

École polytechnique de Louvain

The effects of Coriolis forces on the wake phenomenon within a wind farm: a Large Eddy Simulation study

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Abstract

Enhancing our insight into wind farm physics is key in order to improve wind energy production capabilities thereby facilitating energetic transition. Different phenomena should therefore be studied such as the impact of the wind farm configuration or the wind turbine rotational direction. The behaviour of the wind in the Atmospheric Boundary Layer (ABL) is governed by multiple phenomena such as the Coriolis effect. In this work, the implementation in an in-house developed Navier-Stokes equation solver and the validation of a Coriolis effect model is presented. The validation process is performed against two different test cases in Direct Numerical Simulation (DNS): a rotating channel with $Re_\tau = 194$ and a laminar Ekman layer; and over one wall-modeled Large Eddy Simulation (wmLES) case: the turbulent Ekman layer. The impact of this effect on a small wind farm is then studied. The turbines are modeled with actuator disks. The results indicate a positive impact of the Coriolis Effect on the power output of the wind farm and it also induces a deformation of the wake combined with a deflection of its trajectory. Finally, the last study focuses on the effect of the rotational direction of the wind turbines. Some differences were observed in the wake deformation but no major impacts were noticed in term of energy production. The trajectory of the wake is also slightly influenced by the rotating direction of the turbines.

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Chapter 1

Introduction

Nowadays, the climate change is the center of concern for a lot of people around the world. One of the main reason is the emission of greenhouse gasses which has been growing over the years. Between 1970 and 2014, the greenhouse emissions have increased by 90% all around the world. 25% of these emissions come from the production of electricity and heat [1]. In Belgium, 30% of the electricity is produced by the nuclear power plant and another 30% is produced by gas centrals [2]. It is a fact that those two means of electricity production are controversy for their radioactive wastes or their greenhouse gases emissions. To overcome these issues, some other electricity production technologies should be developed such as the wind turbines, solar energy and biomass. Within Belgium, the solar and the wind energy production have become more and more popular. The number of times we reached the target of, 50% of the electricity consumed was produced by solar and wind energy, was almost 5 days in 2020. In 2020, during 119 hours more than 50% of the electricity consumed was produced by the solar and the wind energies. It is clearly not enough and we cannot rely only on just those two energies even-though, the electricity produced by offshore wind farm has increased by 40% between 2019 and 2020 [3]. This highlights the willingness of the Belgian government to move towards a more renewable energy source. But we are still far from producing 100% of our electricity thru renewable energy. To accelerate the energetic transition, men and engineers need to study further how to optimize the production of energy thru those new sources.

For this transition, the wind energy is considered as one of the new sources. This solution presents a lot of advantages such as unlimited and free renewable energy once the farms are installed. It has a low maintenance cost and does not emit any greenhouse gasses [4]. Some of the downsides of this electricity source are the installation costs, the deterioration of the landscape, the noise and the large dependency on weather conditions which pushes engineers to innovate and find locations with the best wind conditions. One solution is to gather multiple turbines together to form a wind farm and to install them in the sea. This helps in reducing installation and energy transport costs. Even-though the installation of offshore wind farms are more expensive than onshore ones, the wind conditions are usually better as the wind blows harder in a more uniform way [5]. In Belgium, the installation of these wind farms have already started a couple of years ago. At the Belgian coast, already eight wind farms have been installed, for a total production of 6.7 TWh of electricity in 2020 [6].

The position of each turbine is very important within a wind farm in order to avoid too many interactions between each of them. As it is shown on Figure 1.1, when the turbines are aligned with the wind direction, the wakes of the first turbines enter the downstream ones. This has a strong impact on the energy production of the wind farm. In order to increase the energy, differ-

1.1. SCOPE OF THIS THESIS

ent studies are in progress to define the perfect wind farm configuration, the yaw angle and the rotation direction of the wind turbines. These studies are conducted via numerical simulations and it requires a proper atmospheric boundary layer to obtain realistic results.

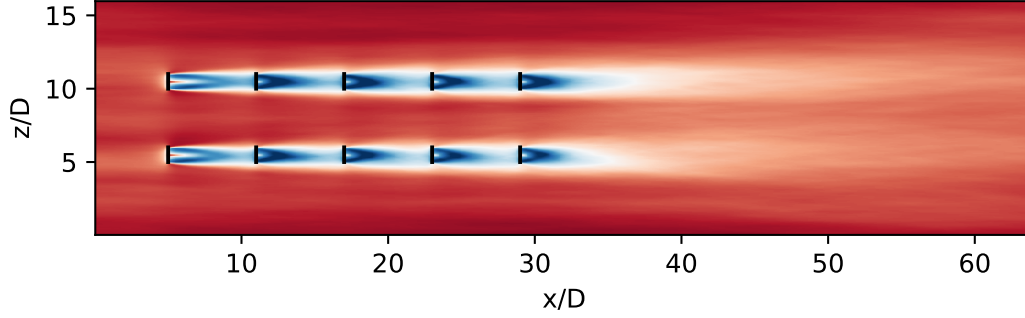


Figure 1.1: Wake interaction in a simulated wind farm. The black lines mark the wind turbines position.

The atmospheric boundary layer (ABL) is the section of air between the surface of the earth and the free atmosphere. This is the lowest layer of the troposphere and it is largely influenced by the presence of the earth surface [7]. Some of the important phenomena that govern the wind in the ABL are the Coriolis effect and the surface heat fluxes which can induce different state of the ABL such as a convective ABL or an unstable ABL. The ABL simulation is one of the ongoing research topic in the scientific world for its multiple physical applications such as wind farm simulations or understanding the pollutant dispersion.

1.1 Scope of this thesis

With a high Reynolds number ($\sim 10^8$), the ABL is a turbulent flow. Its simulation requires the use of the wall-modeled Large Eddy Simulation (wmLES) [8]. The ABL can be divided in two different sublayers. The lowest one is the surface layer which highly depends on the surface roughness. This part is modelled with a wall model and was implemented in BigFlow by Thiry [9]. Above the surface layer is the Ekman layer. In this second layer, the effects of the surface are getting less and less important and it is the Coriolis effect that becomes dominant. It is in this layer that the rotor of the wind turbines are located. This thesis aims at implementing the Coriolis effect in the in-house developed fourth-order finite difference Navier-Stokes equation solver BigFlow.

This implementation is used to simulate the atmospheric boundary layer in which the behaviour of a small wind farm will be studied. The Coriolis effect has a strong impact on the wind direction within the height of the Ekman layer. The variation of the wind direction is the result of the variation of the Coriolis force intensity and direction with the altitude. To optimize the energy production of a wind farm, it is then essential to understand the impact of this effect on the wake and the deformation that it induces. This study is done on a small wind farm with advanced actuator disk models implemented by M. Moens [10].

To date, few studies have analysed the impact of the rotational direction of the wind turbines. Looking in the streamwise direction, the majority of the wind turbines rotate in the clockwise direction in both hemisphere. But the behaviour of the wind is different in each hemisphere due to the opposition of the Coriolis force. A good example is the direction of rotation of a cyclone which is opposed in both hemispheres. Changing the rotational direction of the wind turbines could potentially impact the energy production of a wind farm. In fact, during the summer of 2020, an article from the Journalism for the energy transition [11] was published in Germany stating that inverting the wind turbines direction of rotation in the northern hemisphere could increase the power output by 11% and up to 23% under specific conditions. This article was supported by a study from the Deutsches Zentrum für Luft [12]. Such an increase of output would be extraordinary and could have a strong impact on our energy production and the transition towards renewable energy. In this thesis, a similar study is conducted to try to understand the impact of the rotational direction in an ABL with the Coriolis effect.

1.2 Outlines

This thesis is divided in three consecutive chapters. The Chapter 2 describes the Coriolis effect and its implementation in BigFlow. This implementation is then validated against three reference cases. The first one is a rotating channel, followed by the laminar Ekman layer and the validation process ends with the validation of the wall-modeled Large Eddy Simulation (wmLES) implementation against a Direct Numerical Simulation (DNS) of the Ekman turbulent layer.

We will continue in Chapter 3 with a small validation of the co-current simulation. Then the study of the impact of the Coriolis effect on a wind farm is described. For this study, two different precursors were converged, one with and one without the Coriolis effect. A dedicated section compares the wind conditions in both simulations. After that, the analysis of the wind farm in the different wind conditions can start. The first analysis is the structure and the deformation of the wake. It is followed by an analysis of the power output of the wind farm.

And finally, in Chapter 4, the impact of the rotational direction of the wind turbines will be studied on a small wind farm. Once both clockwise and counter-clockwise simulations are done, we will study the wake deformation, the efficiency of the farm and the angular velocity in the wake and compare the results in both cases.

Chapter 2

Implementation and validation of the Coriolis effect in BigFlow

The Coriolis force is an inertial force, it then appears when a body is in motion. This force is a fictitious force and it appears when the reference frame rotates with respects to the inertial frame. The direction of the force is always perpendicular to the body/fluid motion and it induces a deflection of the trajectory of the body/flow. This effect allows the scientists to explain multiple meteorological phenomena that happen in the atmosphere and in the ocean. Some example could be the direction of the wind, the Ekman layer and the Rossby wave [13]. This force strongly depends on the latitude of the body and the hemisphere. The hemisphere dependency can be visualized by the direction of rotation of the cyclone and anti-cyclone. In the northern hemisphere the cyclone rotates in the counter-clockwise direction and the opposite in the southern hemisphere. This directly results from the orientation of the Coriolis force which is oriented towards the right in the northern hemisphere and towards the left in the southern one.

In this chapter, the solver BigFlow is firstly described and then the implementation of a Coriolis effect model in this solver is presented. BigFlow is the solver used for this thesis. The validation of the implementation is then performed against three different simulations. The first validation test case is a rotating channel, the second one is the simulation of the laminar Ekman layer and the last validation test case is the simulation of the turbulent Ekman layer.

2.1 BigFlow solver

The simulation are done using an in-house developed fourth order finite difference code named BigFlow and developed by the thermodynamics and fluid mechanics lab of UCLouvain. This solver allows to solve the Navier-Stokes equation with different method such as Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and Reynolds Averaged Navier-Stokes Simulation (RANS). For the purpose of this thesis the DNS and LES version of the code are used. In the LES framework, a subgrid scale model is added in the Navier-Stokes equation.

$$\nabla \cdot \mathbf{v} = 0 \quad (2.1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \nabla \cdot (\tau + \tau^{sgs}) + \mathbf{F} \quad (2.2)$$

where $P = p/\rho$, τ represents the shear stress, τ^{sgs} is the stress tensor modeling the effect of the subgrid scales turbulence on the resolved scales and $\mathbf{F}$ is a volumetric force. The grid is a structure grid with a staggered configuration (*cfr*: Figure 2.1). This means that the different velocities and the pressure are computed at different locations in the cell.

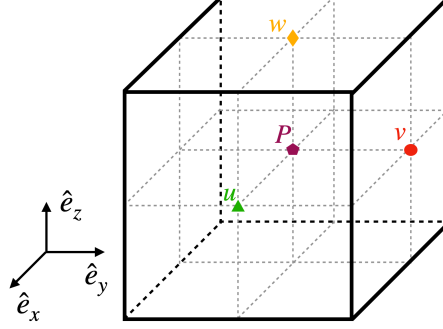


Figure 2.1: Staggered mesh representation

The viscous stress is defined as $\tau_{ij} = 2\nu S_{ij}$ with S_{ij} the strain rate tensor.

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2.3)$$

To model the subgrid scale effects, the Smagorinsky eddy viscosity model was chosen for the purpose of this thesis [14]. This model was implemented by M. Duponcheel during his PhD [15]. With this model, the SGS stress tensor τ^{sgs} is defined as

$$\tau_{ij}^{sgs} = 2\nu^{sgs} S_{ij} \quad (2.4)$$

$$\nu^{sgs} = C\Delta^2 (2S_{ij}S_{ij})^{\frac{1}{2}} \quad (2.5)$$

where C is the Smagorinsky constant sets to 0.027 and Δ is a length scale of the order of the grid spacing. Since the meshes are non uniform Cartesian meshes, Δ is defines as $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$.

In an ABL, the roughness of the ground and the large Reynolds number induced a strong velocity gradient at the wall and a turbulent flow. The simulation of the velocity field in the near wall region by DNS would require a super refine mesh in this region to properly capture the smaller energy-carrying eddies. To avoid such an expensive simulation, a wall model is used. The wall model aims at computing an appropriate wall shear stress based on the friction velocity (Equation 2.6) and a law of the wall. Different law of the wall were implemented by O. Thiry [9] such as the log law (*cfr*: equation 2.7).

$$\tau_w = \rho \frac{\mathbf{u}|\mathbf{u}|}{|\langle \mathbf{u}|\mathbf{u} \rangle|} u_\tau^2 \quad (2.6)$$

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \log \left(\frac{y}{y_0} \right) + C \quad (2.7)$$

where $\langle \cdot \rangle$ stands for the norm of the vector, $|\cdot|$ is the spatial averaging operator, u_τ is the friction velocity, κ is the von Karman constant sets to 0.38 in BigFlow and C is a constant.

2.2. IMPLEMENTATION OF THE CORIOLIS EFFECT INTO BIGFLOW

The last part of BigFlow that must be described is the the stencil reduction near a wall model. In the center of the mesh the differentiation operator is a fourth order operator. In the direction ξ_1 , with a stencil of width $n\Delta_1$ (Δ_1 is the grid spacing in the direction 1 in the computational domain ξ), the differentiation operator can be written as

$$\frac{\delta_n \phi}{\delta_n \xi_1} \triangleq \frac{\phi(\xi_1 + \frac{n}{2}\Delta_1, \xi_2, \xi_3) - \phi(\xi_1 - \frac{n}{2}\Delta_1, \xi_2, \xi_3)}{n\Delta_1} \quad (2.8)$$

$$J_{i,n} \triangleq \frac{\delta_n x_i}{\delta_n \xi_i} \quad (2.9)$$

where ξ is the variable of the mapping transformation and $J_{i,n}$ is the Jacobian of the mapping and without summation over the index, i . With the different notations introduced, the fourth order stencil for the first derivative is given by:

$$\frac{\partial \phi}{\partial x_i} \approx \frac{9}{8} \frac{1}{J_{i,1}} \frac{\delta_1 \phi}{\delta_1 \xi_i} - \frac{1}{8} \frac{1}{J_{i,3}} \frac{\delta_3 \phi}{\delta_3 \xi_i} \quad (2.10)$$

This method can require up to three layers of points behind a boundary. The value of those ghost points is set to respect the value of the boundary. When a wall model is applied, the information contained in the ghost point for the parallel velocity components are not relevant and so cannot be used. The stencil order in the wall normal direction is then reduced to a second order stencil in the cell next to the wall when a wall model is applied. The second order differentiation operator in the direction i can be defined as:

$$\frac{\partial \phi}{\partial x_i} \approx \frac{1}{2} \frac{1}{J_{i,1}} \frac{\delta_1 \phi}{\delta_1 \xi_i} \quad (2.11)$$

This reduction of stencil was also used in the implementation of the Coriolis effect and is described in the next section.

2.2 Implementation of the Coriolis effect into BigFlow

The Coriolis effect is considered as an additional source term in the momentum conservation equation. This force has three different components (one for each direction) with different amplitudes. The momentum conservation equation can be expressed as

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + Fc_i \quad (2.12)$$

where Fc_i is the Coriolis source term in the i -th direction.

This source term is obtained by taking the cross product between the rotation velocity, $\mathbf{\Omega}$, of the reference frame and the fluid velocity, $\mathbf{u}$. It can be expressed as:

$$\mathbf{Fc} = -2\mathbf{\Omega} \times \mathbf{u} \quad (2.13)$$

where, $\mathbf{\Omega} = \Omega \begin{pmatrix} 0 \\ \sin\phi \\ -\cos\phi \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$ as shown in Figure 2.2. The Coriolis source term can be developed and expressed in its vector form.

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$$\mathbf{F}_c = -2\Omega \begin{pmatrix} w \sin\phi + v \cos\phi \\ -u \cos\phi \\ -u \sin\phi \end{pmatrix} \quad (2.14)$$

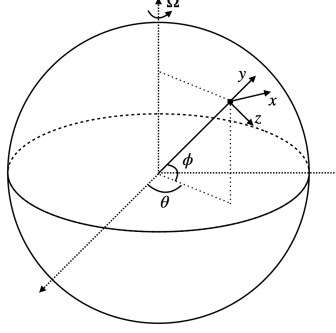


Figure 2.2: Coordinate on a rotating sphere [16]

In order to compute the Coriolis force in the x -direction, it is required to know the velocity v and w at the location of u and the same applies for the computation of the other components of the Coriolis force. Due to the staggered mesh configuration, it is required to interpolate two times each velocity. The first interpolation computes the velocity on the corner of the cell. The second interpolation uses the corner velocities to compute the velocity at the new positions. This manipulation of velocity is represented in Figure 2.3.

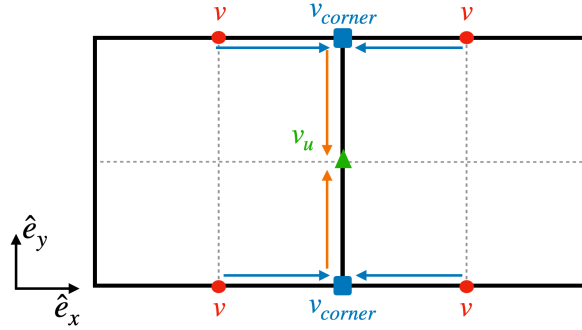


Figure 2.3: Representation of the interpolations to compute the velocity v at the node u (v_u). The blue arrows represent the first interpolation and the orange arrows represent the second interpolation.

The BigFlow code is implemented with fourth order difference schemes. To stay consistent with the solver a fourth order centered interpolation scheme is used which required to know the velocity values two nodes away on each sides of the computation node. The formula used for the first and second interpolations are respectively:

$$f_{i+1/2} = \frac{9(f_i + f_{i+1}) - (f_{i-1} + f_{i+2})}{16} \quad (2.15)$$

$$f_{j-1/2} = \frac{9(f_j + f_{j-1}) - (f_{j-2} + f_{j+1})}{16} \quad (2.16)$$

Similarly to the problem faced to solve the Navier-Stokes equation with a wall model, the fourth order scheme cannot always be use. A second order stencil is then applied for the interpolations in the wall normal direction near the wall model. The Figure 2.4 represents the variations of the stencil order. This reduces the order of precision of the method at the boundary. The Coriolis effect being directly proportional to the velocity, which is very small close to the wall, the order of magnitude of this force is less relevant in that part of the mesh. The general computation should then not be affected by this manipulation.

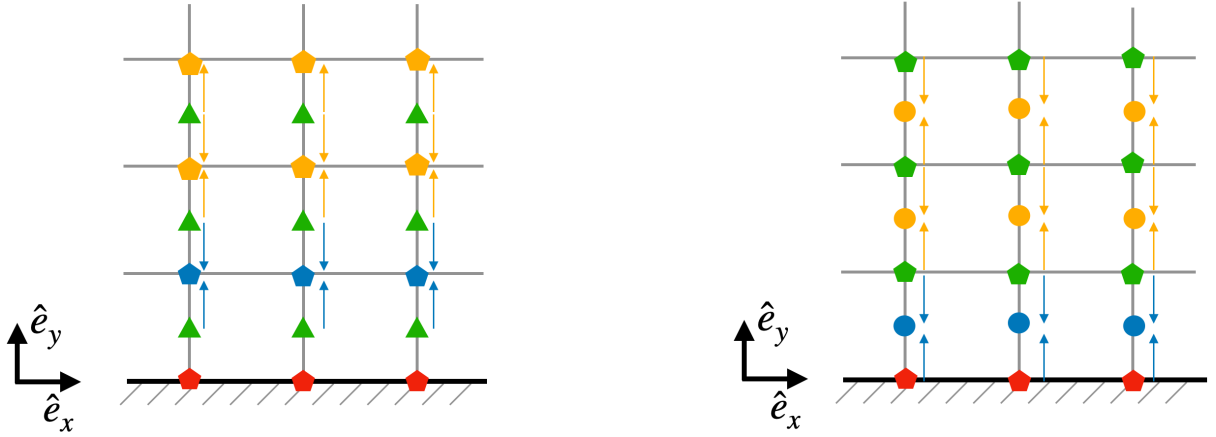


Figure 2.4: Sketch of a xy cut at the middle of the cells in the computational domain. Left is the first interpolation and Right is the second interpolation. Yellow: stencil order 4, Blue: stencil order 2, Green: values used for the interpolation, Red: boundary condition values, Triangle: velocity u , Pentagon: corner velocity and Circle: velocity v at the u location (v_u)

2.3 Validation of the implementation

The validation of the implementation is performed against three different simulations. The goal is to validate the implementation in the Direct Numerical Simulation (DNS) and in the wall-modeled Large Eddy Simulation (wmLES) frameworks, the latter simulation type being the one selected to simulate the turbulent Ekman layer at $Re = 2000$ and the atmospheric boundary layer with the wind turbines. The atmospheric boundary layer is highly turbulent with a Reynolds number equivalent to $\sim 10^6 - 10^8$ [17]. The wmLES framework offers a much cheaper simulation method that models the low scale turbulent structures than the DNS framework for such simulations. It is then important to validate the implementation for this type of simulation as well.

2.3.1 Rotating channel in DNS

This first simulation was based on the simulation made by R. Kristoffersen and H. I. Anderson in their article "*Direct simulations of low-Reynolds-number turbulent flow in a rotating channel*" [18].

2.3. VALIDATION OF THE IMPLEMENTATION

2.3.1.1 Numerical set-up

This validation case is a rotating channel where the flow is driven by a pressure gradient. This simulation is solved at relatively low Reynolds number. The channel is represented by two plates in the y -directions. A no-slip no-through boundary condition for the velocity is applied on those two plates. In the x -direction and the z -direction, periodic boundary conditions are applied. The channel is rotating around the z -axis. The rotation velocity can be written in the following form $\mathbf{\Omega} = (0, 0, \Omega)$. The numerical set-up is depicted in Figure 2.5.

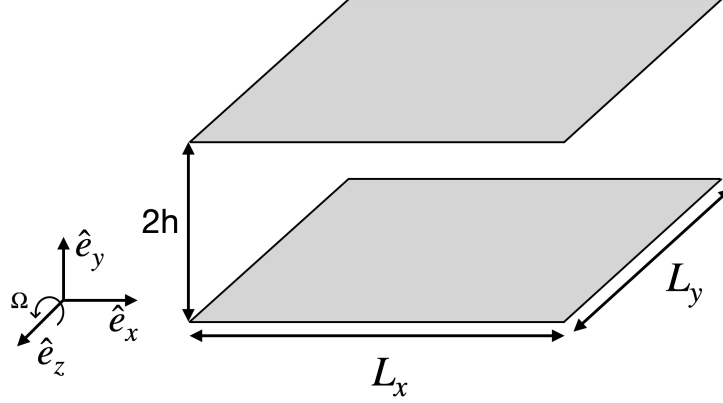


Figure 2.5: Representation of the set-up for the rotating channel validation case.

The streamwise direction is along the x -axis and is defined by the driven pressure gradient. The velocity profile is homogeneous along this direction as well as along the z -axis. Later in this section, some statistics of the simulation are presented and are averaged in time and in the x -direction and z -direction.

The physical dimensionless size of the channel are $\frac{L_x}{h} \times \frac{L_y}{h} \times \frac{L_z}{h} = 4\pi \times 2 \times 2\pi$, the grid dimension is $N_x \times N_y \times N_z = 128 \times 128 \times 128$ and the resolution is constant in the streamwise and z -direction. The resolution in the wall normal direction varies with respect to the height. Next to the wall, the resolution is $\Delta y_{min}^+ \approx 0.5$. The minimum resolution in the wall normal direction is at the center of the channel and worth $\Delta y_{max}^+ \approx 5$. The wall coordinate system, y^+ , is defined as the distance from the wall with respects to the shear velocity and the viscosity.

$$y^+ = \frac{y u_\tau}{\nu} \quad (2.17)$$

The Coriolis parameters is $f = 7.5 [s^{-1}]$ for a rotation speed of $\Omega = 3.75 [s^{-1}]$. Two dimensionless numbers are used to characterize this simulation. The first one, Re_τ , is defined as the ratio of inertial forces to viscous forces.

$$Re_\tau = \frac{u_\tau h}{\nu} \quad (2.18)$$

where ν is the kinematic viscosity and u_τ is the shear velocity. In this simulation, Re_τ is equal to 194 and the normal Reynolds number based on the bulk velocity is close to 3050.

The second dimensionless number is the rotation number, Ro_τ , that characterizes the ratio between the rotation speed and the shear velocity [18].

$$Ro_\tau = \frac{2\Omega h}{u_\tau} \quad (2.19)$$

In this simulation, the rotation number expressed with the shear velocity is 7.8 but it could also be expressed with the bulk velocity (equation (2.20)). In that case, it is worth 0.5.

$$Ro = \frac{2|\Omega|h}{U_m} = |Ro_\tau| \frac{u_\tau}{U_m} \quad (2.20)$$

Those dimensionless numbers allow to write the Navier-Stokes equation with the Coriolis effect in a dimensionless form (equation (2.21) and (2.22)).

$$\nabla \cdot \mathbf{u} = 0 \quad (2.21)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p_{eff} + \frac{1}{Re_\tau} \nabla^2 \mathbf{u} - Ro_\tau \frac{\boldsymbol{\Omega}}{\Omega} \times \mathbf{u} \quad (2.22)$$

where p_{eff} is the dimensionless effective pressure. The effective pressure absorbs the pressure in the channel and the effect of the centrifugal force due to the rotation of the channel.

2.3.1.2 Results analysis

The results include an analysis of the velocity in the wall coordinate system which was also presented in the Kristoffersen and Andersson [18]. The wall coordinate systems are separated in two different coordinate systems which are respectively located at the surface of each wall. Those two walls are called the pressure and suction side. On the pressure side wall, the pressure is higher than the average pressure due to the rotation of the channel while it is the opposite on the suction side. Both walls have their own shear velocities that are denoted $u_{\tau s}$ and $u_{\tau p}$ for the suction and the pressure side wall. Their values were computed at the end of the simulation and are visible at equations (2.23) and (2.24). In a non rotating channel, the shear velocities are equals at both boundary. This shows a first impact of the rotation.

$$u_{\tau s} = 0.7 \quad [m/s] \quad (2.23)$$

$$u_{\tau p} = 1.2 \quad [m/s] \quad (2.24)$$

The average shear velocity can be computed by the root mean square value of the two shear velocities and is worth 0.98. The wall coordinates can be expressed with those two reference shear velocities. The expression of the location along the y -axis in the wall coordinate can be seen in equations (2.25) and (2.26).

$$y_s^+ = (1 + y) \frac{u_{\tau s}}{\nu} \quad (y < 0) \quad (2.25)$$

$$y_p^+ = (1 - y) \frac{u_{\tau p}}{\nu} \quad (y > 0) \quad (2.26)$$

The velocity can also be expressed in the wall coordinate system.

$$u^+ = \frac{u}{u_{\tau s}} \quad (y < 0) \quad (2.27)$$

$$u^+ = \frac{u}{u_{\tau p}} \quad (y > 0) \quad (2.28)$$

To validate the implementation of the Coriolis effect, the results are compared against that from Kristoffersen and Andersson [18]. In their article, the simulation was done over a period of 100 *letot*'s. A *letot* is a Large Eddy Turn Over Time and it is the reference time used to compare two simulations. This time unit can be defined as:

$$letot = \frac{h}{u_\tau} \quad [s] \quad (2.29)$$

2.3. VALIDATION OF THE IMPLEMENTATION

where , u_τ is the RMS value of the two shear velocities. In this case, the simulation was conducted only over a period of 42 time units to reduce the computation cost. Kristoffersen and Andersson used a pseudo-spectral method to simulate the channel. The results presented are averaged in time over a period of 8 letot as it was done in A. Boundin and V. Vanden Hoek master thesis [13]. These results are also averaged in the streamwise and z -directions.

The streamwise velocity is shown in Figure 2.6. On the top figure, the instantaneous velocity is display, and different turbulent structures are visible. The bottom figure represents the average of the velocity in time and in the z -direction along the channel. The average velocity figure shows that the maximum of the streamwise is shifted toward the suction side ($Y = -h$). The average velocity profile of the bottom figure can also be average in the streamwise direction and it is represented in Figure 2.7.

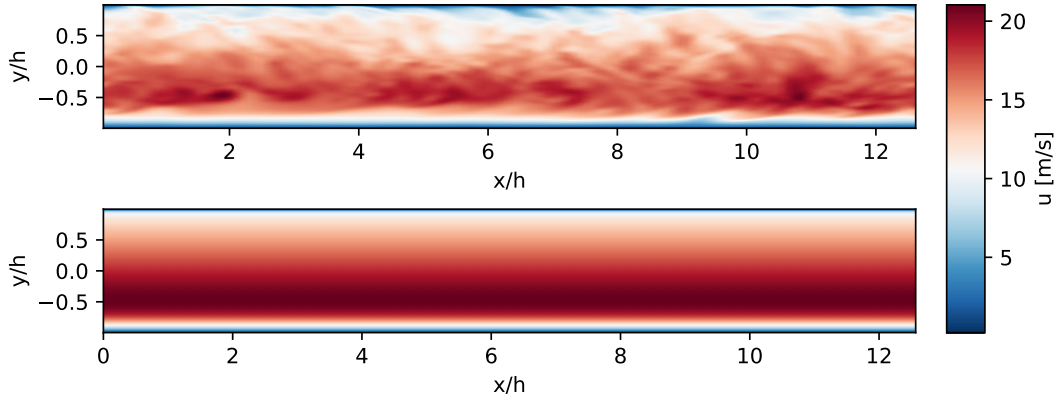


Figure 2.6: Top figure: Instantaneous velocity at $t = 42$ [letot] in the middle of the channel width. Bottom figure: average velocity in time and in the z -direction.

The average velocity profile in the streamwise direction has an asymmetry due to the rotation of the channel (Figure 2.7). The slope of the linear part of the velocity profile directly depends on the rotation velocity and should be equal to -2Ω . This slope is represented in Figure 2.7 and the BigFlow simulation results follow this trend. Based on this velocity profile the mean bulk velocity profile can be calculated and is equal to $15.8u_\tau$.

In Figure 2.8, the mean streamwise velocity profile is represented in wall coordinate systems. The velocity profiles are based on the local friction velocity. They are compared to the law of the wall for a turbulent channel without rotation presented in Graf article [19]. This profile is the same for the two boundaries. It is clear that the rotation of the channel has an impact on the velocity profiles. The velocity on the suction side has increased compared to the non rotating channel and on the pressure side the mean velocity profile has decreased. The suction side exhibits a profile that approaches a linear law of the wall profile with $U^+ = Y^+$. The location of the maximum of the streamwise velocity is shifted from the middle of the channel therefore the velocity profile on the suction side starts to decrease before the middle of the channel. The pressure side presents a more logarithmic profile.

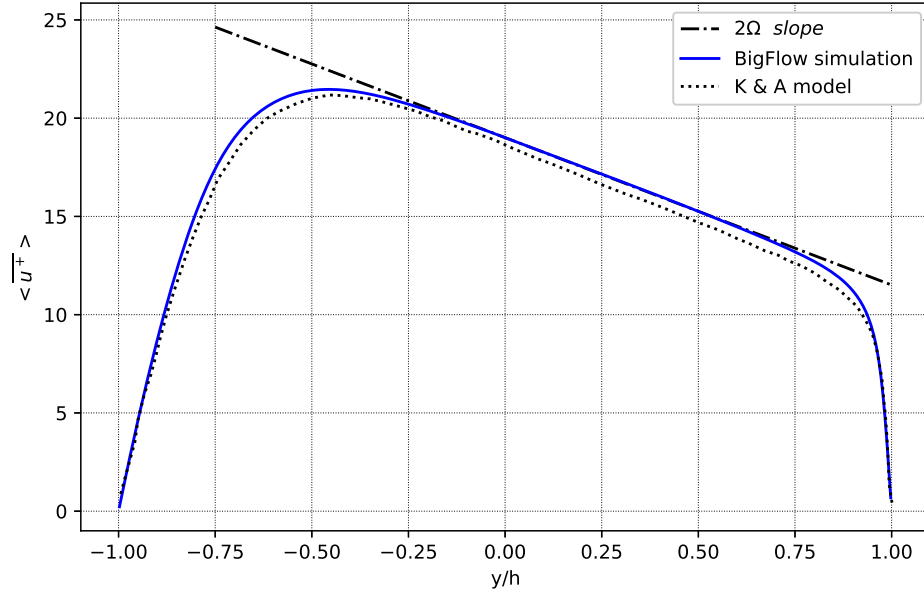


Figure 2.7: Mean streamwise velocity profile in global coordinate system. Dash-dotted Black: slope of -2Ω , Dotted Black: Kristoffersen Andersson results [18] and Solid Blue: BigFlow simulation.

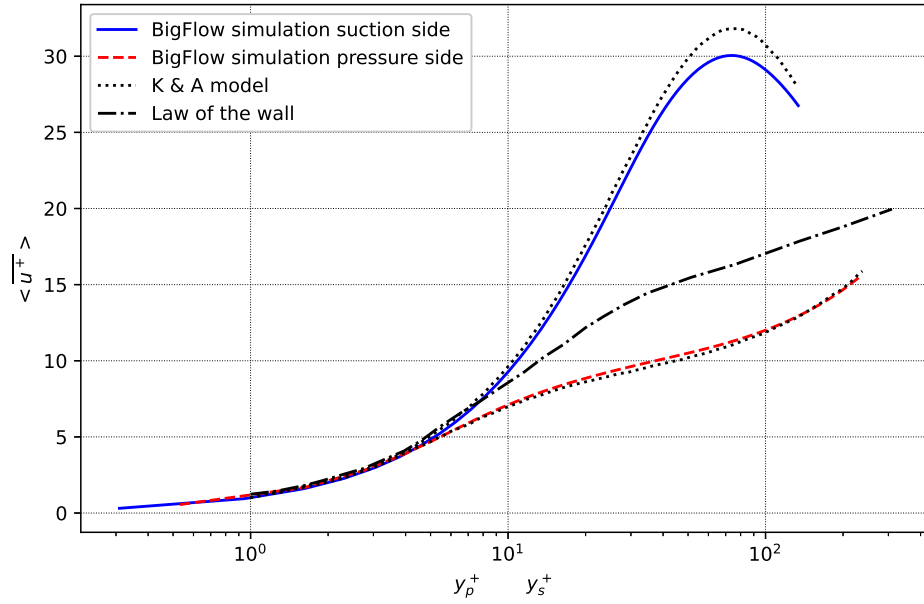


Figure 2.8: Streamwise mean velocity profiles in wall coordinate system. Dashed Red: BigFlow simulation pressure side, Solid Blue: BigFlow simulation suction side, Dotted Black: Kristoffersen and Andersson results [18], Dash-dotted Black: Law of the wall profile for non rotating channel [19] .

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Additionally, the profiles of the non zero elements of the Reynolds stress tensor are shown in Figures 2.9, 2.10, 2.11 and 2.12. In those Figures, u' , v' and w' represent the fluctuating velocity that can be computed with the Reynolds decomposition. The different profiles present a maximum on the pressure side due to the rotation of the channel. The Reynolds stresses are strongly affected by the rotation. As it was mentioned in equation (2.22), the rotation number explicitly enters the Navier-Stokes equation and so directly impact the behaviour of the flow [20].

If the flow is fully established, the Reynolds shear stress ($\overline{u'v'}$) should present a large linear profile at the center of the channel [21]. This linear profile can be seen in Figure 2.9. Based on this profile, it can be assumed that the flow is almost fully established. In the vicinity of the suction wall, the Reynolds shear stress almost completely vanished [22]. This phenomenon was also observed by Kristoffersen and Andersson and according to them, it is due to the nearly complete disappearance of the normal to the wall fluctuating velocities (*cfr*: Figure 2.11). The pick of the Reynolds stress ($u'u'$) normally appearing at each wall is also damped on the suction side (Figure 2.10) which also tends to reduce $\overline{u'v'}$. The total kinematic shear stress is also computed. It sums the Reynolds shear stress and the viscous stress $\frac{\tau}{\rho} = -\langle \overline{u'v'} \rangle + \nu \frac{dU}{dy}$ [23]. This profile is also fully linear with extremum values at the boundary. The null total kinetic shear stress is located at $y/h = -0.5$, while in a non rotating channel, it would be at the middle $y/h = 0$.

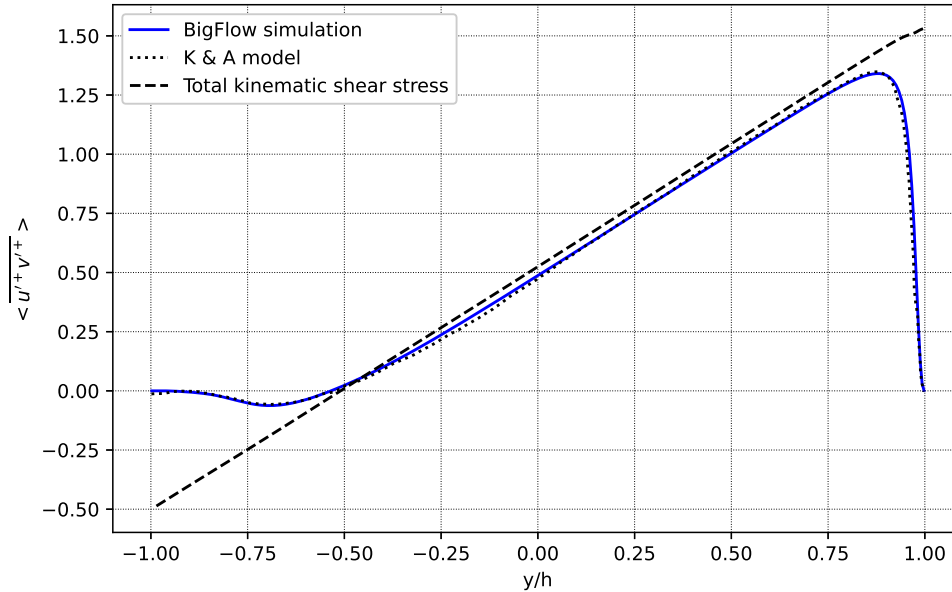


Figure 2.9: Reynolds shear stress in global coordinate system normalized by the global friction velocity. Solid Blue: BigFlow simulation, Dotted Black: Kristoffersen and Andersson results [18], Dashed Black: BigFlow results for $\frac{\tau}{\rho} = -\langle \overline{u'v'} \rangle + \nu \frac{dU}{dy}$.

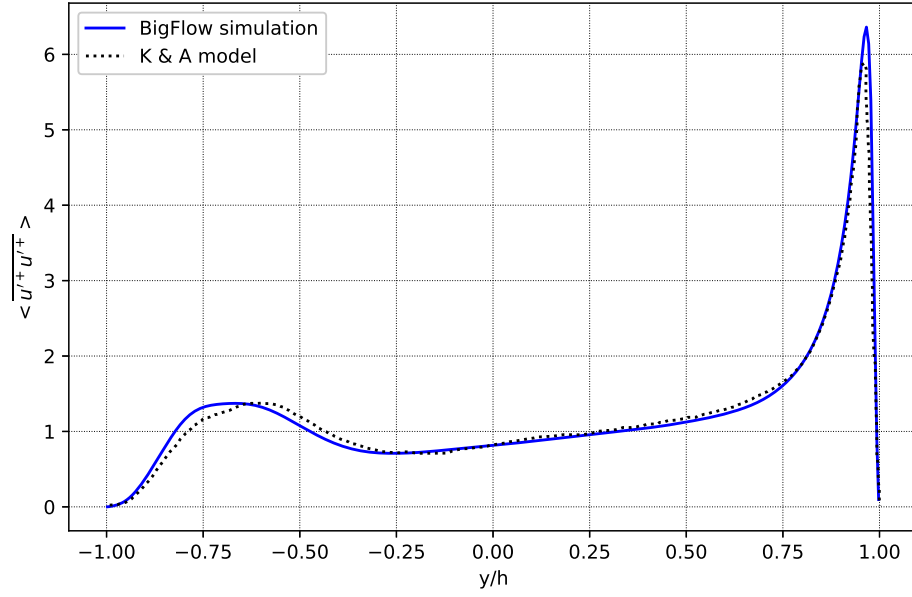


Figure 2.10: Reynolds stress component $\overline{u'u'}$ in global coordinate system normalized by the global friction velocity. Solid Blue: BigFlow simulation, Dotted Black: Kristoffersen and Andersson results [18].

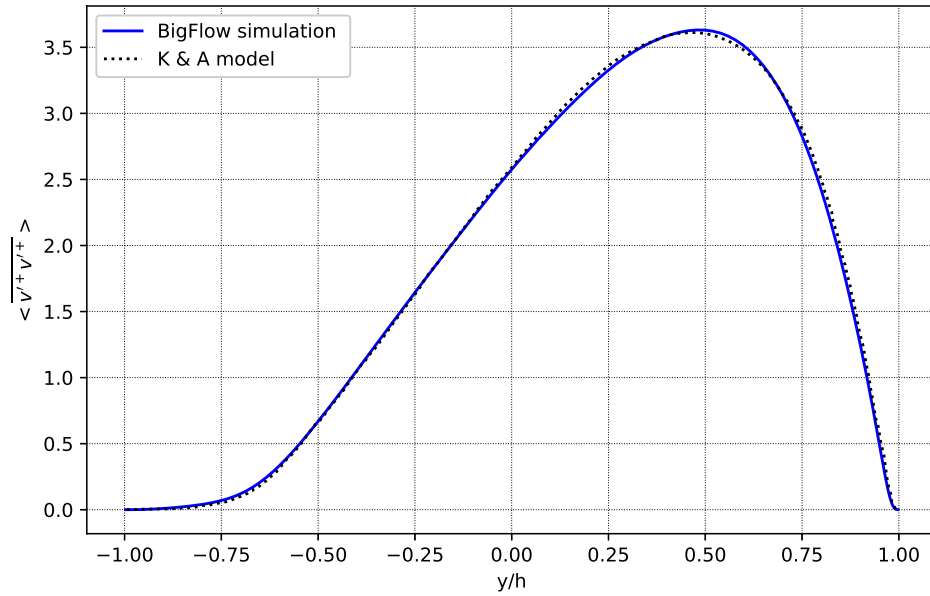


Figure 2.11: Reynolds stress component $\overline{v'v'}$ in global coordinate system normalized by the global friction velocity. Solid Blue: BigFlow simulation, Dotted Black: Kristoffersen and Andersson results [18].

2.3. VALIDATION OF THE IMPLEMENTATION

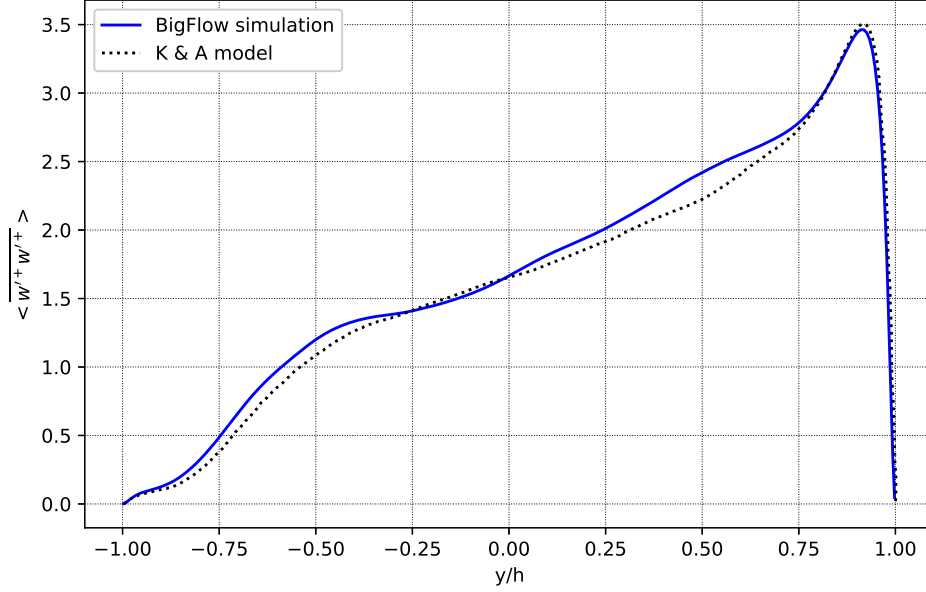


Figure 2.12: Reynolds stress component $\overline{w'w'}$ in global coordinate system normalized by the global friction velocity. Solid Blue: BigFlow simulation, Dotted Black: Kristoffersen and Andersson results [18].

The simulation gives some accurate results in comparison to the simulation performed by Kristoffersen and Andersson [18]. The different velocity profiles and Reynolds stresses follow the same trends as the reference curves with some small oscillations visible at some locations. Those oscillations are most likely due to the comparatively smaller run time of BigFlow simulation. Nevertheless, based on the previous results, the implementation of the Coriolis effect for a channel in Direct Numerical Simulation (DNS) is considered to have been validated.

2.3.2 Laminar and turbulent Ekman layer

Subsequent to the validation of the Coriolis force treated previously on a rotating channel in DNS, the Ekman layer is now studied. This second validation process consists in simulating a semi channel at low Reynolds number. The first study case is the DNS of the laminar Ekman layer, followed by the study of its turbulent counterpart using wall-modeled Large Eddy Simulation (wmLES). In this section, the Ekman layer is first introduced then the different simulations are presented.

2.3.2.1 Ekman layer Theory

The Ekman layer can occur in the atmosphere as well as in the ocean. For the purpose of this thesis, only the atmospheric Ekman layer is simulated and analysed. It is the layer of the atmospheric boundary layer located between the surface layer and the free atmosphere. This layer is characterized by a spiral formed by the streamwise and spanwise velocity profiles variation with respect to the altitude. This spiral will be presented later in the simulation part.

2.3. VALIDATION OF THE IMPLEMENTATION

The Ekman layer is characterized by the equilibrium between three forces. The first one is the force induced by a pressure gradient. This force acts perpendicularly to the isobars. The second force is the Coriolis force and acts perpendicularly to the motion of the fluid. In the vicinity of the ground, the velocity decreases due to the viscosity and creates a vertical gradient of velocity and induces some shear stresses which is the third force. Those three forces are in equilibrium in the Ekman layer. Due to the variation of those forces with the altitude, the direction as well as the magnitude of the flow velocity vary and form the Ekman spiral. The viscosity force and the Coriolis effect are some consequences of the fluid motion and do not generate the wind but modify its direction and intensity. Equations (2.30), (2.31) and (2.32) show the simplification of the three governing equations for this problem [24].

$$-fw = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (2.30)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (2.31)$$

$$fu = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \frac{\partial^2 v}{\partial y^2} \quad (2.32)$$

where $f = 2\Omega \sin\phi$ [s^{-1}] is the Coriolis parameter and the angle is worth $\phi = 90^\circ$ (*cfr*: Figure 2.2). The vertical axis is the y -axis.

In the free atmosphere, the shear stress due to the boundary is null. Therefore, the balance of forces involves only the force due to the pressure gradient and the Coriolis effect. In this upper region, the Coriolis effect induces a force of the same amplitude as the pressure gradient force but in the opposite direction. The velocity of the wind is constant and called geostrophic wind and it travels in parallel to the isobars. This balance results in the equations (2.33) and (2.34) [25].

$$-fW_g = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (2.33)$$

$$fU_g = -\frac{1}{\rho} \frac{\partial p}{\partial z} \quad (2.34)$$

2.3.2.2 Laminar Ekman layer simulation

To pursue the validation of the Coriolis effect implementation the laminar Ekman layer is simulated. This is an interesting simulation because an analytical solution exists for the laminar Ekman layer which is used as the reference curve. It is also a good transition towards the turbulent Ekman layer which is the last simulation. The analytical solution is written in equation (2.35), (2.36) and (2.37) [26].

$$\frac{u(y)}{U_g} = \left(1 - e^{-\frac{y}{\delta}} \cos\left(\frac{y}{\delta}\right)\right) \quad (2.35)$$

$$\frac{v(y)}{U_g} = 0 \quad (2.36)$$

$$\frac{w(y)}{U_g} = U_g e^{-\frac{y}{\delta}} \sin\left(\frac{y}{\delta}\right) \quad (2.37)$$

$$\delta = \sqrt{\frac{2\nu}{f}} \quad (2.38)$$

where δ is the Ekman depth and defines the thickness of the Ekman layer.

2.3. VALIDATION OF THE IMPLEMENTATION

Numerical set-up :

The Figure 2.13 shows the simulation set-up as well as the boundary conditions for the DNS. The numerical set-up of the simulation has been inspired by Coleman et al. article "*A numerical study of the turbulent Ekman layer*" [27]. The dimensionless size of the mesh is:

$$\frac{L_x f}{u_\tau} \times \frac{L_y f}{u_\tau} \times \frac{L_z f}{u_\tau} = 18 \times 9 \times 18 \quad (2.39)$$

And the number of nodes in each direction is:

$$N_x \times N_y \times N_z = 128 \times 128 \times 128 \quad (2.40)$$

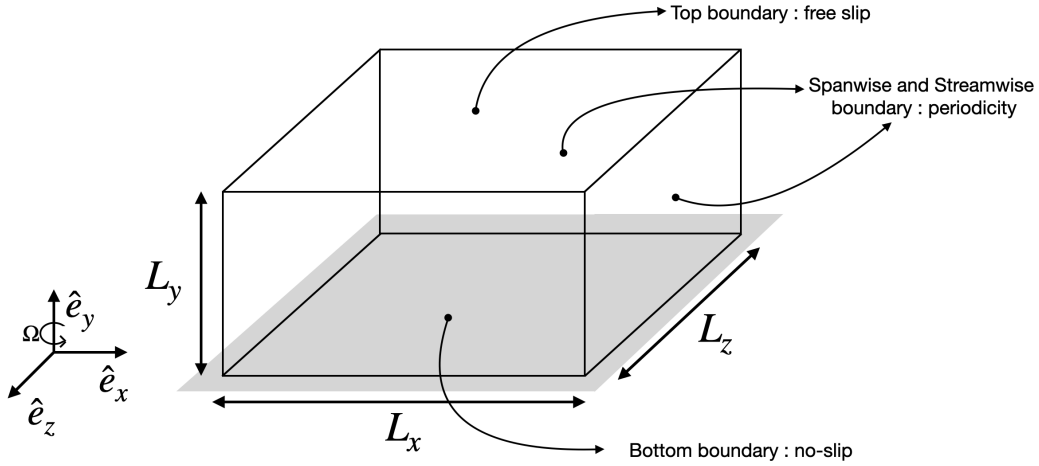


Figure 2.13: Set-up of the laminar Ekman layer simulation with the boundary conditions.

No stretching has been applied in the streamwise (x -direction, $\hat{\mathbf{e}}_x$) and spanwise (z -direction, $\hat{\mathbf{e}}_z$) direction which results in a constant grid space of $\frac{\Delta x f}{u_\tau} = \frac{\Delta z f}{u_\tau} = 0.14$. In the vertical direction, the mesh was stretched with a hyperbolic tangent function (equation (2.41)). This stretching gives a minimum grid spacing in the vertical direction of $\frac{\Delta y_{min} f}{u_\tau} = 0.0022$ and a maximum grid spacing of $\frac{\Delta y_{max} f}{u_\tau} = 0.21$.

$$\frac{y_i}{L_y} = \frac{\tanh((y_{1i} - 1) \gamma)}{\tanh(\gamma)} + 1 \quad (2.41)$$

where y_{1i} is a vector going from 0 to 1 with an equal spacing between each terms and γ is the stretching strength parameter, sets to 3 in this case.

In the streamwise and spanwise directions, periodic boundary conditions are applied to simulate an infinite box. The bottom boundary condition is a no-slip condition ($u = v = w = 0$) and the top boundary is a free slip condition which is the best way to simulate the free atmosphere.

The initial condition is a semi Poiseuille flow (equation (2.42)) in the streamwise direction and no velocity in the two other directions.

$$\frac{u(y)}{U_g} = \frac{y(2L_y - y)}{L_y^2} \quad (2.42)$$

2.3. VALIDATION OF THE IMPLEMENTATION

The simulation can be characterized by its Reynolds number based on the geostrophic wind and the Ekman layer depth:

$$Re = \frac{U_g \delta}{\nu} = \frac{U_g}{\sqrt{\frac{1}{2}\nu f}} = 50 \quad (2.43)$$

Similarly, the Reynolds number can be evaluated using the shear velocity. In this case, the simulation shows a shear velocity of $\frac{u_\tau}{U_g} = 0.1415$.

$$Re_\tau = \frac{u_\tau \delta}{\nu} = \frac{u_\tau}{\sqrt{\frac{1}{2}\nu f}} = 7.08 \quad (2.44)$$

A pressure gradient must be applied in the spanwise direction to counter the Coriolis effect due to the geostrophic wind in the streamwise direction, $(U_g, V_g, W_g) = (1, 0, 0)$ [m/s].

$$\frac{dP}{dz} = -f U_g \quad (2.45)$$

with the Coriolis parameter $f = 2\Omega \sin(\phi) = 0.04$ [s⁻¹].

The simulation has run for 30 inertial oscillation periods. The statistics were done on the five last oscillation periods in time and in the streamwise and spanwise direction thanks to the homogeneity of the velocity profile in those directions.

Results :

The Figure 2.14 represents the results of the simulation with the streamwise and spanwise velocity profiles. The projection of the velocity shows the characteristic Ekman spiral which is also visible in Figure 2.15. At the top, the velocity is purely streamwise. This is coherent with the theory explained earlier and it is called the geostrophic wind.

The mean velocity magnitude profile in wall coordinates is represented in Figure 2.16 as well as the streamwise velocity profile. Close to the bottom boundary the magnitude velocity is a bit bigger than the streamwise component due to the presence of a spanwise velocity component. The spanwise component has a negligible amplitude above $y^+ = 20$.

The velocity magnitude increases with respect to the altitude to reach a stable value around $yf/u_\tau = 2$. The variation of the velocity amplitude is a consequence of the variation of the three equilibrium forces and the bottom boundary. The Coriolis force increase with the velocity magnitude and the shear stress varies with the velocity gradient and is non negligible below $yf/u_\tau = 1.6$. At mid altitude (around $yf/u_\tau = 1$), due to the absence of turbulence, the flow does not feel the drag from the ground surface which causes an acceleration of the flow. The flow becomes supergeostrophic which in an ABL is known as the nocturnal jet [28].

The deviation induced by the Coriolis effect between the velocity close to the ground and the direction of the geostrophic wind formed an angle of $\alpha = 41^\circ$. A later simulation will show that this angle decreases when the Reynolds number increases.

To fully validate the Coriolis effect implementation, the results are compared to the analytical solution presented earlier [26]. The first comparison is done on the Ekman spiral, Figure 2.15. The BigFlow results follow perfectly the analytical solution. In a second time, the velocity profiles as a function of the height are compared to the analytical solution (Figures 2.17 and 2.18). The results are also very accurate and follow the theoretical path. Those results allow the validation of the Coriolis effect implementation for DNS.

2.3. VALIDATION OF THE IMPLEMENTATION

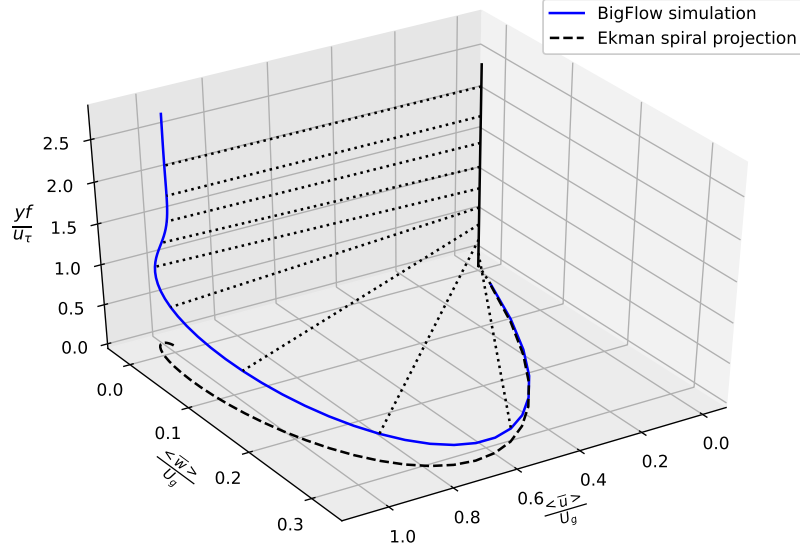


Figure 2.14: Representation of the streamwise and spanwise mean velocity profile. Solid Blue: BigFlow velocity profile, Dashed Black: projection of the velocity on an horizontal plane.

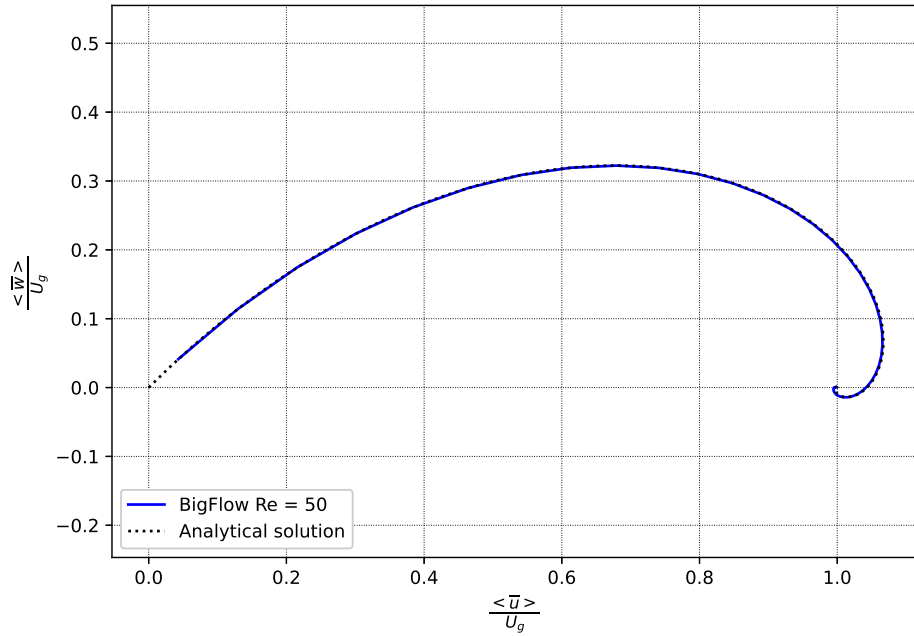


Figure 2.15: Relation between the streamwise and spanwise mean dimensionless velocity profile also known as Ekman spiral. The results are compared to the analytical solution. Solid Blue: BigFlow simulation, Dotted Black: Analytical solution [26].

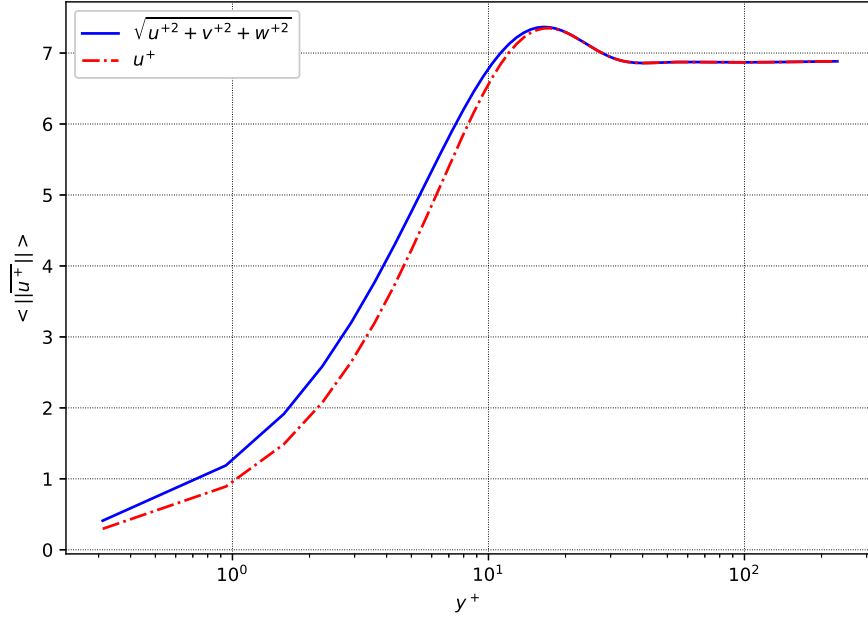


Figure 2.16: Mean velocity magnitude profile in wall coordinates. Solid Blue: BigFlow simulation velocity magnitude, Dash-dotted Red: BigFlow simulation streamwise velocity.

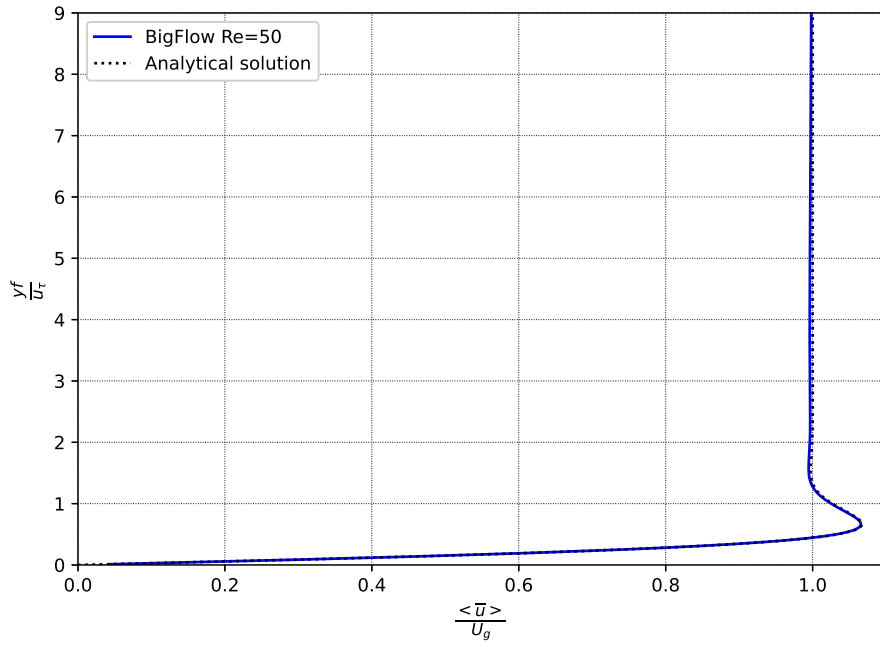


Figure 2.17: Mean streamwise velocity profile in global coordinates. Solid Blue: BigFlow simulation, Dotted Black: analytical solution [26].

2.3. VALIDATION OF THE IMPLEMENTATION

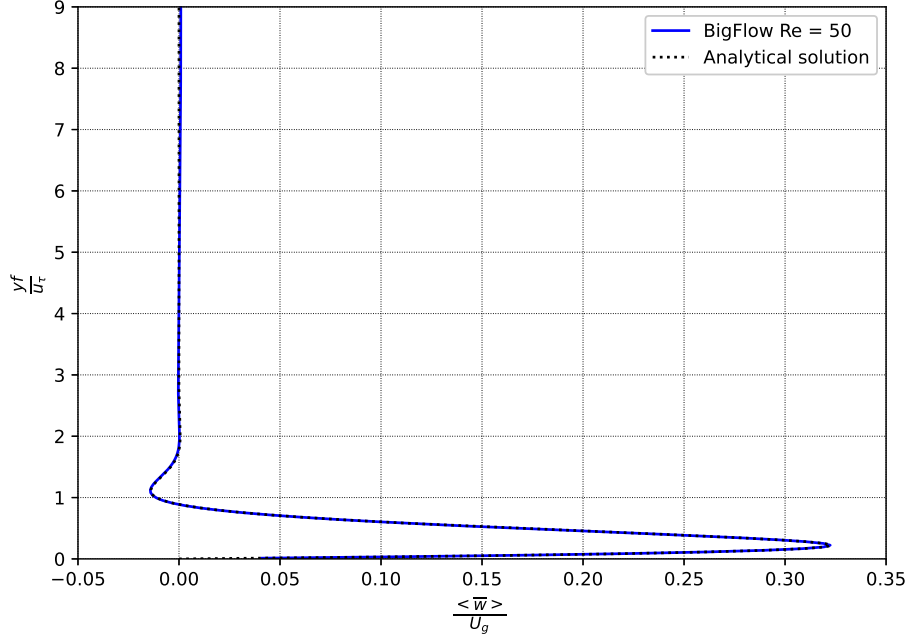


Figure 2.18: Mean spanwise velocity profile in global coordinates. Solid Blue: BigFlow simulation, Dotted Black: analytical solution [26].

2.3.2.3 Turbulent Ekman layer simulation

The turbulent Ekman layer is simulated in wmLES. This section ensures that the Coriolis effect is also correctly implemented when used in a wmLES with the stencil reduction near the wall model. The simulation is compared to a DNS simulation done by M. Ashworth, G. Coleman and R. Johnstone in their article, *DNS of the turbulent Ekman layer at $Re=2000$* [29].

Numerical set-up :

The geometry of the simulation set-up is similar to the one used in the laminar case and is shown in Figure 2.19. The main difference is the bottom boundary condition. Previously, a simple no-slip boundary condition was applied but in this case, due to the higher Reynolds number, a wall model is used with the Reichardt law of the wall (equation (2.46)). This law of the wall connects the laminar sub-layer of the boundary flow with the logarithmic layer [30].

$$u^+ = \frac{\ln(1 + \kappa y^+)}{\kappa} + C \left(1 - e^{\frac{-y^+}{11}} - \frac{y^+}{11} e^{\frac{-y^+}{3}} \right) \quad (2.46)$$

where C and κ are two constants that are set respectively to 7.42 and 0.38 in the BigFlow solver [9].

The dimensionless size of the mesh expressed in term of the Coriolis parameter is $\frac{L_x f}{u_\tau} \times \frac{L_y f}{u_\tau} \times \frac{L_z f}{u_\tau} = 3 \times 3.34 \times 3$. No stretching has been applied in the streamwise direction (x -direction, $\hat{\mathbf{e}}_x$) and spanwise direction (z -direction, $\hat{\mathbf{e}}_z$). The grid spacing in those two directions expressed in wall coordinates is $\Delta x^+ = \Delta z^+ = 94.6$. In the wall normal direction a stretching has been applied with a minimum step size of $\Delta y_{min}^+ = 26.27$. This step size is constant for the first 15% of the mesh and then a stretching is applied to the remaining node to fill up the rest of the

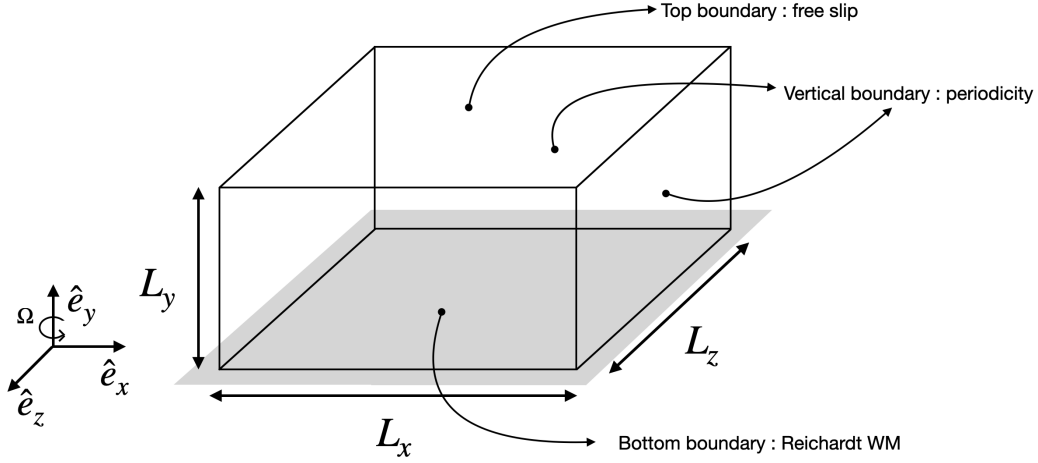


Figure 2.19: Simulation set-up for the turbulent Ekman layer simulation.

height of the mesh. This minimum step size is much bigger than the one used by Coleman et al. ($\Delta y_{min}^+ = 8.1$) thanks to the use of wmLES. It results in a large difference in the grid resolution. In our simulation, the grid is $N_x \times N_y \times N_z = 128 \times 128 \times 128$ while in Coleman et al., it is $N_x \times N_y \times N_z = 1024 \times 200 \times 1792$

The Coriolis parameter is set to: $f = 0.005[s^{-1}]$ and a constant pressure gradient is applied in the spanwise direction with the intensity corresponding to the intensity of the Coriolis effect at the top boundary $(U_g, V_g, W_g) = (1, 0, 0)[m/s]$.

$$\frac{dP}{dz} = -f U_g \quad (2.47)$$

For this simulation, the initial condition was a constant wind in the streamwise direction with a velocity equivalent to the geostrophic wind velocity. To accelerate the convergence process, some small random perturbations were added in the three directions which trigger the turbulence.

Results :

The wind in the ABL keeps on oscillating slowly over time. Those oscillations are called inertial oscillations and have a oscillations period of $Tf = 6.32 \approx 2\pi$ (f is the Coriolis parameter and T is the period). This period is consistent with what was experience by Jiang in his simulation [31]. The convergence time to completely damp those oscillations would results in an unaffordable computation cost, so the results are average over one oscillation period, from $tf = 6.03$ to $tf = 12.5$.

The table 2.1 compares different values from the BigFlow simulation and Coleman et al. simulation. The Reynolds number based on the geostrophic wind was imposed in both simulation while the Reynolds number based on the shear velocity was computed afterward based on the simulation results. It presents a very small difference between the BigFlow simulation and the reference value. This difference could have various explanations such as the variation of the grid resolution, the simulation type or the duration of the statistics and the same reasons could explain the small variation in the angle between the velocity direction close to the bottom boundary and the geostrophic wind. In a turbulent simulation, it is very difficult to obtain the

2.3. VALIDATION OF THE IMPLEMENTATION

exact same results and so small differences are acceptable for the validation process. In this case, the differences are too small to be the results of an implementation problem. The deviation angle can be compared to the angle obtained in the laminar case. In the laminar case the angle was 41° which is more than the double of the angle in the turbulent case. It was shown in Coleman et al. article [29] that an increase of the Reynolds number induces a decreases of the deviation angle. It could be expected then that, in the ABL for the wind farm simulation, this angle decreases even more and would be below 10.

	Coleman et al. [29]	BigFlow
$Re = \frac{G}{\sqrt{0.5} \nu f}$	2000	2000
$Re_\tau = \frac{u_\tau}{\sqrt{0.5} \nu f}$	93.14	89.7
$\frac{u_\tau}{U_g}$	0.04657	0.04485
$\alpha[^\circ]$	15.75	15.5

Table 2.1: Comparison of the parameter in the Coleman et al. simulation and the BigFlow simulation.

The Figure 2.20 offers a first view at the velocity profile and the angle of deviation of the velocity. The projection of the velocity profile on the horizontal plane gives a preview at the Ekman spiral which is better represented in Figure 2.21. In comparison to the laminar case, it is clear that the spiral is flatter because the spanwise velocity component remains smaller.

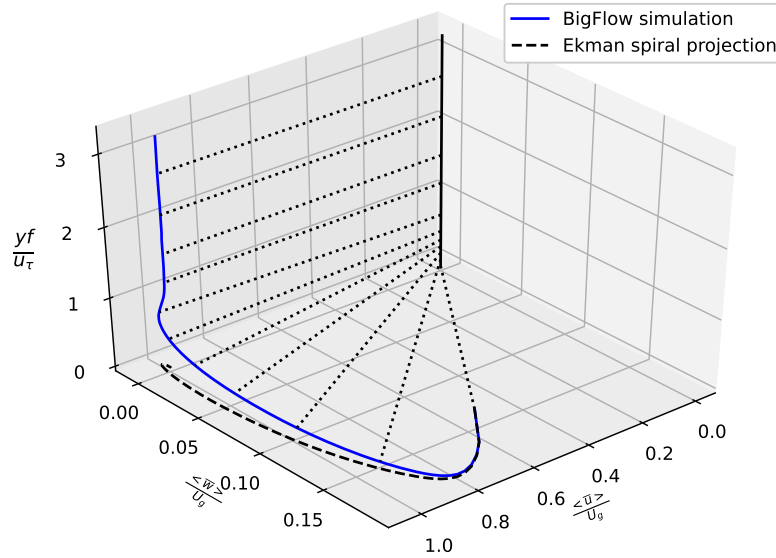


Figure 2.20: Representation of the streamwise and spanwise mean velocity profile. Solid Blue: velocity, Dashed Black: projection of the velocity on an horizontal plane.

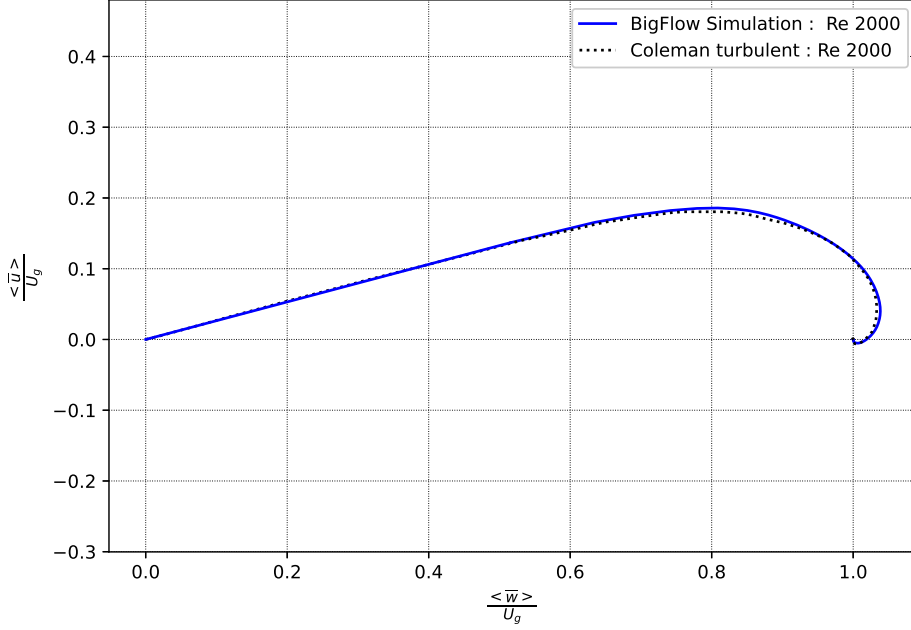


Figure 2.21: Relation between the streamwise and spanwise velocity, also known as the Ekman spiral. Solid Blue: BigFlow simulation, Dotted Black: Coleman et al. simulation [29].

On the streamwise velocity graph (Figure 2.22) the overshoot of velocity is also visible. This graph offers a good comparison of the intensity of the streamwise and spanwise velocity. Since the initial condition was purely streamwise, the spanwise component was generated only by the Coriolis effect. Without this effect, the flow would have stabilized its velocity in the streamwise direction with negligible components in the other directions. It can also be seen that the Coriolis effect affects only the lowest part of the domain. Above a dimensionless height of $\frac{y}{u_\tau} = 1.3$ the spanwise velocity component becomes negligible and only a stable streamwise velocity component remains with an intensity equals to the geostrophic wind. At this altitude, the shear stress induces by the bottom boundary is negligible and the wind is at the equilibrium between the Coriolis force and the pressure gradient. Those two forces have the same amplitude but opposite directions.

Another interesting profile to analyse is the magnitude of the velocity in the global coordinates system (Figure 2.23) and in the wall coordinates system (Figure 2.24). In the first coordinates system, the velocity profile is very similar to the streamwise velocity profile in Figure 2.22. The spanwise velocity component affects only a little bit the intensity of the velocity profile. But this component has a strong impact on the direction of the wind. The second figure highlights the modelization of the wall with the Reichardt law of the wall. The logarithmic law region is clearly visible with a linear profile of the velocity on the logarithmic plot. In the logarithmic sublayer, the Reichardt law of the wall can be approximate by the function in equation (2.48).

$$u^+ = \frac{1}{\kappa} \ln(1 + \kappa y^+) + C \quad (2.48)$$

where, in BigFlow $\kappa = 0.38$ and C is set to 7.42.

2.3. VALIDATION OF THE IMPLEMENTATION

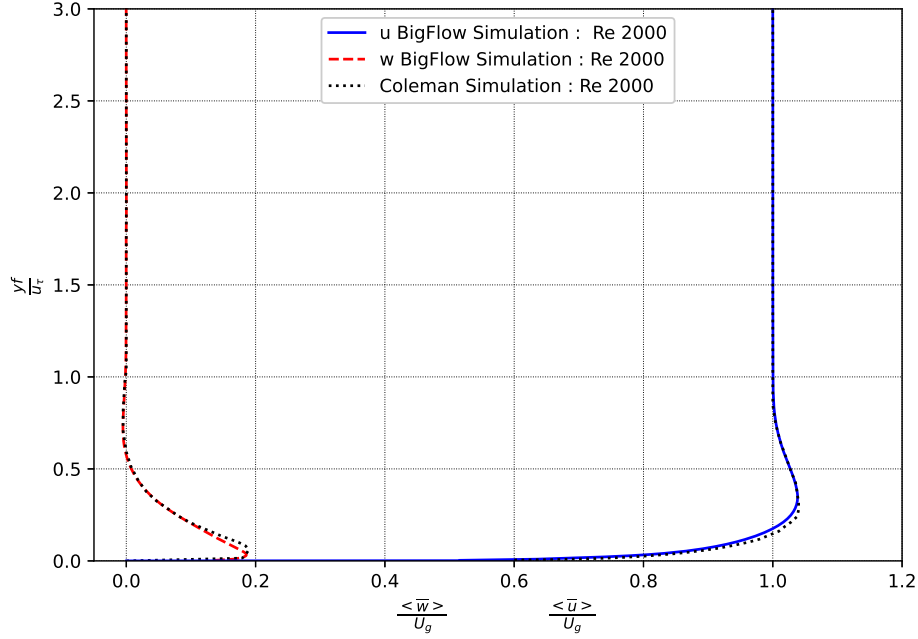


Figure 2.22: Streamwise and spanwise velocity profile in the global coordinates system. Solid Blue: BigFlow simulation streamwise velocity, Dashed Red: BigFlow simulation spanwise velocity, Dotted Black: Coleman et al. simulation [29].

The logarithmic sublayer starts around a $y^+ = 100$. The equation (2.48) can be simplified by taking the κ out of the logarithm. In this case, $\frac{1}{\kappa} = 2.63$ which is very small compared to y^+ in the logarithmic region and can then be neglected which results in the final equation for the logarithmic region (equation (2.49)) [32].

$$u^+ = \frac{1}{\kappa} \ln(y^+) + 4.85 \quad (2.49)$$

This approximation was drawn on the Figure 2.24. On the graph, the approximation of the law of the wall does not fit perfectly the reference curve from [29]. To have an exact fit, the parameters of the wall model such as κ and C should have been tuned. This small variation between the law of the wall approximation and the reference curve could potentially explain the slight differences observed between the same reference curve and the BigFlow results.

All the velocity profiles can be compared to the results found in Coleman et al. article [29]. In Figures 2.21, 2.23, 2.24 and 2.22, the reference profiles are represented by a dotted black curve. On every figures, the velocity profiles close to the wall present some variations compared to the reference curve. These differences were explained earlier and are probably due to the wall model parameters. They could also come from the use of wmLES instead of DNS. But these differences remain very small compared to the general velocity and the upper part of the velocity profiles matches perfectly the the reference curve. It can then be concluded that the BigFlow simulation is accurate in comparison to the simulation performed by Coleman et al. This allows the validation of the implementation of the Coriolis effect in wmLES.

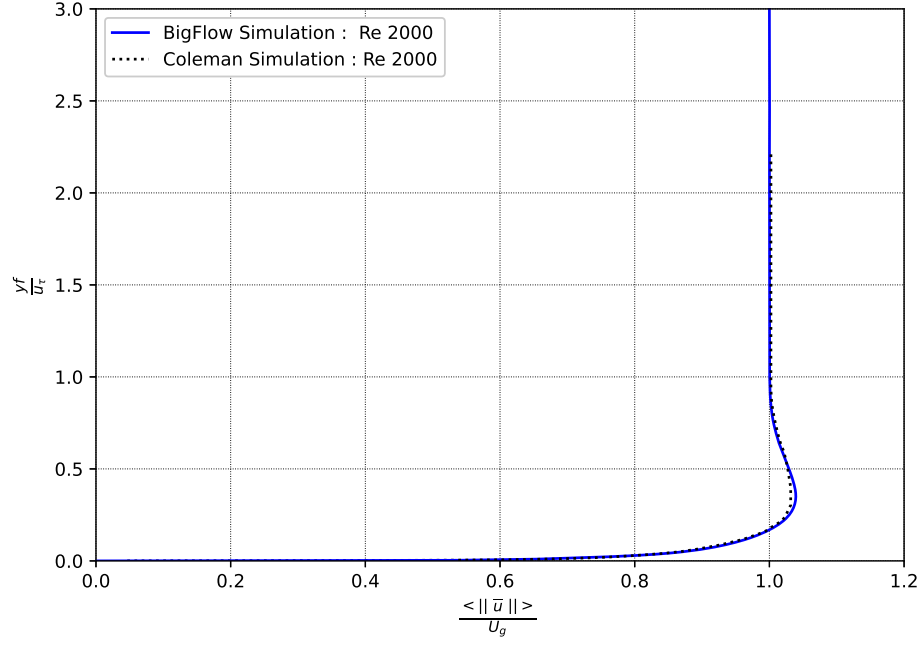


Figure 2.23: Magnitude velocity profile in the global coordinates system. Solid Blue: BigFlow simulation, Dotted Black: Coleman et al. simulation [29].

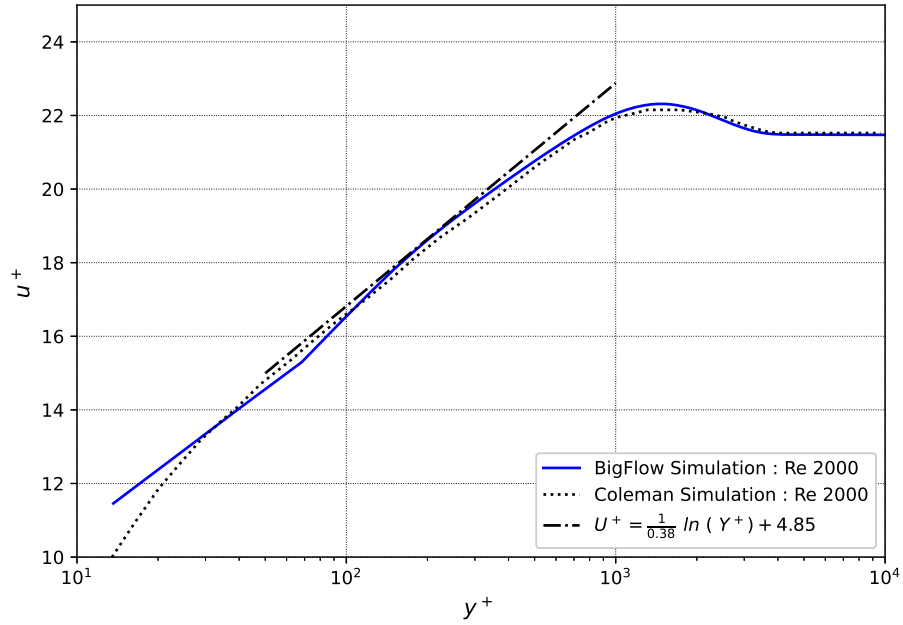


Figure 2.24: Magnitude velocity profile in the wall coordinates system. Solid Blue: BigFlow simulation, Dotted Black: Coleman et al. simulation [29] and Dash-dotted Black: $u^+ = \frac{1}{0.38} \ln(y^+) + 4.85$, wall model approximation.

2.4 Conclusion

The Coriolis effect is very important to simulate an atmospheric boundary layer and so is its implementation. The different simulations presented in the previous section show some accurate results in comparison with the relevant literature. Two DNS were conducted to simulate a rotating channel and a laminar Ekman spiral. The rotating channel presented some small differences compared to the reference curve which were probably caused by a smaller convergence time. While, in the laminar Ekman layer simulation, the results were identical compared to the analytical solution. The last simulation was a wmLES of a turbulent Ekman spiral at Reynolds number equals to 2000. The turbulent Ekman layer presents some variations that remain very small compared to the velocity magnitude and are probably due to a small variation of the law of the wall and not the Coriolis effect implementation. It is then possible to conclude that the Coriolis effect was properly implemented in BigFlow and can be used in further simulations.

Chapter 3

Impact of the Coriolis effect on a wind farm

This chapter aims at evaluating the impact of the Coriolis effect on a wind farm. The Coriolis effect induces a variation of the wind direction with the altitude. This velocity profile induces a slight variation of the yaw angle between the bottom and the top of the blade which could impact the development of the wake. The variation of the development of the wake could then also potentially impact the downstream wind turbines and their energy production. A small wind farm is studied in this chapter for the two different types of atmospheric boundary layer, one with and one without the Coriolis effect. We will start with a presentation of the wind turbine model used in the simulation and then pursue with the validation of the co-current simulation. Then a comparison of the two ABL is done and it ends with an analysis of the wake development and of the power production of the wind farm.

3.1 Wind turbine model

The wind turbine used for this simulation is the NREL 5MW which is developed by the National Renewable Energy Laboratory. This wind turbine model is a relatively classic model with 3 blades oriented at 120° from each other. The hub is at 90 meters altitude and the rotor diameter is 126m. An sketch of the NREL 5MW wind turbine is shown in Figure 3.1. Those turbines are mostly used in offshore wind farm and have a working range between 3 [m/s] and 25 [m/s] with a maximum rotation speed of 12.1 [rpm]. The study range for this thesis is limited to a wind speed around 10 [m/s]. This turbine is also controllable with the rotation speed, the pitch and the yaw angle [33]. It has already been implemented in BigFlow by M. Moens [10] with all the different control options.

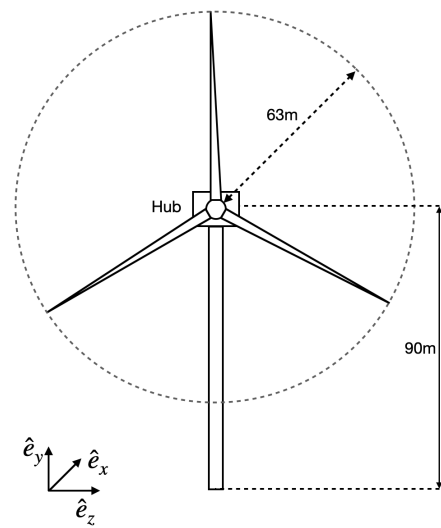


Figure 3.1: Sketch of the design of a NREL-5MW wind turbine.

3.2. VALIDATION OF THE CO-CURRENT SIMULATION WITH NON PERPENDICULAR VELOCITY

Two methods of wind turbine modelling are implemented in BigFlow which are the actuator disk and the actuator lines methods. The goal of an actuator model is to retrieve the forces acting on the blades while keeping the computational cost lower than for a blade resolved simulation. The force is then incorporated in the RHS of the Navier-Stokes equation as a volumetric source term. The blade element theory is the basic principle behind the actuator models. The surface swept by the blades is divided in a serie of radial and azimuthal elements. At the center of each element, the forces are computed based on the local wind velocity, the lift and drag coefficient of the blades.

The actuator disk (AD) with tip-loss correction developped by M. Moens [10] is the wind turbine model chosen for this thesis. The main difference with the actuator lines (AL) is that the AD models an infinite number of blades which forms a circle and the AL model consider each of the three blades separately: they are more accurate yet also more expensive to use as they require a finer grid. The rotor area is divided into multiple elements. The incoming flow velocities are interpolated on every control element of the actuator method. The forces can be computed at the center of each element based on the local velocity field. The computed forces are then interpolated on the simulation mesh to be added in the Navier-Stokes equation as a source term [34]. The modeling of a wind turbine with an infinite number of blades neglects the simulation of the tip vortices which induces an overestimation of the power and of the forces acting at the tip of the blades. A tip correction is then applied to correct the flow simulation between an infinite blade rotor and a 3 blades rotor [35].

The second model is the actuator line method which solves one of the major limitation of the actuator disk method, the ability to properly capture the near-wake. The AL models the three blades independently and is also based on the blade element theory. Instead of computing the forces on the full area swept by the turbine blades, it computes them on three lines located at the real blade locations. Between each time steps these lines rotate like real blades. The tip vortices are then properly modeled without the need of a tip vortices correction and the near wake is more accurate [36].

The actuator disk with tip-loss correction was chosen for this thesis because it is less restrictive on the geometry of the mesh. An actuator disk can give fairly accurate results with only 16 points per diameter of rotor while the actuator lines required a minimum of 32 points per diameter. The simulation set-up used for the wind farm simulation is relatively long. Being able to reduce by two the number of nodes in each direction constitutes a significant advantage that here guided the actuator model choice. A study conducted by F. Trigaux [37] highlights the similarities between the actuator disk and the actuator lines model when used for wind turbines in an atmospheric boundary layer (thus with the turbulence of the wind), and without yaw. For the far wake, the results are very similar this is however not the case when considering a wind turbine with non-turbulent inflow [10]. As this thesis focuses on the impact of the Coriolis effect on a wind farm, the accuracy of the near wake of each wind turbines is not the priority as long as the global wake remains consistent.

3.2 Validation of the co-current simulation with non perpendicular velocity

Later in this thesis, some simulation with wind turbines will be presented. Those simulations require the use of a co-current simulation. This type of simulation have already been used multiple times with a velocity that is perpendicular to the inflow plane. In the case with the Coriolis effect, the velocity is not always perpendicular as the angle varies with the height.

3.2. VALIDATION OF THE CO-CURRENT SIMULATION WITH NON PERPENDICULAR VELOCITY

This type of co-current simulation has been less used and required a validation process to make sure that the non-perpendicular velocity does not affect the co-current simulation and that the velocity profile remains consistent in both simulations.

3.2.1 Co-current simulation explanation

The simulation with the wind turbines required the use of two simulations which run simultaneously, one called the precursor which helps to converge and to maintain the ABL and a second one where the wind turbines are placed. The precursor is a classical ABL simulation like the one used to validate the Coriolis effect implementation with periodic boundary conditions in the streamwise direction. The second simulation (where the wind turbines are placed) cannot have a periodic boundary condition in the streamwise direction to avoid having the wakes of the last wind turbines being re-injected in the first one. Inflow and outflow boundary conditions are then applied in the streamwise direction. The inflow condition of the second simulation is extracted from a plane in the precursor. Those conditions are shown in Figure 3.2.

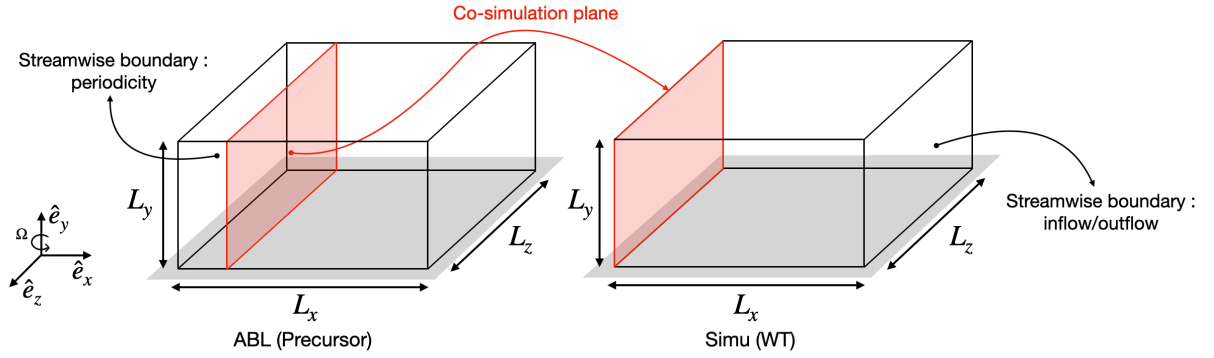


Figure 3.2: Set-up for the co-current simulation validation.

The use of the co-current simulation principle allows to converge an ABL with the Coriolis effect on a domain that is smaller in the longitudinal direction. The use of a precursor simulation offers also a faithful reproduction of the inflow turbulent structures required for the second simulation.

3.2.2 Validation

The set-up of the simulation is shown in Figure 3.2. This set-up consists on two times the same mesh. The red mark on the precursor of the Figure 3.2 represents the plane where the velocity values are extracted and used as the inflow condition of the second simulation.

The set-up size is $L_x \times L_y \times L_z = 1260[m] \times 1008[m] \times 504[m]$ and the grid is $N_x \times N_y \times N_z = 256 \times 128 \times 128$. The rotation rate of the ABL is $\Omega = 7.27 \times 10^{-5}[s^{-1}]$. The Coriolis parameter equals $1.028 \times 10^{-4}[s^{-1}]$ which is equivalent to the Coriolis parameter observed on earth at a latitude of 45 degrees North. To avoid an unnecessary long simulation, the precursor has run first for 20000 times step which is equivalent to 800 seconds. That time is not long enough to reach the convergence but already enough to have a strong spanwise velocity component and a variation of the wind direction in the height, which were the important characteristics for the validation. After the small convergence of the precursor, the co-current simulation runs for 10000 time steps, equivalent to 400 seconds, the initial condition being the same in both simulations.

3.2. VALIDATION OF THE CO-CURRENT SIMULATION WITH NON PERPENDICULAR VELOCITY

The first analysis is done on the average velocity profile in the streamwise and spanwise direction and in time over the 10000 time steps. In a second phase, the analysis is done on the instantaneous velocity but averaged in the spanwise direction. These analysis are done at $x/L_x = 0.4$ of the precursor which corresponds to $x/L_x = 0.2$ for the second ABL. The last validation is a validation of the instantaneous velocity without any averaging. The analysed velocities are taken at $x/L_x = 0.7$ in the precursor and $x/L_x = 0.5$ in the second simulation and at the middle in the spanwise direction in both simulation. The small variation of the x location between both ABL is due to the localisation of the plane on which the velocity is extracted. This whole validation process gives the opportunity to have an overview at different locations in the mesh (close and far from the co-current simulation plane).

The velocity profile of the first analysis are depicted in Figures 3.3 and 3.4. Both velocity profiles are very similar. A very small variation can be seen at $yf/U_\tau = 0.13$ in the spanwise velocity profile. But this variation is too small to have a real impact on the velocity field.

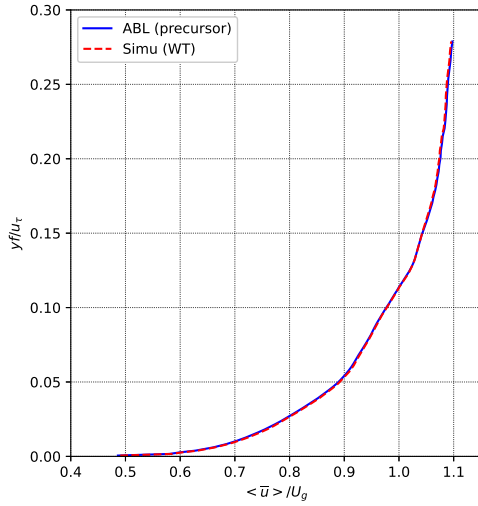


Figure 3.3: Comparison of the streamwise time and space averaged velocity profile. Solid Blue: ABL (precursor), Dashed Red: second simulation (wind turbines).

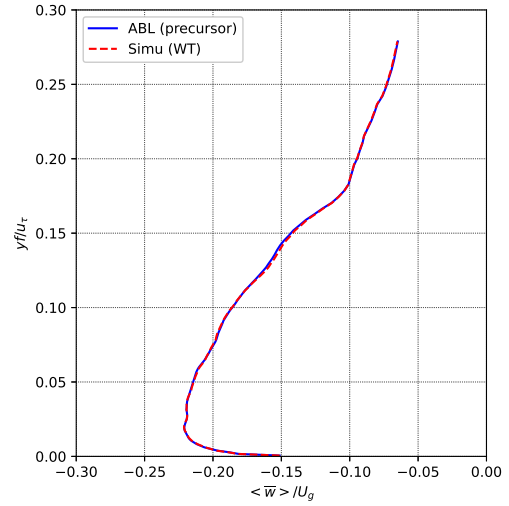


Figure 3.4: Comparison of the spanwise time and space averaged velocity profile. Solid Blue: ABL (precursor), Dashed Red: second simulation (wind turbines).

The second analysis is done close to the co-current simulation plane after 5000 time steps and the two velocity profiles are displayed in Figure 3.5 and Figure 3.6. The velocity profiles cannot be differentiated. As the analysis is done close to the co-current simulation plane it is expected to have some strong similarities. The flow does not have enough distance to grow some new turbulent structures. The aim of the last analysis is to see if with enough distance some big variations occur in the velocity profile and could compromise the co-current simulation.

The last analysis is done further away from the co-current simulation plane and after 10000 time steps. No averaging was done. The different velocity profiles are shown in Figures 3.7 and 3.8. Compared to the previous analysis, more variations occurs between the spanwise velocity profiles while the streamwise velocity profiles remain similar in both simulations. The largest variation observed is around 5%. This variation remains very small and localized so it should not affect the entire simulation.

3.2. VALIDATION OF THE CO-CURRENT SIMULATION WITH NON PERPENDICULAR VELOCITY

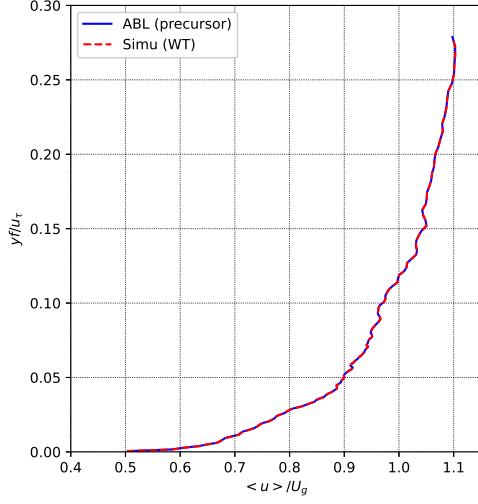


Figure 3.5: Comparison of the streamwise instantaneous velocity profile averaged in the spanwise direction at $x/L_x = 0.2$ and $n = 5000$. Solid Blue: ABL (precursor), Dashed Red: second simulation (wind turbines).

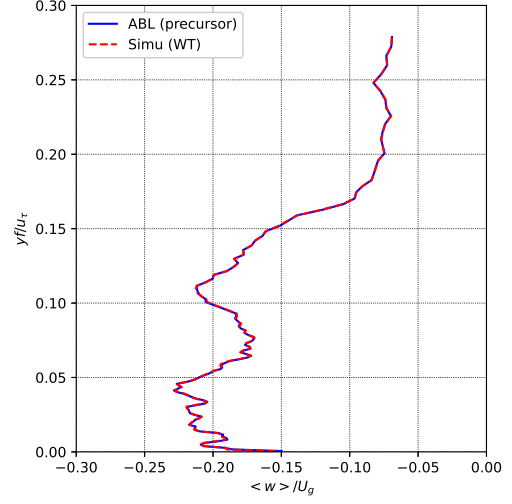


Figure 3.6: Comparison of the spanwise instantaneous velocity profile averaged in the spanwise direction at $x/L_x = 0.2$ and $n = 5000$. Solid Blue: ABL (precursor), Dashed Red: second simulation (wind turbines).

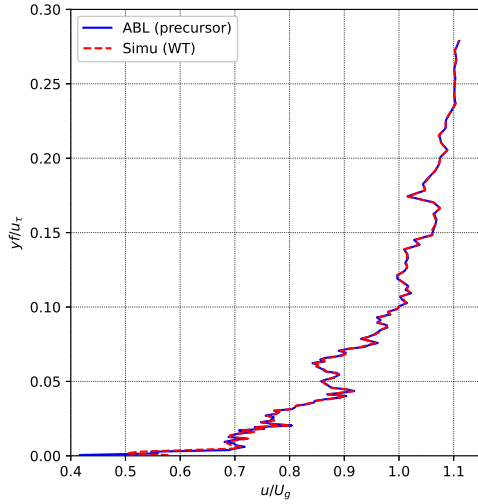


Figure 3.7: Comparison of the streamwise instantaneous velocity profile at $x/L_x = 0.5$, $z/L_z = 0.5$ and $n = 10000$. Solid Blue: ABL (precursor), Dashed Red: second simulation (wind turbines).

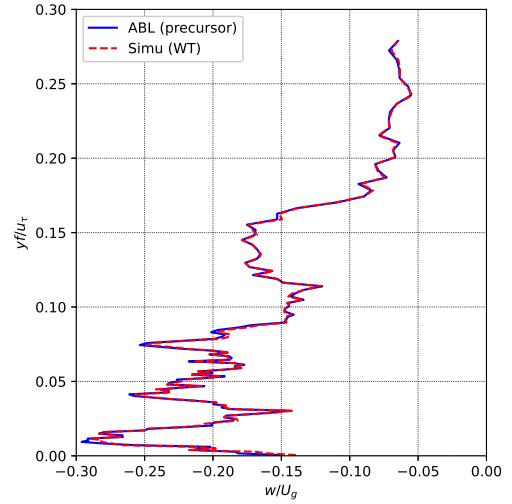


Figure 3.8: Comparison of the spanwise instantaneous velocity profile at $x/L_x = 0.5$, $z/L_z = 0.5$ and $n = 10000$. Solid Blue: ABL (precursor), Dashed Red: second simulation (wind turbines).

3.3. SET-UP FOR THE SIMULATION OF WIND FARMS

From the above results, the co-current simulation is indeed validated for a case where the velocity is not perpendicular to the plane on the entire height of the mesh. In the different studied cases, the velocity profiles in both domain were very similar. It has been seen that, with the distance travelled by the flow, some small variations of the velocity close to the wall could appear between the two domains. Those variations are small in comparison to the velocity and should not have an impact. The average velocity profiles are identical which confirms the validity of the co-current simulation. For the next simulations, the most important is to conserve the velocity profile induced by the Coriolis effect. This profile is well maintained in the second domain which allows the validation of the co-current simulation.

3.3 Set-up for the simulation of wind farms

Two different domains have been used for this simulation which are similar to the set-up used for the validation of the co-current simulation (Figure 3.2). The first one, named the precursor, has a size of $\frac{L_x}{D} \times \frac{L_y}{D} \times \frac{L_z}{D} = 32 \times 8 \times 16$. The grid resolution in the streamwise and spanwise directions is 16 points per wind turbine diameter ($\Delta x = \Delta z = 7.875[m]$). In the vertical direction the resolution is also 16 points per diameter below 200 meters. Above this altitude, a stretching is applied. This organization results in grid size of $N_x \times N_y \times N_z = 512 \times 64 \times 256$ cells. The boundary conditions of the precursor are similar to the one used in the wmLES simulation of the turbulent Ekman layer with a periodicity in the longitudinal direction. The bottom boundary uses a wall model assuming a logarithmic velocity profile and on the top boundary, a free slip is applied.

The second simulation set-up hosts the wind turbines. This mesh is twice longer than the mesh of the precursor. Since the goal of this simulation is also to observe the wake evolution behind a wind farm, it is important to have a long mesh. The mesh size is thus $\frac{L_x}{D} \times \frac{L_y}{D} \times \frac{L_z}{D} = 64 \times 8 \times 16$ and thus $N_x \times N_y \times N_z = 1024 \times 64 \times 256$.

This second simulation hosts a small wind farm made up of 10 wind turbines. The turbines are set in two rows of five turbines with a streamwise distance of separation of 6D and a spanwise distance of separation of 5D. The first turbines are placed 5D away from the inflow condition to avoid any interference with the inflow condition. The last one is thus placed 29D away from the outflow plane which provides a large enough study area as depicted in Figure 3.9. The wind turbines are aligned with the wind direction at the hub height.

The rotation speed and the yaw angle are not kept constant over the whole simulation. The rotation speed is also adjusted with the incoming wind at the hub to have a realistic response to the incoming wind. The controller is described in [10]. The yaw angle is adapted with the wind direction thanks to a yaw controller described in [10]. The yaw controller computes the wind direction error which is the misalignment between the rotor direction and the wind direction, to adjust the wind turbine orientation.

The simulation lasts for 40 minutes with a time step of 0.08 seconds. The time step is chosen in order to keep a relatively small CFL number. The 40 minutes of simulation is equivalent to three flow-through times (a flow-through time being the required time for the flow to go across the entire domain where the velocity at hub height, U_{hub} , is used as the reference velocity, $T = L_x/U_{hub}$). The averages, shown later in this thesis, are done over the 2 last flow-through times.

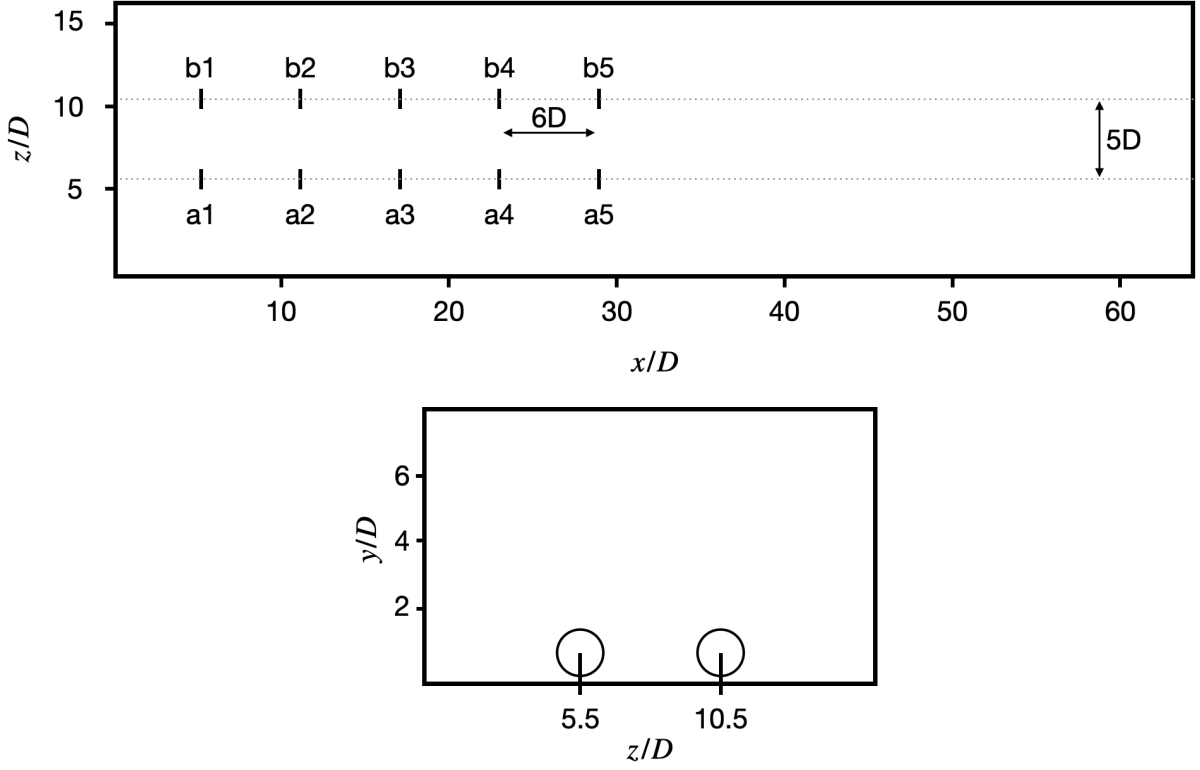


Figure 3.9: Top: top view of the wind farm simulation set-up. Bottom: cross view of the wind farm set-up.

For a good comparison between the simulation with and without the Coriolis effect, the simulation set-ups are the same. The initial condition as well as the precursor could not be exactly the same since they were converged separately. Nevertheless, the two ABL have some strong similarities in term of flow velocities and wind direction at the hub and turbulence intensity. The characteristics of those ABL are detailed in the next section.

3.4 Simulation of atmospheric boundary layers without and with the Coriolis effect

Before running the wind farm simulation an atmospheric boundary layer must be converged. This convergence is done in three consecutive steps. The first step gives a first look at the velocity profile induced by the Coriolis effect. As studied in section 2.3.2, the Coriolis effect induced a variation of the wind direction with respect to the altitude. This first ABL allows the computation of the velocity angle at the hub height to re-orientate the geostrophic wind to have the wind aligned with the wind farm at hub height ($90[m]$). This simulation is done on a very coarse mesh to accelerate the convergence. For the simulation with the Coriolis effect, the set-up is located at 45° of latitude in the northern hemisphere. At this latitude, the Coriolis parameter is equal to $f = 1.0284 \times 10^{-4} [s^{-1}]$ and the rotation speed is $\Omega = 7.2722 \times 10^{-5} [s^{-1}]$.

3.4. SIMULATION OF ATMOSPHERIC BOUNDARY LAYERS WITHOUT AND WITH THE CORIOLIS EFFECT

The second simulation converges the final ABL on a coarse mesh. This simulation converged the ABL with the average wind at hub height oriented with the wind farm. Since the mesh was re-oriented it required some modifications in the Coriolis effect implementation to keep the proper orientation of the Coriolis force by adding some rotation matrix force in the computation process. This simulation ran for approximately 175 flow-through times which corresponds to 1.1 inertial oscillations periods. In some article like the one written by Archer et al. [38], their precursor runs for only 30 flow-through times which seems to be enough in their case. It was only shortly investigated to see if such a short converging time would be enough. In this case, after 30 flow-through times, the turbulence intensity (TI) varied between 4.5 and 5.8 % at the rotor height. The goal is to be as close as possible to 6%. After 70 flow-through times the turbulence intensity stabilized but the inertial oscillation induced some variation of the angle at the hub height. 175 flow-through times was the best combination to have a 0° angle at the hub height between the wind direction and the wind farm orientation and a good TI percentage.

The third simulation starts based on the results of the previous simulation and keeps on converging the results. It runs on a refine mesh that has the same grid resolution and properties as the precursor. This simulation runs for a total of 15 flow-through times which develops some smaller scale turbulent structures.

The convergence of the ABL without the Coriolis effect does not require the first step to compute the wind direction at the hub height which made its convergence process faster.

The roughness of the bottom boundary is set to 0.0028[m] which corresponds to an open sea area or a desert [39]. To study only the effect of the Coriolis force it has been chosen to simulate an offshore wind farm in a neutral atmospheric boundary layer. This choice is made to avoid any parasitic effect that could be confused with the Coriolis effect.

Before analysing the results it is important to examine the characteristics of the two ABL. The table 3.1 summarizes their main characteristics. For a better interpretation of the Coriolis effect on the wind turbine wakes and energy production, it is important that the two ABL are as similar as possible.

Parameters	ABL without Coriolis	ABL with Coriolis
$u_\tau[m/s]$	0.375	0.387
$TI_{hub}[\%]$	6.02	6.14
$\alpha_{geo}[^{\circ}]$	0.05	8.3
$\alpha_{hub}[^{\circ}]$	-0.03 $\pm$ 0.02	0.01 $\pm$ 0.25
$\Delta\alpha[^{\circ}]$	0.02	1.55
$U_g[m/s]$	-	12
$U_{hub}[m/s]$	10.25	10.15

Table 3.1: Comparison of the ABL with the Coriolis effect and without the Coriolis effect.

3.4. SIMULATION OF ATMOSPHERIC BOUNDARY LAYERS WITHOUT AND WITH THE CORIOLIS EFFECT

The turbulence intensity and the velocity of the wind at the hub height were the two major parameters that must be as equivalent as possible in both atmospheres. Having the exact same values is almost impossible but, as it can be seen in the table 3.1, both atmospheres have some strong similarities. The friction velocity in both case is very similar and coherent with [40] and [41]. The Figure 3.10 shows the turbulent intensity with respect to the set-up height. The turbulent intensity is computed based on equation (3.1). At the wind turbines height (between the two dashed black lines) both atmosphere are very comparable.

$$TI = \frac{\sqrt{\frac{1}{3}(u'^2 + v'^2 + w'^2)}}{\sqrt{\overline{u^2} + \overline{v^2} + \overline{w^2}}} \quad (3.1)$$

The Figure 3.11 represents the variation of the angle of the wind. It can be seen that with the Coriolis effect the total variation is a bit bigger than 8 degrees. The wind is perfectly oriented with the wind farm at the hub. The main difference between the two ABL is the wind direction. With the Coriolis effect the wind angle varies between the bottom of the turbine blade and the top of it ($\Delta\alpha = 1.55^\circ$). It is expected that, although quite small, this variation of the wind direction will affect the wind turbines wakes. This is analysed in the next section.

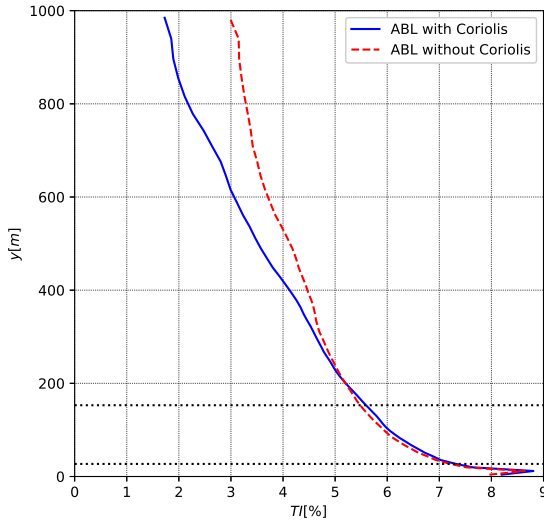


Figure 3.10: Turbulent intensity based on the velocity magnitude. Solid Blue: ABL with the Coriolis effect, Dash-dotted Red: ABL without the Coriolis effect and Dotted Black: delimitation of the area swept by the blades.

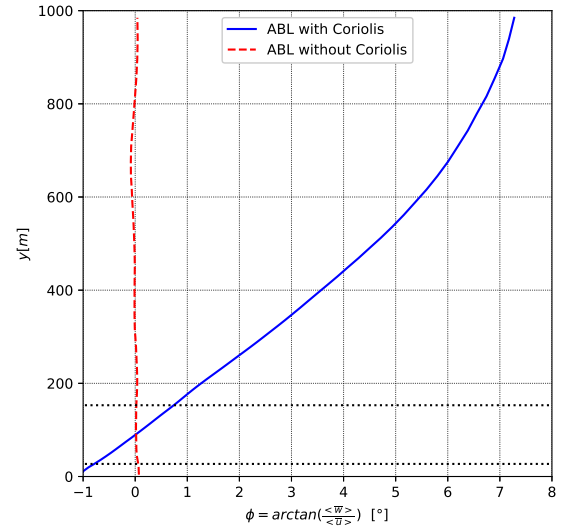


Figure 3.11: Angle of deviation of the wind. Solid Blue: ABL with the Coriolis effect, Dash-dotted Red: ABL without the Coriolis effect and Dotted Black: delimitation of the area swept by the blades.

3.5. INVESTIGATION OF THE WAKE EVOLUTION

The last analysed component is the streamwise velocity profile. This velocity profile is depicted in Figure 3.12. It appears that, at the hub height, both velocity profiles have the same values with a small tolerance. The wind on the upper boundary is different in both simulations in order to keep the same wind intensity at the wind turbines height as shown in table 3.1.

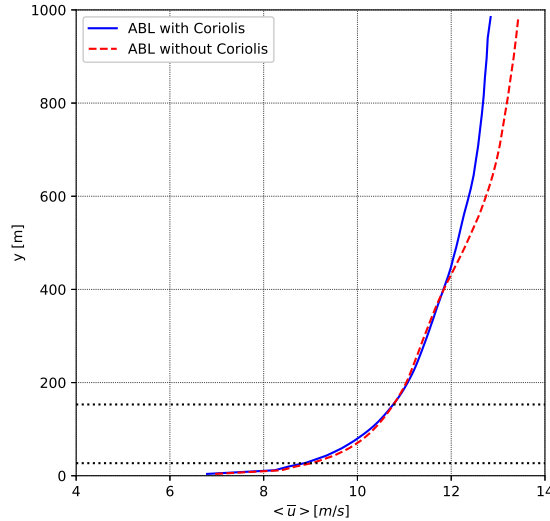


Figure 3.12: Velocity in the wind farm direction. Solid Blue: ABL with the Coriolis effect, Dash-dotted Red: ABL without the Coriolis effect and Dotted Black: Delimitation of the area swept by the blades.

To conclude this section, it has been seen that different simulation parameters must be chosen for each ABL to reach a similar state after convergence. But, at the end, both simulations are very comparable at the wind turbine height. Those similarities allow a good comparison of the performance for the two wind farm simulations in the next sections.

3.5 Investigation of the wake evolution

This section is devoted to investigating the wake evolution. All the results presented are the time-average velocity profiles over 2 flow-through times.

3.5.1 Average streamwise velocity

The mean streamwise velocity profile in the horizontal plane at hub height is shown in Figures 3.13 and 3.14. In both simulations, the wake from the first wind turbine directly enters the second wind turbine and no major deviation appears. The wake directly behind the wind turbines appears to have a similar intensity in both simulations. The black lines define the contour of the zones with an intensity of 0.4, 0.7, 0.8 and 0.9 on both graph. In the first 32 diameters, no differences are observed in the shape of the wake. In the far field behind the wind farm, the wake in the simulation with the Coriolis effect remains separated while, in the simulation without the Coriolis effect, both wakes meet after 32D.

3.5. INVESTIGATION OF THE WAKE EVOLUTION

A second observation is the direction of the wake. In both configuration, the wakes evolve in the streamwise direction. With the Coriolis effect a slight deviation of the wake towards the positive z -direction appears which is not the case without the Coriolis effect. This deviation is smaller than the one observed in [42]. In an other article [43], they state that the deflection of the wake occurs in the far field. In fact, their simulation runs on a much longer mesh than the one used here. In their case, the strong deflection of the wake occurs only 100D behind the last wind turbine. In the present simulation, our mesh is only 35D long behind the last wind turbines which is then clearly not enough to observe a large deflection of the wake. But on a short distance, the deflection is comparable to [43].

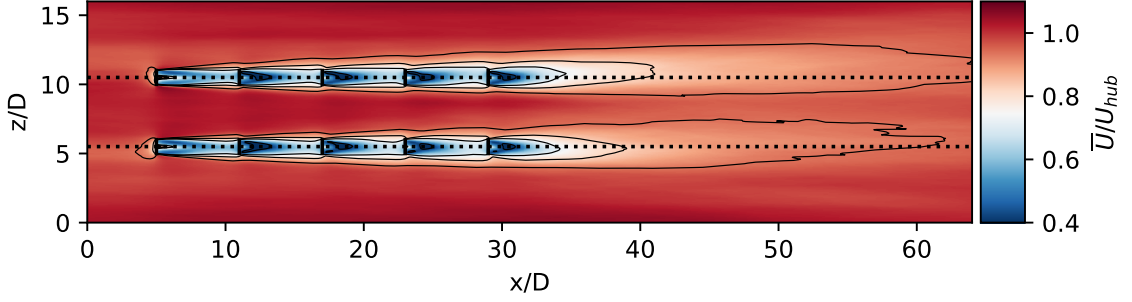


Figure 3.13: Streamwise velocity profile in the horizontal plane at hub height in the simulation with the Coriolis effect.

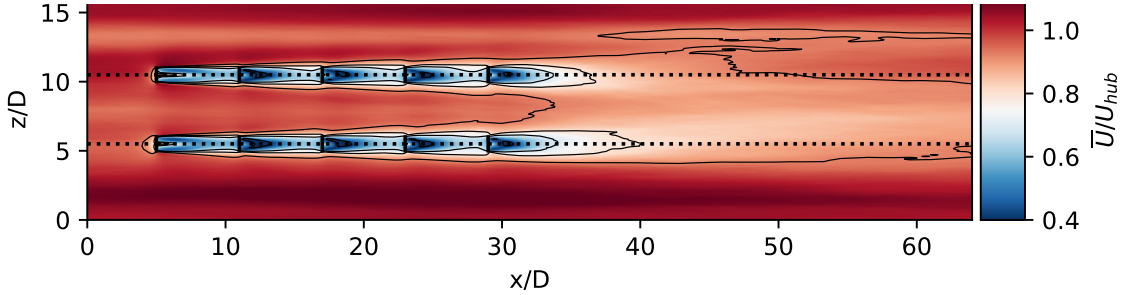


Figure 3.14: Streamwise velocity profile in the horizontal plane at hub height in the simulation without the Coriolis effect.

A last observation of those results could highlight the better spanwise homogeneity of the simulation with the Coriolis effect. Some stries are visible in Figure 3.14. A strie is a region with an higher or lower velocity than its neighborhood. They travel in a pure streamwise direction and they are a consequence of the recycling of the turbulent structures due to the periodic boundary conditions of the precursor. To properly see the stries, the streamwise velocity at 500 meters of altitude is displayed in Figures 3.15 and 3.16. It has been chosen to look at 500 meters altitude to avoid seeing the turbines wakes. The color bar has the same amplitude in both cases. Some strong stries clearly appear in the simulation without the Coriolis effect while, in the simulation with the Coriolis effect, the velocity field is much smoother and homogeneous. Some velocity variations still appear with the Coriolis effect but with a smaller intensity. This effect induces a non negligible spanwise velocity component. The variation of the spanwise velocity component with the altitude, deforms the turbulent structures and induces a mixing in the spanwise direction which is less present without the Coriolis effect. This mixing results in a more homogeneous velocity field.

3.5. INVESTIGATION OF THE WAKE EVOLUTION

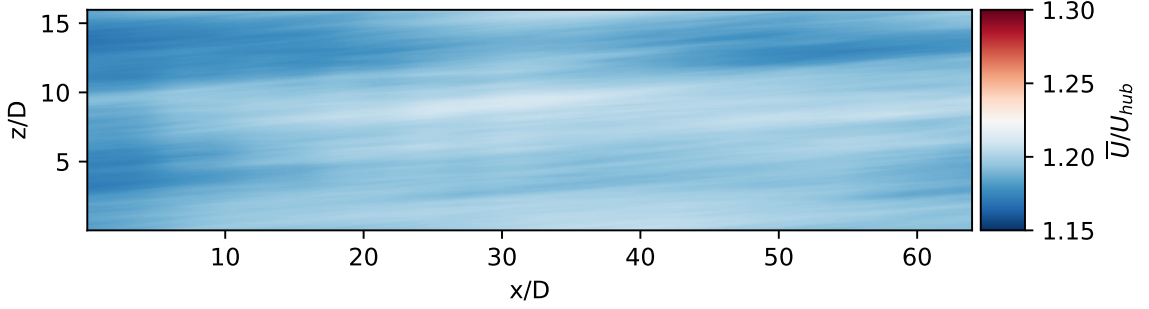


Figure 3.15: Streamwise velocity profile in the horizontal plane at 500 meters of altitude in the simulation with the Coriolis effect.

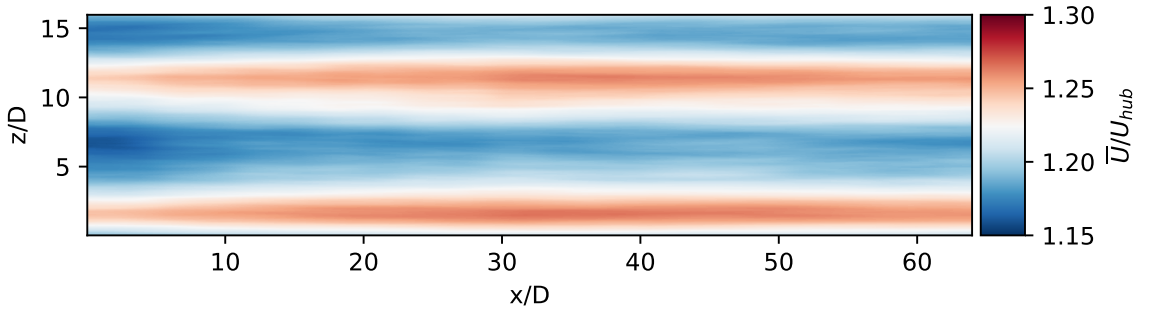


Figure 3.16: Streamwise velocity profile in the horizontal plane at 500 meters of altitude in the simulation without the Coriolis effect.

In a second time, the streamwise velocity is analysed behind the wind turbines. The streamwise velocity profile in vertical plane perpendicularly to the streamwise direction are displayed in Figure 3.17. The distance is expressed from the last wind turbine ($x/D = 29$). Those graphs show the evolution of the wake and its dispersion. From a global point of view, three diameters behind the last wind turbines the wakes appear to be similar in both simulations. In the simulation without the Coriolis effect the wakes diffuse homogeneously in the spanwise direction. With the Coriolis effect the wakes appear to diffuse more in the positive z -direction. This can be seen with the black lines that have a steeper slope on the left side of the wind turbines than on the right side.

A similar effect as the one observed on the horizontal velocity plane can be observed in this case. Without the Coriolis effect, the two wakes start to meet after $12D$. At $15D$, it is even more visible that the black lines which represent some iso-contours, are flat between the two turbine wakes in the case without the Coriolis effect. While, in the simulation with the Coriolis effect, the lines have a U-shape between the two turbine wakes (*cfr*: Figure 3.17). The origin of this difference is unclear. Two hypothesis were raised: either it is due to the Coriolis effect which promotes the diffusion of the wake in one direction and avoid the diffusion in the other directions or it is due to the velocity distribution and the turbulent structures in the simulation. To find out the reason, it would require to converge some new ABL, redo the simulations and see if this effect is still present.

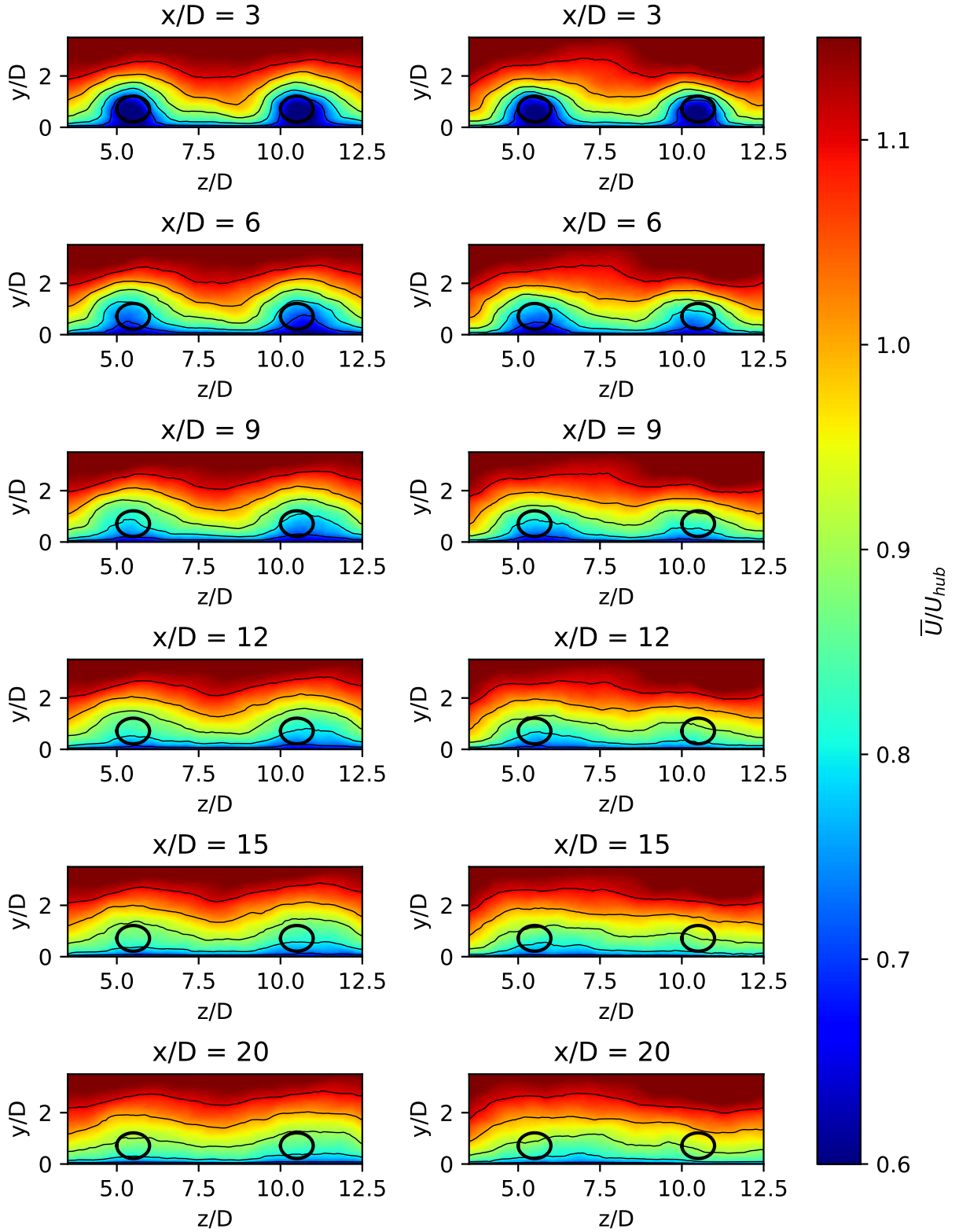


Figure 3.17: Time average streamwise velocity profile in different vertical plane perpendicularly to the stream direction and at different distances (x/D) from the last wind turbine. The horizontal and vertical axis are the distance normalized by the turbine diameter.

3.5. INVESTIGATION OF THE WAKE EVOLUTION

3.5.2 Streamwise velocity deficit

The velocity deficit is computed by subtracting the velocity at a certain distance behind the wind turbine to a reference velocity. The goal of this manipulation is to represent only the wake induced by the wind turbines and to neglect the gradient of velocity induced by the bottom boundary. The reference velocity is the velocity 2D in front of the first wind turbine of each row.

The deficit of velocity in an horizontal plane at hub height and in a vertical plane crossing the hub of the first wind turbines are displayed in Figure 3.18, 2D and 5D behind the turbines. Two diameters behind the turbines, no major differences can be observed. A small asymmetry appears but it cannot be linked to the Coriolis effect. Five diameters behind the wind turbines some differences clearly appear in both the vertical and horizontal profile. The velocity deficit with the Coriolis effect is on every graphs smaller than without the Coriolis effect. This could imply that the wakes recovers faster thanks to the Coriolis effect. It confirms the observation made by Abkar and Porté-Agel in their article [44]. They observed that with a veering incoming wind the wake recovers faster. A difference of around 5% of velocity deficit is observed in their study. In our simulation, the velocity deficit variation at the hub is closer to 3%. The surface heat flux applied by Abkar and Porté-Agel in their simulation is known to increase the veering angle of the wind. They observed a veering angle of 33° while, in our simulation, the veering angle is 8.3° . This much larger angle and the bigger spanwise velocity could explain the fastest wake recovery due to a larger spanwise mixing.

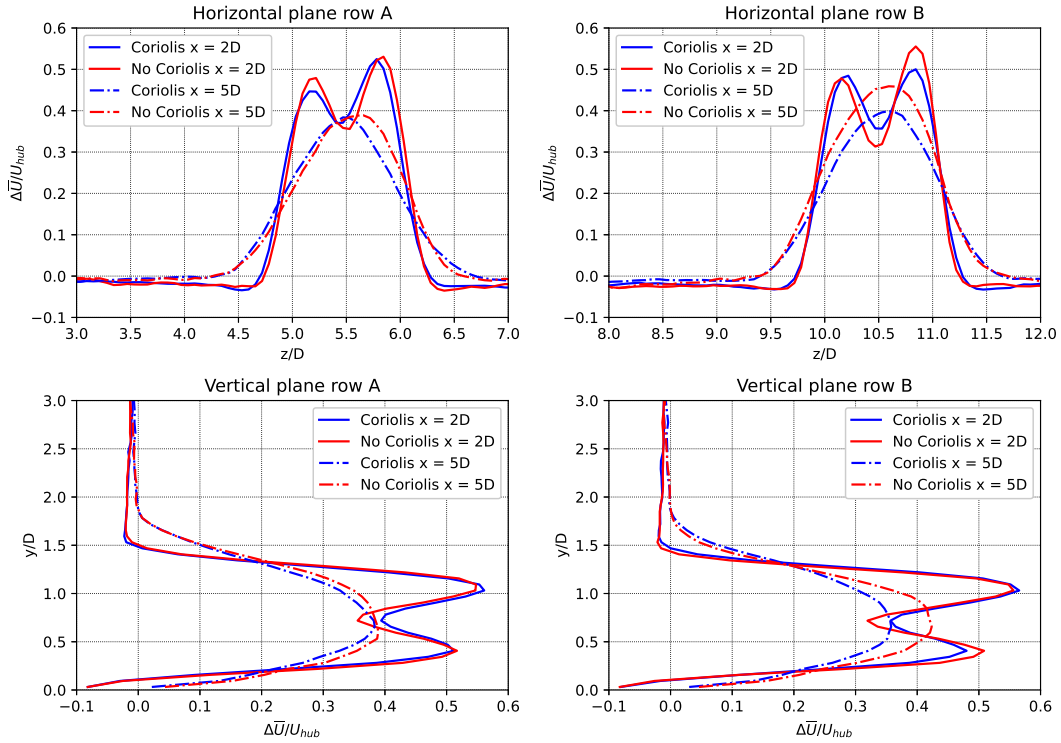


Figure 3.18: Time average streamwise dimensionless velocity deficit behind the first wind turbine. $\Delta U = U_{inflow} - U$ where U_{inflow} is the velocity 2 diameters before the first wind turbine. Top is the horizontal velocity profile at hub height, Bottom is the vertical velocity profile through the hub, Left is the wind turbine in row A and Right is the wind turbine in row B. Blue: simulation with the Coriolis effect, Red: simulation without the Coriolis effect, Solid: 2 diameters behind the first turbine, Dash-dotted: 5 diameters behind the first turbine.

3.5. INVESTIGATION OF THE WAKE EVOLUTION

A two dimensional representation of the velocity deficit behind the first and fifth wind turbines is displayed in Figures 3.20 and 3.21. Those images represent the wake behind the wind turbines and give an inside view at the wake structure and its evolution. In the simulation without the Coriolis effect, behind the first wind turbine, the wake diffuses homogeneously in the spanwise direction. It can be seen that 5D behind the turbine, the iso-contour lines are circular and are concentric. In the simulation with the Coriolis effect, the wake has, at first, also a circular shape that evolves into an ellipse. The ellipse is oriented towards the top right corner. This deformation is the result of the Coriolis effect. The angle of the wind in the region swept by the turbines blade is represented in Figure 3.19. This variation of angle moves the bottom part of the wake towards the negative z -direction and the upper part of the wake towards the positive z -direction. This asymmetry of the wake is not as strong as it could be observed in [44] because of the absence of heat flux but it can still be observed that the wake is deformed. It could be compared to the type of deformations observed by Gebraad et al. [45] in their simulations run on the Simulator fOr Wind Farm Applications (SOWFA).

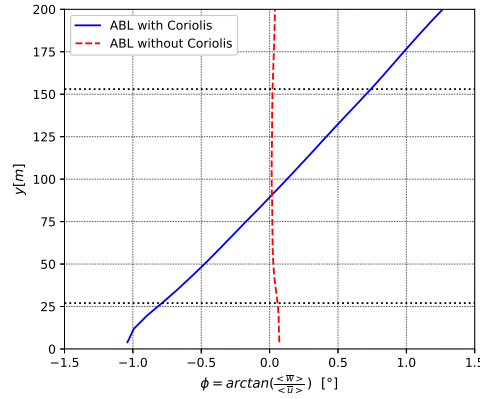


Figure 3.19: Velocity angle of the simulation at the wind turbines height. Solid Blue: simulation with the Coriolis effect, Dashed Red: simulation without the Coriolis effect and Dotted Black: limit of the area swept by the blades.

The evolution of the wakes behind the fifth wind turbine is very interesting because it gives a view of the wake of the last turbine but also the deformation of the wakes of the previous wind turbines. The elliptical shape of the wake of the fifth wind turbine is less visible than behind the first one. One of the reasons is that the wake of the previous wind turbines enter this turbine and it is harder to isolate only the wake of the last wind turbine.

The light blue color visible in Figure 3.21 is the wake of the previous wind turbines that have diffused with the distance. One diameter after the last wind turbine is equivalent to 25 diameters after the first wind turbine. The wakes of the previous turbines take more and more an elliptic shape with the Coriolis effect. It has largely diffused towards the up right while, in the simulation without the Coriolis effect, the wake has more a circular shape and has diffused in every directions. In the simulation with the Coriolis effect, the wake from the wind farm has diffused up to one diameter towards the left and a bit more than two diameters towards the right.

3.6. INVESTIGATION OF THE POWER PRODUCTION

Without the Coriolis effect the diffusion is symmetric and goes up to 1.7 diameter on both side. Those numbers give a more quantitative comparison of the wake diffusion with and without the Coriolis effect.

Figure 3.21 highlights again the fact that the Coriolis effect has an impact on the wake but an impact that could be characterized as slow. In fact, the impact of the Coriolis effect could be observed in the close wake by a deformation of the wake. But it is more apparent behind the last wind turbine when the wake from the upstream turbines has enough distance to diffuse. In their article Abkar et al. [44] observed a strong deformation of the wake three diameters behind the wind turbine while in our simulation, the deformation appears later and is less noticeable. The reason for this difference is probably again the surface heat flux that increases the veering angle of the wind. In our simulation the focus is set on the pure impact of the Coriolis effect without surface heat fluxes. This influence of the heat flux has been shown and explained by Gebraad et al. [45]. In their article they modified the FLORIS wake engineering model to incorporate the Coriolis effect and the surface heat flux effect. In a second time, they looked at the wake of the wind turbines in different atmospheric conditions such as an neutral atmospheric boundary layer and a stable atmospheric boundary layer. They observed a larger wake deformation with the surface heat flux. The surface heat flux generates a much larger veering angle of the wind which results in a larger wake deformation.

3.6 Investigation of the power production

The main goal of a wind turbine is the production of energy. Since the wake is deformed due to the Coriolis effect it could affect the energy production. This section compares the energy production of the two wind farms and try to understand the reasons for those differences. The table 3.2 summarizes the time average power output of the wind farm. In the simulation without the Coriolis effect, a stria enters the row of wind turbines B. This stria has a bigger velocity which results in a bigger power output. To limit the impact of the differences between the two ABL, the normalization is done based on the power output of the first wind turbine of each row.

	Wind farm		Row A		Row B	
Simulation type	$P/P_{ini}[-]$	Variation	$P/P_{ini}[-]$	Variation	$P/P_{ini}[-]$	Variation
Simulation without Coriolis	5.515	-	2.869	-	2.646	-
Simulation with Coriolis	5.847	+ 6.02 %	3.032	+ 5.68 %	2.815	+ 6.38 %

Table 3.2: Comparison of the power output of the two wind farms. The power is normalized with the power of the first wind turbine of each row (P/P_{ini}). The variation is computed based on $P_{Cor}/P_{No\ Cor} - 1$.

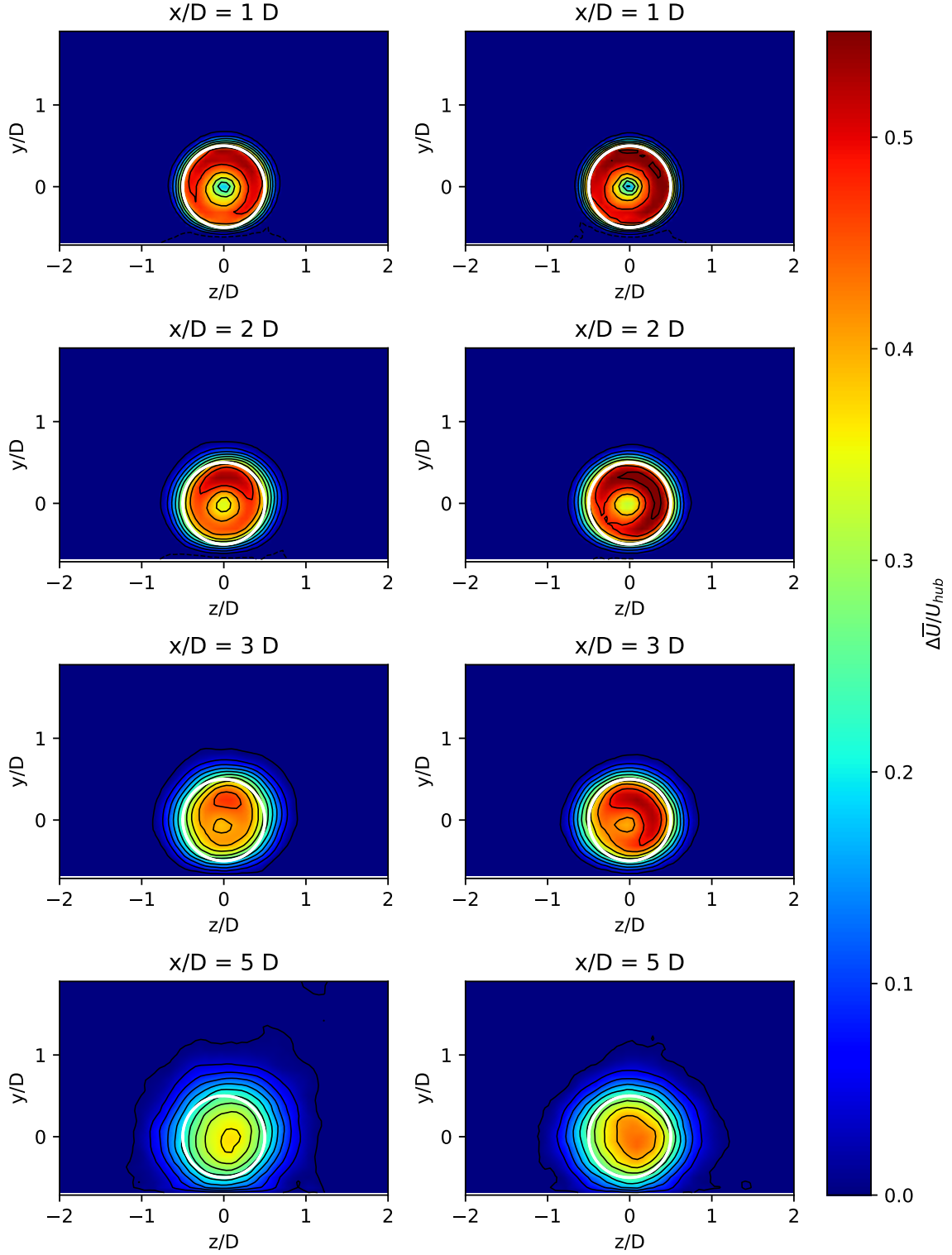


Figure 3.20: Time averaged streamwise velocity deficit behind the first wind turbine. The reference for the distance (dis) is set at the wind turbine location. The white circle represents the position of the wind turbines. $\Delta U = U_{inflow} - U$ where U_{inflow} is the velocity 2 diameters before the first wind turbine.

3.6. INVESTIGATION OF THE POWER PRODUCTION

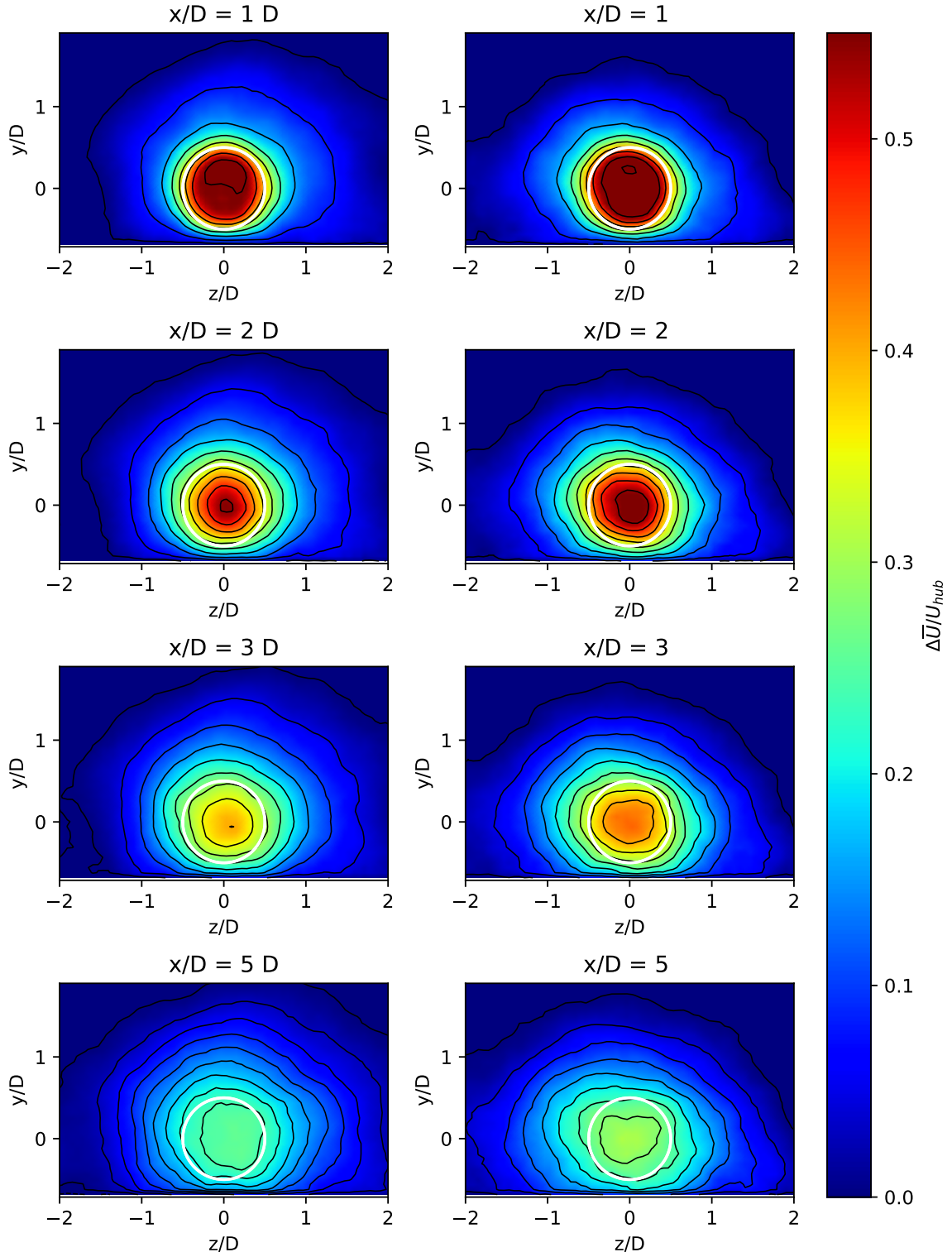


Figure 3.21: Time averaged streamwise velocity deficit behind the fifth wind turbine. The reference for the distance (dis) is set at the wind turbine location. The white circle represents the position of the wind turbines. $\Delta U = U_{inflow} - U$ where U_{inflow} is the velocity 2 diameters before the first wind turbine.

3.6. INVESTIGATION OF THE POWER PRODUCTION

The results show a net increase of the energy production with the Coriolis effect. The power increase over the whole wind farm is 6 %. The result shows a small dissimilarity between each row of the wind farm which can be explained by a small variation of the turbulent intensity and the veering angle of the wind in front of each row. To better understand the origin of the difference between the two wind farms, the power production of each wind turbine must also be analysed. This power is averaged over the two rows of each simulation and depicted in Figure 3.22. Those results are compared with the results obtained by Nouri et al. [46]. For their simulations they used a similar wind farm with 10 wind turbines and the same distance between each turbines.

The general trend of the power output is similar to the reference curve. The power output of the first wind turbines is double than the other wind turbines. This is explained by the recycling of the wake of the front turbines. The wake being a region of lower velocity, when it enters the downstream wind turbines, the flow has less power that could be extracted than the incoming wind in front of the first wind turbine. This results in a decrease of the power extracted between the first and second wind turbine. In the case with the Coriolis effect, the power output of the second wind turbine is slightly bigger than the case without the Coriolis effect. This increase remains for all the downstream wind turbines and varies from 4.6% to 7%. The power output increases slightly between the second and the fifth wind turbines. But this increase is not linked to the Coriolis effect since it is observed in our simulation as well as in Nouri et al. simulation (with/without the Coriolis effect) [46]. In their simulation Nouri et al. observed a total gain of 4.1% which is similar to the 6% observed in our case. Those results confirm the positive impact that the Coriolis effect has on a wind farm.

The difference of power output between the two wind farms can be explained by a better wake recovery with the Coriolis effect. To quantify the wake recovery, the power of the flow has been computed for each row on the surface swept by the turbine blades. The power in the flow is computed based on equation (3.2) where U_{mean} is the averaged streamwise velocity on the turbine area and S_{turb} is the surface swept by the blade.

$$P_{flow} = 0.5 \rho U_{mean}^3 S_{turb} \quad (3.2)$$

The averaged power in the flow is displayed in Figure 3.23 and has been normalized with the initial power in the flow. A strong decrease of power occurs at each wind turbine location and then it increases again slowly. This increase of power in the flow is due to the diffusion of the wake and the mixing with the surrounding wind. It is important that the flow recovers as much energy as possible before the next wind turbine to increase the power output of the wind turbines. The increase of energy production with the Coriolis effect can directly be linked with the power in the flow. In fact, in Figure 3.23, it is clear that the wake in the simulation with the Coriolis effect recover much faster than without the Coriolis effect. The difference goes up to 6.5% of more power in the flow with the Coriolis effect than without. This faster wake recovery can be explained by a better mixing. In fact, as it was explained previously, the spanwise component of the wind, induced by the Coriolis effect, generates a spanwise mixing that breaks the turbulent structures traveling in the streamwise direction. This effect can easily be transposed in a better mixing of the wake with its surroundings since, like the stries, the wake is a region with a variation of the velocity compared to its surroundings. A better mixing is then induced by the Coriolis effect and breaks the structure of the wake which reduces faster the velocity deficit. This effect was already observed in Figure 3.18, where the velocity deficit is smaller after 5D with the Coriolis effect than without it. The flow has then more power that could be converted by the wind turbines into electrical power.

3.7. CONCLUSION

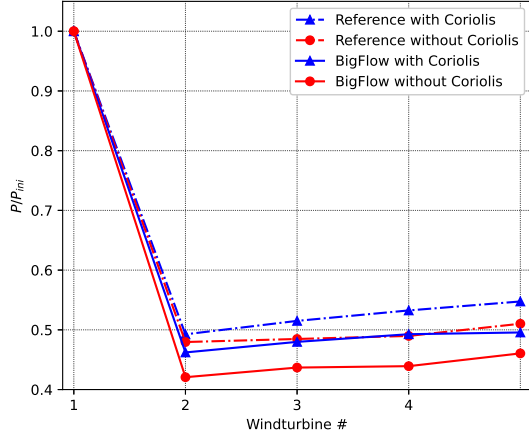


Figure 3.22: Power output of each wind turbines. The average is done in time and between the two rows. The power is normalized by the mean power of the front row. Blue-Triangles: simulation with the Coriolis effect, Red-Circles: simulation without the Coriolis effect, Solid: BigFlow results and Dash-dotted: reference curve [46].

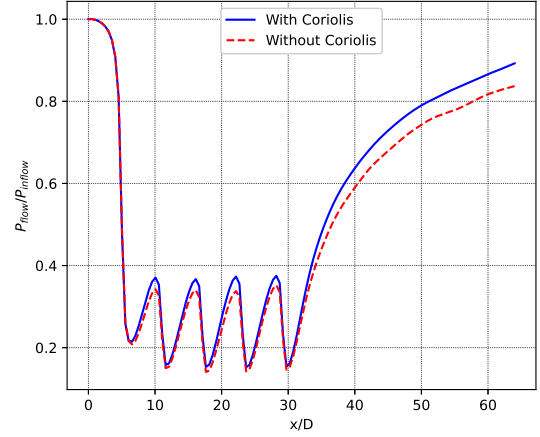


Figure 3.23: Power in the flow in the area swept by the turbine blades. The power is normalized with the inflow power and averaged in time and between the two turbine rows. Solid Blue: simulation with the Coriolis effect and Dashed Red: simulation without the Coriolis effect.

3.7 Conclusion

This chapter focus on the analysis of the Coriolis effect on a small wind farm. To properly interpret the impact of the Coriolis effect it is necessary to have two atmospheric boundary layers with similar characteristics in terms of streamwise velocities and turbulent intensity. On average, the two different simulations were very similar. But when running the simulation with the wind farm, some stries appear in the simulation without the Coriolis effect with a variation of $\pm 1[m/s]$. Those velocity variations affected one of the two wind turbines row. To avoid this influence, all the power output were normalized with the power output of the first wind turbine.

The Coriolis effect influences in a positive way the power output. The energy production increases by 6% under veering incoming wind. This increase is due to the better mixing of the wake. The veering wind also deforms the wake which could influence also the power output. But without the surface heat flux the wake deformation remains small. The Coriolis effect is a slow effect and to properly see some deformations of the wake the analysis must be conducted on a long distance.

Some similarities with the literature have been demonstrated in the results. The increase of power output and the wake deformation were observed by multiple sources. The results obtained previously validate the positive effect of the Coriolis effect on a wind farm and they highlight the importance of the Coriolis effect in the development of a proper wind farm model in an ABL. However, those results cannot certify the positive effect regarding all wind conditions. Since the intensity of the Coriolis force directly depends on the wind speed, to fully validate those results, some simulations should be conducted in different ABL with different wind velocities.

Chapter 4

Impact of the wind turbine rotating direction on a wind farm

In this chapter, the wind turbine rotating direction is analysed. It has been shown that the Coriolis effect induced a variation of the wind direction within the height of the area swept by the turbine blades. The variation of the wind direction could potentially promotes a direction of rotation of the wind turbines. A small wind farm is studied in a neutral boundary layer with the Coriolis effect in the northern hemisphere. Two simulations are presented, one with wind turbines rotating in the clockwise direction (CW) and one with wind turbines rotating counter-clockwise (CCW).

First the numerical set-up is briefly introduced and then the results of the simulation are analysed in three different sections. The first section is a study of the wake evolution and a comparison of the two wind farms and the second analysis compares the power outputs. It ends with an analysis of the angular velocity in the wake.

4.1 Numerical set-up

The numerical set-up used for this simulation is the same set-up as the one used in Chapter 3. The analysed wind farm is a small farm made of 10 wind turbines arranged in two rows of 5 rotors.

In this chapter, the comparison between the two simulations is easier because they run on the same ABL. The turbulent structures at the inflow are expected to be similar. The set-up is again located at 45° of latitude in the northern hemisphere with an earth rotation rate of $\Omega = 7.2722 \times 10^{-5} [s^{-1}]$. In this simulation no stratification is applied and the temperature is constant. The same precursor is used as in the previous chapter with Coriolis. All the details of the velocity and turbulent intensity are presented in section 3.4. The wind blows about $10[m/s]$ at the hub height and is oriented in the wind farm direction. The turbulence intensity at hub height is about 6%

4.2 Investigation of the wake evolution

The rotation of the blade of the wind turbines induces a rotation of the wake which is impacted by the rotating direction of the turbines. This section studies the deformation of the wake and its trajectory. All the results presented are the average in time over four flow-through times, equivalent to an hour of real time.

4.2. INVESTIGATION OF THE WAKE EVOLUTION

4.2.1 Average streamwise velocity

Figures 4.1 and 4.2 represent the time-averaged streamwise velocity at the hub height. The wakes of the two simulations presents some strong similarities in terms of intensities and directions. The observations show a small deviations of the wake in the far field towards the positive z -direction. The orientation of the wind farm is marked by the black dotted line. It appears that in the far wake, around $x/D = 50$, the iso-contour lines are deviated towards the upper part of the figure which corresponds to a deviation of the turbines wakes. This deviation was already observed in the previous chapter and is a results of the Coriolis effect. The deviation of the wake can also be observed on Figure 4.4. This Figure depicted the streamwise velocity profile on a vertical plane perpendicular to the streamwise direction fifteen diameters behind the last wind turbines. The deviation is more apparent at row B ($z/D = 10.5$) where, the center of the wake is shifted by half a diameter towards the right. Nevertheless, this deviation of the wake occurs in the far wake and does not have a real impact inside the wind farm. Even with this deviation, the downstream are largely exposed to the wakes of the upstream turbines.

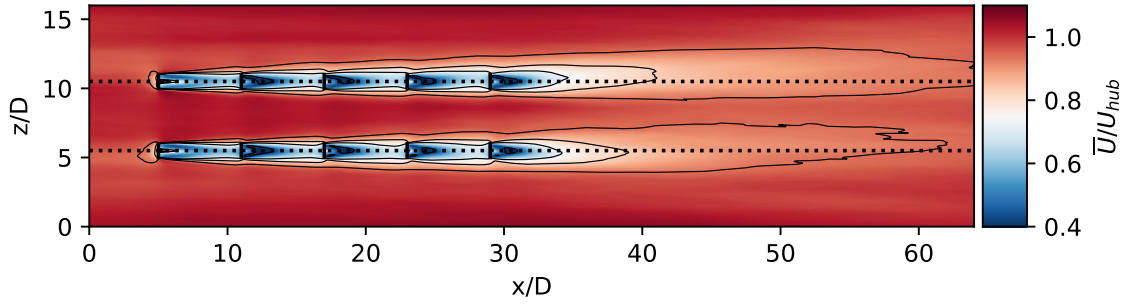


Figure 4.1: Streamwise velocity profile on an horizontal plane at hub height in the CW case.

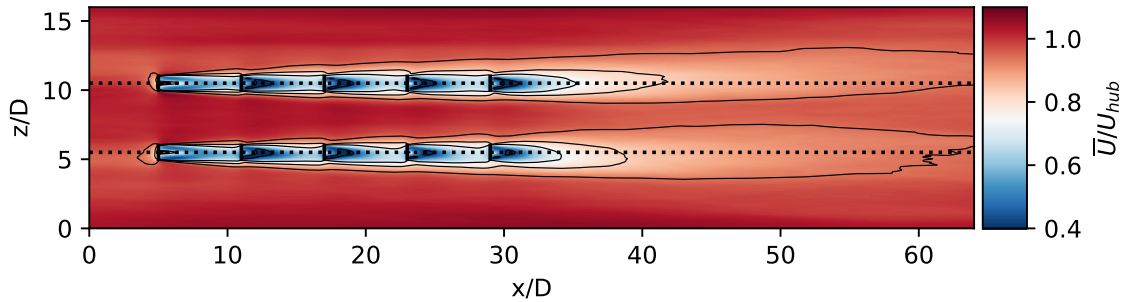


Figure 4.2: Streamwise velocity profile on an horizontal plane at hub height in the CCW case.

In addition to the deviation resulting from the Coriolis effect, the rotational direction induces also a small deviation. The Figure 4.3 depicts the subtraction of the average streamwise velocity from the CCW to the CW case. The blue color represents a larger velocity in the counter-clockwise case and the red color represents a larger velocity of the clockwise case. The wakes of the wind turbines rotating clockwise have a smaller velocity on the right side of the turbines (positive z -direction) than in the counter-clockwise wind turbines simulation. The rotating direction has then an impact on the wake near the turbines. In the far wake, it can be seen that the wake is slightly more deflected toward the positive z -direction when the wind turbines rotate

in the clockwise direction. The deflection of the clockwise rotating wind turbines is then sum up with the deflection induced by the Coriolis effect [46]. In the counter-clockwise rotating wind turbines, this deflection goes against the deflection from the Coriolis effect. In that case, it could be expected that for a very long wind farm, the wakes remain more aligned with the wind farm and so decrease the power output of the farm.

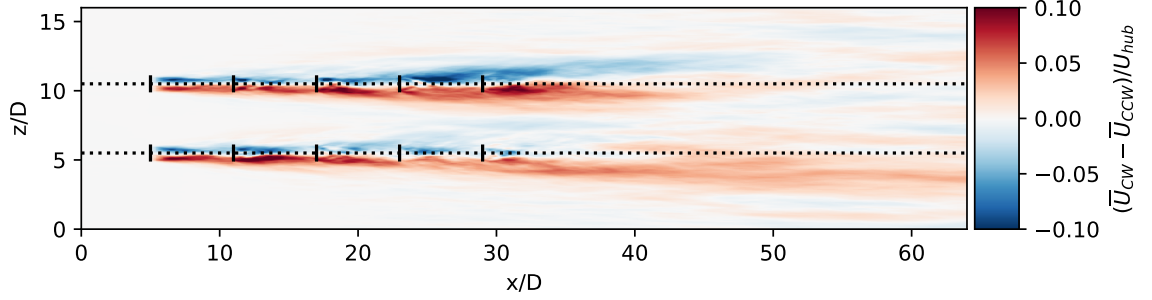


Figure 4.3: Difference of the streamwise velocity at hub height between the CW and CCW case, $(U_{CW} - U_{CCW})/U_{hub}$.

Beside the deviation of the wake, the wake structure is also visible on Figure 4.4. Both wakes remain separated as it was already observed in the previous chapter. This can be seen with the iso-contour lines that goes down and up between the two rows. The structures is not completely similar between the two simulations. In the counter-clockwise rotating wind turbines, the upper part of the wake is slightly more shifted towards the right than in the clockwise case. This wake deformation is analysed in more details in the next section.

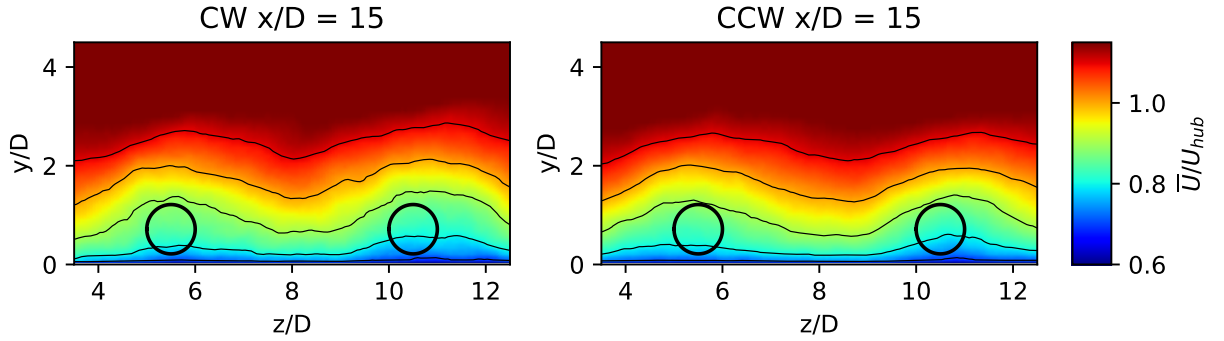


Figure 4.4: Streamwise velocity profile on a vertical plane perpendicular to the streamwise direction at 15 diameters behind the last wind turbines in the CW and CCW case.

4.2.2 Average velocity deficit

Figure 4.5 represents the velocity deficit behind the first wind turbine in an horizontal and vertical plane passing by the middle of the hub. The velocity deficit is computed with the reference velocity being the velocity two diameters in front of the first wind turbine of each row ($\Delta U = U_{inflow} - U$). The velocity deficit is shown at 2 and 5 diameters behind the turbine.

4.2. INVESTIGATION OF THE WAKE EVOLUTION

On the vertical plane, no major differences can be observed. In contrary to what was observed in the simulation with and without Coriolis (*cfr*: Chapter 3), the wake intensities have a very comparable amplitudes regardless of the wind turbine direction of rotation.

On the horizontal plane, more differences appear between the different velocity deficit profiles. Two diameters behind the turbine an asymmetry is visible between the two sides of the rotor. The velocity deficit is bigger on the right side when the turbines rotate in the clockwise direction and the opposite when the turbines rotate in the counter-clockwise direction. This asymmetry of the velocity deficit is the same one as observed in the previous section and could potentially steer the wake in the direction of the largest velocity deficit [46]. It is observed that five diameters behind the turbines, the horizontal velocity deficit profiles still present this asymmetry. This situation is visible in the two rows and it is a direct consequence of the rotating direction. The direction of rotation of the wind turbines influences the orientation of the asymmetry. The reason for this is presented in details later in this chapter.

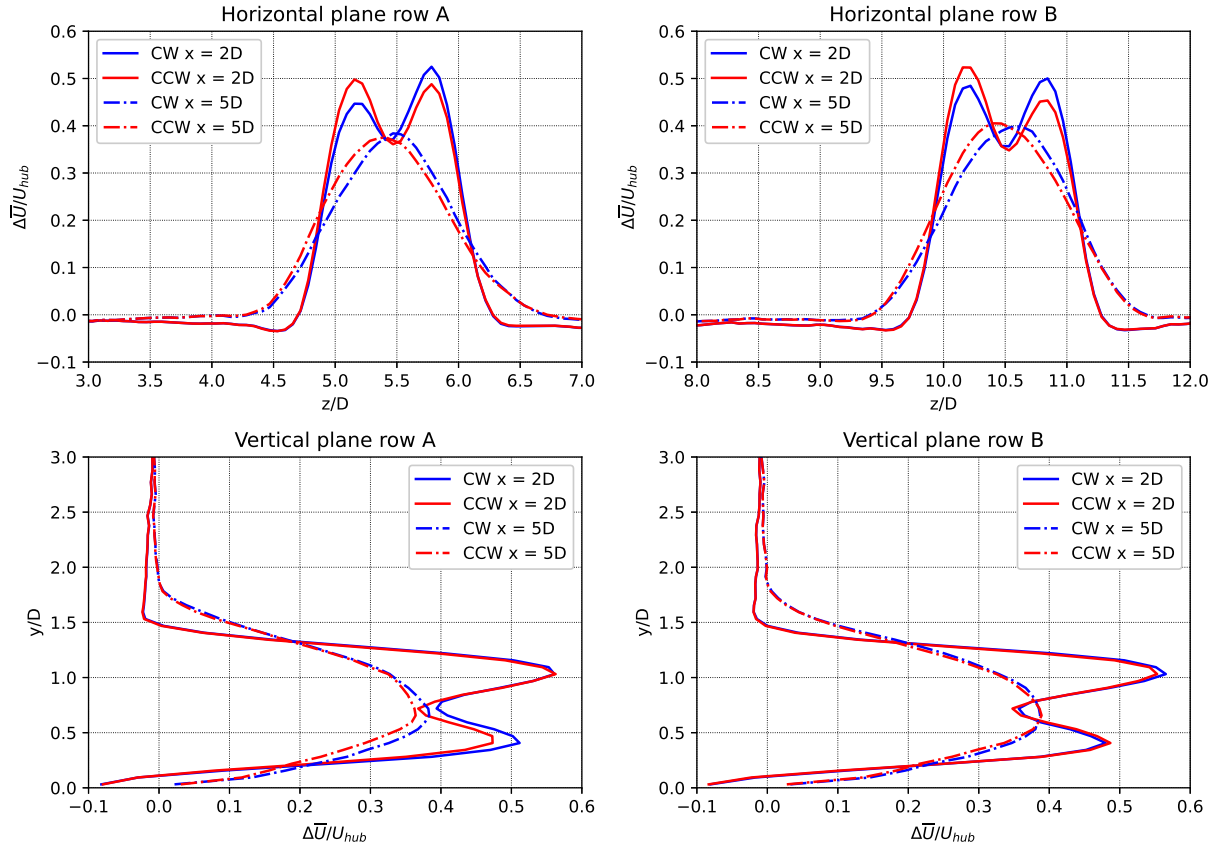


Figure 4.5: Time average streamwise velocity deficit behind the first wind turbines. $\Delta U = U_{inflow} - U$ where U_{inflow} is the velocity 2 diameters before the first wind turbines. Top is the horizontal velocity profile at hub height, Bottom is the vertical velocity profile aligned with the hub, Left is the wind turbine in row A and Right is the wind turbine in row B. Blue: wind turbine rotating in the Clockwise direction, Red: wind turbine rotating in the counter-clockwise direction, Solid: 2 diameters behind the first turbines, Dash-dotted: 5 diameters behind the first turbines.

4.2. INVESTIGATION OF THE WAKE EVOLUTION

In a second phase, the shape of the wake is analysed close to the wind turbines. This analysis is done behind the first and the fifth wind turbines. The objective is first to observe the deformation of the wake of one wind turbine which is done behind the first wind turbine. And then the wake of the entire wind farm is analysed and this is done behind the fifth wind turbine. The Figure 4.6 represents the velocity deficit one, two, three and five diameters behind the first wind turbine. One and two diameters away, no major differences are observed in the shape of the wake. But the earlier observation is seen again, an horizontal asymmetry appears in the wake and is different with regards to the direction of rotation of the wind turbines. A vertical asymmetry is also visible. In both cases a bigger velocity deficit is observed in the upper part of the rotor.

Three diameters behind the turbines, the wakes have different shapes, and it is even more apparent five diameters behind. Both wakes have an elliptical shape due to the Coriolis effect and are oriented towards the upper right. The deformation of the wake of the CCW rotating turbine is more apperant than the one of the CW rotating turbine.

The variation of the shape of the deformation is even more visible behind the fifth wind turbine, as shown on Figure 4.7. The wake of the fifth wind turbines is difficult to identify but it is interesting to analyse the diffusion of the wake of the upstream wind turbines. While in the CW simulation most of the wake seems to be pushed towards the right side of the atmosphere, in the CCW case the upper part is pushed towards the right side but the lower part is pushed towards the left. Those deviations give a more circular shape to the wake of the clockwise rotating turbines and a more elliptical shape to the wake of the counter-clockwise rotating wind turbines.

The differences of the orientation of the wake diffusion is a consequence of both the direction of rotation of the turbines and also the Coriolis effect. As it can be seen on Figure 4.8, the wakes induced by CW rotating wind turbines rotate in the counter-clockwise direction and the opposite for the CCW rotating wind turbines. Due to the variation of the wind direction within the rotor height, the interaction of the wake with the wind depends on its direction of rotation. Figure 4.9 depicts the wind direction within the height of the wind turbine and the wake rotating direction. When the wake turns in the counter-clockwise direction, its spanwise velocity component (velocity component along $\hat{\mathbf{e}}_z$) is directly opposed to the wind direction. The wind in the upper part of the turbine goes towards the right while the upper part of the wake goes towards the left, and the opposite direction in the bottom part. The wake of a wind turbine rotating in the clockwise direction goes then directly against the spanwise component of the wind. The opposite is observed for a CCW rotating wind turbines; the wake rotates in the clockwise direction. Its spanwise velocity component goes in the same direction as the wind induced by the Coriolis effect. In this case, instead of fighting against each other, the wakes and the atmospheric wind reinforce each other. The upper part of the wake is then strongly pushed towards the right and the bottom part is strongly pushed towards the left. All of this results in a larger wake deformation in the CCW case than in the CW wind farm.

The asymmetry in the velocity profile, shown in Figure 4.5 is induced by the rotation of the turbines as well as the gradient of velocity induced by the ground, it is not a direct consequence of the Coriolis. In fact, this asymmetry could also be observed during the simulation without the Coriolis effect. The rotor rotating clockwise induces a wake rotating in the counter-clockwise direction as mentioned previously. As a results, the low speed region of the flow (in the lower part of the rotor) is deviated towards the upper right part of the rotor while the high speed region of the flow (upper part of the rotor) is deviated toward the lower left part of the rotor. Consequently, a larger velocity deficit appears on the upper right part and a smaller velocity

4.2. INVESTIGATION OF THE WAKE EVOLUTION

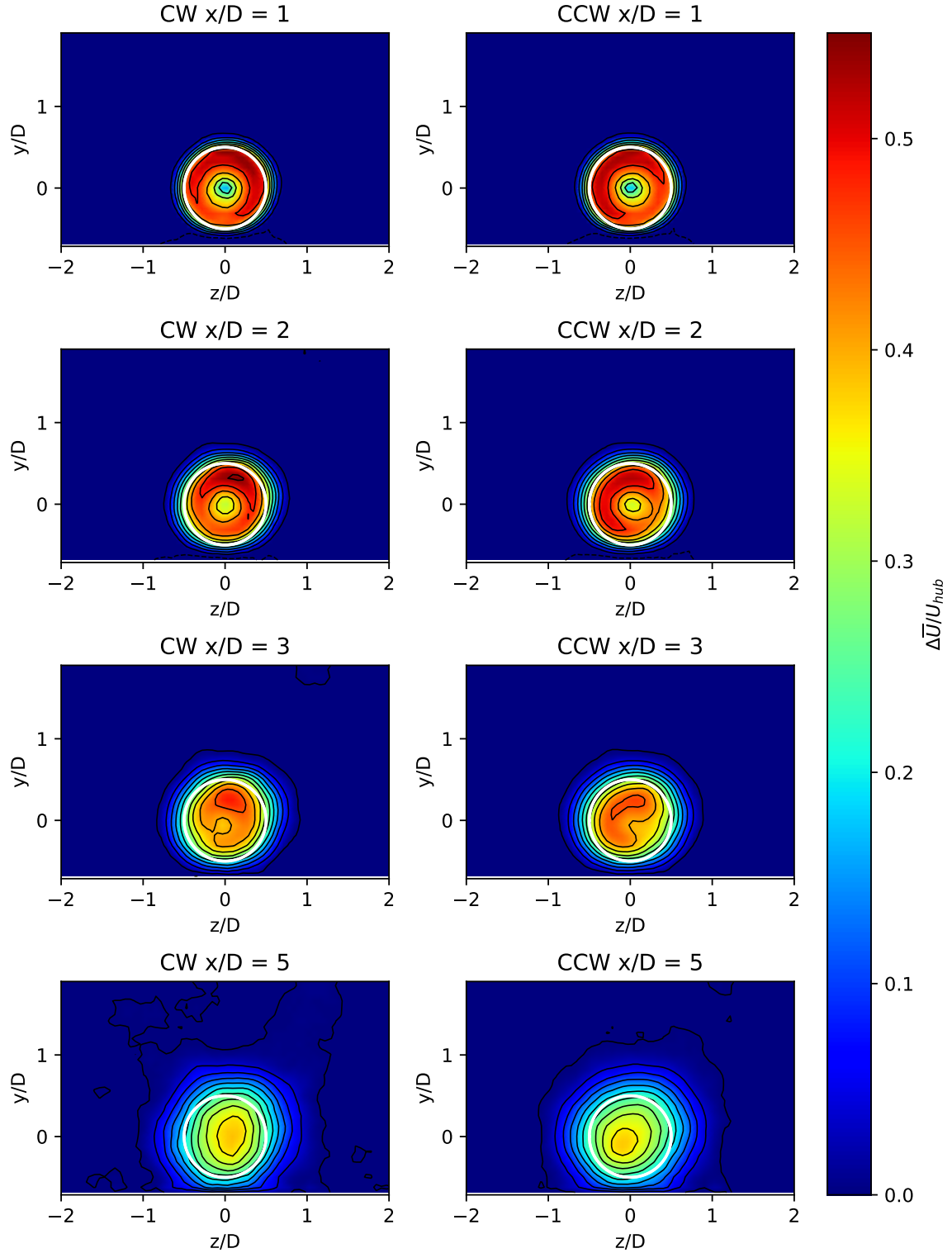


Figure 4.6: Streamwise velocity deficit behind the first wind turbine. The reference for the distance (x/D) is set at the wind turbine location. The white circle represents the position of the wind turbines. $\Delta U = U_{inflow} - U$ where U_{inflow} is the velocity 2 diameters before the first wind turbine. Left column: Clockwise case and Right column: counter-clockwise case.

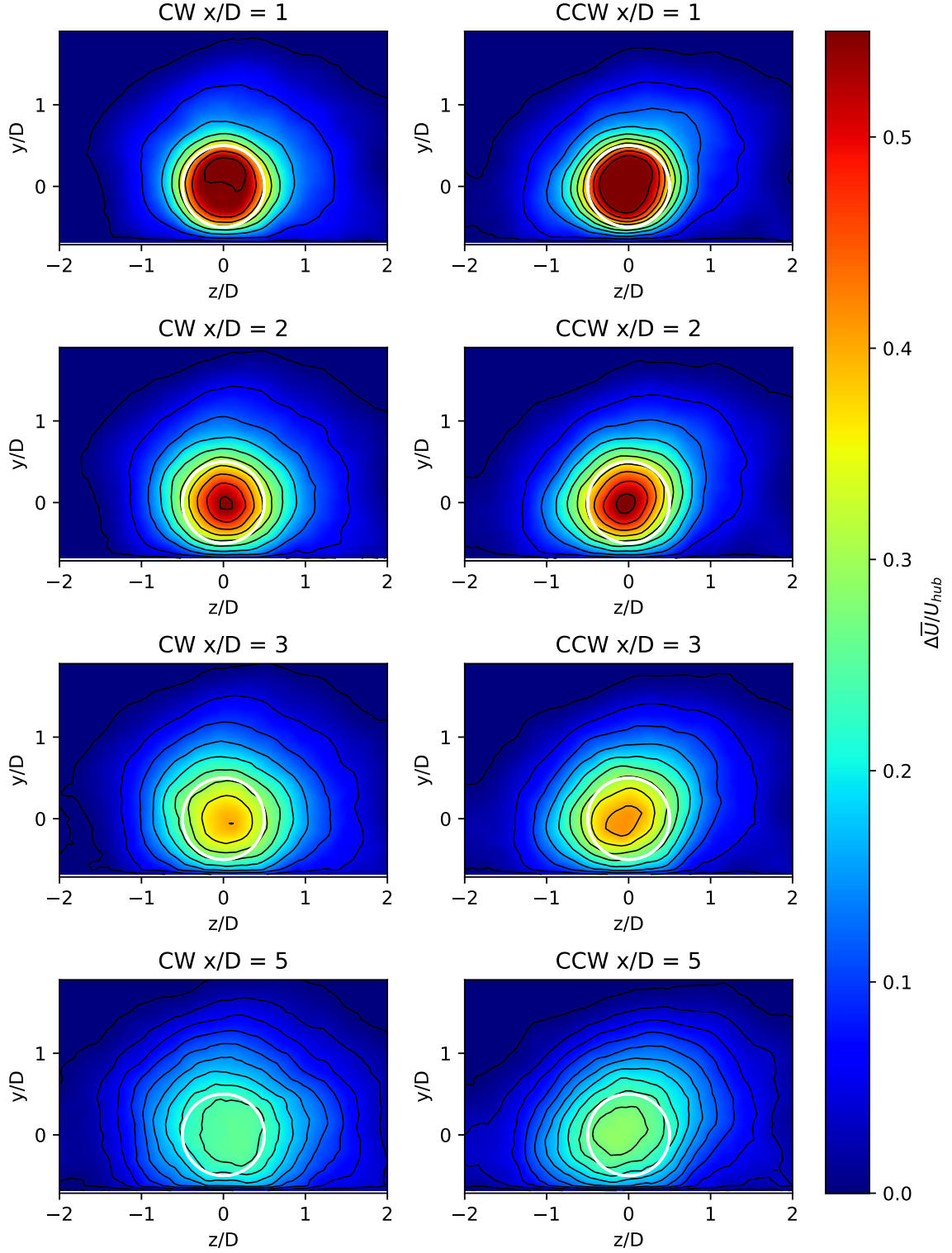


Figure 4.7: Streamwise velocity deficit behind the fifth wind turbine. The reference for the distance (x/D) is set at the wind turbine location. The white circle represents the position of the wind turbines. $\Delta U = U_{inflow} - U$ where U_{inflow} is the velocity 2 diameters before the first wind turbine. Left column: Clockwise case and Right column: counter-clockwise case.

4.2. INVESTIGATION OF THE WAKE EVOLUTION

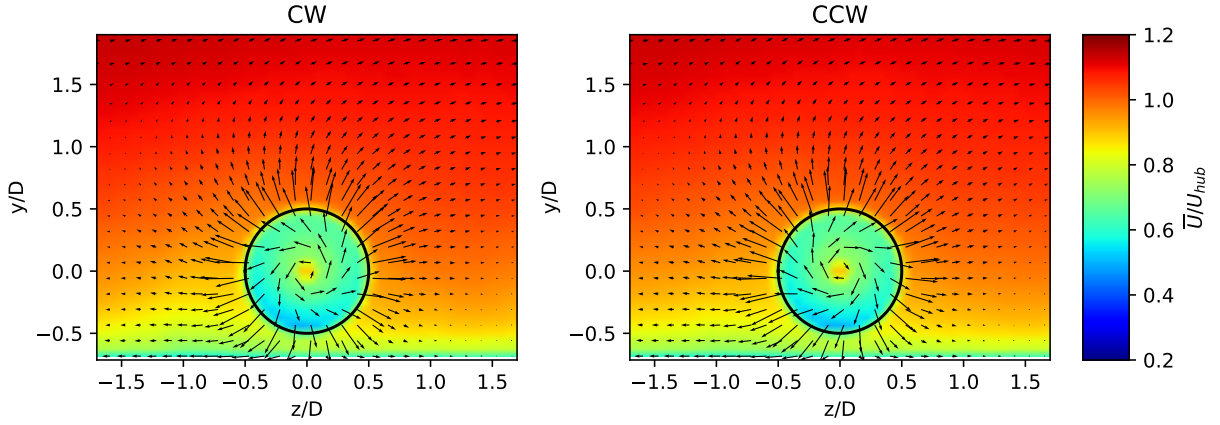


Figure 4.8: Comparison of the rotation of the wake behind a wind turbine with regards to its direction of rotation. Color background: streamwise average velocity, Left: CW case and Right: CCW case.

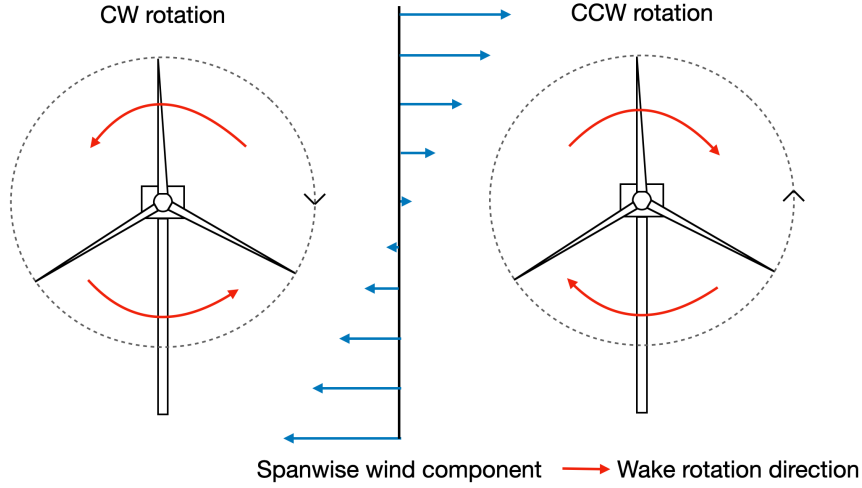


Figure 4.9: Sketch of the spanwise component of the wind velocity and the wake direction with regards to the wind turbine direction of rotation.

deficit appears on the lower left part of the rotor. This results in a deflection of the wake towards the right direction that could be observed previously [47]. The exact opposite deviation of the flow occurs when the turbines rotate in the counter-clockwise direction. Consequently, the larger velocity deficit is located in the upper left part of the wake and the smaller velocity deficit is in the lower right part of the wake.

This difference of the wake is not without consequences. It directly modifies the wind direction and intensity entering the downstream wind turbines. The difference, between the two simulations, of the velocity profile in the normal direction to the turbine blades is shown in Figure 4.10. The wind perceived by the first wind turbines is very similar in both simulations which was expected since they run on the same precursor. But, the flow entering the downstream wind turbines is very different between the two simulations. A difference of $+ and - 5\%$ of the hub streamwise velocity is observed. The flow variation induces a variation of the forces applied on the rotor and results in a variation of the power output of each wind turbines.

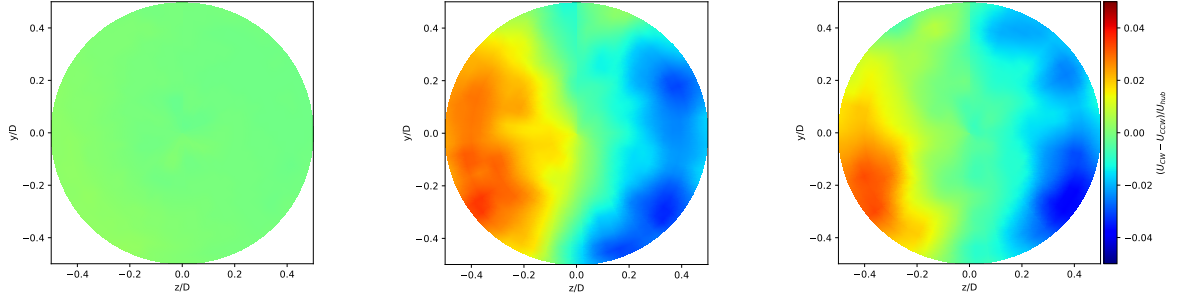


Figure 4.10: Difference of the velocity of the flow in the normal direction to the blades of the wind turbines between the two simulations, $(U_{CW} - U_{CCW})/U_{hub}$. From left to right: velocity on the rotor of the first turbine row A, velocity on the rotor of the second turbine row B and velocity on the rotor of the fifth turbine row A. The colors go from -0.05 to $+0.05$.

4.3 Investigation of the power production

The wind entering the downstream wind turbines has different intensities depending on the rotating direction of the wind turbines. Those differences could have an impact on the power output of the wind farm and the wake recovery. The power output of the two simulation is displayed in table 4.1. For this comparison, the outputs are normalized by the power output of the first wind turbine of each row in the wind farm rotating CW. Normalizing with the same value both simulations allows to see if the rotating direction already affects the power production of the first turbines.

	Wind farm		Row A		Row B	
WT rotating direction	$P/P_{ini}[-]$	Variation	$P/P_{ini}[-]$	Variation	$P/P_{ini}[-]$	Variation
CW	5.865	-	3.035	-	2.825	-
CCW	5.847	-0.32 %	3.032	- 0.3 %	2.815	- 0.35 %

Table 4.1: Comparison of the power output of the two wind farms. The power is normalized with the power of the first wind turbine of each row (P/P_{ini}) in the wind farm rotating CW. The variation is computed by: $P_{CCW}/P_{CW} - 1$.

The total output of the wind farm presents a variation of 0.3% between the two simulations. The analysis of each row separately also shows a slight increase of the power output in favour of the clockwise rotational direction. Those observations are not consistent with those of the simulation done in [12]. Their simulation demonstrated a difference of 11% of energy production in the northern hemisphere in favour of the counter-clockwise rotation and it could go up to a difference of 23% under specific atmospheric conditions. The maximal output is achieved when the wind has a large veering angle due to the stratification. Our simulation set-up is too different from the one in the article to do a proper comparison but it is worth mentioning that the general trends of the results are different.

4.3. INVESTIGATION OF THE POWER PRODUCTION

In the two rows of the simulations, the clockwise rotation wind turbines produced slightly more energy. To understand the variation of energy production, it is interesting to analyse the power output of each individual wind turbine and see if this trend is observed at every turbines. (*cfr*: Figure 4.11). As seen in the table 4.1, one of the rows produced more power than the other row. The slight difference between the two rows can be attributed to a slight variation of the wind speed at the inflow and is unavoidable. The power output of the front row wind turbine is the same in both cases. This shows that for a singular wind turbine, the rotational direction does not have any influence on the power output.

The output of the downstream turbines present some small variations that go up to 1%. The larger power output alternates between the two simulation. Even if on a global point of view, the power output was slightly bigger for the clockwise rotation case, on a more detailed analysis it appears that both directions of rotation of the wind mills have an equivalent power output. The variation of wind direction in a neutral boundary layer is not strong enough to induce a significant variation of power production between the two rotating direction of the turbines.

The power in the flow is also analysed based on the streamwise velocity and it is displayed on Figure 4.12. No difference can be seen in the wake recovery inside the wind farm. A difference is visible in the far wake behind the wind farm, around 15 diameters behind the last wind turbine. This difference can probably be explained by the opposition of the rotation of the wake to the ABL wind direction in the clockwise rotation case as it was explained earlier. This opposition could induce a better wake recovery over the long term by a reduction the angular velocity of the wake and a better mixing. In the counter-clockwise rotation case, the rotation of the wake is supported by the wind direction and so the rotation could remain longer. The angular velocity of the flow is analysed in the next section.

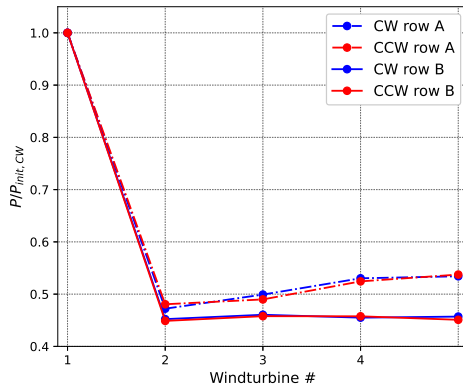


Figure 4.11: Time average power output of each wind turbines. The power is normalized by the mean power of the front row in the CW case ($P/P_{init,CW}$). Blue: CW case, Red: CCW case, Dash-dotted: row A and Dash-Dotted: row B.

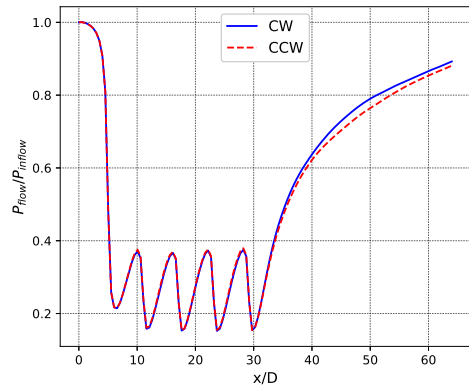


Figure 4.12: Dimensionless power in the flow in the area swept by the turbine blades. The power is normalized with the inflow power and averaged in time and between the two turbine rows. Solid Blue: CW case and Dashed Red: CCW case.

4.4 Investigation of the angular velocity of the wake

To fully validate the explanation of the rotation of the wake and the direction of the wind, one must look at the angular velocity in the wake. It is computed with the equation (4.1), where r is the distance between each point of the study area and the center of the wake and $v_{\perp}$ is the tangential velocity. The results are computed on different circles, at 0.1 diameters of each others, aligned with the rotors in the streamwise direction. The results, displayed later, represent the average of the angular velocity over these circles or a part of these circles. The positive angular velocity rotates in the counter-clockwise direction. A small sketch is shown on Figure 4.13 to describe the computation process.

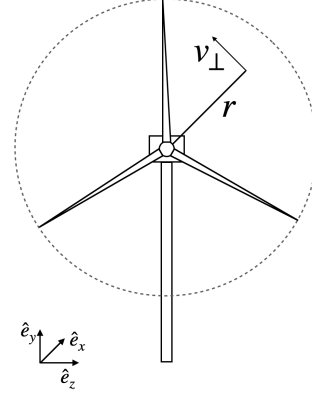


Figure 4.13: Sketch of the computation of the angular velocity.

$$\omega = \frac{v_{\perp}}{r} \quad [s^{-1}] \quad (4.1)$$

The Figure 4.14 displays the absolute value of the angular velocity in the wake. It appears that the wake of the wind farm rotating in the clockwise direction has a smaller angular velocity which is reduced faster than in the counter-clockwise direction. It is even possible to see that the wake between two wind turbines almost lost all of its angular velocity in the CW case while in the CCW case the wake enters the next wind turbine with a non negligible angular velocity. This variation of angular velocities could potentially explain the variation of power output seen in the previous section.

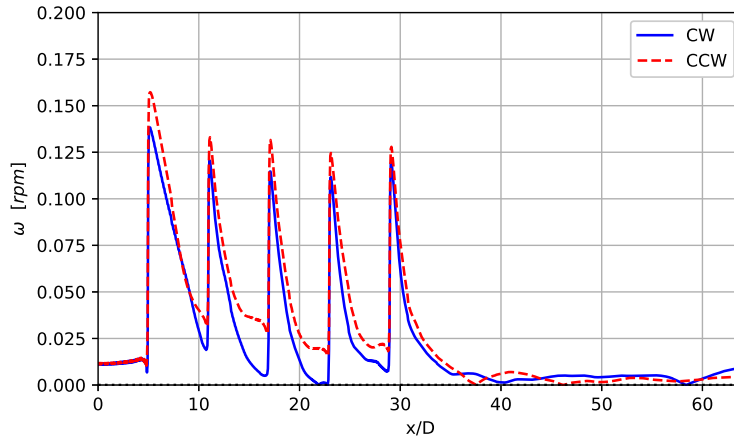


Figure 4.14: Angular velocity of the wake. Solid Blue: CW case and Dashed Red: CCW case.

In a second time, the angular velocity was analysed in the different sectors of the wind turbine wakes. The Figure 4.15 depicts the angular velocity in the upper and lower part of the wake in both wind farms. The angular velocity is smaller in both cases at the bottom of the wake than at the top. The velocity gradient imposed by the ground is the reason for the reduction of the

4.5. CONCLUSION

angular velocity in the lower part of the wake. The Figure 4.16 depicts the angular velocity at the right and left side of the wake. In this case, an opposition is visible between the two simulations. In the CW case, the angular velocity is higher when the blades are going down (right side of the wake) and the angular velocity is smaller when the blades are going up (left side of the wake). The opposite phenomenon is observed in the CCW case. The reason is also probably due to the bottom boundary which impacts the flow trajectory.

Another comparison of the angular velocity can be demonstrated with the velocity deficit. The velocity deficit behind the first wind turbine in the CW case was larger at the top and on the right side of the wake as it can be seen on Figure 4.6. In the CCW case the largest velocity deficit was also in the upper part but on the left side. A larger velocity deficit can potentially be associated with a larger angular velocity and are a consequence of the ground that impacts the flow trajectory and velocity.

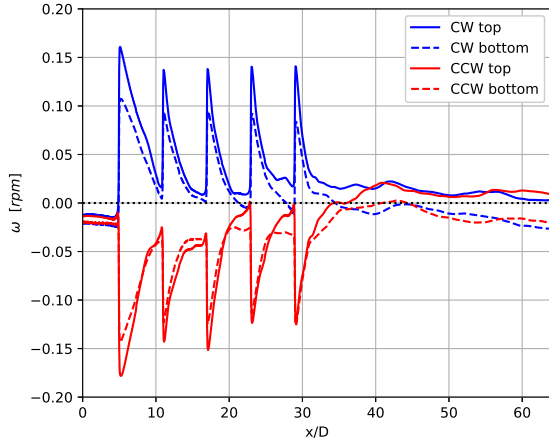


Figure 4.15: Angular velocity of the wake. Blue: CW case, Red: CCW case, Solid: top part of the wake and Dashed: bottom part of the wake.

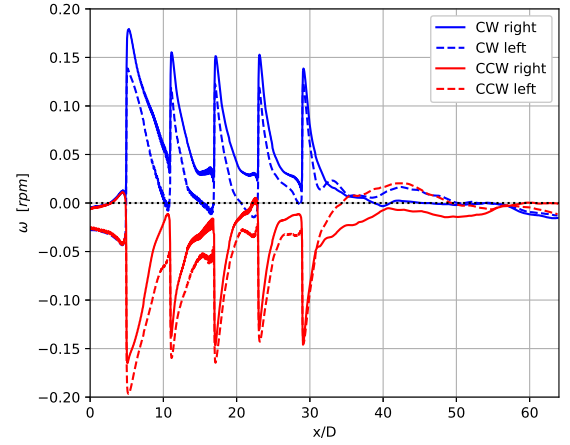


Figure 4.16: Angular velocity of the wake behind wind turbines. Blue: CW case, Red: CCW case, Solid: right part of the wake and Dashed: left part of the wake.

4.5 Conclusion

The analysis shows that the rotational direction influences the wake structure and its evolution. It has been demonstrated that a larger deformation of the wake is encountered when the turbines rotate in the counter-clockwise direction. Nonetheless, no preferential direction of rotation of the wind turbines was found during this study in a neutral boundary layer. A slight difference was observed in favor to the clockwise direction of rotation. But, this difference was below 1% and is then not relevant. The direction of the wake is slightly impacted by the direction of rotation. The deflections of the wakes due to the Coriolis effect and the direction of rotation, sum up in the clockwise case while they are opposed in the counter-clockwise simulation. Finally, it has been shown that the clockwise rotating wind turbines induce a smaller angular velocity in the wake as the wake rotation is opposed to the spanwise wind direction in the boundary layer.

Chapter 5

Conclusion and perspectives

This study aimed at providing a deeper knowledge of the influence of the Coriolis effect on a wind farm and then analysing the influence of the wind turbines direction of rotation. To achieve this study, the Coriolis effect was implemented in BigFlow and validated under different simulation set-up.

The implementation of a Coriolis effect model was the first step described in this thesis. Its implementation was first explained on a mathematical point of view and some implementation details were given such as the reduction of the stencil order at the boundary in the case of a wmLES. The validation of the effect was then explained over three different set-ups. The first two simulations were in DNS and simulate respectively at rotating channel with $Re_\tau = 194$ and the laminar Ekman layer. Those two simulations gave some accurate results that were comparable to the reference curves. The turbulent Ekman layer was then simulated to validate the implementation in a wmLES and compared to a DNS at $Re = 2000$. This last simulation presented some minor differences compared to the reference curve. They could potentially be reduced by tuning the parameters of the wall model. Nevertheless, the differences were small in contrast to the velocity magnitude. It can be concluded that the Coriolis effect was properly implemented in BigFlow

The second part of this manuscript focuses on wind farm simulations. A small wind farm made of 10 wind turbines was simulated in two different boundary layers, one with and one without the Coriolis effect. The major difference in the two ABL was the variation of the wind direction with respect to the altitude. A veering wind was observed with the Coriolis effect while, without it, the direction remains oriented throughout the wind farm direction. A positive impact of the wind direction variation is the reduction of the sties number and their intensity in the ABL. It has been shown that the variation of the spanwise velocity breaks the monotonic streamwise motion of the turbulent structures. This implies a better mixing of the ABL which is an interesting results for future wind farm simulations.

The veering wind induced a deformation the wake behind the turbines. Close to the wind turbines, the wake is slightly deformed and take an elliptical shape. Further downstream the differences between the two simulations are more apparent. It has been shown that the wake diffusion was more deviated towards the right with the Coriolis effect in the northern hemisphere. The comparison with the literature shows a correct deformation of the wake. The power output of the wind farm shows some interesting results. Adding the Coriolis effect increased by 6% the

output of the farm. This increase is due to a fastest wake recovery thanks to a spanwise mixing. However, further investigation should be done on this subject to verify the positive impact of the Coriolis effect under different ABL conditions and wind turbines alignment.

The last topic of this thesis was the analysis of the wind turbines rotational direction in a neutral boundary layer. This study was undertaken to react to a newspaper article [11], based on the simulation performed by Engelberger et al. [12], stating that inverting the direction of rotation of the wind turbines could increase by 11% the power output under specific conditions. Our results were less optimistic than the article. An increase smaller than 1% was observed but in favour of the clockwise direction of rotation in the northern hemisphere. The condition of our simulation and the article simulation were not identical which could explain the difference. The analysis of the direction of rotation also highlights a difference in the wake deformation. The wake of a counter-clockwise rotating wind turbine rotates in the direction promoted by the spanwise component of the wind. The wind then increases the wake deformation. While for a clockwise rotating wind turbine, the wake rotates in the opposite direction and is directly opposed to the spanwise wind component. In that case, the wake is less deformed but over a long distance it appears that the wake recovers faster. In future works, a surface heat flux could be added to analyse the deformation of the wake and then simulate a full 24 hours wind cycle to observe the behaviour of the farm in more realistic conditions. It would be interesting to analyse the impact of the rotational direction of the wind turbines on such a simulation and compare it against the Engelberger et al. simulation [12]. It would be also interesting to analyse the importance of the wake deformations regarding under different surface heat fluxes because, as mentioned in the literature, the surface heat flux could increase the wake deformation induced by the Coriolis effect.

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