Appendix A

Matlab Functions Description

In this appendix, we list the main Matlab functions (or scripts) that we used in this thesis, we briefly explain them and we describe for which purpose they were used. They can be found on the physical CD attached to the printed version of this work.

Some variables taken as input are common for many functions listed below: the initial stock price `S0`, the volatility coefficient `v`, the risk-free interest rate `r`, the underlying stock expected return `rS`, the number of time steps `N`, the number of Monte Carlo random paths `MC`, the number of years simulated `T` and the option strike price `K`. The other variables will be explained when relevant.

A.1 Simulations and Density Functions

This first section describes the functions used to simulate the GBM, HN and Heston processes, to generate the associated return density functions with Monte Carlo simulation, and therefore also how the HN process and the replicating portfolio were simulated with different re-balancing frequencies.

- `GBM_Simulation`

  This function was used to generate figure 2.1. The function is `function y = GBM_Simulation(S0,v,rS,N,T)`. The output is a plot of the simulated stock price under the GBM process and of its expected value.

- `GBM_Distribution`

  This function was used to generate figure 2.2. The function is `function y = GBM_Distribution(v,rS,N,MC,T)`. The output is the physical density of stock returns under the GBM process with a Gaussian kernel under Matlab default bandwidth.

- `GBM_PayoffReplication`

  This function was used to generate figure 3.1. The function is `function y = GBM_PayoffReplication(S0,v,r,rS,N,T,K)`. The output is a plot of the simulated stock price under the GBM process, the option payoff and the replicating portfolio.

- `HN_Simulation`

  This function was used to generate figure 4.2. The function is `function y = HN_Simulation(S0,v,r,rS,T)`. The output is a plot of the simulated stock price under the HN and GBM processes.

- `HN_Distribution`

  This function was used to generate figure 4.3. The function is `function y = HN_Distribution(S0,v,r,rS,MC,T)`. The output is a plot of physical density of stock returns under the HN process with a Gaussian kernel under Matlab default bandwidth, compared with the normal distribution.

- `Heston_Simulation`

  This function was used to generate figure 4.4. The function is `function y = Heston_Simulation(S0,v,rS,N,T)`. The output is a plot of the simulated stock price under the Heston and GBM processes.

- `Heston_Distribution`

  This function was used to generate figure 4.5. The function is `function y = Heston_Distribution(v,rS,N,MC,T)`. The output is a plot of physical density of stock returns under the Hestn process with a Gaussian kernel under Matlab default bandwidth, compared with the normal distribution.

- `HN_PayoffReplication_Simulation`

  This function was used to generate figure 6.9 and the results in table I.1. The function is `function y = HN_PayoffReplication_Simulation(S0,v, r,rS,k,MC,T,K)` where `k` fixes the re-balancing frequency. The functions `Call_HN` and `Delta_HN` (see section A.2) are called to calculate the payoff replication errors. The outputs are different metrics of payoff replication errors as well as a plot of the simulated stock price under the HN process, the option payoff and the replicating portfolio.

A.2 Call Prices, Delta & Models Calibration

This second section describes the functions used to calculate the call option price and Delta under each model, as well as those used to calibrate the models' parameters.

Call Prices & Delta

These functions are used to then calibrate the models' parameters by minimizing the pricing $MSE, as well as to calculate the pricing and hedging errors out-of-sample (see functions in sections A.3 and A.4).

- `Call_BS`

  The function is `function Call = Call_BS(S0,K,T,r,v)`. The output is the price of a call option under the BS model.

- `Delta_BS`

  The function is `function Delta = Delta_BS(S0,K,T,r,v)`. The output is the value of the Delta under the BS model.

- `Call_PBS`

  The function is `function Call = Call_PBS(S0,K,T,r,theta0,theta1,theta2)` where `theta0`, `theta1` and `theta2` are the parameters of the PBS model. The output is the price of a call option under the PBS model.

- `Delta_PBS`

  The function is `function Delta = Delta_PBS(S0,K,T,r,theta0,theta1,theta2)` where `theta0`, `theta1` and `theta2` are the parameters of the PBS model. The output is the value of the Delta under the PBS model.

- `Call_HN`

  This function was coded with the help of a function which can be found in the following link: https://www.mathworks.com/matlabcentral/fileexchange/27644-heston-nandi-option-price. The function is `function Call = Call_HN(S0,K,t,r,h0,b0,b1,b2,omega)` where `r` is the daily risk-free rate, `h0` the initial variance and `b0`, `b1`, `b2` and `omega` the parameters of the model. The output is the price of a call option under the HN model following the combined integrand approach of Rouah and Vainberg (2007).

- `Delta_HN`

  This function was also coded with the help of the link right above. The function is `function Delta = Delta_HN(S0,K,t,r,h0,b0,b1,b2,omega)` where `r` is the daily risk-free rate, `h0` the initial variance and `b0, b1, b2` and `omega` the parameters of the model. The output is the value of the Delta under the HN model.

- `ChFun_Heston`

  This intermediary function is used to calculate the characteristic function of $\log(S_t)$ in the Heston model, which is then used to calculate the Heston call option price and Delta. This function comes from Crisostomo (2014). The function is `function ChFun = ChFun_Heston(S0,V0,theta,kappa,alpha,r,rho,t,w)` where `V0` is the initial variance, `theta, kappa, alpha` and `rho` the parameters of the model and `w` the point at which to evaluate the function. The output is the value of the characteristic function at point `w`.

- `Call_Heston`

  This function comes from Crisostomo (2014). The function is `function Call = Call_Heston(S0,V0,theta,kappa,alpha,r,rho,T,K)` where `V0` is the initial variance and `theta, kappa, alpha` and `rho` the parameters of the model. The function `ChFun_Heston` (see right above) is called and the output is the price of call option under the Heston model.

- `Delta_Heston`

  This function comes from Crisostomo (2014). The function is `function Delta = Delta_Heston(S0,V0,theta,kappa,alpha,r,rho,T,K)` where `V0` is the initial variance and `theta, kappa, alpha` and `rho` the parameters of the model. The function `ChFun_Heston` (see above) is called and the output is the value of the Delta under the Heston model.

Cost Functions & Models Calibration

The cost functions calculate the pricing $MSE and are then used to calibrate the models' parameters with the *lsqnonlin* numerical scheme in the calibration functions.

All the functions load a document `Data.txt` (there exists one for each estimation window of two days) that contains the empirical data and whose columns are, in order: dividend-adjusted stock price, strike price, maturity (in years), mid bid-ask call option price, risk-free rate (annual), implied volatility, bid price, ask price, indice (0 for first day of the window, 1 for second day), maturity (in days), stock price (not adjusted for dividends) and finally moneyness.

- `Cost_BS`

  The function is `function Error = Cost_BS(x)` where `x` is the value of the implied volatility. Given the empirical data and the value of `x`, the output is a vector of pricing errors and the $RMSE under the BS model, calculated by calling the function `Call_BS`.

- `Calibration_BS`

  This script loads the empirical data and, given the initial value and bounds on the implied volatility, calibrates the value of the implied volatility with the *lsqnonlin* numerical scheme, which calls the function `Cost_BS`. The output is the calibrated implied volatility, the $RMSE and the vector of in-sample pricing errors for the estimation window considered.

- `Cost_PBS`

  The function is `function Error = Cost_PBS(x)` where `x` is the vector of parameters of the PBS model: `x = [theta0,theta1,theta2]`. Given the empirical data and the value of `x`, the output is a vector of pricing errors and the $RMSE under the PBS model, calculated by calling the function `Call_PBS`.

- `Calibration_PBS`

  This script loads the empirical data and, given the initial value and bounds on parameters, calibrates the value of the parameters with the *lsqnonlin* numerical scheme, which calls the function `Cost_PBS`. The output is the calibrated parameters (`theta0,theta1,theta2`), the $RMSE and the vector of in-sample pricing errors for the estimation window considered.

- `Cost_HN`

  The function is `function Error = Cost_HN(x)` where `x` is the vector of parameters of the HN model: `x = [h0,b0,b1,omega,persistence]` where `persistence` is given by `b1+b2*omega*omega`. Given the empirical data and the value of `x`, the output is a vector of pricing errors and the $RMSE under the HN model, calculated by calling the function `Call_HN`.

- `Calibration_HN`

  This script loads the empirical data and, given the initial value and bounds on parameters, calibrates the value of the parameters with the *lsqnonlin* numerical scheme, which calls the function `Cost_PBS`. The output is the calibrated parameters (`h0,b0,b1,b2,omega`), the $RMSE and the vector of in-sample pricing errors for the estimation window considered.

- `Cost_Heston`

  The function is `function Error = Cost_Heston(x)` where `x` is the vector of parameters of the HN model: `x = [V0,theta,alpha,rho,2*kappa*theta-alpha*alpha]`. Given the empirical data and the value of `x`, the output is a vector of pricing errors and the $RMSE under the Heston model, calculated by calling the function `Call_Heston`.

- `Calibration_Heston`

  This script loads the empirical data and, given the initial value and bounds on parameters, calibrates the value of the parameters with the *lsqnonlin* numerical scheme, which calls the function `Cost_Heston`. The output is the calibrated parameters (`V0,theta,alpha,rho,kappa`), the $RMSE and the vector of in-sample pricing errors for the estimation window considered.

A.3 Out-of-Sample Pricing Errors

These scripts, given the parameters of the last estimation window, calculate the pricing errors out-of-sample. To do so, in addition to `Data.txt` (see section A.2), the document `Parameters.txt` is loaded, which contains all the calibrated parameters, one row for each estimation window, for 15 rows in total.

- `PricingErrors_BS`

  This scripts loads the empirical data and the parameters of the last estimation window, and calls the function `Call_BS` to calculate the $RMSE and the vector of out-of-sample pricing errors of the window considered as an output.

- `PricingErrors_PBS`

  This scripts loads the empirical data and the parameters of the last estimation window, and calls the function `Call_PBS` to calculate the $RMSE and the vector of out-of-sample pricing errors of the window considered as an output.

- `PricingErrors_HN`

  This scripts loads the empirical data and the parameters of the last estimation window, and calls the function `Call_HN` to calculate the $RMSE and the vector of out-of-sample pricing errors of the window considered as an output.

- `PricingErrors_Heston`

  This scripts loads the empirical data and the parameters of the last estimation window, and calls the function `Call_Heston` to calculate the $RMSE and the vector of out-of-sample pricing errors of the window considered as an output.

A.4 Out-of-Sample Payoff Replication Errors

These scripts, given the parameters of the last estimation window, calculate the payoff replication errors out-of-sample, with or without re-calibration of parameters. To do so, in addition to `Data.txt` (see section A.2) and `Parameters.txt` (see section A.3), the document `PriceDiv.txt` is loaded, which is needed to calculate the payoff replication errors and whose five columns contain the following data, in order: date, stock price (not adjusted for dividends), sum of discounted dividends for each date for the first day of the hedging window, sum of discounted dividends for each date for the second day of the hedging window, daily dividend.

- `Hedging_WithoutRecalibration_BS`

  This scripts loads the empirical data and the parameters of the last estimation window to calculate the $MAE and the vector of out-of-sample payoff replication errors (without re-calibration) of the window considered as an output.

- `Hedging_WithRecalibration_BS`

  This scripts loads the empirical data and the parameters of the last estimation window to calculate the $MAE and the vector of out-of-sample payoff replication errors (with re-calibration) of the window considered as an output.

- `Hedging_WithoutRecalibration_PBS`

  This scripts loads the empirical data and the parameters of the last estimation window and calls the function `Delta_PBS` to calculate the $MAE and the vector of out-of-sample payoff replication errors (without re-calibration) of the window considered as an output.

- `Hedging_WithRecalibration_PBS`

  This scripts loads the empirical data and the parameters of the last estimation window and calls the function `Delta_PBS` to calculate the $MAE and the vector of out-of-sample payoff replication errors (with re-calibration) of the window considered as an output.

- `Hedging_WithoutRecalibration_HN`

  This scripts loads the empirical data and the parameters of the last estimation window and calls the function `Delta_HN` to calculate the $MAE and the vector of out-of-sample payoff replication errors (without re-calibration) of the window considered as an output.

- `Hedging_WithRecalibration_HN`

  This scripts loads the empirical data and the parameters of the last estimation window and calls the function `Delta_HN` to calculate the $MAE and the vector of out-of-sample payoff replication errors (with re-calibration) of the window considered as an output.

- `Hedging_WithoutRecalibration_Heston`

  This scripts loads the empirical data and the parameters of the last estimation window and calls the function `Delta_Heston` to calculate the $MAE and the vector of out-of-sample payoff replication errors (without re-calibration) of the window considered as an output.

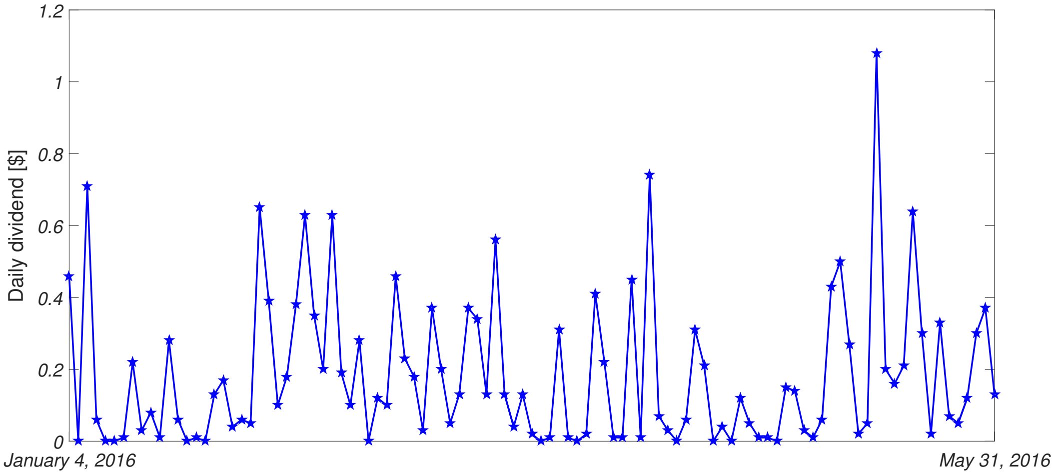
- `Hedging_WithRecalibration_Heston`

  This scripts loads the empirical data and the parameters of the last estimation window and calls the function `Delta_Heston` to calculate the $MAE and the vector of out-of-sample payoff replication errors (with re-calibration) of the window considered as an output.

Appendix B

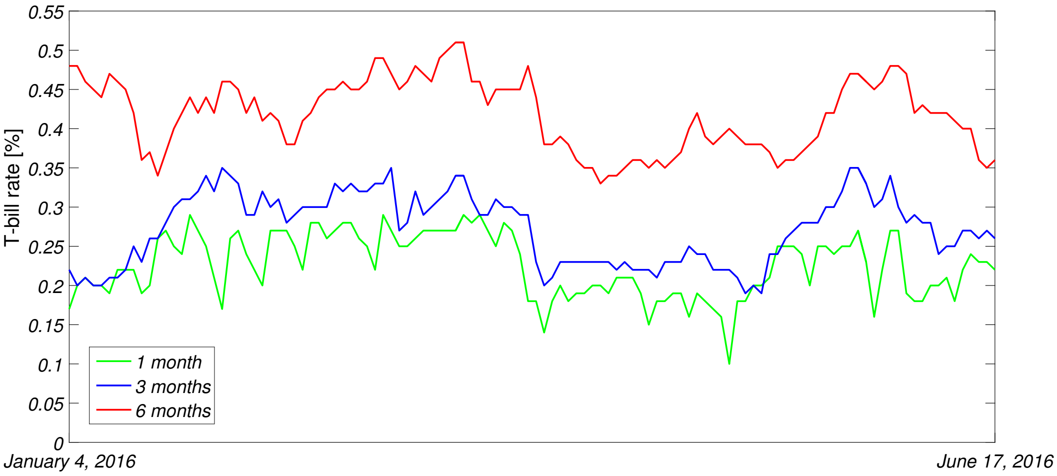
Description of Data

FIGURE B.1: S&P500 index daily dividends from January 4, 2016 to May 31, 2016.



Notes. The time span goes from January 4, 2016, which is the first date of the sample of S&P500 options, to the largest maturity considered for payoff replication, i.e. May 31, 2016. The dividends were obtained on S&P Dow Jones Indices (2016b)

FIGURE B.2: Treasury-bill rates from January 4, 2016 to June 17, 2016.



Notes. The time span goes from January 4, 2016, which is the first date of our sample of options, to the furthest maturity considered for payoff replication for the two samples, i.e. June 17, 2016 for Apple. The T-bill rates were obtained on Board of Governors of the Federal Reserve System (2016).

TABLE B.1: Quantiles of maturity, moneyness and call option price for the S&P500 index and Apple stock.

S&P500

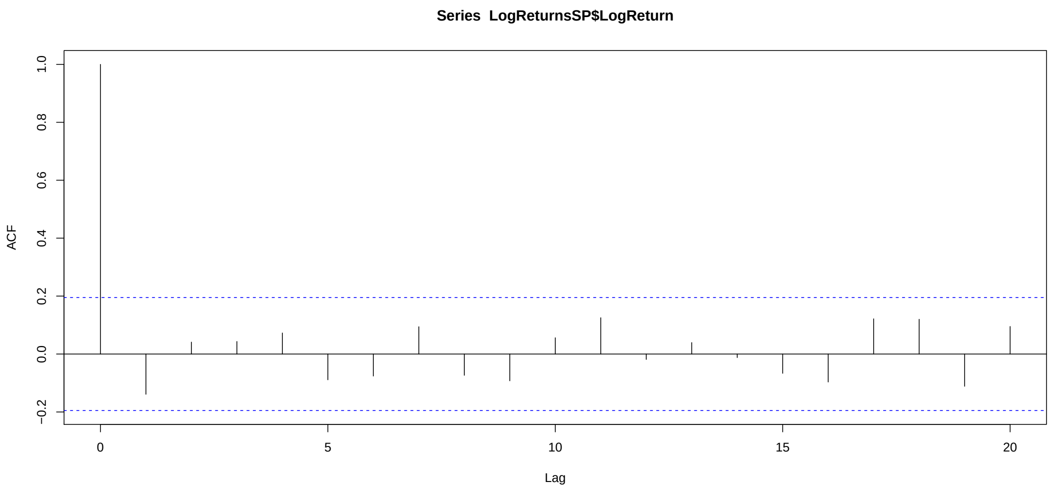
	min	25%	50%	75%	max
Days to maturity	6	30	55	80	105
Moneyness	0.9283	0.9875	0.9967	1.0060	1.0619
Call option price	4.5500	39.8125	55.0000	76.9500	173.7000

Apple

	min	25%	50%	75%	max
Days to maturity	6	30	55	79	115
Moneyness	0.7964	0.9634	1.015	1.0899	1.2760
Call option price	0.3750	2.5188	5.8000	10.8562	22.9500

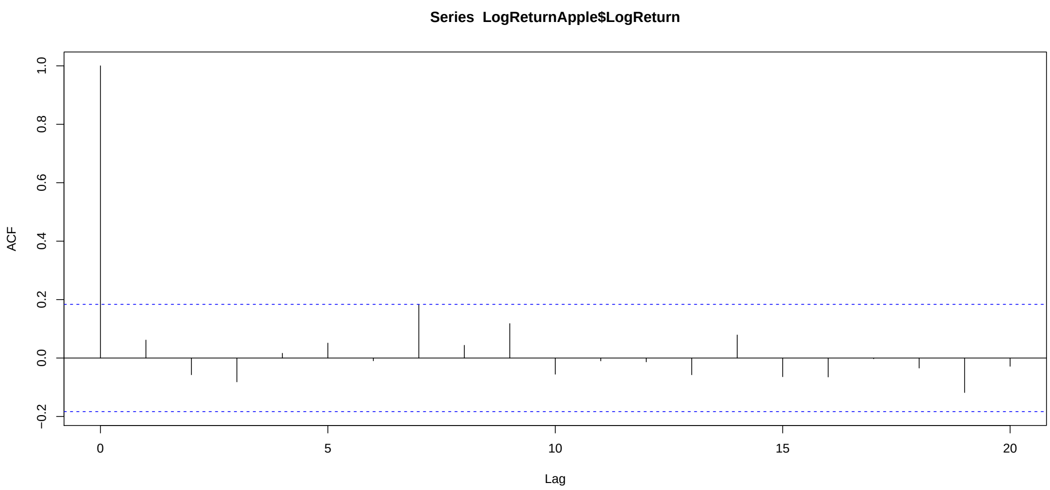
Notes: The two samples of option contracts cover the period January 4, 2016 to March 30, 2016. The moneyness is defined as $\frac{S_t - D_{pv}}{K}$. We report the quantiles from the total sample of options used for each time series. The total sample also includes those options that will have to be eliminated to calculate the payoff replication errors because they have maturity dates extending beyond the time of the analysis.

FIGURE B.3: Auto-correlation function of log-returns of the S&P500 index from January 4 to May 31, 2016.



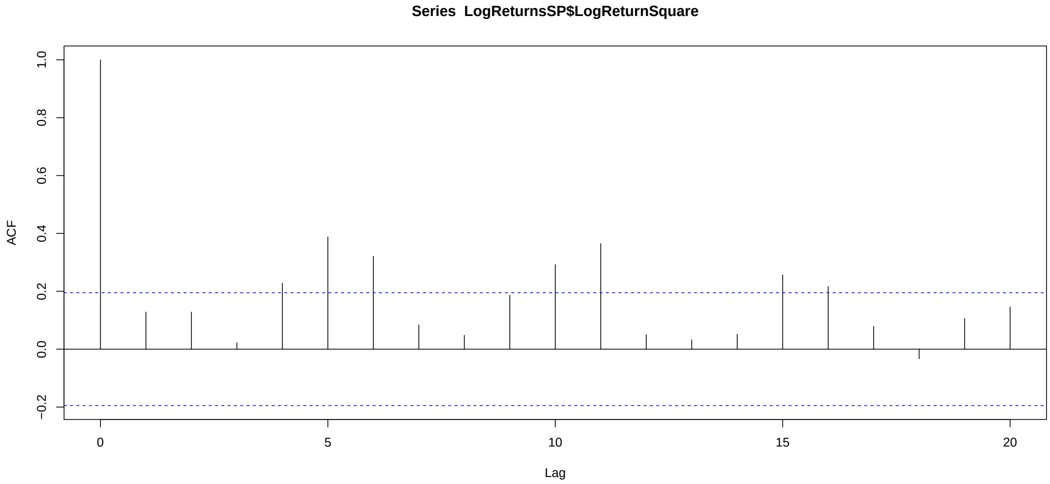
Notes. The time span goes from January 4, 2016, which is the first date of our sample of options, to the largest maturity considered for payoff replication, i.e. May 30, 2016. The blue bands represent the 5% confidence interval for a null auto-correlation.

FIGURE B.4: Auto-correlation function of log-returns of Apple stock from January 4 to June 17, 2016.



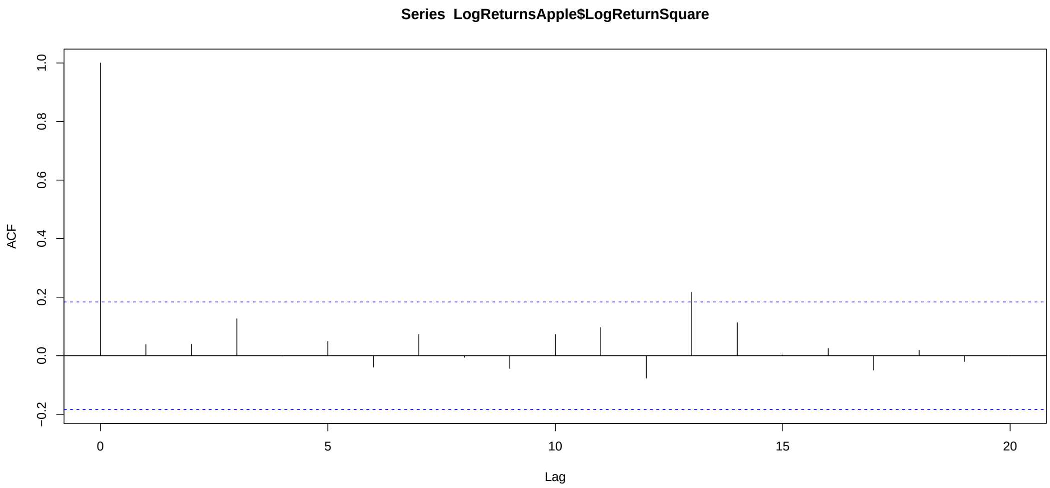
Notes. The time span goes from January 4, 2016, which is the first date of our sample of options, to the largest maturity considered for payoff replication, i.e. June 17, 2016. The blue bands represent the 5% confidence interval for a null auto-correlation.

FIGURE B.5: Auto-correlation function of log-returns squared for the S&P500 index from January 4 to May 31, 2016.



Notes. The time span goes from January 4, 2016, which is the first date of our sample of options, to the largest maturity considered for payoff replication, i.e. May 30, 2016. The blue bands represent the 5% confidence interval for a null auto-correlation.

FIGURE B.6: Auto-correlation function of log-returns squared for Apple stock from January 4 to June 17, 2016.



Notes. The time span goes from January 4, 2016, which is the first date of our sample of options, to the largest maturity considered for payoff replication, i.e. June 17, 2016. The blue bands represent the 5% confidence interval for a null auto-correlation.

Appendix C

Breakdown of Number of Option Contracts

Table C.1: Options elimination criteria applied to the S&P500 index and Apple stock.

	S&P500	**Apple**
Original sample	3000	2930
Option price below \$3/8	-0	-180
Maturity lower than 6 days	-170	-66
Arbitrage condition	-0	-0
Total sample	2830	2684
Hedging criterion	-320	-140
Hedging sub-sample	2510	2544

Notes: The two samples of option contracts cover the period January 4, 2016 to March 30, 2016. The arbitrage condition eliminates options when $C_t < S_t - D_{pv} - Ke^{-r(T-t)}$, but the inequality was not verified on any options of our two samples. The hedging criterion eliminates options with maturity dates that extended beyond the time of the analysis because we did not have all the stock prices available to calculate the hedging errors.

TABLE C.2: Number of option contracts used across moneyness and maturity in the estimation and hedging windows for the S&P500 index.

Estimation windows

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.98	0	21	190	211
0.98<M<1	203	254	168	625
1 <M<1.02	157	146	139	442
M>1.02	0	9	133	142
Total	360	430	630	1420

Hedging windows

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.98	0	24	121	145
0.98<M<1	202	248	49	499
1 <M<1.02	158	142	42	342
M>1.02	0	16	88	104
Total	360	430	300	1090

Notes: The option contracts cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the statistics on the number of contracts used for each moneyness-maturity category in the estimation and hedging windows for the S&P500 index as underlying asset. Each window is composed of two days. The estimation windows are used to calibrate the parameters of our option pricing models, while the hedging windows are used to calculate the payoff replication errors out-of-sample (i.e. using the parameters of the previous estimation window). Our sample contains 30 windows of two days in total, half being used for parameters estimation and half for hedging purposes.

TABLE C.3: Number of option contracts used across moneyness and maturity in the estimation and hedging windows for Apple stock.

Estimation windows

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.94	15	90	141	246
0.94<M<1	122	72	103	297
1<M<1.06	132	86	126	344
M>1.06	59	169	220	448
Total	328	417	590	1335

Hedging windows

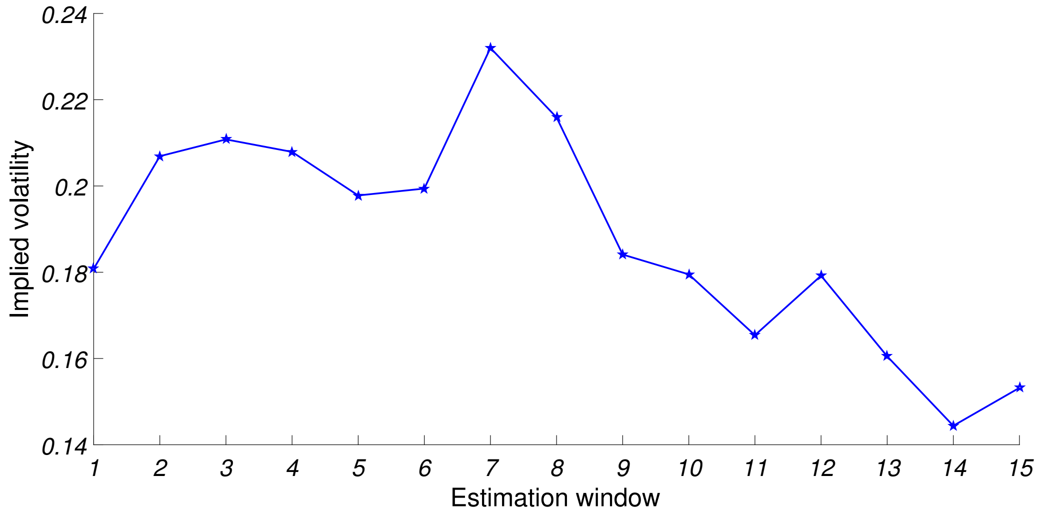
	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.94	20	82	117	219
0.94<M<1	135	69	80	284
1 <M<1.06	141	78	96	315
M>1.06	63	161	167	391
Total	359	390	460	1209

Notes: The option contracts cover the period from January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the statistics on the number of contracts used for each moneyness-maturity category in the estimation and hedging windows for Apple stock as underlying asset. Each window is composed of two days. The estimation windows are used to calibrate the parameters of our option pricing models, while the hedging windows are used to calculate the payoff replication errors out-of-sample (i.e. using the parameters of the previous estimation window). Our sample contains 30 windows of two days in total, half being used for parameters estimation and half for hedging purposes.

Appendix D

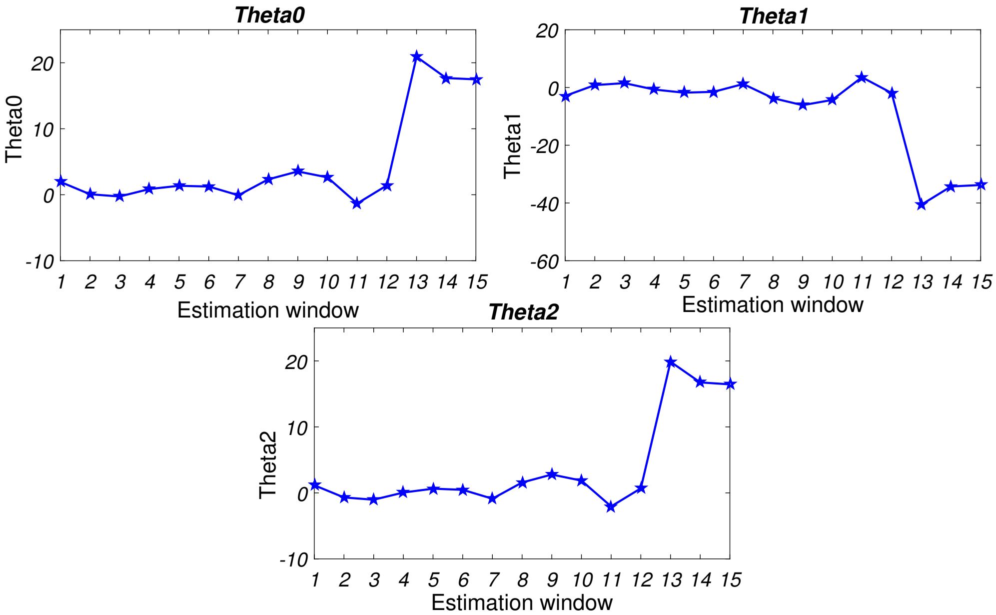
Plots & Summary Statistics of Parameters

FIGURE D.1: Calibrated implied volatility from the S&P500 index.



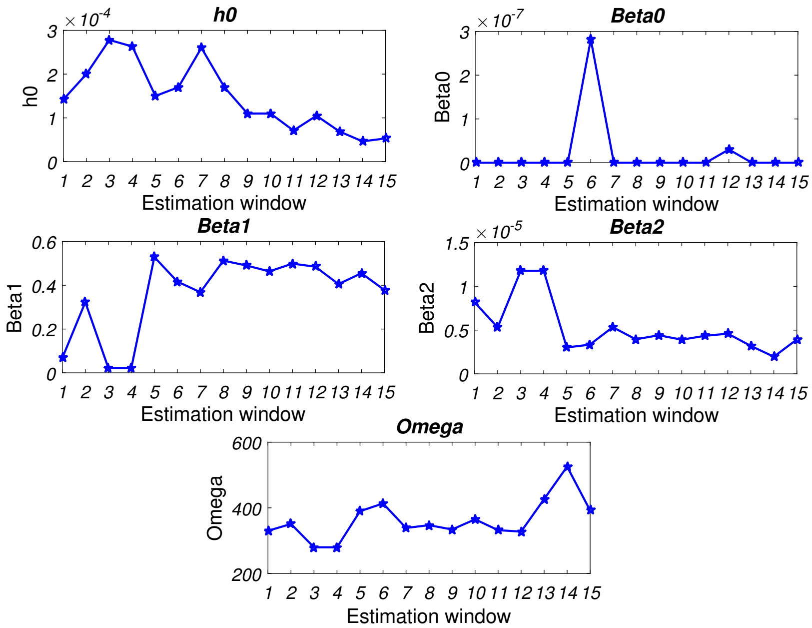
Notes. The implied volatility is calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for the S&P500 index. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

FIGURE D.2: Calibrated PBS parameters from the S&P500 index.



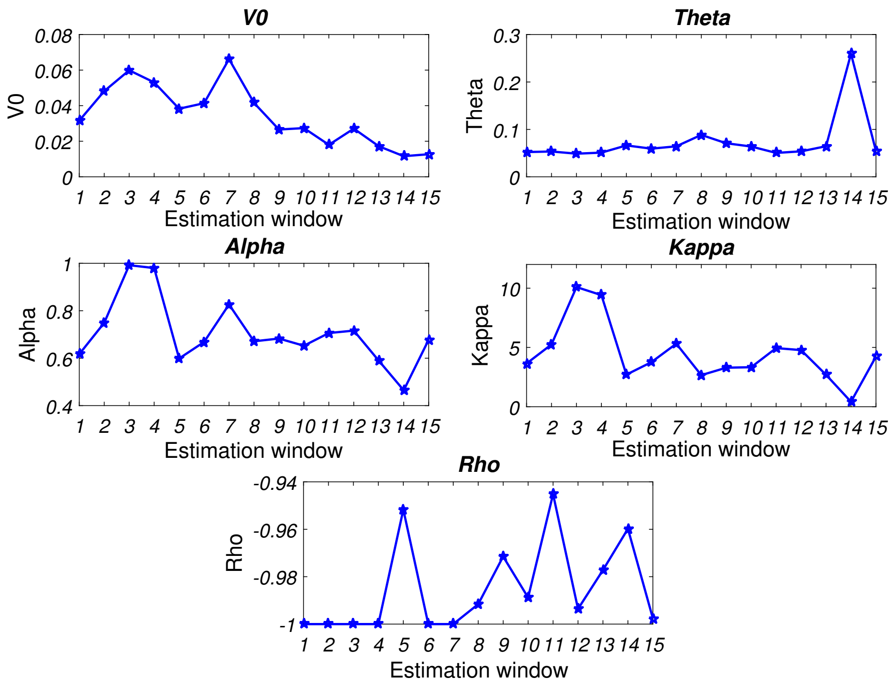
Notes: The PBS parameters are calibrated to option data by minimizing the Dollar Mean Squared Error (\$MSE) between the market and the model call option prices for the S&P500 index. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

FIGURE D.3: Calibrated HN parameters from the S&P500 index.



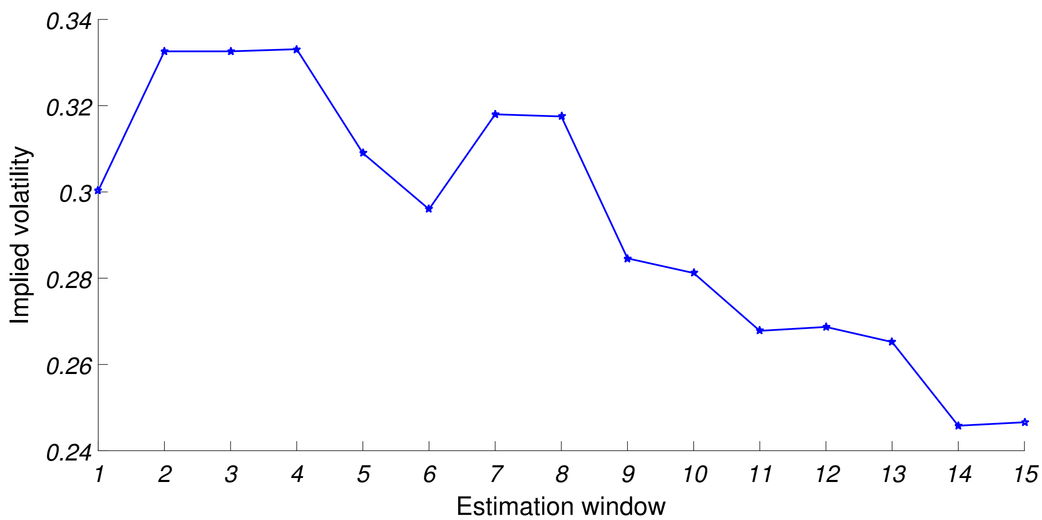
Notes: The HN parameters are calibrated to option data by minimizing the Dollar Mean Squared Error (\$MSE) between the market and the model call option prices for the S&P500 index. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

FIGURE D.4: Calibrated Heston parameters from the S&P500 index.



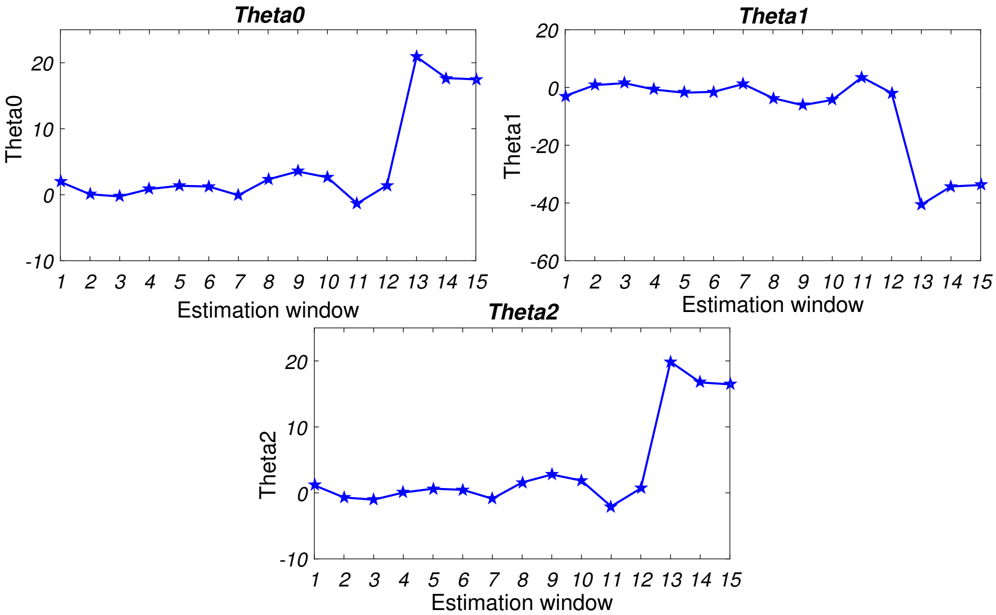
Notes: The Heston parameters are calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for the S&P500 index. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

FIGURE D.5: Calibrated implied volatility from Apple stock.



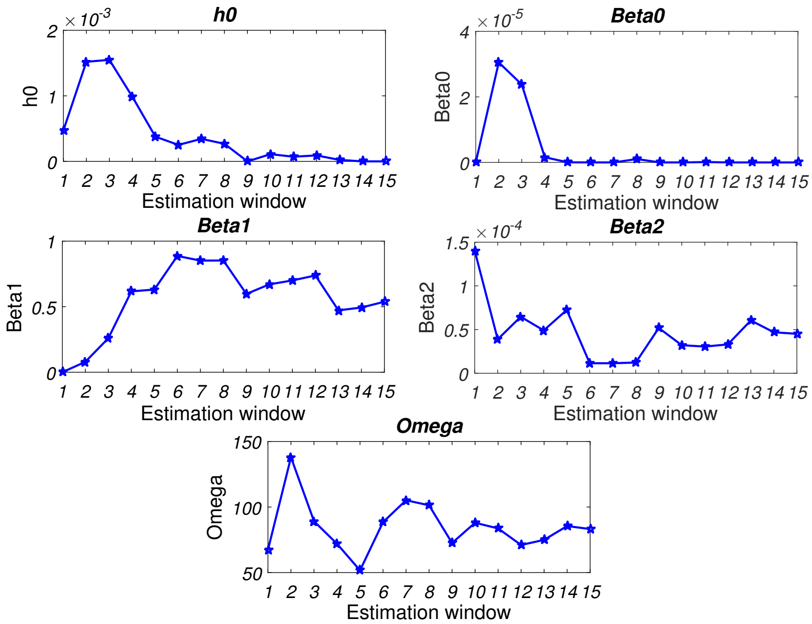
Notes. The implied volatility is calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for Apple stock. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

Figure D.6: Calibrated PBS parameters from Apple stock.



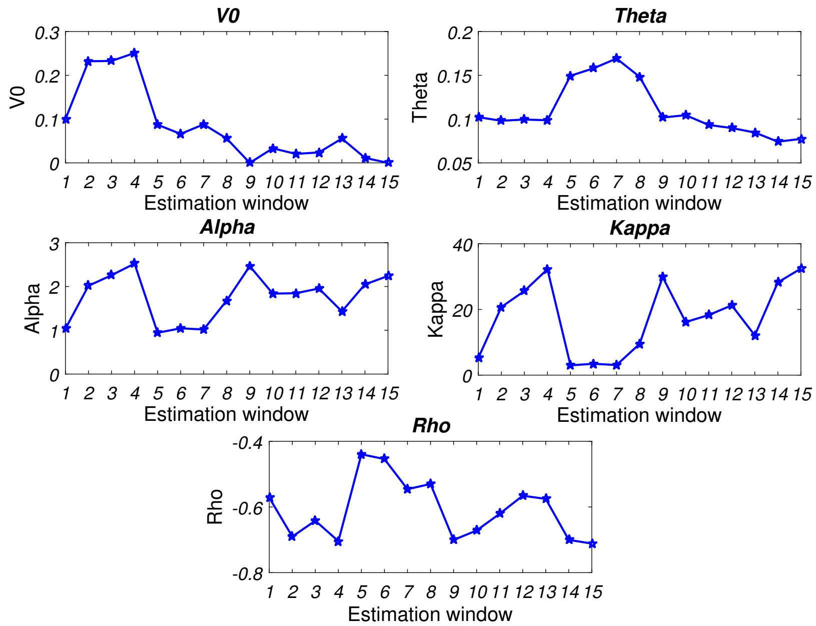
Notes: The PBS parameters are calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for Apple stock. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

Figure D.7: Calibrated HN parameters from Apple stock.



Notes: The HN parameters are calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for Apple stock. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

FIGURE D.8: Calibrated Heston parameters from Apple stock.



Notes: The Heston parameters are calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for Apple stock. The calibration is done on 15 estimation windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

Table D.1: Summary statistics of the calibrated parameters for the S&P500 index.

	Mean	**Std. dev.**	$\left\lvert\frac{\textbf{Std.dev.}}{\textbf{Mean}}\right\rvert$	**Min**	**Max**
Black-Scholes					
σ^*	0.1879	0.0251	0.1335	0.1444	0.2320
Practitioner BS					
θ_0	4.6509	7.4026	1.5916	-1.3290	20.901
θ_1	-8.3009	14.727	1.774	-40.612	3.5491
θ_2	3.8400	7.3049	1.9023	-2.0520	19.8715
Heston-Nandi					
h_0	$1.46{*}10^{-4}$	$7.68{*}10^{-5}$	0.5244	$4.63{*}10^{-5}$	$2.78{*}10^{-4}$
β_0	$2.08{*}10^{-8}$	$7.27{*}10^{-8}$	3.4920	$4.33{*}10^{-14}$	$2.82{*}10^{-7}$
β_1	0.3625	0.1781	0.4912	0.0223	0.5289
β_2	$5.27{*}10^{-6}$	$2.98{*}10^{-6}$	0.5664	$1.96{*}10^{-6}$	$1.18{*}10^{-5}$
ω	362.17	61.8265	0.1707	279.82	525.61
$\beta_1 + \beta_2\omega^2$	0.9768	0.0157	0.0161	0.9442	0.9964
Heston					
V_0	0.0347	0.0170	0.4893	0.0116	0.0662
θ	0.0734	0.0525	0.7155	0.0488	0.2595
α	0.7061	0.1393	0.1973	0.4645	0.9921
κ	4.4352	2.5013	0.5640	0.4157	10.093
ρ	-0.9851	0.0193	0.0196	-1.0000	-0.9452

Notes: The parameters are calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for the S&P500 index. The calibration is done on 15 windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

Table D.2: Summary statistics of the calibrated parameters for Apple stock.

	Mean	Std. dev.	$\left\lvert\frac{\text{Std.dev.}}{\text{Mean}}\right\rvert$	Min	Max
Black-Scholes					
σ^*	0.2933	0.0303	0.1034	0.2458	0.3331
Practitioner BS					
θ_0	1.5538	0.4759	0.3062	0.5317	2.4415
θ_1	-2.2647	1.0049	0.4437	-4.1249	-0.1576
θ_2	0.9973	0.4972	0.4985	-0.0424	1.9154
Heston-Nandi					
h_0	$4.04{*}10^{-4}$	$5.26{*}10^{-4}$	1.3019	$2.99{*}10^{-14}$	$1.54{*}10^{-3}$
β_0	$3.80{*}10^{-6}$	$9.57{*}10^{-6}$	2.5158	$2.22{*}10^{-14}$	$3.05{*}10^{-5}$
β_1	0.5589	0.2670	0.4777	$1.88{*}10^{-7}$	0.8869
β_2	$4.67{*}10^{-5}$	$3.20{*}10^{-5}$	0.6857	$1.15{*}10^{-5}$	$1.40{*}10^{-4}$
ω	84.8064	19.920	0.235	51.692	137.92
$\beta_1 + \beta_2\omega^2$	0.8643	0.0897	0.1038	0.6381	0.9783
Heston					
V_0	0.0838	0.0861	1.0271	0	0.251
θ	0.1099	0.0305	0.2776	0.0742	0.1692
α	1.7563	0.5428	0.3091	0.9434	2.523
κ	17.416	10.894	0.625	2.977	32.435
ρ	-0.6082	0.0908	0.1492	-0.7114	-0.4395

Notes: The parameters are calibrated to option data by minimizing the Dollar Mean Squared Error ($MSE) between the market and the model call option prices for Apple stock. The calibration is done on 15 windows of two days, each separated by two days, on the period January 4, 2016 to March 30, 2016.

Appendix E

Out-of-Sample Payoff Replication Performance

Table E.1: Out-of-sample payoff replication \$MAE across moneyness and maturity.

S&P500

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	18.3521	18.1669	M<0.98	/	27.2593	28.6919
0.98<M<1	10.2086	11.3916	14.4577	0.98<M<1	11.7161	14.5022	23.8324
1<M<1.02	10.0708	12.1788	15.2162	1<M<1.02	11.7352	15.1616	22.7461
M>1.02	/	19.7333	16.3625	M>1.02	/	25.8244	21.8833

Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	15.5544	15.0618	M<0.98	/	27.8024	29.8399
0.98<M<1	10.1493	10.6130	13.2918	0.98<M<1	12.0002	14.6957	25.4196
1<M<1.02	10.1113	11.8978	13.8004	1<M<1.02	11.9224	15.3283	23.7301
M>1.02	/	15.1355	15.2485	M>1.02	/	25.9930	22.7682

Apple

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	1.0186	1.0110	1.0164	M<0.94	0.8515	1.0527	0.9146
0.94<M<1	0.7652	1.0354	1.8044	0.94<M<1	0.7967	1.2089	1.7972
1<M<1.06	1.0185	1.0547	2.1203	1<M<1.06	1.1358	1.2093	2.2971
M>1.06	1.1058	1.3111	1.8128	M>1.06	1.1528	1.3201	2.0585

Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	0.9319	0.9011	0.9689	M<0.94	0.9206	1.0932	0.9407
0.94<M<1	0.7347	0.9468	1.8363	0.94<M<1	0.8163	1.2348	1.8031
1<M<1.06	1.0138	1.0083	2.1993	1<M<1.06	1.1633	1.2278	2.3000
M>1.06	1.0730	1.3097	1.8255	M>1.06	1.1664	1.3030	2.0396

Notes: The two samples of option contracts (S&P500 and Apple) cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the \$MAE for each moneyness-maturity category. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total for both underlying asset sample.

TABLE E.2: Out-of-sample normalized payoff replication $ME across moneyness and maturity.

S&P500

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	0.8184	0.9411	M<0.98	/	0.9224	0.9997
0.98<M<1	0.5011	0.6166	0.7009	0.98<M<1	0.6316	0.8196	0.9793
1<M<1.02	0.3901	0.4794	0.6630	1<M<1.02	0.5368	0.7411	0.9236
M>1.02	/	1	0.5944	M>1.02	/	1	0.7910
Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	0.7185	0.8657	M<0.98	/	0.9304	1
0.98<M<1	0.4673	0.4882	0.5610	0.98<M<1	0.6317	0.7970	0.9869
1<M<1.02	0.3506	0.3417	0.4840	1<M<1.02	0.5289	0.7342	0.9492
M>1.02	/	1	0.3279	M>1.02	/	1	0.8179

Apple

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	0.7801	0.8008	0.9005	M<0.94	0.5128	0.9056	0.8967
0.94<M<1	0.5980	0.5823	0.9631	0.94<M<1	0.6341	0.7414	0.9465
1<M<1.06	0.4490	0.5102	0.9356	1<M<1.06	0.6527	0.5159	0.9323
M>1.06	0.6622	0.5947	0.8145	M>1.06	0.6977	0.5574	0.8557
Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	0.7923	0.8376	0.9265	M<0.94	0.4860	0.9108	0.8940
0.94<M<1	0.5626	0.5927	0.9713	0.94<M<1	0.5887	0.7595	0.9457
1<M<1.06	0.4406	0.5083	0.9532	1<M<1.06	0.6676	0.5088	0.9301
M>1.06	0.6193	0.5822	0.8243	M>1.06	0.7011	0.5442	0.8533

Notes: The two samples of option contracts (S&P500 and Apple) cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the normalized $ME for each moneyness-maturity category. The normalized errors are calculated as the mean errors divided by the mean absolute errors. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total for both underlying asset sample.

Table E.3: Out-of-sample payoff replication \$MAE across moneyness for short-maturity option contracts for the S&P500 index.

		Options	*BS*	*PBS*	*Heston*	*HN*
0.98<M<1	DTM<20	115	11.2308	11.1004	11.8057	11.9669
	DTM<15	79	10.4455	10.3401	10.8603	11.0768
	DTM<10	36	7.1393	7.0973	7.9842	8.1461
1<M<1.02	DTM<20	95	11.3156	11.1650	12.1706	12.1803
	DTM<15	71	10.2749	10.1805	11.1414	11.3747
	DTM<10	34	7.4969	7.6053	8.4599	8.4937

Notes. The S&P500 options cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the \$MAE for short-maturity ITM and OTM options, deeper levels of moneyness being not available for low maturities in our sample. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total over the sample period.

Table E.4: Ranking of models across moneyness and maturity based on the out-of-sample payoff replication \$MAE for Apple stock.

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	1) Heston (0.8515)	1) PBS (0.9011)	1) Heston (0.9146)
	2) HN (+0.0691)	2) BS (+0.1099)	2) HN (+0.0261)
	3) PBS (+0.0113)	3) Heston (+0.0417)	3) PBS (+0.0282)
	4) BS (+0.0867)	4) HN (+0.0405)	4) BS (+0.0475)
0.94<M<1	1) PBS (0.7347)	1) PBS (0.9468)	1) Heston (1.7972)
	2) BS (+0.0315)	2) BS (+0.0886)	2) HN (+0.0059)
	3) Heston (+0.0315)	3) Heston (+0.1735)	3) BS (+0.0013)
	4) HN (+0.0196)	4) HN (+0.0259)	4) PBS (+0.0319)
1<M<1.06	1) PBS (1.0138)	1) PBS (1.0083)	1) BS (2.1203)
	2) BS (+0.0047)	2) BS (+0.0464)	2) PBS (+0.0790)
	3) Heston (+0.1173)	3) Heston (+0.1546)	3) Heston (+0.0978)
	4) HN (+0.0275)	4) HN (+0.0185)	4) HN (+0.0029)
M>1.06	1) PBS (1.0730)	1) HN (1.3030)	1) BS (1.8128)
	2) BS (+0.0328)	2) PBS (+0.0067)	2) PBS (+0.0127)
	3) Heston (+0.0470)	3) BS (+0.0014)	3) HN (+0.2141)
	4) HN (+0.0136)	4) Heston (+0.0090)	4) Heston (+0.0189)

Notes. The Apple stock options cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We rank the models for each moneyness-maturity category according to the payoff replication \$MAE, which are reported in brackets for the best model for each category, while the incremental increase in \$MAE is reported for the second, third and fourth best models. The \$MAE are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total over the sample period.

Table E.5: Out-of-sample normalized payoff replication $MAE across moneyness and maturity.

S&P500

Black-Scholes				*Heston*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	0.7804	0.5400	M<0.98	/	1.0229	0.8281
0.98<M<1	0.4078	0.2319	0.1933	0.98<M<1	0.4539	0.2955	0.3207
1<M<1.02	0.2449	0.1853	0.1572	1<M<1.02	0.2827	0.2317	0.2367
M>1.02	/	0.1654	0.1220	M>1.02	/	0.2194	0.1642
Practitioner BS				*Heston-Nandi*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	0.6240	0.4450	M<0.98	/	1.0339	0.8525
0.98<M<1	0.4045	0.2167	0.1774	0.98<M<1	0.4607	0.2995	0.3411
1<M<1.02	0.2458	0.1815	0.1420	1<M<1.02	0.2857	0.2343	0.2465
M>1.02	/	0.1287	0.1135	M>1.02	/	0.2210	0.1708

Apple

Black-Scholes				*Heston*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	1.7130	1.0723	0.4727	M<0.94	1.3112	1.0710	0.4428
0.94<M<1	0.4922	0.3060	0.3235	0.94<M<1	0.4994	0.3666	0.3231
1<M<1.06	0.2892	0.1639	0.2580	1<M<1.06	0.3143	0.1909	0.2765
M>1.06	0.0914	0.1044	0.1333	M>1.06	0.0968	0.1057	0.1521
Practitioner BS				*Heston-Nandi*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	1.5562	0.9632	0.4355	M<0.94	1.3875	1.1090	0.4571
0.94<M<1	0.4678	0.2805	0.3270	0.94<M<1	0.5009	0.3762	0.3248
1<M<1.06	0.2883	0.1566	0.2677	1<M<1.06	0.3211	0.1941	0.2769
M>1.06	0.0887	0.1041	0.1351	M>1.06	0.0978	0.1043	0.1505

Notes: The two samples of option contracts (S&P500 and Apple) cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the normalized $MAE for each moneyness-maturity category. The normalization is done by dividing the hedging error of one option by its market price. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total for both underlying asset sample.

TABLE E.6: Average out-of-sample normalized payoff replication \$MAE for different moneyness and maturity categories.

		BS	*PBS*	*Heston*	*HN*
Overall	S&P500	0.3028	0.2679	0.4056	0.4146
	Apple	0.3615	0.3426	0.3742	0.3815
		BS	*PBS*	*Heston*	*HN*
DTM⩽30	S&P500	0.3263	0.3251	0.3683	0.3732
	Apple	0.3907	0.3780	0.4068	0.4110
30<DTM <60	S&P500	0.3407	0.2877	0.4424	0.4472
	Apple	0.4116	0.3761	0.4335	0.4459
DTM>60	S&P500	0.2531	0.2195	0.3874	0.4027
	Apple	0.2969	0.2913	0.2986	0.3023
		BS	*PBS*	*Heston*	*HN*
Deep-out-of-the-money	S&P500	0.6602	0.5345	0.9255	0.9432
	Apple	0.7725	0.6993	0.7569	0.7830
Out-of-the-money	S&P500	0.2777	0.2662	0.3567	0.3671
	Apple	0.3739	0.3584	0.3964	0.4006
In-the-money	S&P500	0.1958	0.1898	0.2504	0.2555
	Apple	0.2370	0.2375	0.2606	0.2640
Deep-in-the-money	S&P500	0.1437	0.1211	0.1918	0.1959
	Apple	0.1188	0.1196	0.1289	0.1274

Notes: The two samples of option contracts (S&P500 and Apple) cover the period January 4, 2016 to March 30, 2016. The moneyness is defined as $M = \frac{S_t - D_{pv}}{K}$ and the categories are different for S&P500 and Apple. For S&P500, they are, in order of increasing moneyness: $M \in [\leftarrow, 0.98], [0.98, 1], [1, 1.02], [1.02, \rightarrow]$. For Apple they are: $M \in [\leftarrow, 0.94], [0.94, 1], [1, 1.06], [1.06, \rightarrow]$. DTM refers to Days To Maturity. We report the normalized payoff replication \$MAE, averaged for different categories of moneyness and maturity. The normalization is done by dividing the hedging error of one option by its market price. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total for both underlying asset sample.

Table E.7: Number of option contracts used for out-of-sample payoff replication in function of the end-moneyness.

S&P500

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.98	8	9	/	17
0.98<M<1.03	217	108	26	351
1.03<M<1.08	128	195	108	431
M>1.08	7	118	166	291
Total	360	430	300	1090

Apple

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.85	3	26	30	59
0.85<M<1	101	92	173	366
1<M<1.15	201	154	204	559
M>1.15	54	118	53	225
Total	359	390	460	1209

Notes: The two samples of option contracts cover the period January 4, 2016 to March 30, 2016. M refers to the end-moneyness, defined as $\frac{S_T}{K}$, and DTM to Days To Maturity. We report the statistics on the number of contracts used to calculate the out-of-sample payoff replication errors for each moneyness-maturity category.

Table E.8: Out-of-sample payoff replication \$MAE in function of the end-moneyness.

S&P500

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	15.0351	14.2850	/	M<0.98	14.2477	2.7470	/
0.98<M<1.03	10.4916	13.1134	27.9608	0.98<M<1.03	11.7505	13.7490	28.8934
1.03<M<1.08	9.5791	11.4221	17.2942	1.03<M<1.08	11.7748	16.7577	29.1211
M>1.08	4.3223	13.0386	14.4027	M>1.08	7.1149	17.2844	21.8329
Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	16.7130	12.3837	/	M<0.98	14.2386	6.9505	/
0.98<M<1.03	10.3494	12.0191	22.2067	0.98<M<1.03	11.9973	13.7470	28.8177
1.03<M<1.08	9.6742	10.5440	14.3952	1.03<M<1.08	12.0415	16.7188	30.2988
M>1.08	4.2747	12.4694	13.6337	M>1.08	7.0179	17.7701	23.1020

Apple

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.85	0.1249	0.2295	0.6109	M<0.85	0.2206	0.3113	0.4669
0.85<M<1	1.0716	0.9192	1.5439	0.85<M<1	1.10702	0.9011	1.4294
1<M<1.15	0.8933	1.2613	1.9473	1<M<1.15	0.9691	1.3674	2.2649
M>1.15	0.9034	1.3807	1.6395	M>1.15	0.9281	1.4832	1.7309
Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.85	0.1937	0.2046	0.5870	M<0.85	0.3229	0.3364	0.4998
0.85<M<1	1.0207	0.8342	1.5396	0.85<M<1	1.1225	0.9170	1.4388
1<M<1.15	0.8870	1.2261	2.0140	1<M<1.15	1.0038	1.3888	2.2571
M>1.15	0.8595	1.3376	1.5365	M>1.15	0.9263	1.4696	1.7241

Notes: The two samples of option contracts (S&P500 and Apple) cover the period January 4, 2016 to March 30, 2016. M refers to the end-moneyness, defined as $\frac{S_T}{K}$, and DTM to Days To Maturity. We report the \$MAE for each moneyness-maturity category. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days, by using the parameters estimated on the previous estimation window, also composed of two days. 15 hedging windows are used in total for both underlying asset sample.

Appendix F

Out-of-Sample Pricing Performance

TABLE F.1: Number of option contracts used for out-of-sample pricing across moneyness and maturity.

S&P500

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.98	0	24	182	206
0.98<M<1	202	248	173	623
1<M<1.02	158	142	137	437
M>1.02	0	16	128	144
Total	360	430	620	1410

Apple

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.94	20	82	148	250
0.94<M<1	135	69	100	304
1<M<1.06	141	78	122	341
M>1.06	63	161	230	454
Total	359	390	600	1349

Notes: The two samples of option contracts cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the statistics on the number of contracts used to calculate the out-of-sample pricing errors for each moneyness-maturity category.

TABLE F.2: Out-of-sample pricing \$RMSE across moneyness and maturity.

S&P500

Black-Scholes				*Heston*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	8.0373	10.3846	M<0.98	/	2.3954	4.6803
0.98<M<1	5.57586	5.8640	4.8417	0.98<M<1	3.5738	4.57864	4.4500
1<M<1.02	5.0609	5.0279	6.0960	1<M<1.02	4.1158	4.4463	4.3314
M>1.02	/	8.6172	10.8642	M>1.02	/	2.4544	4.8217
Practitioner BS				*Heston-Nandi*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	2.2924	4.7949	M<0.98	/	2.3255	4.6027
0.98<M<1	5.0643	5.2039	4.4499	0.98<M<1	3.5208	4.6004	4.3516
1<M<1.02	5.1592	4.9945	4.5242	1<M<1.02	3.8894	4.4483	4.2347
M>1.02	/	2.2451	5.0347	M>1.02	/	2.0792	4.4671

Apple

Black-Scholes				*Heston*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	0.3881	0.3945	0.4846	M<0.94	0.3374	0.2824	0.2876
0.94<M<1	0.5634	0.4902	0.4087	0.94<M<1	0.3278	0.3840	0.3769
1<M<1.06	0.5379	0.4301	0.3647	1<M<1.06	0.3126	0.3935	0.3735
M>1.06	0.4963	0.5639	0.6136	M>1.06	0.3548	0.3677	0.3203
Practitioner BS				*Heston-Nandi*			
	DTM⩽30	30<DTM⩽60	DTM>60		DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	0.3735	0.2728	0.2634	M<0.94	0.4247	0.3119	0.2787
0.94<M<1	0.5211	0.4358	0.3662	0.94<M<1	0.3653	0.4047	0.3611
1<M<1.06	0.5117	0.4405	0.3522	1<M<1.06	0.3274	0.4145	0.3630
M>1.06	0.4330	0.3617	0.3030	M>1.06	0.3392	0.3704	0.3236

Notes: The two samples of option contracts (S&P500 and Apple) cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the \$RMSE for each moneyness-maturity category. The errors are calculated out-of-sample on a window of two days by using the parameters estimated on the previous estimation window, also composed of two days. 15 windows are used in total to calculate the out-of-sample pricing errors for both underlying asset sample.

Table F.3: Ranking of models across moneyness and maturity based on the out-of-sample pricing \$RMSE for the S&P500 index.

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.98	/	1) PBS (2.2924) 2) HN (+0.0331) 3) Heston (+0.0699) 4) BS(+5.6419)	1) HN (4.6027) 2) Heston (+0.0776) 3) PBS (+0.1146) 4) BS (+5.5897)
0.98<M<1	1) HN (3.5208) 2) Heston (+0.0530) 3) PBS (+1.4905) 4) BS (+0.5116)	1) Heston (4.57864) 2) HN (+0.0218) 3) PBS (+0.6035) 4) BS (+0.6601)	1) HN (4.3516) 2) PBS (+0.0983) 3) Heston (+0.0001) 4) BS (+0.3917)
1<M<1.02	1) HN (+3.8894) 2) Heston (+0.2264) 3) BS (+0.9451) 4) PBS (+0.0983)	1) Heston (4.4463) 2) HN (+0.0020) 3) PBS (+0.5462) 4) BS (+0.0334)	1) HN (4.2347) 2) Heston (+0.0967) 3) PBS (+0.1928) 4) BS (+1.5718)
M>1.02	/	1) HN (+2.0792) 2) PBS (+0.1659) 3) Heston (+0.2093) 4) BS (+6.1628)	1) HN (4.4671) 2) Heston (+0.3546) 3) PBS (+0.213) 4) BS (+5.8295)

Notes. The S&P500 options cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We rank the models for each moneyness-maturity category according to the \$RMSE, which is reported in brackets for the best model for each category, while the incremental increase in \$RMSE is reported for the second, third and fourth best models. The \$RMSE is calculated out-of-sample on a window of two days by using the parameters estimated on the previous estimation window, also composed of two days. 15 windows are used in total to calculate the out-of-sample pricing errors over the sample period.

Table F.4: Ranking of models across moneyness and maturity based on the out-of-sample pricing $RMSE for Apple stock.

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.94	1) Heston (0.3374)	1) PBS (0.2728)	1) PBS (0.2634)
	2) PBS (+0.0361)	2) Heston (+0.0096)	2) HN (+0.0153)
	3) BS (+0.0146)	3) HN (+0.0295)	3) Heston (+0.0089)
	4) HN (+0.0366)	4) BS (+0.0826)	4) BS (+0.1970)
0.94<M<1	1) Heston (0.3278)	1) Heston (0.3840)	1) HN (0.3611)
	2) HN (+0.0375)	2) HN (+0.0207)	2) PBS (+0.0051)
	3) PBS (+0.1558)	3) PBS (+0.0311)	3) Heston (+0.0107)
	4) BS (+0.0423)	4) BS (+0.0544)	4) BS (+0.0318)
1<M<1.06	1) Heston (0.3126)	1) Heston (0.3935)	1) PBS (0.3522)
	2) HN (+0.0148)	2) HN (+0.021)	2) HN (+0.0108)
	3) PBS (+0.1843)	3) BS (+0.0156)	3) BS (+0.0017)
	4) BS (+0.0262)	4) PBS (+0.0104)	4) Heston (+0.0088)
M>1.06	1) HN (0.3392)	1) PBS (0.3617)	1) PBS (0.3030)
	2) Heston (+0.0156)	2) Heston (+0.0060)	2) Heston (+0.0173)
	3) PBS (+0.0782)	3) HN (+0.0027)	3) HN (+0.0033)
	4) BS (+0.0633)	4) BS (+0.1935)	4) BS (+0.6136)

Notes. The Apple stock options cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We rank the models for each moneyness-maturity category according to the $RMSE, which is reported in brackets for the best model for each category, while the incremental increase in $RMSE is reported for the second, third and fourth best models. The $RMSE is calculated out-of-sample on a window of two days by using the parameters estimated on the previous estimation window, also composed of two days. 15 windows are used in total to calculate the out-of-sample pricing errors over the sample period.

Appendix G

Out-of-Sample Payoff Replication: Linear Regressions

Table G.1: Results of the linear regressions on the out-of-sample payoff replication $AE for the S&P500 index.

Black-Scholes

	Estimate	HAC std. dev.	T-test	p-value
β_0	1427	519.25	2.7482	0.0061
β_1	-2829	1039	-2.7217	0.0066
β_2	1411	521.08	2.7078	0.0069
β_3	20.29	11.0235	1.8406	0.0660
β_4	0.0551	0.1329	0.4143	0.6787

$R^2 = 14.59\%$

Practitioner BS

	Estimate	HAC std. dev.	T-test	p-value
β_0	786.41	368.13	2.1363	0.0329
β_1	-1559	735.83	-2.119	0.0343
β_2	783.22	368.42	2.1259	0.0337
β_3	15.1901	9.8945	1.5352	0.1250
β_4	-0.375	0.2201	-1.7041	0.0886

$R^2 = 10.20\%$

Heston

	Estimate	HAC std. dev.	T-test	p-value
β_0	1992	514.24	3.8737	$1.14*10^{-4}$
β_1	-3912	1033	-3.7884	$1.60*10^{-4}$
β_2	1928	518.77	3.7157	$2.13*10^{-4}$
β_3	45.3608	12.2863	3.6920	$2.34*10^{-4}$
β_4	0.1145	0.4069	0.2814	0.7784

$R^2 = 29.08\%$

Heston-Nandi

	Estimate	HAC std. dev.	T-test	p-value
β_0	1924	545.88	3.5246	$4.42*10^{-4}$
β_1	-3775	1097	-3.4409	$6.02*10^{-4}$
β_2	1858	551.38	3.3697	$7.79*10^{-4}$
β_3	49.8000	12.3375	4.0365	$5.81*10^{-5}$
β_4	-0.0335	0.4254	-0.0789	0.9371

$R^2 = 30.17\%$

Notes. The S&P500 options used for the regression cover the period January 4, 2016 to March 30, 2016. The linear regression equation is: $|\text{Replication Error}|_i = \beta_0 + \beta_1 * \text{Moneyness}_i + \beta_2 * \text{Moneyness}_i^2 + \beta_3 * \text{Maturity}_i + \beta_4 * |\text{Pricing Error}|_i + \varepsilon_i$. 15 windows of two days have been used to calculate the payoff replication and pricing errors out-of-sample, by using the model parameters estimated on the previous estimation window, also composed of two days. The total number of options used is 1090. The parameter estimates are found by Ordinary Least Squares. The homoskedasticity assumption of the error term ε has been tested with the test of Breusch and Pagan (1979), and the assumption of non auto-correlated errors with the Lagrange-multiplier test developed by Breusch (1978) and Godfrey (1978). Both tests rejected the two assumptions for all models with a p-value $< 2.2 * 10^{-16}$. As a result, HAC (Heteroskedasticity and Auto-correlation Consistent) standard deviations of the estimated parameters have been used by resorting to the variance-covariance matrix of Newey and West (1987), with a lag value of 10. The T-test column reports the value of the bilateral Student test for the significance of each parameter: $H0 : \beta_i = 0$, $H1 : \beta_i \neq 0$, $i = 0, ..., 4$. The last column reports the p-value associated to the T-test.

TABLE G.2: Results of the linear regressions on the out-of-sample payoff replication $AE for Apple Stock.

Black-Scholes

	Estimate	HAC std. dev.	T-test	p-value
β_0	-12.8162	4.7234	-2.7133	0.0068
β_1	23.9166	9.0802	2.6339	0.0085
β_2	-10.5339	4.4016	-2.3932	0.0169
β_3	3.8880	1.0172	3.8224	$1.39{*}10^{-4}$
β_4	-0.0866	0.2013	-0.4303	0.6670
$R^2 = 15.65\%$				

Practitioner BS

	Estimate	HAC std. dev.	T-test	p-value
β_0	-13.7402	4.8266	-2.8468	0.0045
β_1	24.9812	9.3826	2.6625	0.0079
β_2	-10.9908	4.5438	-2.4189	0.0157
β_3	4.2964	1.0016	4.2895	$1.93{*}10^{-5}$
β_4	0.5728	0.42856	1.3366	0.1816
$R^2 = 19.05\%$				

Heston

	Estimate	HAC std. dev.	T-test	p-value
β_0	-19.7061	5.3778	-3.6644	$2.59{*}10^{-4}$
β_1	36.8114	10.2641	3.5864	$3.49{*}10^{-4}$
β_2	-16.6132	4.9438	-3.3604	$8.03{*}10^{-4}$
β_3	4.1233	1.1302	3.6483	$2.75{*}10^{-4}$
β_4	0.4076	0.4172	0.9769	0.3288
$R^2 = 17.59\%$				

Heston-Nandi

	Estimate	HAC std. dev.	T-test	p-value
β_0	-19.4104	29.5404	-0.6571	0.5113
β_1	36.3899	107.4473	0.3387	0.7349
β_2	-16.4792	24.9416	-0.6607	0.5089
β_3	4.0626	1.2259	3.3140	$9.47{*}10^{-4}$
β_4	0.4637	0.1284	3.6107	$3.18{*}10^{-4}$
$R^2 = 16.79\%$				

Notes. The Apple stock options used for the regression cover the period January 4, 2016 to March 30, 2016. The linear regression equation is: $|\text{Replication Error}|_i = \beta_0 + \beta_1 * \text{Moneyness}_i + \beta_2 * \text{Moneyness}_i^2 + \beta_3 * \text{Maturity}_i + \beta_4 * |\text{Pricing Error}|_i + \varepsilon_i$. 15 windows of two days have been used to calculate the payoff replication and pricing errors out-of-sample, by using the model parameters estimated on the previous estimation window, also composed of two days. The total number of options used is 1090. The parameter estimates are found by Ordinary Least Squares. The homoskedasticity assumption of the error term ε has been tested with the test of Breusch and Pagan (1979), and the assumption of non auto-correlated errors with the Lagrange-multiplier test developed by Breusch (1978) and Godfrey (1978). Both tests rejected the two assumptions for all models with a p-value $< 2.2 * 10^{-16}$. As a result, HAC (Heteroskedasticity and Auto-correlation Consistent) standard deviations of the estimated parameters have been used by resorting to the variance-covariance matrix of Newey and West (1987), with a lag value of 10. The T-test column reports the value of the bilateral Student test for the significance of each parameter: $H0 : \beta_i = 0$, $H1 : \beta_i \neq 0$, $i = 0, ..., 4$. The last column reports the p-value associated to the T-test.

Appendix H

Payoff Replication Under Parameters Re-Calibration

TABLE H.1: Number of option contracts used for out-of-sample payoff replication with a frequent re-calibration of parameters across moneyness and maturity.

S&P500

	DTM⩽20	20<DTM⩽40	DTM>40	*Total*
M<0.99	33	42	34	109
0.99<M<1	82	69	31	182
1<M<1.01	79	65	29	173
M>1.01	16	4	6	26
Total	210	180	100	490

Apple

	DTM⩽30	30<DTM⩽60	DTM>60	*Total*
M<0.97	3	37	23	63
0.97<M<1	75	25	5	105
1<M<1.03	72	23	7	102
M>1.03	8	64	25	97
Total	158	149	60	367

Notes: The two samples of option contracts cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the statistics on the number of contracts used to calculate the out-of-sample payoff replication errors with a frequent re-calibration of parameters for each moneyness-maturity category. Compared to the number of options used without re-calibration of parameters (in tables C.2 and C.3), we had to eliminate options expiring after the final date of our sample of options, i.e. March 31, 2016, because we can't re-calibrate the parameters after that date. As a result, the definition of our moneyness-maturity categories are different since we have lower maturities and therefore not as large levels of moneyness available.

Table H.2: Comparison of out-of-sample payoff replication errors $MAE across moneyness and maturity for the S&P500 index with and without parameters re-calibration.

Without re-calibration

Black-Scholes

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	10.7015	10.3471	11.9807
0.99<M<1	11.4438	11.3845	13.1827
1<M<1.01	11.4856	12.6424	13.7899
M>1.01	10.4765	6.1586	7.8416

Heston

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	11.8930	13.2223	12.6120
0.99<M<1	11.7706	12.8337	14.6149
1<M<1.01	12.2651	14.1007	15.7297
M>1.01	11.7039	6.5675	12.7337

Practitioner BS

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	10.6849	9.5349	10.3860
0.99<M<1	11.2676	10.6019	11.2685
1<M<1.01	11.3093	12.0056	12.4121
M>1.01	10.4525	7.8325	8.5247

Heston-Nandi

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	12.0982	14.0215	12.3328
0.99<M<1	11.9141	13.8129	14.2470
1<M<1.01	12.2381	14.3977	15.7797
M>1.01	11.8951	7.0141	15.4186

With re-calibration

Black-Scholes

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	11.0682	11.3559	13.4185
0.99<M<1	11.6599	12.4614	15.5543
1<M<1.01	11.8742	14.0939	15.6241
M>1.01	10.7697	5.1811	7.8874

Heston

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	11.9545	14.4294	17.0120
0.99<M<1	11.9322	14.3175	20.2355
1<M<1.01	12.6551	16.3061	20.1468
M>1.01	12.0297	5.3252	15.0239

Practitioner BS

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	10.8492	10.1693	11.3628
0.99<M<1	11.4170	11.4802	12.7600
1<M<1.01	11.6128	13.3020	13.7981
M>1.01	10.5452	7.9783	8.8253

Heston-Nandi

	DTM⩽30	30<DTM⩽60	DTM>60
M<0.99	11.9116	15.0924	16.8148
0.99<M<1	11.8505	14.6993	20.2304
1<M<1.01	12.5790	16.3061	20.5898
M>1.01	12.1110	5.7917	17.1161

Notes: The S&P500 options cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the $MAE for each moneyness-maturity category. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days. Without re-calibration, the parameters of the last estimation window at initiation, also composed of two days, are kept constant until expiration of the option. With re-calibration, we update the parameters throughout the life of the option by using those of the last estimation window, meaning that the re-calibration is done every four days. 14 hedging windows are used in total.

TABLE H.3: Comparison of out-of-sample payoff replication $MAE across moneyness and maturity for Apple stock with and without parameters re-calibration.

Without re-calibration

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.97	0.9644	0.9711	0.9641	M<0.97	0.9848	1.0036	0.9835
0.97<M<1	0.8948	0.8282	1.0906	0.97<M<1	0.7833	1.0393	1.1224
1<M<1.03	1.2238	1.1570	1.5071	1<M<1.03	1.2098	1.4802	1.5008
M>1.03	0.7833	1.6231	1.5897	M>1.03	0.9690	1.7327	1.6440
Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.97	0.9124	0.8971	0.8836	M<0.97	0.9560	1.0815	0.9996
0.97<M<1	0.8746	0.7712	1.0156	0.97<M<1	0.7631	1.1668	1.1558
1<M<1.03	1.2179	1.1352	1.4177	1<M<1.03	1.2241	1.5151	1.5399
M>1.03	0.7887	1.5900	1.5763	M>1.03	0.9973	1.7543	1.6468

With re-calibration

Black-Scholes	DTM⩽30	30<DTM⩽60	DTM>60	*Heston*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.97	1.0090	0.8768	0.8192	M<0.97	0.9617	0.7421	0.5757
0.97<M<1	0.9132	0.8455	1.0961	0.97<M<1	0.8144	0.9877	1.1593
1<M<1.03	1.2295	1.2049	1.6041	1<M<1.03	1.1900	1.4606	1.7845
M>1.03	0.7795	1.6964	1.6587	M>1.03	0.9515	1.8644	1.8264
Practitioner BS	DTM⩽30	30<DTM⩽60	DTM>60	*Heston-Nandi*	DTM⩽30	30<DTM⩽60	DTM>60
M<0.97	0.9658	0.7850	0.7269	M<0.97	0.9575	0.7291	0.5535
0.97<M<1	0.8924	0.7894	1.0088	0.97<M<1	0.8330	1.0250	1.1728
1<M<1.03	1.2222	1.1825	1.5235	1<M<1.03	1.2224	1.5270	1.8461
M>1.03	0.7844	1.6607	1.6345	M>1.03	0.9674	1.8991	1.8590

Notes: The Apple stock options cover the period January 4, 2016 to March 30, 2016. M refers to moneyness, defined as $\frac{S_t - D_{pv}}{K}$, and DTM to Days To Maturity. We report the $MAE for each moneyness-maturity category. The errors are calculated out-of-sample on each of our hedging windows, which are composed of two days. Without re-calibration, the parameters of the last estimation window at initiation, also composed of two days, are kept constant until expiration of the option. With re-calibration, we update the parameters throughout the life of the option by using those of the last estimation window, meaning that the re-calibration is done every four days. 12 hedging windows are used in total.

Appendix I

Under/Over-Hedging: Results from Simulated Data

TABLE I.1: Normalized payoff replication \$ME under simulated data from the HN model.

Volatility = 15%

	0.5 day	1 day	2 days	5 days
μ = -10%	-0.5837	0.2109	0.7606	0.9951
μ = -5%	-0.5896	0.1246	0.7711	0.9910
μ = 5%	-0.6609	-0.0068	0.7407	0.9953
μ = 10%	-0.6449	0.0461	0.7652	0.9964

Volatility = 30%

	0.5 day	1 day	2 days	5 days
μ = -10%	-0.5073	0.0199	0.6542	0.9656
μ = -5%	-0.5319	0.0245	0.6677	0.9598
μ = 5%	-0.6131	0.0194	0.6520	0.9740
μ = 10%	-0.5973	-0.0410	0.6724	0.9851

Volatility = 45%

	0.5 day	1 day	2 days	5 days
μ = -10%	-0.4596	0.0235	0.6480	0.9637
μ = -5%	-0.5413	0.0419	0.5196	0.9637
μ = 5%	-0.5408	-0.0562	0.6589	0.9688
μ = 10%	-0.5689	-0.0388	0.5816	0.9665

Notes: The results were obtained by Monte Carlo simulation with $S_0 = K = 50$ and $T = 0.5$. The normalized errors are calculated as the \$ME divided by the \$MAE. The columns report the errors under different re-balancing frequency of the replicating portfolio. The mean and volatility are reflected by the HN parameters' values as follows: λ is set such that $\mu = r + 252 * k * \lambda * \bar{h}$, β_2 such that $\sigma = \sqrt{252 * k * \bar{h}}$ where $k = 0.5, 1, 2, 5$ depending on the re-balancing frequency, $\bar{h} = \beta_0/(1 - \beta_1 - \beta_2\gamma^2)$, i.e. the long run variance under the HN model, and $h_0 = \bar{h}$. The other parameters' values are: $r = 3\%$, $\beta_0 = 8 * 10^{-6}$, $\beta_1 = 0.45$, $\gamma = 700$. Number of Monte Carlo random paths: 1000.