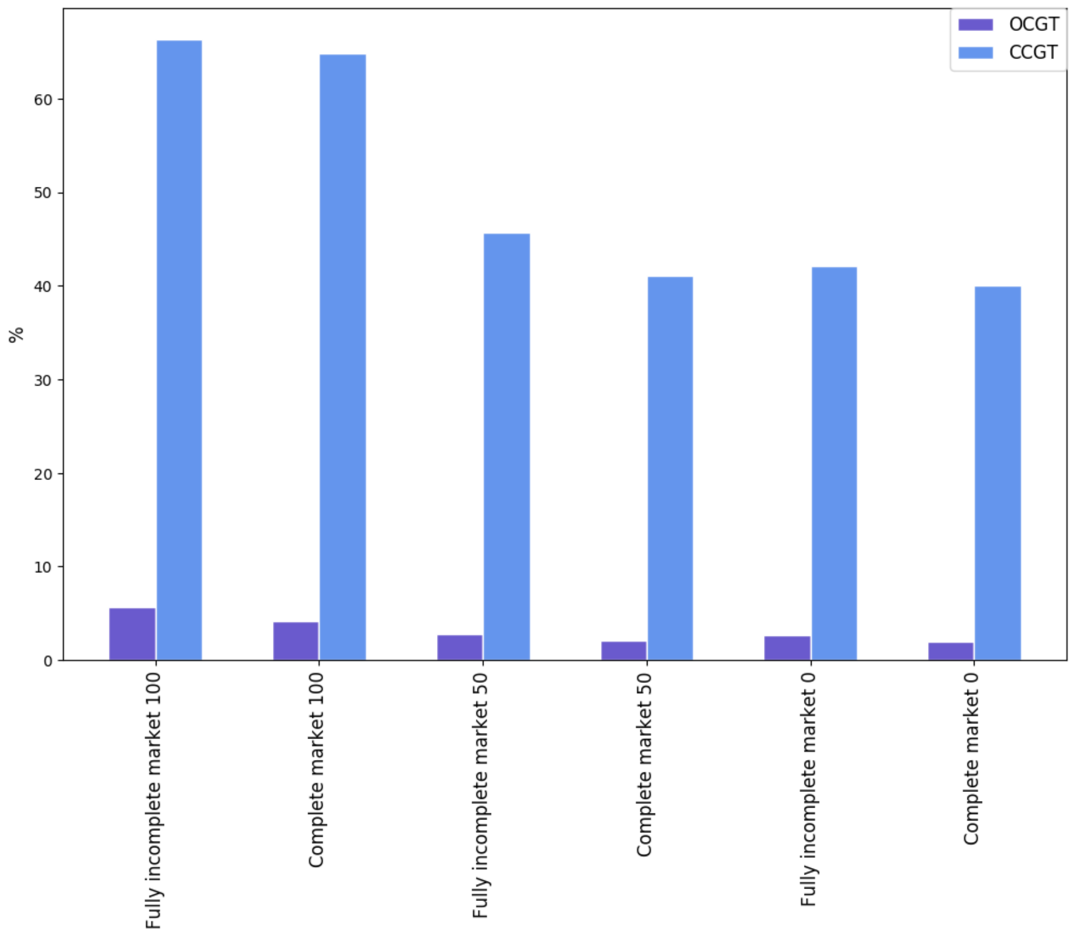

Figure 6.8: Average running hours of OCGT and CCGT

6.2.8 Maximum curtailment

Maximum curtailment is the greatest value of $z_\ell(\omega)$ in all scenarios ω and time segments ℓ. This is described in Figure 6.9.

The capacity market is the best solution to limit curtailment. Restraining curtailment does not work very well in the capacity market when no CO2 emissions are allowed. This is because in the 100 % and the 50 % case, OCGT capacity is built thanks to the capacity remuneration ν, while in the 0 % case, battery storage capacity is built (this will be detailed in further sections). Storage capacity has the drawback that when the battery is discharged it cannot produce any electricity. So, there is no guarantee of having no curtailment, while the OCGT and CCGT plants do not suffer from this disadvantage.

This graph clearly shows the benefit of contracts. Adding a contract in the market leads to less load curtailment. The complete market, representing the case where all the tools to trade risk exists, indicates overall lower value of curtailment than the other cases. Hence a conclusion is that trading risk is good to have lower curtailment values.

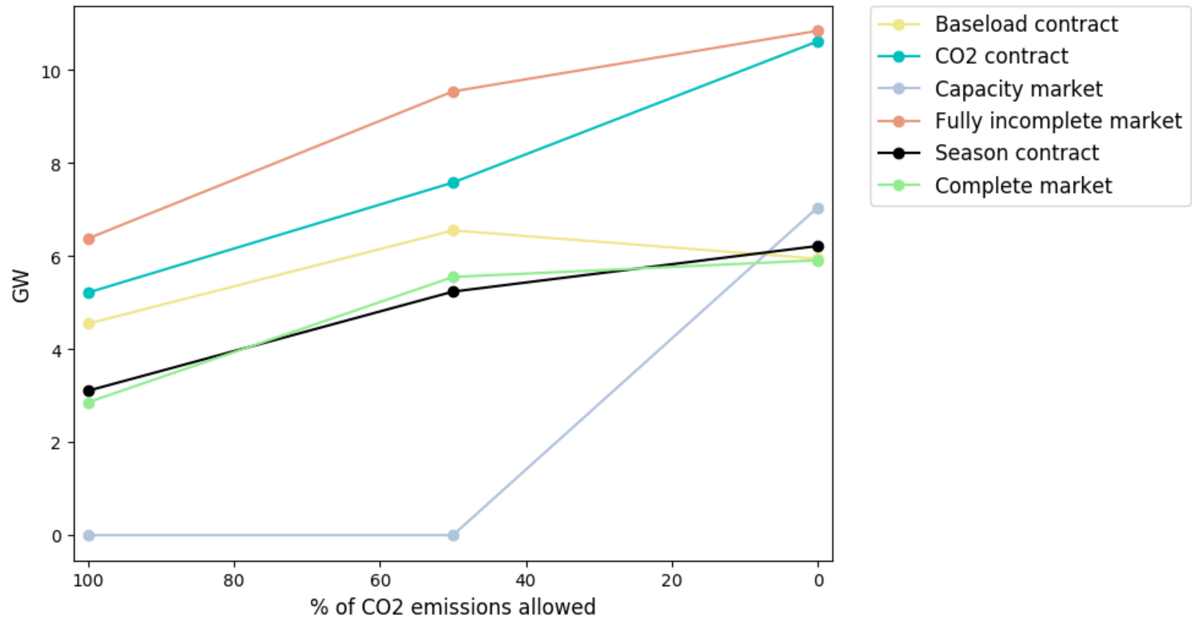

Figure 6.9: Maximum load curtailed

6.2.9 Hedging position

The normalized hedging position of the consumer is defined as

$$\mathbb{E}_P\left[\frac{W_{cons}}{\sum_{\ell \in L} D_\ell(\omega)}\right]$$

This measure is useful when the contract payoff has something to do with an electric price. In that case, it gives information about how many contracts the consumer bought to hedge his risk. If for instance the contract is the average electricity price, normalizing by the demand tells us which percentage of his consumption the consumer wanted to hedge. Figure 6.10 shows the normalized hedging position of the consumer. One can observe that the consumer tends to over-hedge his demand with the baseload contract. By this is meant that he buys more contracts than his average consumption. While with the season contract he nearly hedges all his demand.

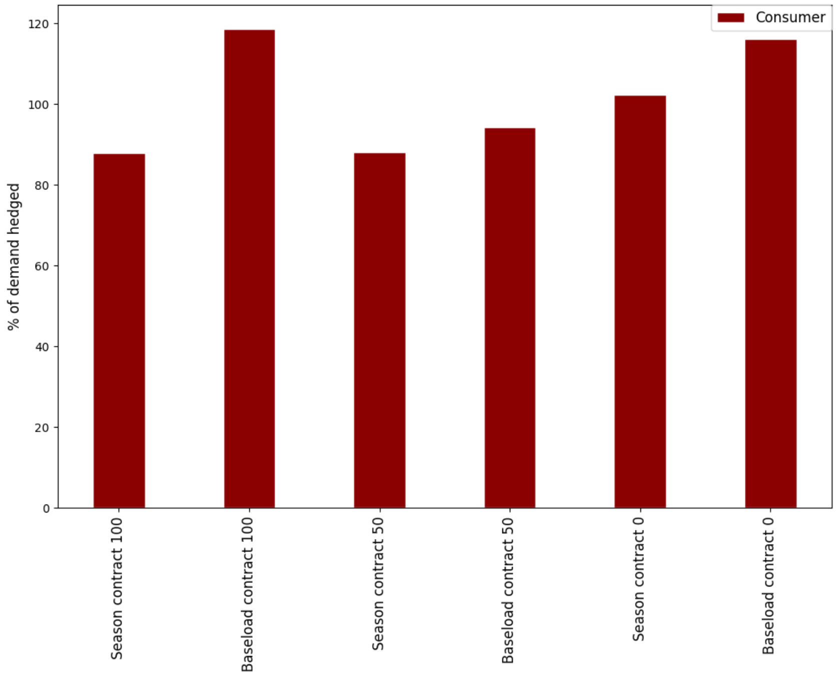

Figure 6.10: Normalized hedging position of the consumer

6.2.10 Sharpe ratios and Risk Adjusted Investment Cost

The Sharpe ratio of the different producers will be analysed together with the Risk Adjusted Investment Cost (RAIC). They will be interpreted in parallel because both of these measures tell something about the risk associated with the investment cost.

For the Sharpe ratio, two cases are distinguished: a Sharpe ratio incorporating the return of the contract (with hedging) and a Sharpe ratio not incorporating this contract return (without hedging).

In section 2.3 the Sharpe ratio is defined as the ratio of the expected return on the volatility of this return: $SR_i = \frac{\mathbb{E}_P[R^i(\omega)]}{\sigma_P[R^i(\omega)]}$.

The return rate of the different producers is defined in the following way:

Definition 11 (Return rate).

$$R^k_{no\_hedge}(\omega) = \frac{\Gamma_k(\omega)}{I_x x_k} \tag{6.1}$$

$$R^k_{hedge}(\omega) = \frac{\Gamma_k(\omega) + W_i(\pi^f_c - \pi_c(\omega))}{I_x x_k + W_i \pi^f_c} \tag{6.2}$$

The next graphs will show the standard deviation of the return rate on the x-axis and the mean of the return rate on the y-axis. The argument was made that all the Sharpe ratios have to be bounded due to the Hansen–Jagannathan bound. This is to be checked in the current section. The portfolio with hedging should have a lower risk profile ($\sigma[R^i(\omega)]$) than without hedging.

This is demonstrated in Figure 6.11. Different producers accept a negative return rate on their average investment. On average these producers are going to lose money by investing into their plant. But since they have an opposite risk compared to the consumer, they can hedge themselves. This results in a hedged Sharpe Ratio that is on the Hansen–Jagannathan bound. A drawback with this situation is that some agents external to the energy sector could buy this contract to speculate.

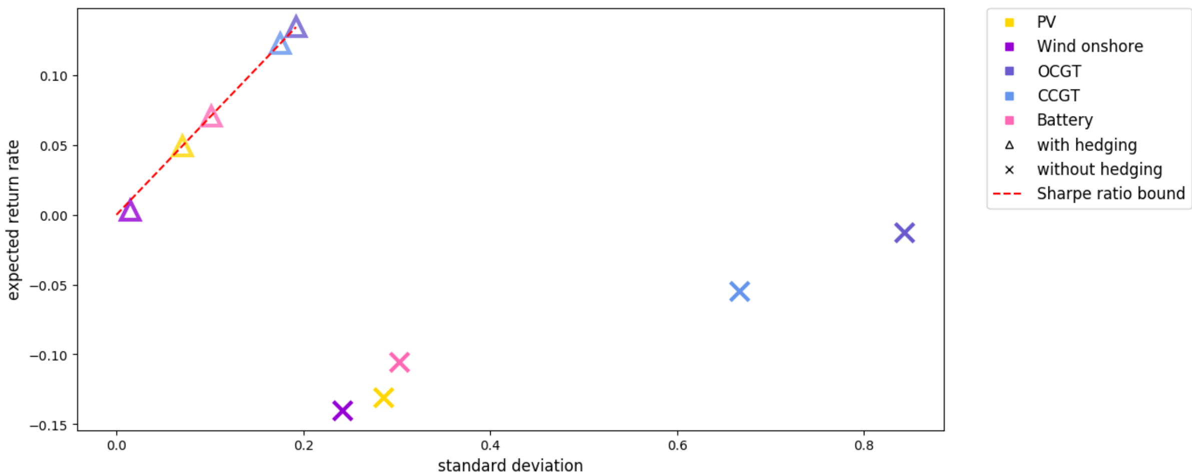

Figure 6.11: Sharpe ratio of the producers in a case of 50% CO2 emissions with the Baseload contract

Choice is made to only show the hedged Sharpe ratio on the future graphs, in order to not overload the graphs.

The Risk Adjusted Investment Cost (RAIC) is also analysed:

Definition 12 (Risk Adjusted Investment Cost (RAIC)).

$$RAIC = \frac{\mathbb{E}_{[P-Q_k]}[\mu_k(\omega)]}{I_k} = \frac{\mathbb{E}_P[\mu_k(\omega)] - \mathbb{E}_{Q_k}[\mu_k(\omega)]}{I_k} \tag{6.3}$$

The RAIC is interesting to analyse because it shows the deviation of the investment due to the risk aversion of the producer. As mentioned in Chapter 3, the modified investment cost of equation (3.7), is the same kind of measure but now with respect to the RN probability measure. A negative value for the RAIC would mean that the producer sees investment cheaper than it really is. Hence, it means he is going to make losses on average by investing in this technology. This number is divided by the investment cost I_k to be able to compare technologies with each other.

Figure 6.12 shows the Sharpe ratios after hedging of the different agents. The case of an incomplete market with no contract and the CO2 contract are compared. The expected return rate of OCGT and CCGT are important if no contract is on the market. They ask an average return of about 70% and 100%. Adding a contract gives the possibility to these agents to hedge themselves and ask a lower expected return rate. Hence this demonstrates the usefulness of a contract. It is also interesting to note that the standard deviation of the renewable technologies is smaller than the one of the conventional technologies. This means that the risk is actually more important for conventional technologies than for renewable technologies.

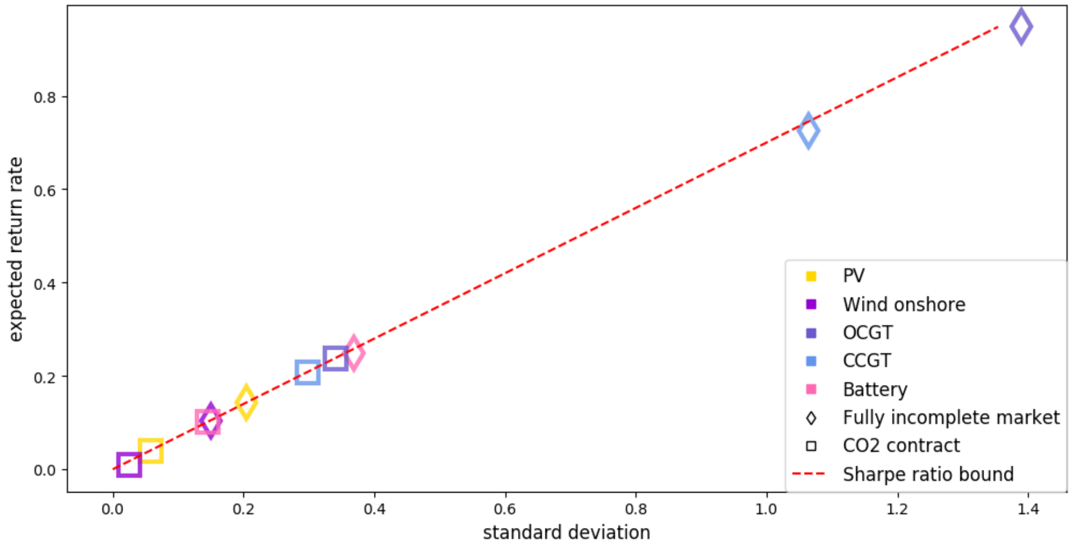

Figure 6.12: Sharpe ratio of the producers in a case with 50% CO_2 emissions

Since the risk position of different producers varies a lot, the decision was made to discuss the renewable producers and the conventional producers separately. The discussion focuses on the 50% case and starts with the renewables.

	pv	windon
Fully incomplete market	14%	11%
Capacity market	15%	11%
Baseload contract	−13%	−14%
CO2 contract	−12%	−12%
Complete market	−26%	−24%

Table 6.6: RAIC of renewables in a case with 50% CO_2 emissions

In Table 6.6, the change in RAIC of the renewable producers is presented. Both have about the same RAIC. The pv producer is the most risk averse for the capacity market and no contracts. This is not surprising since the capacity market is not a big help for renewables. The same conclusion for the incomplete and the capacity market can be drawn from Figure 6.13. Photovoltaic investment is encouraged in a market with a baseload contract and in a market with a CO2 contract. In this case RAIC being negative means the contract allows this producer to invest in a loss-making plant on average. But since his risk has an opposite position compared to the other agents his risk can be traded. One can see in Figure 6.13 that when the adequate tools for trading the risk are on the market, the expected return of the renewable producers are lower, this is also because they can take less risk.

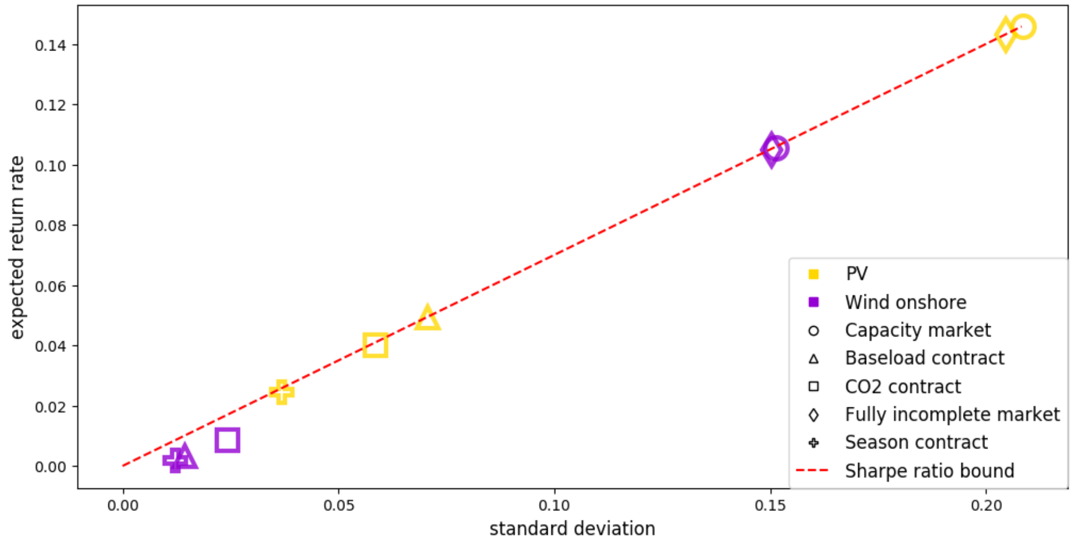

Figure 6.13: Sharpe ratio of the renewable producers in a case with 50% CO2 emissions

The Sharpe ratios of the conventional producers are indicated in Figure 6.14. The information that is given in the graph is that the CO2 contract is already a good tool to hedge the risk compared to the case with no contracts. For conventional producers, the best case is the capacity market. This is because they receive some money in the first stage for offering flexible capacity on the market. The baseload contract is a better tool to hedge risk of conventional producers than the CO2 contract.

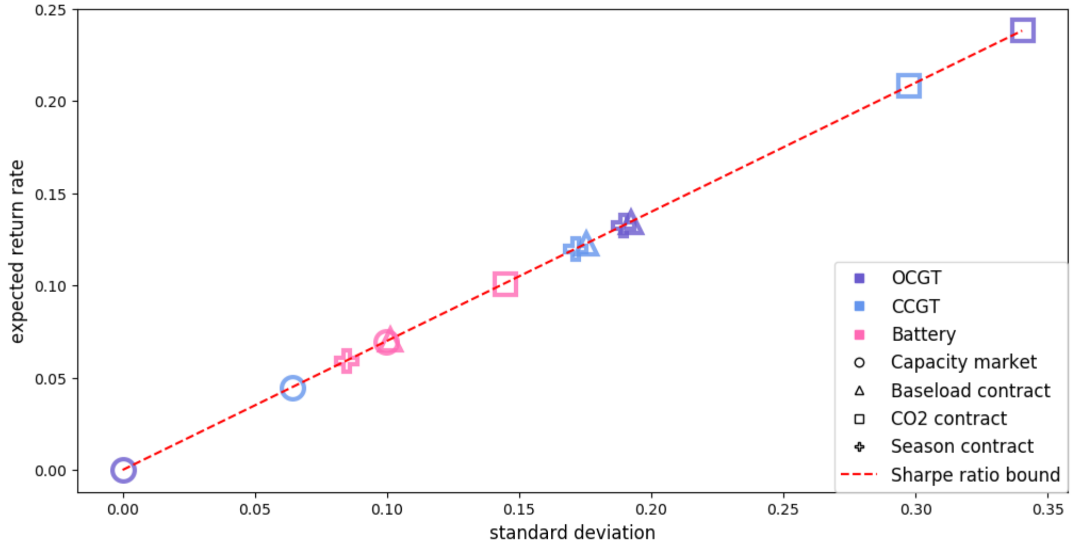

Figure 6.14: Sharpe ratio of the conventional producers in a case with 50% CO2 emissions

	storage capacity
Fully incomplete market	26%
Capacity market	7%
Baseload contract	−10%
CO2 contract	−6%
Complete market	−19%

Table 6.7: RAIC of storage capacity in a case with 50% CO2 emissions

The trend of the RAIC for the battery storage capacity is described in table 6.7. Observe this is about the same as pv's RAIC. This makes sense since the battery storage is linked to the pv production. Indeed pv provides the electricity used to charge the battery. So the risk profile of both agents are more or less the same. The main difference is in the capacity market where the battery is sure to get some money back because of the remuneration of available capacity.

	OCGT	CCGT
Fully incomplete market	95%	73%
Capacity market	0%	4%
Baseload contract	−1%	−5%
CO2 contract	12%	5%
Complete market	−14%	−18%

Table 6.8: RAIC of storage thermal producers in a case with 50% CO2 emissions

The RAICs of the thermal producers are described in Table 6.7. The capacity market is a particular case here. If one looks back at the RAIC formula in Definition 12, one can notice that there are two terms that can be influenced. On the one side there is this risk averse probability $q_k(\omega)$ and on the other side there is the value of capacity $\mu_k(\omega)$. If $q_k(\omega) = p(\omega)$ or if $\mu_k(\omega) = 0$ for all $\omega \in \Omega$ then the RAIC takes a zero value. What happens in the capacity market is that the price of capacity ν is set to the investment cost of OCGT. So, due to the conditions of the Nash equilibrium the plant cannot make any profit by selling electricity. Hence it has to have a zero capacity value $\mu_k(\omega) = 0$. So, the investor in OCGT does not take any risk by building capacity. The same conclusion can be drawn from Figure 6.14.

In the case of 0% CO2 emissions the return rates are on average lower. This can be seen in Figure 6.15. The return rates are now much lower than in the 50% case. This indicates that the uncertainty is also smaller.

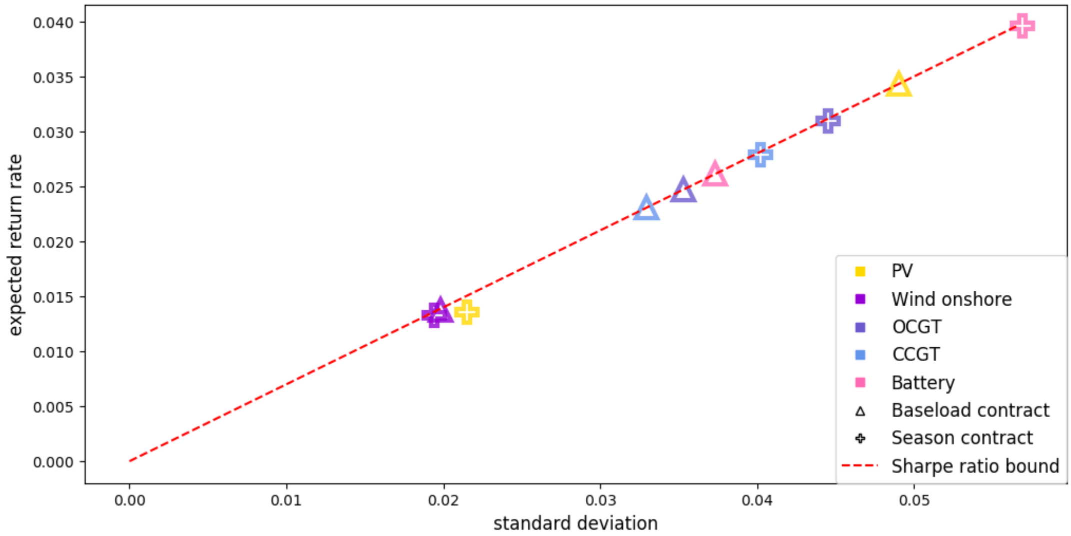

Figure 6.15: Sharpe ratio with 0% CO2 emissions

6.3 88 scenarios

In this section all the uncertainty scenarios described in section 6.1.2 are analysed. So the uncertainty is taken such that $\Omega = \Omega_1 \times \Omega_2 \times \Omega_3$. This results in $|\Omega| = 88$ scenarios.

The problem was solved on an Intel i7 Quad Core at 2.6Ghz and 8GB RAM using GAMS and the Gurobi 8 solver. The computation time was about 30 hours.

A constraint of 50% of CO2 emissions is imposed. In Figure 6.16 the distribution of the return rate of the battery is shown. The case with no contracts asks a higher return rate on average than the case with a risk neutral (RN) agent. The risk aversion of the battery agent can be seen.

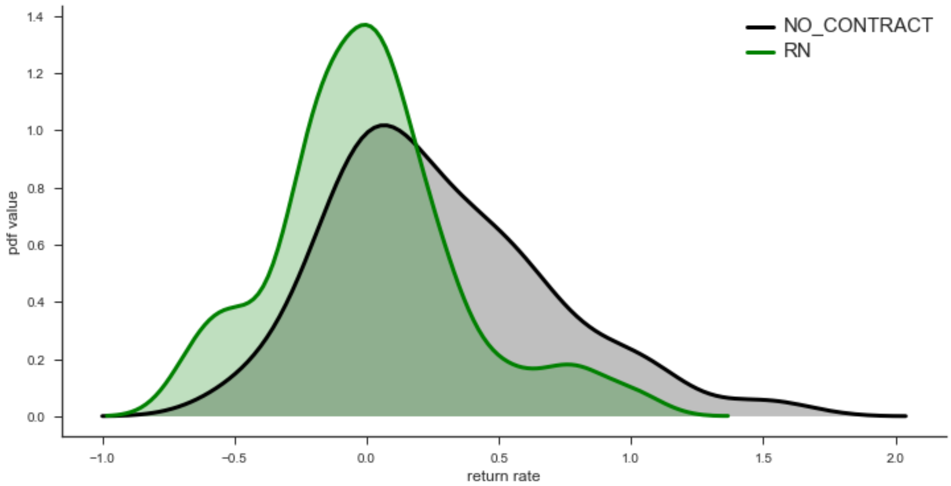

Figure 6.16: Probability density function of the return rate of the battery agent

CHAPTER

7 CONCLUSION

Risk is omnipresent in the energy sector. This is partly due to the arrival of renewables on the market. Consequently, the electricity generation depends on the meteorological conditions. In this context the different agents each have their own risk aversion. Finding the optimal decisions of the agents and their risk aversion is captured in the RCCEP.

Throughout this master thesis, an innovative approach decomposing the problem is proposed to solve the RCCEP. This approach decomposes the RCCEP in 2 parts: a physical part and a financial part. Each of these parts is solved individually and hands over the information to the next part. Even if no guarantee on convergence exists, the algorithm converges in our case.

In Chapter 6 the decomposition algorithm was used to solve the RCCEP with different settings. This gave the possibility to analyse the efficiency of different contracts. The RCCEP was thereafter solved at a larger scale with 88 yearly scenarios.

Questions from the introduction

◇ **Which technology is at the end of the day the most subject to risk?**

Decarbonising the energy sector has huge consequences on the risk perceived by the different technologies. Counter intuitively, it appears that in a world where renewables have an important market share, risk is more important for the conventional producers than for renewables (see Figures 6.14 and 6.13). This is because renewables produce electricity at a cheaper cost than conventional producers. Conventional producers can only produce electricity when demand peaks or when the renewable production is low (if both happen at the same time we end up in a Dunkelflaute case). Hence it is crucial to reduce the risk of these conventional plants for instance through a capacity market.

This risk is more important during the electricity transition than when the energy sector is decarbonised. Comparing the return rates of the producers in different cases of CO2 restrictions (Figures 6.15, 6.14 and 6.13) indicates that, when the CO2 price is uncertain, producers ask a higher payoff. Specific tools are needed to help manage risk in the energy transition.

- **Is there a particular problem with financing renewables?**

  Financing the first unit of renewable energy is not a problem. The problem lies in the cannibalisation effect when several units of renewables are installed as shown in Figure 6.7. The results that are computed in this work assume there is unlimited supply of biogas. This might not be the case. Therefore new simulations should be run with a limited amount of available biogas, but this is out of scope of this master thesis.

- **Which contract could solve this financial issue for risk averse investors?**

  I chapter 6, different contracts were analysed. The ones that yielded the best results were the baseload contract, the seasonal contract and the CO2 contract. These contracts drastically reduce the risk of the different technologies and consequently increase the welfare as illustrated in Figure 6.2.

Further improvements

The uncertainty could be modelled in a more refined way. In this master thesis, uncertainty is modelled through a set Ω of scenarios that can occur. Each scenario represents a one year period, because each year has different meteorological conditions. The agents make their second stage decisions in function of the scenario, in this case one single year. However a plant has a life span of 20 years or even more. Therefore the investment cost of the plant was annualized.

But how should we annualize uncertainty in such cases? Meteorological conditions cannot be limited to one year. Because if the meteorological conditions are unfavourable for an agent one year they will most likely be favourable the next year. But it is very unlikely they would be unfavourable during the whole plant's life span. So our model, by only taking a single year into account, leads to excessive risk averse agents. Hence the uncertainty should be modelled taking multiple years into account. Another idea to solve this issue could be to use the representative days approach described in [14].

On the more technical side, the algorithm was implemented using GAMS and the Gurobi solver. Its speed can be improved for solving larger instances. Most of the computation time is used to load the problem in the solver. An improvement would be to keep the problem in memory and only update the terms that change between iterations. Further, the Physical problem could be solved using algorithms dedicated to stochastic optimisation such as the SDDP algorithm.

On the side of the analysis, more cases with various types of contracts could be analysed. Different contracts can be combined to improve the risk trading.

Last word

Finally, even if some topics may be studied in more depth, this master thesis proposed an approach to solve the RCCEP with multiple yearly scenarios at an hourly resolution. The results of this approach seem to indicate that the thermal technologies are the most at risk.

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APPENDIX

A Coherent Risk Measure

This section will briefly introduce the concept of a coherent risk measure. These explanations are based on material from the book "Lectures on Stochastic Programming: Modeling and Theory" by Shapiro [3].

A risk measure $\rho(\cdot)$ is a function that maps uncertain outcomes random functions $Z(\omega)$ to the extended real line $\bar{\mathbb{R}} = \mathbb{R} \cup \{+\infty\} \cup \{-\infty\}$. We define $\mathcal{Z}$ the space where the random functions $Z(\omega)$ belongs to. For the sake of readability $Z(\omega)$ will be written as Z. The reader can consult [3] for formal details on the assumptions made on this space.

A coherent risk measure is a risk measure respecting 4 axioms: convexity, monotonicity, translation equivariance and positive homogeneity. If one considers Z as a cost the 4 axioms are then the follwing:

- convexity: $\rho(tZ + (1-t)Z^{'}) \leq t\rho(Z) + (1-t)\rho(Z^{'}) \quad \forall Z, Z^{'} \in \mathcal{Z}$ and $t \in [0,1]$
- monotonicity: If $Z, Z^{'} \in \mathcal{Z}$ and $Z \succeq Z^{'}$ then $\rho(Z) \geq \rho(Z^{'})$
- translation equivariance: if $a \in \mathbb{R}$ and $Z \in \mathcal{Z}$ then $\rho(Z + a) = \rho(Z) + a$
- positive homogeneity: if $t > 0$ and $Z \in \mathcal{Z}$ then $\rho(tZ) = t\rho(Z)$

However if $\mathbf{Z}$ is considered as a reward (a surplus or profit) and the risk measure $\varrho(\mathbf{Z})$ is defined as $\varrho(\mathbf{Z}) = \rho(-Z)$ then the axioms of monotonicity and translation equivariance change to:

- monotonicity: If $\mathbf{Z}, \mathbf{Z}^{'} \in \mathcal{Z}$ and $\mathbf{Z} \succeq \mathbf{Z}^{'}$ then $\varrho(\mathbf{Z}) \leq \varrho(\mathbf{Z}')$
- translation equivariance: if $a \in \mathbb{R}$ and $\mathbf{Z} \in \mathcal{Z}$ then $\varrho(\mathbf{Z} + a) = \varrho(\mathbf{Z}) - a$

If Z is considered as a cost, the convexity axiom states that the risk on the average of two cost functions $tZ + (1-t)Z^{'}$ is smaller than the average risk of two cost functions. So it is better in terms of risk to have a diversified portfolio.

The monotonicity axiom states that if the cost in the first cost function is greater or equal than the second cost function then the risk associated to the first cost is also greater or equal than the risk associated to the second cost.

Translation equivariance states that if one adds a fixed cost a, that does not depend on uncertainty, to the cost function Z then this is the same as taking the risk of the cost Z and adding this fixed cost a. Terms that do not depend on uncertainty can thus be removed from the risk measure computations.

Positive homogeneity states that the risk measure is invariant to scaling of the cost function Z.

B Specific KKT conditions and dual of each agent

B.1 KKT and dual of battery agent

The KKT conditions of the battery agent are given by:

$$0 \leq Storage\_cap - e_{\ell}(\omega) \perp \mu_{k,\ell}(\omega) \geq 0 \tag{1}$$

$$0 \leq \sum_{k \in K} y_{k,\ell}(\omega) + z_{\ell}(\omega) - c_{\ell}(\omega) - D_{\ell} \perp \pi_{\ell}(\omega) \geq 0 \tag{2}$$

$$e_{\ell}(\omega) = Dissipation\_factor \cdot e_{\ell-1}(\omega) - \frac{y_{\ell}(\omega)}{\sqrt{\eta}} + \sqrt{\eta} \cdot c_{\ell}(\omega) \qquad \gamma_{\ell}(\omega) \text{ free} \tag{3}$$

$$0 \leq Charge\_cap - y_{\ell}(\omega) \perp \sigma_{\ell}(\omega) \geq 0 \tag{4}$$

$$0 \leq Charge\_cap - c_{\ell}(\omega) \perp \kappa_{\ell}(\omega) \geq 0 \tag{5}$$

$$0 \leq \sigma_{\ell}(\omega) + C_k(\omega) - \pi_{\ell}(\omega) + \frac{\gamma_{\ell}(\omega)}{\sqrt{\eta}} \perp y_{\ell}(\omega) \geq 0 \tag{6}$$

$$0 \leq \kappa_{\ell}(\omega) + C_k(\omega) + \pi_{\ell}(\omega) - \sqrt{\eta} \cdot \gamma_{\ell}(\omega) \perp c_{\ell}(\omega) \geq 0 \tag{7}$$

$$0 \leq \mu_{k,\ell}(\omega) + \gamma_{\ell}(\omega) - Dissipation\_factor \cdot \gamma_{\ell+1}(\omega) \perp e_{\ell}(\omega) \geq 0 \tag{8}$$

$$0 \leq I_{Storage\_cap} - \sum_{\ell \in L} \mathbb{E}_{\bar{Q}}[\mu_{k,\ell}(\omega)] \perp Storage\_cap \geq 0 \tag{9}$$

$$0 \leq I_{charge\_cap} - \sum_{\ell \in L} \mathbb{E}_{\bar{Q}}[\sigma_{\ell}(\omega) + \kappa_{\ell}(\omega)] \perp Charge\_cap \geq 0 \tag{10}$$

The dual would look like this:

$$\min -\mathbb{E}_{\bar{Q}}[\sum_{\ell} D_{\ell}\pi_{\ell}(\omega)] \tag{11}$$

$$\text{s.t. } 0 \le \mu_{k,\ell}(\omega) \tag{12}$$

$$\pi_{\ell}(\omega), \sigma_{\ell}(\omega) \ge 0 \tag{13}$$

$$\gamma_{\ell}(\omega) \text{ free} \tag{14}$$

$$0 \le \sigma_{\ell}(\omega) + C_k(\omega) - \pi_{\ell}(\omega) + \frac{\gamma_{\ell}(\omega)}{\sqrt{\eta}} \qquad y_{\ell}(\omega)\bar{q}(\omega) \tag{15}$$

$$0 \le \kappa_{\ell}(\omega) + C_k(\omega) + \pi_{\ell}(\omega) - \sqrt{\eta}\cdot\gamma_{\ell}(\omega) \qquad c_{\ell}(\omega)\bar{q}(\omega) \tag{16}$$

$$0 \le \mu_{k,\ell}(\omega) + \gamma_{\ell}(\omega) - Dissipation\_factor \cdot \gamma_{\ell+1}(\omega) \qquad e_{\ell}(\omega)\bar{q}(\omega) \tag{17}$$

$$0 \le I_{Storage\_cap} - \sum_{\ell\in L} \mathbb{E}_{\bar{Q}}[\mu_{k,\ell}(\omega)] \qquad Storage\_cap \tag{18}$$

$$0 \le I_{charge\_cap} - \sum_{\ell\in L} \mathbb{E}_{\bar{Q}}[\sigma_{\ell}(\omega) + \kappa_{\ell}(\omega)] \qquad Charge\_cap \tag{19}$$

B.2 KKT conditions of renewable agent

KKT conditions become:

$$0 \le fact_{k,\ell}(\omega)\mu_{k,\ell}(\omega) - \pi_{\ell}(\omega) \perp y_{k,\ell}(\omega) \ge 0 \tag{20}$$

$$0 \le fact_{k,\ell}(\omega)x_k - y_{k,\ell}(\omega) \perp \mu_{k,\ell}(\omega) \ge 0 \tag{21}$$

$$0 \le I_k - \sum_{\ell\in L,\omega\in\Omega} \mathbb{E}_{Q_k}\left[fact_{k,\ell}(\omega)\mu_{k,\ell}(\omega)\right] \perp x_k \ge 0 \tag{22}$$

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