

École polytechnique de Louvain

Transport properties comparison between walking droplets in a rotating bath and electrons in two-dimensional systems with perpendicular magnetic field

Author: **Cyril HANON DE LOUVET**
Supervisor: **Benoît HACKENS**
Readers: **Laurent FRANCIS, Vincent BAYOT**
Academic year 2021–2022
Master [120] in Electrical Engineering

Acknowledgements

First of all, I would like to warmly thank my advisor Benoît Hackens that gave me the great opportunity to work on such a particular subject for an electrical master thesis. On the other hand, I also sincerely thank Olivier Leblanc, who first proposed this subject two years ago and started this research field at UCLouvain. They have both been very supportive and gave me precious advice throughout the year, I am extremely grateful for their dedication to helping me in this work.

I also would like to thank Guilhem Vanlancker and Marc Daman for their expertise in electricity and computer science, as well as Hervé Laurent and Luca Azzola for their expertise in mechanics, without whom the development of a functional setup would probably not have been possible. We had frequent conversation on practical issues raised during the development of the whole setup, and they gave their valuable time to help me overcome them.

I am also thankful to Jean-Charles Godefroid and the Makilab for giving me the chance to work with high quality 3D printers and test the fabrication of different tailor-made baths.

I thank my parents and my siblings for being supportive all over my studies and personal projects. I thank Eleonore Sulmon, for all her love and support over the last years. I thank my friends, the "old" ones, with whom I started these studies five years ago, and the new ones I met at UCLouvain, without whom these years would not have been that memorable.

Abstract

The walking droplet experiment gained interest over the last decades because it is known as the first example of a macroscopic pilot-wave system that exhibits behaviours thought to be exclusive to the microscopic quantum realm. Until now, the walking droplet topics were mainly focused about its behaviour characterization both in fluid mechanics and mathematical modeling, and also about the comparison of its behaviour with particles showing wave-particle duality such as electrons. However, no reported experiments have paid attention to walker transport properties through cavities with input and output leads, that could be comparable to transport properties of electrons. Consequently, the objective of this master thesis is to analyze the transport properties of walking droplets through two types of cavity that undergo a constant angular velocity, analog to magnetic field application on two-dimensional system. The two cavities are an analog quantum ring and a Bunimovich stadium billiard, each interesting for different known phenomena in the electronic world. Indeed, the quantum ring is particularly well known for exhibiting Aharonov-Bohm effect while the Bunimovich stadium billiard is known for showing *scars* in the quantum world, as opposed to the classical world that presents chaotic behaviours.

To fulfill this objective, the experimental setup developed by O. Leblanc [1] was the starting point, mainly modified to include bath rotation control. The current setup includes the following components : the two baths that can be fixed on the membrane of a loudspeaker for the vertical acceleration, a 3-axis accelerometer, a droplet generator, a stepper motor on which the loudspeaker is nested and a fixed camera recording top view images of the droplet horizontal motion, post-processed by tracking algorithms. The imperfect electrical contacts between the stationary and rotating structure shows amplitude variations of $\pm 120[mg]$ that guided the choice of low memory experiments, with a Faraday instability threshold found at $\gamma_F = 4.03[g]$. For both baths, the walking droplet transmission shows a global increase to a maximum transmission probability with the angular velocity increase of the bath, with some transmission fluctuations. The ring-shaped bath fluctuations with angular velocity are thought to be related to particular interactions at the 'T-junction' both to enter or to exit the ring. Yet, the stadium-shaped bath fluctuations are rather due to the orbital radius of the walking droplet that decreases with angular velocity, guiding the walking droplet trajectory and influencing the transmission.

Keywords: Walking droplet, Ring, Aharonov-Bohm, Stadium, Chaotic, Scar

Contents

1	Introduction	1
2	State of the art on walking droplet	5
2.1	History of the walking droplet	5
2.2	Walking droplet physics	8
2.2.1	Walking droplet state	8
2.2.2	Walking droplet parameters	9
3	Magnetic field effect in two-dimensional materials and analogy to walking droplet experiments	15
3.1	Magnetic field effect in two-dimensional systems	15
3.1.1	Lorentz force	15
3.1.2	Quantum world	17
3.2	Rotating bath and analogy to quantum world	20
3.2.1	Principle	20
3.2.2	Walker experiments in rotating bath	21
4	Walker experiment	28
4.1	Experimental setup	28
4.1.1	Electrical connections to the rotating structure	32
4.1.2	Experimental setup control	34
4.1.3	Vertical acceleration	35
4.1.4	Droplet generator	39
4.1.5	Bath rotation	46
4.2	Data acquisition and analysis	47
5	Walkers behaviour and comparison	52
5.1	Data collection and relevant parameters	52
5.1.1	Ring-shaped bath	55
5.1.2	Stadium-shaped bath	62
5.2	Quantum simulations and experimental data	66
5.2.1	Mesoscopic ring cavities	67
5.2.2	Stadium cavity	71
5.3	Comparison of walkers experiments with experimental data and simulations	75
6	Conclusion and perspectives	77
6.1	Discussion on the obtained results	77
6.2	Perspectives for future research	79
A	Experimental setup control	81
B	Quantum simulations	83
C	Classical system simulations	85

List of Figures

2.1	Motion of a single drop in interaction with its own wavefield	5
2.2	Two walkers following a quantized orbital	5
2.3	Collision between two droplets	6
2.4	Self-organizing walkers lattice	6
2.5	Single slit walker experiment	6
2.6	Angle deviation histogram of single and double slit walker experiment	7
2.7	Droplet walking state schematic	8
2.8	Walking states depending on the forcing frequency and forcing acceleration for different droplet diameters	10
2.9	Phase diagram depending on the droplet diameter and the forcing acceleration	10
2.10	Walking and bouncing state depending on the forcing acceleration and frequency for different viscosities	11
2.11	Phase diagram depending on the forcing acceleration and the surface liquid height	12
2.12	Spatio-temporal images of different bouncing modes	14
2.13	Phase diagram depending on the vibration number and the forcing acceleration	14
3.1	Schematic of the Lorentz force effect on a charged particle	16
3.2	Hall effect schematic	17
3.3	Landau levels for non-relativistic and relativistic case	18
3.4	Change of phase of water wave fronts passing through a vortex	20
3.5	Sketch of electronic wavefronts for different magnetic field values . .	20
3.6	Orbital radius of a walker varying with angular velocity of the bath, observations and theoretical model	23
3.7	Regime diagram of a walker in rotating bath	23
3.8	Orbital statistics in rotating bath for a high memory value	24
3.9	Numerical simulations of orbital statistics in rotating bath for a high memory value	24
3.10	Diameter variation between two walkers in a rotating bath	25
3.11	Shift of the diameter separating two walkers in a rotating bath . . .	26
3.12	Schematic of droplet interactions schematic in an HSL	27
3.13	Ferromagnetic order for each droplet well and spin-spin correlation depending on the angular velocity	27

4.1	Experimental setup schematic	29
4.2	Cross-section of a bath lead	30
4.3	3D-printed baths: ring-shaped and stadium-shaped baths	31
4.4	Real experimental setup	32
4.5	Home-made slip ring schematic	33
4.6	Schematic of electrical contact between stationary part and rotating part	34
4.7	3-axis acceleration measurements over time	37
4.8	Power spectrum of acceleration measurements	37
4.9	RC-filter bode response simulation	38
4.10	Acceleration peaks for a complete rotation	38
4.11	Schematic of droplet generator.	39
4.12	Typical sent pulse used in the experimental setup for droplet generation	39
4.13	H-bridge circuit used for polarity inversion of the piezoelectric buzzer	40
4.14	Push-button circuit	41
4.15	Polarity change measured for a 9[V] amplitude	41
4.16	Droplet diameter recording setup.	42
4.17	Schematic 2D top view of the camera recording the droplet	42
4.18	Droplet diameter recording setup scheme.	42
4.19	Droplet diameter algorithm illustrations	44
4.20	Droplet diameter characterization depending on the pulse amplitude and duration	45
4.21	Droplet horizontal velocity histogram at null angular velocity	46
4.22	Horizontal velocity V_W/V_F^ϕ varying with the forcing acceleration for different droplet diameters	46
4.23	Stepper motor control diagram	47
4.24	Calibration step of the tracking algorithm	50
4.25	Image processing illustrations of the tracking algorithm	50
4.26	Calibration of the bath orientation during tracking algorithm due to possible slightly different angular velocity than expected	51
5.1	Number of walker samples for each experiment	53
5.2	Evolution of the transmission probability mean with 90% confidence interval with respect to the number of walker samples	54
5.3	Ring-shaped bath transmission varying with the angular velocity	55
5.4	Empirical probability distribution of the walking droplet's position in the ring-shaped bath	57
5.5	Walker current (TCF and TC) varying with angular velocity	59
5.6	Analogy between Taylor-Couette flow and Free-surface vortex	60
5.7	Tangential velocity component ratio $ B / A $ depending on the angu- lar velocity ratio between the outer and inner cylinder	61
5.8	Schematic of wavefront with destructive interference for a bath with inner rotating cylinder	62

5.9	Walking droplet showing trajectories specific to an interferometer behaviour	62
5.10	Stadium-shaped bath transmission varying with the angular velocity	63
5.11	Empirical probability distribution of the walking droplet's position in the stadium-shaped bath	64
5.12	Probability distribution of the walking droplet's position at $\Omega = 0[\text{rad/s}]$ comparable to $[1]$	65
5.13	Walker current (TCF and TC) varying with angular velocity	66
5.14	Aharonov-Bohm effect on a normal metal ring	68
5.15	Recurrence of a scar pattern on a quantum ring	68
5.16	Transmission conductance for quantum rings	69
5.17	Power spectrum of transmission conductance for quantum rings	69
5.18	Landau levels varying with the magnetic field and Fermi energy in red line.	70
5.19	Local current and LDOS simulations on quantum ring	71
5.20	Recurrence of a scar pattern on quantum ring	72
5.21	Similar ∞ -distribution in a stadium geometry between a walker and the LDOS	72
5.22	Transmission conductance for quantum Bunimovich stadium	73
5.23	Power spectrum of transmission conductance for quantum Bunimovich stadium	73
5.24	Local current and LDOS simulations on quantum stadium	74
5.25	LDOS of the modes illustrating circular electron trajectories	75
A.1	Electrical connections	81
A.2	Front panel of the Labview program " <i>Arduino_Ctrl.vi</i> "	82
A.3	Front panel of the Labview program " <i>MyDAQ_memory_control.vi</i> "	82
C.1	Classical simulations in ring-shaped billiard	85
C.2	Classical simulations in stadium-shaped billiard	86

Nomenclature

List of Abbreviations

2DEG	Two Dimensional Electron Gas
AB	Aharonov-Bohm
ADC	Aharonov-Bohm
AI	Analog Input
AO	Analog Output
BPF	Band-pass filter
FFT	Fast Fourier Transform
FPS	Frames per second
GMM	Gaussian Mixture Model
HSL	Hydrodynamic spin lattices
LDOS	Local Density Of States
LED	Light-Emitting Diode
pdf	probability density function
PI	Proportional Integral
PID	Proportional Integral Derivative
ROI	Region Of Interest
std	standard deviation
TB	Tight-Binding
TC	Transmission Current
TCF	Transmission Current Flow
T-junction	Three-way junction with one way perpendicularly crossing the two others that are aligned (shape of T)

Table 1: List of default parameters

Name	Symbol	Value	[Unit]
Fundamental constants			
Gravitational acceleration	g	9.81	$[m/s^2]$
Elementary charge	e	$1.602 \cdot 10^{-19}$	$[C]$
Particle charge	q	$\pm 1.602 \cdot 10^{-19}$	$[C]$
Plank constant	h	$6.626 \cdot 10^{-34}$	$[J.Hz]$
Silicone oil properties			
Oil density	ρ	956	$[kg/m^3]$
Oil kinematic viscosity	ν	20	$[cSt] = [mm^2/s]$
Oil dynamic viscosity	$\mu = \rho\nu$	$19.12 \cdot 10^{-3}$	$[Pa.s] = [kg/m.s]$
Surface tension	σ	20.6	$[mN/m]$
Droplet properties			
Droplet diameter	D	$[0.82 - 1.10]$	$[mm]$
Droplet radius	$R_0 = D/2$	$[0.41 - 0.55]$	$[mm]$
Droplet mass	$m = \rho \frac{4\pi R_0^3}{3}$	$[2.7 - 6.6] \cdot 10^{-7}$	$[kg]$
Characteristic drop oscillation frequency	$\omega_D = \sqrt{\sigma/(\rho R_0^3)}$	$[361.0, 560.9]$	$[rad/s]$
Vertical oscillations parameters			
Sample rate	N	10000	$[-]$
Frequency of forcing	f_0	80	$[Hz]$
Pulsation of forcing	$\omega_0 = 2\pi f_0$	160π	$[rad/s]$
Liquid height	h_0	$[6 - 7]$	$[mm]$
Acceleration amplitude	γ_m	$[0 - 41.622]$	$[m/s^2]$
Non-dimensional acceleration peak	$\Gamma_m = \gamma_m/g$	$[0 - 4.2]$	$[-]$
Faraday wavelength	λ_F	4.72	$[mm]$
Faraday wavenumber	k_F	1330	$[rad/m]$
Faraday period	T_F	25	$[ms]$
Faraday threshold	γ_F	40.036	$[m/s^2]$
Non-dimensional Faraday threshold	$\Gamma_F = \gamma_F/g$	4.03	$[-]$
Distance to Faraday threshold	$\mathcal{M} = \Gamma_F/(\Gamma_F - \Gamma_m)$	$[0 - (+\infty)]$	$[-]$
Vanishing viscosity wavenumber [2]	$k_0 = k \text{ s.t. } \Gamma_F \rightarrow 0$	1319.5	$[rad/m]$
Free damping [2]	$\gamma_0 = 4\nu k_0^2/\omega_0$	0.277	$[-]$
Memory [2]	$Me = \frac{\mathcal{M}}{2\pi\gamma_0} \left(1 + \frac{\gamma_0^{1/2}}{2} + \frac{\gamma_0}{4}\right)$	$[0 - (+\infty)]$	$[-]$
Faraday phase velocity	$V_F^\phi = \sqrt{\frac{g}{k} + \frac{\sigma k}{\rho}}$	190.8	$[mm/s]$
Walker horizontal velocity	V_W approximation from [3]	$[13 - 18]$	$[mm/s]$
Capillary length	$l_c = \sqrt{\frac{\sigma}{\rho g}}$	1.48	$[mm]$
Vibration number	$\Omega_v = \omega/\omega_D$	$[0.90 - 1.39]$	$[-]$
Bouncing mode	(m, n)	$(-, -)$	$[-]$
Experimental setup (Chapter 5)			
Angular velocity	Ω	$[0 - 0.628]$	$[rad/s]$
Orbital radius	r_W	$[\infty - 11.54]$	$[mm]$
Number of transmitted, reflected walkers	w_t, w_r	$[18 - 115]$	$[\#]$
Total time of reflections, transmissions	τ_t, τ_r	/	$[s]$
Transmission	$T = \frac{w_t}{w_t + w_r}$	$[0 - 1]$	$[-]$
Transmission current flow (TCF)	$I_t = \frac{w_t}{\tau_t}$	$[0 - 0.15]$	$[\#/s]$
Transmission current (TC)	$I = \frac{w_t}{\tau_t + \tau_r}$	$[0 - 0.11]$	$[\#/s]$

Chapter 1

Introduction

Quantum mechanics is a branch of physics that aims to describe the physical properties of matter at the scale of atoms and subatomic particles. It provides the best current mathematical theory to predict the measurements that characterize the micro and nanoscopic particles behaviour. Notable principles include the wave-particle duality, energy quantization, probabilistic behaviour described by the Schrodinger equation in situations where classical physics predicts certainties. While the mathematical theory is widely understood, many physical interpretations, that are part of the quantum foundations branch, have been developed in order to explain and understand how the world behaves according to quantum mechanics in "reality", that is more a matter of philosophy. While often not needed in the scientific realm, they may help to better understand how reality is depicted and "accept" the particle quantum behaviour.

Many interpretations have emerged, leading to debates about their postulates which can often not be verified that is the whole point. As N. David Mermin said in [4] "New interpretations appear every year. None ever disappear.". These interpretations rely on different fundamental statements such as the deterministic or stochastic particles behaviour, the effect of measurements on microscopic system, what can be considered as a microscopic system that affects our apparatuses in a prescribed manner, the principle of locality, etc. [5]. We can cite the Copenhagen interpretation, the many-worlds interpretation, the de Broglie-Bohm theory and many others [6].

Among them, the Copenhagen interpretation was the first general interpretation attempt and is still the most widely spread one. This interpretation starts through 1920's and has evolved considerably over years with the support of Niels Bohr, Werner Heisenberg, Max Born and other scientists. Although not always in agreement with each other, generally accepted ideas support that quantum mechanics is stochastic with probabilities calculated using the Born rule. Moreover, it states that the act of "measuring" an object is irreversible because a particle state consists in the superposition of all possible states at once and when an object is observed, the superposition collapses as the object is forced into one of the states of its wave function [7, 8].

As opposed to the Copenhagen interpretation, the de Broglie–Bohm theory was introduced by Louis de Broglie in 1926 [9] and completed by David Bohm in 1952 [10]. The de Broglie–Bohm theory, also called the pilot-wave model, supports different statements such as a deterministic motion. Indeed, the system of particles is described by its wave function that follows the Schrödinger’s equation, and also by the specification of the actual positions of the particles, that evolve according to the “guiding equation”, which expresses the velocities of the particles in terms of the wave function [11, 12].

While the wave-particle duality had only appeared in quantum mechanics with no similar behaviour at the macroscopic scale, Y. Couder, E. Fort and their team introduced in 2006 a system that couples a macroscopic particle and a wave [13], leading to a pilot-wave system. The system consists in a millimetric silicone oil droplet, called "walker" or "walking droplet", bouncing on a vertically vibrated fluid bath, such that the walker can interact with the surface wave it generates and starts moving at constant velocity on the interface. Experiments have shown very remarkable properties such as wave-like statistics in confined geometries, tunneling, orbital quantization, that can be compared to those attributed to the wave-particle duality in quantum mechanics. Some of them will be discussed : self-organising lattice structures, diffraction and interference patterns for single and double slit experiment (Chapter 2) as well as quantized orbits and orbital level splitting (Chapter 3). These "walkers" rise particular interest in the scientific realm as they offer a privileged view on the wave-particle coupling at a macroscopic scale, and represent a realization of pilot-wave dynamics. Therefore, walkers experiments showing similarities to electron behaviours are support to the de Broglie theory and may help understanding quantum mechanics with a macroscopic system.

These walkers experiments already report peculiar walker properties, but no reported experiments have paid attention to walker transport properties through mesoscopic systems with input and output leads, in order to compare them to electrons transport properties, a major topic in nanoelectronics.

Therefore, this master thesis aims to explore the walker transport properties through two types of cavities and investigate the effect of a magnetic field physical analog for walkers, chosen to be a rotating bath.

The first cavity is analogue to an open quantum ring. In quantum electronics, it is best known for quantum-interference effects such as Aharonov-Bohm effect depending on the magnetic field strength [14], firstly observed by Wabburn et al. [15]. The Aharonov-Bohm oscillations are the oscillations of the conductance due to a change a phase induced by a magnetic field. The change of phase depends on the followed path of the electrons and the magnetic field strength. As in quantum mechanics the paths all have a probability to be taken by the electrons, it results that for a quantum ring, the electrons have a non-null probability to go from input

lead to both ring branches. At the output, both branches have a phase difference due to different path and it results in constructive or destructive interference, leading to a periodic change conductance for an increasing magnetic field.

The second cavity is a Bunimovich stadium billiard with input and output leads. It is a chaotic geometry that showed different behaviours between the classical trajectories and the quantum electron behaviour. Chaotic behaviour is defined as an exponential sensitivity to initial conditions. A classical ball bouncing elastically on the stadium boundaries will visit the entire geometry except for some unstable regular orbits. However, quantum particles behaviour in confined geometries is determined by Schrödinger equation, possibly leading to quantum scars, i.e. patterns resembling stable semiclassical trajectories, even in classically chaotic geometries due to the wavefunction eigenstates.

For both cavities, the electron behaviour is strongly different than that of a classical particle. What will be the closest behaviour to the walker? This master thesis will investigate the subject and attempt to answer the following questions for both cavities :

- *How does the walker transmission compare to quantum particles conductance?*
- *Can the probability distribution of the walker positions be associated to the electrons local density of states (LDOS) or the electron local current?*
- *Can one evidence an Aharonov Bohm interference effect in the transmission properties of walkers, similar to the one observed for electrons on mesoscopic ring?*

This master thesis will answer these questions based on experimental observations of the walkers inside these cavities with a rotating bath. The work will be based on comparison between existing data from articles, quantum simulations of electron behaviours in analog geometries obtained with the *Kwant* library [16] and classical simulations of a particle at constant speed, bouncing on obstacles by a specular reflection, obtained with Julia package [17].

It will be organized according to the following guideline. [Chapter 2](#) will present the growing interest in the walking droplets behaviour in its early stages, followed by the global parameters to reach the walking state, that will be useful for the experimental setup.

Then, [Chapter 3](#) will review the main relevant effects of an applied magnetic field on a two-dimensional electronic system, regarding the properties the walker could exhibit in this framework. The analogy between a magnetic field and a rotating bath will be highlighted. Then, a review of the realized experiments in rotating bath with their electronic analogy will be discussed.

[Chapter 4](#) will present the experimental setup and the methodology followed for the data analysis. Starting from the setup developed by O.Leblanc [1], I describe the main improvements realized during the master thesis. To answer the scientific questions listed above, I first had to add the possibility to rotate the bath with

controllable angular velocity. I also had to devise image analysis procedures allowing to extract transmission probabilities for sets of walking droplets in a rotating bath.

[Chapter 5](#) describes the walker observations, including probability distribution and transmission probability depending on the bath angular velocity in order to compare them to analog existing electron experiments and quantum simulations.

[Appendix B](#) discuss the quantum simulations realized in Python with the library *Kwant*[\[16\]](#) using tight-binding approximation. The simulations are performed on a graphene lattice to get more realistic comparisons. The system dimensions, the Fermi energy and the magnetic field are the main parameters that will be investigated to obtain a relevant analog comparison.

Chapter 2

State of the art on walking droplet

2.1 History of the walking droplet

In 2005, Y. Couder, E. Fort and their team first introduced the concept of "walker" or "walking droplet" in [18, 19]. They created a macroscopic system consisting of a vertically vibrated fluid bath and discovered that a millimetric droplet may bounce on the surface of the bath indefinitely. Under certain conditions, the droplet can even enter in an interesting "walking" state in which the droplet travels at constant speed on the bath surface, being propelled by its own wavefield (Figure 2.1). Plus, two walkers could interact with each other and follow quantized orbitals of diameters which self-adapt to the forcing bath frequency (Figure 2.2).

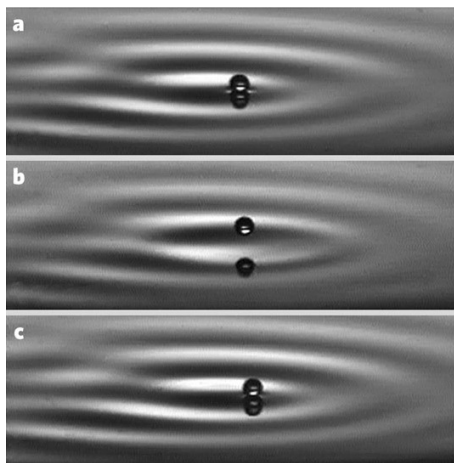


Figure 2.1: **a–c**: Motion of a single drop in interaction with its own wavefield on the liquid surface. Adapted from [19].

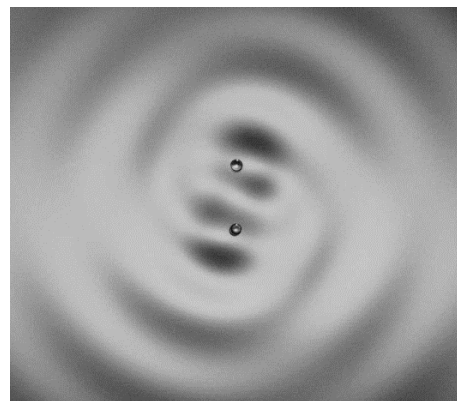


Figure 2.2: Two walkers following a quantized orbital. Adapted from [19].

Following this experiment, interactions between walking droplets [20] have been investigated and they found out that the scattering of two drops could either lead to a repulsive collision or to a mutual capture of two walkers of identical size into a

circular orbit (Figure 2.3). They also observed complex interactions between several walking droplets that exhibit self-organizing lattices behaviours (Figure 2.4).

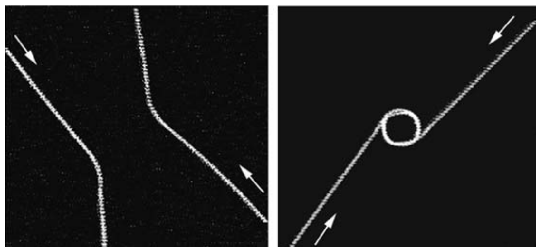


Figure 2.3: Possible results of a collision between two droplets. Left: repulsion behaviour. Right: association and circular trajectory following an orbital. Adapted from [20].

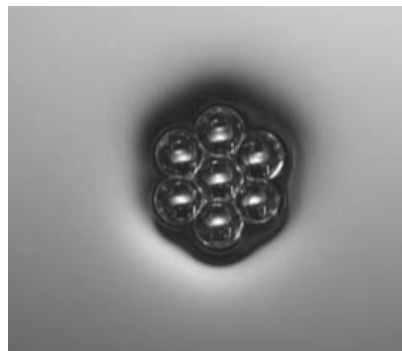


Figure 2.4: Self-organizing walking droplets system forming an hexagonal lattice. Adapted from [20].

The next year, they published another experiment [13] showing new interesting features such as diffraction and interference patterns appearing respectively in a single (Figure 2.5) and a double slit experiment.

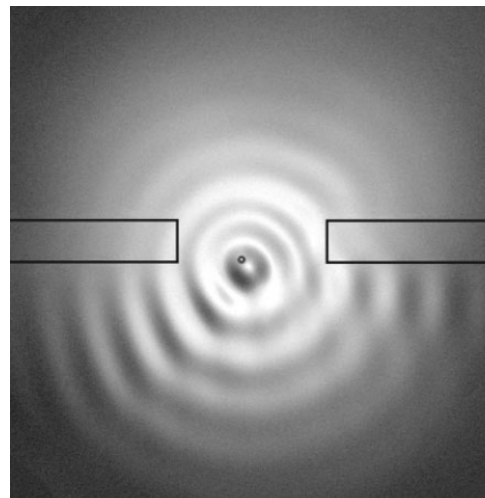
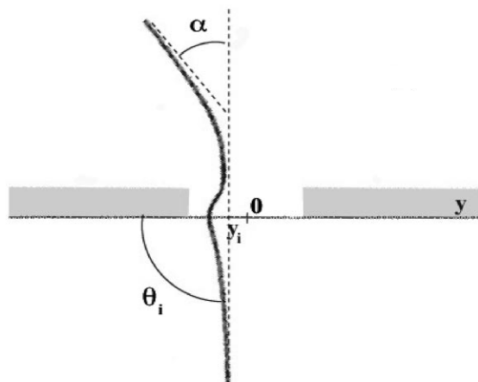


Figure 2.5: Single slit experiment schematic (left) and recorded image (right). θ_i in the incidence angle and α is the deviation angle as a result of the crossing through the slit. Adapted from [13].

These patterns (Figure 2.6) were created from the histogram of the deviations angle α (defined in Figure 2.5), of a large number of successive walkers.

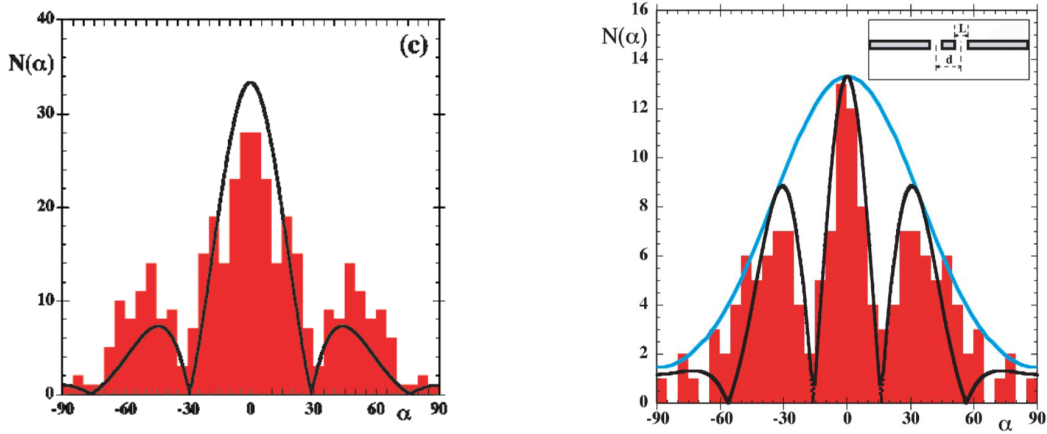


Figure 2.6: Left: angle deviation histogram with 125 single walkers with slit length $L = 6.95[mm]$. Right: angle deviation histogram with 75 single walkers through two slits of width $L = 7.6[mm]$, a distance $d = 14.3[mm]$ apart (inset). Adapted from [13].

These three scientific articles contributed to a growing interest in walking droplets, as they constituted the very first macroscopic experiments demonstrating behaviours related to the wave-particle duality that was until this moment only observed in quantum mechanics. Indeed, the particular behaviour of quantized orbitals is reminiscent of the electron orbitals, quantized by the four quantum numbers: n , l , m , and s . The attractive or repulsive characteristics of the scattering of the two droplets are due to their waves interaction which is a non-local property. This is a feature also met in solid-state physics for example with Cooper pair which is a pair of bound electrons namely due to electron-phonon interaction. Finally, the diffraction and interference pattern of the deviation angle in the single and double slits experiments are comparable to the famous double-slits experiment firstly achieved by Young with light in 1804 [21].

More specifically, it is interesting to remark the analogy of this system and the pilot-wave theory, proposed by de Broglie introduced in the [Chapter 1](#). Indeed, the droplet can be linked to a particle that has interactions with the wave it generated at each bath impact, and the de Broglie-Bohm theory starts from the coupling between a particle being guided by an accompanying wave field pushed in a direction perpendicular to surfaces of constant phase. Plus, the de Broglie-Bohm theory shows diffraction and interference properties similar to the stated walker experiments. Let us now introduce how to reach this macroscopic pilot-wave system.

2.2 Walking droplet physics

2.2.1 Walking droplet state

When a droplet is placed on the surface of a bath of the same liquid, the drop will follow a fusion process called coalescence, that consists of the merge of the droplet and the bath. The droplet, under the action of gravity, approaches the interface and thus crushes the thin film of air that separates them. The intermediate film of air is evacuated to the sides and becomes so thin that the two surfaces will then come into contact at one point due to Van der Waals forces. The air film is broken at this point and the liquid inside the drop will then rapidly merge with the underlying bath to form a bath and equalise the pressures. For the droplet to reach a bounce state that avoids the coalescence phenomenon, the air film needs to be maintained. A way to achieve it was introduced by Y. Couder et al. in [18] that consists in a vertical oscillation of the substrate, namely the liquid bath. When the droplet is placed on a vertically oscillating liquid bath, the vertical acceleration of the liquid bath needs to overcome an acceleration threshold noted γ_m^C , such that the droplet never has time to break the air film until the droplet is released back in the air.

The walking state is a particular state of the bouncing state. The conditions are as follows: when the droplet bounces on the liquid surface, it is accelerated from the edge of a previous waves it had emitted, so that the reaction force has a non-null horizontal component $F(\mathbf{x}_i, t) \propto \nabla\eta|_{(\mathbf{x}_i, x)}$, with $\nabla\eta$ the wave slope at the point of impact $\mathbf{x}_i(t)$ that makes the droplet moves horizontally, schematically illustrated in Figure 2.7.

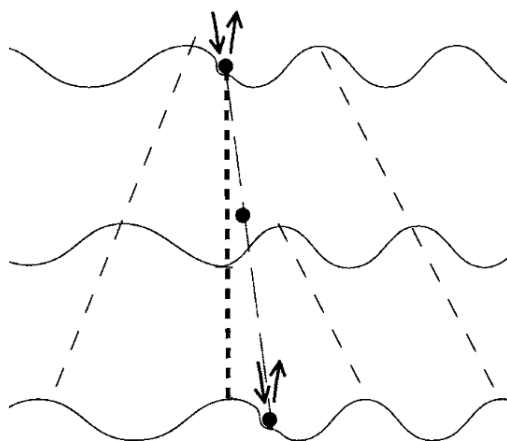


Figure 2.7: Schematic of the droplet walking state. The droplet bounces on the edge of a previous emitted wave.[3]

2.2.2 Walking droplet parameters

Let us now qualitatively introduce the main parameters relative to the walking droplet experiment. A complete analysis of the fluid mechanics is beyond the scope of this thesis.

To enter in the walking state, five main parameters that will be discussed now come into play: the droplet diameter D often in $[mm]$, the liquid properties such as the dynamic viscosity μ in $[Pa.s]$, the liquid bath height, the vertical liquid bath acceleration γ_m in $[m/s^2]$ and the acceleration frequency f_0 in $[Hz]$.

Droplet diameter The droplet diameter influences the transition to bounce state, due to the gravity and the capillary forces. To enter the bounce state, the capillary forces must overcome the gravity forces to minimize droplet deformation from sphere to ellipsoid shape during bath contact and thus higher coalescence probability. The capillary force will be dominant if the droplet diameter $D < 2l_c$, with l_c the capillary length defined as

$$l_c = \sqrt{\frac{\sigma}{\rho g}},$$

where σ is the surface tension, ρ is the oil density and $g = 9.81[m/s^2]$ is the gravitational constant. Droplet of diameter $D < 2l_c \sim 3[mm]$ is the droplet diameter range where interesting experiments were reported. Therefore, a suitable silicone oil that will be used for this walkers experiment is [22], where $\sigma = 20.6[mN/m]$ and $\rho = 956[kg/m^3]$, leading to a capillary length $l_c = 1.48[mm]$

For oil with viscosity $\mu = 50 \cdot 10^{-3}[Pa.s]$, Couder et al. measured the required acceleration threshold to enter in bouncing state depending on the bath frequency, for different droplet diameters (Figure 2.8). Overall, a higher acceleration is needed for larger drop diameter in order to compress and renew the air film at high frequencies or for a wider air film and the acceleration threshold is roughly $\gamma_m^C \propto f_0$. [3, 18]. For viscosity $\mu \leq 20 \cdot 10^{-3}[Pa.s]$, they report stability of the walking state for smaller droplet $D < 1[mm]$ in a frequency range $40 < f_0 < 300[Hz]$. In [23], they characterized a silicone oil of the same viscosity with a forcing frequency $f_0 = 50[Hz]$ that confirms that a higher acceleration is required for larger diameters (Figure 2.9) and prove the walking state is accessible and stable for certain a range of vertical acceleration $[\gamma_W - \gamma_F]$, where W stands for "walking" and F for Faraday instability, explained hereafter.

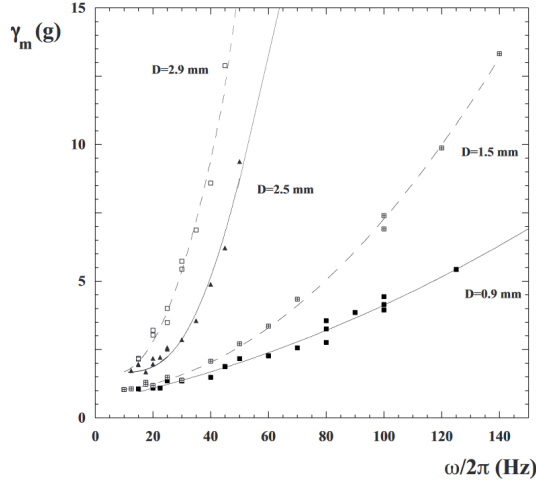


Figure 2.8: The required acceleration thresholds for bouncing states of drops of various sizes as a function of the forcing frequency.[18]

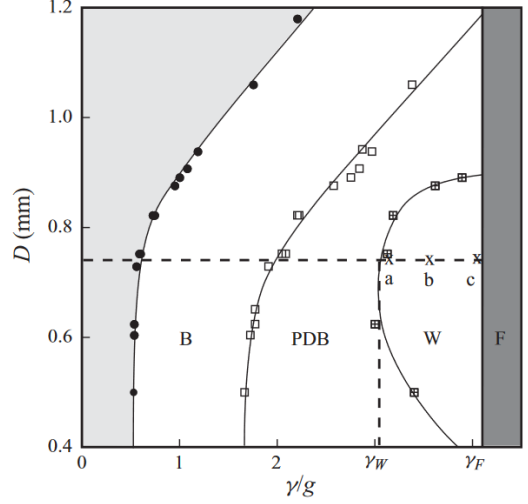


Figure 2.9: Phase diagram of the bouncing states depending on the droplet diameter and the acceleration. *B* stands for bouncing mode, *PDB* period doubled bouncing, *W* walking mode and *F* for Faraday instability. [23]

Viscosity and forcing frequency The viscosity has an important role in the evacuation of the air film as well. Indeed, when the drop lands on the surface the air film resists the drop pressure only in viscous regime, e.g. when the viscous force are dominant over the inertial forces. Different viscosities of silicone oil have been experimentally tested in [3] (Figure 2.10) to analyze the acceleration and frequency range in which the walking state is achieved. In most walking droplets experiments, a viscosity $\mu = 20.10^{-3}[Pa.s]$ is chosen due to its larger frequency range and more accessible acceleration range, with a forcing frequency is $f_0 = 80[Hz]$. Another common choice is $\mu = 50.10^{-3}[Pa.s]$, $f_0 = 50[Hz]$.

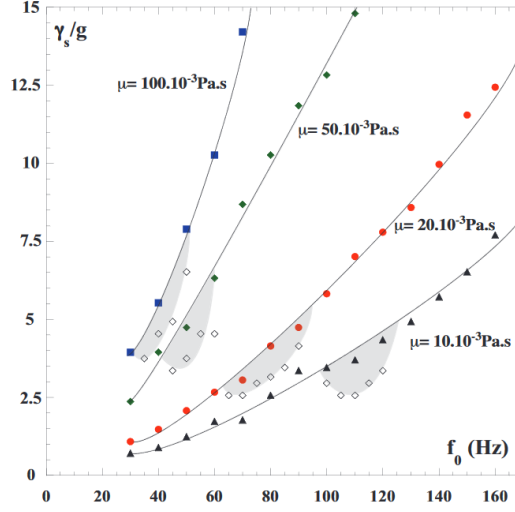


Figure 2.10: The required acceleration thresholds for bouncing states of drops of various viscosity as a function of the forcing frequency.[3]

Surface height and acceleration As far as these parameters are concerned, it is now useful to introduce the Faraday instability phenomenon and Faraday waves that govern the droplet dynamics. Firstly introduced in 1831 by Faraday, the surface of a fluid bath vibrating with vertical oscillation

$$r(t) = A \sin(\omega_0 t),$$

with $\omega_0 = 2\pi f_0$ the pulsation and f_0 the bath frequency, has an acceleration

$$\gamma(t) = -A\omega_0^2 \sin(\omega_0 t) = \gamma_m \sin(\omega_0 t)$$

with $\gamma_m = -A\omega_0^2$ that creates instabilities on the bath surface when the forcing acceleration amplitude is increased beyond a critical value $\gamma_m > \gamma_F$ in which γ_F is called the Faraday threshold. The fluid surface follows a pattern of standing waves oscillating at half the excitation frequency $f_F = f_0/2$ with a Faraday wavelength noted as λ_F . This phenomenon is called the Faraday instability. The waves satisfy the dispersion relation of gravity-capillary waves :

$$\omega_F^2(k_F) = \left(\frac{\omega_0}{2}\right)^2 = \left(gk_F + \frac{\sigma}{\rho}k_F^3\right) \tanh(k_F h_0)$$

with g the gravitational acceleration constant, k_F the Faraday wave vector, σ the surface tension, ρ the mass density and h_0 the height of the liquid surface [23]. With a kinematic viscosity $\nu = \mu/\rho = 20[cSt]$, $\rho = 956[kg/m^3]$, $\mu = 19.12[Pa.s]$, the wave vector and the corresponding Faraday wavelength are respectively:

$$k_F = 1330[rad/m]$$

$$\lambda_F = 4.72[mm]$$

The height of the liquid will stop influencing the generated waves if

$$\tanh(k_F h_0) \approx 1 \iff h_0 > 3[mm]$$

Experimentally, Vandewall et al. [24] built a phase diagram depending on the adimensional forcing amplitude acceleration Γ and the surface height for 20[cSt] silicone oil (Figure 2.11). Let us introduce adimensional acceleration written in upper case Γ :

$$\Gamma_F = \gamma_F/g \quad \text{for the Faraday threshold}$$

$$\Gamma_m = \gamma_m/g \quad \text{for the forcing acceleration (simply noted } \Gamma \text{ in [24])}$$

The surface height threshold at $h_0 = 3[mm]$ is well noticed. The more the surface height decreases, the more the droplet and generated waves will interact with the bath bottom and eventually enters in bouncing state due to too damped emitted wave.

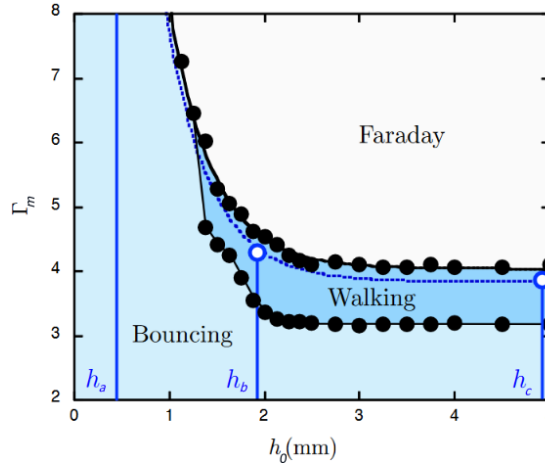


Figure 2.11: Droplet phase diagram with Γ_m the adimensional forcing acceleration amplitude and h_0 the surface liquid height. Experimental data are indicated by circles.[24]

The walking state occurs when the forcing amplitude acceleration is slightly below the Faraday instability. The generated waves can be called Faraday waves for bouncing states since their wavelength is equal to the Faraday wavelength; for walking states, their wavelength slightly differs due to Doppler effect [23]

$$\lambda_W^{th} = \lambda_F \mp V_W T_F$$

where V_W and T_F are the walkers velocity and the Faraday period. The waves are less damped as the acceleration increases allowing the droplet to impact a previous wave and enters the walking state. With the Faraday Threshold and the forcing

acceleration, two important parameters can be defined.

The *distance to Faraday Threshold* $\mathcal{M}$ derives from the Faraday threshold being written as

$$\mathcal{M} = \frac{\gamma_F}{\gamma_F - \gamma_m}$$

Another notation is the dimensionless bath acceleration ratio:

$$\mathcal{G} = \gamma_m / \gamma_F$$

The *Memory* Me has been introduced in [23] as the division of τ , the time constant of the exponential decay of generated waves, and the Faraday period T_F :

$$\begin{aligned} Me &= \frac{\tau}{T_F} \\ &= \frac{\mathcal{M}}{2\pi\gamma_0} \left(1 + \frac{\gamma_0^{1/2}}{2} + \frac{\gamma_0}{4} + O(\gamma_0^{3/2}) \right), \end{aligned} \quad (2.1)$$

where the second statement has been developed in [2] with γ_0 the free damping. The memory can be seen as a measurement of the spatial extension of emitted wavefield. With a kinematic viscosity $\nu = \mu/\rho = 20[cSt]$, $\rho = 956[kg/m^3]$, $\mu = 19.12[Pa.s]$, the damping is $\gamma_0 = 0.277$ and the Faraday threshold is theoretically $\Gamma_F = 4.144$.

As far as the forcing frequency is concerned, a vibration number Ω_v has been introduced in [25] as :

$$\Omega_v = \frac{\omega_0}{\omega_D},$$

with $\omega_D = \sqrt{\sigma/\rho R_0^3}$ the characteristic drop oscillation frequency [26] and R_0 is the droplet radius.

Depending on the forcing acceleration and frequency, bouncing states and walking states may exhibit different periodic modes. They are generally noted as (m, n) , introduced by Gilet and Bush in [27]. In the (m, n) mode, the period of the droplet vertical motion lasts m periods of the bath vertical oscillation, during which the drop contacts the bath n times. Plus, for a same mode, a distinction can be made to separate lower-energy mode in which the drop spends a large fraction of its bouncing period in contact with the bath, noted as $(m, n)^1$, and higher-energy mode in which the contact is relatively short, noted as $(m, n)^2$. Higher-energy mode occurs for higher bath acceleration. As an example, Protière et al.[20] recorded the spatio-temporal droplet vertical behaviour of the $(2, 1)^1$, $(2, 1)^2$ and $(2, 2)$ modes (Figure 2.12), by selecting a vertical line from frames aligned with the droplet center, and arranging this pixels column extracted from each successive frame side by side.

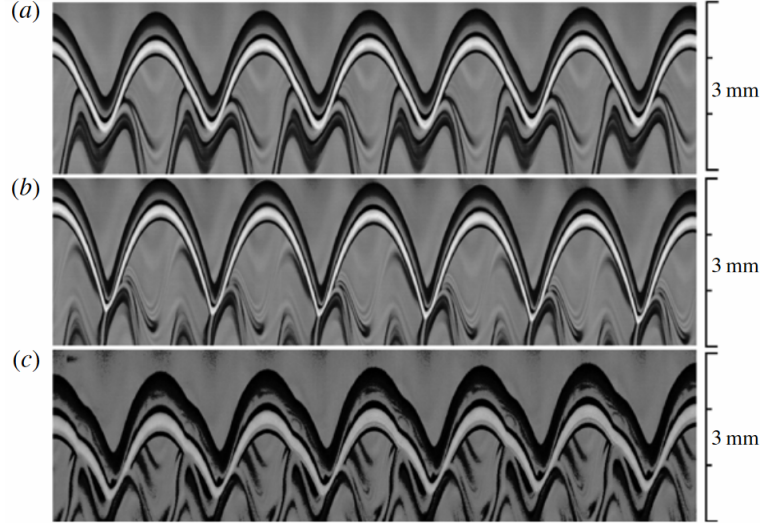


Figure 2.12: Spatio-temporal images silicone oil drops in waking state on a liquid bath vibrating with $f_0 = 50[Hz]$. **(a)** the $(2,1)^1$ mode, **(b)** the $(2,1)^2$ mode, **(c)** the $(2,2)$. Adapted from [20].

The bouncing modes have been explored in [28,29] in which they compared experiment data to theoretical model developed in [25,28]. They found out that the walking state was accessible in the $(2,1)^1$, $(2,1)^2$ the dominant mode, as well as in the $(2,2)$, $(4,2)$, $(4,3)$, modes and chaotic behaviours, characterized by non-periodic droplet motion (Figure 2.13).

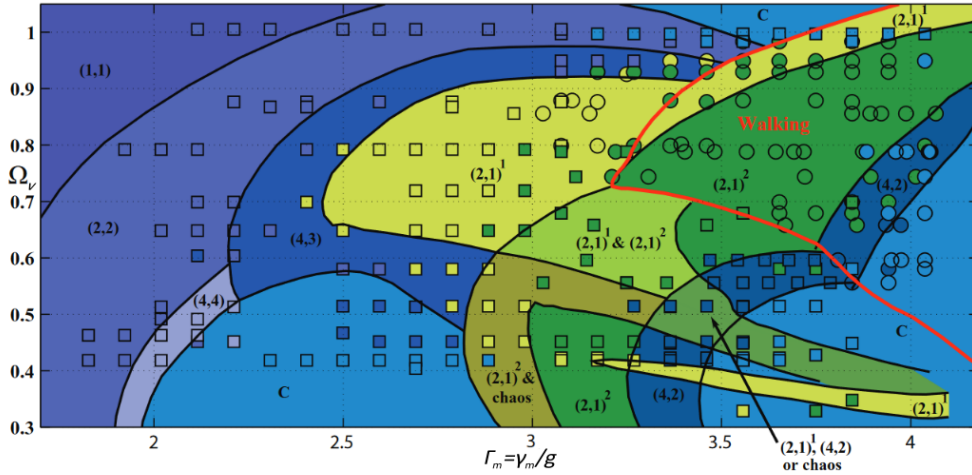


Figure 2.13: Phase diagram indicating the droplet modes depending on the dimensionless forcing acceleration Γ_m and the vibration number Ω_v , with a $20[cSt]$ oil viscosity at $f_0 = 80[Hz]$ and Faraday threshold observed $\Gamma_F = 4.22 \pm 0.05$. The red line and colourful areas correspond to theoretical predictions. Experimental data are presented as square for bouncing states or round markers for walking states. Adapted from [29].

Chapter 3

Magnetic field effect in two-dimensional materials and analogy to walking droplet experiments

In this chapter, a brief review describes the consequences of a magnetic field on a charged particle in the quantum and classic world that are relevant regarding our walking droplet experiment. Then, it will be followed by a comparison to walking droplet experiments with a rotating bath is introduced. As we will see, the case is similar to the application of a magnetic field on a particle.

3.1 Magnetic field effect in two-dimensional systems

3.1.1 Lorentz force

In the classical world, a magnetic field $\mathbf{B}$ induces a component in the force that acts on a charged particle called the Lorentz force noted :

$$\mathbf{F}_{Lorentz} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

with q the particle charge $= \pm e$ (e being the elementary charge $e = 1.6 \cdot 10^{-19}[C]$), $\mathbf{E}$ the electric field, $\mathbf{v}$ the particle velocity and $\mathbf{B}$ the magnetic field. The first component is the electrical force and the second one is the magnetic force which is of interest in our case. Depending on the charge, the trajectory of a particle in a two-dimensional system will follow a circle of radius r (Figure 3.1). The Lorentz force for a particle moving at velocity amplitude v in a plane normal to a magnetic field of amplitude B , becomes for a null electric field :

$$\mathbf{F}_{Lorentz} = |F_{Lorentz}| \hat{\mathbf{u}} \quad \text{where} \quad |F_{Lorentz}| = qvB \quad \text{and} \quad \hat{\mathbf{u}} = \frac{\mathbf{v} \times \mathbf{B}}{|\mathbf{v} \times \mathbf{B}|}$$

For example, the [Figure 3.1](#) gives the Lorentz force direction for initial velocity $\mathbf{v} = v \hat{\mathbf{x}}$ and $\mathbf{B} = B \hat{\mathbf{z}}$. The Lorentz force provides a centripetal force

$$F_{cent} = \frac{mv^2}{r}$$

and equalling both, we obtain the radius r of the circular path :

$$r = \frac{mv}{qB}$$

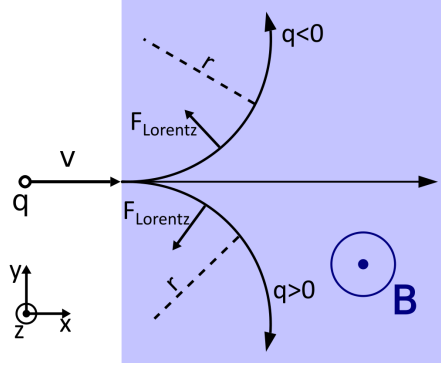


Figure 3.1: Schematic of a curved trajectory of a particle of charge q with velocity $\mathbf{v} = v \hat{\mathbf{x}}$ in a magnetic field $\mathbf{B} = B \hat{\mathbf{z}}$. [30]

Hall effect One of the main important results of the Lorentz force is the Hall effect discovered by Edwin Hall in 1879. This is the production of a voltage difference, called the Hall voltage V_H , across an electrical conductor (a "Hall bar") that is transverse to an electric current in the conductor and to an applied magnetic field normal to the current. The electron flow is bent by the magnetic field, leading to a higher electron concentration on one side of the conductor which results in an electric field and thus a difference of potential V_H :

$$V_H = \frac{I_x B_z}{n_e t e}$$

where I_x is the current intensity, n_e is the electron density, t is the height of the Hall bar ([Figure 3.2](#)).

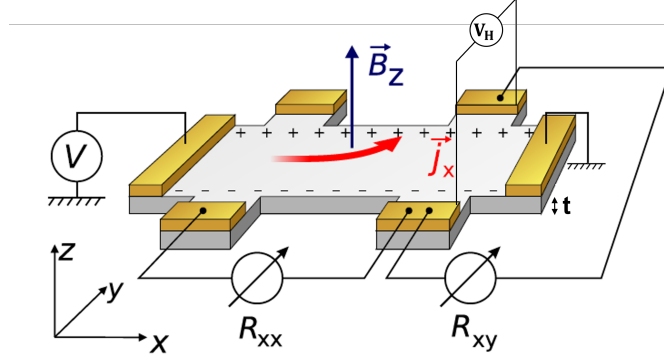


Figure 3.2: Schematic of the Hall effect with a potential difference V creating a current density j_x , constant magnetic field B_z normal to the surface, R_{xx} and R_{xy} respectively the longitudinal and hall resistances. Adapted from [31].

The longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} can be derived as [32]

$$\rho_{xx} = \frac{E_x}{j_x} = \frac{m}{n_e e^2 \tau} \quad \rho_{xy} = \frac{E_y}{j_x} = \frac{B_z}{en_e}$$

where m is the electronic effective mass, e the electron charge and τ the time between two electrons collisions.

3.1.2 Quantum world

At the scale of atoms and subatomic particles, quantum mechanics govern physical quantities such as the position, momentum, energy, spin that are represented by observables [33, 34]. The quantum state $|\Psi(t)\rangle$ evolution of a particle is defined more generally by the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$

where $\hbar = h/2\pi$ is defined with the Plank constant h , t is the time, $\hat{H}$ is an observable or the Hamiltonian operator.

If the solution does not depend on time, the time-independent Schrödinger equation can directly be used :

$$\hat{H} |\Psi\rangle = E |\Psi\rangle$$

Due to quantum superposition property, a quantum state $|\Psi(t)\rangle$ will be a linear combination of the eigenvectors of an observable of the time-independent Schrödinger equation $|\psi_{E_n}\rangle$, in which case they are called eigenstates, and the associated eigenvalues correspond to the value of the observable in these eigenstates.

$$|\Psi(t)\rangle = \sum_n A_n e^{-iE_n t/\hbar} |\psi_{E_n}\rangle$$

To calculate the Hamiltonian of a charged particle in a potential $V(\mathbf{r}, t)$ and a magnetic field $\mathbf{B}$, the magnetic field can be derived from a vector potential $\mathbf{A}$ as :

$$\mathbf{B} = \nabla \times \mathbf{A}$$

and the Schrödinger equation is then :

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \frac{1}{2m} [(-i\hbar \nabla - q\mathbf{A}(\mathbf{r}, t))^2 + qV(\mathbf{r}, t)] |\psi\rangle$$

In the time-independent case and for a potential $V(\mathbf{r}, t) = 0$, the equation is :

$$\frac{1}{2m} [(-i\hbar \nabla - q\mathbf{A}(\mathbf{r}, t))^2] \psi = E\psi$$

The solution of this equation often uses Landau gauge for the vector potential defined as

$$\mathbf{A} = B_z x \hat{\mathbf{y}}$$

Then, the eigenvalues of the operator for non-relativistic particles are found to be quantized (Figure 3.3) :

$$E_n = \hbar\omega_c \left(n + \frac{1}{2}\right) \quad n = 0, 1, 2, \dots$$

Each value of n labels a discrete energy level called a Landau level, separated by an energy gap $\Delta = \hbar\omega_C$ with ω_C the cyclotron frequency

$$\omega_C = \frac{eB_z}{m}$$

For relativistic particles such as electrons flowing in graphene, it can be proven that energies become [35]:

$$E_{\text{graphene}} = \pm \sqrt{2\nu\hbar e B v_F^2}$$

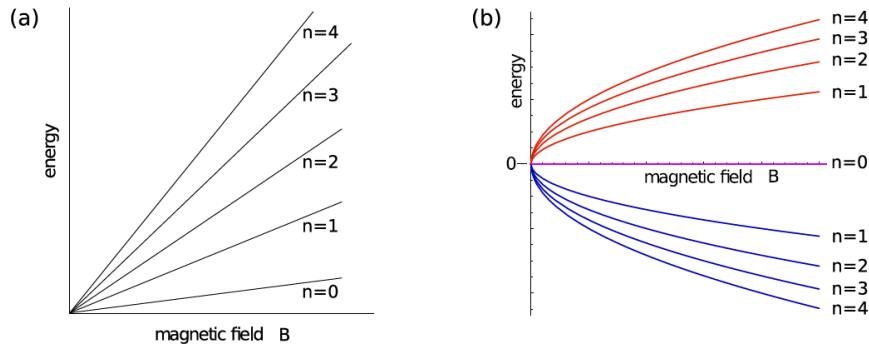


Figure 3.3: Landau levels as a function of the magnetic field, (a) for the non-relativistic case and (b) for a relativistic case such as graphene. Adapted from [36].

Quantum Hall Effect The quantum Hall effect [37] is the quantized version of the Hall effect with a longitudinal resistivity that takes discrete values :

$$\rho_{xy} = \frac{h}{e^2\nu},$$

with ν the filling factor that can be an integer or the result of some allowed fractions [38]. This effect derives from the Landau levels, where discrete energy levels determined by ν are bent on the Hall bar boundaries due to confinement potential. The number of energy levels crossed by the Fermi energy lead to quantized resistivity, and the appearance of edge states.

Aharonov-Bohm effect The Aharonov-Bohm effect [14] is a pure quantum effect resulting from the electron complex phase governed not directly by the magnetic field $\mathbf{B}$ but by its associated potential vector $\mathbf{A}$. This leads to an interesting phenomenon where particles, with charge q travelling along a path P in a region with zero magnetic field $\mathbf{B}$, but a non-zero potential vector $\mathbf{A}$ ($\mathbf{B} = \nabla \times \mathbf{A} = 0$), acquires a phase shift

$$\phi = \frac{q}{\hbar} \int_P \mathbf{A} d\mathbf{x},$$

Therefore, electrons with same input and output leads taking different routes can acquire a phase difference $\Delta\phi$:

$$\Delta\phi = \frac{q\Phi_B}{\hbar}$$

with $\Phi_B = \mathbf{B} \cdot \mathbf{S}$ the magnetic flux, $\mathbf{S}$ the surface $\mathbf{B}$ goes through. The phase shift has thus a period h/q .

This effect explains certain frequencies of the conductance oscillations with respect to varying magnetic field intensity on mesoscopic rings. If the magnetic field $\mathbf{B}$ is perpendicular to the surface S with a flux that has circular boundaries, the conductance G is oscillating with period

$$T_G = \frac{h}{e \pi r^2},$$

where r is the radius of the surface S , generally computed as the mean of the inner and outer radius of the considered mesoscopic ring. For electrons going one more round, this period is doubled and thus some harmonics can also be observed.

The effect of the magnetic field on the electron dynamics in cavities is further developed in [Chapter 5](#), with a discussion of our results.

3.2 Rotating bath and analogy to quantum world

3.2.1 Principle

Walking droplets in rotating baths have been firstly explored by E. Fort et al. [39] influenced by an article from Berry et al. [40] that introduced an analogy between electromagnetic fields and surface waves. Indeed, they showed that water waves passing an irrotational vortex could exhibit Aharonov-Bohm effect varying the vertex strength (Figure 3.4). An irrotational vortex is characterized by fluid particles each travelling according to their own axis, with a flow velocity inversely proportional to the center distance. With the support of this experiment, they illustrated the Aharonov-Bohm effect found in quantum mechanics, associating an irrotational vortex to a magnetic vector potential to analyze how it modifies the wavefront phase of water waves front associated to electronic waves (Figure 3.5). Defining the flux parameter as $\alpha = q\Phi/h$ (Table 3.1) in the quantum mechanics case, they show that the integer value α is equal to the number of wavefront dislocations, i.e. a phase difference of the wavefront equal to a multiple of 2π after passing the vortex. For half-integer α values, the interactions show destructive interference between wave fronts.

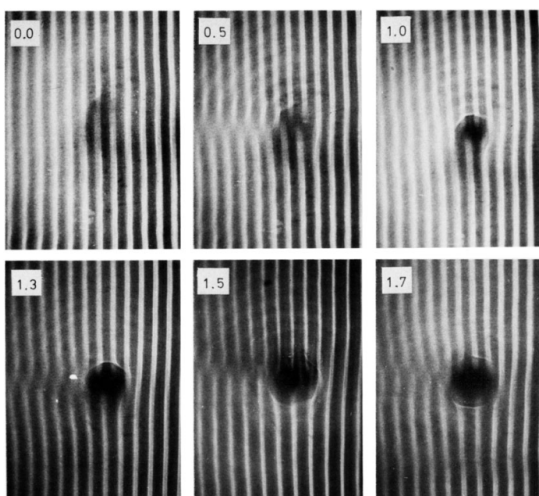


Figure 3.4: Water wave passing through an irrotational vortex leading to wavefront dislocation depending on the vortex strength. Waves are incident from the right and the water is circulating clockwise. Estimated values of the flux parameter α are indicated. Adapted from [40].

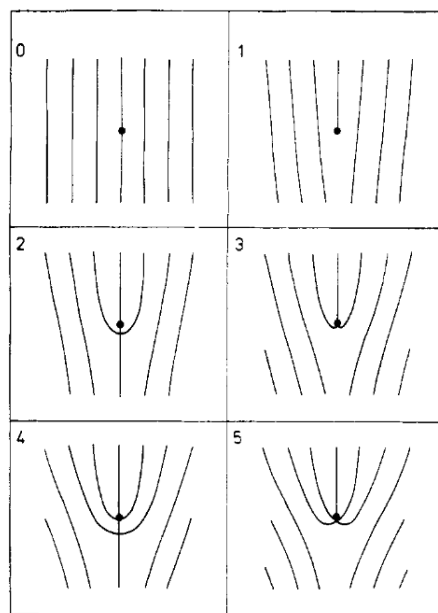


Figure 3.5: Sketch of electronic wavefronts incident from the right with different flux parameter α integer value (top left) corresponding to a magnetic field at the origin and pointing out of the paper.[40]

The analogy has been further explored in [41] where they pointed out common points and differences between quantum mechanics and fluid mechanics (Table 3.1).

The similar expression between a magnetic field $\mathbf{B}$ and the vorticity $\boldsymbol{\omega}$ of the *medium* with their potential vector $\mathbf{A}$ and $\mathbf{U}$ is the starting point for the walking droplet experiment in rotating bath.

Table 3.1: Analogy between quantum mechanics and fluid mechanics adapted from [41], v_ϕ is the phase velocity.

Quantum mechanics	Fluid mechanics
Magnetic field $\mathbf{B}$	Vorticity $\boldsymbol{\omega}$
$\mathbf{B} = \nabla \times \mathbf{A}$	$\boldsymbol{\omega} = \nabla \times \mathbf{U}$
Vector potential $\mathbf{A}$	Flow velocity $\mathbf{U}$
Magnetic flux Φ	Velocity circulation Γ_c
Wave function ψ	Surface displacement η
Flux parameter α	Dislocation parameter α
$\alpha = q\Phi/2\pi\hbar$	$\alpha = k\Gamma_c/2\pi v_\phi$
Dislocated wave is an exact solution	Dislocated wave is approximate solution
Phase is not measurable	Phase is measurable

As the rotation speed of the medium is not well suited for walkers experiment, the first experiment inspired by these articles [39] rather chose to rotate the bath. Indeed, it leads to another interesting analogy between the Lorentz force on charged particles and the Coriolis force on object moving at constant speed. As seen in 3.1.1, the Lorentz force acting on a charged particle in absence of electric field is written :

$$F_{Lorentz} = q\mathbf{v} \times \mathbf{B}$$

In a rotating referential at an angular velocity $\boldsymbol{\Omega}$, an object of mass m moving at speed $\mathbf{v}$ is subject to the Coriolis force equal to

$$F_{Coriolis} = -2m\mathbf{v} \times \boldsymbol{\Omega}$$

If the object moves in a plane with normal vector $\hat{\mathbf{z}}$ with $\mathbf{B} = B \hat{\mathbf{z}}$ or $\boldsymbol{\Omega} = \Omega \hat{\mathbf{z}}$, both these forces lead to circular trajectories with radius found by equalling the respective force with centripetal force $F = mv^2/r$:

$$r_{Lorentz} = \frac{mv}{qB} \quad ; \quad r_{Coriolis} = \frac{v}{2\Omega},$$

with periods :

$$\tau_{Larmor} = \frac{m}{qB} \quad ; \quad \tau_{Coriolis} = \frac{1}{2\Omega}$$

3.2.2 Walker experiments in rotating bath

We will review the results of rotating bath effects on a walker with some proposed analogies to the quantum realm.

Orbitals First walking droplet experiments in rotating bath have been realized by Fort et al. in [39] to investigate their presumed circular trajectory dependence on the rotation speed. They report an orbital radius dependence on the memory parameter. At low memory, a walker behaves more like a classical particle whose trajectory during free movements is controlled by the Coriolis force with radius slightly larger than classical Coriolis radius:

$$r_W = a \left(\frac{V_W}{2\Omega} \right),$$

where a is a fitting parameter found experimentally between 1.2 and 1.5. However, at higher memory, the orbital radius no longer varies continuously with Ω and it become quantized :

$$\frac{r}{\lambda_F} = \frac{b^2}{a}(n + 1/2), \text{ for } n > 0,$$

where $a \approx 1.5$ and $b \approx 0.89$ are fitting parameters. The authors explain the quantization phenomenon as the effect of the high-memory leading to walker interaction with the superposition of its emitted waves and the waves of its past trajectory.

The authors also report an hysteresis phenomenon of the orbital radii leading to an orbital degeneracy. Two different orbital radii may be observed for the same rotation rate, depending on whether Ω is approached from above or below.

Following this article, Harris and Bush [42] conducted an experimental investigation about the memory parameter effect on walker orbital in the same system, which revealed a number of new walker orbital behaviours. They found out that orbital quantization only exists for a finite range of memory (Figure 3.6). As the memory increases, the authors report more exotic states such as wobbling orbits with alternately stationary and drifting orbital centres and then followed by wobble-and-leap dynamics. Finally, in the high limit of the Faraday threshold, they found erratic trajectories, but still characterized by a multimodal probability distribution of the unstable orbital states that reflects its wavelike statistical behaviour (Figure 3.8).

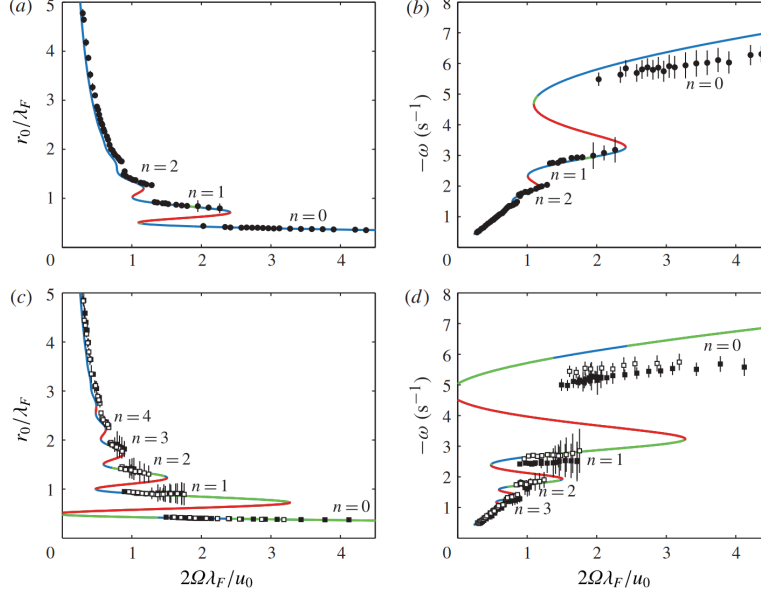


Figure 3.6: Points are the observed dependence of the orbital radius r_0 on the rotation rate of the bath. For (a), (b), (c), (d), Ω is respectively (0.822, 0.922, 0.954, 0.971) reported in [42]. Solid lines are the model prediction developed in [43], blue portions are stable while red and green portions are unstable

In [43, 44], Oza et al. theoretically explained the observations [42] through linear stability analysis, matching the different orbital walker states (Figure 3.7).

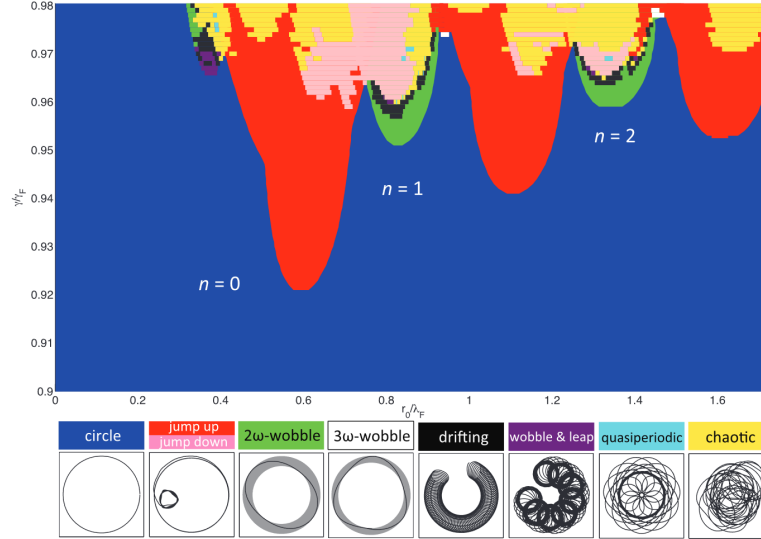


Figure 3.7: Regime diagram with the parameters r_0 and γ/γ_F , respectively the walker's trajectory on the initial orbital radius and the dimensionless vertical acceleration. [43]

The linear stability analysis being limited to stable solutions of the circular

orbits, the same research group then investigated the walker behaviour in the very high memory regime through numerical simulations [45], where they report matching results with [42] (Figure 3.9)

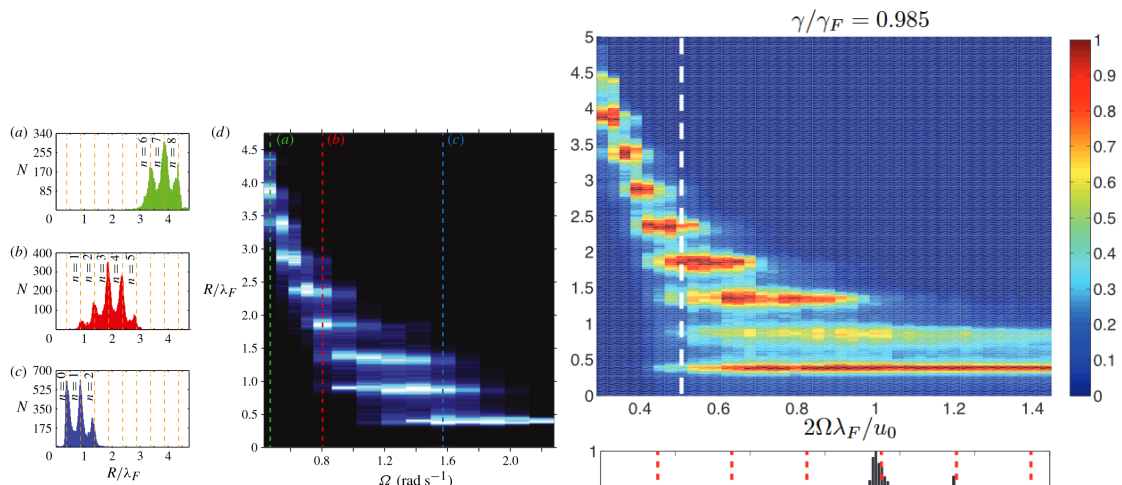


Figure 3.8: Orbital statistics with forcing amplitude $\gamma/\gamma_F = 0.988 \pm 0.003$, for increasing rotation rates (a) $\Omega = 0.47[\text{rad/s}]$, (b) $\Omega = 0.79[\text{rad/s}]$ and (c) $\Omega = 1.57[\text{rad/s}]$. (d) Probability distribution of the radius of curvature R depending on Ω . [42]

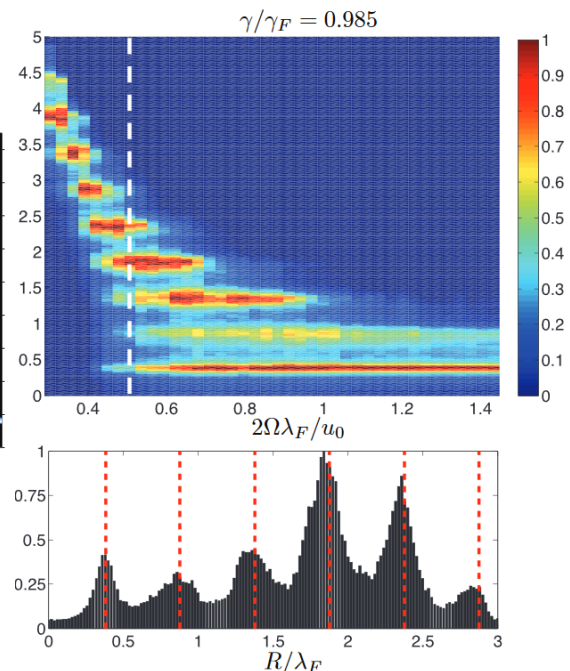


Figure 3.9: Numerical simulations of orbital statistics and probability distribution with forcing amplitude $\gamma/\gamma_F = 0.985$ for dimensionless rotation rate $2\Omega\lambda_F/u_0$. [45]

Quantum analogy

The authors [39] proposed an analogy between the walker's quantized orbits and the Landau levels of an electron, as the quantized Larmor radii r_L is:

$$\frac{r_L}{\lambda_{dB}} = \frac{1}{\pi}(n + 1/2), \text{ for } n > 0,$$

where λ_{dB} is the de Broglie wavelength, that the Faraday wavelength is associated to when looking at the walker radius expression

$$\frac{r}{\lambda_F} = \frac{b^2}{a}(n + 1/2), \text{ for } n > 0,$$

In [45], the authors introduced an analogy for the quasi-periodic orbitals defined by periodic change between unstable solutions based on Coriolis and Lorentz force expression similarity. They compare them to a Rabi oscillation phenomenon, a quantum physical phenomenon occurring for an oscillating force field where an electron periodically alternate between two energy states.

Orbital level splitting for walker bound states

In [20], Protiere et al. report an orbital quantization phenomenon for two walkers in bound states and [46] investigated the walkers bound states in a rotating bath. The orbit diameter separating the walkers in a static bath is discrete and scaled on the Faraday wavelength λ_F :

$$d_n^{orb} = (n - \varepsilon_0) \lambda_F,$$

where for drops bouncing in phase, $n = 1, 2, 3, \dots$ or $n = 1/2, 3/2, 5/2, \dots$ for drops bouncing with opposite phases and $0.15 < \varepsilon_0 < 0.25$ is a shift found experimentally depending on the droplet diameter that was around $D = 0.7[mm]$ [20].

In a rotating bath, Eddit et al [46] observed (Figure 3.10) that the orbital radii of rotating orbits increase with the angular velocity Ω for a same direction of rotation of the bath (noted n^+ state, filled symbols), while the orbits decrease for the opposite case (noted n^- state, empty symbols).

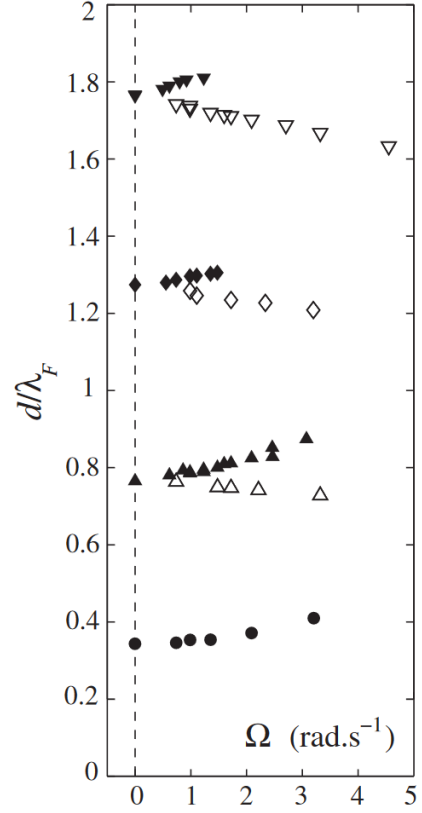


Figure 3.10: Evolution of diameter d between two walkers in bound states depending on the angular velocity Ω . [46]

They developed a model to include shift δ_n (Figure 3.11) from the previous expression :

$$d_n(\Omega) = d_n(0) + \delta_n(\Omega),$$

by including Coriolis force to the balance between the centrifugal force and the wave mediated interaction force of the static case:

$$\frac{\delta}{\lambda_F} = \frac{m_G}{\pi F_0} \Omega V_{W,n} \exp\left(\frac{d_n(0)}{l_v}\right),$$

where $m_G/\pi F_0 = 3[s^2/m]$ with m_G the droplet mass and F_0 the amplitude of the wave mediated interaction force, $V_{W,n}$ is the walkers velocity orbiting on the n^{th} bound state and $l_v \approx 2\lambda_F$ is the characteristic damping length.

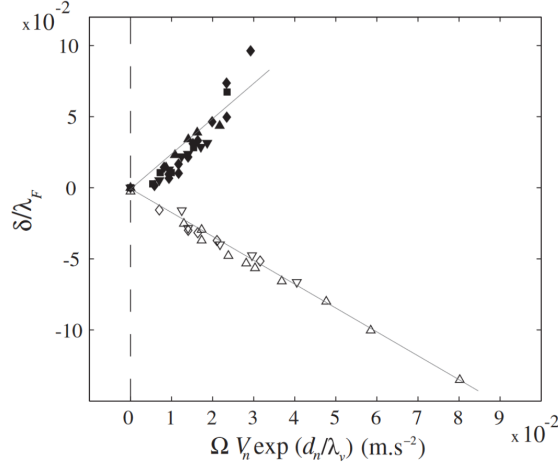


Figure 3.11: Rescaled experimental data and developed model in solid line.[46]

Quantum analogy

The authors present an analogy between the level splitting of the orbitals depending on their $\pm$ state and the Zeeman effect in which energy levels may be split into different energies levels under a magnetic field. The analogy is supported by the expression similarity between the Coriolis potential energy and the magnetic interaction energy:

$$E_{\text{Cor}} = 2\mathbf{L}_n \cdot \boldsymbol{\Omega} \quad ; \quad E_{\text{mag}} = \frac{e}{2m_e} \mathbf{L} \cdot \mathbf{B}$$

respectively leading to a shift distance and a shift frequency written as

$$\delta\omega_n^{\pm} = \pm\alpha_n\boldsymbol{\Omega} \quad ; \quad \delta\omega_E = \frac{e}{2m_e} \frac{\mathbf{L} \cdot \mathbf{B}}{\hbar}$$

where $\omega_n = V_{W,n}/d_n(\Omega)$ and $\omega_E = E/\hbar$ are respectively the characteristic frequency and the rotation frequency.

Hydrodynamic spin lattices

Recently, Sàenz et al. [47] reported a macroscale analogue of microscopic spin systems in the form of hydrodynamic spin lattices (HSLs): a system of in-phase bouncing droplets that influence each other one by one with wave-mediated interactions (Figure 3.12). They introduce an analog magnetization M with spin S and spin-spin correlation χ defined as

$$M(t) = \sum_i S_i(t) \quad ; \quad S(t) = L_z(t)/m \quad ; \quad \chi(t) = \sum_{i \sim j} S_i(t) S_j(t) \quad ;$$

where m is the droplet mass and L_z the angular momentum with respect to the well centre. The correlation is computed from adjacent walkers (written as $i \sim j$). The analog reported spin is thus orbital motion.

They found out spin-spin correlation tunability by imposing bath rotation leading to transition from analog antiferromagnetic (orbital motions in opposite directions from one to the next) to ferromagnetic order (orbital motions in same direction) (Figure 3.13)¹.

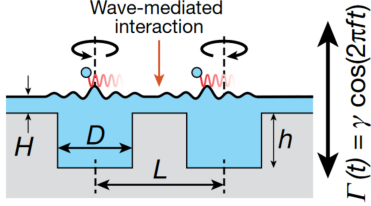


Figure 3.12: Schematic of droplet interactions in an HSL between two droplets in their respective well.[47]

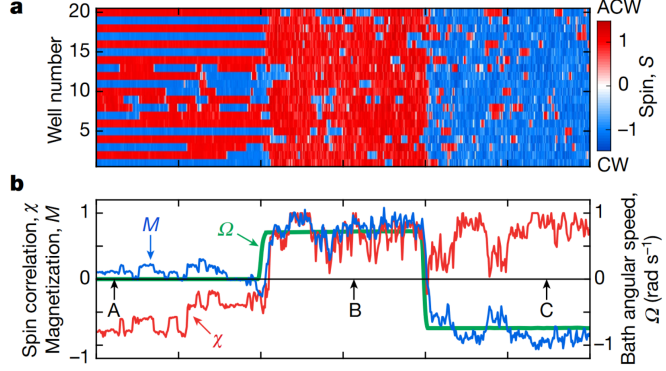


Figure 3.13: (a) The local ferromagnetic order for each droplet well. (b) The spin-spin correlation χ and magnetization M evolution with bath rotations that varies with time $\Omega = (0, 0.7, -0.7)[rad/s]$. [47]

The discussed analogies are summarized in the following Table 3.2.

Table 3.2: Walker analogies with electron properties.

Electron properties	Walkers analogy
Magnetic field B	Angular velocity Ω
Coriolis Force $F_{Coriolis}$	Lorentz Force $F_{Lorentz}$
$F_{Coriolis} = -2m\mathbf{v} \times \boldsymbol{\Omega}$	$F_{Lorentz} = q\mathbf{v} \times \mathbf{B}$
$r_{Lorentz} = \frac{mv}{qB}$	$r_{Coriolis} = a \frac{v}{2\Omega}$
$\tau_{Larmor} = \frac{m}{qB}$	$\tau_{Coriolis} = \frac{1}{2\Omega}$
Larmor radius r_L	Walker orbital radius r
de Broglie wavelength λ_{dB}	Faraday wavelength λ_F
$\frac{r_L}{\lambda_{dB}} = \frac{1}{\pi}(n + 1/2)$	$\frac{r}{\lambda_F} = \frac{b^2}{a}(n + 1/2)$
Magnetic interaction energy E_{mag}	Coriolis potential energy E_{Cor}
$E_{mag} = \frac{e}{2m_e} \mathbf{L} \cdot \mathbf{B}$	$E_{Cor} = 2\mathbf{L}_n \cdot \boldsymbol{\Omega}$
Shift frequency $\delta\omega_E$	Shift distance between two coupled walkers $\delta\omega_n^\pm$
$\delta\omega_E = \frac{e}{2m_e} \frac{\mathbf{L} \cdot \mathbf{B}}{\hbar}$	$\delta\omega_n^\pm = \pm\alpha_n \boldsymbol{\Omega}$
Spin S	Walker 'spin' $S = L_z/m$

¹Illustration videos are available at the following url: [Ferromagnetic order](#) and [Antiferromagnetic order](#)

Chapter 4

Walker experiment

The experimental setup and its validation will be firstly discussed in this chapter. It started from the setup previously developed by O. Leblanc [1] and has been modified to improve some reported points such as the droplet generator, the bath horizontality, and the walker state reported for acceleration amplitude regime measured too low compared to the literature. Plus, the setup includes a stepper motor that allows to set a controllable angular velocity of the bath. The baths include input and output branch to study the transmission probability.

With the walker parameters analysis from 2.2.2, the silicone oil is chosen with viscosity $20[cSt]$ [22], vertically accelerated at $f_0 = 80[Hz]$. The droplet diameter is $D = 0.82[mm] \pm 0.02$, described in 4.1.4. The surface oil height is $h_0 = 6[mm]$ depending on the bath, with wave damping on the boundaries $h_1 \leq 1[mm]$, described in 4.1.

The methodology for data analysis will then be presented, including algorithms developed for droplet diameter estimation and tracking.

4.1 Experimental setup

A schematic of the experimental setup is shown in Figure 4.1, here with the ring-shaped bath(7). The complete electrical connections between components are schematically shown in the Appendix (Figure A.1).

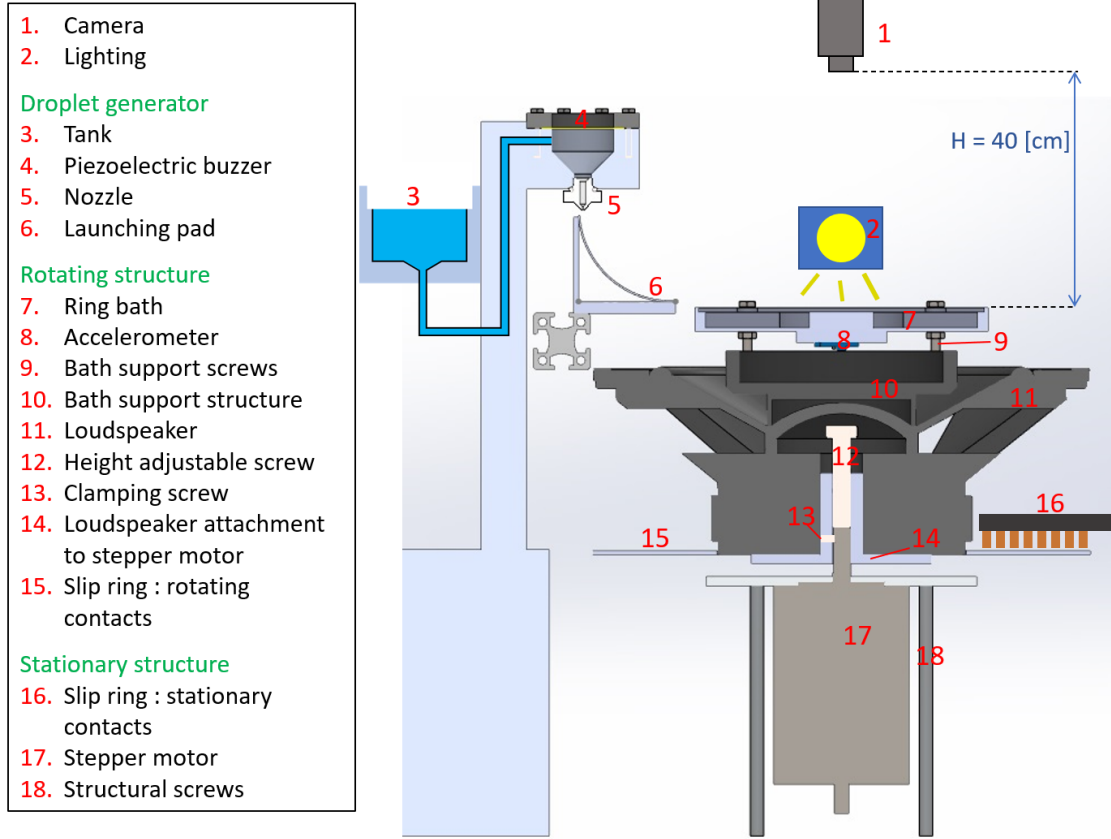


Figure 4.1: Experimental setup schematic

A camera *Sony HDR-PJ10E*[48] is placed vertically in line with the centre of the bath. The camera records at 1920×1080 pixels with a 50FPS framerate. As the camera only aims to analyze the horizontal droplet dynamics to track and store the walker position, two lights are placed with grazing incidence on the bath with the aim of increasing the contrast between the walking droplet and its background including the generated wavefield reflection in the camera lens; it eases the tracking algorithm to work for isolated objects without environment changes. To avoid heating the liquid whose properties vary with temperature, LED lights are chosen over classical incandescent lights that produce more heat, and are placed at a sufficiently long distance $D > 15[cm]$ considering LED technology. Plus, the LEDs have adjustable light intensity which is chosen to be the lowest possible while ensuring a good droplet visualisation. The vertical dynamics and the wave dynamics are not analyzed. The video recordings are post-processed with Python routines, discussed in 4.2.

Rotating structure The two studied baths are 3D printed with Polylactic Acid (PLA), with a precision of $100[\mu m]$, (Figure 4.3). For both baths, the input branch is the left horizontal branch and the output branch is the right horizontal branch.

As the geometry is symmetrical, no return path between output and input branches is added. To always consider the left branch as input, when the droplet positions reach the right output branch, the next positions are 'reversed' such that the right branch becomes the left branch and vice versa. Both baths have height $h_0 = 6[mm]$ to fulfill the oil surface height condition explained in 2.2.2. In practice, a bath is fully filled to overcome the well height by $h_1 \leq 1[mm]$ which is too shallow to support a walking drop (Figure 4.2). This value was used in [13] and the reflections angle were investigated in [49]. The minimal distance between the well and the borders of a bath of is $7[mm]$.

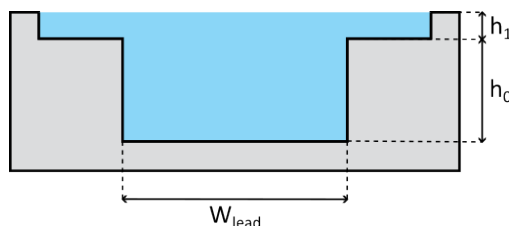


Figure 4.2: Scheme of the cross-section of a bath lead. h_0 and h_1 are respectively the well oil height and the height that overcomes h_0 .

The ring-shaped bath has paths of constant width $W_{cav,r} = W_{lead,r} = 2.5\lambda_F = 14.16 [mm]$ inspired by [50] ("cav" and "r" stands for "cavity" and "ring"). The width is chosen to consider the paths as an analogy to electrical leads, acting as a waveguide for the walker. In [50], they report a maximum longitudinal speed for $W_{lead,r} = 2\lambda_F$ without axial speed. Therefore, the width is chosen slightly bigger to induce a possible small axial speed and thus a slightly higher degree of freedom for the walker trajectory. This is meant to allow the walker to have possible scarring pattern, such as the one observed in [1]. The inner radius is $R_{in} = W_{path,r} = 14.16 [mm]$ and the outer radius is $R_{out} = R_{in} + W_{path,r} = 28.32 [mm]$.

The stadium-shaped bath is inspired by [1, 51]. The stadium has a cavity width $W_{cav,s} = 45 [mm]$ ("s" stands for "stadium") and length $L_{cav,s} = 2W_{cav,s} = 90 [mm]$ (without the input and output leads), where $L_{cav,s} = 2W_{cav,s}$ is chosen for the most chaotic behaviour of ball trajectories [52]. The width $W_{cav,s} = 9.5\lambda_F$ is chosen to test whether the preference pattern for the droplet positions obtained for $W_{cav,s} = 8\lambda_F$ in [1] is robust for higher cavity dimensions. Input and output paths are also added, with a width $W_{lead,s} \approx 2\lambda_F = 9.5 [mm]$. The width is chosen such that the walker initial conditions is overall constant in the stadium.

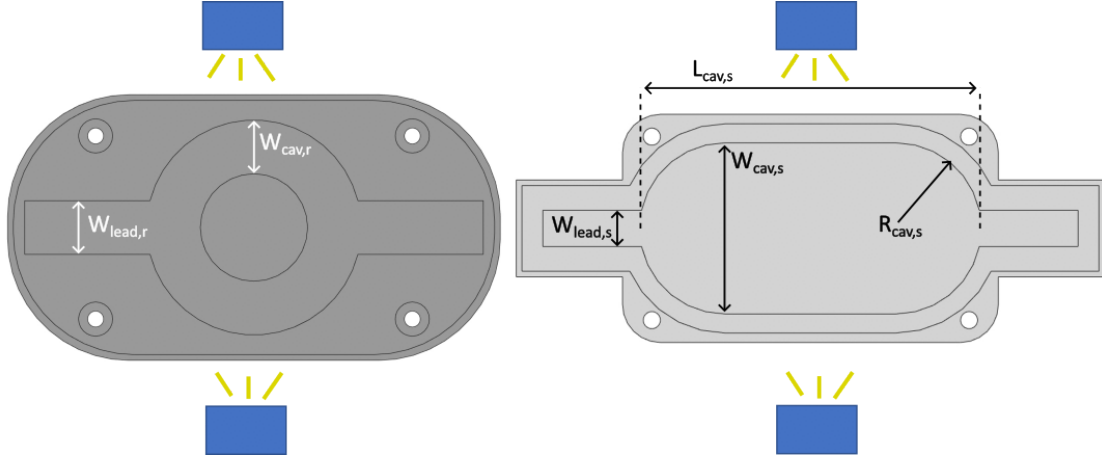


Figure 4.3: The two 3D printed studied baths of the experimental setup. Left: the ring-shaped bath with path width $W = 2.5\lambda_F$. Right: the stadium-shaped bath with leads $W_{lead} = 2\lambda_F, L = 19\lambda_F, W = 9.5\lambda_F$.

A bath structure support has been fabricated in Polyvinyl chloride (PVC) and glued on the membrane of a loudspeaker [53], serving as an electromagnetic shaker. The bath is attached to the PVC bath structure with 4 screws aimed at fixing the horizontality problem encountered in [1]. The loudspeaker membrane applies a vertical acceleration $\gamma(t) = \gamma_m \sin(2\pi f_0 t)$, where $3.2 < \gamma_m < 4.03$ [g] is the acceleration amplitude and $f_0 = 80$ [Hz], described in 4.1.3. The silicone oil is provided by Sigma-Aldrich [22], with viscosity $\nu = 20$ [cSt] at $T = 25$ [°C].

An accelerometer is fixed beyond the bath to optimally measure the silicone oil acceleration. The loudspeaker is fixed on a stepper motor with a dedicated attachment piece in PVC designed to support the loudspeaker, being attached to the motor shaft with a clamping screw for couple transmission and a height adjustable screw to adjust the loudspeaker distance from the stepper motor. For electrical connections between the rotating loudspeaker and the stationary part, a home-made slip ring has been designed, covered in 4.1.5. On the rotating structure side, it is composed of copper rings created from a PCB (with a PCB milling machine) that is attached to the loudspeaker.

Stationary structure The stationary structure is composed of a NEMA 23 stepper motor [54] fixed with four structural screws, and stationary contacts of the slip ring. A stepper motor has been chosen to provide a low-cost motor with fully controllable rotation speed. The four structural screws are used to adjust the stepper motor horizontality. The stationary part of the slip ring is composed of three row of copper brushes in contact with the rotating surface rings.

Droplet generator The droplet generator inspired by [55] is discussed in 4.1.4 and produces oil droplet of tunable diameter $0.8[mm] < D < 1.13[mm]$ generated with a piezoelectric buzzer [56] powered by a voltage source. The oil droplets are launched in the bath with launching pad made of soft plastic to land in one of the bath leads.

The droplet generator, the loudspeaker membrane vertical acceleration, the angular velocity and the data extraction from the accelerometer are all handled from a PC with two NI Labview programs running in parallel. The first one, *"MyDAQ_memory_control.vi"* (front panel in Figure A.3), is dedicated to the vertical acceleration control and Z-axis measurements that runs on a NI myDaq connected with dedicated components that is discussed in 4.1.3. The second one, *"Arduino_Ctrl.vi"* (front panel in Figure A.2), handles a serial communication with an Arduino Uno explained in 4.1.2 (Front panel in Figure A.1).

The Figure 4.4 presents the real setup with components indications.

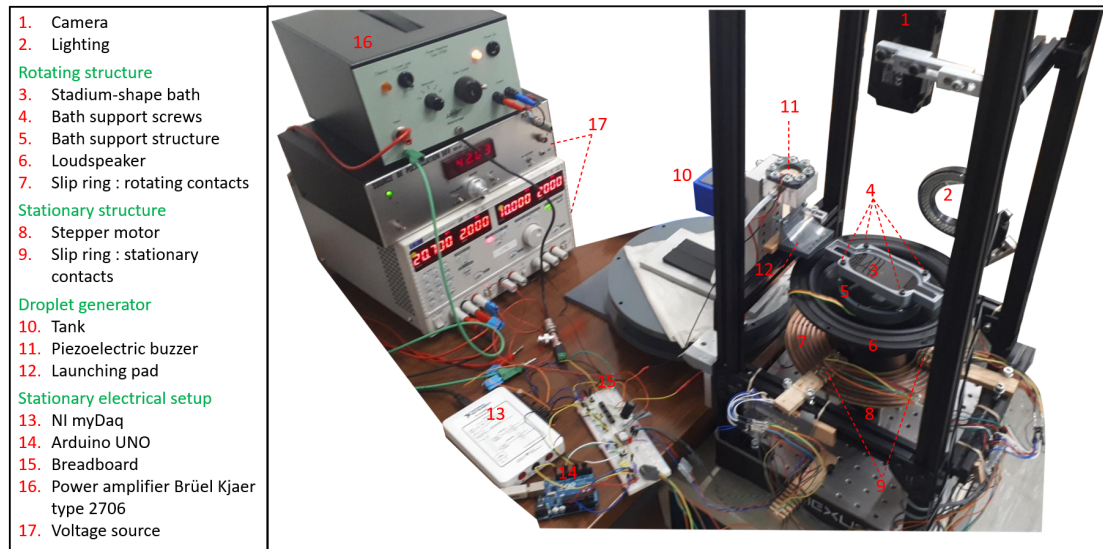


Figure 4.4: Presentation of the real experimental setup with the stadium-shaped bath.

4.1.1 Electrical connections to the rotating structure

The electrical connections to the rotating structure include the loudspeaker power supply that is a voltage source requiring two connections and the accelerometer connections that requires six connections: three for the three-axis accelerometer measurements, two for the voltage source and one for the reference voltage. To ensure electrical connections between the rotating structure and the stationary structure, a home-made slip ring has been created (Figure 4.5).

The rotating contacts consists in a circular PCB plate with a hole in its center to fit with the loudspeaker bottom diameter and attach them to each other. On the circular plate, eight independent conducting surface rings have been patterned with a PCB milling machine, with a little hole to let pass an electrical cable. A schematic of cutting plane is on [Figure 4.6](#). The end of a wire is soldered on each surface ring passing through the little hole from the bottom of the plate to connect the surface ring to the component requiring electrical connections. The stationary contacts is a row of electrical cables placed above each surface ring. The cables are chosen to be composed of a huge number of very small copper wire to look like a brush contact and maximize the electrical contact. To ensure no contact loss during the rotations, three rows of stationary contacts have been created and form 90° angles from each other.

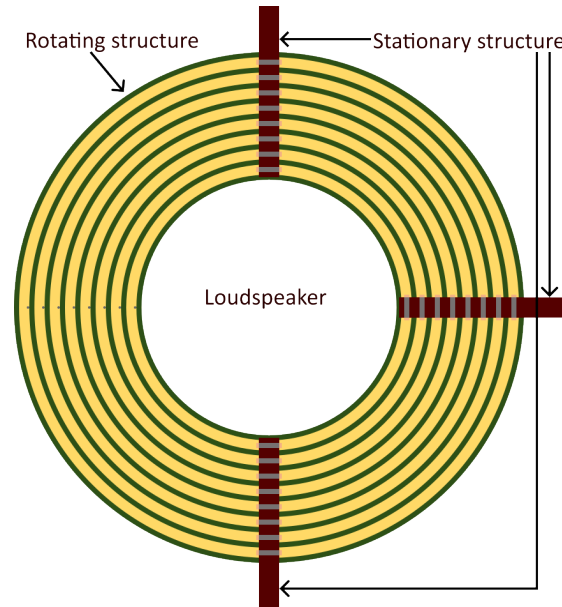


Figure 4.5: Home-made slip ring. Rotating structure consists in a circular plate with eight surface rings that are in independent electrical contact with the loudspeaker and the accelerometer. The stationary structure consists in three rows of eight electrical brushes each in contact with one of the rotating surface rings.

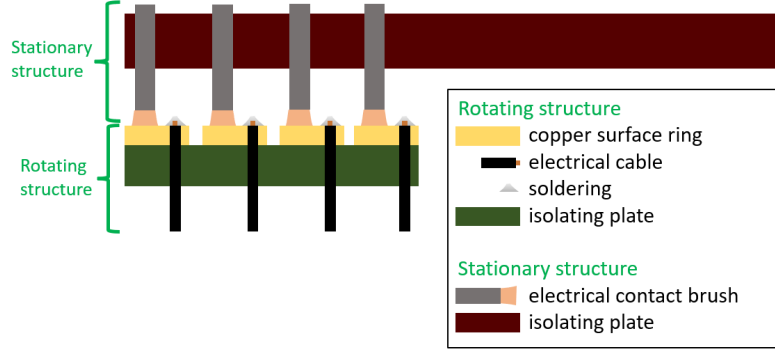


Figure 4.6: Schematic of the slip ring electrical contacts between stationary part and rotating part.

Validation The electrical contacts are verified by measuring the resistances between stationary contacts and rotating contacts for several complete rotations. The resistances showed no peak values that would indicate a bad electrical contact and measured values are constantly $R < 3 \text{ } [\Omega]$. Eventually, the three rows were necessary as the rotating part of the slip ring nested in the loudspeaker was not perfectly horizontal due to some bad fixed contacts at the loudspeaker bottom. As a result the rows of stationary contacts ensure electrical contacts each in turn.

4.1.2 Experimental setup control

The experimental setup control is handled with two Labview programs running in parallel, *"MyDAQ_memory_control.vi"* and *"Arduino_Ctrl.vi"*. The first one is dedicated to the vertical acceleration control and measurement (Z-axis) and will be further discussed in 4.1.3. The second one interacts with the the Arduino Uno microcontroller by implementing a master/slave serial communication between a PC (master) and the Arduino Uno (slave).

Serial communication uses the PC as a transmitter to send data one bit at a time over a single communication line to the Arduino Uno as a receiver. Arduino Uno has two way of communications: parallel or serial communications. While parallel connection offers a higher data rates, it requires a higher number of I/O pins that were not available. Therefore, the serial communication is chosen. The Arduino Uno program handles the droplet generation, the stepper motor control and the three-axis measurements. It starts with default value for each components and then checks for Serial communication events sent by the Labview program and executes what is asked.

In the Labview program user interface, the following features are proposed.

- **Acceleration :**
 - Selection of the desired axis acceleration;

- Possibility to output the mean of a number of samples over time that reduces the noise measure;
- Three-points calibration for each axis : enter the reported Arduino UNO values $[0, 1023]$ (internal conversion of the Arduino with its ADC) for three different orientation, typically $0[g], \pm 1[g]$;
- **Droplet generator** :
 - Choice of the piezoelectric pulse duration (discussed in 4.1.4);
 - 'Generate' button for droplet generation;
- **Stepper motor** : control over target speed $[Hz]$, maximum speed allowed in $[Hz]$, division step mode and acceleration parameters (discussed in 4.1.5).

At each confirmed change of value, an instruction is sent to the Arduino Uno that executes the corresponding task.

4.1.3 Vertical acceleration

The vertical acceleration of the loudspeaker membrane whose bath is fixed is directly ensured by a NI Labview program that communicates with a NI myDAQ. The program controls sinusoidal signal that is sent with one of the two analog output of the NI myDAQ. The signal goes through a power amplifier *Brüel Kjaer type 2706* with a gain set at ~ 4.5 , which is then sent to the loudspeaker by passing through the slip ring.

The stepper motor is fixed on an optical table [57] suitable to absorb vibrations coming from the loudspeaker, and positioned flat with the structural screws using a precise bubble level as verification tool. The loudspeaker is nested on the stepper motor. While the optical table ensures vertical vibration damping, nothing has been used to stabilize the bath in the transverse directions.

The acceleration measurements are performed with 3-axis $\pm 16 [g]$ ADXL326 accelerometer [58]. The Arduino Uno handles accelerometer power supply with its 5 $[V]$ output, and receives the 3-axis acceleration output measurements from the accelerometer that are in the range $[0, 3.3] [V]$ in one of its two analog inputs (AI). The Z-axis (vertical axis) is also measured by the NI Mydaq for an analog voltage measurement, gaining precision. Three capacitors are by default soldered on the accelerometer to add a low-pass filter (R-C filter) at the end of the output voltage. It aims to lower the noise density but limits the bandwidth output voltage at 50 $[Hz]$. These have been removed and replaced by three capacitors $C = 32 [nF]$. The cut-off frequency is :

$$f = \frac{1}{2\pi RC} = 155.4 [Hz],$$

with the ADXL326 resistance $R = 32 \text{ [k}\Omega\text{]}$ to allow the 80 [Hz] measurements while maintaining noise reduction.

User interface The Labview program developed for acceleration driving and Z-axis measurements shows in real-time five different waveforms over time: the voltage measurements from the accelerometer, the vertical acceleration measurements, its associated FFT in logarithmic scale, the found acceleration peaks corresponding to Γ_m , the ratio $\mathcal{G} = \Gamma_m/\Gamma_F$ and the memory parameter.

For the vertical acceleration parameters, the Labview program user has the ability to vary the frequency f_0 , the vertical acceleration amplitude by modifying the sinusoid amplitude A , and the NI myDAQ sample rate. Before starting an experiment, the Z-axis acceleration can be calibrated with a three points voltage measurements for three different orientation, typically 0 and $\pm 1[g]$. The user reports the values in a dedicated calibration window and the program computes a linear regression to convert voltage to acceleration measurements. The linear accelerometer behaviour is assumed due to a small non-linearity 0.3% [58]. As an acceleration setpoint controller has not been successfully implemented yet, the user is then invited to progressively increase the amplitude voltage in order to reach the Faraday threshold Γ_F and report the acceleration value. This way, the program computes the ratio Γ_m/Γ_F and the memory parameter from (2.1).

Validation In practice, the serial communication cannot handle the reading of the three axis acceleration at the same time due to a too low data rate, and gets satisfying results for one axis reading maximum. A way to solve this problem is using parallel communication or to better optimize the Arduino code constantly waiting for events that slows down its main loop. Therefore, this way of measurement was mainly used as a double-check of the NI-myDAQ measurements.

Then, the bath horizontality was tested at null angular velocity by measuring the acceleration axis with the NI myDAQ one at a time (Figure 4.7) and computing the resulting amplitude mean. From the measurements, the tilt angles associated to the rotation matrix $R(\alpha = 0, \beta, \theta) = R_z(\alpha)R_y(\beta)R_x(\theta)$, where $\alpha = 0$ assuming no rotation around the Z-axis, are computed from the mean of the amplitude acceleration Γ_{m,μ_i} where i is the axis measurement :

$$\begin{aligned}\theta &= \arcsin\left(\frac{\Gamma_{m,\mu_Y}}{\Gamma_{m,\mu_Z}}\right) = \arcsin\left(\frac{32.6582}{3743.3397}\right) = 0.5^\circ; \\ \beta &= \arctan\left(\frac{\Gamma_{m,\mu_X}}{\Gamma_{m,\mu_Z}}\right) = \arctan\left(\frac{49.3023}{3743.3397}\right) = 0.75^\circ;\end{aligned}$$

where the angles have well decreased comparing to the ones found by O.Leblanc [1]. The power spectrum of the acceleration measurements (Figure 4.8) reveals an expected main frequency peak at $f = 80[\text{Hz}]$. Frequency peaks at lower values

are present. Some of them are related to $50[Hz]$ and higher harmonics, which are typical electrical noises related to electric hum. The remaining peaks are harmonics of the main $80[Hz]$ peak. The $80[Hz]$ signal is assumed to be perfect for the rest of the experiments, considering a very low total harmonic distortion (THD)

$$THD = 100 \frac{\sqrt{\sum_{h=2}^H v_h^2}}{\sqrt{\sum_{h=1}^H v_h^2}} = 0.8863[\%]$$

where v_h is the amplitude of the Fourier transform at the h harmonic.

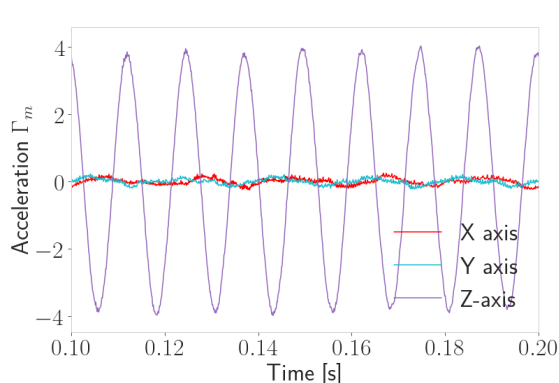


Figure 4.7: 3-axis acceleration measurements over time measured by NI myDAQ.

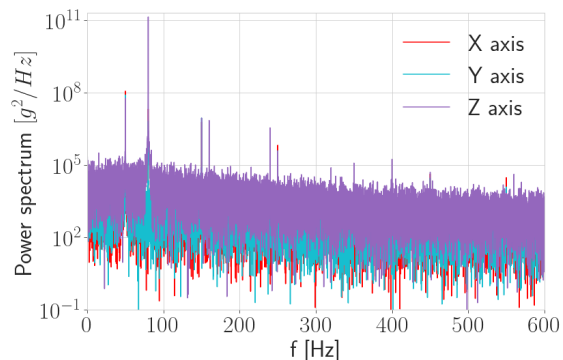


Figure 4.8: Power spectrum of acceleration measurements showing main expected peak at $f = 80[Hz]$ and other peaks at $f = (50, 150, 160)[Hz]$.

However, two problems were then found, and the origin of these ones could unfortunately not be resolved before the results presented in [Chapter 5](#). The first problem could be corrected from the measurements but the second one needs some setup improvements discussed hereafter.

The first problem was Faraday threshold observed slightly too low compared to experimental data presented in [2.2.2](#). The cause of the problem was a too low cut-off frequency of $155.4[Hz]$. Indeed, considering a first-order low pass filter, the cut-off frequency value had an impact on the $80[Hz]$ forcing frequency, confirmed by a Bode plot ([Figure 4.9](#)) realized with LT spice simulation. To solve the problem of the measured accelerations for the results of [Chapter 5](#), these were post-processed by applying a digital bandpass butterworth filter ($f_{low} = 60[Hz]$, $f_{high} = 100[Hz]$, $order = 3$) to isolate the $80[Hz]$ frequency and multiplied by a gain G that previously attenuated them, computed from LT spice simulation :

$$G = 10^{(1.0207745/20)} = 1.1247$$

The second problem was acceleration amplitude variations with the rotation ([Figure 4.10](#)). The origin of the problem is thought to be the series resistance between the voltage amplifier *Brüel Kjaer type 2706* and the loudspeaker input, involved by the slip ring, whose impact was underestimated during the validation

step of the electrical connections 4.1.1. The series resistance R_s has been measured at 8 different points, varying between $1.1[\Omega]$ and $2.9[\Omega]$. As the loudspeaker impedance R_{lp} is $8[\Omega]$, it will inevitably impact (voltage divider circuit) its input voltage by a range :

$$V_{lp} = V_{in} G_{amp} \frac{R_{lp}}{R_{lp} + R_s} \in [5.14, 4.29] [V],$$

where $V_{in} = 1.3[V]$ is the NI myDAQ output and $G_{amp} = 4.5$ is the voltage amplifier. To support this explanation, as mentioned previously in 4.1.1, the three rows alternately ensure electrical contacts, and one can clearly notice the three steps of acceleration measurements (Figure 4.10) due to different electrical contacts between the rows and the rotating rings.

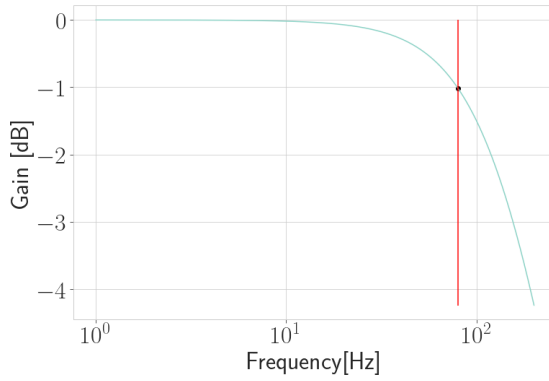


Figure 4.9: Bode response of RC-filter from LT spice simulation, where $R = 32[k\Omega]$ and $C = 32[nF]$. The red line corresponds to the forcing frequency $80[Hz]$

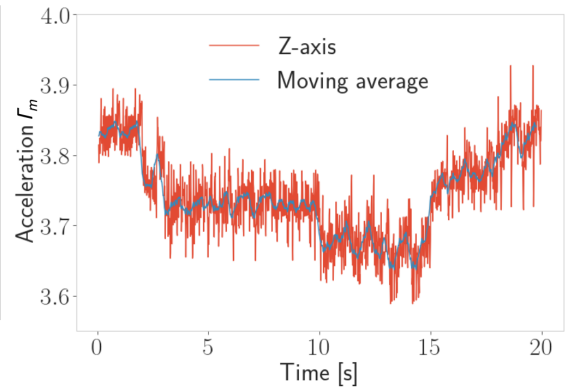


Figure 4.10: Acceleration peaks for a complete rotation with a period of $20[s]$.

Due to this problem, the acceleration amplitudes of Chapter 5 were chosen at

$$\Gamma_m \in [3.52, 3.76],$$

fluctuating around $\Gamma_m = 3.64$ giving a dimensionless ratio and memory

$$\mathcal{G} \in [0.87, 0.93] \quad ; \quad Me \in [6.06, 11.47]$$

This choice of low acceleration amplitude is motivated by a possible high behaviour change of the droplet between very high memory and high memory, and a more constant behaviour for a lower memory range. With the corrections, the Faraday threshold was consistent, and found at

$$\Gamma_F = 4.03$$

Improvement propositions To solve this problem, two solutions are proposed. The first one is more rapid and low-cost, consisting in the implementation of an amplitude acceleration setpoint controller (such as PI or PID controller) that should be reactive enough to counteract the resistance changes. The second one is the electrical contact improvements. For this, two possibilities : the change of the slip ring by a professional one that handles all the connections, or the stationary electrical contacts improvement by a more sophisticated system including springs that maintain a constant electrical contact. The professional slip ring was not primarily envisaged due to too long delivery times compared to this Master thesis date submission, and the accessibility to create one in the electronic lab of the UCLouvain NAPS pole where the thesis was realized.

4.1.4 Droplet generator

A droplet generator had been fabricated in the framework of the O. Leblanc master thesis [1] inspired by [55] (Figure 4.11). The authors [55] report droplet generation with a highly reproducible diameter control with size range $0.5 - 1.4$ [mm].

The droplet generation principle relies on a piezoelectric deformation induced by a short polarity change that sends a pressure pulse to the oil filling the fluid chamber, generating a droplet at the nozzle output. The droplet lands ideally tangentially on a launchpad (not shown in the figure) that deviates its vertical trajectory to a horizontal trajectory towards the bath. The droplet diameter is determined by the amplitude V_s and the duration t of a voltage square pulse sent to the piezoelectric terminals, as well as the height difference Δh between the surface level of the fluid reservoir and the nozzle output. A characterization of the droplet diameter depending on the amplitude and pulse duration parameters is presented afterwards and a typical pulse sent by the Arduino Uno is on the Figure 4.12.

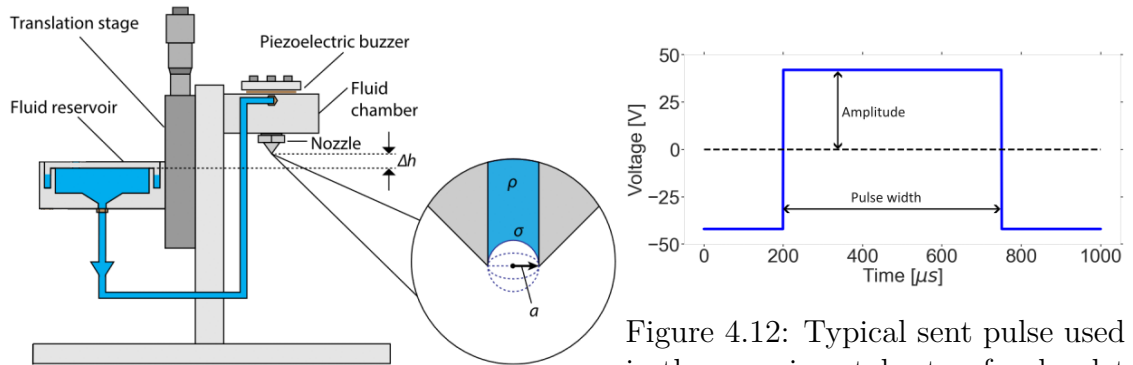


Figure 4.11: Schematic of droplet generator.

Figure 4.12: Typical sent pulse used in the experimental setup for droplet generation, $V_s = 42$ [V], $t = 750$ [μs].

The setup is similar to the one presented on Figure 4.11 except that the translation stage is static and positioned to get $\Delta h = 0$ [mm] for a reservoir fully filled. The nozzle diameter is $D = 1$ [mm], in light of the results [55].

The electrical components used in [1] for driving the piezoelectric buzzer are a voltage source $V_s(0 - 50 [V])$, a Arduino Uno microcontroller and a bistable relay driven by the Arduino. However, O. Leblanc reported a minimum pulse duration achieved by the bistable relay of $1000 [\mu s]$ which often leads to multiple droplets generation, as shown in [55] that reported single droplet generation for a duration range $400 - 1000 [\mu s]$. Therefore, the bistable relay has been replaced by an H-bridge circuit (Figure 4.13).

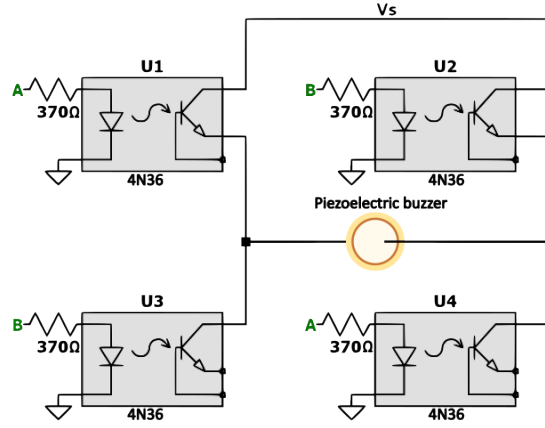


Figure 4.13: H-bridge circuit implemented for polarity inversion of the piezoelectric buzzer.

Two digital output pins of the Arduino Uno are connected to the H-bridge circuit, labeled **A** and **B**, where $A = \bar{B}$, which allows for the rapid reversal of the polarity of the applied voltage to the piezoelectric buzzer. The sent pulses width can be controlled within the range $3 - 16383 [\mu s]$ [59] to high precision as the delay is accomplished by an integer number of processor internal clock periods. This one is *ATmega328P*[60] that has a $16 [MHz]$ clock speed giving a time resolution of $0.0625 [\mu s]$. During a pulse step, the **A** and **B** pins respectively receive voltage values $0 - 5 - 0[V]$ and inversely $5 - 0 - 5[V]$.

Four optocouplers *4N36* [61] are used as NPN transistors in the H-bridge circuit. The optocoupler works as a NPN transistor except that it is a NPN phototransistor with a base current generated from the emission of light from a photodiode, the whole circuit in one package. In our case, they are preferred over classical NPN transistors because they create a safe connection between high voltage equipment and microcontrollers with a means of complete electrical insulation, preventing electrical surge on the microcontroller and the PC side that can cause damage. The *4N36* transistor has a typical turning-on time $t_{on} = 7.5 [\mu s]$ and turning-off time $t_{off} = 5.7 [\mu s]$, that could lead to a slightly different effective pulse duration $\Delta t \approx 1.4 [\mu s]$. The optocouplers are turned on with forward current $I_F = 10 [mA]$ and forward voltage $V_F = 1.3[V]$. A resistance is added to get a voltage drop

$\Delta V = 3.7 \text{ [V]}$ and reach the prescribed forward voltage:

$$R = \frac{\Delta V}{I_F} = \frac{3.7}{0.010} = 370 \text{ } [\Omega]$$

The droplet generation is handled either with the "*Arduino_Ctrl.vi*" program with a 'Generate' button or manually with a push-button circuit (Figure 4.14). The push button activation is detected with a pin transition state from LOW (0 [V]) to HIGH (5 [V]).

The polarity change has been measured for an amplitude 9[V] by the NI myDaq (Figure 4.15) due to AI limited to ± 10 [V]. The sample rate was set at its highest possible, being $200[kS/s]$, that gives a period resolution $5[\mu s]$. The pulse duration taken between 90% of 9[V] is measured $t = 755[\mu s]$; for 99% of 9[V], $t = 750[\mu s]$, that testifies the right polarity behaviour.

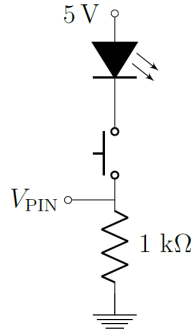


Figure 4.14: Push-button circuit.

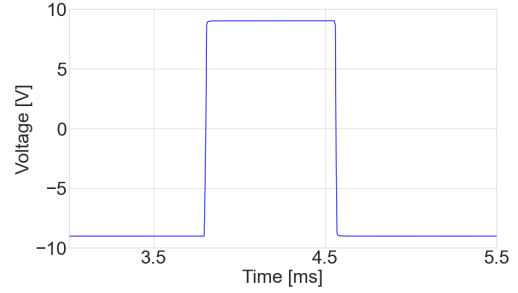


Figure 4.15: Polarity change measured for a 9[V] amplitude. Pulse duration for 90% of the amplitude is $755[\mu s]$ with time resolution $5[\mu s]$.

Droplet diameter characterization The camera [48] is placed horizontally to record the generated droplets at the nozzle output (real setup in Figure 4.16 and scheme in Figure 4.18). Two lights are placed at the sides and a dimension reference pattern is used to measure the droplet diameter in post-processing. The reference is placed just behind the nozzle output to consider the droplet in the same plane (Figure 4.17). The droplet distance h from the plane has been evaluated to deviate from a maximum $h_{max} = 3 \text{ [mm]}$ before reaching a container. The camera is placed at a distance $H = 25[cm]$. Therefore, taking the droplet diameter $\approx 1[mm]$, the distance between the camera and the droplet center is $24.65 < h_2 < 24.95[cm]$. As h_2 is not measured in practice, the following approximation is assumed for the worst case:

$$\frac{h_2}{H} = \frac{24.7}{25} = 0.988 \approx 1 \Rightarrow \frac{d}{D} = 1$$

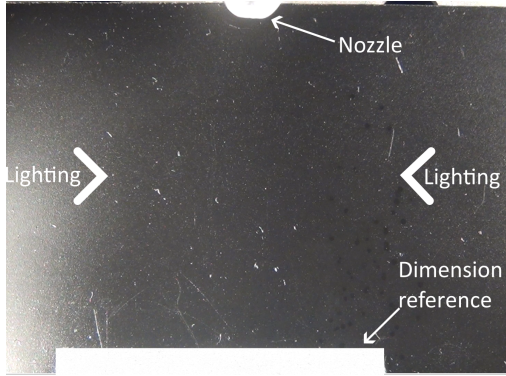


Figure 4.16: Droplet diameter recording setup from the camera point of view. The dimension reference is a white marker 3[cm] long.

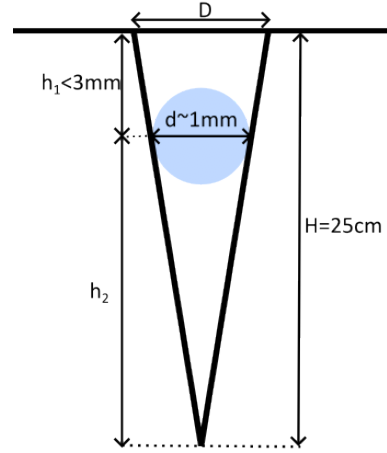


Figure 4.17: Schematic 2D top view of the camera recording the droplet, where D is the projected droplet diameter, d is the real diameter.

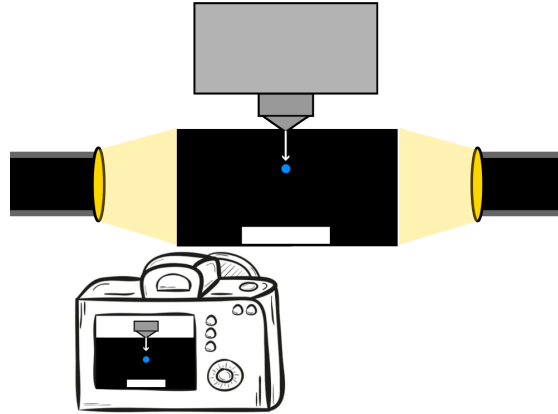


Figure 4.18: Droplet diameter recording setup scheme.

To evaluate the droplet diameter, the post-processing algorithm has been developed.

Algorithm 1

1. Calibration from first frame $I[0]$
 - ROI window selection for droplet detections
 - Dimension reference selection
 - Pixels to millimeters ratio computation
 2. Instance creation of a Gaussian Mixture Model (GMM) [62] with parameters $history = 200$; $varThreshold = 60$. Used for foreground/background segmentation.
 3. Reading of next frames $I[n]$ for $n = 1, \dots, N$ where $N = \text{total number of frames}$.
For each frame :
 - Frame cropping at ROI selection
 - Grayscale conversion
 - Gaussian blurring : kernel size $(x = 5, y = 5)$ and $\sigma_x = \sigma_y = 15$
 - Closing morphological operator : Ellipse kernel size $(x = 4, y = 7)$, 5 iterations
 - GMM application (Figure 4.19 Center)
 - Edge detection : XOR operation with eroded image (Ellipse kernel size $(x = 5, y = 5)$) (Figure 4.19 Right)
 - Contours detection [63] (Figure 4.19 Left)
 - Edge widths computation inside detected contours in pixels unit.
 - Edge widths filter : weighted mean and standard deviation of maximum histogram value + values at two bins on each side of the maximum value
 4. Global mean computation of found means and standard deviations, and conversion to millimeters unit.
-

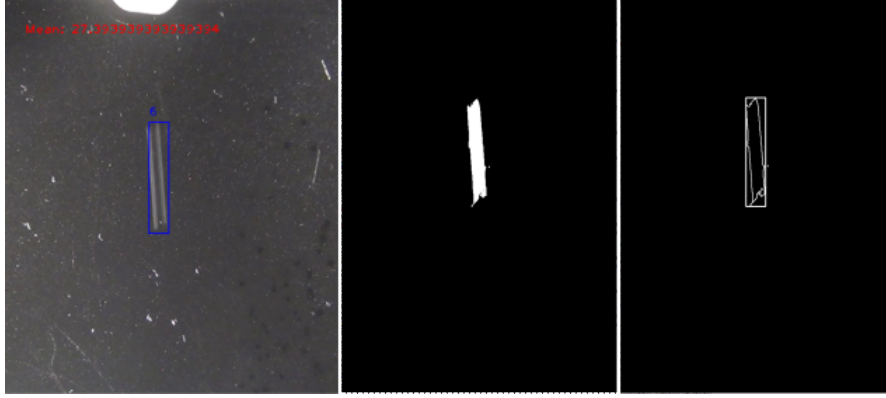


Figure 4.19: Algorithm 1 example. Left : frame with detected contour drawn as a blue rectangle and a display of the pixel width mean. Center: Image result after GMM application. Right: Image result after edge detection with detected contours displayed.

To select an appropriate range of parameters, a first constant $42 [V]$ was sent to select the duration range leading to one droplet generation, found at $750 - 1150[\mu s]$. Then, different amplitude pulses V_s have been tested for $750, 950$ and $1150[\mu s]$ to determine the one-droplet generation range of these durations. The droplet diameter is then characterized for these pulse durations and going from the lowest pulse amplitude boundary to the highest one in 1 volt increments. For each choice of parameters, 30 droplets have been generated and the reported means and standard deviation are on [Figure 4.20](#). Despite different ranges from [\[55\]](#), the results are relevant regarding the parameters, with an increasing diameter for higher pulse amplitude and pulse time. The droplets mean diameter range in $0.82 - 1.10[mm]$ and the final chosen parameters value are $V_s = 42 [V], t = 750 [\mu s]$ for a droplet diameter mean $D = 0.82[mm]$.

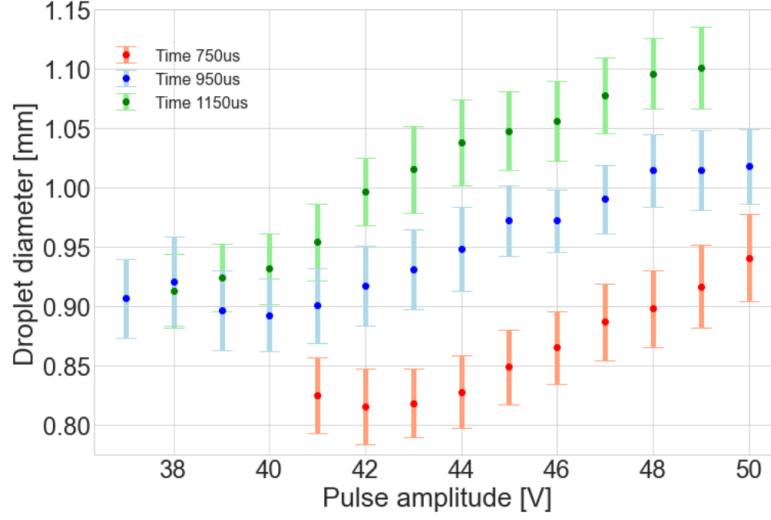


Figure 4.20: Measured droplet diameters for three different pulse durations. Dots correspond to mean diameters and intervals show the standard deviation. The boundary of the parameters (pulse amplitude and pulse duration) is determined between no-droplet generation behaviour and multiple droplet generation behaviour.

Due to an interaction between the droplet and the launchpad before reaching the bath, its diameter could be modified. To verify this, a droplet was generated with a $750[\mu s]$ pulse duration and a $42[V]$ pulse amplitude. Its horizontal velocity has been measured in the stadium-shaped bath (data acquisition is discussed in 4.2) without rotation at $\Gamma_m = 3.83 \pm 0.01$ (Figure 4.21) and compared to the diameter characterization realized in [3] (Figure 4.22). The velocity histogram informs a maximum occurrence for $V_W/V_F^\phi = 0.076$ where V_W is the walker horizontal velocity and $V_F^\phi = 190.8 [m/s]$ is the Faraday wave velocity. The values are reported on Figure 4.22 with red lines, the droplet diameter $D = 0.82[mm]$ fits with the reported observed values.

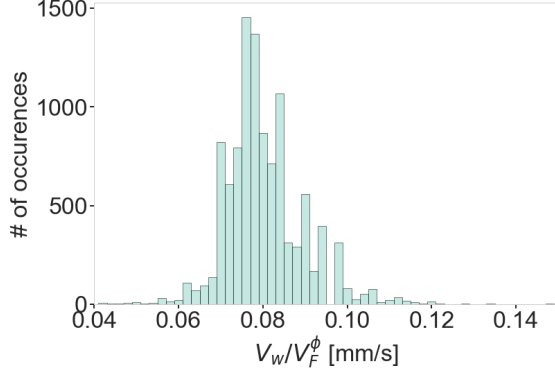


Figure 4.21: Droplet horizontal velocity histogram for $\Gamma_m = 3.83 \pm 0.01$ with bin size fixed at $\approx 1[mm]$ (after pixel to $[mm]$ conversion), and $V_F^\phi = 190.8[mm/s]$ the Faraday wave velocity and null angular velocity.

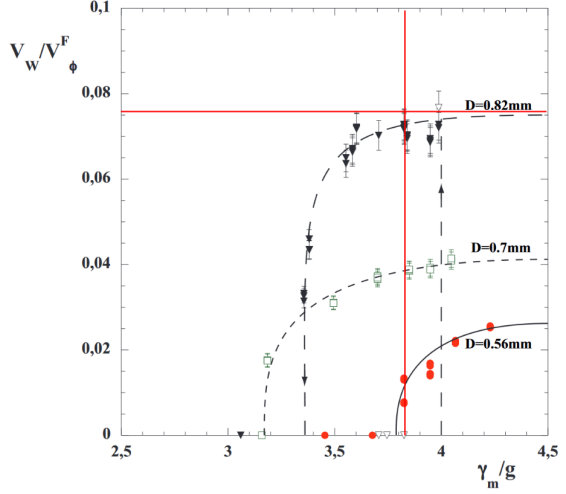


Figure 4.22: Horizontal velocity V_W/V_F^ϕ for three walker sizes depending on the vertical acceleration Γ_m . The red lines corresponds to the droplet measurements of Figure 4.21. Adapted from [3].

4.1.5 Bath rotation

The bath rotation is ensured with a stepper motor NEMA 23 [54]. It is a 8-wires stepper motor with a 1.8° step angle that has been configured in series to decrease power heat compared to a parallel configuration.

The "*Arduino_Ctrl.vi*" Labview program controls the Arduino Uno controller that drives the stepper motor with a stepper motor driver *DRV8885*[64] (Figure 4.23). The *DRV8885* is chosen for its 1/32 microstepping mode, reaching a 0.05625° step angle to produce smooth rotations. The current limit is set to a maximum $2.1[A]$ to prevent damage to the stepper motor. In practice, the current limit is reduced to optimise the stepper motor noise. The Labview program handles acceleration to the chosen target rotation speed and deceleration to null rotation speed.

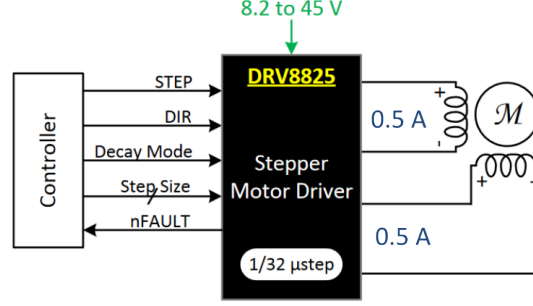


Figure 4.23: Stepper motor control diagram with typical current $0.5[A]$ and $1/32$ microstepping mode. Adapted from [64].

Validation Three different target rotation speeds were tested $\Omega = (0.01, 0.05, 0.1)[Hz]$ and the period manually measured was correct. Plus, the number of pulses the Arduino sends to the stepper driver motor has been verified to correspond to the rotation speed.

4.2 Data acquisition and analysis

The camera [48] is placed with a height $H = 40[cm]$ away from the surface height bath. The distance is chosen as a trade-off between the droplet position spatial resolution and the reduction of the dimensions distortion trying to reach Fraunhofer approximation

$$H \gg w_{bath}/2 \Leftrightarrow 40[cm] \gg 8.9[cm]$$

The spatial resolution is preferred over the dimensions distortion because only the droplet positions are evaluated and the bath boundaries are generated from the video recording to match the bath with droplet positions. In practice, the droplet positions and bath would be slightly horizontally stretched.

Regarding the speed computation from 4.1.4, the dimension distortions could have an impact on the evaluated walker speed as the pixel to millimeters ratio is not constant over the bath. However, the speed is evaluated as the distance travelled by the walker between five frames due to a spatial resolution of $\Delta x = 0.134[mm]$ too high to compute the distance between each frame. The mean distance traveled by the walker between five frames is $x = 1.49[mm]$ and so the Fraunhofer approximation for this distance is verified $40[cm] \gg 0.149[cm]$.

Tracking algorithm The droplet positions are tracked with a second algorithm explained below. The input lead of each bath is by default the left one. The algorithm handles the droplet positions tracking while keeping the bath view horizontal with respect to the leads.

Due to the symmetry of the baths, when the droplet reaches the right lead (output), this one is changed as input lead to reiterate the experiment from this new start. This process is applied after the tracking algorithm.

The algorithm starts with a bath calibration step that defines the rotation center of the bath and rotate it to get horizontal leads. The second step is the droplet analysis calibration step that takes measurements of the coordinates of the initial position of the droplet and the bath dimensions to recreate the bath boundaries in the probability distribution computation presented in [Chapter 5](#). Then the tracking algorithm starts with image processing steps and tracker update, with a possible re-orientation calibration every 50 frames due to imperfect angular velocity explained in the next validation step¹.

¹Video of the tracking algorithm on ring-shaped bath for an angular velocity $\Omega = 0.126[\text{rad/s}]$ is available at the link [Tracking algorithm](#)

Algorithm 2

1. Bath calibration from the first frame $I[0]$
 - Rotation center selection
 - ROI window selection containing the bath
 - Median blurring : kernel size 5
 - Canny edge detection [65]
 - Statistical Hough Transform [66]: lines detection
 - Bath orientation angle computation from the mean of detected lines of the previous step
 2. Droplet analysis calibration
 - Instance creation of a Channel and Spatial Reliability Tracker (CSRT) tracker [62]
 - ROI window selection from the left lead upper corner to the right lead lower corner. Compute the input/output leads coordinates and the bath rotation center.
 - ROI window selection of the walker and tracker initialization
 - Masks creations for bath isolation
 3. Reading of next frames $I[n]$ for $n = 1, \dots, N$ where $N =$ total number of frames.
For each frame :
 - Frame rotation of angle computed from rotation speed.
 - Grayscale conversion
 - Dilation morphological operator: : Ellipse kernel size $(x = 3, y = 3)$, 4 iterations
 - Tracker updateEvery 50 frame :
 - Adaptive threshold [67] on both leads for edges detection
 - Statistical Hough Transform [66]: lines detection on masked image isolating the bath leads.
 - Bath orientation angle correction within the $[-1, 1]$ degree range from the mean of detected line of the previous step.
-

Validation The following illustration images are for the ring-shaped bath and same results were obtained for the stadium-shaped bath.

The calibration step is good at detecting the bath outer boundaries lines, by fine-tuning the Hough transform parameters (Figure 4.24). Sometimes, some undesired lines could also be detected (for example for the electric cables situated

at the top right in the left Figure 4.24), modifying the correct mean angle line. In such a case, the mean was manually set with the correct lines.

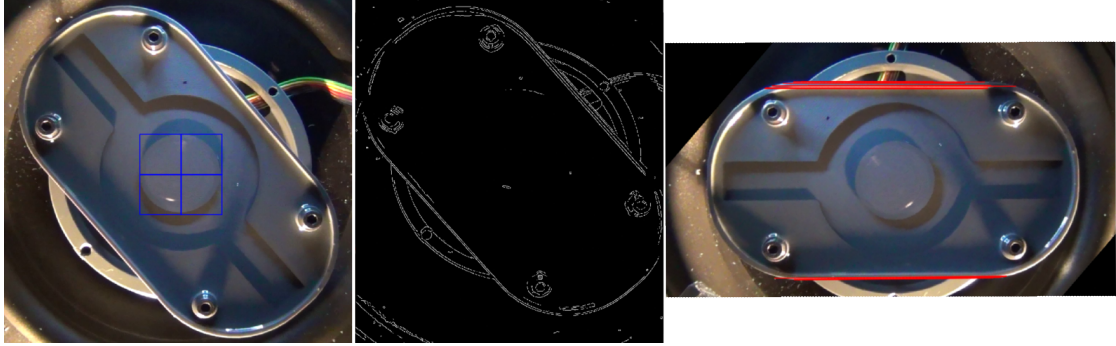


Figure 4.24: Calibration step. Left: rotation center selection. Center: Result of Canny edge detection. Right: Frame rotation by the mean angle of detected red lines from Statistical Hough Line transform.

The CSRT tracker is the most relevant tracker among the proposed OpenCV trackers (Python library used) for a precise walker tracking. Indeed, compared to other OpenCV trackers, it implements a Discriminative Correlation Filter with Channel and Spatial Reliability (DCF-CSR), that shows improved tracking of non-rectangular objects. While operating at a comparatively lower FPS (25 FPS), it gives the highest accuracy for object tracking among the models [68]. In practice, the walking droplet being sufficiently isolated due to a darker background, a good horizontal illumination and some image processing (Figure 4.25) no tracking failure were detected.

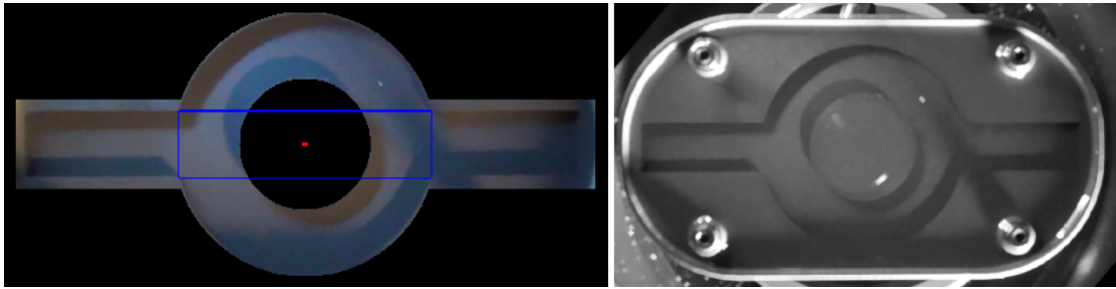


Figure 4.25: Image processing for tracking algorithm. Left: Mask application over initial grayscale frame, rotation center as red point and input/output lead joined with a blue rectangle (used to recover the bath boundaries). Right: Grayscale frame provided to the tracker after dilation operator.

The frame rotation is implemented to keep bath images static while the bath is rotating, as if the camera was rotating with the bath. While the frame rotation works for a precise input rotation speed, it turned out that during experiments, the real rotation speed could sometimes be very slightly lower than expected. After

investigation, the reason was a possible too low adjusted current limit of the stepper motor (aimed at reducing the motor noise) and some stepper motor steps could be missed due to a too low current level.

In the validation step of 4.1.5, the period measurements should have been more precisely realized. Due to our lack of previous experience with the stepper motor, the period was considered correct due to the measured pulses sent by the Arduino.

Finally, the real rotation speed was evaluated at

$$\Omega_{real} \in [\Omega_{input} - 2\%, \Omega_{input}]$$

In the algorithm, the problem is skirted by adding a small rotation angle correction every 50 frames by detecting the bath orientation (Figure 4.26) with Hough Transforms detecting the lead edges.

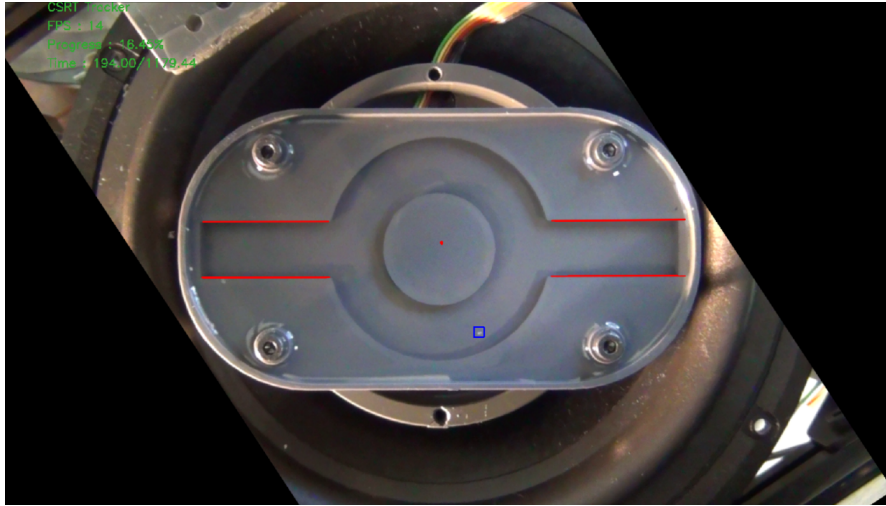


Figure 4.26: Droplet tracking during the bath angle correction. The droplet tracker is the blue rectangle surrounding the droplet. The four red lines are the ones detected by the Statistical Hough Transform to possibly correct the bath orientation. At the top left, information in green are provided in the order: the used tracker model, the FPS the algorithm is processing, the total progress in % and in progress time of the video.

Chapter 5

Walkers behaviour and comparison

This chapter presents the experimental results of both baths, including transmission properties and probability distributions compared to simulations and experimental data of electron behaviour in mesoscopic cavities, as well as simulations of classical particles ('balls') at constant speed with specular reflection in a billiard. As explained in the [Chapter 4](#), the dimensionless acceleration ratio is $\mathcal{G} \in [0.87, 0.93]$ which determines the corresponding memory $Me \in [6.06, 11.47]$ for the walking droplets (as a reminder, this parameter quantifies the spatial extension of the wavefield associated with the droplet). This parameter was chosen constant over the experiments that were rather towards an investigation of the angular velocity effect, analog to the magnetic field. Indeed, the study of the memory effect was not suitable with this setup due to the lack of acceleration setpoint controller (possible setup improvements presented in [4.1.3](#)). The Faraday instability threshold is measured at $\gamma_F = 4.03[g]$, and the angular velocity varies in the range $[0 - 5.655] [rad/s]$ that is the value above which transmission barely varies around 1 due to walkers following the edges, explained hereafter. For this angular velocity range, the circular trajectory of the walking droplet with free velocity $V_W \approx 15.5[m/s]$ (mean velocity measured during experiments with bath in rotation, slightly higher than the result previously found [4.1.4](#)) has a radius $r_W \in [\infty, 21.24][mm]$ due to Coriolis force.

5.1 Data collection and relevant parameters

Each experiment used a single walker that was recorded between 13 and 20 minutes, giving a range of about 30-80 samples of transmitted or reflected walkers between the leads ([Figure 5.1](#)). Each time a walker reaches a lead, input or output, the image analysis algorithm considers the latter lead as the new input lead for the next walker trajectory. This is made possible considering the baths symmetry with respect to input-output leads. Plus, this allows to keep the same walker for the experiment as it is rotating and decrease the dimensional constraints that a return path connecting the output to the input would bring to the current setup.

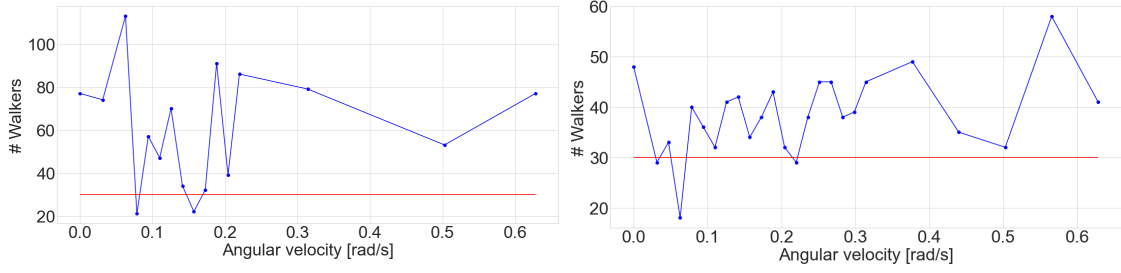


Figure 5.1: Number of walker samples for each experiment, transmitted or reflected depending on the angular velocity $\Omega[\text{rad/s}]$. The red line is the 30 samples arbitrary limit. Left: Ring-shaped bath. Right: Stadium-shaped bath.

The walker velocity V_W can be divided into a cross-radial and a radial component with respect to the bath rotation center. A co-rotating walker has a cross-radial component in the same orientation as the linear velocity of the bath, and in the opposite orientation for a counter-rotating walker. In order to prove the significance of the statistical results given the sample size, a 90% confidence interval is computed for the transmission probability, described hereafter. For some experiments, it could happen that a counter-rotating walker was 'trapped' in the cavity, decreasing the number of samples of the experiment. Analyzing the mean convergence, it was found that a 30 samples of walkers, either transmitted or reflected, was reasonable and giving consistent results. For the experiments with lower number of samples, their mean convergence (Figure 5.2) aims to still prove the significance of the corresponding results.

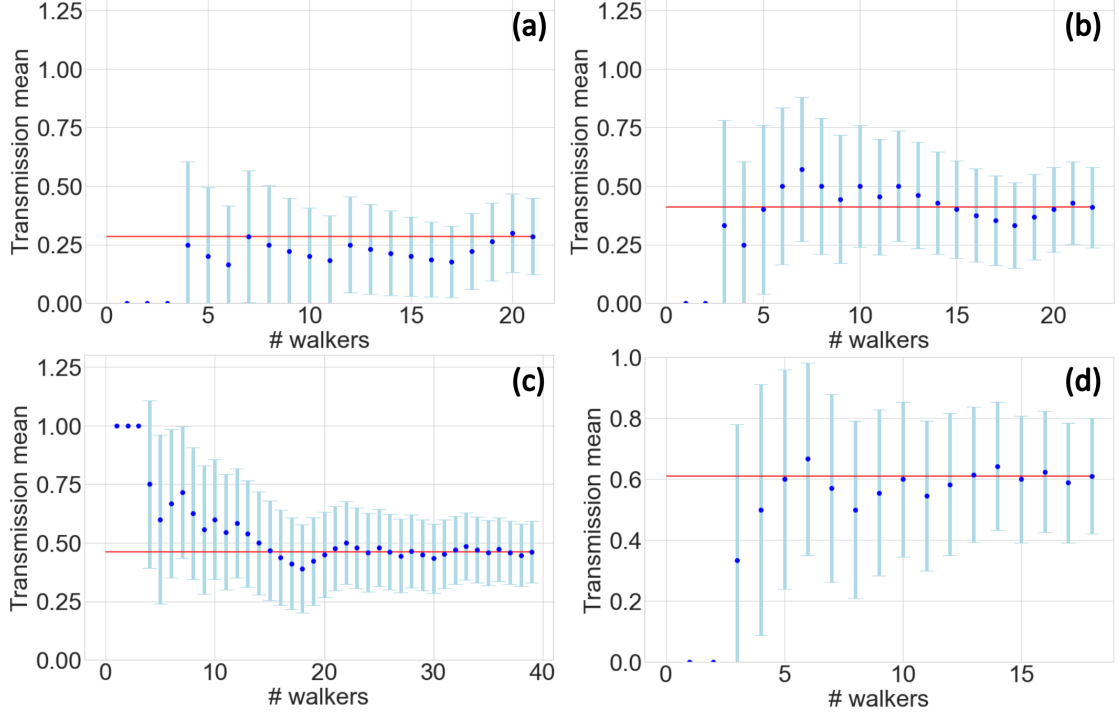


Figure 5.2: Evolution of the transmission probability mean with 90% confidence interval with respect to the number of walker samples ($\#$ walkers). **(a-c)** ring-shaped bath respectively for $\Omega = (0.079, 0.157, 0.204)[rad/s]$, **(c)** has a number of samples > 30 for comparison with **(a-b)** convergence. **(d)** stadium-shaped bath for $\Omega = 0.062[rad/s]$

We now state w_t and w_r respectively the number of transmitted and reflected walkers, and τ_t and τ_r the total time respectively taken by the w_t and w_r walkers to reach output and input lead. A transmitted walker goes from input to output lead, and a reflected walker from input to input lead. Let's now describe the different features that will be analyzed in analogy with the behaviour of electrons :

- **Transmission** $[-]$: the walker transmission probability between the input and output lead, computed as

$$T = \frac{w_t}{w_t + w_r}$$

- **Transmission current *flow*** (TCF) $[\#/s]$: the walker current between input and output lead, computed as

$$I_t = \frac{w_t}{\tau_t},$$

- **Transmission current** (TC) $[\#/s]$: the 'total' walker current computed as

$$I = \frac{w_t}{\tau_t + \tau_r}$$

5.1.1 Ring-shaped bath

The transmission (Figure 5.3) is globally increasing until reaching ~ 1 , with some fluctuations at low angular velocities. As angular velocity increases, the transmission starts at very low values $T \sim 0.12$ for $\Omega = 0[\text{rad/s}]$, to then have a fluctuating behaviour for $\Omega \in [0, 0.2][\text{rad/s}]$ and eventually tends towards 1 for $\Omega \in [0.2, -][\text{rad/s}]$. The radius of the circular walker trajectory is computed as

$$r_W = a \frac{V_W}{2\Omega};$$

where $a = 1.5$ is a fitting parameter [42]. In the walker experiments, due to the fluctuating vertical accelerations, the walker horizontal velocity could also fluctuate. The mean velocity was recorded at $V_W = 15.5[\text{mm/s}]$, slightly higher than the one found in 4.1.4. Still, this velocity was also obtained in [3] for droplet diameter $D = 0.82[\text{mm}]$ in the acceleration range of this experiment. One can already notice that the transmission tends towards 1 before the walker circular trajectory reaches a radius $r_W \approx W$ the path width (Figure 5.3 red dashed lines).

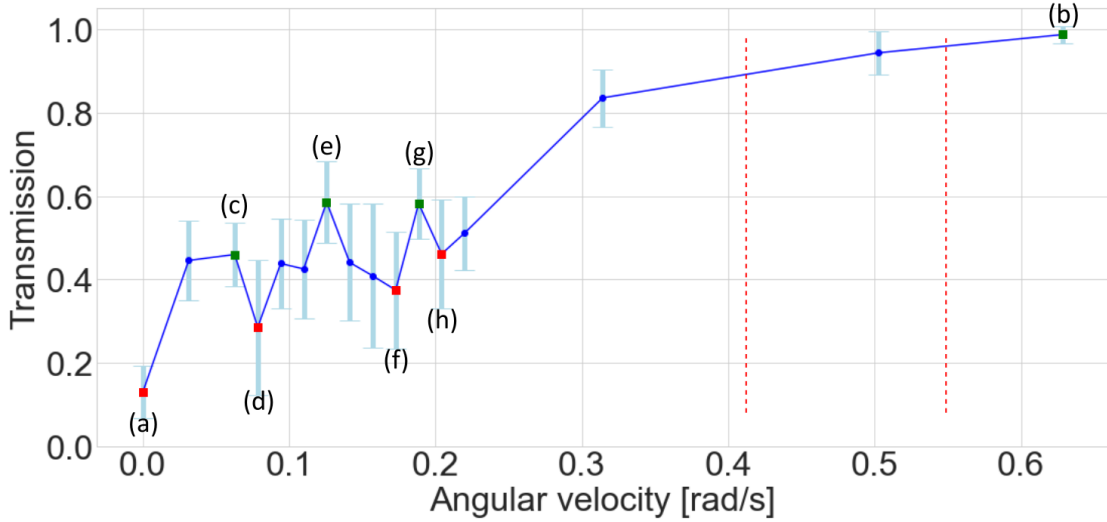


Figure 5.3: Ring-shaped bath transmission varying with the angular velocity $\Omega[\text{rad/s}]$. Dots are measured transmission and the 90%-confidence interval is in light blue. Green and red square markers respectively correspond to local maxima and minima analyzed in the probability distributions (Figure 5.4) with the corresponding letter. Red dashed lines corresponds to the radius $r_W = (2W_{cav,r}, 1.5W_{cav,r})$ of the walker circular trajectory, where $W_{cav,r}$, is the ring width. The radius is computed for a mean free walking velocity $V_W = 15.5[\text{mm/s}]$ and a fitting parameter $a = 1.5$ [42].

To analyze the transmission behaviour, the probability distribution of the walker's position (Figure 5.4) is computed for the local extrema as well as for the null and highest angular velocities (green and red markers). The probability

distribution is obtained from a two-dimensional histogram of the walker's position tracked by the algorithm 2. The retrieved positions of the tracking algorithm are expressed in pixel coordinates. With the calibration step, the dimensions of the bath are measured in pixels and knowing the corresponding dimensions in millimetres, the pixel coordinates can be converted in millimetres. The histogram is computed with an isometric pixel bin size $b_p = 8$ ($b_{p,x} = b_{p,y}$ with x and y the axis). The value is chosen such that the bin size converted in millimeters $b_{mm} \approx D$ the droplet diameter ($b_{mm} = 8\Delta x = 1.04[mm]$ where $\Delta x \approx 0.13[mm]$ is the spatial resolution (4.2) corresponding to 1 pixel width).

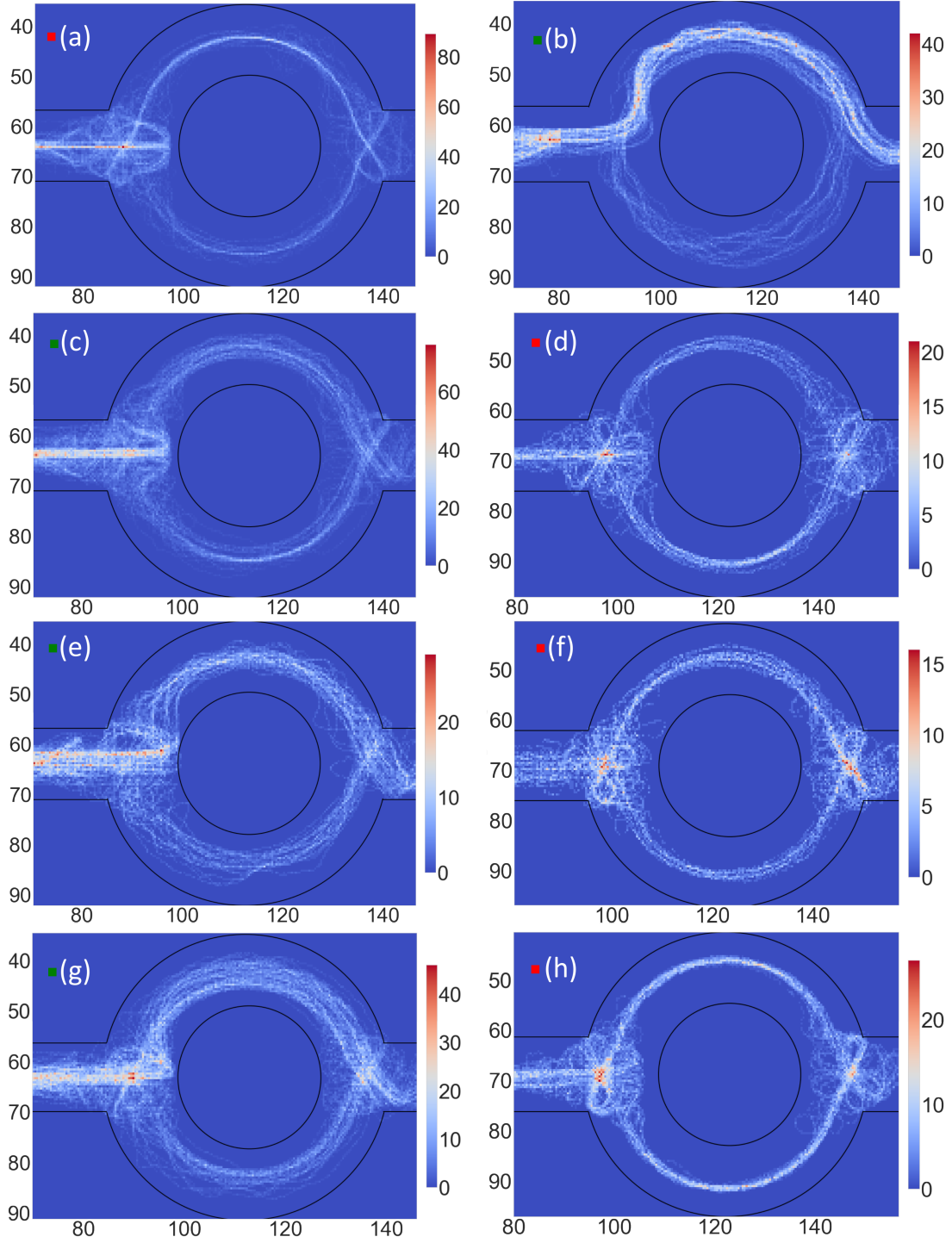


Figure 5.4: Empirical probability distribution of the walking droplet's position, with input/output lead respectively on the left/right side. **(a-b)** corresponds to minimum and maximum angular velocity $\Omega = (0, 0.6283)[rad/s]$. **(c-h)** alternately corresponds to transmission local maxima in left column and minima in right column, $\Omega = (0.0628, 0.0785, 0.1257, 0.1728, 0.1885, 0.2042)[rad/s]$.

At null angular velocity (Figure 5.4(a)), the walker penetrates into the cavity with a trajectory almost perfectly perpendicular to the inner circle and its reflection pushes it back very often in the input lead. At high angular velocity (Figure 5.4(b)), the walker has no reflection with the inner circle and follows the upper path. This is explained by the Coriolis force, analog to the Lorentz force, that linearly increases with the angular velocity and forces the walkers to move along the edges more and more. The Coriolis force is dominant over the T-junction interactions starting around $\Omega \approx 0.25[\text{rad/s}]$, stated as the high angular velocity regime (illustrated in Figure 5.4(b)). The regime of low angular velocity is consequently $\Omega < 0.25[\text{rad/s}]$.

In the regime of low angular velocities ($\Omega < 0.25[\text{rad/s}]$), the transmission reaches some local maxima and minima (Figure 5.3(c-f)). The two main factors that are thought to be considered are the walker interaction with its own wavefield on the ring path where it did not pass through, and the bath boundaries influence at the "T-junction".

The first factor was considered thinking Aharonov-Bohm effect could arise (explained in 3.2.1) due to interactions between the emitted waves in the upper and the lower path that would have a change of relative phase. This would possibly lead to constructive/destructive interference that could respectively increase or decrease the walker velocity and attract or repel it at the entry of the output lead. However, this mechanism can be set aside for the following reason. The fluid is enclosed between two rotating cylinders (assuming the outer cylinder perfect, without considering the leads), a configuration leading to Taylor–Couette flow. In such a configuration, the azimuthal velocity component of the fluid is computed as [69]:

$$v_\theta = Ar + \frac{B}{r},$$

where

$$A = \Omega_{in} \frac{\mu - \eta^2}{1 - \eta^2} \quad ; \quad B = \Omega_{in} R_{in}^2 \frac{1 - \mu}{1 - \eta^2}$$

with

$$\mu = \frac{\Omega_{out}}{\Omega_{in}} \quad ; \quad \eta = \frac{R_{in}}{R_{out}}$$

for Ω_{in} , Ω_{out} , R_{in} and R_{out} respectively being the angular velocity of the inner cylinder, angular velocity of the outer cylinder, inner radius, outer radius. In this bath, the flow simply becomes a Couette flow as $\Omega_{in} = \Omega_{out}$ (single rigid piece), that leads to $B = 0$ and $v_\theta = Ar$, i.e. a rotational flow.

Comparing to the Aharonov-Bohm effect observations that were realized in [40, 41], the authors inform that the flow needs to be irrotationnal to observe a such effect, characterized by azimuthal velocity $v_\theta = \frac{\Gamma}{2\pi r}$ where Γ is the fluid circulation. Therefore, the Aharonov-Bohm effect is not thought to play a role in this setup.

The second factor seems way more important considering the probability distributions. For the local maxima (Figure 5.4(c,e,g)), the walker encounters less interactions with the T-junction's walls, both to enter the ring and to leave through the output lead, improving the walker flow. In the local minima cases (Figure 5.4(d,f,h)), the walker undergoes more reflections at the T-junction where the distribution shows walker interactions with the inner circle and the lead corners. This seems to show that certain angular velocity are prone to lead to some resonant reflections between these boundaries. A total reflection of the walker is either realized with a reflection from the inner circle at the start or with a reflection at the T-junction output that sends the walker back to the entry.

To analyze the role of the reflections with boundaries once the walker has been transmitted through the first T-junction, the TCF and the TC (Figure 5.5) can give an insight about the time spent by the walker inside the cavity. Both follow the global transmission behaviour except for some points where the currents have a larger behaviour difference between them ($\Omega = (0.0314, 0.0785)[rad/s]$). For these ones, the dwell time of reflected walkers were significant, meaning that they could be 'trapped' a certain time in the cavity before coming back to the entry. Except for the outliers, the transmission local maxima and minima faithfully corresponds to smaller and larger dwell times for the walkers, as indicated by the qualitative picture given by the T-junction probability distribution (Figure 5.4).

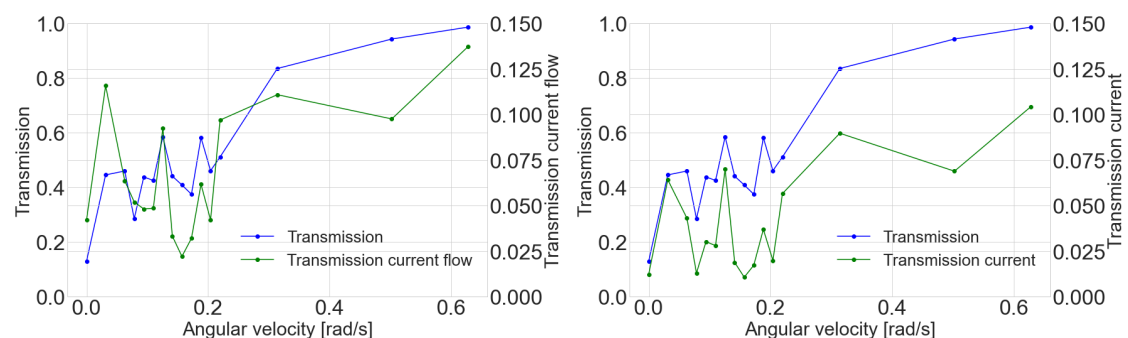


Figure 5.5: Walker current varying with angular velocity. Left: TCF compared to transmission. Right: TC compared to transmission.

Now that the transmission behaviour has been discussed, we can take a look at some walkers trajectory behaviours. First, the walker trajectory inside the ring paths is well analog the trajectory in a wave guide reported in [50], where the walker follows the path with a small degree of freedom in the radial direction. The distribution width variation inside the ring path for the different angular velocities is however thought to be due to an imperfect setup. The two possible reasons are a rotation center not perfectly set during the tracking algorithm 2 that slightly distorts the positions with respect to the measured bath dimensions in the calibration step, and the noise coming from the stepper motor that makes the bath vibrate slightly.

Then, at the T-junction, one can notice the walker could go beyond the lead corners boundaries. Whereas the previous imperfection might suggest that it is due to a distortion of the walker's position, this behaviour was 'really' observed (by eye). It is similar to tunneling effect already observed for walkers [70–72], where a walker can travel across a submerged barrier. The difference here is that the barrier has a limited width of 7[mm] followed by a wall, forcing the droplet to return in the bath

Aharonov-Bohm effect perspectives Although the Aharonov-Bohm effect is not thought to lead to the observed transmission fluctuations in the bath considered here, some suggestions are proposed to try to highlight this effect. As a reminder, Aharonov-Bohm effect would be observable if the emitted waves travel through two paths that would change their relative phase. Such results can be reached with a free vortex, also called “bath-tub” vortex, due to the advection of the emitted waves by the fluid velocity. The waves going in the direction of the vortex will have a faster phase velocity while the waves in the opposite direction will have a lower one. At first sight, walker experiments are not well suited for being realized in a free vortex for practical reasons: the bath should be constituted of a hole at its bottom where the oil can flows out while keeping the oil height surface constant. Plus, considering the surface oil needs to be horizontal, a practical realization of this setup can not easily be implemented.

Another suggestion is presented here consisting in a Taylor-Couette flow in a different configuration than the setup that is analog to an irrotational flow (investigated in [73]). The irrotational flow being characterized by a tangential fluid velocity $v_\theta \propto 1/r$, the Taylor-Couette flow can have similar behaviour if the tangential fluid velocity $v_\theta = Ar + \frac{B}{r}$ is such that $B \gg A$ (Figure 5.6).

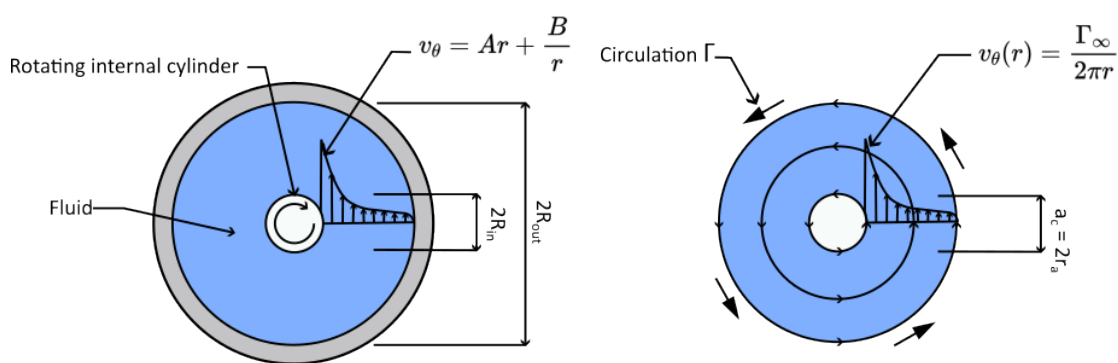


Figure 5.6: Analogy between Taylor-Couette flow (left) and Free-surface vortex (right), adapted from [73].

Setting the outer radius as twice the inner radius (current dimensions), it is possible to maximize the ratio $|B|/|A|$ with the ratio $\Omega_{out}/\Omega_{in} = (R_{in}/R_{out})^2 = 0.25$

(Figure 5.7). However, it would require the outer cylinder to rotate, which is not practical for a ring-shaped bath with leads. A setup that could be more easily achievable would be a rotating inner cylinder while the outer one is static. For such a case, the condition is thought to be still acceptable as $\Omega_{out}/\Omega_{in} = 0$ results in $|B| = 802.02|A|$ (red line on Figure 5.7).

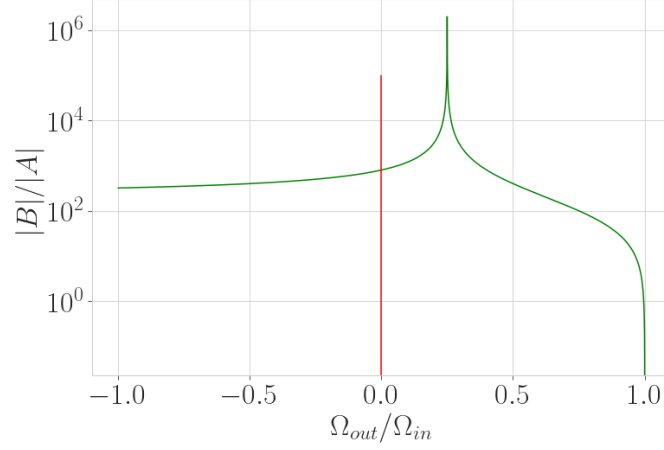


Figure 5.7: Tangential velocity component ratio $|B|/|A|$ depending on the angular velocity ratio between the outer and inner cylinder Ω_{out}/Ω_{in} . Vertical line is for $\Omega_{out}/\Omega_{in} = 0$, leading to $|B| = 802.02|A|$.

Regarding the walking droplet parameters, the system should be set at memory sufficiently high such that waves emitted from the walker can interfere coming from the two different paths (Figure 5.8). That way, the waves can interfere. Such a case could possibly lead to walker change of dynamics. Plus, a recent walkers experiment [74] reported an interferometer behaviour of the walkers (Figure 5.9) with a similar geometry (but without rotating inner cylinder) that reinforce this setup suggestion.

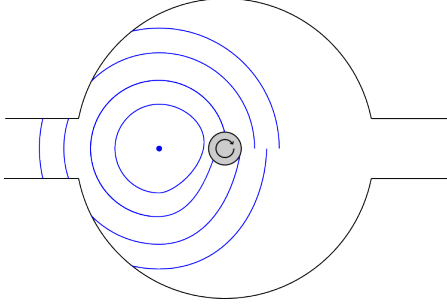


Figure 5.8: Schematic of wavefront emitted by a walker coming from the left with destructive interference for a bath with inner rotating cylinder. The displacement parameter represented is $\alpha = 1.5$.

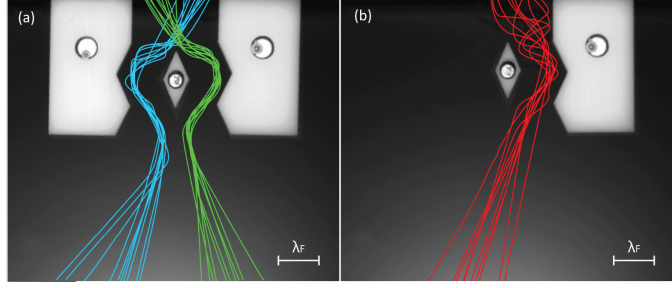


Figure 5.9: **(a)** Symmetric setup, where a droplet shows trajectories specific to an interferometer behaviour. **(b)** When one of the barriers is removed, the symmetry of the system is broken and no such trajectory is found. The system is adapted from the interferometer setup imagined in [75].

Then, the dimensions of the inner cylinder, namely its radius and its height in the bath, could be modified. The optimal inner cylinder would have a minimal radius to decrease the required wavefield spreadness and so the high memory constraint. Regarding the height, it is a trade-off between a good Taylor-Couette flow (high height) and a decrease of the waves damping (low height) to reach a similar case of [40] and allows better interaction between waves of different phase. The transmission of the walker and/or its emitted waves through damping walls gained interest over the last years due to interesting interaction behaviour [47, 76, 77]. However, a high height is considered preferable since the irrotational flow is crucial to observe Aharonov-Bohm effect.

The analogy between Taylor-Couette flow and free-surface vortex should be further investigated in the framework of fluid mechanics and is beyond the scope of this master thesis.

5.1.2 Stadium-shaped bath

The transmission (Figure 5.10) is also globally increasing until reaching ~ 1 , with some fluctuations both at low and high angular velocities. As opposed to the ring-shaped bath, the transmission at null angular velocity is higher, as we could expect. Indeed, the walker does not have here a first obstacle that can lead to perpendicular reflection such as the inner circle of the ring-shaped bath.

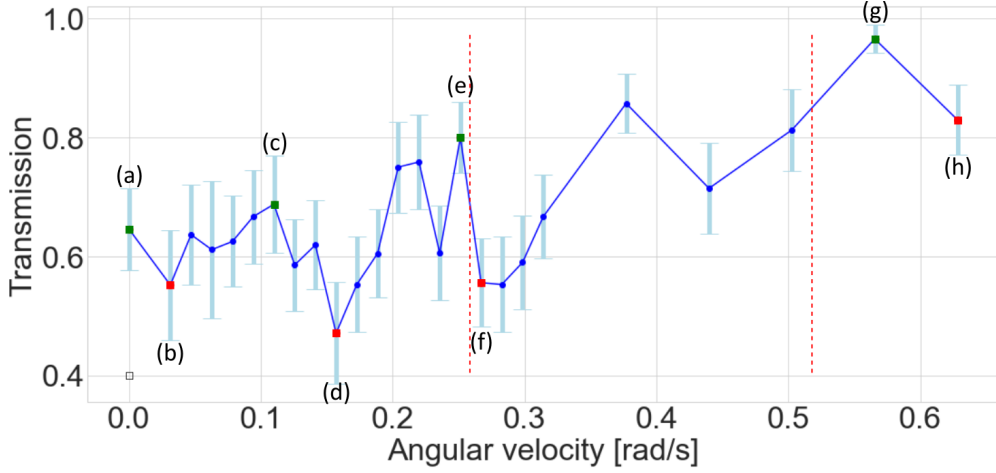


Figure 5.10: Stadium-shaped bath transmission varying with the angular velocity $\Omega[\text{rad/s}]$. Dots are measured transmission and the 90%-confidence interval is in light blue. Green and red square markers respectively correspond to local maxima and minima analyzed in the probability distributions (Figure 5.11) with the corresponding letter. Red dashed lines corresponds to the radius $r_W = (W_{cav,s}, W_{cav,s}/2)$ of the walker circular trajectory where W is the stadium width.

The transmission behaviour can again be investigated with the probability distribution of the walker's position (Figure 5.11). The walkers enter the bath with an initial trajectory angle $\in \pm 40^\circ$ with the lead orientation. The walkers trajectory is guided by the Coriolis force, with circular trajectories more and more visible as the angular velocity Ω increases. The Coriolis force seems to be dominant over the initial walker trajectory initial trajectory angle starting around $\Omega = 0.25[\text{rad/s}]$ and the initial range of angle of 80° decreases as well with the Coriolis force. As the angular velocity increases, the radius of the walker orbital decreases leading to one, two or three reflections on the upper boundary before reaching or not the output lead, that varies the transmission. The local maxima and minima depend on the orbital radius that either sends the walker in the output lead direction or slightly next to it which increases the chance of being sent back to the input.

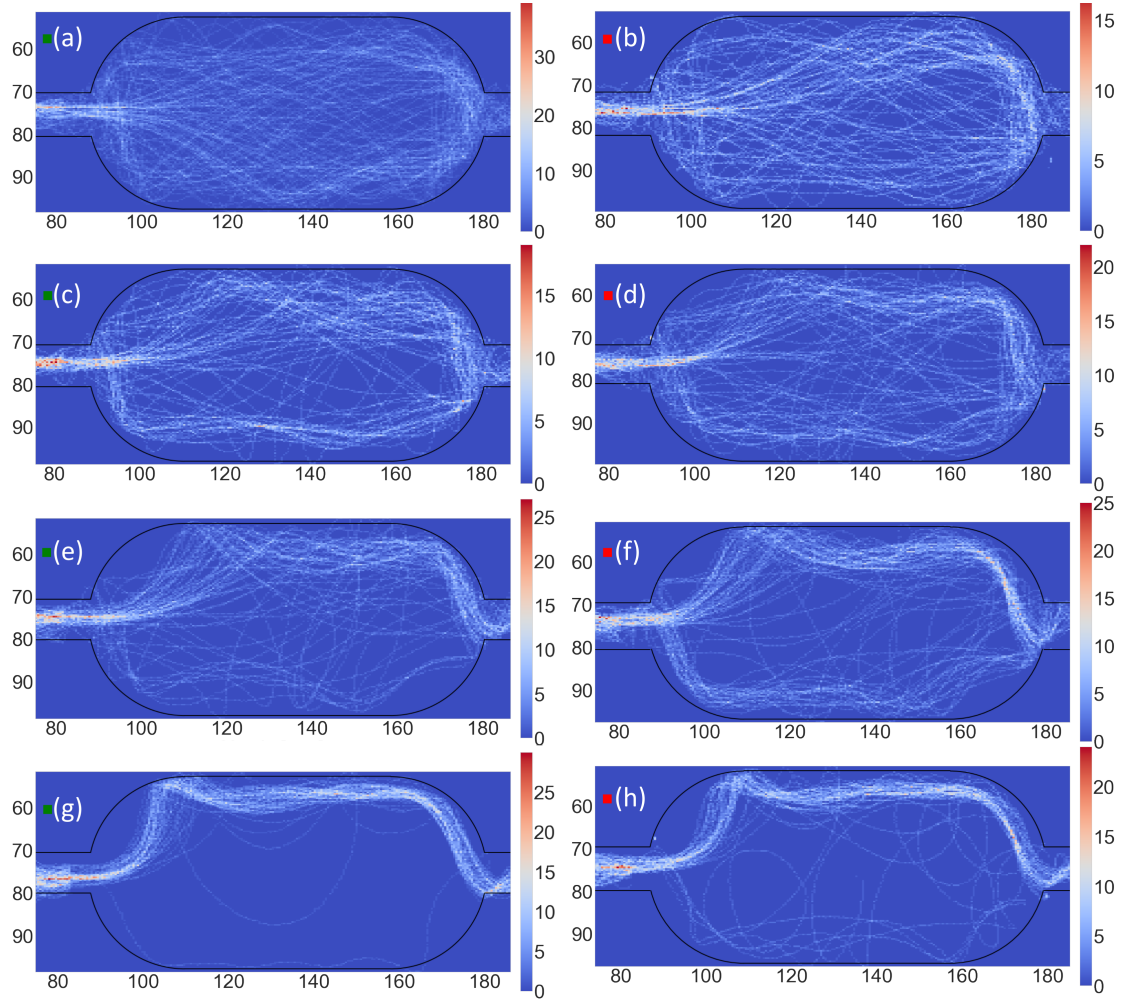


Figure 5.11: Empirical probability distribution of the walking droplet's position in the stadium-shaped bath, with input/output lead respectively on the left/right side. **(a-h)** alternately corresponds to transmission local maxima in the left column and minima in the right column $\Omega = (0, 0.0314, 0.1100, 0.1571, 0.2513, 0.2670, 0.5655, 0.6283)[rad/s]$.

At null angular velocity, the result diverges from the one found by O. Leblanc [1] that is here more comparable to the classical chaotic case, showing no preferential pattern. The main parameters that could explain this result are the higher cavity dimensions, the addition of the leads, the memory and the height $h_1 < 1[mm]$ of the oil fill of the edges that have impact on the wave damping and thus the walker reflections. The memory can be raised for null angular velocity (4.1.3). With $h_1 \approx 1[mm]$, it was tested by gradually increasing the vertical acceleration until reaching the Faraday threshold but no similar pattern have shown up. Then different height h_1 of the filling edges surface have been tested at constant memory of the setup ($Me \approx 7$) from 1 to 0[mm].

Similar ∞ -pattern (Figure 5.12) could be obtained for $h_1 = 0[mm]$ (no damping) but not as robust as reported in [1] that was obtained with wave damping $h_1 < 1[mm]$. Thus, the walker reflections are influenced by the wave damping possibly leading to a higher preferential pattern probability. The angle reflection of walkers on filling edges were investigated in [49] for a surface height $h_1 = 0.42 \pm 0.03[mm]$ that showed a non-specular reflection behaviour. Particularly, the authors of [49] indicate that the reflection angle range is significantly lower than the incidence angle range, and thus 'stabilize' the reflections.

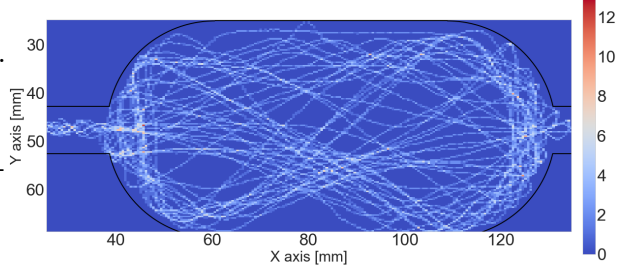


Figure 5.12: Empirical probability distribution of the walking droplet's position in the stadium-shaped bath, $\Omega = 0[rad/s]$, $Me \approx 7$, comparable to [1].

It seems the cavity dimensions in [1] were suitable to present "scarring effects" where walkers have stable trajectory due to a stable reflection angle sending it right at the next expected quarter circle boundary. Still, the ∞ -pattern found for our dimensions indicate a change on the non-specular reflections behaviour. Research about the surface height effect of the edges on the reflection angle is missing and could lead to interesting results as the one reported here. The robustness of ∞ -pattern is lower than the one reported [1] and the reason is thought to be due to walker change of dynamics seeing the influence of the lead at the four quarter circle of the stadium and the missing characterization of the walkers reflection when the silicone oil does not fill the area away from the main cavity.

Again, we can look at the TCF (Figure 5.13 Left) and the TC (Figure 5.13 Right). While the TC follows well the transmission behaviour, the TCF shows more distinctions with maxima and minima sometimes at different positions than the transmission ones.

The different TCF optima are explained by the walker orbital radius. For high transmission and low TCF, it means the transmission was sometimes realized after having missed the first direct trajectory (only reflections on the upper boundary and directly reaching the output lead) and following orbitals that sends it back to the output after one or more rotations in the clockwise direction (well illustrated in Figure 5.11(c)). On the other hand, a low transmission and a high TCF means that the transmission was not very likely but when the walker was transmitted, it was quick and thus probably directly from input to output with some possible reflections on the upper/lower boundaries.

Regarding the similarity of shape between the TC and the transmission, one can suggest that the dwell time of reflected walkers seems to rebalance the TCF to the transmission shape behaviour.

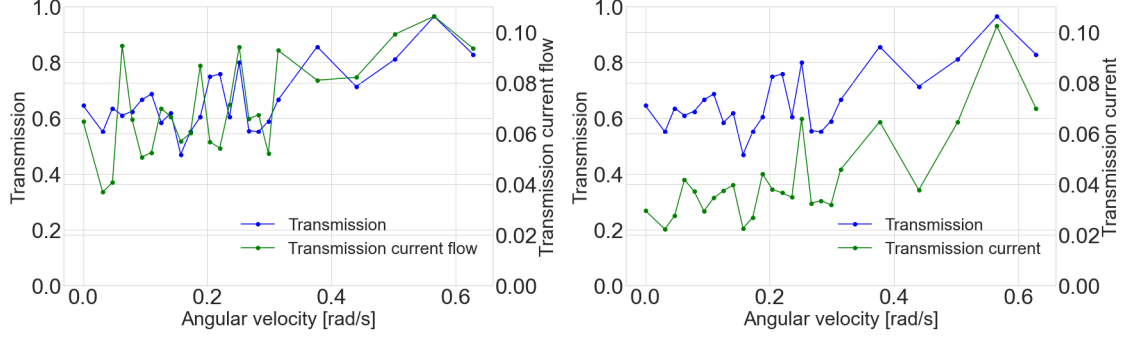


Figure 5.13: Walker current varying with angular velocity. Left: TCF. Right: TC.

5.2 Quantum simulations and experimental data

Quantum simulations are realized with the *Kwant* package [16] using tight-binding approximation with ballistic system. The simulations are focused on the conductance variations with the magnetic field and the local density of states (LDOS) and local current at certain magnetic field value comparable to the walkers experiment results. As no scarring patterns were observed (except at null angular velocity in the stadium with a ∞ -distribution), the LDOS and local current simulations were produced at high magnetic field, to compare the circular trajectory of walkers and electrons. Further information about simulation parameters are provided in [Appendix B](#).

The simulations are computed on two graphene cavities, one with a dimensional constraint (explained in 5.2.1 and 5.2.2) and another one which is a mesoscopic graphene cavities. Indeed, mesoscopic cavities have dimensions suitable to explain electron dynamics both classically and quantum mechanically, i.e. following 'semi-classical' behaviour¹, a case that could be analog to the walkers behaviour dynamics.

Classically, part of electrons dynamics in ballistic mesoscopic devices can be explained by billiard models, where electrons travel along straight line over the table and collide elastically with the walls defining the cavity until they disappear into one of the leads. Ballistic mesoscopic devices are characterized by an electron mean free path² longer than the cavity dimensions, feasible by reducing the density

¹A semiclassical system is described on one part quantum mechanically and on the other classically.

²The electron mean free path is the average distance over which the electron travels before being scattered

of impurities. Diffusive behaviour, as opposed to ballistic, is described with a mean free path shorter than the cavity dimensions leading to random electrons trajectories due to scattering. In ballistic systems, fluctuations are observed in the device electrical conductance, originating from device boundaries reflection [78].

On the other hand, quantum mechanics can capture the wave-like electron behaviour and describe carriers interference when the phase coherence length³ is longer than the cavity dimensions.

Mesoscopic cavities are at the crossing point between both worlds, which is convenient to develop scarring effect (a.k.a. "scarred wavefunctions" or "scars"), where some semiclassical orbit are privileged, even in the case of chaotic geometries. Indeed, in the framework of classical behaviours, scars are not expected for chaotic geometries and a uniform distribution of electrons trajectories should be observed. On the contrary, quantum mechanics predict scarring phenomena, even in quantum analogs of classically chaotic systems.

Semiclassical analysis shows that scarring patterns can be periodic with the Fermi energy of the system. On graphene, the bandstructure can be assumed linear for low energy

$$E(\mathbf{k}) = \pm \hbar v_{Fe} |\Delta \mathbf{k}|$$

where v_{Fe} is the Fermi velocity and $|\Delta \mathbf{k}|$ is the magnitude of the wave vector from the Dirac points. The resulting period for a given length of the periodic orbit L_{orb} is $\Delta E = \hbar v_{Fe} / L_{orb}$ [79].

5.2.1 Mesoscopic ring cavities

Mesoscopic ring cavities were firstly known to exhibit Aharonov-Bohm (AB) effect [15, 80–86], as explained in 3.1 (Figure 5.14) with an expected magnetoconductance period $T_G = h / (e \pi r^2)$ where r is the mean radius of the quantum ring. Scarring effect could also be observed experimentally by means of scanning probes techniques such as scanning gate microscopy that can capture charge-carrier wave functions concentration [81, 82, 84, 86] by measuring the change of local conductance (ΔG) with movable voltage gate (Figure 5.15). The scars periodicity is also shown on Figure 5.15 for a characteristic $\Delta E = 0.0115[eV]$ leading to L_{orb} comparable to twice the ring arm's width.

³The phase coherence length and time are the length and time for which an electron can travel before losing its phase coherence.

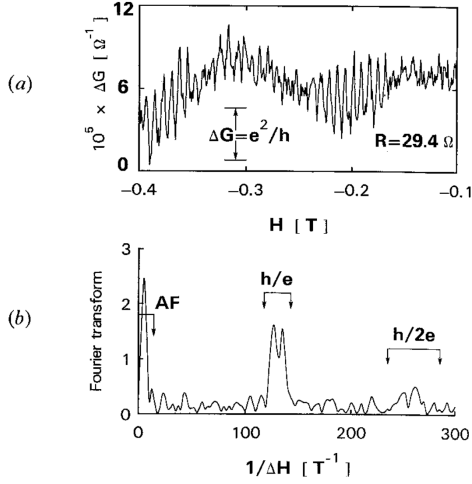


Figure 5.14: AB effect. **(a)** Magnetoresistance of a metal quantum ring showing main h/e period. **(b)** Fourier transform of (a) with two peaks at the two first expected harmonics h/e and $h/2e$. [80]

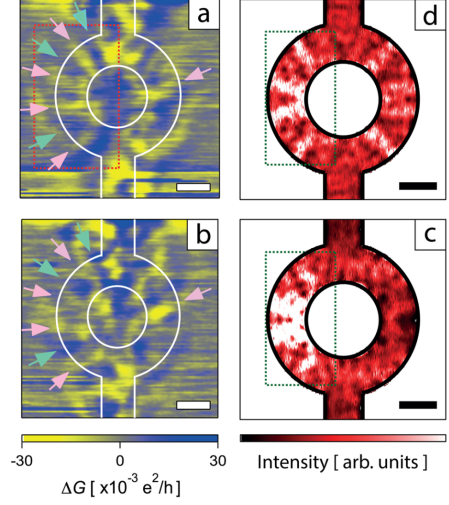


Figure 5.15: **(a, b)** Recurrence of a scar pattern for two different Fermi energies separated by $\Delta E = 0.0115[eV]$. **(c, d)** are corresponding LDOS simulations. [86]

Quantum simulations results

Quantum simulations are realized on a mesoscopic graphene quantum ring (named QR_1) with inner radius $R_{in} = 200[nm]$ and outer radius $R_{out} = 400[nm]$ and on another graphene quantum ring (named QR_2) whose dimensions fulfill the constraint :

$$\frac{\lambda_{Fe}}{W_q} = \frac{\lambda_F}{W_b}$$

where λ_{Fe} is the Fermi wavelength, λ_F is the Faraday wavelength of walkers, W_q and $W_b = W_{cav,r}$ designate the ring arm width in the quantum simulations ("q" for quantum) and in the experiments ("b" for "bath"), inspired by [1]. The QR_1 aims to get close to existing experimental quantum ring while QR_2 is meant to have quantum simulations comparable to the experimental results obtained with walking droplets in Section 5.1.

In graphene, the Fermi energy is expressed as

$$E_{Fe} = \hbar v_{Fe} k_{Fe} = 2\pi \hbar v_{Fe} / \lambda_{Fe}$$

where k_{Fe} and λ_{Fe} are the Fermi wave vector and Fermi wavelength related to each other as $k_{Fe} = 2\pi / \lambda_{Fe}$. Quantum simulations are realized with tight-binding approximation with hopping parameter $t = 2.8[eV]$ and graphene lattice constant $a = 0.142[nm]$ with a possible scaling factor $s = (10, 1)$ (explained in Appendix B). The Fermi energy is set at $E_{Fe} = 0.1t$ for the QR_1 to restrict to number of propagating modes to 4 and $E_{Fe} = 0.3t$ for the QR_2 as a trade-off between reasonable computation time (small Fermi energy leads to high Fermi wavelength

and thus large cavity dimensions) and a moderate number of propagating modes equal to 10. Both are also chosen to fulfill the condition $E < 0.5t$, and assume a linear energy with the wave vector close to Dirac points. The transmission conductance (Figure 5.16) is computed with a magnetic field increasing from 0 to the value corresponding to the cyclotron radius lower than half the cavity arm width. Its expression on graphene is given by

$$r_c = \frac{m^* v_{Fe}}{eB} \quad \text{where} \quad m^* = \frac{\hbar k_{Fe}}{v_{Fe}} \text{ is the effective mass,}$$

The different ranges of value of the conductance in each cavity is explained by the difference in number of modes, higher for the QR_2 . Both conductance have an oscillating behaviour and the QR_2 shows a step at $B \approx 0.725[T]$.

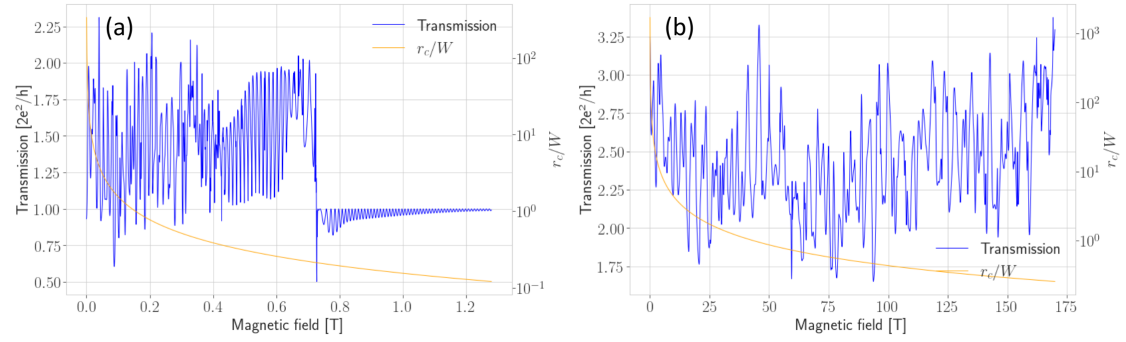


Figure 5.16: Transmission conductance $[2e^2/h]$ and cyclotron orbit radius ratio r_c/W for (a) QR_1 and (b) QR_2 .

The Fourier transform (Figure 5.17) reveals peaks close to AB oscillations frequency. The QR_1 has a slightly higher main frequency obtained for a mean ring radius $r = 330[nm]$ instead of $300[nm]$ but is still consistent, while the QR_2 a peak perfectly aligned with the first AB harmonic.

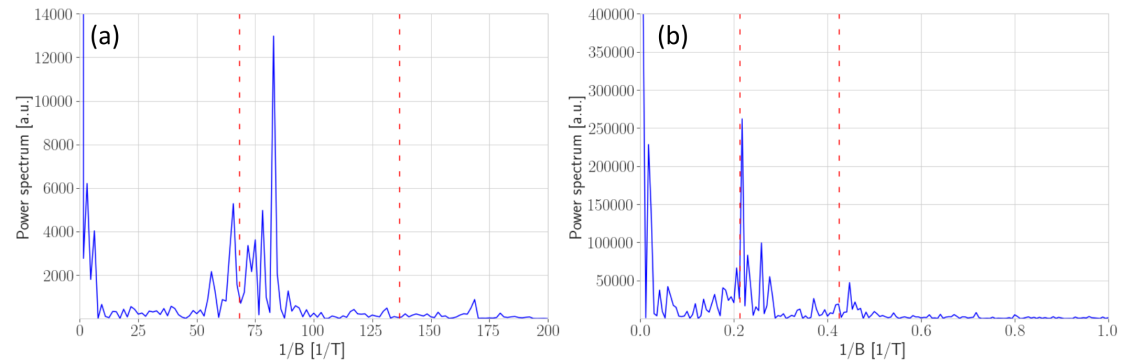


Figure 5.17: Power spectrum of transmission conductance and AB two first harmonics in red dashed line for (a) QR_1 and (b) QR_2 .

The transmission step of the QR_1 at $B \approx 0.725[T]$ is explained by Landau levels. A Landau at level n on graphene takes the expression

$$E_n = \pm v_{Fe} \sqrt{2\hbar e} \sqrt{n B_z}$$

and we can notice (Figure 5.18) that it is the value where the number of Landau levels below the Fermi energy goes from 2 to 1. It means that the Fermi energy only crosses one Landau level on the graphene edge due to potential confinement, leading to one electron channel and thus a mean conductance $\sim 1[2e^2/h]$ taking the electron spin into account. This effect is responsible for the integer quantum Hall effect. Oscillations above this value are thought to be due to quantum tunneling from an edge channel along the outer circular boundary to a 'trapped' edge channel along the inner circular boundary, but could not be verified in simulation.

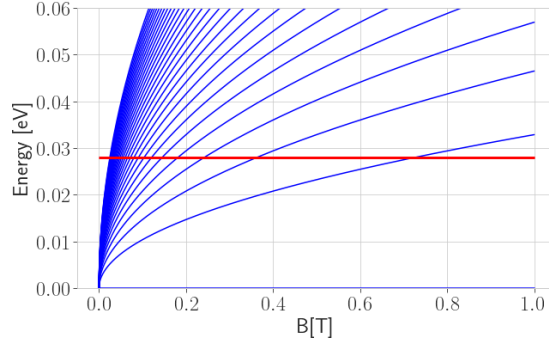


Figure 5.18: Landau levels varying with the magnetic field and Fermi energy in red line.

The local density of states (LDOS) and local current in the QR_2 (Figure 5.19) are computed for magnetic field leading to cyclotron orbit radius $r_c = W/2$ and $r_c = W/5$. It should be noticed that the very small cavity dimensions of the QR_2 leads to very high magnetic field required to reach these cyclotron orbit radius. The goal of the QR_2 is not meant to be realistic but rather to provide a qualitative comparison with the walkers experiment.

For $r_c = W/2$, the local current in the upper path is globally higher due to the Lorentz force and some trajectories follow the edge at the top and the inner circle at the bottom. The LDOS indicates two high density circular zones at the beginning of the upper path, spaced by a distance $\approx W$ the path width, that demonstrate the electron circular trajectory of radius $r_c = W/2$.

For $r_c = W/5$, the local current is practically only present in the upper path. At this value, the number of Landau levels below the Fermi energy is 2, leading to two edge channels. These ones are not clearly visible and rather form only one channel. Again, the LDOS demonstrate the electron circular trajectory of radius $r_c = W/5$ with the distance between some higher density circular zones $\approx 2W/5$.

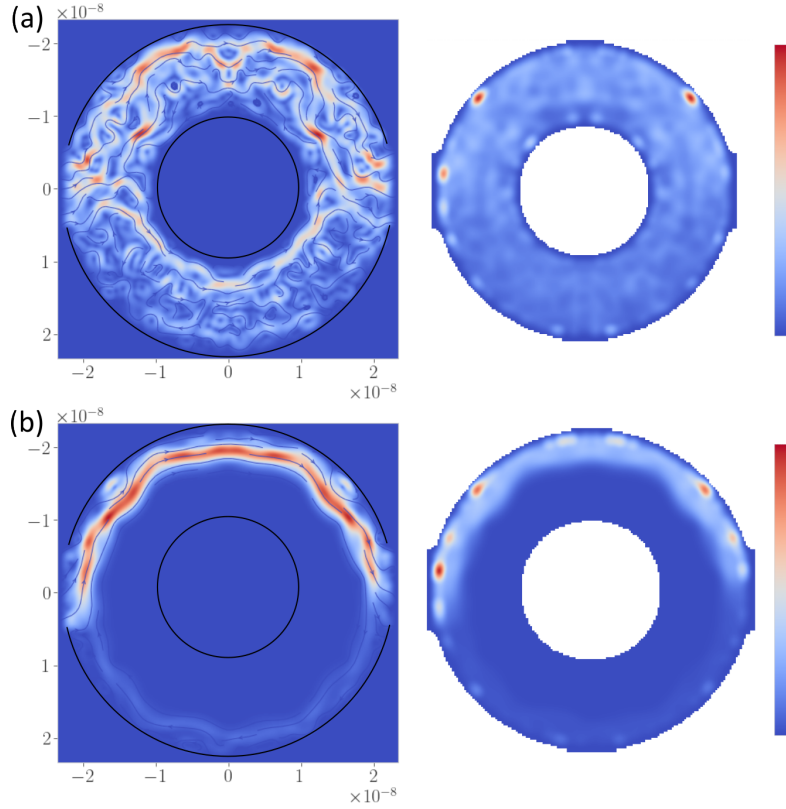


Figure 5.19: Simulation results on QR_2 . Left column : the local current. Right column: LDOS. **(a)** line results are for magnetic field $B = 166.27[T]$, $r = W/2$, **(b)** line results are for magnetic field $B = 415.67[T]$, $r = W/2$.

5.2.2 Stadium cavity

Regarding the stadium cavity, research have been mainly focused on scarring effect both on closed and open (addition of leads) stadium [51, 78, 79, 87–89] due to its chaotic geometry. Quantum behaviours are reported in [1, 90] with scarring pattern periodicity with Fermi energy. Akis et. al [51] also reported scarring pattern periodicity but varying a perpendicular magnetic field instead of Fermi energy. They show a magnetoconductance periodicity at low magnetic field (Figure 5.20), indicating that only a few regular orbits dominate the quantum transport. Indeed, they report rectangular scar pattern (Figure 5.20 (b-c)) with a magnetic period corresponding to the magnetoconductance period (Figure 5.20 (a)). Plus, the authors point out the importance of the leads positions that change the electrons initial condition, prone to lead to different scarring effect [51, 78]. Eventually, the contact and width of leads play an important role, as pointed out in [89], due to the selection of the electron modes able to propagate in the cavity, possibly leading to resonant states (scarring effects).

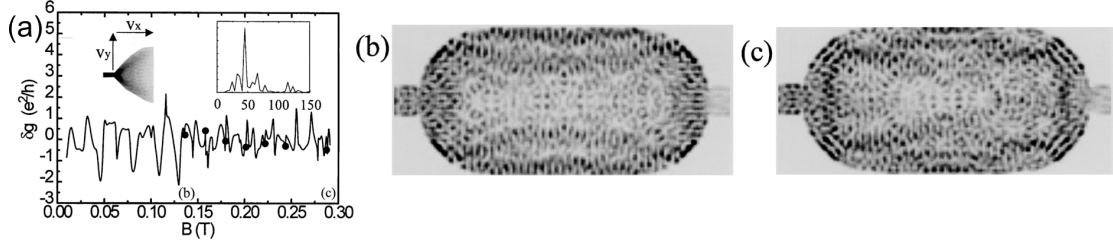


Figure 5.20: **(a)** Magnetoconductance of open stadium cavity where dots correspond to rectangular scars (Inset : Fourier transform showing a large frequency peak). **(b, c)** Recurrence of the rectangular scar pattern for two different magnetic fields $B = 0.135[T]$ and $B = 0.288[T]$, showing a rectangular scar in both cases. [51]

Regarding walkers experiment, O. Leblanc reported [1] preferred "probable positions" of the droplet inside a closed stadium geometry, found to be analog to a scarring pattern obtained in a stadium cavity with quantum simulations, adding an input lead at the top (Figure 5.21). The walker behaviour is explained by the influence of the billiard borders through its coupling to its pilot-wave properties in the walking regime. Similar scarred wavefunctions were reported in the Fig. 2 of [87] and on graphene simulations [79].

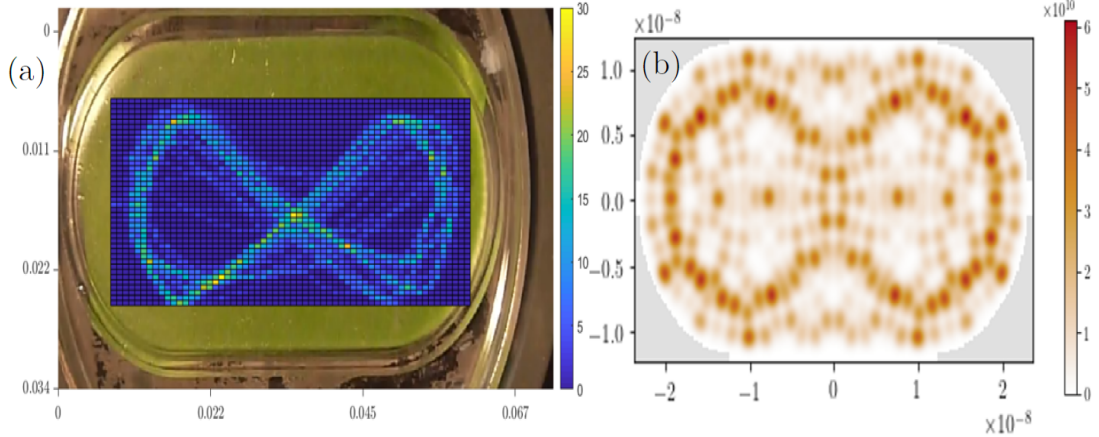


Figure 5.21: Similar ∞ -distribution in a stadium geometry between **(a)** Experimental observation with the walking droplet and **(b)** LDOS simulation.

Quantum simulations results

Quantum simulations are realized on a mesoscopic graphene stadium (named S_1) with width $W = 200[nm]$ and length $L = 2W$ and on another graphene quantum stadium (named S_2) whose dimensions fulfill the same constraint as in [1]:

$$\frac{\lambda_{Fe}}{R_q} = \frac{\lambda_F}{R_b}$$

where R_q and $R_b = R_{cav,s}$ designate the semicircle radii in the quantum simulations and in the experiments.

As for the ring system, the Fermi energy is set at $E_{Fe} = 0.1t$ for the S_1 to restrict to number of propagating modes to 4 and $E_{Fe} = 0.3t$ for the S_2 leading to a number of propagating modes equal to 10. The transmission conductance (Figure 5.22) is computed with a magnetic field increasing from 0 to the value corresponding to the cyclotron radius lower than half the cavity semicircle radius. The different ranges of value of the conductance in each cavity is as well explained by the difference in number of modes, higher for the S_2 .

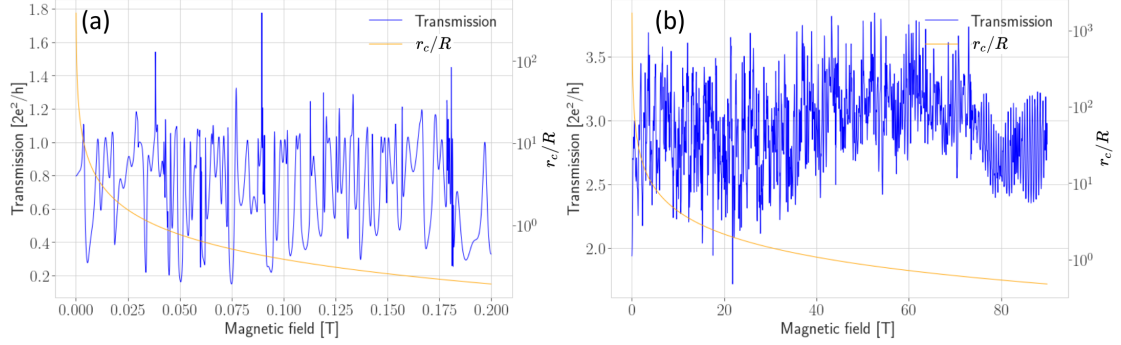


Figure 5.22: Transmission conductance $[2e^2/h]$ and cyclotron orbit radius ratio r_c/W for (a) S_1 and (b) S_2 .

Both conductance exhibit at first sight a fluctuating behaviour. Looking at the Fourier transform (Figure 5.23), no single peak value are found as reported in [51], but several ones for both cavities. The periodicity of several magnetoconductance frequencies peak could be investigated to search for corresponding recurring scars (as in [51]), but was not further explored seeing the walkers experimental results.

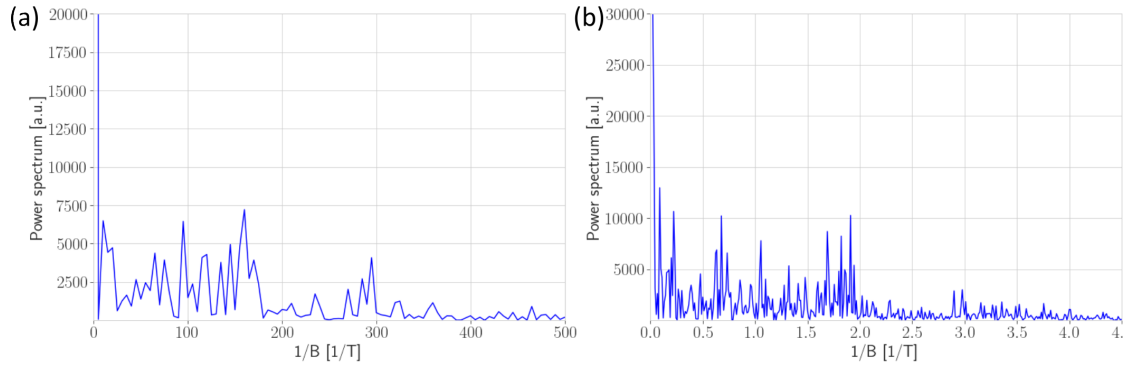


Figure 5.23: Power spectrum of transmission conductance for (a) S_1 and (b) S_2 .

The LDOS and local current of the S_2 (Figure 5.19) are computed for magnetic field leading to cyclotron orbit radius $r = R/2$ and $r = R/4$. The goal of the QR_2 is also here not meant to be realistic but rather to provide a qualitative comparison with the walkers experiment.

For $r_c = W/2$, the local current is bent in the upper part, and some circular trajectories are more visible in the lower part with the expected radius $r_c = W/2$. The LDOS indicates as well some circular trajectory.

For $r_c = W/4$, the local current is practically only present in the upper path along the boundary and showing some circular trajectory with an expected radius $r_c = W/4$ on the upper straight boundary zone. Here, the LDOS is not very illustrative of the cyclotron orbital.

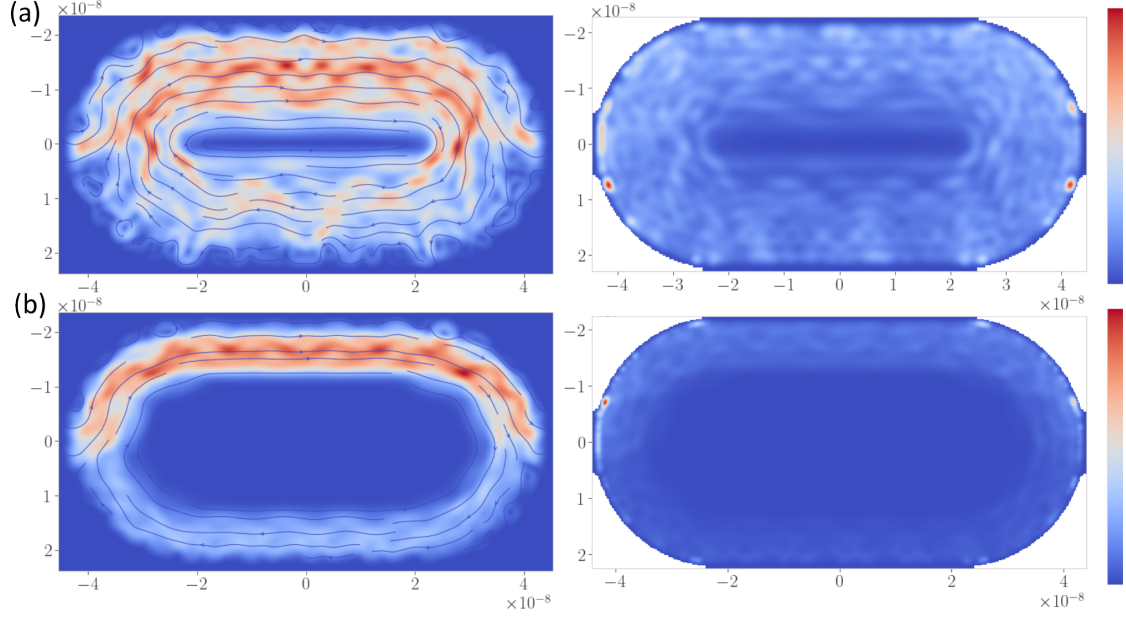


Figure 5.24: Simulation results on S_2 . Left column : the local current. Right column: the LDOS. **(a)** line results are for magnetic field $B = 85.70[T]$, $r_c = W/2$, **(b)** line results are for magnetic field $B = 171.41[T]$, $r_c = W/4$.

As a double-check of the right circular trajectories radius, the LDOS of the propagating modes was computed, and the most illustrative ones (Figure 5.25) well satisfy the expectations.

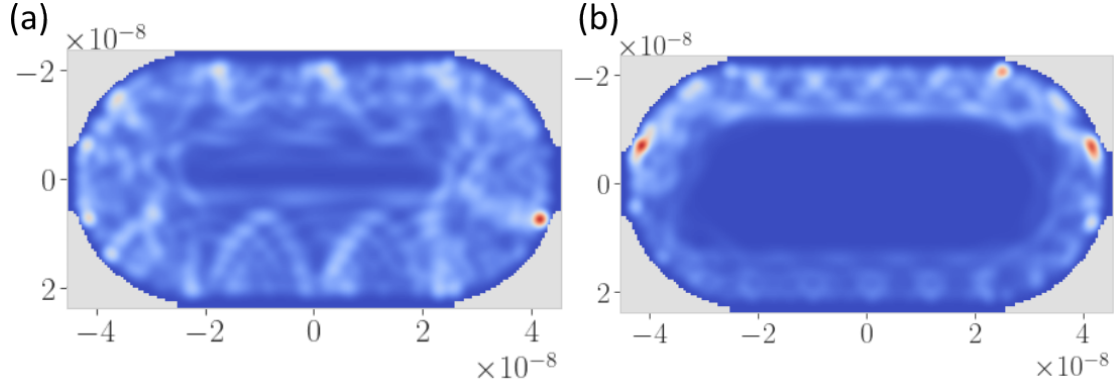


Figure 5.25: LDOS of modes illustrating circular electron trajectories **(a)** for magnetic field $B = 85.70[T]$, $r = W/2$, **(b)** for magnetic field $B = 171.41[T]$, $r = W/4$.

5.3 Comparison of walkers experiments with experimental data and simulations

We will now address a comparison between the walkers experiment (5.1) and the experimental data and simulations (5.2).

Transmission behaviour

The transmission behaviours in the walker experiments is expressed as a probability of being transmitted and the analog conductance is computed in *Kwant* with the Landauer-Buttiker formalism:

$$G_{ab} = \frac{2e^2}{h} S_{ab}$$

where a, b labels two leads and S_{ab} is the scattering matrix element giving the transmission between lead a and b . The scattering matrix is computed as

$$S_{ab} = \sum_{n \in a, m \in b} |t_{nm}|^2,$$

where t_{nm} is the transmission coefficient from mode n of lead a to mode m of lead b .

The transmission behaviours of walker and electron in these cavities have been shown to radically differ. On the ring geometry, electron conductance exhibit AB oscillations while on the stadium geometry, the transmission has several frequency peak at different locations. On the other side, the walker transmission shows oscillating behaviour that are rather thought to be due to the reflections on bath boundaries in both case, influenced by the angular velocity.

In electronic system of such geometries, a lot of parameters come at play namely the superposition of the electron modes, the Fermi energy, the magnetic field leading to different effects such as Landau levels quantization, electron phase shift, Lorentz force, etc. On the other hand, a walker is a macroscopic system associating a particle and a wave, but the wave has a constant wavelength, i.e. the Faraday wavelength. Setting aside the physical oil parameters, the most important remaining one is the memory compared to the cavity dimensions, possibly leading to wave-like behaviours. The limitation of the walking droplets in such a system regarding the electron dynamics is the impossibility to superimpose modes due to low memory experiments preventing interference with previously emitted waves, that could lead to more complex transmission behaviours.

Still, the walker experiment in the stadium-shaped bath has shown behaviour similar to magnetic focusing effect observed on electronic systems[91] where the transmission has local maxima for cyclotron orbital radius leading the electron trajectory in the output lead after zero or more reflection on a boundary. Indeed, the walker transmission was shown to have periodic extrema above a certain angular velocity where trajectories were mainly influenced by the number of reflections on the upper boundary due to the walker cyclotron radius varying with the angular velocity of the bath.

Walker's position distribution compared to local current and LDOS

Both local current and LDOS were not shown to be comparable at low magnetic field. Indeed, walkers in ring-shaped and stadium-shaped bath did not exhibit scar pattern in the whole cavity (except for null angular velocity in the stadium-shaped bath discussed in [Chapter 6](#)), which are a key property of analog electronic system. Also, the T-junction peculiar behaviour of walkers for transmission local minima was not observed in simulations.

At strong magnetic field ($r_c \leq W/2$ and $r_c \leq R/2$ for the ring and stadium geometries), the local current has shown more and more similar behaviours as the magnetic field increases. Indeed, at very high magnetic field, the electron dynamics can be governed by one conducting channel due to Landau levels. This case is analog to walkers in bath with high angular velocity, where walkers follow the device edges before reaching the output. The LDOS shows less similarity with the walkers dynamics. It can capture precise cyclotron orbit which was not observed for walkers whose trajectory tended to follow the boundary.

Although the walkers trajectories do not follow classical trajectory of a bouncing ball in a billiard model with specular reflections, simulations of such model are in [Appendix C](#) as a support to the reflection non-specular behaviour of walkers [49].

Chapter 6

Conclusion and perspectives

This work was realized in the continuity of the master thesis realized by O. Leblanc[1] where a first experimental setup was developed at *UCLouvain*. Some reported improvements have been realized and a controllable angular velocity has been set, bringing in turn new possible improvements described in 4.1.3.

6.1 Discussion on the obtained results

Let us remind the initial questions of this master thesis to answer them:

- *How does the walker transmission compare to quantum particles conductance?*

From the walkers observations compared with electron simulations and experimental data, the transmission behaviours are shown to radically differ in such geometries. Indeed, for electronic system, a lot of parameters come at play. The Fermi energy defines the propagating modes and the electron dynamics is governed by the superpositions of these modes. The magnetic field has many consequences on the electron dynamics such as Landau levels, phase shift of the electron waves, etc. On the other side, a walker is a macroscopic system associating a particle and a wave at constant Faraday wavelength, and its main important parameter to consider to reach wave-like behaviours in cavity is the memory compared to the cavity dimensions [1, 92–95], setting aside the physical oil parameter. The current setup was not suitable to work at high memory (4.1.3), leading to walkers transmission mainly governed by boundaries reflections.

- *Can the probability distribution of the walker positions be associated to the electrons density or the electron local current?*

For both baths, the local current at strong magnetic field ($r_c \leq W/2$ and $r_c \leq R/2$ for the ring and stadium geometries) showed behaviours similar to the distribution of walker's positions at high angular velocity. In such a case, the walkers trajectories are dominated by the Coriolis force and electrons by the

Lorentz force, that have similar expression respectively with the angular velocity and magnetic field. Plus, as magnetic field increases in electronic systems, the number of Landau levels crossed by the Fermi energy decreases until 1, as it was observed in simulations. This case can be analog to the walkers experiment in both geometries as trajectories were shown to follow the boundary edges, analog to a single edge electron channel following the cavity boundary.

On the other hand, the LDOS is less comparable to the walkers dynamics. The LDOS is suitable to observe precise cyclotron orbit which was not observed for walkers whose trajectories tended to follow the boundary.

Regarding these results, one can state that the local current is more suitable for a qualitative comparison with the walker experiments oriented toward transmission. Indeed, the local current can often give more appealing results and insights about the electron flow that can be analog to the walkers flow. On the other hand, the LDOS here is less comparable to the walker experiments and is thought to be more relevant for closed cavity, to illustrate resonant eigenstates resulting in scarring effect.

- *Can one evidence an Aharonov Bohm interference effect in the transmission properties of walkers, similar to the one observed for electrons on mesoscopic ring?*

As discussed in 5.1, the Aharonov-Bohm effect was firstly considered in the ring-shaped bath due to oscillating transmission characteristics. However, this effect is not thought to be responsible of this observation. Indeed, the relative phase accumulation at the origin of the Aharonov-Bohm effect was observed for wavefront passing through an irrotational vortex [41] that modify their phase leading to interference depending on the wavefront dislocation parameter (itself depending on the vortex strength). Yet, the current setup leads to a rotational vortex, which should therefore not exhibit such an effect. A proposition of setup suitable for the AB observation was put forward, with some connections to a recent research showing walker interferometer behaviours [74].

Apart from the stated questions, the stadium geometry showed a more chaotic behaviour of the walker at null angular velocity, as opposed to the results of O.Leblanc [1]. The proposed reasons are the change of walkers dynamics due to the addition of the leads and the higher cavity dimensions not suitable for scarring effect with edges filled by oil, considering the angle reflections stability observed in [49]. The surface height of the oil filling edges was investigated and it has been shown that it influences the non-specular reflection behaviour of walkers, that could possibly lead to ∞ — walker’s position distribution found in [1]. The perspectives of this observation are discussed hereafter.

6.2 Perspectives for future research

Walkers experiments in rotating bath gained interest due to new interesting behaviours where some analogies to electron dynamics were proposed [20, 39, 42–47]. In this master thesis, an analogy between the walkers transmission probability and the conductance was investigated in such a system but showed different behaviours. Although the transmission behaviours are different, the walker’s position distribution at high angular velocity showed behaviours analog to electrons under high magnetic field leading to one-dimensional electron channels, responsible of the Quantum Hall effect.

This work opens up many perspectives for future work, being the first one to investigate walker transmission properties between an input and output lead. As previously pointed out (5.3), the walkers transmission behaviour in the stadium geometry was similar to conductance behaviour resulting from magnetic focusing effect observed on electronic systems[91]. Therefore, one could imagine a bath analog to the cavity reported in [91] with input and output lead in the same plane and examine the walker transmission walker probability dependence on the angular velocity.

Then, as already mentioned, Aharonov-Bohm effect could be investigated in another proposed setup (5.1), including a rotating inner cylinder that results in a vortex approaching irrotational flow, suitable to observe such an effect [41]. Aharonov-Bohm effect is the result of interference between waves of different phase, and a recent research showing walker interferometer behaviours [74] support the proposed setup.

Eventually, for the stadium geometry, chaotic behaviour of the walker was reported for oil filling the bath edges with height surface $0 < h < 1[mm]$, but ‘preferred positions’ of the walkers were found for $h = 0[mm]$, with ∞ -pattern comparable to [1]. Therefore, the question of the dependency of scarring effect with the oil filling height at the bath edges could be further explored. Other dimensions could be tested to identify the oil height leading to scarring effect, and perhaps find a relationships between both. An analogy could be proposed between the surface height on the borders and the depletion length on the borders of an electronic system. The surface height on the borders could be seen as the border potential, and the depletion length characterizes the narrowness of the potential seen by electrons on the system that differs according to the considered material (graphene, GaAs/AlGaAs heterostructures, ...)

Appendices

Appendix A

Experimental setup control

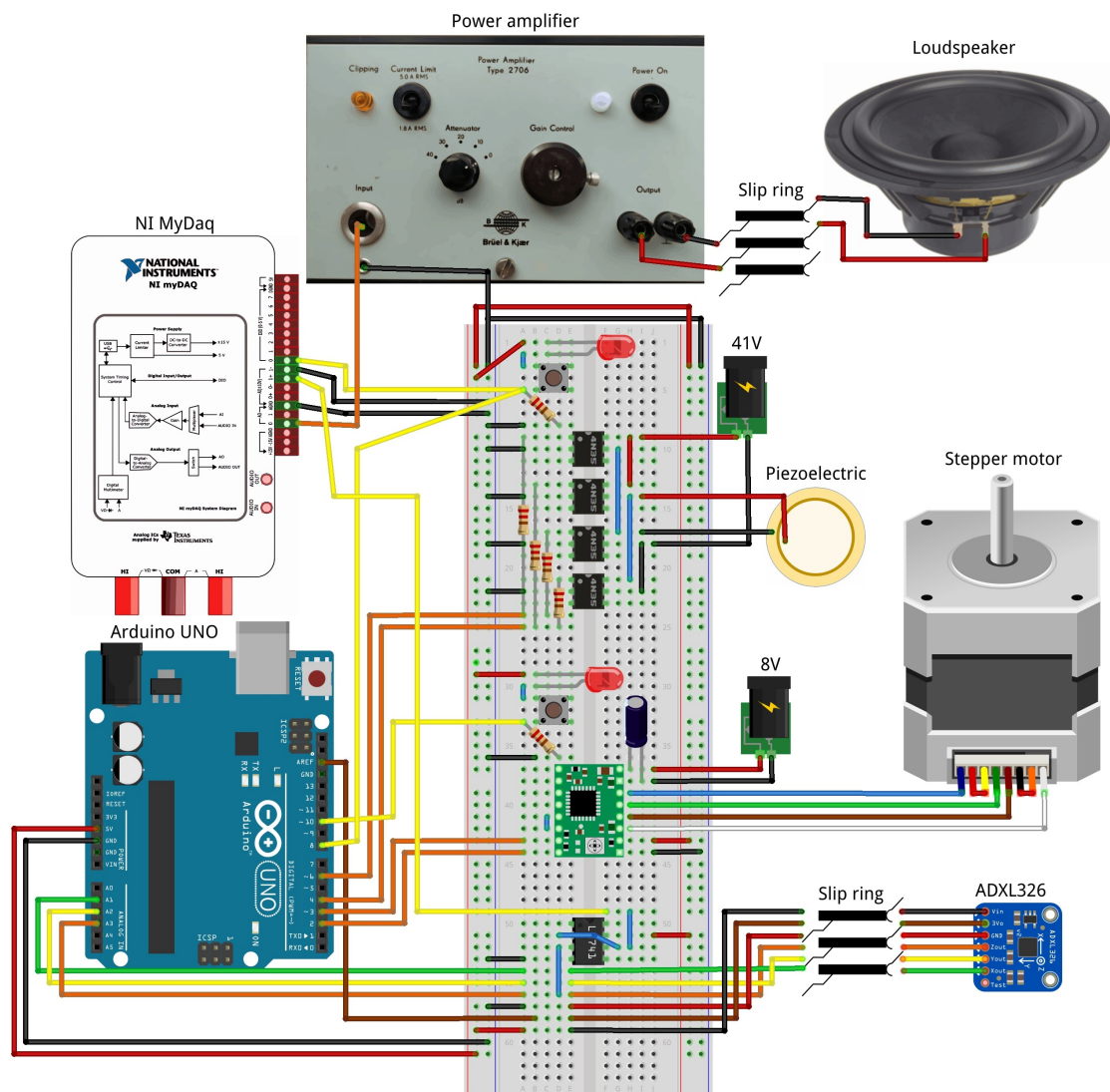


Figure A.1: Electrical connections

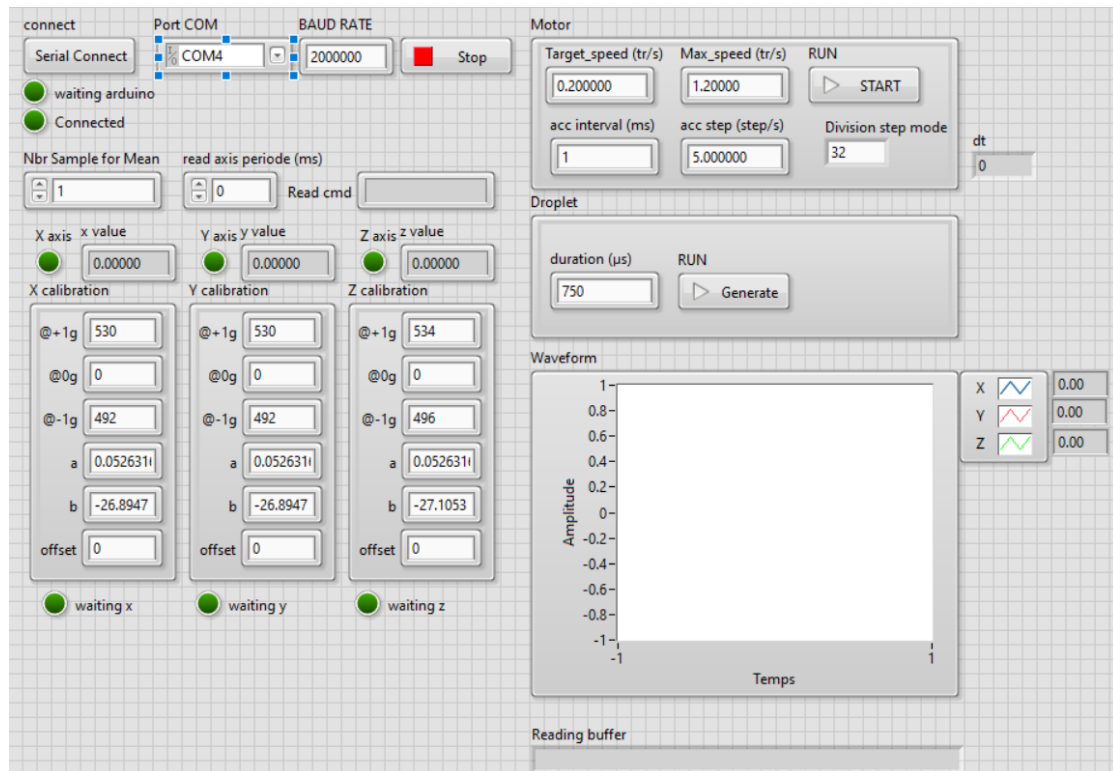


Figure A.2: Front panel of the Labview program "Arduino_Ctrl.vi"

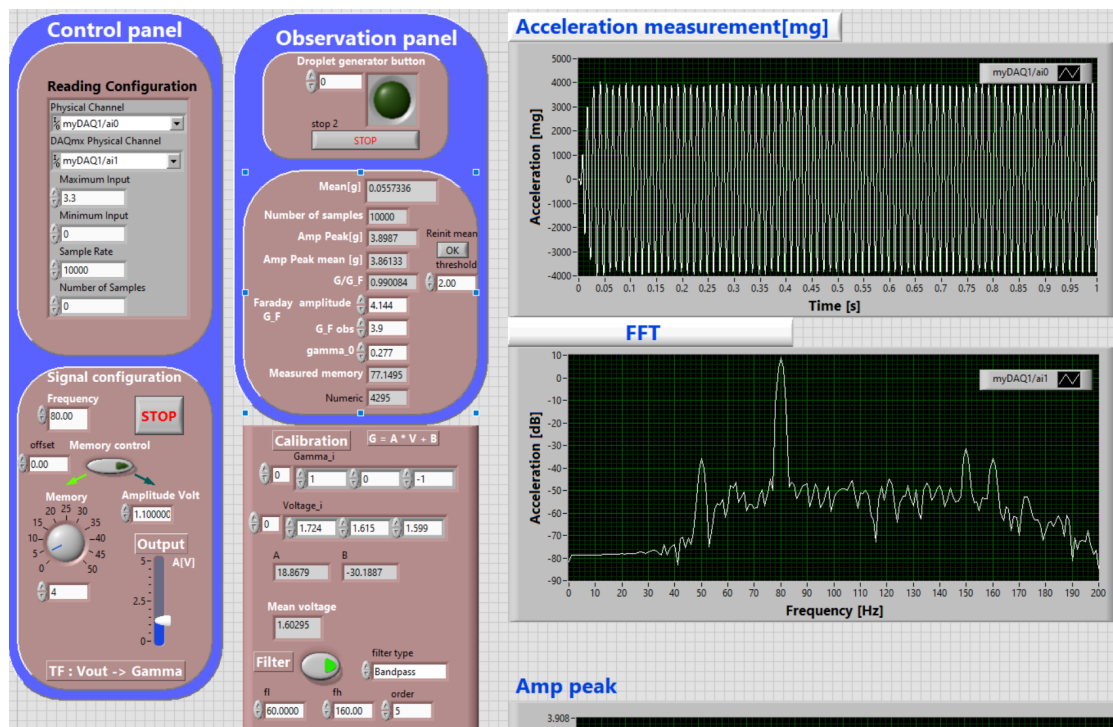


Figure A.3: Front panel of the Labview program "MyDAQ_memory_control.vi" directly applying a BPF. Missing parts are other waveforms : Accelerations peaks, G , Raw output voltage of accelerometer

Appendix B

Quantum simulations

Simulations were performed in the tight-binding framework using the *Kwant* package [16]. The tight-binding framework can capture the main physical characteristics we want to calculate, and *Kwant* is oriented toward electronic transport, providing tools to create custom system lattice with leads and compute the conductance, the local current, the LDOS, etc. The tight-binding Hamiltonian can be expressed in the second quantization formalism as [96]

$$\hat{\mathcal{H}} = \sum_i U(\mathbf{r}_i) \hat{c}_i^\dagger \hat{c}_i - \sum_{\langle i,j \rangle} t_0 \hat{c}_i^\dagger \hat{c}_j$$

where $t_0 = 2.8[eV]$ is the hopping parameter, $\langle i, j \rangle$ is the restriction over the nearest neighboring atoms indices, $\hat{c}_i^\dagger$ and $\hat{c}_i$ are respectively the creation and annihilation operators on site i and $U(\mathbf{r}_i)$ is the potential on site i . In our simulations, no potential was applied on the sites and the Hamiltonian becomes

$$\hat{\mathcal{H}} = - \sum_{\langle i,j \rangle} t \hat{c}_i^\dagger \hat{c}_j$$

where $t = t_0$. In the hexagonal lattice of graphene, the energy dispersion at low energies can be approximated linearly as a function of the wave vector k written as

$$E(\mathbf{k}) = \pm \frac{3at}{2} |\mathbf{k}|$$

where $a = a_0 = 0.142[nm]$ is the lattice constant equal to carbon-carbon distance. At low energy, it was shown that a (reasonable) scaling of the system, useful to decrease the number of sites and thus the computation time, kept the electron dynamics unchanged [96]. It is defined with a scale factor s as

$$a = a_0 \times s \quad \text{and} \quad t = t_0/s$$

The magnetic field B can be added in the tight-binding Hamiltonian with a modification of hopping parameter t , becoming

$$t = t_0 \exp \left(-j \frac{e}{h} B (x_i - x_j) \frac{y_i + y_j}{2} \right)$$

and the resulting Landau levels on graphene take the form

$$E_n = v_F \sqrt{2\hbar e} \sqrt{nB}$$

where $v_F = 3at/(2\hbar)$ is the Fermi velocity and $n = 0, \pm 1, \pm 2, \dots$ designates the Landau level label where $+$ and $-$ respectively stands for electrons and holes.

Within the Landauer-Buttiker formalism the conductance is computed as:

$$G_{ab} = \frac{2e^2}{h} S_{ab}$$

where a, b labels two leads and S_{ab} is the scattering matrix element giving the transmission between lead a and b . The scattering matrix is computed as

$$S_{ab} = \sum_{n \in a, m \in b} |t_{nm}|^2,$$

where t_{nm} is the transmission coefficient from mode n of lead a to mode m of lead b .

For our simulations, the [Table B.1](#) summarises the set parameters. As explained in the main text, the simulations were realized on two dimensions for each geometry, hence the two possible values for some parameters. When two parameters are mentioned, they correspond to the order 1, 2 of the main text (for example: QR_1 and QR_2). The magnetic field was increased until reaching a cyclotron orbit radius $r_c \leq R_q/2$ or $r_c \leq W_q/2$ respectively for the ring and the stadium.

Name	Expression	Value	[Unit]
Planck's constant	$\hbar$	$6.626069 \cdot 10^{-34}$	[J.s]
Electric charge of the electron	e	$1.602 \cdot 10^{-19}$	[C]
Scaling factor	s	(10, 1)	[−]
Hopping parameter	$t = t_0/s$	(0.28, 2.8)	[eV]
Lattice parameter	$a = a_0 \cdot s$	(1.42, 0.142)	[nm]
Fermi energy	$E_F = (0.1, 0.3)t$	(0.027, 0.84)	[eV]
Fermi velocity	$v_F = 3at/(2\hbar)$	$0.906 \cdot 10^5$	[m/s]
Fermi wave vector	$k_F = E_F/(\hbar v_F)$	$(4.695 \cdot 10^7, 4.695 \cdot 10^8)$	[rad/m]
Fermi wavelength	$\lambda_F = 2\pi/k_F$	$(1.338 \cdot 10^{-7}, 1.338 \cdot 10^{-8})$	[m]
Cyclotron orbit radius	$r_c = E_F/(v_F B)$	$[+\infty, R_q/2 \text{ or } W_q/2]$	[m]
Magnetic field	$B = E_F/(v_F r_c)$	range set from r_c	[T]
Ring			
Width of the leads	$W_{lead,r}$	$(200 \cdot 10^{-9}, 2.5\lambda_F)$	[m]
Inner radius	R_{in}	$(200 \cdot 10^{-9}, 2.5\lambda_F)$	[m]
Outer radius	R_{out}	$(400 \cdot 10^{-9}, 5\lambda_F)$	[m]
Stadium			
Width of the leads	$W_{lead,s}$	$(200 \cdot 10^{-9}, 2\lambda_F)$	[m]
Stadium semicircle radius	$R_q = W_q/2$	$(200 \cdot 10^{-9}, 4.75\lambda_F)$	[m]
Stadium length	L_q	$(800 \cdot 10^{-9}, 19\lambda_F)$	[m]

Table B.1: Simulation parameters

Appendix C

Classical system simulations

Ring billiard classical simulations obtained with Julia package [17] for a magnetic particle. The initial trajectory angle formed with the horizontal axis is simulated with a Gaussian distribution of mean = 0 and standard deviation $\sigma = 45^\circ$.

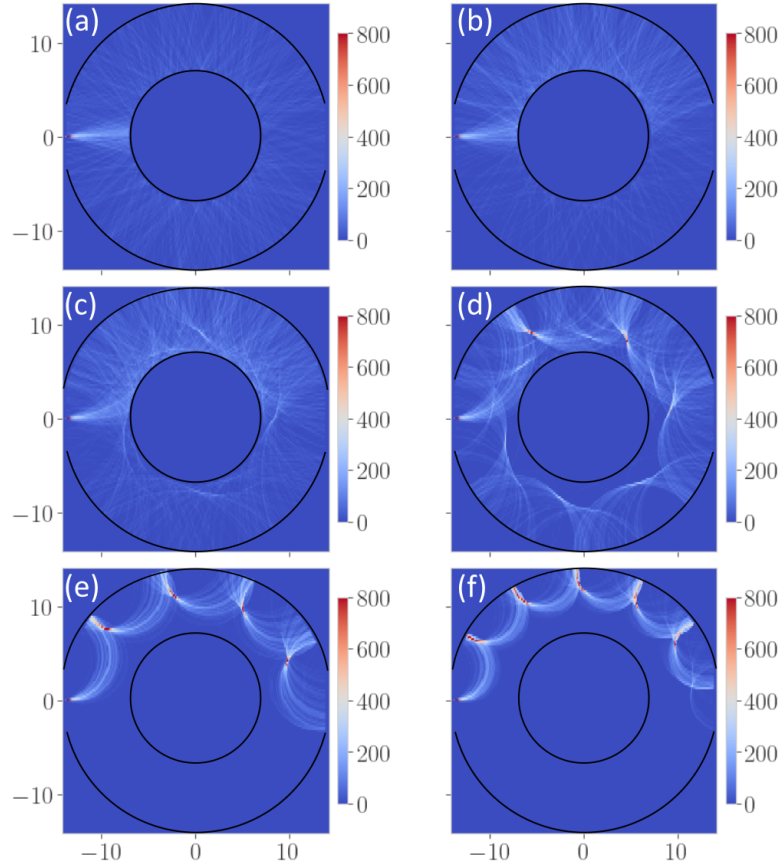


Figure C.1: **(a-f)** Classical simulations in ring-shaped billiard with cyclotron orbits $r_c = (\infty, 0.5, 0.66, 1, 4, 9)R_{in}[mm]$, with $R_{in} = 2.5\lambda_F$ the inner circle radius.

Stadium billiard classical simulations for a magnetic particle [17]

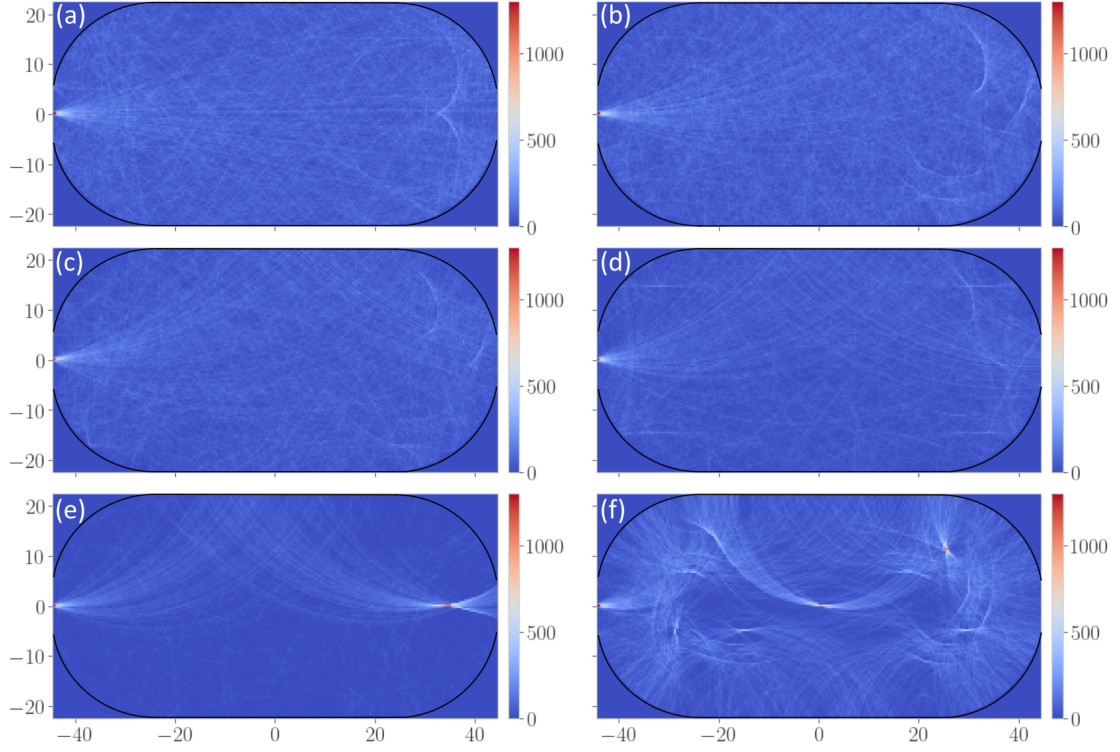


Figure C.2: **(a-f)** Classical simulations in stadium-shaped billiard with cyclotron orbits $r_c = (\infty, 1, 2, 3, 4, 5)R_{stadium}[mm]$, with $R_{stadium} = 4.75\lambda_F$ the semi-circle radius.

Bibliography

- [1] O. Leblanc, “Comparison between the walking droplet and the electron behaviour inside a chaotic stadium cavity,” Master’s thesis, Ecole polytechnique de Louvain, Université catholique de Louvain, 2020. [Online]. Available: <http://hdl.handle.net/2078.1/thesis:25162>
- [2] L. Tadrist, J.-B. Shim, T. Gilet, and P. Schlagheck, “Faraday instability and subthreshold Faraday waves: surface waves emitted by walkers,” *Journal of Fluid Mechanics*, vol. 848, pp. 906–945, 2018. [Online]. Available: https://www.cambridge.org/core/product/identifier/S0022112018003580/type/journal_article
- [3] S. Protiere, “Gouttes rebondissantes: une association onde-particule à échelle macroscopique,” p. 141, 2007.
- [4] N. D. Mermin, “Commentary: Quantum mechanics: Fixing the shifty split,” *Physics Today*, vol. 65, no. 7, pp. 8–10, 2012, publisher: American Institute of Physics. [Online]. Available: <https://physicstoday.scitation.org/doi/10.1063/PT.3.1618>
- [5] J. Ismael, “Quantum Mechanics,” in *The Stanford Encyclopedia of Philosophy*, fall 2021 ed., E. N. Zalta, Ed. Metaphysics Research Lab, Stanford University, 2021. [Online]. Available: <https://plato.stanford.edu/archives/fall2021/entries/qm/>
- [6] W. contributors, “Interpretations of quantum mechanics,” 2022, page Version ID: 1090726888. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Interpretations_of_quantum_mechanics&oldid=1090726888
- [7] J. Faye, “Copenhagen Interpretation of Quantum Mechanics,” in *The Stanford Encyclopedia of Philosophy*, winter 2019 ed., E. N. Zalta, Ed. Metaphysics Research Lab, Stanford University, 2019. [Online]. Available: <https://plato.stanford.edu/archives/win2019/entries/qm-copenhagen/>
- [8] W. contributors, “Copenhagen interpretation,” 2022, page Version ID: 1088931089. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Copenhagen_interpretation&oldid=1088931089
- [9] L. De Broglie, *Ondes et mouvements*. Gauthier-Villars, 1926, vol. 1.

- [10] D. Bohm, “A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I,” *Physical Review*, vol. 85, no. 2, pp. 166–179, 1952, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRev.85.166>
- [11] S. Goldstein, “Bohmian Mechanics,” in *The Stanford Encyclopedia of Philosophy*, fall 2021 ed., E. N. Zalta, Ed. Metaphysics Research Lab, Stanford University, 2021. [Online]. Available: <https://plato.stanford.edu/archives/fall2021/entries/qm-bohm/>
- [12] W. contributors, “De Broglie–Bohm theory,” 2022, page Version ID: 1088317097. [Online]. Available: https://en.wikipedia.org/w/index.php?title=De_Broglie%E2%80%93Bohm_theory&oldid=1088317097
- [13] Y. Couder and E. Fort, “Single-Particle Diffraction and Interference at a Macroscopic Scale,” *Physical Review Letters*, vol. 97, no. 15, p. 154101, 2006, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.97.154101>
- [14] Y. Aharonov and D. Bohm, “Significance of Electromagnetic Potentials in the Quantum Theory,” *Physical Review*, vol. 115, no. 3, pp. 485–491, 1959. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRev.115.485>
- [15] R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, “Observation of the Aharonov-Bohm Oscillations in Normal-Metal Rings,” *Physical Review Letters*, vol. 54, no. 25, pp. 2696–2699, 1985. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.54.2696>
- [16] C. W. Groth, M. Wimmer, A. R. Akhmerov, and X. Waintal, “Kwant: a software package for quantum transport,” *New Journal of Physics*, vol. 16, no. 6, p. 063065, 2014, publisher: IOP Publishing. [Online]. Available: <https://doi.org/10.1088/1367-2630/16/6/063065>
- [17] G. Datseris, “DynamicalBilliards.jl: An easy-to-use, modular and extendable julia package for dynamical billiard systems in two dimensions.” *The Journal of Open Source Software*, vol. 2, no. 19, p. 458, 2017. [Online]. Available: <https://doi.org/10.21105/joss.00458>
- [18] Y. Couder, E. Fort, C.-H. Gautier, and A. Boudaoud, “From Bouncing to Floating: Noncoalescence of Drops on a Fluid Bath,” *Physical Review Letters*, vol. 94, no. 17, p. 177801, 2005, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.94.177801>
- [19] Y. Couder, S. Protière, E. Fort, and A. Boudaoud, “Walking and orbiting droplets,” *Nature*, vol. 437, no. 7056, pp. 208–208, 2005, number: 7056 Publisher: Nature Publishing Group. [Online]. Available: <https://www.nature.com/articles/437208a>

- [20] S. Protière, A. Boudaoud, and Y. Couder, “Particle–wave association on a fluid interface,” *Journal of Fluid Mechanics*, vol. 554, no. -1, p. 85, 2006. [Online]. Available: http://www.journals.cambridge.org/abstract_S0022112006009190
- [21] T. Young, “I. The Bakerian Lecture. Experiments and calculations relative to physical optics,” *Philosophical Transactions of the Royal Society of London*, vol. 94, pp. 1–16, 1804, publisher: Royal Society. [Online]. Available: <https://royalsocietypublishing.org/doi/abs/10.1098/rstl.1804.0001>
- [22] “Silicone oil 20cSt 25°C.” [Online]. Available: <http://www.sigmaaldrich.com/>
- [23] A. Eddi, E. Sultan, J. Moukhtar, E. Fort, M. Rossi, and Y. Couder, “Information stored in Faraday waves: the origin of a path memory,” *Journal of Fluid Mechanics*, vol. 674, pp. 433–463, 2011. [Online]. Available: https://www.cambridge.org/core/product/identifier/S0022112011000176/type/journal_article
- [24] N. Vandewalle, B. Filoux, and M. Hubert, *Bragg’s reflection for walking droplets in 1D crystals*, 2019.
- [25] J. Moláček and J. W. M. Bush, “Drops bouncing on a vibrating bath,” *Journal of Fluid Mechanics*, vol. 727, pp. 582–611, 2013, publisher: Cambridge University Press. [Online]. Available: <https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/drops-bouncing-on-a-vibrating-bath/441A614F657E800EA06F6C08674CCE67>
- [26] L. Rayleigh *et al.*, “On the capillary phenomena of jets,” *Proc. R. Soc. London*, vol. 29, no. 196-199, pp. 71–97, 1879.
- [27] T. Gilet and J. W. M. Bush, “Chaotic bouncing of a droplet on a soap film,” *Physical Review Letters*, vol. 102, no. 1, p. 014501, 2009. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.102.014501>
- [28] J. Moláček and J. W. M. Bush, “Drops walking on a vibrating bath: towards a hydrodynamic pilot-wave theory,” *Journal of Fluid Mechanics*, vol. 727, pp. 612–647, 2013. [Online]. Available: https://www.cambridge.org/core/product/identifier/S0022112013002802/type/journal_article
- [29] O. Wind-Willassen, J. Moláček, D. M. Harris, and J. W. M. Bush, “Exotic states of bouncing and walking droplets,” *Physics of Fluids*, vol. 25, no. 8, p. 082002, 2013. [Online]. Available: <http://aip.scitation.org/doi/10.1063/1.4817612>
- [30] “Lorentz force,” 2022, page Version ID: 1092538384. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Lorentz_force&oldid=1092538384
- [31] N. Moreau, “LELEC 2710 - Exercise sessions : Simulations using the KWANT package.”

- [32] R. Karplus and J. Luttinger, “Hall effect in ferromagnetics,” *Physical Review*, vol. 95, no. 5, p. 1154, 1954.
- [33] C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Mécanique Quantique 1*, hermann ed., 1973.
- [34] —, *Mécanique Quantique 2*, hermann ed., 1997.
- [35] V. B. Berestetskii, L. P. Pitaevskii, and E. M. Lifshitz, *Quantum Electrodynamics: Volume 4*. Elsevier, 2012.
- [36] M. O. Goerbig, “Quantum Hall Effects,” 2009, arXiv:0909.1998 [cond-mat]. [Online]. Available: <http://arxiv.org/abs/0909.1998>
- [37] K. von Klitzing, “The quantized Hall effect,” *Reviews of Modern Physics*, vol. 58, no. 3, pp. 519–531, 1980. [Online]. Available: <https://link.aps.org/doi/10.1103/RevModPhys.58.519>
- [38] D. C. Tsui, H. L. Stormer, and A. C. Gossard, “Two-Dimensional Magnetotransport in the Extreme Quantum Limit,” *Physical Review Letters*, vol. 48, no. 22, pp. 1559–1562, 1982, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.48.1559>
- [39] E. Fort, A. Eddi, and A. Boudaoud, “Path-memory induced quantization of classical orbits,” *PNAS*, 2010. [Online]. Available: <https://www.pnas.org/doi/10.1073/pnas.1007386107>
- [40] M. V. Berry, R. G. Chambers, M. D. Large, C. Upstill, and J. C. Walmsley, “Wavefront dislocations in the Aharonov-Bohm effect and its water wave analogue,” *European Journal of Physics*, vol. 1, no. 3, pp. 154–162, 1980, publisher: IOP Publishing. [Online]. Available: <https://doi.org/10.1088/0143-0807/1/3/008>
- [41] C. Coste, F. Lund, and M. Umeki, “Scattering of dislocated wave fronts by vertical vorticity and the Aharonov-Bohm effect. I. Shallow water,” *Physical Review E*, vol. 60, no. 4, pp. 4908–4916, 1999, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevE.60.4908>
- [42] D. M. Harris and J. W. M. Bush, “Droplets walking in a rotating frame: from quantized orbits to multimodal statistics,” *Journal of Fluid Mechanics*, vol. 739, pp. 444–464, 2014, publisher: Cambridge University Press. [Online]. Available: <https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/abs/droplets-walking-in-a-rotating-frame-from-quantized-orbits-to-multimodal-statistics/29896D9B22B6B6D17328F82A35EEAE96>

- [43] A. U. Oza, D. M. Harris, R. R. Rosales, and J. W. M. Bush, “Pilot-wave dynamics in a rotating frame: on the emergence of orbital quantization,” *Journal of Fluid Mechanics*, vol. 744, pp. 404–429, 2014. [Online]. Available: https://www.cambridge.org/core/product/identifier/S0022112014000500/type/journal_article
- [44] A. U. Oza, R. R. Rosales, and J. W. M. Bush, “A trajectory equation for walking droplets: hydrodynamic pilot-wave theory,” *Journal of Fluid Mechanics*, vol. 737, pp. 552–570, 2013. [Online]. Available: https://www.cambridge.org/core/product/identifier/S0022112013005818/type/journal_article
- [45] A. U. Oza, O. Wind-Willassen, D. M. Harris, R. R. Rosales, and J. W. M. Bush, “Pilot-wave hydrodynamics in a rotating frame: Exotic orbits,” *Physics of Fluids*, vol. 26, no. 8, p. 082101, 2014. [Online]. Available: <http://aip.scitation.org/doi/10.1063/1.4891568>
- [46] A. Eddi, J. Moukhtar, S. Perrard, E. Fort, and Y. Couder, “Level Splitting at Macroscopic Scale,” *Physical Review Letters*, vol. 108, no. 26, p. 264503, 2012, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.108.264503>
- [47] P. J. Sáenz, G. Pucci, S. E. Turton, A. Goujon, R. R. Rosales, J. Dunkel, and J. W. M. Bush, “Emergent order in hydrodynamic spin lattices,” *Nature*, vol. 596, no. 7870, pp. 58–62, 2021. [Online]. Available: <https://www.nature.com/articles/s41586-021-03682-1>
- [48] “HDR-PJ10E Specifications | Sony UK.” [Online]. Available: <https://www.sony.co.uk/electronics/support/memory-camcorders-hdr-pj-series/hdr-pj10e/specifications>
- [49] G. Pucci, P. J. Sáenz, L. M. Faria, and J. W. M. Bush, “Non-specular reflection of walking droplets,” *Journal of Fluid Mechanics*, vol. 804, p. R3, 2016. [Online]. Available: https://www.cambridge.org/core/product/identifier/S0022112016005371/type/journal_article
- [50] B. Filoux, M. Hubert, P. Schlagheck, and N. Vandewalle, “Walking droplets in linear channels,” *Physical Review Fluids*, vol. 2, no. 1, p. 013601, 2017. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevFluids.2.013601>
- [51] R. Akis, D. K. Ferry, and J. P. Bird, “Wave Function Scarring Effects in Open Stadium Shaped Quantum Dots,” *Physical Review Letters*, vol. 79, no. 1, pp. 123–126, 1997. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.79.123>
- [52] J. L. Hansen and P. Cvitanovic, “Lyapunov Exponents for the Stadium Billiard,” 1995.

- [53] “55-5670 - Woofer with Paper/Kevlar Fiber Cone and Rubber Surround, Die Cast, 8", 80W RMS at 8 Ohm.” [Online]. Available: <https://www.newark.com/mcm-audio-select/55-5670/8-die-cast-woofer-8-ohm-rubbber/dp/95Y2936>
- [54] “NEMA 23, M60STH series.” [Online]. Available: https://www.motioncontrolproducts.co.uk/m/products/2/6/nema_23_high_torque_stepper_motors/
- [55] D. M. Harris, T. Liu, and J. W. M. Bush, “A low-cost, precise piezoelectric droplet-on-demand generator,” *Experiments in Fluids*, vol. 56, no. 4, p. 83, 2015. [Online]. Available: <https://doi.org/10.1007/s00348-015-1950-6>
- [56] “Piezoelectric buzzer MCABT-455-RC.” [Online]. Available: <https://fr.farnell.com/multicomp/mcabt-455-rc/element-audio-piezo-2-8khz-35mm/dp/2433035>
- [57] “Thorlabs - B3030A Nexus Breadboard.” [Online]. Available: <https://www.thorlabs.com/thorproduct.cfm?partnumber=B3030A>
- [58] “ADXL326.” [Online]. Available: <https://www.analog.com/media/en/technical-documentation/data-sheets/ADXL326.pdf>
- [59] “delayMicroseconds() - Arduino Reference.” [Online]. Available: <https://www.arduino.cc/reference/en/language/functions/time/delaymicroseconds/>
- [60] “ATmega328P datasheet.” [Online]. Available: https://ww1.microchip.com/downloads/en/DeviceDoc/Atmel-7810-Automotive-Microcontrollers-ATmega328P_Datasheet.pdf
- [61] “Optocoupler Datasheet 4N35 4N36 4N37.” [Online]. Available: <https://www.vishay.com/docs/81181/4n35.pdf>
- [62] “OpenCV: Gaussian Mixture-based Background/Foreground Segmentation : cv::BackgroundSubtractorMOG2 - Class Reference.” [Online]. Available: https://docs.opencv.org/3.4/d7/d7b/classcv_1_1BackgroundSubtractorMOG2.html
- [63] “OpenCV: Contours Detection - Structural Analysis and Shape Descriptors.” [Online]. Available: https://docs.opencv.org/3.4/d3/dc0/group__imgproc__shape.html#ga17ed9f5d79ae97bd4c7cf18403e1689a
- [64] “DRV8885 data sheet | TI.com.” [Online]. Available: <https://www.makerguides.com/wp-content/uploads/2019/02/DRV8825-Datasheet.pdf>
- [65] “OpenCV: Canny Edge Detection.” [Online]. Available: https://docs.opencv.org/3.4/da/d22/tutorial_py_canny.html

- [66] “OpenCV: Hough transform probabilistic.” [Online]. Available: https://docs.opencv.org/3.4/dd/d1a/group__imgproc__feature.html#ga8618180a5948286384e3b7ca02f6feeb
- [67] “OpenCV: Adaptive threshold.” [Online]. Available: https://docs.opencv.org/3.4/d7/d1b/group__imgproc__misc.html#ga72b913f352e4a1b1b397736707afcd3
- [68] “Object Tracking using OpenCV (C++/Python),” 2017. [Online]. Available: <https://learnopencv.com/object-tracking-using-opencv-cpp-python/>
- [69] A. Davey, “The growth of taylor vortices in flow between rotating cylinders,” *Journal of Fluid Mechanics*, vol. 14, no. 3, pp. 336–368, 1962.
- [70] A. Nachbin, P. A. Milewski, and J. W. Bush, “Tunneling with a hydrodynamic pilot-wave model,” *Physical Review Fluids*, vol. 2, no. 3, p. 034801, 2017.
- [71] L. Tadriss, T. Gilet, P. Schlagheck, and J. W. Bush, “Predictability in a hydrodynamic pilot-wave system: Resolution of walker tunneling,” *Physical Review E*, vol. 102, no. 1, p. 013104, 2020.
- [72] K. Papatryfonos, M. Ruelle, C. Bourdiol, A. Nachbin, J. W. Bush, and M. Labousse, “Hydrodynamic superradiance in wave-mediated cooperative tunneling,” *Communications Physics*, vol. 5, no. 1, pp. 1–7, 2022.
- [73] S. Mulligan, G. De Cesare, J. Casserly, and R. Sherlock, “Understanding turbulent free-surface vortex flows using a taylor-couette flow analogy,” *Scientific reports*, vol. 8, no. 1, pp. 1–14, 2018.
- [74] V. Frumkin, D. Darrow, J. W. Bush, and W. Struyve, “Real surreal trajectories in pilot-wave hydrodynamics,” *Physical Review A*, vol. 106, no. 1, p. L010203, 2022.
- [75] B.-G. Englert, M. O. Scully, G. Süssmann, and H. Walther, “Surrealistic bohm trajectories,” *Zeitschrift für Naturforschung A*, vol. 47, no. 12, pp. 1175–1186, 1992.
- [76] K. Papatryfonos, M. Ruelle, C. Bourdiol, A. Nachbin, J. W. M. Bush, and M. Labousse, “Hydrodynamic superradiance in wave-mediated cooperative tunneling,” *Communications Physics*, vol. 5, no. 1, pp. 1–7, 2022, number: 1 Publisher: Nature Publishing Group. [Online]. Available: <https://www.nature.com/articles/s42005-022-00918-y>
- [77] L. Tadriss, T. Gilet, P. Schlagheck, and J. W. M. Bush, “Predictability in a hydrodynamic pilot-wave system: Resolution of walker tunneling,” *Physical Review E*, vol. 102, no. 1, p. 013104, 2020, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevE.102.013104>

- [78] C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. F. Hopkins, and A. C. Gossard, “Conductance fluctuations and chaotic scattering in ballistic microstructures,” *Physical Review Letters*, vol. 69, no. 3, pp. 506–509, 1992, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.69.506>
- [79] L. Huang, Y.-C. Lai, D. K. Ferry, S. M. Goodnick, and R. Akis, “Relativistic Quantum Scars,” *Physical Review Letters*, vol. 103, no. 5, p. 054101, 2009. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.103.054101>
- [80] S. Washburn and R. A. Webb, “Aharonov-Bohm effect in normal metal quantum coherence and transport,” *Advances in Physics*, vol. 35, no. 4, pp. 375–422, 1986, publisher: Taylor & Francis _eprint: <https://doi.org/10.1080/00018738600101921>. [Online]. Available: <https://doi.org/10.1080/00018738600101921>
- [81] B. Hackens, F. Martins, T. Ouisse, H. Sellier, S. Bollaert, X. Wallart, A. Cappy, J. Chevrier, V. Bayot, and S. Huant, “Imaging and controlling electron transport inside a quantum ring,” *Nature Physics*, vol. 2, no. 12, pp. 826–830, 2006. [Online]. Available: <http://www.nature.com/articles/nphys459>
- [82] F. Martins, B. Hackens, M. G. Pala, T. Ouisse, H. Sellier, X. Wallart, S. Bollaert, A. Cappy, J. Chevrier, V. Bayot, and S. Huant, “Imaging Electron Wave Functions Inside Open Quantum Rings,” *Physical Review Letters*, vol. 99, no. 13, p. 136807, 2007, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.99.136807>
- [83] J. Wurm, M. Wimmer, H. U. Baranger, and K. Richter, “Graphene rings in magnetic fields: Aharonov–Bohm effect and valley splitting,” *Semiconductor Science and Technology*, vol. 25, no. 3, p. 034003, 2010. [Online]. Available: <https://iopscience.iop.org/article/10.1088/0268-1242/25/3/034003>
- [84] D. Cabosart, S. Faniel, F. Martins, B. Brun, A. Felten, V. Bayot, and B. Hackens, “Imaging coherent transport in a mesoscopic graphene ring,” *Physical Review B*, vol. 90, no. 20, p. 205433, 2014. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevB.90.205433>
- [85] J. Dauber, M. Oellers, F. Venn, A. Epping, K. Watanabe, T. Taniguchi, F. Hassler, and C. Stampfer, “Aharonov-Bohm oscillations and magnetic focusing in ballistic graphene rings,” *Physical Review B*, vol. 96, no. 20, p. 205407, 2017, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevB.96.205407>
- [86] D. Cabosart, A. Felten, N. Reckinger, A. Iordanescu, S. Toussaint, S. Faniel, and B. Hackens, “Recurrent Quantum Scars in a Mesoscopic Graphene Ring,” *Nano Letters*, vol. 17, no. 3, pp. 1344–1349, 2017,

- publisher: American Chemical Society. [Online]. Available: <https://doi.org/10.1021/acs.nanolett.6b03725>
- [87] E. J. Heller, “Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits,” *Physical Review Letters*, vol. 53, no. 16, pp. 1515–1518, 1984, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevLett.53.1515>
 - [88] Y. Okubo, J. P. Bird, Y. Ochia, D. K. Ferry, K. Ishibashi, Y. Aoyagi, and T. Sugano, “Magnetically induced suppression of phase breaking in ballistic mesoscopic billiards,” *Physical Review B*, vol. 55, no. 3, pp. 1368–1371, 1997. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevB.55.1368>
 - [89] R. Akis, J. P. Bird, and D. K. Ferry, “The persistence of eigenstates in open quantum dots,” *Applied Physics Letters*, vol. 81, no. 1, pp. 129–131, 2002, publisher: American Institute of Physics. [Online]. Available: <https://aip.scitation.org/doi/abs/10.1063/1.1490404>
 - [90] S. Tomsovic and E. J. Heller, “Long-time semiclassical dynamics of chaos: The stadium billiard,” *Physical Review E*, vol. 47, no. 1, pp. 282–299, 1993. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevE.47.282>
 - [91] K. E. Aidala, R. E. Parrott, T. Kramer, E. Heller, R. Westervelt, M. P. Hanson, and A. C. Gossard, “Imaging magnetic focusing of coherent electron waves,” *Nature Physics*, vol. 3, no. 7, pp. 464–468, 2007.
 - [92] D. M. Harris, J. Moukhtar, E. Fort, Y. Couder, and J. W. M. Bush, “Wavelike statistics from pilot-wave dynamics in a circular corral,” *Physical Review E*, vol. 88, no. 1, p. 011001, 2013, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevE.88.011001>
 - [93] T. Gilet, “Quantumlike statistics of deterministic wave-particle interactions in a circular cavity,” *Physical Review E*, vol. 93, no. 4, p. 042202, 2016, publisher: American Physical Society. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevE.93.042202>
 - [94] P. J. Sáenz, T. Cristea-Platon, and J. W. M. Bush, “A hydrodynamic analog of Friedel oscillations,” *Science Advances*, vol. 6, no. 20, p. eaay9234, 2020, publisher: American Association for the Advancement of Science. [Online]. Available: <https://www.science.org/doi/10.1126/sciadv.aay9234>
 - [95] P. J. Sàenz, T. Cristea-Platon, and J. W. M. Bush, “Statistical projection effects in a hydrodynamic pilot-wave system,” *Nature Physics*, vol. 14, no. 3, pp. 315–319, 2018, number: 3 Publisher: Nature Publishing Group. [Online]. Available: <https://www.nature.com/articles/s41567-017-0003-x>

- [96] M.-H. Liu, P. Rickhaus, P. Makk, E. Tóvári, R. Maurand, F. Tkatschenko, M. Weiss, C. Schönenberger, K. Richter *et al.*, “Scalable tight-binding model for graphene,” *Physical review letters*, vol. 114, no. 3, p. 036601, 2015.

UNIVERSITÉ CATHOLIQUE DE LOUVAIN
École polytechnique de Louvain

Rue Archimède, 1 bte L6.11.01, 1348 Louvain-la-Neuve, Belgique | www.uclouvain.be/epl