

Topology optimization of ironless synchrocyclotrons

Dissertation presented by
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for obtaining the Master's degree in
Electromechanical engineering

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Academic year 2017-2018

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1.1 Abstract

Cyclotrons are particle accelerators that have a wide range of applications, in physics or nuclear medicine. Their role in hadron therapy is especially very promising for the next decades. This type of radiation therapy consists in treating tumors by irradiation through accelerated ions (up to now mainly protons). It offers many advantages over classical radiation therapy such as higher efficiency and more accurate covering of the tumor. The hadrons are generally accelerated using cyclotrons or synchrotrons¹.

A serious drawback encountered by cyclotron technology is the difficulty to vary the beam energy. Minervini *et al.* from the Plasma Science and Fusion Center (PSFC) of the Massachusetts Institute of Technology (MIT) developed several models of a new type of cyclotrons, the *ironless synchrocyclotron*, that could overcome this issue. This kind of cyclotron, as its name suggests, is designed without the heavy iron yoke traditionally used. This feature gives important advantages, the most important one being the possibility of having an output beam energy that is more easily variable. Indeed, without the presence of saturated iron, the magnetic field varies linearly with the current in the wire. By changing the current level, it will be possible to choose the beam energy. Minervini *et al.* developed several models in [4], [3] and [5] but never optimized them in term of quantity of conductor.

In this master thesis we develop a topology optimization tool for the design and the optimization of ironless synchrocyclotrons. Topology optimization is a powerful method that consists in finding the optimal material distribution in a given domain, subject to some constraints without any presupposed solution. Basically mostly used for mechanical applications, topology optimization applications on electromagnetic devices received more attention during the last years [30]. The topology optimization tool is used to reduce the conductor quantity of the model designed by the MIT. An ironless equivalent to the superconducting synchrocyclotron (S2C2) of Ion Beam Applications (IBA) is also developed. In addition, the layout of a potential conductor for this ironless S2C2 is proposed.

We begin this introducing chapter with a summary of the main uses of particle accelerators in the industry. Afterwards the basics of the accelerator technology are explained. Finally, we expose the main motives of the concept of ironless synchrocyclotrons and present the difficulties that this technology could raise.

¹Explanations about the different types of accelerators are given in Section 1.3.2

1.2 Cyclotrons in the industry

Accelerated particles are used in many different fields. Large accelerators are mostly employed for scientific research, for instance in particle physics. Smaller accelerators (for example cyclotrons) are used in medical applications such as hadron therapy and radioisotope production.

In this first section, we expose the main uses of the cyclotrons that will be treated in this master thesis.

1.2.1 Hadron therapy

Different methods exist for cancer treatments, such as surgery, radiation therapy and chemotherapy. They are used alone or together, depending on the type of tumor and its location.

Conventional radiation therapy exists since the beginning of the XX^e century and the beginning of X-rays development. It consists in exposing the tumor to ionizing radiation which kills the malignant cells. The radiation damages the DNA of the concerned cells, leading to their death. The cancerous cells having a reduced ability to repair damaged DNA, they are more vulnerable to the treatment than the surrounding healthy cells.

Hadron therapy² is a type of radiation therapy. It allows the treatment of certain types of cancer, for instance with tumors that are localized in places where surgery is impossible and where classical radiation therapy (with x-rays) would be too invasive, for example if the tumor is situated close or in critical organs such as the brain. Certain categories of cancers are also best treated with hadron therapy such as pediatric cancers or ophthalmic tumors.

The evolution of the absorbed dose by the tissue with the depth of penetration is different in proton therapy than in other types of radiation therapy such as X-rays or electrons (as shown on Fig 1.1). For the X-rays and electrons, only tissues next to the skin absorb a significant dose of radiation, but with protons (or other hadrons) the absorbed dose increases with depth, until the so-called *Bragg peak*. Beyond this peak, it falls suddenly to zero.

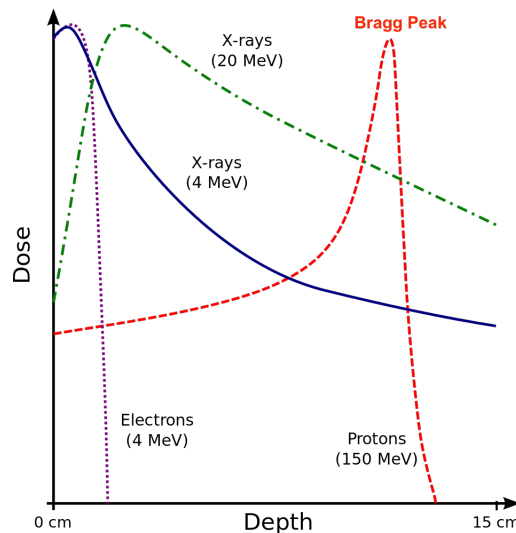


Figure 1.1: Dose in function of the depth for different radiation therapy methods

²"Hadron" is a generic term defined by Wikipedia as: "A composite particle made of quarks held together by the strong force", in practice it covers particles such as neutrons, protons but also ions as carbon

The hadron curve has the advantage to limit the radiation on healthy surrounding cells and allows to reach deeper parts of a tumor. Thus, it reduces up to 50% the risks of radiation induced cancer^[32]. By varying the energy of the beam, it is possible to reach different depths and thus to provide an irradiation that follows very precisely the shape of the tumor.

Different particles can be used for hadron therapy. Up to now, due to lower costs of the technology, protons are by far the most used particles. But beside proton therapy, a growing interest is put on heavier ions and more precisely on carbon. The *relative biological effectiveness*^[3] of proton therapy is between 1.1 and 2 times larger than for classical radiation therapy. But with carbon therapy it can reach significantly higher RBE, between 2 and 5 larger^[17] than for classical radiation therapy. Like proton therapy, carbon therapy is aimed at the precise treatments of specific types of cancers, the ones that can not be removed with surgery or those containing radiation resistant cells. The use of carbon also limits the scattering of the beam, allowing a more precise application of the irradiation.

The goal of this master thesis is to design an ironless cyclotrons for proton therapy but as explained here under, this type of architecture would also open the field to multi-particles cyclotrons.

Hadron therapy facility

A hadron therapy facility is composed of four principal systems:

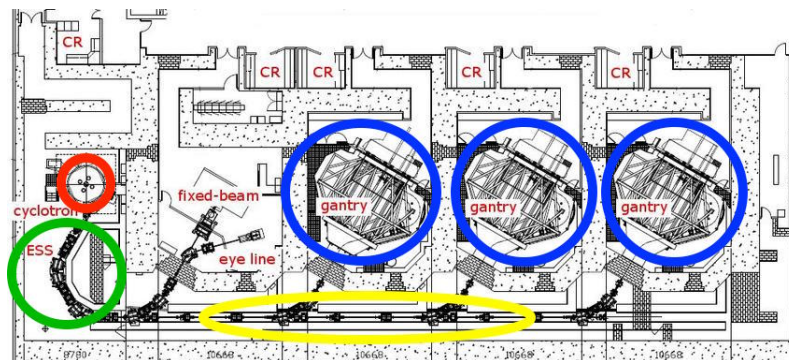


Figure 1.2: Proton therapy structure (here from IBA) ^[24]

- Acceleration of the hadron (red)
- Energy selection system (green)
- Beam transport (yellow)
- Gantry and irradiation (blue)

The **accelerator** is an isochronous cyclotron, a synchrocyclotron or a synchrotron^[4]. In the case of an IBA proton therapy facility, it is an isochronous cyclotron or a synchrocyclotron. These types of accelerators are up to now used as fixed energy machines.

The second step is an **energy selection system**.

The depth of irradiation of a proton beam depends on the energy of that beam. For the covering of a tumor in three dimensions, a varying beam energy is thus needed but as stated just here above, the cyclotrons have a fixed beam energy. To remediate this issue, an energy selection

³The relative biological effectiveness (RBE) is the ratio that measures the efficiency of an ionizing radiation technique.

⁴The differences between these types of accelerators are explained in Section ^[1.3.2]

system is placed at the output of the accelerator. This system is based on *degraders*, that are obstacles slowing down the beam and hence degrading its energy. These degraders have the advantage to be simple, and they completely uncouple the accelerator and the output energy level. But they have also other drawbacks. The first one is obviously the loss in beam intensity. In the worst cases (for lowest energy level), only 0.2 % of the beam current is used [1]. The second one is that the impact between the beam and the degrader or collimator provokes an important neutron production. The system then produces more radioactivity and needs an improved shielding system. Ultimately, the degraders and collimators induce a beam scattering, which will make the tumor painting less precise and could damage surrounding cells. The opportunity of avoiding the use of degraders is one of the main motive of an ironless design, as explained in Section 1.4

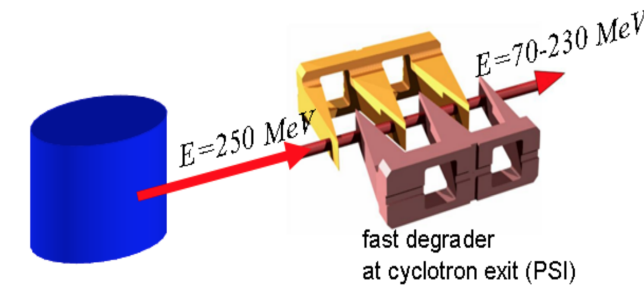


Figure 1.3: Energy degrader [24]

The third step of the proton therapy is the **beam transportation** until the irradiation room and the last stage is the treatment of the patient in the **gantry**. These two steps are composed of bending magnets that allow to freely choose the angle for the tumor irradiation.

1.2.2 Production of medical isotopes

Another main use of cyclotrons is the radioisotope production for medical imaging. The main techniques are *SPECT* (Single Photon Emission Computed Tomography) and *PET* (Positron Emission Tomography). This latter consists in the injection of a *tracer* that allows the 3D imaging of a metabolism or an activity of an organ. The tracer is composed of radioactive atoms that emit positrons which produce photons by annihilation. The tracer is often ^{18}F , incorporated in a glucose molecule. The tumors consume a lot of glucose and then will be highlighted by a high concentration of tracers and a high emission of photons.

The production of ^{18}F is usually done by the bombardment of enriched ^{18}O water with protons accelerated by cyclotrons.

The radioactivity period of ^{18}F lasts less than 2 hours. It thus develops a need of cyclotrons located very close to the hospitals or the activity.

Cyclotrons for radioisotope production are generally smaller than those for proton therapy (between 18 and 70 MeV for the formers in comparison with $\approx 230 \text{ MeV}$ for the latters) and are in practice much more numerous. Since the magnetic field levels are lower, there is no fundamental need of superconducting magnets as used in this report. In this thesis, we will only concentrate our work on cyclotrons for proton therapy.

1.3 Basics of accelerator technology

After this introduction about the practical uses of cyclotrons, it could be useful to introduce the basic physics underlying accelerator technology. A first idea of the different existing types of

accelerators will be helpful for the understanding of the rest of this master thesis.

There are two main architectures of electromagnetic accelerators⁵: the **linear accelerator (linac)** and the **cyclotron**.

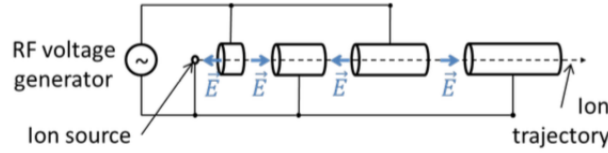


Figure 1.4: Scheme of a linear particles accelerator [28]

In **linear particles accelerators (linac)**, series of RF-cavities producing an oscillating electric field accelerate linearly an ion beam. These devices are mostly used in X-ray generation, as particle injectors for higher-energy accelerators and in particle physics for the lightest particles (electrons and positrons). They have the advantage to produce a fairly constant and high intensity output but their huge size is a main drawback.

In order to reduce the size of the accelerators and reach higher levels of energy, physicists imagined a circular accelerator, the **cyclotron**. Its basic principle is that a particle moving in a constant magnetic field will follow a circular trajectory with a frequency which is independent of the particle level of energy. Only the radius will increase as the particle gains in energy. The expression of the constant orbit frequency is (derivation in Section 1.3.1):

$$f = \frac{qB}{2\pi m} \quad (1.1)$$

Where q is the charge of the accelerated particle, m its mass and B the magnetic field in the midplane.

1.3.1 Classical cyclotron

The first cyclotron was invented by Lawrence in 1930 and is called the classical cyclotron. A scheme of this cyclotron is shown on Fig 1.5. It consists of two principal magnets (composed of coils surrounding an iron yoke) that produce a mainly vertical magnetic field and two "dees"⁶ producing a mainly horizontal electric field. The magnetic field is responsible for the beam focusing and the orbital movement of the ions. The radio-frequency (RF) oscillating electric field is in turn responsible for the ion acceleration. Each time the particle crosses the gap between the dees, it is accelerated. In this master thesis we will only focus on the magnetic field production. An introduction to the working principle of the dees is given in Appendix B.1. More details are available in [12].

The basic principle of a cyclotron is the equilibrium between centripetal Lorentz forces and the centrifugal effect, the former pushing the electron inward and the latter pushing the electron outward. This balance between the amplitude of the two forces is written as:

⁵Other types of accelerators such as electrostatic accelerators also exist. Their kinetic energy is limited by the maximal voltage. Electrostatic accelerators can then only reach low energy levels and are thus used in small scale application. On the contrary, electromagnetic accelerators allow the particle to pass through the same accelerating field multiple time without the need of high voltage. They produce significantly higher levels of energy and represent the large majority of the present accelerators.

⁶In reference to their D-shape.

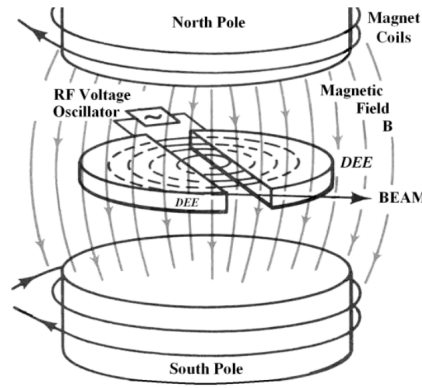


Figure 1.5: Schematic of a classical cyclotron [2]

$$\underbrace{m \frac{v^2}{r}}_{\text{Centrifugal force}} = \underbrace{qvB_z}_{\text{Lorentz force}} \Rightarrow \omega = \frac{qB}{m} \quad (1.2)$$

Which gives the result of Eq 1.1

Beam focusing

A main issue in the cyclotron design is the beam focusing. The ions have to be focused in the three dimensions in order to avoid their loss on the dees or the magnets. To this end, Lawrence added iron shims to its cyclotron, which produced a slightly bumped field, decreasing in amplitude with the radius. This resulting magnetic field then has a small B_r component outside of the midplane, generating a restoring force F_{res} that focuses the ion in its orbit. The focusing of the beam through iron-shaping is still used nowadays. In our ironless design, additional coils (shaping coils) will play this role.

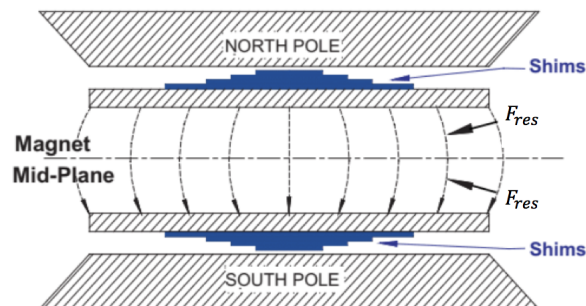


Figure 1.6: Cross-sectional view of a classical cyclotron equipped with shims for field shaping and beam focusing [12]

The vertical component of this restoring force is [2]:

$$F_z = qv_\theta B_r = -m\omega^2 n z \quad (1.3)$$

Where n is called the *field index* and is defined by:

$$n = -\frac{r}{B} \frac{dB}{dr} \quad (1.4)$$

This force produces an oscillation of the particles around their orbit, with a frequency:

$$\nu_z = \omega\sqrt{n} \quad (1.5)$$

The same phenomenon occurs in the horizontal direction, with a frequency of:

$$\nu_r = \omega\sqrt{1-n} \quad (1.6)$$

If $0 < n < 1$ the beam undergoes a stable motion in both directions.

ν_r and ν_z are called *betatron tunes* and are primordial values in the design of a cyclotron. They will be discussed in detail in Section 5.4.3. The complete derivation of the results here above is given in [12].

Isochronism

Another important issue in cyclotron design is called *isochronism*. The beam has to keep a constant orbit frequency in order to stay synchronized with the accelerating electrical field of the RF. The orbit frequency is defined by Eq 1.1, we see that the magnetic field B has to stay constant to keep the orbit frequency constant. But as explained in the previous section, the magnetic field is designed to decrease with the radius for the focusing and will then produce a decreasing revolution frequency. **Isochronism is in fact competing with beam focusing.**

Moreover, the mass of the particle varies with the radius. Indeed, as the speed of the particles increases, relativistic effects are no longer negligible and the mass of the particle evolves as [2]:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 \cdot \gamma \quad (1.7)$$

Where v is the speed of the particle, c is the speed of light and γ is called the Lorentz factor. When its energy raises, the mass of the particles increases following Eq. 1.7 and its orbit frequency falls.

1.3.2 Type of cyclotrons

In order to cope with these effects and reach high energy levels, two kinds of cyclotrons have been developed: the **synchrocyclotron** and the **isochronous cyclotron**.

- **Synchrocyclotron**

One idea to overcome the drawbacks of the classical cyclotron is to reduce the frequency of the accelerating field of the RF for increasing radius of revolution of the hadron. The particle orbit frequency can then stay synchronized with the RF and the beam can be accelerated at relativistic levels of energy. The main disadvantage of this type of cyclotron is that the output is pulsed (only one train of bunches of particles can be fully accelerated before the following one is injected). The average output intensity is then largely lower than that of other types of cyclotrons.

This technology is used for the S2C2 synchrocyclotron of IBA and will be used for the development of the ironless synchrocyclotrons treated in this master thesis.

- **Isochronous cyclotron**

The isochronous cyclotron (also called AVF⁷ cyclotron) overcomes the main drawbacks of the synchrocyclotron, its low average output intensity. The idea of the isochronous

⁷Azimuthal Varying Field

cyclotron is to maintain a constant revolution frequency by increasing the magnetic field with the radius in order to counteract the effects of the mass increase and keep the orbit frequency constant (Eq [1.1](#)).

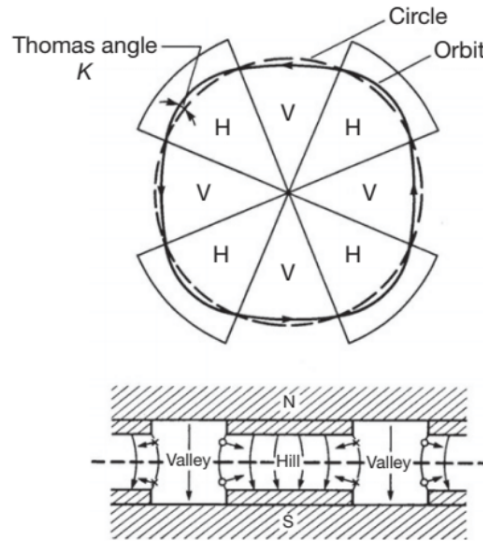


Figure 1.7: Schematic of a radial-sector-type isochronous cyclotron [\[19\]](#)

As increasing magnetic field with the radius defocuses the beam, the isochronous cyclotron has another magnetic structure. It is based on hills and valleys, as visible on Fig. [1.7](#). The particle does not undergo a perfectly circular orbit anymore but the hills and valleys create a vertical magnetic force at each entrance or each exit from a hill sector. This kind of cyclotron necessitates difficult and very precise designs but allows the main RF to stay constant. It is then possible to accelerate ions at a constant rate and thus to produce a continuous wave (CW) beam output.

- **Synchrotron**

A third type of circular accelerator is used in particle therapy, the **synchrotron**.

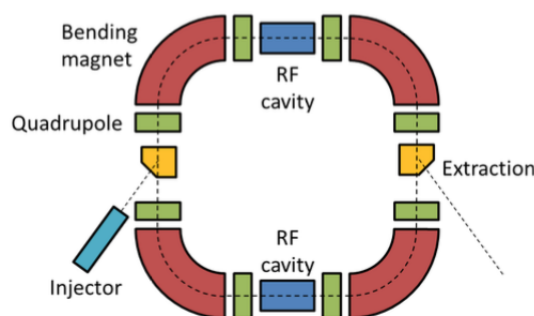


Figure 1.8: Schematic of synchrotron [\[19\]](#)

In a synchrotron, a beam is accelerated in a fixed orbit trajectory. The device is made of RF accelerating cavities, bending and focusing magnets. Since the speed of the particles increases, the revolution frequency increases and following Eq. [1.1](#) the magnetic field has to increase. Hence both magnetic field and RF will be pulsed. The results is that, as with the synchrocyclotron, the synchrotron will not produce a CW output. The output intensity

is thus quite low.

Other drawbacks are that the synchrotrons are slow because the magnets can not be pulsed fast and that it is difficult to extract the beam. This extraction is then done on several turns of the particle and it is difficult to control precisely the extracted beam intensity.

Besides, a main advantage of the synchrotron is the varying beam energy level. The particle can be extracted at any turn and the energy level will depend on the number of turns done.

The lack of this possibility to change the energy level constitutes one of the main drawback of the synchrocyclotron and isochronous cyclotron. In the cyclotrons, the iron yoke fixes the magnetic field level and it is then not possible to change the energy level. By removing the iron yoke, it is possible to overcome this issue and reach different energy levels.

1.4 Motive for ironless synchrocyclotron

The goal of this master thesis is the design of ironless synchrocyclotrons. As their name suggests, these cyclotrons are based on the synchrocyclotron principle but contain no iron. The yoke is replaced by extra coils, the shielding and the shaping coils, acting for the stray field reduction and the shaping of the magnetic field in the midplane respectively.

Even if some models have been designed, no prototype of ironless synchrocyclotrons has been built up to now. This type of cyclotron would have a lot of advantages but could also raise important issues. The main pros and cons of this technology have been summarized by the MIT in [31]. They are exposed and explained hereunder.

Advantages

- **Scalable energy** (by adjusting coil current): As already explained, the development of cyclotrons able to vary the beam energy is nowadays one of the major challenge in the hadron therapy industry. These energy variations are an essential aspect of the hadron therapy as they are required to irradiate different depths in the body and really cover all the layers of the tumor.

Nowadays, the practical way to obtain these different energy levels is through the use of *degraders*. But these degraders are the source of secondary radiation (intense neutron production) that considerably increases the radiation shielding needed. And since the beam loss is large in degraders, a larger production is needed from the ion source, which will increase further the radiation intensit.

Some cyclotrons able to produce different levels of energy already exist. They require a fine tuning of all the parameters (current in the coils, radio frequency,...) that take a long time (from some minutes up to some hours). They are thus only used for scientific and research purposes but they are not able to change the beam energy fast enough for proton therapy [8].

On the contrary ironless cyclotrons would allow to vary easily and quickly the output energy level. Since only windings are used for the magnetic field generation, the relation between the current in the coils and the magnetic field is linear. The output beam kinetic momentum is proportional to the magnitude of this magnetic field. Variations in energy level are then readily obtained, only by adjusting coil current.

⁸The variation of energy has to be fast enough so that a treatment session does not last too long. We have to keep in mind that a complete treatment requires about 20 sessions and that owners of the cyclotron need enough patients to be cost efficient.

The output beam energy would vary between the minimum permitted by the subsystems (RF system, injection and extraction systems,...) and the maximum allowed by the coil design. The RF system has also to be redesigned to vary with the magnetic field level.

An interesting property of iron-free designs is that, with appropriate frequency and current scaling, the particle trajectory is independent of the output beam energy. Extraction of the beam can then be done at the same radius for different levels of energy.

In comparison, because of their complex structure, iron-free isochronous cyclotron would be much more complex to design [4]. This is the reason why this master thesis only covers the synchrocyclotron option.

- **Reduced weight:** The removing of the heavy iron yoke could reduce the total weight of an order of magnitude [4]. The weight of the extra coils (more their additional support structure) and of the cryostat represents only a small fraction of the total weight of the iron that they replace (See in the MIT design [3]).

The reduction of the weight of the cyclotron would simplify the development of new applications that require transportability. It also substantially reduces the weight and dimensions of the gantry for gantry-mounted cyclotrons.

- **Larger midplane and axial bore clear spaces:** The iron yoke, due to its important volume, limits the clear spaces and thus the access to the center of the cyclotron. Without iron, the access would be easier. It develops the possibility of using interchangeable devices (injection/RF/extraction) for different ion types (protons, carbon,...).
- **Variables species in a single device:** The opportunity to **vary the output energy** and the presence **larger clear spaces** that allow change of ion source, RF and extraction system would permit the use of different ions in a single device. For example carbon and proton could be accelerated in the same synchrocyclotron⁹ what could really develop the use of hadron therapy in general.

Disadvantages

- **Higher AC losses in conductor:** Since the magnetic field will vary during a treatment, the AC losses in the conductor will be higher. More expensive technologies (other superconducting materials, other cooling methods) could be needed to evacuate these losses. This subject will be discussed more deeply in Chapter 6.
- **Less radiation shielding:** One of the other main role of iron is the radiation shielding. Neutrons are produced by the impact between accelerated particles and air, RF or magnet. This radiation has to be contained and without iron, other neutron shielding methods have to be developed.
- **Large radius shielding coils:** Shielding coils are used for the stray field limitation. These coils have a larger radius than the previously used ones (only main coils). The size of the cryostat increases and conduction cooling by cryocoolers could then be more difficult. Others (and more expensive) cooling techniques have to be designed. This issue will be discussed in Chapter 6.
- **Difficult extraction:** Without iron, it is difficult to produce the large field gradients needed for the extraction. None extraction technique has been developed for the ironless case up to now (details in Section 7.1.1).
- **Higher price:** Due to higher ac losses, the types of conductor and superconducting materials required could be more expensive than for previous superconducting synchrocyclotrons.

⁹But obviously not at the same time.

The quantity of superconductor is also higher, due to the (expensive) shielding and shaping coils that replace the (cheap) iron.

1.5 Master thesis plan

This first chapter introduced the different uses of cyclotrons in the industry, presented the main concepts of cyclotrons technology and explained the motivations of an ironless synchrocyclotron.

The rest of this master thesis is organized as follows: The following chapter describes and exposes the specifications of the different models. The modeling tools that have been developed (and their validation) are described in the third chapter. The fourth chapter defines the mathematical representation of the problem. It also explains post-processing steps applied on the solution of the topology optimization. The main results are presented and discussed in the fifth chapter. It will be shown that the optimization tool is able to produce a design equivalent to the model of the MIT but using considerably less conductor. In addition, topology of an ironless equivalent of the superconducting synchrocyclotron S2C2 of IBA will be developed. In the last chapter, the conductor of the ironless S2C2 is designed. This last part has been made with the help of the researchers of the Plasma Science and Fusion Center of the MIT that developed the first ironless synchrocyclotrons.

Throughout this report, we mainly concentrate on coils topology and magnetic field design. Thermal and structural matters are also introduced for the conductor design. The injection, extraction or acceleration devices are not developed and could be interesting subjects for further investigation in the field of ironless synchrocyclotrons.

2.1 Objectives

This master thesis aims at the development of an optimization tool for the design of ironless synchrocyclotrons. This tool will take as input an objective midplane magnetic field and some specifications. It will produce as output the topology of the synchrocyclotron minimizing the conductor quantity but still respecting the specifications.

Three main objectives are handled throughout this report:

1. Optimization of the MIT ironless design

As previously explained, the PSFC of MIT already designed three ironless synchrocyclotron models in [4], [3] and [5]. These three models have been made by fixing the number of coils, their horizontal position and their current density. The current direction, the height and vertical position of each coil has been optimized to follow as close as possible the magnetic field profile of the iron model. The model has not been optimized in term of conductor minimization. As explained in the previous chapter, the superconducting windings are expensive and a reduction of the conductor quantity could lead to significant reduction of the overall cost¹.

The first objective is to challenge the results of the MIT ironless design in [4]. It will be shown that using topology optimization, it is possible to design an ironless synchrocyclotron producing the same midplane magnetic field profile and respecting the same specifications but to reduce significantly the superconductor quantity.

The model that we follow in [4] is the first one that they designed. This model has been particularly chosen because it is the only one that has the same current density in all the coils. Indeed, in this work we will take as hypothesis that the current density has to be the same in all the coils of each model. It will greatly simplify the topology optimization. Of course, as explained later, further improvement could be made by developing the possibility of using different current densities in different coils.

¹Up to now, its potential high price has been one of the major obstacle in the development of ironless synchrocyclotrons.

2. S2C2 ironless equivalent

The S2C2 of IBA is a superconducting synchrocyclotron, currently used in Proteus One, the main proton therapy of the company. The topology of an ironless equivalent of this synchrocyclotron will be developed. Since the S2C2 is the only superconducting synchrocyclotron of IBA, it has the most suitable architecture for an ironless design. The specifications of the model developed are generally directly taken from the original S2C2 but for some parameters, they are adapted to the ironless situation. These specifications are discussed in the following sections.

3. Design of the conductor

The last part is focused on the conductor design. The goal is to prove by the design of a potential conductor that the topologies of the preceding sections are practically achievable. For this part of the report, we will especially concentrate on the ironless S2C2 equivalent but the reasonings are easily applicable on other models.

The two basic models, the ironless synchrocyclotron from the MIT and the S2C2 of IBA are shown on Fig. 2.1 and Fig. 2.2.

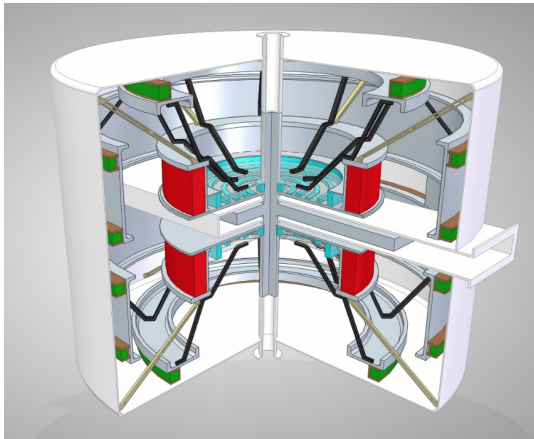


Figure 2.1: Ironless synchrocyclotron developed at the MIT

^a

^a<http://news.mit.edu/2015/making-cancer-treatment-more-accessible-0604>

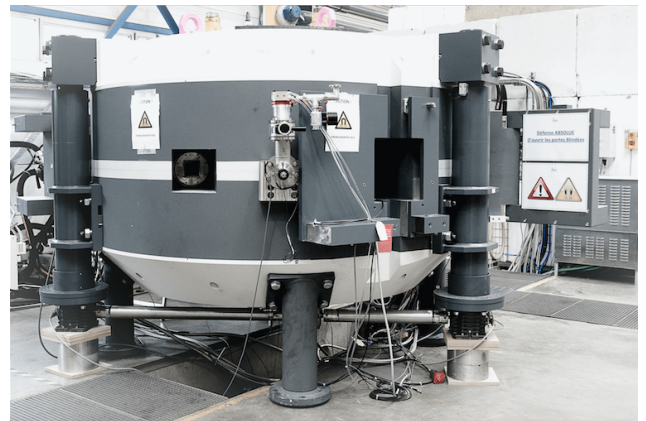


Figure 2.2: S2C2 of IBA

^a

^a<https://iba-worldwide.com/proton-therapy/proton-therapy-solutions/proteus-one>

2.2 Specifications of the models

For the optimization of the MIT model, the specifications from the ironless reference design [4] have been taken directly. For the ironless S2C2, some specifications have been redefined, since the ironless situation considerably differs from the original design. Nevertheless, for the sake of comparability, the dimensions, the required beam energy, the midplane field profile and stray field upper limits are the same than in the original S2C2. Only the current is left as a free parameter. Further improvement could be made with the introduction of new (and more appropriate to the ironless case) specifications.

2.2.1 Dimensions

We assume a rotational symmetry and a midplane symmetry. All the specifications are given on a quarter of cross section of the cyclotron.

For the dimensional specifications, the problem is divided into 3 different zones:

- Zone I: Part of the midplane, called the Good Field Region (GFR), extending from the center to the extraction radius. This is the zone where the objective magnetic field is fixed.
- Zone II: Design zone where the conductor will be distributed. The zone is of course slightly separated from the GFR in order to leave some space for the beam, the cryostat and structural elements. This zone is divided into cells of equal size.
- Zone III: External zone called the Low Field Region (LFR). This zone is internally bounded and a maximal magnitude of magnetic field is defined.

Fig. 2.3 displays the different zones and dimensions of the problem. The values are given in Table 2.1 for both models. It has to be noted that the definition of the low field region slightly differs between our two models. The LFR of the ironless S2C2 will be elliptic-shaped while the one of MIT model will be circular.

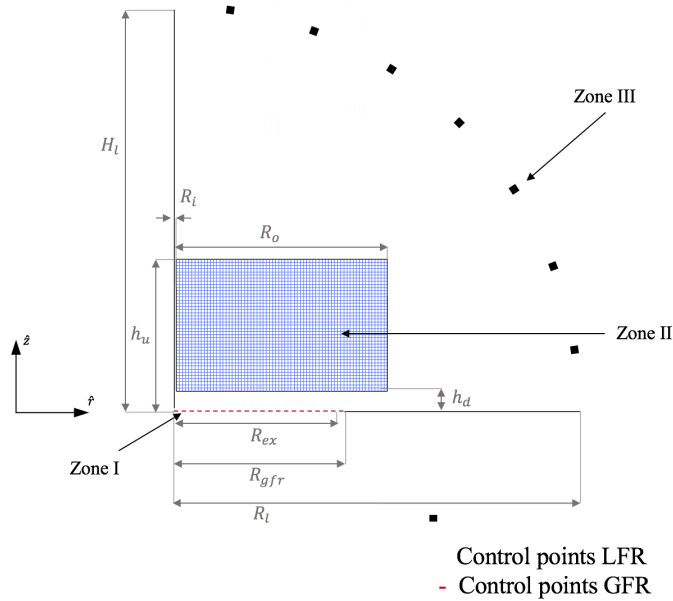


Figure 2.3: Dimensions and different zones and of a quarter of cross-section

In practice, the constraints on the magnetic field are defined at some control points, in the GFR and at the border of the LFR. It is at these points that the resulting magnetic field is checked. The rest of the domain is called the free zone, and no constraints are put on it.

The values of the geometrical parameters are:

Symbol	Quantity		MIT	S2C2
R_l	Radius Low Field Region	[m]	2	1.54
H_l	Height Low Field Region	[m]	2	2
h_d	Half height midplane space	[m]	0.995	0.1
h_u	Half height synchrocyclotron	[m]	0.727	0.78
R_{ex}	Extraction radius	[m]	0.3	0.45
R_{gfr}	Radius Good Field Region	[m]	0.31	0.46
R_o	Radius synchrocyclotron	[m]	1.0245	1.25
R_i	Minimal inner coil radius	[m]	0.047	0.04

Table 2.1: Dimensions of the ironless synchrocyclotron

Note:

- We imposed a minimal coil radius of 4 cm (4.7 cm) for the ironless S2C2 (MIT model). This value is limited by the wire design (explained in Chapter 6) and has been chosen following advice from engineers of IBA. In practice this radius could be reduced further, by using smaller coils with a different design or material and with a different current density. Nevertheless, since we decided to only use one current density, we will not consider this option in our designs.
- The radius of the GFR is slightly larger than the radius of extraction. In practice only the magnetic field before extraction matters but we observed that the matching with the objective field was better if we the GFR a little bit further than the extraction radius.

2.2.2 Magnetic Field

In the midplane, we assume the B_r component of the magnetic field to be negligible. The objective magnetic field is thus only described in terms of B_z . The objective field profiles used in this report are shown on Fig. 2.4 and Fig 2.5.

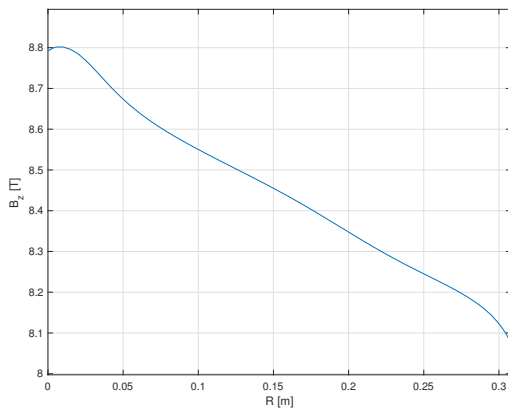


Figure 2.4: Objective midplane magnetic field (MIT)

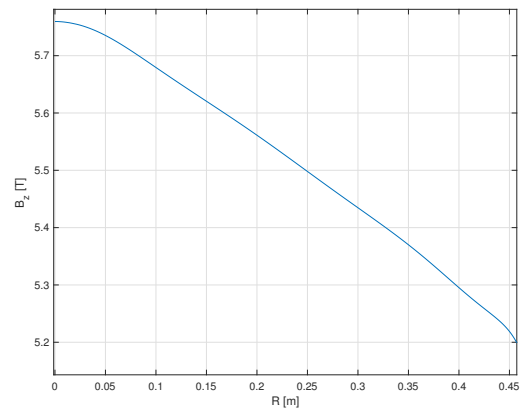


Figure 2.5: Objective midplane magnetic field (S2C2)

The specifications on the stray field of our two models are given in Table 2.2

Control point		MIT	S2C2
Stray field @ 2 m midplane radius	[G]	209	-
Stray field @ 1.54 m midplane radius	[G]	-	500
Stray field @ 2 m on-axis distance	[G]	416	500

Table 2.2: Stray field specifications

The two sets of specifications on the magnetic field in the Low Field Region are defined differently for each model but they express the same idea, more field is tolerated in the LFR above the cyclotron than in the midplane.

2.2.3 Beam energy

The beam energy is the energy level of the protons at the end of the acceleration. The nominal values for the two models are:

$$\begin{aligned} \text{MIT:} & \quad 253 \text{ MeV} \\ \text{S2C2:} & \quad 230 \text{ MeV} \end{aligned}$$

These values mainly depend on the magnitude of the magnetic field value at the extraction radius.

2.2.4 Current density

The current densities for the two models are:

$$\text{MIT:} \quad 235.9 \text{ A/mm}^2 \quad \text{S2C2:} \quad 55 \text{ A/mm}^2$$

For the MIT, this value is a specification and comes directly from the original model. On the contrary, we can choose the current density for the ironless S2C2. Its value directly comes from the limitations on the design of the conductor. Its derivation will be done in Chapter 6.

2.2.5 Additional specification S2C2

The full specifications for the ironless S2C2 are given in Table C.1 of Appendix C. They have been established in collaboration with IBA and are mostly the same than the specifications of the original S2C2, with some adaptations for the ironless case. The most important features have been highlighted in this chapter but Table C.1 adds information on the required treatment characteristics, the superconductor materials, and other important points.

In the development of a topology optimization program, several modeling tools are needed. We have for example to calculate the magnetic field produced by the coils, to compute their inductance, the magnetic energy stored,... The modeling tools and the optimization tool have been developed in MATLAB. The reasons of this choice is that we wanted an easy and home-made program for which we could understand all the different steps and their implications.

The MATLAB tool uses GMSH for the mesh generation. Indeed, our design is based on a finite element model where each element represents one cell of the topology optimization (explanations are provided in the next chapter). Moreover, the finite element solver GETDP is used as a validation tool and GMSH also serves for the visualization of some results. Finally, we also used Opera 2D and Opera 3D for some specific computations and validation that will be detailed later.

The rest of this chapter describes the main modeling tools that have been developed and exposes the assumptions that are made for the modeling of the problem.

Validation example

The modeling tools will be validated on the simple example shown on Fig. 3.1. It corresponds to two symmetric coils of dimensions $R_1 = 0.5\text{ m}$, $R_2 = 0.8\text{ m}$, $Z_1 = 0.1\text{ m}$ and $Z_2 = 0.3\text{ m}$. These are roughly the dimensions of the main coils in our models. A current density of 235.9 A/mm^2 is put in each coils. Since a rotational symmetry is assumed, the computation is only done in one half plane, as represented on Fig 3.1.

3.1 General assumptions

3.1.1 Filament method

The magnetic field is calculated by integration of the Biot-Savart law on circular coils with non negligible rectangular cross section. The exact solution of such an integral does not have a fully analytical expression and its numerical computation is very slow. We decided here to solve a semi-analytical method that computes the magnetic field produced by punctual coils. We thus need a division of the current density in an equivalent distribution of punctual conductors.

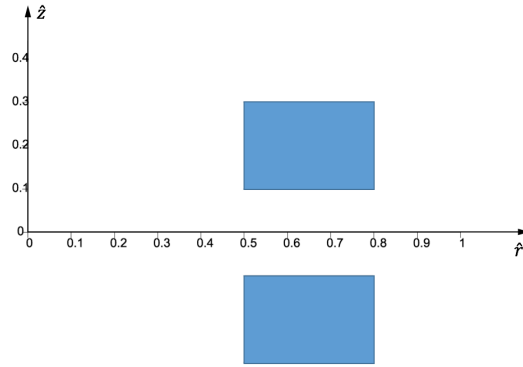
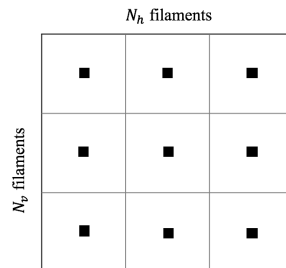


Figure 3.1: Validation coils

This modeling method is called the *filament method*. This method allows an easy and fast calculation of the magnetic field, with minimal error. Each coil is divided in a number of filaments (N_v in the vertical direction and N_h in the horizontal direction). An equivalent current $I_f = \frac{I_{tot}}{N_h \cdot N_v}$ is put in each of these filaments. I_{tot} is the current in the coil: $I_{tot} = J_{op} \cdot A_{coil}$ where J_{op} is the operating current density and A_{coil} the cross-section area of the coil.

The magnetic field is then computed using Biot-Savart law with this punctual distribution of the current. The details of this calculus are described in the next section.


 Figure 3.2: Division of one element in $N_v \cdot N_h$ filament

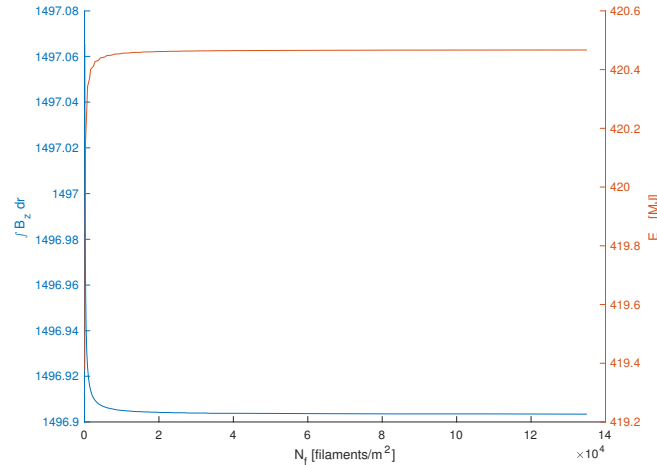
In practice, the number of filaments is computed from one single value, the filament density N_f (in number of filaments/ m^2). N_v and N_h are then derived from this value in function of the length of the height and the width of the coil.

Determination of the filaments density

The value of the filaments density N_f will allow us to choose the accuracy of the results that we will produce. We want to have enough filaments to produce precise results but we want to avoid too high number of filaments in order to limit the computation time. It has not to be forgotten that the parameter N_f is only a numerical value used in our simulations. It is independent on the real number of windings in the coil.

Fig. 3.3 shows the evolution of the magnetic stored energy E_m and of the integral of the magnetic field in the GFR $\int B_z dr$ for different values of the filaments density in our validation coils. We observe that for low values of N_f , these results are variable but that they reach constant values for growing N_f .

We chose to work with a value of $N_f = 4 \cdot 10^4$ filaments/ m^2 . This corresponds approximatively to one filament every half centimeter.


 Figure 3.3: Evolution of $\int Bdr$ and E_m with the N_f

3.2 Modeling tools

3.2.1 Magnetic field

For the computation of the magnetic field, since the problem is totally linear we want to find a matrix M_i such as:

$$B_i = M_i \cdot J \quad (3.1)$$

where i could be the two directions in the considered plane ($i = \hat{\mathbf{r}}$ or $i = \hat{\mathbf{z}}$)¹.

As already announced, this matrix is derived using the Biot-Savart law with the filament method. The other options would have been for example to run the finite element solver GETDP for every computation, or to compute it multiple times in different situations and examine the resulting magnetic field to compute the matrix M_i .

Our choice of Biot-Savart is based on several reasons. First, since we only need the value of the magnetic field at some fixed control nodes, a whole computation of the magnetic field in the entire design region is not needed. Secondly, since the problem is symmetric, linear and has a quite simple geometry, the utilization of a more complex method is not useful. Simplicity and rapidity have thus been chosen.

Thanks to the linearity of the problem, the calculation of the magnetic field density at one point is done by addition of the magnetic field generated by all the current loops (all the filaments). For each coil above the midplane, the symmetrical one under the midplane is considered (with current in the same direction in both coils).

As already stated, a rotational symmetry is assumed and cylindrical coordinates are thus used. The situation for one coil is represented on Fig 3.4. The details of the Biot-Savart integration are based on a reasoning from IBA [33], the main steps and results are summarized hereunder.

Using the Biot-Savart law for the magnetic potential at point $\vec{x}$, we have:

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \int \int \frac{\vec{J}(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' \quad (3.2)$$

¹As explained below J is considered here to have only one component, $\hat{\theta}$

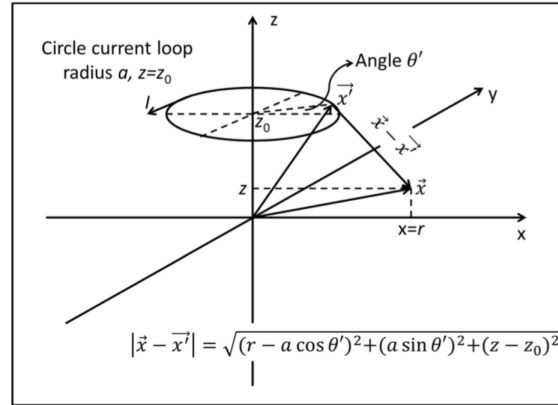


Figure 3.4: Circular current loop[33]

Where $\vec{J} = J\vec{e}_\theta$ is the current density vector and $\vec{x}'$ represents the integration variables (to be integrated over the circle current loop represented on Fig 3.4).

The magnitude of the current is expressed in term of delta-functions, a current I is put in the filaments (of coordinates $[r_0, z_0]$) and nothing elsewhere:

$$J = I\delta(r - r_0)\delta(z - z_0) \quad (3.3)$$

The expression of the integral, using the definition of the current and the expression of $|\vec{x} - \vec{x}'|$ from Fig 3.4 becomes:

$$A_\theta(r, z) = \frac{\mu_0 I}{2\pi} \int_{-\pi}^{\pi} \frac{\cos\theta}{\sqrt{r^2 + r_0^2 + (z - z_0)^2 - 2r r_0 \cos\theta}} d\theta \quad (3.4)$$

After some calculation described in [33], the final expression of the magnetic potential is:

$$A_\theta(r, z) = \frac{\mu_0 I}{2\pi k} \sqrt{\frac{r_0}{r}} \left[(2 - k^2)K(k) - 2E(k) \right] \quad (3.5)$$

With the parameter k defined as:

$$k^2 = \frac{4r_0 r}{(r_0 + r)^2 + (z - z_0)^2} \quad (3.6)$$

$K(k)$ and $E(k)$ are the complete elliptic integrals of the first kind and the second kind respectively

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad (3.7)$$

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta \quad (3.8)$$

The magnetic field is then obtained using:

$$\vec{\nabla} \times \vec{B} = \vec{A} \quad (3.9)$$

Which gives:

$$B_r(r, z) = \frac{\mu_0 I}{4\pi} \frac{k}{r\sqrt{r_0 r}} \left[K(k) - \left(\frac{1 - \frac{k^2}{2}}{1 - k^2} \right) E(k) \right] (z - z_0) \quad (3.10)$$

$$B_z(r, z) = \frac{\mu_0 I}{8\pi} \frac{k}{\sqrt{r_0 r}} \left[\left(1 + \frac{r_0}{r} \right) \frac{k^2}{1 - k^2} E(k) + 2 \left(K(k) - \frac{E(k)}{1 - k^2} \right) \right] \quad (3.11)$$

The MATLAB tool uses these expressions for the magnetic field derivation, and the complete elliptic functions are derived numerically.

The magnetic field profile in the midplane produced by our validation coils is shown on Fig. 3.5. It is computed with the filament method (with $N_f = 4 \cdot 10^4$ filaments/ m^2) and in GETDP. We observe a good matching between both results, excepted at the origin. This is due to the way the problem is modeled in GETDP (as explained in Appendix D.1) and is thus not of major importance.

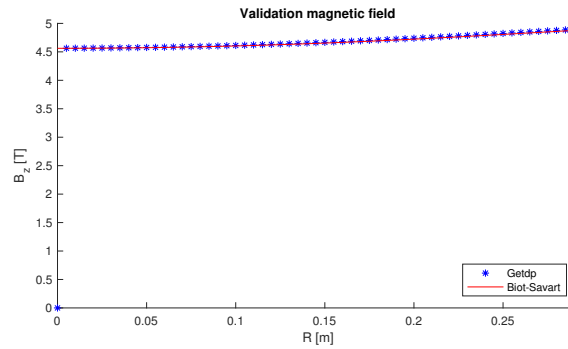


Figure 3.5: Validation of the magnetic field

3.2.2 Inductances

Thanks to the tool computing the magnetic field, it is possible to derive the self- and mutual inductance of the coils. These are required to obtain the amount of energy stored in the cyclotron. We have to know the amount of stored energy since it is the quantity of heat that could be released in case of quench².

The definition of the **self-inductance** and **mutual inductance** are:

$$L = \frac{\psi}{I} \quad M_{12} = M_{21} = \frac{\psi_{12}}{I_1} = \frac{\psi_{21}}{I_2} \quad (3.12)$$

Where ψ is the magnetic flux produced and enclosed by the wire and I the current that it carries. ψ_{12} is the magnetic flux produced by coil 1 that is enclosed by coil 2.

ψ is defined by:

$$\psi = \oint_S \bar{B} d\bar{S} = \oint_{\Gamma} \bar{A} d\bar{l} \quad (3.13)$$

As shown in Eq 3.13, it is possible to express the magnetic flux in function of the magnetic potential using Stoke's theorem. It is this method that will be used for the computation of the inductances.

²Phenomenon that occurs when superconducting magnets lose their superconductivity. It could potentially lead to their destruction. More explanations in Section 6.1.3

The self- and mutual inductance are computed using the filament method. The parameters used in their definition are shown on Fig 3.6 and Fig 3.7

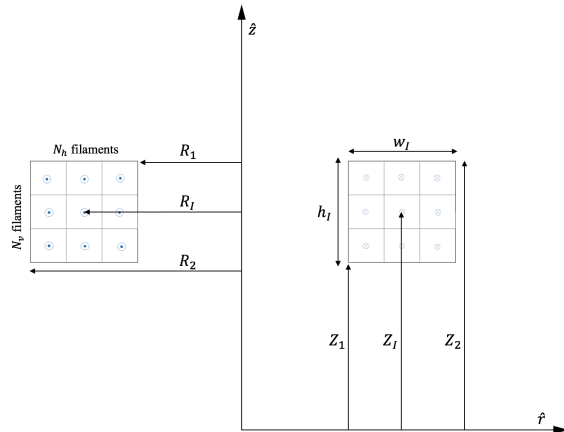


Figure 3.6: Self inductance structure

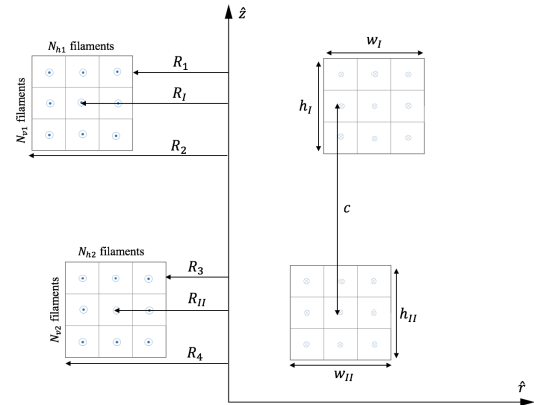


Figure 3.7: Mutual inductance structure

All the coils are considered as being in series. The inductances are first computed for each filament and then averaged to form the total inductance of the coil.

Magnetic field on filaments

Because the filament method consists in a repartition of punctual conductors, the magnetic field (and magnetic potential) can not be computed on the conductor themselves, because their magnitude would be infinite. In this thesis, for the computation of the self inductance, the magnetic potential has thus been calculated on the inner side of each filament section as visible on Fig. 3.8. For the mutual inductance between two filaments there is no need of offset, as the magnetic field is measured on another filament than the filament creating the field that we are looking to.

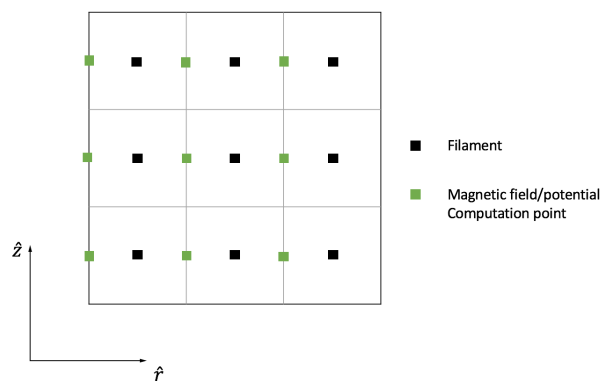


Figure 3.8: Points where the field is measured in an coil with 9 filaments

This hypothesis consists in practice in neglecting the inner inductance of the filament section.

In all this report, when the magnetic field on a filament is required, we use this method and compute it at the same z-coordinate but on the inner side of the filament surface.

Self-inductance

The method used to compute the self inductance of an element consists in the addition of the self- and mutual inductances between all the filaments of one coil. The resulting inductance is then divided by the total number of filaments to the square, to obtain an average value.

The expression of the self-inductance is:

$$L = N_1^2 \cdot \frac{\sum_{g=1}^{N_v} \sum_{h=1}^{N_h} \sum_{s=1}^{N_v} \sum_{l=1}^{N_h} L_f(g, h, s, l)}{N_v^2 \cdot N_f^2} \quad (3.14)$$

L_f is the mutual inductance between two filaments of the coil:

$$L_f = \frac{\mu_0 \sqrt{R_{11} R_{22}}}{k} \left[(2 - k^2) K(k) - 2E(k) \right] \quad (3.15)$$

$$k^2 = \frac{4R_{11}R_{22}}{(R_{11} + R_{22})^2 + z^2} \quad (3.16)$$

In these equations, $E(k)$ and $K(k)$ are again the first and second complete elliptic integrals.

The different variables used are defined as (using the notations of Fig. 3.6, $[R_{11}, Z_{11}]$ and $[R_{22}, Z_{22}]$ being the coordinates of the filaments):

$$R_{11} = R_1 + \frac{w_I}{N_h} \left(h - \frac{1}{2} \right), \quad R_{22} = R_1 + \frac{w_I}{N_h} \left(l - \frac{1}{2} \right) \quad (3.17)$$

$$Z_{11} = Z_1 + \frac{h_I}{N_z} \left(g - \frac{1}{2} \right), \quad Z_{22} = Z_1 + \frac{h_I}{N_z} \left(s - \frac{1}{2} \right) \quad (3.18)$$

$$h_I = Z_2 - Z_1, \quad w_I = R_2 - R_1 \quad z = Z_{22} - Z_{11} \quad (3.19)$$

If $R_{11} = R_{22}$ and $Z_{11} = Z_{22}$, it is the self inductance of a filament that is computed and an offset has to be put:

$$R_d = R_{22} - \frac{w_I}{2 \cdot N_h} \quad (3.20)$$

$$L_f = \frac{\mu_0 \sqrt{R_{11} R_d}}{k} \left[(2 - k^2) K(k) - 2E(k) \right] \quad (3.21)$$

$$k^2 = \frac{4R_{11}R_d}{(R_{11} + R_d)^2} \quad (3.22)$$

Fig 3.9 shows the error percentage produced by the filaments method. These results have not been computed for our validation example, because no reference value of its inductances exists. We instead used a coil of $R_1 = 50 \text{ cm}$, $R_2 = 75 \text{ cm}$, $h_I = 12.5 \text{ cm}$ and $N = 1$. The reference value is $L = 2.8693 \mu H$ of L is taken from [37].

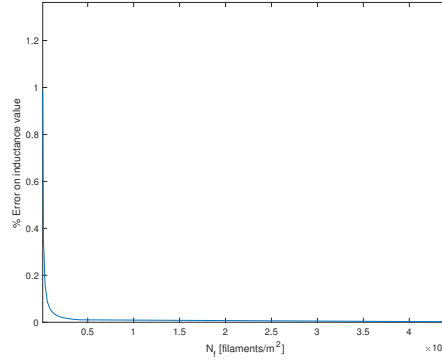


Figure 3.9: % of error in inductance computation in function of filament density N_f

The filament method provides highly precise results. The error rapidly falls to negligible values for higher number of filaments. For our conductor density ($N_f = 4 \cdot 10^4$ filaments/m²), we have an error of 0.003 %.

Mutual inductance

For the computation of the mutual inductances, the method used is:

$$M = \frac{N_1 N_2 \sum_{g=1}^{N_{v1}} \sum_{h=1}^{N_{h1}} \sum_{s=1}^{N_{v2}} \sum_{l=1}^{N_{h2}} M_{12}(g, h, s, l)}{N_{h1} \cdot N_{h2} \cdot N_{v1} \cdot N_{v2}} \quad (3.23)$$

With:

$$M_{12} = \frac{\mu_0 \sqrt{R_{11} R_{22}}}{k} \left[(2 - k^2) K(k) - 2E(k) \right] \quad (3.24)$$

$$k^2 = \frac{4R_{11}R_{22}}{(R_{11} + R_{22})^2 + z^2} \quad (3.25)$$

Using the notations of Fig. [3.7](#):

$$R_{11} = R_1 + \frac{w_I}{N_{h1}} \left(h - \frac{1}{2} \right) \quad R_{22} = R_3 + \frac{w_{II}}{N_{h2}} \left(l - \frac{1}{2} \right) \quad (3.26)$$

$$w_I = R_2 - R_1, \quad w_{II} = R_4 - R_3 \quad h_I = Z_2 - Z_1, \quad h_{II} = Z_4 - Z_3 \quad (3.27)$$

$$z = c + \frac{h_I}{N_{v1}} \left(g - \frac{1}{2} \right) + \frac{h_{II}}{N_{v2}} \left(s - \frac{1}{2} \right) - \frac{h_I + h_{II}}{2} \quad (3.28)$$

For the validation, it was not possible to use our validation coils neither. We used coils of dimensions $R_I = 30$ cm, $R_{II} = 70$ cm, $w_I = w_{II} = h_I = h_{II} = 20$ cm, $c = 0$ cm, $N_1 = 500$ and $N_2 = 500$. The value of the reference mutual inductance from [\[15\]](#) is $M = 70.77152437$ mH.

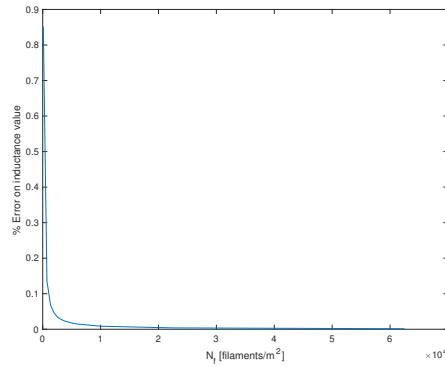


Figure 3.10: % of error in mutual inductance computation in function of filament density N_f

Again, the error quickly plummets to negligible values for higher number of filaments. For our conductor density ($N_f = 4 \cdot 10^4$ filaments/m²), we have an error of 0.002751 %.

3.2.3 Magnetic energy stored

The magnetic energy of a multi coils system (e.g. an ironless synchrocyclotron) can be computed using the value of the inductance of each coil, the mutual inductances and the operating current.

We have the general expression:

$$E_m = \sum_{m,n=1}^N I_m L_{m,n} I_n \quad (3.29)$$

where $L_{m,n}$ is the mutual inductance between the coil m and the coil n (or the self inductance if $m = n$).

We developed a MATLAB function calculating the stored magnetic energy. It has been validated with Opera3D on the validation coils. We obtained for the two cases:

$$\text{Opera3D: } E_m = 423.48 \text{ MJ} \quad \text{MATLAB tool: } E_m = 420.47 \text{ MJ}$$

Giving an error of about 0.7 %.

From the total energy of a multi-coils system using the same current in all the coils, it is easy to derive the value of the total inductance with the expression:

$$L_{tot} = \frac{2E_m}{I^2} \quad (3.30)$$

3.3 Software support

3.3.1 Optimization algorithm

For the topology optimization itself, the MATLAB function `linprog` is used with the `dual-simplex` algorithm.

Generally speaking, the most used algorithms for topology optimization are MMA (Method of Moving Asymptotes) and SLP (Sequential Linear Programming). These methods are efficient in computing solutions for non-linear problems with non-linear constraints but no optimization function using them has been written in the MATLAB library. Since the case study only contains linear calculation and linear constraints, a simpler algorithm may be used. Rewriting the MMA and SLP would be less efficient than using functions given by the MATLAB library. `linprog` is the function that is the most appropriate for this optimization. It solves linear problems with linear boundary conditions, as in our case.

Three algorithms are available with this function. Their computation time are ³:

- `dual-simplex` $\Rightarrow$ Computation time: 2.4 seconds
- `interior-point` $\Rightarrow$ Computation time: 50.11 seconds
- `interior-point-legacy` $\Rightarrow$ Not used because according to MATLAB documentation:
"interior-point-legacy is similar to `interior-point` but can be slower, less robust or use more memory."

The `dual-simplex` solves the problem significantly faster, it has thus been chosen for our resolution.

3.3.2 GMSH / GetDP

As already explained, in this project GMSH is used as a mesh generator and for the visualization of the different results. We use GETDP as a validation tool of the results.

The details about the problem definition in GETDP are given in Appendix [D.1](#). Two important features are also highlighted:

- **Infinite elements** have been used. Their implementation in GMSH is explained in Appendix [D.1.2](#)
- The **imposition of the current** to GETDP after the computation of its repartition by the MATLAB tool is also detailed in Appendix [D.1.3](#)

³These tests have been made for the iron-free S2C2 with `meshsize` = 0.005, $N_f = 1$, $J_{op} = 112A/mm^2$ and $\delta_B = 3 \cdot 10^{-3}$ (See next Chapter)

4.1 Introduction to topology optimization

Topology optimization is an optimization method that consists in finding the optimal material distribution in a given domain on which constraints are applied. The main advantage of topology optimization over classical shape optimization is that no pre-defined solution has to be assumed. The domain is divided in multiple *cells* over which the material will be distributed to minimize a given cost function Φ_{obj} .

While some basics of the theory of topology optimization exist since the beginning of the XX^e century, this method has been developed and precisely defined in the 90's and 2000's, particularly by Sigmund *et al.* [29] and Rozvany [13]. Increasingly powerful softwares have broadened the possibilities offered by topology optimization during the last years.

This optimization method was originally mainly used in mechanics for structure design. The resulting structures are generally lighter and more resistant than the original ones, but their architecture is often complex, as shown in Fig 4.1. The choice of additional constraints that increase manufacturability is today one of the main matter in topology optimization.

For some specific applications, the piece can be made with 3D printing that allows to achieve complex structures. For these applications, the development of 3D printing technology highly increases the potential of topology optimization.

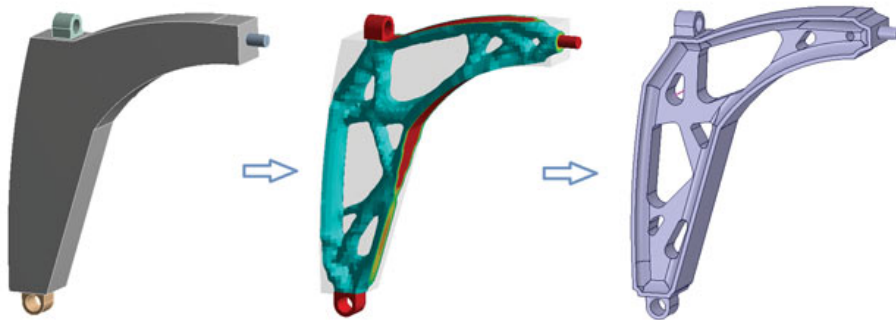


Figure 4.1: Topology optimization applied on an automobile upper control-arm [7]

In this master thesis, we will use topology optimization for electromagnetic design. This domain

received much less attention than the mechanical one. The these of Labbe [30] constitutes a good introduction to the domain.

Topology optimization is a 0-1 method, it means that the objective is to determine if a material is in a cell or not. But in order to allow the use of gradient-based algorithm, this assumption is relaxed and intermediates values are tolerated during the optimization. The result will then contain intermediate material if this material could optimize the objective function better than solid materials. Post-processing of the solution and its repartition in cells containing only solid material is very important. This subject will be detailed in Section 4.3.1

To summarize, the main advantage of topology optimization is the complete independence from an initial design, while its main drawbacks are mesh dependence and difficulties to deal with complex objective functions and constraints.

This chapter describes the principal steps of the derivation of our solution. The first section defines the mathematical representation of the topology optimization problem. The second section explains the post-processing steps of the topology optimization result. The last part studies the numerical parameters used in the simulations and their influence on the resulting topology. A value for each of them is chosen for both models.

4.2 Problem definition

4.2.1 General hypotheses

The current density will be the same in every cell. This means in practice that the coils will all be connected in series. It is often the case in reality, as with our two examples, the MIT ironless synchrocyclotron and the S2C2 of IBA. Some improvement could still be done by putting different current densities in the different coils, as explained in Section 7.1.2

As explained in the previous chapter, the cells are the (rectangular) elements of the design zone in the mesh generated with GMSH.

4.2.2 Optimization variables

The optimization variables are the proportions of material in each cell. These materials are here only the superconducting material (positive current and negative current) and the air. Since the sum of all the proportions of material can not exceed 1, the quantity of air can be deduced from the quantity of superconductor. Indeed, the proportion of air in cell k is $\rho_{ka} = 1 - |\rho_{kp} + \rho_{kn}|$. Where ρ_{kp} and ρ_{kn} ¹ are respectively the proportion of positive and negative current superconducting material in cell k .

4.2.3 Objective function

The goal of the optimization is to minimize the superconducting material quantity (and in consequence the overall cost) that produces a given magnetic field profile in the midplane. If we assume that each cell has the same cross section, that the radius and the cross-section of the conductors are constant over the circle and that the optimization will not lead to a mix of

¹Note that ρ_{kn} is defined as positive

positive and negative conductor², the objective function can be expressed as:

$$\Phi_{obj} = \sum_k r_k (\rho_{kp} + \rho_{kn}) \quad (4.1)$$

Where r_k is the radial position of cell k .

The quantity of superconducting material is directly derived from the objective function:

$$V_c = 2\pi \cdot \Phi_{obj} \cdot A_e \cdot m^3 \quad (4.2)$$

Where A_e is the cross section area of a cell.

4.2.4 Constraints

The constraints of the problem can be divided in two different groups, firstly the physical limits on the optimization variables and secondly the magnetic constraints. These magnetic constraints include the values of the magnetic field in the good field region (GFR) and in the low field region (LFR). They are expressed in a way that preserves the linearity of the problem.

Optimization variables

The proportion of positive and negative conductor in each cell can not be negative or exceed 1. This physical limitation gives the constraints:

$$0 \leq \rho_{kp} \leq 1 \quad (4.3)$$

$$0 \leq \rho_{kn} \leq 1 \quad (4.4)$$

In each cell k .

Magnetic field distribution

Since no iron is used in our design, the magnetic field will evolve linearly with the current. As already stated here above, it is possible to find a matrix M_i linking the optimization variable with the magnetic field at some control points:

$$B_i = M_i \cdot J \quad (4.5)$$

Where i is the direction of the magnetic field.

The current J_k in cell k is expressed in terms of the operating current density and of the conductor proportions in the cell:

$$J_k = J_{op} \cdot (\rho_{pk} - \rho_{nk}) \quad (4.6)$$

Where J_{op} is defined as the operating current density when the cell is full of conductor³.

The matrices M_i are computed using the Biot-Savart law. Using them and the definition of J , it is possible to find the value of the magnetic field at given control points in function of the

²Because it would not minimize the conductor quantity.

³Note that since in our case of varying energy synchrocyclotrons, when referring to the *operating conditions* we will automatically refer to the maximal energy conditions expect if the contrary is explicitly stated.

optimization variables ρ_{pk} and ρ_{nk} . There are two magnetic optimization constraints, the first one forces the magnetic field in the Good Field Region to match the required profile while the second one prevents the magnetic field in the Low Field Region to exceed a given value.

The magnetic field constraint in the **Good Field Region** is:

$$B_{zg}^*(1 - \delta_B) < B_{zg} < B_{zg}^*(1 + \delta_B) \quad (4.7)$$

We only check the z component of the field since only this one matters in beam focusing and acceleration (B_r is supposed to be null in the midplane region). An infinitesimal deviation δ_B is still allowed from the given magnetic field.

In the **Low Field Region**, the field has to be lower than a given maximal value, which gives the constraint:

$$B_{rl} \leq B_l^*/\sqrt{2} \quad (4.8)$$

$$B_{zl} \leq B_l^*/\sqrt{2} \quad (4.9)$$

Note:

The above formulation does not express the exact non-linear constraint on the control points of the LFR which is $|\vec{B}_l| \leq B_l^*$ but is a linearization of this constraint. This approximation is conservative but will greatly simplify the optimization algorithm.

4.3 Post-processing

A theoretical example of a typical result of topology optimization is shown on Fig. 4.2. We observe that the shape of the coils is complex and that some cells contain intermediate material. In practice, such coils are very difficult to manufacture. Some simplification has to be done and the intermediate material has to be removed.

This master thesis develops two main methods to produce structures that are easier to manufacture. The first one consists in replacing the coils by equivalents having a rectangular cross-section and the second one consists in replacing the coils by multiple rectangular cross-sectional layers. This section will expose the different steps of the post-processing of the coils.

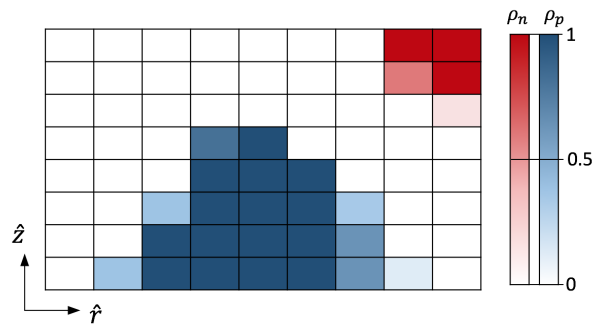


Figure 4.2: Example of result of the topology optimization

4.3.1 Intermediate material

The first step after the topology optimization is the intermediate material post-processing. After the topology optimization some cells are full of material ($\rho = 0$ or $\rho = 1$) but some cells contain intermediate material ($0 < \rho < 1$). In our cases, these intermediate cells usually represent only between 0.02 % and 0.03 % of the total number of cells (for both of our models).

If the number of different types of materials is large it may be really difficult to treat these intermediate materials but in our case the problem is quite simple. Indeed, since we assume that the algorithm will not mix ρ_p and ρ_n in the same cell, the only decision we have to make is to choose a value for the threshold ρ_{th} . If ρ is above ρ_{th} , the cell will be considered as full of material and if ρ is under ρ_{th} it will be considered as empty (full of air). The same ρ_{th} is used for cells containing positive and negative current.

The choice of ρ_{th} is made by finding the value that will minimize the error on the midplane magnetic field. There is no fixed threshold that always minimizes the magnetic field error and in our MATLAB tool, a value of ρ_{th} is found for each situation. A set of different ρ_{th} values (here 40) is tested, the error on the magnetic field is measured and the best candidate is chosen. A method of trial-and-error has been chosen since the error in function ρ_{th} is not monotonic⁴ what would make optimization too difficult.

The respect of the conditions in the LFR is also checked and if no values of ρ_{th} could make these conditions to remain respected, an error is raised.

4.3.2 Rectangular cross-section

The second step in the post-processing of the coils is their replacement by geometries that are easier to manufacture. In the first option, each coil cross-section is replaced by a unique rectangle that has:

- The same **cross-section area** than the original coil.
- The same **gravity center** than the original coil.

And that respects:

- The general geometrical constraints (dimensions of the design zone).
- No overlap between coils (and respect of a minimal distance between them).

The choice of the width (and consequently of the length since the area is fixed) of each rectangle is made by minimization of the elements that are part of the original coil but not of the rectangle (in other words by the maximization of the overlap between the original coil cross-section and rectangular cross-section). The situation is shown on Fig 4.3. The algorithm used for this minimization is a golden-section search.

The coils may not overlap themselves. For this purpose, a small algorithm (described in Appendix E.2) has been integrated to the function that creates the rectangles. It takes as argument a minimal space between coils (that can be 0, but not be negative) and adapts the shape of the coils (but not their area) to avoid overlaps.

This approximation of complex cross-sections by rectangles will of course have consequences on the accuracy of the midplane magnetic field.

⁴A typical evolution of the total error on the magnetic field in function of ρ_{th} is shown in Appendix E.1

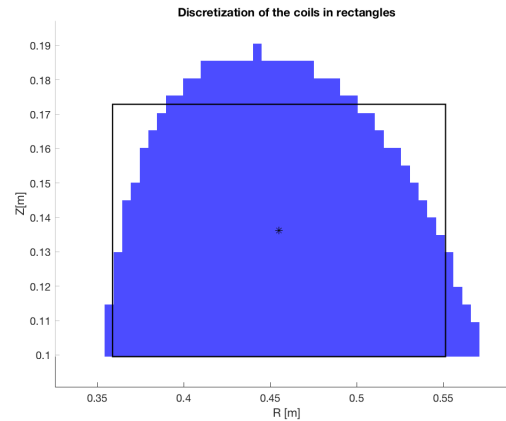


Figure 4.3: Discretization of the result of the topology optimization in rectangles

At this step of the post-processing, the user may also decide to divide the coils in an integer number of conductors having fixed dimensions. For example, if the user only wants to use conductors having a width W_c and a height H_c , the coils dimensions are rounded to the nearest integer multiple of W_c and H_c . If an overlap between the two coils is created due to this discretization, the width of the smallest coil between the two is reduced of one W_c (assuming that overlaps occurs only horizontally, what in our models is by far the most common case).

4.3.3 Multi-layered cross-section

In order to obtain a more accurate fit of the original coils, we may replace them not by single rectangular coils but by multiple layers of rectangular coils having different widths. Each coil is divided in layers of fixed height h_c (parameter chosen by the user). The remaining part of the coil (if its height is smaller than h_c) is designed using the same procedure than the single rectangular coils here above.

Each layer has the same area and the same gravity center than the part of the original coil that it is covering. The situation is shown on Fig 4.4. The width of the layer is then directly derived from h_c and the area of the layer.

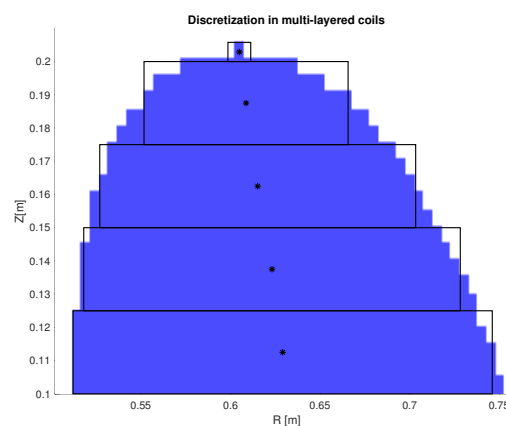


Figure 4.4: Discretization of the result of the topology optimization in rectangles

As we will see later, this option improves the quality of the midplane magnetic field but the coils generated with this technique are more difficult to manufacture.

4.3.4 Optimization of the position

The discretization of the coils in rectangular equivalents generally reduces the accuracy of the midplane magnetic field. To improve our solution, we use a Nelder-Mead algorithm that finds the position of the coils minimizing the value of the error on the midplane magnetic field.

Nelder-Mead algorithm

The Nelder-Mead algorithm is a simplex algorithm often used to optimize multi-variable functions. This algorithm is normally used with unbounded domains. Here, the coils may not go outside of the design zone and may not overlap themselves. The method for the addition of these bounds to the original algorithm is described in [23].

The problem is defined as finding R_c and Z_c in order to minimize:

$$\min_{R_c, Z_c} \left(\frac{\int_{\Gamma} B_{zg} dl - \int_{\Gamma} B_{zg}^* dl}{\int_{\Gamma} B_{zg}^* dl} \right) \quad (4.10)$$

R_c and Z_c are the coordinates of the coils centers. It is important to note that the area of the coils cross section stay constant during the optimization. The results of this optimization are shown in the next chapter for our two models.

The optimization of the position of the coils could lead to overlaps between them. The method to remove these overlaps has been implemented in two different ways. The description of these algorithms is done in Appendix E.3.

4.4 Values of the numerical parameters

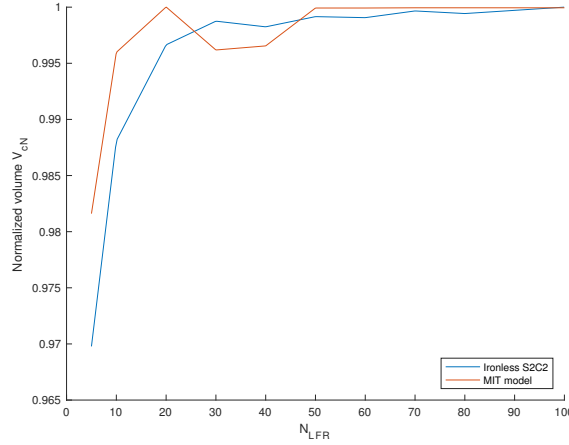
Different numerical parameters have been introduced up to now. In this section, we will justify the values that are used for each of our models.

4.4.1 Number of control points

The number of control points in the LFR and the GFR has to be chosen carefully. For a good modeling of the situation, it is important to ensure that the position and the number of control points do not influence the final topology. We will thus compute the evolution of the conductor quantity V_c in function of the number of control points.

Determination of N_{LFR}^*

Fig. 4.5 shows the evolution of the normalized conductor volume V_{cN} with N_{LFR}^* for our two models (V_c is normalized with the conductor quantity obtained for $N_{LFR}^*=100$). We observe that after $N_{LFR}^* = 60$, the variations in topology become negligible. It is the value that will be used throughout the rest of this report for both cases.

Figure 4.5: Evolution of V_{cN} with N_{LFR}^*

Determination of N_{GFR}^*

To determine the control points in the GFR, we simply took all the points that are given in the files containing the objective magnetic fields (until R_{gfr}). We have for the MIT model (S2C2) $N_{GFR}^* = 800$ ($N_{GFR}^* = 457$).

4.4.2 Determination of δ_B

The parameter δ_B is introduced here above and gives the tolerance on B_{zg}^* , the magnetic field at the control points of the GFR. Generally in cyclotron design, there are no predefined values of this tolerance (since as explained in the next chapter, the betatron tunes will be more important) but we still want to be as precise as possible.

The choice of δ_B is made by analyzing its influence on the conductor quantity V_c . The situation for the MIT model and the ironless S2C2 are shown on Fig 4.6 and Fig 4.7 respectively. We observe that for too low values of δ_B the optimization cannot fulfill the constraints and finally diverges (domain full of conductor). For higher tolerances, the conductor quantity becomes lower. Indeed, since the algorithm seeks at minimizing the conductor quantity, it will use the maximal allowable deviation with respect to B_z to reduce some more the conductor quantity. We finally choose:

$$\text{MIT model: } \delta_B = 0.01 \text{ T} \quad \text{S2C2 equivalent: } \delta_B = 0.003 \text{ T}$$

4.4.3 Determination of meshsize

The refinement of the mesh is another important parameter. We do not want its value to have an influence on the resulting topology. The value of the parameter `meshsize` is linked with the number of elements in the vertical and horizontal direction of the domain with respectively:

$$NbDivr = \text{floor}[(R_o - R_i)/\text{meshsize}] \quad NbDivz = \text{floor}[(h_u - h_d)/\text{meshsize}] \quad (4.11)$$

For this analysis we used the case of the ironless S2C2 and again, we computed the final conductor quantity V_c for different values of `meshsize`. The result is shown on Fig. 4.8. We observed that

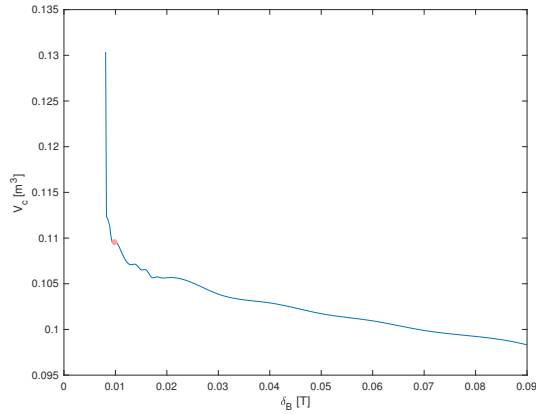


Figure 4.6: Evolution of the conductor quantity V_c with the tolerance value δ_B for the MIT model

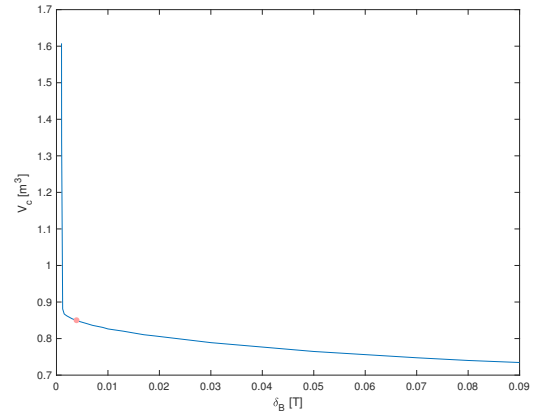


Figure 4.7: Evolution of the conductor quantity V_c with the tolerance value δ_B for the ironless S2C2

for large elements, this quantity varies significantly. Afterwards, when `meshsize` reaches lower values, the conductor quantity levels off and becomes independent on the refinement of the mesh. We want the value of `meshsize` to be small enough to be in this part, but not too small to avoid useless computations. We choose for our model a value of `meshsize` = 0.005.

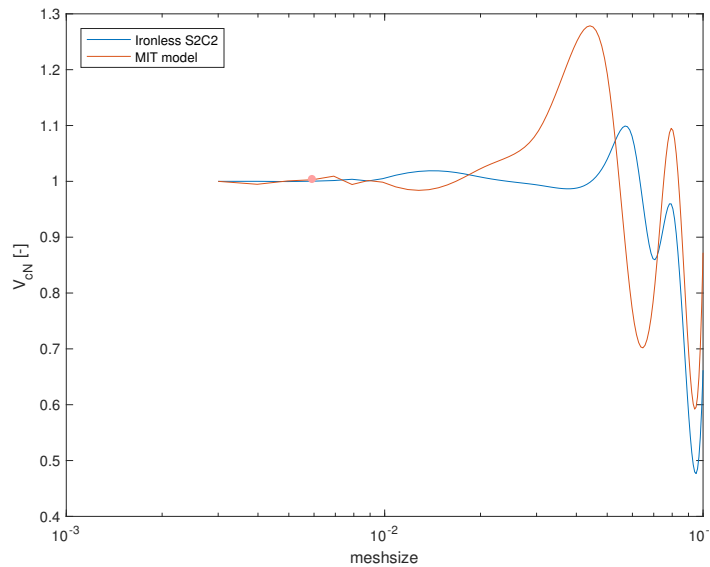


Figure 4.8: % of error between the field for different values of meshsize

If we put one filament per element, we observe that the filament density N_f corresponding to our mesh refinement `meshsize` = 0.005 is approximatively equal to $N_f = 4 \cdot 10^4$ cond/m² (which was the value chosen in the previous chapter). We have for the MIT model $N_f = 3.9219 \cdot 10^4$ cond/m² and for the ironless S2C2 $N_f = 3.9542 \cdot 10^4$ cond/m². It is these filaments densities that will finally be used in our simulations.

These two values (N_f and `meshsize`) are quite conservative. This choice is due to the fact that we are aiming at developing tools giving accurate results in all possible situations and not to develop the fastest possible program.

This chapter will summarize and explain the main results. As explained previously, the topology and the magnetic considerations will be analyzed for both designs. However, the thermal and mechanical analyses will only be done on the ironless S2C2 (in the next chapter).

The first section is dedicated to the derivation of the topology. Afterwards, the magnetic field in the different regions is studied. The third section shows the computation of the stored magnetic energy and the fourth section describes the energy of the beam at the different radii. Finally, the last part studies the influence of some design parameters on our results.

The main outcomes will be exposed for both our model but sometimes an accent is put on one case or the other, if the concerned results are more pertinent.

5.1 Numerical parameters

The numerical parameters used in our simulations have been defined in Chapter 3 and 4 and are summarized in Table 5.1.

	N_f [cond/m ²]	δ_B [T]	N_{LFR}^*	meshsize
MIT model	$3.9219 \cdot 10^4$	0.01	60	0.005
S2C2	$3.9542 \cdot 10^4$	0.003	60	0.005

Table 5.1: Numerical values used

5.2 Main results

A summary of the main results is given in Table 5.2. Throughout the following chapters, we will discuss each of them and compare them with the values of the original models.

As already stated, the current density was given for the MIT but was not specified for our ironless S2C2. This parameter has a strong influence on the topology and of course on several other important values. The discussion on its influence on the results is done in Section 5.7.1 and the derivation of the value used in this case is explained by the conductor design described in Chapter 6.

Specifications		MIT	Optimization	S2C2	Ironless S2C2
Beam Energy	[MeV]	253	250.2	230	226.6
Beam extraction radius	[cm]	30	30	45	43.92
Overall current density	[A/mm ²]	235.9	235.9	56	55
Central magnetic field density	[T]	8.8	8.8	5.7	5.7
Peak magnetic field on superconducting winding	[T]	12.4	13.9	5.59	7.55
Superconductor quantity	[m ³]	0.1996	0.1155	0.2841	0.8852
Stored magnetic energy	[MJ]	30.3	17.14	8.6	22.15

Table 5.2: Main specifications and results of the S2C2 ironless equivalent

5.3 Topology

To have a better understanding of our resulting topologies, we will expose all the steps of their post-processing. For each step, the topology will be shown, with its corresponding midplane magnetic field.

5.3.1 Results of the topology optimization

The results of the topology optimization are shown in blue and red (depending on the current direction) on Fig 5.1 and Fig 5.2. We observe on Fig. 5.3 and Fig. 5.4 that the midplane magnetic field produced by the topology optimization closely fits the objective field B_z in both cases.

5.3.2 Post-processing of the coils

Rectangular cross-sections

The two models and their discretization in coils having a rectangular cross-section are shown on Fig 5.1 and Fig 5.2 for the MIT model and the ironless S2C2 respectively. Their corresponding midplane magnetic field is also given in Fig 5.3 and Fig 5.4. The '*' symbols are the gravity centers of each coil. We took here a minimal distance between the coils of 0 cm. Indeed, following advice from the MIT, the coils can touch each other.

For the rectangular cross-section, we also have the option to divide the coils in an integer number of conductors of given dimensions (W_c and H_c). It is not used for the MIT model since the windings are not designed in the original article. For the ironless S2C2 on the contrary, we here divide the coils in an integer number of windings of dimensions $W_c = H_c = 6.5 \text{ mm}$ (details on the conductor layout are given in the next chapter). It changes a bit the volume of conductor (we have after discretization $V_c = 0.880 \text{ m}^3$) instead of ($V_c = 0.855 \text{ m}^3$) and has a significant influence on the midplane magnetic field (B_z after discretization is shown on Fig. 5.5). The deviation with respect to the objective field will be reduced later by the optimization of the position.

Multi-layered cross sections

The divisions in multi-layered coils of our two designs are shown on Fig 5.6 and Fig 5.7 with their corresponding midplane magnetic field on Fig 5.8 and Fig 5.9. We chose here an arbitrary height

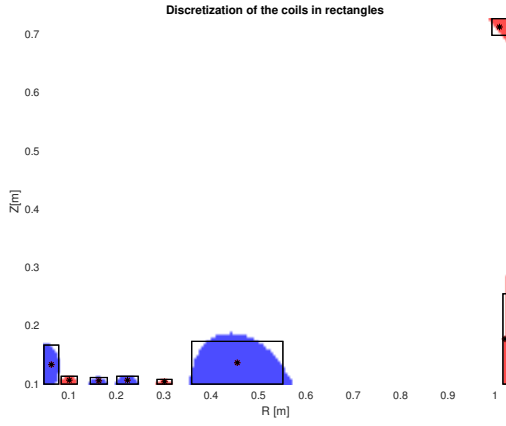


Figure 5.1: MIT model: Cross-section produced by topology optimization and discretization in rectangular coils

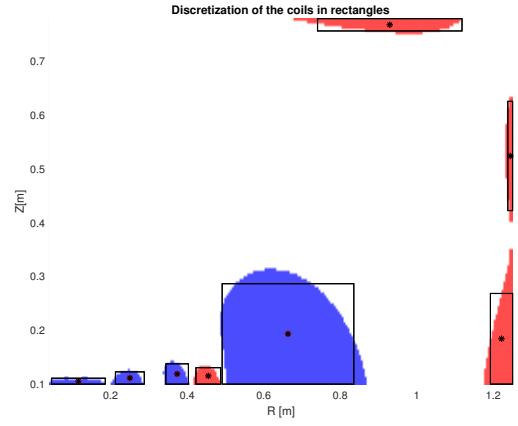


Figure 5.2: Ironless S2C2: Cross-section produced by the topology optimization and discretization in rectangular coils

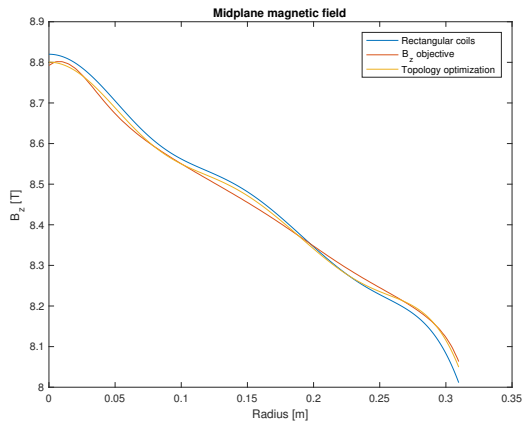


Figure 5.3: MIT model: Midplane B_z in the different situations

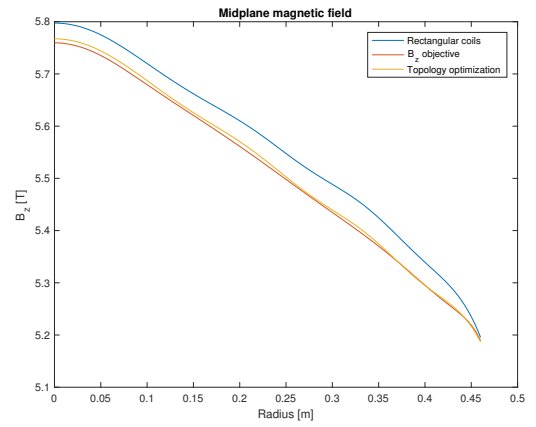


Figure 5.4: Ironless S2C2: Midplane B_z in the different situations

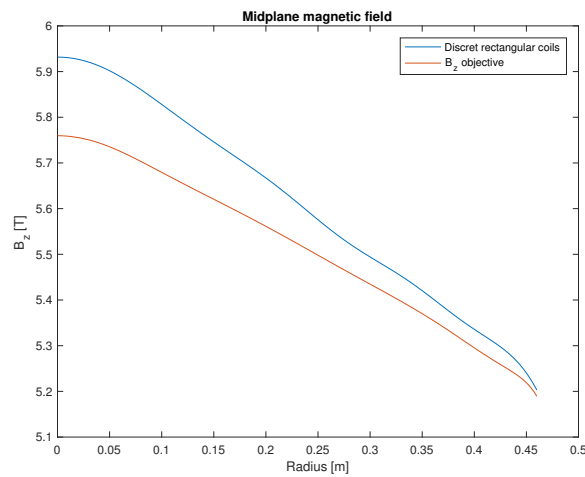


Figure 5.5: Ironless S2C2: Midplane B_z after discretization in cables of $W_c = H_c = 6.5 \text{ mm}$

of coils of $h_c = 2.5 \text{ cm}$. The computation could of course have been done for lower (or higher) height of coils, leading to a better (or worse) matching with the objective magnetic field.

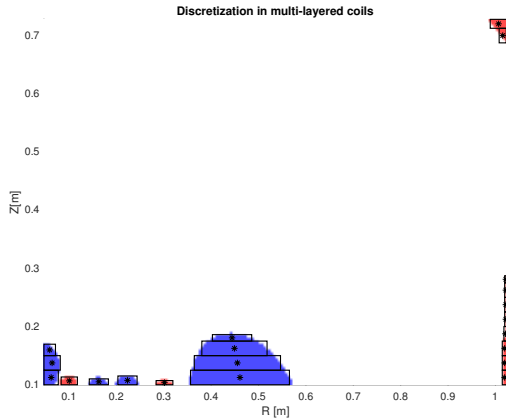


Figure 5.6: MIT model: Cross-section of the multi-layered coils

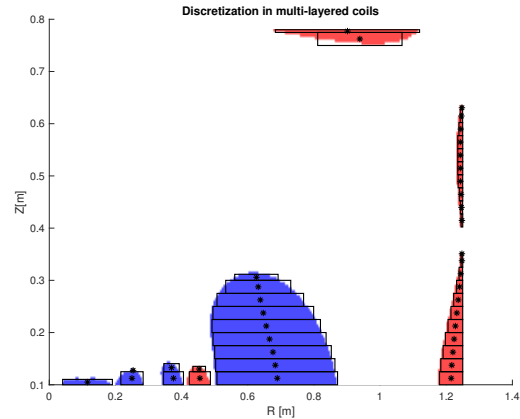


Figure 5.7: Ironless S2C2: Cross-section of the multi-layered coils

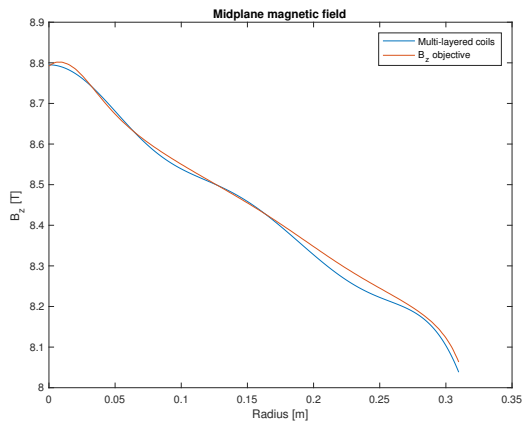


Figure 5.8: MIT model: Midplane B_z for the multi-layered coils

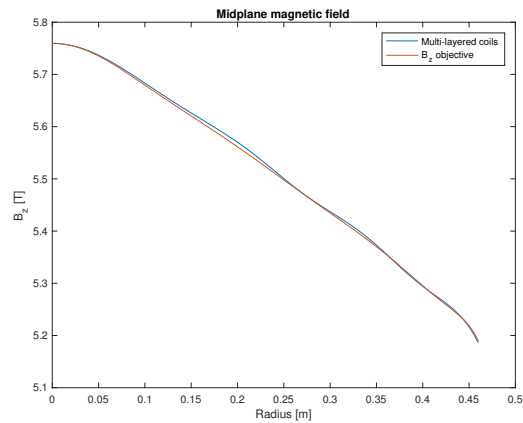


Figure 5.9: Ironless S2C2: Midplane B_z for the multi-layered coils

We observe that the division in multiple layers improves the quality of the magnetic field in comparison with the rectangular coils. Unfortunately, this improvement comes at the cost of a topology that is more difficult to manufacture.

Optimization of the position of the coils

After the discretization of the coils, their position is optimized again in order to reduce the error on the magnetic field. Fig. 5.10 and Fig. 5.11 show the coils before and after optimization of their position. The final midplane magnetic fields are compared with the objective ones in Fig. 5.12 and Fig. 5.13. We observe that this optimization strongly improves our results. For the MIT model the displacement of the coils is small, but it takes larger proportions for the ironless S2C2. We observe in particular that the shaping coils move away from the midplane. This is done to counteract the increase in volume due to the discretization of the coils in an integer number of conductors.

Since the position optimization of rectangular coils already produces really good results, we decided to base the rest of this master thesis on the models obtained with this method. More

complex topologies, as the multi-layered coils, would dramatically increase the manufacturing issues without increasing significantly the quality of the midplane magnetic field.

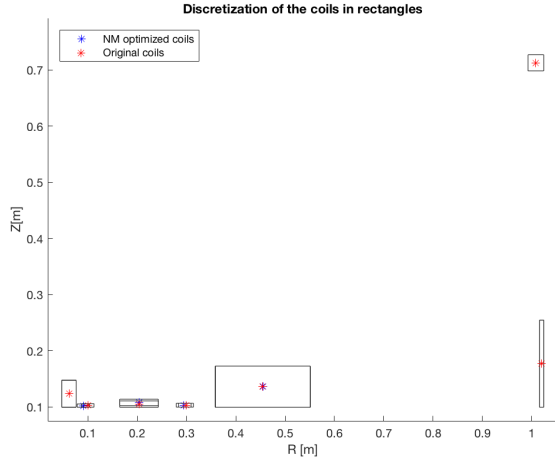


Figure 5.10: MIT model: Optimization of the position of the coils

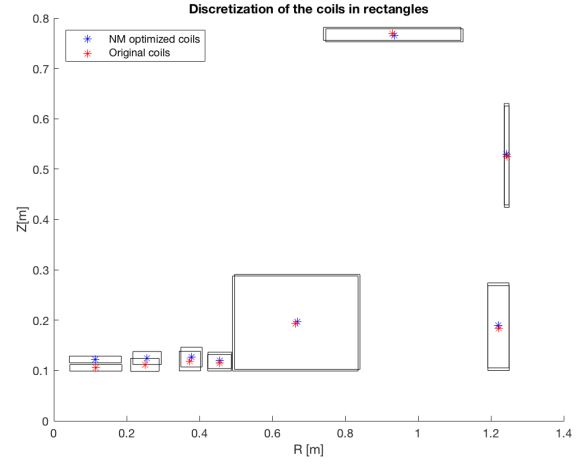


Figure 5.11: Ironless S2C2: Optimization of the position of the coils

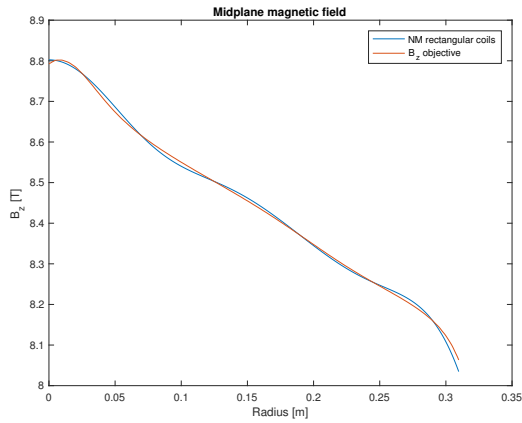


Figure 5.12: MIT model: Midplane B_z after optimization of the position

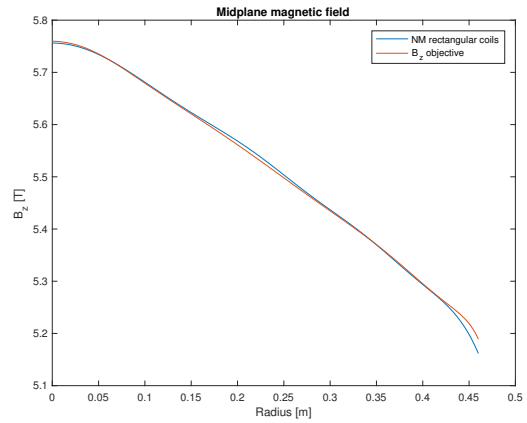


Figure 5.13: Ironless S2C2: Midplane B_z after optimization of the position

As explained in the previous chapter, two methods have been developed for the Nelder-Mead optimization. Both algorithms are described in Appendix [E.3](#).

5.3.3 Summary topology

Table [5.3](#) and [5.4](#) show the main characteristics of the two models. R_c and Z_c correspond to the coordinates of the coils' centers in the first quadrant. ΔR and ΔZ are the width and height of each coil.

MIT model

The coils of the MIT model are described in Table [5.3](#).

Coil	R_c [m]	Z_c [m]	ΔR [m]	ΔZ [m]	J [A/mm^2]
1	1.0089	0.7119	0.0312	0.0302	-235.9
2	1.0201	0.1771	0.0087	0.1552	-235.9
3	0.4558	0.1362	0.1925	0.0734	235.9
4	0.0630	0.1328	0.0319	0.0663	235.9
5	0.0995	0.1063	0.0340	0.0135	-235.9
6	0.1625	0.1078	0.0358	0.0114	235.9
7	0.2257	0.1079	0.0454	0.0135	235.9
8	0.2899	0.1053	0.0315	0.0081	-235.9

Table 5.3: MIT model: Coils parameters (1-2: Shielding coils, 3: Main coil, 4-8: Shaping coils)

The final topology, and the original one from MIT are shown on Fig 5.14 and Fig. 5.15. Their corresponding volume are:

$$\begin{aligned} \text{MIT model} \quad V_c &= 0.11549 \, m^3 \\ \text{Original model} \quad V_c &= 0.1996 \, m^3 \end{aligned}$$

We observe that the conductor quantity is strongly reduced in comparison with the original model. This confirms the interest of using topology optimization for such a design of ironless synchrocyclotron. The comparison between the two topologies is shown on Fig 5.14 and Fig 5.15

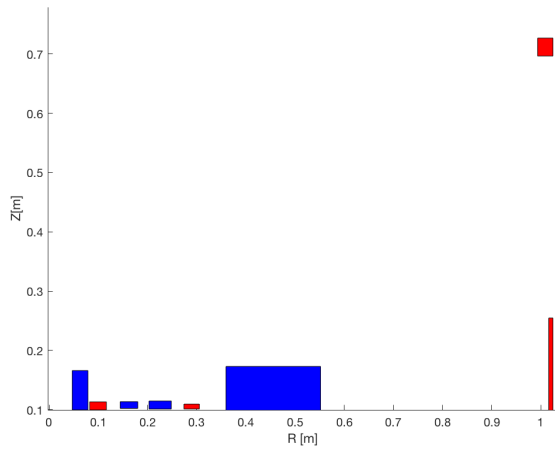


Figure 5.14: MIT model - Final topology

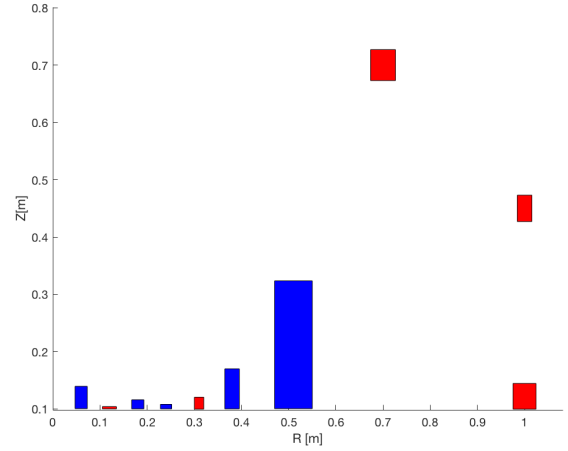


Figure 5.15: MIT model - Original topology

Ironless S2C2

For the S2C2 equivalent, the conductors are described in Table 5.4. The number of windings in each coil is also given, using square conductors with sides of $W_c = H_c = 6.5$ mm. N_r is the number of windings in the horizontal direction and N_z in the vertical direction in each coil. Of course the total number of windings in one coil is equal to $N_r \cdot N_z$.

Coil	R_c [m]	Z_c [m]	ΔR [m]	ΔZ [m]	I_{op} [A]	N_r	N_z
1	0.9352	0.7657	0.377	0.026	-2750	58	4
2	1.2433	0.5297	0.013	0.2015	-2750	2	31
3	1.2205	0.1897	0.0585	0.1690	-2750	9	26
4	0.6683	0.1965	0.3445	0.1885	2750	53	29
5	0.3781	0.1267	0.0585	0.0390	2750	9	6
6	0.4557	0.1203	0.065	0.0325	-2750	10	5
7	0.2563	0.1249	0.078	0.0260	2750	12	4
8	0.1141	0.1219	0.143	0.0130	2750	22	2

Table 5.4: Ironless S2C2: Coils parameters (1-3: Shielding coils, 4: Main coil, 5-8: Shaping coils)

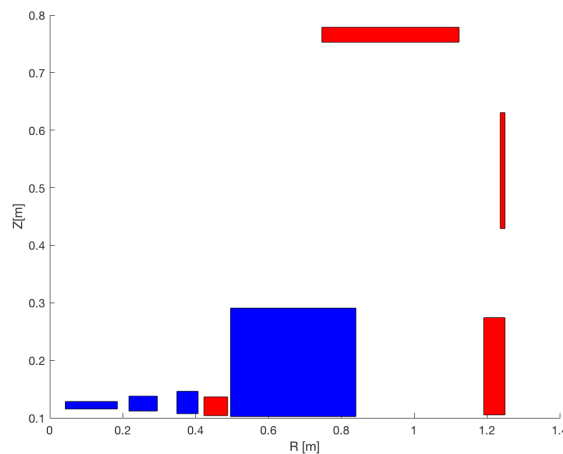


Figure 5.16: Ironless S2C2 - Final topology

5.4 Magnetic field

5.4.1 Midplane magnetic field

The magnetic field profiles in the midplane for the final models are shown on Fig 5.12 and Fig 5.13.

Following the different steps of the post-processing, we first observe that the results of the topology optimization produces a magnetic field very close to the objective. The discretization of the coils in rectangular equivalents has then a negative impact and we lose some precision. But accuracy is recovered thanks to the position optimization. The inaccurate matching with the objective field is not a major issue since the most important properties are the conditions on the betatron tunes (See Section 5.4.3).

We call midplane error the presence of B_r components in the midplane region. These are often caused by a small displacement of one coil and a partial loss of symmetry. Due to Lorentz forces, the B_r components provoke a vertical displacement of all the hadrons, and in many cases, the loss of the beam. It is then primordial to avoid these errors. We observe on Fig F.1 (from Appendix F.1) that they are negligible in our solution.

5.4.2 Stray field

The values of the stray field at the specification points are summarized in Table 5.5

	Specifications	MIT result	Optimization	S2C2	Ironless S2C2
Stray field @ 1.54 m mid-plane radius [G]	-	-	-	500	453
Stray field @ 2 m mid-plane radius [G]	209	4	147	-	-
Stray field @ 2 m on-axis distance [G]	416	13	296	500	283

Table 5.5: Stray field: Specifications and results

The stray fields of our two models are shown on Fig 5.17 and 5.18

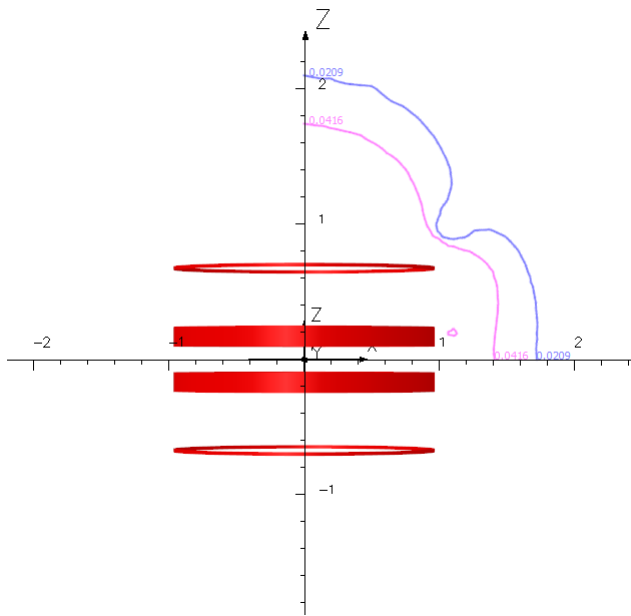


Figure 5.17: Optimized MIT model - Stray field at $B = 209$ G and $B = 416$ G

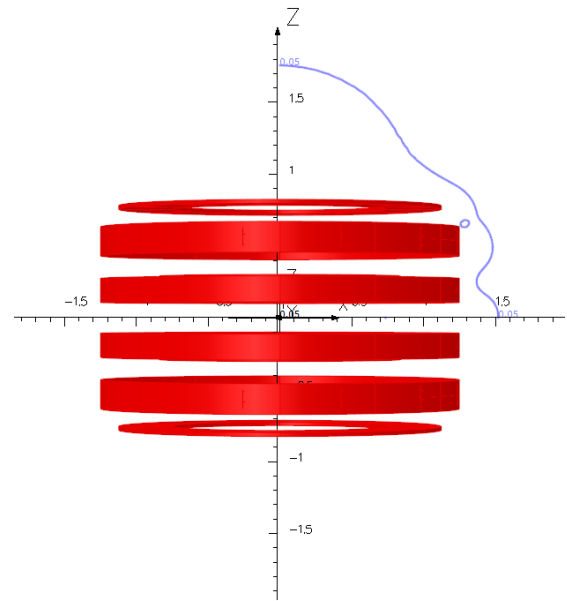


Figure 5.18: Ironless S2C2 - Stray field at $B = 500$ G

We observe that in both cases, the magnitude of the stray field respects the specifications (from Chapter 2). No control points have been put at the exact location of these specifications but the magnetic field stays under the given maximal values, which shows the robustness of the method.

Since the beginning of this work, we chose to follow the same specifications than the original MIT model and not to take their results as specifications. If we force our model to take as specification the stray field that they obtained (see *MIT result* in Table 5.5), we simply observe an increase of the conductor volume ($V_c = 0.1335 \text{ m}^3$) due to the larger shielding coils needed, but we still stay under the volume of their result ($V_c = 0.1996 \text{ m}^3$).

5.4.3 Betatron tune

As already explained in the introduction about physics of accelerators, the beam does not follow an exactly circular orbit but oscillates in the vertical and the horizontal direction as shown on Fig 5.22:

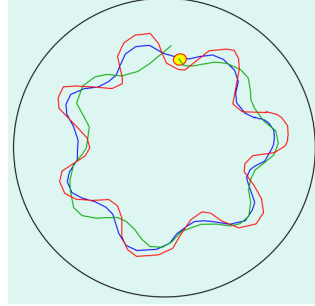


Figure 5.19: Oscillations around the orbit in the horizontal direction

The frequency of the oscillations in the vertical and the horizontal direction are [2]:

$$\nu_z = \omega \sqrt{n} \quad \text{and} \quad \nu_r = \omega \sqrt{1-n} \quad (5.1)$$

Where $0 < n < 1$

$$n = -\frac{r}{B} \frac{dB}{dr} \quad (5.2)$$

These frequencies are called betatron tunes and vary with the radius. This evolution has to respect some conditions in order to avoid resonance and beam loss. The progression of the betatron tunes in our two models are shown on Fig. 5.20 and Fig. 5.21.

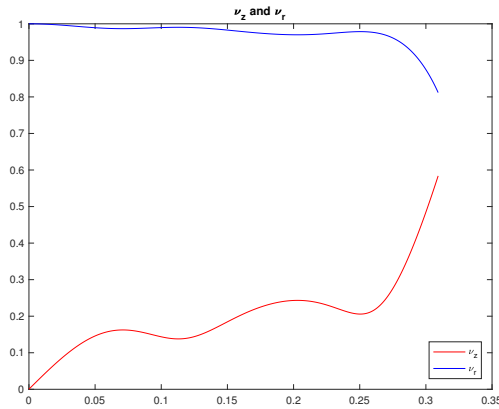


Figure 5.20: MIT model: Evolution of the betatron tunes with the radius

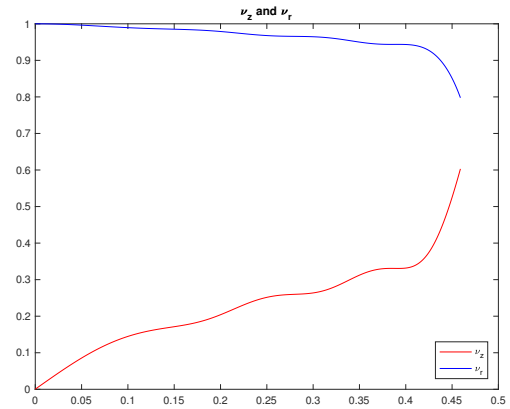


Figure 5.21: Ironless S2C2: Evolution of the betatron tunes with the radius

Conditions of stability

In order to have a stable focusing of the beam, ν_z and ν_r have to remain real which means that the field index has to respect $0 < n < 1$.

This condition expresses the fact that we want $\frac{dB}{dr}$ to be negative, i.e. a decreasing magnetic field with the radius. This is essential for the beam stability (as explained in Section 1.3). More details are given in [12] and [2].

Walkinshaw resonance

The beam crosses multiple resonance points and their exhaustive study is a complex topic, out of the scope of this work. Only one point has to be taken into account at our level, the Walkinshaw resonance because the beam can not cross it. A cyclotron unable to avoid this resonance would just not be operational.

A Walkinshaw resonance happens when $\nu_r - 2\nu_z = l$ (with $l = 0, 1, 2, \dots$). At this point, the beam becomes unstable i.e. its horizontal kinetic energy converts in vertical kinetic energy¹ and the beam is lost in the engine. Hence, it is primordial that this resonance takes place at a radius larger than the extraction radius. In the case of the ironless S2C2, the first resonance ($\nu_r = 2\nu_z$) takes place at:

$$\text{S2C2: } R_w = 43.92 \text{ cm}$$

In the first model of the MIT [\[4\]](#) on which this work is based, the Walkinshaw resonance had not been taken into account; it will thus not be analyzed in this work.

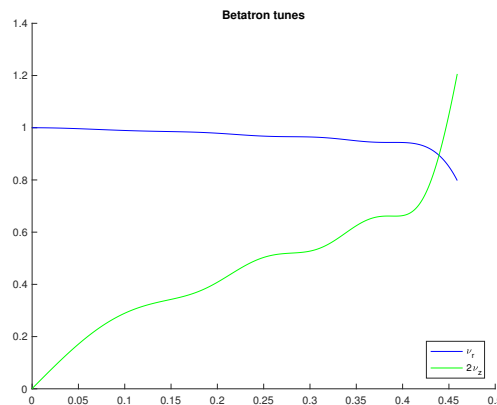


Figure 5.22: Walkinshaw resonance at $\nu_r = 2\nu_z$ for the optimized MIT model

The objective field has been designed to have a Walkinshaw resonance at $R_w = 45$ cm. This resonance corresponding with an increase of the amplitude of $\frac{dB}{dr}$, it will happen at the position of steep change of the magnetic field shape (for example the angle around the extraction radius). In the midplane magnetic field profile produced by our ironless S2C2, we observe a fall earlier than in the objective profile. This is the reason of our value of R_w .

Our MATLAB tool is not able to reproduce exactly the objective field because this field has been created with iron pieces. Especially the steep angle at the pole is very difficult to obtain only with coils that are placed at more than 10 cm of the midplane (to leave space for supporting structure and cryostat). In the S2C2, a piece of iron is put at 7.5 mm of the midplane at R_{ex} and allows thus very steep changes of field².

Two options exist to solve this problem. We could **change the objective field** and limit the derivative of the field at the pole. ν_z (which is dependent on the magnitude of $\frac{dB}{dr}$) would grow slower and it would push the Walkinshaw resonance at a larger radius. The second option is simply to **extract the beam earlier**. The beam would not cross the Walkinshaw resonance and

¹Detailed explanations about beam dynamics crossing a Walkinshaw resonance are available in [\[14\]](#)

²Generally speaking the production of abrupt variation of the field is one of the main difficulty of the ironless model, for example for the extraction system (see Section [7.1.1](#)).

we would not have to care about a perfect matching between the ironless field and the objective field in the last centimeters.

Since we do not want to change arbitrarily the objective profile, the second option is chosen here. The main drawback of this choice is that we have to take into account that the output beam energy will be slightly lower, since the particles will do less turns.

Total "lost" beam energy represents $\Delta T = 3.4$ MeV. It is not a major since it is small in comparison with the total extraction beam energy (and that, since it will be a variable energy cyclotron, the use at full energy will only represent a very small fraction of the total use). In general, the extraction system also allows to move further the Walkinshaw resonance radius [28]. The extraction method for ironless synchrocyclotron is still not well defined so at our stage, we will not put too much attention to the extraction. Of course, it would be an interesting investigation subject in further development (see Section 7.1.1).

5.4.4 Magnetic field on conductors

The value of the magnetic field on the conductors is especially important for the design of the superconducting wire. As it will be shown in the next chapter, too high values of the magnetic field increase the ac losses and could limit the current density that may be used. The maximal values of the magnetic field on the conductors for each models are:

$$\begin{aligned} \text{MIT model: } B_{max} &= 13.9 \text{ T} \\ \text{Ironless S2C2: } B_{max} &= 7.55 \text{ T} \end{aligned}$$

The amplitude of magnetic field on the conductor is shown on Fig 5.23 and 5.24.

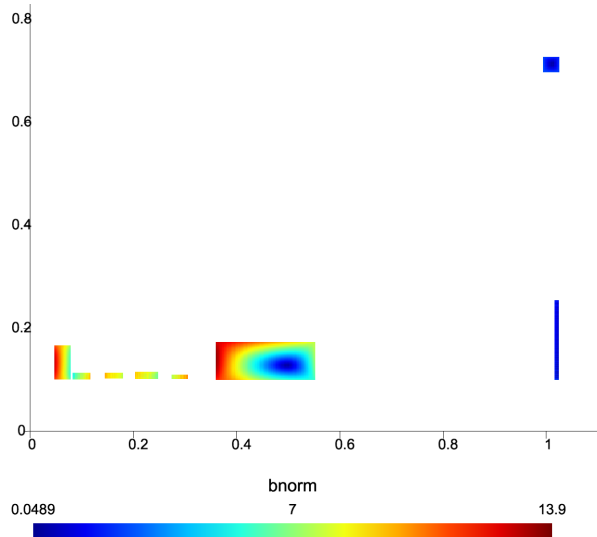


Figure 5.23: MIT model: Magnetic field on the conductors

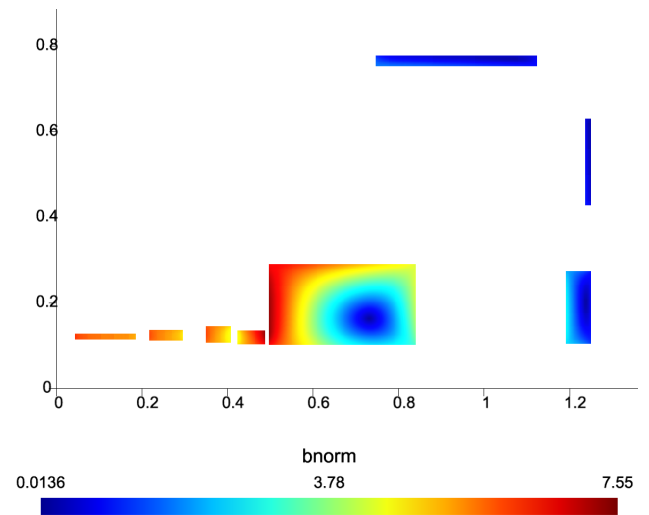


Figure 5.24: Ironless S2C2: Magnetic field on the conductors

5.5 Stored magnetic energy

The magnetic energy stored in the coils is a key parameter in the design of an electromagnetic device. Indeed, the quantity of energy stored is the quantity of heat that could be released as

heat in case of quench of the magnet. The total magnetic energy stored is computed thanks to the value of the self- and mutual inductances between the coils, obtained with the methods of Section 3.2.2

The values of the stored magnetic energy in our two models are:

$$\text{MIT model: } E_m = 17.14 \text{ MJ} \quad \text{Ironless S2C2: } E_m = 22.15 \text{ MJ}$$

For the MIT, less energy is stored in our optimized model in comparison with the original one. This is due to the lower quantity of conductor, which reduces the value of the inductance. This reduction of the stored energy is another interest of reducing the conductor quantity (beside the cost reduction). Indeed, the magnetic energy has to be evacuated quickly in case of quench what is not always easy in term of conductor design and quench protection (explanations in the Chapter 6).

For the ironless S2C2, we observe of course that this value is largely higher than in the classical S2C2 (with iron yoke), where this value only reaches 8.6 MJ. It is normal since significantly more conductor is used (larger inductance). It means of course that the conductors and the quench protection devices have to be redesigned to be able to evacuate this extra amount of energy.

As previously explained, the quantity of magnetic energy stored is independent of N_w (the number of windings per m^2)³. From the magnetic energy stored we can also compute the value of the total inductance. Its value is in contrast dependent on N_w .

For the **MIT model**, we do not know the final number of windings so we do not know the current in these windings neither. The inductance is computed in term of windings density (N_w in windings/ m^2). The current in the windings I_{op} is obtained with $I_{op} = \frac{J_{op}}{N_w}$ where J_{op} is the operating current density. The inductance is:

$$L_{tot} = \frac{2E_m}{I_{op}^2} = N_w^2 \cdot 0.616 \quad [nH] \quad (5.3)$$

For the **ironless S2C2** on the contrary, the number of windings and operating current I_{op} are known (see design of the conductor in the next chapter). We can thus derive the inductance value:

$$L_{tot} = 5.86 \quad [H] \quad (5.4)$$

5.6 Beam energy

The beam energy will define the maximal depth that can be reached by the protons. In our ironless synchrocyclotron, this energy is variable and depends on the current density. This section will only present the situation at beam output energy. Section 6.2.1 goes further in the analysis of the variable energy situation.

First of all, we have to remind some important values:

- $m_0 = 1.67262e^{-27}$ kg: proton rest mass

³The value of N_w , in windings/ m^2 is of course different from the value of N_f , in filaments/ m^2 . The first one is a physical value while the second one is only a numerical parameter used in the simulations.

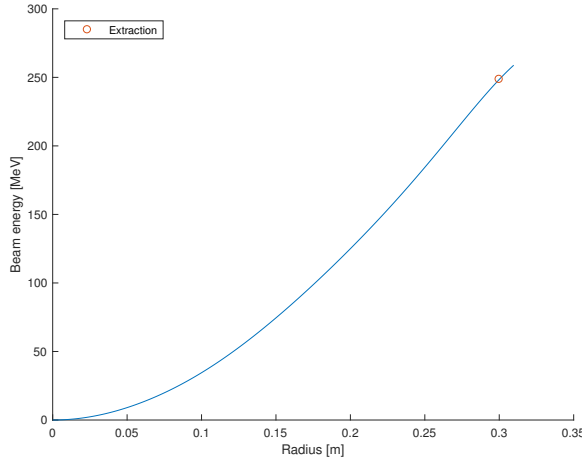


Figure 5.25: MIT model: Evolution of the beam energy with the radius

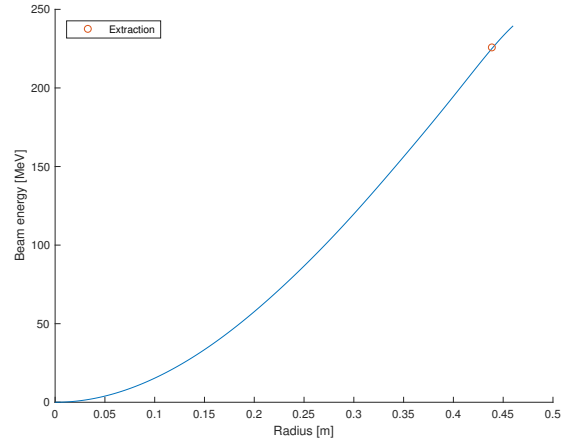


Figure 5.26: Ironless S2C2: Evolution of the beam energy with the radius

- $q = 1.60218e^{-19}$ C: proton electric charge
- $c = 299792458$ m/s: speed of the light
- $E_0(= m_0c^2) = 938.27$ MeV: rest energy of a proton

The beam energy can be evaluated at each radius by first computing the momentum of the hadrons:

$$p = mv = qrB(r) \quad (5.5)$$

This equation comes from the basic equation of the cyclotron Eq 1.2. The total energy of the particles is defined as $W^2 = p^2c^2 + E_0^2$ and the beam energy as $T = W - E_0$.

We then ultimately obtain the beam energy:

$$T(r) = \sqrt{\frac{p^2(r)c^2}{q^2} + E_0^2} - E_0 \quad [MeV] \quad (5.6)$$

Note that we divided here by q in order to have [MeV] units⁴. The evolutions of the beam energy with the radius and the extraction point are shown on Fig. 5.25 and Fig. 5.26 for the MIT model and the S2C2 respectively. The extraction radius and beam energy at the output for each model are:

$$\begin{array}{ll} \text{MIT model: } R_{ex} = 30 \text{ cm} & T_{ex} = 250.2 \text{ MeV} \\ \text{Ironless S2C2: } R_{ex} = 43.92 \text{ cm} & T_{ex} = 226.6 \text{ MeV} \end{array}$$

5.7 Influence of design parameters

In this section, we observe the influence of several design parameters on the characteristics of the ironless S2C2. Some of these parameters (such as the dimensions of the cyclotron) have been fixed arbitrarily (following the original S2C2) and their value could be improved for the ironless

⁴With E_0 already expressed in [MeV]

case. This analysis could serve as a first basis for a future improvement of the specifications. In this section we only take the case of the ironless S2C2 since changing the specifications of the MIT model would not make sense (for the sake of comparability between both models).

We will begin with the analysis of the influence of current density on the volume of conductor and stored energy. Afterwards we will look at the influence of the Low Field Region radius and half height of the cyclotron on the volume of conductor.

5.7.1 Current density

With the optimized MIT design, the current density has to be the same than in the original model ($J_{op} = 235.9 \text{ A/mm}^2$) but for the ironless S2C2 we may choose another value than in the original S2C2. This section will expose the impact of the current density on the important values of the ironless S2C2.

The variations of the stored magnetic energy and conductor quantity with the current density are shown on Fig 5.27 and Fig 5.28 respectively. Table 5.6 shows three examples of topologies for different current densities.

We particularly observe that for too low values of the current, the solution diverges i.e. the entire design zone is filled of conductor trying to respect the constraints on the Good and Low Field Regions. The result of the topology optimization at $J_{op} = 27 \text{ A/mm}^2$ is shown on Fig. F.2 in Appendix F.2. The domain is almost full of conductor and it is not possible anymore to transform the coils in rectangular equivalents.

We also observe that the stored magnetic energy decreases with increasing current densities. This magnetic energy is proportional to the current density to the square and to the inductance of the cyclotron. The shape of the evolution of E_m means that the inductance here plays a predominant role.

The value of the current density that we used throughout this report ($J_{op} = 55 \text{ A/mm}^2$) is chosen following thermal and structural considerations, that will be explained in the next chapter.

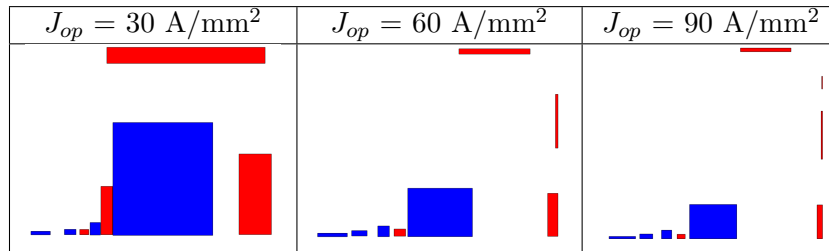


Table 5.6: Cross section of the ironless S2C2 for different values of J_{op}

5.7.2 Radius of the LFR

One easy way to reduce further the conductor quantity and thus also the stored magnetic energy is by reducing the constraints on the Low Field Region. Fig 5.29 shows the evolution of the objective function with R_l , the LFR radius, for the iron-free S2C2. Beside the progressive disappearance of the shielding coils, the main and shaping coils also tail off when R_l increase. This is due to the fact that in normal functioning mode, these coils need extra volume to counteract the field produced by the shielding coils.

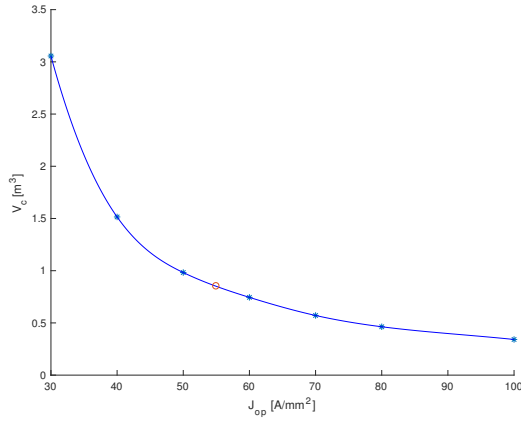


Figure 5.27: Evolution of V_c with the current density J_{op}

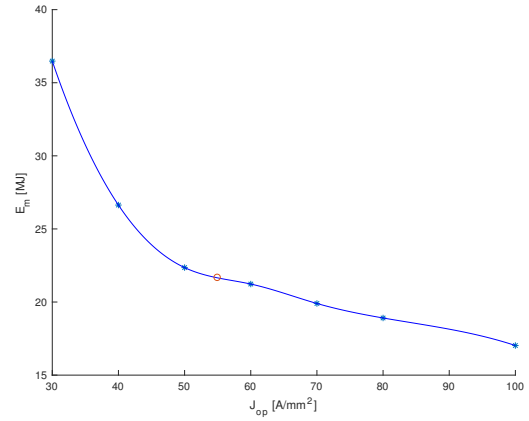


Figure 5.28: Evolution of E_m with the current density J_{op}

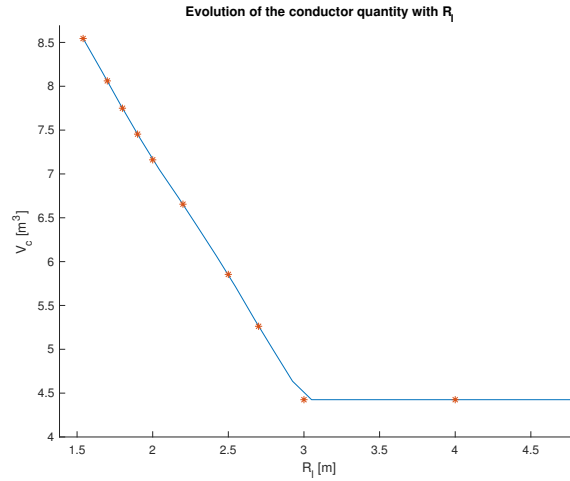


Figure 5.29: Evolution of the conductor quantity with the radius of the LFR (S2C2 model)

We observe on Fig 5.29 that, after a decreasing period, the conductor quantity values remain constant. It is the situation without shielding coils (after a radius of $R_l \approx 3$ m). Note that we kept the ratio between the LFR radius in the midplane (R_l) and the LFR height on the z-axis (H_l) of $H_l/R_l = 2/1.54$.

Of course, even if an increase of the LFR radius reduces the superconductor quantity, we still have to respect some limits fixed by the maximal field that auxiliary devices can stand. These physical limits are the reasons of the value chosen here ($R_l = 1.54$ m).

5.7.3 Height of the cyclotron

Another parameter that could be adapted is the height of the cyclotron (in fact we will rather speak in term of *half height*, from the midplane to the top of the upper coil, since the model is symmetric). Its has to remain between some limits. The cyclotron may not be too high and too cumbersome but not too small (some space has to be left between the coils for structural elements,...) but we could change the value that we put here (half height of cyclotron of $h_u = 78$ cm).

The influence of the half-height h_u on the conductor quantity V_c is shown on Fig 5.30.

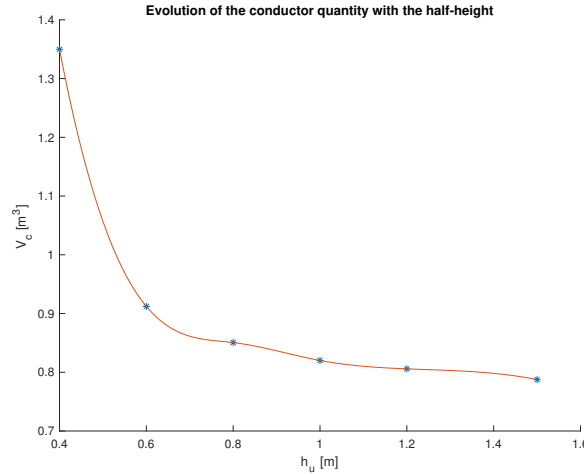


Figure 5.30: Evolution of the conductor quantity and the magnetic stored energy with h_u , the half-height of the cyclotron

We observe that after a steep descent, the conductor quantity decreases slowly for higher h_u . At low h_u , the coils are really close to each other and need extra volume to counteract the effects of the coils with a current in different direction. The value that we took as specification ($h_u = 78$ cm) is in the angle of the curve and seems thus to be a right choice. This height is large enough to avoid the high quantities of conductor due to too high closeness between the coils but it is not too large to have a unnecessarily high cyclotron.

Note:

The other geometrical parameters are all restricted by physical considerations. For example R_i , the minimal coil diameter is limited by the physical layout of the coil, h_d the half-height of the bore has to be greater than a given value to leave space for the supporting structure, the cryostat and the RF. R_o , the radius of the cyclotron is limited by the low field region on one side and by the fixed extraction radius on the other side.

CHAPTER 6

CONDUCTOR DESIGN

After the previous results, a natural outcome is to assess the feasibility of our solution by adding a more physical point of view to the problem. It will be the goal of this chapter. We will design a feasible conductor layout for the ironless S2C2 and it will lead to the derivation of some results used previously (current density, stored magnetic energy,...). The following developments would not have been very interesting for the MIT model since its current density was already fixed.

We will begin this chapter with a short summary of the most important concepts of superconductivity that will be used for the following calculations. Afterwards, we will compute the losses and find the type of cooling method and superconducting material that would be the most suitable to our situation. The following part will expose the details of the conductor layout and the reasoning that has led to the results. We will finish with a computation of the main mechanical stresses in the different components and a first sketch of a potential supporting structure.

The following development has been made thanks to advice and help from the designers of the first ironless synchrocyclotron models (of the Plasma Science and Fusion Center at MIT). The design of superconducting wires is a tough task and requires an important expertise. The goal of this section is thus, based on some assumptions, to produce a feasible (and efficient) solution but not to find the optimal conductor design.

6.1 Introduction to superconductivity

The first choice that has to be made is about the type of conductor that will be used. It is clear that given the magnitude of the magnetic field and the current density required, a superconductor is needed, no normal conductor could stand such conditions.

Superconductivity is a phenomenon occurring in some materials, characterized by the absence of any electrical resistance when the material operates under a given critical temperature. The absence of resistance means that if a current is induced in such a conductor, it will never stop. Such materials have of course numerous application possibilities in fields where high currents and high magnetic fields are needed, as for example energy storage, nuclear fusion or accelerators.

In this introductory section, we will begin by explaining the concepts of critical surface. Afterwards, we will show the typical layout of a superconducting cable and explain the current sharing

process. We will finish with a introduction to quench protection design.

6.1.1 Critical surface

The superconducting state is in general dependent on three variables: B_{op} the operating magnetic field on the winding, T_{op} the operating temperature and J_{op} the operating current density in the winding. If one of these parameters increases too much, the material may loose its superconductivity and become resistive. The limit between the superconductor and the resistive state (also called the *normal* state) is called the critical surface. Beside its dependence on B_{op} , J_{op} and T_{op} , the critical surface may also be dependent on the strain ϵ (we will explain it further, for now we only consider the dependence on the three usual values).

The critical surfaces of the two most common superconductors, NbTi and Nb₃Sn, are shown on Fig. 6.1 and Fig. 6.2 respectively [10].

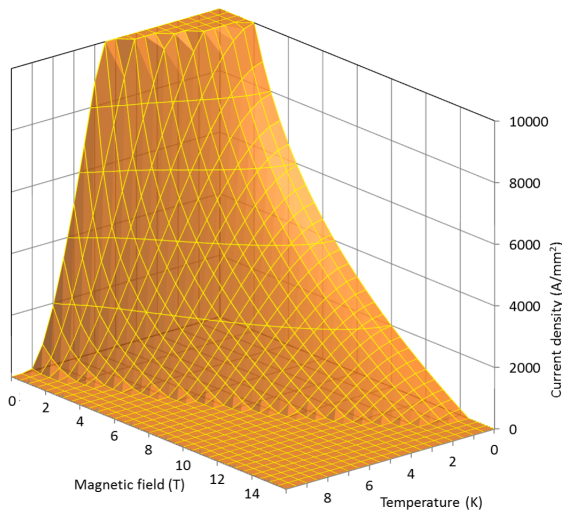


Figure 6.1: Critical surface of NbTi

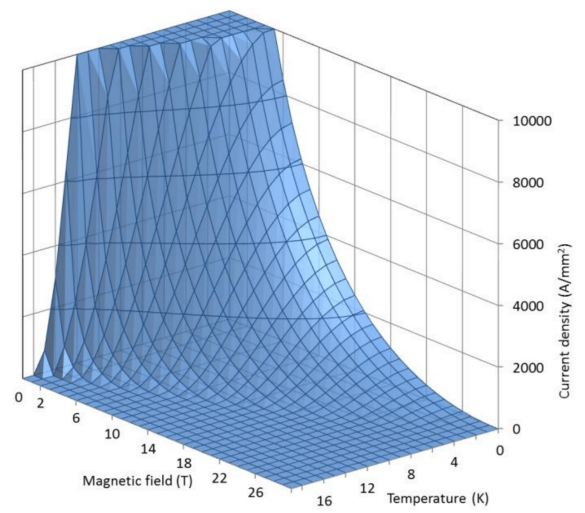


Figure 6.2: Critical surface of Nb₃Sn

J_c , T_c and B_c are called the critical current, critical temperature and critical magnetic field; they are the limits of the superconducting mode at one operating point.

If the operating point crosses the critical surface the conductor becomes resistive. This situation is called a *quench* of the coil. It can be due to multiple reasons as for example AC losses, leaks in the cryostat, friction in the materials,... The reasons are numerous and sometimes unpredictable. The windings are then designed to operate at states that respect some margins against the critical surface (see Section 6.4.3) and protection devices are added in case of too strong mishaps.

In practice, the critical surfaces are parametrized and it is this parametrization that is used for the design of superconductors. The one that we use in this report is called the Summers parametrization [10] and is the most widely used¹.

This parametrization for NbTi is:

$$B_{c2}(T) = B_{c20} \left[1 - \left(\frac{T}{T_{c0}} \right)^{1.7} \right] \quad (6.1)$$

¹The derivation of critical surfaces parametrizations is complex and still subject to intense research, for example at MIT or at the CERN in Europe.

$$T_c(B) = T_{c0} \left[1 - \left(\frac{B}{B_{c20}} \right) \right]^{1/1.7} \quad (6.2)$$

$$J_c = J_{c,ref} \cdot \frac{C_0}{B} \left[\frac{B}{B_{c2(T)}} \right]^\alpha \left[1 - \frac{B}{B_{c2(T)}} \right]^\beta \left[1 - \left(\frac{T}{T_{c0}} \right)^{1.7} \right]^\gamma \quad (6.3)$$

The resulting critical surface is shown on Fig 6.3. The parameters used are given in Table 6.1. The parametrization of the critical surface of Nb₃Sn is given in Appendix G.1.

B_{c20}	[T]	14.5
T_{c0}	[K]	9.2
$J_{c,ref}$	[A/mm ²]	3000
C_0	[T]	27
α		0.63
β		1
γ		2.3

Table 6.1: Value parameters

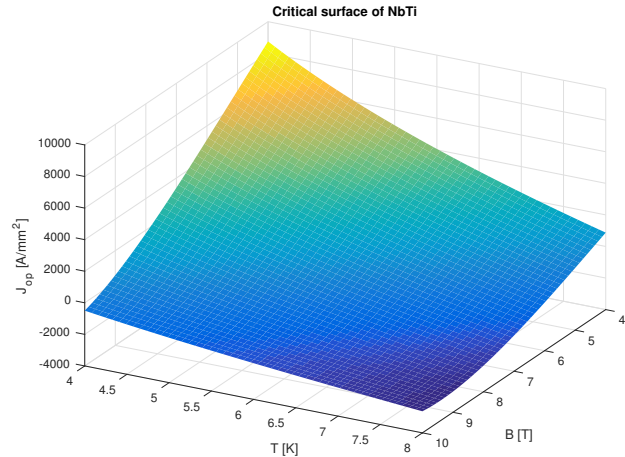


Figure 6.3: Parametrization of NbTi critical surface

6.1.2 Composition of a superconducting wire

The superconducting windings are of course not made only of superconducting material, they contain several different components. In this sections, we will present the most important ones and explain the reasons of their presence.

Superconducting materials have a high resistivity in normal state compared to normal conductors. If a superconductors quenches and enters in the normal state, a lot of heat is generated by Joule losses. Since obviously we want to limit these losses (to allow a faster recovery of the superconducting conditions), superconducting windings are always built as a composite material containing superconductor filaments in a normal conductor matrix, called the *stabilizer* (often copper or aluminium). This normal conductor matrix also helps the manufacture of the strands and protects the superconducting filaments.

In addition to the normal conductors and the superconductors, superconducting windings often contains structural (for example in stainless steel) and insulations layers. Coolant (mostly helium) is sometimes also directly integrated in the windings, as it will be shown later, in our case with Cable-in-Conduit conductors (CICC).

The superconducting windings are thus generally made of composite strands (having a diameter in the mm-range), containing superconducting filaments (having a diameter in the μm -range). An example of one strand is shown on Fig. 6.4. Multiple strands are then gathered in larger windings with the addition of the above mentioned materials, depending on the type of wire (additional pure copper strands, structural elements, insulation, coolant,...).

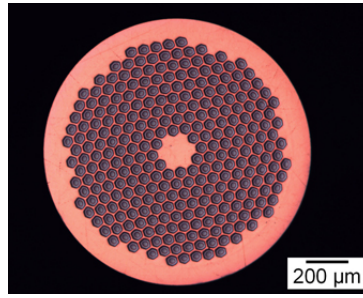


Figure 6.4: Example of wire strand [9]

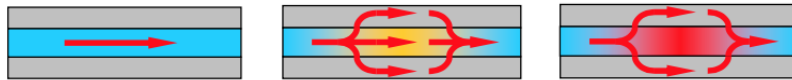


Figure 6.5: Repartition of the current during a quench [27]

In the following sections, when we will refer to the stabilizer we will speak about copper because it is by far the most used stabilizer (and the one used in our case). But it has to be noted that other conductors are also sometimes used.

Current sharing process

The transition to normal state is called the *current sharing process* and is illustrated on Fig. 6.5. T_{cs} is the current-sharing temperature, defined as the temperature for which the operating current I_{op} is equal to the critical current I_c (We have $I_c(T_{cs}) = I_{op}$).

If we operate under T_{cs} , all the current stays in the superconducting filament. If the temperature increases between T_{cs} and T_c , the superconductor filament becomes resistive but still carries the critical current. A voltage along the winding is developed and the rest of the current will thus flow in the stabilizer. It will provoke heat production by Joule dissipation.

If this power generation is too high and the cooling too low, the temperature will keep on growing and the critical current will decrease. More current will thus flow in the stabilizer, leading to higher heat generation and temperature. After the critical temperature T_c , the critical current becomes equal to zero and all the current flows in the stabilizer.

This process, and especially the concept of current sharing temperature will be of great use for the design of superconducting windings.

Customarily, the evolution of the critical current with the temperature is approximated by a linear relation [20]:

$$I_c(T) = \frac{T_c - T}{T_c - T_{cs}} \quad (6.4)$$

6.1.3 Quench protection

A quench occurs when, in the current sharing process, the heat generation due to Joule losses is higher than the cooling of the wire. In these conditions, the superconductor enters completely in

the normal zone and all the current flows in the stabilizer, generating high Joule losses that are not evacuated fast enough. The temperature of the wire thus increases very fast.

In the design of a superconducting magnet, particular attention has to be put on quench protection. Indeed, even if the magnet is well designed to avoid quenches in normal operating conditions, sudden events could happen (movement of the coils, electrical problems,...) and provoke a quench of the coils. In this case, large heat quantities are released, which could damage or destroy the conductors (and auxiliary devices by overcurrent). It is then important to detect quenches on time and to install protection devices.

To have a first idea of what happens during a quench, it is useful to analyze the lumped element model of the cyclotron (on Fig. 6.6). In superconducting mode the switch S is closed, the inductance L represents the cyclotron coils, R_{quench} is null and thus no current passes through R_{dump} .

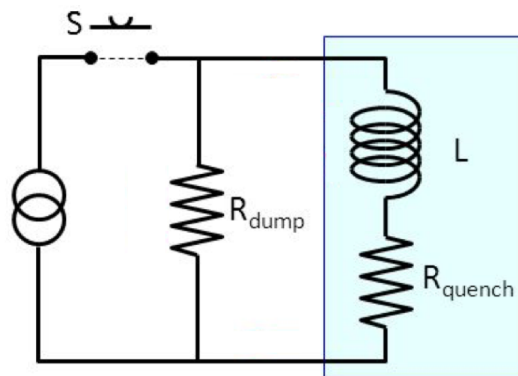


Figure 6.6: Circuit representation of the cyclotron during a quench

When a quench occurs, R_{quench} (the resistivity of the wires) increases quickly. The goal of a quench protection device is to cut the current as fast as possible in order to limit the Joule losses and the overheating of the wires. For this purpose, the power supply is disconnected and a dump resistor R_{dump} is connected in parallel. All the energy stored in the inductance will be discharged in these resistors (R_{quench} and R_{dump}) and the current will follow a decreasing exponential:

$$I(t) = I_{op} \cdot e^{-t \frac{R}{L}} \quad (6.5)$$

The protection system aims to reduce the time constant, which is in the form of $\tau_{dump} = \frac{L}{R}$ (with $R = R_{quench} + R_{dump}$). The dump resistor R_{dump} has to be sized carefully. Indeed, the resistor has to be large enough to reduce the time constant (and limit the temperature increase) but it should not be too large because it would in this case induce a voltage exceeding the maximum allowed and provoke a voltage breakdown. The maximal voltages allowed are generally around 1-2 kV [27].

In addition to the dump resistor R_{dump} , another method that is widely used in superconductors are the **coil heaters**. This method consists in heating the entire coil in case of quench to make it all resistive. It will increase the value of R_{quench} and reduce the time constant of discharge. This method is often used in combination with the addition of a dump resistor, for example in the original S2C2 or in the Large Hadron Collider at the CERN. Unfortunately, for reasons that will be explained later, this method can not be applied in our case. The sizing of our quench protection system will thus only be based on the value of R_{dump} .

Maximal hot-spot temperature

During a quench, the important parameter that has to be controlled is the maximal hot-spot temperature. This temperature has to remain under a given value to limit the mechanical stresses due to fast thermal expansion of the components (and overpressure of helium, as it will be explained later). In practice, as explained by Bottura in [21] this temperature is not highly affected by the heat transport in the wires. It is then possible to compute the maximal temperature, assuming adiabatic conditions [8] by writing a simple heat balance during a quench between heat generation and heat capacity of the components of the wire.

$$\overbrace{\frac{\rho_{cu}(T)}{f_{cu}} J_{op}^2(t) dt}^{\text{Joule Losses}} = \overbrace{\sum_i f_i C_i(T) dT}^{\text{Enthalpy of the components}} \quad (6.6)$$

The f_i 's are the fractions of each material of the cable (f_{cu} is the fraction of copper), the C_i 's are the heat capacities of the different materials and ρ_{cu} is the resistivity of copper.

Eq. 6.6 means that all the energy release by Joule losses in the stabilizer has to be absorbed by the specific enthalpy of the different components. $J_{op}(t)$ is the current density that follows a decreasing exponential as in Eq 6.5.

We define a γ function as:

$$\gamma(T) = \frac{\sum_i f_i C_i(T)}{\rho_{cu}(T)} \quad (6.7)$$

Afterwards, from Eq 6.6 we isolate all the terms dependent on the temperature on one side the the ones dependent on the time on the other side. By integrating we obtain the following expression:

$$\int_{T_0}^{T_{max}} \gamma(T) dT = I(T_0, T_{max}) = \frac{1}{f_{cu}} \int_0^\infty J_{op}(t)^2 dt \quad (6.8)$$

What gives, using the exponential definition of J_{op} :

$$\Rightarrow I(T_0, T_{max}) = \frac{1}{f_{cu}} J_{op}^2 (\tau_{det} + \frac{\tau_{dump}}{2}) \quad (6.9)$$

Where τ_{det} and τ_{dump} [2] are the detection delay and the dump time constant respectively. Several quench detectors are placed along the winding and trigger the quench protection system in case of too high voltage rise (due to the resistance introduced by the quench). The detection delay τ_{det} is the time between the initiation of the quench and the activation of the protection system.

$I(T_0, T_{max})$ only depends on the value of the maximal hot-spot temperature (the objective value of T_{max} is generally chosen by the designer). τ_{det} is a fixed value, given by the quench detection system used.

Using 6.9 we can derive the value of τ_{dump} (all the other values are known). It is then possible to compute the value of V_{dump} :

$$V_{dump} = -L \frac{dI}{dt} = \frac{2E_m}{I_{op} \cdot \tau_{dump}} \quad (6.10)$$

²As already stated, $\tau_{dump} = \frac{L}{R_{dump} + R_{quench}}$

Ultimately the dump resistance using simply the Ohmic law

$$R_{dump} = \frac{V_{dump}}{I_{dump}} \quad (6.11)$$

In practice, the limiting value will be the dump voltage V_{dump} . We can indeed not put a too high voltage on the superconducting magnets without inducing voltage breakdowns in the helium or from one winding to another. We see on Eq. 6.10 that the stored magnetic energy E_m has a direct influence on V_{dump} . It confirms the interest of having a value of E_m as small as possible.

6.2 Choice of the cooling method

After this short introduction to the basic concepts of superconductivity, we may begin with the design of the conductor of the ironless S2C2. The first step is the evaluation of the losses that have to be evacuated. The conductor has to be able to evacuate these losses without entering the normal zone.

6.2.1 Computation of the losses

The losses are mainly AC losses due to the variations of the magnetic field and current during the treatment of the patients. In order to estimate their value, we will analyze more precisely the required beam energy and its variations during one treatment. We first have to find the scaling laws between the required output beam energy, the current in the wires and the magnetic field. Afterwards, we will compute what are the AC losses caused by a treatment and take this value as basis for the design of our conductor. We will finish this section by choosing the cooling method that best suits our situation.

Variable beam energy

1. Scaling laws

It is possible to find coefficients between the different values and their evolution with the output beam energy. The derivation of the scaling coefficients is done by the MIT in [5]. We briefly recall the main results.

Using the basic equation of the cyclotron Eq 1.2 and the definition of γ ³, we obtain the general expression of the *rigidity* at radius r :

$$B_z(r)r = \frac{m_0 c}{q} \sqrt{\gamma(r)^2 - 1} \quad (6.12)$$

Thanks to this value, we can compute K_B the magnetic field scaling coefficient defined by:

$$K_B(K_T) = \frac{B_z(R_{ex}, K_T)}{B_z(R_{ex}, 1)} = \frac{I_{op}(K_T)}{I_{op}(1)} = \sqrt{\frac{\gamma(R_{ex}, K_T)^2 - 1}{\gamma(R_{ex}, 1)^2 - 1}} \quad (6.13)$$

³Here $\gamma(r) = \frac{1}{\sqrt{1 - \left(\frac{v(r)}{c}\right)^2}}$ is the Lorentz factor expressing the relativistic mass change of a moving object. It has not to be confounded with the function $\gamma(T)$ defined in the previous section for quench protection.

Where K_T is the beam energy scaling coefficient defined by:

$$T(K_T, R_{ex}) = K_T T_0(R_{ex}) \quad (6.14)$$

T being the beam energy. We also have the stored magnetic energy in the synchrocyclotron varying as:

$$E_m(K_T) = K_B(K_T)^2 \cdot E_m(1) \quad (6.15)$$

The beam stability conditions are still guaranteed at every beam energy levels. Indeed, since the focusing values ν_r and ν_z are dependent on the derivatives of the magnetic field and not on the magnetic field itself, they keep the same value in spite of a change in output beam energy.

2. Beam energy modulation

Using these scaling laws, it is possible to analyze the evolution of the magnetic field (and of the current density) with the required evolution of the beam energy during a real treatment. We here use the description of a treatment done in the cyclotron's specifications in Table C.1 from Appendix C.

For a good treatment of the tumor in the three dimensions, the beam has to vary its energy to reach different depths in the body. The time of painting has to remain short. At each session, the tumor is covered four times and the patient undergoes around 20 sessions in a complete cure. Hence the treatment has to be fast enough to treat multiple patients and be cost-efficient.

On the other hand, too fast treatments would produce too fast changing magnetic fields and higher AC losses. The synchrocyclotron would need a more complex cooling system, what could increase its acquisition price. For our validation, we took 40 seconds for the total ramp in one direction (evolution of the beam energy from its lowest value to its highest value).

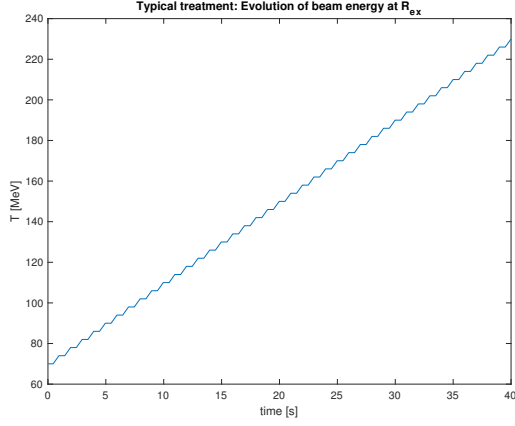
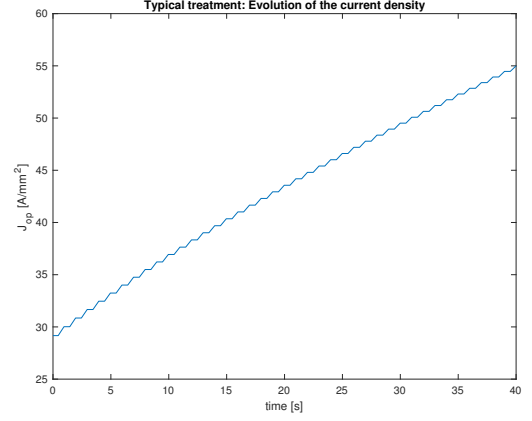
With $T_{min} = 70MeV$ and a $T_{max} = 230MeV$, we divided the painting in steps of $\Delta T = 4MeV$ in a changing time interval of $\Delta t_d = 0.5s$. The beam will cover the tumor horizontally in each layer during a time interval of $\Delta t_h = 0.5s$.

The evolution the beam energy and of the current that has to be put are presented on Fig 6.7 and Fig 6.8. The curves are made of stairs since the evolution is based on steps. For a complete cycle, the ramp is repeated downward. In our specifications, a treatment session contains two complete cycles.

During a rising ramp the voltage at the current source is positive each time the current is increasing, an null elsewhere. We have thus a positive input of energy when the current is not constant and this energy is stored as magnetic energy. During a falling ramp, the large majority of this magnetic energy is reinjected on the grid (the cyclotron acts then as a electricity storer) and the rest is lost in the wires, in particular through AC losses.

AC losses

The exhaustive derivation of all the AC losses is complex and out of the scope of this work. However, making some simple assumptions it is possible to have a first estimation of their importance, allowing us to develop a realistic design of the superconducting wire.

Figure 6.7: Evolution of $T(R_{ex})$ with t Figure 6.8: Evolution of J_{op} with t

The main AC losses in a superconductor have three components: hysteresis, coupling and eddy losses (these types of losses are described in [36]). The coupling and eddy losses are proportional to $\frac{dB}{dt}$. Since the duration of one ramp is relatively large, we neglect these losses and assume the hysteresis losses to be predominant [3]. These losses are only dependent on the initial and final value of the magnetic field but not on the rate of variation.

For the hysteresis losses computation, we neglect the constant level parts in the evolution of the current (without change of current density, there are no hysteresis losses). We then assume the current to increase constantly between $J_{min} = 29.17 \text{ A/mm}^2$ and $J_{max} = 55 \text{ A/mm}^2$ in 20 seconds.

We can evaluate the losses using the Kim-Anderson approximation of the critical surface [26]:

$$J_c(B) = J_o \frac{B_o}{B + B_o} \quad (6.16)$$

Where J_o and B_o are fitting parameters for the superconducting material.

The expression of the hysteresis losses is then derived, using the magnetization $M = \frac{2d_f J_c(B)}{3\pi}$ from [36].

$$Q_h = \int_{B_1}^{B_2} M dB = \frac{2d_f}{3\pi} \int_{B_1}^{B_2} J_c(B) dB = \frac{2d_f}{3\pi} \int_{B_1}^{B_2} \frac{J_o B_o}{B + B_o} dB = \frac{2}{3\pi} d_f B_o J_o \ln \left(\frac{B_0 + B_f}{B_0 + \alpha B_f} \right) \quad (6.17)$$

This equation gives us the losses per unit volume of superconducting material at a given point in the domain.

The values of fitting parameters are $B_o = 0.0219 \text{ T}$ and $J_o = 5.62 \cdot 10^{11} \text{ A/m}^2$ [24]. We have also $\alpha = \frac{B_i}{B_f}$ where B_i and B_f are the local initial and final magnetic field. Since we want to find an upper value of the hysteresis losses, we will compute them at the highest field point. We then have $B_i = 4 \text{ T}$ and $B_f = 7.55 \text{ T}$. The parameters and values used to compute the AC losses are summarized in Table 6.2:

⁴The computation of the fitting parameters is available in Appendix G.3

Conductor	NbTi
B_0 [T]	0.0219
J_0 [A/m ²]	$5.62 \cdot 10^{11}$
d_f [μ m]	12
α	0.53
B_f [T]	7.55

Table 6.2: Hysteresis losses

And we obtain losses in the superconductor during one ramp of:

$$Q_h = 17865 \text{ J/m}^3 = 17.865 \text{ mJ/cm}^3$$

We here used the fact that the superconductor was NbTi, choice that will be justified later.

Note:

The parameter d_f , used in the above computation, is the superconducting filament diameter. Indeed, as explained here above, the wires are not made of full superconductor but of thin superconducting filaments embedded in a copper matrix. The subdivision in very thin strands of superconducting material reduces the flux jumping and the field distortion, two problems that appear in superconducting wires (these two issues are explained in [36]). The maximal diameter of these strands to avoid flux jumping is easily computed using [35]. The computation in our case is shown in Appendix G.5. We obtain a maximal diameter of $d < 212.62 \mu\text{m}$. This value is however largely above what is usually done in the industry and, following advice from the MIT, we chose a customary value of $d_f = 12 \mu\text{m}$. Since we are significantly under the maximal limit, we do not have to care about flux jumping.

6.2.2 Comparison between cooling methods

For the design of the superconducting wire, we first have to choose a cooling method. Thanks to the evaluation of the AC losses during one ramp, we can now estimate the total power that has to be evacuated during a complete treatment and choose the most appropriate cooling technique.

For this estimation, we make strong assumptions:

- **Heat transfer** through the conductor is **instantaneous**, which means that the all AC losses can be evacuated directly by the cooling system.
- **Heat evacuation** is only done through the cooling system (i.e. we neglect the variations of enthalpy of other materials and consider their temperature to stay constant).
- **Heat generation** (in superconducting mode) is **uniform** over the conductor and is approximated by the value calculated here above (hysteresis losses at the **highest field point**).

These assumptions will only serve for the choice of the cooling method. For the design of the conductor we will use a more precise model. Three main cooling options have been identified: the **cryocoolers**, the **helium bath** and the **cable-in-conduit conductors (CICC's)**. The first one consists in a *dry* cooling system, where the heat is only evacuated by conduction. In a helium bath, all the conductors are immersed in helium at saturation. The heat is then evacuated

by evaporation of this helium. For the third method, cable-in-conduit, helium flows directly in the wires, which allows a significantly higher exchange surface between the conductors and the wires.

1. Basic **cryocoolers** (an example of cryocooler is shown in Appendix [G.2](#)), are dry cooling systems used for example by the S2C2. They extract the heat by conduction and have a cooling power of about 1 W at 4.5 K. We can estimate the heat evacuation power to assess the feasibility of this solution. Using the AC losses value and the volume of superconductor in the S2C2, we obtain a total heat generation during one ramp of:

$$Q_{tot} = f_{sc} \cdot V_c \cdot Q_h = 1.4635 \quad kJ \quad (6.18)$$

We used here the value $f_{sc} = 0.0925$ which is the fraction of superconductor material in the conductor. The value used here will be explained later. It gives a required cooling power of:

$$P_h = \frac{Q_{tot}}{t_{ramp}} = 36.5884 \quad W \quad (6.19)$$

Where t_{ramp} is the duration of a ramp (40 seconds in our case).

We thus observe that the required cooling power is too high for this cooling method, or the number of cryocoolers would be too high. To have an idea, the S2C2 uses 4 cryocoolers, which is already a high number in comparison with other applications.

2. To assess the **helium bath** option, we have to measure the quantity of helium that has to be evaporated. Using the properties of helium at saturation [\[18\]](#), we have $\rho_{he} = 0.11 \text{ g/cm}^3$ and $\Delta h_{vap} = 18.5 \text{ J/g}$.

The losses for a complete treatment are equal to $Q_{treat} = 4 \cdot Q_{tot} = 5.8541 \text{ kJ}$ (as stated in the last section, 2 cycles are needed, thus 4 ramps. The total heat released is equal to 4 times Q_{tot} above calculated). The volume of helium needed is:

$$V_{he} = \frac{Q_{treat}}{\Delta h_{vap} \cdot \rho_{he}} = 2.8767 \quad \text{liters} \quad (6.20)$$

This value is quite high but does not seem unfeasible neither. In fact, after discussion with the designers of the first ironless models at MIT, they said that to assess the helium bath option, the assumption that the heat transfer is instantaneous is quite strong. With a coil of our dimensions, it could take seconds up to minutes (due mostly to the insulation between conductors) and a high difference of temperature between the center of the coil and the helium would be needed. This high temperature could provoke a quench and this is the reason why another method has to be used. The heat transfer in the winding pack is complex and its study is out of the scope of this work.

3. Therefore, we chose to use the last option, **cable-in-conduit conductors** (CICC).

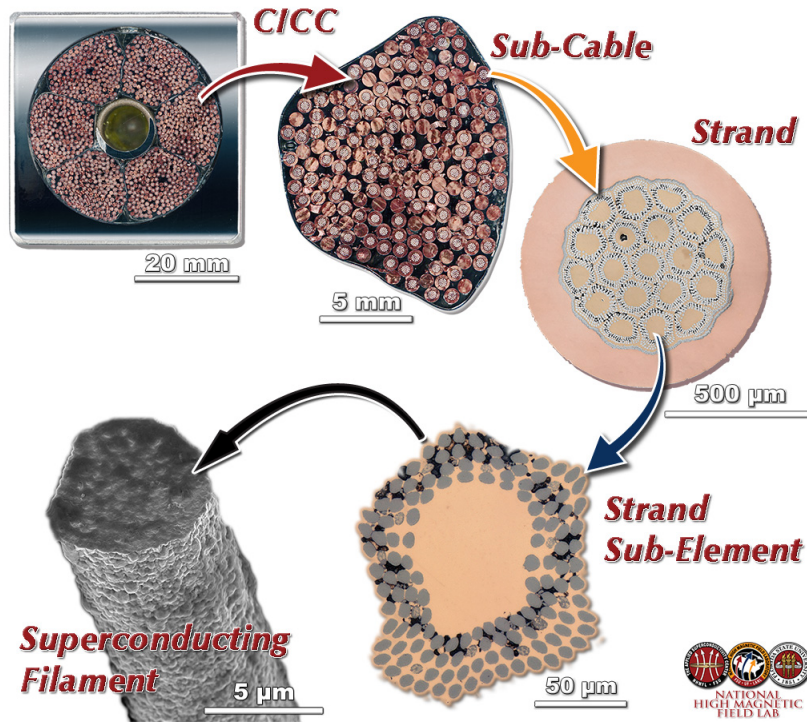


Figure 6.9: Cable-in-Conduit conductor (example: ITER)

An example of cable-in-conduit conductor is shown on Fig 6.9. It consists in multiple strands of copper containing superconducting filaments immersed in supercritical helium⁵. The presence of helium reduces the proportion of superconducting material in the cross-section and thus lowers the overall engineering current density. Besides, helium provides a larger *heat sink* than the sole enthalpy of conductor material⁶. This type of conductor is thus generally used in situations with changing magnetic fields and high AC losses such as, for example, in ITER.

6.3 Choice of the superconducting material

After the cooling method, we have to choose the superconducting material that will be used in our cyclotron. A curve comparing different superconducting critical surfaces for a fixed current density is shown in Appendix G.4. Between all the superconductors presented, NbTi has the worst critical surface (lower B_c and T_c) but it has the important characteristic to be the only one that is ductile. Since this property is a serious advantage for the manufacturing of the coils, NbTi is by far the superconducting material that has been the most used in accelerators [35].

Beside NbTi, the use of other materials has been developed throughout the last years. The most mature of them is Nb₃Sn. This material is already used in some cyclotrons (MeVion - Monarch

⁵All the existing CICC are not made with supercritical helium. Sometimes helium is put in its superfluidity state, a state encountered at very low temperature (≈ 2.18 K) and low pressure. It is characterized by a fluid with almost no viscosity and a very high heat conductivity. These specificities are of course very interesting for a cooling fluid. But since the fluid dynamics of superfluid helium between the strands is not totally understood up to now, we used a more classical form of helium in CICC, its supercritical state. More information about superfluid helium in CICC is given in [21].

⁶At cryogenic temperatures the heat capacity of copper and superconductors is very small and tends toward 0 at the absolute 0 K. Helium is the only known component that has a significant heat capacity at very low temperatures [20].

250) and in important projects (ITER, improvement of LHC at the CERN. It can stand higher current densities in higher magnetic fields. But Nb_3Sn has an important drawback: it is brittle. Its critical surface is dependent on the strain that the wire is suffering, what makes its design more complex.

Because of its brittleness, manufacturing of windings based on Nb_3Sn is significantly more difficult than using NbTi windings. Different methods exist that consist in first creating the cable containing the different components (Nb and Sn separated) and then to provoke a reaction (around 700 °C) to produce the Nb_3Sn . Different method exist (RRP, PIT, Bronze route) and are described in [25].

The dependence of the critical current on the strain also raises several issues (Fig 6.10 shows the evolution, following the Bordini parametrization of the critical surface [6]). We observe that the critical current is strongly degraded for high strains. Unfortunately, important strains occur frequently due to the high thermal shock that the windings undergo between the high reaction temperature and the cryogenic low temperatures. Strains can reach up to $\epsilon = -0.7\%$. It means that the windings have to be prestressed by the supporting structure and the design is made much more complicated⁷.

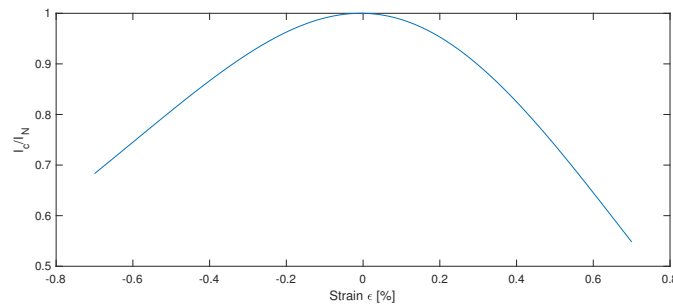


Figure 6.10: Dependence of the critical current of Nb_3Sn on the strain ϵ (Bordini parametrization)

Other, more efficient, materials are today under development. These materials have a large critical surface and allow thus higher currents, magnetic field and/or temperature. Examples are MgB_2 or HTS (High Temperature Superconductors) such as YBCO or BSCO. But despite promising possibilities, these materials are only at prototype state and a more mature technology (NbTi or Nb_3Sn) has to be chosen here.

The choice of the superconducting material is finally reduced to NbTi or Nb_3Sn . In order to take this decision, we developed one model for each case.

Since the critical surface of Nb_3Sn is larger, it will be possible to use higher current densities and thus to reduce the conductor quantity. We will compute the conductor quantity for each model and eventually look at the difference of price between both models. Following [10], we take a difference of price per meter of winding of $\text{Price}_{\text{Nb}_3\text{Sn}} = 5 \cdot \text{Price}_{\text{NbTi}}$ taking into account the price of the material itself and manufacturing costs. In conclusion, if the use of Nb_3Sn allows us to reduce the superconductor quantity by 5 times or more we will use it. If not, the we will prefer NbTi .

⁷Strictly speaking, the critical current of NbTi is also dependent on the strain but this dependence is significantly lower than for Nb_3Sn . In practice, it is almost always neglected.

Ironless S2C2 using Nb₃Sn

The layout of the conductor using Nb₃Sn is given in Table 6.3. The different characteristics of the winding that are exposed will be explained in the following sections. For the sake of comparability, we took the same geometrical characteristics than for the NbTi model. The maximal engineering current density that may be used is:

$$J_{op} = 65 \text{ A/mm}^2$$

Which is of course higher than the one used with the NbTi option. The topology of the ironless S2C2 obtained for this current density is shown on Fig. 6.11.

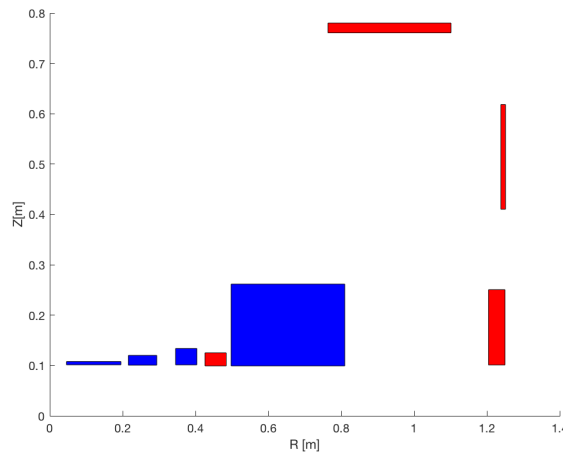


Figure 6.11: Topology of the ironless S2C2 made of Nb₃Sn

We finally obtain a volume of conductor of:

$$\text{NbTi : } V_c = 0.885 \text{ m}^3 \quad \text{Nb}_3\text{Sn : } V_c = 0.665 \text{ m}^3$$

The volume reduction is significant but not large enough to justify the use of Nb₃Sn.

Furthermore, we observe on Table 6.3 that the temperature margin is very large, what means that significantly higher losses could be evacuated. In other word, the design in Nb₃Sn does not take advantage of all the potential of the material and is then not economically efficient (but further increase of the current density is here restricted by the high dump voltage: $V_{dump} = 2052 \text{ V}$).

6.4 Conductor design

The design of CICC and the derivation of the optimal proportions of stabilizer, superconductor and coolant is a tricky exercise. The designer's goal is to maximize the current density with a wire design that will still satisfy the requirements of stability, protection and cost [39]. In this section, we will present a feasible layout to prove our solution is practically achievable. Some optimization should be needed if the cyclotron had to be developed further.

Table 6.3 summarizes the characteristics of the windings. The inputs are in the first part and the outputs in the second. The following sections will expose the different steps leading to these values.

6.4.1 General hypotheses

We designed our conductor with the *CICC Design Tool*⁸, an Excel spreadsheet often used by the team of the MIT for the dimensioning of cable-in-conduit conductors. This spreadsheet is joint to the present work.

The *CICC Design Tool* is based on several assumptions:

- The **temperature of the helium** is considered as **uniform over the cable cross section**.
- The **current density** is considered as **uniform over the strand cross section**.
- **Heat production** is supposed to happen on an infinite length of wire.
- The **heat transfer coefficient h** is considered as **constant**. Its value comes from [21].

These assumptions are discussed more deeply in [21] and [8]. The analysis is made in a 0D geometrical domain. Indeed, the two first assumptions allow to neglect variations on the cross-section and the hypothesis of infinite length allows to neglect longitudinal variations. These conclusion come from the analysis of the time constant of the different phenomena, in [8]. The conclusions are that the flow of helium is significantly slower than the heat transfer from the cable to the helium (what allows to only treat the case locally, in one cross section) and that due to the insulation, the heat transfer from one windings to the surrounding ones is also slow (what allows in first approximation to neglect the influence of the other cables).

The following design is based on the conditions at the highest field point. This choice is conservative and it would have been more optimal to develop different designs for the different coils or type of coil (shielding/main/shaping). But these multiple designs would have led to different current densities, what is against one of the basic assumption used in this work. Nevertheless, the introduction of different current densities would be an interesting improvement of the model.

The reasoning consists in making varying several inputs and check the different outputs to verify that they do not exceed given limits. Three phenomena have to be particularly checked:

- **Margins on the operating conditions:** The operating conditions and their corresponding critical values are derived. We will check if the margins on all the variables are sufficient for a safe operation of the synchrocyclotron.
- **Quench protection:** The quench protection is designed. We want our operating conditions to allow a safe quench protection system.
- **Stability:** The stability of a superconductor to external disturbance is its ability to evacuate a given energy input in steady state operating mode without losing its superconductivity. As we will explain further, in our case it will not be a limiting factor but for the sake of completeness, the concept will be briefly introduced at the end of this section.

The main inputs and their range of possible values are described in the first section. Afterwards, the outputs are presented, with the margins and values that they have to respect. This second section will cover one by one the three important phenomena just mentioned.

⁸Developed by A. E. Long, J. V. Minervini and A. Radovsky, *Massachusetts Institute of Technology* 1993, Updates in 1999 & 2015

Inputs			
Conductor		NbTi	Nb ₃ Sn
Geometric parameters			
d_{strand}	mm	0.75	0.75
f_{Cu}		0.67	0.67
N_{comp}		32	32
N_{cu}		16	16
N_{tot}		48	48
W_c	mm	6.5	6.5
H_c	mm	6.5	6.5
T_w	mm	0.5	0.5
T_{ins}	mm	0.3	0.3
Operating conditions			
I_{op}	A	2750	3300
T_{in}	K	4.5	4.5
P_{avg}	atm	2.6	2.6
Quench protection			
t_{det}	sec	0.5	0.5
T_{hot}	K	150	150
Outputs			
Geometric parameters			
A_{env}	mm^2	50.41	50.41
A_{ss}	mm^2	11.61	11.61
A_{ins}	mm^2	8.58	8.58
A_{he}	mm^2	9.01	9.01
A_{cu}	mm^2	16.54	16.54
A_{sc}	mm^2	4.67	4.67
f_{he}		0.2982	0.2982
Performance characteristics			
B_{op}	T	7.55	7.69
Q_{loss}	mJ/cm^3	71.46	308.84
T_{op}	K	4.547	4.702
J_c	A/mm^2	1571	1665
J_{op}	A/mm^2	589.46	707.36
J_{wp}	A/mm^2	55	65
T_c	K	6.15	11.26
T_{cs}	K	5.551	8.475
$dT = T_{cs} - T_{op}$	K	1.004	3.773
J_{op}/J_c		0.38	0.42
Quench protection			
E_{em}	MJ	22.15	20.1
R_{dump}	Ω	0.67	0.62
t_{dump}	sec	8.99	5.94
V_{dump}	V	1792	2052
Stability			
h	W/m^2K	1000	1000
I_{op}/I_{lim}		1.24	0.74

Table 6.3: Conductors lay-out

Strand diameter	d_{strand}	mm	0.75
Fraction copper in composite strands	f_{Cu}		0.67
Number of composite strands	N_{comp}		32
Number of pure Cu strands	N_{cu}		16
Width conductor	W_c	mm	6.5
Height conductor	H_c	mm	6.5
Thickness wall	T_w	mm	0.5
Thickness insulation	T_{ins}	mm	0.3

Table 6.4: Geometrical inputs

6.4.2 Inputs *CICC Design Tool*

1. Geometric parameters

The geometrical inputs are summarized in Table 6.4.

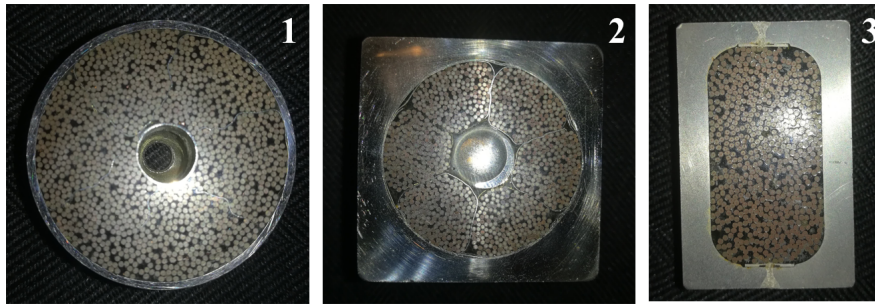


Figure 6.12: Different types of conductor (designed by PSFC at MIT)

1-Round conduit/Round conductor 2-Square conduit/Round conductor 3-Rectangle conduit/Rectangle conductor

Different designs of CICC are shown on Fig 6.12. They all come from the ITER Tokamak⁹. The conductor configuration that we chose is a **rectangular conductor** with a **rectangular conduit** (3 on Fig. 6.12). This layout has the advantage to provide high mechanical stability between the windings (due to the rectangular stainless steel jacket) and a high current density (due to the rectangular conduit that allows a high filling factor).

We decided not to put a central core. In some CICC, the center of the cable contains a central core carrying only helium (1-2 on Fig 6.12). This additional flow may serve to increase further the heat sink when the losses are too high. It also has a role in the evacuation of pressure peaks and as an extra structural element. Since in our case the losses are easily evacuated using a more *classical* design, we do not need the extra central core.

The diameter of the strands d_{strand} has been fixed to 0.75 mm¹⁰. Usually, this value is between 0.7 mm and 1 mm. It has to be small enough to allow a high number of strands per m^2 and in consequence a large contact surface between the strands and helium. But it may of course not be too small for evident manufacture reasons [11].

⁹These conductors are of course significantly larger than the ones used in our ironless synchrocyclotron. Nevertheless, the layout has more or less the same shape.

¹⁰The diameter of the strands d_{strand} has not to be confounded with the diameter of the superconducting filaments d_f . The diameter of the latter is significantly lower than the one of the former. These two values are thus independent from each other.

Operating current	I_{op}	A	2750
Inlet helium temperature	T_{in}	K	4.5

Table 6.5: Inputs operating conditions

Detection delay	t_{det}	A	2750
Hot-spot temperature	T_{hot}	K	150

Table 6.6: Inputs quench protection

There are two different types of strands, strands made only of copper and others composite strands made of superconducting filaments embedded in a copper matrix. The number of copper and composite strands (N_{comp} , N_{cu}), their diameter (d_{strand}) and the dimensions of the conductor (W_c , H_c) are of course correlated by dimensional constraints. The remaining space of the conduit is called the void fraction f_{he} (part of the cable that contains the helium), an important parameter that will be discussed later.

The thickness of the stainless steel wall T_w is also an important parameter. When the cyclotron is in operation, the windings undergo large mechanical stresses due to thermal compression and electromagnetic forces (See section 6.5). It is the steel jacket that will support all these stresses (since the strands are strongly tight in the conduit, all the stresses that they will bear will be reverberated on the steel jacket). This steel layer has to be large and strong enough to withstand these efforts, but not too large since it would reduce the average current density and the void fraction f_{he} .

T_{ins} is the thickness of insulation. We here took a customary value that allows sufficient insulation but does not take too much space.

The last important parameter is f_{Cu} the copper fraction in the composite strands. Its value is generally between 0.2 and 0.67. These limits are mostly due to mechanical reasons and quench protection considerations [11]. We here chose a value of $f_{Cu} = 0.67$, in order to limit the quench protection dump voltage (explanations in the following sections).

2. Operating conditions

The operating conditions of our conductor are summarized in Table 6.6. The operating current I_{op} is one of the most important input. It has to be chosen to allow a maximal current density but still respect the different margins that will be exposed later. It has an influence on the operating conditions and the margins but of course also on the dimensioning of the protection system.

The inlet temperature T_{in} is the temperature of the helium when it enters in the cable (or when it has been cooled down when the helium is stagnant). $T_{in} = 4.5$ K is a customary value. The average pressure P_{avg} has to be high enough to put the helium in its supercritical state but not too high for evident mechanical reasons. We also took P_{avg} as a customary value.

3. Quench protection

The values that have been chosen for t_{det} and T_{hot} are again usual values in the design of CICC.

The quench detection delay t_{det} is dependent on the quench detection system. This system triggers the quench protection in case of voltage rise in the conductor (due to appearance of a resistive zone due to the quench). They are difficult to design and the detection delay

Envelope area	A_{env}	mm^2	50.41
Steel jacket area	A_{ss}	mm^2	11.61
Insulation area	A_{ins}	mm^2	8.58
Helium flow area	A_{he}	mm^2	9.01
Area of copper	A_{cu}	mm^2	16.54
Area of superconducting filaments	A_{sc}	mm^2	4.67
Cable space void fraction	f_{he}		0.2982
Superconductor fraction	f_{sc}		0.0926

Table 6.7: Conductor geometric parameters

time is often a trade-off between a fast and efficient detection system and a robust system that does not trigger the protection system if it is not needed.

The maximal value of the hot-spot temperature T_{hot} is chosen to limit the pressure rise during a quench.

6.4.3 Outputs *CICC Design Tool*

1. Geometric characteristics

The first type of outputs are the geometric characteristics. Table 6.7 is a summary of their values. They have been directly computed using the geometric parameters taken as input.

The most important one is the cable space void fraction f_{he} . Its value is limited by mechanical constraints and is often between $f_{he} = 30\%$ and $f_{he} = 40\%$. If the void fraction is too large, the strands will move significantly in the conduit, what could damage them or lead to inaccuracies in the shape of the field. If on the contrary, there is not enough void space, the strands will be compressed on each other, what could also damage them. Furthermore, it would limit the helium quantity and reduce the available heat sink.

2. Performance characteristics

The second type of outputs are the performance characteristics. These are the operating conditions and critical values that derive from the inputs. They allow to evaluate the margins of our operating point and, if necessary, to adapt the geometrical and operating inputs to respect the necessary values.

The performances characteristics and critical values of our windings are summarized in Table 6.8. The margins are computed and compared to the objective values in Table 6.9. The schematic representation of the following reasonings is given in Appendix ??.

First, the engineering current density¹¹ J_{op} is computed from the operating current input and the total cross section area of the conductor. This current density is sent as input to the topology optimization tool and will give as output the topology, the magnetic energy stored and the peak magnetic field on the conductor B_{op} . Using the value of the latter and the reasoning of section 6.2, we obtain the value of the AC losses Q_{loss} .

Subsequently, the operating temperature T_{op} is deduced from the AC losses. Neglecting the variation of enthalpy of the other components¹², we first compute the increase of enthalpy

¹¹The engineering current density is the average current on the conductor cross section. It is the value used in the previous chapters and is defined as $J_{op} = f_{sc} \cdot J_{sc} = \frac{I_{op}}{A_{env}}$.

¹²As already stated, at cryogenic temperatures there is a factor of two to three orders of magnitude between the enthalpy of the helium and the one of the other materials constituting the cables [20].

Peak magnetic field	B_{op}	T	7.55
AC losses	Q_{loss}	mJ/cm^3	71.46
Operating temperature	T_{op}	K	4.547
Critical current density	J_c	A/mm^2	1571
Operating current density in superconductor	J_{sc}	A/mm^2	589.46
Engineering current density	J_{op}	A/mm^2	55
Critical temperature	T_c	K	6.15
Current sharing temperature	T_{cs}	K	5.551
Critical magnetic field	B_c	T	9.436
Critical current on load line	J_{max}	A/mm^2	731.28

Table 6.8: Performances characteristics

of helium:

$$h_{final} = h_{initial} + \frac{1}{\rho_{initial}} \cdot Q_{loss} \cdot \frac{A_{sc}}{A_{he}} \quad (6.21)$$

In Eq. 6.21, $\rho_{initial}$ is the density of helium and is used to convert the volume losses in mass losses. The resulting heat has also to be multiplied by the factor $\frac{A_{sc}}{A_{he}}$ because the heat is generated in the filaments but we are looking for the quantity of heat that will be absorbed per units mass of helium. As stated at the beginning of this section, we also considered the situation as adiabatic (time constant of helium renewal significantly higher than time constant of heat generation) so analysis is only made locally. Finally, using the thermodynamic properties of supercritical helium, we can find the temperature $T_{op} = T(h_{final}, p_{avg})$ as a function of the enthalpy and the pressure.

This temperature, in addition with the maximal magnetic field will allow us to find the critical current J_c , magnetic field B_c and temperature T_c using the Summers parametrization (Eq. 6.1, Eq. 6.2 and Eq. 6.3).

For the computation of T_{cs} , we use the linear expression of the critical current from Section 6.1.2:

$$T_{cs} = T_c - \frac{J_{op}}{J_c} (T_c - T_{op}) \quad (6.22)$$

Finally, J_{max} is the current that would produce a quench if we follow the load line¹³, i.e. if we take into account the increase in maximal magnetic field B_{max} in function of the increase of the current. It is found using the intersection between the loadline and the critical surface.

It is now possible to compute the different **margins**, using the critical and operating conditions just derived. These margins are illustrated on Fig. 6.13¹⁴.

As in the rest of the report, these margins have been calculated for the point undergoing the highest magnetic field, as it is the most endangered location. A more rigorous analysis would consist in taking several points but we assumed that if it was good for the worst one, it should be good for the others.

¹³The load line is defined by the scalar k in $B_{max} = k \cdot J_{op}$ where B_{max} is the maximal field on the conductors.

¹⁴This figure is only illustrative, the situation corresponding to our situation is shown on Fig. 6.14.

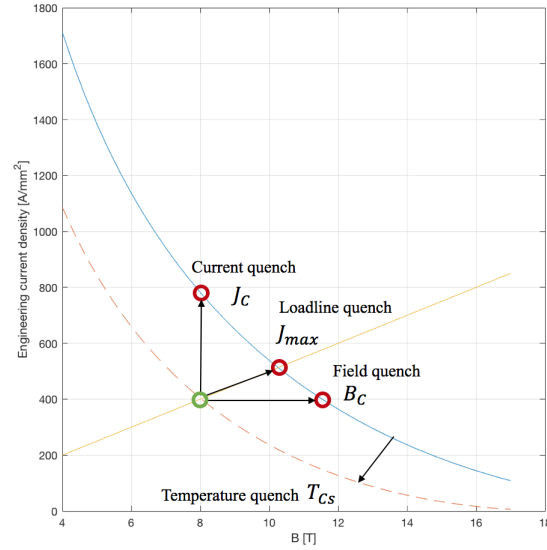


Figure 6.13: Critical surfaces and load line: Different margins

The most important margin in our case is the temperature margin $\Delta T = T_{cs} - T_{op} = 1.004K$, measuring the distance of the operating temperature from the current sharing situation. This margin allows to compute the amount of losses that can be done before reaching the current sharing situation. It will thus be possible to calculate what would be the number of treatments using the *same* supercritical helium i.e. without changing the helium or cooling it. Indeed, we may not warm up the helium after T_{cs} since it would certainly quench the magnet. By a simple equality between the helium enthalpy and the losses, we obtain:

$$\rho_{he} \cdot [h_{he}(T_{cs}) - h_{he}(T_{op})] \cdot A_{he} = n \cdot Q_{loss} \cdot A_{sc} \quad (6.23)$$

The maximal number of treatments is **n = 56 treatments**. This result is only theoretical. Of course the helium will certainly be cooled down before reaching a temperature of T_{cs} , but this value gives us a more intuitive comprehension of the temperature margins; and shows that ours is sufficiently large.

The other margins are shown in Table 6.9 and compared with the usual values from [16].

	J_{op}/J_c	B_{op}/B_c	J_{op}/J_{max}	$T_{cs} - T_{op}$
Objective margins	0.5	0.75	0.85	1
Results	0.38	0.8	0.811	1.004

Table 6.9: Different margins

We observe that our margins are quite close to the usual values. It means that our design is economical and takes advantage of the maximum of the properties of NbTi.

In our situation, the margin on the temperature is the most important. For example the margin on the magnetic field is somewhat too small but it is not a major issue. Indeed, in our case a sudden peak of magnetic field is unlikely to happen in comparison with a peak of temperature due to too intensive use of the cyclotron or external reasons (friction, leak in the cryostat,...).

Our situation is represented on Fig 6.14.

Stored magnetic energy	E_m	MJ	22.15
Hot spot temperature	T_{hot}	K	150
Detection delay time	t_{det}	sec	0.5
Dump resistance	R_{dump}	Ω	0.65
Dump time constant	t_{dump}	sec	8.99
Dump resistance	V_{dump}	V	1792

Table 6.10: Values for the quench protection

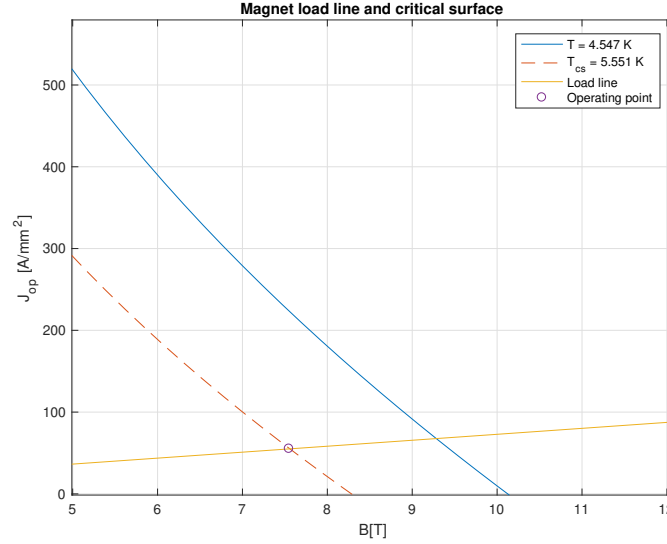


Figure 6.14: Critical surface, load line and operating point

3. Quench protection:

The results of our quench protection system are shown in Table 6.10. In CICC's the hot-spot temperature is mostly chosen to limit the overpressure in case of quench.

As explained in the theoretical introduction about quench protection, the first step is to compute the value of the function $\Gamma(T_0, T_{hot}) = \int_{T_0}^{T_{hot}} \frac{\sum_i f_i C_i(T)}{\rho_{cu}(T)} dT$.

In our case, we used a correlation depending on the geometrical inputs and the hot-spot temperature. This correlation is given in [11] and neglects the enthalpy provided by the helium. Indeed, at higher temperatures (around T_{hot}) the enthalpies of the solid materials become predominant in comparison with the one of helium. In our case, we have:

$$\Gamma(T_0, T_{hot}) = \int_{T_0}^{T_{hot}} \frac{\sum_i f_i C_i(T)}{\rho_{cu}(T)} dT = 1.38 \cdot 10^5 \quad (6.24)$$

The values of t_{dump} , R_{dump} and V_{dump} are derived using Eq. 6.9, Eq. 6.11 and Eq. 6.10 respectively, using the operating current I_{op} , the stored magnetic energy E_m and the geometrical characteristics.

The value of the dump voltage V_{dump} value is generally around 1kV [27] but can go up to 2kV [10]. If the value obtained is too high, I_{op} and the geometrical characteristics have to be adapted (E_m is a direct consequence of these input parameters).

In our case, V_{dump} stays in the possible range of solutions but some improvement to reduce its value could be done if the design would have to be developed further.

We only designed the quench protection system using a dump resistance R_{dump} . Another option explained in Section 6.1.3 is to use coil heaters. Unfortunately, in our case the access to the strands is really limited and this solution may not be used here. It is generally applied on devices with "solid" conductors with an easy access to all the coils.

4. Stability characteristics

The stability is often a limiting factor in the design of CICC's. The idea behind stability is that we want our conductor to operate at the maximal current density possible but to be still able to evacuate a given quantity of losses. The concept of cryostability has been introduced by Stelky [21] for general superconductors and adapted to the case of CICC's by Minervini *et al.* [22]. The model is based on the two concepts of **limiting current** and **energy margin**. We quickly present the simplest version of the theory, in 0D and independent on the time.

The first concept used is the limiting current:

$$\underbrace{\frac{\rho_{cu} I_{op}^2}{A_{cu}}}_{\text{Joule Losses}} \leq \underbrace{P_{st} h_q (T_c - T_{op})}_{\text{Maximal heat transfer}} \Rightarrow I_{lim} = \sqrt{\frac{A_{cu} P_{st} h_q (T_c - T_{op})}{\rho_{cu}}} \quad (6.25)$$

If I_{op} is below the limiting current, the energy margin is equal to the maximal heat sink available (defined as the enthalpy of the different materials between T_{op} and T_{cs}). This region is called *well-cooled*. The intuitive understanding is the following: we may warm up all the conductor until T_{cs} because after the current sharing temperature, the highest possible heat generation (maximal Joule losses, when all the current is in the copper) can still be evacuated by the helium. On the contrary, if the current is too high the conductor enters in the *ill-cooled* region (the Joules losses will be too high to be evacuated by the helium). The heat sink is thus be significantly reduced (we may not bring all the conductor to T_{cs}) and depends on the duration of the heat pulse.

In this simplified model we make two strong assumptions: we consider the difference between the *strand temperature* and the *helium temperature*. In addition we assumed the temperature of helium to be constant.

In practice, for our situation, stability will not be a major issue and we may go above the limiting current (as shown in Table 6.11). This is due to the fact that the heat generation takes place on a large time scale (significantly higher than the usual time scale of heat exchanges between the strands and the helium i.e. $\tau_r \approx 1ms$ [8]). We may thus consider that the strands will always be in equilibrium with the surrounding helium and we do not take into account any temperature difference between them.

It has to be noted that even if Eq. 6.25 ensure sufficient cooling to evacuate the losses, it is based on a very strong assumption, the temperature of helium is considered as constant. We saw in the previous sections that in practice the temperature increases with the losses, what will reduce significantly the maximal heat transfer and the limiting current. A cable-in-conduit conductor undergoing too high or too long heat pulses will always quench, since the available helium is no infinite.

More sophisticated models have been developed (among others by Minervini [22] from the MIT). Since in our case stability is not a key issue, they will not be detailed here. More explanations are given [21], [8] and [38].

Heat transfer coefficient	h	W/m^2K	1000
Fraction of limiting current	I_{op}/I_{lim}		1.24

Table 6.11: Stability characteristics

In this computations, we took a constant value for the heat transfer coefficient of $h = 1000 \text{ W/m}^2 \cdot K$. In practice, this value evolves in the different situations, its evolution follows complex paths and its study is largely out of the scope of this work. Nevertheless it has been shown that an average value could already give accurate results [21].

6.5 Mechanical stresses

In this last section, we will compute the mechanical stresses in the conductors and design a potential supporting structure. The evaluation of the mechanical stresses is an important matter and it can be a limitation on the maximal current density that can be used. We have to take into account two effects:

1. **Electromagnetic forces:** The coils undergo significant Lorentz forces due to the high fields and current densities of the system.
2. **Thermal stresses:** The changes of temperature are strong between room temperature and cryogenics temperature provoking considerable mechanical stresses. Furthermore, the materials of the supporting structure and the conductors have different thermal expansion coefficients, what will induce additional stresses.

We will begin this section by presenting a potential supporting structure. Subsequently, the forces in the conductors and in the supporting structure are computed and compared to the maximal values allowed.

6.5.1 Design of the supporting structure

The design of this supporting structure (often called the *former*) is a complex exercise. The following assumptions have been made to model the situation and develop the design of the supporting structure. This structure has not been optimized and more investigation is required to obtain a more accurate and efficient solution.

Model assumptions

- We model the coils as **isotropic**, having a Young modulus dominated by the steel of the conduit (we took here 316 Stainless Steel) and equivalent to $E_{coils} = \frac{E_{ss}}{f_{ss}}$, where $f_{ss} = \frac{A_{env}}{A_{ss}}$. The Poisson coefficient and thermal expansion properties are considered as the same as those of steel. This is of course a coarse representation of the reality.
- In practice, the coils are not glued on the supporting surface, they may slide on it. It is very difficult to model sliding faces between two bodies in contact with Opera3D so in our simulation, the coils and the supporting structure are glued together. It will produce local effects and stresses that can be neglected in this first analysis. Of course for further development, a more representative model of the reality would be needed.

The supporting structure is shown on Fig. 6.15. It is made of a Stainless Steel 316 plate (in blue), supporting the main and the shaping coils. The shielding coils are in turn supported by

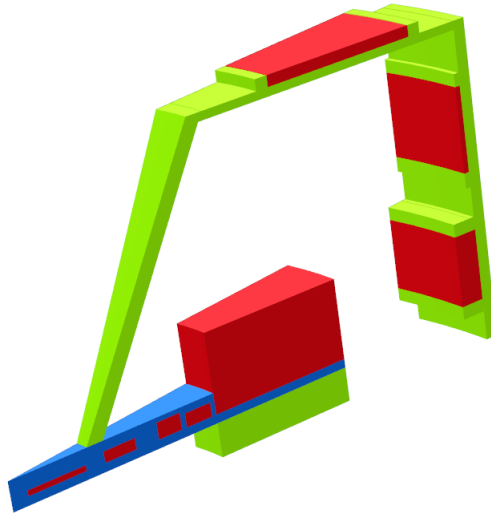


Figure 6.15: Coils with supporting structure (1/36th rotational symmetry)

an aluminium (Al-6061-T6) structure. The idea was to use a stronger support for the main coils and the shaping coils since they undergo higher fields and thus higher stresses. In addition, the walls of the CICC's are also made of 316-SS, and so they have the same thermal expansion coefficient than their supporting structure what will reduce the stresses. For the outer part, we chose aluminium for economical reasons. These two particular materials (316-SS and Al-6061-T6) have been chosen following advice of MIT but other options are of course possible.

The properties of the different material used in our model are given in Table 6.12. E is the Young modulus, ν the Poisson coefficient and ϵ the strain due to thermal compression between room and cryogenic temperatures¹⁵.

	E GPa	ν	ϵ
316 Stainless Steel	205	0.25	-0.00312
Al-6061-T6	81	0.33	-0.00429
Coils	47	0.25	-0.00312

Table 6.12: Properties of the materials used

6.5.2 Evaluation of the mechanical stresses

The two critical values that have to be checked are the hoop stresses in the coils and the von Mises stresses in the supporting structure.

Throughout the following points, we will analyze the different stresses and compare them with the maximal values allowed (from [3]). These maximal values are summarized in Table 6.13. S_{uts} is the ultimate strength, S_y the yield strength and S_m is the stress limit.

¹⁵These values are given by the MIT. <https://bit.ly/2KZRxs3> gives almost the same values.

	S_{uts} MPa	S_y MPa	S_m MPa
316 Stainless Steel	1759	862	575
Al-6061-T6	496	362	165

Table 6.13: Maximal stresses in different materials

Hoop stresses in the coils

The hoop stresses (also called azimuthal stresses) are the stresses occurring in the azimuthal direction of a coil. They are due to the tendency of a coil to expand itself due to the Lorentz forces caused by the interaction of the magnetic field with the current that they are carrying. It is of course very important to have coils able to stand the hoop stresses, without damaging the different material. This is one of the main roles of the stainless steel conduit. The expression of the hoop stresses $\sigma_{\theta\theta}$ in function of the Cartesian stresses σ_{xx} , σ_{yy} and σ_{xy} is:

$$\sigma_{\theta\theta} = \sigma_{xx} \cdot \sin\left(\tan^{-1}\left(\frac{y}{x}\right)\right) + \sigma_{yy} \cdot \cos\left(\tan^{-1}\left(\frac{y}{x}\right)\right) + \sigma_{xy} \cdot \sin\left(2 \cdot \tan^{-1}\left(\frac{y}{x}\right)\right) \quad (6.26)$$

The result, computed with Opera3D is shown on Fig. 6.16

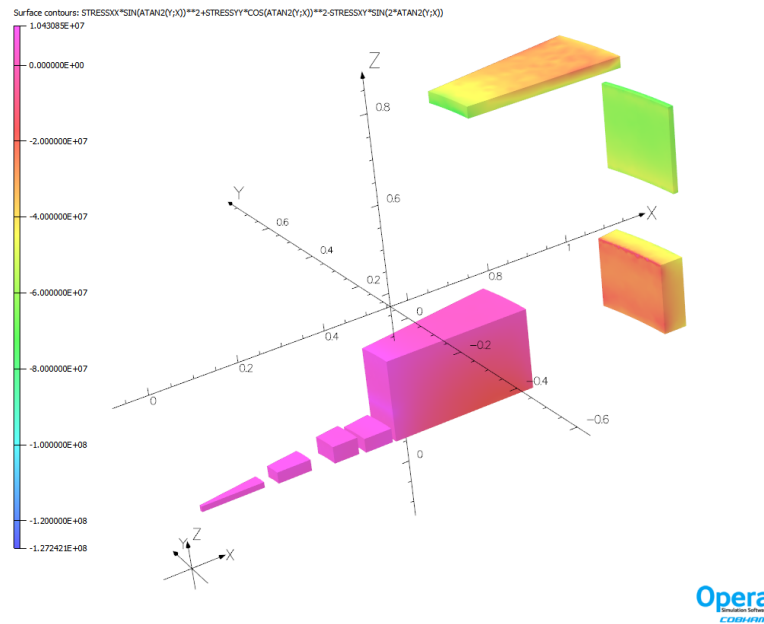


Figure 6.16: Hoop stresses (1/36th rotational symmetry)

We observe that the maximal value is $\sigma_{\theta\theta} = 10.43$ MPa. In Opera, we modeled the conductors as isotropic but in practice only the steel (316-SS) will support the mechanical stresses. We thus have to multiply the obtained value by the factor $f_{ss} = \frac{A_{env}}{A_{ss}}$ to observe what are the real stresses in the 316-SS part of the conductor. Using the conductor layout from the previous section, we obtain $f_{ss} = 4.34$ and a maximal stress value of $\sigma_M = 45.27$ MPa, what is still significantly under the stress limit of Table 6.13

Stresses in the structure

In addition to the forces in the coils, we have to verify that the mechanical stresses in the supporting structure will not cause the material to undergo plastic deformation. For this purpose, we use the von Mises criterion and observe if the mechanical stresses do not exceed the maximal allowable value. The von Mises criterion is defined as:

$$\sigma_v = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} < S_m \quad (6.27)$$

With S_m being the stress limit and σ_1 , σ_2 and σ_3 being the principal stresses. The result of the von Mises stresses in the supporting structure are shown on Fig. 6.17 and Fig. 6.18 for the aluminium and stainless steel part of the supporting structure respectively.

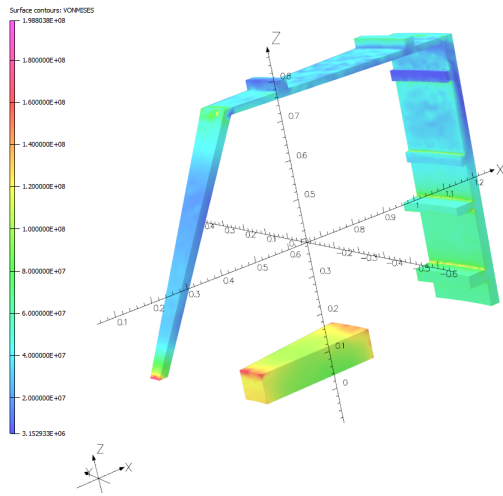


Figure 6.17: von Mises stresses in Al-6061-T6 (1/36th rotational symmetry)

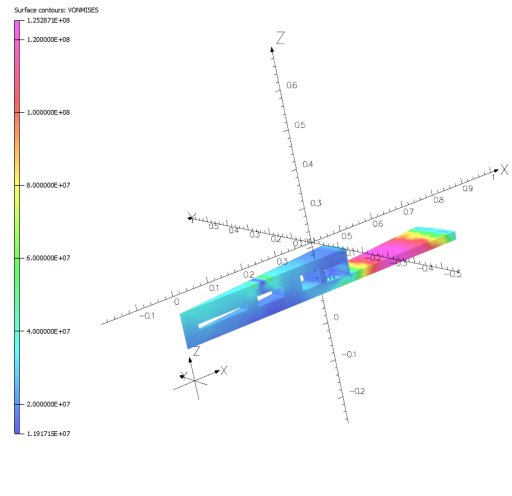


Figure 6.18: von Mises stresses in 316-SS (1/36th rotational symmetry)

From Fig. 6.17, we see that the stresses in the Al-6061-T6 are mostly under the stress limit S_m . However, some locations undergo higher stresses but they are mostly next to the coils and due to the fact that the coils are modeled as "glued" to the structure. Since it is not the case in reality, these stresses will disappear. The only important locations are at the joint between the 316-SS plate and the aluminium structure supporting the shielding coils. It undergoes stresses of about $\sigma = 198.8 \text{ MPa}$ what is above the stress limit of $S_m = 165 \text{ MPa}$. This place will thus have to be reinforced in future designs.

On the contrary, with maximal values of about $\sigma = 125.8 \text{ MPa}$ (Fig. 6.18) the stresses in 316-SS stay largely under the maximal values of $S_m = 575 \text{ MPa}$. It would be possible to use more 316-SS in the structure, we only used Al-6061-T6 for economical reasons. The design of the supporting structure is a trade off between strength of the structure and economies on material quantity and quality.

This master thesis had three objectives. The first one was to develop a topology optimization tool able to design ironless synchrocyclotrons with minimal conductor quantity, the second one was to design an ironless equivalent to the S2C2 of IBA and the last one was to propose a potential conductor lay-out for the case of the ironless S2C2. These three objectives have been successfully fulfilled.

Firstly, it is shown that topology optimization is a powerful tool for the design and optimization of ironless synchrocyclotrons. We observe that using topology optimization, it is possible to obtain synchrocyclotrons respecting the same specifications than the original models but using considerably less superconductor. The topology was first optimized in order to minimize the conductor quantity. Afterwards, some manufacture constraints have been added and a rectangular cross section was given to the coils. The position of the coils has then been optimized to improve the accuracy on the midplane magnetic field. At the end, it has been shown that the conductor volume of the resulting model is reduced of 35-40 % in comparison with the reference model from the MIT.

Secondly, we propose an ironless equivalent to the S2C2. This model respects the specifications of the S2C2 in terms of dimensions and magnetic field and does not have the heavy iron yoke.

A potential conductor has also been designed for the case of the ironless S2C2. It can stand the conditions of varying magnetic field and current density necessary to the treatments. The AC losses have first been evaluated for a typical treatment situation. Subsequently the most appropriate cooling method and superconducting material have been identified: cable-in-conduit conductors (CICC) using superconducting NbTi filaments. The CICC is designed so that our device can function in operating conditions and withstand typical AC losses. Finally, a quench protection system has been designed.

Furthermore, a potential supporting structure is proposed. This example shows that it is possible to stand the mechanical stresses due to thermal expansion and electromagnetic forces using materials such as aluminium and stainless steel. Some improvement of this design is still needed to make it more resistant and economical.

This master thesis is only a small step in the development of ironless synchrocyclotrons. It provides an efficient and systematic tool giving the topology of synchrocyclotrons with minimal conductor quantity. Nevertheless it could be significantly improved. Several enhancement possibilities are introduced in throughout the thesis. The most important ones are summarized here under.

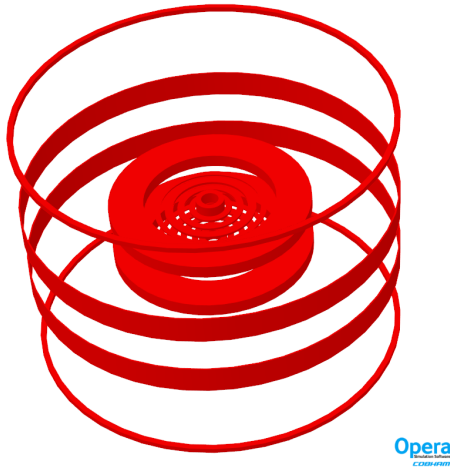


Figure 7.1: Final design MIT model

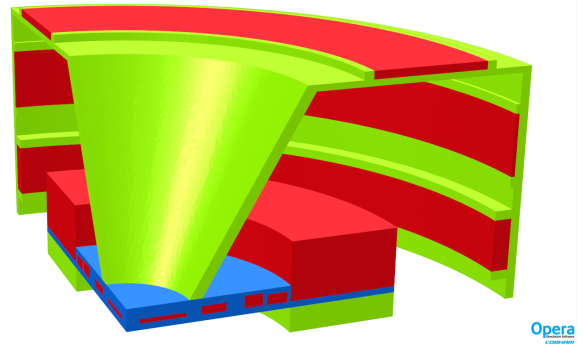


Figure 7.2: Final design ironless S2C2 with its supporting structure (1/4th rotational symmetry)

The first step could be the use of several different current densities for each type of coils, more adapted to the operating conditions of these coils (discussion in Section 7.1.2). Other improvements concern the cable layout, protection system and supporting structure. We only proposed potential designs, they could also be optimized to reach higher current densities and reduce the overall costs.

On a longer term prospective, some important difficulties remain in the development of ironless synchrocyclotrons. The next important step is the design of an adapted (and ironless) extraction system (discussion on the existing possibilities in 7.1.1). Another issue would be its high price. Indeed, in ironless synchrocyclotrons we replace cheap iron with expensive superconductors, what will certainly increase the price of a treatment that is already quite expensive.

In conclusion, ironless synchrocyclotrons are feasible and have a clear interest in terms of variable energy and weight reduction. Topology optimization is a powerful tool that may be used to design synchrocyclotrons minimizing the conductor quantity. Cable-in-conduit conductors is an efficient option for the windings of these ironless synchrocyclotrons as they allow sufficient current density and can stand high AC losses. Nevertheless, more investigation is still needed, especially to develop a reduce the price of the conductor and develop an ironless extraction system.

The final ironless synchrocyclotrons developed in this master thesis are shown on Fig. 7.1 and Fig. 7.2 for the MIT model and the ironless S2C2 (with its supporting structure).

7.1 Perspectives

7.1.1 Extraction

Extraction of hadrons from cyclotrons is not an easy task. Multiple techniques [34] have been developed and their adaptation to ironless devices remains an important unknown.

The MIT already discussed the possibility of using **stripping extraction** [5]. It consists in accelerating H^- particles instead of protons and to remove the additional e^- at the end of the acceleration, giving to the particle another charge sign and thus another direction in the magnetic field. The deviated beam is then collected and extracted. Unfortunately, it has been shown that in high magnetic fields like in our ironless designs (≈ 10 T), the particles of H^- are magnetically

stripped¹ before the end of the acceleration and the beam is lost in the cyclotron. This option has then been abandoned.

Another option could be **regenerative extraction**, as in the S2C2 synchrocyclotron of IBA. This is a passive extraction technique consisting in provoking a resonance of the beam by putting the horizontal betatron tune at $2\nu_r = 2$. The distance between following orbits increases (due to larger horizontal oscillations) and the beam is ejected in the extraction channel. The field bump needed to increase the horizontal betatron tune until a value of $\nu_r = 1$ is obtained with a piece of iron called the *regenerator*. This regenerator is positioned very close to the midplane in order to obtain a sharp field bump and a fast and precise extraction. The scheme of the extraction system is shown on Fig 7.3. This method also presents its difficulties for an ironless equivalent. As explained in Section 5.4.3, it is very difficult to obtain sharp edges of magnetic field only with coils. Even if the problem could be solved for the regenerator, it would remain very complicated to design the corrector bars and the extraction channel. Not enough space is available for the installation of superconducting dipoles (because of the space that the cryostat takes) and the use of permanent magnets will provoke the loss of variable energy possibilities, one of the key interest of ironless cyclotrons.

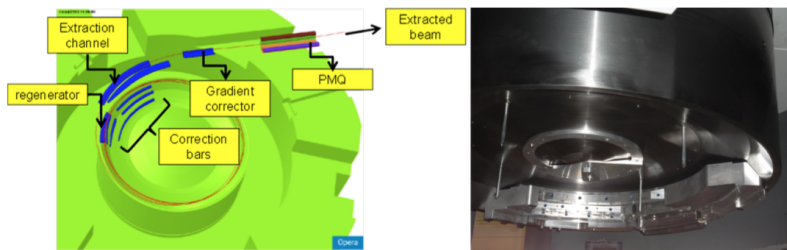


Figure 7.3: Regenerative extraction structure of the S2C2 [34]

More details about these techniques and others extraction methods are available in [34].

7.1.2 Multiple current densities

An important improvement, that could lower the price of the ironless synchrocyclotrons, is the use of different current densities for each type of coils. This is the option chosen by the MIT in their two other designs ([3] and [5]). It would allow to use conductors that are more adapted to the operating conditions. Indeed, the different types of coils undergo very distinct conditions of magnetic field. It is thus for example not optimal to design the conductor of the shielding coil taking as operating conditions the conditions at the highest magnetic field point. Some superconducting material could certainly be spared. The use of several current sources could also maybe give a better fit of the midplane magnetic field (different current densities in different shaping coils).

The number of current densities is nevertheless limited. Current sources consume lot of the heat extraction power of the cryostat and a too high number of sources would reduce its efficiency.

¹They loose the additional e^- .

CHAPTER 8

ACKNOWLEDGEMENT

I am a truly grateful to my thesis supervisor, Bruno Dehez who provided me continuous support and help. I really appreciated to discover the fields of accelerators and superconductivity and to apply knowledges acquired throughout my studies on this real life cases. Bruno considerably helped me in the different parts of the thesis but also gave me the opportunity to go further in my research and fully benefit of the possibilities that my subject offered with, for example, the organization of the three weeks visit at the Massachusets Institute of Technology.

I would also like to thank the members of IBA, without whom this thesis would not have been possible. I want especially to thank Eric Forton for the organization of this thesis, the coordination with UCL and the help and advice in the organization of the stay at the MIT. I also really appreciated the hours spent with Willem Kleeven in discussions on the functioning of cyclotrons, the challenges and stakes of this field. I learned a lot from him and I am truly grateful for its time and investment. Lastly, I want to thank Jarno van de Walle for his insightful advice and help.

I wish also the address my gratefulness to Alexey Radovinsky, Leslie Bromberg, Joseph Minervini and Phil Michael that welcomed me for a visit of three weeks at the Plasma Science and Fusion Center. Most precisely I thank Alexey for his daily explanatiosn and advice, Leslie for the organization of the stay, Joseph for sharing his huge experience in the field of superconductivity and Philip for his advice and attention.

Other people at UCL really helped me in my research. I thank Christophe De Gréef and François Henrotte for their help concerning GETDP and GMSH and also Luc Piraux for the discussion on superconductivity and CICC's conductors.

Thank you also to my grandfather Yayo for the hours spent in proofreading and correction of the English.

Finally, I am very grateful to my family and friends that supported me during these fives years at EPL .

General functioning

This chapter exposes briefly the different steps of the program. The main functions will be described here. Many others auxiliary functions are also used and are well commented in the program itself.

The simulation is launched with the `main` script. The program is based on a GMSH mesh and can thus only run if the software is installed. Moreover, the paths to GMSH, the `.geo` and `.msh` file have to be updated in the command line run by the `main` script.

A.1 Load of parameters of the model

- `parameters.model` is 'S2C2' or 'MIT' depending on the case that we want to model.
- `solution_type(parameters,parameters.type)` loads all the parameters corresponding to the case that we want to model. These parameters are the different dimensions, constraints, numerical values,... It is in this function that we can change the objective magnetic field and the specifications of the model.
- `controlpanel` is the control panel of all the actions that have to be done during the simulation and of values that the user wants to compute. Most of the values are saved after computation so they do not have to be recomputed at each run of the program. It is possible to choose which ones have to be recomputed or loaded thanks to this function. If nothing is changed, a default set is activated.
- `checkconditions` checks whether the options chosen by the user in control panel are consistent and send an error message if not. For example, it is not possible to proceed to the topology optimization if the magnetic field matrices have not been computed, or it is not possible to compute the energy if we do not know yet the inductances.
- `generatefiles` charges the files where the different values will be stored (or have already been stored).
- `writeParameters` writes the parameters in a `.geo` file that is read by GMSH and GETDP.

A.2 Mesh generation and extraction

The mesh (stored in a `.msh` file) is automatically generated by GMSH through a MATLAB command. Afterwards, it is extracted from the `.msh` and stored in arrays thanks to:

- `readnodes` extracts the nodes coordinates from the `.msh` file.
- `readelement` extracts the elements from the `.msh` file.

A.3 Computation of the magnetic field matrices

- `matrices_MagneticField` computes the matrices that will give the magnetic field at the required points (control points in the GFR and the LFR for example) in function of the current density.

A.4 Topology optimization

- `TopologyOptimisation` proceeds to the topology optimization. Its output is the conductor proportion in each cell.
- `intermediate_material` treats the results of the topology optimization. The cells containing intermediate materials are filled or emptied in order to have a topology that minimizes the error on the magnetic field in the GFR (and still respects the constraints in the LFR).
- `findcoils` identifies the different coils and the belonging of each elements to one of them.

A.5 Coils post-processing

When the coils have been identified, different options are available to post-process them (or not). This first step consists in the coils discretization in rectangles or multi-layer rectangles.

- `createrectcoils` creates the new rectangular coils.
- `createrectcoils_layer` creates the multilayered coils.

A.5.1 Main functions

- `postprocess_rectcoils` is the main function that post-processes the rectangular coils and computes the different values characterizing the system (See [A.6](#)). All the following functions are called by this main function (except those that are independent of coils type as `walkinshaw_resonance` and `beamenergy`). Since the multi-layers option is not analyzed further in this report, no `postprocess` function has been written for this type of coils.

A.5.2 Options post-processing

- `newcoils_discret` divides the coils in an integer number of conductors of dimensions W_c x H_c .

- `NelderMeadMethod1` and `NelderMeadMethod2` optimize the position of the coils using a Nelder-Mead algorithm following the 1st and 2th method respectively (see Appendix [E.3](#)). This step may take a significant time (up to 20 minutes in some cases). The result is automatically saved after computation so it is possible to load the previous solution afterwards and avoid to run again the function.
- `aftertreatment` removes the overlaps between the coils that may appear after the Nelder-Mead optimization (if 1st method is used. Details in Section [E.3](#)).

A.6 Characterization of the system

- `inductance_d` (nd) computes the self- and mutual inductances of all the coils. Function purely geometrical (i.e. a mutual inductance between two coils having different signs of current is considered as positive). `inductance_d` is used when the coils have been divided in an integer number of conductors and `inductance_nd` in the other case.

A.6.1 Magnetic energy

- `magneticenergy_d` (nd) computes the magnetic energy stored in the multicoils system.

A.6.2 Walkinshaw resonance

- `walkinshaw_resonance` computes the walkinshaw resonance radius. Computes and plots the evolution of the betatron tunes.

A.6.3 Beam energy

- `beamenergy` computes and plots the evolution of the energy of the beam. Computes the energy of the beam at the extraction radius.

A.6.4 Treatment analysis

- `variablebeamenergy` computes and plot the evolution of the energy of the beam, the current density and the stored energy during the treatment of a patient (following the specifications).

A.6.5 Evaluation of the AC losses

- `computeAClosses` computes the AC losses thanks with the reasoning used in this report. Function independent of the others (not run by `main`).

APPENDIX B

BASICS CYCLOTRONS

B.1 Working principles of the dees

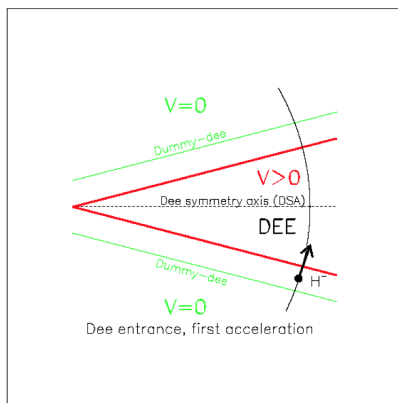


Figure B.1: Dee at the entrance of the ion

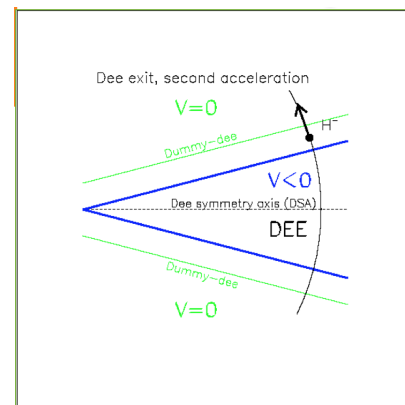


Figure B.2: Dee at the exit of the ion

Fig [B.1](#) and [B.2](#) represent the acceleration of an ion H^- (for protons the sign is inverted). The voltage in the dee follows a sinusoid. When the ion enters, the dee has a positive voltage and attracts the ion. When the ion goes out, the dee has a negative voltage and pushes the ion out.

APPENDIX C

PROBLEM SPECIFICATIONS

Extracted beam		
Particle		Proton
Beam current	nA	100
Emittance		TBD
Energy spread		TBD
Energy setpoint		TBD
Extraction efficiency	%	>20
Dimensions		
Magnet full height	m	<1.75
Magnet radius	m	<1.25
Magnetic field		
Extraction radius	cm	45
Center magnetic field	T	5.7
Stray field @ 1.54 m midplane radius	G	500
Stray field @ 2 m midplane radius	G	500
Maximal field on RotCo	G	30
Maximal field on cryocoolers	G	500
Cryogenics		
Cooling method		TBD
RF		
Frequency range	MHz	93-63
Voltages	kV	<15
Repetition rate	kHz	1
Ion source		
		Internal PIG source
Ramping schedule		
Energy range	MeV	70-230
Horizontal covering duration	sec	0.5
Transition between energy level duration	sec	0.5
Duration ramp	sec	40
Nbr ramp/treatment		4 (2 complete cycles)
Cooling period after treatment	min	5
Material		
		NbTi/Nb3Sn

Minimal half-height warm bore		
	cm	10
Reproducibility of the magnetic field		
		$1e - 01$

Table C.1: General specifications of the ironless S2C2

The specifications TBD (To Be Defined) are left free at our level but are important points that have to be defined for a further development of this work.

D.1 GetDP / GMSH

D.1.1 Boundary conditions

Since we only model a quarter of the cross section of the synchrocyclotron, we have to impose boundary conditions that will express the symmetries.

The problem is solved in GETDP in function of the magnetic potential $\vec{A}$. This technique allows to solve the electromagnetic equations with only one component at each point of the domain¹. The magnetic field is then computed by GETDP using the definition of the magnetic potential: $\vec{B} = \nabla \times \vec{A}$.

We seek to find $\vec{A}$, solution of the following problem:

$$\left\{ \begin{array}{ll} \nabla \times \left(\frac{1}{\mu} \nabla \times \vec{A} \right) = \vec{J} & \text{on } \Omega \\ \frac{\partial \vec{A}}{\partial \vec{n}} = 0 & \text{on } \Gamma_2 \\ \vec{A} = 0 & \text{on } \Gamma_1 \end{array} \right. \quad (\text{D.1})$$

We impose a Dirichlet boundary condition on Γ_1 which will cancel the $\vec{B}_r$ component on the z-axis and a Neumann boundary condition in the midplane, which will also cancel the $\vec{B}_r$ component in the midplane.

D.1.2 Infinite elements

For the modeling of an infinite domain, it is better practice to use infinite elements. For this purpose, we have to define a new physical part (here the domain **I**) in the `.geo` file on the border we want to be infinite. By rewriting the Jacobian of the `.pro` as explained here under, it is possible to create virtually infinite elements. We can observe on Fig [D.3](#) close iso-values of the magnetic field that are typical of infinite elements.

The new jacobian definition is:

¹Thanks to the rotational symmetry, only the A_θ component will be different of 0.

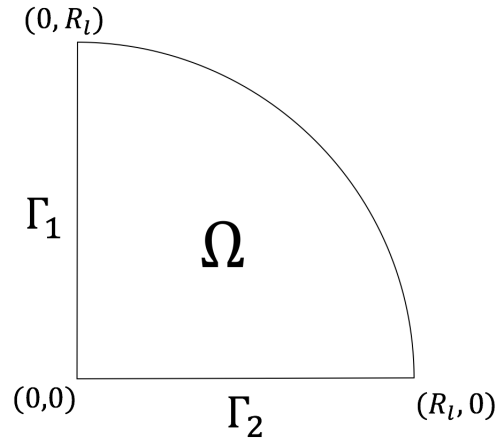


Figure D.1: Domain as defined in GETDP

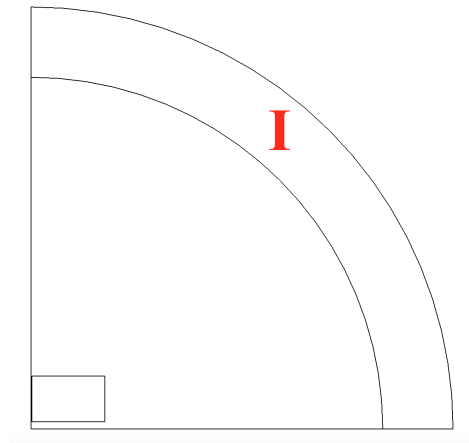
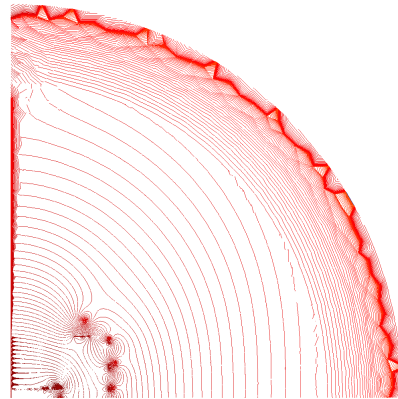


Figure D.2: Definition of a new physical surface with infinite elements (Zone I)

Figure D.3: Isovalues of the magnetic field B (Ironless S2C2: $J_{op}=200$ [A/mm²], Logarithmic scale)

```

Jacobian{
  {Name JVol ;
    Case{
      {Region Infinity;
        Jacobian VolAxisSphShell {R_int, R_ext};}
      {Region All ; Jacobian VolAxi ;}}
    }
  {Name JSur ;
    Case{
      {Region All ; Jacobian Sur ;}
    }
  {Name JLin ;
    Case{
      {Region All ; Jacobian Lin ;}
    }
  }
}

```

D.1.3 Imposition of the current

The conductor repartition in each element is computed with the MATLAB tool and will be different for different optimization parameters. It is then impossible to predefine a physical part in our `.geo` where the current will be put. The easiest way to overcome this issue is to rewrite the `.msh` file after the computation of the current distribution by the MATLAB tool. The idea is to impose new physical parts in the `.msh` file, corresponding to positive current and negative current.

The second part of this file contains the mesh connection. The elements are described in the form:

```

$Elements
number-of-elements
elm-number elm-type number-of-tags < tag > ... node-number-list
$EndElements

```

- `elm-type` represents the type of the finite element (geometry and order).
- `number-of-tags` represents the number of parameters of the element.
- `<tags>` represent the list of these parameters (with a size of `number-of-tags`)
 - The first parameter is the reference to the physical entity that contains the element.
 - The second parameter is the reference to the geometrical entity of the element
 - If the mesh contains partition, the third parameter is the reference of the mesh partition
- `node-number-list` represent the nodes list of the element (in order)

The imposition of the current is amde by changing the first parameter in the tag description and put the number reference of two other physical parts, one for positive current (in y-axis direction) and one for negative current (in y-axis direction).

Afterwards, the current is imposed in the new physical parts in the `Function` part of the `.pro` file.

E.1 Threshold determination

Typical variation of the norm of the error on the magnetic field following the value of ρ_{th} is shown on Fig E.1. (Here ironless S2C2, `meshsize` = 0.005, $J = 112$ [A/mm²], $N_f = 1$, $\delta_B = 0.003$)

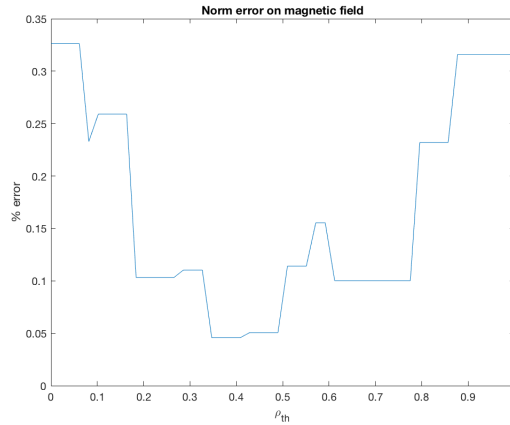


Figure E.1: % of error in the magnetic field

An optimization algorithm would be difficult to implement because the function is not monotonic. Here, we only test a range of different values of ρ_{th} and choose the one that will minimize the overall error on the field.

E.2 Rectangle discretization: overlaps

The rectangular cross-sectional coils are first created without paying attentions to the overlaps. Afterwards, all the overlaps highlighted and the concerned coils are deformed (extended) to remove them. The algorithm can only treat "horizontal" overlaps, between coils that are side by side. This is not an important limitation since overlaps in the vertical direction (between coils

one above the other) almost never happen, excepted for very low current densities (and huge coils), what is not our case here. The steps followed to removed the overlaps are:

1. **Identifications of the overlaps:**

All the coils that overlap themselves are identified with the corresponding Δr the horizontal widths of the overlaps.

2. **The width of the overlapping coils is reduced:**

The idea is to remove the same are of cross section for each of the two coils. Coil 1 will loose $\Delta r_1 = \frac{H_2}{H_2+H_1} \cdot \Delta r$ and Coil 2 will loose $\Delta r_2 = \frac{H_1}{H_2+H_1} \cdot \Delta r$. The "lost areas" are equal for the two coils: $\Delta r_1 \cdot H_1 = \Delta r_2 \cdot H_2$.

3. **The height of the coils is increased:**

Since we want to preserve the area of the coils cross sections, the lost surface is added vertically on each coil, above and/or under the coil (depending on the limitations of the design zone, the coil may of course not go out of the design zone).

4. **The gravity centers are updated to the new dimensions of the coils**

E.3 Nelder-Mead algorithm: overlaps

Two functions have been implemented for the optimization of the position, both using a Nelder-Mead algorithm. The first one uses the MATLAB function `fminsearchbnd`¹ and the second is based on the function `Nelder_Mead_MultiDimMatrixForm`².

The Nelder-Mead algorithm is originally a non bounded algorithm. In our case we need two types of bounds on our variable, the coordinates of the coils centers.

1. **Limits design zone:** the first type of bounds is **constant** and represents the geometrical limits on the design zone.
2. **Non-overlap between coils:** the second type of bounds is **dynamic** and depends on the position of the coils at a given iteration of the optimization.

Elimination of overlaps

One general method is used in both functions to remove the overlaps after the discretization in rectangles. In contrast with the algorithm used in the coils discretization, we do not change the shape (and the area neither) of the coils.

For example if there is an overlap between Coil 1 and Coil 2, we compute the $\vec{\Delta}$, the vector of shortest displacement that has to be done by the coils to remove this overlap. Afterwards, we calculate **error1** and **error2** the error on the midplane magnetic field done if Coil 1 moves of $\vec{\Delta}$ or Coil 2 moves of $-\vec{\Delta}$ respectively. The final displacement is a weighting of the the two errors. Coil 1 moves of $\frac{\text{error2}}{\text{error1}+\text{error2}} \cdot \vec{\Delta}$ and Coil 2 moves of $-\frac{\text{error1}}{\text{error1}+\text{error2}} \cdot \vec{\Delta}$ (so the coil provoking the highest error will have a lower displacement). The total displacement is equal to $\vec{\Delta}$ and the overlap is removed. This reasoning could of course be improved by optimizing the weighting of the displacement that minimizes the error on the magnetic field. This method would be more

¹Copyright (c) 2006, John D'Errico All rights reserved.

²Copyright (c) 2014, Michael Mathew All rights reserved.

time consuming and since the results are already of good quality, it has not been implemented here.

Other methods using deformations of the coils have been tested but gave worse results in term of final error on the midplane magnetic field.

E.3.1 1st method

In the first method we took the MATLAB function as a black box. This function takes the limits of the design zone as bounds³ but we do not put any constraints for the overlaps.

The result is then post-processed and, if there is an overlap, the position of the coils is adapted following the procedure of overlap elimination exposed here above.

The error on the midplane magnetic field decreases and reaches a plateau after many iterations. The stopping criterion used here is the number of iterations. We put it to `max iterations = 200` for both models (MIT) and (S2C2). We see on Fig.E.2 that the error is almost constant around this number of iterations.

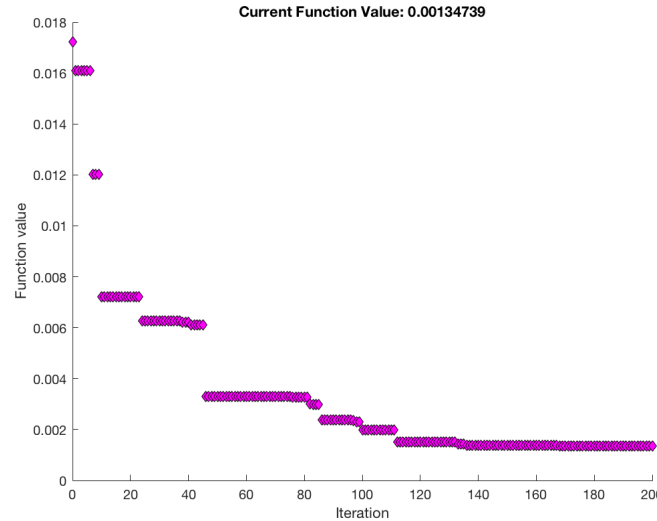


Figure E.2: Evolution of the error on the midplane magnetic field with the iterations (for ironless S2C2)

E.3.2 2th method

In the second method, we changed the function itself. The geometrical limits of the design zone have been added following [23] and are checked at each iteration of the Nelder-Mead algorithm.

Moreover, overlaps between the coils are checked at each iteration. If an overlap occurs, the coils are moved away from each other following the procedure described here above.

The stopping criterion in this function is on the standard deviation of the error on the magnetic

³Of course the bounds are adapted for each coil depending on its size.

field. When this standard deviation becomes lower than 0.0003⁴ we stop the iterations. We observed that going to smaller values does not change significantly the result. The function generally do between 70 and 100 iterations depending on the case that we model.

E.3.3 Comparison

Generally speaking, the 1st method produces better results. This method is used for the MIT model (because there is a risk of overlap between the two coils having the smallest radii) and for the ironless S2C2 when the coils are relatively large and that there are overlaps to remove. In contrast, the 2th method produces better results for the S2C2 when there are no overlaps (higher current densities and smaller coils). Its result has thus been chosen here.

Strictly speaking, the two methods are not comparable since their stopping criteria are different. The elaboration of one single method that produces the best result in every situation could be a good improvement of this work.

The 1st method is thus used for the MIT model and for the S2C2 when the current density is low. For the ironless S2C2, with a current of $J_{op} = 55 \text{ A/m}^2$ there are no overlaps. The comparison between the coils produced by the two methods and their resulting magnetic field and shown on Fig. E.3 and Fig. E.5 for the 1st method and Fig. E.4 and Fig. E.6 for the 2th method.

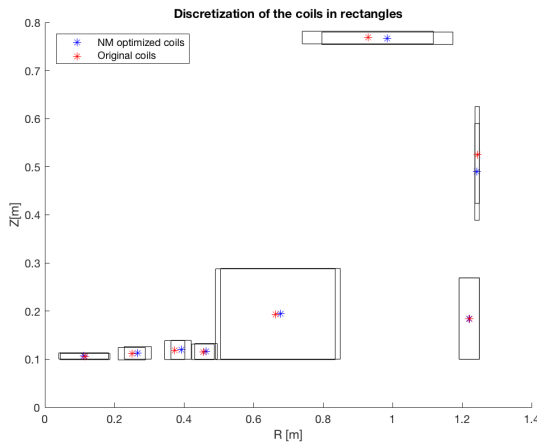


Figure E.3: Coils position: 1st method

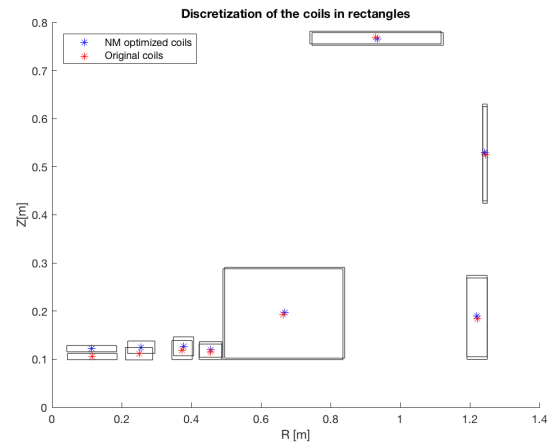


Figure E.4: Coils position: 2th method

⁴This value is small and requires an important number of iterations, so a long computation time. Since we wanted to have a precise design, we chose to keep this small value but for faster run of the program, a higher standard deviation already gives good results.

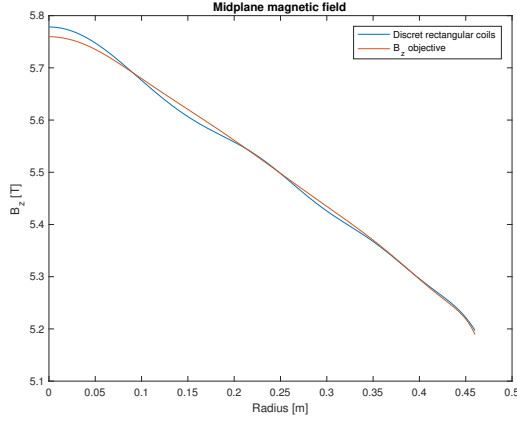


Figure E.5: Midplane magnetic field B_z :
1st method

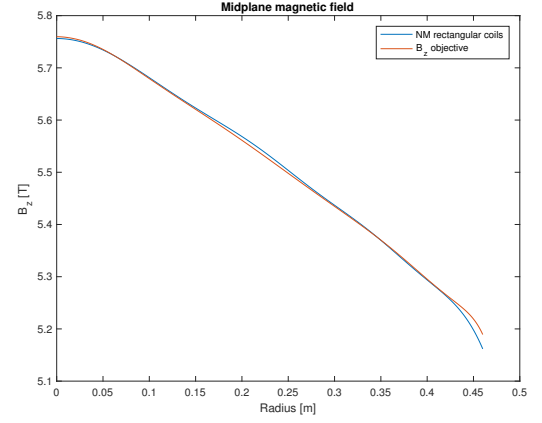


Figure E.6: Midplane magnetic field B_z 2th method

The two field are quite good but the one produced with the 2th method is better. We observe the the motions of the coils are different but follow the same logic. In the 1st method the coils mostly move to the right while in the 2th method, they mostly move upwards. The two displacements have the same goal, reduce the magnetic field close to the origin, where it was too high after the discretization in an integer number of conductors (See Fig. [5.5](#)).

APPENDIX F OPTIMIZATION RESULTS

F.1 Midplane error

Fig F.1 shows the midplane errors (B_r in the midplane), for the MIT model. The situation is very similar for the ironless S2C2. These errors are negligible.

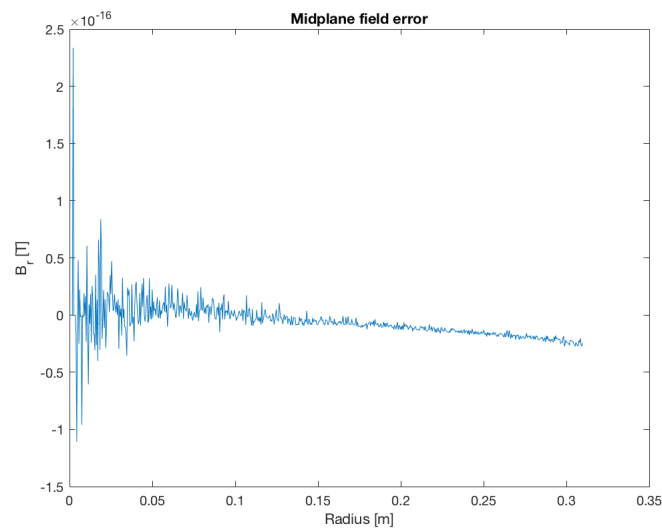


Figure F.1: Midplane field error

F.2 Topology of the S2C2 for $J_{op} = 27 \text{ A/mm}^2$

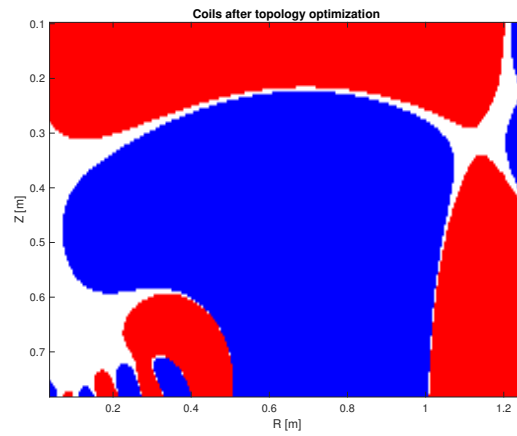


Figure F.2: Ironless S2C2: Topology for $J_{op} = 27 \text{ A/mm}^2$

APPENDIX G

DESIGN OF THE CONDUCTOR

G.1 Critical surface parametrization of Nb₃Sn

The parametrization of the critical surface of Nb₃Sn that has been used in this report is:

- $T_{c0} = 16$ [K]
- $B_{c20} = 29.38$ [T]
- $a = 0.96$
- $C_0 = 213000$ [A/mm²]

$$B_{c2}(T) = B_{c20} \cdot (1 - t^{1.52}) \quad (\text{G.1})$$

$$C(t) = C_0 \cdot (1 - t^{1.52})^a \cdot (1 - t^2)^a \quad (\text{G.2})$$

$$J_c = \frac{C(t)}{B} \cdot b^{0.5} \cdot (1 - b)^2 \quad (\text{G.3})$$

Where $t = \frac{T}{T_{c0}}$ and $b = \frac{B}{B_{c2}(t)}$

Mettre nouvelle critical surface avec epsilon

G.2 Cryocooler

A cryocooler is a *dry* cooling system, used in many superconducting accelerators as, for example, the S2C2. A model is shown on Fig [G.1](#).

The left side is in contact with the cooled object and heat is extracted by conduction. The cryocooler itself is cooled by a Stirling cycle.

G.3 Kim-Anderson fitting parameters

The Kim-Anderson approximation suggests a representation of the critical current curve as:



Figure G.1: Cryocooler

$$J_c(B) = J_o \frac{B_o}{B + B_o} \quad (\text{G.4})$$

The two parameters J_o and B_o are computed using the parametrization of the critical surfaces. We use the `fminsearch` function of MATLAB to find the values of J_0 and B_0 that minimize the norm of the distance between the critical surface and the curve from Eq. [G.4](#). Our result is shown Fig. [G.2](#).

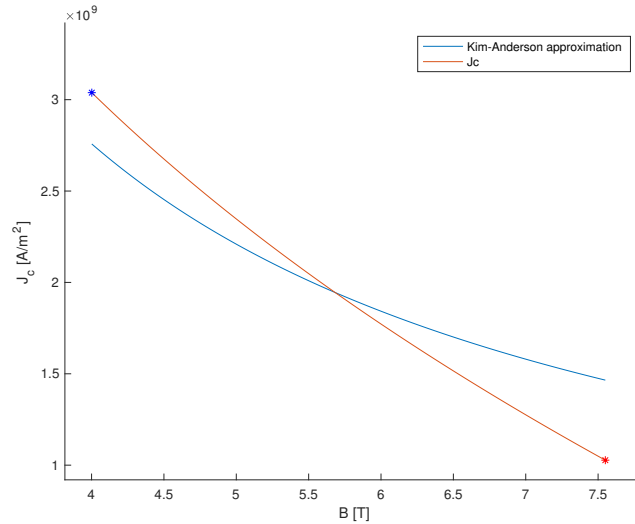


Figure G.2: Kim-Anderson approximation of the critical curve

The fit is not very good at this magnetic field level (we obtained significantly better results for other magnetic fields), but it is sufficient at our level. If further investigation has to be done, some improvement could be necessary. We may also integrate directly the parametrization of the critical surface/

G.4 Superconductor material

Critical fields and temperatures for different superconducting material are shown on Fig [G.3](#). From all the different superconductors shown, only NbTi is ductile.

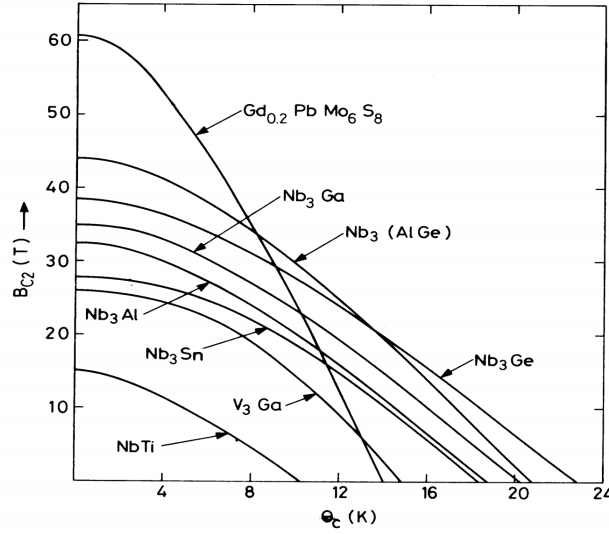


Figure G.3: Critical field and temperature different superconductors [35]

G.5 Maximal filament diameter

The superconducting wires are subjects to a phenomenon called *flux-jumping*, that causes a lot of power dissipation. This phenomenon is explained in [36].

The filament will be stable against a flux jumping if [36]:

$$d \leq \frac{2}{J_c} \left(\frac{3\gamma C(\theta_c - \theta_o)}{\mu_o} \right)^{\frac{1}{2}} \quad (\text{G.5})$$

The parameters for NbTi are:

- J_c critical current density = $10.272 \cdot 10^8 \text{ A/m}^2$
- γC specific heat/volume = $3400 [J \cdot m^{-3} K^{-1}]$
- θ_c critical temperature = $6.15 [K]$

And this give a maximal diameter of :

$$d < 212.62 \mu m$$

This phenomenon is the main reason of the very small diameter of superconducting material in superconductors.

G.6 Quench protection: Correlations used

The two following correlations are based on [11] and are used in the *CICC Design Tool*. The first one gives the resistivity of copper in function of RRR (Residual-resistivity ratio, ratio between the resistivity at room temperature and at 0 K), the magnetic field and the operating temperature:

$$\rho_{cu} = \frac{1.5 \cdot 10^{-8}}{RRR - 1} \cdot (1 + 0.3046 \cdot B_{op} - (0.000613) \cdot B_{op}^2) \quad (\text{G.6})$$

And the second one gives the integral of the $\gamma(T)$ function, used to design the quench protection system.

$$I(T_0, T_{hot}) = \int_{T_0}^{T_{hot}} \frac{\sum_i f_i C_i(T)}{\rho_{cu}(T)} dT = 1 \cdot 10^4 \left(16.8 - \frac{1363}{T_{hot} + \frac{4173}{T_{hot} + 47.89}} \right) \cdot \left(1 + 0.802 \left(\frac{A_{ss}}{A_{cu}} \right) \right) \quad (G.7)$$

BIBLIOGRAPHY

- [1] Gerbershagen A., Baumgarten C., Kiselev D., van der Meer R., Risters Y., and Schippers M. "Measurements and simulations of boron carbide as degrader material for proton therapy". *Phys. Med. Biol.* 61 (2016) N337–N348, 2016.
- [2] Goto A., Tachikawa T., Jongen Y., and Schillo M. "Cyclotrons". *Comprehensive Biomedical Physics (MEPH)*, Chapter 6.12, 2013.
- [3] Radovinsky A., Minervini J.V., Winklehner D., Bromberg L., and Michael P.C. "Superconducting cyclotrons for hadron radiotherapy". *EUCAS 2017*, EUCAS17-2LO1-02, Sep 2017.
- [4] Radovinsky A., Minervini J. V., Miller C. E., Bromberg L., Philip C. M., and Maggiore M. "Superconducting magnets for ultra light and magnetically shielded, compact cyclotrons for medical, scientific, and security applications". *IEEE Transactions on Applied Superconductivity*, vol 24:pg 1–5, 2014.
- [5] Radovinsky A.L., Minervini J.V., Michael P.C., Bromberg L., and Miller C.E. "Variable energy acceleration in a single iron-free synchrocyclotron". PSFC/RR-13-9, Sep 2013.
- [6] Bordini. B. "An exponential scaling law for the strain dependence of the nb3sn critical current density". *Superconducting Science Technology*, vol 26 075014, 2013.
- [7] Dhruv B. Additive manufacturing. <http://www.padtinc.com/blog/additive-mfg/additive-manufacturing-back-to-the-future>, January 28 2016. Accessed: 12-05-2018.
- [8] Seeber B. "Handbook of applied superconductivity volume 1: Fundamental theory, basic hardware and low-temperature science and technology". *Institute of Physics Publishing Bristol and Philadelphia*, 1998.
- [9] CERN. "Un nouveau fil supraconducteur pour les accélérateurs du futur". <http://cds.cern.ch/record/990379>, Oct 23, 2006. Accessed: 9-05-2018.
- [10] Schoerling D. "Mini-workshop on superconducting magnets". *European Organization for Nuclear Research (CERN)*, 2017.
- [11] Long A. E. "Optimization of a cable-in-conduit superconductor design using a spreadsheet-based computational model". *Massachusetts Institute of Technology*, Bachelor's thesis, May 1993.
- [12] Antaya T.A. Friesel D.L. "Medical cyclotrons". *Review of Accelerator Science and Technology*, Vol 2, pg 1-24, 2009.

- [13] Rozvany G.I.N. "Aims, scope, methods, history and unified terminology of computer-aided topology optimization in structural mechanics". *Struct. Multidiscip. Optim.*, vol 21:pg 90–108, Apr 2001.
- [14] Lee S.Y.and Ng K.Y.and Liu H.and Chao H.-C. "Beam dynamics in crossing a walkinshaw resonance". *Department of Physics Indiana University - Bloomington Fermi National Accelerator Laboratory*, March 14, 2013.
- [15] Slobodan I.B. and Cevdet A. "New analytic-numerical solutions for the mutualinductance of two coaxial circular coils with rectangular cross section in air". *IEEE Transactions on magnetics*, vol. 42, No 6, June 2006.
- [16] Weisend J.G II. "Superconducting magnets for accelerators". *National Superconducting Cyclotron Lab, Michigan State University*.
- [17] Willkens J. "Therapy with protons and ion beams". http://www.desy.de/~garutti/LECTURES/BioMedical/Lecture2_TherapyProtonsIons.pdf, 2013. Accessed: 25-05-2018.
- [18] Jensen J.E., Tuttle W.A., Stewart R.B., Brechna H., and Prodell A.G. "cryogenic data notebook". <https://www.bnl.gov/magnets/staff/gupta/cryogenic-data-handbook/Section2.pdf>, August 1980.
- [19] Craddock M. K. and Symon K. R. "Cyclotrons and fixed-field alternating-gradient accelerators". *Reviews of Accelerator Science and Technology*, Vol. 1, pp 65-97, 2008.
- [20] Bottura L. "Cable stability". *CAS - CERN Accelerator School: Course on Superconductivity for Accelerators*, vol.CERN-2014-005 pp.401-451, 2013.
- [21] Bottura L. "Stability and protection of cicc's - an updated designer's view". *Cryogenics*, Vol 38, pg 491-502, February 20, 1999.
- [22] Bottura L. "Stability, protection and ac loss of cable-in-conduit conductors - a designer's approach". *Fusion Engineering and Design, Elsevier*, vol 20, May 1993.
- [23] Guyon F. Luersen M.A., Le Riche R. "A constrained, globalized, and bounded nelder-mead method for engineering optimization". *Springer*, vol. 27, pp 43-54, May 2004.
- [24] Schippers M. "Transport and energy setting of particle beams". *Paul Scherrer Institut*, May 2015.
- [25] Wilson M. "Superconductivity for accelerators". *JUAS*, Lecture 1, February 2017.
- [26] Seidel P. Applied superconductivity. *Wiley*, 2015.
- [27] Ferracin P.(CERN). "Superconducting magnets, section iii". *Joint Universities Accelerator School, Archamps*, 2017.
- [28] Sahoo N. Rath A. K. "Particle radiotherapy: Emerging technology for treatment of cancer". *Springer*, March 2016.
- [29] Bendsoe M Sigmund O. "Topology optimization: Theory, methods, and applications". *Springer*, 2003.
- [30] Labbe T. "*Topology optimization methods for the design of electromagnetic actuators*". PhD thesis, Université catholique de Louvain, November 24 2011.
- [31] Minervini J. V., Radovinsky A., Miller C. E., Michael P., Bromberg L, and Antaya T.and Maggioro M. "Design options for highly compact, superconducting cyclotrons and gantry magnets for hadron therapy". *Beam Dynamics Meets Magnets – II*, Bad Zurzach, Switzerland, December 1-4, 2014.

- [32] Kleeven W. "IBA proton therapy systems: Accelerators, beamlines and gantry technology". http://www.bvsabr.be/js/tinyce/plugins/moxiemanager/data/files/20170626/IBA_WKleeven.pdf.
- [33] Kleeven W. "Fields from concentric circular current loops". IBA internal report, February 22, 2017.
- [34] Kleeven W. and Zarembo S. "Cyclotrons: Magnetic design and beam dynamics". *CERN Yellow Reports: School Proceedings*, Vol. 1/2017, CERN-2017-004-SP, 2017.
- [35] Ferracin P.(CERN) Wilson M. (Consultant). "Superconducting magnets, lecture 1". *Joint Universities Accelerator School, Archamps*, 2017.
- [36] Ferracin P.(CERN) Wilson M.(Consultant). "Superconducting magnets, lecture 2". *Joint Universities Accelerator School, Archamps*, 2017.
- [37] Dignan Y. and Han K. S. "Self-inductance of air-core circular coils with rectangular cross section". *IEEE Transactions on applied magnetics*, Vol. 23, No. 6, November 1987.
- [38] Iwasa Y. "Case studies in superconducting magnets design and operational issues". *Kluwer Academic Publishers*, 2002.
- [39] Iwasa Y. "Superconducting magnets lecture 6: Low-temperature superconductors". https://ocw.mit.edu/courses/nuclear-engineering/22-68j-superconducting-magnets-spring-2003/lecture-notes/lec6rev_3apr03_1.pdf, April 3, 2003. Accessed: 20-05-2018.

